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TRAUTWINE

THE CIVIL ENGINEER'S REFERENCE-BOOK

(formerly "Pocket-Book")

BY

JOHN C. TRAUTWINE

AND

JOHN C. TRAUTWINE, JR.

CIVIL ENGINEERS

EDITED BY

JOHN C. TRAUTWINE 3d, C.E.

180th THOUSAND

21st EDITION

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PREFACE

TO FIRST EDITION, 1872.

SHOULD experts in engineering complain that they do not find anything of interest in this volume, the writer would merely remind them that it was not his intention that they should. The book has been prepared for *young* members of the profession; and one of the leading objects has been to elucidate, in plain English, a few important elementary principles which the savants have enveloped in such a haze of mystery as to render pursuit hopeless to any but a confirmed mathematician.

Comparatively few engineers are good mathematicians; and in the writer's opinion, it is fortunate that such is the case; for nature rarely combines high mathematical talent, with that practical tact, and observation of outward things, so essential to a successful engineer.

There have been, it is true, brilliant exceptions; but they are very rare. But few even of those who have been tolerable mathematicians when young, can, as they advance in years, and become engaged in business, spare the time necessary for retaining such accomplishments.

Nearly all the scientific principles which constitute the foundation of civil engineering are susceptible of complete and satisfactory explanation to any person who *really* possesses only so much elementary knowledge of arithmetic and natural philosophy as is *supposed* to be taught to boys of twelve or fourteen in our public schools.*

* Let two little boys weigh each other on a platform scale. Then when they balance each other on their board see-saw, let them see (and measure for themselves) that the lighter one is farther from the fence-rail on which their board is placed, in the same proportion as the heavier boy outweighs the lighter one. They will then have learned the grand principle of the *lever*. Then let them measure and see that the light one see-saws farther than the heavy one, in the same proportion; and they will have acquired the principle of *virtual velocities*. Explain to them that *equality of moments* means nothing more than that when

The little that is beyond this, might safely be intrusted to the savants. Let them work out the results, and give them to the engineer *in intelligible language.* We could afford to take their words for it, because such things are their specialty; and because we know that they are the best qualified to investigate them. On the same principle we intrust our lives to our physician, or to the captain of the vessel at sea. Medicine and seamanship are their respective specialties.

If there is any point in which the writer may hope to meet the approbation of proficients, it is in the accuracy of the tables. The pains taken in this respect have been very great. Most of the tables have been entirely recalculated expressly for this book; and one of the results has been the detection of a great many errors in those in common use. He trusts that none will be found exceeding one, or sometimes two, in the last figure of any table in which great accuracy is required. There are many errors to that amount,

---

they seat themselves at their measured distances on their see-saw, *they balance each other.* Let them see that the weight of the heavy boy, when multiplied by his distance in feet from the fence-rail amounts to just as much as the weight of the light one when multiplied by his distance. Explain to them that each of the amounts is in *foot-pounds.* Tell them that the lightest one, because he see-saws so much faster than the other, will bump against the ground just as hard as the heavy one; and that this means that *their momentums are equal.* The boys may then go in to dinner, and probably puzzle their big lout of a brother who has just passed through college with high honors. They will not forget what they have learned, for they learned it *as play,* without any ear-pulling, spanking, or keeping in. Let their bats and balls, their marbles, their swings, &c, once become their philosophical apparatus, and children may be taught (*really* taught) many of the most important principles of engineering before they can read or write. It is the ignorance of these principles, so easily taught even to children, that constitutes what is popularly called "THE PRACTICAL ENGINEER;" which, in the great majority of cases, means simply an ignoramus, who blunders along without knowing any other reason for what he does, than that he has seen it done so before. And it is this same ignorance that causes employers to prefer this practical man to one who is conversant with principles. They, themselves, were spanked, kept in, &c, when boys, because they could not master leverage, equality of moments, and virtual velocities, enveloped in x's, p's, Greek letters, square-roots, cube-roots, &c, and they naturally set down any man as a fool who could. They turn up their noses at science, not dreaming that the word means simply, *knowing why.* And it must be confessed that they are not altogether without reason; for the savants appear to prepare their books with the express object of preventing purchasers, (they have but few *readers,*) from learning why.

especially where the recalculation was very tedious, and where, consequently, interpolation was resorted to. They are too small to be of practical importance. He knows, however, the almost impossibility of avoiding larger errors *entirely;* and will be glad to be informed of any that may be detected, except the final ones alluded to, that they may be corrected in case another edition should be called for. Tables which are absolutely reliable, possess an intrinsic value that is not to be measured by money alone. With this consideration the volume has been made a trifle larger than would otherwise have been necessary, in order to admit the stereotyped sines and tangents from his book on railroad curves. These have been so thoroughly compared with standards prepared independently of each other, that the writer believes them to be absolutely correct.

In order to reduce the volume to pocket-size, smaller type has been used than would otherwise have been desirable.

Many abbreviations of common words in frequent use have been introduced, such as abut, cen, diag, hor, vert, pres, &c, instead of abutment, center, diagonal, horizontal, vertical, pressure, &c. They can in no case lead to doubt; while they appreciably reduce the thickness of the volume.

Where prices have been added, they are placed in footnotes. They are intended merely to give an approximate or comparative idea of value; for constant fluctuations prevent anything farther.

The addresses of a few manufacturing establishments have also been inserted in notes, in the belief that they might at times be found convenient. They have been given without the knowledge of the proprietors.

The writer is frequently asked to name good elementary books on civil engineering; but regrets to say that there are very few such in our language. "Civil Engineering," by Prof. Mahan of West Point; "Roads and Railroads," by the late Prof. Gillespie; and the "Handbook of Railroad Construction," by Mr. George L. Vose, Civ. Eng. of Boston, are the best. The writer has reason to know that a new edition of the last, now in press, will be far

superior to all predecessors; and better adapted to the wants of the young engineer than any book that has appeared.

Many of Weale's series are excellent. Some few of them are behind the times; but it is to be hoped that this may be rectified in future editions. Among pocket-books, Haswell, Hamilton's Useful Information, Henck, Molesworth, Nystrom, Weale, &c, abound in valuable matter.

The writer does not include Rankine, Moseley, and Weisbach, because, although their books are the productions of master-minds, and exhibit a profundity of knowledge beyond the reach of ordinary men, yet their language also is so profound that very few engineers can read them. The writer himself, having long since forgotten the little higher mathematics he once knew, cannot. To him they are but little more than striking instances of how completely the most simple facts may be buried out of sight under heaps of mathematical rubbish.

There is no table of errata, because no errors are known to exist except two or three of a single letter in spelling; and which will probably escape notice.

JOHN C. TRAUTWINE.

PHILADELPHIA, *November* 13th, 1871.

FROM PREFACE

OF TWENTIETH EDITION, 1918.

As in our preceding editions, all new work and all revisions have been the subject of our personal attention, and "scissors-and-paste" methods have been scrupulously avoided.

As in all cases heretofore, every rule and every formula and every description of methods, etc., can be readily understood by anyone, engineer or layman, understanding the use of common and decimal fractions, of roots and powers, of logarithms, and of elementary plane trigonometry. On the other hand, one who is not possest of this very meagre stock of mathematical knowledge will hardly approach engineering problems, even as an amateur . . .

. . . Extraordinary precautions have been taken for the protection of our readers against the occurrence of typographical and other errors. In this, as in previous editions, special attention has been given to the matter of typography, which, like other steps in manufacture, has been under our own direct personal control. This includes the preparation of illustrations . . .

The manuscript was thoroly checkt before it was sent to the printer. The first proof was minutely read by ourselves, as well as by the printer. In this work we used a new apparatus, of our own invention, to facilitate the verification of punctuation, of bold and italic characters, etc. Another device of our own was used in comparing successive proofs, to detect any accidental shifting of type matter.. . .

JOHN C. TRAUTWINE JR.
JOHN C. TRAUTWINE 3D.

Philadelphia, *August, 1918.*

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MATHEMATICS.

MATHEMATICAL SYMBOLS.

$+$ **Plus, positive, add.** 1.414 + means 1.414 + other decimals.
$-$ Minus, negative, subtract.
$\pm$ Plus or minus, positive or negative. Thus, $\sqrt{a^2} = \pm a$.
$\mp$ Minus or plus.
$\times$ Multiplied by, times. Thus, $x \times y = x.y = x\,y$; $3 \times 4 = 12$.
$\div$: / Divided by. Thus, $a \div b = a : b = a/b = \frac{a}{b}$
: : Proportion. Thus, $a : b :: c : d$, as a is to b, so is c to d.
$=$ Equals, is equal to
$>$ Is greater than. Thus, $6 > 5$.
$<$ Is less than. Thus, $5 < 6$.
$\neq$ Is not equal to
$\gtrless$ Is greater or less than.
$\ngtr$ Is not greater than.
$\nless$ Is not less than.
$\geqq$ Is equal to or greater than.
$\leqq$ Is equal to or less than.
$\propto$ Is proportional to, varies with.
∞ Infinity.
$\perp$ Is perpendicular to.
$\angle$ $\measuredangle$ Angle.
$\sim$ Is similar to.
$\parallel$ Is parallel to.
$\sqrt{}$ $\sqrt{}$ Root of. Thus, $\sqrt[2]{a}$ or $\sqrt{a}$ = square root of a, $\sqrt[3]{a}$ = 3d or cube root of a. $\sqrt[n]{a}$ = nth root of a.
( ) Parenthesis. [ ] Brackets. { } Braces. —— Vinculum. Quantities enclosed or covered by the symbol are to be taken together.
$\because$ Since, because.
$\therefore$ Hence, therefore.
° Degrees.
′ Minutes of arc,* feet.
″ Seconds of arc,* inches.
′ ″ ‴ etc. Prime, second, third, etc. Distinguishing accents. Thus, a' a prime; a'', a second, etc.
$\pi = \frac{\text{Circumference}}{\text{Diameter}} = 3.14159265+$, arc of semicircle, or 180°.
E, Modulus of elasticity.
e ϵ, Base of Napierian, natural or hyperbolic logarithms = 2.718281828.
g, Acceleration of gravity = approximately 32.2 feet per second per second = approximately 9.81 meters per second per second.

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*Minutes and seconds of time, formerly also denoted by ′ and ″, are now denoted by m and s, or by min and sec, respectively.

THE GREEK ALPHABET.

This alphabet is inserted for the benefit of those who have occasion to consult scientific works in which Greek letters are used, and who find it inconvenient to memorize the letters

Greek letters. Capital	Small	Name.	Approximate equivalent.	Commonly used to designate
Α	α	Alpha	a	Angles, Coefficients.
Β	β	Beta	b	" "
Γ	γ	Gamma	g	" " Specific gravity.
Δ	δ	Delta	d	" " Density, Variation.
Ε	ε	Epsilon	e (short)	Base of hyperbolic logarithms = 2.7182818. Eccentricity in conic sections.
Ζ	ζ	Zeta	z	Co-ordinates, Coefficients.
Η	η	Eta	e (long)	" "
Θ	θ ϑ	Theta	th	Angles.
Ι	ι	Iota	i	
Κ	κ	Kappa	k	
Λ	λ	Lambda	l	Angles, Coefficients, Latitude.
Μ	μ	Mu	m	" "
Ν	ν	Nu	n	"
Ξ	ξ	Xi	x	Co-ordinates.
Ο	ο	Omicron	o (short)	
Π	π	Pi	p	Circumference ÷ diameter.*
Ρ	ρ	Rho	r	Radius, Ratio.
Σ	σ ς	Sigma	s	Distance (space).†
Τ	τ	Tau	t	Temperature, Time.
Υ	υ	Upsilon	u or y	
Φ	φ	Phi	ph	Angles, Coefficients.
Χ	χ	Chi	ch	
Ψ	ψ	Psi	ps	Angles
Ω	ω	Omega	o (long)	Angular velocities.

* The small letter π (*pi*) is universally employed to designate the number of times ($= 3.14159265\ldots$) the diameter of a circle is contained in the circumference, or the radius in the semi circumference. In the circular measure of angles, an angle is designated by the number of times the radius of any circle is contained in an arc of the same circle subtending that angle. π then stands for an angle of 180° (= two right angles), because, in any circle, $\pi \times$ radius = the semi-circumference

The capital letter Π (*pi*) is used by some mathematical writers to indicate the product obtained by multiplying together the numbers 1, 2, 3, 4, 5 . . . etc., up to any given point. Thus, $\Pi 4 = 1 \times 2 \times 3 \times 4 = 24$.

† The capital letter Σ (*sigma*) is used to designate a *sum*. Thus, in a system of parallel forces, if we call each of the forces (irrespective of their amounts) F, then their resultant, which is equal to the (algebraic) sum of the forces, may be written $R = \Sigma F$.

ARITHMETIC.

FACTORS AND MULTIPLES.

(1) Factors of any number, n, are numbers whose product is $= n$. Thus, 17 and 4 are factors of 68, so also are 34 and 2; also 17, 2, and 2.

(2) A prime number, or **prime,** is a number which has no factors, except itself and 1; as 2, 3, 5, 19, 233.

(3) A common factor, common divisor or **common measure,** of two or more numbers, is a number which exactly divides each of them. Thus, 3 is a common divisor of 6, 12, and 18

(4) The highest common factor or **greatest common divisor,** of two or more numbers, is called their **H. C. F.** or their **G. C. D.** Thus, 6 is the H. C. F. of 6, 12, and 18.

(5) To find the H. C. F. of two or more numbers; find the prime factors of each, and multiply together those factors which are common to all, taking each factor only once. Thus, required the H. C. F. of 78, 126, and 234.

$$\begin{array}{ll} 78 & 2 \times 3 \times 13 \\ 126 & 2 \times 3 \times 3 \times 7 \\ 234 & 2 \times 3 \times 3 \times 13 \end{array}$$

and H. C. F. $= 2 \times 3 = 6$.

(6) To find the H. C. F. of two large numbers; divide the greater by the less; then the less by the remainder, A; A by the second remainder, B; B by the third remainder, C; and so on until there is no remainder. The last divisor is the H. C. F. Thus, required the H. C. F. of 575 and 782.

```
575)782(1
    575
  A 207)575(2
        414
      B 161)207(1
            161
          C 46)161(3
               138
             D 23)46(2      H. C. F. = D = 23.
                  46
                   0
```

(7) A common multiple of two or more numbers is a number which is exactly divisible by each of them.

(8) The least common multiple of two or more numbers is called their **L. C. M.**

(9) To find the L. C. M. of two or more numbers; find the prime factors of each. Multiply the factors together, taking each as many times as it is contained in that number in which it is oftenest repeated. Thus, required the L. C. M. of 7, 30, and 48.

$$\begin{aligned} 7 &= 7 \\ 30 &= 2 \times 3 \times 5 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

$$\text{L. C. M.} = 7 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 1680.$$

(10) To find the L. C. M. of two large numbers; find the H. C. F., as above; and, by means of it, find the other factors. Then find the product of the factors, as before. Thus, required the L. C. M. of 575 and 782. As above,

H. C. F. $= 23$; $\frac{575}{23} = 25$; and $\frac{782}{23} = 34$. Hence,

$$\begin{aligned} 575 &= 23 \times 25 \\ 782 &= 23 \times 34 \end{aligned}$$

and L. C. M. $= 23 \times 25 \times 34 = 19{,}550$.

FRACTIONS.

(1) A common denominator of two or more fractions is a common multiple of their denominators.

(2) The least common denominator, or L. C. D., of two or more fractions is the L. C. M. of their denominators.

(3) To reduce to a common denominator. Let

N = the new numerator of any fraction
n = its old numerator
d = its old denominator
C = the common denominator

Then

$$N = n\frac{C}{d}$$

Thus, $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$. C = L. C. M. of denominators = 24.

$$\frac{3}{4} = \frac{3\times\frac{24}{4}}{4\times\frac{24}{4}} = \frac{3\times 6}{4\times 6} = \frac{18}{24};\ \frac{5}{6} = \frac{5\times 4}{6\times 4} = \frac{20}{24};\ \frac{7}{8} = \frac{7\times 3}{8\times 3} = \frac{21}{24}.$$

If none of the denominators have a common factor, then C=the product of all the denominators, $\frac{C}{d}$ = the product, P, of all the *other* denominators, and N = P n

Thus, $\frac{2}{3}, \frac{1}{4}, \frac{5}{7}$. C = 84

$$\frac{2}{3} = \frac{2\times 4\times 7}{84} = \frac{56}{84},\ \frac{1}{4} = \frac{1\times 3\times 7}{84} = \frac{21}{84};\ \frac{5}{7} = \frac{5\times 3\times 4}{84} = \frac{60}{84}.$$

(4) Addition and Subtraction. If necessary, reduce the fractions to a common denominator, the lower the better. Add or subtract the numerators. Thus,

$$\frac{1}{2}+\frac{1}{2} = \frac{2}{2} = 1;\ \frac{3}{4}+\frac{1}{4} = \frac{4}{4} = 1;\ \frac{3}{4}+\frac{5}{9} = \frac{27}{36}+\frac{20}{36} = \frac{47}{36} = 1\frac{11}{36};$$

$$\frac{3}{4}+\frac{7}{8} = \frac{6}{8}+\frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}.$$

$$\frac{3}{4}-\frac{1}{4} = \frac{2}{4} = \frac{1}{2};\ \frac{3}{4}-\frac{5}{9} = \frac{27}{36}-\frac{20}{36} = \frac{7}{36};\ \frac{7}{8}-\frac{3}{4} = \frac{7}{8}-\frac{6}{8} = \frac{1}{8}.$$

(5) Multiplication. Multiply together the numerators, also the denominators, cancelling where possible. Thus.

$$\frac{1}{2}\times\frac{1}{2} = \frac{1}{4};\ \frac{3}{4}\times\frac{1}{4} = \frac{3}{16};\ \frac{3}{4}\times\frac{5}{9}\times\frac{2}{3} = \frac{5}{18};$$

$$3\frac{4}{7}\times 1\frac{2}{3} = \frac{25}{7}\times\frac{5}{3} = \frac{125}{21} = 5\frac{20}{21};\ \frac{2}{3}\times\frac{3}{5} = \frac{2}{5};$$

$$\frac{3}{4}\text{ of }\frac{1}{2}\text{ of }\frac{5}{3}\text{ of }\frac{7}{5} = \frac{3}{4}\times\frac{1}{2}\times\frac{5}{3}\times\frac{7}{5} = \frac{7}{8}.$$

(6) Division. Invert the divisor and multiply. Thus,

$$\frac{1}{2}\div\frac{1}{2} = \frac{1}{2}\times\frac{2}{1} = \frac{2}{2} = 1;\ \frac{3}{4}\div\frac{1}{4} = \frac{3}{4}\times\frac{4}{1} = \frac{3}{1} = 3;$$

$$\frac{3}{4}\div\frac{5}{9} = \frac{3}{4}\times\frac{9}{5} = \frac{27}{20} = 1\frac{7}{20};$$

$$3\frac{4}{7}\div 1\frac{2}{3} = \frac{25}{7}\div\frac{5}{3} = \frac{25}{7}\times\frac{3}{5} = \frac{5}{7}\times\frac{3}{1} = \frac{15}{7} = 2\frac{1}{7};$$

$$5\div\frac{7}{8} = 5\times\frac{8}{7} = \frac{40}{7} = 5\frac{5}{7}.$$

(7) A fraction is said to be in its **lowest terms**, or to be **simplified**, when its numerator and denominator have no common factor. Thus,

$$\frac{34}{85}\text{ simplified } = \frac{2}{5}.$$

(8) To reduce to lowest terms. Divide numerator and denominator by their H. C. F. Thus, required the lowest terms of $\frac{34}{85}$.

$$\text{H. C. F. of 34 and 85} = 17;\ \text{and } \frac{34}{85} = \frac{34\div 17}{85\div 17} = \frac{2}{5}.$$

DECIMALS.

(9) Multiplication. The product has as many decimal places as the factors combined. Thus,

Factors: $100 \times 3 \times 3.5 \times 0.004 \times 465.21 = 1953.882000$

Number of decimal places: $0 + 0 + 1 + 3 + 2 = 6$

(10) Division. The number of decimal places in the quotient = those in the dividend minus those in the divisor. Thus,

$$\frac{5.125}{4.1} = 1.25;\ \frac{5}{4} = \frac{5.00}{4} = 1.25;\ \frac{3}{4} = \frac{3.00}{4} = 0.75;\ \frac{0.42}{0.0021} = \frac{0.4200}{0.0021} = 200;$$

When the divisor is a fraction or a mixed number, we may multiply both divisor and dividend by the least power of 10 which will make the divisor a whole number. Thus,

$$\frac{2.679454}{0.0062} = \frac{26,794.54}{62} = 432.17.$$

(11) To reduce a common fraction to decimal form; divide the numerator by the denominator. Thus, $\frac{32}{40} = 0.8$; $1\frac{3}{5} = \frac{8}{5} = 1.6$.

Table 1. Decimal equivalents of common fractions.

8ths	16ths	32ds	64ths		8ths	16ths	32ds	64ths	
			1	.015625				33	.515625
		1	2	.03125			17	34	.53125
			3	.046875				35	.546875
	1	2	4	.0625		9	18	36	.5625
			5	.078125				37	.578125
		3	6	.09375			19	38	.59375
			7	.109375				39	.609375
1	2	4	8	.125	5	10	20	40	.625
			9	.140625				41	.640625
		5	10	.15625			21	42	.65625
			11	.171875				43	.671875
	3	6	12	.1875		11	22	44	.6875
			13	.203125				45	.703125
		7	14	.21875			23	46	.71875
			15	.234375				47	.734375
2	4	8	16	.25	6	12	24	48	.75
			17	.265625				49	.765625
		9	18	.28125			25	50	.78125
			19	.296875				51	.796875
	5	10	20	.3125		13	26	52	.8125
			21	.328125				53	.828125
		11	22	.34375			27	54	.84375
			23	.359375				55	.859375
3	6	12	24	.375	7	14	28	56	.875
			25	.390625				57	.890625
		13	26	.40625			29	58	.90625
			27	.421875				59	.921875
	7	14	28	.4375		15	30	60	.9375
			29	.453125				61	.953125
		15	30	.46875			31	62	.96875
			31	.484375				63	.984375
4	8	16	32	.5	8	16	32	64	1.

(12) To reduce a decimal fraction to common form. Supply the denominator (1), and reduce the resulting fraction to its lowest terms. Thus:

$$0.25 = \frac{0.25}{1.00} = \frac{25}{100} = \frac{1}{4};\quad 0.75 = \frac{75}{100} = \frac{3}{4};\quad 0.890625 = \frac{890625}{1000000} = \frac{57}{64}.$$

(13) **Recurring, circulating, or repeating decimals** are those in which certain digits, or series of digits, recur indefinitely. Thus, $\frac{1}{3} = 0.3333....$, and so on; $\frac{10}{7} = 1.428571428571.....$, and so on. Recurring decimals may be indicated thus: $0.\bar{3}$, $1.\overline{428571}$; or thus: 0.*3, 1.*428571.

RATIO AND PROPORTION.

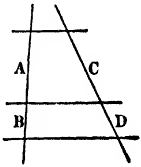
(1) **Ratio.** The ratio of two quantities, as A and B, is expressed by their quotient, $\frac{A}{B}$ or $\frac{B}{A}$. Thus, the ratio of 10 to 5 is $= \frac{10}{5} = 2$; the ratio of 5 to 10 is $= \frac{5}{10} = \frac{1}{2} = 0.5$.

(2) **Duplicate ratio** is the ratio of the *squares* of numbers. Thus, $\frac{A^2}{B^2}$ is the duplicate ratio of A and B.

(3) **Proportion** is equality of ratios. Thus, $\frac{10}{5} = \frac{12}{6} = \frac{1266}{633} = 2$. In the figure, which represents segments, A, B, C, and D, between parallel lines; A : B : : C : D, or $\frac{A}{B} \; \frac{C}{D}$.

(4) The first and fourth terms, A and D, are called the **extremes**, and the second and third, B and C, are called the **means**. The first term, A or C, of each ratio, is called the **antecedent**, and the second term, B or D, is called the **consequent**. D is called the **fourth proportional** of A, B, and C.

(5) In a proportion, A : B = C : D, we have:



Product of extremes = product of means. A D = B C.

Alternation. $\frac{A}{B} = \frac{C}{D}$; $\frac{A}{C} = \frac{B}{D}$.

Inversion. $\frac{B}{A} = \frac{D}{C}$; $\frac{B}{D} = \frac{A}{C}$; $\frac{D}{B} = \frac{C}{A}$.

Composition. $\frac{A+B}{A} = \frac{C+D}{C}$; $\frac{A+B}{B} = \frac{C+D}{D}$.

Division. $\frac{A-B}{A} = \frac{C-D}{C}$; $\frac{A-B}{B} = \frac{C-D}{D}$.

Composition and division. $\frac{A+B}{A-B} = \frac{C+D}{C-D}$.

We have, also:

$$\frac{mA}{mB} = \frac{A}{B} = \frac{C}{D} = \frac{nA}{nB} = \frac{nC}{nD};\quad \frac{mA}{nB} = \frac{mC}{nD};\quad \frac{A^n}{B^n} = \frac{C^n}{D^n};\quad \frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \frac{\sqrt[n]{C}}{\sqrt[n]{D}}.$$

(6) If, in the proportion, A : B = C : D, we have B = C = m, then A : m = m : D, or $\frac{A}{m} = \frac{m}{D}$ or $m^2 = AD$, or $m = \sqrt{AD}$.

(7) In such cases, m is called the **mean proportional** between A and D, and D is called the **third proportional** of A and m.

A continued proportion is a series of equal ratios, as

A : B = C : D = E : F, etc. = R; or $\frac{A}{B} = \frac{C}{D} = \frac{E}{F}$, etc. = R

In continued proportion,

$$\frac{A+C+E+\text{etc.}}{B+D+F+\text{etc.}} = \frac{A}{B} = \frac{C}{D} = \frac{E}{F}\text{ etc.} = R$$

If $\frac{A}{B} = \frac{C}{D}$; $\frac{A'}{B'} = \frac{C'}{D'}$; $\frac{A''}{B''} = \frac{C''}{D''}$; then $\frac{A\,A'\,A''}{B\,B'\,B''} = \frac{C\,C'\,C''}{D\,D'\,D''}$ etc.

(8) Let A, B, and C be any three numbers. Then

$$\frac{A}{C} = \frac{A}{B}\cdot\frac{B}{C} \text{ and } \frac{A}{B} = \frac{A}{C}\cdot\frac{C}{B}.$$

* 0.*3, 1.*428571, etc., standing for 0.3333...., 1.428571428571..... etc.

(9) Reciprocal or inverse proportion. Two quantities are said to be *reciprocally* or *inversely* proportional, when the ratio $\frac{A}{B}$ of two values, A and B, of the one, is = the reciprocal, $\frac{B'}{A'}$, of the ratio of the two corresponding values of the other. Thus, let A = a velocity of 2 miles per hour, and B = 3 miles per hour. Then the *hours required per mile* are respectively, $A' = \frac{1}{A} = \frac{1}{2}$, and $B' = \frac{1}{B} = \frac{1}{3}$. Here $A : B = B' : A'$, or $\frac{A}{B} = \frac{B'}{A'}$, or $\frac{2}{3} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} = 1 \div \frac{A'}{B'}$.

(10) If two variable numbers, A and B, are reciprocally proportional, so that $A' : B' = B'' : A''$, the product, $A' A''$, of any two values of one of the numbers is equal to the product, $B' B''$ of the two corresponding values of the other.

(11) The application of proportion to practical problems is sometimes called the **rule of three.** Thus: **single rule of three.** If 3 men lay 10,000 bricks in a certain time, how many could 6 men lay in the same time?

As 3 men are to 6 men, so are 10,000 bricks to 20,000 bricks; or, 10,000 bricks $\times \frac{6}{3} = 20{,}000$ bricks.

If 3 men require 10 hours to lay a certain number of bricks, how many hours would 6 men require to lay the same number?

As 6 men are to 3 men, so are 10 hours to 5 hours; or, 10 hours $\times \frac{3}{6} = 5$ hours.

(12) Double rule of three.

If 3 men can lay 4,000 bricks in 2 days, how many men can lay 12,000 bricks in 3 days? Here 4,000 bricks require 3 men 2 days, or 6 man-days, and 12,000 bricks will require $6 \times \frac{12{,}000}{4{,}000} = 6 \times 3 = 18$ man-days; and, as the work is to be done in 3 days, $\frac{18}{3} = 6$ men will be required.

PROGRESSION.

(1) Arithmetical Progression. A series of numbers is said to be in arithmetical progression when each number differs from the preceding one by the same amount. Thus, −2, −1, 0, 1, 2, 3, 4, etc., where difference = 1; or 4, 3, 2, 1, 0, −1, −2, etc., where difference = −1; or −4, −2, 0, 2, 4, 6, 8, 10, where difference = 2; or 1½, 1¼, 1, ¾, ½, ¼, 0, −¼, −½, etc., where difference = −¼.

(2) In any such series the numbers are called terms. Let a be the first term, l the last term, d the common difference, n the number of terms, and s the sum of the terms. Then

Required	Given	
l	$a\ d\ n$	$l = a + (n - 1)\,d$
l	$a\ d\ s$	$l = -\frac{1}{2}d \pm \sqrt{2\,d\,s + (a - \frac{1}{2}d)^2}$
s	$a\ d\ n$	$s = \frac{1}{2}n\left[2\,a + (n - 1)\,d\right]$
a	$d\ l\ s$	$a = \frac{1}{2}d \pm \sqrt{(l + \frac{1}{2}d)^2 - 2\,d\,s}$
n	$a\ d\ s$	$n = \frac{d - 2\,a \pm \sqrt{(2\,a - d)^2 + 8\,d\,s}}{2\,d}$
n	$d\ l\ s$	$n = \frac{2\,l + d \pm \sqrt{(2\,l + d)^2 - 8\,d\,s}}{2\,d}$

(3) Geometrical Progression. A series of numbers is said to be in geometrical progression when each number stands to the preceding one in the same ratio. Thus: $\frac{1}{9}$, $\frac{1}{3}$, 1, 3, 9, 27, 81, etc., where ratio = 3; or 48, 24, 12, 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, etc., where ratio = $\frac{1}{2}$; or $\frac{27}{32}$, $1\frac{11}{16}$, $3\frac{3}{8}$, $6\frac{3}{4}$, $13\frac{1}{2}$, 27, etc., where ratio = 2.

(4) Let a be the first term, l the last term, r the constant ratio, n the number of terms, and s the sum of the terms. Then:

Required	Given	
l	$a\ r\ n$	$l = a r^{n-1}$*
l	$a\ r\ s$	$l = \dfrac{a + (r-1)s}{r}$
l	$r\ n\ s$	$l = \dfrac{(r-1)s r^{n-1}}{r^n - 1}$
s	$a\ n\ l$	$s = \dfrac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}$
s	$r\ n\ l$	$s = \dfrac{l r^n - l}{r^n - r^{n-1}}$
r	$a\ n\ l$	$r = \sqrt[n-1]{l \div a}$

PERMUTATION, Etc.

(1) **Permutation** shows in how many positions any number of things can be arranged in a row. To do this, multiply together all the numbers used in counting the things. Thus, in how many positions in a row can 9 things be placed? Here,

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880 \text{ positions. Ans.}$$

(2) **Combination** shows how many combinations of a few things can be made out of a greater number of things. To do this, first set down that number which indicates the greater number of things; and after it a series of numbers, diminishing by 1, until there are *in all* as many as the number of the few things that are to form each combination. Then beginning under the last one, set down said number of few things; and going backward, set down another series, also diminishing by 1, until arriving under the first of the upper numbers. Multiply together all the upper numbers to form one product; and all the lower ones to form another. Divide the upper product by the lower one.

Ex. How many combinations of 4 figures each, can be made from the 9 figures 1, 2, 3, 4, 5, 6, 7, 8, 9, or from 9 any things?

$$\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = \frac{3024}{24} = 126 \text{ combinations. Ans.}$$

(3) **Alligation** shows the value of a mixture of different ingredients, when the quantity and value of each of these last is known.

Ex. What is the value of a pound of a mixture of 20 lbs of sugar worth 15 cts per lb; with 30 lbs worth 25 cts per lb?

lbs.		cts.		cts.
20	×	15	=	300
30	×	25	=	750
50 lbs.				1050 cts.

Therefore, $\dfrac{1050}{50} = 21$ cts. Ans.

PERCENTAGE, INTEREST, ANNUITIES.

Percentage.

(1) Ratio is often expressed by means of the word "per" Thus, we speak of a grade of 105.6 feet per mile, *i. e.*, per 5280 feet. When the two numbers in the ratio refer to quantities of the same kind and denomination, the ratio is often expressed as a per*centage* (per*hundred*age). Thus, a grade of 105.6 feet per mile,

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* Equations involving powers and roots are conveniently solved by means of logarithms.

or per 5280 feet, is equivalent to a grade of 0.02 foot per foot,* or 2 feet per 100 feet, or simply (since both dimensions are in feet) 2 per 100, or 2 per "cent."

(2) One-fiftieth, or 1 per 50, is plainly equal to two hundredths, or 2 per *hundred*, or 2 per *cent*. Similarly ¼ = 25 per cent., ¾ = 3 × 25 per cent. = 75 per cent., etc. Hence, to reduce a ratio to the form of percentage, divide 100 times* the first term by the second. Thus, in a concrete of 1 part cement to 2 of sand and 5 of broken stone, there are 8 parts in all, and we have, by weight—†

Cement	$\frac{1}{8}$ = 0.125 =	12.5 per cent. of the whole.			
Sand	$\frac{2}{8}$ = 0.250	25.0	"	"	
Stone	$\frac{5}{8}$ = 0.625	62.5	"	"	
Concrete	$\frac{8}{8}$ = 1.000	100.0	"	"	

(3) Percentage is of very wide application in money matters, payment for service in such matters being often based upon the amount of money involved. Thus, a purchasing or selling agent may be paid a brokerage or commission which forms a certain percentage of the money value of the goods bought or sold; the premium paid for insurance is a percentage upon the value of the goods insured, etc.

Interest.

(4) Interest is hire or rental paid for the loan of money. The sum loaned is called the **principal**, and the number of cents paid annually for the loan of each dollar, or of dollars per hundred dollars, is called the **rate** of interest. The rate is always stated as a percentage.

(5) If the interest is paid to the lender as it accrues, the money is said to be at **simple interest**; but if the interest is periodically added to the principal, so that it also earns interest, the money is said to be at **compound interest**, and the interest is said to be compounded.

Simple Interest.

(6) At the end of a year, the interest on the principal, P, at the rate, r, is = P r, and the **amount**, A, or sum of principal and interest, is

$$A = P + P\,r = P\,(1 + r).$$

(7) At the end of a number, n, of years, the interest is = P r n (see right-hand side of Fig. 1), and

$$A = P + P\,r\,n = P\,(1 + r\,n).$$

Thus, let P = \$865.32, r = 3 per cent., or 0.03, n = 1 year, 3 months and 10 days = 1 year and 100 days = $1\frac{100}{365}$ years = 1.274 years. Then A = P (1 + r n) = \$865.32 × (1 + 0.03 × 1.274) = \$865.32 × 1.03822 = \$898.39.

(8) For the **present worth, principal, or capitalization,** P, of the amount, A, we have

$$P = \frac{A}{1 + r\,n}$$

Thus, for the sum, P, which, in 1 year, 3 months, 10 days, at 3 per cent. simple interest, will amount to \$898.39, we have P = $\frac{898.39}{1 + 0.03 \times 1.274}$ = \$865.32.

(9) In commercial business, interest is commonly **calculated approximately** by taking the year as consisting of 12 months of 30 days each. Then, at 6 per cent., the interest for 2 months, or 60 days, = 1 per cent.; 1 month, or 30 days, = ½ per cent.; 6 days = 0.1 per cent. Thus, required the interest on \$1264.35 for 5 months, 28 days, at 5 per cent.

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* A *fraction*, as $\frac{1}{8}$, $\frac{5}{16}$, etc., or its decimal equivalent, as 0.125, 0.3125, etc., is compared with *unity* or *one*; but in *percentage* the first term of the ratio is compared with *one hundred* units of the second term. Mistakes often occur through neglect of this distinction. Thus, 0.06 (six per cent. or six per hundred) is sometimes mis-read six one-hundredths of one per cent. or six one-hundredths per cent.

† For proportions by volume, see pp 935 and 943.

Principal		\$1264.35
Interest, 2 mos,	1 per cent	12.64
" 2 mos,	1 "	12.64
" 1 mo,	$\frac{1}{2}$ "	6.32
" 20 days,	$\frac{1}{3}$ "	4.21
" 6 days,	0.1 "	1.26
" 2 days,	$\frac{1}{30}$ "	0.42
Interest at 6 per cent		\$37.49
Deduct one-sixth		6.25
Interest at 5 per cent		\$31.24

Equation of Payments.

(10) A owes B \$1200; of which \$400 are to be paid in 3 months; \$500 in 4 months; and \$300 in 6 months; all bearing interest until paid; but it has been agreed to pay all at once. Now, at what time must this payment be made so that neither party shall lose any interest?

\$		months.		
400	×	3	=	1200
500	×	4	=	2000
300	×	6	=	1800
1200				5000

Average time $= \frac{5000}{1200} = 4\frac{1}{6}$ months. Ans.

Compound Interest.

(11) Interest is usually compounded annually, semi-annually, or quarterly. If it is compounded annually, then (see left side of Fig. 1)

at the end of 1 year $A = P(1+r)$

" " 2 years $A = P(1+r)(1+r) = P(1+r)^2$

" " n years $A = P(1+r)^n$; and

$$P = \frac{A}{(1+r)^n} = A(1+r)^{-n}$$

$$\frac{A}{P} = (1+r)^n$$

(12) If the interest is compounded q times per year, we have

$$\frac{A}{P} = \left(1 + \frac{r}{q}\right)^{qn}$$

(13) The principal, P, is sometimes called the **present worth** or **present value** of the amount, A. Thus, in the following table, \$1.00 is the present worth of \$2.191 due in 20 years at 4 per cent. compound interest, etc etc.

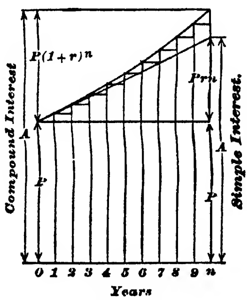

Fig. 1.

Table 2. Compound Interest.

Amount of $1 at Compound Interest.

Years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	6½ per cent.
1	1 030	1.035	1.040	1 045	1.050	1.055	1.060	1.065
2	1.061	1.071	1.082	1.092	1.103	1.113	1.124	1.134
3	1.093	1.109	1.125	1.141	1.158	1.174	1.191	1.208
4	1.126	1.148	1.170	1.193	1.216	1.239	1.262	1.286
5	1.159	1.188	1.217	1.246	1.276	1.307	1.338	1.370
6	1.194	1.229	1.265	1.302	1.340	1.379	1.419	1.459
7	1.230	1.272	1.316	1.361	1.407	1.455	1.504	1.554
8	1.267	1.317	1.369	1.422	1.477	1.535	1.594	1.655
9	1.305	1 363	1.423	1 486	1 551	1.619	1.689	1.763
10	1.344	1.411	1.480	1.553	1.629	1.708	1.791	1.877
11	1.384	1.460	1.539	1.623	1.710	1.802	1.898	1.999
12	1.426	1.511	1.601	1.696	1.796	1.901	2 012	2.129
13	1.469	1.564	1.665	1.772	1.886	2 006	2.133	2.267
14	1.513	1.619	1.732	1.852	1.980	2.116	2 261	2 415
15	1.558	1.675	1.801	1.935	2.079	2.232	2.397	2.572
16	1.605	1.734	1.873	2.022	2.183	2.355	2.540	2.739
17	1.653	1.795	1.948	2 113	2.292	2.485	2 693	2.917
18	1.702	1.858	2.026	2.208	2.407	2.621	2 854	3 107
19	1.754	1.923	2.107	2.308	2.527	2 766	3 026	3.309
20	1.806	1.990	2.191	2.412	2.653	2.918	3.207	3.524
21	1.860	2.059	2 279	2 520	2 786	3.078	3.400	3.753
22	1.916	2.132	2.370	2.634	2 925	3.248	3.604	3.997
23	1.974	2.206	2.465	2.752	3 072	3.426	3.820	4.256
24	2.033	2.283	2 563	2.876	3.225	3.615	4.049	4.533
25	2.094	2.363	2.666	3.005	3 386	3.813	4.292	4.828
26	2.157	2.446	2.772	3.141	3.556	4.023	4 549	5.141
27	2.221	2.552	2 883	3.282	3.733	4.244	4.822	5.476
28	2 288	2.620	2.999	3.430	3.920	4.478	5 112	5.832
29	2.357	2 712	3.119	3.584	4.116	4.724	5.418	6.211
30	2.427	2.807	3.243	3.745	4.322	4.984	5.743	6.614
31	2 500	2.905	3.373	3.914	4.538	5.258	6.088	7.044
32	2.575	3.007	3.508	4.090	4.765	5.547	6.453	7.502
33	2 652	3.112	3.648	4.274	5 003	5.852	6.841	7.990
34	2.732	3.221	3.794	4.466	5 253	6.174	7.251	8.509
35	2.814	3.334	3.946	4.667	5.516	6.514	7.686	9.062
36	2.898	3.450	4.104	4.877	5.792	6.872	8.147	9.651
37	2.985	3.571	4.268	5.097	6.081	7.250	8.636	10.279
38	3 075	3.696	4.439	5.326	6 385	7.649	9.154	10.947
39	3.167	3.825	4 616	5.566	6 705	8.069	9.704	11.658
40	3.262	3.959	4.801	5.816	7.040	8 513	10.286	12 416

Compound interest on M dollars, at any rate r for n years = M × compound interest on $1 at same rate, r, and for n years.

Annuity, Sinking Fund, Amortization, Depreciation.

(14) Under "Interest" we deal with cases where a certain sum or "principal," P, paid once for all, is allowed to accumulate either simple or compound interest; but in many cases equal periodical payments or appropriations, called **annuities**, are allowed to accumulate, each earning its own interest, usually compound.

(15) Thus, a sum of money is set aside annually to accumulate compound interest and thus form a **sinking fund**, in order to extinguish a debt. In this way, the cost of engineering works is frequently paid virtually in instalments. This process is called **amortization.**

(16) In estimating the operating expenses of engineering works, an allowance is made for **depreciation.** In calculating this allowance, we estimate or assume the life-time, n, of the plant, and find that annuity, p, which, at an assumed rate, r, of compound interest, will, in the time n, amount to the cost of the plant, and thus provide a fund by means of which the plant may be replaced when worn out or superseded.

(17) The **present worth, present value,** or **capitalization,** W, Fig. 2, of an annuity, p, for a given number, n, of years, is that sum which, if now placed at compound interest at the assumed rate, r, will, at the end of that time, reach the same amount, A, as will be reached by that annuity.

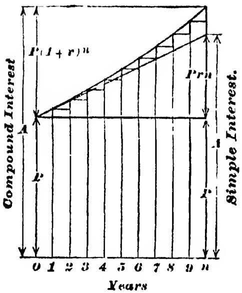

Fig. 1.

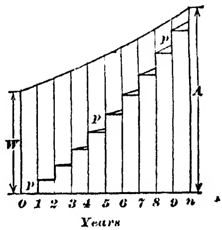

Fig. 2.

(18) Equations for Compound Interest and Annuities. (See Figs. 1 and 2.)

P = principal; r = rate of interest; n = number of years;
A = amount; p = annuity; W = present worth.

The interest is supposed to be compounded, and the annuities to be set aside, at the *end* of each year.

Compound Interest.

(1) **The amount, A,** *of \$1, at the end of n years, see* **(11)**, *is* $A = (1+r)^n$.

(2) Since the present worth of $(1+r)^n$, due in n years, is \$1, see (1), it follows, by proportion, that **the present worth, W,** of \$1, due in n years, is $W = \dfrac{1}{(1+r)^n} = (1+r)^{-n}$.

Annuities.

(3) In n years, an annuity of \$$r$ will amount to $(1+r)^n - 1$.* Hence, the **amount, A, of an annuity** of \$1, at the end of n years, is

$$A = \frac{(1+r)^n - 1}{r}.$$

---

*In the case of compound interest on \$1, the rate, r, may be regarded as an annuity, earning its interest; and, at the end of n years, the amount of the several annuities (each = the annual interest, r, on the \$1 principal) with the interests earned by them, is = the amount, $(1+r)^n$, of \$1 in n years at rate, r, minus the \$1 principal itself; or, amount of annuity = $(1+r)^n - 1$.

(4) For **the present worth, W, of an annuity** of \$1 for n years, we have, from Equations (1) and (3):

$$(1+r)^n : 1 = \frac{(1+r)^n - 1}{r} : \mathrm{W}. \quad \text{Hence, } \mathrm{W} = \frac{(1+r)^n - 1}{(1+r)^n\, r} = \frac{1 - \dfrac{1}{(1+r)^n}}{r}$$

See Table 3.

(5) The annuity for n years, which \$1 will purchase, is

$$p = \frac{1}{\mathrm{W}}* = \frac{r}{1 - \dfrac{1}{(1+r)^n}}$$

(6) The annuity which, in n years, will amount to \$1, is

$$p' = p - r = \frac{1}{\mathrm{W}} - r = \frac{r}{1 - \dfrac{1}{(1+r)^n}} - r = \frac{r}{(1+r)^n - 1}\dagger$$

See Table 4.

Table 3. Present Value of Annuity of \$1000. See Equation (4).

	Rate of Interest (Compound).							
Years.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
5	4,646	4,580	4,515	4,452	4,390	4,329	4,268	4,212
10	8,752	8,530	8,316	8,111	7,913	7,722	7,538	7,360
15	12,381	11,938	11,517	11,218	10,740	10,380	10,037	9,712
20	15,589	14,877	14,212	13,590	13,008	12,462	11,950	11,470
25	18,424	17,413	16,482	15,622	14,828	14,094	13,414	12,783
30	20,930	19,600	18,392	17,292	16,289	15,372	14,534	13,765
35	23,145	21,487	20,000	18,664	17,461	16,374	15,391	14,498
40	25,103	23,115	21,355	19,793	18,401	17,159	16,045	15,046
45	26,833	24,519	22,495	20,720	19,156	17,774	16,548	15,456
50	28,362	25,730	23,456	21,482	19,762	18,256	16,932	15,762
100	36,614	31,599	27,655	24,505	21,950	19,848	18,096	16,618

(19) In comparing the merits of proposed systems of improvement, it is usual to add, to the operating expenses and to the cost of ordinary repairs and maintenance, (1) the interest on the cost, (2) an allowance for depreciation, and sometimes (3) an annuity to form a sinking fund for the extinction of the debt incurred by construction. The capitalization of the total annual expense, thus obtained, is then *regarded as the true first cost of the construction. All* the elements of cost are thus reduced to a common basis, and the several propositions become properly comparable.

(20) Thus, in estimating, in 1899,‡ the cost of improving the water supply of Philadelphia, the rate, r, of interest was assumed at 3 per cent, and depreciation was assumed as below. Under "Life" is given the assumed life-time of each class of structure or apparatus, and under "Annuity" the sum which must be set aside annually in order to replace, at the expiration of that life, \$1,000 of the corresponding value.

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* Because, $\underset{\text{Equation (4)}}{\overset{\text{Present worth}}{\mathrm{W}} : \overset{\text{Annuity}}{\$1.00}} \;::\; \underset{\text{Equation (5)}}{\overset{\text{Present worth}}{\$1.00} : \overset{\text{Annuity}}{p}}$. Hence, $p = \dfrac{1}{\mathrm{W}}$.

† Because, $\underset{\text{Equation (3)}}{\overset{\text{Annuity}}{r} : \overset{\text{Amount}}{(1+r)^n - 1}} \;::\; \underset{\text{Equation (6)}}{\overset{\text{Annuity}}{p'} : \overset{\text{Amount}}{\$1.00}}$. Hence, $p' = \dfrac{r}{(1+r)^n - 1}$.

‡ Report by Rudolph Hering, Samuel M. Gray and Joseph M. Wilson.

Structures, Apparatus, etc.	Life, in years	Annuity $
Masonry conduits, filter beds, reservoirs	Indefinite	0.00
Permanent buildings	100	1.65
Cast iron pipe, railroad side-tracks	80	3.11
Steel pipe, valves, blow-offs, and gates	35	16.54
Engines and pumps	30	21.02
Boilers, electric light plants, tramways and equipment, iron fences	20	37.22
Telephone lines, sand-washer, and regulating apparatus	10	87.24

(**21**) Calculated upon this basis, two projects, each designed to furnish 450 million gallons per day, compared as follows.

Unfiltered Water, by Aqueduct.

First Cost.

Storage reservoirs		$30,900,000
Aqueducts		47,730,000
Distribution		3,555,000
Distributing reservoir		1,000,000
Total		$83,185,000

Annual.

Interest on $83,185,000		$2,495,550
Depreciation		198,640
Operation and Maintenance.		
Analyses and inspection	$41,620	
Ordinary repairs	49,150	
Pumping and wages	140,770	
		231,540
		$2,925,730

River Water, taken within City Limits and Filtered.

First Cost.

Filter plants		$23,174,680
Mains		10,980,000
Total		$34,154,680

Annual.

Interest on $34,154,680		$1,024,640
Depreciation		205,540
Operation and Maintenance.		
Pumping	$1,216,021	
Filtration	525,600	
		1,741,621
		$2,971,801

It will be noticed that, although the first cost of the filtration project was much less than half that of the aqueduct project, its large proportion of perishable parts made its charge for depreciation somewhat greater, while its cost for operation and maintenance was more than seven times as great, and its total annual charge a little greater.

Table 4. Annuity required to redeem $1000. See Equation (6).

Rate of Interest (Compound).

Years.	1 per cent.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
5	196.04	192.16	190.24	188.36	186.49	184.63	180.98	177.39
10	95.58	91.33	89.25	87.23	85.24	83.29	79.50	75.87
15	62.12	57.83	55.77	53.77	51.82	49.94	46.34	42.96
20	45.42	41.15	39.14	37.22	35.36	33.58	30.24	27.18
25	35.41	31.22	29.27	27.43	25.67	24.01	20.95	18.23
30	28.75	24.65	22.78	21.02	19.37	17.83	15.05	12.65
35	24.00	20.00	18.20	16.54	15.00	13.58	11.07	8.97
40	20.46	16.55	14.84	13.26	11.83	10.52	8.28	6.46
45	17.71	13.91	12.27	10.79	9.45	8.26	6.26	4.70
50	15.51	11.82	10.26	8.87	7.63	6.55	4.78	3.44
60	12.24	8.77	7.35	6.13	5.09	4.20	2.83	1.88
70	9.93	6.67	5.40	4.34	3.46	2.74	1.70	1.03
80	8.22	5.16	4.03	3.11	2.38	1.81	1.03	0.578
90	6.91	4.05	3.04	2.26	1.66	1.21	0.627	0.318
100	5.87	3.20	2.31	1.65	1.16	0.808	0.383	0.177

DUODENAL OR DUODENARY NOTATION.*

(1) In the Arabic system of notation 10 is taken as the base, but in duodenal notation 12, or "a dozen," is the base. While 10 is divisible only by 5, and (once only) by 2, 12 is divisible twice by 2, and once by 3, by 4, and by 6. This accounts for the popularity of the dozen as a basis of enumeration; of weights, as in the Troy pound of 12 ounces; of measures, as in the foot of 12 inches; the year of 12 months, and the half day of 12 hours; and of coinage, as in the British shilling of 12 pence.

(2) The duodenal notation uses the dozen (12), the gross ($12^2 = 144$), and the great gross ($12^3 = 12$ gross $= 1728$), as the decimal system uses the ten (10), the hundred ($10^2 = 100$), and the thousand ($10^3 = 10$ hundred $= 1000$). Two arbitrary single characters, such as T and E, represent ten and eleven respectively; the symbol 10 represents a dozen; 11 represents thirteen, and so on. Thus, the numerals of the two systems compare as follows:

Decimal	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...	20	21	22	23	24	25	36	48	60
Duodenal	1	2	3	4	5	6	7	8	9	T	E	10	11	12	...	18	19	1T	1E	20	21	30	40	50

Decimal	72	84	96	99	100	108	109	110	111	112	113	117	118	119	120	121	122
Duodenal	60	70	80	83	84	90	91	92	93	94	95	99	9T	9E	T0	T1	T2

Decimal	129	130	131	132	133	138	140	141	142	143	144	288	1728	20736	etc.
Duodenal	T9	TT	TE	E0	E1	E6	E8	E9	ET	EE	100	200	1000	10000	etc.

(3) Duodecimals. Areas of rectangular figures, the sides of which are expressed in feet and inches, are still sometimes found by a method called "Duodecimals," in which the products are in square feet, in twelfths of a square foot (each equal to 12 square inches) and in square inches; but, by means of our table of "Inches, reduced to decimals of a foot," page 221, the sides may be taken in feet and decimals of a foot, and the multiplication thus more conveniently performed, after which the decimal fraction of a foot in the product may, if desired, be converted into square inches by multiplying by 144.

*See Elements of Mechanics, by the late John W. Nystrom.

Table of Reciprocals of Numbers. See p. 52.

No.	Reciprocal.	No	Reciprocal.	No.	Reciprocal.	No	Reciprocal.
1	1.000000000	56	.017857143	111	.009009009	166	.006024096
2	0.500000000	57	.017543860	112	.008928571	167	.005988024
3	333333333	58	.017241379	113	.008849558	168	.005952381
4	250000000	59	.016949153	114	.008771930	169	.005917160
5	.200000000	60	.016666667	115	.008695652	170	.005882353
6	.166666667	61	.016393443	116	.008620690	171	.005847953
7	.142857143	62	.016129032	117	.008547009	172	005813953
8	.125000000	63	.015873016	118	.008474576	173	.005780347
9	.111111111	64	.015625000	119	.008403361	174	.005747126
10	.100000000	65	.015384615	120	.008333333	175	.005714286
11	.090909091	66	.015151515	121	.008264463	176	.005681818
12	.083333333	67	.014925373	122	.008196721	177	.005649718
13	.076923077	68	.014705882	123	.008130081	178	.005617978
14	.071428571	69	.014492754	124	.008064516	179	.005586592
15	.066666667	70	.014285714	125	.008000000	180	.005555556
16	.062500000	71	.014084507	126	.007936508	181	.005524862
17	.058823529	72	.013888889	127	.007874016	182	.005494505
18	.055555556	73	.013698630	128	.007812500	183	.005464481
19	.052631579	74	.013513514	129	.007751938	184	.005434783
20	.050000000	75	.013333333	130	.007692308	185	.005405405
21	.047619048	76	.013157895	131	.007633588	186	.005376344
22	.045454545	77	.012987013	132	.007575758	187	.005347594
23	.043478261	78	.012820513	133	.007518797	188	.005319149
24	.041666667	79	.012658228	134	.007462687	189	.005291005
25	.040000000	80	.012500000	135	.007407407	190	.005263158
26	.038461538	81	.012345679	136	.007352941	191	.005235602
27	.037037037	82	.012195122	137	.007299270	192	.005208333
28	.035714286	83	.012048193	138	007246377	193	.005181347
29	.034482759	84	.011904762	139	.007194245	194	.005154639
30	.033333333	85	.011764706	140	.007142857	195	.005128205
31	.032258065	86	.011627907	141	.007092199	196	.005102041
32	.031250000	87	.011494253	142	.007042254	197	.005076142
33	.030303030	88	.011363636	143	.006993007	198	.005050505
34	.029411765	89	.011235955	144	.006944444	199	.005025126
35	.028571429	90	.011111111	145	.006896552	200	.005000000
36	.027777778	91	.010989011	146	.006849315	201	.004975124
37	.027027027	92	.010869565	147	.006802721	202	.004950495
38	.026315789	93	.010752688	148	.006756757	203	.004926108
39	.025641026	94	.010638298	149	.006711409	204	.004901961
40	.025000000	95	.010526316	150	.006666667	205	.004878049
41	.024390244	96	.010416667	151	.006622517	206	.004854369
42	.023809524	97	.010309278	152	.006578947	207	004830918
43	.023255814	98	.010204082	153	.006535948	208	004807692
44	.022727273	99	.010101010	154	.006493506	209	004784689
45	.022222222	100	.010000000	155	.006451613	210	004761905
46	.021739130	101	009900990	156	.006410256	211	.004739336
47	.021276596	102	.009803922	157	.006369427	212	.004716981
48	.020833333	103	.009708738	158	.006329114	213	.004694836
49	.020408163	104	.009615385	159	.006289308	214	.004672897
50	.020000000	105	.009523810	160	.006250000	215	.004651163
51	.019607843	106	.009433962	161	006211180	216	.004629630
52	.019230769	107	.009345794	162	.006172840	217	.004608295
53	.018867925	108	.009259259	163	.006134969	218	.004587156
54	.018518519	109	.009174312	164	006097561	219	.004566210
55	.018181818	110	.009090909	165	.006060606	220	.004545455

Table of Reciprocals of Numbers.—(*Continued*) See p 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
221	.004524887	276	.003623188	331	.003021148	386	.002590674
222	.004504505	277	.003610108	332	.003012048	387	.002583979
223	.004484305	278	.003597122	333	.003003003	388	.002577320
224	.004464286	279	.003584229	334	.002994012	389	.002570694
225	.004444444	280	.003571429	335	.002985075	390	.002564103
226	.004424779	281	.003558719	336	.002976190	391	.002557545
227	.004405286	282	.003546099	337	.002967359	392	.002551020
228	.004385965	283	.003533569	338	.002958580	393	.002544529
229	.004366812	284	.003521127	339	.002949853	394	.002538071
230	.004347826	285	.003508772	340	.002941176	395	.002531646
231	.004329004	286	.003496503	341	.002932551	396	.002525253
232	.004310345	287	.003484321	342	.002923977	397	.002518892
233	.004291845	288	.003472222	343	.002915452	398	.002512563
234	.004273504	289	.003460208	344	.002906977	399	.002506266
235	.004255319	290	.003448276	345	.002898551	400	.002500000
236	.004237288	291	.003436426	346	.002890173	401	.002493766
237	.004219409	292	.003424658	347	.002881844	402	.002487562
238	.004201681	293	.003412969	348	.002873563	403	.002481390
239	.004184100	294	.003401361	349	.002865330	404	.002475248
240	.004166667	295	.003389831	350	.002857143	405	.002469136
241	.004149378	296	.003378378	351	.002849003	406	.002463054
242	.004132231	297	.003367003	352	.002840909	407	.002457002
243	.004115226	298	.003355705	353	.002832861	408	.002450980
244	.004098361	299	.003344482	354	.002824859	409	.002444988
245	.004081633	300	.003333333	355	.002816901	410	.002439024
246	.004065041	301	.003322259	356	.002808989	411	.002433090
247	.004048583	302	.003311258	357	.002801120	412	.002427184
248	.004032258	303	.003300330	358	.002793296	413	.002421308
249	.004016064	304	.003289474	359	.002785515	414	.002415459
250	.004000000	305	.003278689	360	.002777778	415	.002409639
251	.003984064	306	.003267974	361	.002770083	416	.002403846
252	.003968254	307	.003257329	362	.002762431	417	.002398082
253	.003952569	308	.003246753	363	.002754821	418	.002392344
254	.003937008	309	.003236246	364	.002747253	419	.002386635
255	.003921569	310	.003225806	365	.002739726	420	.002380952
256	.003906250	311	.003215434	366	.002732240	421	.002375297
257	.003891051	312	.003205128	367	.002724796	422	.002369668
258	.003875969	313	.003194888	368	.002717391	423	.002364066
259	.003861004	314	.003184713	369	.002710027	424	.002358491
260	.003846154	315	.003174603	370	.002702703	425	.002352941
261	.003831418	316	.003164557	371	.002695418	426	.002347418
262	.003816794	317	.003154574	372	.002688172	427	.002341920
263	.003802281	318	.003144654	373	.002680965	428	.002336449
264	.003787879	319	.003134796	374	.002673797	429	.002331002
265	.003773585	320	.003125000	375	.002666667	430	.002325581
266	.003759398	321	.003115265	376	.002659574	431	.002320186
267	.003745318	322	.003105590	377	.002652520	432	.002314815
268	.003731343	323	.003095975	378	.002645503	433	.002309469
269	.003717472	324	.003086420	379	.002638522	434	.002304147
270	.003703704	325	.003076923	380	.002631579	435	.002298851
271	.003690037	326	.003067485	381	.002624672	436	.002293578
272	.003676471	327	.003058104	382	.002617801	437	.002288330
273	.003663004	328	.003048780	383	.002610966	438	.002283105
274	.003649635	329	.003039514	384	.002604167	439	.002277904
275	.003636364	330	.003030303	385	.002597403	440	.002272727

Table of Reciprocals of Numbers.—(*Continued.*) See p. 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal
441	.002267574	496	002016129	551	.001814882	606	.001650165
442	.002262443	497	.002012072	552	.001811594	607	.001647446
443	.002257336	498	.002008032	553	.001808318	608	.001644737
444	.002252252	499	.002004008	554	.001805054	609	.001642036
445	.002247191	500	.002000000	555	.001801802	610	.001639344
446	.002242152	501	.001996008	556	.001798561	611	.001636661
447	.002237136	502	.001992032	557	.001795332	612	.001633987
448	.002232143	503	.001988072	558	.001792115	613	.001631321
449	.002227171	504	.001984127	559	.001788909	614	.001628664
450	.002222222	505	.001980198	560	.001785714	615	.001626016
451	.002217295	506	.001976285	561	.001782531	616	.001623377
452	.002212389	507	.001972387	562	.001779359	617	.001620746
453	.002207506	508	.001968504	563	.001776199	618	.001618123
454	.002202643	509	.001964637	564	.001773050	619	.001615509
455	.002197802	510	001960784	565	.001769912	620	.001612903
456	.002192982	511	.001956947	566	.001766784	621	.001610306
457	.002188184	512	.001953125	567	.001763668	622	.001607717
458	.002183406	513	.001949318	568	001760563	623	.001605136
459	.002178649	514	.001945525	569	001757469	624	.001602564
460	.002173913	515	.001941748	570	.001754386	625	.001600000
461	.002169197	516	.001937984	571	.001751313	626	.001597444
462	.002164502	517	.001934236	572	001748252	627	.001594896
463	.002159827	518	.001930502	573	.001745201	628	.001592357
464	.002155172	519	.001926782	574	.001742160	629	.001589825
465	.002150538	520	.001923077	575	.001739130	630	.001587302
466	.002145923	521	.001919386	576	.001736111	631	.001584786
467	.002141328	522	.001915709	577	.001733102	632	.001582278
468	.002136752	523	.001912046	578	.001730104	633	.001579779
469	.002132196	524	.001908397	579	.001727116	634	.001577287
470	.002127660	525	.001904762	580	.001724138	635	001574803
471	.002123142	526	.001901141	581	.001721170	636	.001572327
472	.002118644	527	.001897533	582	.001718213	637	.001569859
473	.002114165	528	.001893939	583	001715266	638	.001567398
474	.002109705	529	.001890359	584	001712329	639	.001564945
475	.002105263	530	.001886792	585	.001709402	640	.001562500
476	.002100840	531	.001883239	586	001706485	641	.001560062
477	.002096436	532	.001879699	587	.001703578	642	.001557632
478	.002092050	533	.001876173	588	.001700680	643	.001555210
479	.002087683	534	.001872659	589	.001697793	644	.001552795
480	.002083333	535	.001869159	590	.001694915	645	.001550388
481	.002079002	536	.001865672	591	.001692047	646	.001547988
482	.002074689	537	.001862197	592	.001689189	647	.001545595
483	.002070393	538	.001858736	593	.001686341	648	.001543210
484	.002066116	539	.001855288	594	.001683502	649	.001540832
485	.002061856	540	.001851852	595	.001680672	650	.001538462
486	.002057613	541	.001848429	596	.001677852	651	.001536098
487	.002053388	542	.001845018	597	.001675042	652	.001533742
488	.002049180	543	.001841621	598	.001672241	653	.001531394
489	.002044990	544	.001838235	599	.001669449	654	.001529052
490	.002040816	545	.001834862	600	.001666667	655	.001526718
491	.002036660	546	.001831502	601	.001663894	656	.001524390
492	.002032520	547	.001828154	602	.001661130	657	.001522070
493	.002028398	548	.001824818	603	.001658375	658	.001519757
494	.002024291	549	.001821494	604	.001655629	659	.001517451
495	.002020202	550	.001818182	605	.001652893	660	.001515152

Table of Reciprocals of Numbers.—(*Continued.*) See p. 52.

No.	Reciprocal.	No.	Reciprocal	No.	Reciprocal.	No.	Reciprocal.
661	.001512859	716	.001396648	771	.001297017	826	.001210654
662	.001510574	717	.001394700	772	.001295337	827	.001209190
663	.001508296	718	.001392758	773	.001293661	828	.001207729
664	.001506024	719	.001390821	774	.001291990	829	.001206273
665	.001503759	720	.001388889	775	.001290323	830	.001204819
666	.001501502	721	.001386963	776	.001288660	831	.001203369
667	.001499250	722	.001385042	777	.001287001	832	.001201923
668	.001497006	723	.001383126	778	.001285347	833	.001200480
669	.001494768	724	.001381215	779	.001283697	834	.001199041
670	.001492537	725	.001379310	780	.001282051	835	.001197605
671	.001490313	726	.001377410	781	.001280410	836	.001196172
672	.001488095	727	.001375516	782	.001278772	837	.001194743
673	.001485884	728	.001373626	783	.001277139	838	.001193317
674	.001483680	729	.001371742	784	.001275510	839	.001191895
675	.001481481	730	.001369863	785	.001273885	840	.001190476
676	.001479290	731	.001367989	786	.001272265	841	.001189061
677	.001477105	732	.001366120	787	.001270648	842	.001187648
678	.001474926	733	.001364256	788	.001269036	843	.001186240
679	.001472754	734	.001362398	789	.001267427	844	.001184834
680	.001470588	735	.001360544	790	.001265823	845	.001183432
681	.001468429	736	.001358696	791	.001264223	846	.001182033
682	.001466276	737	.001356852	792	.001262626	847	.001180638
683	.001464129	738	.001355014	793	.001261034	848	.001179245
684	.001461988	739	.001353180	794	.001259446	849	.001177856
685	.001459854	740	.001351351	795	.001257862	850	.001176471
686	.001457726	741	.001349528	796	.001256281	851	.001175088
687	.001455604	742	.001347709	797	.001254705	852	.001173709
688	.001453488	743	.001345895	798	.001253133	853	.001172333
689	.001451379	744	.001344086	799	.001251564	854	.001170960
690	.001449275	745	.001342282	800	.001250000	855	.001169591
691	.001447178	746	.001340483	801	.001248439	856	.001168224
692	.001445087	747	.001338688	802	.001246883	857	.001166861
693	.001443001	748	.001336898	803	.001245330	858	.001165501
694	.001440922	749	.001335113	804	.001243781	859	.001164144
695	.001438849	750	.001333333	805	.001242236	860	.001162791
696	.001436782	751	.001331558	806	.001240695	861	.001161440
697	.001434720	752	.001329787	807	.001239157	862	.001160093
698	.001432665	753	.001328021	808	.001237624	863	.001158749
699	.001430615	754	.001326260	809	.001236094	864	.001157407
700	.001428571	755	.001324503	810	.001234568	865	.001156069
701	.001426534	756	.001322751	811	.001233046	866	.001154734
702	.001424501	757	.001321004	812	.001231527	867	.001153403
703	.001422475	758	.001319261	813	.001230012	868	.001152074
704	.001420455	759	.001317523	814	.001228501	869	.001150748
705	.001418440	760	.001315789	815	.001226994	870	.001149425
706	.001416431	761	.001314060	816	.001225490	871	.001148106
707	.001414427	762	.001312336	817	.001223990	872	.001146789
708	.001412429	763	.001310616	818	.001222494	873	.001145475
709	.001410437	764	.001308901	819	.001221001	874	.001144165
710	.001408451	765	.001307190	820	.001219512	875	.001142857
711	.001406470	766	.001305483	821	.001218027	876	.001141553
712	.001404494	767	.001303781	822	.001216545	877	.001140251
713	.001402525	768	.001302083	823	.001215067	878	.001138952
714	.001400560	769	.001300390	824	.001213592	879	.001137656
715	.001398601	770	.001298701	825	.001212121	880	.001136364

Table of Reciprocals of Numbers.—(*Continued.*) See below.

No.	Reciprocal	No.	Reciprocal.	No.	Reciprocal.	No	Reciprocal
881	.001135074	911	.001097695	941	.001062699	971	.001029866
882	.001133787	912	.001096491	942	.001061571	972	.001028807
883	.001132503	913	.001095290	943	.001060445	973	.001027749
884	.001131222	914	.001094092	944	.001059322	974	.001026694
885	.001129944	915	.001092896	945	.001058201	975	.001025641
886	.001128668	916	.001091703	946	.001057082	976	.001024590
887	.001127396	917	.001090513	947	.001055966	977	.001023541
888	.001126126	918	.001089325	948	.001054852	978	.001022495
889	.001124859	919	.001088139	949	.001053741	979	.001021450
890	.001123596	920	.001086957	950	.001052632	980	.001020408
891	.001122334	921	.001085776	951	.001051525	981	.001019368
892	.001121076	922	.001084599	952	.001050420	982	.001018330
893	.001119821	923	.001083424	953	.001049318	983	.001017294
894	.001118568	924	.001082251	954	.001048218	984	.001016260
895	.001117318	925	.001081081	955	.001047120	985	.001015228
896	.001116071	926	.001079914	956	.001046025	986	.001014199
897	.001114827	927	.001078749	957	.001044932	987	.001013171
898	.001113586	928	.001077586	958	.001043841	988	.001012146
899	.001112347	929	.001076426	959	.001042753	989	.001011122
900	.001111111	930	.001075269	960	.001041667	990	.001010101
901	.001109878	931	.001074114	961	.001040583	991	.001009082
902	.001108647	932	.001072961	962	.001039501	992	.001008065
903	.001107420	933	.001071811	963	.001038422	993	.001007049
904	.001106195	934	.001070664	964	.001037344	994	.001006036
905	.001104972	935	.001069519	965	.001036269	995	.001005025
906	.001103753	936	.001068376	966	.001035197	996	.001004016
907	.001102536	937	.001067236	967	.001034126	997	.001003009
908	.001101322	938	.001066098	968	.001033058	998	.001002004
909	.001100110	939	.001064963	969	.001031992	999	.001001001
910	.001098901	940	.001063830	970	.001030928	1000	.001000000

RECIPROCALS.

(**a**) **The reciprocal of a number is** the quantity obtained by dividing unity or 1 by that number. In other words, if n be any number, then Recip $n = \frac{1}{n}$. Thus, Recip 40 $= \frac{1}{40} = 0.025$, Recip 0.4 $= \frac{1}{0.4} = 2.5$, etc., etc.

Hence, Recip $\frac{a}{b} = \frac{b}{a}$, because Recip $\frac{a}{b} = 1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$.

Thus, since 1 yard = 36 inches, 1 inch = $\frac{1}{36}$ yard = .027777778 yard, for Recip 36 = .027777778. Again, 1 foot head of water gives a pressure of .4335 lbs. per square inch. Hence a pressure of 1 lb per square inch corresponds to a head of $\frac{1}{.4335}$ feet = 2.306805 feet, for Recip .4335 = 2.306805. (See **h**, below.)

(**b**) It follows that if any number in the column headed "No." be taken as the denominator of a common fraction whose numerator is 1, the corresponding reciprocal is the value of that fraction expressed in decimals.* Thus, $\frac{1}{32}$ = .03125. Hence, **to reduce a common fraction to decimal form,** multiply the reciprocal of the denominator by the numerator. Thus, $\frac{17}{32}$ = .53125, because Recip 32 = .03125, and .03125 × 17 = .53125.

(**c**) Conversely, if the reciprocal of a number n be taken as a number, then the number n itself becomes the reciprocal. In other words, Recip $\frac{1}{n} = n$. Thus, Recip 0.025 = Recip $\frac{1}{40}$ = 40; Recip 2.5 = Recip $\frac{1}{0.4}$ = 0.4, etc., etc.

* The numbers 2 and 5, and their powers and products, are the only ones whose reciprocals can be *exactly* expressed in decimals.

(d) The product of any number by its own reciprocal is equal to unity or 1; or, $n \times \frac{1}{n} = \frac{n}{n} = 1$.

(e) Any number, $a \times$ Recip of a number, $n = a \times \frac{1}{n} = \frac{a}{n}$.

Hence, **to avoid the labor of dividing,** we may *multiply* by the *reciprocal* of the divisor. Thus,
$200 \div 48750 = 200 \times$ Recip $48750 = 200 \times .0000205128$ (see **h.** below) $= .004102564\ldots$

(f) Any number, $a \div$ Recip of a number, $n = a \div \frac{1}{n} = a\,n$.

Hence, $\quad a \div$ Recip $a = a \div \frac{1}{a} = a \times \frac{a}{1} = a^2$

Thus $\quad$ Recip $2 = 0.5$, and $\frac{2}{\text{Recip } 2} = \frac{2}{0.5} = 4 = 2^2$.

(g) The numbers in the foregoing table extend from 1 to 1000, but the reciprocals of **multiples of these numbers by 10** may be taken from the table by adding one cipher to the left of the reciprocal (after the decimal point) for each cipher added to the number. Thus,

Recip 390 = .002564103,
Recip 3900 = .0002564103;
Recip 39000 = .00002564103;

and the reciprocals of **numbers containing decimals** may be taken from the table by shifting the decimal point in the tabular reciprocal one place to the right for each decimal place in the number. Thus,

Recip 227 = .004405286;
Recip 22.7 = .04405286;
Recip 2.27 = .4405286,
Recip .227 = 4.405286;
Recip .0227 = 44.05286.

(h) The reciprocal of a **number of more than three figures** may be taken from the table approximately by interpolation. Thus, to find Recip 236.4:

Recip 236 = .004237288
Recip 237 = .004219409

Differences: 1, .000017879, $236.4 - 236 = 0.4$.
Then, $0.4 \times .000017879 = .000007152$,

and Recip 236 = .004237288
minus .000007152

= Recip 236.4 = .004230136 by interpolation.

The correct reciprocal is .004230118.

(i) The reciprocals of numbers not in the table may be conveniently found **by means of logarithms.** Thus, to find the Recip $236.4 = \frac{1}{236.4}$:

Log 1 = 0.000000
Subtract Log 236.4 = 2.373647

$\bar{3}.626353$ = Log 0.00423012
Recip 236.4 = 0.00423012.

To find Recip $\frac{8424}{236.4} = \frac{236.4}{8424}$,

Log 236.4 = 2.373647
Subtract Log 8424 = 3.925518

$\bar{2}.448129$ = Log 0.0280627.
Recip $\frac{8424}{236.4} = 0.0280627$.

(j) Position of the decimal point. For the Nos 10, 100, 1000, etc., the number of the decimal place occupied by the first significant figure in the reciprocal is equal to the number of *ciphers* in the No; but for all other Nos it is equal to the number of the *digits* in the integral portion of the No. Thus: Recip 143.7 = .0069..., etc. Here the number of digits in the integral portion (143) of the No. is 3, and the first significant figure (6) of the reciprocal occupies the *third* decimal place.

Square Roots and Cube Roots of Numbers from .1 to 28.

No errors.

No.	Square.	Cube.	Sq Rt.	C. Rt.	No.	Sq. Rt.	C Rt.	No.	Sq Rt.	C. Rt.
.1	.01	001	.316	.464	.7	2.387	1.786	.4	3 661	2 375
.15	.0225	.0034	.387	.531	.8	2.408	1 797	6	3 688	2 387
.2	.04	008	.447	.585	.9	2.429	1 807	8	3 715	2 399
.25	.0625	.0156	.500	.630	6	2.449	1 817	14.	3 742	2 410
.3	.09	.027	.548	669	.1	2.470	1 827	.2	3 768	2 422
.35	.1225	.0429	.592	.705	.2	2 490	1.837	.4	3.795	2.433
4	.16	.064	.633	737	.3	2 510	1.847	.6	3.821	2.444
45	.2025	.0911	.671	.766	4	2 530	1.857	8	3 847	2 455
5	25	.125	.707	.794	5	2 550	1.866	15	3 873	2 466
55	3025	.1664	.742	.810	6	2 569	1.876	.2	3 899	2 477
6	36	.216	.775	.843	7	2.588	1.885	.4	3 921	2 488
.65	.4225	.2746	.806	.866	8	2 608	1 895	.6	3 950	2 499
.7	49	.343	837	.888	9	2 627	1 904	.8	3 975	2 509
.75	5625	.4219	.866	.909	7	2 646	1 913	16	4	2 520
.8	.64	.512	.894	.928	1	2 665	1 922	.2	4 025	2 530
.85	.7225	.6141	.922	.947	2	2 683	1 931	.4	4 050	2 541
.9	.81	.729	.949	.965	.3	2 702	1.940	.6	4 074	2 551
.95	.9025	.8574	975	.983	4	2 720	1 949	.8	4 099	2 561
1.	1 000	1 000	1 000	1 000	5	2 739	1 957	17	4 123	2 571
.05	1.103	1 158	1 025	1 016	6	2 757	1 966	.2	4 147	2.581
1.1	1.210	1.331	1 049	1 032	.7	2 775	1 975	.4	4 171	2.591
.15	1.323	1 521	1 072	1.048	8	2 793	1 983	.6	4 195	2.601
1.2	1.440	1.728	1 095	1 063	9	2 811	1 992	.8	4 219	2 611
25	1.563	1 953	1.118	1 077	8.	2 828	2 000	18	4 243	2 621
1.3	1.690	2 197	1 140	1.091	1	2 846	2.008	.2	4 266	2 630
.35	1.823	2 460	1 162	1 105	.2	2 864	2 017	.4	4 290	2 640
1.4	1 960	2 744	1 83	1 119	3	2 881	2 025	.6	4 313	2 650
.45	2 103	3 049	1 204	1.132	4	2.898	2 033	.8	4.336	2 659
1.5	2.250	3 375	1 225	1 145	5	2.915	2 041	19.	4 359	2 668
.55	2.403	3 724	1.245	1 157	.6	2.933	2 049	.2	4 382	2 678
1.6	2.560	4.096	1 265	1 170	.7	2 950	2 057	.4	4.405	2 687
.65	2 723	4 492	1 285	1.182	8	2.966	2.065	.6	4 427	2 696
1 7	2 890	4.913	1 304	1.193	.9	2 983	2 072	.8	4 450	2 705
.75	3 063	5 359	1.323	1.205	9	3	2 080	20	4 472	2 714
1.8	3 240	5.832	1.342	1.216	.1	3 017	2 088	.2	4 494	2 723
.85	3 423	6 332	1.360	1.228	.2	3 033	2 095	.4	4 517	2 732
1.9	3 610	6.859	1.378	1 239	.3	3 050	2 103	.6	4 539	2.741
.95	3 803	7.415	1.396	1.249	.4	3 066	2 110	.8	4 561	2 750
2.	4.000	8 000	1.414	1.260	.5	3 082	2 118	21	4.583	2.759
.1	4.410	9.261	1.449	1.281	6	3 098	2 125	.2	4 604	2 768
.2	4 840	10 65	1 483	1.301	.7	3 114	2 133	.4	4 626	2 776
.3	5 290	12 17	1 517	1.320	.8	3.130	2 140	6	4 648	2 785
.4	5 760	13.82	1 549	1 339	.9	3.146	2 147	.8	4 669	2.794
.5	6.250	15 63	1.581	1.357	10	3 162	2 151	22	4 690	2 802
.6	6.760	17.58	1 612	1.375	.1	3 178	2 162	.2	4 712	2.810
.7	7.290	19.68	1 643	1,392	.2	3.194	2 169	4	4 733	2 819
.8	7.840	21.95	1 673	1 409	.3	3 209	2 176	.6	4.754	2.827
.9	8.410	24 39	1 703	1 426	.4	3 225	2.183	.8	4 775	2.836
3.	9.	27.	1.732	1.442	.5	3 240	2.190	23.	4.796	2.844
.1	9 61	29.79	1 761	1 458	.6	3 256	2.197	.2	4.817	2.852
.2	10.24	32 77	1 789	1 474	.7	3 271	2 204	4	4 837	2.860
.3	10 89	35.94	1.817	1 489	.8	3 286	2 210	.6	4.858	2 868
.4	11.56	39.30	1.844	1 504	.9	3 302	2 217	.8	4 879	2 876
.5	12 25	42.88	1.871	1 518	11.	3 317	2 224	24.	4 899	2 884
.6	12 96	46 66	1.897	1.533	.1	3 332	2 231	.2	4 919	2 892
.7	13.69	50.65	1.924	1.547	.2	3 347	2 237	.4	4 940	2 900
.8	14 44	54 87	1.949	1 560	.3	3 362	2 244	.6	4 960	2 908
.9	15.21	59.32	1.975	1 574	.4	3 376	2 251	8	4 980	2 916
4.	16.	64	2.	1.587	.5	3.391	2 257	25	5	2 924
.1	16.81	68.92	2 025	1 601	.6	3 406	2.264	.2	5 020	2 932
.2	17.64	74 09	2.049	1 613	.7	3.421	2 270	4	5 040	2 940
.3	18.49	79.51	2 074	1 626	.8	3.435	2 277	.6	5 060	2 947
.4	19.36	85.18	2 098	1 639	.9	3.450	2 283	8	5 079	2 955
.5	20.25	91.13	2 121	1 651	12.	3.464	2 289	26	5 099	2 964
.6	21 16	97 34	2.145	1 663	.1	3 479	2 296	.2	5 119	2.970
.7	22 09	103 8	2.168	1 675	.2	3.493	2 302	4	5.138	2.978
.8	23.04	110 6	2.191	1.687	.3	3 507	2 308	6	5.158	2.985
.9	24 01	117 6	2.214	1.698	.4	3 521	2.315	.8	5 177	2 993
5.	25.	125.	2 236	1.710	.5	3 536	2 321	27.	5 196	3 000
.1	26.01	132 7	2.258	1.721	.6	3 550	2.327	2	5 215	3 007
.2	27 04	140 6	2 280	1 732	.7	3.564	2 333	.4	5 235	3.015
.3	28 09	148 9	2 302	1.744	.8	3 578	2 339	6	5.254	3 022
.4	29 16	157.5	2.324	1 754	.9	3 592	2 345	.8	5.273	3.029
.5	30.25	166.4	2.345	1 765	13.	3.606	2 351	28.	5.292	3.037
.6	31.36	175.6	2 366	1.776	.2	3.633	2 363	.2	5.310	3.044

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000.

REMARK ON THE FOLLOWING TABLE. Wherever the effect of a fifth decimal in the roots would be to add 1 to the fourth and final decimal in the table, the addition has been made. No errors.

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
1	1	1	1.0000	1.0000	61	3721	226981	7.8102	3.9365
2	4	8	1.4142	1.2599	62	3844	238328	7.8740	3.9579
3	9	27	1.7321	1.4422	63	3969	250047	7.9373	3.9791
4	16	64	2.0000	1.5874	64	4096	262144	8.0000	4
5	25	125	2.2361	1.7100	65	4225	274625	8.0623	4.0207
6	36	216	2.4495	1.8171	66	4356	287496	8.1240	4.0412
7	49	343	2.6458	1.9129	67	4489	300763	8.1854	4.0615
8	64	512	2.8284	2.0000	68	4624	314432	8.2462	4.0817
9	81	729	3.0000	2.0801	69	4761	328509	8.3066	4.1016
10	100	1000	3.1623	2.1544	70	4900	343000	8.3666	4.1213
11	121	1331	3.3166	2.2240	71	5041	357911	8.4261	4.1408
12	144	1728	3.4641	2.2894	72	5184	373248	8.4853	4.1602
13	169	2197	3.6056	2.3513	73	5329	389017	8.5440	4.1793
14	196	2744	3.7417	2.4101	74	5476	405224	8.6023	4.1983
15	225	3375	3.8730	2.4662	75	5625	421875	8.6603	4.2172
16	256	4096	4.0000	2.5198	76	5776	438976	8.7178	4.2358
17	289	4913	4.1231	2.5713	77	5929	456533	8.7750	4.2543
18	324	5832	4.2426	2.6207	78	6084	474552	8.8318	4.2727
19	361	6859	4.3589	2.6684	79	6241	493039	8.8882	4.2908
20	400	8000	4.4721	2.7144	80	6400	512000	8.9443	4.3089
21	441	9261	4.5826	2.7589	81	6561	531441	9.	4.3267
22	484	10648	4.6904	2.8020	82	6724	551368	9.0554	4.3445
23	529	12167	4.7958	2.8439	83	6889	571787	9.1104	4.3621
24	576	13824	4.8990	2.8845	84	7056	592704	9.1652	4.3795
25	625	15625	5.0000	2.9240	85	7225	614125	9.2195	4.3968
26	676	17576	5.0990	2.9625	86	7396	636056	9.2736	4.4140
27	729	19683	5.1962	3.0000	87	7569	658503	9.3274	4.4310
28	784	21952	5.2915	3.0366	88	7744	681472	9.3808	4.4480
29	841	24389	5.3852	3.0723	89	7921	704969	9.4340	4.4647
30	900	27000	5.4772	3.1072	90	8100	729000	9.4868	4.4814
31	961	29791	5.5678	3.1414	91	8281	753571	9.5394	4.4979
32	1024	32768	5.6569	3.1748	92	8464	778688	9.5917	4.5144
33	1089	35937	5.7446	3.2075	93	8649	804357	9.6437	4.5307
34	1156	39304	5.8310	3.2396	94	8836	830584	9.6954	4.5468
35	1225	42875	5.9161	3.2711	95	9025	857375	9.7468	4.5629
36	1296	46656	6.0000	3.3019	96	9216	884736	9.7980	4.5789
37	1369	50653	6.0828	3.3322	97	9409	912673	9.8489	4.5947
38	1444	54872	6.1644	3.3620	98	9604	941192	9.8995	4.6104
39	1521	59319	6.2450	3.3912	99	9801	970299	9.9499	4.6261
40	1600	64000	6.3246	3.4200	100	10000	1000000	10.	4.6416
41	1681	68921	6.4031	3.4482	101	10201	1030301	10.0499	4.6570
42	1764	74088	6.4807	3.4760	102	10404	1061208	10.0995	4.6723
43	1849	79507	6.5574	3.5034	103	10609	1092727	10.1489	4.6875
44	1936	85184	6.6332	3.5303	104	10816	1124864	10.1980	4.7027
45	2025	91125	6.7082	3.5569	105	11025	1157625	10.2470	4.7177
46	2116	97336	6.7823	3.5830	106	11236	1191016	10.2956	4.7326
47	2209	103823	6.8557	3.6088	107	11449	1225043	10.3441	4.7475
48	2304	110592	6.9282	3.6342	108	11664	1259712	10.3923	4.7622
49	2401	117649	7.0000	3.6593	109	11881	1295029	10.4403	4.7769
50	2500	125000	7.0711	3.6840	110	12100	1331000	10.4881	4.7914
51	2601	132651	7.1414	3.7084	111	12321	1367631	10.5357	4.8059
52	2704	140608	7.2111	3.7325	112	12544	1404928	10.5830	4.8203
53	2809	148877	7.2801	3.7563	113	12769	1442897	10.6301	4.8346
54	2916	157464	7.3485	3.7798	114	12996	1481544	10.6771	4.8488
55	3025	166375	7.4162	3.8030	115	13225	1520875	10.7238	4.8629
56	3136	175616	7.4833	3.8259	116	13456	1560896	10.7703	4.8770
57	3249	185193	7.5498	3.8485	117	13689	1601613	10.8167	4.8910
58	3364	195112	7.6158	3.8709	118	13924	1643032	10.8628	4.9049
59	3481	205379	7.6811	3.8930	119	14161	1685159	10.9087	4.9187
60	3600	216000	7.7460	3.9149	120	14400	1728000	10.9545	4.9324

TABLE of Squares, Cubes Square Roots, and Cube Roots of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No	Square.	Cube.	Sq. Rt.	C. Rt.
121	14641	1771561	11.	4.9461	186	34596	6434856	13.6382	5.7083
122	14884	1815848	11.0454	4.9597	187	34969	6539203	13.6748	5.7185
123	15129	1860867	11.0905	4.9732	188	35344	6644672	13.7113	5.7287
124	15376	1906624	11.1355	4.9866	189	35721	6751269	13.7477	5.7388
125	15625	1953125	11.1803	5.	190	36100	6859000	13.7840	5.7489
126	15876	2000376	11.2250	5.0133	191	36481	6967871	13.8203	5.7590
127	16129	2048383	11.2694	5.0265	192	36864	7077888	13.8564	5.7690
128	16384	2097152	11.3137	5.0397	193	37249	7189057	13.8924	5.7790
129	16641	2146689	11.3578	5.0528	194	37636	7301384	13.9284	5.7890
130	16900	2197000	11.4018	5.0658	195	38025	7414875	13.9642	5.7989
131	17161	2248091	11.4455	5.0788	196	38416	7529536	14	5.8088
132	17424	2299968	11.4891	5.0916	197	38809	7645373	14.0357	5.8186
133	17689	2352637	11.5326	5.1045	198	39204	7762392	14.0712	5.8285
134	17956	2406104	11.5758	5.1172	199	39601	7880599	14.1067	5.8383
135	18225	2460375	11.6190	5.1299	200	40000	8000000	14.1421	5.8480
136	18496	2515456	11.6619	5.1426	201	40401	8120601	14.1774	5.8578
137	18769	2571353	11.7047	5.1551	202	40804	8242408	14.2127	5.8675
138	19044	2628072	11.7473	5.1676	203	41209	8365427	14.2478	5.8771
139	19321	2685619	11.7898	5.1801	204	41616	8489664	14.2829	5.8868
140	19600	2744000	11.8322	5.1925	205	42025	8615125	14.3178	5.8964
141	19881	2803221	11.8743	5.2048	206	42436	8741816	14.3527	5.9059
142	20164	2863288	11.9164	5.2171	207	42849	8869743	14.3875	5.9155
143	20449	2924207	11.9583	5.2293	208	43264	8998912	14.4222	5.9250
144	20736	2985984	12	5.2415	209	43681	9129329	14.4568	5.9345
145	21025	3048625	12.0416	5.2536	210	44100	9261000	14.4914	5.9439
146	21316	3112136	12.0830	5.2656	211	44521	9393931	14.5258	5.9533
147	21609	3176523	12.1244	5.2776	212	44944	9528128	14.5602	5.9627
148	21904	3241792	12.1655	5.2896	213	45369	9663597	14.5945	5.9721
149	22201	3307949	12.2066	5.3015	214	45796	9800344	14.6287	5.9814
150	22500	3375000	12.2474	5.3133	215	46225	9938375	14.6629	5.9907
151	22801	3442951	12.2882	5.3251	216	46656	10077696	14.6969	6
152	23104	3511808	12.3288	5.3368	217	47089	10218313	14.7309	6.0092
153	23409	3581577	12.3693	5.3485	218	47524	10360232	14.7648	6.0185
154	23716	3652264	12.4097	5.3601	219	47961	10503459	14.7986	6.0277
155	24025	3723875	12.4499	5.3717	220	48400	10648000	14.8324	6.0368
156	24336	3796416	12.4900	5.3832	221	48841	10793861	14.8661	6.0459
157	24649	3869893	12.5300	5.3947	222	49284	10941048	14.8997	6.0550
158	24964	3944312	12.5698	5.4061	223	49729	11089567	14.9332	6.0641
159	25281	4019679	12.6095	5.4175	224	50176	11239424	14.9666	6.0732
160	25600	4096000	12.6491	5.4288	225	50625	11390625	15.	6.0822
161	25921	4173281	12.6886	5.4401	226	51076	11543176	15.0333	6.0912
162	26244	4251528	12.7279	5.4514	227	51529	11697083	15.0665	6.1002
163	26569	4330747	12.7671	5.4626	228	51984	11852352	15.0997	6.1091
164	26896	4410944	12.8062	5.4737	229	52441	12008989	15.1327	6.1180
165	27225	4492125	12.8452	5.4848	230	52900	12167000	15.1658	6.1269
166	27556	4574296	12.8841	5.4959	231	53361	12326391	15.1987	6.1358
167	27889	4657463	12.9228	5.5069	232	53824	12487168	15.2315	6.1446
168	28224	4741632	12.9615	5.5178	233	54289	12649337	15.2643	6.1534
169	28561	4826809	13.	5.5288	234	54756	12812904	15.2971	6.1622
170	28900	4913000	13.0384	5.5397	235	55225	12977875	15.3297	6.1710
171	29241	5000211	13.0767	5.5505	236	55696	13144256	15.3623	6.1797
172	29584	5088448	13.1149	5.5613	237	56169	13312053	15.3948	6.1885
173	29929	5177717	13.1529	5.5721	238	56644	13481272	15.4272	6.1972
174	30276	5268024	13.1909	5.5828	239	57121	13651919	15.4596	6.2058
175	30625	5359375	13.2288	5.5934	240	57600	13824000	15.4919	6.2145
176	30976	5451776	13.2665	5.6041	241	58081	13997521	15.5242	6.2231
177	31329	5545233	13.3041	5.6147	242	58564	14172488	15.5563	6.2317
178	31684	5639752	13.3417	5.6252	243	59049	14348907	15.5885	6.2403
179	32041	5735339	13.3791	5.6357	244	59536	14526784	15.6205	6.2488
180	32400	5832000	13.4164	5.6462	245	60025	14706125	15.6525	6.2573
181	32761	5929741	13.4536	5.6567	246	60516	14886936	15.6844	6.2658
182	33124	6028568	13.4907	5.6671	247	61009	15069223	15.7162	6.2743
183	33489	6128487	13.5277	5.6774	248	61504	15252992	15.7480	6.2828
184	33856	6229504	13.5647	5.6877	249	62001	15438249	15.7797	6.2912
185	34225	6331625	13.6015	5.6980	250	62500	15625000	15.8114	6.2996

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt	No.	Square.	Cube.	Sq Rt.	C. Rt.
251	63001	15813251	15.8430	6.3080	316	99856	31554496	17.7764	6.8113
252	63504	16003008	15.8745	6.3164	317	100489	31855013	17.8045	6.8185
253	64009	16194277	15.9060	6.3247	318	101124	32157432	17.8326	6.8256
254	64516	16387064	15.9374	6.3330	319	101761	32461759	17.8606	6.8328
255	65025	16581375	15.9687	6.3413	320	102400	32768000	17.8885	6.8399
256	65536	16777216	16	6.3496	321	103041	33076161	17.9165	6.8470
257	66049	16974593	16.0312	6.3579	322	103684	33386248	17.9444	6.8541
258	66564	17173512	16.0624	6.3661	323	104329	33698267	17.9722	6.8612
259	67081	17373979	16.0935	6.3743	324	104976	34012224	18	6.8683
260	67600	17576000	16.1245	6.3825	325	105625	34328125	18.0278	6.8753
261	68121	17779581	16.1555	6.3907	326	106276	34645976	18.0555	6.8824
262	68644	17984728	16.1864	6.3988	327	106929	34965783	18.0831	6.8894
263	69169	18191447	16.2173	6.4070	328	107584	35287552	18.1108	6.8964
264	69696	18399744	16.2481	6.4151	329	108241	35611289	18.1384	6.9034
265	70225	18609625	16.2788	6.4232	330	108900	35937000	18.1659	6.9104
266	70756	18821096	16.3095	6.4312	331	109561	36264691	18.1934	6.9174
267	71289	19034163	16.3401	6.4393	332	110224	36594368	18.2209	6.9244
268	71824	19248832	16.3707	6.4473	333	110889	36926037	18.2483	6.9313
269	72361	19465109	16.4012	6.4553	334	111556	37259704	18.2757	6.9382
270	72900	19683000	16.4317	6.4633	335	112225	37595375	18.3030	6.9451
271	73441	19902511	16.4621	6.4713	336	112896	37933056	18.3303	6.9521
272	73984	20123648	16.4924	6.4792	337	113569	38272753	18.3576	6.9589
273	74529	20346417	16.5227	6.4872	338	114244	38614472	18.3848	6.9658
274	75076	20570824	16.5529	6.4951	339	114921	38958219	18.4120	6.9727
275	75625	20796875	16.5831	6.5030	340	115600	39304000	18.4391	6.9795
276	76176	21024576	16.6132	6.5108	341	116281	39651821	18.4662	6.9864
277	76729	21253933	16.6433	6.5187	342	116964	40001688	18.4932	6.9932
278	77284	21484952	16.6733	6.5265	343	117649	40353607	18.5203	7
279	77841	21717639	16.7033	6.5343	344	118336	40707584	18.5472	7.0068
280	78400	21952000	16.7332	6.5421	345	119025	41063625	18.5742	7.0136
281	78961	22188041	16.7631	6.5499	346	119716	41421736	18.6011	7.0203
282	79524	22425768	16.7929	6.5577	347	120409	41781923	18.6279	7.0271
283	80089	22665187	16.8226	6.5654	348	121104	42144192	18.6548	7.0338
284	80656	22906304	16.8523	6.5731	349	121801	42508549	18.6815	7.0406
285	81225	23149125	16.8819	6.5808	350	122500	42875000	18.7083	7.0473
286	81796	23393656	16.9115	6.5885	351	123201	43243551	18.7350	7.0540
287	82369	23639903	16.9411	6.5962	352	123904	43614208	18.7617	7.0607
288	82944	23887872	16.9706	6.6039	353	124609	43986977	18.7883	7.0674
289	83521	24137569	17	6.6115	354	125316	44361864	18.8149	7.0740
290	84100	24389000	17.0294	6.6191	355	126025	44738875	18.8414	7.0807
291	84681	24642171	17.0587	6.6267	356	126736	45118016	18.8680	7.0873
292	85264	24897088	17.0880	6.6343	357	127449	45499293	18.8944	7.0940
293	85849	25153757	17.1172	6.6419	358	128164	45882712	18.9209	7.1006
294	86436	25412184	17.1464	6.6494	359	128881	46268279	18.9473	7.1072
295	87025	25672375	17.1756	6.6569	360	129600	46656000	18.9737	7.1138
296	87616	25934336	17.2047	6.6644	361	130321	47045881	19.	7.1204
297	88209	26198073	17.2337	6.6719	362	131044	47437928	19.0263	7.1269
298	88804	26463592	17.2627	6.6794	363	131769	47832147	19.0526	7.1335
299	89401	26730899	17.2916	6.6869	364	132496	48228544	19.0788	7.1400
300	90000	27000000	17.3205	6.6943	365	133225	48627125	19.1050	7.1466
301	90601	27270901	17.3494	6.7018	366	133956	49027896	19.1311	7.1531
302	91204	27543608	17.3781	6.7092	367	134689	49430863	19.1572	7.1596
303	91809	27818127	17.4069	6.7166	368	135424	49836032	19.1833	7.1661
304	92416	28094464	17.4356	6.7240	369	136161	50243409	19.2094	7.1726
305	93025	28372625	17.4642	6.7313	370	136900	50653000	19.2354	7.1791
306	93636	28652616	17.4929	6.7387	371	137641	51064811	19.2614	7.1855
307	94249	28934443	17.5214	6.7460	372	138384	51478848	19.2873	7.1920
308	94864	29218112	17.5499	6.7533	373	139129	51895117	19.3132	7.1984
309	95481	29503629	17.5784	6.7606	374	139876	52313624	19.3391	7.2048
310	96100	29791000	17.6068	6.7679	375	140625	52734375	19.3649	7.2112
311	96721	30080231	17.6352	6.7752	376	141376	53157376	19.3907	7.2177
312	97344	30371328	17.6635	6.7824	377	142129	53582633	19.4165	7.2240
313	97969	30664297	17.6918	6.7897	378	142884	54010152	19.4422	7.2304
314	98596	30959144	17.7200	6.7969	379	143641	54439939	19.4679	7.2368
315	99225	31255875	17.7482	6.8041	380	144400	54872000	19.4936	7.2432

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt	No.	Square.	Cube.	Sq. Rt.	C. Rt.
381	145161	55306341	19.5192	7.2495	446	198916	88716536	21.1187	7.6403
382	145924	55742968	19.5448	7.2558	447	199809	89314623	21.1424	7.6460
383	146689	56181887	19.5704	7.2622	448	200704	89915392	21.1660	7.6517
384	147456	56623104	19.5959	7.2685	449	201601	90518849	21.1896	7.6574
385	148225	57066625	19.6214	7.2748	450	202500	91125000	21.2132	7.6631
386	148996	57512456	19.6469	7.2811	451	203401	91733851	21.2368	7.6688
387	149769	57960603	19.6723	7.2874	452	204304	92345408	21.2603	7.6744
388	150544	58411072	19.6977	7.2936	453	205209	92959677	21.2838	7.6801
389	151321	58863869	19.7231	7.2999	454	206116	93576664	21.3073	7.6857
390	152100	59319000	19.7484	7.3061	455	207025	94196375	21.3307	7.6914
391	152881	59776471	19.7737	7.3124	456	207936	94818816	21.3542	7.6970
392	153664	60236288	19.7990	7.3186	457	208849	95443993	21.3776	7.7026
393	154449	60698457	19.8242	7.3248	458	209764	96071912	21.4009	7.7082
394	155236	61162984	19.8494	7.3310	459	210681	96702579	21.4243	7.7138
395	156025	61629875	19.8746	7.3372	460	211600	97336000	21.4476	7.7194
396	156816	62099136	19.8997	7.3434	461	212521	97972181	21.4709	7.7250
397	157609	62570773	19.9249	7.3496	462	213444	98611128	21.4942	7.7306
398	158404	63044792	19.9499	7.3558	463	214369	99252847	21.5174	7.7362
399	159201	63521199	19.9750	7.3619	464	215296	99897344	21.5407	7.7418
400	160000	64000000	20.	7.3681	465	216225	100544625	21.5639	7.7473
401	160801	64481201	20.0250	7.3742	466	217156	101194696	21.5870	7.7529
402	161604	64964808	20.0499	7.3803	467	218089	101847563	21.6102	7.7584
403	162409	65450827	20.0749	7.3864	468	219024	102503232	21.6333	7.7639
404	163216	65939264	20.0998	7.3925	469	219961	103161709	21.6564	7.7695
405	164025	66430125	20.1246	7.3986	470	220900	103823000	21.6795	7.7750
406	164836	66923416	20.1494	7.4047	471	221841	104487111	21.7025	7.7805
407	165649	67419143	20.1742	7.4108	472	222784	105154048	21.7256	7.7860
408	166464	67917312	20.1990	7.4169	473	223729	105823817	21.7486	7.7915
409	167281	68417929	20.2237	7.4229	474	224676	106496424	21.7715	7.7970
410	168100	68921000	20.2485	7.4290	475	225625	107171875	21.7945	7.8025
411	168921	69426531	20.2731	7.4350	476	226576	107850176	21.8174	7.8079
412	169744	69934528	20.2978	7.4410	477	227529	108531333	21.8403	7.8134
413	170569	70444997	20.3224	7.4470	478	228484	109215352	21.8632	7.8188
414	171396	70957944	20.3470	7.4530	479	229441	109902239	21.8861	7.8243
415	172225	71473375	20.3715	7.4590	480	230400	110592000	21.9089	7.8297
416	173056	71991296	20.3961	7.4650	481	231361	111284641	21.9317	7.8352
417	173889	72511713	20.4206	7.4710	482	232324	111980168	21.9545	7.8406
418	174724	73034632	20.4450	7.4770	483	233289	112678587	21.9773	7.8460
419	175561	73560059	20.4695	7.4829	484	234256	113379904	22.	7.8514
420	176400	74088000	20.4939	7.4889	485	235225	114084125	22.0227	7.8568
421	177241	74618461	20.5183	7.4948	486	236196	114791256	22.0454	7.8622
422	178084	75151448	20.5426	7.5007	487	237169	115501303	22.0681	7.8676
423	178929	75686967	20.5670	7.5067	488	238144	116214272	22.0907	7.8730
424	179776	76225024	20.5913	7.5126	489	239121	116930169	22.1133	7.8784
425	180625	76765625	20.6155	7.5185	490	240100	117649000	22.1359	7.8837
426	181476	77308776	20.6398	7.5244	491	241081	118370771	22.1585	7.8891
427	182329	77854483	20.6640	7.5302	492	242064	119095488	22.1811	7.8944
428	183184	78402752	20.6882	7.5361	493	243049	119823157	22.2036	7.8998
429	184041	78953589	20.7123	7.5420	494	244036	120553784	22.2261	7.9051
430	184900	79507000	20.7364	7.5478	495	245025	121287375	22.2486	7.9105
431	185761	80062991	20.7605	7.5537	496	246016	122023936	22.2711	7.9158
432	186624	80621568	20.7846	7.5595	497	247009	122763473	22.2935	7.9211
433	187489	81182737	20.8087	7.5654	498	248004	123505992	22.3159	7.9264
434	188356	81746504	20.8327	7.5712	499	249001	124251499	22.3383	7.9317
435	189225	82312875	20.8567	7.5770	500	250000	125000000	22.3607	7.9370
436	190096	82881856	20.8806	7.5828	501	251001	125751501	22.3830	7.9423
437	190969	83453453	20.9045	7.5886	502	252004	126506008	22.4054	7.9476
438	191844	84027672	20.9284	7.5944	503	253009	127263527	22.4277	7.9528
439	192721	84604519	20.9523	7.6001	504	254016	128024064	22.4499	7.9581
440	193600	85184000	20.9762	7.6059	505	255025	128787625	22.4722	7.9634
441	194481	85766121	21.	7.6117	506	256036	129554216	22.4944	7.9686
442	195364	86350888	21.0238	7.6174	507	257049	130323843	22.5167	7.9739
443	196249	86938307	21.0476	7.6232	508	258064	131096512	22.5389	7.9791
444	197136	87528384	21.0713	7.6289	509	259081	131872229	22.5610	7.9843
445	198025	88121125	21.0950	7.6346	510	260100	132651000	22.5832	7.9896

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED)

No.	Square.	Cube.	Sq Rt.	C. Rt.	No.	Square.	Cube.	Sq Rt.	C. Rt.
511	261121	133432831	22.6053	7.9948	576	331776	191102976	24.	8.3203
512	262144	134217728	22.6274	8.	577	332929	192100033	24.0208	8.3251
513	263169	135005697	22.6495	8.0052	578	334084	193100552	24.0416	8.3300
514	264196	135796744	22.6716	8.0104	579	335241	194104539	24.0624	8.3348
515	265225	136590875	22.6936	8.0156	580	336400	195112000	24.0832	8.3396
516	266256	137388096	22.7156	8.0208	581	337561	196122941	24.1039	8.3443
517	267289	138188413	22.7376	8.0260	582	338724	197137368	24.1247	8.3491
518	268324	138991832	22.7596	8.0311	583	339889	198155287	24.1454	8.3539
519	269361	139798359	22.7816	8.0363	584	341056	199176704	24.1661	8.3587
520	270400	140608000	22.8035	8.0415	585	342225	200201625	24.1868	8.3634
521	271441	141420761	22.8254	8.0466	586	343396	201230056	24.2074	8.3682
522	272484	142236648	22.8473	8.0517	587	344569	202262003	24.2281	8.3730
523	273529	143055667	22.8692	8.0569	588	345744	203297472	24.2487	8.3777
524	274576	143877824	22.8910	8.0620	589	346921	204336469	24.2693	8.3825
525	275625	144703125	22.9129	8.0671	590	348100	205379000	24.2899	8.3872
526	276676	145531576	22.9347	8.0723	591	349281	206425071	24.3105	8.3919
527	277729	146363183	22.9565	8.0774	592	350464	207474688	24.3311	8.3967
528	278784	147197952	22.9783	8.0825	593	351649	208527857	24.3516	8.4014
529	279841	148035889	23.	8.0876	594	352836	209584584	24.3721	8.4061
530	280900	148877000	23.0217	8.0927	595	354025	210644875	24.3926	8.4108
531	281961	149721291	23.0434	8.0978	596	355216	211708736	24.4131	8.4155
532	283024	150568768	23.0651	8.1028	597	356409	212776173	24.4336	8.4202
533	284089	151419437	23.0868	8.1079	598	357604	213847192	24.4540	8.4249
534	285156	152273304	23.1084	8.1130	599	358801	214921799	24.4745	8.4296
535	286225	153130375	23.1301	8.1180	600	360000	216000000	24.4949	8.4343
536	287296	153990656	23.1517	8.1231	601	361201	217081801	24.5153	8.4390
537	288369	154854153	23.1733	8.1281	602	362404	218167208	24.5357	8.4437
538	289444	155720872	23.1948	8.1332	603	363609	219256227	24.5561	8.4484
539	290521	156590819	23.2164	8.1382	604	364816	220348864	24.5764	8.4530
540	291600	157464000	23.2379	8.1433	605	366025	221445125	24.5967	8.4577
541	292681	158340421	23.2594	8.1483	606	367236	222545016	24.6171	8.4623
542	293764	159220088	23.2809	8.1533	607	368449	223648543	24.6374	8.4670
543	294849	160103007	23.3024	8.1583	608	369664	224755712	24.6577	8.4716
544	295936	160989184	23.3238	8.1633	609	370881	225866529	24.6779	8.4763
545	297025	161878625	23.3452	8.1683	610	372100	226981000	24.6982	8.4809
546	298116	162771336	23.3666	8.1733	611	373321	228099131	24.7184	8.4856
547	299209	163667323	23.3880	8.1783	612	374544	229220928	24.7386	8.4902
548	300304	164566592	23.4094	8.1833	613	375769	230346397	24.7588	8.4948
549	301401	165469149	23.4307	8.1882	614	376996	231475544	24.7790	8.4994
550	302500	166375000	23.4521	8.1932	615	378225	232608375	24.7992	8.5040
551	303601	167284151	23.4734	8.1982	616	379456	233744896	24.8193	8.5086
552	304704	168196608	23.4947	8.2031	617	380689	234885113	24.8395	8.5132
553	305809	169112377	23.5160	8.2081	618	381924	236029032	24.8596	8.5178
554	306916	170031464	23.5372	8.2130	619	383161	237176659	24.8797	8.5224
555	308025	170953875	23.5584	8.2180	620	384400	238328000	24.8998	8.5270
556	309136	171879616	23.5797	8.2229	621	385641	239483061	24.9199	8.5316
557	310249	172808693	23.6008	8.2278	622	386884	240641848	24.9399	8.5362
558	311364	173741112	23.6220	8.2327	623	388129	241804367	24.9600	8.5408
559	312481	174676879	23.6432	8.2377	624	389376	242970624	24.9800	8.5453
560	313600	175616000	23.6643	8.2426	625	390625	244140625	25.	8.5499
561	314721	176558481	23.6854	8.2475	626	391876	245314376	25.0200	8.5544
562	315844	177504328	23.7065	8.2524	627	393129	246491883	25.0400	8.5590
563	316969	178453547	23.7276	8.2573	628	394384	247673152	25.0599	8.5635
564	318096	179406144	23.7487	8.2621	629	395641	248858189	25.0799	8.5681
565	319225	180362125	23.7697	8.2670	630	396900	250047000	25.0998	8.5726
566	320356	181321496	23.7908	8.2719	631	398161	251239591	25.1197	8.5772
567	321489	182284263	23.8118	8.2768	632	399424	252435968	25.1396	8.5817
568	322624	183250432	23.8328	8.2816	633	400689	253636137	25.1595	8.5862
569	323761	184220009	23.8537	8.2865	634	401956	254840104	25.1794	8.5907
570	324900	185193000	23.8747	8.2913	635	403225	256047875	25.1992	8.5952
571	326041	186169411	23.8956	8.2962	636	404496	257259456	25.2190	8.5997
572	327184	187149248	23.9165	8.3010	637	405769	258474853	25.2389	8.6043
573	328329	188132517	23.9374	8.3059	638	407044	259694072	25.2587	8.6088
574	329476	189119224	23.9583	8.3107	639	408321	260917119	25.2784	8.6132
575	330625	190109375	23.9792	8.3155	640	409600	262144000	25.2982	8.6177

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
641	410881	263374721	25.3180	8.6222	706	498436	351895816	26.5707	8.9043
642	412164	264609288	25.3377	8.6267	707	499849	353393243	26.5895	8.9085
643	413449	265847707	25.3574	8.6312	708	501264	354894912	26.6083	8.9127
644	414736	267089984	25.3772	8.6357	709	502681	356400829	26.6271	8.9169
645	416025	268336125	25.3969	8.6401	710	504100	357911000	26.6458	8.9211
646	417316	269586136	25.4165	8.6446	711	505521	359425431	26.6646	8.9253
647	418609	270840023	25.4362	8.6490	712	506944	360944128	26.6843	8.9295
648	419904	272097792	25.4558	8.6535	713	508369	362467097	26.7021	8.9337
649	421201	273359449	25.4755	8.6579	714	509796	363994344	26.7208	8.9378
650	422500	274625000	25.4951	8.6624	715	511225	365525875	26.7395	8.9420
651	423801	275894451	25.5147	8.6668	716	512656	367061696	26.7582	8.9462
652	425104	277167808	25.5343	8.6713	717	514089	368601813	26.7769	8.9503
653	426409	278445077	25.5539	8.6757	718	515524	370146232	26.7955	8.9545
654	427716	279726264	25.5734	8.6801	719	516961	371694959	26.8142	8.9587
655	429025	281011375	25.5930	8.6845	720	518400	373248000	26.8328	8.9628
656	430336	282300416	25.6125	8.6890	721	519841	374805361	26.8514	8.9670
657	431649	283593393	25.6320	8.6934	722	521284	376367048	26.8701	8.9711
658	432964	284890312	25.6515	8.6978	723	522729	377933067	26.8887	8.9752
659	434281	286191179	25.6710	8.7022	724	524176	379503424	26.9072	8.9794
660	435600	287496000	25.6905	8.7066	725	525625	381078125	26.9258	8.9835
661	436921	288804781	25.7099	8.7110	726	527076	382657176	26.9444	8.9876
662	438244	290117528	25.7294	8.7154	727	528529	384240583	26.9629	8.9918
663	439569	291434247	25.7488	8.7198	728	529984	385828352	26.9815	8.9959
664	440896	292754944	25.7682	8.7241	729	531441	387420489	27.	9.
665	442225	294079625	25.7876	8.7285	730	532900	389017000	27.0185	9.0041
666	443556	295408296	25.8070	8.7329	731	534361	390617891	27.0370	9.0082
667	444889	296740963	25.8263	8.7373	732	535824	392223168	27.0555	9.0123
668	446224	298077632	25.8457	8.7416	733	537289	393832837	27.0740	9.0164
669	447561	299418309	25.8650	8.7460	734	538756	395446904	27.0924	9.0205
670	448900	300763000	25.8844	8.7503	735	540225	397065375	27.1109	9.0246
671	450241	302111711	25.9037	8.7547	736	541696	398688256	27.1293	9.0287
672	451584	303464448	25.9230	8.7590	737	543169	400315553	27.1477	9.0328
673	452929	304821217	25.9422	8.7634	738	544644	401947272	27.1662	9.0369
674	454276	306182024	25.9615	8.7677	739	546121	403583419	27.1846	9.0410
675	455625	307546875	25.9808	8.7721	740	547600	405224000	27.2029	9.0450
676	456976	308915776	26.	8.7764	741	549081	406869021	27.2213	9.0491
677	458329	310288733	26.0192	8.7807	742	550564	408518488	27.2397	9.0532
678	459684	311665752	26.0384	8.7850	743	552049	410172407	27.2580	9.0572
679	461041	313046839	26.0576	8.7893	744	553536	411830784	27.2764	9.0613
680	462400	314432000	26.0768	8.7937	745	555025	413493625	27.2947	9.0654
681	463761	315821241	26.0960	8.7980	746	556516	415160936	27.3130	9.0694
682	465124	317214568	26.1151	8.8023	747	558009	416832723	27.3313	9.0735
683	466489	318611987	26.1343	8.8066	748	559504	418508992	27.3496	9.0775
684	467856	320013504	26.1534	8.8109	749	561001	420189749	27.3679	9.0816
685	469225	321419125	26.1725	8.8152	750	562500	421875000	27.3861	9.0856
686	470596	322828856	26.1916	8.8194	751	564001	423564751	27.4044	9.0896
687	471969	324242703	26.2107	8.8237	752	565504	425259008	27.4226	9.0937
688	473344	325660672	26.2298	8.8280	753	567009	426957777	27.4408	9.0977
689	474721	327082769	26.2488	8.8323	754	568516	428661064	27.4591	9.1017
690	476100	328509000	26.2679	8.8366	755	570025	430368875	27.4773	9.1057
691	477481	329939371	26.2869	8.8408	756	571536	432081216	27.4955	9.1098
692	478864	331373888	26.3059	8.8451	757	573049	433798093	27.5136	9.1138
693	480249	332812557	26.3249	8.8493	758	574564	435519512	27.5318	9.1178
694	481636	334255384	26.3439	8.8536	759	576081	437245479	27.5500	9.1218
695	483025	335702375	26.3629	8.8578	760	577600	438976000	27.5681	9.1258
696	484416	337153536	26.3818	8.8621	761	579121	440711081	27.5862	9.1298
697	485809	338608873	26.4008	8.8663	762	580644	442450728	27.6043	9.1338
698	487204	340068392	26.4197	8.8706	763	582169	444194947	27.6225	9.1378
699	488601	341532099	26.4386	8.8748	764	583696	445943744	27.6405	9.1418
700	490000	343000000	26.4575	8.8790	765	585225	447697125	27.6586	9.1458
701	491401	344472101	26.4764	8.8833	766	586756	449455096	27.6767	9.1498
702	492804	345948408	26.4953	8.8875	767	588289	451217663	27.6948	9.1537
703	494209	347428927	26.5141	8.8917	768	589824	452984832	27.7128	9.1577
704	495616	348913664	26.5330	8.8959	769	591361	454756609	27.7308	9.1617
705	497025	350402625	26.5518	8.9001	770	592900	456533000	27.7489	9.1657

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 — (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt	No.	Square.	Cube.	Sq. Rt.	C. Rt.
771	594441	458314011	27.7669	9.1696	836	698896	584277056	28.9137	9.4204
772	595984	460099648	27.7849	9.1736	837	700569	586376253	28.9310	9.4241
773	597529	461889917	27.8029	9.1775	838	702244	588480472	28.9482	9.4279
774	599076	463684824	27.8209	9.1815	839	703921	590589719	28.9655	9.4316
775	600625	465484375	27.8388	9.1855	840	705600	592704000	28.9828	9.4354
776	602176	467288576	27.8568	9.1894	841	707281	594823321	29.	9.4391
777	603729	469097433	27.8747	9.1933	842	708964	596947688	29.0172	9.4429
778	605284	470910952	27.8927	9.1973	843	710649	599077107	29.0345	9.4466
779	606841	472729139	27.9106	9.2012	844	712336	601211584	29.0517	9.4503
780	608400	474552000	27.9285	9.2052	845	714025	603351125	29.0689	9.4541
781	609961	476379541	27.9464	9.2091	846	715716	605495736	29.0861	9.4578
782	611524	478211768	27.9643	9.2130	847	717409	607645423	29.1033	9.4615
783	613089	480048687	27.9821	9.2170	848	719104	609800192	29.1204	9.4652
784	614656	481890304	28	9.2209	849	720801	611960049	29.1376	9.4690
785	616225	483736625	28.0179	9.2248	850	722500	614125000	29.1548	9.4727
786	617796	485587656	28.0357	9.2287	851	724201	616295051	29.1719	9.4764
787	619369	487443403	28.0535	9.2326	852	725904	618470208	29.1890	9.4801
788	620944	489303872	28.0713	9.2365	853	727609	620650477	29.2062	9.4838
789	622521	491169069	28.0891	9.2404	854	729316	622835864	29.2233	9.4875
790	624100	493039000	28.1069	9.2443	855	731025	625026375	29.2404	9.4912
791	625681	494913671	28.1247	9.2482	856	732736	627222016	29.2575	9.4949
792	627264	496793088	28.1425	9.2521	857	734449	629422793	29.2746	9.4986
793	628849	498677257	28.1603	9.2560	858	736164	631628712	29.2916	9.5023
794	630436	500566184	28.1780	9.2599	859	737881	633839779	29.3087	9.5060
795	632025	502459875	28.1957	9.2638	860	739600	636056000	29.3258	9.5097
796	633616	504358336	28.2135	9.2677	861	741321	638277381	29.3428	9.5134
797	635209	506261573	28.2312	9.2716	862	743044	640503928	29.3598	9.5171
798	636804	508169592	28.2489	9.2754	863	744769	642735647	29.3769	9.5207
799	638401	510082399	28.2666	9.2793	864	746496	644972544	29.3939	9.5244
800	640000	512000000	28.2843	9.2832	865	748225	647214625	29.4109	9.5281
801	641601	513922401	28.3019	9.2870	866	749956	649461896	29.4279	9.5317
802	643204	515849608	28.3196	9.2909	867	751689	651714363	29.4449	9.5354
803	644809	517781627	28.3373	9.2948	868	753424	653972032	29.4618	9.5391
804	646416	519718464	28.3549	9.2986	869	755161	656234909	29.4788	9.5427
805	648025	521660125	28.3725	9.3025	870	756900	658503000	29.4958	9.5464
806	649636	523606616	28.3901	9.3063	871	758641	660776311	29.5127	9.5501
807	651249	525557943	28.4077	9.3102	872	760384	663054848	29.5296	9.5537
808	652864	527514112	28.4253	9.3140	873	762129	665338617	29.5466	9.5574
809	654481	529475129	28.4429	9.3179	874	763876	667627624	29.5635	9.5610
810	656100	531441000	28.4605	9.3217	875	765625	669921875	29.5804	9.5647
811	657721	533411731	28.4781	9.3255	876	767376	672221376	29.5973	9.5683
812	659344	535387328	28.4956	9.3294	877	769129	674526133	29.6142	9.5719
813	660969	537367797	28.5132	9.3332	878	770884	676836152	29.6311	9.5756
814	662596	539353144	28.5307	9.3370	879	772641	679151439	29.6479	9.5792
815	664225	541343375	28.5482	9.3408	880	774400	681472000	29.6648	9.5828
816	665856	543338496	28.5657	9.3447	881	776161	683797841	29.6816	9.5865
817	667489	545338513	28.5832	9.3485	882	777924	686128968	29.6985	9.5901
818	669124	547343432	28.6007	9.3523	883	779689	688465387	29.7153	9.5937
819	670761	549353259	28.6182	9.3561	884	781456	690807104	29.7321	9.5973
820	672400	551368000	28.6356	9.3599	885	783225	693154125	29.7489	9.6010
821	674041	553387661	28.6531	9.3637	886	784996	695506456	29.7658	9.6046
822	675684	555412248	28.6705	9.3675	887	786769	697864103	29.7825	9.6082
823	677329	557441767	28.6880	9.3713	888	788544	700227072	29.7993	9.6118
824	678976	559476224	28.7054	9.3751	889	790321	702595369	29.8161	9.6154
825	680625	561515625	28.7228	9.3789	890	792100	704969000	29.8329	9.6190
826	682276	563559976	28.7402	9.3827	891	793881	707347971	29.8496	9.6226
827	683929	565609283	28.7576	9.3865	892	795664	709732288	29.8664	9.6262
828	685584	567663552	28.7750	9.3902	893	797449	712121957	29.8831	9.6298
829	687241	569722789	28.7924	9.3940	894	799236	714516984	29.8998	9.6334
830	688900	571787000	28.8097	9.3978	895	801025	716917375	29.9166	9.6370
831	690561	573856191	28.8271	9.4016	896	802816	719323136	29.9333	9.6406
832	692224	575930368	28.8444	9.4053	897	804609	721734273	29.9500	9.6442
833	693889	578009537	28.8617	9.4091	898	806404	724150792	29.9666	9.6477
834	695556	580093704	28.8791	9.4129	899	808201	726572699	29.9833	9.6513
835	697225	582182875	28.8964	9.4166	900	810000	729000000	30.	9.6549

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq Rt.	C. Rt	No.	Square.	Cube.	Sq. Rt.	C. Rt.
901	811801	731432701	30.0167	9.6585	951	904401	860085351	30.8383	9.8339
902	813604	733870808	30.0333	9.6620	952	906304	862801408	30.8545	9.8374
903	815409	736314327	30.0500	9.6656	953	908209	865523177	30.8707	9.8408
904	817216	738763264	30.0666	9.6692	954	910116	868250664	30.8869	9.8443
905	819025	741217625	30.0832	9.6727	955	912025	870983875	30.9031	9.8477
906	820836	743677416	30.0998	9.6763	956	913936	873722816	30.9192	9.8511
907	822649	746142643	30.1164	9.6799	957	915849	876467493	30.9354	9.8546
908	824464	748613312	30.1330	9.6834	958	917764	879217912	30.9516	9.8580
909	826281	751089429	30.1496	9.6870	959	919681	881974079	30.9677	9.8614
910	828100	753571000	30.1662	9.6905	960	921600	884736000	30.9839	9.8648
911	829921	756058031	30.1828	9.6941	961	923521	887503681	31.	9.8683
912	831744	758550528	30.1993	9.6976	962	925444	890277128	31.0161	9.8717
913	833569	761048497	30.2159	9.7012	963	927369	893056347	31.0322	9.8751
914	835396	763551944	30.2324	9.7047	964	929296	895841344	31.0483	9.8785
915	837225	766060875	30.2490	9.7082	965	931225	898632125	31.0644	9.8819
916	839056	768575296	30.2655	9.7118	966	933156	901428696	31.0805	9.8854
917	840889	771095213	30.2820	9.7153	967	935089	904231063	31.0966	9.8888
918	842724	773620632	30.2985	9.7188	968	937024	907039232	31.1127	9.8922
919	844561	776151559	30.3150	9.7224	969	938961	909853209	31.1288	9.8956
920	846400	778688000	30.3315	9.7259	970	940900	912673000	31.1448	9.8990
921	848241	781229961	30.3480	9.7294	971	942841	915498611	31.1609	9.9024
922	850084	783777448	30.3645	9.7329	972	944784	918330048	31.1769	9.9058
923	851929	786330467	30.3809	9.7364	973	946729	921167317	31.1929	9.9092
924	853776	788889024	30.3974	9.7400	974	948676	924010424	31.2090	9.9126
925	855625	791453125	30.4138	9.7435	975	950625	926859375	31.2250	9.9160
926	857476	794022776	30.4302	9.7470	976	952576	929714176	31.2410	9.9194
927	859329	796597983	30.4467	9.7505	977	954529	932574833	31.2570	9.9227
928	861184	799178752	30.4631	9.7540	978	956484	935441352	31.2730	9.9261
929	863041	801765089	30.4795	9.7575	979	958441	938313739	31.2890	9.9295
930	864900	804357000	30.4959	9.7610	980	960400	941192000	31.3050	9.9329
931	866761	806954491	30.5123	9.7645	981	962361	944076141	31.3209	9.9363
932	868624	809557568	30.5287	9.7680	982	964324	946966168	31.3369	9.9396
933	870489	812166237	30.5450	9.7715	983	966289	949862087	31.3528	9.9430
934	872356	814780504	30.5614	9.7750	984	968256	952763904	31.3688	9.9464
935	874225	817400375	30.5778	9.7785	985	970225	955671625	31.3847	9.9497
936	876096	820025856	30.5941	9.7819	986	972196	958585256	31.4006	9.9531
937	877969	822656953	30.6105	9.7854	987	974169	961504803	31.4166	9.9565
938	879844	825293672	30.6268	9.7889	988	976144	964430272	31.4325	9.9598
939	881721	827936019	30.6431	9.7924	989	978121	967361669	31.4484	9.9632
940	883600	830584000	30.6594	9.7959	990	980100	970299000	31.4643	9.9666
941	885481	833237621	30.6757	9.7993	991	982081	973242271	31.4802	9.9699
942	887364	835896888	30.6920	9.8028	992	984064	976191488	31.4960	9.9733
943	889249	838561807	30.7083	9.8063	993	986049	979146657	31.5119	9.9766
944	891136	841232384	30.7246	9.8097	994	988036	982107784	31.5278	9.9800
945	893025	843908625	30.7409	9.8132	995	990025	985074875	31.5436	9.9833
946	894916	846590536	30.7571	9.8167	996	992016	988047936	31.5595	9.9866
947	896809	849278123	30.7734	9.8201	997	994009	991026973	31.5753	9.9900
948	898704	851971392	30.7896	9.8236	998	996004	994011992	31.5911	9.9933
949	900601	854670349	30.8058	9.8270	999	998001	997002999	31.6070	9.9967
950	902500	857375000	30.8221	9.8305	1000	1000000	1000000000	31.6228	10.

To find the square or cube of any whole number ending with ciphers. First, omit all the final ciphers. Take from the table the square or cube (as the case may be) of the rest of the number. To this *square* add *twice* as many ciphers as there were final ciphers in the original number. To the *cube* add *three* times as many as in the original number. Thus, for 90500^2; $905^2 = 819025$. Add twice 2 ciphers, obtaining 8190250000. for 90500^3 $905^3 = 741217625$. Add 3 times 2 ciphers, obtaining 741217625000000.

Square Roots and Cube Roots of Numbers from 1000 to 10000.

No errors.

Num.	Sq. Rt	Cu Rt	Num	Sq Rt	Cu. Rt	Num	Sq Rt	Cu Rt	Num.	Sq Rt.	Cu. Rt.
1005	31.70	10.02	1405	37.48	11.20	1805	42.49	12.18	2205	46.96	13.02
1010	31.78	10.03	1410	37.55	11.21	1810	42.54	12.19	2210	47.01	13.03
1015	31.86	10.05	1415	37.62	11.23	1815	42.60	12.20	2215	47.06	13.04
1020	31.94	10.07	1420	37.68	11.24	1820	42.66	12.21	2220	47.12	13.05
1025	32.02	10.08	1425	37.75	11.25	1825	42.72	12.22	2225	47.17	13.05
1030	32.09	10.10	1430	37.82	11.27	1830	42.78	12.23	2230	47.22	13.06
1035	32.17	10.12	1435	37.88	11.28	1835	42.84	12.24	2235	47.28	13.07
1040	32.25	10.13	1440	37.95	11.29	1840	42.90	12.25	2240	47.33	13.08
1045	32.33	10.15	1445	38.01	11.31	1845	42.95	12.26	2245	47.38	13.09
1050	32.40	10.16	1450	38.08	11.32	1850	43.01	12.28	2250	47.43	13.10
1055	32.48	10.18	1455	38.14	11.33	1855	43.07	12.29	2255	47.49	13.11
1060	32.56	10.20	1460	38.21	11.34	1860	43.13	12.30	2260	47.54	13.12
1065	32.63	10.21	1465	38.28	11.36	1865	43.19	12.31	2265	47.59	13.13
1070	32.71	10.23	1470	38.34	11.37	1870	43.24	12.32	2270	47.64	13.14
1075	32.79	10.24	1475	38.41	11.38	1875	43.30	12.33	2275	47.70	13.15
1080	32.86	10.26	1480	38.47	11.40	1880	43.36	12.34	2280	47.75	13.16
1085	32.94	10.28	1485	38.54	11.41	1885	43.42	12.35	2285	47.80	13.17
1090	33.02	10.29	1490	38.60	11.42	1890	43.47	12.36	2290	47.85	13.18
1095	33.09	10.31	1495	38.67	11.43	1895	43.53	12.37	2295	47.91	13.19
1100	33.17	10.32	1500	38.73	11.45	1900	43.59	12.39	2300	47.96	13.20
1105	33.24	10.34	1505	38.79	11.46	1905	43.65	12.40	2305	48.01	13.21
1110	33.32	10.35	1510	38.86	11.47	1910	43.70	12.41	2310	48.06	13.22
1115	33.39	10.37	1515	38.92	11.49	1915	43.76	12.42	2315	48.11	13.23
1120	33.47	10.38	1520	38.99	11.50	1920	43.82	12.43	2320	48.17	13.24
1125	33.54	10.40	1525	39.05	11.51	1925	43.87	12.44	2325	48.22	13.25
1130	33.62	10.42	1530	39.12	11.52	1930	43.93	12.45	2330	48.27	13.26
1135	33.69	10.43	1535	39.18	11.54	1935	43.99	12.46	2335	48.32	13.27
1140	33.76	10.45	1540	39.24	11.55	1940	44.05	12.47	2340	48.37	13.28
1145	33.84	10.46	1545	39.31	11.56	1945	44.10	12.48	2345	48.43	13.29
1150	33.91	10.48	1550	39.37	11.57	1950	44.16	12.49	2350	48.48	13.30
1155	33.99	10.49	1555	39.43	11.59	1955	44.22	12.50	2355	48.53	13.30
1160	34.06	10.51	1560	39.50	11.60	1960	44.27	12.51	2360	48.58	13.31
1165	34.13	10.52	1565	39.56	11.61	1965	44.33	12.53	2365	48.63	13.32
1170	34.21	10.54	1570	39.62	11.62	1970	44.38	12.54	2370	48.68	13.33
1175	34.28	10.55	1575	39.69	11.63	1975	44.44	12.55	2375	48.73	13.34
1180	34.35	10.57	1580	39.75	11.65	1980	44.50	12.56	2380	48.79	13.35
1185	34.42	10.58	1585	39.81	11.66	1985	44.55	12.57	2385	48.84	13.36
1190	34.50	10.60	1590	39.87	11.67	1990	44.61	12.58	2390	48.89	13.37
1195	34.57	10.61	1595	39.94	11.68	1995	44.67	12.59	2395	48.94	13.38
1200	34.64	10.63	1600	40.00	11.70	2000	44.72	12.60	2400	48.99	13.39
1205	34.71	10.64	1605	40.06	11.71	2005	44.78	12.61	2405	49.04	13.40
1210	34.79	10.66	1610	40.12	11.72	2010	44.83	12.62	2410	49.09	13.41
1215	34.86	10.67	1615	40.19	11.73	2015	44.89	12.63	2415	49.14	13.42
1220	34.93	10.69	1620	40.25	11.74	2020	44.94	12.64	2420	49.19	13.43
1225	35.00	10.70	1625	40.31	11.76	2025	45.00	12.65	2425	49.24	13.43
1230	35.07	10.71	1630	40.37	11.77	2030	45.06	12.66	2430	49.30	13.44
1235	35.14	10.73	1635	40.44	11.78	2035	45.11	12.67	2435	49.35	13.45
1240	35.21	10.74	1640	40.50	11.79	2040	45.1	12.68	2440	49.40	13.46
1245	35.28	10.76	1645	40.56	11.80	2045	45.22	12.69	2445	49.45	13.47
1250	35.36	10.77	1650	40.62	11.82	2050	45.28	12.70	2450	49.50	13.48
1255	35.43	10.79	1655	40.68	11.83	2055	45.33	12.71	2460	49.60	13.50
1260	35.50	10.80	1660	40.74	11.84	2060	45.39	12.72	2470	49.70	13.52
1265	35.57	10.82	1665	40.80	11.85	2065	45.44	12.73	2480	49.80	13.54
1270	35.64	10.83	1670	40.87	11.86	2070	45.50	12.74	2490	49.90	13.55
1275	35.71	10.84	1675	40.93	11.88	2075	45.55	12.75	2500	50.00	13.57
1280	35.78	10.86	1680	40.99	11.89	2080	45.61	12.77	2510	50.10	13.59
1285	35.85	10.87	1685	41.05	11.90	2085	45.66	12.78	2520	50.20	13.61
1290	35.92	10.89	1690	41.11	11.91	2090	45.72	12.79	2530	50.30	13.63
1295	35.99	10.90	1695	41.17	11.92	2095	45.77	12.80	2540	50.40	13.64
1300	36.06	10.91	1700	41.23	11.93	2100	45.83	12.81	2550	50.50	13.66
1305	36.12	10.93	1705	41.29	11.95	2105	45.88	12.82	2560	50.60	13.68
1310	36.19	10.94	1710	41.35	11.96	2110	45.93	12.83	2570	50.70	13.70
1315	36.26	10.96	1715	41.41	11.97	2115	45.99	12.84	2580	50.79	13.72
1320	36.33	10.97	1720	41.47	11.98	2120	46.04	12.85	2590	50.89	13.73
1325	36.40	10.98	1725	41.53	11.99	2125	46.10	12.86	2600	50.99	13.75
1330	36.47	11.00	1730	41.59	12.00	2130	46.15	12.87	2610	51.09	13.77
1335	36.54	11.01	1735	41.65	12.02	2135	46.21	12.88	2620	51.19	13.79
1340	36.61	11.02	1740	41.71	12.03	2140	46.26	12.89	2630	51.28	13.80
1345	36.67	11.04	1745	41.77	12.04	2145	46.31	12.90	2640	51.38	13.82
1350	36.74	11.05	1750	41.83	12.05	2150	46.37	12.91	2650	51.48	13.84
1355	36.81	11.07	1755	41.89	12.06	2155	46.42	12.92	2660	51.58	13.86
1360	36.88	11.08	1760	41.95	12.07	2160	46.48	12.93	2670	51.67	13.87
1365	36.95	11.09	1765	42.01	12.09	2165	46.53	12.94	2680	51.77	13.89
1370	37.01	11.11	1770	42.07	12.10	2170	46.58	12.95	2690	51.87	13.91
1375	37.08	11.12	1775	42.13	12.11	2175	46.64	12.96	2700	51.96	13.92
1380	37.15	11.13	1780	42.19	12.12	2180	46.69	12.97	2710	52.06	13.94
1385	37.22	11.15	1785	42.25	12.13	2185	46.74	12.98	2720	52.15	13.96
1390	37.28	11.16	1790	42.31	12.14	2190	46.80	12.99	2730	52.25	13.98
1395	37.35	11.17	1795	42.37	12.15	2195	46.85	13.00	2740	52.35	13.[illegible]
1400	37.42	11.19	1800	42.43	12.16	2200	46.90	13.01	2750	52.44	14.0[illegible]

Square Roots and Cube Roots of Numbers from 1000 to 10000
—(CONTINUED.)

Num.	Sq. Rt	Cu. Rt	Num.	Sq. Rt	Cu. Rt	Num.	Sq. Rt	Cu. Rt	Num.	Sq. Rt	Cu. Rt.
2760	52.54	14.03	3550	59.58	15.25	4340	65.88	16.31	5130	71.62	17.25
2770	52.63	14.04	3560	59.67	15.27	4350	65.95	16.32	5140	71.69	17.26
2780	52.73	14.06	3570	59.75	15.28	4360	66.03	16.34	5150	71.76	17.27
2790	52.82	14.08	3580	59.83	15.30	4370	66.11	16.35	5160	71.83	17.28
2800	52.92	14.09	3590	59.92	15.31	4380	66.18	16.36	5170	71.90	17.29
2810	53.01	14.11	3600	60.00	15.33	4390	66.26	16.37	5180	71.97	17.30
2820	53.10	14.13	3610	60.08	15.34	4400	66.33	16.39	5190	72.04	17.31
2830	53.20	14.14	3620	60.17	15.35	4410	66.41	16.40	5200	72.11	17.32
2840	53.29	14.16	3630	60.25	15.37	4420	66.48	16.41	5210	72.18	17.34
2850	53.39	14.18	3640	60.33	15.38	4430	66.56	16.42	5220	72.25	17.35
2860	53.48	14.19	3650	60.42	15.40	4440	66.63	16.44	5230	72.32	17.36
2870	53.57	14.21	3660	60.50	15.41	4450	66.71	16.45	5240	72.39	17.37
2880	53.67	14.23	3670	60.58	15.42	4460	66.78	16.46	5250	72.46	17.38
2890	53.76	14.24	3680	60.66	15.44	4470	66.86	16.47	5260	72.53	17.39
2900	53.85	14.26	3690	60.75	15.45	4480	66.93	16.49	5270	72.59	17.40
2910	53.94	14.28	3700	60.83	15.47	4490	67.01	16.50	5280	72.66	17.41
2920	54.04	14.29	3710	60.91	15.48	4500	67.08	16.51	5290	72.73	17.42
2930	54.13	14.31	3720	60.99	15.49	4510	67.16	16.52	5300	72.80	17.44
2940	54.22	14.33	3730	61.07	15.51	4520	67.23	16.53	5310	72.87	17.45
2950	54.31	14.34	3740	61.16	15.52	4530	67.31	16.55	5320	72.94	17.46
2960	54.41	14.36	3750	61.24	15.54	4540	67.38	16.56	5330	73.01	17.47
2970	54.50	14.37	3760	61.32	15.55	4550	67.45	16.57	5340	73.08	17.48
2980	54.59	14.39	3770	61.40	15.56	4560	67.53	16.58	5350	73.14	17.49
2990	54.68	14.41	3780	61.48	15.58	4570	67.60	16.59	5360	73.21	17.50
3000	54.77	14.42	3790	61.56	15.59	4580	67.68	16.61	5370	73.28	17.51
3010	54.86	14.44	3800	61.64	15.60	4590	67.75	16.62	5380	73.35	17.52
3020	54.95	14.45	3810	61.73	15.62	4600	67.82	16.63	5390	73.42	17.53
3030	55.05	14.47	3820	61.81	15.63	4610	67.90	16.64	5400	73.48	17.54
3040	55.14	14.49	3830	61.89	15.65	4620	67.97	16.66	5410	73.55	17.55
3050	55.23	14.50	3840	61.97	15.66	4630	68.04	16.67	5420	73.62	17.57
3060	55.32	14.52	3850	62.05	15.67	4640	68.12	16.68	5430	73.69	17.58
3070	55.41	14.53	3860	62.13	15.69	4650	68.19	16.69	5440	73.76	17.59
3080	55.50	14.55	3870	62.21	15.70	4660	68.26	16.70	5450	73.82	17.60
3090	55.59	14.57	3880	62.29	15.71	4670	68.34	16.71	5460	73.89	17.61
3100	55.68	14.58	3890	62.37	15.73	4680	68.41	16.73	5470	73.96	17.62
3110	55.77	14.60	3900	62.45	15.74	4690	68.48	16.74	5480	74.03	17.63
3120	55.86	14.61	3910	62.53	15.75	4700	68.56	16.75	5490	74.09	17.64
3130	55.95	14.63	3920	62.61	15.77	4710	68.63	16.76	5500	74.16	17.65
3140	56.04	14.64	3930	62.69	15.78	4720	68.70	16.77	5510	74.23	17.66
3150	56.12	14.66	3940	62.77	15.79	4730	68.77	16.79	5520	74.30	17.67
3160	56.21	14.67	3950	62.85	15.81	4740	68.85	16.80	5530	74.36	17.68
3170	56.30	14.69	3960	62.93	15.82	4750	68.92	16.81	5540	74.43	17.69
3180	56.39	14.71	3970	63.01	15.83	4760	68.99	16.82	5550	74.50	17.71
3190	56.48	14.72	3980	63.09	15.85	4770	69.07	16.83	5560	74.57	17.72
3200	56.57	14.74	3990	63.17	15.86	4780	69.14	16.85	5570	74.63	17.73
3210	56.66	14.75	4000	63.25	15.87	4790	69.21	16.86	5580	74.70	17.74
3220	56.75	14.77	4010	63.32	15.89	4800	69.28	16.87	5590	74.77	17.75
3230	56.83	14.78	4020	63.40	15.90	4810	69.35	16.88	5600	74.83	17.76
3240	56.92	14.80	4030	63.48	15.91	4820	69.43	16.89	5610	74.90	17.77
3250	57.01	14.81	4040	63.56	15.93	4830	69.50	16.90	5620	74.97	17.78
3260	57.10	14.83	4050	63.64	15.94	4840	69.57	16.92	5630	75.03	17.79
3270	57.18	14.84	4060	63.72	15.95	4850	69.64	16.93	5640	75.10	17.80
3280	57.27	14.86	4070	63.80	15.97	4860	69.71	16.94	5650	75.17	17.81
3290	57.36	14.87	4080	63.87	15.98	4870	69.79	16.95	5660	75.23	17.82
3300	57.45	14.89	4090	63.95	15.99	4880	69.86	16.96	5670	75.30	17.83
3310	57.53	14.90	4100	64.03	16.01	4890	69.93	16.97	5680	75.37	17.84
3320	57.62	14.92	4110	64.11	16.02	4900	70.00	16.98	5690	75.43	17.85
3330	57.71	14.93	4120	64.19	16.03	4910	70.07	17.00	5700	75.50	17.86
3340	57.79	14.95	4130	64.27	18.04	4920	70.14	17.01	5710	75.56	17.87
3350	57.88	14.96	4140	64.34	16.06	4930	70.21	17.02	5720	75.63	17.88
3360	57.97	14.98	4150	64.42	16.07	4940	70.29	17.03	5730	75.70	17.89
3370	58.05	14.99	4160	64.50	16.08	4950	70.36	17.04	5740	75.76	17.90
3380	58.14	15.01	4170	64.58	16.10	4960	70.43	17.05	5750	75.83	17.92
3390	58.22	15.02	4180	64.65	16.11	4970	70.50	17.07	5760	75.89	17.93
3400	58.31	15.03	4190	64.73	16.12	4980	70.57	17.08	5770	75.96	17.94
3410	58.40	15.05	4200	64.81	16.13	4990	70.64	17.09	5780	76.03	17.95
3420	58.48	15.07	4210	64.88	16.15	5000	70.71	17.10	5790	76.09	17.96
3430	58.57	15.08	4220	64.96	16.16	5010	70.78	17.11	5800	76.16	17.97
3440	58.65	15.10	4230	65.04	16.17	5020	70.85	17.12	5810	76.22	17.98
3450	58.74	15.11	4240	65.12	16.19	5030	70.92	17.13	5820	76.29	17.99
3460	58.82	15.12	4250	65.19	16.20	5040	70.99	17.15	5830	76.35	18.00
3470	58.91	15.14	4260	65.27	16.21	5050	71.06	17.16	5840	76.42	18.01
3480	58.99	15.15	4270	65.35	16.22	5060	71.13	17.17	5850	76.49	18.02
3490	59.08	15.17	4280	65.42	16.24	5070	71.20	17.18	5860	76.55	18.03
3500	59.16	15.18	4290	65.50	16.25	5080	71.27	17.19	5870	76.62	18.04
3510	59.25	15.20	4300	65.57	16.26	5090	71.34	17.20	5880	76.68	18.05
3520	59.33	15.21	4310	65.65	16.27	5100	71.41	17.21	5890	76.75	18.06
3530	59.41	15.23	4320	65.73	16.29	5110	71.48	17.22	5900	76.81	18.07
3540	59.50	15.24	4330	65.80	16.30	5120	71.55	17.24	5910	76.88	18.08

Square Roots and Cube Roots of Numbers from 1000 to 10000
—(CONTINUED.)

Num.	Sq. Rt	Cu. Rt	Num.	Sq Rt	Cu Rt	Num.	Sq Rt	Cu Rt	Num	Sq. Rt	Cu R
5920	76 94	18 09	6710	81.91	18 86	7500	86 60	19 57	8290	91 05	20 24
5930	77 01	18 10	6720	81 98	18 87	7510	86 66	19 58	8300	91 10	20 25
5940	77.07	18 11	6730	82 04	18 88	7520	86 72	19 59	8310	91 16	20 26
5950	77 14	18 12	6740	82 10	18 89	7530	86 78	19 60	8320	91 21	20 26
5960	77.20	18 13	6750	82 16	18 90	7540	86 83	19 61	8330	91 27	20 27
5970	77 27	18 14	6760	82 22	18 91	7550	86 89	19 62	8340	91 32	20 28
5980	77 33	18 15	6770	82 28	18 92	7560	86 95	19 63	8350	91 38	20.29
5990	77 40	18 16	6780	82 34	18 93	7570	87 01	19 64	8360	91.43	20 30
6000	77.46	18 17	6790	82 40	18 94	7580	87 06	19 64	8370	91.49	20 30
6010	77 52	18 18	6800	82 46	18 95	7590	87 12	19 65	8380	91 54	20.31
6020	77 59	18 19	6810	82 52	18 95	7600	87 18	19 66	8390	91 60	20.32
6030	77.65	18 20	6820	82 58	18 96	7610	87 24	19 67	8400	91 65	20.33
6040	77 72	18 21	6830	82 64	18 97	7620	87 29	19 68	8410	91.71	20 34
6050	77 78	18 22	6840	82 70	18 98	7630	87 35	19 69	8420	91 76	20.34
6060	77 85	18 23	6850	82 76	18 99	7640	87 41	19.70	8430	91 82	20 35
6070	77 91	18 24	6860	82 83	19 00	7650	87 46	19 70	8440	91 87	20 36
6080	77 97	18 25	6870	82 89	19 01	7660	87 52	19 71	8450	91 92	20 37
6090	78 04	18 26	6880	82 95	19 02	7670	87 58	19 72	8460	91 98	20 38
6100	78.10	18 27	6890	83 01	19 03	7680	87 64	19 73	8470	92 03	20 38
6110	78 17	18 28	6900	83 07	19 04	7690	87 69	19 74	8480	92 09	20 39
6120	78.23	18 29	6910	83 13	19 05	7700	87 75	19 75	8490	92 14	20 40
6130	78 29	18 30	6920	83 19	19 06	7710	87 81	19 76	8500	92 20	20 41
6140	78 36	18 31	6930	83 25	19 07	7720	87 86	19 76	8510	92 25	20 42
6150	78 42	18 32	6940	83 31	19 07	7730	87 92	19 77	8520	92 30	20 42
6160	78 49	18 33	6950	83 37	19 08	7740	87 98	19 78	8530	92 36	20 43
6170	78 55	18 34	6960	83 43	19 09	7750	88 03	19 79	8540	92 41	20.44
6180	78.61	18 35	6970	83 49	19 10	7760	88 09	19 80	8550	92 47	20.45
6190	78 68	18 36	6980	83 55	19 11	7770	88 15	19 81	8560	92 52	20.46
6200	78 74	18 37	6990	83 61	19 12	7780	88 20	19 81	8570	92 57	20 46
6210	78 80	18 38	7000	83 67	19 13	7790	88 26	19 82	8580	92.63	20 47
6220	78 87	18 39	7010	83 73	19 14	7800	88 32	19 83	8590	92 68	20 48
6230	78 93	18 40	7020	83 79	19 15	7810	88 37	19 84	8600	92 74	20 49
6240	78 99	18 41	7030	83 85	19 16	7820	88 43	19 85	8610	92 79	20 50
6250	79 06	18 42	7040	83 90	19 17	7830	88 49	19 86	8620	92 84	20.50
6260	79 12	18 43	7050	83.96	19 17	7840	88 54	19 87	8630	92 90	20.51
6270	79 18	18 44	7060	84 02	19 18	7850	88 60	19 87	8640	92 95	20 52
6280	79 25	18 45	7070	84 08	19 19	7860	88 66	19 88	8650	93.01	20.53
6290	79 31	18 46	7080	84 14	19 20	7870	88 71	19 89	8660	93 06	20 54
6300	79 37	18 47	7090	84 20	19 21	7880	88 77	19 90	8670	93 11	20 54
6310	79 44	18 48	7100	84 26	19 22	7890	88 83	19.91	8680	93 17	20 55
6320	79 50	18 49	7110	84 32	19 23	7900	88 88	19 92	8690	93 22	20.56
6330	79 56	18 50	7120	84 38	19 24	7910	88 94	19 92	8700	93 27	20 57
6340	79 62	18 51	7130	84 44	19 25	7920	88 99	19 93	8710	93.33	20 57
6350	79 69	18 52	7140	84 50	19 26	7930	89 05	19 94	8720	93 38	20 58
6360	79 75	18 53	7150	84 56	19 26	7940	89 11	19 95	8730	93 43	20 59
6370	79 81	18 54	7160	84 62	19.27	7950	89 16	19 96	8740	93 49	20 60
6380	79 87	18 55	7170	84 68	19 28	7960	89 22	19 97	8750	93 54	20 61
6390	79 94	18.56	7180	84 73	19 29	7970	89.27	19.97	8760	93 59	20 61
6400	80 00	18 57	7190	84 79	19 30	7980	89 33	19.98	8770	93 65	20 62
6410	80 06	18 58	7200	84 85	19 31	7990	89.39	19.99	8780	93 70	20.63
6420	80.12	18 59	7210	84 91	19 32	8000	89 44	20 00	8790	93 75	20 64
6430	80 19	18 60	7220	84 97	19 33	8010	89 50	20 01	8800	93 81	20.65
6440	80 25	18 60	7230	85 03	19 34	8020	89 55	20 02	8810	93 86	20 65
6450	80 31	18 61	7240	85 09	19 35	8030	89 61	20 02	8820	93 91	20 66
6460	80 37	18 62	7250	85 15	19 35	8040	89 67	20 03	8830	93 97	20 67
6470	80 44	18.63	7260	85 21	19 36	8050	89 72	20 04	8840	94 02	20 68
6480	80 50	18.64	7270	85 26	19 37	8060	89 78	20 05	8850	94 07	20 68
6490	80 56	18 65	7280	85 32	19 38	8070	89 83	20.06	8860	94 13	20 69
6500	80.62	18 66	7290	85 38	19 39	8080	89 89	20.07	8870	94 18	20.70
6510	80 68	18 67	7300	85 44	19 40	8090	89 94	20.07	8880	94.23	20.71
6520	80 75	18 68	7310	85 50	19 41	8100	90 00	20.08	8890	94.29	20.72
6530	80 81	18.69	7320	85 56	19 42	8110	90 06	20 09	8900	94 34	20.72
6540	80.87	18.70	7330	85 62	19 43	8120	90.11	20 10	8910	94 39	20.73
6550	80 93	18 71	7340	85 67	19 43	8130	90.17	20 11	8920	94.45	20.74
6560	80 99	18 72	7350	85 73	19 44	8140	90 22	20 12	8930	94 50	20.75
6570	81.06	18 73	7360	85.79	19.45	8150	90 28	20 12	8940	94 55	20.75
6580	81.12	18.74	7370	85 85	19.46	8160	90 33	20 13	8950	94 60	20 76
6590	81.18	18 75	7380	85 91	19 47	8170	90 39	20 14	8960	94 66	20.77
6600	81 24	18 76	7390	85.97	19 48	8180	90 44	20.15	8970	94 71	20 78
6610	81.30	18 77	7400	86.02	19 49	8190	90 50	20 16	8980	94 76	20 79
6620	81.36	18 78	7410	86 08	19 50	8200	90 55	20 17	8990	94 82	20.79
6630	81 42	18 79	7420	86.14	19.50	8210	90 61	20 17	9000	94 87	20 80
6640	81 49	18 80	7430	86 20	19 51	8220	90 66	20.18	9010	94 92	20.81
6650	81.55	18 81	7440	86 26	19 52	8230	90 72	20.19	9020	94.97	20 82
6660	81 61	18 81	7450	86.31	19 53	8240	90 77	20.20	9030	95.03	20 82
6670	81 67	18 82	7460	86 37	19 54	8250	90 83	20 21	9040	95 08	20 83
6680	81.73	18 83	7470	86 43	19 55	8260	90.88	20 21	9050	95.13	20.84
6690	81.79	18 84	7480	86 49	19 56	8270	90 94	20.22	9060	95 18	20.85
6700	81.85	18.85	7490	86.54	19.57	8280	90.99	20.23	9070	95.24	20.86

Square Roots and Cube Roots of Numbers from 1000 to 10000 —(Continued.)

Num	Sq Rt	Cu Rt	Num.	Sq. Rt.	Cu. Rt	Num	Sq Rt	Cu. Rt	Num.	Sq. Rt.	Cu. Rt.
9080	95 29	20 86	9320	96.54	21.04	9550	97.72	21.22	9780	98.89	21.39
9090	95.34	20 87	9330	96 59	21.05	9560	97 78	21 22	9790	98.94	21.39
9100	95 39	20 88	9340	96.64	21.06	9570	97 83	21.23	9800	98 99	21.40
9110	95 45	20 89	9350	96 70	21.07	9580	97 88	21 24	9810	99.05	21.41
9120	95 50	20 89	9360	96 75	21.07	9590	97.93	21 25	9820	99 10	21 41
9130	95.55	20 90	9370	96 80	21.08	9600	97.98	21.25	9830	99.15	21 42
9140	95 60	20 91	9380	96 85	21.09	9610	98 03	21.26	9840	99 20	21.43
9150	95.66	20 92	9390	96 90	21.10	9620	98 08	21 27	9850	99 25	21 44
9160	95 71	20 92	9400	96 95	21.10	9630	98 13	21.28	9860	99.30	21 44
9170	95 76	20 93	9410	97.01	21 11	9640	98 18	21 28	9870	99 35	21.45
9180	95 81	20 94	9420	97.06	21.12	9650	98 23	21.29	9880	99.40	21 46
9190	95 86	20.95	9430	97 11	21 13	9660	98 29	21.30	9890	99 45	21.47
9200	95 92	20 95	9440	97 16	21.13	9670	98 34	21 30	9900	99 50	21 47
9210	95.97	20 96	9450	97.21	21 14	9680	98 39	21.31	9910	99.55	21 48
9220	96 02	20.97	9460	97.26	21 15	9690	98.44	21 32	9920	99.60	21 49
9230	96 07	20 98	9470	97.31	21 16	9700	98 49	21.33	9930	99 65	21.49
9240	96.12	20 98	9480	97.37	21.16	9710	98 54	21.33	9940	99 70	21.50
9250	96 18	20 99	9490	97.42	21 17	9720	98 59	21.34	9950	99 75	21.51
9260	96 23	21 00	9500	97.47	21.18	9730	98.64	21 35	9960	99 80	21.52
9270	96 28	21.01	9510	97.52	21.19	9740	98 69	21 36	9970	99.85	21.52
9280	96.33	21 01	9520	97.57	21 19	9750	98.74	21.36	9980	99.90	21.53
9290	96 38	21 02	9530	97 62	21 20	9760	98 79	21 37	9990	99 95	21.54
9300	96.44	21 03	9540	97.67	21.21	9770	98.84	21 38	10000	100.00	21.54
9310	96 49	21.04									

To find Square or Cube Roots of large numbers not contained in the column of numbers of the table.

Such roots may sometimes be taken at once from the table, by merely regarding the columns of powers as being columns of numbers; and those of numbers as being those of roots. Thus, if the *sq rt* of 25281 is reqd, first find that number in the column of *squares;* and opposite to it, in the column of numbers, is its sq rt 159 For the *cube rt* of 857375, find that number in the column of *cubes;* and opposite to it, in the col of numbers, is its cube rt 95. When the exact number is not contained in the column of squares, or cubes, as the case may be, we may use instead the number nearest to it, if no great accuracy is reqd But when a considerable degree of accuracy is necessary, the following very correct methods may be used.

For the square root.

This rule applies both to whole numbers, and to those which are *partly* (not wholly) decimal First, in the foregoing manner, take out the tabular number, which is nearest to the given one; and also its tabular sq rt. Mult this tabular number by 3, to the prod add the given number. Call the sum A. Then mult the given number by 3, to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 946 53. Here we find the nearest tabular number to be 947; and its tabular sq rt 30.7734 Hence,

$$\left.\begin{array}{l} 947 = \text{tab num} \\ \quad 3 \\ \hline 2841 \\ 946.53 = \text{given num.} \\ \hline 3787.53 = \text{A.} \end{array}\right\} \text{ and } \left\{\begin{array}{l} 946.53 = \text{given num.} \\ \quad 3 \\ \hline 2839\ 59 \\ 947 \quad = \text{tab num.} \\ \hline 3786.59 = \text{B.} \end{array}\right.$$

	A.		B.		Tab root.		Reqd root.
Then	3787.53	:	3786.59	: :	30.7734	:	30.7657 +.

The root as found by actual mathematical process is also 30.7657 +.

For the cube root.

This rule applies both to whole numbers, and to those which are *partly* decimal. First take out the tabular number which is nearest to the given one; and also *its* tabular cube rt. Mult this tabular number by 2; and to the prod add the given number Call the sum A. Then mult the given number by 2; and to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 7368. Here we find the nearest tabular number (in the column of cubes) to be 6859; and its tabular cube rt 19. Hence,

$$\left.\begin{array}{l} 6859 = \text{tab num.} \\ \quad 2 \\ \hline 13718 \\ 7368 = \text{given num.} \\ \hline 21086 = \text{A.} \end{array}\right\} \text{ and } \left\{\begin{array}{l} 7368 = \text{given num.} \\ \quad 2 \\ \hline 14736 \\ 6859 = \text{tab num.} \\ \hline 21595 = \text{B.} \end{array}\right.$$

	A.		B.		Tab Root.		Reqd Rt.
Then, as	21086	:	21595	: :	19	:	19 4585

The root as found by correct mathematical process is 19.4588. The engineer rarely requires even

this degree of accuracy; for his purposes, therefore, this process is greatly preferable to the ordinary laborious one.

To find the square root of a number which is wholly decimal.

Very simple, and correct to the third numeral figure inclusive. If the number does not contain at least five figures, *counting from the first numeral, and including it*, add one or more ciphers to make five If, after that, the *whole number* is not separable into twos, add another cipher to make it so. Then beginning at the first numeral figure, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one, take out its tabular sq rt, move the decimal point of this tabular root to the left, *half* as many places as the finally modified *decimal* number has figures.

Ex. What is the sq rt of the decimal 002? Here, in order to have at least five decimal figures, counting from the first numeral (2), and including it, add ciphers thus, 00 20,00 0. But, as it is not now separable into twos, add another cipher, thus, .00,20,00,00 Then beginning at the first numeral (2), assume this decimal to be the whole number 200000. The nearest to this in the table is 199809; and the sq rt of this is 447 Now the decimal number as finally modified, namely, 00,20,00,00, has eight figures, one half of which is 4, therefore, move the decimal point of the root 447, four places to the left, making it .0447 This is the reqd sq rt of 002, correct to the third numeral 7 included.

To find the cube root of a number which is wholly decimal.

Very simple, and correct to the third numeral inclusive

If the number does not contain at least five figures, counting from the first numeral, and including it, add one or more ciphers to make five If, after that, the number is not separable into threes, add one or more ciphers to make it so. Then beginning at the first numeral, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one, and take out its tabular cub rt Move the decimal point of this rt to the left, one third as many places as the finally modified *decimal* number has figures

Ex What is the cube rt of the decimal 002? Here, in order to have at least five figures, counting from the first numeral (2), and including it, add ciphers thus, 002 000 0 But as it is not now separable into threes, add two more ciphers to make it so, thus 002,000,000 Then beginning with the first numeral (2), assume the decimal to be the whole number 2000000 The nearest cube to this in the table in the column of cubes, is 2000376, and its tabular cube rt as found in the col of numbers is 126 Now, the decimal number as finally modified namely, 002 000 000, has nine figures; one-third of which is 3, therefore, move the decimal point of the root 126, three places to the left, making it .126. This is the reqd cube rt of the decimal .002, correct to the third numeral 6 included.

Fifth roots and fifth powers.

Power	No or Root	Power	No or Root	Power	No or Root	Power	No or Root	Power	No or Root	Power.	No or Root.
0000100	1	000112	170	001219	.335	077760	.60	695688	.93	8 11368	1 52
		000164	175	001544	.340	081460	.61	.733904	.94	8 66171	1 54
0000110	102	000189	180	001888	.345	091613	.62	.773781	95	9 23896	1 56
		000217	185	005252	350	099244	63	815373	.96	9 84658	1 58
000012	104	000248	190	005638	.355	107374	64	858734	.97	10 4858	1 60
		000282	195	006047	360	116029	65	.903921	.98	11 1577	1 62
.0000134	.106	000320	200	006478	365	125233	.66	950990	99	11 8637	1 64
		000362	205	006934	370	135012	.67	1	1	12 6049	1 66
0000147	108	000408	210	007416	375	145393	68	1 10408	1 02	13 3828	1 68
0000161	110	000459	215	007924	.380	.156403	.69	1 21665	1 04	14 1986	1 70
0000176	112	.000515	.220	008459	385	168070	70	1 33823	1 06	15 0537	1 72
0000193	114	000577	225	009022	390	180423	71	1 46933	1 08	15 9495	1 74
.0000210	116	000644	230	.009616	.395	193492	72	1 61051	1 10	16 8874	1 76
0000229	118	000717	.235	010240	.400	207307	73	1 76234	1 12	17 8690	1 78
0000249	120	000796	210	.011586	.41	221901	74	1 92541	1 14	18 8957	1.80
0000270	122	000883	.245	013069	.42	237305	.75	2 10034	1 16	19 9690	1.82
0000293	124	000977	250	.014701	.43	254653	76	2 28775	1.18	21 0906	1.84
0000318	.126	001078	255	016492	.44	270678	.77	2 48832	1 20	22 2620	1.86
0000344	.128	001188	.260	018453	45	288717	.78	2 70271	1.22	23.4849	1.88
0000371	130	001307	265	020596	46	307706	79	2 93163	1 24	24.7610	1.90
0000401	.132	001435	.270	022935	47	327680	.80	3 17580	1 26	26 0919	1.92
0000432	134	001573	.275	·025480	.48	348678	.81	3.43597	1.28	27 4795	1.94
.0000465	.136	001721	.280	028248	.49	370740	.82	3 71293	1 30	28 9255	1.96
.0000500	.138	001880	285	.031250	.50	393904	83	4 00746	1.32	30 4317	1.98
.0000538	.140	002051	.290	034503	.51	418212	.84	4.32040	1.34	32 0000	2 00
.0000577	.142	.002234	.295	.038020	52	.443705	.85	4 65259	1.36	36.2051	2.05
.0000619	.144	002430	.300	041820	53	470427	.86	5 00490	1 38	40.8410	2.10
.0000663	.146	002639	.305	045917	.54	498421	.87	5 37824	1.40	45.9401	2.15
0000710	.148	002863	.310	050328	.55	527732	.88	5 77353	1.42	51 5363	2.20
0000754	.150	.003101	.315	.055073	.56	558406	.89	6 19174	1.44	57.6650	2.25
.0000895	.155	.003355	.320	.060169	.57	.590490	.90	6 63383	1 46	64 3634	2.30
.000105	.160	003626	.325	.065636	.58	624032	.91	7 10082	1.48	71 6703	2.35
.000122	.165	.003914	.330	.071492	.59	.659082	.92	7.59375	1.50	79.6262	2.40

Continued on next page.

Fifth roots and fifth powers—(Continued).

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root	Power.	No. or Root	Power.	No. or Root
88.2735	2 45	2824 75	4 90	85873	9 70	2609193	19 2	20511149	29 0	459165024	54
97 6562	2 50	2971 84	4 95	90392	9 80	2747949	19 4	21228253	29 2	503284375	55
107.820	2 55	3125 00	5 00	95099	9 90	2892547	19 6	21965275	29 4	550731776	56
118 814	2 60	3450 25	5 10	100000	10 0	3043168	19 8	22722628	29.6	601692057	57
130.686	2 65	3802 04	5 20	110408	10 2	3200000	20 0	23500728	29 8	656356768	58
143.489	2 70	4181 95	5 30	121665	10 4	3363232	20 2	24300000	30 0	714924299	59
157.276	2 75	4591 65	5.40	133823	10 6	3533059	20 4	26393634	30 5	777600000	60
172 104	2 80	5032 84	5 50	146933	10 8	3709677	20 6	28629151	31 0	844596301	61
188.029	2 85	5507 32	5 60	161051	11 0	3893289	20 8	31013642	31 5	916132832	62
205 111	2 90	6016.92	5 70	176234	11 2	4084101	21 0	33554432	32 0	992436543	63
223.414	2 95	6563 57	5 80	192541	11 4	4282322	21 2	36259082	32 5	1073741824	64.
243.000	3 00	7149.24	5 90	210034	11 6	4488166	21 4	39135393	33 0	1160290625	65
263 936	3 05	7776 00	6 00	228776	11 8	4701850	21 6	42191410	33 5	1252332576	66
286.292	3 10	8415 96	6 10	248832	12 0	4923597	21 8	45435424	34 0	1350125107	67
310.136	3 15	9161 33	6 20	270271	12 2	5153632	22 0	48875980	34 5	1453933568	68.
335 544	3 20	9924 37	6 30	293163	12 4	5392186	22 2	52521875	35 0	1564031349	69.
362 591	3 25	10737	6 40	317580	12 6	5639493	22 4	56382167	35 5	1680700000	70.
391.354	3 30	11603	6 50	343597	12 8	5895793	22 6	60466176	36 0	1804229351	71.
421.419	3 35	12523	6 60	371293	13 0	6161327	22 8	64783487	36 5	1934917632	72.
454 354	3 40	13501	6 70	400746	13 2	6436343	23 0	69343957	37 0	2073071593	73.
488 760	3 45	14539	6 80	432040	13 4	6721093	23 2	74157715	37 5	2219006624	74.
525.219	3 50	15640	6 90	465259	13 6	7015834	23 4	79235168	38 0	2373046875	75
563.822	3 55	16807	7 00	500490	13 8	7320825	23 6	84587005	38 5	2535525376	76
601 662	3 60	18012	7 10	537824	14 0	7636332	23 8	90224199	39 0	2706784157	77
647 835	3 65	19319	7 20	577353	14 2	7962624	24 0	96158012	39 5	2887174368	78
693 440	3 70	20731	7 30	619174	14 4	8299976	24 2	102400000	40 0	3077056399	79.
741 577	3 75	22190	7 40	663383	14 6	8648666	24 4	108962013	40 5	3276800000	80
792 352	3 80	23730	7 50	710082	14 8	9008978	24 6	115856201	41 0	3486784401	81.
845 870	3 85	25355	7 60	759375	15 0	9381200	24 8	123095020	41 5	3707398432	82
902 242	3 90	27068	7 70	811368	15 2	9765625	25 0	130691232	42 0	3939040643	83
961.580	3 95	28872	7 80	866171	15 4	10162550	25 2	138657910	42 5	4182119424	84
1024 00	4.00	30771	7 90	923896	15 6	10572278	25 4	147008443	43 0	4437053125	85
1089 62	4 05	32768	8 00	981658	15 8	10995116	25 6	155756538	43 5	4704270176	86
1158 56	4 10	34868	8 10	1048576	16 0	11431377	25 8	164916224	44 0	4984209207	87.
1230 95	4 15	37074	8 20	1115771	16 2	11881376	26 0	174501858	44 5	5277319168	88
1306 91	4 20	39390	8 30	1186367	16 4	12345437	26 2	184528125	45 0	5584059449	89
1386 58	4 25	41821	8 40	1260493	16.6	12823886	26 4	195010045	45 5	5904900000	90
1470 08	4 30	44371	8 50	1338278	16 8	13317055	26 6	205962976	46 0	6240321451	91.
1557.57	4 35	47043	8 60	1419857	17 0	13825281	26 8	217402615	46 5	6590815232	92.
1649 16	4 40	49842	8 70	1505366	17 2	14348907	27 0	229345007	47 0	6956883693	93.
1745 02	4 45	52773	8 80	1594917	17 4	14888280	27 2	241806543	47 5	7339040224	94.
1845 28	4 50	55841	8 90	1688742	17 6	15443752	27 4	254803968	48 0	7737809375	95
1950.10	4 55	59049	9 00	1786899	17 8	16015681	27 6	268354383	48 5	8153726976	96
2059 63	4 60	62403	9 10	1889568	18 0	16604430	27 8	282475249	49 0	8587340257	97
2174 08	4 65	65908	9 20	1996903	18 2	17210368	28 0	297184391	49 5	9039207968	98
2293 45	4 70	69569	9 30	2109061	18 4	17834868	28 2	312500000	50 0	9509900499	99
2418 07	4 75	73390	9 40	2226203	18 6	18475309	28 4	345025251	51		
2548 04	4 80	77378	9 50	2348493	18 8	19135075	28 6	380204032	52		
2683.54	4.85	81537	9 60	2476099	19 0	19813557	28 8	418195493	53.		

Square roots of fifth powers of numbers, $\sqrt{n^5}$,

or $5/2$ powers of numbers, $n^{5/2}$.

See table, page 69.

The column headed "12 n" facilitates the use of the table in cases where, for instance, the quantity is given in *inches*, and where it is desired to obtain the $5/2$ power of the same quantity in *feet*. Thus, suppose we have a $1/2$ inch pipe, and we require the $5/2$ power of the diameter in feet. Find $1/2$ (the diameter, in inches) in the column headed "12 n," opposite which, in the column headed "n," is 0.041666 (the diameter, in feet), and, in column headed "$n^{5/2}$," 0.00035 (the $5/2$ power of the diameter, 0.041666, in feet).

Values of n, ending in 0 or in 5, are *exact* values. All others end in repeating decimals. Thus: n = 0.052083 signifies n = 0.0520833333 . . . . .

Square roots of fifth powers of numbers. See page 65.

12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$
¼	0.020833	0.000063	22	1.8333	4.5510	84	7.000	129.64	468	39	9499
⅜	0.031250	0.000173	23	1.9166	5.0859	85	7.083	133.53	480	40	10119
½	0.041666	0.000351	24	2.0000	5.6569	86	7.166	137.50	492	41	10764
⅝	0.052083	0.000619	25	2.0833	6.2617	87	7.250	141.53	504	42	11432
¾	0.062500	0.000977	26	2.1666	6.9100	88	7.333	145.63	516	43	12125
⅞	0.072916	0.001436	27	2.2500	7.5938	89	7.416	149.80	528	44	12842
1	0.083333	0.002005	28	2.3333	8.3165	90	7.500	154.05	540	45	13584
⅛	0.093750	0.002691	29	2.4166	9.0791	91	7.583	158.36	552	46	14351
¼	0.104166	0.003502	30	2.5000	9.8821	92	7.666	162.75	564	47	15144
⅜	0.114583	0.001144	31	2.5833	10.726	93	7.750	167.21	576	48	15964
½	0.125000	0.005524	32	2.6666	11.612	94	7.833	171.74	588	49	16807
⅝	0.135416	0.006748	33	2.7500	12.541	95	7.916	176.34	600	50	17678
¾	0.145833	0.008122	34	2.8333	13.513	96	8.000	181.02	612	51	18575
⅞	0.156250	0.009651	35	2.9166	14.528	97	8.083	185.77	624	52	19499
2	0.166666	0.011340	36	3.0000	15.588	98	8.166	190.60	636	53	20450
¼	0.187500	0.015223	37	3.0833	16.694	99	8.250	195.49	648	54	21428
½	0.208333	0.019811	38	3.1666	17.845	100	8.333	200.47	660	55	22434
¾	0.229166	0.025141	39	3.2500	19.042	102	8.50	210.64	672	56	23468
3	0.250000	0.031250	40	3.3333	20.286	105	8.75	226.47	684	57	24529
¼	0.270833	0.038173	41	3.4166	21.578	108	9.00	243.00	696	58	25615
½	0.291666	0.045943	42	3.5000	22.918	111	9.25	260.23	708	59	26738
¾	0.312500	0.054592	43	3.5833	24.306	114	9.50	278.17	720	60	27885
4	0.333333	0.064150	44	3.6666	25.744	117	9.75	296.83	732	61	29062
¼	0.354166	0.074618	45	3.7500	27.232	120	10.0	316.23	744	62	30268
½	0.375000	0.086115	46	3.8333	28.770	126	10.5	357.25	756	63	31503
¾	0.395833	0.098578	47	3.9166	30.359	132	11.0	401.31	768	64	32768
5	0.416666	0.11207	48	4.0000	32.000	138	11.5	448.48	780	65	34063
¼	0.437500	0.12660	49	4.0833	33.693	144	12.0	498.83	792	66	35388
½	0.458333	0.14222	50	4.1666	35.438	150	12.5	552.43	804	67	36744
¾	0.479166	0.15893	51	4.2500	37.237	156	13.0	609.34	816	68	38130
6	0.500000	0.17678	52	4.3333	39.089	162	13.5	669.63	828	69	39548
½	0.541666	0.21594	53	4.4166	40.996	168	14.0	733.37	840	70	40996
7	0.583333	0.25989	54	4.5000	42.957	174	14.5	800.61	852	71	42476
½	0.625000	0.30882	55	4.5833	44.973	180	15.0	871.42	864	72	43988
8	0.666666	0.36289	56	4.6666	47.045	186	15.5	945.87	876	73	45531
½	0.708333	0.42227	57	4.7500	49.174	192	16.0	1024.0	888	74	47106
9	0.750000	0.48714	58	4.8333	51.359	198	16.5	1105.9	900	75	48714
½	0.791666	0.55764	59	4.9166	53.602	204	17.0	1191.6	912	76	50354
10	0.833333	0.63394	60	5.0000	55.902	210	17.5	1281.1	924	77	52027
½	0.875000	0.71618	61	5.0833	58.260	216	18.0	1374.6	936	78	53732
11	0.916666	0.80451	62	5.1666	60.677	222	18.5	1472.1	948	79	55471
½	0.958333	0.89907	63	5.2500	63.154	228	19.0	1573.6	960	80	57243
12	1.000000	1.00000	64	5.3333	65.690	234	19.5	1679.1	972	81	59049
½	1.041666	1.1074	65	5.4166	68.286	240	20	1788.9	984	82	60888
13	1.083333	1.2215	66	5.5000	70.943	252	21	2020.9	996	83	62762
½	1.125000	1.3424	67	5.5833	73.660	264	22	2270.2	1008	84	64669
14	1.166666	1.4702	68	5.6666	76.440	276	23	2537.0	1020	85	66611
½	1.208333	1.6050	69	5.7500	79.281	288	24	2821.8	1032	86	68588
15	1.250000	1.7469	70	5.8333	82.185	300	25	3125.0	1044	87	70599
½	1.291666	1.8962	71	5.9166	85.152	312	26	3446.9	1056	88	72645
16	1.333333	2.0528	72	6.0000	88.182	324	27	3788.0	1068	89	74727
½	1.375000	2.2170	73	6.0833	91.276	336	28	4148.5	1080	90	76843
17	1.416666	2.3887	74	6.1666	94.434	348	29	4528.9	1092	91	78996
½	1.458333	2.5683	75	6.2500	97.656	360	30	4929.5	1104	92	81184
18	1.500000	2.7557	76	6.3333	100.94	372	31	5350.6	1116	93	83408
½	1.541666	2.9510	77	6.4166	104.30	384	32	5792.6	1128	94	85668
19	1.583333	3.1545	78	6.5000	107.72	396	33	6255.8	1140	95	87965
½	1.625000	3.3662	79	6.5833	111.20	408	34	6740.6	1152	96	90298
20	1.666666	3.5861	80	6.6666	114.76	420	35	7247.2	1164	97	92668
½	1.708333	3.8144	81	6.7500	118.37	432	36	7776.0	1176	98	95075
21	1.750000	4.0513	82	6.8333	122.06	444	37	8327.3	1188	99	97519
½	1.791666	4.2968	83	6.9166	125.82	456	38	8901.4	1200	100	100000

Logarithms.

(**1**) Tables of logarithms greatly facilitate multiplication and division and the finding of powers and roots of numbers.*

(**2**) The tables pp. 78 to 91 contain the **common, decimal** or **Briggs logarithms** of numbers. The *common* logarithm of a number is the exponent or index of that number as a power of 10. See (18). Thus: $1000 = 10^3$, and log 1000 (logarithm of 1000) = 3 00 000. Similarly, $28.7 = 10^{1.45\ 788}$, and log 28.7 = 1.45 788.

(**3**) In general, let A and B be any two numbers, and k any exponent. Then, from the principle of powers and roots, we have

$$(1)\ \log AB = \log A + \log B; \qquad (2)\ \log \frac{A}{B} = \log A - \log B;$$

$$(3)\ \log A^k = k\,(\log A); \qquad (4)\ \log \sqrt[k]{A} = \frac{\log A}{k}$$

In other words:—
(1) Log of product = sum of logs of factors.
(2) Log of quotient = log of dividend — log of divisor.
Or log of fraction = log of numerator — log of denominator.
(3) Log of power = log of number, multiplied by exponent.
(4) Log of root = log of number, divided by exponent.

(**4**) From what has been said, it follows that

Log 100	=	log 10^2	=	2.00 000	Log 0.1	=	log 10^{-1}	=	$\bar{1}$.00 000†
Log 10	=	log 10^1	=	1.00 000	Log 0.01	=	log 10^{-2}	=	$\bar{2}$.00 000
Log 1	=	log 10^0	=	0.00 000†	Log 0.001	=	log 10^{-3}	=	$\bar{3}$.00 000

(**5**) Each common logarithm is a mixed number, consisting of an *integral* portion, called the **characteristic** or **index** (*preceding* the decimal point), and a *fractional* portion, called the **mantissa** (*following* the decimal point). The table gives only the *mantissa* of each log, the characteristic being found as explained below. The *mantissa* is always *positive*. The characteristic is equal to the number of the place of the first significant figure in the number, counting from the *unit* place as *zero*, and is positive for whole or mixed numbers, and negative for fractions. Thus:

$$\log \underset{4\,3\,2\,1\,0}{2\,4\,6\,0\,0} = 4.39\ 094; \quad \log \underset{0\ \ 1\,2\,3\,4\,5}{0\,.\,0\,0\,0\,0\,4} = \bar{5}.60\ 206.$$

For example:

log 2870 = 3.45 788		log 0.287 = $\bar{1}$.45 788‡	
" 287 = 2.45 788		" 0.0287 = $\bar{2}$.45 788	
" 28.7 = 1.45 788		" 0.00287 = $\bar{3}$.45 788	
" 2.87 = 0.45 788		" 0.000287 = $\bar{4}$.45 788	

It will be noticed that the mantissa remains constant for any given combination of significant figures in a number, wherever the decimal point in the number be placed; while the characteristic depends solely upon the position of the decimal point in the number.

(**6**) Let the number be resolved into two factors, one of which is an integer power of 10, while the other is greater than 1 and less than 10. Then the index of the power of 10 is the characteristic of the logarithm, and the logarithm of the other factor is the mantissa. Thus, $2870 = 1000 \times 2.87 = 10^3 \times 2.87$, and the logarithm of 2870 (3.45 788) is the sum of the exponent 3 (or 3.00 000) and the log (0.45 788) of 2.87.‡

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* Logarithms not being exact quantities, operations performed with them are subject to some inaccuracy, especially where a logarithm is multiplied by a large number, the existing error being thus magnified. Logarithms of only five places in the mantissa usually suffice for calculations with numbers of four or five places. Greater accuracy is obtained by the use of tables of logarithms carried out to seven places.

† Log 1 = log $\frac{10}{10}$ = log 10—log 10 = 1—1 = 0; or $1 = 10^0$.

Log 0.1 = log $\frac{1}{10}$ = log 1 — log 10 = 0 — 1 = $\bar{1}$.0; or $0.1 = 10^{-1}$.

‡ 0 287 = 2.87 ÷ 10. Hence, log 0.287 = log 2.87 − log 10 = 0.45 788 − 1, which, for convenience, is written $\bar{1}$.45 788. See (16). Similarly, log 0.0287 = log 2.87 − log 100 = 0.45 788 − 2 = $\bar{2}$.45 788

(7) To find the logarithm of a number. The short table on pages 78, 79 gives logs of numbers up to 1000. The longer table, pages 80 to 91, gives

(1) The mantissa for *each* number from	1000 to 1750
(2) The mantissa for each *even* number from	1750 to 3750
(3) The mantissa for each *fifth* number from	3750 to 10000

(8) Logs of **numbers intermediate** of those given in the tables are found by simple proportion. The procedure necessary in these cases is explained in the examples given in connection with the tables, but it will often be found sufficiently accurate to use the log of the nearest number given in the table, neglecting interpolation.

The antilogarithm, formerly called the **num log** (*numerus logarithmi*), is the **number corresponding** to a given logarithm. Thus, log. 2 = 0.30 103, and antilog 0.30 103 = 2. Usually written: "log $^{-1}$ 0.30 103 = 2."

(9) Multiplication. To multiply together two or more numbers, add together their logs and find the antilog of their sum. See Proportion (11) below.

(10) Division. Subtract the log of the divisor from that of the dividend, and find the antilog of the remainder. See Proportion (11) below.

The **reciprocal** of any number, n, $= \frac{1}{n}$. See page 52. Thus, recip 2 = $\frac{1}{2}$ = 0.5. Hence, log recip n = $\log \frac{1}{n} = \log 1 - \log n = 0 - \log n$.

Similarly, log recip $\frac{m}{n} = \log \frac{1}{m \div n} = 0 - \log \frac{m}{n}$.

Thus, $\log \frac{6.3023}{23057} = 0 - \log \frac{23057}{6.3023} = 0 - 3.56\ 331 = \bar{4}.43\ 669.$

Since $n^{2-1} = n^1 = \frac{n^2}{n}$, $n^{1-1} = n^0 = \frac{n}{n} = 1$, $n^{0-1} = n^{-1} = \frac{1}{n}$, and $n^{0-2} = n^{-2} = \frac{1}{n^2}$ it follows that $\log n^{-1} = \log \frac{1}{n}$ = log recip n, $\log n^{-2} = \log \frac{1}{n^2}$ = log recip n^2, etc.

(11) Proportion. Example. 6.3023 : 290.19 = 1260.7 : ?

Multiply Nos,	Log 290.19	= 2.46 269
Add Logs.	" 1260.7	= 3.10 062
	Log 290.19 × 1260.7	= 5.56 331
Divide Nos,	Log 6.3023	= 0.79 950
Subtract Log.	Log 58051	= 4.76 381

The true value is 58019.05 +

(12) Instead of subtracting the log of the divisor, we may add its **cologarithm** or **arithmetical complement,** which is log of reciprocal of divisor, = 0 — log divisor = 10 — log divisor — 10. Thus:

$$\frac{1523}{3.332 \times 8.655} = ?$$

Log 1523	= 3.18 270
Colog 3.332 = 10 — log 3.332 — 10 = 10 — 0.52 270 — 10	= 9.47 730 — 10
Colog 8.655 = 10 — log 8.655 — 10 = 10 — 0.93 727 — 10	= 9.06 273 — 10
Sum of logs and cologs	= 21.72 273 — 20
	= Log 52.813 = 1.72 273

The true value is 52.8114 +

(13) Involution, or finding powers of numbers. Multiply log of given number by the exponent of the required power, and find the antilog of the product. Thus: $36^3 = ?$

Log 36 = 1.55 630. 1.55 630 × 3 = 4.66 890. Antilog 4.66 890 = 46656.

(14) Evolution, or finding roots of numbers. Divide log of given number by exponent of required root, and find antilog of quotient. Thus: $\sqrt[3]{46656} = ?$ Log 46656 = 4.66 890. 4.66 890 ÷ 3 = 1.55 630. Antilog 1.55 630 = 36.

(15) Involution and evolution of fractions. Negative characteristics. See page 72.

Examples. Required: (1) 0.48^3; (2) $\sqrt[3]{0.48} = 0.48^{1/3}$.

Log $0.48 = \log(48/100) = \log 48 - \log 100 = 1.68\,124 - 2 = 2.68\,124 - 3 = -0.31\,876 = 0.68\,124 - 1$; usually written $\bar{1}.68\,124$.

(1) $\text{Log } 0.48^3 = 3 \log 0.48 = 3(-0.31\,876) = 3(0.68\,124 - 1)$
$= -0.95\,628 \qquad = 2.04\,372 - 3$

Each $= \bar{1}.04\,372$. The corresponding No. is 0.1106. $0.48^3 = 0.1106$.

(2) $\text{Log } 0.48^{1/3} = (\log 0.48)/3 = (0.68\,124 - 1)/3 = (2.68\,124 - 3)/3$
$= (-0.31\,876)/3 = 0.22\,708 - 0.33\,333 = 0.89\,375 - 1$
$= -0.10\,625 \qquad = -0.10\,625 = \bar{1}.89\,375$

Each $= \bar{1}.89\,375$. The corresponding No. is 0.7830. $\sqrt[3]{0.48} = 0.7830$.

(**16**) To avoid inconvenience from the use of **negative characteristics**, it is customary to **modify** them by adding 10 to them, afterward deducting each such 10 from the sum, etc., of the logarithms. Thus: in multiplying or dividing 7425 by 0.25, we have

		Multiplying	Dividing
either	log 7425	3.87 070	3.87 070
	log 0.25	$\bar{1}.39\,794$	$\bar{1}.39\,794$
		3.26 864	4.47 276
or	log 7425	3.87 070	3.87 070
	modified log 0.25	9.39 794 − 10	9.39 794 − 10
		13.26 864 − 10	6.47 276 + 10
		3.26 864	4.47 276

In most cases the actual process of deducting the added tens may be neglected, the nature of the work usually being such that an error so great as that arising from such neglect could hardly pass unnoticed.

(**17**) **To divide a modified logarithm**, add to it such a multiple of 10 as will make the sum exceed the true log by 10 times the divisor. Thus: to divide log 0.00048 by 3. Log 0.00048 = $\bar{4}.68\,124$, which, divided by 3, = $\bar{2}.89\,375$. See (15).

Log 0.00048	= $\bar{4}.68\,124$	
Modified log 0.00048	= 6.68 124	− 10
Add 2 × 10	20	− 20
Dividing by	3) 26.68 124	− 30
we obtain	8.89 375	− 10, which is $\bar{2}.89\,375$ modified.

(**18**) Except 1, any number can (like 10) be made the base of a system of logarithms. The base of the **hyperbolic**, **Napierian**, or **natural logarithms**, much used in steam engineering, is

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \ldots = 2.71\,828 +$$

and is called ϵ (epsilon) or e.

$M = \log_{10} e$ (common log e) $= 0.43\,429$; $\frac{1}{M} = \log_e 10$ (hyperbolic log 10) $= 2.30\,259$.

For any number, n,

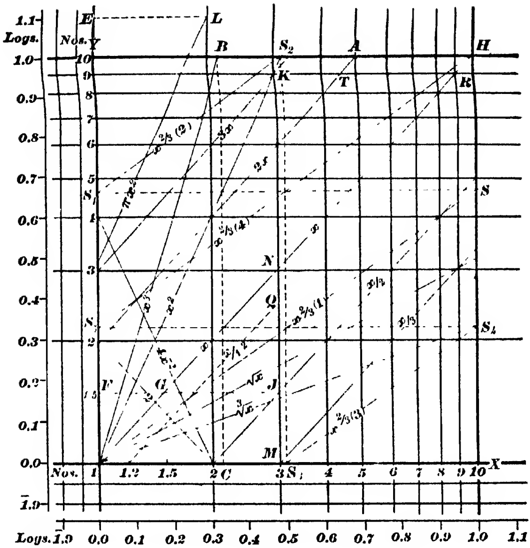
$$\log_e n = \frac{\log_{10} n}{M} = 2.30259 \log_{10} n; \quad \log_{10} n = M \log_e n = 0.43429 \log_e n$$

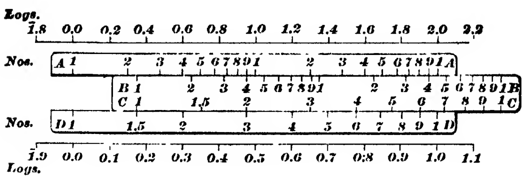
(**19**) Whatever may be the base chosen for a system of logs, the **mantissas** of the logs of any given numbers bear a **constant ratio** to each other. Thus, in any system of logs, log 4 is always $= 2 \times \log 2$, and $= \frac{2}{3} \times \log 8$, etc., etc.

(**20**) **Logarithmic sines, tangents**, etc. of angles are the logs of the sines, tangents, etc. of those angles. Thus, sin 30° = 0.5000000, and log sin 30° = log. 0.5 = $\bar{1}.69\,897$, usually written 9.69 897 − 10, or simply 9.69 897.

For tables of logarithmic sines, tangents, etc., see pp. 1029, etc.

(**21**) Since no power of a positive number can be negative, negative numbers properly have no logs; but **operations with negative numbers** can nevertheless be performed by means of logs, by treating all the numbers as positive and taking care to use the proper sign, + or −, in the result. Thus: required the 3rd power of (−2). Log 2 = 0.30 103; and $3 \times 0.30\,103 = 0.90\,309 = \log 8$; but $(-2)^3$ must be negative. Hence $(-2)^3 = -8$.





The Logarithmic Chart and the Slide Rule.

(1) By means of a logarithmic chart or diagram (often miscalled logarithmic cross-section paper) logarithmic operations are performed graphically, and by means of the slide rule mechanically, without reference to the logarithms themselves *. But see †, p 76. Their use greatly facilitates many hydraulic and other engineering computations.

(*) The ratio between the mantissas of the logs of any given numbers being constant for all systems of logs, the ratio between the distances laid off on the chart or slide rule is the same for all systems, and the use of the chart or rule is independent of the system of logs used.

(2) **The logarithmic chart** consists primarily of a square,* on the sides of which the distances marked 1–2, 1–3, etc., are laid off by scale according to the logs (0.30 103, 0.47 712, etc.) of 2, 3, etc. Ordinary "squared" or **cross section paper** may of course be used for logarithmic plotting, by plotting on it the *logs* instead of their *Nos.* Lines representing Nos. may be drawn in their proper places as desired.

(3) As ordinarily constructed,‡ the **slide rule** consists essentially of four scales, A, B, C, and D, see (17), scales A and D being placed on the "rule," while B and C are placed upon the sliding piece, or "slide" As in the logarithmic chart, see (2), the scales are divided logarithmically (see figure), but *marked* with the *numbers* corresponding to the logs. Scales A and B are equal, as are also scales C and D, but a given length on A or B represents a logarithm twice as great as on C or D. See (4). Hence, each number marked on A is the *square* of the coinciding number marked on D.

(4) A single logarithmic scale is usually numbered from 1 to 10, or from 10 to 100; but it may be taken as representing any series embracing the numbers from 10^n to 10^{n+1}; as from 0.1 to 1.0 ($n = -1$); or from 1.0 to 10.0 ($n = 0$); or from 10.0 to 100.0 ($n = 1$); or—etc., etc. Here n and $n + 1$ are the **characteristics** of the corresponding logarithms.

A single scale would therefore serve for all values, from 0 to infinity; but for convenience several contiguous scales are sometimes added, as in the log chart*.

When a line reaches the limit of a square, the next square may be entered* or the same square may be re-entered at a point directly opposite. Thus, in the case of line $x^{2/3}$ ($= \sqrt[3]{x^2}$).

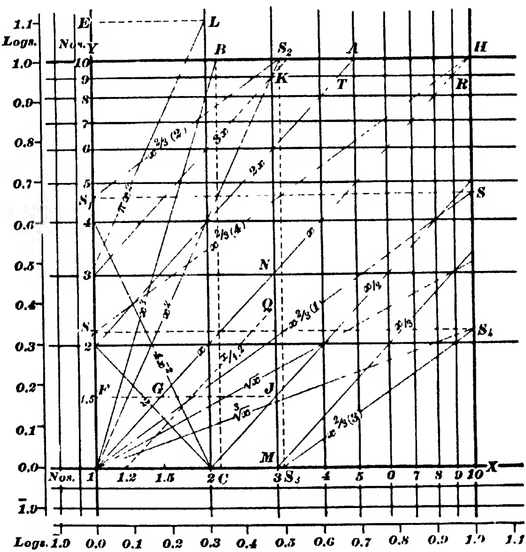
Line marked $x^{2/3}$	between	corresponds to values of	
		x from	$x^{2/3}$ from
(1)	1 and S	1 to 10	1 to 4.64
(2)	S_1 and S_2	10 to 31.62	4.64 to 10
(3)	S_3 and S_4	31.62 to 100	10 to 21.54
(4)	S_5 and H	100 to 1000	21.54 to 100

Note that the numbers, marked on any given scale, must be taken as 10 times the corresponding numbers marked in the next scale preceding, and the characteristics therefore as being greater by 1, and *vice versa*. Thus, in our figure, log 1.5 + log 2 = 1-1.5 + 1-2 = log 3 = distance 1-M. But log 15 + log 20 = (1-1 5 + 1-10) + (1-2 + 1-10), so that the characteristic of the resulting log is greater by 2, and the 3 representing the product of 15 and 20 is really in the second square to the right of that shown. In finding *roots* of numbers, remember that multiplying or dividing the *number* by 0.1, 10, 100, etc. (*i. e.*, changing the *characteristic* of its log), changes also the *mantissa* of the log of its *root*. Thus, $\sqrt[3]{2.7} = 1.39..$, (log = 0.14 379); but $\sqrt[3]{27} = 3$, (log = 0 47 712) and $\sqrt[3]{270} = 6.46..$, (log = 0.81 023). The chart or rule gives *all* such possible roots, and care must be taken to select the proper one. Most operations exceed the limits of one scale, and facility in using either instrument depends largely upon the ability to pass readily and correctly from one scale to another. This ability is best gained by practice, aided by a thorough grasp of the principles involved. Where several successive operations are to be performed, a sliding runner or marker (furnished with each slide rule) is used, in order to avoid error in shifting the slide. Detailed instructions are usually furnished with the slide rule.

(*) A common form of chart has four or more similar squares joined together. See (4). Our figure represents one complete square, with portions of adjoining squares For actual use, both charts and slide rules are, of course, much more finely subdivided than in our figures, which are given merely to illustrate the principles. Carefully engraved charts are published by Mr. John R. Freeman, Providence, R. I.

(‡) Other forms embodying the same principle are: The "Reaction Scale and General Slide Rule," by W. H. Breithaupt, M. Am. Soc. C. E.; Sexton's Omnimeter or Circular Slide Rule, by Thaddeus Norris; The Goodchild Computing Chart; The Thacher Calculating Machine or Cylindrical Slide Rule; The Cox Computers, designed for special formulas; and the Pocket Calculator, issued by "The Mechanical Engineer," London.

(5) **Multiplication and division.** For example, $2 \times 1.5 = ?$ On 1-X, in the chart, or on C or D, in the slide rule, the distance 1-1.5 represents by scale the logarithm (0.17 609) of 1.5, and 1-2 represents the logarithm (0.30 103) of 2. If now we add these two distances together, by laying off 1-2 from 1.5 on 1-X of the chart, or by placing the slide as in the figure, we obtain the distance 1-3 = .47 712 the mantissa of log 3 or of log (2×1.5).* Conversely, to divide 3 by 2, we graphically or mechanically subtract 1-2 from 1-3.



(6) **In the logarithmic chart,** the scales of both axes, 1-X and 1-Y, being equal, a line 1-H, marked x, bisecting the square and forming an angle of 45° with each axis (tan 45° = 1),† will bisect also the intersections of all *equal* co-ordinates. Thus, points in the line x, immediately over 2, 3, 4, etc., in 1-X, are also opposite 2, 3, 4, etc., respectively, on 1-Y. See (4).

(7) If lines 2-A, 3-K, etc. (marked $2x$, $3x$, etc.), parallel to *and above* 1-H, be drawn through 2, 3, etc., on 1-Y, then points in such lines, immediately over any number, x, in 1-X, will be respectively opposite the

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(*) In the slide rule, with the slide as shown, *each* number on D is = $1.5 \times$ the coinciding number on C.

(†) In discussing tangents of angles on log chart, we refer to the actual measured distances, as shown on the equally divided scales of *logs* in our figures, and not to the *numbers*, which, for mere convenience, are *marked* on the chart. Thus, in line 1-B, $\tan C\,1\,B = \frac{C\,B}{1\,C} = \frac{1.0}{0.33}$, not $\frac{10}{2.15}$.

numbers giving the products $2x$, $3x$, etc., on 1-Y; while similar lines, drawn *below* 1-H and through 2, 3, etc, on 1-X, give values of $\frac{x}{2}$, $\frac{x}{3}$, etc., respectively If these lines $\frac{x}{2}$, $\frac{x}{3}$, etc, be produced downward, they will cut 1-Y (produced) at 0.5 (= ½), 0.33 (= ⅓), etc., respectively * See (**4**).

(**8**) **Powers and roots.** If a line x^2 be drawn through 1, at an angle S_2 1-X, whose tangent, $\frac{S_3-S_2}{1-S_3}$ is 2, it will give values of x^2. Thus, the vertical through 3, on 1-X, cuts the line x^2 opposite 9 (= 3^2) on 1-Y. Similarly, line x^3 (tangent = 3) gives values of x^3; and line $\sqrt[3]{x}$ (tangent = ⅓) gives values of $x^{\frac{1}{3}}$ or $\sqrt[3]{x}$. See (**4**).

(**9**) Any equation of the form $y = C x^n$ in which $\log y = \log C + n \log x$, (such as: area of circle = π radius²), is represented, on a logarithmic chart, by a straight line so drawn that the tangent T of its angle with 1-X is = n, and intersecting 1-Y at that point which represents the value C. Thus, the line marked πx^2, (tangent = 2) is a line of squares, and, being drawn through π (= 3.14. . .) on 1-Y, it gives values of πx^2. Thus, for a circle of radius 2, we find, in the line πx^2 over 2, a point L opposite E, or 12.57. . . , the area of such circle.† Conversely, having area = 12.57. . ., we obtain, from the diagram, radius = 2.

(**10**) If a chart is to be used for solving many equations of a single kind, such as $y = C x^n$, where C is a variable coefficient, and n a constant exponent, parallel lines, forming the proper angle with 1-X, should be permanently ruled across the sheet at short intervals.

(**11**) For any log, as 1-3 (= log 3), we may substitute its equal, M-N or 3-N, extending to the central diagonal line 1-H, marked x; and then, since, for instance, 1-1.2 = N-Q, 1-3 = N-K, etc., we may *add* any log (as 1-3) by moving *upward* from line x (as from N to K) or to the *right*, and *subtract* any log (as 1-1.2) by moving *downward* (as from N to Q) or to the *left*. This facilitates the performance of a series of operations.

Thus:

To multiply 1.5 by 2 (= 3), by 3 (= 9), and divide by 2 (= 4.5).

F-G = 1-F = log 1.5. Add G-J = 1-2 = log 2; sum = F-J = log 3 = 1-3 = M-N. Add N-K = 1-3 = log 3; sum = M-K = log 9 = 1-9 = 9-R. Subtract R-T = 1-2 = log 2; remainder = 9-T = log 4.5.

For an example of the application of this principle to engineering problems, see "Diagrams for proportioning wooden beams and posts," by Carl S. Fogh, "Engineering News", Sept. 27, 1894.

(**12**) **Negative exponents.** If x is in the *divisor*, the line will slope in the opposite direction, or downward from left to right. Thus, line 4-2 leaving 1-Y, at 4, and forming, with 1-X, the angle X, 2, 4, with tangent $= \frac{-.602 \ldots}{-.301 \ldots} = -2$, represents the equation: $y = \frac{4}{x^2} = 4\,x^{-2}$

(**13**) If the lines of products, powers, and roots, $C\,x$, x^n, and $\sqrt[n]{x}$, etc., be drawn at angles whose tangents are less by 1 than those of the angles formed by the corresponding lines in our figure, the results may be read directly from oblique lines drawn parallel to 2-2. Lines ($C\,x$) giving multiples and sub-multiples of the *first* power of x then become *horizontal* lines (tan = 0).‡

(**14**) **Powers and roots by the slide rule.** Scales C and D being twice as large as scales A and B, these scales, with their ends coinciding, form a table of squares and of square roots. See (**3**). By moving the slide we solve equations of the forms $y = (C\,x)^2$ and $y = C\,x^2$ Thus, with the

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(*) In each of these lines, the product of the two numbers at its ends is = 10 Thus, in line 2-A. 2 × 5 = 10; in 3-K, 3 × 3.33 . . . = 10, etc. The chart thus furnishes a table of **reciprocals.**

(†) Even with full-size charts and slide rules for actual use, accuracy is not to be expected beyond the third or fourth significant figure.

(‡) A chart of this kind, prepared by Major Wm. H. Bixby, U. S. A., after the method of Léon Lalanne, Corps de Ponts et Chaussées, France, is published by Messrs. John Wiley & Sons, New York. Price, 25 cents.

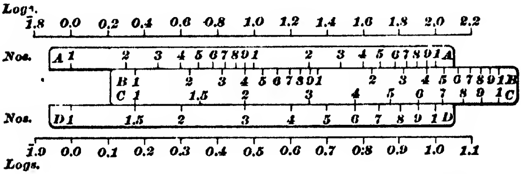
slide as shown, each number on A is = the square of (1.5 × the coinciding number on C); while, with 1 on B opposite 1.5 on A, each number on A is = 1.5 × the square of the coinciding number on C

(15) Since $x^3 = x^2 \times x$, we find cubes or third powers by placing the slide with 1 on B opposite x^2 on A (*i. e.*, opposite x on D), see (**3**), and reading x^3 from A opposite x on B. Thus, 1.5^3 = ? Place 1 on B opposite 1.5 on D; *i. e.*, opposite 1.5^2 (= 2.25) on A. Then, on A, opposite 1.5 on B, find 3.375 = 1.5^3. Or, turn the slide end for end. Place 1.5 on B opposite 1.5 on D, *i. e.*, opposite 1.5^2 = 2.25 on A. Then, adding log 1.5 (on B) to log 2.25 on A, we find 3.375 (= 1.5^3) on A opposite 1 on B.

(16) Conversely, to find $\sqrt[3]{x}$, we shift the slide (in its normal position) until we find, on B, opposite x on A, the same number as we have on D opposite 1 on C, and this number will be $\sqrt[3]{x}$. Or, turn the slide end for end,* place 1 on C opposite x on A, and find, on B, a number which coincides with its equal on D. This number is = $\sqrt[3]{x}$. See also (**17**). (**18**).

(17) On the back of the slide is usually placed a scale of logs (see scale shown below the rule in figure) and two scales of angles, marked "S" and "T" respectively, for finding sines of angles greater than 0° 34′ . . . ″, and tangents of angles between 5° 42′ . . . ″ and 45°.

(18) Placing 1 on C opposite any number x on D (with slide in its normal position), log x is read from the scale of logs by means of an index on the back of the rule. The logs may be used in finding powers and roots.



(19) To find the **sine or tangent** of an angle a; bring a, on scale S or T, as the case may be, opposite the **index** on back, and read the natural (not logarithmic) sine or tangent opposite 10 at the end of A or D: sines on B, and tangents on C. Or, invert the slide, placing S under A, and T over D, with the ends of the scales coinciding. Then the numbers on A and D are the sines and tangents, respectively, of the angles on S and T.

Caution. Sines of angles less than 5° 45′ . . . ″ are less than 0.1.
" " " " " 90° " " " 1.0.
Tangents " " betw. 5° 42′ . . . ″ and 45° are betw 0.1 and 1.0.

(20) On the back of the rule is usually printed a table of ratios of numbers in common use, for convenience in operating with the slide rule. Thus:

$\frac{\text{diameter}}{\text{circumference}} = \frac{113}{355}$; $\frac{\text{U. S. gallons}}{\text{pounds}} = \frac{3}{25}$ (in a given quantity of water).

(21) Soaping the edges of the slide and the groove in which it runs, will often cure sticking, which is apt to be very annoying. If the slide is too loose, the groove may be deepened, and small springs, cut from narrow steel tape, inserted between it and the edge of the slide.

(*) With the slide thus reversed, and with the ends of the scales coinciding, the numbers on A and B are **reciprocals** (page 52), as are also those on C and D.

LOGARITHMS OF NUMBERS TO 1000

For use of logarithms, see pp. 70, 71 and 72.

For five-place logarithms of numbers to 10,000, see pp. 80 to 91.

No.	0	1	2	3	4	5	6	7	8	9	No.
0		0000	3010	4771	6021	6990	7782	8451	9031	9542	**0**
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788	1
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624	2
3	4771	4914	5052	5185	5315	5441	5563	5682	5798	5911	3
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902	4
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709	5
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388	6
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976	7
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494	8
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	**10**
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	11
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	12
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	13
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	14
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	15
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	16
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	17
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	18
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	**20**
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	21
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	22
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	23
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	24
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	25
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	26
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	27
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	28
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	29
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	**30**
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	31
32	5052	5065	5079	5092	5105	5119	5132	5145	5159	5172	32
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	33
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	34
35	5441	5453	5465	5478	5490	5502	5515	5527	5539	5551	35
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	36
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	37
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	38
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	39
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	**40**
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	41
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	42
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	43
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	44
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	45
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	46
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	47
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	48
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	49
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	**50**

LOGARITHMS OF NUMBERS TO 1000. (Continued.)

For use of logarithms, see pp. 70, 71 and 72.
For five-place logarithms of numbers to 10,000, see pp. 80 to 91.

No.	0	1	2	3	4	5	6	7	8	9	No.
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	50
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	51
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	52
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	53
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	54
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	55
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	56
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	57
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	58
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	59
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	60
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	61
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	62
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	63
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	64
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	65
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	66
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	67
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	68
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	69
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	70
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	71
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	72
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	73
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	74
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	75
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	76
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	77
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	78
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	79
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	80
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	81
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	82
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	83
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	84
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	85
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	86
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	87
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	88
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	89
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	90
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	91
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	92
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	93
94	9731	9736	9741	9745	9750	9754	9759	9764	9768	9773	94
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	95
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	96
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	97
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	98
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	99
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	100

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif	No.	Log.	Dif	No.	Log.	Dif	No.	Log.	Dif	No.	Log.	Dif
1000	00000	43	**1050**	02119	41	**1100**	04139	40	**1150**	06070	38	**1200**	07918	36
01	043	44	51	160	42	01	—179	39	51	—108	37	01	954	36
02	—087	43	52	—202	41	02	218	40	52	145	38	02	990	37
03	130	43	53	—243	41	03	—258	39	53	—183	38	03	08027	36
04	173	44	54	284	41	04	—297	39	54	—221	37	04	—063	36
05	—217	43	55	325	41	05	336	40	55	258	38	05	—099	36
06	—260	43	56	366	41	06	—376	39	56	—296	37	06	—135	36
07	—303	43	57	407	42	07	—415	39	57	333	38	07	—171	36
08	346	43	58	—449	41	08	—454	39	58	—371	37	08	—207	36
09	389	43	59	—490	41	09	493	39	59	408	38	09	—243	36
1010	432	43	**1060**	—531	41	**1110**	532	39	**1160**	—446	37	**1210**	—279	35
11	475	43	61	—572	40	11	571	39	61	483	38	11	314	36
12	518	43	62	612	41	12	610	40	62	—521	37	12	350	36
13	—561	43	63	653	41	13	—650	39	63	—558	37	13	386	36
14	—604	43	64	694	41	14	—689	38	64	595	38	14	—422	36
15	—647	42	65	—735	41	15	727	39	65	—633	37	15	—458	35
16	689	43	66	—776	40	16	766	39	66	—670	37	16	493	36
17	732	43	67	816	41	17	805	39	67	707	37	17	529	36
18	—775	42	68	857	41	18	844	39	68	744	37	18	—565	35
19	817	43	69	—898	41	19	883	39	69	781	38	19	600	36
1020	860	43	**1070**	938	41	**1120**	—922	39	**1170**	—819	37	**1220**	—636	36
21	—903	42	71	—979	40	21	—961	38	71	—856	37	21	—672	35
22	945	43	72	03019	41	22	999	39	72	—893	37	22	707	36
23	—988	42	73	—060	40	23	05038	39	73	—930	37	23	—743	35
24	01030	42	74	100	41	24	—077	38	74	—967	37	24	778	36
25	072	43	75	—141	40	25	115	39	75	07004	37	25	—814	35
26	—115	42	76	181	41	26	—154	38	76	—041	37	26	849	35
27	157	42	77	—222	40	27	192	39	77	—078	37	27	884	36
28	199	43	78	—262	40	28	—231	38	78	—115	36	28	—920	35
29	—242	42	79	302	40	29	269	39	79	151	37	29	955	36
1030	—284	42	**1080**	342	41	**1130**	—308	38	**1180**	188	37	**1230**	—991	35
31	—326	42	81	—383	40	31	346	39	81	—225	37	31	09026	35
32	—368	42	82	—423	40	32	—385	38	82	—262	36	32	061	35
33	410	42	83	—463	40	33	—423	38	83	298	37	33	096	36
34	452	42	84	—503	40	34	461	39	84	335	37	34	—132	35
35	494	42	85	—543	40	35	—500	38	85	—372	36	35	—167	35
36	—536	42	86	—583	40	36	—538	38	86	408	37	36	—202	35
37	—578	42	87	—623	40	37	576	38	87	445	37	37	—237	35
38	—620	42	88	—663	40	38	614	38	88	—482	36	38	272	35
39	—662	41	89	—703	40	39	652	38	89	518	37	39	307	35
1040	703	42	**1090**	—743	39	**1140**	690	39	**1190**	—555	36	**1240**	342	35
41	745	42	91	782	40	41	—729	38	91	591	37	41	377	35
42	—787	41	92	822	40	42	—767	38	92	—628	36	42	412	35
43	828	42	93	862	40	43	—805	38	93	664	36	43	447	35
44	870	42	94	—902	39	44	—843	38	94	700	37	44	482	35
45	—912	41	95	941	40	45	—881	37	95	—737	36	45	—517	35
46	953	42	96	981	40	46	918	38	96	773	36	46	—552	35
47	—995	41	97	04021	39	47	956	38	97	809	37	47	—587	34
48	02036	42	98	060	40	48	994	38	98	—846	36	48	621	35
49	—078	41	99	—100	39	49	06032	38	99	—882	36	49	656	35

Example:
To find Log. 11826:
Log. 11830 = 07298
Dif. = 10 **36**
Log. 11820 = 07262
11826 — 11820 = **6**
Dif. for **6** under **36** = 22
Log. 11826 = 07262 + 22 = 07284

	44	**43**	**42**	**41**	**40**	**39**	**38**	**37**	**36**	**35**	**34**	
1	4	4	4	4	4	4	4	4	4	4	3	**1**
2	9	9	8	8	8	8	8	7	7	7	7	**2**
3	13	13	13	12	12	12	11	11	11	11	10	**3**
4	18	17	17	16	16	16	15	15	14	14	14	**4**
5	22	22	21	21	20	20	19	19	18	18	17	**5**
6	26	26	25	25	24	23	23	22	22	21	20	**6**
7	31	30	29	29	28	27	27	26	25	25	24	**7**
8	35	34	34	33	32	31	30	30	29	28	27	**8**
9	40	39	38	37	36	35	34	33	32	32	31	**9**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
1250	09691	35	**1300**	11394	34	**1350**	13033	33	**1400**	14613	31	**1450**	16137	30
51	—726	34	01	—428	33	51	—066	32	01	—644	31	51	—167	30
52	760	35	02	461	33	52	—098	32	02	—675	31	52	—197	30
53	795	35	03	494	34	53	—130	32	03	—706	31	53	—227	29
54	—830	34	04	—528	33	54	—162	32	04	—737	31	54	256	30
55	864	35	05	561	33	55	—194	32	05	—768	31	55	286	30
56	—899	35	06	594	34	56	—226	32	06	—799	30	56	316	30
57	—934	34	07	—628	33	57	—258	32	07	829	31	57	—346	30
58	968	35	08	—661	33	58	—290	32	08	860	31	58	—376	30
59	10003	34	09	—694	33	59	—322	32	09	891	31	59	—406	29
1260	037	35	**1310**	727	33	**1360**	—354	32	**1410**	—922	31	**1460**	435	30
61	—072	34	11	760	33	61	—386	32	11	—953	30	61	465	30
62	—106	34	12	793	33	62	—418	32	12	983	31	62	—495	29
63	140	35	13	826	34	63	—450	31	13	15014	31	63	524	30
64	—175	34	14	—860	33	64	481	32	14	—045	31	64	554	30
65	209	34	15	—893	33	65	513	32	15	—076	30	65	—584	29
66	243	35	16	—926	33	66	545	32	16	106	31	66	613	30
67	—278	34	17	—959	33	67	—577	32	17	—137	31	67	643	30
68	—312	34	18	—992	32	68	—609	31	18	—168	30	68	—673	29
69	346	34	19	12024	33	69	640	32	19	198	31	69	702	30
1270	380	35	**1320**	057	33	**1370**	672	32	**1420**	—229	30	**1470**	—732	29
71	—415	34	21	090	33	71	—704	31	21	259	31	71	761	30
72	—449	34	22	123	33	72	735	32	22	—290	30	72	—791	29
73	—483	34	23	—156	33	73	767	32	23	320	31	73	820	30
74	—517	34	24	—189	33	74	—799	31	24	—351	30	74	—850	29
75	551	34	25	—222	32	75	830	32	25	381	31	75	879	30
76	585	34	26	254	33	76	—862	31	26	412	30	76	—909	29
77	619	34	27	287	33	77	893	32	27	442	31	77	938	29
78	653	34	28	—320	32	78	—925	31	28	—473	30	78	967	30
79	687	34	29	352	33	79	956	32	29	503	31	79	—997	29
1280	—721	34	**1330**	385	33	**1380**	—988	31	**1430**	—534	30	**1480**	17026	30
81	—755	34	31	—418	32	81	14019	32	31	—564	30	81	—056	29
82	—789	34	32	450	33	82	—051	31	32	594	31	82	—085	29
83	—823	34	33	483	33	83	082	32	33	—625	30	83	114	29
84	-857	33	34	—516	32	84	—114	31	34	—655	30	84	143	30
85	890	34	35	548	33	85	—145	31	35	685	30	85	—173	29
86	924	34	36	—581	32	86	176	32	36	715	31	86	—202	29
87	—958	34	37	613	33	87	—208	31	37	—746	30	87	231	29
88	—992	33	38	—646	32	88	—239	31	38	—776	30	88	260	29
89	11025	34	39	678	32	89	270	31	39	806	30	89	289	30
1290	—059	34	**1340**	710	33	**1390**	301	32	**1440**	836	30	**1490**	—319	29
91	—093	33	41	—743	32	91	—333	31	41	866	31	91	—348	29
92	126	34	42	775	33	92	—364	31	42	—897	30	92	—377	29
93	—160	33	43	—808	32	93	395	31	43	—927	30	93	—406	29
94	193	34	44	—840	32	94	426	31	44	—957	30	94	435	29
95	—227	34	45	872	33	95	457	32	45	—987	30	95	464	29
96	—261	33	46	—905	32	96	—489	31	46	16017	30	96	493	29
97	—294	33	47	—937	32	97	—520	31	47	—047	30	97	522	29
98	327	34	48	—969	32	98	—551	31	48	—077	30	98	551	29
99	—361	33	49	13001	32	99	—582	31	49	—107	30	99	580	29

Example:
To find Log. 12605.
Log 12610 = 10072
Dif. = 10 **35**
Log. 12600 = 10037
12605 — 12600 = **5**
Dif. for **5** under **35** = 18
Log. 12605 = 10037 + 18 = 10055

	35	34	33	32	31	30	29	
1	4	3	3	3	3	3	3	**1**
2	7	7	7	6	6	6	6	**2**
3	11	10	10	10	9	9	9	**3**
4	14	14	13	13	12	12	12	**4**
5	18	17	17	16	16	15	15	**5**
6	21	20	20	19	19	18	17	**6**
7	25	24	23	22	22	21	20	**7**
8	28	27	26	26	25	24	23	**8**
9	32	31	30	29	28	27	26	**9**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 1366=1354507
" 1367=1357685

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif	No.	Log.	Dif	No.	Log.	Dif.	No.	Log.	Dif	No.	Log.	Dif.
1500	17609	29	**1550**	19033	28	**1600**	20412	— 27	**1650**	21748	27	**1700**	23045	— 25
01	638	29	51	061	28	01	439	27	51	—775	26	01	070	26
02	—667	29	52	089	28	02	466	27	52	801	26	02	—096	25
03	—696	29	53	117	28	03	493	27	53	827	27	03	121	26
04	—725	29	54	145	28	04	520	28	54	—854	26	04	—147	25
05	—754	28	55	173	28	05	—548	27	55	—880	26	05	172	26
06	782	29	56	—201	28	06	—575	27	56	906	26	06	—198	25
07	811	29	57	—229	28	07	—602	27	57	932	26	07	223	26
08	840	29	58	—257	28	08	—629	27	58	958	27	08	—249	25
09	—869	29	59	—285	27	09	—656	27	59	—985	26	09	274	26
1510	—898	28	**1560**	312	28	**1610**	—683	27	**1660**	22011	— 26	**1710**	—300	25
11	926	29	61	340	28	11	—710	27	61	—037	26	11	325	25
12	955	29	62	368	28	12	—737	26	62	063	26	12	350	26
13	—984	29	63	—396	28	13	763	27	63	089	26	13	—376	25
14	18013	— 28	64	—424	27	14	790	27	64	115	26	14	401	25
15	041	29	65	451	28	15	817	27	65	141	26	15	426	26
16	—070	29	66	479	28	16	841	27	66	167	27	16	—452	25
17	—099	28	67	—507	28	17	871	27	67	—194	26	17	477	25
18	127	29	68	—535	27	18	—898	27	68	—220	26	18	502	26
19	—156	28	69	562	28	19	—925	27	69	—246	26	19	—528	25
1520	184	29	**1570**	—590	28	**1620**	—952	26	**1670**	—272	26	**1720**	—553	25
21	—213	28	71	—618	27	21	978	27	71	—298	26	21	578	25
22	241	29	72	645	28	22	21005	27	72	—324	26	22	603	26
23	—270	28	73	—673	27	23	—032	27	73	—350	26	23	—629	25
24	298	29	74	700	28	24	—059	26	74	—376	25	24	—654	25
25	—327	28	75	728	28	25	085	27	75	401	26	25	—679	25
26	355	29	76	—756	27	26	112	27	76	427	26	26	704	25
27	—381	28	77	783	28	27	—139	26	77	453	26	27	729	25
28	412	29	78	—811	27	28	165	27	78	479	26	28	754	25
29	—441	28	79	838	28	29	192	27	79	505	26	29	779	26
1530	469	29	**1580**	—866	27	**1630**	—219	26	**1680**	—531	26	**1730**	—805	25
31	—498	28	81	893	28	31	245	27	81	—557	26	31	—830	25
32	—526	28	82	—921	27	32	272	27	82	—583	25	32	—855	25
33	554	29	83	948	28	33	—299	26	83	608	26	33	—880	25
34	—583	28	84	—976	27	34	325	27	84	634	26	34	—905	25
35	—611	28	85	20003	— 27	35	—352	26	85	—660	26	35	—930	25
36	639	28	86	030	28	36	378	27	86	—686	26	36	—955	25
37	667	29	87	—058	27	37	—405	26	87	—712	25	37	—980	25
38	—696	28	88	085	27	38	431	27	88	737	26	38	24005	— 25
39	—724	28	89	112	28	39	—458	26	89	—763	26	39	—030	25
1540	752	28	**1590**	—140	27	**1640**	484	27	**1690**	—789	25	**1740**	—055	25
41	780	28	91	167	27	41	—511	26	91	814	26	41	—080	25
42	808	29	92	194	28	42	537	27	92	840	26	42	—105	25
43	—837	28	93	—222	27	43	—564	26	93	—866	25	43	—130	25
44	—865	28	94	—249	27	44	590	27	94	891	26	44	—155	25
45	—893	28	95	276	27	45	—617	26	95	—917	26	45	—180	24
46	—921	28	96	303	27	46	—643	26	96	—943	25	46	204	25
47	949	28	97	330	28	47	669	27	97	968	26	47	229	25
48	977	28	98	—358	27	48	—696	26	98	—994	25	48	254	25
49	19005	28	99	—385	27	49	722	26	99	23019	26	49	—279	25

Example:
To find Log. 15414:
Log. 15420 = 18808
Dif. = 10 **28**
Log. 15410 = 18780
15414 — 15410 = **4**
Dif. for **4** under **28** = 11
Log. 15414 =
18780 + 11 = 18791

	29	**28**	**27**	**26**	**25**	**24**	
1	3	3	3	3	8	2	**1**
2	6	6	5	5	5	5	**2**
3	9	8	8	8	8	7	**3**
4	12	11	11	10	10	10	**4**
5	15	14	14	13	13	12	**5**
6	17	17	16	16	15	14	**6**
7	20	20	19	18	18	17	**7**
8	23	22	22	21	20	19	**8**
9	26	25	24	23	23	22	**9**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 1562=1936810
" 1563=1939590

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif	No.	Log	Dif	No.	Log.	Dif	No.	Log.	Dif.	No.	Log.	Dif.
1750	21304	49	**1850**	26717	47	**1950**	29003	45	**2050**	31175	43	**2150**	33244	40
52	353	50	52	761	47	52	—048	44	52	—218	42	52	284	41
51	—403	49	54	—811	47	54	092	45	54	260	42	54	—325	40
56	452	50	56	—858	47	56	—137	44	56	302	43	56	—365	40
58	—502	49	58	—905	46	58	181	45	58	—345	42	58	405	40
1760	551	50	**1860**	951	47	**1960**	—226	41	**2060**	—387	42	**2160**	445	41
62	—601	49	62	—998	47	62	—270	44	62	—429	42	62	—486	40
64	—650	49	64	27045	46	64	314	44	64	—471	42	64	—526	40
66	699	49	66	09[illegible]	47	66	358	45	66	513	42	66	—566	40
68	748	49	68	—138	46	68	—403	44	68	555	42	68	—606	40
1770	797	49	**1870**	184	47	**1970**	—447	44	**2070**	597	42	**2170**	—646	40
72	846	49	72	—231	46	72	—491	44	72	—639	42	72	—686	40
74	895	49	74	—277	46	74	—535	44	74	—681	42	74	—726	40
76	944	49	76	323	47	76	—579	44	76	—723	42	76	—766	40
78	993	49	78	—370	46	78	—623	44	78	—765	41	78	—806	40
1780	25042	49	**1880**	—416	46	**1980**	—667	43	**2080**	806	42	**2180**	—846	39
82	—091	48	82	—462	46	82	710	44	82	848	42	82	885	40
84	139	49	84	508	46	84	754	44	84	—890	41	84	925	40
86	188	49	86	554	46	86	—798	44	86	931	42	86	965	40
88	237	48	88	600	46	88	—842	43	88	973	42	88	34005	39
1790	285	49	**1890**	646	46	**1990**	885	44	**2090**	32015	41	**2190**	044	40
92	—334	48	92	692	46	92	—929	41	92	056	42	92	084	40
94	382	49	94	—738	46	94	—973	43	94	—098	41	94	—124	39
96	—431	48	96	—784	46	96	30016	44	96	139	42	96	168	40
98	—479	43	98	—830	45	98	—060	43	98	—181	41	98	—203	39
1800	527	48	**1900**	875	46	**2000**	—103	43	**2100**	—222	41	**2200**	242	40
02	575	49	02	921	46	02	146	41	02	268	42	02	—282	39
04	—624	48	04	—967	45	04	—190	43	04	—305	41	04	321	40
06	—672	48	06	28012	46	06	233	43	06	—346	41	06	—361	39
08	—720	48	08	—058	45	08	276	44	08	387	41	08	—400	39
1810	—768	48	**1910**	103	46	**2010**	—320	43	**2110**	428	41	**2210**	439	40
12	—816	48	12	—149	45	12	—363	43	12	469	41	12	—479	39
14	—864	48	14	194	46	14	—406	43	14	510	42	14	—518	39
16	—912	47	16	—240	45	16	449	43	16	—552	41	16	—557	39
18	959	48	18	—285	45	18	492	43	18	—593	41	18	596	39
1820	26007	48	**1920**	330	45	**2020**	535	43	**2120**	—634	41	**2220**	635	39
22	—055	47	22	375	46	22	578	43	22	—675	40	22	674	39
24	102	48	24	—421	45	24	621	43	24	715	41	21	713	40
26	150	48	26	—466	45	26	—664	43	26	756	41	26	—753	39
28	—198	47	28	—511	45	28	—707	43	28	797	41	28	—792	38
1830	245	48	**1930**	—556	45	**2030**	—750	42	**2130**	—838	41	**2230**	830	39
32	—293	47	32	—601	45	32	792	43	32	—879	40	32	869	39
34	—340	47	34	—646	45	34	835	43	34	919	41	34	908	39
36	387	48	36	—691	41	36	—878	42	36	960	41	36	947	39
38	—435	47	38	735	45	38	920	43	38	33001	40	38	986	39
1840	—482	47	**1940**	780	45	**2040**	963	43	**2140**	041	41	**2240**	35025	39
42	—529	47	42	—825	45	42	31006	42	42	—082	40	42	—064	38
44	576	47	44	—870	44	44	048	43	44	122	41	44	102	39
46	623	47	46	914	45	46	—091	42	46	—163	40	46	—141	39
48	670	47	48	—959	44	48	—133	42	48	203	41	48	—180	38

To find Log. 18117
Log. 18120 = 25816
Dif. 20 **48**
Log. 18100 = 25768
18117 — 18100 = **17**
Under **48**
Dif. for 10 = 24
" " 7 = 17
" " **17** = 41
Log. 18117 =

	50	49	48	47	46	45	44	43	42	41	40	39	38	
1	3	2	2	2	2	2	2	2	2	2	2	2	2	**1**
2	5	5	5	5	5	5	4	4	4	4	4	4	4	**2**
3	8	7	7	7	7	7	7	6	6	6	6	6	6	**3**
4	10	10	10	9	9	9	9	9	8	8	8	8	8	**4**
5	13	12	12	12	12	11	11	11	11	10	10	10	10	**5**
6	15	15	14	14	14	14	13	13	13	12	12	12	11	**6**
7	18	17	17	16	16	16	15	15	15	14	14	14	13	**7**
8	20	20	19	19	18	18	18	17	17	16	16	16	15	**8**
9	23	22	22	21	21	20	20	19	19	18	18	18	17	**9**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
2250	35218	39	**2350**	37107	37	**2450**	38917	35	**2550**	40654	34	**2650**	42325	32
52	—257	38	52	—144	37	52	952	35	52	688	34	52	357	33
54	295	39	54	—181	37	54	987	36	54	722	34	54	390	33
56	—334	38	56	—218	36	56	39023	35	56	756	34	56	—423	32
58	372	39	58	254	37	58	058	36	58	790	34	58	455	33
2260	—411	38	**2360**	291	37	**2460**	—094	35	**2560**	—824	34	**2660**	488	33
62	449	39	62	—328	37	62	—129	35	62	—858	34	62	—521	32
64	—488	38	64	—365	36	64	164	35	64	—892	34	64	553	33
66	—526	38	66	401	37	66	199	36	66	—926	34	66	586	33
68	564	39	68	438	37	68	—235	35	68	—960	33	68	—619	32
2270	—603	38	**2370**	—475	36	**2470**	—270	35	**2570**	993	34	**2670**	651	33
72	—641	38	72	511	37	72	—305	35	72	41027	34	72	—684	32
74	679	38	74	548	37	74	—340	35	74	—061	34	74	716	33
76	717	38	76	—585	36	76	375	35	76	—095	33	76	—749	32
78	755	38	78	621	37	78	410	35	78	128	34	78	781	32
2280	793	39	**2380**	—658	36	**2480**	445	35	**2580**	—162	34	**2680**	813	33
82	—832	38	82	694	37	82	480	35	82	—196	33	82	—846	32
84	—870	38	84	—731	36	84	515	35	84	229	34	84	878	33
86	—908	38	86	767	36	86	550	35	86	—263	33	86	—911	32
88	—946	38	88	803	37	88	585	35	88	296	34	88	—943	32
2290	—984	37	**2390**	—840	36	**2490**	—620	35	**2590**	—330	33	**2690**	975	33
92	36021	38	92	876	36	92	—655	35	92	363	34	92	43008	32
94	059	38	94	912	37	94	—690	34	94	—397	33	94	—040	32
96	097	38	96	—949	36	96	724	35	96	430	34	96	—072	32
98	135	38	98	—985	36	98	759	35	98	—464	33	98	104	32
2300	—173	38	**2400**	38021	36	**2500**	794	35	**2600**	497	34	**2700**	136	33
02	—211	37	02	057	36	02	—829	34	02	—531	33	02	—169	32
04	248	38	04	093	37	04	863	35	04	564	33	04	—201	32
06	—286	38	06	—130	36	06	898	35	06	597	34	06	—233	32
08	—324	37	08	—166	36	08	—933	34	08	—631	33	08	—265	32
2310	361	38	**2410**	—202	36	**2510**	967	35	**2610**	664	33	**2710**	—297	32
12	—399	37	12	—238	36	12	40002	35	12	697	34	12	—329	32
14	436	38	14	—274	36	14	—037	34	14	—731	33	14	—361	32
16	—474	37	16	—310	36	16	071	35	16	—764	33	16	—393	32
18	511	38	18	—346	36	18	—106	34	18	—797	33	18	—425	32
2320	—549	37	**2420**	—382	35	**2520**	140	35	**2620**	830	33	**2720**	—457	32
22	586	38	22	417	36	22	—175	34	22	863	33	22	—489	32
24	—624	37	24	453	36	24	—209	34	24	896	33	24	—521	32
26	—661	37	26	489	36	26	243	35	26	929	34	26	—553	31
28	698	38	28	—525	36	28	—278	34	28	—963	33	28	584	32
2330	—736	37	**2430**	—561	35	**2530**	312	34	**2630**	—996	33	**2730**	616	32
32	—773	37	32	596	36	32	346	35	32	42029	33	32	648	32
34	810	37	34	632	36	34	—381	34	34	—062	33	34	—680	32
36	847	37	36	—668	35	36	—415	34	36	—095	32	36	—712	31
38	884	38	38	703	36	38	449	34	38	127	33	38	743	32
2340	—922	37	**2440**	—739	36	**2540**	483	35	**2640**	160	33	**2740**	775	32
42	—959	37	42	—775	35	42	—518	34	42	193	33	42	—807	31
44	—996	37	44	810	36	44	—552	34	44	226	33	44	838	32
46	37038	37	46	—846	35	46	—586	34	46	—259	33	46	870	32
48	—070	37	48	881	36	48	—620	34	48	—292	33	48	—902	31

To find Log. 23335:

Log. 23340 = 36810
Dif. 20 **37**
Log. 23320 = 36773
23335 — 23320 = **15**
Under **37**
Dif. for 10 = 19
" " 5 = 9
" " **15** = 28
Log. 23335 =
36773 + 28 = 36801.

	39	38	37	36	35	34	33	32	31	
1	2	2	2	2	2	2	2	2	2	**1**
2	4	4	4	4	4	3	3	3	3	**2**
3	6	6	6	5	5	5	5	5	5	**3**
4	8	8	7	7	7	7	7	6	6	**4**
5	10	10	9	9	9	9	8	8	8	**5**
6	12	11	11	11	11	10	10	10	9	**6**
7	14	13	13	13	12	12	12	11	11	**7**
8	16	15	15	14	14	14	13	13	12	**8**
9	18	17	17	16	16	15	15	14	14	**9**
10	20	19	19	18	18	17	17	16	16	**10**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
2750	43933	32	**2850**	45484	31	**2950**	46982	30	**3050**	48430	28	**3150**	49831	28
52	—965	31	52	—515	30	52	47012	29	52	458	29	52	—859	27
54	996	32	54	545	31	54	041	29	54	—487	28	54	886	28
56	44028	31	56	—576	30	56	070	30	56	515	29	56	—914	27
58	059	32	58	606	31	58	—100	29	58	—544	28	58	941	28
2760	—091	31	**2860**	—637	30	**2960**	129	30	**3060**	572	29	**3160**	—969	27
62	122	32	62	—667	30	62	—159	29	62	—601	28	62	996	28
64	—154	31	64	697	31	64	—188	29	64	—629	28	64	50024	27
66	185	32	66	—728	30	66	217	29	66	657	29	66	051	28
68	—217	31	68	—758	30	68	246	30	68	—686	28	68	—079	27
2770	—248	31	**2870**	788	30	**2970**	—276	29	**3070**	—714	28	**3170**	—106	27
72	279	32	72	818	31	72	—305	29	72	742	28	72	133	28
74	—311	31	74	—849	30	74	334	29	74	770	29	74	—161	27
76	—342	31	76	—879	30	76	363	29	76	—799	28	76	188	27
78	373	31	78	909	30	78	392	30	78	—827	28	78	215	28
2780	404	32	**2880**	939	30	**2980**	—422	29	**3080**	855	28	**3180**	—243	27
82	—436	31	82	969	31	82	—451	29	82	883	28	82	270	27
84	—467	31	84	46000	30	84	—480	29	84	911	29	84	297	28
86	498	31	86	—030	30	86	—509	29	86	—940	28	86	—325	27
88	529	31	88	—060	30	88	538	29	88	—968	28	88	—352	27
2790	560	32	**2890**	—090	30	**2990**	567	29	**3090**	—996	28	**3190**	879	27
92	—592	31	92	—120	30	92	596	29	92	49024	28	92	406	27
94	—623	31	94	—150	30	94	625	29	94	052	28	94	433	28
96	—654	31	96	—180	30	96	654	29	96	080	28	96	—461	27
98	—685	31	98	—210	30	98	683	29	98	108	28	98	—488	27
2800	—716	31	**2900**	—240	30	**3000**	712	29	**3100**	136	28	**3200**	—515	27
02	—747	31	02	—270	30	02	741	29	02	164	28	02	542	27
04	—778	31	04	—300	30	04	—770	29	04	192	28	04	569	27
06	—809	31	06	—330	29	06	—799	29	06	220	28	06	596	27
08	—840	31	08	359	30	08	—828	29	08	248	28	08	623	28
2810	—871	31	**2910**	389	30	**3010**	—857	28	**3110**	276	28	**3210**	—651	27
12	—902	30	12	419	30	12	885	29	12	—304	28	12	—678	27
14	932	31	14	—449	30	14	914	29	14	—332	28	14	—705	27
16	963	31	16	—479	30	16	943	29	16	—360	28	16	—732	27
18	994	31	18	—509	29	18	—972	29	18	—388	27	18	—759	27
2820	45025	31	**2920**	538	30	**3020**	48001	28	**3120**	415	28	**3220**	—786	27
22	—056	30	22	568	30	22	029	29	22	443	28	22	—813	27
24	086	31	24	—598	29	24	058	29	24	471	28	24	—840	26
26	117	31	26	627	30	26	—087	29	26	—499	28	26	866	27
28	—148	31	28	657	30	28	—116	28	28	—527	27	28	893	27
2830	—179	30	**2930**	—687	29	**3030**	144	29	**3130**	554	28	**3230**	920	27
32	209	31	32	716	30	32	—173	29	32	582	28	32	947	27
34	—240	31	34	746	30	34	—202	28	34	—610	28	34	974	27
36	—271	30	36	—776	29	36	230	29	36	—638	27	36	51001	27
38	301	31	38	805	30	38	—259	28	38	665	28	38	—028	27
2840	—332	30	**2940**	—835	29	**3040**	287	29	**3140**	—693	28	**3240**	—055	26
42	362	31	42	864	30	42	—316	28	42	—721	27	42	081	27
44	—393	30	44	—894	29	44	344	29	44	748	28	44	108	27
46	423	31	46	923	30	46	—373	28	46	—776	27	46	—135	27
48	—454	30	48	—953	29	48	401	29	48	803	28	48	—162	26

To find Log. 29019:
Log. 29020 = 46270
Dif. 20 **30**
Log. 29000 = 46240
29019 — 29000 = **19**
Under **30**
Dif. for 10 = 15
" " 9 = 14
" " **19** = 29
Log 29019 =
46240 + 29 = 46269.

	32	31	30	29	28	27	26	
1	2	2	2	1	1	1	1	**1**
2	3	3	3	3	3	3	3	**2**
3	5	5	5	4	4	4	4	**3**
4	6	6	6	6	6	5	5	**4**
5	8	8	8	7	7	7	7	**5**
6	10	9	9	9	8	8	8	**6**
7	11	11	11	10	10	9	9	**7**
8	13	12	12	12	11	11	10	**8**
9	14	14	14	13	13	12	12	**9**
10	16	16	15	15	14	14	13	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 3128=4952667
" 3130=4955443

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
3250	51188	27	**3350**	52504	26	**3450**	53782—	25	**3550**	55023—	24	**3650**	56229	24
52	215	27	52	530	26	52	807	25	52	047	25	52	253	24
54	—242	26	54	556	26	54	832	25	54	—072	24	54	—277	24
56	268	27	56	582	26	56	857	25	56	096	25	56	—301	23
58	295	27	58	608	26	58	882	26	58	—121	24	58	324	24
3260	—322	26	**3360**	—634	26	**3460**	—908	25	**3560**	—145	24	**3660**	348	24
62	348	27	62	—660	26	62	—933	25	62	169	25	62	—372	24
64	375	27	64	—686	25	64	—958	25	64	—194	24	64	—396	23
66	—402	26	66	711	26	66	—983	25	66	218	24	66	419	24
68	428	27	68	737	26	68	54008—	25	68	242	25	68	—443	24
3270	—455	26	**3370**	—763	26	**3470**	—033	25	**3570**	—267	24	**3670**	—467	23
72	481	27	72	—789	26	72	—058	25	72	291	24	72	490	24
74	—508	26	74	—815	25	74	—083	25	74	315	25	74	—514	24
76	534	27	76	840	26	76	—108	25	76	—340	24	76	—538	23
78	—561	26	78	—866	26	78	—133	25	78	364	24	78	561	24
3280	587	27	**3380**	—892	25	**3480**	—158	25	**3580**	388	25	**3680**	—585	23
82	—614	26	82	917	26	82	—183	25	82	—413	24	82	608	24
84	640	27	84	943	26	84	—208	25	84	—437	24	84	—632	24
86	—667	26	86	—969	25	86	—233	25	86	461	24	86	—656	23
88	693	27	88	994	26	88	—258	25	88	485	24	88	679	24
3290	—720	26	**3390**	53020—	26	**3490**	—283	24	**3590**	509	25	**3690**	—703	23
92	—746	26	92	—046	25	92	307	25	92	—534	24	92	726	24
94	772	27	94	071	26	94	332	25	94	—558	24	94	—750	23
96	—799	26	96	—097	25	96	357	25	96	—582	24	96	773	24
98	825	26	98	122	26	98	—382	25	98	606	24	98	—797	23
3300	851	27	**3400**	—148	25	**3500**	—407	25	**3600**	630	24	**3700**	820	24
02	—878	26	02	173	26	02	—432	24	02	654	24	02	—844	23
04	904	26	04	—199	25	04	456	25	04	678	25	04	867	24
06	930	27	06	224	26	06	481	25	06	—703	24	06	—891	23
08	—957	26	08	—250	25	08	—506	25	08	—727	24	08	—914	23
3310	—983	26	**3410**	275	26	**3510**	—531	24	**3610**	—751	24	**3710**	937	24
12	52009	26	12	—301	25	12	555	25	12	—775	24	12	—961	23
14	035	26	14	326	26	14	580	25	14	—799	24	14	984	24
16	061	27	16	—352	25	16	—605	25	16	—823	24	16	57008—	23
18	—088	26	18	377	26	18	—630	24	18	—847	24	18	—031	23
3320	—114	26	**3420**	—403	25	**3520**	654	25	**3620**	—871	24	**3720**	054	24
22	—140	26	22	428	25	22	—679	25	22	—895	24	22	—078	23
24	166	26	24	453	26	24	—704	24	24	—919	24	24	—101	23
26	192	26	26	—479	25	26	728	25	26	—943	24	26	124	24
28	218	26	28	504	25	28	—753	24	28	—967	24	28	—148	23
3330	244	26	**3430**	529	26	**3530**	777	25	**3630**	—991	24	**3730**	—171	23
32	270	27	32	—555	25	32	802	25	32	56015—	23	32	194	23
34	—297	26	34	580	25	34	—827	24	34	038	24	34	217	24
36	—323	26	36	605	26	36	851	25	36	062	24	36	—241	23
38	—349	26	38	—631	25	38	—876	24	38	086	24	38	—264	23
3340	—375	26	**3440**	—656	25	**3540**	900	25	**3640**	110	24	**3740**	287	23
42	—401	26	42	681	25	42	—925	24	42	—134	24	42	310	24
44	—427	26	44	706	26	44	949	25	44	—158	24	44	—334	23
46	—453	26	46	—732	25	46	—974	24	46	—182	23	46	—357	23
48	—479	25	48	—757	25	48	998	25	48	205	24	48	—380	23

To find Log. 36114:
Log. 36120 = 55775
Log. 36100 = 55751
Dif. 20 **24**
36114 — 36100 = **14**
Under **24**
Dif. for 10 = 12
" " 4 = 5
" " **14** = 17
Log. 36114 =
55751 + 17 = 55768.

	27	26	25	24	23	
1	1	1	1	1	1	**1**
2	3	3	3	2	2	**2**
3	4	4	4	4	3	**3**
4	5	5	5	5	5	**4**
5	7	7	6	6	6	**5**
6	8	8	8	7	7	**6**
7	10	9	9	8	8	**7**
8	11	10	10	10	9	**8**
9	12	12	11	11	10	**9**
10	14	13	13	12	12	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 3490 = 5428254
" 3492 = 5430742

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
3750	57403	58	**4000**	60206	— 54	**4250**	62839	51	**4500**	65321	48	**4750**	67669	46
55	—461	58	05	260	54	55	—890	51	05	369	49	55	715	46
60	—519	57	10	314	55	60	—941	51	10	—418	48	60	—761	45
65	576	58	15	—369	54	65	—992	51	15	—466	48	65	806	46
70	634	58	20	—423	54	70	63043	— 51	20	—514	48	70	—852	45
75	—692	57	25	—477	54	75	—094	50	25	—562	48	75	897	46
80	749	58	30	—531	53	80	144	51	30	—610	48	80	—943	45
85	—807	57	35	584	54	85	195	51	35	—658	48	85	988	46
90	—864	57	40	638	54	90	—246	50	40	—706	47	90	68034	— 45
95	921	57	45	—692	54	95	296	51	45	753	48	95	—079	45
3800	978	57	**4050**	—746	53	**4300**	—347	50	**4550**	801	48	**4800**	124	45
05	58035	57	55	799	54	05	397	51	55	—849	47	05	169	46
10	092	57	60	—853	53	10	—448	50	60	896	48	10	—215	45
15	149	57	65	906	53	15	498	50	65	944	48	15	—260	45
20	206	57	70	959	54	20	548	51	70	—992	47	20	—305	45
25	263	57	75	61013	— 53	25	—599	50	75	66039	48	25	—350	45
30	—320	57	80	066	53	30	—649	50	80	—087	47	30	—395	45
35	—377	56	85	119	53	35	—699	50	85	—134	47	35	—440	45
40	433	57	90	172	53	40	—749	50	90	181	48	40	—485	44
45	—490	56	95	225	53	45	—799	50	95	—229	47	45	529	45
3850	546	56	**4100**	278	53	**4350**	—849	50	**4600**	—276	47	**4850**	574	45
55	602	57	05	331	53	55	—899	50	05	—323	47	55	—619	45
60	—659	56	10	384	53	60	—949	49	10	370	47	60	—664	44
65	—715	56	15	—437	53	65	998	50	15	417	47	65	708	45
70	771	56	20	—490	52	70	64048	50	20	464	47	70	—753	44
75	827	56	25	542	53	75	—098	49	25	511	47	75	797	45
80	883	56	30	595	53	80	147	50	30	558	47	80	—842	44
85	939	56	35	—648	52	85	—197	49	35	—605	47	85	886	45
90	—995	56	40	700	52	90	246	50	40	—652	47	90	—931	44
95	59051	— 55	45	752	53	95	—296	49	45	—699	46	95	975	45
3900	106	56	**4150**	—805	52	**4400**	345	50	**4650**	745	47	**4900**	69020	— 44
05	162	56	55	857	52	05	—395	49	55	—792	47	05	—064	44
10	—218	55	60	909	53	10	—444	49	60	—839	46	10	108	44
15	273	56	65	—962	52	15	493	49	65	885	47	15	152	45
20	—329	55	70	62014	— 52	20	542	49	70	—932	46	20	—197	44
25	—384	55	75	—066	52	25	591	49	75	978	47	25	—241	44
30	439	55	80	—118	52	30	640	49	80	67025	— 46	30	—285	44
35	494	56	85	—170	51	35	689	49	85	—071	46	35	—329	44
40	—550	55	90	221	52	40	738	49	90	117	47	40	—373	44
45	—605	55	95	273	52	45	787	49	95	—164	46	45	—417	44
3950	—660	55	**4200**	—325	52	**4450**	836	49	**4700**	—210	46	**4950**	—461	43
55	—715	55	05	—377	51	55	—885	48	05	—256	46	55	504	44
60	—770	54	10	428	52	60	933	49	10	302	46	60	548	44
65	824	55	15	—480	51	65	982	49	15	348	46	65	—592	44
70	879	55	20	531	52	70	65031	— 48	20	394	46	70	—636	43
75	—934	54	25	—583	51	75	079	49	25	440	46	75	679	44
80	988	55	30	634	51	80	—128	48	30	486	46	80	—723	44
85	60043	— 54	35	685	52	85	176	49	35	—532	46	85	—767	43
90	097	55	40	—737	61	90	—225	48	40	—578	46	90	810	44
95	—152	54	45	—788	51	95	—273	48	45	—624	45	95	—854	43

	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	
1	1.2	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.9	**1**
2	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	**2**
3	3.5	3.4	3.4	3.3	3.2	3.2	3.1	3.1	3.0	2.9	2.9	2.8	2.8	2.7	2.6	2.6	**3**
4	4.6	4.6	4.5	4.4	4.3	4.2	4.2	4.1	4.0	3.9	3.8	3.8	3.7	3.6	3.5	3.4	**4**
5	5.8	5.7	5.6	5.5	5.4	5.3	5 2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	**5**
6	7 0	6.8	6.7	6.6	6.5	6.4	6.2	6.1	6.0	5.9	5.8	5.6	5.5	5.4	5.3	5.2	**6**
7	8.1	8.0	7.8	7.7	7.6	7.4	7.3	7.1	7.0	6.9	6.7	6.6	6.4	6.3	6.2	6.0	**7**
8	9.3	9.1	9.0	8.8	8.6	8.5	8.3	8.2	8.0	7.8	7.7	7.5	7.4	7.2	7.0	6.9	**8**
9	10.4	10.3	10.1	9.9	9.7	9.5	9.4	9 2	9.0	8.8	8.6	8.5	8.3	8.1	7.9	7.7	**9**
10	11.6	11.4	11.2	11.0	10.8	10.6	10.4	10.2	10.0	9.8	9.6	9.4	9.2	9.0	8.8	8.6	**10**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
5000	69897	43	**5250**	72016	41	**5500**	74036	40	**5750**	75967	38	**6000**	77815	36
05	940	44	55	057	42	05	—076	39	55	76005	37	05	851	36
10	—984	43	60	—099	41	10	115	40	60	042	38	10	887	37
15	70027	43	65	—140	41	15	—155	39	65	—080	38	15	—924	36
20	070	41	70	181	41	20	—194	39	70	—118	37	20	—960	36
25	—114	43	75	222	41	25	233	40	75	155	38	25	—996	36
30	—157	43	80	263	41	30	—273	39	80	—193	37	30	78032	36
35	—200	43	85	304	42	35	—312	39	85	230	38	35	—068	36
40	243	4.3	90	—346	41	40	—351	39	90	—268	37	40	—104	36
45	286	43	95	—387	41	45	390	39	95	305	38	45	—140	36
5050	329	43	**5300**	—428	41	**5550**	429	39	**5800**	—343	37	**6050**	—176	35
55	372	43	05	—469	40	55	468	39	05	380	38	55	211	36
60	415	43	10	509	41	60	507	40	10	—418	37	60	247	36
65	—458	43	15	550	41	65	—547	39	15	—455	37	65	283	36
70	—501	43	20	591	41	70	—586	38	20	492	38	70	—319	36
75	—544	42	25	—632	41	75	624	39	25	—530	37	75	—355	35
80	586	43	30	—673	40	80	663	39	30	—567	37	80	390	36
85	629	43	35	713	41	85	702	39	35	604	37	85	426	36
90	—672	42	40	754	41	90	741	39	40	641	37	90	—462	35
95	714	43	45	—795	40	95	780	39	45	678	38	95	497	36
5100	757	43	**5350**	835	41	**5600**	—819	39	**5850**	—716	37	**6100**	—533	36
05	—800	42	55	—876	40	05	—858	38	55	—753	37	05	—569	35
10	842	43	60	916	41	10	896	39	60	—790	37	10	604	36
15	—885	42	65	—957	40	15	—935	39	65	—827	37	15	—640	35
20	—927	42	70	997	41	20	—974	38	70	—864	37	20	675	36
25	969	43	75	73038	40	25	75012	39	75	—901	37	25	—711	35
30	71012	42	80	078	41	30	—051	38	80	—938	37	30	746	35
35	054	42	85	—119	40	35	089	39	85	—975	37	35	781	36
40	096	43	90	—159	40	40	—128	38	90	77012	36	40	—817	35
45	—139	42	95	199	40	45	166	39	95	048	37	45	852	36
5150	—181	42	**5400**	239	41	**5650**	—205	38	**5900**	085	37	**6150**	—888	35
55	—223	42	05	—280	40	55	243	39	05	—122	37	55	—923	35
60	—265	42	10	—320	40	60	—282	38	10	—159	36	60	958	35
65	307	42	15	—360	40	65	—320	38	15	195	37	65	993	36
70	349	42	20	—400	40	70	358	39	20	232	37	70	79029	35
75	391	42	25	—440	40	75	—397	38	25	—269	36	75	—064	35
80	—433	42	30	—480	40	80	—435	38	30	305	37	80	—099	35
85	—475	42	35	—520	40	85	473	38	35	342	37	85	—134	35
90	—517	42	40	—560	40	90	511	38	40	—379	36	90	169	35
95	—559	41	45	—600	40	95	549	38	45	415	37	95	204	35
5200	600	42	**5450**	—640	39	**5700**	587	39	**5950**	—452	36	**6200**	239	35
05	642	42	55	679	40	05	—626	38	55	488	37	05	274	35
10	—684	41	60	719	40	10	—664	38	60	—525	36	10	309	35
15	725	42	65	759	40	15	—702	38	65	561	36	15	344	35
20	767	42	70	—799	39	20	—740	38	70	597	37	20	379	35
25	—809	41	75	838	40	25	—778	37	75	—634	36	25	—414	35
30	850	42	80	878	40	30	815	38	80	670	36	30	—449	35
35	—892	41	85	—918	39	35	853	38	85	706	37	35	—484	34
40	933	42	90	957	40	40	891	38	90	—743	36	40	518	35
45	—975	41	95	—997	39	45	929	38	95	—779	36	45	553	35

	44	**43**	**42**	**41**	**40**	**39**	**38**	**37**	**36**	**35**	**34**	
1	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	**1**
2	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	**2**
3	2.6	2.6	2.5	2.5	2.4	2.3	2.8	2.2	2.2	2.1	2.0	**3**
4	3.5	3.4	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.7	**4**
5	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5	3.4	**5**
6	5.3	5.2	5.0	4.9	4.8	4.7	4.6	4.4	4.3	4.2	4.1	**6**
7	6.2	6.0	5.9	5.7	5.6	5.5	5.3	5.2	5.0	4.9	4.8	**7**
8	7.0	6.9	6.7	6.6	6.4	6.2	6.1	5.9	5.8	5.6	5.4	**8**
9	7.9	7.7	7.6	7.4	7.2	7.0	6.8	6.7	6.5	6.3	6.1	**9**
10	8.8	8.6	8.4	8.2	8.0	7.8	7.6	7.4	7.2	7.0	6.8	**10**

To find Log. 58636:
Log 58650 = 76827
Log. 58600 = 76790
Dif. 50 **37**
58636 — 58600 = **36**
Under **37**
Dif. for 10 × 3 = 22.0
" " 6 = 4.4
" " **36** = 26.4
Log. 58636 =
76790 + 26 = 76816

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
6250	79588	35	**6500**	81291	34	**6750**	82930	33	**7000**	84510	31	**7250**	86034	30
55	—623	34	05	—325	33	55	—963	32	05	—541	31	55	—064	30
60	657	35	10	358	33	60	—995	32	10	—572	31	60	—094	30
65	692	35	15	391	34	65	83027	32	15	—603	31	65	—124	29
70	—727	34	20	—425	33	70	—059	32	20	—634	31	70	153	30
75	761	35	25	458	33	75	—091	32	25	—665	31	75	183	30
80	—796	35	30	491	34	80	—123	32	30	—696	30	80	213	30
85	—831	34	35	—525	33	85	—155	32	35	726	31	85	—243	30
90	865	35	40	—558	33	90	—187	32	40	757	31	90	—273	30
95	—900	34	45	—591	33	95	—219	32	45	788	31	95	—303	29
6300	934	35	**6550**	624	33	**6800**	—251	32	**7050**	—819	31	**7300**	332	30
05	—969	34	55	657	33	05	—283	32	55	—850	30	05	362	30
10	80003	34	60	690	33	10	—315	32	60	880	31	10	—392	29
15	037	35	65	723	34	15	—347	31	65	911	31	15	421	30
20	—072	34	70	—757	33	20	378	32	70	—942	31	20	451	30
25	106	34	75	—790	33	25	410	32	75	—973	30	25	—481	29
30	140	35	80	—823	33	30	442	32	80	85003	31	30	510	30
35	—175	34	85	—856	33	35	—474	32	85	—034	31	35	540	30
40	—209	34	90	—889	32	40	—506	31	90	—065	30	40	—570	29
45	243	34	95	921	33	45	537	32	95	095	31	45	599	30
6350	277	35	**6600**	954	33	**6850**	569	32	**7100**	—126	30	**7350**	—629	29
55	—312	34	05	987	33	55	—601	31	05	156	31	55	658	30
60	—346	34	10	82020	33	60	632	32	10	—187	30	60	—688	29
65	—380	34	15	—053	33	65	664	32	15	217	31	65	717	30
70	—414	34	20	—086	33	70	—696	31	20	—248	30	70	—747	29
75	448	34	25	—119	32	75	727	32	25	278	31	75	776	30
80	482	34	30	151	33	80	—759	31	30	—309	30	80	—806	29
85	516	34	35	184	33	85	790	32	35	339	31	85	835	29
90	550	34	40	—217	32	90	—822	31	40	—370	30	90	864	30
95	584	31	45	249	33	95	853	32	45	400	31	95	—894	29
6400	—618	34	**6650**	282	33	**6900**	—885	31	**7150**	—431	30	**7400**	923	30
05	—652	34	55	—315	32	05	916	32	55	—461	30	05	—953	29
10	—686	34	60	347	33	10	—948	31	60	491	31	10	—982	29
15	—720	34	65	380	33	15	979	32	65	—522	30	15	87011	29
20	—754	33	70	—413	32	20	84011	31	70	—552	30	20	040	30
25	787	34	75	445	33	25	—042	31	75	582	30	25	—070	29
30	821	34	80	—478	32	30	073	32	80	612	31	30	—099	29
35	—855	34	85	510	33	35	—105	31	85	—643	30	35	128	29
40	—889	33	90	—543	32	40	—136	31	90	—673	30	40	157	29
45	922	34	95	575	32	45	167	31	95	703	30	45	186	30
6450	—956	34	**6700**	607	33	**6950**	198	32	**7200**	733	30	**7450**	—216	29
55	—990	33	05	—640	32	55	—230	31	05	763	31	55	—245	29
60	81023	34	10	672	33	60	—261	31	10	—794	30	60	—274	29
65	—057	33	15	—705	32	65	292	31	15	—824	30	65	—303	29
70	090	34	20	—737	32	70	323	31	20	—854	30	70	332	29
75	—124	34	25	769	33	75	354	32	25	—884	30	75	361	29
80	—158	33	30	—802	32	80	—386	31	30	—914	30	80	390	29
85	—191	33	35	—834	32	85	—417	31	35	—944	30	85	419	29
90	224	34	40	—866	32	90	—448	31	40	—974	30	90	448	29
95	—258	33	45	898	32	95	—479	31	45	86004	30	95	477	29

To find Log. 63023.
Log. 63050 = 79969
Log. 63000 = 79934
Dif. 50 **35**
63023 — 63000 = **23**
Under **35**
Dif. for 10 × 2 = 14.0
" " 3 = 2.1
" " **23** = 16.1
Log. 63023 =
79934 + 16 = 79950.

	35	34	33	32	31	30	29	
1	0 7	0.7	0.7	0.6	0.6	0.6	0.6	**1**
2	1.4	1.4	1.3	1.3	1.2	1 2	1.2	**2**
3	2.1	2.0	2.0	1.9	1.9	1.8	1.7	**3**
4	2.8	2.7	2.6	2.6	2.5	2.4	2.3	**4**
5	3.5	3.4	3.3	3.2	3.1	3.0	2.9	**5**
6	4.2	4.1	4.0	3.8	3.7	3.6	3.5	**6**
7	4.9	4.8	4.6	4.5	4.3	4.2	4.1	**7**
8	5.6	5.4	5.3	5.1	5.0	4.8	4.6	**8**
9	6.3	6.1	5.9	5.8	5.6	5.4	5.2	**9**
10	7.0	6.8	6.6	6.4	6.2	6.0	5.8	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log.7400 = 8692317
" 7405 = 8695251

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
7500	87506	29	**7750**	88930	28	**8000**	90309	27	**8250**	91645	27	**8500**	92942	25
05	535	29	55	958	28	05	336	27	55	—672	26	05	967	26
10	—564	29	60	986	28	10	363	27	60	698	26	10	—993	25
15	—593	29	65	89014	28	15	390	27	65	724	27	15	93018	26
20	—622	29	70	042	28	20	417	28	70	—751	26	20	—044	25
25	—651	28	75	070	28	25	—445	27	75	—777	26	25	069	26
30	679	29	80	—098	28	30	—472	27	80	803	26	30	—095	25
35	708	29	85	—126	28	35	—499	27	85	829	26	35	120	26
40	737	29	90	—154	28	40	—526	27	90	855	27	40	—146	25
45	—766	29	95	—182	27	45	—553	27	95	—882	26	45	171	26
7550	—795	28	**7800**	209	28	**8050**	—580	27	**8300**	—908	26	**8550**	—197	25
55	823	29	05	237	28	55	—607	27	05	—934	26	55	222	25
60	852	29	10	265	28	60	—634	26	10	960	26	60	247	26
65	—881	29	15	—293	28	65	660	27	15	986	26	65	—273	25
70	—910	28	20	—321	27	70	687	27	20	92012	26	70	298	25
75	938	29	25	348	28	75	714	27	25	038	27	75	323	26
80	—967	29	30	376	28	80	741	27	30	—065	26	80	—349	25
85	—996	28	35	—404	28	85	768	27	35	—091	26	85	374	25
90	88024	29	40	—432	27	90	—795	27	40	—117	26	90	399	26
95	—053	28	45	459	28	95	—822	27	45	—143	26	95	—425	25
7600	081	29	**7850**	—487	28	**8100**	—849	26	**8350**	—169	26	**8600**	—450	25
05	—110	28	55	—515	27	05	875	27	55	—195	26	05	475	25
10	138	29	60	542	28	10	902	27	60	—221	26	10	500	26
15	—167	28	65	—570	27	15	—929	27	65	—247	26	15	—526	25
20	195	29	70	597	28	20	—956	26	70	—273	25	20	—551	25
25	—224	28	75	625	28	25	982	27	75	298	26	25	—576	25
30	252	29	80	—653	27	30	91009	27	80	324	26	30	601	25
35	—281	28	85	680	28	35	—036	26	85	350	26	35	626	25
40	309	29	90	—708	27	40	062	27	90	376	26	40	651	25
45	—338	28	95	735	28	45	089	27	95	402	26	45	676	26
7650	366	29	**7900**	—763	27	**8150**	—116	26	**8400**	—428	26	**8650**	—702	25
55	—395	28	05	790	28	55	142	27	05	—454	26	55	—727	25
60	—423	28	10	—818	27	60	169	27	10	—480	25	60	—752	25
65	451	29	15	845	28	65	—196	26	15	505	26	65	—777	25
70	—480	28	20	—873	27	70	222	27	20	531	26	70	—802	25
75	—508	28	25	—900	27	75	—249	26	25	—557	26	75	—827	25
80	536	28	30	927	28	80	275	27	30	—583	26	80	—852	25
85	564	29	35	—955	27	85	—302	26	35	—609	25	85	—877	25
90	—593	28	40	982	27	90	328	27	40	634	26	90	—902	25
95	—621	28	45	90009	28	95	--355	26	45	—660	26	95	—927	25
7700	649	28	**7950**	—037	27	**8200**	381	27	**8450**	—686	25	**8700**	—952	25
05	677	28	55	064	27	05	—408	26	55	711	26	05	—977	25
10	705	29	60	091	28	10	434	27	60	737	26	10	94002—	25
15	—734	28	65	—119	27	15	—461	26	65	—763	25	15	—027	25
20	—762	28	70	—146	27	20	487	27	70	788	26	20	—052	25
25	—790	28	75	173	27	25	—514	26	75	—814	26	25	—077	24
30	—818	28	80	200	27	30	—540	26	80	—840	25	30	101	25
35	846	28	85	227	28	35	566	27	85	865	26	35	126	25
40	874	28	90	—255	27	40	—593	26	90	—891	25	40	151	25
45	902	28	95	—282	27	45	619	26	95	916	26	45	—176	25

To find Log. 83678:
Log. 83700 = 92273
Log. 83650 = 92247
Dif. 50 **26**
83678 — 83650 = **28**
Under **26**
Dif. for 10 × 2 = 10.0
" " 8 = 4.2
" " **28** = 14.2
Log. 83678 =
92247 + 14 = 92261.

	29	**28**	**27**	**26**	**25**	**24**	
1	0.6	0.6	0.5	0.5	0.5	0.5	**1**
2	1.2	1.1	1.1	1.0	1.0	1.0	**2**
3	1.7	1.7	1.6	1.6	1.5	1.4	**3**
4	2.3	2.2	2.2	2.1	2.0	1.9	**4**
5	2.9	2.8	2.7	2.6	2.5	2.4	**5**
6	3.5	3.4	3.2	3.1	3.0	2.9	**6**
7	4.1	3.9	3.8	3.6	3.5	3.4	**7**
8	4.6	4.5	4.3	4.2	4.0	3.8	**8**
9	5.2	5.0	4.9	4.7	4.5	4.3	**9**
10	5.8	5.6	5.4	5.2	5.0	4.8	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 8325 = 9203842
" 8330 = 9206450

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
8750	94201	25	**9000**	95424	24	**9250**	96614	24	**9500**	97772	23	**9750**	98900	23
55	—226	24	05	448	24	55	—638	23	05	795	23	55	—923	22
60	250	25	10	472	25	60	661	24	10	818	23	60	—945	22
65	275	25	15	—497	24	65	—685	23	15	—841	23	65	967	22
70	—300	25	20	—521	24	70	—708	23	20	—864	22	70	989	23
75	—325	24	25	—545	24	75	731	24	25	886	23	75	99012	22
80	349	25	30	—569	24	80	—755	23	30	909	23	80	—034	22
85	374	25	35	—593	24	85	778	24	35	932	23	85	056	22
90	—399	25	40	—617	24	90	—802	23	40	—955	23	90	078	22
95	—424	24	45	—641	24	95	—825	23	45	—978	22	95	100	23
8800	448	25	**9050**	—665	24	**9300**	848	24	**9550**	98000	23	**9800**	—123	22
05	—473	25	55	—689	24	05	—872	23	55	023	23	05	—145	22
10	—498	24	60	—713	24	10	—895	23	60	—046	22	10	—167	22
15	522	25	65	—737	24	15	918	24	65	068	23	15	189	22
20	—547	24	70	—761	24	20	—942	23	70	091	23	20	211	22
25	571	25	75	—785	24	25	—965	23	75	—114	23	25	233	22
30	596	25	80	—809	23	30	988	23	80	—137	22	30	255	22
35	—621	24	85	832	24	35	97011	24	85	159	23	35	277	23
40	645	25	90	856	24	40	—035	23	90	—182	22	40	—300	22
45	—670	24	95	880	24	45	—058	23	95	204	23	45	—322	22
8850	694	25	**9100**	904	24	**9350**	081	23	**9600**	227	23	**9850**	—344	22
55	—719	24	05	—928	24	55	104	24	05	—250	22	55	—366	22
60	743	25	10	—952	24	60	—128	23	10	272	23	60	—388	22
65	—768	24	15	—976	23	65	—151	23	15	—295	23	65	—410	22
70	792	25	20	999	24	70	—174	23	20	—318	22	70	—432	22
75	—817	24	25	96023	24	75	197	23	25	340	23	75	—454	22
80	841	25	30	047	24	80	220	23	30	—363	22	80	—476	22
85	—866	24	35	—071	24	85	243	24	35	385	23	85	—498	22
90	890	25	40	—095	23	90	—267	23	40	—408	22	90	—520	22
95	—915	24	45	118	24	95	—290	23	45	430	23	95	—542	22
8900	939	24	**9150**	142	24	**9400**	—313	23	**9650**	—453	22	**9900**	—564	21
05	963	25	55	—166	24	05	—336	23	55	475	23	05	585	22
10	—988	24	60	—190	23	10	—359	23	60	—498	22	10	607	22
15	95012	24	65	213	24	15	382	23	65	520	23	15	629	22
20	036	25	70	—237	24	20	405	23	70	—543	22	20	651	22
25	—061	24	75	—261	23	25	428	23	75	565	23	25	673	22
30	085	24	80	284	21	30	451	23	80	—588	22	30	—695	22
35	109	25	85	—308	24	35	474	23	85	—610	22	35	—717	22
40	—134	24	90	—332	23	40	497	23	90	632	23	40	—739	21
45	158	24	95	355	24	45	520	23	95	—655	22	45	760	22
8950	182	25	**9200**	—379	23	**9450**	543	23	**9700**	677	23	**9950**	782	22
55	—207	24	05	402	24	55	566	23	05	—700	22	55	804	22
60	—231	24	10	—426	24	60	589	23	10	—722	22	60	—826	22
65	255	24	15	—450	23	65	612	23	15	744	23	65	—848	22
70	279	24	20	473	24	70	—635	23	20	—767	22	70	—870	21
75	303	25	25	—497	23	75	—658	23	25	—789	22	75	891	22
80	—328	24	30	520	24	80	—681	23	30	811	23	80	913	22
85	—352	24	35	—544	23	85	—704	23	35	—834	22	85	—935	22
90	—376	24	40	567	24	90	—727	22	40	—856	22	90	—957	22
95	400	24	45	—591	23	95	749	23	45	878	22	95	978	22

To find Log. 95544:
Log. 95550 = 98023
Log. 95500 = 98000
Dif. 50 **23**
95544 - 95500 = **44**
Under **23**
Dif. for 10 × 4 = 18.0
" " 4 = 1.8
" " **44** = 19.8
Log. 95544 =
98000 + 20 = 98020.

	25	24	23	22	21	
1	0.5	0.5	0.5	0.4	0.4	**1**
2	1.0	1.0	0.9	0.9	0.8	**2**
3	1.5	1.4	1.4	1.3	1.3	**3**
4	2.0	1.9	1.8	1.8	1.7	**4**
5	2.5	2.4	2.3	2.2	2.1	**5**
6	3.0	2.9	2.8	2.6	2.5	**6**
7	3.5	3.4	3.2	3.1	2.9	**7**
8	4.0	3.8	3.7	3.5	3.4	**8**
9	4.5	4.3	4.1	4.0	3.8	**9**
10	5.0	4.8	4.6	4.4	4.2	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 9600 = 9822712
" 9605 = 9824974

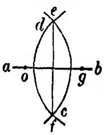
GEOMETRY.

Lines, Figures, Solids, defined. Strictly speaking a geometrical **line** is simply length, or distance. The lines we draw on paper have not only length, but breadth and thickness, still they are the most convenient symbol we can employ for denoting a geometrical line. **Straight** lines are also called **right** lines. A **vertical** line is one that points toward the center of the earth; and a **horizontal** one is at right angles to a vert one. A **plane figure** is merely any flat surface or area entirely enclosed by lines either straight or curved; which are called its outline, boundary, circumf, or periphery. We often confound the outline with the fig itself as when we speak of drawing circles, squares, &c, for we actually draw only their outlines. Geometrically speaking, a fig has length and breadth only; no thickness. **A solid** is any body; it has length, breadth, and thickness.

Geometrically similar figs or solids, are not necessarily of the same **size**; but only of precisely the same **shape**. Thus, any two squares are, scientifically speaking, similar to each other; so also any two circles, cubes, &c, no matter how different they may be in size. When they are not only of the same shape, but of the same size, they are said to be **similar**, and **equal.**

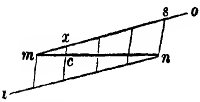
The **quantities of lines** are to each other simply as their **lengths**; but the quantities, or areas, or surfaces of similar **figures**, are as, or in proportion to, the **squares** of any one of the corresponding lines or sides which enclose the figures, or which may be drawn upon them; and the quantities, or solidities of similar **solids**, are as the **cubes** of any of the corresponding lines which form their edges, or the figures by which they are enclosed.

Rem.—Simple as the following operations appear, it is only by care, and good instruments, that they are made to give accurate results. Several of them can be much better performed by means of a metallic triangle having one perfectly accurate right angle. In the field, the tape-line, chain, or a measuring rod will take the place of the dividers and ruler used indoors.



To divide a given line, *a b*, into two equal parts.

From its ends *a* and *b* as centers, and with any rad greater than one-half of *a b*, describe the arcs *c* and *d*, and join *e f*. If the line *a b* is very long, first lay off equal dists *a o* and *b g*, each way from the ends, so as to approach conveniently near to each other, and then proceed as if *o g* were the line to be divided. Or measure *a b* by a scale, and thus ascertain its center.



To divide a given line, *m n*, into any given number of equal parts.

From *m* and *n* draw any two parallel lines *m o* and *n a*, to an indefinite dist, and on them, from *m* and *n* step off the reqd number of equal parts of any convenient length; finally, join the corresponding points thus stepped off. Or only one line, as *m o* may be drawn and stepped off, as to *s*; then join *s n*, and draw the other short lines parallel to it.

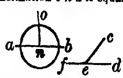
To divide a given line, *m n*, into two parts which shall have a given proportion to each other.

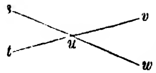
This is done on the same principle as the last; thus, let the proportion be as 1 to 3. First draw any line *m o*; and with any convenient opening of the dividers make *m x* equal to one step; and *x s* equal to three steps. Join *s n*: and parallel to it draw *x c*. Then *m c* is to *c n* as 1 is to 3.

ANGLES.

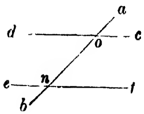
Angles. When two straight, or right lines meet each other at any inclination, the inclination is called an **angle**; and is measured by the degrees contained in the arc of a circle described from the point of meeting as a center. Since all circles, whether large or small, are supposed to be divided into 360 degrees, it follows that any number of degrees of a small circle will measure the same degree of inclination as will the same number of a large one.

When two straight lines, as *o n* and *a b*, meet in such a manner that the inclination *o n a* is equal to the inclination *o n b*, then the two lines are said to be **perpendicular** to each other; and the angles *o n a* and *o n b*, are called **right angles**; and are each measd by, or are equal to, 90°, or one-fourth part of the circumf of a circle. Any angle, as *c e d*, smaller than a right angle, is called **acute** or sharp; and one *c e f*, larger than a right angle, is called **obtuse**, or blunt. When one line meets another, as in the first Fig on opposite page, the two angles on the same side of either line are called **contiguous**, or **adjacent.** Thus, *v u s* and *s u w* are adjacent; also *t u s* and *t u w*, *s u t* and *s u v*, *w u t* and *w u v*. The sum of two adjacent angles is always equal to two right angles; or to 180°. Therefore, if we know the number of degrees contained in one of them, and subtract it from 180°, we obtain the other



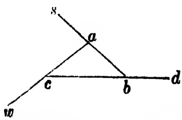


When two straight lines *cross* each other, forming four angles, either pair of those angles which point in exactly opposite directions are called **opposite**, or **vertical angles**: thus, the pair *s u t* and *v u w* are opposite angles; also the pair *s u v* and *t u w*. The opposite angles of any pair are always equal to each other.



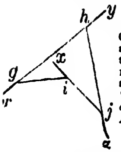
When a straight line *a b* crosses two parallel lines *c d*, *e f*, the **alternate** angles which form a kind of Z are equal to each other. Thus, the angles *d o n* and *o n f* are equal as are also *c o n* and *o n e*. Also the sum of the two internal angles on the same side of *a b*, is equal to two right angles, or 180°; thus, *c o n* + *o n f* = 180°; also *d o n* + *o n e* = 180°.

An interior angle,



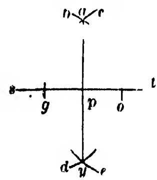
In any fig. is any angle formed *inside* of that fig, by the meeting of two of its sides, as the angles *c a b*, *a b c*, *b c a*, of this triangle All the interior angles of any straight-lined figure of any number of sides whatever, are together equal to twice as many right angles minus four, as the figure has sides Thus, a triangle has 3 sides, twice that number is 6; and 6 right angles, or 6 × 90° = 540°, from which take 4 right angles, or 360°; and there remain 180°, which is the number of degrees in every plane, or straight lined triangle. This principle furnishes an easy means of testing our measurements of the angles of any fig. for if the sum of all our measurements does not agree with the sum given by the rule, it is a proof that we have committed some error.

An exterior angle



Of any straight-lined figure, is any angle, as *a b d*, formed by the meeting of any side, as *a b*, with the prolongation of an adjacent side, as *c b*; so likewise the angles *c a s* and *b c w*. All the exterior angles of any straight-lined fig, no matter how many sides it may have, amount to 360°; but, in the case of a *re-entering* angle, as *g i j*, the interior angle, *g i j*, exceeds 180°, and the "exterior" angle, *g i x*, being = 180° − interior angle, is *negative*. Thus *a b d* + *b c w* + *c a s* = 360°; and *y h j* + *z j i* − *g i x* + *i g w* = 360°. Angles, as *a*, *b*, *c*, *g*, *h*, and *j*, which point *outward*, are called **salient.**

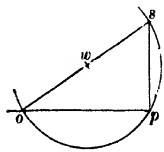
From any given point, *p*, on a line *s t*, to draw a perp, *p a*.



From *p*, with any convenient opening of the dividers, step off the equals *p o*, *p g* From *o* and *g* as centers, with any opening greater than half *o g*, describe the two short arcs *b* and *c*; and join *a p*. Or still better, describe four arcs, and join *a y*.

Or from *p* with any convenient scale describe two short arcs through *g* and *a*, either one of them with a radius 3, and the other with a rad 4. Then from *g* with rad 5 describe the arc *b*. Join *p a*

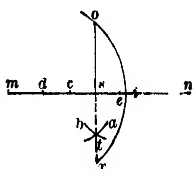
If the point *p* is at one end of the line, or very near it,



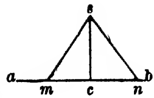
Extend the line, if possible, and proceed as above. But if this cannot be done, then from any convenient point, *w*, open the dividers to *p*, and describe the semicircle, *s p o*, through *o w* draw *o w s*, join *p s*.

Or use the last foregoing process with rads 3, 4, and 5.

From a given point, *o*, to let fall a perp *o s*, to a given line, *m n*.

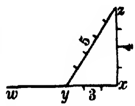


From *o*, measure to the line *m n*, any two equal dists, *o c*, *o e*; and from *c* and *e* as centers, with any opening greater than half of *c e*, describe the two arcs *a* and *b*; join *o t*. Or from any point, as *d* on the line, open the dividers to *o*, and describe the arc *o g*; make *i x* equal to *i o*; and join *o x*.



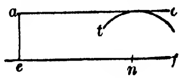
If the line, *a b*, is on the ground,

And a perp is reqd to be drawn from *c*, first measure off any two equal dists, *c m*, *c n*. At *m* and *n*, hold the ends of a piece of string, tape line, or chain, *m s n*; then tighten out the string, &c, as shown by *m s n*; *s* being its center. Then will *s c* be the reqd perp. Or if the perp *x z* is to be drawn from the *end* of the line *w x*, first measure *x y* upon the line, and equal to three feet, then holding the end of a tape-line at *x*, and its nine feet mark at *y*, hold the four feet mark at *z*, keeping *x z* and *z y* equally stretched. Then *x z* will be the reqd perp, because 3, 4, and 5, make the sides of a right-angled triangle Instead of 3, 4, and 5, any multiples of those numbers may be used, such as 6, 8, and 10; or 9, 12, 15, &c: also instead of feet, we may use yards, chains, &c.



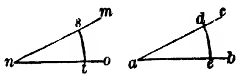
Through a given point, *a*, to draw a line, *a c*, parallel to another line, *e f*.

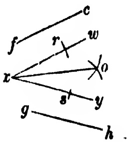
With the perp dist, *a e*, from any point, *n*, in *e f*, describe an arc, *t*; draw *a c* just touching the arc.



At any point, *a*, in a line *a b*, to make an angle *c a b*, equal to a given angle, *m n o*.

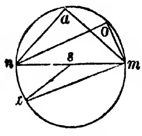
From *n* and *a*, with any convenient rad, describe the arcs *s t*, *d e*; measure *s t*, and make *e d* equal to it; through *a d* draw *a c*.



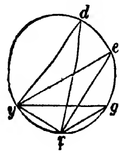


To bisect, or divide any angle, *w x y*, into two equal parts.

From *x* set off any two equal dists, *x r*, *x s*. From *r* and *s* with any rad describe two arcs intersecting, as at *o*; and join *o x*. If the two sides of the angle do not meet, as *c f* and *g h*, either first extend them until they do meet; or else draw lines *x w*, and *x y*, parallel to them, and at equal dists from them, so as to meet; then proceed as before.



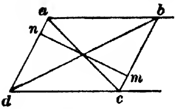
All angles, as *n a m*, *n o m*, at the circumf of a semicircle, and standing on its diam *n m*, are right angles; or, as it is usually expressed, **all angles in a semicircle are right angles.** An angle *n s x* at the center of a circle, is twice as great as an angle *n m x* at the circumf, when both stand upon the same arc *n x*.



All angles, as *y d p*, *y e p*, *y g p*, at the circumf of a circle, and standing upon the same arc, as *y p*, are equal to each other; or, as usually expressed, **all angles in the same segment of a circle are equal.**

The **complement** of an angle is what it lacks of 90°. Thus, the complement of 80° is 90° — 80° = 10°; and that of 210° is 90° — 210° = — 120°. The **supplement** of an angle is what it lacks of 180°. Thus, the supplement of 80° is 180° — 80° = 100°; and that of 210° is 180° — 210° = — 30°. But ordinarily we may neglect the signs + and —, before complements and supplements, and call the complement of an angle its *diff* from 90°; and the supplement its *diff* from 180°.

Angles in a Parallelogram.



A parallelogram is any four-sided straight-lined figure whose opposite sides are equal, as *a b c d*; or a square, &c. Any line drawn across a parallelogram between 2 opposite angles, is called a *diagonal*, as *a c*, or *b d*. A diag divides a parallelogram into two equal parts; as does also any line *m n* drawn through the center of either diag; and moreover, the line *m n* itself is div into two equal parts by the diag. Two diags bisect each other; they also divide the parallelogram into four triangles of equal areas. The sum of the two angles at the ends of any one side is = 180°; thus, *d a b* + *a b c* = *a b c* + *b c d* = 180°, and the sum of the four angles, *d a b, a b c, b c d, c d a* = 360°.

The sum of the squares of the four sides is equal to the sum of the squares of the two diags.

To reduce Minutes and Seconds to Degrees and decimals of a Degree, etc.

In any given angle—

Number of degrees = Number of minutes ÷ 60.
= Number of seconds ÷ 3600.

Number of minutes = Number of degrees × 60.
= Number of seconds ÷ 60.

Number of seconds = Number of degrees × 3600.
= Number of minutes × 60.

Table of Minutes and Seconds in Decimals of a Degree, and of Seconds in Decimals of a Minute.

(The* columns of Mins and Degs answer equally for Secs and Mins.)

Mins.	Deg.	Mins.	Deg.	Mins.	Deg.	Secs.	Deg.	Secs.	Deg.	Secs.	Deg.
In each equivalent, **the last digit repeats indefinitely.** See * below											
1	0.016	21	0.350	41	0.683	1	0.00027	21	0.00583	41	0.01138
2	0.033	22	0.366	42	0.700	2	0.00055	22	0.00611	42	0.01166
3	0.050	23	0.383	43	0.716	3	0.00083	23	0.00638	43	0.01194
4	0.066	24	0.400	44	0.733	4	0.00111	24	0.00666	44	0.01222
5	0.083	25	0.416	45	0.750	5	0.00138	25	0.00694	45	0.01250
6	0.100	26	0.433	46	0.766	6	0.00166	26	0.00722	46	0.01277
7	0.116	27	0.450	47	0.783	7	0.00194	27	0.00750	47	0.01305
8	0.133	28	0.466	48	0.800	8	0.00222	28	0.00777	48	0.01333
9	0.150	29	0.483	49	0.816	9	0.00250	29	0.00805	49	0.01361
10	0.166	30	0.500	50	0.833	10	0.00277	30	0.00833	50	0.01388
11	0.183	31	0.516	51	0.850	11	0.00305	31	0.00861	51	0.01416
12	0.200	32	0.533	52	0.866	12	0.00333	32	0.00888	52	0.01444
13	0.216	33	0.550	53	0.883	13	0.00361	33	0.00916	53	0.01472
14	0.233	34	0.566	54	0.900	14	0.00388	34	0.00944	54	0.01500
15	0.250	35	0.583	55	0.916	15	0.00416	35	0.00972	55	0.01527
16	0.266	36	0.600	56	0.933	16	0.00444	36	0.01000	56	0.01555
17	0.283	37	0.616	57	0.950	17	0.00472	37	0.01027	57	0.01583
18	0.300	38	0.633	58	0.966	18	0.00500	38	0.01055	58	0.01611
19	0.316	39	0.650	59	0.983	19	0.00527	39	0.01083	59	0.01638
20	0.333	40	0.666	60	1.000	20	0.00555	40	0.01111	60	0.01666
Secs.	Min.	Secs.	Min.	Secs.	Min.	Secs.	Deg.	Secs.	Deg.	Secs.	Deg.

* Each equivalent is a **repeating decimal,** thus:

2 minutes = 0.0333333 . . . . degree
7 " = 0.1166666 . . . "
12 " = 0.2000000 . . . . "

12 seconds = 0.2000000 . . . . minute
1 second = 0.0002777 . . . . degree
50 seconds = 0.0138888 . . . . "

Approximate Measurement of Angles.

(1) **The four fingers of the hand,** held at right angles to the arm and at arm's length from the eye, cover about 7 degrees And an angle of 7° corresponds to about 12.2 feet in 100 feet; or to 36 6 feet in 100 yards; or to 645 feet in a mile.

(2) **By means of a two-foot rule,** either on a drawing or between distant objects in the field. If the inner edges of a common two-foot rule be opened to the extent shown in the column of inches, they will be inclined to each other at the angles shown in the column of angles Since an opening of ⅛ inch (up to 19 inches or about 105°) corresponds to from about ½° to 1°, no great accuracy is to be expected, and beyond 105° still less, for the liability to error then increases very rapidly as the opening becomes greater Thus, the last ⅛ inch corresponds to about 12°.

Angles for openings intermediate of those given may be calculated to the nearest minute or two, by simple proportion, up to 23 inches of opening, or about 147°.

Table of Angles corresponding to openings of a 2-foot rule.

(Original). Correct.

Ins.	Deg min.	Ins.	Deg. min.	Ins.	Deg min.	Ins.	Deg. min.	Ins.	Deg. min.	Ins.	Deg min.
¼	1 12	4¼	20 24	8¼	40 13	12¼	61 23	16¼	85 14	20¼	115 5
	1 48		21		40 51		62 5		86 3		116 12
½	2 24	½	21 37	½	41 29	½	62 47	½	86 52	½	117 20
	3 00		22 13		42 7		63 28		87 41		118 30
¾	3 36	¾	22 50	¾	42 46	¾	64 11	¾	88 31	¾	119 40
	4 11		23 27		43 24		64 53		89 21		120 52
1	4 47	5	24 8	9	44 3	13	65 35	17	90 12	21	122 6
	5 23		24 39		44 42		66 18		91 3		123 20
¼	5 58	¼	25 16	¼	45 21	¼	67 1	¼	91 54	¼	124 36
	6 34		25 53		45 59		67 44		92 46		125 54
½	7 10	½	26 30	½	46 38	½	68 28	½	93 38	½	127 14
	7 46		27 7		47 17		69 12		94 31		128 35
¾	8 22	¾	27 44	¾	47 56	¾	69 55	¾	95 24	¾	129 59
	8 58		28 21		48 35		70 38		96 17		131 25
2	9 34	6	28 58	10	49 15	14	71 22	18	97 11	22	132 53
	10 10		29 35		49 54		72 6		98 5		134 24
¼	10 46	¼	30 11	¼	50 34	¼	72 51	¼	99 00	¼	135 58
	11 22		30 49		51 13		73 36		99 55		137 35
½	11 58	½	31 26	½	51 53	½	74 21	½	100 51	½	139 16
	12 34		32 3		52 33		75 6		101 48		141 1
¾	13 10	¾	32 40	¾	53 13	¾	75 51	¾	102 45	¾	142 51
	13 46		33 17		53 53		76 36		103 43		144 46
3	14 22	7	33 54	11	54 34	15	77 22	19	104 41	23	146 48
	14 58		34 33		55 14		78 8		105 40		148 58
¼	15 34	¼	35 10	¼	55 55	¼	78 54	¼	106 39	¼	151 17
	16 10		35 47		56 35		79 40		107 40		153 48
½	16 46	½	36 25	½	57 16	½	80 27	½	108 41	½	156 31
	17 22		37 3		57 57		81 14		109 43		159 48
¾	17 59	¾	37 41	¾	58 38	¾	82 2	¾	110 46	¾	163 27
	18 35		38 19		59 19		82 49		111 49		168 18
4	19 12	8	38 57	12	60 00	16	83 37	20	112 53	24	180 00
	19 48		39 35		60 41		84 26		113 58		

(3) **With the same table, using feet instead of inches.** From the given point measure 12 *feet* toward * each object, and place marks. Measure the distance in *feet* between these marks. Suppose the first column in the table to be *feet* instead of inches. Then opposite the distance in *feet* will be the angle.

⅛ foot = 1.5 inches.

1 in. = .083 ft.	4 ins. = .333 ft.	7 ins. = .583 ft.	10 ins = .833 ft.
2 ins. = .167 ft.	5 ins. = .416 ft.	8 ins. = .667 ft.	11 ins = .917 ft.
3 ins. = .25 ft.	6 ins. = .5 ft.	9 ins. = .75 ft.	12 ins. = 1 0 ft.

(4) **Or, measure toward * each object** 100 or any other number of feet, and place marks. Measure the distance in feet between the marks. Then

$$\text{Sine of } \textit{half} \text{ the angle} = \frac{\textit{half} \text{ the distance between the marks}}{\text{the distance measured toward one of the objects}}.$$

Find this sine in the table pp. 98, etc.; take out the corresponding angle and multiply it by 2

(5) **See** last paragraph of foot-note, pp 152 and 153.

* If it is inconvenient to measure *toward* the objects, measure directly *from* them.

PLANE TRIGONOMETRY.

For illustrations, see p 97 *d*.

1. Usual measure of angles. If a circle be divided into 360 equal arcs, each of these arcs, or the angle at center, subtended by it, is called a **degree** (°). Each degree is subdivided into 60 **minutes** (′) and each minute into 60 **seconds** (″). **A right angle** is an angle of 90°.

2. Circular measure of angles. The arc whose length is equal to the radius, or the angle at center subtended by such arc, is called a **radian.** Since the semi-circumf (see p 161) is = π × radius = π radians = 180°, we have

$$\textbf{1 radian} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3.14159+} = 57.2957795\ldots^\circ. \quad \text{Log} = 1.758\ 1226.$$

$$\left.\begin{array}{l} 1 \text{ degree} = 0.017\ 453\ 292\ 520 \text{ radian;} \\ 1 \text{ minute} = 0.000\ 290\ 888\ 209 \text{ radian;} \\ 1 \text{ second} = 0.000\ 004\ 848\ 137 \text{ radian;} \end{array}\right\} \text{see table, p 185.}$$

3. Ratio of arc to radius. In any angle, the length of the arc, in terms of the radius, may be called **the arc** of the angle. Thus, in Figs **1*** and 3*, arc $A = ZB \,/\, OZ$. For other angular ratios, see ¶¶ 6 and 10.

4. Positive and negative angles. In Fig 1*, suppose a radius, as OZ, to sweep around the whole or part of the circle to the *left* or *counter-clockwise*, as indicated by the arrow. Angles or arcs, as ZB, BC, ZBC, CDE, $ZBCDE$, *thus* described, are considered **positive,** e g, arc $ZBCDE$ = +280°; while angles described by a radius travelling in the *opposite* or *clockwise* direction, as ZE, EDC, $ZEDCB$, are considered **negative,** e g, arc ZE = −80°.

5. Complement, Supplement, etc. For any angle, A, we have:
complement $A = 90° - A$; **supplement** $A = 180° - A$;
explement $A = 360° - A$. See ¶ 7.

6. Angular functions, or **angular ratios,** are the ratios between the sides of a right-triangle, Fig 2*. The principal ratios of an angle, A, are the **sine** (sin A), **cosine** (cos A), **tangent†** (tan A), **cosecant** (csc A,) **secant** (sec A), and **cotangent** (cot A or ctn A). For other ratios, see ¶¶ 3 and 10.

In Fig 2*

$$\textbf{Sin A} = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}; \quad \textbf{csc A} = \frac{1}{\sin A} = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}};$$

$$\textbf{Cos A} = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}; \quad \textbf{sec A} = \frac{1}{\cos A} = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}};$$

$$\textbf{Tan A} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}; \quad \textbf{cot A} = \frac{1}{\tan A} = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}}$$

7. If we represent the denominator, in each of the ratios of ¶ 6, by a line of unit length, then the **length of the line** representing the numerator gives the value of the ratio. Thus, in Fig 3*, let radius OB = 1, angle ZOU = 90°, ZOB = any angle, A, and let MB and ZB' be perpendicular to OZ. Then:

$$\textbf{sin A} = MB; \quad \textbf{cos A} = OM = U'B; \quad \textbf{tan A} = ZB';$$
$$\textbf{csc A} = OB''; \quad \textbf{sec A} = OB'; \quad \textbf{cot A} = UB''.$$

The sine (etc), thus expressed, was formerly called the **natural sine** (etc), or sine (etc) **to radius 1.**

Note that

$$\begin{array}{l} \cos A = \sin(90^\circ - A) = \sin(\text{complement } A) \\ \cot A = \tan(90^\circ - A) = \tan(\text{complement } A) \\ \csc A = \sec(90^\circ - A) = \sec(\text{complement } A). \end{array}$$

See ¶ 13.

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* **For illustrations,** see p 97 *d*.

† This use of the word "tangent" was suggested by its use in geometry where it denotes any line touching a curve without intersecting it. In the location of railroad curves, the name "tangent" is often given to the "apex distance," which is the trigonometrical tangent of half the curve.

8. Positive and negative signs. Fig 4*. Suppose the circle divided into four quadrants, I, II, III and IV, beginning on the right of and above the center, and progressing to the left, or counter-clockwise. Then, vertical or horizontal lines, measured upward from, or to the right of, the horizontal and vertical diameters, respectively, are considered positive; while such lines measured downward from, or to the left of, the same diams, are considered negative; but the radius, in whatever quadrant it may lie, is always considered pos when measured from the center outward. Hence,

Sine (and cosecant) *positive* in *upper* quadrants;
Cosine (and secant) " " *right-hand* "
Tangent (and cotangent) " " *first and third* " or:

In quadrant	**I**	**II**	**III**	**IV**
Including angles from	0° to 90°	90° to 180°	180° to 270°	270° to 360°
Sine and cosecant	+	+	—	—
Tangent and Cotangent	+	—	+	—
Secant and Cosine	+	—	—	+

9. Numerical values of functions of certain angles.

	0°	30°	45°	60°	90°	120°	180°	270°	360°
Sine	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	0	-1	0
Tangent	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0
Secant	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	∞	-2	-1	∞	1
Cosine	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	0	1
Sine^2	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	0	1	0
Tangent^2	0	$\frac{1}{3}$	1	3	∞	3	0	∞	0
Secant^2	1	$\frac{4}{3}$	2	4	∞	4	1	∞	1
Cosine^2	1	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	1	0	1

For equations between angular functions, see ¶¶ 14 to 19.

10. Other Functions. See also ¶¶ 3 and 6.

In Fig 3*, BN = chord 2 A = 2 sin A ; and **chord A** = 2 sin (A/2).*

The versed sine, MZ, Fig 3, of an angle, A, is vers A = 1 — cos A. It is much used in connection with the location of railroad curves.

The coversed sine (covers), $U'U$, = 1 — sin A, Fig 3, of an angle, A, is the versed sine of the complement, BOU, of A.

By their definitions, vers A (= 1 — cos A) and covers A (= 1 — sin A) are always pos. In the 4 quadrants, Fig 4*, their values change as follows:

Quadrant	**I**	**II**	**III**	**IV**
Including angles from	0° to 90°	90° to 180°	180° to 270°	270° to 360°
Versed sine	0 to 1	1 to 2	2 to 1	1 to 0
Coversed sine	1 to 0	0 to 1	1 to 2	2 to 1

The external secant (exsec) $B B'$, Fig 3, of an angle, A, = sec A — 1. Like the versed sine, it is used in the location of railroad curves.

* **For illustrations**, see p 97 *d*.

Table of sines, tans, cotans, cosines.

For logarithmic sines, etc., see pp 143 *a*, etc.

11. Table of Natural Angular Ratios. The table, pp 98–142, contains the natural sines, tangents, cotangents and cosines of angles from 0° to 90°, progressing by single minutes. Functions for intermediate angles can be found, in nearly all cases, with sufficient approximation, by simple proportion. **For functions not given in the table,** we have:

Secant $A = 1/\cos A$; Versed sine $A = 1 - \cos A$;
Cosecant $A = 1/\sin A$; Coversed sine $A = 1 - \sin A$;
Chord $A = 2 \sin (A/2)$; External secant $A = \sec A - 1 = (1/\cos A) - 1$.

12. Angle as function of ratio. The arc (or angle) whose sin, tan, etc, is f, is called, by continental writers, arc sin f, arc tan f, etc, or anti-sin f, anti-tan f, etc, and, by English writers, $\sin^{-1} f$, $\tan^{-1} f$, etc.

Thus, let A = the angle; $f' = \sin A$; $f'' = \cos A$. Then

$$A = \text{arc}\sin f' = \text{anti-sin} f' = \sin^{-1} f'$$
$$= \text{arc}\cos f'' = \text{anti-cos} f'' = \cos^{-1} f''.$$

Example. Let $A = 30°$ Then $f' = \sin 30° = 0.5$; $f'' = \cos 30° = 0.866\ldots$, and

$$30° = \text{arc}\sin 0.5 = \text{anti-sin}\ 0.5 = \sin^{-1} 0.5$$
$$= \text{arc}\cos 0.866\ldots = \text{anti-cos}\ 0.866\ldots = \cos^{-1} 0.866\ldots$$

Functions of Supplements and Complements.

13. Putting A = any angle, and F = "any trig function of," we have the following convenient rules, see Fig 5: (For vers and covers, see p 97*a*, ¶ 10.)

$F\,[\pm A \pm$ (an *even* multiple of 90°)] numerically = *same* function of A;
$F\,[\pm A \pm$ (an *odd* multiple of 90°)] numerically = *co-named* function of A;
the sign being determined, in each case, by considering the quadrants in which the angles lie. (See ¶ 8.) Thus, numerically.

$$\begin{aligned}
\sin A &= \sin (180° + A) = \sin (180° - A) = \sin (A - 180°) \\
&= \cos (\ 90° + A) = \cos (\ 90° - A) = \cos (A - \ 90°); \\
\cos A &= \cos (180° + A) = \cos (180° - A) = \cos (A - 180°) \\
&= \sin (\ 90° + A) = \sin (\ 90° - A) = \sin (A - \ 90°); \\
\tan A &= \tan (180° + A) = \tan (180° - A) = \tan (A - 180°) \\
&= \cot (\ 90° + A) = \cot (\ 90° - A) = \cot (A - \ 90°).
\end{aligned}$$

Properties of the trigonometric functions, Figs 6, 7, 8.

14. For any angle, A, we have:

$$\tan A = \frac{\sin A}{\cos A}; \qquad \sin^2 A + \cos^2 A = 1;$$

$$1 + \tan^2 A = \sec^2 A; \qquad 1 + \cot^2 A = \csc^2 A;$$

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}; \quad \sec A = \frac{1}{\cos A}; \quad \csc A = \frac{1}{\sin A} = \frac{\sec A}{\tan A}.$$

From Fig 4, $\sin(-A) = -\sin A$; $\tan(-A) = -\tan A$; and $\cos(-A) = \cos A$.

15. For any two angles, A and B, it may be shown that

$$\sin (A + B) = \sin A \,.\, \cos B + \cos A \,.\, \sin B;$$
$$\text{and } \cos (A + B) = \cos A \,.\, \cos B - \sin A \,.\, \sin B;$$

whence, dividing by $\dfrac{\cos A \ \cos B}{\cos A \,.\, \cos B}$, we obtain

$$\tan (A + B) = \frac{\sin (A + B)}{\cos (A + B)} = \frac{\tan A + \tan P}{1 - \tan A \tan B}.$$

16. Putting $A = B$, in the formulas for $(A + B)$, we have

$$\mathbf{\sin 2A} = 2 \sin A \,.\, \cos A;$$
$$\mathbf{\cos 2A} = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1;$$
$$\mathbf{\tan 2A} = \frac{2 \tan A}{1 - \tan^2 A}.$$

and, changing B to $-B$, we have

$$\mathbf{\sin (A - B)} = \sin A \,.\, \cos B - \cos A \,.\, \sin B;$$
$$\mathbf{\cos (A - B)} = \cos A \,.\, \cos B + \sin A \,.\, \sin B;$$
$$\mathbf{\tan (A - B)} = \frac{\tan A - \tan B}{1 + \tan A \,.\, \tan B}.$$

17. Since $\cos 2A = 1 - 2\sin^2 A$, we have: (putting $A/2$ for A) $\cos A = 1 - 2\sin^2\frac{A}{2}$; or $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$; and, since $\cos 2A = 2\cos^2 A - 1$, we have $2\cos^2\frac{A}{2} = \cos A + 1$, or $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

and hence $\tan\frac{A}{2} = \frac{\sin A/2}{\cos A/2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$.

18. Formulas transforming a sum or difference into a product. Combining the formulas for $\sin(A \pm B)$ and $\cos(A \pm B)$, we have, finally:

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cdot\cos\frac{A+B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$$

See also ¶ 20, for formulas used in solution of triangles.

19. Any function of an angle may be expressed **in terms of any other.** Thus, Figs 6, 7, 8:

in terms of the **sine:**

$$\cos^2 A = 1 - \sin^2 A; \quad \text{hence, } \cos A = \sqrt{1 - \sin^2 A};$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}};$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}};$$

in terms of the **tangent:**

$$\sec^2 A = 1 + \tan^2 A; \quad \text{hence, } \sec A = \sqrt{1+\tan^2 A};$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{1+\tan^2 A}};$$

$$\sin A = \frac{\tan A}{\sec A} = \frac{\tan A}{\sqrt{1+\tan^2 A}}.$$

20. Formulas used in the solution of triangles. Fig 9. Let a, b, c be the sides of any plane triangle, and A, B, C the angles opposite a, b, c respectively. Then

$$\frac{\sin A}{\sin B} = \frac{a}{b};$$

See also pp 148 to 156.
See Case 2, p 150.

$$a^2 = b^2 + c^2 - 2bc\cos A;$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cdot\tan\frac{A+B}{2};$$

$$\tan\frac{A}{2} = \frac{r}{s-a}; \text{ where } s = \frac{a+b+c}{2};$$

$$\text{and } r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

= radius of inscribed circle.

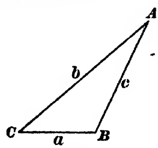

Fig. 9.

$$\text{Area of the triangle} = \frac{ab\sin C}{2} = rs = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{\sin A \sin C\, b^2}{2\sin B}.$$

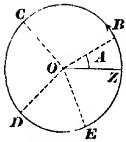

Fig. 1.

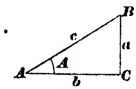

Fig. 2.

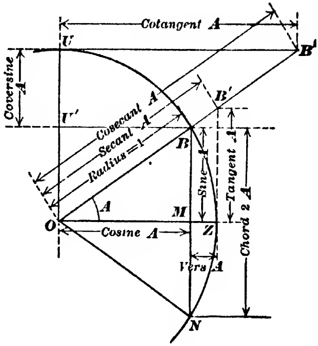

Fig. 3.

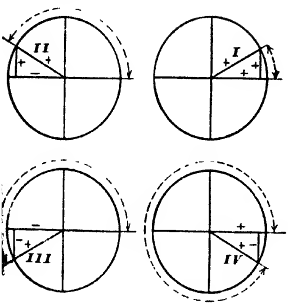

Fig. 4.

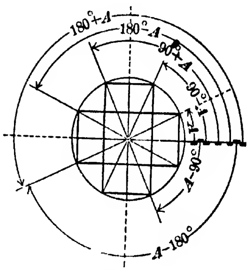

Fig. 5.

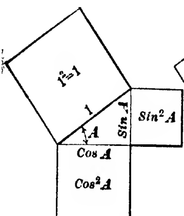

Fig. 6

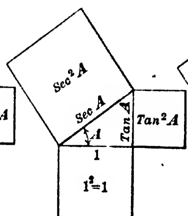

Fig. 7

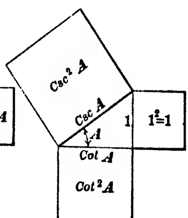

Fig. 8

Table of

Natural Sines, Tangents, Cotangents and Cosines.

Pages 98 to 142.

TABLE OF SINES, TANGENTS, COTANGENTS AND COSINES. **(For logarithmic sines, etc.** see pp 143 *a* **etc.)**

0 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0000000	·000000	Infinite.	1·000000	60
1	·0002909	·000291	3437·746	1·000000	59
2	·0005818	·000582	1718·873	·9999998	58
3	·0008727	·000872	1145·915	·9999996	57
4	·0011636	·001163	859·4363	·9999993	56
5	·0014544	·001454	687·5488	·9999989	55
6	·0017453	·001745	572·9572	·9999985	54
7	·0020362	·002036	491·1060	·9999979	53
8	·0023271	·002327	429·7175	·9999973	52
9	·0026180	·002618	381·9709	·9999966	51
10	·0029089	·002908	343·7737	·9999958	50
11	·0031998	·003199	312·5213	·9999949	49
12	·0034907	·003490	286.4777	·9999939	48
13	·0037815	·003781	264·4408	·9999928	47
14	·0040724	·004072	245.5519	·9999917	46
15	·0043633	·004363	229·1816	·9999905	45
16	·0046542	·004654	214·8576	·9999892	44
17	·0049451	·004945	202·2187	·9999878	43
18	·0052360	·005236	190·9841	·9999863	42
19	·0055268	·005526	180·9322	·9999847	41
20	·0058177	·005817	171·8854	·9999831	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 89.

0 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0061086	·006108	163·7001	·9999813	39
22	·0063995	·006399	156·2590	·9999795	38
23	·0066904	·006690	149·4650	·9999776	37
24	·0069813	·006981	143·2371	·9999756	36
25	·0072721	·007272	137·5075	·9999736	35
26	·0075630	·007563	132·2185	·9999714	34
27	·0078539	·007854	127·3213	·9999692	33
28	·0081448	·008145	122·7739	·9999668	32
29	·0084357	·008436	118·5401	·9999644	31
30	·0087265	·008726	114·5886	·9999619	30
31	·0090174	·009017	110·8920	·9999593	29
32	·0093083	·009308	107·4264	·9999567	28
33	·0095992	·009599	104·1709	·9999539	27
34	·0098900	·009890	101·1069	·9999511	26
35	·0101809	·010181	98·21794	·9999482	25
36	·0104718	·010472	95·48947	·9999452	24
37	·0107627	·010763	92·90848	·9999421	23
38	·0110535	·011054	90·46333	·9999389	22
39	·0113444	·011345	88·14357	·9999357	21
40	·0116353	·011636	85·93979	·9999323	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 89.

0 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0119261	·011927	83·84350	·9999289	19
42	·0122170	·012217	81·84704	·9999254	18
43	·0125079	·012508	79·94343	·9999218	17
44	·0127987	·012799	78·12634	·9999181	16
45	·0130896	·013090	76·39000	·9999143	15
46	·0133805	·013381	74·72916	·9999105	14
47	·0136713	·013672	73·13899	·9999065	13
48	·0139622	·013963	71·61507	·9999025	12
49	·0142530	·014254	70·15334	·9998984	11
50	·0145439	·014545	68·75008	·9998942	10
51	·0148348	·014836	67·40185	·9998900	9
52	·0151256	·015127	66·10547	·9998856	8
53	·0154165	·015418	64·85800	·9998812	7
54	·0157073	·015709	63·65674	·9998766	6
55	·0159982	·016000	62·49915	·9998720	5
56	·0162890	·016291	61·38290	·9998673	4
57	·0165799	·016582	60·30582	·9998625	3
58	·0168707	·016873	59·26587	·9998577	2
59	·0171616	·017164	58·26117	·9998527	1
60	·0174524	·017455	57·28996	·9998477	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 89.

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0174524	·017455	57·28996	·9998477	60
1	·0177432	·017746	56·35059	·9998426	59
2	·0180341	·018037	55·44151	·9998374	58
3	·0183249	·018328	54·56130	·9998321	57
4	·0186158	·018619	53·70858	·9998267	56
5	·0189066	·018910	52·88211	·9998213	55
6	·0191974	·019201	52·08067	·9998157	54
7	·0194883	·019492	51·30315	·9998101	53
8	·0197791	·019783	50·54850	·9998044	52
9	·0200699	·020074	49·81572	·9997986	51
10	·0203608	·020365	49·10388	·9997927	50
11	·0206516	·020656	48·41208	·9997867	49
12	·0209424	·020947	47·73950	·9997807	48
13	·0212332	·021238	47·08534	·9997745	47
14	·0215241	·021529	46·44886	·9997683	46
15	·0218149	·021820	45·82935	·9997620	45
16	·0221057	·022111	45·22614	·9997556	44
17	·0223965	·022402	44·63859	·9997492	43
18	·0226873	·022693	44·06611	·9997426	42
19	·0229781	·022984	43·50812	·9997360	41
20	·0232690	·023275	42·96407	·9997292	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg 88.

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0235598	·023566	42·43346	·9997224	39
22	·0238506	·023857	41·91579	·9997156	38
23	·0241414	·024148	41·41058	·9997086	37
24	·0244322	·024439	40·91741	·9997015	36
25	·0247230	·024730	40·43583	·9996943	35
26	·0250138	·025021	39·96546	·9996871	34
27	·0253046	·025312	39·50589	·9996798	33
28	·0255954	·025603	39·05677	·9996724	32
29	·0258862	·025894	38·61773	·9996649	31
30	·0261769	·026185	38·18845	·9996573	30
31	·0264677	·026477	37·76861	·9996497	29
32	·0267585	·026768	37·35789	·9996419	28
33	·0270493	·027059	36·95600	·9996341	27
34	·0273401	·027350	36·56265	·9996262	26
35	·0276309	·027641	36·17759	·9996182	25
36	·0279216	·027932	35·80055	·9996101	24
37	·0282124	·028223	35·43128	·9996020	23
38	·0285032	·028514	35·06954	·9995937	22
39	·0287940	·028805	34·71511	·9995854	21
40	·0290847	·029097	34·36777	·9995770	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 88.

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0293755	·029388	34·02730	·9995684	19
42	·0296662	·029679	33·69350	·9995599	18
43	·0299570	·029970	33·36619	·9995512	17
44	·0302478	·030261	33·04517	·9995424	16
45	·0305385	·030552	32·73026	·9995336	15
46	·0308293	·030843	32·42129	·9995247	14
47	·0311200	·031135	32·11809	·9995157	13
48	·0314108	·031426	31·82051	·9995066	12
49	·0317015	·031717	31·52839	·9994974	11
50	·0319922	·032008	31·24157	·9994881	10
51	·0322830	·032299	30·95992	·9994788	9
52	·0325737	·032591	30·68330	·9994693	8
53	·0328644	·032882	30·41158	·9994598	7
54	·0331552	·033173	30·14461	·9994502	6
55	·0334459	·033464	29·88229	·9994405	5
56	·0337366	·033755	29·62449	·9994308	4
57	·0340274	·034047	29·37110	·9994209	3
58	·0343181	·034338	29·12200	·9994110	2
59	·0346088	·034629	28·87708	·9994009	1
60	·0348995	·034920	28·63625	·9993908	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 88.

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0348995	·034920	28·63625	·9993908	60
1	·0351902	·035212	28·39939	·9993806	59
2	·0354809	·035503	28·16642	·9993704	58
3	·0357716	·035794	27·93723	·9993600	57
4	·0360623	·036085	27·71174	·9993495	56
5	·0363530	·036377	27·48985	·9993390	55
6	·0366437	·036668	27·27148	·9993284	54
7	·0369344	·036959	27·05655	·9993177	53
8	·0372251	·037250	26·84498	·9993069	52
9	·0375158	·037542	26·63669	·9992960	51
10	·0378065	·037833	26·43160	·9992851	50
11	·0380971	·038124	26·22963	·9992740	49
12	·0383878	·038416	26·03073	·9992629	48
13	·0386785	·038707	25·83482	·9992517	47
14	·0389692	·038998	25·64183	·9992404	46
15	·0392598	·039290	25·45170	·9992290	45
16	·0395505	·039581	25·26436	·9992176	44
17	·0398411	·039872	25·07975	·9992060	43
18	·0401318	·040164	24·89782	·9991944	42
19	·0404224	·040455	24·71851	·9991827	41
20	·0407131	·040746	24·54175	·9991709	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0410037	·041038	24·36750	·9991590	39
22	·0412944	·041329	24·19571	·9991470	38
23	·0415850	·041621	24·02632	·9991350	37
24	·0418757	·041912	23·85927	·9991228	36
25	·0421663	·042203	23·69453	·9991106	35
26	·0424569	·042495	23·53205	·9990983	34
27	·0427475	·042786	23·37177	·9990859	33
28	·0430382	·043078	23·21366	·9990734	32
29	·0433288	·043369	23·05767	·9990609	31
30	·0436194	·043660	22·90376	·9990482	30
31	·0439100	·043952	22·75189	·9990355	29
32	·0442006	·044243	22·60201	·9990227	28
33	·0444912	·044535	22·45409	·9990098	27
34	·0447818	·044826	22·30809	·9989968	26
35	·0450724	·045118	22·16398	·9989837	25
36	·0453630	·045409	22·02171	·9989706	24
37	·0456536	·045701	21·88125	·9989573	23
38	·0459442	·045992	21·74256	·9989440	22
39	·0462347	·046284	21·60563	·9989306	21
40	·0465253	·046575	21·47040	·9989171	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87.

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0468159	·046867	21.33685	·9989035	19
42	·0471065	·047158	21·20494	·9988899	18
43	·0473970	·047450	21·07466	·9988761	17
44	·0476876	·047741	20·94596	·9988623	16
45	·0479781	·048033	20·81882	·9988484	15
46	·0482687	·048325	20·69322	·9988344	14
47	·0485592	·048616	20·56911	·9988203	13
48	·0488498	·048908	20·44648	·9988061	12
49	·0491403	·049199	20·32530	·9987919	11
50	·0494308	·049491	20·20555	·9987775	10
51	·0497214	·049782	20·08719	·9987631	9
52	·0500119	·050074	19·97021	·9987486	8
53	·0503024	·050366	19·85459	·9987340	7
54	·0505929	·050657	19·74029	·9987194	6
55	·0508835	·050949	19·62729	·9987046	5
56	·0511740	·051241	19·51558	·9986898	4
57	·0514645	·051532	19·40513	·9986748	3
58	·0517550	·051824	19·29592	·9986598	2
59	·0520455	·052116	19·18793	·9986447	1
60	·0523360	·052407	19·08113	·9986295	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526264	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	.053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0697565	·069926	14·30066	·9975641	60
1	·0700467	·070219	14·24113	·9975437	59
2	·0703368	·070511	14·18209	·9975233	58
3	·0706270	·070803	14·12353	·9975028	57
4	·0709171	·071096	14·06545	·9974822	56
5	·0712073	·071388	14·00785	·9974615	55
6	·0714974	·071680	13·95071	·9974408	54
7	·0717876	·071973	13·89404	·9974199	53
8	·0720777	·072265	13·83782	·9973990	52
9	·0723678	·072558	13·78206	·9973780	51
10	·0726580	·072850	13·72673	·9973569	50
11	·0729481	·073143	13·67185	·9973357	49
12	·0732382	·073435	13·61740	·9973145	48
13	·0735283	·073727	13·56339	·9972931	47
14	·0738184	·074020	13·50979	·9972717	46
15	·0741085	·074312	13·45662	·9972502	45
16	·0743986	·074605	13·40386	·9972286	44
17	·0746887	·074897	13·35151	·9972069	43
18	·0749787	·075190	13·29957	·9971851	42
19	·0752688	·075482	13·24803	·9971633	41
20	·0755589	·075775	13·19688	·9971413	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85.

4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0758489	·076068	13·14612	·9971193	39
22	·0761390	·076360	13·09575	·9970972	38
23	·0764290	·076653	13.04576	·9970750	37
24	·0767190	·076945	12·99616	·9970528	36
25	·0770091	·077238	12·94692	·9970304	35
26	·0772991	·077531	12·89805	·9970080	34
27	·0775891	·077823	12·84955	·9969854	33
28	·0778791	·078116	12·80141	·9969628	32
29	·0781691	·078409	12·75363	·9969401	31
30	·0784591	·078701	12·70620	·9969173	30
31	·0787491	·078994	12·65912	·9968915	29
32	·0790391	·079287	12·61239	·9968715	28
33	·0793290	·079579	12·56599	·9968485	27
34	·0796190	·079872	12·51994	·9968254	26
35	·0799090	.080165	12·47422	·9968022	25
36	·0801989	·080458	12·42883	·9967789	24
37	·0804889	·080750	12·38376	·9967555	23
38	·0807788	·081043	12·33902	·9967321	22
39	·0810687	·081336	12·29460	·9957085	21
40	·0813587	·081629	12·25050	·9966849	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85.

4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0816486	·081922	12·20671	·9966612	19
42	·0819385	·082215	12·16323	·9966374	18
43	·0822284	·082507	12·12006	·9966135	17
44	·0825183	·082800	12·07719	·9965895	16
45	·0828082	·083093	12·03462	·9965655	15
46	·0830981	·083386	11·99234	·9965414	14
47	·0833880	·083679	11·95037	·9965172	13
48	·0836778	·083972	11·90868	·9964929	12
49	·0839677	·084265	11·86728	·9964685	11
50	·0842576	·084558	11·82616	·9964440	10
51	·0845474	·084851	11·78533	·9964195	9
52	·0848373	·085144	11·74477	·9963948	8
53	·0851271	·085437	11·70450	·9963701	7
54	·0854169	·085730	11·66449	·9963453	6
55	·0857067	·086023	11·62476	·9963204	5
56	·0859966	·086316	11·58529	·9962954	4
57	·0862864	·086609	11·54609	·9962704	3
58	·0865762	·086902	11·50715	·9962452	2
59	·0868660	·087195	11·46847	·9962200	1
60	·0871557	·087488	11·43005	·9961947	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0871557	·087488	11·43005	·9961947	60
1	·0874455	·087781	11·39188	·9961693	59
2	·0877353	·088074	11·35397	·9961438	58
3	·0880251	·088368	11·31630	·9961183	57
4	·0883148	·088661	11·27888	·9960926	56
5	·0886046	·088954	11·24171	·9960669	55
6	·0888943	·089247	11·20478	·9960411	54
7	·0891840	·089540	11·16808	·9960152	53
8	·0894738	·089834	11·13163	·9959892	52
9	·0897635	·090127	11·09541	·9959631	51
10	·0900532	·090420	11·05943	·9959370	50
11	·0903429	·090713	11·02367	·9959107	49
12	·0906326	·091007	10·98815	·9958844	48
13	·0909223	·091300	10·95285	·9958580	47
14	·0912119	·091593	10·91777	·9958315	46
15	·0915016	·091887	10·88292	·9958049	45
16	·0917913	·092180	10·84828	·9957783	44
17	·0920809	·092473	10·81387	·9957515	43
18	·0923706	·092767	10·77967	·9957247	42
19	·0926602	·093060	10·74568	·9956978	41
20	·0929499	·093354	10·71191	·9956708	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0932395	·093647	10·67834	·9956437	39
22	·0935291	·093940	10·64499	·9956165	38
23	·0938187	·094234	10·61184	·9955892	37
24	·0941083	·094527	10·57889	·9955620	36
25	·0943979	·094821	10·54615	·9955345	35
26	·0946875	·095114	10·51360	·9955070	34
27	·0949771	·095408	10·48126	·9954794	33
28	·0952666	·095701	10·44911	·9954517	32
29	·0955562	·095995	10·41715	·9954240	31
30	·0958458	·096289	10·38539	·9953962	30
31	·0961353	·096582	10·35382	·9953683	29
32	·0964248	·096876	10·32244	·9953403	28
33	·0967144	·097169	10·29125	·9953122	27
34	·0970039	·097463	10·26024	·9952840	26
35	·0972934	·097757	10·22942	·9952557	25
36	·0975829	·098050	10·19878	·9952274	24
37	·0978724	·098344	10·16833	·9951990	23
38	·0981619	·098638	10·13805	·9951705	22
39	·0984514	·098932	10·10795	·9951419	21
40	·0987408	·099225	10·07803	·9951132	20
′	Cosine.	Cotan.	Tang.	Sine..	′

Deg. 84.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0990303	·099519	10·04828	·9950844	19
42	·0993197	·099813	10·01871	·9950556	18
43	·0996092	·100107	9·989305	·9950266	17
44	·0998986	·100400	9·960072	·9949976	16
45	·1001881	·100694	9·931008	·9949685	15
46	·1004775	·100988	9·902112	·9949393	14
47	·1007669	·101282	9·873382	·9949101	13
48	·1010563	·101576	9·844816	·9948807	12
49	·1013457	·101870	9·816414	·9948513	11
50	·1016351	·102164	9·788173	·9948217	10
51	·1019245	·102458	9·760092	·9947921	9
52	·1022138	·102752	9·732171	·9947625	8
53	·1025032	·103046	9·704407	·9947327	7
54	·1027925	·103339	9·676800	·9947028	6
55	·1030819	·103634	9·649347	·9946729	5
56	·1033712	·103928	9·622048	·9946428	4
57	·1036605	·104222	9·594902	·9946127	3
58	·1039499	·104516	9·567906	·9945825	2
59	·1042392	·104810	9·541061	·9945523	1
60	·1045285	·105104	9·514364	·9945219	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60
1	·1048178	·105398	9·487814	·9944914	59
2	·1051070	·105692	9·461411	·9944609	58
3	·1053963	·105986	9·435153	·9944303	57
4	·1056856	·106280	9·409038	·9943996	56
5	·1059748	·106575	9·383066	·9943688	55
6	·1062641	·106869	9·357235	·9943379	54
7	·1065533	·107163	9·331545	·9943070	53
8	·1068425	·107457	9·305993	·9942760	52
9	·1071318	·107751	9·280580	·9942448	51
10	·1074210	·108046	9·255303	·9942136	50
11	·1077102	·108340	9·230162	·9941823	49
12	·1079994	·108634	9·205156	·9941510	48
13	·1082885	·108929	9·180283	·9941195	47
14	·1085777	·109223	9·155543	·9940880	46
15	·1088669	·109517	9·130934	·9940563	45
16	·1091560	·109812	9·106456	·9940246	44
17	·1094452	·110106	9·082107	·9939928	43
18	·1097343	·110401	9·057886	·9939610	42
19	·1100234	·110695	9·033793	·9939290	41
20	·1103126	·110989	9·009826	·9938969	40
′	Cosine.	Cotan.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1106017	·111284	8·985984	·9938648	39
22	·1108908	·111578	8·962266	·9938326	38
23	·1111799	·111873	8·938672	·9938003	37
24	·1114689	·112168	8·915200	·9937679	36
25	·1117580	·112462	8·891850	·9937355	35
26	·1120471	·112757	8·868620	·9937029	34
27	·1123361	·113051	8·845510	·9936703	33
28	·1126252	·113346	8·822518	·9936375	32
29	·1129142	·113641	8·799644	·9936047	31
30	·1132032	·113935	8·776887	·9935719	30
31	·1134922	·114230	8·754246	·9935389	29
32	·1137812	·114525	8·731719	·9935058	28
33	·1140702	·114819	8·709307	·9934727	27
34	·1143592	·115114	8·687008	·9934395	26
35	·1146482	·115409	8·664822	·9934062	25
36	·1149372	·115703	8·642747	·9933728	24
37	·1152261	·115998	8·620783	·9933393	23
38	·1155151	·116293	8·598929	·9933057	22
39	·1158040	·116588	8·577183	·9932721	21
40	·1160929	·116883	8·555546	·9932384	20
′	Cosine.	Cotan.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1163818	·117178	8·534017	·9932045	19
42	·1166707	·117473	8·512594	·9931706	18
43	·1169596	·117767	8·491277	·9931367	17
44	·1172485	·118062	8·470065	·9931026	16
45	·1175374	118357	8·448957	·9930685	15
46	·1178263	·118652	8·427953	·9930342	14
47	·1181151	·118947	8·407051	·9929999	13
48	·1184040	·119242	8·386251	·9929655	12
49	·1186928	·119537	8·365553	·9929310	11
50	·1189816	·119832	8·344955	·9928965	10
51	·1192704	·120127	8·324457	·9928618	9
52	·1195593	·120423	8·304058	·9928271	8
53	·1198481	·120718	8·283757	·9927922	7
54	·1201368	·121013	8·263554	·9927573	6
55	·1204256	·121308	8·243448	·9927224	5
56	·1207144	·121603	8·223488	·9926873	4
57	·1210031	·121898	8·203523	·9926521	3
58	·1212919	·122194	8·183704	·9926169	2
59	·1215806	·122489	8·163978	·9925816	1
60	·1218693	·122784	8·144346	·9925462	0
′	Cosine.	Cotan.	Tang.	Sine.	′

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60
1	·1221581	·123079	8·124807	·9925107	59
2	·1224468	·123375	8·105359	·9924751	58
3	·1227355	·123670	8·086004	·9924394	57
4	·1230241	·123965	8·066739	·9924037	56
5	·1233128	·124261	8·047564	·9923679	55
6	·1236015	·124556	8·028479	·9923319	54
7	·1238901	·124852	8·009483	·9922959	53
8	·1241788	·125147	7·990575	·9922599	52
9	·1244674	·125442	7·971755	·9922237	51
10	·1247560	·125738	7·953022	·9921874	50
11	·1250446	·126033	7·934375	·9921511	49
12	·1253332	·126329	7·915815	·9921147	48
13	·1256218	·126624	7·897339	·9920782	47
14	·1259104	·126920	7·878948	·9920416	46
15	·1261990	·127216	7·860642	·9920049	45
16	·1264875	·127511	7·842419	·9919682	44
17	·1267761	·127807	7·824279	·9919314	43
18	·1270646	·128103	7·806221	·9918944	42
19	·1273531	·128398	7·788245	·9918574	41
20	·1276416	·128694	7·770350	·9918204	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1279302	·128990	7·752536	·9917832	39
22	·1282186	·129285	7·734802	·9917459	38
23	·1285071	·129581	7·717148	·9917086	37
24	·1287956	·129877	7·699573	·9916712	36
25	·1290841	·130173	7·682076	·9916337	35
26	·1293725	·130469	7·664658	·9915961	34
27	·1296609	·130764	7·647317	·9915584	33
28	·1299494	·131060	7·630053	·9915206	32
29	·1302378	·131356	7·612865	·9914828	31
30	·1305262	·131652	7·595754	·9914449	30
31	·1308146	·131948	7·578717	·9914069	29
32	·1311030	·132244	7·561756	·9913688	28
33	·1313913	·132540	7·544869	·9913306	27
34	·1316797	·132836	7·528057	·9912923	26
35	·1319681	·133132	7·511317	·9912540	25
36	·1322564	·133128	7·494651	·9912155	24
37	·1325447	·133724	7·478057	·9911770	23
38	·1328330	·134020	7·461535	·9911384	22
39	·1331213	·134316	7·445085	·9910997	21
40	·1334096	·134612	7·428706	·9910610	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1336979	·134909	7·412397	·9910221	19
42	·1339862	·135205	7·396159	·9909832	18
43	·1342744	·135501	7·379990	·9909442	17
44	·1345627	·135797	7·363891	·9909051	16
45	·1348509	·136094	7·347861	·9908659	15
46	·1351392	·136390	7·331898	·9908266	14
47	·1354274	·136686	7·316004	·9907873	13
48	·1357156	·136983	7·300178	·9907478	12
49	·1360038	·137279	7·284418	·9907083	11
50	·1362919	·137575	7·268725	·9906687	10
51	·1365801	·137872	7·253098	·9906290	9
52	·1368683	·138168	7·237537	·9905893	8
53	·1371564	·138465	7·222042	·9905494	7
54	·1374445	·138761	7·206611	·9905095	6
55	·1377327	·139058	7·191245	·9904694	5
56	·1380208	·139354	7·175943	·9904293	4
57	·1383089	·139651	7·160705	·9903891	3
58	·1385970	·139947	7·145530	·9903489	2
59	·1388850	·140244	7·130419	·9903085	1
60	·1391731	·140540	7·115369	·9902681	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60
1	·1048178	·105398	9·487814	·9944914	59
2	·1051070	·105692	9·461411	·9944609	58
3	·1053963	·105986	9·435153	·9944303	57
4	·1056856	·106280	9·409038	·9943996	56
5	·1059748	·106575	9·383066	·9943688	55
6	·1062641	·106869	9·357235	·9943379	54
7	·1065533	·107163	9·331545	·9943070	53
8	·1068425	·107457	9·305993	·9942760	52
9	·1071318	·107751	9·280580	·9942448	51
10	·1074210	·108046	9·255303	·9942136	50
11	·1077102	·108340	9·230162	·9941823	49
12	·1079994	·108634	9·205156	·9941510	48
13	·1082885	·108929	9·180283	·9941195	47
14	·1085777	·109223	9·155543	·9940880	46
15	·1088669	·109517	9·130934	·9940563	45
16	·1091560	·109812	9·106456	·9940246	44
17	·1094452	·110106	9·082107	·9939928	43
18	·1097343	·110401	9·057886	·9939610	42
19	·1100234	·110695	9·033793	·9939290	41
20	·1103126	·110989	9·009826	·9938969	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1106017	·111284	8·985984	·9938648	39
22	·1108908	·111578	8·962266	·9938326	38
23	·1111799	·111873	8·938672	·9938003	37
24	·1114689	·112168	8·915200	·9937679	36
25	·1117580	·112462	8·891850	·9937355	35
26	·1120471	·112757	8·868620	·9937029	34
27	·1123361	·113051	8·845510	·9936703	33
28	·1126252	·113346	8·822518	·9936375	32
29	·1129142	·113641	8·799644	·9936047	31
30	·1132032	·113935	8·776887	·9935719	30
31	·1134922	·114230	8·754246	·9935389	29
32	·1137812	·114525	8·731719	·9935058	28
33	·1140702	·114819	8·709307	·9934727	27
34	·1143592	·115114	8·687008	·9934395	26
35	·1146482	·115409	8·664822	·9934062	25
36	·1149372	·115703	8·642747	·9933728	24
37	·1152261	·115998	8·620783	·9933393	23
38	·1155151	·116293	8·598929	·9933057	22
39	·1158040	·116588	8·577183	·9932721	21
40	·1160929	·116883	8·555546	·9932384	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1163818	·117178	8·534017	·9932045	19
42	·1166707	·117473	8·512594	·9931706	18
43	·1169596	·117767	8·491277	·9931367	17
44	·1172485	·118062	8·470065	·9931026	16
45	·1175374	118357	8·448957	·9930685	15
46	·1178263	·118652	8·427953	·9930342	14
47	·1181151	·118947	8·407051	·9929999	13
48	·1184040	·119242	8·386251	·9929655	12
49	·1186928	·119537	8·365553	·9929310	11
50	·1189816	·119832	8·344955	·9928965	10
51	·1192704	·120127	8·324457	·9928618	9
52	·1195593	·120423	8·304058	·9928271	8
53	·1198481	·120718	8·283757	·9927922	7
54	·1201368	·121013	8·263554	·9927573	6
55	·1204256	·121308	8·243448	·9927224	5
56	·1207144	·121603	8·223488	·9926873	4
57	·1210031	·121898	8·203523	·9926521	3
58	·1212919	·122194	8·183704	·9926169	2
59	·1215806	·122489	8·163978	·9925816	1
60	·1218693	·122784	8·144346	·9925462	0
′	Cosine	Cotan.	Tang.	Sine.	′

Deg. 83.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60
1	·1221581	·123079	8·124807	·9925107	59
2	·1224468	·123375	8·105359	·9924751	58
3	·1227355	·123670	8·086004	·9924394	57
4	·1230241	·123965	8·066739	·9924037	56
5	·1233128	·124261	8·047564	·9923679	55
6	·1236015	·124556	8·028479	·9923319	54
7	·1238901	·124852	8·009483	·9922959	53
8	·1241788	·125147	7·990575	·9922599	52
9	·1244674	·125442	7·971755	·9922237	51
10	·1247560	·125738	7·953022	·9921874	50
11	·1250446	·126033	7·934375	·9921511	49
12	·1253332	·126329	7·915815	·9921147	48
13	·1256218	·126624	7·897339	·9920782	47
14	·1259104	·126920	7·878948	·9920416	46
15	·1261990	·127216	7·860642	·9920049	45
16	·1264875	·127511	7·842419	·9919682	44
17	·1267761	·127807	7·824279	·9919314	43
18	·1270646	·128103	7·806221	·9918944	42
19	·1273531	·128398	7·788245	·9918574	41
20	·1276416	·128694	7·770350	·9918204	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1279302	·128990	7·752536	·9917832	39
22	·1282186	·129285	7·734802	·9917459	38
23	·1285071	·129581	7·717148	·9917086	37
24	·1287956	·129877	7·699573	·9916712	36
25	·1290841	·130173	7·682076	·9916337	35
26	·1293725	·130469	7·664658	·9915961	34
27	·1296609	·130764	7·647317	·9915584	33
28	·1299494	·131060	7·630053	·9915206	32
29	·1302378	·131356	7·612865	·9914828	31
30	·1305262	·131652	7·595754	·9914449	30
31	·1308146	·131948	7·578717	·9914069	29
32	·1311030	·132244	7·561756	·9913688	28
33	·1313913	·132540	7·544869	·9913306	27
34	·1316797	·132836	7·528057	·9912923	26
35	·1319681	·133132	7·511317	·9912540	25
36	·1322564	·133428	7·494651	·9912155	24
37	·1325447	·133724	7·478057	·9911770	23
38	·1328330	·134020	7·461535	·9911384	22
39	·1331213	·134316	7·445085	·9910997	21
40	·1334096	·134612	7·428706	·9910610	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1336979	·134909	7·412397	·9910221	19
42	·1339862	·135205	7·396159	·9909832	18
43	·1342744	·135501	7·379990	·9909442	17
44	·1345627	·135797	7·363891	·9909051	16
45	·1348509	·136094	7·347861	·9908659	15
46	·1351392	·136390	7·331898	·9908266	14
47	·1354274	·136686	7·316004	·9907873	13
48	·1357156	·136983	7·300178	·9907478	12
49	·1360038	·137279	7·284418	·9907083	11
50	·1362919	·137575	7·268725	·9906687	10
51	·1365801	·137872	7·253098	·9906290	9
52	·1368683	·138168	7·237537	·9905893	8
53	·1371564	·138465	7·222042	·9905494	7
54	·1374445	·138761	7·206611	·9905095	6
55	·1377327	·139058	7·191245	·9904694	5
56	·1380208	·139354	7·175943	·9904293	4
57	·1383089	·139651	7·160705	·9903891	3
58	·1385970	·139947	7·145530	·9903489	2
59	·1388850	·140244	7·130419	·9903085	1
60	·1391731	·140540	7·115369	·9902681	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1391731	·140540	7·115369	·9902681	60
1	·1394612	·140837	7·100382	·9902275	59
2	·1397492	·141134	7·085457	·9901869	58
3	·1400372	·141430	7·070593	·9901462	57
4	·1403252	·141727	7·055790	·9901055	56
5	·1406132	·142024	7·041048	·9900646	55
6	·1409012	·142321	7·026366	·9900237	54
7	·1411892	·142617	7·011744	·9899826	53
8	·1414772	·142914	6·997180	·9899415	52
9	·1417651	·143211	6·982678	·9899003	51
10	·1420531	·143508	6·968233	·9898590	50
11	·1423410	·143805	6·953847	·9898177	49
12	·1426289	·144102	6·939519	·9897762	48
13	·1429168	·144399	6·925248	·9897347	47
14	·1432047	·144696	6·911035	·9896931	46
15	·1434926	·144993	6·896879	·9896514	45
16	·1437805	·145290	6·882780	·9896096	44
17	·1440684	·145587	6·868737	·9895677	43
18	·1443562	·145884	6·854750	·9895258	42
19	·1446440	·146181	6·840819	·9894838	41
20	·1449319	·146478	6·826943	·9894416	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1452197	·146775	6·813122	·9893994	39
22	·1455075	·147072	6·799356	·9893572	38
23	·1457953	·147369	6·785644	·9893148	37
24	·1460830	·147667	6·771986	·9892723	36
25	·1463708	·147964	6·758382	·9892298	35
26	·1466585	·148261	6·744831	·9891872	34
27	·1469463	·148559	6·731334	·9891445	33
28	·1472340	·148856	6·717889	·9891017	32
29	·1475217	·149153	6·704496	·9890588	31
30	·1478094	·149451	6·691156	·9890159	30
31	·1480971	·149748	6·677867	·9889728	29
32	·1483848	·150045	6·664630	·9889297	28
33	·1486724	·150343	6·651444	·9888865	27
34	·1489601	·150640	6·638310	·9888432	26
35	·1492477	·150938	6·625225	·9887998	25
36	·1495353	·151235	6·612191	·9887564	24
37	·1498230	·151533	6·599208	·9887128	23
38	·1501106	·151830	6·586273	·9886692	22
39	·1503981	·152128	6·573389	·9886255	21
40	·1506857	·152426	6·560553	·9885817	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1509733	·152723	6·547767	·9885378	19
42	·1512608	·153021	6·535029	·9884939	18
43	·1515484	·153319	6·522339	·9884498	17
44	·1518359	·153617	6·509698	·9884057	16
45	·1521234	·153914	6·497104	·9883615	15
46	·1524109	·154212	6·484558	·9883172	14
47	·1526984	·154510	6·472059	·9882728	13
48	·1529858	·154808	6·459607	·9882284	12
49	·1532733	·155106	6·447201	·9881838	11
50	·1535607	·155404	6·434842	·9881392	10
51	·1538482	·155701	6·422530	·9880945	9
52	·1541356	·155999	6·410263	·9880497	8
53	·1544230	·156297	6·398042	·9880048	7
54	·1547104	·156595	6·385866	·9879599	6
55	·1549978	·156893	6·373735	·9879148	5
56	·1552851	·157191	6·361650	·9878697	4
57	·1555725	·157490	6·349609	·9878245	3
58	·1558598	·157788	6·337612	·9877792	2
59	·1561472	·158086	6·325660	·9877338	1
60	·1564345	·158384	6·313751	·9876883	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1564345	·158384	6·313751	·9876883	60
1	·1567218	·158682	6·301886	·9876428	59
2	·1570091	·158980	6·290065	·9875972	58
3	·1572963	·159279	6·278286	·9875514	57
4	·1575836	·159577	6·266551	·9875057	56
5	·1578708	·159875	6·254858	·9874598	55
6	·1581581	·160174	6·243208	·9874138	54
7	·1584453	·160472	6·231600	·9873678	53
8	·1587325	·160770	6·220034	·9873216	52
9	·1590197	·161069	6·208510	·9872754	51
10	·1593069	·161367	6·197027	·9872291	50
11	·1595940	·161666	6·185586	·9871827	49
12	·1598812	·161964	6·174186	·9871363	48
13	·1601683	·162263	6·162827	·9870897	47
14	·1604555	·162561	6·151508	·9870431	46
15	·1607426	·162860	6·140230	·9869964	45
16	·1610297	·163159	6·128992	·9869496	44
17	·1613167	·163457	6·117794	·9869027	43
18	·1616038	·163756	6·106636	·9868557	42
19	·1618909	·164055	6·095517	·9868087	41
20	·1621779	·164353	6·084438	·9867615	40
′	Cosine.	Cotan.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1624650	·164652	6·073397	·9867143	39
22	·1627520	·164951	6·062396	·9866670	38
23	·1630390	·165250	6·051434	·9866196	37
24	·1633260	·165548	6·040510	·9865722	36
25	·1636129	·165847	6·029624	·9865246	35
26	·1638999	·166146	6·018777	·9864770	34
27	·1641868	·166445	6·007967	·9864293	33
28	·1644738	·166744	5·997195	·9863815	32
29	·1647607	·167043	5·986461	9863336	31
30	·1650476	·167342	5·975764	·9862856	30
31	·1653345	·167641	5·965104	·9862375	29
32	·1656214	·167940	5·954481	·9861894	28
33	·1659082	·168239	5·943895	·9861412	27
34	·1661951	·168539	5·933345	·9860929	26
35	·1664819	·168838	5·922832	·9860445	25
36	·1667687	·169137	5·912355	·9859960	24
37	·1670556	·169436	5·901913	·9859475	23
38	·1673423	·169735	5·891508	·9858988	22
39	·1676291	·170035	5·881138	·9858501	21
40	·1679159	·170334	5·870804	·9858013	20
′	Cosine.	Cotan.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1682026	·170633	5·860505	·9857524	19
42	·1684894	·170933	5·850241	·9857035	18
43	·1687761	·171232	5·840011	·9856544	17
44	·1690628	·171532	5·829817	·9856053	16
45	·1693495	·171831	5·819657	·9855561	15
46	·1696362	·172130	5·809531	·9855068	14
47	·1699228	·172430	5·799440	·9854574	13
48	·1702095	·172730	5·789382	·9854079	12
49	·1704961	·173029	5·779358	·9853583	11
50	·1707828	·173329	5·769368	·9853087	10
51	·1710694	·173628	5·759412	·9852590	9
52	·1713560	·173928	5·749488	·9852092	8
53	·1716425	·174228	5·739598	·9851593	7
54	·1719291	·174527	5·729741	·9851093	6
55	·1722156	·174827	5·719917	·9850593	5
56	·1725022	·175127	5·710125	·9850091	4
57	·1727887	·175427	5·700366	·9849589	3
58	·1730752	·175727	5·690639	·9849086	2
59	·1733617	·176027	5·680944	·9848582	1
60	·1736482	·176327	5·671281	·9848078	0
′	Cosine.	Cotan.	Tang.	Sine.	′

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1736482	·176327	5·671281	·9848078	60
1	·1739346	·176626	5·661650	·9847572	59
2	·1742211	·176926	5·652051	·9847066	58
3	·1745075	·177226	5·642483	·9846558	57
4	·1747939	·177527	5·632947	·9846050	56
5	·1750803	·177827	5·623442	·9845542	55
6	·1753667	·178127	5·613968	·9845032	54
7	·1756531	·178427	5·604524	·9844521	53
8	·1759395	·178727	5·595112	·9844010	52
9	·1762258	·179027	5·585730	·9843498	51
10	·1765121	·179327	5·576378	·9842985	50
11	·1767984	·179628	5·567057	·9842471	49
12	·1770847	·179928	5·557766	·9841956	48
13	·1773710	·180228	5·548505	·9841441	47
14	·1776573	·180529	5·539274	·9840924	46
15	·1779435	·180829	5·530072	·9840407	45
16	·1782298	·181129	5·520900	·9839889	44
17	·1785160	·181430	5·511757	·9839370	43
18	·1788022	·181730	5·502644	·9838850	42
19	·1790884	·182031	5·493560	·9838330	41
20	·1793746	·182331	5·484505	·9837808	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1796607	·182632	5·475478	·9837286	39
22	·1799469	·182933	5·466481	·9836763	38
23	·1802330	·183233	5·457512	·9836239	37
24	·1805191	·183534	5·448571	·9835715	36
25	·1808052	·183835	5·439659	·9835189	35
26	·1810913	·184135	5·430775	·9834663	34
27	·1813774	·184436	5·421918	·9834136	33
28	·1816635	·184737	5·413090	·9833608	32
29	·1819495	·185038	5·404290	·9833079	31
30	·1822355	·185339	5·395517	·9832549	30
31	·1825215	·185639	5·386771	·9832019	29
32	·1828075	·185940	5·378053	·9831487	28
33	·1830935	·186241	5·369363	·9830955	27
34	·1833795	·186542	5·360699	·9830422	26
35	·1836654	·186843	5·352062	·9829888	25
36	·1839514	·187144	5·343452	·9829353	24
37	·1842373	·187446	5·334869	·9828818	23
38	·1845232	·187747	5·326313	·9828282	22
39	·1848091	·188048	5·317783	·9827744	21
40	·1850949	·188319	5·309279	·9827206	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1853808	·188650	5·300801	·9826668	19
42	·1856666	·188952	5·292350	·9826128	18
43	·1859524	·189253	5·283925	·9825587	17
44	·1862382	·189554	5·275525	·9825046	16
45	·1865240	·189855	5·267151	·9824504	15
46	·1868098	·190157	5·258803	·9823961	14
47	·1870956	·190458	5·250480	·9823417	13
48	·1873813	·190760	5·242183	·9822873	12
49	·1876670	·191061	5·233911	·9822327	11
50	·1879528	·191363	5·225664	·9821781	10
51	·1882385	·191664	5·217442	·9821234	9
52	·1885241	·191966	5·209245	·9820686	8
53	·1888098	·192268	5·201073	·9820137	7
54	·1890954	·192569	5·192926	·9819587	6
55	·1893811	·192871	5·184803	·9819037	5
56	·1896667	·193173	5·176705	·9818485	4
57	·1899523	·193474	5·168631	·9817933	3
58	·1902379	·193776	5·160581	·9817380	2
59	·1905234	·194078	5·152555	·9816826	1
60	·1908090	·194380	5·144554	·9816272	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1908090	·194380	5·144554	·9816272	60
1	·1910945	·194682	5·136576	·9815716	59
2	·1913801	·194984	5·128622	·9815160	58
3	·1916656	·195286	5·120692	·9814603	57
4	·1919510	·195588	5·112785	·9814045	56
5	·1922365	·195890	5·104902	·9813486	55
6	·1925220	·196192	5·097042	·9812927	54
7	·1928074	·196494	5·089206	·9812366	53
8	·1930928	·196796	5·081392	·9811805	52
9	·1933782	·197098	5·073602	·9811243	51
10	·1936636	·197400	5·065835	·9810680	50
11	·1939490	·197703	5·058090	·9810116	49
12	·1942344	·198005	5·050369	·9809552	48
13	·1945197	·198307	5·042670	·9808986	47
14	·1948050	·198610	5·034993	·9808420	46
15	·1950903	·198912	5·027339	·9807853	45
16	·1953756	·199214	5·019707	·9807285	44
17	·1956609	·199517	5·012098	·9806716	43
18	·1959461	·199819	5·004511	·9806147	42
19	·1962314	·200122	4·996945	·9805576	41
20	·1965166	·200424	4·989402	·9805005	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1968018	·200727	4·981881	·9804433	39
22	·1970870	·201030	4·974381	·9803860	38
23	·1973722	·201332	4·966903	·9803286	37
24	·1976573	·201635	4·959447	·9802712	36
25	·1979425	·201938	1·952012	·9802136	35
26	·1982276	·202240	4·944599	·9801560	34
27	·1985127	·202543	4·937206	·9800985	33
28	·1987978	·202846	4·929835	·9800405	32
29	·1990829	·203149	4·922485	·9799827	31
30	·1993679	·203452	4·915157	·9799247	30
31	·1996530	·203755	4·907849	·9798667	29
32	·1999380	·204058	4·900562	·9798086	28
33	·2002230	·204361	4·893295	·9797504	27
34	·2005080	·204664	4·886049	·9796921	26
35	·2007930	·204967	4·878824	·9796337	25
36	·2010779	·205270	4·871620	·9795752	24
37	·2013629	·205573	4·864435	·9795167	23
38	·2016478	·205876	4·857271	·9794581	22
39	·2019327	·206180	4·850128	·9793994	21
40	·2022176	·206483	4·843004	·9793406	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
11	·2025024	·206786	4·835901	·9792818	19
42	·2027873	·207090	4·828817	·9792228	18
43	·2030721	·207393	1·821753	·9791638	17
44	·2033569	·207696	4·814709	·9791047	16
45	·2036418	·208000	4·807685	·9790455	15
16	·2039265	·208303	4·800680	·9789862	14
47	·2042113	·208607	4·793695	·9789268	13
48	·2044961	·208910	4·786730	·9788674	12
49	2047808	·209214	4·779783	·9788079	11
50	·2050655	·209518	4·772856	·9787483	10
51	·2053502	·209821	4·765949	·9786886	9
52	·2056349	·210125	4·759060	·9786288	8
53	·2059195	·210429	4·752190	·9785689	7
54	·2062042	·210733	4·745340	·9785090	6
55	·2064888	·211036	4·738508	·9784490	5
56	·2067734	·211340	4·731695	·9783889	4
57	·2070580	·211644	4·724901	·9783287	3
58	·2073426	·211948	4·718125	·9782684	2
59	·2076272	·212252	4·711368	·9782080	1
60	·2079117	·212556	4·704630	·9781476	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2079117	·212556	4·704630	·9781476	60
1	·2081962	·212860	4·697910	·9780871	59
2	·2084807	·213164	4·691208	·9780265	58
3	·2087652	·213468	4·684524	·9779658	57
4	·2090497	·213773	4·677859	·9779050	56
5	·2093341	·214077	4·671212	·9778441	55
6	·2096186	·214381	4·664583	·9777832	54
7	·2099030	·214685	4·657972	·9777222	53
8	·2101874	·214990	4·651378	·9776611	52
9	·2104718	·215294	4·644803	·9775999	51
10	·2107561	·215598	4·638245	·9775386	50
11	·2110405	·215903	4·631705	·9774773	49
12	·2113248	·216207	4·625183	·9774159	48
13	·2116091	·216512	4·618678	·9773544	47
14	·2118934	·216816	4·612190	·9772928	46
15	·2121777	·217121	4·605720	·9772311	45
16	·2124619	·217425	4·599268	·9771693	44
17	·2127462	·217730	4·592832	·9771075	43
18	·2130304	·218035	4·586414	·9770456	42
19	·2133146	·218340	4·580012	·9769836	41
20	·2135988	·218644	4·573628	·9769215	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2138829	·218949	4·567261	·9768593	39
22	·2141671	·219254	4·560911	·9767970	38
23	·2144512	·219559	4·554577	·9767347	37
24	·2147353	·219864	4·548260	·9766723	36
25	·2150194	·220169	4·541960	·9766098	35
26	·2153035	·220474	4·535677	·9765472	34
27	·2155876	·220779	4·529410	·9764845	33
28	·2158716	·221084	4·523160	·9764217	32
29	·2161556	·221389	4·516926	·9763589	31
30	·2164396	·221694	4·510708	·9762960	30
31	·2167236	·221999	4·504507	·9762330	29
32	·2170076	·222305	4·498322	·9761699	28
33	·2172915	·222610	4·492153	·9761067	27
34	·2175754	·222915	4·486000	·9760435	26
35	·2178593	·223221	4·479863	·9759802	25
36	·2181432	·223526	4·473742	·9759168	24
37	·2184271	·223831	4·467637	·9758533	23
38	·2187110	·224137	4·461548	·9757897	22
39	·2189948	·224442	4·455475	·9757260	21
40	·2192786	·224748	4·449418	·9756623	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2195624	·225054	4·443376	·9755985	19
42	·2198462	·225359	4·437350	·9755345	18
43	·2201300	·225665	4·431339	·9754706	17
44	·2204137	·225971	4·425343	·9754065	16
45	·2206974	·226276	4·419364	·9753423	15
46	·2209811	·226582	4·413399	·9752781	14
47	·2212648	·226888	4·407450	·9752138	13
48	·2215485	·227194	4·401516	·9751494	12
49	·2218321	·227500	4·395597	·9750849	11
50	·2221158	·227806	4·389694	·9750203	10
51	·2223994	·228112	4·383805	·9749556	9
52	·2226830	·228418	4·377931	·9748909	8
53	·2229666	·228724	4·372073	·9748261	7
54	·2232501	·229030	4·366229	·9747612	6
55	·2235337	·229336	4·360400	·9746962	5
56	·2238172	·229642	4·354586	·9746311	4
57	·2241007	·229949	4·348786	·9745660	3
58	·2243842	·230255	4·343001	·9745008	2
59	·2246676	·230561	4·337231	·9744355	1
60	·2249511	·230868	4·331475	·9743701	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg 77.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2249511	·230868	4·331475	·9743701	60
1	·2252345	·231174	4·325734	·9743046	59
2	·2255179	·231481	4·320007	·9742390	58
3	·2258013	·231787	4·314295	·9741734	57
4	·2260846	·232094	4·308597	·9741077	56
5	·2263680	·232400	4·302913	·9740419	55
6	·2266513	·232707	4·297244	·9739760	54
7	·2269346	·233014	4·291588	·9739100	53
8	·2272179	·233320	4·285947	·9738439	52
9	·2275012	·233627	4·280319	·9737778	51
10	·2277844	·233934	4·274706	·9737116	50
11	·2280677	·234241	4·269107	·9736453	49
12	·2283509	·234547	4·263521	·9735789	48
13	·2286341	·234854	4·257950	·9735124	47
14	·2289172	·235161	4·252392	·9734458	46
15	·2292004	·235468	4·246848	·9733792	45
16	·2294835	·235775	4·241317	·9733125	44
17	·2297666	·236082	4·235800	·9732457	43
18	·2300497	·236390	4·230297	·9731789	42
19	·2303328	·236697	4·224808	·9731119	41
20	·2306159	·237004	4·219331	·9730449	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine	′
21	·2308989	·237311	4·213869	·9729777	39
22	·2311819	·237618	4·208419	·9729105	38
23	·2314649	·237926	4·202983	·9728432	37
24	·2317479	·238233	4·197560	·9727759	36
25	·2320309	·238541	4·192151	·9727084	35
26	·2323138	·238848	4·186754	·9726409	34
27	·2325967	·239156	4·181371	·9725733	33
28	·2328796	·239463	4·176001	·9725056	32
29	·2331625	·239771	4·170644	·9724378	31
30	·2334454	·240078	4·165299	·9723699	30
31	·2337282	·240386	4·159968	·9723020	29
32	·2340110	·240694	4·154650	·9722339	28
33	·2342938	·241001	4·149344	·9721658	27
34	·2345766	·241309	4·144051	·9720976	26
35	·2348594	·241617	4·138771	·9720294	25
36	·2351421	·241925	4·133504	·9719610	24
37	·2354248	·242233	4·128249	·9718926	23
38	·2357075	·242541	4·123007	·9718240	22
39	·2359902	·242849	4·117778	·9717554	21
40	·2362729	·243157	4·112561	·9716867	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2365555	·243465	4·107356	·9716180	19
42	·2368381	·243773	4·102164	·9715491	18
43	·2371207	·244081	4·096985	·9714802	17
44	·2374033	·244390	4·091817	·9714112	16
45	·2376859	·244698	4·086662	·9713421	15
46	·2379684	·245006	4·081519	·9712729	14
47	·2382510	·245315	4·076389	·9712036	13
48	·2385335	·245623	4·071270	·9711343	12
49	·2388159	·245932	4·066164	·9710649	11
50	·2390984	·246240	4·061070	·9709953	10
51	·2393808	·246519	4·055987	·9709258	9
52	·2396633	·246857	4·050917	·9708561	8
53	·2399457	·247166	4·045859	·9707863	7
54	·2402280	·247475	4·040812	·9707165	6
55	·2405104	·247783	4·035777	·9706466	5
56	·2407927	·248092	4·030755	·9705766	4
57	·2410751	·248401	4·025744	·9705065	3
58	·2413574	·248710	4·020744	·9704363	2
59	·2416396	·249019	4·015757	·9703660	1
60	·2419219	·249328	4·010780	·9702957	0
′	Cosine.	Cotang.	Tang.	Sine.	′

14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2419219	·249328	4.010780	·9702957	60
1	·2422041	·249637	4·005816	·9702253	59
2	·2424863	·249946	4·000863	·9701548	58
3	·2427685	·250255	3·995922	·9700842	57
4	·2430507	·250564	3·990992	·9700135	56
5	·2433329	·250873	3·986073	·9699428	55
6	·2436150	·251182	3·981166	·9698720	54
7	·2438971	·251491	3·976271	·9698011	53
8	·2441792	·251801	3·971386	·9697301	52
9	·2444613	·252110	3·966513	·9696591	51
10	·2447433	·252420	3·961651	·9695879	50
11	·2450254	·252729	3·956801	·9695167	49
12	·2453074	·253038	3·951961	·9694453	48
13	·2455894	·253348	3·947133	·9693740	47
14	·2458713	·253658	3·942315	·9693025	46
15	·2461533	·253967	3·937509	·9692309	45
16	·2464352	·254277	3·932714	·9691593	44
17	·2467171	·254587	3·927929	·9690875	43
18	·2469990	·254896	3·923156	·9690157	42
19	·2472809	·255206	3·918393	·9689438	41
20	·2475627	·255516	3·913642	·9688719	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 75.

14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2478445	·255826	3·908901	·9687998	39
22	·2481263	·256136	3·904171	·9687277	38
23	·2484081	·256446	3·899451	·9686555	37
24	·2486899	·256756	3·894742	·9685832	36
25	·2489716	·257066	3·890044	·9685108	35
26	·2492533	·257376	3·885357	·9684383	34
27	·2495350	·257686	3·880680	·9683658	33
28	·2498167	·257997	3·876014	·9682931	32
29	·2500984	·258307	3·871358	·9682204	31
30	·2503800	·258617	3·866713	·9681476	30
31	·2506616	·258928	3·862078	·9680748	29
32	·2509432	·259238	3·857453	·9680018	28
33	·2512248	·259548	3·852839	·9679288	27
34	·2515063	·259859	3·848235	·9678557	26
35	·2517879	·260169	3·843642	·9677825	25
36	·2520694	·260480	3·839059	·9677092	24
37	·2523508	·260791	3·834486	·9676358	23
38	·2526323	·261101	3·829923	·9675624	22
39	·2529137	·261412	3·825370	·9674888	21
40	·2531952	·261723	3·820828	·9674152	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 75.

14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2534766	·262034	3·816295	·9673415	19
42	·2537579	·262345	3·811773	·9672678	18
43	·2540393	·262656	3·807260	·9671939	17
44	·2543206	·262967	3·802758	·9671200	16
45	·2546019	·263278	3·798266	·9670459	15
46	·2548832	·263589	3·793783	·9669718	14
47	·2551645	·263900	3·789310	·9668977	13
48	·2554458	·264211	3·784848	·9668234	12
49	·2557270	·264522	3·780395	·9667490	11
50	·2560082	·264833	3·775951	·9666746	10
51	·2562894	·265145	3·771518	·9666001	9
52	·2565705	·265456	3·767094	·9665255	8
53	·2568517	·265768	3·762680	·9664508	7
54	·2571328	·266079	3·758276	·9663761	6
55	·2574139	·266390	3·753881	·9663012	5
56	·2576950	·266702	3·749496	·9662263	4
57	·2579760	·267014	3·745120	·9661513	3
58	·2582570	·267325	3·740754	·9660762	2
59	·2585381	·267637	3·736398	·9660011	1
60	·2588190	·267949	3·732050	·9659258	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 75.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2588190	·267949	3·732050	·9659258	60
1	·2591000	·268261	3·727713	·9658505	59
2	·2593810	·268572	3·723384	·9657751	58
3	·2596619	·268884	3·719065	·9656996	57
4	·2599428	·269196	3·714756	·9656240	56
5	·2602237	·269508	3·710455	·9655484	55
6	·2605045	·269820	3·706164	·9654726	54
7	·2607853	·270132	3·701883	·9653968	53
8	·2610662	·270444	3·697610	·9653209	52
9	·2613469	·270757	3·693346	·9652449	51
10	·2616277	·271069	3·689092	·9651689	50
11	·2619085	·271381	3·684847	·9650927	49
12	·2621892	·271694	3·680611	·9650165	48
13	·2624699	·272006	3·676384	·9649402	47
14	·2627506	·272318	3·672166	·9648638	46
15	·2630312	·272631	3·667957	·9647873	45
16	·2633118	·272943	3·663757	·9647108	44
17	·2635925	·273256	3·659566	·9646341	43
18	·2638730	·273569	3·655384	·9645574	42
19	·2641536	·273881	3·651211	·9644806	41
20	·2644342	·274194	3·647046	·9644037	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2647147	·274507	3·642891	·9643268	39
22	·2649952	·274820	3·638744	·9642497	38
23	·2652757	·275133	3·634606	·9641726	37
24	·2655561	·275445	3·630477	·9640954	36
25	·2658366	·275758	3·626356	·9640181	35
26	·2661170	·276071	3·622244	·9639407	34
27	·2663973	·276385	3·618141	·9638633	33
28	·2666777	·276698	3·614046	·9637858	32
29	·2669581	·277011	3·609960	·9637081	31
30	·2672384	·277324	3·605883	·9636305	30
31	·2675187	·277637	3·601814	·9635527	29
32	·2677989	·277951	3·597754	·9634748	28
33	·2680792	·278264	3·593702	·9633969	27
34	·2683594	·278578	3·589659	·9633189	26
35	·2686396	·278891	3·585624	·9632408	25
36	·2689198	·279205	3·581597	·9631626	24
37	·2692000	·279518	3·577579	·9630843	23
38	·2694801	·279832	3·573569	·9630060	22
39	·2697602	·280145	3·569568	·9629275	21
40	·2700403	·280459	3·565574	·9628490	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2703204	·280773	3·561590	·9627704	19
42	·2706004	·281087	3·557613	·9626917	18
43	·2708805	·281401	3·553644	·9626130	17
44	·2711605	·281715	3·549684	·9625342	16
45	·2714404	·282029	3·545732	·9624552	15
46	·2717204	·282343	3·541788	·9623762	14
47	·2720003	·282657	3·537852	·9622972	13
48	·2722802	·282971	3·533925	·9622180	12
49	·2725601	·283285	3·530005	·9621387	11
50	·2728400	·283599	3·526093	·9620594	10
51	·2731198	·283914	3·522190	·9619800	9
52	·2733997	·284228	3·518294	·9619005	8
53	·2736794	·284543	3·514407	·9618210	7
54	·2739592	·284857	3·510527	·9617413	6
55	·2742390	·285172	3·506655	·9616616	5
56	·2745187	·285486	3·502791	·9615818	4
57	·2747984	·285801	3·498935	·9615019	3
58	·2750781	·286115	3·495087	·9614219	2
59	·2753577	·286430	3·491247	·9613418	1
60	·2756374	·286745	3·487414	·9612617	0
′	Cosine.	Cotang.	Tang.	Sine.	′

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2756374	·286745	3·487414	·9612617	60
1	·2759170	·287060	3·483589	·9611815	59
2	·2761965	·287375	3·479772	9611012	58
3	·2764761	·287690	3·475963	·9610208	57
4	·2767556	·288005	3·472161	·9609403	56
5	·2770352	·288320	3·468367	·9608598	55
6	·2773147	·288635	3·464581	·9607792	54
7	·2775941	·288950	3·460802	·9606984	53
8	·2778736	·289265	3·457031	·9606177	52
9	·2781530	·289580	3·453267	·9605368	51
10	·2784324	·289896	3·449512	·9604558	50
11	·2787118	·290211	3·445763	·9603748	49
12	·2789911	·290526	3·442022	·9602937	48
13	·2792704	·290842	3·438289	·9602125	47
14	·2795497	·291157	3·434563	·9601312	46
15	·2798290	·291473	3·430844	·9600499	45
16	·2801083	·291789	3·427133	·9599684	44
17	·2803875	·292104	3·423429	·9598869	43
18	·2806667	·292420	3·419733	·9598053	42
19	·2809459	·292736	3·416044	·9597236	41
20	·2812251	·293052	3·412362	·9596418	40
	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2815042	·293368	3·408688	·9595600	39
22	·2817833	·293683	3·405021	·9594781	38
23	·2820624	·293999	3·401361	·9593961	37
24	·2823415	·294316	3·397708	·9593140	36
25	·2826205	·294632	3·394063	·9592318	35
26	·2828995	·294948	3·390424	·9591496	34
27	·2831785	·295264	3·386793	·9590672	33
28	·2834575	·295580	3·383169	·9589848	32
29	·2837364	·295897	3·379553	·9589023	31
30	·2840153	·296213	3·375943	·9588197	30
31	·2842942	·296529	3·372340	·9587371	29
32	·2845731	·296846	3·368745	·9586543	28
33	·2848520	·297163	3 365156	·9585715	27
34	·2851308	·297479	3·361575	·9584886	26
35	·2854096	·297796	3·358000	·9584056	25
36	·2856884	·298112	3·354433	·9583226	24
37	·2859671	·298429	3·350872	·9582394	23
38	·2862458	·298746	3·347319	·9581562	22
39	·2865246	·299063	3·343772	·9580729	21
40	·2868032	·299380	3·340232	·9579895	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73.

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2870819	·299697	3·336699	·9579060	19
42	·2873605	·300014	3·333173	·9578225	18
43	·2876391	·300331	3·329654	·9577389	17
44	·2879177	·300648	3·326141	·9576552	16
45	·2881963	·300965	3·322636	·9575714	15
46	·2884748	·301283	3·319137	·9574875	14
47	·2887533	·301600	3·315645	·9574035	13
48	·2890318	·301917	3·312159	·9573195	12
49	·2893103	·302235	3·308681	·9572354	11
50	·2895887	·302552	3·305209	·9571512	10
51	·2898671	·302870	3·301743	·9570669	9
52	·2901455	·303187	3·298285	·9569825	8
53	·2904239	·303505	3·294833	·9568981	7
54	·2907022	·303823	3·291387	·9568136	6
55	·2909805	·304141	3·287948	·9567290	5
56	·2912588	·304458	3·284516	·9566443	4
57	·2915371	·304776	3·281090	·9565595	3
58	·2918153	·305094	3·277671	·9564747	2
59	·2920935	·305412	3·274258	·9563898	1
60	·2923717	·305730	3·270852	·9563048	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73.

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2923717	·305730	3·270852	·9563048	60
1	·2926499	·306048	3·267452	·9562197	59
2	·2929280	·306367	3·264059	·9561345	58
3	·2932061	·306685	3·260672	·9560492	57
4	·2934842	·307003	3·257292	·9559639	56
5	·2937623	·307321	3·253918	·9558785	55
6	·2940403	·307640	3·250550	·9557930	54
7	·2943183	·307958	3·247189	·9557074	53
8	·2945963	·308277	3·243834	·9556218	52
9	·2948743	·308595	3·240486	·9555361	51
10	·2951522	·308914	3·237143	·9554502	50
11	·2954302	·309233	3·233807	·9553643	49
12	·2957081	·309551	3·230478	·9552784	48
13	·2959859	·309870	3·227154	·9551923	47
14	·2962638	·310189	3·223837	·9551062	46
15	·2965416	·310508	3·220526	·9550199	45
16	·2968194	·310827	3·217221	·9549336	44
17	·2970971	·311146	3·213922	·9548473	43
18	·2973749	·311465	3·210630	·9547608	42
19	·2976526	·311784	3·207344	·9546743	41
20	·2979303	·312103	3·204063	·9545876	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72.

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2982079	·312422	3·200789	·9515009	39
22	·2984856	·312742	3·197521	·9544141	38
23	·2987632	·313061	3·194259	·9543273	37
24	·2990408	·313381	3·191003	·9542403	36
25	·2993184	·313700	3·187754	·9541533	35
26	·2995959	·314020	3·184510	·9540662	34
27	·2998734	·314339	3·181272	·9539790	33
28	·3001509	·314659	3·178040	·9538917	32
29	·3004284	·314979	3·174814	·9538044	31
30	·3007058	·315298	3·171594	·9537170	30
31	·3009832	·315618	3·168380	·9536294	29
32	·3012606	·315938	3·165172	·9535418	28
33	·3015380	·316258	3·161970	·9534542	27
34	·3018153	·316578	3·158774	·9533664	26
35	·3020926	·316898	3·155584	·9532786	25
36	·3023699	·317218	3·152399	·9531907	24
37	·3026471	·317538	3·149220	·9531027	23
38	·3029244	·317859	3·146047	·9530146	22
39	·3032016	·318179	3·142880	·9529264	21
40	·3034788	·318499	3·139719	·9528382	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3037559	·318820	3·136563	·9527499	19
42	·3040331	·319140	3·133414	·9526615	18
43	·3043102	·319461	3·130270	·9525730	17
44	·3045872	·319781	3·127131	·9524844	16
45	·3048643	·320102	3·123999	·9523958	15
46	·3051413	·320423	3·120872	·9523071	14
47	·3054183	·320744	3·117750	·9522183	13
48	·3056953	·321064	3·114635	·9521294	12
49	3059723	·321385	3·111525	·9520404	11
50	·3062492	·321706	3·108421	·9519514	10
51	·3065261	·322027	3·105322	·9518623	9
52	·3068030	·322348	3·102229	·9517731	8
53	·3070798	·322670	3·099141	·9516838	7
54	·3073566	·322991	3·096059	·9515944	6
55	·3076334	·323312	3·092983	·9515050	5
56	·3079102	·323633	3·089912	·9514154	4
57	·3081869	·323955	3·086846	·9513258	3
58	·3084636	·324276	3·083786	·9512361	2
59	·3087403	·324598	3·080732	·9511464	1
60	·3090170	·324919	3·077683	·9510565	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg 72.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3090170	·324919	3·077683	·9510565	60
1	·3092936	·325241	3·074640	·9509666	59
2	·3095702	·325563	3·071602	·9508766	58
3	·3098468	·325884	3·068569	·9507865	57
4	·3101234	·326206	3·065542	·9506963	56
5	·3103999	·326528	3·062520	·9506061	55
6	·3106764	·326850	3·059503	·9505157	54
7	·3109529	·327172	3·056492	·9504253	53
8	·3112294	·327494	3·053487	·9503348	52
9	·3115058	·327816	3·050486	·9502443	51
10	·3117822	·328138	3·047491	·9501536	50
11	·3120586	·328461	3·044501	·9500629	49
12	·3123349	·328783	3·041517	·9499721	48
13	·3126112	·329105	3·038538	·9498812	47
14	·3128875	·329428	3·035564	·9497902	46
15	·3131638	·329750	3·032595	·9496991	45
16	·3134400	·330073	3·029632	·9496080	44
17	3137163	·330395	3·026673	·9495168	43
18	·3139925	·330718	3·023720	·9494255	42
19	·3142686	·331041	3·020772	·9493341	41
20	·3145448	·331363	3·017830	·9492426	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3148209	·331686	3·014892	·9491511	39
22	·3150969	·332009	3·011960	·9490595	38
23	·3153730	·332332	3·009033	·9489678	37
24	·3156490	·332655	3·006110	·9488760	36
25	·3159250	·332978	3·003193	·9487842	35
26	·3162010	·333302	3·000282	·9486922	34
27	·3164770	·333625	2·997375	·9486002	33
28	·3167529	·333948	2·994473	·9485081	32
29	·3170288	·334271	2·991576	·9484159	31
30	·3173047	·334595	2·988685	·9483237	30
31	·3175805	·334918	2·985798	·9482313	29
32	·3178563	·335242	2·982916	·9481389	28
33	·3181321	·335566	2·980040	·9480464	27
34	·3184079	·335889	2·977168	·9479538	26
35	·3186836	·336213	2·974301	·9478612	25
36	·3189593	·336537	2·971439	·9477684	24
37	·3192350	·336861	2·968583	·9476756	23
38	·3195106	·337185	2·965731	·9475827	22
39	·3197863	·337509	2·962884	·9474897	21
40	·3200619	·337833	2·960042	·9473966	20
′	Cosine	Cotang	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3203374	·338157	2·957205	·9473035	19
42	·3206130	·338481	2·954372	·9472103	18
43	·3208885	·338805	2·951545	·9471170	17
44	·3211640	·339129	2·948722	·9470236	16
45	·3214395	·339454	2·945905	·9469301	15
46	·3217149	·339778	2·943092	·9468366	14
47	·3219903	·340103	2·940284	·9467430	13
48	·3222657	·340427	2·937480	·9466493	12
49	·3225411	·340752	2·934682	·9465555	11
50	·3228164	·341077	2·931888	·9464616	10
51	·3230917	·341401	2·929099	·9463677	9
52	·3233670	·341726	2·926315	·9462736	8
53	·3236422	·342051	2·923535	·9461795	7
54	·3239174	·342376	2·920761	·9460854	6
55	·3241926	·342701	2·917990	·9459911	5
56	·3244678	·343026	2·915225	·9458968	4
57	·3247429	·343351	2·912464	·9458023	3
58	·3250180	·343677	2·909708	·9457078	2
59	·3252931	·344002	2·906957	·9456132	1
60	·3255682	·344327	2·904210	·9455186	0
′	Cosine.	Cotang.	Tang.	Sine.	′

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3255682	·344327	2·904210	·9455186	60
1	·3258432	·344653	2·901468	·9454238	59
2	·3261182	·344978	2·898731	·9453290	58
3	·3263932	·345304	2·895998	·9452341	57
4	·3266681	·345629	2·893270	·9451391	56
5	·3269430	·345955	2·890546	·9450441	55
6	·3272179	·346281	2·887827	·9449489	54
7	·3274928	·346606	2·885113	·9448537	53
8	·3277676	·346932	2·882403	·9447584	52
9	·3280424	·347258	2·879697	·9446630	51
10	·3283172	·347584	2·876997	·9445675	50
11	·3285919	·347910	2·874300	·9444720	49
12	·3288666	·348236	2·871608	·9443764	48
13	·3291413	·348563	2·868921	·9442807	47
14	·3294160	·348889	2·866238	·9441849	46
15	·3296906	·349215	2·863560	·9440890	45
16	·3299653	·349542	2·860886	·9439931	44
17	·3302398	·349868	2·858216	·9438971	43
18	·3305144	·350195	2·855551	·9438010	42
19	·3307889	·350521	2·852891	·9437048	41
20	·3310634	·350848	2·850234	·9436085	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3313379	·351175	2·847583	·9435122	39
22	·3316123	·351501	2·844935	·9434157	38
23	·3318867	·351828	2·842292	·9433192	37
24	·3321611	·352155	2·839653	·9432227	36
25	·3324355	·352482	2·837019	·9431260	35
26	·3327098	·352809	2·834389	·9430293	34
27	·3329841	·353136	2·831763	·9429324	33
28	·3332584	·353464	2·829142	·9428355	32
29	·3335326	·353791	2·826525	·9427386	31
30	·3338069	·354118	2·823912	·9426415	30
31	·3340810	·354446	2·821304	·9425444	29
32	·3343552	·354773	2·818700	·9424471	28
33	·3346293	·355101	2·816100	·9423498	27
34	·3349034	·355428	2·813504	·9422525	26
35	·3351775	·355756	2·810913	·9421550	25
36	·3354516	·356084	2·808326	·9420575	24
37	·3357256	·356411	2·805743	·9419598	23
38	·3359996	·356739	2·803164	·9418621	22
39	·3362735	·357067	2·800590	·9417644	21
40	·3365475	·357395	2·798019	·9416665	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3368214	·357723	2·795453	·9415686	19
42	·3370953	·358051	2·792891	·9414705	18
43	·3373691	·358380	2·790333	·9413724	17
44	·3376429	·358708	2·787780	·9412743	16
45	·3379167	·359036	2·785230	·9411760	15
46	·3381905	·359365	2·782685	·9410777	14
47	·3384642	·359693	2·780144	·9409793	13
48	·3387379	·360022	2·777606	·9408808	12
49	·3390116	·360350	2·775073	·9407822	11
50	·3392852	·360679	2·772544	·9406835	10
51	·3395589	·361008	2·770019	·9405848	9
52	·3398325	·361337	2·767499	·9404860	8
53	·3401060	·361666	2·764982	·9403871	7
54	·3403796	·361994	2·762469	·9402881	6
55	·3406531	·362324	2·759960	·9401891	5
56	·3409265	·362653	2·757456	·9400899	4
57	·3412000	·362982	2·754955	·9399907	3
58	·3414734	·363311	2·752458	·9398914	2
59	·3417468	·363640	2·749966	·9397921	1
60	·3420201	·363970	2·747477	·9396926	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3420201	·363970	2·747477	·9396926	60	21	·3477540	·370903	2·696118	·9375858	39	41	·3532027	·377536	2·648753	·9355468	19
1	·3422935	·364299	2·744992	·9395931	59	22	·3480267	·371234	2·693714	·9374846	38	42	·3534748	·377868	2·646423	·9354440	18
2	·3425668	·364629	2·742512	·9394935	58	23	·3482994	·371565	2·691314	·9373833	37	43	·3537469	·378201	2·644096	·9353412	17
3	·3428400	·364958	2·740035	·9393938	57	24	·3485720	·371896	2·688919	·9372820	36	44	·3540190	·378533	2·641774	·9352382	16
4	·3431133	·365288	2·737562	·9392940	56	25	·3488447	·372227	2·686526	·9371806	35	45	·3542910	·378866	2·639454	·9351352	15
5	·3433865	·365618	2·735093	·9391942	55	26	·3491173	·372559	2·684138	·9370790	34	46	·3545630	·379198	2·637139	·9350321	14
6	·3436597	·365948	2·732628	·9390943	54	27	·3493898	·372890	2·681753	·9369774	33	47	·3548350	·379531	2·634827	·9349289	13
7	·3439329	·366277	2·730167	·9389943	53	28	·3496624	·373221	2·679372	·9368758	32	48	·3551070	·379864	2·632518	.9348257	12
8	·3442060	·366607	2 727710	·9388942	52	29	·3499349	·373553	2·676995	·9367740	31	49	·3553789	·380197	2·630213	·9347223	11
9	·3444791	·366937	2·725256	·9387940	51	30	·3502074	·373884	2·674621	·9366722	30	50	·3556508	·380530	2·627912	·9346189	10
10	·3447521	·367268	2·722807	·9386938	50	31	·3504798	·374216	2·672251	·9365703	29	51	·3559226	·380863	2·625614	·9345154	9
11	·3450252	·367598	2·720362	·9385934	49	32	·3507523	·374547	2·669885	·9364683	28	52	·3561944	·381196	2·623319	·9344119	8
12	·3452982	·367928	2·717920	·9384930	48	33	.3510246	·374879	2·667522	·9363662	27	53	·3564662	·381529	2·621028	·9343082	7
13	·3455712	·368258	2·715482	·9383925	47	34	·3512970	·375211	2·665163	·9362641	26	54	·3567380	·381862	2·618741	·9342045	6
14	·3458441	·368589	2·713048	·9382920	46	35	·3515693	·375543	2·662808	·9361618	25	55	·3570097	·382196	2·616457	·9341007	5
15	·3461171	·368919	2·710618	·9381913	45	36	·3518416	·375875	2·660456	·9360595	24	56	·3572814	·382529	2·614176	·9339968	4
16	·3463900	·369250	2·708192	·9380906	44	37	·3521139	·376207	2·658108	·9359571	23	57	·3575531	·382863	2·611899	·9338928	3
17	·3466628	·369580	2·705769	·9379898	43	38	·3523862	·376539	2·655764	·9358547	22	58	·3578248	·383196	2·609625	·9337888	2
18	·3469357	·369911	2·703351	·9378889	42	39	·3526584	·376871	2·653423	·9357521	21	59	·3580964	·383530	2·607355	·9336846	1
19	·3472085	·370242	2·700936	·9377880	41	40	·3529306	·377203	2·651086	·9356495	20	60	·3583679	·383864	2·605089	·9335804	0
20	·3474812	·370572	2·698525	·9376869	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

21 Deg.　　　　21 Deg.　　　　21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3583679	·383864	2·605089	·9335804	60
1	·3586395	·384197	2·602825	·9334761	59
2	·3589110	·384531	2·600565	9333718	58
3	·3591825	·384865	2·598309	·9332673	57
4	·3594540	·385199	2·596056	·9331628	56
5	·3597254	·385533	2·593806	·9330582	55
6	·3599968	·385867	2·591560	·9329535	54
7	·3602682	·386202	2·589317	·9328488	53
8	·3605395	·386536	2·587078	·9327439	52
9	·3608108	·386870	2·584842	·9326390	51
10	·3610821	·387205	2·582609	·9325340	50
11	·3613534	·387539	2·580380	·9324290	49
12	·3616246	·387874	2·578153	·9323238	48
13	·3618958	·388209	2·575931	·9322186	47
14	·3621669	·388543	2·573711	·9321133	46
15	·3624380	·388878	2·571495	·9320079	45
16	·3627091	·389213	2·569283	·9319024	44
17	·3629802	·389548	2·567073	·9317969	43
18	·3632512	·389883	2·564867	·9316912	42
19	·3635222	·390218	2·562664	·9315855	41
20	·3637932	·390554	2·560464	·9314797	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine	′
21	·3640641	·390889	2·558268	·9313739	39
22	·3643351	·391224	2·556075	·9312679	38
23	·3646059	·391560	2·553885	·9311619	37
24	·3648768	·391895	2·551699	·9310558	36
25	·3651476	·392231	2·549516	·9309496	35
26	·3654184	·392567	2·547335	·9308434	34
27	·3656891	·392902	2·545159	·9307370	33
28	·3659599	·393238	2·542985	·9306306	32
29	·3662306	·393574	2·540815	·9305241	31
30	·3665012	·393910	2·538647	·9304176	30
31	·3667719	·394246	2·536483	·9303109	29
32	·3670425	·394582	2·534323	·9302042	28
33	·3673130	·394918	2·532165	·9300974	27
34	3675836	·395255	2·530011	·9299905	26
35	·3678541	·395591	2·527859	·9298835	25
36	·3681246	·395928	2·525711	·9297765	24
37	·3683950	·396264	2·523566	·9296694	23
38	·3686654	·396601	2·521424	·9295622	22
39	·3689358	·396937	2·519286	·9294549	21
40	·3692061	·397274	2·517150	·9293475	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3694765	·397611	2·515018	·9292401	19
42	·3697468	·397948	2·512889	·9291326	18
43	·3700170	·398285	2·510762	·9290250	17
44	·3702872	·398622	2·508639	·9289173	16
45	·3705574	·398959	2·506519	·9288096	15
46	·3708276	·399296	2·504403	·9287017	14
47	·3710977	·399634	2·502289	·9285938	13
48	·3713678	·399971	2·500178	·9284858	12
49	·3716379	·400308	2·498070	·9283778	11
50	·3719079	·400646	2·495966	·9282696	10
51	·3721780	·400984	2·493864	·9281614	9
52	·3724479	·401321	2·491766	·9280531	8
53	·3727179	·401659	2·489670	·9279447	7
54	·3729878	·401997	2·487578	·9278363	6
55	·3732577	·402335	2·485488	·9277277	5
56	·3735275	·402673	2·483402	·9276191	4
57	·3737973	·403011	2·481319	·9275104	3
58	·3740671	·403349	2·479238	·9274016	2
59	·3743369	·403687	2·477161	·9272928	1
60	·3746066	·404026	2·475086	·9271839	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 68.　　　　Deg. 68.　　　　Deg. 68.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3746066	·404026	2·475086	·9271839	60	21	·3802634	·411149	2·432204	·9248782	39	41	·3856377	·417967	2·392531	·9226503	19
1	·3748763	·404364	2·473015	·9270748	59	22	·3805324	·411489	2·430193	·9247676	38	42	·3859060	·418309	2·390576	·9225381	18
2	·3751459	·404703	2·470947	·9269658	58	23	·3808014	·411830	2·428186	·9246568	37	43	·3861744	·418650	2·388625	·9224258	17
3	·3754156	·405041	2·468881	·9268566	57	24	·3810704	·412170	2·426181	·9245460	36	44	·3864427	·418992	2·386675	·9223134	16
4	·3756852	·405380	2·466819	·9267474	56	25	·3813393	·412510	2·424180	·9244351	35	45	·3867110	·419334	2·384729	·9222010	15
5	·3759547	·405719	2·464759	·9266380	55	26	·3816082	·412851	2·422181	·9243242	34	46	·3869792	·419676	2·382785	·9220884	14
6	·3762243	·406057	2·462703	·9265286	54	27	·3818770	·413191	2·420185	·9242131	33	47	·3872474	·420019	2·380844	·9219758	13
7	·3764938	·406396	2·460649	·9264192	53	28	·3821459	·413532	2·418191	·9241020	32	48	·3875156	·420361	2·378906	·9218632	12
8	·3767632	·406735	2·458598	·9263096	52	29	·3824147	·413872	2·416201	·9239908	31	49	·3877837	·420703	2·376970	·9217504	11
9	·3770327	·407074	2·456551	·9262000	51	30	·3826834	·414213	2·414213	·9238795	30	50	·3880518	·421046	2·375037	·9216375	10
10	·3773021	·407413	2·454506	·9260902	50	31	·3829522	·414554	2·412228	·9237682	29	51	·3883199	·421388	2·373106	·9215246	9
11	·3775714	·407753	2·452464	·9259805	49	32	·3832209	·414895	2·410246	·9236567	28	52	·3885880	·421731	2·371179	·9214116	8
12	·3778408	·408092	2·450425	·9258706	48	33	·3834895	·415236	2·408267	·9235452	27	53	·3888560	·422073	2·369254	·9212986	7
13	·3781101	·408431	2·448389	·9257606	47	34	·3837582	·415577	2·406290	·9234336	26	54	·3891240	·422416	2·367331	·9211854	6
14	·3783794	·408771	2·446355	·9256506	46	35	·3840268	·415918	2·404316	·9233220	25	55	·3893919	·422759	2·365411	·9210722	5
15	·3786486	·409110	2·444325	·9255405	45	36	·3842953	·416259	2·402345	·9232102	24	56	·3896598	·423102	2·363494	·9209589	4
16	·3789178	·409450	2·442298	·9254303	44	37	·3845639	·416601	2·400377	·9230984	23	57	·3899277	·423445	2·361580	·9208455	3
17	·3791870	·409790	2·440273	·9253201	43	38	·3848324	·416942	2·398411	·9229865	22	58	·3901955	·423788	2·359668	·9207320	2
18	·3794562	·410129	2·438251	·9252097	42	39	·3851008	·417284	2·396449	·9228745	21	59	·3904633	·424131	2·357759	·9206185	1
19	·3797253	·410469	2·436233	·9250993	41	40	·3853693	·417625	2·394488	·9227624	20	60	·3907311	·424474	2·355852	·9205049	0
20	·3799944	·410809	2·434217	·9249888	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3907311	·424474	2·355852	·9205049	60
1	·3909989	·424818	2·353948	·9203912	59
2	·3912666	·425161	2·352046	·9202774	58
3	·3915343	·425505	2·350148	·9201635	57
4	·3918019	·425848	2·348251	·9200496	56
5	·3920695	·426192	2·346358	·9199356	55
6	·3923371	·426536	2·344467	·9198215	54
7	·3926047	·426880	2·342578	·9197073	53
8	·3928722	·427223	2·340692	·9195931	52
9	·3931397	·427568	2·338809	·9194788	51
10	·3934071	·427912	2·336928	·9193644	50
11	·3936745	·428256	2·335050	·9192499	49
12	·3939419	·428600	2·333174	·9191353	48
13	·3942093	·428944	2·331301	·9190207	47
14	·3944766	·429289	2·329431	·9189060	46
15	·3947439	·429633	2·327563	·9187912	45
16	·3950111	·429978	2·325697	·9186763	44
17	·3952783	·430323	2·323834	·9185614	43
18	·3955455	·430668	2·321974	·9184464	42
19	·3958127	·431012	2·320116	·9183313	41
20	·3960798	·431357	2·318260	·9182161	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3963468	·431703	2·316407	·9181009	39
22	·3966139	·432048	2·314557	·9179855	38
23	·3968809	·432393	2·312709	·9178701	37
24	·3971479	·432738	2·310863	·9177546	36
25	·3974148	·433084	2·309020	·9176391	35
26	·3976818	·433429	2·307180	·9175234	34
27	·3979486	·433775	2·305342	·9174077	33
28	·3982155	·434120	2·303506	·9172919	32
29	·3984823	·434466	2·301673	·9171760	31
30	·3987491	·434812	2·299842	·9170601	30
31	·3990158	·435158	2·298014	·9169440	29
32	·3992825	·435504	2·296188	·9168279	28
33	·3995492	·435850	2·294365	·9167118	27
34	·3998158	·436196	2·292544	·9165955	26
35	·4000825	·436542	2·290725	·9164791	25
36	·4003490	·436889	2·288909	·9163627	24
37	·4006156	·437235	2·287095	·9162462	23
38	·4008821	·437582	2·285284	·9161297	22
39	·4011486	·437928	2·283475	·9160130	21
40	·4014150	·438275	2·281669	·9158963	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4016814	·438622	2·279865	·9157795	19
42	·4019478	·438969	2·278063	·9156626	18
43	·4022141	·439316	2·276264	·9155456	17
44	·4024804	·439663	2·274467	·9154286	16
45	·4027467	·440010	2·272672	·9153115	15
46	·4030129	·440357	2·270880	·9151943	14
47	·4032791	·440705	2·269090	·9150770	13
48	·4035453	·441052	2·267303	·9149597	12
49	·4038114	·441400	2·265518	·9148422	11
50	·4040775	·441747	2·263735	·9147247	10
51	·4043436	·442095	2·261955	·9146072	9
52	·4046096	·442443	2·260177	·9144895	8
53	·4048756	·442791	2·258401	·9143718	7
54	·4051416	·443139	2·256628	·9142540	6
55	·4054075	·443487	2·254857	·9141361	5
56	·4056734	·443835	2·253088	·9140181	4
57	·4059393	·444183	2·251322	·9139001	3
58	·4062051	·444531	2·249558	·9137819	2
59	·4064709	·444880	2·247796	·9136637	1
60	·4067366	·445228	2·246036	·9135455	0
′	Cosine.	Cotang.	Tang.	Sine.	′

24 Deg. | 24 Deg. | 24 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4067366	·445228	2·246036	·9135455	60	21	·4123096	·452568	2·209611	·9110438	39	41	·4176028	·459596	2·175822	·9086297	19
1	·4070024	·445577	2·244279	·9134271	59	22	·4125745	·452918	2·207901	·9109238	38	42	·4178671	·459948	2·174155	·9085082	18
2	·4072681	·445926	2·242524	·9133087	58	23	·4128395	·453269	2·206193	·9108038	37	43	·4181313	·460301	2·172491	·9083866	17
3	·4075337	·446274	2·240772	·9131902	57	24	·4131044	·453620	2·204487	·9106837	36	44	·4183956	·460653	2·170828	·9082649	16
4	·4077993	·446623	2·239021	·9130716	56	25	·4133693	·453970	2·202784	·9105635	35	45	·4186597	·461006	2·169167	·9081432	15
5	·4080649	·446972	2·237273	·9129529	55	26	·4136342	·454321	2·201083	·9104432	34	46	·4189239	·461359	2·167509	·9080214	14
6	·4083305	·447321	2·235528	·9128342	54	27	·4138990	·454672	2·199384	·9103228	33	47	·4191880	·461711	2·165852	·9078995	13
7	·4085960	·447670	2·233784	·9127154	53	28	·4141638	·455023	2·197687	·9102024	32	48	·4194521	·462064	2·164198	·9077775	12
8	·4088615	·448020	2·232043	·9125965	52	29	·4144285	·455375	2·195992	·9100819	31	49	·4197161	·462417	2·162546	·9076554	11
9	·4091269	·448369	2·230304	·9124775	51	30	·4146932	·455726	2·194299	·9099613	30	50	·4199801	·462771	2·160895	·9075333	10
10	·4093923	·448718	2·228567	·9123584	50	31	·4149579	·456077	2·192609	·9098406	29	51	·4202441	·463124	2·159247	·9074111	9
11	·4096577	·449068	2·226833	·9122393	49	32	·4152226	·456429	2·190921	·9097199	28	52	·4205080	·463477	2·157601	·9072888	8
12	·4099230	·449417	2·225100	·9121201	48	33	·4154872	·456780	2·189234	·9095990	27	53	·4207719	·463831	2·155957	·9071665	7
13	·4101883	·449767	2·223370	·9120008	47	34	·4157517	·457132	2·187551	·9094781	26	54	·4210358	·464184	2·154315	·9070440	6
14	·4104536	·450117	2·221643	·9118815	46	35	·4160163	·457483	2·185869	·9093572	25	55	·4212996	·464538	2·152675	·9069215	5
15	·4107189	·450467	2·219917	·9117620	45	36	4162808	·457835	2·184189	·9092361	24	56	·4215634	·464891	2·151037	·9067989	4
16	·4109841	·450817	2·218194	·9116425	44	37	·4165453	·458187	2·182511	·9091150	23	57	·4218272	·465245	2·149402	·9066762	3
17	·4112492	·451167	2·216473	·9115229	43	38	·4168097	·458539	2·180836	·9089938	22	58	·4220909	·465599	2·147768	·9065535	2
18	·4115144	·451517	2·214754	·9114033	42	39	·4170741	·458891	2·179163	·9088725	21	59	·4223546	·465953	2·146136	·9064307	1
19	·4117795	·451867	2·213037	·9112835	41	40	·4173385	·459243	2·177492	·9087511	20	60	·4226183	·466307	2·144506	·9063078	0
20	·4120445	·452217	2·211323	·9111637	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 65. | Deg. 65. | Deg. 65.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4226183	·466307	2·144506	·9063078	60
1	·4228819	·466661	2·142879	·9061848	59
2	·4231455	·467016	2·141253	·9060618	58
3	·4234090	·467370	2·139630	·9059386	57
4	·4236725	·467725	2·138008	·9058154	56
5	·4239360	·468079	2·136389	·9056922	55
6	·4241994	·468434	2·134771	·9055688	54
7	·4244628	·468789	2·133155	·9054454	53
8	·4247262	·469143	2·131542	·9053219	52
9	·4249895	·469498	2·129930	·9051983	51
10	·4252528	·469853	2·128321	·9050746	50
11	·4255161	·470209	2·126713	·9049509	49
12	·4257793	·470564	2·125108	·9048271	48
13	·4260425	·470919	2·123504	·9047032	47
14	·4263056	·471275	2·121903	·9045792	46
15	·4265687	·471630	2·120303	·9044551	45
16	·4268318	·471986	2·118705	·9043310	44
17	·4270949	·472342	2·117110	·9042068	43
18	·4273579	·472697	2·115516	·9040825	42
19	·4276208	·473053	2·113924	·9039582	41
20	·4278838	·473409	2·112334	·9038338	40
′	Cosine.	Cotang.	Tang.	Sine.	′

25 Deg

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4281467	·473765	2·110747	·9037093	39
22	·4284095	·474122	2·109161	·9035847	38
23	·4286723	·474478	2·107577	·9034600	37
24	·4289351	·474834	2·105995	·9033353	36
25	·4291979	·475191	2·104415	·9032105	35
26	·4294606	·475548	2·102836	·9030856	34
27	·4297233	·475904	2·101260	·9029606	33
28	·4299859	·476261	2·099686	·9028356	32
29	·4302485	·476618	2·098114	·9027105	31
30	·4305111	·476975	2·096543	·9025853	30
31	·4807736	·477332	2·094975	·9024600	29
32	·4310361	·477689	2·093408	·9023347	28
33	·4312986	·478047	2·091843	·9022092	27
34	·4315610	·478404	2·090280	·9020838	26
35	·4318234	·478762	2·088720	·9019582	25
36	·4320857	·479119	2·087161	·9018325	24
37	·4323481	·479477	2·085603	·9017068	23
38	·4326103	·479835	2·084048	·9015810	22
39	·4328726	·480193	2·082495	·9014551	21
40	·4331348	·480551	2·080943	·9013292	20
′	Cosine	Cotang.	Tang.	Sine.	′

25 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4333970	·480909	2·079394	·9012031	19
42	·4336591	·481267	2·077846	·9010770	18
43	·4339212	·481625	2·076300	·9009508	17
44	·4341832	·481984	2·074756	·9008246	16
45	·4344453	·482342	2·073214	·9006982	15
46	·4347072	·482701	2·071674	·9005718	14
47	·4349692	·483060	2·070135	·9004453	13
48	·4352311	·483418	2·068599	·9003188	12
49	·4354930	·483777	2·067064	·9001921	11
50	·4357548	·484136	2·065531	·9000654	10
51	·4360166	·484495	2·064000	·8999386	9
52	·4362784	·484855	2·062471	·8998117	8
53	·4365401	·485214	2·060944	·8996848	7
54	·4368018	·485573	2·059418	·8995578	6
55	·4370634	·485933	2·057895	·8994307	5
56	·4373251	·486293	2·056373	·8993035	4
57	·4375866	·486652	2·054853	·8991763	3
58	·4378482	·487012	2·053334	·8990489	2
59	·4381097	·487372	2·051818	·8989215	1
60	·4383711	·487732	2·050303	·8987940	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 64.

Deg. 64.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4383711	·487732	2·050303	·8987940	60
1	·4386326	·488092	2·048791	·8986665	59
2	·4388940	·488453	2·047280	·8985389	58
3	·4391553	·488813	2·045770	·8984112	57
4	·4394166	·489173	2·044263	·8982834	56
5	·4396779	·489534	2 042757	·8981555	55
6	·4399392	·489894	2·041254	·8980276	54
7	·4402004	·490255	2·039751	·8978996	53
8	·4404615	·490616	2·038251	·8977715	52
9	·4407227	·490977	2·036753	·8976433	51
10	·4409838	·491338	2·035256	·8975151	50
11	·4412448	·491699	2·033761	·8973868	49
12	·4415059	492061	2·032268	·8972584	48
13	·4417668	·492422	2·030776	·8971299	47
14	·4420278	·492783	2·029287	·8970014	46
15	·4422887	·493145	2·027799	·8968727	45
16	·4425496	·493507	2·026313	·8967440	44
17	·4428104	·493868	2·024828	·8966153	43
18	·4430712	·494230	2·023346	·8964864	42
19	·4433319	·494592	2·021865	·8963575	41
20	·4435927	·494954	2·020386	·8962285	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4438534	·495317	2·018908	·8960994	39
22	·4441140	·495679	2·017433	·8959703	38
23	·4443746	·496041	2·015959	·8958411	37
24	·4446352	·496404	2·014486	·8957118	36
25	·4448957	·496766	2·013016	·8955824	35
26	·4451562	·497129	2·011547	·8954529	34
27	·4454167	·497492	2·010080	·8953234	33
28	·4456771	·497855	2·008615	·8951938	32
29	·4459375	·498218	2·007151	·8950641	31
30	·4461978	·498581	2·005689	·8949344	30
31	·4464581	·498944	2·004229	·8948045	29
32	·4467184	·499308	2·002771	·8946746	28
33	·4469786	·499671	2·001314	·8945446	27
34	·4472388	·500035	1·999859	·8944146	26
35	·4474990	·500398	1·998405	·8942844	25
36	·4477591	·500762	1·996953	·8941542	24
37	·4480192	·501126	1·995503	·8940240	23
38	·4482792	·501490	1·994055	·8938936	22
39	·4485392	·501854	1·992608	·8937632	21
40	·4487992	·502218	1·991163	·8936326	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4490591	·502583	1·989720	·8935021	19
42	·4493190	·502947	1·988278	·8933714	18
43	·4495789	·503312	1·986838	·8932406	17
44	·4498387	·503676	1·985400	·8931098	16
45	·4500984	·504041	1·983963	·8929789	15
46	·4503582	·504406	1·982528	·8928480	14
47	·4506179	·504771	1·981095	·8927169	13
48	·4508775	·505136	1·979663	·8925858	12
49	·4511372	·505501	1·978233	·8924546	11
50	·4513967	·505866	1·976805	·8923234	10
51	·4516563	·506232	1·975378	·8921920	9
52	·4519158	·506597	1·973953	·8920606	8
53	·4521753	·506963	1·972529	·8919291	7
54	·4524347	·507329	1·971107	·8917975	6
55	·4526941	·507694	1·969687	·8916659	5
56	·4529535	·508060	1·968268	·8915342	4
57	·4532128	·508426	1·966851	·8914024	3
58	·4534721	·508792	1·965436	·8912705	2
59	·4537313	·509159	1·964022	·8911385	1
60	·4539905	509525	1·962610	·8910065	0
′	Cosine.	Cotang.	Tang.	Sine	′

Deg. 63.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4539905	·509525	1·962610	·8910065	60
1	·4542497	·509891	1·961200	·8908744	59
2	·4545088	·510258	1·959791	·8907423	58
3	·4547679	·510625	1·958383	·8906100	57
4	·4550269	·510991	1·956978	·8904777	56
5	·4552859	·511358	1·955573	·8903453	55
6	·4555449	·511725	1·954171	·8902128	54
7	·4558038	·512093	1·952770	·8900803	53
8	·4560627	·512460	1·951371	·8899476	52
9	·4563216	·512827	1·949973	·8898149	51
10	·4565804	·513195	1·948577	·8896822	50
11	·4568392	·513562	1·947182	·8895493	49
12	·4570979	·513930	1·945789	·8894164	48
13	·4573566	·514298	1·944398	·8892834	47
14	·4576153	·514665	1·943008	·8891503	46
15	·4578739	·515033	1·941620	·8890171	45
16	·4581325	·515401	1·940233	·8888839	44
17	·4583910	·515770	1·938848	·8887506	43
18	·4586496	·516138	1·937464	·8886172	42
19	·4589080	·516506	1·936082	·8884838	41
20	·4591665	·516875	1·934702	·8883503	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine	′
21	·4594248	·517244	1·933323	·8882166	39
22	·4596832	·517612	1·931945	·8880830	38
23	·4599415	·517981	1·930569	·8879492	37
24	·4601998	·518350	1·929195	·8878154	36
25	·4604580	·518719	1·927822	·8876815	35
26	·4607162	·519089	1·926451	·8875475	34
27	·4609744	·519458	1·925081	·8874134	33
28	·4612325	·519827	1·923713	·8872793	32
29	·4614906	·520197	1·922347	·8871451	31
30	·4617486	·520567	1·920982	·8870108	30
31	·4620066	·520936	1·919618	·8868765	29
32	·4622646	·521306	1·918256	·8867420	28
33	·4625225	·521676	1·916896	·8866075	27
34	·4627804	·522046	1·915537	·8864730	26
35	·4630382	·522417	1·914179	·8863383	25
36	·4632960	·522787	1·912823	·8862036	24
37	·4635538	·523157	1·911469	·8860688	23
38	·4638115	·523528	1·910116	·8859339	22
39	·4640692	·523899	1·908764	·8857989	21
40	·4643269	·524269	1·907414	·8856639	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4645845	·524640	1·906066	·8855288	19
42	·4648420	·525011	1·904719	·8853936	18
43	·4650996	·525382	1·903373	·8852584	17
44	·4653571	·525754	1·902029	·8851230	16
45	·4656145	·526125	1·900687	·8849876	15
46	·4658719	·526496	1·899346	·8848522	14
47	·4661293	·526868	1·898006	·8847166	13
48	·4663866	·527240	1·896668	·8845810	12
49	·4666439	·527612	1·895332	·8844453	11
50	·4669012	·527983	1·893997	·8843095	10
51	·4671584	·528356	1·892663	·8841736	9
52	·4674156	·528728	1·891331	·8840377	8
53	·4676727	·529100	1·890000	·8839017	7
54	·4679298	·529472	1·888671	·8837656	6
55	·4681869	·529845	1·887343	·8836295	5
56	·4684439	·530217	1·886017	·8834933	4
57	·4687009	·530590	1·884692	·8833569	3
58	·4689578	·530963	1·883369	·8832206	2
59	·4692147	·531336	1·882047	·8830841	1
60	·4694716	·531709	1·880726	·8829476	0
′	Cosine.	Cotang.	Tang.	Sine.	′

28 Deg. 28 Deg. 28 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4694716	·531709	1·880726	·8829476	60	21	·4748564	·539570	1·853325	·8800633	39	41	·4799683	·547106	1·827799	·8772858	19
1	·4697284	·532082	1·879407	·8828110	59	22	·4751124	·539946	1·852035	·8799251	38	42	·4802235	·547484	1·826537	·8771462	18
2	·4699852	·532455	1·878089	·8826743	58	23	·4753683	·540322	1·850747	·8797869	37	43	·4804786	·547862	1·825276	·8770064	17
3	·4702419	·532829	1·876773	·8825376	57	24	·4756242	·540698	1·849461	·8796486	36	44	·4807337	·548240	1·824017	·8768666	16
4	·4704986	·533202	1·875458	·8824007	56	25	·4758801	·541074	1·848176	·8795102	35	45	·4809888	·548618	1·822759	·8767268	15
5	·4707553	533576	1·874145	·8922638	55	26	·4761359	·541450	1·846892	·8793717	34	46	·4812438	·548997	1·821502	·8765868	14
6	·4710119	·533950	1·872833	·8821269	54	27	·4763917	·541826	1·845609	·8792332	33	47	·4814987	·549375	1·820247	·8764468	13
7	·4712685	·534324	1·871523	·8819898	53	28	·4766474	·542202	1·844328	·8790946	32	48	·4817537	·549754	1·818993	·8763067	12
8	·4715250	·534698	1·870214	·8818527	52	29	·4769031	·542579	1·843049	·8789559	31	49	·4820086	·550133	1·817740	·8761665	11
9	·4717815	·535072	1·868906	·8817155	51	30	·4771588	·542955	1·841770	·8788171	30	50	·4822634	·550512	1·816489	·8760263	10
10	·4720380	·535446	1·867600	·8815782	50	31	·4774144	·543332	1·840494	·8786783	29	51	·4825182	·550891	1·815239	·8758859	9
11	·4722944	·535820	1·866295	·8814409	49	32	·4776700	·543709	1·839218	·8785394	28	52	·4827730	·551270	1·813990	·8757455	8
12	·4725508	·536195	1·864992	·8813035	48	33	·4779255	·544086	1·837944	·8784004	27	53	·4830277	·551650	1·812743	·8756051	7
13	·4728071	·536569	1·863690	·8811660	47	34	·4781810	·544463	1·836671	·8782613	26	54	·4832824	·552029	1·811496	·8754645	6
14	·4730634	·536944	1·862389	·8810284	46	35	·4784364	·544840	1·835399	·8781222	25	55	·4835370	·552409	1·810252	·8753239	5
15	·4733197	·537319	1·861090	·8808907	45	36	·4786919	·545217	1·834129	·8779830	24	56	·4837916	·552789	1·809008	·8751832	4
16	·4735759	·537694	1·859792	·8807530	44	37	·4789472	·545595	1·832861	·8778437	23	57	·4840462	553168	1·807766	·8750425	3
17	·4738321	·538069	1·858496	·8806152	43	38	·4792026	·545972	1·831593	·8777043	22	58	·4843007	·553548	1·806525	·8749016	2
18	·4740882	·538444	1·857201	·8804774	42	39	·4794579	·546350	1·830327	·8775649	21	59	·4845552	·553928	1·805286	·8747607	1
19	·4743443	·538819	1·855908	·8803394	41	40	·4797131	·546728	1·829062	·8774254	20	60	·4848096	·554309	1·804047	·8746197	0
20	·4746004	·539195	1·854615	·8802014	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 61. Deg. 61. Deg. 61.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4848096	·554309	1·804047	·8746197	60
1	·4850640	·554689	1·802810	·8744786	59
2	·4853184	·555069	1·801575	·8743375	58
3	·4855727	·555450	1·800340	·8741963	57
4	·4858270	·555831	1·799107	·8740550	56
5	·4860812	·556211	1·797875	·8739137	55
6	·4863354	·556592	1·796645	·8737722	54
7	·4865895	·556973	1·795416	·8736307	53
8	·4868436	·557355	1·794188	·8734891	52
9	·4870977	·557736	1·792961	·8733475	51
10	·4873517	·558117	1·791736	·8732058	50
11	·4876057	·558499	1·790512	·8730640	49
12	·4878597	·558881	1·789289	·8729221	48
13	·4881136	·559262	1·788067	·8727801	47
14	·4883674	·559644	1·786847	·8726381	46
15	·4886212	·560026	1·785628	·8724960	45
16	·4888750	·560409	1·784410	·8723538	44
17	·4891288	·560791	1·783194	·8722116	43
18	·4893825	·561173	1·781979	·8720693	42
19	·4896361	·561556	1·780765	·8719269	41
20	·4898897	·561939	1·779552	·8717844	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4901433	·562321	1·778340	·8716419	39
22	·4903968	·562704	1·777130	·8714993	38
23	·4906503	·563087	1·775921	·8713566	37
24	·4909038	·563471	1·774714	·8712138	36
25	·4911572	·563854	1·773507	·8710710	35
26	·4914105	·564237	1·772302	·8709281	34
27	·4916638	·564621	1·771098	·8707851	33
28	.4919171	·565005	1·769895	·8706420	32
29	·4921704	·565388	1·768694	·8704989	31
30	·4924236	·565772	1·767494	·8703557	30
31	·4926767	·566156	1·766295	·8702124	29
32	·4929298	·566541	1·765097	·8700691	28
33	·4931829	·566925	1·763900	·8699256	27
34	·4934359	·567309	1·762705	·8697821	26
35	·4936889	·567694	1·761511	·8696386	25
36	·4939419	·568079	1·760318	·8694949	24
37	·4941948	·568463	1·759126	·8693512	23
38	·4944476	·568848	1·757936	·8692074	22
39	·4947005	·569233	1·756747	·8690636	21
40	·4949532	·569619	1·755559	·8689196	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4952060	·570004	1·754372	·8687756	19
42	·4954587	·570389	1·753186	·8686315	18
43	·4957113	·570775	1·752002	·8684874	17
44	·4959639	·571161	1·750819	·8683431	16
45	·4962165	·571547	1·749637	·8681988	15
46	·4964690	·571933	1·748456	·8680544	14
47	·4967215	·572319	1·747276	·8679100	13
48	·4969740	·572705	1·746098	·8677655	12
49	·4972264	·573091	1·744921	·8676209	11
50	·4974787	·573478	1·743745	·8674762	10
51	·4977310	·573864	1·742570	·8673314	9
52	·4979833	.574251	1·741396	·8671866	8
53	·4982355	·574638	1·740224	·8670417	7
54	·4984877	·575025	1·739053	·8668967	6
55	·4987399	·575412	1·737883	·8667517	5
56	·4989920	·575799	1·736714	·8666066	4
57	·4992441	·576187	1·735546	·8664614	3
58	·4994961	·576574	1·734380	·8663161	2
59	·4997481	·576962	1·733214	·8661708	1
60	·5000000	·577350	1·732050	·8660254	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60

30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5000000	·577350	1·732050	·8660254	60
1	·5002519	·577738	1·730887	·8658799	59
2	·5005037	·578126	1·729726	·8657344	58
3	·5007556	·578514	1·728565	·8655887	57
4	·5010073	·578902	1·727406	·8654430	56
5	·5012591	·579291	1·726247	·8652973	55
6	·5015107	·579679	1·725090	·8651514	54
7	·5017624	·580068	1·723934	·8650055	53
8	·5020140	·580457	1·722779	·8648595	52
9	·5022655	·580846	1·721626	·8647134	51
10	·5025170	·581235	1·720473	·8645673	50
11	·5027685	·581624	1·719322	·8644211	49
12	·5030199	·582013	1·718172	·8642748	48
13	·5032713	·582403	1·717023	·8641284	47
14	·5035227	·582793	1·715875	·8639820	46
15	·5037740	·583182	1·714728	·8638355	45
16	·5040252	·583572	1·713582	·8636889	44
17	·5042765	·583962	1·712438	·8635423	43
18	·5045276	·584352	1·711294	·8633956	42
19	·5047788	·584743	1·710152	·8632488	41
20	·5050298	·585133	1·709011	·8631019	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 59.

30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5052809	·585524	1·707871	·8629549	39
22	·5055319	·585914	1·706732	·8628079	38
23	·5057828	·586305	1·705595	·8626608	37
24	·5060338	·586696	1·704458	·8625137	36
25	·5062846	·587087	1·703323	·8623664	35
26	·5065355	·587478	1·702189	·8622191	34
27	·5067863	·587870	1·701055	·8620717	33
28	·5070370	·588261	1·699923	·8619243	32
29	·5072877	·588653	1·698792	·8617768	31
30	·5075384	·589045	1·697663	·8616292	30
31	·5077890	·589436	1·696534	·8614815	29
32	·5080396	·589828	1·695406	·8613337	28
33	·5082901	·590221	1·694280	·8611859	27
34	·5085406	·590613	1·693155	·8610380	26
35	·5087910	·591005	1·692030	·8608901	25
36	·5090414	·591398	1·690907	·8607420	24
37	·5092918	·591791	1·689785	·8605939	23
38	·5095421	·592183	1·688664	·8604457	22
39	·5097924	·592576	1·687544	·8602975	21
40	·5100426	·592969	1·686426	·8601491	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 59.

30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5102928	·593363	1·685308	·8600007	19
42	·5105429	·593756	1·684191	·8598523	18
43	·5107930	·594150	1·683076	·8597037	17
44	·5110431	·594543	1·681962	·8595551	16
45	·5112931	·594937	1·680848	·8594064	15
46	·5115431	·595331	1·679736	·8592576	14
47	·5117930	·595725	1·678625	·8591088	13
48	·5120429	·596119	1·677515	·8589599	12
49	·5122927	·596514	1·676406	·8588109	11
50	·5125425	·596908	1·675298	·8586619	10
51	·5127923	·597303	1·674192	·8585127	9
52	·5130420	·597697	1·673086	·8583635	8
53	·5132916	·598092	1·671981	·8582143	7
54	·5135413	·598487	1·670878	·8580649	6
55	·5137908	·598882	1·669775	·8579155	5
56	·5140404	·599278	1·668674	·8577660	4
57	·5142899	·599673	1·667574	·8576164	3
58	·5145393	·600069	1·666474	·8574668	2
59	·5147887	·600464	1·665376	·8573171	1
60	·5150381	·600860	1·664279	·8571673	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 59.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5150381	·600860	1·664279	·8571673	60
1	·5152874	·601256	1 663183	·8570174	59
2	·5155367	·601652	1·662088	·8568675	58
3	·5157859	·602049	1·660994	·8567175	57
4	·5160351	·602445	1·659901	·8565674	56
5	·5162842	·602841	1·658809	·8564173	55
6	·5165333	·603238	1·657718	·8562671	54
7	·5167824	·603635	1·656629	·8561168	53
8	·5170314	·604032	1·655540	·8559664	52
9	·5172804	·604429	1·654452	·8558160	51
10	·5175293	·604826	1·653366	·8556655	50
11	·5177782	·605224	1·652280	·8555149	49
12	·5180270	·605621	1·651196	·8553643	48
13	·5182758	·606019	1·650112	·8552135	47
14	·5185246	·606417	1·649030	·8550627	46
15	·5187733	·606814	1·647949	·8549119	45
16	·5190219	·607213	1·646868	·8547609	44
17	·5192705	·607611	1·645789	·8546099	43
18	·5195191	·608009	1·644711	·8544588	42
19	·5197676	·608408	1·643633	·8543077	41
20	·5200161	·608806	1·642557	·8541564	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5202646	·609205	1·641482	·8540051	39
22	·5205130	·609604	1·640408	·8538538	38
23	·5207613	·610003	1·639335	·8537023	37
24	·5210096	·610402	1·638263	·8535508	36
25	·5212579	·610801	1·637191	·8533992	35
26	·5215061	·611201	1·636121	·8532475	34
27	·5217543	·611601	1·635052	·8530958	33
28	·5220024	·612000	1·633984	·8529440	32
29	·5222505	·612400	1·632917	·8527921	31
30	·5224986	·612800	1·631851	·8526402	30
31	·5227466	·613201	1·630786	·8524881	29
32	·5229945	·613601	1·629722	·8523360	28
33	·5232424	·614001	1·628659	·8521839	27
34	·5234903	·614402	1·627597	·8520316	26
35	·5237381	·614803	1·626536	·8518793	25
36	·5239859	·615204	1·625476	·8517269	24
37	·5242336	·615605	1·624417	·8515745	23
38	·5244813	·616006	1·623359	·8514219	22
39	·5247290	·616407	1·622302	·8512693	21
40	·5249766	·616809	1·621246	·8511167	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5252241	·617210	1·620192	·8509639	19
42	·5254717	·617612	1·619138	·8508111	18
43	·5257191	·618014	1·618085	·8506582	17
44	·5259665	·618416	1·617033	·8505053	16
45	·5262139	·618818	1·615982	·8503522	15
46	·5264613	·619221	1·614932	·8501991	14
47	·5267085	·619623	1·613882	·8500459	13
48	·5269558	·620026	1·612834	·8498927	12
49	·5272030	·620429	1·611787	·8497394	11
50	·5274502	·620832	1·610741	·8495860	10
51	·5276973	·621235	1·609696	·8494325	9
52	·5279443	·621638	1·608652	·8492790	8
53	·5281914	·622041	1·607609	·8491254	7
54	·5284383	·622445	1·606567	·8489717	6
55	·5286853	·622848	1·605526	·8488179	5
56	·5289322	·623252	1·604485	·8486641	4
57	·5291790	·623656	1·603446	·8485102	3
58	·5294258	·624060	1·602408	·8483562	2
59	·5296726	·624465	1·601370	·8482022	1
60	·5299193	·624869	1·600334	·8480481	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

′	Sine.	Tang.	Cotang.	Cosine.	
0	·5299193	·624869	1·600334	·8480481	60
1	·5301659	·625273	1·599299	·8478939	59
2	·5304125	·625678	1·598264	·8477397	58
3	·5306591	·626083	1·597231	·8475853	57
4	·5309057	·626488	1·596198	·8474309	56
5	·5311521	·626893	1·595167	·8472765	55
6	·5313986	·627298	1·594136	·8471219	54
7	·5316450	·627704	1·593107	·8469673	53
8	·5318913	·628109	1·592078	·8468126	52
9	·5321376	·628515	1·591050	·8466579	51
10	·5323839	·628921	1·590023	·8465030	50
11	·5326301	·629327	1·588997	·8463481	49
12	·5328763	·629733	1·587973	·8461932	48
13	·5331224	·630139	1·586949	·8460381	47
14	·5333685	·630546	1·585926	·8458830	46
15	·5336145	·630953	1·584904	·8457278	45
16	·5338605	·631359	1·583883	·8455726	44
17	·5341066	·631766	1·582862	·8454172	43
18	·5343523	·632173	1·581843	·8452618	42
19	·5345982	·632581	1·580825	·8451064	41
20	·5348440	632988	1·579807	·8449508	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5350898	·633395	1·578791	·8447952	39
22	·5353355	·633803	1·577776	·8446395	38
23	·5355812	·634211	1·576761	·8444838	37
24	·5358268	·634619	1·575747	·8443279	36
25	·5360724	·635027	1·574735	·8441720	35
26	·5363179	·635435	1·573723	·8440161	34
27	·5365634	·635844	1·572712	·8438600	33
28	·5368089	·636252	1·571702	·8437039	32
29	·5370543	·636661	1·570693	·8435477	31
30	·5372996	·637070	1·569685	·8433914	30
31	·5375449	·637479	1·568678	·8432351	29
32	·5377902	·637888	1·567672	·8430787	28
33	·5380354	·638297	1·566666	·8429222	27
34	·5382806	·638707	1·565662	·8427657	26
35	·5385257	·639116	1·564659	·8426091	25
36	·5387708	·639526	1·563656	·8424524	24
37	·5390158	·639936	1·562654	·8422956	23
38	·5392608	·640346	1·561654	·8421388	22
39	·5395058	·640756	1·560654	·8419819	21
40	·5397507	·641167	1·559655	·8418249	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5399955	·641577	1·558657	·8416679	19
42	·5402403	·641988	1·557660	·8415108	18
43	·5404851	·642399	1·556663	·8413536	17
44	·5407298	·642810	1·555668	·8411963	16
45	·5409745	·643221	1·554674	·8410390	15
46	·5412191	·643632	1·553680	·8408816	14
47	·5414637	·644044	1·552688	·8407241	13
48	5417082	·644456	1·551696	·8405666	12
49	.5419527	·644867	1·550705	·8404090	11
50	·5421971	·645279	1·549715	·8402513	10
51	·5424415	·645691	1·548726	·8400936	9
52	·5426859	·646104	1·547738	·8399357	8
53	·5429302	·646516	1·546751	·8397778	7
54	·5431744	·646929	1·545764	·8396199	6
55	·5434187	·647341	1·544779	·8394618	5
56	·5436628	·647754	1·543794	·8393037	4
57	·5439069	·648167	1·542810	·8391455	3
58	·5441510	·648580	1·541828	·8389873	2
59	·5443951	·648994	1·540846	·8388290	1
60	·5446390	·649407	1·539865	·8386706	0
′	Cosine.	Cotang.	Tang.	Sine.	′

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5446390	·649407	1·539865	·8386706	60
1	·5448830	·649821	1·538884	·8385121	59
2	·5451269	·650235	1·537905	·8383536	58
3	·5453707	·650649	1·536927	·8381950	57
4	·5456145	·651063	1·535949	·8380363	56
5	·5458583	·651477	1·534972	·8378775	55
6	·5461020	·651891	1·533996	·8377187	54
7	·5463456	·652306	1·533021	·8375598	53
8	·5465892	·652721	1·532047	·8374009	52
9	·5468328	·653136	1·531074	·8372418	51
10	·5470763	·653551	1·530102	·8370827	50
11	·5473198	·653966	1·529130	·8369236	49
12	·5475632	·654381	1·528160	·8367643	48
13	·5478066	·654797	1·527190	·8366050	47
14	·5480499	·655212	1·526221	·8364456	46
15	·5482932	·655628	1·525253	·8362862	45
16	·5485365	·656044	1·524286	·8361266	44
17	·5487797	·656460	1·523320	·8359670	43
18	·5490228	·656877	1·522354	·8358074	42
19	·5492659	·657293	1·521389	·8356476	41
20	·5495090	·657710	1·520426	·8354878	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5497520	·658127	1·519463	·8353279	39
22	·5499950	·658544	1·518501	·8351680	38
23	·5502379	·658961	1 517540	·8350080	37
24	·5504807	·659378	1·516579	·8348479	36
25	·5507236	·659796	1·515620	·8346877	35
26	·5509663	·660213	1·514661	·8345275	34
27	·5512091	·660631	1·513703	·8343672	33
28	·5514518	·661049	1·512746	·8342068	32
29	·5516944	·661467	1·511790	·8340463	31
30	·5519370	·661885	1·510835	·8338858	30
31	·5521795	·662304	1·509880	·8337252	29
32	·5524220	·662722	1·508927	·8335646	28
33	·5526645	·663141	1·507974	·8334038	27
34	·5529069	·663560	1·507022	·8332430	26
35	·5531492	·663979	1·506071	8330822	25
36	·5533915	·664398	1·505121	·8329212	24
37	·5536338	·664817	1·504171	·8327602	23
38	·5538760	·665237	1·503222	·8325991	22
39	·5541182	·665657	1·502275	·8324380	21
40	·5543603	·666076	1·501328	·8322768	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5546024	·666496	1·500382	·8321155	19
42	·5548444	·666917	1·499436	·8319541	18
43	·5550864	·667337	1·498492	·8317927	17
44	·5553283	·667758	1·497548	·8316312	16
45	·5555702	·668178	1·496605	·8314696	15
46	·5558121	·668599	1·495663	·8313080	14
47	5560539	·669020	1·494722	·8311463	13
48	·5562956	·669441	1·493782	·8309845	12
49	·5565373	·669863	1·492842	·8308226	11
50	·5567790	·670284	1·491903	·8306607	10
51	·5570206	·670706	1·490965	·8304987	9
52	·5572621	·671128	1·490028	·8303366	8
53	·5575036	·671550	1·489092	·8301745	7
54	·5577451	·671972	1·488157	·8300123	6
55	·5579865	·672394	1·487222	·8298500	5
56	·5582279	·672816	1·486288	·8296877	4
57	·5584692	·673239	1·485355	·8295252	3
58	·5587105	·673662	1·484423	·8293628	2
59	·5589517	·674085	1·483491	·8292002	1
60	·5591929	·674508	1·482561	·8290376	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56

Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5591929	·674508	1·482561	·8290376	60
1	·5594340	·674931	1·481631	·8288749	59
2	·5596751	·675355	1·480702	·8287121	58
3	·5599162	·675779	1·479773	·8285493	57
4	·5601572	·676202	1·478846	·8283864	56
5	·5603981	·676626	1·477919	·8282234	55
6	·5606390	·677050	1·476993	·8280603	54
7	·5608798	·677475	1·476068	·8278972	53
8	·5611206	·677899	1·475144	·8277340	52
9	·5613614	·678324	1·474221	·8275708	51
10	·5616021	·678749	1·473298	·8274074	50
11	·5618428	·679174	1·472376	·8272440	49
12	·5620834	·679599	1·471455	·8270806	48
13	·5623239	·680024	1·470535	·8269170	47
14	·5625645	·680450	1·469615	·8267534	46
15	·5628049	·680875	1·468696	·8265897	45
16	·5630453	·681301	1·467778	·8264260	44
17	·5632857	·681727	1·466861	·8262622	43
18	·5635260	·682153	1·465945	·8260983	42
19	·5637663	·682580	1·465029	·8259343	41
20	·5640066	·683006	1·464114	·8257703	40
′	Cosine.	Cotang.	Tang.	Sine.	′

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5642467	·683433	1·463200	·8256062	39
22	·5644869	·683860	1·462287	·8254420	38
23	·5647270	·684287	1·461374	·8252778	37
24	·5649670	·684714	1·460463	·8251135	36
25	·5652070	·685141	1·459552	·8249491	35
26	·5654469	·685569	1·458642	·8247847	34
27	·5656868	·685996	1·457732	·8246202	33
28	·5659267	·686424	1·456824	·8244556	32
29	·5661665	·686852	1·455916	·8242909	31
30	·5664062	·687281	1·455009	·8241262	30
31	·5666459	·687709	1·454102	·8239614	29
32	·5668856	·688137	1·453197	·8237965	28
33	·5671252	·688566	1·452292	·8236316	27
34	·5673648	·688995	1·451388	·8234666	26
35	·5676043	·689424	1·450485	·8233015	25
36	·5678437	·689853	1·449582	·8231364	24
37	·5680832	·690283	1·448680	·8229712	23
38	·5683225	·690712	1·447779	·8228059	22
39	·5685619	·691142	1·446879	·8226405	21
40	·5688011	·691572	1·445980	·8224751	20
′	Cosine.	Cotang.	Tang.	Sine.	′

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5690403	·692002	1·445081	·8223096	19
42	·5692795	·692432	1·444183	·8221440	18
43	·5695187	·692863	1·443286	·8219784	17
44	·5697577	·693293	1·442389	·8218127	16
45	·5699968	·693724	1·441494	·8216469	15
46	·5702357	·694155	1·440599	·8214811	14
47	·5704747	·694586	1·439704	·8213152	13
48	·5707136	·695018	1·438811	·8211492	12
49	·5709524	·695449	1·437918	·8209832	11
50	·5711912	·695881	1·437026	·8208170	10
51	·5714299	·696313	1·436135	·8206509	9
52	·5716686	·696745	1·435245	·8204846	8
53	·5719073	·697177	1·434355	·8203183	7
54	·5721459	·697609	1·433466	·8201519	6
55	·5723844	.698042	1·432578	·8199854	5
56	·5726229	·698474	1·431690	·8198189	4
57	·5728614	·698907	1.430803	·8196523	3
58	·5730998	·699340	1.429917	·8194856	2
59	·5733381	·699774	1.429032	·8193189	1
60	·5735764	·700207	1.428148	·8191520	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55. Deg. 55. Deg. 55

35 Deg.

′	Sine.	Tang	Cotang.	Cosine.	′
0	·5735764	·700207	1·428148	·8191520	60
1	·5738147	·700641	1·427264	·8189852	59
2	·5740529	·701074	1·426381	·8188182	58
3	·5742911	·701508	1·425498	·8186512	57
4	·5745292	·701943	1·424617	·8184841	56
5	·5747672	·702377	1·423736	·8183169	55
6	·5750053	·702811	1·422856	·8181497	54
7	·5752432	·703246	1·421976	·8179824	53
8	·5754811	·703681	1·421097	·8178151	52
9	·5757190	·704116	1·420220	·8176476	51
10	·5759568	·704551	1·419342	·8174801	50
11	·5761946	·704986	1·418466	·8173125	49
12	·5764323	·705422	1·417590	·8171449	48
13	·5766700	·705858	1·416715	·8169772	47
14	·5769076	·706294	1·415840	·8168094	46
15	·5771452	·706730	1·414967	·8166416	45
16	·5773827	·707166	1·414094	·8164736	44
17	·5776202	·707602	1·413222	·8163056	43
18	·5778576	·708039	1·412350	·8161376	42
19	·5780950	·708476	1·411479	·8159695	41
20	·5783323	·708913	1·410609	8158013	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·5785696	·709350	1·409740	·8156330	39
22	·5788069	·709787	1·408871	·8154647	38
23	·5790440	·710225	1·408003	·8152963	37
24	·5792812	·710663	1·407136	·8151278	36
25	.5795183	·711100	1·406270	·8149593	35
26	·5797553	·711539	1·405404	·8147906	34
27	·5799923	·711977	1·404539	·8146220	33
28	·5802292	·712415	1·403674	·8144532	32
29	·5804661	·712854	1·402811	·8142844	31
30	·5807030	·713293	1·401948	·8141155	30
31	·5809397	·713732	1·401086	·8139466	29
32	·5811765	·714171	1·400224	·8137775	28
33	·5814132	·714610	1·399363	·8136084	27
34	·5816498	·715050	1·398503	·8134393	26
35	·5818864	·715490	1·397644	·8132701	25
36	·5821230	·715929	1·396785	·8131008	24
37	·5823595	·716369	1·395927	·8129314	23
38	·5825959	·716810	1·395069	·8127620	22
39	·5828323	·717250	1·394213	·8125925	21
40	·5830687	·717691	1·393357	·8124229	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5833050	·718131	1·392501	·8122532	19
42	·5835412	·718572	1·391647	·8120835	18
43	·5837774	·719014	1·390793	·8119137	17
44	·5840136	·719455	1·389940	·8117439	16
45	·5842497	·719897	1·389087	·8115740	15
46	·5844857	·720338	1·388235	·8114040	14
47	·5847217	·720780	1·387384	·8112339	13
48	·5849577	·721222	1·386534	·8110638	12
49	·5851936	·721665	1·385684	·8108936	11
50	·5854294	·722107	1·384835	·8107234	10
51	·5856652	·722550	1·383986	·8105530	9
52	·5859010	·722993	1·383139	·8103826	8
53	·5861367	·723436	1·382292	·8102122	7
54	·5863724	·723879	1·381445	·8100416	6
55	·5866080	·724322	1·380600	·8098710	5
56	·5868435	·724766	1·379755	·8097004	4
57	·5870790	·725210	1·378910	·8095296	3
58	·5873145	·725654	1·378067	·8093588	2
59	·5875499	·726098	1·377224	·8091879	1
60	·5877853	·726542	1·376381	·8090170	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5877853	·726542	1·376381	·8090170	60
1	·5880206	·726987	1·375540	·8088460	59
2	·5882558	·727431	1·374699	·8086749	58
3	·5884910	·727876	1·373859	·8085037	57
4	·5887262	·728321	1·373019	·8083325	56
5	5889613	·728767	1·372180	·8081612	55
6	·5891964	·729212	1·371342	·8079899	54
7	·5894314	·729658	1·370504	·8078185	53
8	·5896663	·730104	1·369667	·8076470	52
9	·5899012	·730550	1·368831	·8074754	51
10	·5901361	·730996	1·367995	·8073038	50
11	·5903709	·731442	1·367161	·8071321	49
12	·5906057	·731889	1·366326	·8069603	48
13	·5908404	·732336	1·365493	·8067885	47
14	·5910750	·732783	1·364660	·8066166	46
15	·5913096	·733230	1·363827	·8064446	45
16	·5915442	·733677	1·362996	·8062726	44
17	·5917787	·734125	1·362165	·8061005	43
18	·5920132	·734573	1·361335	·8059283	42
19	·5922476	·735021	1·360505	·8057560	41
20	·5924819	·735469	1·359676	·8055837	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5927163	·735917	1·358848	·8054113	39
22	·5929505	·736366	1·358020	·8052389	38
23	·5931847	·736814	1·357193	·8050664	37
24	·5934189	·737263	1·356367	·8048938	36
25	·5936530	·737712	1·355541	·8047211	35
26	·5938871	·738162	1·354716	·8045484	34
27	·5941211	·738611	1·353891	·8043756	33
28	·5943550	·739061	1·353068	·8042028	32
29	·5945889	·739511	1·352244	·8040299	31
30	·5948228	·739961	1·351422	·8038569	30
31	·5950566	·740411	1·350600	·8036838	29
32	·5952904	·740861	1·349779	·8035107	28
33	·5955241	·741312	1·348958	·8033375	27
34	·5957577	·741763	1·348139	·8031642	26
35	·5959913	·742214	1·347319	·8029909	25
36	·5962249	·742665	1·346501	·8028175	24
37	·5964584	·743117	1.345683	·8026440	23
38	·5966918	·743568	1.344865	·8024705	22
39	·5969252	·744020	1.344049	·8022969	21
40	·5971586	·744472	1.343233	·8021232	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5973919	·744924	1·342417	·8019495	19
42	·5976251	·745377	1·341602	·8017756	18
43	·5978583	·745829	1·340788	·8016018	17
44	·5980915	·746282	1·339975	·8014278	16
45	·5983246	·746735	1·339162	·8012538	15
46	·5985577	·747188	1·338350	·8010797	14
47	·5987906	·747642	1·337538	·8009056	13
48	·5990236	·748095	1·336727	·8007314	12
49	·5992565	·748549	1·335917	·8005571	11
50	·5994893	·749003	1·335107	·8003827	10
51	·5997221	·749457	1·334298	·8002083	9
52	·5999549	·749911	1·333490	·8000338	8
53	·6001876	·750366	1·332682	·7998593	7
54	·6004202	·750821	1·331875	·7996847	6
55	·6006528	·751276	1·331068	·7995100	5
56	·6008854	·751731	1·330262	·7993352	4
57	·6011179	·752186	1·329457	·7991604	3
58	·6013503	·752642	1·328652	·7989855	2
59	·6015827	·753098	1·327848	·7988105	1
60	·6018150	·753554	1·327044	·7986355	0
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6018150	·753554	1·327044	·7986355	60
1	·6020473	·754010	1·326242	·7984604	59
2	·6022795	·754466	1·325439	·7982853	58
3	.6025117	·754923	1·324638	·7981100	57
4	·6027439	·755379	1·323837	·7979347	56
5	·6029760	·755836	1·323036	·7977594	55
6	·6032080	·756294	1·322237	·7975839	54
7	·6034400	·756751	1·321437	·7974084	53
8	·6036719	·757209	1·320639	·7972329	52
9	·6039038	·757666	1·319841	·7970572	51
10	·6041356	·758124	1·319044	·7968815	50
11	·6043674	·758582	1·318247	·7967058	49
12	·6045991	·759041	1·317451	·7965299	48
13	·6048308	·759499	1·316555	·7963540	47
14	·6050624	·759958	1·315861	·7961780	46
15	·6052940	·760417	1.315066	·7960020	45
16	·6055255	·760876	1·314273	·7958259	44
17	·6057570	·761336	1.313480	·7956497	43
18	·6059884	·761795	1.312687	·7954735	42
19	·6062198	·762255	1.311895	.7952972	41
20	·6064511	·762715	1.311104	·7951208	40
′	Cosine.	Cotang.	Tang.	Sine.	′

37 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6066824	·763175	1·310314	·7949444	39
22	·6069136	·763636	1·309523	·7947678	38
23	·6071447	·764096	1·308734	·7945913	37
24	·6073758	·764557	1·307945	·7944146	36
25	·6076069	·765018	1·307157	·7942379	35
26	·6078379	·765480	1·306369	·7940611	34
27	·6080689	·765941	1·305582	·7938843	33
28	·6082998	·766403	1·304796	·7937074	32
29	·6085306	·766864	1·304010	·7935304	31
30	·6087614	·767327	1·303225	·7933533	30
31	·6089922	·767789	1·302440	·7931762	29
32	·6092229	·768251	1·301656	·7929990	28
33	·6094535	·768714	1·300873	·7928218	27
34	·6096841	·769177	1·300090	·7926445	26
35	·6099147	·769640	1·299308	·7924671	25
36	·6101452	·770103	1·298526	·7922896	24
37	·6103756	·770567	1·297745	·7921121	23
38	·6106060	·771030	1·296964	·7919345	22
39	·6108363	·771494	1·296185	·7917569	21
40	·6110666	·771958	1·295405	·7915792	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 52.

37 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6112969	·772423	1·294627	·7914014	19
42	·6115270	·772887	1·293848	·7912235	18
43	·6117572	·773352	1·293071	·7910456	17
44	·6119873	·773817	1·292294	·7908676	16
45	·6122173	·774282	1·291517	·7906896	15
46	·6124473	·774748	1·290742	·7905115	14
47	·6126772	·775213	1·289966	·7903333	13
48	·6129071	·775679	1·289192	·7901550	12
49	·6131369	·776145	1·288418	·7899767	11
50	·6133666	·776611	1·287644	·7897983	10
51	·6135964	·777078	1·286871	·7896198	9
52	·6138260	·777544	1·286099	·7894413	8
53	·6140556	·778011	1·285327	·7892627	7
54	·6142852	·778478	1·284556	·7890841	6
55	·6145147	·778946	1·283786	·7889054	5
56	·6147442	·779413	1·283016	·7887266	4
57	·6149736	·779881	1·282246	·7885477	3
58	·6152029	·780349	1·281477	·7883688	2
59	·6154322	·780817	1·280709	·7881898	1
60	·6156615	·781285	1·279941	·7880108	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 52.

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6156615	·781285	1·279941	·7880108	60
1	·6158907	·781754	1·279174	·7878316	59
2	·6161198	·782222	1·278407	·7876524	58
3	·6163489	·782691	1·277641	·7874732	57
4	·6165780	·783161	1·276876	·7872939	56
5	·6168069	·783630	1·276111	·7871145	55
6	·6170359	·784100	1·275347	·7869350	54
7	·6172648	·784570	1·274583	·7867555	53
8	·6174936	·785040	1·273820	·7865759	52
9	·6177224	·785510	1·273057	·7863963	51
10	·6179511	·785980	1·272295	·7862165	50
11	·6181798	·786451	1·271534	·7860367	49
12	·6184084	·786922	1·270773	·7858569	48
13	·6186370	·787393	1·270013	·7856770	47
14	·6188655	·787864	1·269253	·7854970	46
15	·6190939	·788336	1·268494	·7853169	45
16	·6193224	788808	1·267735	·7851368	44
17	·6195507	·789280	1·266977	·7849566	43
18	·6197790	·789752	1·266219	·7847764	42
19	·6200073	·790224	1·265462	·7845961	41
20	·6202355	·790697	1·264706	·7844157	40
′	Cosine.	Cotang.	Tang.	Sine.	′

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6204636	·791170	1·263950	·7842352	39
22	·6206917	·791643	1·263195	·7840547	38
23	·6209198	·792116	1·262440	·7838741	37
24	·6211478	·792590	1·261686	·7836935	36
25	·6213757	·793064	1·260932	·7835127	35
26	·6216036	·793537	1·260179	·7833320	34
27	·6218314	·794012	1·259426	·7831511	33
28	·6220592	·794486	1·258674	·7829702	32
29	·6222870	·794961	1·257923	·7827892	31
30	·6225146	·795435	1·257172	·7826082	30
31	·6227423	·795911	1·256421	·7824270	29
32	·6229698	·796386	1·255672	·7822459	28
33	·6231974	·796861	1·254922	·7820646	27
34	·6234248	·797337	1·254174	·7818833	26
35	·6236522	·797813	1·253426	·7817019	25
36	·6238796	·798289	1·252678	·7815205	24
37	·6241069	·798765	1·251931	·7813390	23
38	·6243342	·799242	1·251184	·7811574	22
39	·6245614	·799719	1·250438	·7809757	21
40	·6247885	·800196	1·249693	·7807940	20
′	Cosine.	Cotang.	Tang.	Sine.	′

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6250156	·800673	1·248948	·7806123	19
42	·6252427	·801151	1·248204	·7804304	18
43	·6254696	·801628	1·247460	·7802485	17
44	·6256966	·802106	1·246716	·7800665	16
45	·6259235	·802584	1·245974	·7798845	15
46	·6261503	·803063	1·245232	·7797024	14
47	·6263771	·803541	1·244490	·7795202	13
48	·6266038	·804020	1·243749	·7793380	12
49	·6268305	·804499	1·243008	·7791557	11
50	·6270571	·804979	1·242268	·7789733	10
51	·6272837	·805458	1·241529	·7787909	9
52	·6275102	·805938	1·240790	·7786084	8
53	·6277366	·806418	1·240051	·7784258	7
54	·6279631	·806898	1·239313	·7782431	6
55	·6281894	·807378	1·238576	·7780604	5
56	·6284157	·807859	1·237839	·7778777	4
57	·6286420	·808340	1·237103	·7776949	3
58	·6288682	·808821	1·236367	·7775120	2
59	·6290943	·809302	1·235631	·7773290	1
60	·6293204	·809784	1·234897	·7771460	0
′	Cosine.	Cotang.	Tang.	Sine.	′

39 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6293204	·809784	1·234897	·7771460	60
1	·6295464	·810265	1·234162	·7769629	59
2	·6297724	·810747	1·233429	·7767797	58
3	·6299983	·811230	1·232696	·7765965	57
4	·6302242	·811712	1·231963	·7764132	56
5	·6304500	·812195	1·231231	·7762298	55
6	·6306758	·812678	1·230499	·7760464	54
7	·6309015	·813161	1·229768	·7758629	53
8	·6311272	·813644	1·229038	·7756794	52
9	·6313528	·814128	1·228308	·7754957	51
10	·6315784	·814611	1·227578	·7753121	50
11	·6318039	·815095	1·226849	·7751283	49
12	·6320293	·815580	1·226121	·7749445	48
13	·6322547	·816064	1·225393	·7747606	47
14	·6324800	·816549	1·224665	·7745767	46
15	·6327053	·817034	1·223938	·7743926	45
16	·6329306	·817519	1·223212	·7742086	44
17	·6331557	·818004	1·222486	·7740244	43
18	·6333809	·818490	1·221761	·7738402	42
19	·6336059	·818976	1·221036	·7736559	41
20	·6338310	·819462	1·220312	·7734716	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 50.

39 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6340559	·819948	1·219588	·7732872	39
22	·6342808	·820435	1·218865	·7731027	38
23	·6345057	·820922	1·218142	·7729182	37
24	·6347305	·821409	1·217419	·7727336	36
25	·6349553	·821896	1·216698	·7725489	35
26	·6351800	·822384	1·215976	·7723642	34
27	·6354046	·822871	1·215256	·7721794	33
28	·6356292	·823359	1·214535	·7719945	32
29	·6358537	·823847	1·213816	·7718096	31
30	·6360782	·824336	1·213097	·7716246	30
31	·6363026	·824825	1·212378	·7714395	29
32	·6365270	·825314	1·211660	·7712544	28
33	·6367513	·825803	1·210942	·7710692	27
34	·6369756	·826292	1·210225	·7708840	26
35	·6371998	·826782	1·209508	·7706986	25
36	·6374240	·827271	1·208792	·7705132	24
37	·6376481	·827762	1·208076	·7703278	23
38	·6378721	·828252	1·207361	·7701423	22
39	·6380961	·828742	1·206646	·7699567	21
40	·6383201	·829233	1·205932	·7697710	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 50.

39 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6385440	·829724	1·205219	·7695853	19
42	·6387678	·830216	1·204505	·7693996	18
43	·6389916	·830707	1·203793	·7692137	17
44	·6392153	·831199	1·203081	·7690278	16
45	·6394390	·831691	1·202369	·7688418	15
46	·6396626	·832183	1·201658	·7686558	14
47	·6398862	·832675	1·200947	·7684697	13
48	·6401097	·833168	1·200237	·7682835	12
49	·6403332	·833661	1·199527	·7680973	11
50	·6405566	·834154	1·198818	·7679110	10
51	·6407799	·834648	1·198109	·7677246	9
52	·6410032	·835141	1·197401	·7675382	8
53	·6412264	·835635	1·196693	·7673517	7
54	·6414496	·836129	1·195986	·7671652	6
55	·6416728	·836624	1·195279	·7669785	5
56	·6418958	·837118	1·194573	·7667918	4
57	·6421189	·837613	1·193867	·7666051	3
58	·6423418	·838108	1·193162	·7664183	2
59	·6425647	·838604	1·192457	·7662314	1
60	·6427876	·839099	1·191753	·7660444	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 50.

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6427876	·839099	1·191753	·7660444	60
1	·6430104	·839595	1·191049	·7658574	59
2	·6432332	·840091	1.190346	·7656704	58
3	·6434559	·840587	1·189643	·7654832	57
4	·6436785	·841084	1·188941	·7652960	56
5	·6439011	·841581	1·188239	·7651087	55
6	·6441236	·842078	1·187538	·7649214	54
7	·6443461	·842575	1·186837	·7647340	53
8	·6445685	·843073	1·186136	·7645465	52
9	·6447909	·843570	1·185437	·7643590	51
10	·6450132	·844068	1·184737	·7641714	50
11	·6452355	·844567	1·184038	·7639838	49
12	·6454577	·845065	1·183340	·7637960	48
13	·6456798	·845564	1·182642	·7636082	47
14	·6459019	·846063	1·181944	·7634204	46
15	·6461240	·846562	1·181247	·7632325	45
16	·6463460	·847062	1·180551	·7630445	44
17	·6465679	·847561	1·179855	·7628564	43
18	·6467898	·848061	1·179159	·7626683	42
19	·6470116	·848561	1·178464	·7624802	41
20	·6472334	·849062	1·177769	·7622919	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49.

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6474551	·849563	1·177075	·7621036	39
22	·6476767	·850064	1·176382	·7619152	38
23	·6478984	·850565	1·175688	·7617268	37
24	·6481199	·851066	1·174996	·7615383	36
25	·6483414	·851568	1·174303	·7613497	35
26	·6485628	·852070	1·173612	·7611611	34
27	·6487842	·852572	1·172920	·7609724	33
28	·6490056	·853075	1·172229	·7607837	32
29	·6492268	·853577	1·171539	·7605949	31
30	·6494480	·854080	1·170849	·7604060	30
31	·6496692	·854583	1·170160	·7602170	29
32	·6498903	·855087	1·169471	·7600280	28
33	·6501114	·855591	1·168782	·7598389	27
34	·6503324	·856095	1·168094	·7596498	26
35	·6505533	·856599	1·167407	·7594606	25
36	·6507742	·857103	1·166720	·7592713	24
37	·6509951	·857608	1·166033	·7590820	23
38	·6512158	·858113	1·165347	·7588926	22
39	·6514366	·858618	1·164661	·7587031	21
40	·6516572	·859124	1·163976	·7585136	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg 49.

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6518778	·859629	1·163291	·7583240	19
42	·6520984	·860135	1·162607	·7581343	18
43	6523189	·860641	1·161923	·7579446	17
44	·6525394	·861148	1·161240	·7577548	16
45	·6527598	·861655	1·160557	·7575650	15
46	·6529801	·862162	1·159874	·7573751	14
47	·6532004	862669	1·159192	7571851	13
48	·6534206	·863176	1·158511	·7569951	12
49	·6536408	·863684	1·157830	·7568050	11
50	·6538609	·864192	1·157149	·7566148	10
51	·6540810	·864700	1·156469	·7564246	9
52	·6543010	·865209	1·155789	·7562343	8
53	·6545209	·865718	1·155110	·7560439	7
54	·6547408	·866227	1·154431	·7558535	6
55	·6549607	·866736	1·153753	·7556630	5
56	·6551804	·867246	1·153075	·7554724	4
57	·6554002	·867755	1·152397	·7552818	3
58	·6556198	·868265	1·151721	·7550911	2
59	·6558395	·868776	1·151044	·7549004	1
60	·6560590	·869286	1·150368	·7547096	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49.

41 Deg

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6560590	·869286	1·150368	·7547096	60
1	·6562785	·869797	1·149692	·7545187	59
2	·6564980	·870308	1·149017	·7543278	58
3	·6567174	·870820	1·148342	·7541368	57
4	·6569367	·871331	1·147668	·7539457	56
5	·6571560	·871843	1·146994	·7537546	55
6	·6573752	·872355	1·146321	·7535634	54
7	·6575944	·872868	1·145648	·7533721	53
8	·6578135	·873380	1·144976	·7531808	52
9	·6580326	·873893	1·144304	·7529894	51
10	·6582516	·874406	1·143632	·7527980	50
11	·6584706	·874920	1·142961	·7526065	49
12	·6586895	·875433	1·142290	·7524149	48
13	·6589083	·875947	1·141620	·7522233	47
14	·6591271	·876462	1·140950	·7520316	46
15	·6593458	·876976	1·140281	·7518398	45
16	·6595645	·877491	1·139612	·7516480	44
17	·6597831	·878006	1·138944	·7514561	43
18	·6600017	·878521	1·138276	·7512641	42
19	·6602202	·879037	1·137608	·7510721	41
20	·6604386	·879552	1·136941	·7508800	40
′	Cosine.	Cotang.	Tang.	Sine.	′

41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6606570	·880068	1·136274	·7506879	39
22	·6608754	·880585	1·135608	·7504957	38
23	·6610936	·881101	1·134942	·7503034	37
24	·6613119	·881618	1·134277	·7501111	36
25	·6615300	·882135	1·133612	·7499187	35
26	·6617482	·882653	1·132947	·7497262	34
27	·6619662	·883170	1·132283	·7495337	33
28	·6621842	·883688	1·131620	·7493411	32
29	·6624022	·884206	1·130957	·7491484	31
30	·6626200	·884725	1·130294	·7489557	30
31	·6628379	·885244	1·129632	·7487629	29
32	·6630557	·885763	1·128970	·7485701	28
33	·6632734	·886282	1·128308	·7483772	27
34	·6634910	·886801	1·127647	·7481842	26
35	·6637087	·887321	1·126987	·7479912	25
36	·6639262	·887841	1·126327	·7477981	24
37	·6641437	·888361	1·125667	·7476049	23
38	·6643612	·888882	1·125008	·7474117	22
39	·6645785	·889403	1·124349	·7472184	21
40	·6647959	·889924	1·123690	·7470251	20
′	Cosine.	Cotang.	Tang.	Sine.	′

41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6650131	·890445	1·123032	·7468317	19
42	·6652304	·890967	1·122375	·7466382	18
43	·6654475	·891489	1·121718	·7464446	17
44	·6656646	·892011	1·121061	·7462510	16
45	·6658817	·892534	1·120405	·7460574	15
46	·6660987	·893056	1·119749	·7458636	14
47	·6663156	·893579	1·119094	·7456699	13
48	·6665325	894103	1·118439	·7454760	12
49	·6667493	·894626	1·117784	·7452821	11
50	·6669661	·895150	1·117130	·7450881	10
51	·6671828	·895674	1·116476	·7448941	9
52	·6673994	·896199	1·115823	·7446999	8
53	·6676160	·896723	1·115170	·7445058	7
54	·6678326	·897248	1·114518	·7443115	6
55	·6680490	·897773	1·113866	·7441173	5
56	·6682655	·898299	1·113214	·7439229	4
57	·6684818	·898825	1·112563	·7437285	3
58	·6686981	·899351	1·111912	·7435340	2
59	·6689144	·899877	1·111262	·7433394	1
60	·6691306	·900404	1·110612	·7431448	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 48. Deg. 48. Deg. 48.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6691306	·900404	1·110612	·7431448	60
1	·6693468	·900930	1·109963	·7429502	59
2	·6695628	·901458	1·109314	·7427554	58
3	·6697789	·901985	1·108665	·7425606	57
4	·6699948	·902513	1·108017	·7423658	56
5	·6702108	·903041	1·107369	·7421708	55
6	·6704266	·903569	1·106721	·7419758	54
7	·6706424	·904097	1·106075	·7417808	53
8	·6708582	·904626	1·105428	·7415857	52
9	·6710739	·905155	1·104782	·7413905	51
10	·6712895	·905685	1·104136	·7411953	50
11	·6715051	·906214	1·103491	·7410000	49
12	·6717206	·906744	1·102846	·7408046	48
13	·6719361	·907274	1·102201	·7406092	47
14	·6721515	·907805	1·101557	·7404137	46
15	·6723668	·908336	1·100914	·7402181	45
16	·6725821	·908867	1·100270	·7400225	44
17	·6727973	·909398	1·099628	·7398268	43
18	·6730125	·909930	1·098985	·7396311	42
19	·6732276	·910461	1·098343	·7394353	41
20	·6734427	·910994	1·097702	·7392394	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6736577	·911526	1·097060	·7390435	39
22	·6738727	·912059	1·096420	·7388475	38
23	·6740876	·912592	1·095779	·7386515	37
24	·6743024	·913125	1·095139	·7384553	36
25	·6745172	·913659	1·094500	·7382592	35
26	·6747319	·914192	1·093861	·7380629	34
27	·6749466	·914727	1·093222	·7378666	33
28	·6751612	·915261	1·092584	·7376703	32
29	·6753757	·915796	1·091946	·7374738	31
30	·6755902	·916331	1·091308	·7372773	30
31	·6758046	·916866	1·090671	·7370808	29
32	·6760190	·917402	1·090034	·7368842	28
33	·6762333	·917937	1·089398	·7366875	27
34	·6764476	·918474	1·088762	·7364908	26
35	·6766618	·919010	1·088126	·7362940	25
36	·6768760	·919547	1·087491	·7360971	24
37	·6770901	·920084	1·086857	·7359002	23
38	·6773041	·920621	1·086222	·7357032	22
39	·6775181	·921159	1·085588	·7355061	21
40	·6777320	·921696	1·084955	·7353090	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6779459	·922235	1.084322	·7351118	19
42	·6781597	·922773	1·083689	·7349146	18
43	·6783734	·923312	1·083057	·7347173	17
44	·6785871	·923851	1·082425	·7345199	16
45	·6788007	·924391	1·081793	·7343225	15
46	·6790143	·924930	1·081162	·7341250	14
47	·6792278	·925470	1·080532	·7339275	13
48	·6794413	·926010	1·079901	·7337299	12
49	·6796547	·926550	1·079271	·7335322	11
50	·6798681	·927091	1·078642	·7333345	10
51	·6800813	·927632	1·078013	·7331367	9
52	·6802946	·928173	1·077384	·7329388	8
53	·6805078	·928715	1·076756	·7327409	7
54	·6807209	·929257	1·076128	·7325429	6
55	·6809339	·929799	1·075500	·7323449	5
56	·6811469	·930342	1·074873	·7321467	4
57	·6813599	·930884	1·074246	·7319486	3
58	·6815728	·931428	1·073620	·7317503	2
59	·6817856	·931971	1·072994	·7315521	1
60	·6819984	·932515	1·072368	·7313537	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6819984	·932515	1·072368	·7313537	60
1	·6822111	·933059	1·071743	·7311553	59
2	·6824237	·933603	1·071118	·7309568	58
3	·6826363	·934147	1·070494	·7307583	57
4	·6828489	·934692	1·069870	·7305597	56
5	·6830613	·935238	1·069246	·7303610	55
6	·6832738	·935783	1·068623	·7301623	54
7	·6834861	·936329	1·068000	·7299635	53
8	·6836984	·936875	1·067377	·7297646	52
9	·6839107	·937421	1·066755	·7295657	51
10	·6841229	·937968	1·066134	·7293668	50
11	·6843350	·938515	1·065512	·7291677	49
12	·6845471	·939062	1·064891	·7289686	48
13	·6847591	·939610	1·064271	·7287695	47
14	·6849711	·940157	1·063651	·7285703	46
15	·6851830	·940706	1·063031	·7283710	45
16	·6853948	·941254	1·062411	·7281716	44
17	·6856066	·941803	1·061792	·7279722	43
18	·6858184	·942352	1·061174	·7277728	42
19	·6860300	·942901	1·060556	·7275732	41
20	·6862416	·943451	1·059938	·7273736	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6864532	·944001	1·059320	·7271740	39
22	·6866647	·944551	1·058703	·7269743	38
23	·6868761	·945102	1·058086	·7267745	37
24	·6870875	·945653	1·057470	·7265747	36
25	·6872988	·946204	1·056854	·7263748	35
26	·6875101	·946755	1·056238	·7261748	34
27	·6877213	·947307	1·055623	·7259748	33
28	·6879325	·947859	1·055008	·7257747	32
29	·6881435	·948411	1·054394	·7255746	31
30	·6883546	·948964	1·053780	·7253744	30
31	·6885655	·949517	1·053166	·7251741	29
32	·6887765	·950070	1·052553	·7249738	28
33	·6889873	·950624	1·051940	·7247734	27
34	·6891981	·951178	1·051327	·7245729	26
35	·6894089	·951732	1·050715	·7243724	25
36	·6896195	·952287	1·050103	·7241719	24
37	·6898302	·952842	1·049492	·7239712	23
38	·6900407	·953397	1·048880	·7237705	22
39	·6902512	·953952	1·048270	·7235698	21
40	·6904617	·954508	1·047659	·7233690	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6906721	·955064	1·047049	·7231681	19
42	·6908824	·955620	1·046440	·7229671	18
43	·6910927	·956177	1·045831	·7227661	17
44	·6913029	·956734	1·045222	·7225651	16
45	·6915131	·957291	1·044613	·7223640	15
46	·6917232	·957849	1·044005	·7221628	14
47	·6919332	·958407	1·043397	·7219615	13
48	·6921432	·958965	1·042790	·7217602	12
49	·6923531	·959524	1·042183	·7215589	11
50	·6925630	·960082	1·041576	·7213574	10
51	·6927728	·960642	1·040970	·7211559	9
52	·6929825	·961201	1·040364	·7209544	8
53	·6931922	·961761	1·039758	·7207528	7
54	·6934018	·962321	1·039153	·7205511	6
55	·6936114	·962881	1·038548	·7203494	5
56	·6938209	·963442	1·037944	·7201476	4
57	·6940304	·964003	1·037340	·7199457	3
58	·6942398	·964565	1·036736	·7197438	2
59	·6944491	·965126	1·036133	·7195418	1
60	·6946584	·965688	1·035530	·7193398	0
′	Cosine.	Cotang.	Tang.	Sine.	′

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6946584	·965688	1·035530	·7193398	60
1	·6948676	·966251	1·034927	·7191377	59
2	·6950767	·966813	1·034325	·7189355	58
3	6952858	·967376	1·033723	·7187333	57
4	·6954949	·967939	1·033122	·7185310	56
5	·6957039	·968503	1·032520	·7183287	55
6	·6959128	·969067	1·031919	·7181263	54
7	·6961217	·969631	1·031319	·7179238	53
8	·6963305	·970196	1·030719	·7177213	52
9	·6965392	·970761	1·030119	·7175187	51
10	·6967479	·971326	1·029520	·7173161	50
11	·6969565	·971891	1·028921	·7171134	49
12	·6971651	·972457	1·028322	·7169106	48
13	·6973736	·973023	1·027724	·7167078	47
14	·6975821	·973590	1·027126	·7165049	46
15	·6977905	·974156	1·026528	·7163019	45
16	·6979988	·974724	1·025931	·7160989	44
17	·6982071	·975291	1·025334	·7158959	43
18	·6984153	·975859	1·024738	·7156927	42
19	·6986234	·976427	1·024141	·7154895	41
20	·6988315	·976995	1·023546	·7152863	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6990396	·977564	1·022950	·7150830	39
22	·6992476	·978133	1·022355	·7148796	38
23	·6994555	·978702	1·021760	·7146762	37
24	·6996633	·979272	1·021166	·7144727	36
25	·6998711	·979842	1·020572	·7142691	35
26	·7000789	·980412	1·019978	·7140655	34
27	·7002866	·980983	1·019385	·7138618	33
28	·7004942	·981554	1·018792	·7136581	32
29	·7007018	·982125	1·018199	·7134543	31
30	·7009093	·982697	1·017607	·7132504	30
31	·7011167	·983269	1·017015	·7130465	29
32	·7013241	·983841	1·016423	·7128426	28
33	·7015314	·984414	1·015832	·7126385	27
34	·7017387	·984987	1·015241	·7124344	26
35	·7019459	·985560	1·014651	·7122303	25
36	·7021531	·986133	1·014061	·7120260	24
37	·7023601	.986707	1·013471	·7118218	23
38	·7025672	.987282	1·012881	·7116174	22
39	·7027741	.987856	1·012292	·7114130	21
40	·7029811	.988431	1·011703	·7112086	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·7031879	·989006	1·011115	·7110041	19
42	·7033947	·989582	1·010527	·7107995	18
43	·7036014	·990158	1·009939	·7105948	17
44	·7038081	·990734	1·009352	·7103901	16
45	·7040147	·991311	1·008764	·7101854	15
46	·7042213	·991888	1·008178	·7099806	14
47	·7044278	·992465	1·007591	·7097757	13
48	·7046342	·993042	1·007005	·7095707	12
49	·7048406	·993620	1·006420	·7093657	11
50	·7050469	·994199	1·005834	·7091607	10
51	·7052532	·994777	1·005249	·7089556	9
52	·7054594	·995356	1·004665	·7087504	8
53	·7056655	·995935	1·004080	·7085451	7
54	·7058716	·996515	1·003496	·7083398	6
55	·7060776	·997095	1·002913	·7081345	5
56	·7062835	·997675	1·002329	·7079291	4
57	·7064894	·998256	1·001746	·7077236	3
58	·7066953	·998837	1·001164	·7075180	2
59	·7069011	·999418	1·000581	·7073124	1
60	·7071068	1·00000	1·000000	·7071068	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45

Tables of

Logarithmic Sines, Cosines, Tangents
and Cotangents,

with Introduction.

(55 pages in all.)

LOGARITHMIC TRIGONOMETRIC FUNCTIONS.

(Logarithms of trigonometric ratios.)

(1) **The "logarithmic" sine, tangent, etc.** (log sin, log tan, etc.), of an arc is the *logarithm* of the *natural* sin, tang, etc., of the arc. Thus:

$$\begin{aligned} \text{nat sin } 37^\circ &= 0.60182; \text{ and} \\ \text{log sin } 37^\circ &= \text{log } 0.60182 = \bar{1}.779\,463 \\ &= 0.779\,463 - 1 \\ &= 9.779\,463 - 10. \end{aligned}$$

In tables, in order to avoid negatives, the *third* form (log sin 37° = 9 779 463 — 10) is used; but the " — 10" is omitted, because understood.

*TWO TABLES.

We give **two tables** of logarithmic trigonometric functions, viz: a **"main table,"** pp 143 *l* to 146 *b*

and a

"special table," pp 143 *c* to 143 *k*.

The main table gives these functions **for each minute of the quadrant** from 0° to 90°.

The special table gives them **at smaller intervals, for small and large angles** (from 0° to 2°, and from 88° to 90°), in which certain functions change so rapidly that one-minute intervals, between arcs, give values of the functions differing too widely for satisfactory interpolation. See ¶¶ 7 to 9.

Main Table.

(2) **The main table,** pp. 143 *l* to 146 *b*, gives, to 6 decimal places, the **log sin, log cos, log tan and log cot**†

of the arc for each minute of the quadrant, or from 0° 0′ to 89° 60′ (90°). See also (6), below.

(3) For arcs from 0° to 45°, **read downward,** as in the left-hand column of minutes, using the titles at the *heads* of the columns. For arcs from 45° to 90°, **read upward,** as in the right-hand column, using the titles at the *feet* of the columns.

(4) **Example,** to find log sin 7° 34′. At head of page [143s] on the left, in bold type, we find 7°; and, under it, in the first column, or column of minutes (reading *downward*), we find 34; opposite which, in the next or 2d column, *under* "Sine," we find 9.119 519, the reqd log sin of 7° 34′.

(5) **Example,** to find log cos 82° 26′. At foot of page [143s], on the right, in bold type, we find 82°; and, over this, in the right-hand column, or column of minutes (reading *upward*), we find 26; opposite which, in the second column from the left, *over* "Cosine," we find 9.119 519, the required log cos 82° 26′.

(6) **For arcs between 90° and 180°,** we have the relations:

$$\sin a = \sin (180^\circ - a); \qquad \cos a = 0 - \cos (180^\circ - a);$$
$$\tan a = 0 - \tan (180^\circ - a); \qquad \cot a = 0 - \cot (180^\circ - a).$$

Hence, log sin, log cos, log tan and log cot of an angle, a, between 90° and 180°, are numerically the same as log sin, log cos, log tan and log cot, respectively, of the **supplementary angle,** 180° — a. In the main table, these functions may be found directly from the table, by using the headings, in bold type, in the lower left-hand and upper right-hand corners of the pages.

For degs on left or r't (resp) of page, use left or r't hand (resp) min col.
For degs at top or ft (resp) of page, use headgs at top or ft (resp) of page.

Thus: log sin 172° 26′ = 9.119 519;
log cos 172° 26′ = 9.996 202.

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† **Secant** $a = \frac{1}{\cos a}$; log sec a = 0 — log cos a. **Cosecant** $a = \frac{1}{\sin a}$; log cosecant a = 0 — log sin a.

(7) For arcs intermediate of those given in the main table (pp 143 *l* to 146 *b*); if the arc is not between 0° and 2°, or between 88° and 92°, or between 178° and 180° (see ¶¶ 8 and 9), **interpolate,** by means of the column headed D.1" (difference of function for 1 second of angle), as in the following examples:

Given	Required	Table gives D.1" = 15.87, and 9.118 567 =	Result.
a = 7° 33′ 12″	log sin a	log sin 7° 33′ 0″	9.118 567 + 12 D. 1″ = 9.118 757;
a = 82° 26′ 48″	log cos a	log cos 82° 27′ 0″	9.118 567 + 12 D. 1″ = 9.118 757;
log sin a = 9.118 757	a	log sin 7° 33′ 0″	7° 33′ 0″ + $\frac{190^*}{\text{D. 1}''}$ = 7° 33′ 12″;
log cos a = 9.118 757	a	log cos 82° 27′ 0″	82° 27′ 0″ − $\frac{190^*}{\text{D. 1}''}$ = 82° 26′ 48″

Special Table, pp. 143 *c* to 143 *k*.

(8) For the log sin, tan, cot, of arcs from 0° 0′ to 1° 60′ (2° 0′); and for the log cos, cot, tan, of arcs from 88° 0′ to 89° 60′ (90° 0′); the differences, between successive logs, vary so rapidly that the use of the column D. 1″, with intervals of 1 minute between tabular arcs, would give insufficiently accurate results.

(9) For such cases we give the **special 5-place table,** pp. 143 *c* to 143 *k*, with smaller intervals between arcs, viz:

for arcs from	interval	pages
0° 0′ to 0° 18′	1 second	143 *c* – *e*
0° 18′ to 0° 30′	3 seconds	143 *f* – *g*
0° 30′ to 0° 40′	5 "	143 *h*
0° 40′ to 0° 60′	10 "	143 *j*
1° 0′ to 1° 60′	30 "	143 *k*

(10) Tangents of angles near 90°, and cotangents of angles near 0°, are not given in the special table; but we have:

log tan a = 0 − log cot a; log cot a = 0 − log tan a.

(11) With functions to only five decimal places, we have:

in arcs from 0° to 0° 18′; log sin a = log tan a = 0 − log cot a;
" " " 89° 42′ to 90°; log cos a = log cot a = 0 − log tan a.

(12) For arcs from 0° 0′ to 0° 18′ (1 second intervals) of course no interpolation is required for whole seconds. For the other angles, named above, the greatest **error, due to interpolation,** is less than 1 in the 5th place. In the main table, log sin 2° 0′ 30″, the error, due to interpolation, is nearly 0.4 in the 5th place.

Examples of interpolation in use of special table.

	Table gives	
Given, a = 1° 48′ 10″	log sin 1° 48′ 30″ =	8.49 908
Required, log sin a	log sin 1° 48′ 0″ =	8.49 708
	diffs 0° 0′ 30″	0.00 200

Result = log sin a = 8.49 708 + $\frac{10}{30}$ × 200 = 8.49 775.

	Table gives	
Given, log sin a = 8.49 775	8.49 908 =	log sin 1° 48′ 30″
Required, a	8.49 708 =	log sin 1° 48′ 0″
	diffs 0.00 200	0° 0′ 30″

Result = a = 1° 48′ 0″ + $\frac{67\dagger}{200}$ × 30″ = 1° 48′ 10″.

* 9.118 757 − 9.118 567 = 0.000 190.

† 67 = 8.49 775 − 8.49 708.

Log sine = Log tangent = 0 − Log cotangent.							
0°	0′	1′	2′	3′	4′	5′	
0″	−∞	6.46373	6.76476	6.94085	7.06579	7.16270	60″
1″	4.68557	6.47090	6.76836	6.94325	7.06759	7.16414	59″
2″	4.98660	6.47797	6.77193	6.94565	7.06939	7.16558	58″
3″	5.16270	6.48492	6.77548	6.94803	7.07118	7.16702	57″
4″	5.28763	6.49175	6.77900	6.95039	7.07296	7.16845	56″
5″	5.38454	6.49849	6.78248	6.95275	7.07474	7.16987	55″
6″	5.46373	6.50512	6.78595	6.95509	7.07651	7.17130	54″
7″	5.53067	6.51165	6.78938	6.95742	7.07827	7.17271	53″
8″	5.58866	6.51808	6.79278	6.95973	7.08003	7.17413	52″
9″	5.63982	6.52442	6.79616	6.96204	7.08177	7.17553	51″
10″	5.68557	6.53067	6.79952	6.96433	7.08351	7.17694	50″
11″	5.72697	6.53683	6.80285	6.96661	7.08525	7.17834	49″
12″	5.76476	6.54291	6.80615	6.96888	7.08698	7.17973	48″
13″	5.79952	6.54890	6.80943	6.97113	7.08870	7.18112	47″
14″	5.83170	6.55481	6.81268	6.97338	7.09041	7.18250	46″
15″	5.86167	6.56064	6.81591	6.97561	7.09211	7.18389	45″
16″	5.88969	6.56639	6.81911	6.97783	7.09381	7.18526	44″
17″	5.91602	6.57207	6.82230	6.98004	7.09551	7.18663	43″
18″	5.94085	6.57767	6.82545	6.98224	7.09719	7.18800	42″
19″	5.96433	6.58320	6.82859	6.98443	7.09887	7.18937	41″
20″	5.98660	6.58866	6.83170	6.98660	7.10055	7.19072	40″
21″	6.00779	6.59406	6.83479	6.98877	7.10222	7.19208	39″
22″	6.02800	6.59939	6.83786	6.99093	7.10388	7.19343	38″
23″	6.04730	6.60465	6.84091	6.99307	7.10553	7.19478	37″
24″	6.06579	6.60985	6.84394	6.99521	7.10718	7.19612	36″
25″	6.08351	6.61499	6.84694	6.99733	7.10882	7.19746	35″
26″	6.10055	6.62007	6.84993	6.99944	7.11046	7.19879	34″
27″	6.11694	6.62509	6.85289	7.00155	7.11209	7.20012	33″
28″	6.13273	6.63006	6.85584	7.00364	7.11371	7.20145	32″
29″	6.14797	6.63496	6.85876	7.00572	7.11533	7.20277	31″
30″	6.16270	6.63982	6.86167	7.00779	7.11694	7.20409	30″
31″	6.17694	6.64462	6.86455	7.00986	7.11854	7.20540	29″
32″	6.19072	6.64936	6.86742	7.01191	7.12014	7.20671	28″
33″	6.20409	6.65406	6.87027	7.01395	7.12174	7.20802	27″
34″	6.21705	6.65870	6.87310	7.01599	7.12333	7.20932	26″
35″	6.22964	6.66330	6.87591	7.01801	7.12491	7.21062	25″
36″	6.24188	6.66785	6.87870	7.02003	7.12648	7.21191	24″
37″	6.25378	6.67235	6.88147	7.02203	7.12805	7.21320	23″
38″	6.26536	6.67680	6.88423	7.02403	7.12962	7.21449	22″
39″	6.27664	6.68121	6.88697	7.02602	7.13118	7.21577	21″
40″	6.28763	6.68557	6.88969	7.02800	7.13273	7.21705	20″
41″	6.29836	6.68990	6.89240	7.02997	7.13428	7.21833	19″
42″	6.30882	6.69418	6.89509	7.03193	7.13582	7.21960	18″
43″	6.31904	6.69841	6.89776	7.03388	7.13736	7.22087	17″
44″	6.32903	6.70261	6.90042	7.03582	7.13889	7.22213	16″
45″	6.33879	6.70676	6.90306	7.03776	7.14042	7.22339	15″
46″	6.34833	6.71088	6.90568	7.03968	7.14194	7.22465	14″
47″	6.35767	6.71496	6.90829	7.04160	7.14346	7.22590	13″
48″	6.36682	6.71900	6.91088	7.04351	7.14497	7.22715	12″
49″	6.37577	6.72300	6.91346	7.04541	7.14647	7.22840	11″
50″	6.38454	6.72697	6.91602	7.04730	7.14797	7.22964	10″
51″	6.39315	6.73090	6.91857	7.04919	7.14947	7.23088	9″
52″	6.40158	6.73479	6.92110	7.05106	7.15096	7.23212	8″
53″	6.40985	6.73865	6.92362	7.05293	7.15244	7.23335	7″
54″	6.41797	6.74248	6.92612	7.05479	7.15392	7.23458	6″
55″	6.42594	6.74627	6.92861	7.05664	7.15540	7.23580	5″
56″	6.43376	6.75003	6.93109	7.05849	7.15687	7.23702	4″
57″	6.44145	6.75376	6.93355	7.06032	7.15833	7.23824	3″
58″	6.44900	6.75746	6.93599	7.06215	7.15979	7.23946	2″
59″	6.45643	6.76112	6.93843	7.06397	7.16125	7.24067	1″
60″	6.46373	6.76476	6.94085	7.06579	7.16270	7.24188	0″
	59′	58′	57′	56′	55′	54′	89°
Log cosine = Log cotangent = 0 − Log tangent.							

Log sine = Log tangent = 0 − Log cotangent.							
0°	6′	7′	8′	9′	10′	11′	
0″	7.24188	7.30882	7.36682	7.41797	7.46373	7.50512	60″
1″	7.24308	7.30986	7.36772	7.41877	7.46445	7.50578	59″
2″	7.24428	7.31089	7.36862	7.41957	7.46517	7.50643	58″
3″	7.24548	7.31191	7.36952	7.42037	7.46589	7.50709	57″
4″	7.24668	7.31294	7.37042	7.42117	7.46661	7.50774	56″
5″	7.24787	7.31396	7.37132	7.42197	7.46733	7.50840	55″
6″	7.24906	7.31498	7.37221	7.42277	7.46805	7.50905	54″
7″	7.25024	7.31600	7.37310	7.42356	7.46876	7.50970	53″
8″	7.25142	7.31702	7.37399	7.42435	7.46948	7.51035	52″
9″	7.25260	7.31803	7.37488	7.42515	7.47019	7.51100	51″
10″	7.25378	7.31904	7.37577	7.42594	7.47090	7.51165	50″
11″	7.25495	7.32005	7.37666	7.42673	7.47162	7.51230	49″
12″	7.25612	7.32106	7.37754	7.42751	7.47233	7.51294	48″
13″	7.25728	7.32206	7.37842	7.42830	7.47303	7.51359	47″
14″	7.25845	7.32306	7.37930	7.42908	7.47374	7.51423	46″
15″	7.25961	7.32406	7.38018	7.42987	7.47445	7.51488	45″
16″	7.26076	7.32506	7.38106	7.43065	7.47515	7.51552	44″
17″	7.26192	7.32606	7.38193	7.43143	7.47586	7.51616	43″
18″	7.26307	7.32705	7.38280	7.43221	7.47656	7.51680	42″
19″	7.26421	7.32804	7.38368	7.43299	7.47726	7.51744	41″
20″	7.26536	7.32903	7.38454	7.43376	7.47797	7.51808	40″
21″	7.26650	7.33001	7.38541	7.43454	7.47867	7.51872	39″
22″	7.26764	7.33100	7.38628	7.43531	7.47936	7.51936	38″
23″	7.26877	7.33198	7.38714	7.43608	7.48006	7.51999	37″
24″	7.26991	7.33296	7.38801	7.43685	7.48076	7.52063	36″
25″	7.27104	7.33393	7.38887	7.43762	7.48145	7.52126	35″
26″	7.27216	7.33491	7.38972	7.43839	7.48215	7.52190	34″
27″	7.27329	7.33588	7.39058	7.43916	7.48284	7.52253	33″
28″	7.27441	7.33685	7.39144	7.43992	7.48353	7.52316	32″
29″	7.27552	7.33782	7.39229	7.44069	7.48422	7.52379	31″
30″	7.27664	7.33879	7.39314	7.44145	7.48491	7.52442	30″
31″	7.27775	7.33975	7.39400	7.44221	7.48560	7.52505	29″
32″	7.27886	7.34071	7.39484	7.44297	7.48629	7.52568	28″
33″	7.27997	7.34167	7.39569	7.44373	7.48698	7.52631	27″
34″	7.28107	7.34263	7.39654	7.44449	7.48766	7.52693	26″
35″	7.28217	7.34359	7.39738	7.44524	7.48835	7.52756	25″
36″	7.28327	7.34454	7.39822	7.44600	7.48903	7.52818	24″
37″	7.28437	7.34549	7.39907	7.44675	7.48971	7.52881	23″
38″	7.28546	7.34644	7.39990	7.44750	7.49039	7.52943	22″
39″	7.28655	7.34739	7.40074	7.44825	7.49108	7.53005	21″
40″	7.28763	7.34833	7.40158	7.44900	7.49175	7.53067	20″
41″	7.28872	7.34928	7.40241	7.44975	7.49243	7.53129	19″
42″	7.28980	7.35022	7.40324	7.45050	7.49311	7.53191	18″
43″	7.29088	7.35116	7.40408	7.45124	7.49379	7.53253	17″
44″	7.29196	7.35209	7.40491	7.45199	7.49446	7.53315	16″
45″	7.29303	7.35303	7.40573	7.45273	7.49513	7.53376	15″
46″	7.29410	7.35396	7.40656	7.45347	7.49581	7.53438	14″
47″	7.29517	7.35489	7.40739	7.45421	7.49648	7.53499	13″
48″	7.29623	7.35582	7.40821	7.45495	7.49715	7.53561	12″
49″	7.29730	7.35675	7.40903	7.45569	7.49782	7.53622	11″
50″	7.29836	7.35767	7.40985	7.45643	7.49849	7.53683	10″
51″	7.29942	7.35860	7.41067	7.45716	7.49916	7.53744	9″
52″	7.30047	7.35952	7.41149	7.45790	7.49982	7.53805	8″
53″	7.30152	7.36044	7.41230	7.45863	7.50049	7.53866	7″
54″	7.30257	7.36135	7.41312	7.45936	7.50115	7.53927	6″
55″	7.30362	7.36227	7.41393	7.46009	7.50182	7.53988	5″
56″	7.30467	7.36318	7.41474	7.46082	7.50248	7.54049	4″
57″	7.30571	7.36409	7.41555	7.46155	7.50314	7.54109	3″
58″	7.30675	7.36500	7.41636	7.46228	7.50380	7.54170	2″
59″	7.30779	7.36591	7.41716	7.46300	7.50446	7.54230	1″
60″	7.30882	7.36682	7.41797	7.46373	7.50512	7.54291	0″
	53′	52′	51′	50′	49′	48′	89°
Log cosine = Log cotangent = 0 — Log tangent.							

Log sine = Log tangent = 0 – Log cotangent.							
0°	12′	13′	14′	15′	16′	17′	
0″	7.54291	7.57767	7.60985	7.63982	7.66784	7.69417	60″
1″	7.54351	7.57822	7.61037	7.64030	7.66830	7.69460	59″
2″	7.54411	7.57878	7.61089	7.64078	7.66875	7.69502	58″
3″	7.54471	7.57934	7.61140	7.64126	7.66920	7.69545	57″
4″	7.54531	7.57989	7.61192	7.64174	7.66965	7.69587	56″
5″	7.54591	7.58044	7.61243	7.64222	7.67010	7.69630	55″
6″	7.54651	7.58100	7.61294	7.64270	7.67055	7.69672	54″
7″	7.54711	7.58155	7.61346	7.64318	7.67100	7.69714	53″
8″	7.54771	7.58210	7.61397	7.64366	7.67145	7.69757	52″
9″	7.54830	7.58265	7.61448	7.64414	7.67190	7.69799	51″
10″	7.54890	7.58320	7.61499	7.64461	7.67235	7.69841	50″
11″	7.54949	7.58375	7.61550	7.64509	7.67279	7.69883	49″
12″	7.55009	7.58430	7.61601	7.64557	7.67324	7.69925	48″
13″	7.55068	7.58485	7.61652	7.64604	7.67369	7.69967	47″
14″	7.55127	7.58539	7.61703	7.64652	7.67413	7.70009	46″
15″	7.55186	7.58594	7.61754	7.64699	7.67458	7.70051	45″
16″	7.55245	7.58649	7.61805	7.64747	7.67502	7.70093	44″
17″	7.55304	7.58703	7.61855	7.64794	7.67547	7.70135	43″
18″	7.55363	7.58758	7.61906	7.64842	7.67591	7.70177	42″
19″	7.55422	7.58812	7.61957	7.64889	7.67636	7.70219	41″
20″	7.55481	7.58866	7.62007	7.64936	7.67680	7.70261	40″
21″	7.55539	7.58921	7.62058	7.64983	7.67724	7.70302	39″
22″	7.55598	7.58975	7.62108	7.65030	7.67768	7.70344	38″
23″	7.55656	7.59029	7.62158	7.65078	7.67813	7.70386	37″
24″	7.55715	7.59083	7.62209	7.65125	7.67857	7.70427	36″
25″	7.55773	7.59137	7.62259	7.65172	7.67901	7.70469	35″
26″	7.55831	7.59191	7.62309	7.65218	7.67945	7.70510	34″
27″	7.55889	7.59245	7.62359	7.65265	7.67989	7.70552	33″
28″	7.55948	7.59299	7.62409	7.65312	7.68033	7.70593	32″
29″	7.56006	7.59352	7.62459	7.65359	7.68077	7.70635	31″
30″	7.56064	7.59406	7.62509	7.65406	7.68121	7.70676	30″
31″	7.56121	7.59459	7.62559	7.65452	7.68165	7.70718	29″
32″	7.56179	7.59513	7.62609	7.65499	7.68208	7.70759	28″
33″	7.56237	7.59566	7.62659	7.65546	7.68252	7.70800	27″
34″	7.56295	7.59620	7.62709	7.65592	7.68296	7.70841	26″
35″	7.56352	7.59673	7.62758	7.65639	7.68340	7.70883	25″
36″	7.56410	7.59726	7.62808	7.65685	7.68383	7.70924	24″
37″	7.56467	7.59780	7.62857	7.65731	7.68427	7.70965	23″
38″	7.56524	7.59833	7.62907	7.65778	7.68470	7.71006	22″
39″	7.56582	7.59886	7.62956	7.65824	7.68514	7.71047	21″
40″	7.56639	7.59939	7.63006	7.65870	7.68557	7.71088	20″
41″	7.56696	7.59992	7.63055	7.65916	7.68601	7.71129	19″
42″	7.56753	7.60045	7.63104	7.65962	7.68644	7.71170	18″
43″	7.56810	7.60097	7.63153	7.66009	7.68687	7.71211	17″
44″	7.56867	7.60150	7.63203	7.66055	7.68731	7.71251	16″
45″	7.56924	7.60203	7.63252	7.66101	7.68774	7.71292	15″
46″	7.56980	7.60255	7.63301	7.66146	7.68817	7.71333	14″
47″	7.57037	7.60308	7.63350	7.66192	7.68860	7.71374	13″
48″	7.57094	7.60360	7.63399	7.66238	7.68903	7.71414	12″
49″	7.57150	7.60413	7.63448	7.66284	7.68946	7.71455	11″
50″	7.57206	7.60465	7.63496	7.66330	7.68989	7.71496	10″
51″	7.57263	7.60517	7.63545	7.66375	7.69032	7.71536	9″
52″	7.57319	7.60570	7.63594	7.66421	7.69075	7.71577	8″
53″	7.57375	7.60622	7.63643	7.66467	7.69118	7.71617	7″
54″	7.57431	7.60674	7.63691	7.66512	7.69161	7.71658	6″
55″	7.57488	7.60726	7.63740	7.66558	7.69204	7.71698	5″
56″	7.57544	7.60778	7.63788	7.66603	7.69247	7.71739	4″
57″	7.57599	7.60830	7.63837	7.66649	7.69289	7.71779	3″
58″	7.57655	7.60882	7.63885	7.66694	7.69332	7.71819	2″
59″	7.57711	7.60934	7.63933	7.66739	7.69375	7.71859	1″
60″	7.57767	7.60985	7.63982	7.66784	7.69417	7.71900	0″
	47′	46′	45′	44′	43′	42′	89°
Log cosine = Log cotangent = 0 – Log tangent.							

0°	Log sin	D 1″	Log tan*	
18′	7.71900	40	7.71900	42′
3″	7.72020	40	7.72021	57″
6″	7.72140	40	7.72141	54″
9″	7.72260	40	7.72261	51″
12″	7.72380	40	7.72380	48″
15″	7.72499	40	7.72499	45″
18″	7.72618	40	7.72618	42″
21″	7.72736	39	7.72737	39″
24″	7.72854	39	7.72855	36″
27″	7.72972	39	7.72973	33″
30″	7.73090	39	7.73090	30″
33″	7.73207	39	7.73207	27″
36″	7.73324	39	7.73324	24″
39″	7.73440	39	7.73441	21″
42″	7.73557	39	7.73557	18″
45″	7.73673	39	7.73673	15″
48″	7.73788	38	7.73789	12″
51″	7.73904	38	7.73904	9″
54″	7.74019	38	7.74019	6″
57″	7.74133	38	7.74134	3″
19′	7.74248	38	7.74248	41′
3″	7.74362	38	7.74363	57″
6″	7.74476	38	7.74476	54″
9″	7.74589	38	7.74590	51″
12″	7.74703	38	7.74703	48″
15″	7.74815	38	7.74816	45″
18″	7.74928	37	7.74929	42″
21″	7.75040	37	7.75041	39″
24″	7.75153	37	7.75153	36″
27″	7.75264	37	7.75265	33″
30″	7.75376	37	7.75377	30″
33″	7.75487	37	7.75488	27″
36″	7.75598	37	7.75599	24″
39″	7.75709	37	7.75709	21″
42″	7.75819	37	7.75820	18″
45″	7.75929	37	7.75930	15″
48″	7.76039	37	7.76040	12″
51″	7.76148	36	7.76149	9″
54″	7.76258	36	7.76258	6″
57″	7.76367	36	7.76367	3″
20′	7.76475	36	7.76476	40′
3″	7.76584	36	7.76585	57″
6″	7.76692	36	7.76693	54″
9″	7.76800	36	7.76801	51″
12″	7.76908	36	7.76908	48″
15″	7.77015	36	7.77016	45″
18″	7.77122	36	7.77123	42″
21″	7.77229	36	7.77230	39″
24″	7.77335	35	7.77336	36″
27″	7.77442	35	7.77442	33″
30″	7.77548	35	7.77549	30″
33″	7.77654	35	7.77654	27″
36″	7.77759	35	7.77760	24″
39″	7.77864	35	7.77865	21″
42″	7.77969	35	7.77970	18″
45″	7.78074	35	7.78075	15″
48″	7.78179	35	7.78179	12″
51″	7.78283	35	7.78284	9″
54″	7.78387	35	7.78388	6″
57″	7.78491	35	7.78492	3″
21′	7.78594		7.78595	39′
	Log cos	D 1″	Log cot*	89°

0°	Log sin	D 1″	Log tan*	
21′	7.78594	34	7.78595	39′
3″	7.78698	34	7.78698	57″
6″	7.78801	34	7.78801	54″
9″	7.78903	34	7.78904	51″
12″	7.79006	34	7.79007	48″
15″	7.79108	34	7.79109	45″
18″	7.79210	34	7.79211	42″
21″	7.79312	34	7.79313	39″
24″	7.79414	34	7.79415	36″
27″	7.79515	34	7.79516	33″
30″	7.79616	34	7.79617	30″
33″	7.79717	34	7.79718	27″
36″	7.79818	33	7.79819	24″
39″	7.79918	33	7.79919	21″
42″	7.80018	33	7.80019	18″
45″	7.80118	33	7.80119	15″
48″	7.80218	33	7.80219	12″
51″	7.80317	33	7.80318	9″
54″	7.80417	33	7.80418	6″
57″	7.80516	33	7.80517	3″
22′	7.80615	33	7.80615	38′
3″	7.80713	33	7.80714	57″
6″	7.80812	33	7.80812	54″
9″	7.80910	33	7.80911	51″
12″	7.81008	33	7.81009	48″
15″	7.81105	32	7.81106	45″
18″	7.81203	32	7.81204	42″
21″	7.81300	32	7.81301	39″
24″	7.81397	32	7.81398	36″
27″	7.81494	32	7.81495	33″
30″	7.81591	32	7.81591	30″
33″	7.81687	32	7.81688	27″
36″	7.81783	32	7.81784	24″
39″	7.81879	32	7.81880	21″
42″	7.81975	32	7.81976	18″
45″	7.82070	32	7.82071	15″
48″	7.82166	32	7.82167	12″
51″	7.82261	32	7.82262	9″
54″	7.82356	32	7.82357	6″
57″	7.82451	32	7.82452	3″
23′	7.82545	31	7.82546	37′
3″	7.82639	31	7.82640	57″
6″	7.82733	31	7.82734	54″
9″	7.82827	31	7.82828	51″
12″	7.82921	31	7.82922	48″
15″	7.83015	31	7.83016	45″
18″	7.83108	31	7.83109	42″
21″	7.83201	31	7.83202	39″
24″	7.83294	31	7.83295	36″
27″	7.83387	31	7.83388	33″
30″	7.83479	31	7.83480	30″
33″	7.83571	31	7.83572	27″
36″	7.83663	31	7.83664	24″
39″	7.83755	31	7.83756	21″
42″	7.83847	31	7.83848	18″
45″	7.83939	30	7.83940	15″
48″	7.84030	30	7.84031	12″
51″	7.84121	30	7.84122	9″
54″	7.84212	30	7.84213	6″
57″	7.84303	30	7.84304	3″
24′	7.84393		7.84394	36′
	Log cos	D 1″	Log cot*	89°

* Log cot A = 0 – log tan A; Log tan A = 0 – log cot A.

0°	Log sin	D 1″	Log tan*		0°	Log sin	D 1″	Log tan*	
24′	7.84393		7.84394	36′	27′	7.89509		7.89510	33′
3″	7.84484	30	7.84485	57″	3″	7.89589	27	7.89590	57″
6″	7.84574	30	7.84575	54″	6″	7.89669	27	7.89670	54″
9″	7.84664	30	7.84665	51″	9″	7.89749	27	7.89751	51″
12″	7.84754	30	7.84755	48″	12″	7.89829	27	7.89830	48″
15″	7.84843	30	7.84845	45″	15″	7.89909	27	7.89910	45″
18″	7.84933	30	7.84934	42″	18″	7.89988	27	7.89990	42″
21″	7.85022	30	7.85023	39″	21″	7.90068	26	7.90069	39″
24″	7.85111	30	7.85112	36″	24″	7.90147	26	7.90149	36″
27″	7.85200	30	7.85201	33″	27″	7.90226	26	7.90228	33″
30″	7.85289	30	7.85290	30″	30″	7.90305	26	7.90307	30″
33″	7.85377	30	7.85379	27″	33″	7.90384	26	7.90386	27″
36″	7.85466	29	7.85467	24″	36″	7.90463	26	7.90464	24″
39″	7.85554	29	7.85555	21″	39″	7.90542	26	7.90543	21″
42″	7.85642	29	7.85643	18″	42″	7.90620	26	7.90622	18″
45″	7.85730	29	7.85731	15″	45″	7.90698	26	7.90700	15″
48″	7.85817	29	7.85819	12″	48″	7.90777	26	7.90778	12″
51″	7.85905	29	7.85906	9″	51″	7.90855	26	7.90856	9″
54″	7.85992	29	7.85993	6″	54″	7.90933	26	7.90934	6″
57″	7.86079	29	7.86080	3″	57″	7.91010	26	7.91012	3″
25′	7.86166	29	7.86167	35′	28′	7.91088	26	7.91089	32′
3″	7.86253	29	7.86254	57″	3″	7.91165	26	7.91167	57″
6″	7.86340	29	7.86341	54″	6″	7.91243	26	7.91244	54″
9″	7.86426	29	7.86427	51″	9″	7.91320	26	7.91321	51″
12″	7.86512	29	7.86513	48″	12″	7.91397	26	7.91399	48″
15″	7.86598	29	7.86600	45″	15″	7.91474	26	7.91475	45″
18″	7.86684	29	7.86685	42″	18″	7.91551	26	7.91552	42″
21″	7.86770	29	7.86771	39″	21″	7.91627	26	7.91629	39″
24″	7.86856	29	7.86857	36″	24″	7.91704	26	7.91705	36″
27″	7.86941	28	7.86942	33″	27″	7.91780	25	7.91782	33″
30″	7.87026	28	7.87027	30″	30″	7.91857	25	7.91858	30″
33″	7.87111	28	7.87113	27″	33″	7.91933	25	7.91934	27″
36″	7.87196	28	7.87197	24″	36″	7.92009	25	7 92010	24″
39″	7.87281	28	7.87282	21″	39″	7.92085	25	7.92086	21″
42″	7.87366	28	7.87367	18″	42″	7.92160	25	7.92162	18″
45″	7.87450	28	7.87451	15″	45″	7.92236	25	7.92237	15″
48″	7.87534	28	7.87535	12″	48″	7.92311	25	7.92313	12″
51″	7.87618	28	7.87619	9″	51″	7.92387	25	7.92388	9″
54″	7.87702	28	7.87703	6″	54″	7.92462	25	7.92463	6″
57″	7.87786	28	7.87787	3″	57″	7.92537	25	7.92539	3″
26′	7.87870	28	7.87871	34′	29′	7.92612	25	7.92613	31′
3″	7.87953	28	7.87954	57″	3″	7.92687	25	7.92688	57″
6″	7.88036	28	7.88038	54″	6″	7.92761	25	7.92763	54″
9″	7.88119	28	7.88121	51″	9″	7.92836	25	7.92838	51″
12″	7.88202	28	7.88204	48″	12″	7.92910	25	7.92912	48″
15″	7.88285	28	7.88286	45″	15″	7.92985	25	7.92986	45″
18″	7.88368	27	7.88369	42″	18″	7.93059	25	7.93060	42″
21″	7.88450	27	7.88452	39″	21″	7.93133	25	7.93134	39″
24″	7.88533	27	7.88534	36″	24″	7.93207	25	7.93208	36″
27″	7.88615	27	7.88616	33″	27″	7.93281	25	7.93282	33″
30″	7.88697	27	7.88698	30″	30″	7.93354	25	7.93356	30″
33″	7.88779	27	7.88780	27″	33″	7.93428	24	7.93429	27″
36″	7.88860	27	7.88862	24″	36″	7.93501	24	7.93503	24″
39″	7.88942	27	7.88943	21″	39″	7.93575	24	7.93576	21″
42″	7.89023	27	7.89025	18″	42′	7.93648	24	7.93649	18″
45″	7.89105	27	7.89106	15″	45″	7.93721	24	7.93722	15″
48″	7.89186	27	7.89187	12″	48″	7.93794	24	7.93795	12″
51″	7.89267	27	7.89268	9″	51″	7.93867	24	7.93868	9″
54″	7.89347	27	7 89349	6″	54″	7.93939	24	7.93941	6″
57″	7.89428	27	7.89429	3″	57″	7.94012	24	7.94013	3″
27′	7.89509	27	7.89510	33′	30′	7.94084	24	7.94086	30′
	Log cos	D 1″	Log cot*	89°		Log cos	D 1″	Log cot*	89°

* Log cot A = 0 − log tan A: Log tan A = 0 − log cot A.

0°	Log sin	D 1″	Log tan*	
30′	7.94084	24.1	7.94086	30′
5″	7.94205	24.0	7.94206	55″
10″	7.94325	24.0	7.94326	50″
15″	7.94445	23.9	7.94446	45″
20″	7.94564	23.8	7.94566	40″
25″	7.94683	23.8	7.94685	35″
30″	7.94802	23.7	7.94804	30″
35″	7.94921	23.6	7.94922	25″
40″	7.95039	23.6	7.95040	20″
45″	7.95157	23.5	7.95158	15″
50″	7.95274	23.4	7.95276	10″
55″	7.95391	23.4	7.95393	5″
31′	7.95508	23.3	7.95510	29′
5″	7.95625	23.3	7.95627	55″
10″	7.95741	23.2	7.95743	50″
15″	7.95857	23.1	7.95859	45″
20″	7.95973	23.1	7.95974	40″
25″	7.96088	23.0	7.96090	35″
30″	7.96203	23.0	7.96205	30″
35″	7.96318	22.9	7.96320	25″
40″	7.96432	22.8	7.96434	20″
45″	7.96546	22.8	7.96548	15″
50″	7.96660	22.7	7.96662	10″
55″	7.96774	22.7	7.96776	5″
32′	7.96887	22.6	7.96889	28′
5″	7.97000	22.5	7.97002	55″
10″	7.97113	22.5	7.97114	50″
15″	7.97225	22.4	7.97227	45″
20″	7.97337	22.4	7.97339	40″
25″	7.97449	22.3	7.97451	35″
30″	7.97560	22.2	7.97562	30″
35″	7.97672	22.2	7.97673	25″
40″	7.97782	22.1	7.97784	20″
45″	7.97893	22.1	7.97895	15″
50″	7.98003	22.0	7.98005	10″
55″	7.98114	22.0	7.98116	5″
33′	7.98223	21.9	7.98225	27′
5″	7.98333	21.8	7.98335	55″
10″	7.98442	21.8	7.98444	50″
15″	7.98551	21.7	7.98553	45″
20″	7.98660	21.7	7.98662	40″
25″	7.98768	21.6	7.98770	35″
30″	7.98876	21.6	7.98878	30″
35″	7.98984	21.5	7.98986	25″
40″	7.99092	21.5	7.99094	20″
45″	7.99199	21.4	7.99201	15″
50″	7.99306	21.4	7.99308	10″
55″	7.99413	21.3	7.99415	5″
34′	7.99520	21.3	7.99522	26′
5″	7.99626	21.2	7.99628	55″
10″	7.99732	21.2	7.99734	50″
15″	7.99838	21.1	7.99840	45″
20″	7.99943	21.1	7.99946	40″
25″	8.00049	21.0	8.00051	35″
30″	8.00154	21.0	8.00156	30″
35″	8.00259	20.9	8.00261	25″
40″	8.00363	20.9	8.00365	20″
45″	8.00467	20.8	8.00470	15″
50″	8.00571	20.8	8.00574	10″
55″	8.00675	20.7	8.00677	5″
35′	8.00779		8.00781	25′
	Log cos	D 1″	Log cot*	89°

0°	Log sin	D 1″	Log tan*	
35′	8.00779	20.7	8.00781	25′
5″	8.00882	20.6	8.00884	55″
10″	8.00985	20.5	8.00987	50″
15″	8.01088	20.5	8.01090	45″
20″	8.01190	20.5	8.01193	40″
25″	8.01293	20.4	8.01295	35″
30″	8.01395	20.4	8.01397	30″
35″	8.01497	20.3	8.01499	25″
40″	8.01598	20.3	8.01600	20″
45″	8.01699	20.2	8.01702	15″
50″	8.01801	20.2	8.01803	10″
55″	8.01901	20.1	8.01904	5″
36′	8.02002	20.1	8.02004	24′
5″	8.02102	20.0	8.02105	55″
10″	8.02203	20.0	8.02205	50″
15″	8.02303	20.0	8.02305	45″
20″	8.02402	19.9	8.02405	40″
25″	8.02502	19.9	8.02504	35″
30″	8.02601	19.8	8.02604	30″
35″	8.02700	19.8	8.02703	25″
40″	8.02799	19.7	8.02801	20″
45″	8.02898	19.7	8.02900	15″
50″	8.02996	19.6	8.02998	10″
55″	8.03094	19.6	8.03097	5″
37′	8.03192	19.5	8.03194	23′
5″	8.03290	19.5	8.03292	55″
10″	8.03387	19.4	8.03390	50″
15″	8.03484	19.4	8.03487	45″
20″	8.03581	19.4	8.03584	40″
25″	8.03678	19.3	8.03681	35″
30″	8.03775	19.3	8.03777	30″
35″	8.03871	19.2	8.03874	25″
40″	8.03967	19.2	8.03970	20″
45″	8.04063	19.1	8.04066	15″
50″	8.04159	19.1	8.04162	10″
55″	8.04255	19.1	8.04257	5″
38′	8.04350	19.0	8.04353	22′
5″	8.04445	19.0	8.04448	55″
10″	8.04540	18.9	8.04543	50″
15″	8.04635	18.9	8.04638	45″
20″	8.04729	18.9	8.04732	40″
25″	8.04824	18.8	8.04826	35″
30″	8.04918	18.8	8.04921	30″
35″	8.05012	18.7	8.05014	25″
40″	8.05105	18.7	8.05108	20″
45″	8.05199	18.7	8.05202	15″
50″	8.05292	18.6	8.05295	10″
55″	8.05385	18.6	8.05388	5″
39′	8.05478	18.5	8.05481	21′
5″	8.05571	18.5	8.05574	55″
10″	8.05663	18.5	8.05666	50″
15″	8.05756	18.4	8.05758	45″
20″	8.05848	18.4	8.05851	40″
25″	8.05940	18.3	8.05943	35″
30″	8.06031	18.3	8.06034	30″
35″	8.06123	18.3	8.06126	25″
40″	8.06214	18.2	8.06217	20″
45″	8.06305	18.2	8.06308	15″
50″	8.06396	18.2	8.06399	10″
55″	8.06487	18.1	8.06490	5″
40′	8.06578		8.06581	20′
	Log cos	D 1″	Log cot*	89°

* Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

0°	Log sin	D 1″	Log tan*	
0′	8.06578	18.1	8.06581	20′
10″	8.06758	18.0	8.06761	50″
20″	8.06938	17.9	8.06941	40″
30″	8.07117	17.8	8.07120	30″
40″	8.07295	17.7	8.07299	20″
50″	8.07473	17.7	8.07476	10″
1′	8.07650	17.6	8.07653	19′
10″	8.07826	17.6	8.07829	50″
20″	8.08002	17.5	8.08005	40″
30″	8.08176	17.4	8.08180	30″
40″	8.08350	17.3	8.08354	20″
50″	8.08524	17.3	8.08527	10″
2′	8.08696	17.2	8.08700	18′
10″	8 08868	17.1	8.08872	50″
20″	8.09040	17.1	8.09043	40″
30″	8.09210	17.0	8.09214	30″
40″	8.09380	16.9	8.09384	20″
50″	8.09550	16.9	8.09553	10″
3′	8.09718	16.8	8.09722	17′
10″	8.09886	16.7	8.09890	50″
20″	8.10054	16.7	8.10057	40″
30″	8.10220	16.6	8.10224	30″
40″	8.10386	16.5	8.10390	20″
50″	8.10552	16.5	8.10555	10″
4′	8.10717	16.4	8.10720	16′
10″	8.10881	16.4	8.10884	50″
20″	8.11044	16.3	8.11048	40″
30″	8.11207	16.2	8.11211	30″
40″	8.11370	16.2	8.11373	20″
50″	8.11531	16.1	8.11535	10″
5′	8.11693	16.1	8.11696	15′
10″	8.11853	16.0	8.11857	50″
20″	8.12013	15.9	8.12017	40″
30″	8.12172	15.9	8.12176	30″
40″	8.12331	15.8	8.12335	20″
50″	8.12489	15.8	8.12493	10″
6′	8.12647	15.7	8.12651	14′
10″	8.12804	15.6	8.12808	50″
20″	8.12961	15.6	8.12965	40″
30″	8.13117	15.5	8.13121	30″
40″	8.13272	15.5	8.13276	20″
50″	8.13427	15.4	8.13431	10″
7′	8.13581	15.4	8.13585	13′
10″	8.13735	15.3	8.13739	50″
20″	8.13888	15.3	8.13892	40″
30″	8.14041	15.2	8.14045	30″
40″	8.14193	15.2	8.14197	20″
50″	8.14344	15.1	8.14348	10″
8′	8.14495	15.1	8.14500	12′
10″	8.14646	15.0	8.14650	50″
20″	8.14796	15.0	8.14800	40″
30″	8.14945	14.9	8.14950	30″
40″	8.15094	14.9	8.15099	20″
50″	8.15243	14.8	8 15247	10″
9′	8.15391	14.8	8.15395	11′
10″	8.15538	14.7	8.15543	50″
20″	8.15685	14.6	8.15690	40″
30″	8.15832	14.6	8.15836	30″
40″	8.15978	14.6	8.15982	20″
50″	8.16123	14.5	8.16128	10″
0′	8.16268		8.16273	10′
	Log cos	D 1″	Log cot*	89°

0°	Log sin	D 1″	Log tan*	
50′	8.16268	14.5	8.16273	60″
10″	8.16413	14.4	8.16417	50″
20″	8.16557	14.4	8.16561	40″
30″	8.16700	14.3	8.16705	30″
40″	8.16843	14.3	8.16848	20″
50″	8.16986	14.2	8.16991	10″
51′	8.17128	14.2	8.17133	9′
10″	8.17270	14.1	8.17275	50″
20″	8.17411	14.1	8.17416	40″
30″	8.17552	14.0	8.17557	30″
40″	8.17692	14.0	8.17697	20″
50″	8.17832	13.9	8.17837	10″
52′	8.17971	13.9	8.17976	8′
10″	8.18110	13.9	8.18115	50″
20″	8.18249	13.8	8.18254	40″
30″	8.18387	13.8	8.18392	30″
40″	8.18525	13.7	8.18530	20″
50″	8.18662	13.7	8.18667	10″
53′	8.18798	13.6	8.18804	7′
10″	8.18935	13.6	8.18940	50″
20″	8.19071	13.6	8.19076	40″
30″	8.19206	13.5	8.19212	30″
40″	8.19341	13.5	8.19347	20″
50″	8.19476	13.4	8.19481	10″
54′	8.19610	13.4	8.19616	6′
10″	8.19744	13.3	8.19749	50″
20″	8.19877	13.3	8.19883	40″
30″	8.20010	13.3	8.20016	30″
40″	8.20143	13.2	8.20149	20″
50″	8.20275	13.2	8.20281	10″
55′	8.20407	13.1	8.20413	5′
10″	8.20538	13.1	8.20544	50″
20″	8.20669	13.1	8.20675	40″
30″	8.20800	13.0	8.20806	30″
40″	8.20930	13.0	8.20936	20″
50″	8.21060	12.9	8.21066	10″
56′	8.21189	12.9	8.21195	4′
10″	8.21319	12.9	8.21324	50″
20″	8.21447	12.8	8.21453	40″
30″	8.21576	12.8	8.21581	30″
40″	8.21703	12.8	8.21709	20″
50″	8.21831	12.7	8.21837	10″
57′	8.21958	12.7	8.21964	3′
10″	8.22085	12.6	8.22091	50″
20″	8.22211	12.6	8.22217	40″
30″	8.22337	12.6	8.22343	30″
40″	8.22463	12.5	8.22469	20″
50″	8.22588	12.5	8.22595	10″
58′	8.22713	12.5	8.22720	2′
10″	8.22838	12.4	8.22844	50″
20″	8.22962	12.4	8.22968	40″
30″	8.23086	12.4	8.23092	30″
40″	8.23210	12.3	8.23216	20″
50″	8.23333	12.3	8.23339	10″
59′	8.23456	12.3	8.23462	1′
10″	8.23578	12.2	8.23585	50″
20″	8.23700	12.2	8.23707	40″
30″	8.23822	12.1	8.23829	30″
40″	8.23944	12.1	8.23950	20″
50″	8.24065	12.1	8.24071	10″
60′	8.24186		8.24192	0′
	Log cos	D 1″	Log cot*	89°

* Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

1°	Log sin	D 1"	Log tan*	
0'	8.24186	12.01	8.24192	60'
0'30"	8.24546	11.92	8.24553	59'30"
1'	8.24903	11.81	8.24910	59'
1'30"	8.25258	11.72	8.25265	58'30"
2'	8.25609	11.62	8.25616	58'
2'30"	8.25958	11.53	8.25965	57'30"
3'	8.26304	11.44	8.26312	57'
3'30"	8.26648	11.35	8.26655	56'30"
4'	8.26988	11.27	8.26996	56'
4'30"	8.27326	11.18	8.27334	55'30"
5'	8.27661	11.10	8.27669	55'
5'30"	8.27994	11.01	8.28002	54'30"
6'	8.28324	10.92	8.28332	54'
6'30"	8.28652	10.84	8.28660	53'30"
7'	8.28977	10.76	8.28986	53'
7'30"	8.29300	10.68	8.29309	52'30"
8'	8.29621	10.60	8.29629	52'
8'30"	8.29939	10.53	8.29947	51'30"
9'	8.30255	10.45	8.30263	51'
9'30"	8.30568	10.37	8.30577	50'30"
0'	8.30879	10.30	8.30888	50'
0'30"	8.31188	10.23	8.31198	49'30"
1'	8.31495	10.16	8.31505	49'
1'30"	8.31800	10.09	8.31809	48'30"
2'	8.32103	10.02	8.32112	48'
2'30"	8.32403	9.95	8.32413	47'30"
3'	8.32702	9.88	8.32711	47'
3'30"	8.32998	9.81	8.33008	46'30"
4'	8.33292	9.75	8.33302	46'
4'30"	8.33585	9.68	8.33595	45'30"
5'	8.33875	9.61	8.33886	45'
5'30"	8.34164	9.55	8.34174	44'30"
6'	8.34450	9.50	8.34461	44'
6'30"	8.34735	9.43	8.34746	43'30"
7'	8.35018	9.37	8.35029	43'
7'30"	8.35299	9.31	8.35310	42'30"
8'	8.35578	9.25	8.35590	42'
8'30"	8.35856	9.19	8.35867	41'30"
9'	8.36132	9.13	8.36143	41'
9'30"	8.36405	9.08	8.36417	40'30"
0'	8.36678	9.02	8.36689	40'
0'30"	8.36948	8.96	8.36960	39'30"
1'	8.37217	8.91	8.37229	39'
1'30"	8.37484	8.86	8.37497	38'30"
2'	8.37750	8.80	8.37762	38'
2'30"	8.38014	8.74	8.38026	37'30"
3'	8.38276	8.69	8.38289	37'
3'30"	8.38537	8.64	8.38550	36'30"
4'	8.38796	8.58	8.38809	36'
4'30"	8.39054	8.54	8.39067	35'30"
5'	8.39310	8.49	8.39323	35'
5'30"	8.39565	8.44	8.39578	34'30"
6'	8.39818	8.39	8.39832	34'
6'30"	8.40070	8.34	8.40083	33'30"
7'	8.40320	8.30	8.40334	33'
7'30"	8.40569	8.24	8.40583	32'30"
8'	8.40816	8.20	8.40830	32'
8'30"	8.41062	8.16	8.41077	31'30"
9'	8.41307	8.10	8.41321	31'
9'30"	8.41550	8.07	8.41565	30'30"
0'	8.41792		8.41807	30'
	Log cos	D 1"	Log cot*	88°

1°	Log sin	D 1"	Log tan*	
30'	8.41792	8.02	8.41807	30'
30'30"	8.42032	7.97	8.42048	29'30"
31'	8.42272	7.93	8.42287	29'
31'30"	8.42510	7.89	8.42525	28'30"
32'	8.42746	7.84	8.42762	28'
32'30"	8.42982	7.81	8.42997	27'30"
33'	8.43216	7.76	8.43232	27'
33'30"	8.43448	7.71	8.43464	26'30"
34'	8.43680	7.67	8.43696	26'
34'30"	8.43910	7.64	8.43927	25'30"
35'	8.44139	7.60	8.44156	25'
35'30"	8.44367	7.56	8.44384	24'30"
36'	8.44594	7.52	8.44611	24'
36'30"	8.44820	7.47	8.44837	23'30"
37'	8.45044	7.44	8.45061	23'
37'30"	8.45267	7.40	8.45285	22'30"
38'	8.45489	7.36	8.45507	22'
38'30"	8.45710	7.33	8.45728	21'30"
39'	8.45930	7.29	8.45948	21'
39'30"	8.46149	7.25	8.46167	20'30"
40'	8.46366	7.22	8.46385	20'
40'30"	8.46583	7.18	8.46602	19'30"
41'	8.46799	7.14	8.46817	19'
41'30"	8.47013	7.11	8.47032	18'30"
42'	8.47226	7.08	8.47245	18'
42'30"	8.47439	7.04	8.47458	17'30"
43'	8.47650	7.01	8.47669	17'
43'30"	8.47860	6.97	8.47880	16'30"
44'	8.48069	6.94	8.48089	16'
44'30"	8.48278	6.91	8.48298	15'30"
45'	8.48485	6.88	8.48505	15'
45'30"	8.48691	6.84	8.48711	14'30"
46'	8.48896	6.81	8.48917	14'
46'30"	8.49101	6.78	8.49121	13'30"
47'	8.49304	6.74	8.49325	13'
47'30"	8.49506	6.71	8.49528	12'30"
48'	8.49708	6.68	8.49729	12'
48'30"	8.49908	6.66	8.49930	11'30"
49'	8.50108	6.62	8.50130	11'
49'30"	8.50307	6.59	8.50329	10'30"
50'	8.50504	6.56	8.50527	10'
50 30"	8.50701	6.54	8.50724	9'30"
51'	8.50897	6.50	8.50920	9'
51'30"	8.51092	6.47	8.51115	8'30"
52'	8.51287	6.44	8.51310	8'
52'30"	8.51480	6.42	8.51503	7'30"
53'	8.51673	6.39	8.51696	7'
53'30"	8.51864	6.36	8.51888	6'30"
54'	8.52055	6.33	8.52079	6'
54'30"	8.52245	6.30	8.52269	5'30"
55'	8.52434	6.28	8.52459	5'
55'30"	8.52623	6.25	8.52647	4'30"
56'	8.52810	6.22	8.52835	4'
56'30"	8.52997	6.20	8.53022	3'30"
57'	8.53183	6.17	8.53208	3'
57'30"	8.53368	6.15	8.53393	2'30"
58'	8.53552	6.12	8.53578	2'
58'30"	8.53736	6.09	8.53762	1'30"
59'	8.53919	6.07	8.53945	1'
59'30"	8.54101	6.04	8.54127	0'30"
60'	8.54282		8.54308	0'
	Log cos	D 1"	Log cot*	88°

* Log cot A = 0 – log tan A; Log tan A = 0 — log cot A.

"	'	Sine.	D 1"	Cosine.	Tang.	Cotang.	'
0	0	Inf. neg.		ten	Inf. neg.	Inf. pos.	60
60	1	6.463726		ten	6.463726	13.536274	59
120	2	764756		ten	.764756	.235244	58
180	3	6 940847		ten	6 940847	13.059153	57
240	4	7.065786		ten	7.065786	12.934214	56
300	5	.162696		ten	.162696	.837304	55
360	6	.241877	.02	9.999999	.241878	.758122	54
420	7	.308824	.00	.999999	.308825	.691175	53
480	8	.366816	.00	.999999	.366817	.633183	52
540	9	.417968	.00	.999999	.417970	.582030	51
600	10	.463726	.02	.999998	.463727	.536273	50
660	11	7.505118	.00	9.999998	7.505120	12.494880	49
720	12	.542906	.02	.999997	.542909	.457091	48
780	13	.577668	.00	.999997	.577672	.422328	47
840	14	.609853	.02	.999996	.609857	.390143	46
900	15	.639816	.00	.999996	.639820	.360180	45
960	16	.667845	.02	.999995	.667849	.332151	44
1020	17	.694173	.00	.999995	.694179	.305821	43
1080	18	.718997	.02	.999994	.719003	.280997	42
1140	19	.742478	.02	.999993	.742484	.257516	41
1200	20	.764754	.00	.999993	.764761	.235239	40
1260	21	7.785943	.02	9.999992	7.785951	12 214049	39
1320	22	.806146	.02	.999991	.806155	.193845	38
1380	23	.825451	.02	.999990	.825460	.174540	37
1440	24	.843934	.02	.999989	.843944	.156056	36
1500	25	.861662	.00	.999989	.861674	.138326	35
1560	26	.878695	.02	.999988	.878708	.121292	34
1620	27	.895085	.02	.999987	.895099	.104901	33
1680	28	.910879	.02	.999986	.910894	.089106	32
1740	29	.926119	.02	.999985	.926134	.073866	31
1800	30	.940842	.03	.999983	.940858	.059142	30
1860	31	7.955082	.02	9.999982	7.955100	12.044900	29
1920	32	.968870	.02	.999981	.968889	.031111	28
1980	33	.982233	.02	.999980	.982253	.017747	27
2040	34	7.995198	.02	.999979	7.995219	12 004781	26
2100	35	8 007787	.03	.999977	8.007809	11.992191	25
2160	36	.020021	.02	.999976	.020044	.979956	24
2220	37	.031919	.02	.999975	.031945	.968055	23
2280	38	.043501	.03	.999973	043527	.956473	22
2340	39	.054781	.02	.999972	.054809	.945191	21
2400	40	.065776	.02	.999971	.065806	.934194	20
2460	41	8.076500	.03	9.999969	8.076531	11.923469	19
2520	42	.086965	.02	.999968	.086997	.913003	18
2580	43	.097183	.03	.999966	.097217	.902783	17
2640	44	.107167	.03	.999964	.107203	.892797	16
2700	45	.116926	.02	.999963	.116963	.883037	15
2760	46	.126471	.03	.999961	.126510	.873490	14
2820	47	.135810	.03	.999959	.135851	.864149	13
2880	48	.144953	.02	.999958	.144996	.855004	12
2940	49	.153907	.03	.999956	.153952	.846048	11
3000	50	.162681	.03	.999954	.162727	.837273	10
3060	51	8.171280	.03	9.999952	8.171328	11.828672	9
3120	52	.179713	.03	.999950	.179763	.820237	8
3180	53	.187985	.03	999948	.188036	.811964	7
3240	54	.196102	.03	.999946	.196156	803844	6
3300	55	.204070	.03	999944	.204126	795874	5
3360	56	.211895	.03	999942	.211958	.788047	4
3420	57	.219581	.03	.999940	.219641	.780359	3
3480	58	.227134	.03	.999938	.227195	.772805	2
3540	59	.234557	.03	.999936	.234621	.765379	1
3600	60	8.241855	.03	9.999934	8.241921	11.758079	0
"	'	Cosine.	D 1"	Sine.	Cotang.	Tang.	'

For intermediate values, see pp 143 c to 143 j, inclusive.

For intermediate values, see pp 143 c to 143 j, inclusive.

For intermediate values, see p 143 k.

"	'	Sine.	D 1".	Cosine.	Tang.	Cotang.	'
3600	0	8.241855	.03	9.999934	8.241921	11.758079	60
3660	1	.249033	.05	.999932	.249102	.750898	59
3720	2	.256094	.03	999929	.256165	.743835	58
3780	3	.263042	.03	.999927	263115	.736885	57
3840	4	.269881	.05	.999925	.269956	.730044	56
3900	5	.276614	.03	.999922	.276691	.723309	55
3960	6	.283243	.03	.999920	.283323	.716677	54
4020	7	.289773	.05	.999918	.289856	.710144	53
4080	8	.296207	.03	.999915	.296292	.703708	52
4140	9	.302546	.05	.999913	.302634	.697366	51
4200	10	.308794	.05	.999910	.308884	691116	50
4260	11	8.314954	.03	9.999907	8.315046	11.684954	49
4320	12	.321027	.05	.999905	.321122	678878	48
4380	13	.327016	.05	.999902	.327114	.672886	47
4440	14	.332924	.03	.999899	.333025	.666975	46
4500	15	.338753	.05	.999897	.338856	.661144	45
4560	16	.344504	.05	.999894	.344610	.655390	44
4620	17	.350181	.05	.999891	.350289	649711	43
4680	18	355783	.05	.999888	.355895	.644105	42
4740	19	361315	.05	.999885	361430	638570	41
4800	20	.366777	.05	.999882	366895	.633105	40
4860	21	8 372171	.05	9.999879	8.372292	11.627708	39
4920	22	377499	.05	.999876	.377622	.622378	38
4980	23	382762	.05	.999873	.382889	.617111	37
5040	24	.387962	.05	.999870	.388092	.611908	36
5100	25	393101	.05	.999867	393234	.606766	35
5160	26	398179	.05	.999864	.398315	.601685	34
5220	27	403199	.05	.999861	.403338	.596662	33
5280	28	.408161	.07	.999858	.408304	.591696	32
5340	29	.413068	.05	.999854	.413213	.586787	31
5400	30	.417919	.05	.999851	.418068	.581932	30
5460	31	8.422717	.07	9.999848	8.422869	11.577131	29
5520	32	.427462	.05	.999844	.427618	.572382	28
5580	33	.432156	.05	.999841	.432315	.567685	27
5640	34	436800	.07	.999838	.436962	.563038	26
5700	35	.441394	.05	.999834	.441560	.558440	25
5760	36	.445941	.07	.999831	.446110	.553890	24
5820	37	.450440	.05	.999827	.450613	.549387	23
5880	38	.454893	.07	.999824	.455070	.544930	22
5940	39	.459301	.07	.999820	.459481	.540519	21
6000	40	.463665	.05	.999816	.463849	.536151	20
6060	41	8.467985	.07	9.999813	8.468172	11.531828	19
6120	42	.472263	.07	.999809	.472454	527546	18
6180	43	.476496	.07	.999805	.476693	.523307	17
6240	44	.480698	.07	.999801	.480892	.519108	16
6300	45	.484848	.05	.999797	.485050	.514950	15
6360	46	.488963	.07	.999794	489170	.510830	14
6420	47	.493040	.07	.999790	.493250	.506750	13
6480	48	.497078	.07	.999786	.497293	.502707	12
6540	49	.501080	.07	.999782	.501298	.498702	11
6600	50	.505045	.07	.999778	.505267	.494733	10
6660	51	8.508974	.08	9.999774	8.509200	11.490800	9
6720	52	.512867	.07	.999769	.513098	.486902	8
6780	53	.516726	.07	.999765	.516961	.483039	7
6840	54	.520551	.07	.999761	.520790	.479210	6
6900	55	.524343	.07	.999757	524586	.475414	5
6960	56	.528102	.08	.999753	.528349	.471651	4
7020	57	.531828	.07	.999748	.532080	.467920	3
7080	58	.535523	.07	.999744	.535779	.464221	2
7140	59	.539186	.08	.999740	539447	.460553	1
7200	60	8.542819		9.999735	8.543084	11.456916	0
"	'	Cosine.	D 1".	Sine.	Cotang.	Tang.	'

For intermediate values, see p 143 k.

Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
8 542819	60.05	9.999735	.07	8 543084	60.12	11.456916	60
.546422	59 55	.999731	.08	.546691	59 62	.453309	59
.549995	59.07	.999726	.07	550268	59 15	.449732	58
.553539	58.58	999722	.08	.553817	58 65	.446183	57
.557054	58 10	.999717	.07	.557336	58.20	.442664	56
.560540	57 65	.999713	.08	.560828	57 72	.439172	55
.563999	57.20	.999708	.07	.564291	57 27	.435709	54
.567431	56.75	.999704	.08	.567727	56 83	.432273	53
.570836	56.30	.999699	.08	.571137	56 38	.428863	52
.574214	55 87	.999694	.08	.574520	55 95	.425480	51
.577566	55.43	.999689	.07	.577877	55.52	.422123	50
8.580892	55.02	9.999685	.08	8.581208	55 10	11.418792	49
.584193	54.60	.999680	.08	.584514	54 68	.415486	48
.587469	54 20	.999675	.08	.587795	54 27	.412205	47
.590721	53 78	.999670	08	.591051	53.87	.408949	46
.593948	53.40	.999665	.08	.594283	53 48	.405717	45
.597152	53.00	.999660	.08	.597492	53 08	.402508	44
.600332	52.62	.999655	.08	.600677	52 70	.399323	43
.603489	52 23	.999650	.08	.603839	52.32	.396161	42
.606623	51.85	.999645	.08	.606978	51 93	.393022	41
.609734	51.48	.999640	.08	.610094	51 58	.389906	40
8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
.615891	50 77	.999629	.08	.616262	50 85	.383738	38
.618937	50 42	.999624	.08	.619313	50.50	.380687	37
.621962	50.05	.999619	.08	.622343	50.15	.377657	36
.624965	49.72	.999614	.10	.625352	49 80	.374648	35
.627948	49.38	.999608	.08	.628340	49 47	.371660	34
.630911	49.05	.999603	.10	.631308	49 13	.368692	33
.633854	48 70	.999597	.08	.634256	48 80	.365744	32
.636776	48 40	.999592	.10	.637184	48.48	.362816	31
.639680	48.05	.999586	.08	.640093	48.15	.359907	30
8.642563	47.75	9.999581	.10	8.642982	47 85	11.357018	29
.645428	47.43	.999575	.08	.645853	47.52	.354147	28
.648274	47.13	.999570	.10	.648704	47.22	.351296	27
.651102	46.82	.999564	.10	.651537	46.92	.348463	26
.653911	46.52	.999558	.08	.654352	46.62	.345648	25
.656702	46.22	.999553	.10	.657149	46.32	.342851	24
.659475	45.92	.999547	.10	.659928	46 02	.340072	23
.662230	45.63	.999541	.10	.662689	45.73	.337311	22
.664968	45.35	.999535	.10	.665433	45 45	.334567	21
.667689	45.07	.999529	.08	.668160	45 17	.331840	20
8.670393	44.78	9.999524	.10	8.670870	44 88	11.329130	19
.673080	44 52	.999518	.10	.673563	44 60	.326437	18
.675751	44.23	.999512	.10	.676239	44.35	.323761	17
.678405	43 97	.999506	.10	.678900	44.07	.321100	16
.681043	43 70	.999500	.12	.681544	43 80	.318456	15
.683665	43.45	.999493	.10	.684172	43 53	.315828	14
.686272	43.18	.999487	.10	.686784	43 28	.313216	13
.688863	42 92	.999481	.10	.689381	43 03	.310619	12
.691438	42 67	.999475	.10	.691963	42.77	.308037	11
.693998	42.42	.999469	.10	.694529	42.53	.305471	10
8.696543	42.17	9.999463	.12	8.697081	42 27	11.302919	9
.699073	41.93	.999456	.10	.699617	42.03	.300383	8
.701589	41.68	.999450	.12	.702139	41.78	.297861	7
.704090	41.45	.999443	.10	.704646	41.57	.295354	6
.706577	41.20	.999437	.10	.707140	41.30	.292860	5
.709049	40.97	.999431	.12	.709618	41.08	.290382	4
.711507	40.75	.999424	.10	.712083	40 85	.287917	3
.713952	40.52	.999418	.12	.714534	40.63	.285466	2
.716383	40.28	.999411	.12	.716972	40.40	.283028	1
8.718800		9.999404		8.719396		11.280604	0
Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.718800	40.07	9 999404	.10	8.719396	40.17	11.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38 78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38 55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38 37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37 77	9 999329	.12	8.745207	37 87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37 68	.252521	48
13	.749055	37 37	.999315	.12	.749740	37 48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	.753528	36 98	.999301	.12	.754227	37 10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36 92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36 73	.241332	43
18	.760151	36 43	.999279	.12	.760872	36 55	.239128	42
19	.762337	36 23	.999272	.12	.763065	36 35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36 18	.234754	40
21	8.766675	35.88	9 999257	.12	8 767417	36 02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35 82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35 65	.228273	37
24	.773101	35 37	.999235	.13	.773866	35 48	.226134	36
25	.775223	35 17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34 83	.999212	.12	.780222	34 97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34 80	.217680	32
29	.783605	34 50	.999197	.13	.784408	34 63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34 18	9.999181	.12	8.788554	34 32	11.211446	29
32	.789787	34 02	.999174	.13	.790613	34 15	.209387	28
33	.791828	33.85	.999166	.13	.792662	33 98	.207338	27
34	.793859	33 70	.999158	.13	.794701	33 83	.205299	26
35	.795881	33 55	.999150	.13	.796731	33 68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33 52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33 37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33 22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8 808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32 63	.189317	18
43	.811726	32.35	.999086	.15	812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821843	31.62	.999044	.13	.822298	31 78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31 63	.175795	11
50	.825130	81.35	.999027	.13	.826103	31.48	.173897	10
51	8.827011	81.22	9.999019	.15	8.827992	31.37	11.172008	9
52	.828884	81.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.13	.835471	30.83	.164529	5
56	.836297	30.55	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30 30	.998958	.13	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.15	.842825	30.32	.157175	1
60	8.843585		9.998941		8.844644		11.155356	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8 843585	30 03	9.998941	.15	8 844644	30 18	11.155356	60
1	.845387	29 93	998932	.15	.846455	30.08	.153545	59
2	847183	29.80	.998923	.15	.848260	29 95	.151740	58
3	.848971	29 67	998914	.15	.850057	29 82	.149943	57
4	850751	29 57	.998905	15	.851846	29 70	.148154	56
5	852525	29 43	.998896	.15	.853628	29 58	.146372	55
6	854291	29 30	.998887	.15	.855403	29 47	.144597	54
7	856049	29 20	.998878	.15	.857171	29 35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29 23	.141068	52
9	859546	28 95	.998860	.15	.860686	29 12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29 00	.137567	50
11	8 863014	28 73	9.998841	.15	8 864173	28.88	11 135827	49
12	.864738	28 62	.998832	.15	.865906	28 77	134094	48
13	.866455	28.50	.998823	.17	.867632	28 65	132368	47
14	868165	28.38	.998813	.15	869351	28 55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28 17	.998795	.17	.872770	28 32	.127230	44
17	.873255	28 05	998785	.15	.874469	28 22	125531	43
18	.874938	27 95	998776	.17	.876162	28 12	123838	42
19	.876615	27 83	.998766	.15	.877849	28 00	122151	41
20	.878285	27.73	.998757	.17	879529	27 88	.120471	40
21	8.879949	27.63	9.998747	.15	8 881202	27 78	11 118798	39
22	.881607	27.52	998738	.17	.882869	27 68	.117131	38
23	.883258	27 42	998728	.17	.884530	27 58	115470	37
24	.884903	27 32	.998718	.17	.886185	27 47	113815	36
25	.886542	27 20	.998708	15	.887833	27 38	.112167	35
26	.888174	27 12	.998699	.17	.889476	27 27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27 17	.108888	33
28	891421	26 90	.998679	.17	.892742	27 07	.107258	32
29	893035	26.80	.998669	.17	.894366	26 97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26 60	9 998649	.17	8 897596	26 78	11 102404	29
32	.897842	26 50	998639	.17	.899203	26 67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26 58	.099197	27
34	.901017	26 32	998619	.17	.902398	26 48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26 38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26 28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26 20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26 10	.091281	22
39	.908853	25.85	.998568	17	.910285	26 02	089715	21
40	.910404	25.75	.998558	.17	.911846	25 92	088154	20
41	8.911949	25.65	9.998548	.18	8 913401	25 83	11 086599	19
42	.913488	25.57	.998537	.17	914951	25 73	.085049	18
43	.915022	25 47	.998527	.18	.916495	25 63	.083505	17
44	.916550	25 38	.998516	.17	.918034	25.57	.081966	16
45	.918073	25 30	.998506	.18	.919568	25 47	.080432	15
46	.919591	25 20	.998495	.17	.921096	25 38	.078904	14
47	.921103	25 12	.998485	.18	.922619	25 28	.077381	13
48	.922610	25 03	.998474	.17	.924136	[illegible]	.075864	12
49	.924112	24.95	.998464	.18	.925649	25 12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25 03	072844	10
51	8.927100	24.78	9.998442	.18	8 928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24 87	.069845	8
53	.930068	24 60	.998421	.18	.931647	24 78	.068353	7
54	.931544	24 52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24 35	.998388	.18	.936093	24 53	.063907	4
57	.935942	24 27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24 20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		11.058048	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.940296	24.03	9.998344	.18	8 941952	24.20	11.058048	60
1	.941738	23 93	.998333	.18	943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23 98	.053705	57
4	.946034	23 70	.998300	.18	.947734	23 90	.052266	56
5	.947456	23.63	.998289	.20	.949168	23.82	.050832	55
6	.948874	23 55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23 52	.045144	51
10	.954499	23.25	.998232	.20	.956267	23.45	.043733	50
11	8.955894	23.17	9 998220	.18	8.957674	23.35	11.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23 22	.039527	47
14	.960052	22 95	.998186	.20	.961866	23.15	.038134	46
15	.961429	22 87	.998174	.18	.963255	23.07	.036745	45
16	.962801	22 82	.998163	.20	.964639	23 00	.035361	44
17	.964170	22 73	.998151	.20	.966019	22 92	.033981	43
18	.965534	22 65	.998139	.18	.967394	22 87	.032606	42
19	.966893	22 60	.998128	.20	.968766	22 78	.031234	41
20	.968249	22 52	.998116	.20	.970133	22.72	.029867	40
21	8.969600	22 45	9.998104	.20	8.971496	22 65	11.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	972289	22 32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22 23	.998068	.20	.975560	22 43	.024440	36
25	.974962	22 18	.998056	.20	.976906	22.37	.023094	35
26	.976293	22 10	.998044	.20	.978248	22 30	.021752	34
27	.977619	22 03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21 97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21 90	.998008	.20	.982251	22 10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.03	.016423	30
31	8.982883	21 77	9.997984	.20	8.984899	21 97	11.015101	29
32	.984189	21 72	.997972	.22	.986217	21 92	.013783	28
33	.985491	21 63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21 57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.52	.997935	.22	.990149	21.70	.009851	25
36	.989374	21 43	.997922	.20	.991451	21.65	.008549	24
37	.990660	21 38	.997910	.22	.992750	21.58	.007250	23
38	.991943	21 32	.997897	.20	.994045	21.53	.005955	22
39	.993222	21 25	.997885	.22	.995337	21.45	.004663	21
40	.994497	21 18	.997872	.20	.996624	21.40	.003376	20
41	8 995768	21.13	9.997860	.22	8.997908	21.33	11 002092	19
42	997036	21.05	.997847	.20	8 999188	21.28	11 000812	18
43	.998299	21 02	.997835	.22	9.000465	21.22	10.999535	17
44	8 999560	20 93	.997822	.22	.001738	21 15	.998262	16
45	9.000816	20 88	.997809	.20	.003007	21 08	.996993	15
46	.002069	20 82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20 75	.997784	.22	.005534	20.97	.994466	13
48	.004563	20 70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	.007044	20.57	.997745	.22	.009298	20.80	.990702	10
51	9.008278	20 53	9.997732	.22	9.010546	20.73	10.989454	9
52	.009510	20 45	.997719	.22	.011790	20 68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20 57	.985732	6
55	.013182	20.30	.997680	.22	.015502	20.50	.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60	9.019235		9.997614		9.021620		10.978380	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19.52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19.40	9.997466	.23	9.034791	19.63	10.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9.043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18.83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.25	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	
54	.079676	17.38	.996843	.27	.082833	17.63	.917167	
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	
60	9.085894		9.996751		9.089144		10.910856	
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	.129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60	9.143555		9.995753		9.147803		10.852197	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.143555	14.97	9.995753	.30	9.147803	15.25	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	.148026	14.82	.995664	.30	.152363	15.10	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	.152451	14.65	.995573	.30	.156877	14.97	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	.156830	14.50	.995482	.30	.161347	14.82	.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	.161164	14.35	.995390	.30	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.32	9.166654	14.63	10.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.32	.170157	14.53	.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.08	.995203	.32	.174499	14.38	.825501	30
31	9.170547	14.03	9.995184	.32	9.175362	14.37	10.824638	29
32	.171380	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.93	.995108	.32	.178799	14.27	.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	.178072	13.80	.995013	.33	.183059	14.13	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.08	10.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	14.00	.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.87	.808538	10
51	9.187092	13.52	9.994798	.32	9.192294	13.83	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	.190325	13.42	.994720	.33	.195606	13.73	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.65	.801106	1
60	9.194332		9.994620		9.199713		10.800287	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.194332	13 28	9 994620	.33	9 199713	13 60	10.800287	60
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13 57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13 52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13 40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9 994398	.35	9.208619	13.35	10.791381	49
12	.203797	13 00	.994377	.33	.209420	13 33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13 28	.788982	46
15	.206131	12.92	.994316	.35	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13 22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12 83	.994233	.35	214989	13 18	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13 13	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13 07	.781074	36
25	.213818	12.68	.994108	.35	.219710	13 03	.780290	35
26	.214579	12.65	.994087	.35	.220492	13 00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13 00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12 97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12 95	.777170	31
30	.217609	12.57	994003	.35	.223607	12.92	.776393	30
31	9.218363	12.55	9 993982	.37	9.224382	12 90	10.775618	29
32	.219116	12 53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12 85	.774071	27
34	.220618	12 48	.993918	.35	.226700	12.85	.773300	26
35	.221367	12.47	.993897	.37	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.35	.228239	12 80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12 77	.770993	23
38	.223606	12 38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.35	.231302	12.72	.768698	20
41	9 225833	12.33	9.993768	.37	9.232065	12 68	10.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12 28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12 27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.35	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12 20	.993638	.37	.236614	12 57	.763386	13
48	.230984	12 18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12 15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12 12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12 07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12 03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60	9.239670		9.993351		9 246319		10.753681	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.239670	11 93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240386	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11 85	.993262	.37	.249264	12.23	.750736	56
5	.243237	11.83	.993240	.38	.249998	12.20	.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.37	.252920	12.13	.747080	51
10	.246775	11.72	.993127	.38	.253648	12.10	.746352	50
11	9.247478	11.72	9.993104	.38	9.254374	12 10	10.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12 05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12 03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.02	.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11 60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11 57	.992921	.38	.260146	11 95	.739854	41
20	.253761	11.53	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11 48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	.257211	11.45	.992783	.40	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.38	.265138	11 82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.40	.267261	11.77	.732739	31
30	.260633	11.35	.992666	.38	.267967	11.73	.732033	30
31	9.261314	11.33	9.992643	.40	9.268671	11.73	10.731329	29
32	.261994	11.32	.992619	.38	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.40	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.38	.270779	11.67	.729221	26
35	.264027	11.27	.992549	.40	.271479	11.65	.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.38	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274260	11 58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11.57	.725036	20
41	9.268065	11.15	9 992406	.40	9.275658	11 55	10.724842	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11 40	.718826	11
50	.274049	10 98	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	.277337	10.90	.992069	.42	.285268	11.32	.714732	5
56	.277991	10.90	.992044	.40	.285947	11 28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60	9.280599		9.991947		9.288652		10.711348	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.280599	10 82	9.991947	.42	9 288652	11 23	10.711348	60
1	.281248	10.82	.991922	.42	.289326	11 22	.710674	59
2	.281897	10 78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	991873	.42	290671	11 18	.709329	57
4	.283190	10 77	.991848	.42	.291342	11 18	.708658	56
5	.283836	10 73	.991823	.40	.292013	11.15	.707987	55
6	.284480	10 73	.991799	.42	.292682	11 13	.707318	54
7	.285124	10 70	.991774	.42	.293350	11 12	.706650	53
8	.285766	10 70	.991749	42	.294017	11.12	.705983	52
9	.286408	10 67	991724	.42	.294684	11 08	.705316	51
10	.287048	10.67	.991699	.42	.295349	11 07	.704651	50
11	9.287688	10 63	9 991674	.42	9.296013	11.07	10.703987	49
12	.288326	10 63	.991649	42	.296677	11 03	.703323	48
13	.288964	10 60	.991624	.42	.297339	11 03	.702661	47
14	.289600	10 60	.991599	.42	.298001	11.02	.701999	46
15	.290236	10 57	991574	.42	.298662	11 00	.701338	45
16	.290870	10 57	991549	.42	.299322	10 97	.700678	44
17	.291504	10 55	.991524	.43	.299980	10 97	.700020	43
18	.292137	10 52	.991498	.42	.300638	10 95	.699362	42
19	.292768	10 52	.991473	.42	.301295	10 93	.698705	41
20	.293399	10 50	.991448	.43	.301951	10 93	.698049	40
21	9.294029	10 48	9.991422	.42	9.302607	10 90	10.697393	39
22	.294658	10 47	.991397	.42	.303261	10 88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10 88	.696086	37
24	.295913	10 43	.991346	.42	.304567	10 85	.695433	36
25	.296539	10 42	.991321	.43	.305218	10 85	.694782	35
26	.297164	10 40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10 82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10 35	.991218	.42	.307816	10.78	.692184	31
30	.299655	10.35	.991193	.43	.308463	10 77	.691537	30
31	9.300276	10 32	9.991167	43	9.309109	10 75	10.690891	29
32	.300895	10 32	.991141	.43	.309754	10 75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10 72	.689601	27
34	.302132	10 27	.991090	.43	.311042	10.72	.688958	26
35	.302748	10 27	.991064	43	.311685	10 70	.688315	25
36	.303364	10 25	.991038	.43	.312327	10 68	.687673	24
37	.303979	10 23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10 23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10 20	990960	.43	.314247	10.63	.685753	21
40	.305819	10.18	.990934	.43	.314885	10.63	.685115	20
41	9.306430	10.18	9 990908	.43	9.315523	10 60	10.684477	19
42	.307041	10 15	.990882	.45	.316159	10 60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10 13	.990829	.43	.317430	10.57	.682570	16
45	.308867	10 12	.990803	.43	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10 08	.990750	.43	319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	.311893	10.03	.990671	.43	.321222	10.48	.678778	10
51	9.312495	10.03	9.990645	.45	9.321851	10.47	10.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9 98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	.314897	9.97	.990538	.45	.324358	10.42	.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10 40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10 37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60	9.317879		9.990404		9.327475		10.672525	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.317879	9.90	9.990404	.43	9.327475	10 33	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9 87	.990351	.45	.328715	10 32	.671285	58
3	.319658	9 85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10 28	.670047	56
5	.320840	9 83	.990270	.45	.330570	10 28	.669430	55
6	.321430	9 82	.990243	.47	.331187	10 27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10 25	.668197	53
8	.322607	9 78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9 77	.990161	.45	.333038	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9 334259	10.20	10.665741	49
12	.324950	9 73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9 72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9 72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9 68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	337311	10 13	.662689	44
17	.327862	9 67	.989942	.45	.337919	10 13	.662081	43
18	.328442	9 65	.989915	.47	.338527	10 10	.661473	42
19	.329021	9 63	.989887	.45	.339133	10 10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9 60	.989804	.45	.340948	10 07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9 58	.989749	.47	.342155	10 03	.657845	36
25	.332478	9 55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9 55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9 52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9 53	.989637	.45	.344558	9 98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9 48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9 47	.989525	.47	.346949	9 93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9 43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9 88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9 87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9 37	.989328	.47	.351106	9 85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9 35	9.989271	.47	9.352287	9 82	10 647713	19
42	.342119	9.33	.989243	.48	.352876	9 82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9 30	989186	.48	.354053	9.78	.645947	16
45	.343797	9 30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9 28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9 75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9 73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9 73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9 72	.642434	10
51	9.347134	9 22	9.988985	.48	9.358149	9.70	10 641851	9
52	.347687	9 22	.988956	.48	.358731	9 70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9 67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9 63	.638368	3
58	.350992	9 13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9 62	.637218	1
60	9.352088		9.988724		9.363364		10.636636	0
′	Cosine.	D. 1″.	Sine,	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.352088	9.12	9.988724	.48	9.363364	9 60	10.636636	60
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	50	364515	9 58	.635485	58
3	.353726	9.08	.988636	48	.365090	9 57	.634910	57
4	.354271	9 07	.988607	48	.365664	9 55	.634336	56
5	.354815	9.05	.988578	.50	.366237	9 55	.633763	55
6	.355358	9.05	.988548	.48	.366810	9 53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9 52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9 52	.632047	52
9	.356984	9 00	.988460	.50	.368524	9 50	.631476	51
10	.357524	9.00	.988430	.48	.369094	9 48	.630906	50
11	9.358064	8.98	9.988401	.50	9.369663	9.48	10 630337	49
12	.358603	8 97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8 95	.988342	.50	.370799	9 47	.629201	47
14	.359678	8 95	.988312	.50	.371367	9.43	.628633	46
15	.360215	8.95	.988282	.50	.371933	9 43	.628067	45
16	.360752	8 92	.988252	.48	.372499	9.42	.627501	44
17	361287	8 92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8 90	.988193	.50	.373629	9 40	.626371	42
19	.362356	8 88	.988163	.50	.374193	9 38	.625807	41
20	.362889	8 88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8 87	9.988103	.50	9 375319	9.37	10 624681	39
22	.363954	8 85	.988073	.50	.375881	9 35	624119	38
23	.364485	8 85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8 83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8 82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8 82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8 78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8 80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8 77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.27	.619646	30
31	9.368711	8 75	9.987801	.50	9 380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.72	.987740	.50	.382020	9.25	.617980	27
34	.370285	8 72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8 70	.987679	.50	.383129	9.22	.616871	25
36	.371330	8 70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8 68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10 613562	19
42	.374452	8 63	.987465	.52	.386987	9 15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8 60	.987403	.52	.388084	9 12	.611916	16
45	.376003	8 60	.987372	.52	.388631	9 12	.611369	15
46	.376519	8 60	.987341	.52	.389178	9 10	610822	14
47	.377035	8 57	.987310	.52	.389724	9 10	.610276	13
48	.377549	8 57	.987279	.52	.390270	9 08	.609730	12
49	.378063	8 57	.987248	.52	.390815	9 08	.609185	11
50	.378577	8.53	.987217	.52	.391360	9.05	.608640	10
51	9·379089	8.53	9.987186	.52	9.391903	9 07	10.608097	9
52	.379601	8.53	.987155	.52	.392447	9 03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9 03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	.381134	8 48	.987061	.52	.394073	9.02	.605927	5
56	.381643	8.48	.987030	53	.394614	9 00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8 45	986967	.52	.395694	8 98	.604306	2
59	.383168	8 45	.986936	.53	.396233	8.97	.603767	1
60	9.383675		9 986904		9.396771		10.603229	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	.412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.18	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60	9.412996		9.984944		9.428052		10.571948	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.412996	7.85	9.984944	.57	9.428052	8.43	10.571948	60
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.80	.984774	.57	.430573	8.37	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	.417684	7.77	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.33	10.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.28	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.57	9.438554	8.23	10.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.22	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	.425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.58	.983911	.60	.442988	8.18	.557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.15	10.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	.429170	7.55	.983735	.58	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.58	.447870	8.10	.552130	20
41	9.431879	7.50	9.983523	.60	9.448356	8.08	10.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	.433675	7.45	.983381	.60	.450294	8.05	.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.43	.983273	.58	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	.435908	7.42	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.42	9.983166	.60	9.453187	8.02	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	9.440338		9.982842		9.457496		10.542504	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.440338	7 33	9.982842	.62	9.457496	7.95	10.542504	60
1	.440778	7 33	982805	60	.457973	7 93	542027	59
2	.441218	7 33	.982769	.60	.458449	7 93	541551	58
3	.441658	7 30	.982733	62	.458925	7.92	.541075	57
4	.442096	7 32	.982696	60	.459400	7 92	.540600	56
5	.442535	7 30	.982660	60	.459875	7 90	.540125	55
6	.442973	7 28	.982624	62	.460349	7 90	.539651	54
7	.443410	7 28	982587	.60	.460823	7 90	539177	53
8	.443847	7 28	982551	62	.461297	7 88	.538703	52
9	.444284	7 27	982514	.62	.461770	7 87	538230	51
10	.444720	7 25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9 982441	.62	9 462715	7 85	10 537285	49
12	.445590	7.25	982401	.62	.463186	7 87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7 83	536342	47
14	.446459	7.23	.982331	.62	.464128	7 85	535872	46
15	.446893	7.22	982294	.62	.464599	7 83	.535401	45
16	.447326	7 22	982257	.62	.465069	7 83	.534931	44
17	.447759	7.20	.982220	62	.465539	7 82	.534461	43
18	.448191	7.20	982183	.62	.466008	7 82	.533992	42
19	.448623	7 18	.982146	.62	.466477	7 80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7 80	.533055	40
21	9.449485	7.17	9 982072	62	9.467413	7 78	10 532587	39
22	.449915	7.17	982035	.62	.467880	7 78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7 78	531653	37
24	.450775	7.15	.981961	.62	.468814	7 77	531186	36
25	.451204	7 13	.981924	.63	.469280	7.77	530720	35
26	.451632	7.13	.981886	.62	.469746	7 75	530254	34
27	.452060	7 13	.981849	.62	.470211	7 75	.529789	33
28	.452488	7 12	.981812	.63	.470676	7 75	.529324	32
29	.452915	7 12	.981771	.62	.471141	7 73	528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7 72	10 527931	29
32	.454194	7.08	.981662	.62	.472532	7 72	.527468	28
33	.454619	7 08	.981625	.63	.472995	7 70	.527005	27
34	.455044	7 08	.981587	.63	.473457	7 70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7 70	526081	25
36	.455893	7.05	.981512	.63	.474381	7 68	.525619	24
37	.456316	7 05	.981474	.63	.474842	7 68	.525158	23
38	.456739	7 05	.981436	.62	.475303	7 67	.524697	22
39	.457162	7 03	.981399	.63	.475763	7 67	.524237	21
40	.457584	7 03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7 65	10 523317	19
42	.458427	7 02	.981285	.63	.477142	7 65	.522858	18
43	.458848	7 00	.981247	.63	.477601	7 63	522399	17
44	.459268	7 00	.981209	.63	.478059	7 63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7 63	.521483	15
46	.460108	6 98	.981133	.63	.478975	7 62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7 62	.520568	13
48	.460946	6 97	.981057	.63	.479889	7 60	.520111	12
49	.461364	6 97	.981019	.63	.480345	7 60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7 60	.519199	10
51	9.462199	6.95	9 980942	.63	9.481257	7 58	10 518748	9
52	.462616	6.93	.980904	.63	.481712	7 58	.518288	8
53	.463032	6 93	.980866	.65	.482167	7 57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6 92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7 55	.516471	4
57	.464694	6 90	.980712	.65	.483982	7 55	.516018	3
58	.465108	6 90	.980673	.63	.484435	7 53	.515565	2
59	.465522	6 88	.980635	.65	.484887	7.53	.515113	1
60	9.465935		9.980596		9.485339		10.514661	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9 465935	6.88	9 980596	.63	9 485339	7 53	10.514661	60
1	.466348	6 88	.980558	.65	485791	7 52	.514209	59
2	466761	6 87	.980519	.65	.486242	7.52	513758	58
3	467173	6 87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6 85	.980442	.65	.487143	7.50	.512857	56
5	467996	6 85	.980403	.65	.487593	7.50	.512407	55
6	468407	6 83	.980364	.65	.488043	7 48	.511957	54
7	.468817	6 83	.980325	.65	.488492	7 48	.511508	53
8	.469227	6 83	.980286	.65	.488941	7 48	.511059	52
9	.469637	6 82	.980247	.65	.489390	7 47	.510610	51
10	470046	6 82	.980208	.65	.489838	7.47	.510162	50
11	9 470455	6 80	9 980169	.65	9.490286	7 45	10.509714	49
12	.470863	6 80	.980130	.65	.490733	7 45	.509267	48
13	.471271	6 80	.980091	65	.491180	7 45	.508820	47
14	.471679	6 78	.980052	.67	.491627	7 43	.508373	46
15	.472086	6 77	.980012	.65	.492073	7 43	.507927	45
16	472492	6 77	.979973	.65	.492519	7 43	.507481	44
17	472898	6 77	979934	.65	.492965	7 42	.507035	43
18	.473304	6.77	.979895	.67	.493110	7 40	506590	42
19	.473710	6 75	.979855	65	.493854	7 42	.506146	41
20	.474115	6 73	.979816	.67	.494299	7 40	.505701	40
21	9 474519	6 73	9 979776	.65	9 494743	7 38	10 505257	39
22	.474923	6 73	.979737	.67	.495186	7 40	.504814	38
23	.475327	6 72	.979697	.65	.495630	7 38	.504370	37
24	475730	6 72	.979658	.67	496073	7 37	.503927	36
25	476133	6.72	.979618	.65	.496515	7 37	.503485	35
26	476536	6 70	.979579	.67	.496957	7 37	.503043	34
27	.476938	6 70	.979539	.67	.497399	7 37	.502601	33
28	.477340	6 68	.979499	.67	.497841	7 35	.502159	32
29	477741	6 68	.979459	.65	.498282	7 33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7 33	10.500837	29
32	478942	6 67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6 65	.979300	67	.500042	7 32	.499958	27
34	.479741	6.65	.979260	67	.500481	7 32	.499519	26
35	.480140	6 65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6 62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6 62	.979059	.67	.502672	7 28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6 60	9.978979	.67	9 503546	7 27	10.496454	19
42	.482921	6 58	.978939	68	.503982	7 27	.496018	18
43	.483316	6 60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6 58	.978858	.68	.504854	7 25	.495146	16
45	.484107	6 57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6 57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6 57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6 55	.978696	.68	506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.23	.492973	11
50	.486075	6 53	.978615	.68	.507460	7 22	.492540	10
51	9.486467	6.55	9.978574	.68	9 507893	7 22	10.492107	9
52	.486860	6 52	.978533	.67	.508326	7 22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6 52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6 50	978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	978329	.68	.510485	7.18	.489515	3
58	.489204	6 48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	9.489982		9.978206		9.511776		10.488224	0
′	Cosine.	D 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.483943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.35	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	29
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6.95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60	9.512642		9.975670		9.536972		10.463028	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.512642	6 12	9.975670	.72	9.536972	6 83	10.463028	60
1	.513009	6 10	.975627	.73	.537382	6 83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6 82	.461798	57
4	.514107	6 08	.975496	.73	.538611	6 82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6 82	.460980	55
6	.514837	6.08	.975408	.72	.539429	6 80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6 80	.460163	53
8	.515566	6 07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6 80	.459347	51
10	.516294	6.05	.975233	.73	541061	6 78	.458939	50
11	9 516657	6 05	9 975189	.73	9 541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	541875	6 77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6 03	.975057	.73	.542688	6 77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6 75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6 75	.456095	43
18	.519190	6 02	.974880	.73	.544310	6 75	.455690	42
19	.519551	6 00	.974836	.73	.544715	6 73	.455285	41
20	.519911	6 00	.974792	.73	.545119	6 75	.454881	40
21	9.520271	6.00	9.974748	.75	9 545524	6 73	10.454476	39
22	.520631	5 98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6 73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6 72	.453265	36
25	.521707	5.98	.974570	.75	.547138	6.70	.452862	35
26	.522066	5 97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6 70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6 70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6 68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6 67	.449648	27
34	.524920	5 92	.974167	.75	.550752	6 68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6 65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6 65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6 65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6 65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6 63	10.446452	19
42	.527753	5 87	.973807	.77	.553946	6 63	.446054	18
43	.528105	5 88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5 85	.973671	.77	.555139	6 62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6 60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6 60	.443275	11
50	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
51	9.530915	5.83	9.973398	.77	9.557517	6 60	10.442483	9
52	.531265	5.82	.973352	.75	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5 82	.973261	.77	.558703	6 57	.441297	6
55	.532312	5 82	.973215	.77	.559097	6 57	.440903	5
56	.532661	5.80	.973169	.75	.559491	6 57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6 57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6 55	.439327	1
60	9.534052		9.972986		9.561066		10.438934	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	.535783	5.77	.972755	.77	.563028	6.52	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.78	9.565373	6.50	10.434627	49
12	.538194	5.73	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	.539223	5.70	.972291	.77	.566932	6.47	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.68	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.47	.431127	40
21	9.541272	5.68	9.972011	.78	9.569261	6.45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
41	9.548024	5.58	9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	.549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.28	.416200	1
60	9.554329		9.970152		9.584177		10.415823	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60	9.573575		9.967166		9.606410		10.393590	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.573575	5 22	9.967166	.85	9.606410	6 05	10.393590	60
1	.573888	5 20	.967115	.85	.606773	6 07	.393227	59
2	.574200	5 20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5 20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5 20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5 18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6 03	.391050	53
8	.576069	5 17	.966756	.85	.609312	6 03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6 03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6.03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6 02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6 00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6 02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6 00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5 15	.966292	.87	.612561	6 00	.387439	43
18	.579162	5 13	.966240	.87	.612921	6 00	.387079	42
19	.579470	5 12	.966188	.87	.613281	6 00	.386719	41
20	.579777	5.13	.966136	85	.613641	5 98	.386359	40
21	9.580085	5.12	9 966085	.87	9.614000	5 98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5 97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	581618	5.10	.965824	87	.615793	5 97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5 10	.965720	.87	.616509	5 97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5 95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9.617582	5 95	10.382418	29
32	.583449	5 08	.965511	.88	.617939	5 93	.382061	28
33	.583754	5 07	.965458	.87	.618295	5 95	.381705	27
34	.584058	5 05	.965406	.88	.618652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5 93	.380636	24
37	.584968	5 07	.965248	.88	.619720	5 93	.380280	23
38	.585272	5 03	.965195	.87	.620076	5.93	.379024	22
39	.585574	5 05	.965143	.88	.620432	5 92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5 92	10 378858	19
42	.586482	5 02	.964984	.88	.621497	5 92	.378503	18
43	.586783	5 03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.03	.964826	.88	.622561	5 90	.377439	15
46	.587688	5 02	.964773	.88	.622915	5 90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5 90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5 88	.376377	12
49	.588590	5 00	.964613	.88	.623976	5 90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5 88	.375670	10
51	9.589190	4.98	9 964507	.88	9.624683	5 88	10.375317	9
52	.589489	5 00	.964454	.90	.625036	5 87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5 88	.374612	7
54	.590088	4 98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5 87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5 87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5.87	.373203	3
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	.591580	4.97	.964080	.90	.627501	5 85	.372499	1
60	9.591878		9 964026		9.627852		10.372148	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.591878	4 97	9.964026	.90	9.627852	5.85	10.372148	60
1	.592176	4 95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593363	4 93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4 93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4 92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9 963434	.92	9.631704	5 82	10.368296	49
12	.595432	4 92	.963379	.90	.632053	5 82	.367947	48
13	.595727	4 90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4 90	.963271	.90	.632750	5 82	.367250	46
15	.596315	4 90	.963217	.90	.633099	5 80	.366901	45
16	.596609	4 90	.963163	.92	.633447	5 80	.366553	44
17	.596903	4 88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4 90	.963054	.92	.634143	5 78	.365857	42
19	.597490	4 88	.962999	.90	.634490	5 80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4 88	9 962890	.90	9 635185	5 78	10.364815	39
22	.598368	4 87	.962836	.92	.635532	5 78	.364468	38
23	.598660	4 87	.962781	.90	.635879	5 78	.364121	37
24	.598952	4 87	.962727	.92	.636226	5 77	.363774	36
25	.599244	4 87	.962672	.92	.636572	5 78	.363428	35
26	.599536	4 85	.962617	.92	.636919	5 77	.363081	34
27	.599827	4 85	.962562	.90	.637265	5 77	.362735	33
28	.600118	4 85	.962508	92	.637611	5.75	.362389	32
29	.600409	4 85	.962453	.92	637956	5 77	.362044	31
30	.600700	4 83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4 83	9 962343	.92	9.638647	5.75	10.361353	29
32	.601280	4 83	.962288	.92	.638992	5 75	.361008	28
33	.601570	4 83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4 82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4 82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4 82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4 80	.961957	.92	.641060	5 73	.358940	22
39	.603305	4 82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4 80	9 961791	.93	9.642091	5.72	10.357909	19
42	.604170	4 78	.961735	.92	.642434	5 72	.357566	18
43	.604457	4 80	.961680	.93	.642777	5 72	.357223	17
44	.604745	4 78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.72	.356537	15
46	.605319	4 78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4 77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4 78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4 77	.961290	.92	.645174	5.70	.354826	10
51	9.606751	4 75	9.961235	.93	9.645516	5.68	10.354484	9
52	.607036	4 77	.961179	.93	.645857	5.70	.354143	8
53	.607322	4 75	.961123	.93	.646199	5.68	.353801	7
54	.607607	4 75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.75	.961011	.93	.646881	5.68	.353119	5
56	.608177	4 73	.960955	.93	.647222	5.67	.352778	4
57	.608461	4 73	.960899	.93	.647562	5.68	.352438	3
58	.608745	4 73	.960843	.95	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.93	.648243	5.67	.351757	1
60	9.609313		9.960730		9.648583		10.351417	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.93	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	9.625948		9.957276		9.668673		10.331327	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327	60
1	.626219	4.52	.957217	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	.628647	4.48	.956684	.98	.671963	5.47	.328037	50
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763	40
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	.98	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4.38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260	20
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
51	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710	9
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
55	.640544	4.33	.953968	1.03	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	9.641842		9.953660		9.688182		10.311818	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.641842	4.32	9.953660	1 02	9.688182	5.33	10.311818	60
1	.642101	4.32	.953599	1.03	.688502	5 32	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5 33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5 33	.310857	57
4	.642877	4 30	.953413	1.02	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5 33	.310217	55
6	.643393	4.28	.953290	1.03	.690103	5 33	.309897	54
7	.643650	4.30	.953228	1 03	.690423	5 32	.309577	53
8	.643908	4 28	.953166	1 03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1 03	.691062	5 32	.308938	51
10	.644423	4.28	953042	1 03	.691381	5.32	.308619	50
11	9.644680	4.27	9.952980	1 03	9.691700	5.32	10.308300	49
12	.644936	4 28	.952918	1 05	.692019	5.32	.307981	48
13	.645193	4.28	952855	1 03	.692338	5 30	.307662	47
14	.645450	4 27	.952793	1 03	.692656	5 32	.307344	46
15	.645706	4.27	.952731	1 03	.692975	5 30	.307025	45
16	.645962	4 27	.952669	1 05	.693293	5 32	.306707	44
17	.646218	4.27	.952606	1 03	.693612	5 30	.306388	43
18	.646474	4 25	.952544	1 05	693930	5 30	.306070	42
19	.646729	4.25	.952481	1 03	.694248	5 30	.305752	41
20	.646984	4.27	.952419	1 05	.694566	5 28	.305434	40
21	9.647240	4 23	9.952356	1 03	9.694883	5 30	10.305117	39
22	.647494	4.25	.952294	1 05	695201	5 28	.304799	38
23	.647749	4 25	.952231	1 05	.695518	5 30	.304482	37
24	.648004	4 23	.952168	1 03	.695836	5 28	.304164	36
25	.648258	4 23	.952106	1 05	.696153	5 28	.303847	35
26	.648512	4.23	.952043	1 05	.696470	5 28	.303530	34
27	.648766	4 23	.951980	1 05	.696787	5 27	.303213	33
28	.649020	4 23	.951917	1 05	.697103	5 28	.302897	32
29	.649274	4 22	.951854	1 05	.697420	5.27	.302580	31
30	.649527	4.23	.951791	1.05	.697736	5 28	.302264	30
31	9.649781	4.22	9.951728	1 05	9.698053	5 27	10.301947	29
32	.650034	4 22	.951665	1 05	.698369	5 27	.301631	28
33	.650287	4 20	.951602	1.05	.698685	5 27	.301315	27
34	.650539	4.22	.951539	1 05	.699001	5 25	.300999	26
35	.650792	4 20	.951476	1 07	.699316	5 27	.300684	25
36	.651044	4.22	.951412	1 05	.699632	5 25	.300368	24
37	.651297	4.20	.951349	1 05	.699947	5 27	.300053	23
38	.651549	4.18	.951286	1 07	.700263	5 25	.299737	22
39	.651800	4.20	.951222	1 05	.700578	5 25	.299422	21
40	.652052	4.20	.951159	1.05	.700893	5.25	.299107	20
41	9.652304	4.18	9.951096	1.07	9.701208	5 25	10.298792	19
42	.652555	4 18	.951032	1.07	.701523	5 23	.298477	18
43	.652806	4 18	.950968	1 05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5 23	.297848	16
45	653308	4.17	.950841	1.05	.702466	5 25	.297534	15
46	.653558	4.17	.950778	1.07	.702781	5 23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5 23	.296278	11
50	.654558	4.17	.950522	1.07	.704036	5.23	.295964	10
51	9.654808	4.17	9.950458	1.07	9.704350	5.22	10.295650	9
52	.655058	4.15	.950394	1.07	.704663	5 22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5 23	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5 22	.294397	5
56	.656054	4.13	.950138	1.07	.705916	5 20	.294084	4
57	656302	4.15	.950074	1.07	.706228	5 22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60	9.657047		9.949881		9.707166		10.292834	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.657047	4.13	9.949881	1.08	9.707166	5.20	10.292834	60
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.07	.708726	5.18	.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	.660746	4.08	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	.661970	4.07	.948584	1.08	.713386	5.17	.286614	40
21	9.662214	4.08	9.948519	1.08	9.713696	5.15	10.286304	39
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	38
23	.662703	4.05	.948388	1.08	.714314	5.17	.285686	37
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	36
25	.663190	4.05	.948257	1.08	.714933	5.15	.285067	35
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.08	.715860	5.13	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.15	.283832	31
30	.664406	4.03	.947929	1.10	.716477	5.13	.283523	30
31	9.664648	4.05	9.947863	1.10	9.716785	5.13	10.283215	29
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.08	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.12	.718017	5.13	.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.03	.947467	1.10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.13	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	.666824	4.02	.947269	1.10	.719555	5.12	.280445	20
41	9.667065	4.00	9.947203	1.12	9.719862	5.12	10.280138	19
42	.667305	4.02	.947136	1.10	.720169	5.12	.279831	18
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	17
44	.667786	4.02	.947004	1.12	.720783	5.10	.279217	16
45	.668027	4.00	.946937	1.10	.721089	5.12	.278911	15
46	.668267	3.98	.946871	1.12	.721396	5.10	.278604	14
47	.668506	4.00	.946804	1.10	.721702	5.12	.278298	13
48	.668746	4.00	.946738	1.12	.722009	5.10	.277991	12
49	.668986	3.98	.946671	1.12	.722315	5.10	.277685	11
50	.669225	3.98	.946604	1.10	.722621	5.10	.277379	10
51	9.669464	3.98	9.946538	1.12	9.722927	5.08	10.277073	9
52	.669703	3.98	.946471	1.12	.723232	5.10	.276768	8
53	.669942	3.98	.946404	1.12	.723538	5.10	.276462	7
54	.670181	3.97	.946337	1.12	.723844	5.08	.276156	6
55	.670419	3.98	.946270	1.12	.724149	5.08	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.10	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.97	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.95	.946002	1.12	.725370	5.07	.274630	1
60	9.671609		9.945935		9.725674		10.274326	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.671609	3 97	9.945935	1.12	9.725674	5 08	10 274326	60
1	.671847	3 95	.945868	1 13	.725979	5.08	.274021	59
2	.672084	3 95	.945800	1 12	.726284	5 07	.273716	58
3	.672321	3 95	.945733	1 12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1 13	.726892	5 05	.273108	56
5	.672795	3 95	.945598	1.12	.727197	5 07	.272803	55
6	.673032	3 93	.945531	1.12	.727501	5 07	.272499	54
7	.673268	3 95	.945464	1 13	.727805	5 07	.272195	53
8	.673505	3 93	.945396	1 13	.728109	5 05	.271891	52
9	.673741	3.93	.945328	1 12	.728412	5 07	.271588	51
10	.673977	3 93	.945261	1.13	.728716	5 07	.271284	50
11	9.674213	3.92	9.945193	1 13	9.729020	5 05	10 270980	49
12	.674448	3 93	.945125	1 12	.729323	5 05	.270677	48
13	.674684	3.92	.945058	1 13	.729626	5 05	.270374	47
14	.674919	3 93	.944990	1 13	.729929	5 07	.270071	46
15	.675155	3.92	.944922	1 13	.730233	5 03	.269767	45
16	.675390	3 90	.944854	1 13	.730535	5 05	.269465	44
17	.675624	3 92	.944786	1 13	.730838	5.05	.269162	43
18	.675859	3 92	.944718	1 13	.731141	5.05	.268859	42
19	.676094	3 90	.944650	1 13	.731444	5 03	.268556	41
20	.676328	3.90	.944582	1 13	.731746	5 03	.268254	40
21	9.676562	3 90	9.944514	1 13	9.732048	5 05	10 267952	39
22	.676796	3 90	.944416	1 15	.732351	5 03	267649	38
23	.677030	3.90	.944377	1 13	.732653	5 03	.267347	37
24	.677264	3 90	.944309	1 13	.732955	5 03	.267045	36
25	.677498	3 88	.944241	1 15	.733257	5 02	.266743	35
26	.677731	3 88	.944172	1.13	.733558	5 03	.266442	34
27	.677964	3 88	.944104	1 13	.733860	5 03	266140	33
28	.678197	3 88	.944036	1 15	.734162	5 02	.265838	32
29	.678430	3 88	.943967	1 13	.734463	5 02	.265537	31
30	.678663	3 87	.943899	1 15	.734764	5 03	.265236	30
31	9 678895	3 88	9.943830	1 15	9.735066	5 02	10.264934	29
32	.679128	3 87	.943761	1.13	.735367	5 02	.264633	28
33	.679360	3 87	.943693	1 15	.735668	5 02	.264332	27
34	.679592	3.87	.943624	1 15	.735969	5 00	.264031	26
35	.679824	3 87	.943555	1 15	.736269	5 02	.263731	25
36	.680056	3 87	.943486	1 15	.736570	5 00	.263430	24
37	.680288	3 85	.943417	1 15	.736870	5 02	.263130	23
38	.680519	3 85	.943348	1 15	.737171	5 00	.262829	22
39	.680750	3 87	.943279	1 15	.737471	5 00	.262529	21
40	.680982	3.85	.943210	1 15	.737771	5.00	.262229	20
41	9.681213	3 83	9.943141	1 15	9 738071	5 00	10 261929	19
42	.681443	3 85	.943072	1 15	.738371	5 00	.261629	18
43	.681674	3 85	.943003	1.15	.738671	5 00	.261329	17
44	.681905	3 83	.942934	1.17	.738971	5 00	.261029	16
45	.682135	3 83	.942864	1.15	.739271	4 98	.260729	15
46	.682365	3 83	.942795	1 15	.739570	5 00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3 83	.942656	1 15	.740169	4 98	.259831	12
49	.683055	3 82	.942587	1 17	.740468	4 98	.259532	11
50	.683284	3 83	.942517	1.15	.740767	4 98	.259233	10
51	9.683514	3 82	9.942448	1 17	9.741066	4 98	10.258934	9
52	.683743	3 82	.942378	1 17	.741365	4 98	.258635	8
53	683972	3 82	.942308	1.15	.741664	4 97	.258336	7
54	.684201	3 82	.942239	1 17	.741962	4.98	.258038	6
55	.684430	3 80	.942169	1.17	.742261	4.97	.257739	5
56	.684658	3 82	.942099	1 17	.742559	4 98	.257441	4
57	.684887	3.80	.942029	1 17	.742858	4 97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4 97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	9.685571		9.941819		9.743752		10 256248	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.685571	3.80	9.941819	1.17	9.743752	4.97	10.256248	60
1	.685799	3.80	.941749	1.17	.744050	4.97	.255950	59
2	.686027	3.78	.941679	1.17	.744348	4.95	.255652	58
3	.686254	3.80	.941609	1.17	.744645	4.97	.255355	57
4	.686482	3.78	.941539	1.17	.744943	4.95	.255057	56
5	.686709	3.78	.941469	1.18	.745240	4.97	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.77	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.18	.746132	4.95	.253868	52
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571	51
10	.687843	3.77	.941117	1.18	.746726	4.95	.253274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.93	10.252977	49
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681	48
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384	47
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087	46
15	.688972	3.77	.940763	1.17	.748209	4.93	.251791	45
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311	40
21	9.690323	3.75	9.940338	1.18	9.749985	4.93	10.250015	39
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719	38
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424	37
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.20	.751167	4.92	.248833	35
26	.691444	3.73	.989982	1.18	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243	33
28	.691892	3.72	.939840	1.20	.752052	4.92	.247948	32
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653	31
30	.692339	3.72	.939697	1.20	.752642	4.92	.247358	30
31	9.692562	3.72	9.939625	1.18	9.752937	4.90	10.247063	29
32	.692785	3.72	.939554	1.20	.753231	4.92	.246769	28
33	.693008	3.72	.939482	1.20	.753526	4.90	.246474	27
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180	26
35	.693453	3.72	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	.694564	3.70	.938980	1.20	.755585	4.88	.244415	20
41	9.694786	3.68	9.938908	1.20	9.755878	4.90	10.244122	19
42	.695007	3.70	.938836	1.22	.756172	4.88	.243828	18
43	.695229	3.68	.938763	1.20	.756165	4.90	.243535	17
44	.695450	3.68	.938691	1.20	.756759	4.88	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.88	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.22	.757638	4.88	.242362	13
48	.696334	3.67	.938402	1.20	.757931	4.88	.242069	12
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776	11
50	.696775	3.67	.938258	1.22	.758517	4.88	.241483	10
51	9.696995	3.67	9.938185	1.20	9.758810	4.87	10.241190	9
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898	8
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605	7
54	.697654	3.67	.937967	1.20	.759687	4.87	.240313	6
55	.697874	3.67	.937895	1.22	.759979	4.88	.240021	5
56	.698094	3.65	.937822	1.22	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.22	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852	1
60	9.698970		9.937531		9.761439		10.238561	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.698970	3.65	9.937531	1.22	9.761439	4.87	10.238561	60
1	.699189	3.63	.937458	1.22	.761731	4.87	.238269	59
2	.699407	3.65	.937385	1.22	.762023	4.85	.237977	58
3	.699626	3.63	.937312	1.23	.762314	4.87	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.62	.936946	1.23	.763770	4.85	.236230	52
9	.700933	3.63	.936872	1.22	.764061	4.85	.235939	51
10	.701151	3.62	.936799	1.23	.764352	4.85	.235648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.83	10.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.85	.235067	48
13	.701802	3.62	.936578	1.22	.765224	4.83	.234776	47
14	.702019	3.62	.936505	1.23	.765514	4.85	.234486	46
15	.702236	3.60	.936431	1.23	.765805	4.83	.234195	45
16	.702452	3.62	.936357	1.22	.766095	4.83	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936186	1.23	.766965	4.83	.233035	41
20	.703317	3.60	.936062	1.23	.767255	4.83	.232745	40
21	9.703533	3.60	9.935988	1.23	9.767545	4.82	10.232455	39
22	.703749	3.58	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.58	.935840	1.23	.768124	4.83	.231876	37
24	.704179	3.60	.935766	1.23	.768414	4.82	.231586	36
25	.704395	3.58	.935692	1.23	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.25	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.23	.769281	4.83	.230719	33
28	.705040	3.57	.935469	1.23	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.25	.769860	4.80	.230140	31
30	.705469	3.57	.935320	1.23	.770148	4.82	.229852	30
31	9.705683	3.58	9.935246	1.25	9.770437	4.82	10.229563	29
32	.705898	3.57	.935171	1.23	.770726	4.82	.229274	28
33	.706112	3.57	.935097	1.25	.771015	4.80	.228985	27
34	.706326	3.55	.935022	1.23	.771303	4.82	.228697	26
35	.706539	3.57	.934948	1.25	.771592	4.80	.228408	25
36	.706753	3.57	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.55	.934798	1.25	.772168	4.82	.227832	23
38	.707180	3.55	.934723	1.23	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	.707606	3.55	.934574	1.25	.773033	4.80	.226967	20
41	9.707819	3.55	9.934499	1.25	9.773321	4.78	10.226679	19
42	.708032	3.55	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.55	.934349	1.25	.773896	4.80	.226104	17
44	.708458	3.53	.934274	1.25	.774184	4.78	.225816	16
45	.708670	3.53	.934199	1.27	.774471	4.80	.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.78	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.78	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.80	.224667	12
49	.709518	3.53	.933898	1.27	.775621	4.78	.224379	11
50	.709730	3.52	.933822	1.25	.775908	4.78	.224092	10
51	9.709941	3.53	9.933747	1.27	9.776195	4.78	10.223805	9
52	.710153	3.52	.933671	1.25	.776482	4.77	.223518	8
53	.710364	3.52	.933596	1.27	.776768	4.78	.223232	7
54	.710575	3.52	.933520	1.25	.777055	4.78	.222945	6
55	.710786	3.52	.933445	1.27	.777842	4.77	.222658	5
56	.710997	3.52	.933369	1.27	.777628	4.78	.222372	4
57	.711208	3.52	.933293	1.27	.777915	4.77	.222085	3
58	.711419	3.50	.933217	1.27	.778201	4.78	.221799	2
59	.711629	3.50	.933141	1.25	.778488	4.77	.221512	1
60	9.711839		9.933066		9.778774		10.221226	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.711839	3.52	9.933066	1.27	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	.712889	3.48	.932685	1.27	.780203	4.77	.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713308	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.47	9.932228	1.28	9.781916	4.75	10.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.27	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	.716017	3.45	.931537	1.28	.784479	4.75	.215521	40
21	9.716224	3.47	9.931460	1.28	9.784764	4.73	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	.717053	3.43	.931152	1.28	.785900	4.73	.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	.718085	3.43	.930766	1.30	.787319	4.73	.212681	30
31	9.718291	3.43	9.930688	1.28	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	.719114	3.43	.930378	1.30	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	22
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	21
40	.720140	3.42	.929989	1.30	.790151	4.72	.209849	20
41	9.720345	3.40	9.929911	1.30	9.790434	4.70	10.209566	19
42	.720549	3.42	.929833	1.30	.790716	4.72	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.72	.208437	15
46	.721366	3.40	.929521	1.32	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	12
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	11
50	.722181	3.40	.929207	1.30	.792974	4.70	.207026	10
51	9.722385	3.38	9.929129	1.32	9.793256	4.70	10.206744	9
52	.722588	3.38	.929050	1.30	.793538	4.68	.206462	8
53	.722791	3.38	.928972	1.32	.793819	4.70	.206181	7
54	.722994	3.38	.928893	1.30	.794101	4.70	.205899	6
55	.723197	3.38	.928815	1.32	.794383	4.68	.205617	5
56	.723400	3.38	.928736	1.32	.794664	4.70	.205336	4
57	.723603	3.37	.928657	1.32	.794946	4.68	.205054	3
58	.723805	3.37	.928578	1.32	.795227	4.68	.204773	2
59	.724007	3.38	.928499	1.32	.795508	4.68	.204492	1
60	9.724210		9.928420		9.795789		10.204211	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.724210	3.37	9.928420	1.30	9.795789	4.68	10.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.35	.928025	1.32	.797194	4.67	.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.33	.798596	4.68	.201404	50
11	9.726426	3.33	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	.727228	3.33	.927231	1.33	.799997	4.67	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.65	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.67	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	.729223	3.32	.926431	1.33	.802792	4.67	.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	.730217	3.30	.926029	1.33	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.35	9.804466	4.65	10.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4.65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	.731206	3.30	.925626	1.35	.805580	4.65	.194420	25
36	.731404	3.30	.925545	1.33	.805859	4.63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1.37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.62	.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924491	1.37	.809471	4.62	.190529	11
50	.734157	3.27	.924409	1.35	.809748	4.62	.190252	10
51	9.734353	3.27	9.924328	1.37	9.810025	4.62	10.189975	9
52	.734549	3.25	.924246	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924164	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924083	1.37	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.37	.811134	4.60	.188866	5
56	.735330	3.25	.923919	1.37	.811410	4.62	.188590	4
57	.735525	3.23	.923837	1.37	.811687	4.62	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.62	.188036	2
59	.735914	3.25	.923673	1.37	.812241	4.60	.187759	1
60	9.736109		9.923591		9.812517		10.187483	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.736109	3.23	9.923591	1.37	9.812517	4.62	10.187483	60
1	.736303	3.25	.923509	1.37	.812794	4.60	.187206	59
2	.736498	3.23	.923427	1.37	.813070	4.62	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.38	.813899	4.62	.186101	55
6	.737274	3.22	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.38	.814452	4.60	.185548	53
8	.737661	3.23	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	.738048	3.22	.922768	1.37	.815280	4.58	.184720	50
11	9.738241	3.22	9.922686	1.38	9.815555	4.60	10.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.60	.184169	48
13	.738627	3.22	.922520	1.37	.816107	4.58	.183893	47
14	.738820	3.22	.922438	1.38	.816382	4.60	.183618	46
15	.739013	3.22	.922355	1.38	.816658	4.58	.183342	45
16	.739206	3.20	.922272	1.38	.816933	4.60	.183067	44
17	.739398	3.20	.922189	1.38	.817209	4.58	.182791	43
18	.739590	3.22	.922106	1.38	.817484	4.58	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.60	.182241	41
20	.739975	3.20	.921940	1.38	.818035	4.58	.181965	40
21	9.740167	3.20	9.921857	1.38	9.818310	4.58	10.181690	39
22	.740359	3.18	.921774	1.38	.818585	4.58	.181415	38
23	.740550	3.20	.921691	1.40	.818860	4.58	.181140	37
24	.740742	3.20	.921607	1.38	.819135	4.58	.180865	36
25	.740934	3.18	.921524	1.38	.819410	4.57	.180590	35
26	.741125	3.18	.921441	1.40	.819684	4.58	.180316	34
27	.741316	3.20	.921357	1.38	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.40	.820234	4.57	.179766	32
29	.741699	3.17	.921190	1.38	.820508	4.58	.179492	31
30	.741889	3.18	.921107	1.40	.820783	4.57	.179217	30
31	9.742080	3.18	9.921023	1.40	9.821057	4.58	10.178943	29
32	.742271	3.18	.920939	1.38	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.18	.920688	1.40	.822154	4.58	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.15	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.17	.920352	1.40	.823251	4.55	.176749	21
40	.743792	3.17	.920268	1.40	.823524	4.57	.176476	20
41	9.743982	3.15	9.920184	1.42	9.823798	4.57	10.176202	19
42	.744171	3.17	.920099	1.40	.824072	4.55	.175928	18
43	.744361	3.15	.920015	1.40	.824345	4.57	.175655	17
44	.744550	3.15	.919931	1.42	.824619	4.57	.175381	16
45	.744739	3.15	.919846	1.40	.824893	4.55	.175107	15
46	.744928	3.15	.919762	1.42	.825166	4.55	.174834	14
47	.745117	3.15	.919677	1.40	.825439	4.57	.174561	13
48	.745306	3.13	.919593	1.42	.825713	4.55	.174287	12
49	.745494	3.15	.919508	1.40	.825986	4.55	.174014	11
50	.745683	3.13	.919424	1.42	.826259	4.55	.173741	10
51	9.745871	3.15	9.919339	1.42	9.826532	4.55	10.173468	9
52	.746060	3.13	.919254	1.42	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.40	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.12	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.53	.171830	3
58	.747187	3.12	.918745	1.43	.828442	4.55	.171558	2
59	.747374	3.13	.918659	1.42	.828715	4.53	.171285	1
60	9.747562		9.918574		9.828987		10.171013	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.747562	3.12	9.918574	1.42	9.828987	4.55	10.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.53	.170740	59
2	.747936	3.12	.918404	1.43	.829532	4.55	.170468	58
3	.748123	3.12	.918318	1.42	.829805	4.53	.170195	57
4	.748310	3.12	.918233	1.43	.830077	4.53	.169923	56
5	.748497	3.10	.918147	1.42	.830349	4.53	.169651	55
6	.748683	3.12	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.10	.917976	1.42	.830893	4.53	.169107	53
8	.749056	3.12	.917891	1.43	.831165	•4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	.749429	3.10	.917719	1.42	.831709	4.53	.168291	50
11	9.749615	3.10	9.917634	1.43	9.831981	4.53	10.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.08	.917462	1.43	.832525	4.52	.167475	47
14	.750172	3.10	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.08	.917290	1.43	.833068	4.52	.166932	45
16	.750543	3.10	.917204	1.43	.833339	4.53	.166661	44
17	.750729	3.08	.917118	1.43	.833611	4.52	.166389	43
18	.750914	3.08	.917032	1.43	.833882	4.53	.166118	42
19	.751099	3.08	.916946	1.45	.834154	4.52	.165846	41
20	.751284	3.08	.916859	1.43	.834425	4.52	.165575	40
21	9.751469	3.08	9.916773	1.43	9.834696	4.52	10.165304	39
22	.751654	3.08	.916687	1.45	.834967	4.52	.165033	38
23	.751839	3.07	.916600	1.43	.835238	4.52	.164762	37
24	.752023	3.08	.916514	1.45	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.43	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.45	.836051	4.52	.163949	34
27	.752576	3.07	.916254	1.45	.836322	4.52	.163678	33
28	.752760	3.07	.916167	1.43	.836593	4.52	.163407	32
29	.752944	3.07	.916081	1.45	.836864	4.50	.163136	31
30	.753128	3.07	.915994	1.45	.837134	4.52	.162866	30
31	9.753312	3.05	9.915907	1.45	9.837405	4.50	10.162595	29
32	.753495	3.07	.915820	1.45	.837675	4.52	.162325	28
33	.753679	3.07	.915733	1.45	.837946	4.50	.162054	27
34	.753862	3.07	.915646	1.45	.838216	4.52	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.50	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.47	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.52	.160703	22
39	.754778	3.03	.915210	1.45	.839568	4.50	.160432	21
40	.754960	3.05	.915123	1.47	.839838	4.50	.160162	20
41	9.755143	3.05	9.915035	1.45	9.840108	4.50	10.159892	19
42	.755326	3.03	.914948	1.47	.840378	4.50	.159622	18
43	.755508	3.03	.914860	1.45	.840648	4.48	.159352	17
44	.755690	3.03	.914773	1.47	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.45	.841187	4.50	.158813	15
46	.756054	3.03	.914598	1.47	.841457	4.50	.158543	14
47	.756236	3.03	.914510	1.47	.841727	4.48	.158273	13
48	.756418	3.03	.914422	1.47	.841996	4.50	.158004	12
49	.756600	3.03	.914334	1.47	.842266	4.48	.157734	11
50	.756782	3.02	.914246	1.47	.842535	4.50	.157465	10
51	9.756963	3.02	9.914158	1.47	9.842805	4.48	10.157195	9
52	.757144	3.03	.914070	1.47	.843074	4.48	156926	8
53	.757326	3.02	.913982	1.47	.843343	4.48	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.50	.156388	6
55	.757688	3.02	.913806	1.47	.843882	4.48	.156118	5
56	.757869	3.02	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.00	.913630	1.48	.844420	4.48	.155580	3
58	.758230	3.02	913541	1.47	.844689	4.48	.155311	2
59	.758411	3.00	.913453	1.47	.844958	4.48	.155042	1
60	9.758591		9.913365		9.845227		10.154773	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.758591	3.02	9.913365	1.48	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.47	.154504	59
2	.758952	3.00	.913187	1.47	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.47	.846302	4.47	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	3.00	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.98	.912744	1.48	.847108	4.47	.152892	53
8	.760031	3.00	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.98	.912566	1.48	.847644	4.48	.152356	51
10	.760390	2.98	.912477	1.48	.847913	4.47	.152087	50
11	9.760569	2.98	9.912388	1.48	9.848181	4.47	10.151819	49
12	.760748	2.98	.912299	1.48	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.48	.848717	4.48	.151283	47
14	.761106	2.98	.912121	1.50	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.48	.849254	4.47	.150746	45
16	.761464	2.97	.911942	1.48	.849522	4.47	.150478	44
17	.761642	2.98	.911853	1.50	.849790	4.45	.150210	43
18	.761821	2.97	.911763	1.48	.850057	4.47	.149943	42
19	.761999	2.97	.911674	1.50	.850325	4.47	.149675	41
20	.762177	2.98	.911584	1.48	.850593	4.47	.149407	40
21	9.762356	2.97	9.911495	1.50	9.850861	4.47	10.149139	39
22	.762534	2.97	.911405	1.50	.851129	4.45	.148871	38
23	.762712	2.95	.911315	1.48	.851396	4.47	.148604	37
24	.762889	2.97	.911226	1.50	.851664	4.45	.148336	36
25	.763067	2.97	.911136	1.50	.851931	4.47	.148069	35
26	.763245	2.95	.911046	1.50	.852199	4.45	.147801	34
27	.763422	2.97	.910956	1.50	.852466	4.45	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.47	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	.763954	2.95	.910686	1.50	.853268	4.45	.146732	30
31	9.764131	2.95	9.910596	1.50	9.853535	4.45	10.146465	29
32	.764308	2.95	.910506	1.52	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.50	.854069	4.45	.145931	27
34	.764662	2.93	.910325	1.50	.854336	4.45	.145664	26
35	.764838	2.95	.910235	1.52	.854603	4.45	.145397	25
36	.765015	2.93	.910144	1.50	.854870	4.45	.145130	24
37	.765191	2.93	.910054	1.52	.855137	4.45	.144863	23
38	.765367	2.95	.909963	1.50	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.52	.855671	4.45	.144329	21
40	.765720	2.93	.909782	1.52	.855938	4.43	.144062	20
41	9.765896	2.93	9.909691	1.50	9.856204	4.45	10.143796	19
42	.766072	2.92	.909601	1.52	.856471	4.43	.143529	18
43	.766247	2.93	.909510	1.52	.856737	4.45	.143263	17
44	.766423	2.92	.909419	1.52	.857004	4.43	.142996	16
45	.766598	2.93	.909328	1.52	.857270	4.45	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.43	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.43	.142197	13
48	.767124	2.93	.909055	1.52	.858069	4.45	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.43	.141664	11
50	.767475	2.90	.908873	1.53	.858602	4.43	.141398	10
51	9.767649	2.92	9.908781	1.52	9.858868	4.43	10.141132	9
52	.767824	2.92	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.90	.908599	1.53	.859400	4.43	.140600	7
54	.768173	2.92	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.90	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.92	.908324	1.52	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.42	.139270	2
59	.769045	2.90	.908049	1.52	.860995	4.43	.139005	1
60	9.769219		9.907958		9.861261		10.138739	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.769219	2 90	9.907958	1.53	9.861261	4 43	10 138739	60
1	.769393	2 88	.907866	1.53	.861527	4 42	.138473	59
2	.769566	2.90	.907774	1.53	.861792	4 43	.138208	58
3	.769740	2 88	.907682	1.53	.862058	4 42	.137942	57
4	.769913	2 90	.907590	1 53	.862323	4 43	.137677	56
5	.770087	2 88	.907498	1 53	.862589	4 42	.137411	55
6	.770260	2 88	.907406	1.53	.862854	4 42	.137146	54
7	.770433	2 88	.907314	1.53	.863119	4 43	.136881	53
8	.770606	2.88	.907222	1 55	.863385	4 42	.136615	52
9	.770779	2 88	.907129	1 53	.863650	4 42	.136350	51
10	.770952	2.88	.907037	1.53	.863915	4.42	.136085	50
11	9.771125	2 88	9 906945	1.55	9 864180	4 42	10.135820	49
12	.771298	2 87	.906852	1 53	.864445	4.42	.135555	48
13	.771470	2 88	.906760	1 55	.864710	4 42	.135290	47
14	.771643	2 87	.906667	1.53	.864975	4 42	.135025	46
15	.771815	2 87	.906575	1.55	.865240	4 42	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4 42	.134495	44
17	.772159	2 87	.906389	1.55	.865770	4 42	.134230	43
18	.772331	2 87	.906296	1 53	.866035	4 42	.133965	42
19	.772503	2 87	.906204	1.55	.866300	4 40	.133700	41
20	.772675	2.87	.906111	1.55	.866564	4.42	.133436	40
21	9.772847	2 85	9.906018	1.55	9.866829	4 42	10.133171	39
22	.773018	2.87	.905925	1.55	.867094	4 40	.132906	38
23	.773190	2 85	.905832	1.55	.867358	4 42	.132642	37
24	.773361	2 87	.905739	1.57	.867623	4.40	.132377	36
25	.773533	2.85	.905645	1 55	.867887	4 42	.132113	35
26	.773704	2 85	.905552	1.55	.868152	4 40	.131848	34
27	.773875	2 85	.905459	1 55	.868415	4.40	.131584	33
28	.774046	2 85	.905366	1.57	.868680	4 42	.131320	32
29	.774217	2 85	.905272	1 55	.868945	4 40	.131055	31
30	.774388	2.83	.905179	1.57	.869209	4.40	.130791	30
31	9.774558	2 85	9.905085	1.55	9.869473	4 40	10.130527	29
32	.774729	2 83	.904992	1 57	.869737	4 40	.130263	28
33	.774899	2.85	.904898	1 57	.870001	4 40	.129999	27
34	.775070	2 83	.904804	1.55	.870265	4 40	.129735	26
35	.775240	2 83	.904711	1 57	.870529	4 40	.129471	25
36	.775410	2 83	.904617	1 57	.870793	4.40	.129207	24
37	.775580	2 83	.904523	1 57	.871057	4.40	.128943	23
38	.775750	2 83	.904429	1 57	.871321	4 40	.128679	22
39	.775920	2.83	.904335	1 57	.871585	4 40	.128415	21
40	.776090	2.82	.904241	1.57	.871849	4 38	.128151	20
41	9.776259	2 83	9.904147	1.57	9.872112	4 40	10.127888	19
42	.776429	2 82	.904053	1.57	.872376	4 40	.127624	18
43	.776598	2 83	.903959	1 58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4 40	.127097	16
45	.776937	2 82	.903770	1.57	.873167	4 38	.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2 82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4 40	.125780	11
50	.777781	2.82	.903298	1.58	.874484	4.38	.125516	10
51	9.777950	2 82	9.903203	1.58	9.874747	4.38	10.125253	9
52	.778119	2 80	.903108	1.57	.875010	4 38	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4 40	.124727	7
54	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2 80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.78	.902539	1.58	.876589	4.38	.123411	2
59	.779295	2.80	902444	1.58	.876852	4.37	.123148	1
60	9.779463		9 902349		9.877114		10.122886	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.779463	2.80	9.902349	1.60	9.877114	4 38	10.122886	60
1	.779631	2.78	.902253	1.58	.877377	4 38	.122623	59
2	.779798	2.80	.902158	1.58	.877640	4 38	.122360	58
3	.779966	2.78	.902063	1.60	.877903	4 37	.122097	57
4	.780133	2 78	.901967	1.58	.878165	4 38	.121835	56
5	.780300	2 78	.901872	1 60	.878428	4 38	.121572	55
6	.780467	2.78	.901776	1.58	.878691	4 37	.121309	54
7	.780634	2.78	.901681	1.60	.878953	4 38	.121047	53
8	.780801	2.78	.901585	1.58	.879216	4 37	.120784	52
9	.780968	2.77	.901490	1 60	.879478	4 38	.120522	51
10	.781134	2.78	.901394	1.60	.879741	4 37	.120259	50
11	9.781301	2 78	9.901298	1 60	9.880003	4 37	10.119997	49
12	.781468	2 77	.901202	1 60	.880265	4 38	.119735	48
13	.781634	2 77	.901106	1 60	.880528	4 37	.119472	47
14	.781800	2 77	.901010	1 60	.880790	4.37	.119210	46
15	.781966	2 77	.900914	1 60	.881052	4 37	.118948	45
16	.782132	2 77	.900818	1 60	.881314	4 38	.118686	44
17	.782298	2 77	.900722	1 60	.881577	4 37	.118423	43
18	.782464	2.77	.900626	1 62	.881839	4 37	.118161	42
19	.782630	2 77	.900529	1 60	.882101	4 37	.117899	41
20	.782796	2.75	.900433	1 60	.882363	4.37	.117637	40
21	9.782961	2 77	9.900337	1 62	9 882625	4 37	10.117375	39
22	.783127	2.75	.900240	1 60	.882887	4 35	.117113	38
23	.783292	2.77	.900144	1 62	.883148	4.37	.116852	37
24	.783458	2.75	.900047	1 60	.883410	4.37	.116590	36
25	.783623	2.75	.899951	1 62	.883672	4 37	.116328	35
26	.783788	2.75	.899854	1 62	.883934	4 37	.116066	34
27	.783953	2 75	.899757	1 62	.884196	4 35	.115804	33
28	.784118	2.73	.899660	1 60	.884457	4.37	.115543	32
29	.784282	2.75	.899564	1 62	.884719	4.35	.115281	31
30	.784447	2.75	.899467	1.62	.884980	4.37	.115020	30
31	9.784612	2.73	9.899370	1 62	9.885242	4 37	10.114758	29
32	784776	2 75	.899273	1 62	.885504	4 35	.114496	28
33	.784941	2 73	.899176	1 63	.885765	4.35	.114235	27
34	.785105	2.73	.899078	1.62	.886026	4 37	.113974	26
35	.785269	2.73	.898981	1 62	.886288	4 35	.113712	25
36	.785433	2 73	.898884	1.62	.886549	4 37	.113451	24
37	.785597	2 73	.898787	1 63	886811	4 35	.113189	23
38	.785761	2 73	.898689	1.62	.887072	4 35	.112928	22
39	.785925	2 73	.898592	1 63	.887333	4 35	.112667	21
40	.786089	2.72	.898494	1.62	.887594	4.35	.112406	20
41	9.786252	2.73	9.898397	1.63	9.887855	4 35	10.112145	19
42	.786416	2 72	.898299	1 62	.888116	4.37	.111884	18
43	.786579	2.72	.898202	1 63	.888378	4 35	.111622	17
44	.786742	2 73	.898104	1 63	.888639	4 35	.111361	16
45	.786906	2.72	.898006	1 63	.888900	4 35	.111100	15
46	.787069	2 72	.897908	1 63	.889161	4 33	.110839	14
47	.787232	2.72	.897810	1 63	889421	4 35	.110579	13
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2 72	.897614	1 63	.889943	4 35	.110057	11
50	.787720	2.72	.897516	1 63	.890204	4.35	.109796	10
51	9.787883	2.70	9.897418	1 63	9.890465	4 33	10.109535	9
52	.788045	2.72	.897320	1 63	.890725	4.35	.109275	8
53	.788208	2.70	.897222	1 65	.890986	4 35	.109014	7
54	788370	2.70	.897123	1.63	.891247	4.33	.108753	6
55	.788532	2 70	.897025	1 65	.891507	4.35	.108493	5
56	.788694	2.70	.896926	1 63	.891768	4.33	.108232	4
57	.788856	2.70	.896828	1.65	.892028	4.35	.107972	3
58	.789018	2.70	.896729	1 63	.892289	4.33	.107711	2
59	.789180	2.70	.896631	1.65	.892549	4.35	.107451	1
60	9.789342		9.896532		9.892810		10.107190	0
′	Cosine.	D 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.789342	2.70	9.896532	1.65	9.892810	4.33	10.107190	60
1	.789504	2.68	.896433	1.63	.893070	4.35	.106930	59
2	.789665	2.70	.896335	1.65	.893331	4.33	.106669	58
3	.789827	2.68	.896236	1.65	.893591	4.33	.106409	57
4	.789988	2.68	.896137	1.65	.893851	4.33	.106149	56
5	.790149	2.68	.896038	1.65	.894111	4.35	.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.33	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.68	.895741	1.67	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	.790954	2.68	.895542	1.65	.895412	4.33	.104588	50
11	9.791115	2.67	9.895443	1.67	9.895672	4.33	10.104328	49
12	.791275	2.68	.895343	1.65	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.65	.896192	4.33	.103808	47
14	.791596	2.68	.895145	1.67	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.67	.896712	4.32	.103288	45
16	.791917	2.67	.894945	1.65	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.67	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.67	.897491	4.33	.102509	42
19	.792397	2.67	.894646	1.67	.897751	4.32	.102249	41
20	.792557	2.65	.894546	1.67	.898010	4.33	.101990	40
21	9.792716	2.67	9.894446	1.67	9.898270	4.33	10.101730	39
22	.792876	2.65	.894346	1.67	.898530	4.32	.101470	38
23	.793035	2.67	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	.793354	2.67	.894046	1.67	.899308	4.33	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.68	.899827	4.33	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.68	.900346	4.32	.099654	31
30	.794150	2.63	.893544	1.67	.900605	4.32	.099395	30
31	9.794308	2.65	9.893444	1.68	9.900864	4.33	10.099136	29
32	.794467	2.65	.893343	1.67	.901124	4.32	.098876	28
33	.794626	2.63	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.63	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.65	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.63	.892940	1.68	.902160	4.33	.097840	24
37	.795259	2.63	.892839	1.67	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.70	.902938	4.32	.097062	21
40	.795733	2.63	.892536	1.68	.903197	4.32	.096803	20
41	9.795891	2.63	9.892435	1.68	9.903456	4.30	10.096544	19
42	.796049	2.62	.892334	1.68	.903714	4.32	.096286	18
43	.796206	2.63	.892233	1.68	.903973	4.32	.096027	17
44	.796364	2.62	.892132	1.70	.904232	4.32	.095768	16
45	.796521	2.63	.892030	1.68	.904491	4.32	.095509	15
46	.796679	2.62	.891929	1.70	.904750	4.30	.095250	14
47	.796836	2.62	.891827	1.68	.905008	4.32	.094992	13
48	.796993	2.62	.891726	1.70	.905267	4.32	.094733	12
49	.797150	2.62	.891624	1.68	.905526	4.32	.094474	11
50	.797307	2.62	.891523	1.70	.905785	4.30	.094215	10
51	9.797464	2.62	9.891421	1.70	9.906043	4.32	10.093957	9
52	.797621	2.60	.891319	1.70	.906302	4.30	.093698	8
53	.797777	2.62	.891217	1.70	.906560	4.32	.093440	7
54	.797934	2.62	.891115	1.70	.906819	4.30	.093181	6
55	.798091	2.60	.891013	1.70	.907077	4.32	.092923	5
56	.798247	2.60	.890911	1.70	.907336	4.30	.092664	4
57	.798403	2.62	.890809	1.70	.907594	4.32	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.30	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60	9.798872		9.890503		9.908369		10.091631	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.798872	2 60	9.890503	1.72	9.908369	4.32	10.091631	60
1	.799028	2 60	.890400	1 70	.908628	4 30	.091372	59
2	.799184	2 58	.890298	1 72	.908886	4.30	.091114	58
3	.799339	2.60	.890195	1.70	.909144	4 30	.090856	57
4	.799495	2 60	.890093	1.72	.909402	4 30	.090598	56
5	.799651	2 58	.889990	1.70	.909660	4 30	.090340	55
6	.799806	2 60	.889888	1.72	.909918	4 32	.090082	54
7	.799962	2 58	.889785	1.72	.910177	4 30	.089823	53
8	800117	2 58	.889682	1.72	.910435	4 30	.089565	52
9	.800272	2 58	.889579	1.70	.910693	4 30	.089307	51
10	.800427	2.58	.889477	1.72	.910951	4 30	.089049	50
11	9.800582	2.58	9.889374	1 72	9.911209	4.30	10.088791	49
12	.800737	2.58	.889271	1 72	.911467	4 30	.088533	48
13	.800892	2 58	.889168	1 73	.911725	4 28	.088275	47
14	.801047	2 57	.889064	1 72	.911982	4 30	.088018	46
15	.801201	2 58	.888961	1.72	.912240	4 30	.087760	45
16	.801356	2 58	.888858	1 72	.912498	4.30	.087502	44
17	.801511	2 57	.888755	1 73	.912756	4 30	.087244	43
18	.801665	2 57	.888651	1 72	.913014	4.28	.086986	42
19	.801819	2 57	.888548	1 73	.913271	4 30	.086729	41
20	.801973	2 58	.888444	1.72	.913529	4.30	.086471	40
21	9.802128	2 57	9.888341	1 73	9.913787	4.28	10.086213	39
22	.802282	2 57	.888237	1 72	.914044	4 30	.085956	38
23	.802436	2 55	.888134	1 73	.914302	4.30	.085698	37
24	.802589	2 57	.888030	1 73	.914560	4 28	.085440	36
25	.802743	2 57	.887926	1.73	.914817	4 30	.085183	35
26	.802897	2 55	.887822	1.73	.915075	4 28	.084925	34
27	.803050	2 57	.887718	1.73	.915332	4 30	.084668	33
28	.803204	2.55	.887614	1.73	.915590	4 28	.084410	32
29	.803357	2.67	.887510	1.73	.915847	4 28	.084153	31
30	.803511	2.55	.887406	1 73	.916104	4 30	.083896	30
31	9.803664	2 55	9.887302	1 73	9 916362	4 28	10.083638	29
32	.803817	2 55	.887198	1 75	.916619	4 30	.083381	28
33	.803970	2 55	.887093	1.73	.916877	4.28	.083123	27
34	.804123	2 55	.886989	1 73	.917134	4 28	.082866	26
35	.804276	2 55	.886885	1 75	.917391	4 28	.082609	25
36	.804428	2 55	.886780	1.73	.917648	4.30	.082352	24
37	.804581	2 55	.886676	1 75	.917906	4.28	.082094	23
38	.804734	2 53	.886571	1.75	.918163	4 28	.081837	22
39	.804886	2 55	.886466	1 73	.918420	4 28	.081580	21
40	.805039	2 53	886362	1.75	.918677	4.28	.081323	20
41	9 805191	2 53	9.886257	1.75	9 918934	4 28	10.081066	19
42	.805343	2 53	.886152	1 75	.919191	4 28	.080809	18
43	.805495	2 53	.886047	1.75	919448	4 28	.080552	17
44	.805647	2 53	.885942	1.75	.919705	4 28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4 28	.080038	15
46	.805951	2 53	.885732	1 75	.920219	4 28	.079781	14
47	.806103	2 52	.885627	1 75	.920476	4 28	.079524	13
48	.806254	2 53	.885522	1 77	.920733	4 28	.079267	12
49	.806406	2 52	.885416	1 75	920990	4.28	.079010	11
50	.806557	2.53	.885311	1 77	.921247	4.27	.078753	10
51	9.806709	2 52	9 885205	1 75	9 921503	4 28	10.078497	9
52	.806860	2.52	.885100	1.77	.921760	4 28	.078240	8
53	.807011	2 53	.884994	1 75	.922017	4.28	.077983	7
54	.807163	2 52	.884889	1 77	.922274	4.27	.077726	6
55	.807314	2.52	.884783	1 77	.922530	4.28	.077470	5
56	.807465	2 50	884677	1 75	.922787	4.28	.077213	4
57	.807615	2 52	.884572	1 77	.923044	4 27	.076956	3
58	.807766	2 52	.884466	1 77	.923300	4 28	.076700	2
59	.807917	2 50	.884360	1.77	.923557	4 28	.076443	1
60	9.808067		9 884254		9.923814		10 076186	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.808067	2.52	9.884254	1.77	9.923814	4.27	10.076186	60
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	.809569	2.48	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.50	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	.811061	2.48	.882121	1.78	.928940	4.27	.071060	40
21	9.811210	2.47	9.882014	1.78	9.929196	4.27	10 070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.78	.930220	4.25	.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	.812544	2.47	.881046	1.80	.931499	4.27	.068501	30
31	9.812692	2.47	9.880938	1.80	9.931755	4.25	10.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	.813283	2.45	.880505	1.80	.932778	4.25	.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	.814019	2.45	.879963	1.80	.934056	4.25	065944	20
41	9.814166	2.45	9.879855	1.82	9.934311	4.27	10.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	.814753	2.45	.879420	1.82	.935333	4.27	.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	.815485	2.45	.878875	1.82	.936611	4.25	.063389	10
51	9.815632	2.43	9.878766	1.83	9.936866	4.25	10 063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	9.816943		9.877780		9.939163		10.060837	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

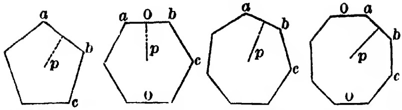
′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817	56
5	.817668	2.42	.877230	1.83	.940439	4.25	.059561	55
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306	54
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796	52
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541	51
10	.818392	2.40	.876678	1.83	.941713	4.25	.058287	50
11	9.818536	2.42	9.876568	1.85	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.40	.875571	1.87	.944262	4.25	.055738	40
21	9.819976	2.40	9.875459	1.85	9.944517	4.23	10.055483	39
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229	38
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974	37
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719	36
25	.820550	2.38	.875014	1.85	.945535	4.25	.054465	35
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955	33
28	.820979	2.38	.874680	1.87	.946299	4.25	.053701	32
29	.821122	2.38	.874568	1.87	.946554	4.23	.053446	31
30	.821265	2.37	.874456	1.87	.946808	4.25	.053192	30
31	9.821407	2.38	9.874344	1.87	9.947063	4.25	10.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428	27
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173	26
35	.821977	2.38	.873896	1.87	.948081	4.23	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156	22
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901	21
40	.822688	2.37	.873335	1.87	.949353	4.25	.050647	20
41	9.822830	2.37	9.873223	1.88	9.949608	4.23	10.050392	19
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138	18
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884	17
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629	16
45	.823397	2.37	.872772	1.88	.950625	4.23	.049375	15
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121	14
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867	13
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358	11
50	.824104	2.35	.872208	1.88	.951896	4.23	.048104	10
51	9.824245	2.35	9.872095	1.90	9.952150	4.25	10.047850	9
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595	8
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341	7
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087	6
55	.824808	2.35	.871641	1.88	.953167	4.23	.046833	5
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579	4
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325	3
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071	2
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817	1
60	9.825511		9.871073		9.954437		10.045563	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	
0	9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60
1	.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
8	.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
11	9.827049	2.33	9.869818	1.90	9.957231	4.23	10.042769	49
12	.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
16	.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
17	.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
21	9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
24	.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
28	.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
30	.829683	2.30	.867631	1.92	.962052	4.23	.037948	30
31	9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
33	.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
35	.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
37	.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
38	.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
40	.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
41	9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
43	.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
44	.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
46	.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9.832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
55	.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
56	.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
57	.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
58	.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
59	.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
60	9.833783		9.864127		9.969656		10.030344	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60	9.841771		9.856934		9.984837		10.015163	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2.12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60	9.849485		9.849485		10.000000		10.000000	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

POLYGONS.



Pentagon. 5 *sides*. **Hexagon.** 6 *sides*. **Heptagon.** 7 *sides*. **Octagon.** 8 *sides*.

Any straight-sided fig is called a polygon. If all the sides and angles are equal, it is a **regular** polygon; if not, it is **irregular**. Of course the number of polygons is infinite.

Table of Regular Polygons.

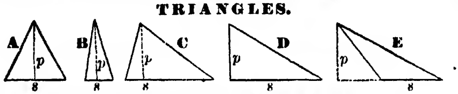
Number of Sides.	Name of Polygon.	**Area** = (square of one side) mult by	**Radius** of circumscribing circle = side mult by	**Interior angle** *a b c* contained between two sides.	**Angle at cen.** subtended by a side.
3	Equilateral triangle.	.433013	.577350	60°	120°
4	Square.	1.000000	.707107	90°	90°
5	Pentagon.	1.720477	.850651	108°	72°
6	Hexagon.	2.598076	1.000000	120°	60°
7	Heptagon.	3.633912	1.152382	128° 34 2857′	51° 25.7143
8	Octagon.	4 828427	1.306563	135°	45°
9	Nonagon.	6 181824	1 461902	140°	40°
10	Decagon.	7.694209	1.618034	144°	36°
11	Undecagon.	9 365640	1.774733	147° 16.3636′	32° 43.6364′
12	Dodecagon	11.196152	1.931854	150°	30°

Area of any regular polygon = length of one side, *a b* × perp *p* drawn from cen of fig to cen of side × half the number of sides.

Sum of interior angles, a b c, etc, **of any polygon, regular or irregular** = 180° × (number of sides — 2)

Angle at cen subtended by a side, in any regular polygon = 360° ÷ number of sides

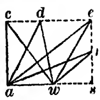
TRIANGLES.



We speak here of **plane** triangles only; or those having **straight** sides.

A triangle is equilateral when all its sides are equal, as A, **isosceles** when only two sides are equal, as B, **scalene** when all the sides are unequal, as C, D and E **acute-angled** when all its angles are acute, or each less than 90°, as A, B, and C, **right-angled** when it contains a right angle, as D **obtuse-angled** when it contains an obtuse angle, or one greater than 90°, as E

All the three angles of any triangle are equal to two right angles, or 180°, therefore, if we know two of them, we can find the third by subtracting their sum from 180°. **All triangles which have equal bases,** and equal perp heights, have also equal areas, thus the areas of *a w c*, *a u d*, and *a w e*, are equal to each other. **The area of any triangle** is equal to half that of any parallelogram which has an equal base, and an equal perp height. **The areas of triangles which have equal bases,** but diff perp heights, are to each other as, or in proportion to, their perp heights, thus the triangle *a w n*, with a perp height *s n*, equal to but one-half that (*s e*) of the three other triangles, but with the same base *a w*, has also but half the area of either of those others.



Area of any triangle, Figs A, B, C, D, E, = half the base, S, × the height, or perp dist *p* to the opposite angle Any side may be taken as the base of a triangle; but the perp height must always be measured from the side so assumed; to do which, the side must sometimes be prolonged, as in Fig E, but the prolongation is not to be considered as a part of the base

Area of any equilateral triangle = .433013 × square of one side.

To find area, having the three sides.

Add them together; div the sum by 2; from the half sum, subtract each side separately; mult the half sum and the three remainders continuously together; take the sq rt of the prod.

Ex.—The three sides = 20, 30, 40 ft. Here $20 + 30 + 40 = 90$; and $\frac{90}{2} = 45$. And $45 - 20 = 25$ $45 - 30 = 15$; and $45 - 40 = 5$. And $45 \times 25 \times 15 \times 5 = 84375$· and the sq rt of 84875 is 290.47 sq ft area reqd.

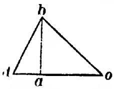
To find area, having one side and the 2 angles at its ends.

Add the 2 angles together; take the sum from 180°, the rem will be the angle opp the given side. Find the nat sine of this angle; also find the nat sines of the other angles, and mult them together. Then as the nat sine of the single angle, is to the prod of the nat sines of the other 2 angles, so is the *square* of the given side to *double* the reqd area.

To find area, having two sides, and the included angle.

Mult together the two sides, and the nat sine of the included angle; div by 2.

Ex —Sides 650 ft and 980 ft, included angle 69° 20'. By the table we find the nat sine .9356; therefore, $\frac{650 \times 980 \times .9356}{2} = 297988.6$ square ft area.



To find area, having the three angles and the perp height, *a b*.

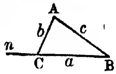
Find the nat sines of the three angles; mult together the sines of the angles *d* and *o*; div the sine of the angle *b* by the prod; mult the quot by the square of the perp height *a b*; div by 2.

To find any side, as *d o*, having the three angles, *d*, *b* and *o*, and the area.

(Sine of *d* × sine of *o*) : sine of *b* :: **twice** the area : **square** of *d o*.

The perp height of an equilateral triangle is equal to one side × .866025 Hence one of its sides is equal to the perp height div by .866025 or to perp height × 1.1547. Or, **to find a side**, mult the sq rt of its area by 1.51967. The side of an equilateral triangle, mult by .658037 = side of a square of the same area; or mult by .742517 it gives the diam of a circle of the same area.

The following apply to any plane triangle, whether oblique or right-angled:



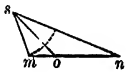
1. The three angles amount to 180°, or two right angles.

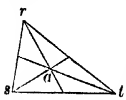
2. Any exterior angle, as A C *n*, is equal to the two interior and opposite ones, A and B.

3. The greater side is opposite the greater angle.

4. The sides are as the sines of the opposite angles. Thus, the side *a* is to the side *b* as the sine of A is to the sine of B.

5. If any angle as *s* be bisected by a line *s o*, the two parts *m o*, *o n* of the opposite side *m n* will be to each other as the other two sides *s m*, *s n*, or, *m o* : *o n* :: *s m* : *s n*.

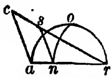




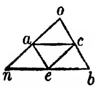
6. If lines be drawn from each angle *r s t* to the center of the opposite side, they will cross each other at one point, *a*, and the short part of each of the lines will be the third part of the whole line. Also, *a* is the **cen of grav** of the triangle.

7. If lines be drawn bisecting the three angles, they will meet at a point perpendicularly equidistant from each side, and consequently the **center of the greatest circle** that can be drawn in the triangle.

8. If a line *s n* be drawn parallel to any side *c a*, the two triangles *r s n*, *r c a*, will be similar.

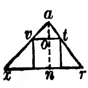


9. To divide any triangle *a c r* into two equal parts by a line *s n* parallel to any one of its sides *c a*. On either one of the other sides, as *a r*, as a diam, describe a semicircle *a o r*; and find its middle *o*. From *r* (opposite *c a*), with radius *r o*, describe the arc *o n*. From *n* draw *n s*, parallel to *c a*.



10. To find the greatest parallelogram that can be drawn in any given triangle *o n b*. Bisect the three sides at *a c e*, and join *a c*, *a e*, *e c*. Then either *a e b c*, *a e c o*, or *a c e n*, each equal to half the triangle, will be the reqd parallelogram. Any of these parallelograms can plainly be converted into a rectangle of equal area, and the greatest that can be drawn in the triangle.

10½. If a line *a c* bisects any two sides *o b*, *o n*, of a triangle, it will be parallel to the third side *n b*, and half as long as it.



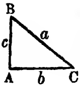
11. To find the greatest square that can be drawn in any triangle *a x r*. From an angle as *a* draw a perp *a n* to the opposite side *x r*, and find its length. Then *v n*, or a side *v t* of the square $= \frac{x\,r \times a\,n}{x\,r + a\,n}$

Rem.—If the triangle is such that two or three such perps can be drawn, then two or three equal squares may be found.

Right-angled Triangles.

All the foregoing apply also to right-angled triangles; but what follow apply to them only.

Call the right angle A, and the others B and C; and call the sides respectively opposite to them a, b, and c. Then is



$$a = \frac{c}{\text{Sine C}} = c \times \text{Sec B} = \frac{b}{\text{Cosine C}} = b \times \text{Sec C} = \sqrt{b^2 + c^2}$$

$b = a \times \text{Sine B} = a \times \text{Cos C} = c \times \text{Cot C} = c \times \text{Tang B}.$

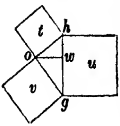
$c = a \times \text{Sine C} = a \times \text{Cos B} = b \times \text{Tang C}.$

Also Sine of $C = \frac{c}{a}$; Cos $C = \frac{b}{a}$; Tang $C = \frac{c}{b}$.

And Sine of $B = \frac{b}{a}$; Cos $B = \frac{c}{a}$; Tang $B = \frac{b}{c}$.

And Sine of A or $90^\circ = 1$. Cos $A = 0$. Tang A = infinity. Sec A = infinity.

1. If from the right angle o a line $o\,w$ be drawn perp to the hypothenuse or long side $h\,g$, then the two small triangles $o\,w\,h$, $o\,w\,g$, and the large one $o\,h\,g$, will be similar. Or $g\,w : w\,o :: w\,o : w\,h$; and $g\,w \times w\,h = w\,o^2$.



2. A line drawn from the right angle to the **center** of the long side will be half as long as said side.

3. If on the three sides $o\,h$, $o\,g$, $g\,h$ we draw three squares t, u, v, or three circles, or triangles, or any other three figs that are similar, then the area of the largest one is equal to the sum of the areas of the two others.

4. In a triangle whose sides are as 3, 4, and 5 (as are those of the triangle A B C), the angles are very approximately 90°; $53^\circ\ 7'\ 48.38''$; and $36^\circ\ 52'\ 11.62''$. Their Sines, 1.; .8; and .6. Their Tangs, infinity; 1.3333; and .75.

5. One whose sides are as 7, 7, and 9.9, has very approx one angle of 90° and two of 45° each, near enough for all practical purposes.

PROBLEMS IN SURVEYING.

Plane trigonometry teaches how to find certain unknown parts of plane, or straight-sided triangles, by means of other parts which are known; and thus enables us to measure inaccessible distances, &c. A triangle consists of six parts, namely, three sides, and three angles; and if we know any three of these, (except the three angles, and in the ambiguous case under "Case 2,") we can find the other three. The following four cases include the whole subject; the student should commit them to memory.

Case 1. Having any two angles, and one side, to find the other sides and angle.

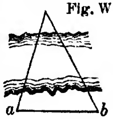


Fig. W

Add the two angles together; and subtract their sum from 180°; the rem will be the third angle. And for the sides, as

Sine of the angle opp the given side : Sine of the angle opp the reqd side :: given side : reqd side.

Use the side thus found, as the given one; and in the same manner find the third side.

Case 2. Having two sides, $b\,a$, $a\,c$, Fig X, and the angle $a\,b\,c$, opposite to one of them, to find the other side and angles.

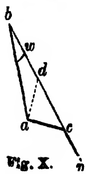


Fig. X.

Side $a\,c$ opp the given angle $a\,b\,c$: The other given side $b\,a$ :: Sine of the given angle $a\,b\,c$: Sine of angle $b\,d\,a$ or $b\,c\,a$ opposite the other given side $b\,a$.

Having found the sine, take out the corresponding angle from the table of nat sines, but, in doing so, if the side $a\,c$ opp the given angle is shorter than the other given side $b\,a$, bear in mind that an angle and its supplement have the same sine. Thus, in Fig X, the sine, as found above, is opp the angle $b\,c\,a$ in the table. But $a\,c$, if *shorter than* $b\,a$, can evidently be laid off in the opp direction, $a\,d$, in which case $b\,d\,a$ is the supplement of $b\,c\,a$. If $a\,c$ is as long as, or longer than, $b\,a$, there can be no doubt; for in that case it cannot be drawn toward b, but only toward n, and the angle $b\,c\,a$ will be found at once in the table, opp the sine as found above.

When the two angles, *a b c*, *b c a*, have been found, find the remaining side by Case 1.
For the remaining angle, *b a c*, add together the angle *a b c* first given, and the one, *b c a*, found as above. Deduct their sum from 180°.

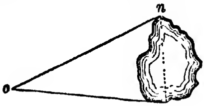
Case 3. Having two sides, and the angle included between them.

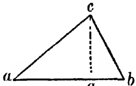
Take the angle from 180°; the rem will be the sum of the two unknown angles. Div this sum by 2; and find the nat tang of the quot. Then as

The sum of the two given sides : Their diff : : Tang of half the sum of the two unknown angles : Tang of half their diff.

Take from the table of nat tang, the angle opposite this last tang. Add this angle to the half sum of the two unknown angles, and it will give the angle opp the longest given side; and subtract it from the same half sum, for the angle opp the shortest given side. Having thus found the angles, find the third side by Case 1.

As a practical example of the use of Case 3, we can ascertain the dist *n m* across a deep pond, by measuring two lines *n o* and *m o*; and the angle *n o m*. From these data we may calculate *n m*; or by drawing the two sides, and the angle on paper, by a scale, we can afterward measure *n m* on the drawing.





Case 4. Having the three sides.

To find the three angles; upon one side *a b* as a base, draw (or suppose to be drawn) a perp *c g* from the opposite angle *c*. Find the diff between the other two sides, *a c* and *c b*; also their sum. Then, as

The base : Sum of the other two sides : : Diff of other two sides : Diff of the two parts *a g* and *b g*, of the base.

Add *half* this diff of the parts, to *half* the base *a b*; the sum will be the longest part *a g*; which taken from the whole base, gives the shortest part *g b*. By this means we get in each of the small triangles *a c g* and *c g b*, two sides, (namely, *a c* and *a g*; and *c b* and *g b*;) and an angle (namely, the right angle *c g a*, or *c g b*) opposite to one of the given sides. Therefore, use Case 2 for finding the angles *a* and *b*. When that is done, take their sum from 180°, for the angle *a c b*.

Or, 2d mode; call **half** the sum of the three sides, **s**; and call the two sides which form either angle, **m** and **n**. Then the nat sine of **half** that angle will be equal to $\sqrt{\dfrac{(s-m)\times(s-n)}{m\times n}}$.

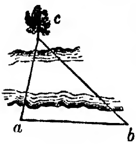


Fig. 1.

Ex. 1. To find the dist from *a* to an inaccessible object *c*.

Measure a line *a b*; and from its ends measure the angles *c a b* and *c b a*. Thus having found one side and two angles of the triangle *a b c*, calculate *a c* by means of Case 1. Or if extreme accuracy is not reqd, draw the line *a b* on paper to any convenient scale; then by means of a protractor lay off the angles *c a b*, *c b a*; and draw *a c* and *c b*; then measure *a c* by the same scale.

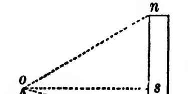
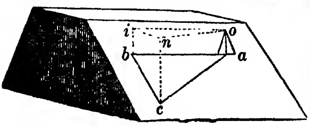


Fig. 2.

Ex. 2. To find the height of a *vertical* object, *n a*.

Place the instrument for measuring angles, at any convenient spot *o*; also meas the dist *o a*; or if *o a* cannot be actually measd in consequence of some obstacle, calculate it by the same process as *a c* in Fig 1. Then, first directing the instrument horizontally,* as *o s*, measure the angle of depression, *s o a*, say 12°; also the angle *s o n*, say 30°. These two angles added together, give the angle *a o n*, 42°. Now, in the small triangle *o s a* we have the angle *o s a* equal to 90°, because *a n* is vert, and *o s* hor; and since the three angles of any triangle are equal to 180°, if we subtract the angles *o s a* (90°), and *s o a* (12°) from 180°, the rem (78°) will be the angle *o a s* or *o a n*. Therefore, in the triangle *o n a*, we have one side *o a*; and two angles *a o n*, and *o a n*, to calculate the side *a n* by Case 1.

* **Angles and dists on sloping ground must be measured horizontally.** The graduated hor circle of the instrument evidently measures the angle between two objects horizontally, no matter how much higher one of them may be than the other; one perhaps requiring the telescope of the instrument to be directed upward toward it; and the other downward. If, therefore, the sides of triangles lying upon sloping ground, are not also measd hor, there can be no accordance between the two. Thus



Rem. If, as in Fig 3, it should be necessary to ascertain the *vert* height $a\,n$ from a point o, entirely above it, then both the angles measd at o, namely, $s\,o\,n$, and $s\,o\,a$, will be angles of depression, or *below* the hor line $o\,s$ assumed to measure them from. In this case we have the side $o\,a$ as before; the angle $n\,o\,a = s\,o\,a - s\,o\,n$; and the angle $o\,a\,n = 180^\circ - \left(o\,s\,a\ (90^\circ),\text{ and } s\,o\,a;\right)$ to calculate $a\,n$ by Case I.

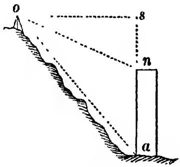


Fig. 3.

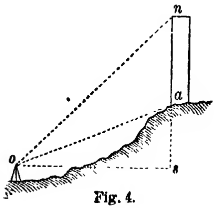


Fig. 4.

Or if, as in Fig 4, the observations are to be taken from a point o, entirely below the object $a\,n$, then both the angles $s\,o\,a$, $s\,o\,n$, will be angles of elevation, or *above* the assumed hor line $o\,s$. Here we have in the triangle $o\,n\,a$, the given side $o\,a$ as before; the angle $a\,o\,n = s\,o\,n - s\,o\,a$; and the angle $o\,n\,a = 180^\circ - \left(o\,s\,n\ (90^\circ),\text{ and } n\,o\,s,\right)$ to calculate $a\,n$ by Case 1

If the object $a\,n$, as in Fig 5, instead of being *vert*, is *inclined*, and instead of its vert height we wish to find its length $a\,n$, we must first ascertain its angle $y\,t\,i$ of inclination to the horizon, to which angle each of the angles $o\,s\,n$ will be equal. To find this angle $y\,t\,i$, suspend a plumb line $i\,y$, of any convenient known length, from the object $a\,n$, and measure also $y\,t$ horizontally Then say as

$$y\,t : i\,y :: 1 : \text{nat tang of angle } y\,t\,i.$$

From the table of nat tangs take out the angle $y\,t\,i$ found opposite this nat tang, and use it for the angles $o\,s\,n$ or $o\,s\,a$; instead of the 90° of Figs 3 and 4. Also when the object inclines, the side $a\,o$ of the triangle must be measd in line, or in range with the inclination. If the object, as the rock $a\,n$, Fig 6, is curved or irregular, a pole $a\,s$ may be planted sloping in the direction $a\,n$; and

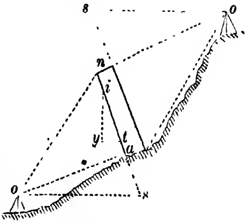


Fig. 5.

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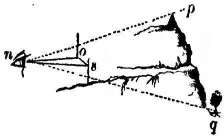
in the triangle $a\,b\,c$, upon sloping ground, the instrument at o measures the hor angle $s\,o\,n$, and not the angle $b\,a\,c$. Therefore, the side which corresponds with this hor angle $s\,o\,n$, is the hor dist $t\,n$; and not the sloping dist $b\,c$. In other words, when sides and angles are on sloping ground, we do not seek their *actual* measures; but their *hor* ones. This remark applies to all surveying for farms railroads, triangulations of countries, &c, &c; and the want of a strict attention to it, is one cause of the small errors, almost unavoidable, (and fortunately, of but trifling consequence in practice), which occur in all ordinary field operations.

When a sextant is used, angles between objects at diff altitudes, as p and q, may be measd hor, by first planting two vert rods o and s, in range with the objects; and then taking the hor angle $o\,n\,s$, subtended by the rods.

Angles may be measd without any inst, thus: Measure 100 ft toward each object, and drive stakes; measure the dist across from one stake to the other. Half this dist will be the sine of *half* the angle to a rad of 100; and if we move the decimal point two places to the left, we get the *nat sine* of this one half of the angle to a rad of 1, as in the tables. Thus, suppose the dist to be 80.64 feet, then 40.32 is the sine of half the angle; and .4032 will be the nat sine, opposite to which in the table of nat sines we find the angle 23° 47′; which mult by 2 gives 57° 34′, the reqd angle. If obstacles prevent measuring *toward* the objects, we may measure directly *from* them; because, when two lines intersect, the opposite angles are equal. A rough measurement may be made by sticking three pins vert, and a few ins apart, into a small piece of board, nailed hor to the top of a post. The pins would occupy the positions $n\,o\,s$, of the last figure. Pencil-lines may then be drawn, connecting the pin-holes; and the angle be measd with a protractor. By nailing a piece of board vert to a tree, and then drawing upon it a short hor line, by means of a pocket carpenters' spirit-level, vert angles of elevation and depression may be taken roughly in the same way In this way the writer has at times availed himself of the outer door of a house, by opening it until it pointed toward some mountain-peak, the dist of which he knew approximately; but of the height of



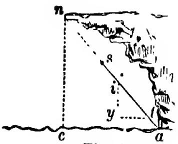


Fig. 6.

its angle *y t i* of inclination with the horizon found as before; in which case the dist *a n* is calculated. Or if the vert height *c n* is sought, the point *c* may first be found by sighting upward along a plumb-line held above the head.

Ex. 3. To find the approximate height, *s x*, of a mountain,

Of which, perhaps, only the very summit, *x*, is visible above interposing forests, or other obstacles, but the dist, *m i*, of which is known. In this case, first direct the instrument hor, as *m h*; and then measure the angle *i m x*. Then in the triangle *i m x* we have one side *m i*, the meas'd angle *i m x*, and the angle *m i x* (90°), to find *i x* by Case 1. But to this *i x* we must add *i o*, equal to the height *y m* of the instrument above the ground, and also *o s*. Now, *o s* is *apparently* due entirely to the curvature of the earth, which is equal to very nearly 8 ins, or .667 ft in one mile, and increases as the squares of the dists, being 4 times 8 ins in 2 miles, 9 times 8 ins in 3 miles, &c. But this is somewhat diminished by the refraction of the atmosphere; which varies with temperature, moisture, &c, but **always** tends to make the object *x* appear higher than it actually is. At an average, this deceptive elevation amounts to about $\frac{1}{7}$th part of the curvature of the earth; and like the latter, it varies with the squares of the dists. Consequently if we subtract $\frac{1}{7}$ part from 8 ins, or .667 ft, we have at once the combined effect of curvature and refraction for one mile, equal to 6.857 ins, or .5714 ft, and for other dists, as shown in the following table, by the use of which we avoid the necessity of making *separate* allowances for curvature and refraction.

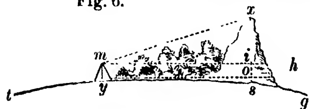


Fig. 7.

Table of allowances to be added for curvature of the earth; and for refraction; combined.

Dist. in yards.	Allow. feet	Dist. in miles.	Allow. feet	Dist. in miles.	Allow. feet.	Dist. in miles	Allow. feet.
100	.002	¼	.036	6	20.6	20	229
150	.004	½	.143	7	28.0	22	277
200	.007	¾	.321	8	36.6	25	357
300	.017	1	.572	9	46.3	30	514
400	.030	1¼	.893	10	57.2	35	700
500	.046	1½	1.29	11	69.2	40	915
600	.066	1¾	1.75	12	82.3	45	1158
700	.090	2	2.29	13	96.6	50	1429
800	.118	2½	3.57	14	112	55	1729
900	.149	3	5.14	15	129	60	2058
1000	.185	3½	7.00	16	146	70	2801
1200	.266	4	9.15	17	165	80	3659
1500	.415	4½	11.6	18	185	90	4631
2000	.738	5	14.3	19	206	100	5717

Hence, if a person whose eye is 5.14 ft, or 112 ft above the sea, sees an object just at the sea's horizon, that object will be about 3 miles, or 14 miles distant from him.

A horizontal line is not a level one, for a straight line cannot be a level one. The curve of the earth, as exemplified in an expanse of quiet water, is level. In Fig 7, if we suppose the curved line *t y s g* to represent the surface of the sea, then the points *t y s* and *g* are on a level with each other. They need not be equidistant from the center of the earth, for the sea at the poles is about 13 miles nearer it than at the equator; yet its surface is everywhere on a level. **Up, and down,** refer to sea level. **Level** means parallel to the curvature of the sea; and **horizontal** means tangential to a level.

Ex. 4. If the inaccessible vert height *c d*, Fig 8,

is so situated that we cannot reach it at all, then place the instrument for measuring angles, at any convenient spot *n*; and in range between *n* and *d*, plant two staffs, whose tops *o* and *i* shall range precisely with *n*, though they need not be on the same level or hor line with it. Measure *n o*: also from *n* measure the angles *o n d* and *o n c*. Then move the instrument to the precise spot previously

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which he had no idea. For allowance for curvature and refraction see above Table.

A triangle whose sides are as 3, 4, and 5, is right angled; and one whose sides are as 7, 7; and 9.9; contains 1 right angle; and 2 angles of 45° each. As it is frequently necessary to lay down angles of 45° and 90° on the ground, these proportions may be used for the purpose, by shaping a portion of a tape-line or chain into such a triangle, and driving a stake at each angle.

occupied by the top o of the staff; and from o measure the angles $i\,o\,d$ and $d\,o\,c$. This being done, subtract the angle $i\,o\,c$ from 180°; the rem will be the angle $c\,o\,n$. Consequently in the triangle $n\,o\,c$, we have one side $n\,o$, and two angles, $c\,n\,o$ and $c\,o\,n$, to find by Case 1 the side $o\,c$. Again, take the angle $i\,o\,d$ from 180°; the remainder will be the angle $n\,o\,d$, so that in the triangle $d\,n\,o$ we have one side $n\,o$, and the two angles $d\,n\,o$ and $n\,o\,d$, to find by Case 1 the side $o\,d$. Finally, in the triangle $c\,o\,d$, we have two sides $c\,o$ and $o\,d$, and their included angle $c\,o\,d$, to find $c\,d$, the reqd vert height.

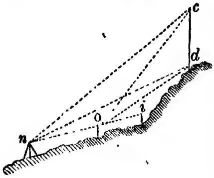


Fig. 8.

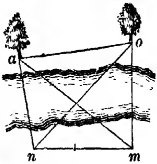


Fig. 9.

Rem. If $c\,d$ were in a valley, or on a hill, and the observations reqd to be made from either higher or lower ground, the operation would be precisely the same.

Ex. 5. See Ex 10.

To find the dist *a o*, Fig 9, between two entirely inaccessible objects,

Measure a side $n\,m$; at n measure the angles $a\,n\,m$ and $o\,n\,m$; also at m measure the angles $o\,m\,n$, and $a\,m\,n$. This being done, we have in the triangle $a\,n\,m$, one side $n\,m$, Fig 9, and the angles $a\,n\,m$, and $n\,m\,a$; hence, by Case 1, we can calculate the side $a\,n$. Again, in the triangle $o\,m\,n$ we have one side $n\,m$, and the two angles $o\,m\,n$, and $m\,n\,o$; hence, by Case 1, we can calculate the side $n\,o$. This being done, we have in the triangle $a\,n\,o$, two sides $a\,n$, and $n\,o$; and their included angle $a\,n\,o$; hence, by Case 3, we can calculate the side $a\,o$, which is the reqd dist. It is plain that in this manner we may obtain also the position or direction of the inaccessible line $a\,o$; for we can calculate the angle $n\,a\,o$; and can therefrom deduce that of $a\,o$; and thus be enabled to run a line parallel to it, if required. By drawing $n\,m$ on paper by a scale, and laying down the four measd angles, the dist $a\,o$ may be measd upon the drawing by the same scale.

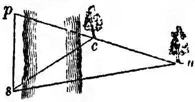


Fig. 10.

If the position of the inaccessible dist $c\,n$, Fig 10, be such that we can place a stake p in line with it, we may proceed thus. Place the instrument at any suitable point s, and take the angles $p\,s\,c$ and $c\,s\,n$. Also find the angle $c\,p\,s$, and measure the dist $p\,s$. Then in the triangle $p\,s\,c$ find $s\,c$ by Case 1; again, the exterior angle $n\,c\,s$, being equal to the two interior and opposite angles $c\,p\,s$, and $p\,s\,c$, we have in the triangle $c\,s\,n$, one side and two angles to find $c\,n$ by Case 1.

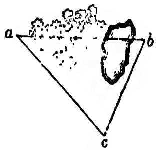


Fig. 11.

Ex. 6. To find a dist *a b*, Fig 11, of which the ends only are accessible.

From a and b, measure any two lines $a\,c$, $b\,c$, meeting at c; also measure the angle $a\,c\,b$. Then in the triangle $a\,b\,c$ we have two sides, and the included angle, to find the third side $a\,b$ by Case 3.

Ex. 7. To find the vert height *o m*, of a hill, above a given point *i*.

Place the instrument at i; measure $a\,m$. Directing the instrument hor, as $a\,n$, take the angle $n\,a\,m$. Then, since $a\,n\,m$ is 90° Fig 12, we have one side $a\,m$, and two angles, $n\,a\,m$ and $a\,n\,m$, to find $n\,m$ by Case 1. Add $n\,o$, equal to $a\,i$, the height of the instrument. Also, if the hill is a long one, add for curvature of the earth, and for refraction, as explained in Example 3, Fig 7. Or the instrument may be placed at the top of the hill; and an angle of depression measured; instead of the angle of elevation $n\,a\,m$.

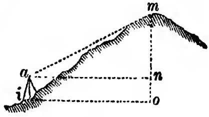


Fig. 12.

Rem. 1. It is plain, that if the height $o\,m$ be previously known, and we wish to ascertain the dist from its summit m to any point i, the same measurement as before, of the angle $n\,a\,m$, will enable us to calculate $a\,m$ by Case 1. So in Ex. 2, if the height $n\,a$ be known, the angles measd in that example, will enable us to compute the dist $a\,o$; so also in Figs 3, 4, 5, and 7; in all of which the process is so plain as to require no further explanation.

Rem. 2. The height of a vert object **by means of its shadow.** Plant one end of a straight stick vert in the ground; and measure its shadow; also measure the length of the shadow of the object. Then, as the length of the shadow of the stick is to the length of the stick above

ground, so is the length of the shadow of the object, to its height. If the object is inclined, the stick must be equally inclined.

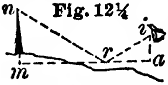


Fig. 12¼

Rem. 3. Or the height of a vert object mn, Fig 12¼, whose distance rm is known, may be found **by its reflection** in a vessel of water, or in a piece of looking glass placed perfectly horizontal at r; for as ra is to the height ai of the eye above the reflector r, so is rm to the height mn of the object above r.

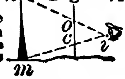


Fig. 12½

Rem. 4. Or let oc, Fig 12½, **be a planted pole,** or a rod held vert by an assistant. Then stand at a proper dist back from it, and keeping the eyes steady, let marks be made at o and c, where the lines of sight in and im strike the rod. Then as ic is to co, so is im to mn.

The following examples may be regarded as substitutes for strict trigonometry · and will at times be useful, in case a table of sines, &c, is not at hand for making trigonometrical calculations.

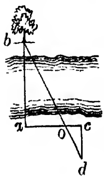


Fig. 13.

Ex. 8. To find the dist ab, **of which one end only is accessible.**

Drive a stake at any convenient point a, from a lay off any angle bac. In the line ac, at any convenient point c, drive a stake, and from c lay off an angle acd, equal to the angle bac. In the line cd, at any convenient point, as d, drive a stake. Then, standing at d, and looking at b, place a stake o in range with db, and at the same time in the line ac. Measure ao, oc, and cd; then, from the principle of similar triangles, as

$$oc : cd :: ao : ab.$$

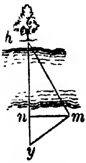


Fig. 14.

Or thus:

Fig 14, nh being the dist, place a stake at n; and lay off the angle hnm 90°. At any convenient dist nm, place a stake m Make the angle $hmy = 90°$; and place a stake at y, in range with hn. Measure ny and nm; then, from the principle of similar triangles, as

$$ny : nm :: nm : nh.$$

Or thus, Fig 14. Lay off the angle $hnm = 90°$, placing a stake m, at any convenient dist nm. Measure nm. Also measure the angle nmh. Find nat tang of nmh by Table Mult this nat tang by nm. The prod will be nh.

Or thus. Lay off angle $hnm = 90°$. From m measure the angle nmh, and lay off angle nmy equal to it, placing a stake at y in range with hn. Then is $ny = nh$.

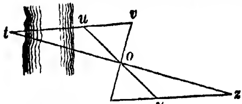


Fig. 15.

Or thus, without measuring any angle;

tu being the dist. Make uv of any convenient length, in range with tu. Measure any vo; and ox equal to it, in range Measure uo; and oy equal to it in range. Place a stake z in range with both xy, and to. Then will yz be both equal to tu, and parallel to it.

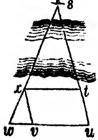


Fig. 16.

Or thus, without measuring any angle.

Drive two stakes t and u, in range with the object s. From t lay off any convenient dist tx, in any direction From u lay off uw parallel to tx, placing w in range with xs. Make uv equal to tx. Measure wv, vx, and xt. Then, as

$$wv : vx :: xt : ts.$$

Or thus. At a lay off angle $oac = 5° 43'$. Lay off oc at right angles to ao. Measure oc. Then $ao = 10\,oc$, too long only 1 part in 935.6, or 5.643 feet in a mile, or .1069 foot (full 1¼ inches) in 100 feet.

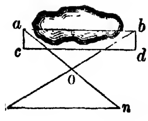


Fig. 17.

Ex. 9. To find the dist *a b*, of which the ends only are accessible.

From *a* lay off the angle *b a c*; and from *b*, the angle *a b d* each 90°. Make *a c* and *b d* equal to each other. Then, *c d* = *a b*. Or *a b* may be considered as the dist across the river in Figs 15, 13, or 14; and be ascertained in the same way. Or measure any dist, Fig 17, *a o*, and make *o n* in line and equal to it. Also measure *b o*; and make *o m* in line and equal to it. Then will *m n* be both parallel to *a b*, and equal to it.

Ex. 10. See Ex. 4. **To find the entirely inaccessible dist *y z*, and also its direction.**

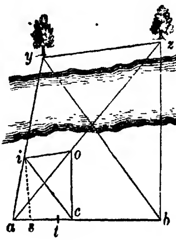


Fig. 18.

At any two convenient points *a* and *b*, from each of which and *z* can be seen, drive stakes. Then we have the four rners of a four sided figure, in which are given the directions ' three of its sides, and of its two diags. These data enable us lay out on the ground, the small four-sided fig *a c o i*, exactly milar to the large one. Thus, in the line *a b* place a stake ; and make *c o* parallel to *b z*; *o* being at the same time in inge of the diag *a z*. Also, from *c* make *c i* parallel to *b y*; being at the same time in range of *a y*. Then will *i o* be in ie same direction as *y z*, or parallel to it. Measure *a c*, *a b*, id *i o*; then evidently, from the principle of similar figures, as

***a c* : *a b* : : *i o* : *y z*.**

If *y z* were a visible line such as a fence or road, we could om *a* divide it into any required portions. Thus, if we wish place a stake halfway between *y* and *z*, first place one half-ay between *i* and *o*, then standing at *a*, by means of signals, ace a person in range on *y z*. Or, to find along *a b*, a point *t* erp to *y z* at *y*, first make *o i s* = 90°, and measure *a s*. Then,

***o i* : *a s* : : *y z* : *a t*.**

Ex. 11. To find the position of a point, *n*, Fig 19,

y means of two angles a n b *and* b n c, *taken from* n *to the three objects* a b c, *whose positions and dists apart are known*

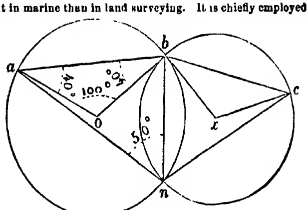


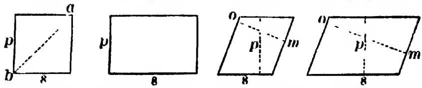
Fig. 19.

The use of this problem is more frequent in marine than in land surveying. It is chiefly employed or determining the position *n* of a oat from which soundings are being aken along a coast. As the boat moves from point to point to take resh soundings, it becomes necessary make a fresh observation at each oint, in order to define its position n the chart. An observation consists n the measurement by a sextant of ie two angles *a n b*, *b n c*, to the sig-als *a b c*, previously arranged on the hore. When practicable, this method hould be rejected; and the observa-ons taken to the boat at the same istant, by two observers on shore, at wo of the stations. The boat to show signal at the proper moment. The ost expeditious mode of fixing the oint *n* upon the map, is to draw three nes, forming the two angles, and ex-nded indefinitely, on a piece of trans-arent paper. Place the paper upon the map, and move it about until the three lines pass through ie three stations, then prick through the point *n* wherever it happens to come.

Instead of the transparent paper, an instrument called a *station pointer* may be used when there re many points to be fixed.

But the position of the point *n* can be found more correctly by describing two circles, as in Fig 19, ach of which shall pass through *n* and two of the station points. The question is to find the centers and *x* of two such circles. This is very simple. We know that the angle *a o b* at the center of a circle is vice as great as any angle *a n b* at the circumf of the same circle, when both are subtended by the ame chord *a b*. Consequently, if the angle *a n b*, observed from the boat is say, 50°, the angle *a o b* must be 100°. And, since the three angles of every plane triangle are equal to 180°, the two angles *a b* and *o b a* are together equal to 180° — 100° = 80°. And, since the two sides *a o* and *b o* are qual (being radii of the same circle), therefore, the angles *o a b* and *o b a* are equal, and each equal to $\frac{80}{2}$ = 40°. Consequently, on the map we have only to lay down at *a* and *b*, two angles of 40°; the oint *o* of intersection will be the center of the circle *a b n*. Proceed in the same way with the angle *n c*, to find the center *x*. Then the intersection of the two circles at *n* will be the point sought.

PARALLELOGRAMS.



A PARALLELOGRAM is any figure of four straight sides, the opposite ones of which are parallel. There are but four, as in the above figs. The rhombus, like the rhombohedron, Fig 3, p 195, is sometimes called "rhomb." In the square and rhombus all the four sides are equal; in the rectangle and rhomboid only the opposite ones are equal. In any parallelogram the four angles amount to four right angles, or 360°; and any two diagonally opposite angles are equal to each other; hence, having one angle given, the other three can readily be found. In a square, or a rhombus, a diag divides each of two angles into two equal parts; but in the two other parallelograms it does not.

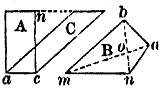
To find the area of any parallelogram.

Multiply any side, as S, by the perp height, or dist p to the opposite side. **Or,** multiply together two sides and nat sine of their included angle.

The diag a b of any square is equal to one side mult by 1.41421, and a side is equal to $\frac{\text{diagonal}}{1.41421}$, **or,** to diag mult by .707107.

The side of a square equal in area to a given circle, is equal to diam × .886227.

The side of the greatest square, *that can be inscribed in a given circle,* is equal to diam × .707107

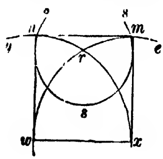


The side of a square mult by 1.51967 gives the **side of an equilateral triangle** of the same area. **All parallelograms** as A and C, **which have equal bases,** $a\ c$, and equal perp heights $n\ c$, have also equal areas, and the area of each is twice that of a triangle having the same base, and perp height. **The area of a square inscribed in a circle** is equal to twice the square of the rad

In every parallelogram, the 4 squares drawn on its sides have a united area equal to that of the two squares drawn on its 2 diags. **If a larger square** be drawn on the diag $a\ b$ of a smaller square its area will be twice that of said smaller square. **Either diag of any parallelogram** divides it into two equal triangles, and the 2 diags div it into 4 triangles of equal areas. **The two diags of any parallelogram** divide each other into two equal parts. **Any line drawn through the center of a diag** divides the parallelogram into two equal parts

Remark 1.—The area of any fig whatever as B **that is enclosed by four straight lines,** may be found thus. Mult together the two diags $a\ m$, $n\ b$; and the nat sine of the least angle $a\ o\ b$; or $n\ o\ m$, formed by their intersection. Div the product by 2. This is useful in land surveying when obstacles, as is often the case, make it difficult to measure the sides of the fig or field; while it may be easy to measure the diags; and after finding their point of intersection o, to measure the required angle. **But if the fig is to be drawn,** the **parts** $o\ a$, $o\ b$, $o\ n$, $o\ m$ of the diags must also be measd.

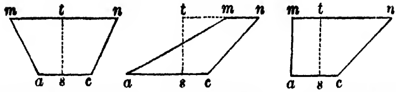
Rem. 2.—The sides of a parallelogram, triangle, and many other figs may be found, when only the area and angles are given, thus: Assume some particular one of its sides to be of the length 1; and calculate what its area would be if that were the case. Then as the sq rt of the area thus found is to this side 1, so is the sq rt of the actual given area, to the corresponding actual side of the fig.



On a given line $w\ x$, to draw a square $w\ x\ n\ m$.

From w and x, with rad $w\ x$, describe the arcs $x\ r\ y$ and $w\ r\ e$. From their intersection r, and with rad equal to ½ of $w\ x$, describe $s\ s\ s$. From w and x draw $w\ n$ and $x\ m$ tangential to $s\ s\ s$, and ending at the other arcs; join $n\ m$.

TRAPEZOIDS.

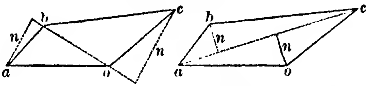


A trapezoid *a c n m*, is any figure with four straight sides, only two of which, as *ac* and *n m*, are parallel.

To find the area of any trapezoid.

Add together the two parallel sides, *a c* and *m n*; mult the sum by the perp dist *s t* between them; div the prod by 2. See the following rules for trapeziums, which are all equally applicable to trapezoids; also see Remarks after Parallelograms.

TRAPEZIUMS.



A trapezium *a b c o*, is any fig with four straight sides, of which no two are parallel.

To find the area of any trapezium, having given the diag *b o*, or *a c*, between either pair of opposite angles; and also the two perps, *n*, *n*, from the other two angles.

Add together these two perps; mult the sum by the diag; div the prod by 2.

Having the four sides; and either pair of opposite angles, as *a b c*, *a o c*; or *b a o*, and *b c o*.

Consider the trapezium as divided into two triangles, in each of which are given two sides and the included angle. Find the area of each of these triangles as directed under the preceding head "Triangles," and add them together.

Having the four angles, and either pair of opposite sides.

Begin with one of the sides, and the two angles at its ends. If the sum of these two angles exceeds 180°, subtract each of them from 180°, and make use of the rems instead of the angles themselves. Then consider this side and its two adjacent angles (or the two rems, as the case may be) as those of a triangle; and find its area as directed for that case under the preceding head "Triangle." Do the same with the other given side, and its two adjacent angles, (or their rems, as the case may be.) Subtract the least of the areas thus found, from the greatest, the rem will be the reqd area.

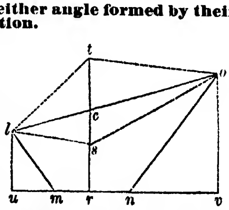
Having three sides; and the two included angles.

Mult together the middle side, and one of the adjacent sides, mult the prod by the nat sine of their included angle; call the result *a*. Do the same with the middle side and its other adjacent side, and the nat sine of the other included angle, call the result *b*. Add the two angles together; find the diff between their sum and 180°, whether greater or less, find the nat sine of this diff; mult together the two given sides which are *opposite* one another, mult the prod by the nat sine just found; call the result c. Add together the results *a* and *b*; then, if the sum of the two given angles is *less* than 180°, subtract *c* from the sum of *a* and *b*; *half* the rem will be the area of the trapezium. But if the sum of the two given angles be *greater* than 180°, add together the three results *a*, *b*, and *c*; *half* their sum will be the area.

Having the two diagonals, and either angle formed by their intersection.

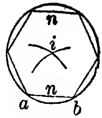
See Remarks after Parallelograms.

In railroad measurements



Of excavation and embankment, the trapezium *l m n o* frequently occurs; as well as the two 5-sided figures *l m n o t* and *l m n o s*; in all of which *m n* represents the roadway; *r s*, *r c*, and *r t* the center-depths or heights; *l u* and *o v* the side-depths or heights, as given by the level; *l m* and *n o* the side-slopes.

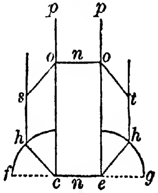
The same general rule for area applies to all three of these figs; namely, mult the extreme hor width *u v* by *half* the center depth *r s*, *r c*, or *r t*, as the case may be. Also mult one *fourth* of the width of roadway *m n*, by the *sum* of the two side-depths *l u* and *o v*. Add the two prods together; the sum is the reqd area. This rule applies whether the two side-slopes *m l* and *n o* have the same angle of inclination or not. **In railroad work, etc., the mid way hor width, center depth, and side depths of a prismoid are respectively = The half sums of the corresponding end ones, and thus can be found without actual measurement.**



To draw a hexagon, each side of which shall be equal to a given line, *a b*.

From *a* and *b*, with rad *a b*, describe the two arcs; from their intersection, *i*, with the same rad, describe a circle; around the circumf of which, step off the same rad.

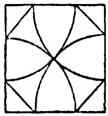
Side of a hexagon $= nn \times .57735$.



To draw an octagon, with each side equal to a given line, *c e*.

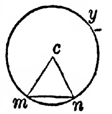
From *c* and *e* draw two perps, *c p*, *e p*. Also prolong *c e* toward *f* and *g*; and from *c* and *e*, with rad equal *c e*, draw the two quadrants; and find their centers *h h*; join *c h*, and *e h*; draw *h s* and *h t* parallel to *c p*; and make each of them equal to *c e*; make *c o*, and *e o*, each equal to *h h*; join *o o*, *o s*, and *o t*.

Side of an octagon $= nn \times .41421354$.



To draw an octagon in a given square.

From each corner of the square, and with a rad equal to half its diag, describe the four arcs; and join the points at which they cut the sides of the square.



To draw any regular polygon, with each side equal to *m n*.

Div 360 degrees by the number of sides; take the quot from 180°; div the rem by 2. This will give the angle *c m n*, or *c n m*. At *m* and *n* lay down these angles by a protractor: the sides of these angles will meet at a point, *c*, from which describe the circle *m n y*; and around its circumf step off dists equal to *m n*.

In any circle, *m n y*, to draw any regular polygon.

Div 360° by the number of sides; the quot will be the angle *m c n*, at the center. Lay off this angle by a protractor; and its chord *m n* will be one side; which step off around the circumf.

To reduce any polygon, as *a b c d e f a*, to a triangle of the same area.

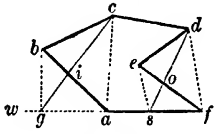


Fig. 1.

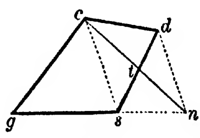
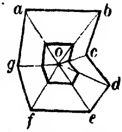


Fig. 2.

If we produce the side *f a* toward *w*; and draw *b g* parallel to *a c*, and join *g c*, we get equal triangles *a c b*, and *a c g*, both on the same base *a c*; and both of the same perp height, inasmuch as they are between the two parallels *a c* and *g b*. But the part *a c i* forms a portion of both these triangles, or in other words, *is common to both*. Therefore, if it be taken away from both triangles, the remaining parts, *i c b* of one of them, and *i g a* of the other, are also equal. Therefore, if the part *i c b* be left off from the polygon, and the part *i g a* be taken into it, the polygon *g f e d c i g* will have the same area as *a f e d c b a*; but it will have but five sides, while the other has six. Again, if *e s* be drawn parallel to *d f*, and *d s* joined, we have upon the same base *e s*, and between the same parallels *e s* and *d f*, the two equal triangles *e s d*, and *e s f*, with the part *e o s* common to both; and consequently the remaining part *e o d* of one, and *o s f* of the other, are equal. Therefore, if *o s f* be left off from the polygon, and *s o d* be taken into it, the new polygon *g s d c g*, Fig 2, will have the same area as *g f e d c g*; but it has but four sides, while the other has five. Finally, if *g s*, Fig 2, be extended toward *n*; and *d n* drawn parallel to *c s*; and *c n* joined, we have on the same base *c s*, and between the same parallels *c s* and *d n*, the two equal triangles *c s n*, and *c s d*, with the part *c s t* common to both. Therefore, if we leave out *c d t*, and take in *s t n*, we have the triangle *g n c* equal to the polygon *g s d c g*, Fig 2; or to *a f e d c b a*, Fig 1.

This simple method is applicable to polygons of any number of sides.

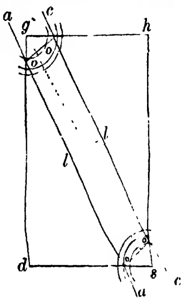


To reduce a large fig, *a b c d e f g*, to a smaller similar one.

From any interior point *o*, which had better be near the center, draw lines to all the angles *a*, *b*, *c*, &c. Join these lines by others parallel to the sides of the fig. If it should be reqd to enlarge a small fig, draw, from any point *o* within it, lines extending beyond its angles; and join these lines by others parallel to the sides of the small fig.

To reduce a map to one on a smaller scale.

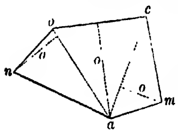
The best method is by dividing the large map into squares by faint lines, with a very soft lead pencil, and then drawing the reduced map upon a sheet of smaller squares. A pair of proportional dividers will assist much in fixing points intermediate of the sides of the squares. If the large map would be injured by drawing and rubbing out the squares, threads may be stretched across it to form the squares.



In a rectangular fig, *g h s d*,

Representing an open panel, to find the points *o o o o* in its sides; and at equal dists from the angles *g*, and *s*; for inserting a diag piece *o o o o*, of a given width *l l*, measured at right angles to its length. From *g* and *s* as centers, describe several concentric arcs, as in the Fig. Draw upon transparent paper, two parallel lines *a a*, *c c*, at a distance apart equal to *l l*, and placing these lines on top of the panel, move them about until it is shown by the arcs that the four dists *g o*, *g o*, *s o*, *s o* are equal. Instead of the transparent paper, a strip of common paper, of the width *l l* may be used.

REM. Many problems which would otherwise be very difficult, may be thus solved with an accuracy sufficient for practical purposes, **by means of transparent paper.**

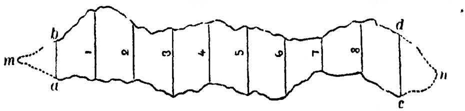


To find the area of any irregular polygon, *a n b c m*.

Div it into triangles, as *a n b*, *a m c*, and *a b c*, in each of which find the perp dist *o*, between its base *a b*, *a c*, or *b c*; and the opposite angle *n*, *m*, or *a*, mult each base by its perp dist; add all the prods together, div by 2.

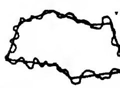
To find approx the area of a long irregular fig, as *a b c d*.

Between its ends *a b*, *c d*,



space off *equal* dists, (the shorter they are the more accurate will be the result,) through which draw the intermediate parallel lines 1, 2, 3, &c, across the breadth of the fig. Measure the lengths of these intermediate lines; add them together, to the sum add *half* the sum of the two end breadths *a b* and *c d*. Mult the entire sum by one of the equal spaces between the parallel lines. The prod will be the area. This rule answers as well if either one or both the ends terminate in points, as at *m* and *n*. In the last of these cases, both *a b* and *c d* will be included in the intermediate lines, and half the two end breadths will be 0, or nothing.

To find the area of any irregular figure.



Draw around it lines which shall enclose within them (as nearly as can be judged by the eye) as much space not belonging to the figure as they exclude space belonging to it. The area of the simplified figure thus formed, being in this manner rendered equal to that of the complicated one, may be calculated by dividing it into triangles, &c. By using a piece of fine thread, the proper position for the new boundary lines may be found, before drawing them in.

Areas of irregular figures may be found from a drawing, by laying upon it a piece of transparent paper carefully ruled into small squares, each of a given area, say 10, 20, or 100 sq. ft. each; and by first counting the whole squares, and then adding the fractions of squares.

CIRCLES.

A circle is the area included within a curved line, called the **circumference,** every point of which is equidistant from a fixed point, within the circle, called its **center.** Any straight line, passing entirely across the circle through its center, is called a **diameter** of the circle. Any straight line, extending from the center to the circumference, is called a **radius.**

The ratio of the circumference to the diameter, or of the semi-circumference to the radius, = circumf ÷ diam, is called π (**pi**).

$$\pi = 4\ (1 - 1/3 + 1/5 - 1/7 + 1/9 - \text{etc.}).$$

π is incommensurable; *i. e.*, it cannot be exactly stated arithmetically. It has been determined to several hundred places of decimals.

Approximate values of π; $\pi = x + x/a$:

x	a	x	a
3.141 592 653 6	— 307,788,000,000	355 ÷ 113	— 11,776,700
3.141 592 65	+ 875,085,000	377 ÷ 120	— 42,447
3.141 593	— 9,069,000	360 ÷ 114.6	+ 13,576
3.141 6	427,637	22 ÷ 7	— 2,485

Functions of π.

		Logarithm.			Logarithm.
π =	3.14159265	0.497 1499	$2/\pi$ =	0.636620	1.803 8801
$\pi/2$ =	1.57079632	0.196 1199	$3/\pi$ =	0.954930	1.979 9714
$\pi/3$ =	1.04719755	0.020 0286	π^2 =	9.869604	0.994 2997
$\pi\sqrt{2}$ =	4.442883	0.647 6649	π^3 =	31.00628	1.491 4496
$\pi/\sqrt{2}$ =	2.221442	0.346 6349	$\sqrt{\pi}$ =	1.772454	0.248 5749
$1/\pi$ =	0.318310	1.502 8501	$1/\sqrt{\pi}$ =	0.564190	1.751 4251

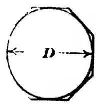
For multiples of π, see "Circumf," in tables, pp. 163 to 178.

Radius, R; Diameter, D; Circumference, C; Area, A.

$$R = \frac{D}{2} = \frac{C}{2\pi} = \sqrt{\frac{A}{\pi}}, \qquad D = 2R = \frac{C}{\pi} = 2\sqrt{\frac{A}{\pi}};$$

$$C = 2\pi R = \pi D = 2\sqrt{\pi A}; \qquad A = \pi R^2 = \frac{\pi}{4} D^2 = \frac{C^2}{4\pi} = \frac{DC}{4}.$$

RELATIONS OF CIRCLE TO OTHER FIGURES.



Circumscribed polygon, regular or irregular.

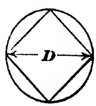
$$\frac{\text{Area of circle}}{\text{Area of polygon}} = \frac{\text{circumference of circle}}{\text{circumference of polygon}}.$$

Equilateral triangle of equal area. D = diam of circle.

Side of triangle = $D\sqrt{\pi} \div \sqrt[4]{3}$ = 1.34677 D.

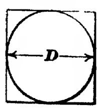
Square of equal area.

Side of square = 0.88623 × diam of circle.
Diam of circle = 1.12838 × side of square.



Inscribed square. D = diam of circle.

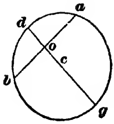
Side of square = $D\sqrt{\frac{1}{2}}$ = 0.7071 D.
Area of square = 2 × radius² of circle.
Radius of circle = ½ × diagonal of square
= $\sqrt{\frac{1}{2}}$ × side of square = 0.7071 × side.



Circumscribed square.

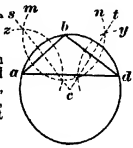
A = area of square; a = area of circle.

$$A = \frac{4}{\pi} a = 1.2732\, a; \qquad a = \frac{\pi}{4} A = 0.7854\, A.$$

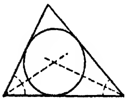


PROBLEMS IN CIRCLES.

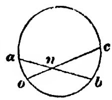
To find the diameter, $d\ g$, and the center, c, of a given circle.
Draw any chord $a\ b$; and from its middle, o, draw, at right angles to it, a diam $d\ g$; find the center c of this diam. Or see below.



To describe a circle through any three points, $a\ b\ d$, not in a straight line; *i. e.* through the three angles of a triangle.
Join the points by the lines $a\ b$, $b\ d$. From b, with any convenient rad, draw the arc $m\ n$; and from a and d, with the same rad, draw arcs y and z; then two lines, $c\ s$, $c\ t$, drawn through the intersections of these arcs, will bisect $a\ b$ and $b\ d$ perpendicularly, and will meet at the center, c, of the circle.



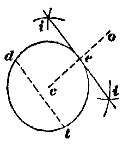
To inscribe a circle in a triangle, draw two lines bisecting any two of the angles. Where these lines meet is the center of the circle.



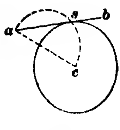
If **any two chords**, as $a\ b$, $o\ c$, cross each other, as at n; then as $o\ n : n\ b :: a\ n : n\ c$. Hence, $n\ b \times a\ n = o\ n \times n\ c$. That is, the product of the two segments of one of the chords is = the product of the two segments of the other chord.

Tangents.

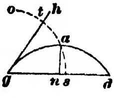
Here the "tangents" are merely straight lines, touching the circle, without cutting it; not trigonometrical tangents.



To draw a tangent, $i\ e\ i$, to a circle, through any given point, e, in its circumf.
Through the center c and the given point e, draw $c\ o$; make $e\ o = e\ c$; from c and o, with any rad greater than half of $o\ c$, describe the two pairs of arcs, i and i; join their intersections $i\ i$.
Or from e lay off any two equal distances $e\ d$, $e\ t$; and draw $i\ i$ parallel to $d\ t$.



To draw a tang, $a\ s\ b$, to a circle, from a point, a, which is outside of the circle.
Join $a\ c$, c being the center of the circle. On $a\ c$, describe a semi-circle; through the intersection, s, draw $a\ s\ b$.



To draw a tang, $g\ h$, from a circular arc, $g\ a\ d$, of which $n\ a$ is the rise. With rad $g\ a$, describe an arc, $s\ a\ o$. Make $t\ a = s\ a$. Through t draw $g\ h$.

TABLE 1 OF CIRCLES.
Diameters in units and eighths, &c.

Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
1-64	.049087	.00019	3. ½	10.9956	9.6211	10⅛	31.8086	80.516	19¼	60.4757	291.04
1-32	.098175	.00077	9-16	11.1919	9.9678	¼	32.2013	82.516	⅜	60.8684	294.83
3-64	.147262	.00173	⅝	11.3883	10.321	⅜	32.5940	84.541	½	61.2611	298.65
1-16	.196350	.00307	11-16	11.5846	10.680	½	32.9867	86.590	⅝	61.6538	302.49
3-32	.294524	.00690	¾	11.7810	11.045	⅝	33.3794	88.664	¾	62.0465	306.35
⅛	.392699	.01227	13-16	11.9773	11.416	¾	33.7721	90.763	⅞	62.4392	310.24
5-32	.490874	.01917	⅞	12.1737	11.793	⅞	34.1648	92.886	20.	62.8319	314.16
3-16	.589049	.02761	15-16	12.3700	12.177	11.	34.5575	95.033	⅛	63.2246	318.10
7-32	.687223	.03758	4.	12.5664	12.566	⅛	34.9502	97.205	¼	63.6173	322.06
¼	.785398	.04909	1-16	12.7627	12.962	¼	35.3429	99.402	⅜	64.0100	326.05
9-32	.883573	.06213	⅛	12.9591	13.364	⅜	35.7356	101.62	½	64.4026	330.06
5-16	.981748	.07670	3-16	13.1554	13.772	½	36.1283	103.87	⅝	64.7953	334.10
11-32	1.07992	.09281	¼	13.3518	14.186	⅝	36.5210	106.14	¾	65.1880	338.16
⅜	1.17810	.11045	5-16	13.5481	14.607	¾	36.9137	108.43	⅞	65.5807	342.25
13-32	1.27627	.12962	⅜	13.7445	15.033	⅞	37.3064	110.75	21.	65.9734	346.36
7-16	1.37445	.15033	7-16	13.9408	15.466	12.	37.6991	113.10	⅛	66.3661	350.50
15-32	1.47262	.17257	½	14.1372	15.904	⅛	38.0918	115.47	¼	66.7588	354.66
½	1.57080	.19635	9-16	14.3335	16.349	¼	38.4845	117.86	⅜	67.1515	358.84
17-32	1.66897	.22166	⅝	14.5299	16.800	⅜	38.8772	120.28	½	67.5442	363.05
9-16	1.76715	.24850	11-16	14.7262	17.257	½	39.2699	122.72	⅝	67.9369	367.28
19-32	1.86532	.27688	¾	14.9226	17.721	⅝	39.6626	125.19	¾	68.3296	371.54
⅝	1.96350	.30680	13-16	15.1189	18.190	¾	40.0553	127.68	⅞	68.7223	375.83
21-32	2.06167	.33824	⅞	15.3153	18.665	⅞	40.4480	130.19	22	69.1150	380.13
11-16	2.15984	.37122	15-16	15.5116	19.147	13.	40.8407	132.73	⅛	69.5077	384.46
23-32	2.25802	.40574	5.	15.7080	19.635	⅛	41.2334	135.30	¼	69.9004	388.82
¾	2.35619	.44179	1-16	15.9043	20.129	¼	41.6261	137.89	⅜	70.2931	393.20
25-32	2.45437	.47937	⅛	16.1007	20.629	⅜	42.0188	140.50	½	70.6858	397.61
13-16	2.55254	.51849	3-16	16.2970	21.135	½	42.4115	143.14	⅝	71.0785	402.04
27-32	2.65072	.55914	¼	16.4934	21.648	⅝	42.8042	145.80	¾	71.4712	406.49
⅞	2.74889	.60132	5-16	16.6897	22.166	¾	43.1969	148.49	⅞	71.8639	410.97
29-32	2.84707	.64504	⅜	16.8861	22.691	⅞	43.5896	151.20	23	72.2566	415.48
15-16	2.94524	.69029	7-16	17.0824	23.221	14.	43.9823	153.94	⅛	72.6493	420.00
31-32	3.04342	.73708	½	17.2788	23.758	⅛	44.3750	156.70	¼	73.0420	424.56
1.	3.14159	.78540	9-16	17.4751	24.301	¼	44.7677	159.48	⅜	73.4347	429.13
1-16	3.33794	.88664	⅝	17.6715	24.850	⅜	45.1604	162.30	½	73.8274	433.74
⅛	3.53429	.99402	11-16	17.8678	25.406	½	45.5531	165.13	⅝	74.2201	438.36
3-16	3.73064	1.1075	¾	18.0642	25.967	⅝	45.9458	167.99	¾	74.6128	443.01
¼	3.92699	1.2272	13-16	18.2605	26.535	¾	46.3385	170.87	⅞	75.0055	447.69
5-16	4.12334	1.3530	⅞	18.4569	27.109	⅞	46.7312	173.78	24.	75.3982	452.39
⅜	4.31969	1.4849	15-16	18.6532	27.688	15.	47.1239	176.71	⅛	75.7909	457.11
7-16	4.51604	1.6230	6.	18.8496	28.274	⅛	47.5166	179.67	¼	76.1836	461.86
½	4.71239	1.7671	⅛	19.2423	29.465	¼	47.9093	182.65	⅜	76.5763	466.64
9-16	4.90874	1.9175	¼	19.6350	30.680	⅜	48.3020	185.66	½	76.9690	471.44
⅝	5.10509	2.0739	⅜	20.0277	31.919	½	48.6947	188.69	⅝	77.3617	476.26
11-16	5.30144	2.2365	½	20.4204	33.183	⅝	49.0874	191.75	¾	77.7544	481.11
¾	5.49779	2.4053	⅝	20.8131	34.472	¾	49.4801	194.83	⅞	78.1471	485.98
13-16	5.69414	2.5802	¾	21.2058	35.785	⅞	49.8728	197.93	25	78.5398	490.87
⅞	5.89049	2.7612	⅞	21.5984	37.122	16.	50.2655	201.06	⅛	78.9325	495.79
15-16	6.08684	2.9483	7.	21.9911	38.485	⅛	50.6582	204.22	¼	79.3252	500.74
2.	6.28319	3.1416	⅛	22.3838	39.871	¼	51.0509	207.39	⅜	79.7179	505.71
1-16	6.47953	3.3410	¼	22.7765	41.282	⅜	51.4436	210.60	½	80.1106	510.71
⅛	6.67588	3.5466	⅜	23.1692	42.718	½	51.8363	213.82	⅝	80.5033	515.72
3-16	6.87223	3.7583	½	23.5619	44.179	⅝	52.2290	217.08	¾	80.8960	520.77
¼	7.06858	3.9761	⅝	23.9546	45.664	¾	52.6217	220.35	⅞	81.2887	525.84
5-16	7.26493	4.2000	¾	24.3473	47.173	⅞	53.0144	223.65	26.	81.6814	530.93
⅜	7.46128	4.4301	⅞	24.7400	48.707	17.	53.4071	226.98	⅛	82.0741	536.05
7-16	7.65763	4.6664	8.	25.1327	50.265	⅛	53.7998	230.33	¼	82.4668	541.19
½	7.85398	4.9087	⅛	25.5254	51.849	¼	54.1925	233.71	⅜	82.8595	546.35
9-16	8.05033	5.1572	¼	25.9181	53.456	⅜	54.5852	237.10	½	83.2522	551.55
⅝	8.24668	5.4119	⅜	26.3108	55.088	½	54.9779	240.53	⅝	83.6449	556.76
11-16	8.44303	5.6727	½	26.7035	56.745	⅝	55.3706	243.98	¾	84.0376	562.00
¾	8.63938	5.9396	⅝	27.0962	58.426	¾	55.7633	247.45	⅞	84.4303	567.27
13-16	8.83573	6.2126	¾	27.4889	60.132	⅞	56.1560	250.95	27.	84.8230	572.56
⅞	9.03208	6.4918	⅞	27.8816	61.862	18.	56.5487	254.47	⅛	85.2157	577.87
15-16	9.22843	6.7771	9.	28.2743	63.617	⅛	56.9414	258.02	¼	85.6084	583.21
3.	9.42478	7.0686	⅛	28.6670	65.397	¼	57.3341	261.59	⅜	86.0011	588.57
1-16	9.62113	7.3662	¼	29.0597	67.201	⅜	57.7268	265.18	½	86.3938	593.96
⅛	9.81748	7.6699	⅜	29.4524	69.029	½	58.1195	268.80	⅝	86.7865	599.37
3-16	10.0138	7.9798	½	29.8451	70.882	⅝	58.5122	272.45	¾	87.1792	604.81
¼	10.2102	8.2958	⅝	30.2378	72.760	¾	58.9049	276.12	⅞	87.5719	610.27
5-16	10.4065	8.6179	¾	30.6305	74.662	⅞	59.2976	279.81	28.	87.9646	615.75
⅜	10.6029	8.9462	⅞	31.0232	76.589	19.	59.6903	283.53	⅛	88.3573	621.26
7-16	10.7992	9.2806	10.	31.4159	78.540	⅛	60.0830	287.27	¼	88.7500	626.80

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam.	Circumf.	Area.
28⅜	89.1427	632.36	38.	119.381	1134.1	47⅝	149.618	1781.4	57¼	179.856	2574.2
½	89.5354	637.94	⅛	119.773	1141.6	¾	150 011	1790.8	⅜	180.249	2585.4
⅝	89.9281	643.55	¼	120.166	1149.1	⅞	150.404	1800.1	½	180.642	2596.7
¾	90.3208	649.18	⅜	120.559	1156.6	48	150.796	1809 6	⅝	181.034	2608.0
⅞	90.7135	654.84	½	120.951	1164.2	⅛	151.189	1819.0	¾	181.427	2619.4
29.	91.1062	660 52	⅝	121.344	1171.7	¼	151.582	1828 5	⅞	181.820	2630.7
⅛	91.4989	666.23	¾	121.737	1179.3	⅜	151.975	1837.9	58	182.212	2642 1
¼	91.8916	671.96	⅞	122.129	1186.9	½	152.367	1847.5	⅛	182.605	2653.5
⅜	92 2843	677.71	39.	122 522	1194.6	⅝	152.760	1857.0	¼	182.998	2664 9
½	92.6770	683.49	⅛	122.915	1202.3	¾	153.153	1866.5	⅜	183.390	2676 4
⅝	93.0697	689.30	¼	123.308	1210 0	⅞	153.545	1876.1	½	183.783	2687 8
¾	93.4624	695.13	⅜	123 700	1217 7	49.	153.938	1885.7	⅝	184.176	2699 3
⅞	93.8551	700.98	½	124.093	1225 4	⅛	154.331	1895.4	¾	184 569	2710 9
30.	94.2478	706.86	⅝	124 486	1233.2	¼	154.723	1905 0	⅞	184 961	2722.4
⅛	94.6405	712 76	¾	124 878	1241 0	⅜	155.116	1914 7	59	185 354	2734.0
¼	95.0332	718.69	⅞	125 271	1248 8	½	155.509	1924 4	⅛	185.747	2745.6
⅜	95.4259	724 64	40	125 664	1256 6	⅝	155.902	1934.2	¼	186.139	2757.2
½	95.8186	730 62	⅛	126.056	1264.5	¾	156 294	1943.9	⅜	186 532	2768.8
⅝	96.2113	736.62	¼	126.449	1272.4	⅞	156.687	1953 7	½	186.925	2780.5
¾	96.6040	742 64	⅜	126.842	1280.3	50.	157.080	1963 5	⅝	187 317	2792.2
⅞	96 9967	748.69	½	127.235	1288.2	⅛	157.472	1973 3	¾	187.710	2803 9
31.	97.3894	754 77	⅝	127.627	1296 2	¼	157.865	1983 2	⅞	188 103	2815.7
⅛	97.7821	760 87	¾	128.020	1304 2	⅜	158.258	1993.1	60	188 496	2827.4
¼	98.1748	766 99	⅞	128.413	1312.2	½	158 650	2003.0	⅛	188 888	2839 2
⅜	98 5675	773.14	41	128.805	1320 3	⅝	159 043	2012 9	¼	189.281	2851.0
½	98.9602	779.31	⅛	129 198	1328.3	¾	159.436	2022 8	⅜	189 674	2862 9
⅝	99.3529	785.51	¼	129.591	1336.4	⅞	159 829	2032.8	½	190 066	2874 8
¾	99.7456	791.73	⅜	129 983	1341 5	51.	160 221	2042.8	⅝	190.459	2886.6
⅞	100.138	797.98	½	130 376	1352 7	⅛	160 614	2052.8	¾	190 852	2898.6
32.	100.531	804 25	⅝	130 769	1360 8	¼	161 007	2062.9	⅞	191 244	2910 5
⅛	100.924	810.54	¾	131.161	1369 0	⅜	161.399	2073 0	61.	191.637	2922.5
¼	101.316	816.86	⅞	131.554	1377 2	½	161.792	2083.1	⅛	192.030	2934.5
⅜	101.709	823.21	42.	131.947	1385 4	⅝	162.185	2093.2	¼	192 423	2946 5
½	102.102	829.58	⅛	132.340	1393 7	¾	162.577	2103.3	⅜	192 815	2958.5
⅝	102 494	835.97	¼	132 732	1402 0	⅞	162 970	2113.5	½	193 208	2970.6
¾	102 887	842.39	⅜	133 125	1410.3	52.	163.363	2123.7	⅝	193.601	2982.7
⅞	103.280	848 83	½	133.518	1418 6	⅛	163 756	2133 9	¾	193.993	2994 8
33.	103.673	855.30	⅝	133.910	1427.0	¼	164 148	2144.2	⅞	194.386	3006.9
⅛	104.065	861.79	¾	134.303	1435.4	⅜	164.541	2154.5	62.	194.779	3019.1
¼	104.458	868.31	⅞	134.696	1443 8	½	164 934	2164 8	⅛	195.171	3031.3
⅜	104.851	874.85	43.	135.088	1452.2	⅝	165.326	2175.1	¼	195 564	3043.5
½	105.243	881.41	⅛	135 481	1460.7	¾	165 719	2185.4	⅜	195 957	3055.7
⅝	105.636	888.00	¼	135.874	1469.1	⅞	166.112	2195.8	½	196.350	3068.0
¾	106.029	894.62	⅜	136.267	1477.6	53.	166 504	2206.2	⅝	196.742	3080.3
⅞	106.421	901.26	½	136 659	1486.2	⅛	166 897	2216 6	¾	197.135	3092.6
34.	106.814	907.92	⅝	137.052	1494.7	¼	167 290	2227.0	⅞	197.528	3104.9
⅛	107.207	914 61	¾	137.445	1503.3	⅜	167 683	2237.5	63.	197 920	3117.2
¼	107.600	921.32	⅞	137.837	1511.9	½	168.075	2248.0	⅛	198 313	3129.6
⅜	107.992	928.06	44.	138.230	1520.5	⅝	168.468	2258.5	¼	198.706	3142.0
½	108.385	934.82	⅛	138 623	1529.2	¾	168.861	2269 1	⅜	199.098	3154.5
⅝	108.778	941.61	¼	139.015	1537.9	⅞	169.253	2279.6	½	199.491	3166.9
¾	109.170	948.42	⅜	139 408	1546.6	54.	169.646	2290 2	⅝	199 884	3179.4
⅞	109.563	955.25	½	139 801	1555.3	⅛	170 039	2300.8	¾	200 277	3191.9
35.	109.956	962.11	⅝	140 194	1564.0	¼	170 431	2311.5	⅞	200.669	3204.4
⅛	110.348	969.00	¾	140 586	1572.8	⅜	170.824	2322.1	64.	201.062	3217.0
¼	110.741	975.91	⅞	140.979	1581.6	½	171.217	2332.8	⅛	201.455	3229.6
⅜	111.134	982.84	45.	141.372	1590.4	⅝	171 609	2343.5	¼	201.847	3242.2
½	111.527	989 80	⅛	141.764	1599.3	¾	172.002	2354.3	⅜	202.240	3254.8
⅝	111.919	996 78	¼	142.157	1608.2	⅞	172.395	2365.0	½	202.633	3267 5
¾	112.312	1003 8	⅜	142.550	1617.0	55.	172.788	2375.8	⅝	203.025	3280.1
⅞	112.705	1010.8	½	142.942	1626.0	⅛	173.180	2386.6	¾	203.418	3292.8
36.	113.097	1017 9	⅝	143.335	1634.9	¼	173.573	2397.5	⅞	203.811	3305.6
⅛	113.490	1025 0	¾	143.728	1643.9	⅜	173.966	2408.3	65.	204.204	3318.3
¼	113 883	1032 1	⅞	144.121	1652.9	½	174 358	2419.2	⅛	204.596	3331.1
⅜	114.275	1039.2	46.	144.513	1661.9	⅝	174.751	2430.1	¼	204.989	3343.9
½	114.668	1046.3	⅛	144.906	1670.9	¾	175.144	2441.1	⅜	205.382	3356.7
⅝	115 061	1053.5	¼	145.299	1680.0	⅞	175.536	2452.0	½	205.774	3369.6
¾	115.454	1060.7	⅜	145.691	1689.1	56.	175.929	2463.0	⅝	206.167	3382.4
⅞	115 846	1068.0	½	146.084	1698.2	⅛	176.322	2474 0	¾	206.560	3395.3
37.	116.239	1075.2	⅝	146.477	1707.4	¼	176.715	2485.0	⅞	206.952	3408.2
⅛	116.632	1082.5	¾	146.869	1716.5	⅜	177.107	2496.1	66.	207.345	3421.2
¼	117 024	1089.8	⅞	147.262	1725.7	½	177.500	2507.2	⅛	207.738	3434.2
⅜	117.417	1097.1	47.	147.655	1734.9	⅝	177.893	2518.3	¼	208.131	3447.2
½	117.810	1104.5	⅛	148.048	1744.2	¾	178.285	2529.4	⅜	208.523	3460.2
⅝	118.202	1111 8	¼	148.440	1753.5	⅞	178.678	2540.6	½	208.916	3473.2
¾	118.596	1119.2	⅜	148.833	1762.7	57.	179.071	2551.8	⅝	209.309	3486.3
⅞	118.988	1126.7	½	149.226	1772.1	⅛	179.463	2563.0	¾	209.701	3499.4

TABLE 1 OF CIRCLES.
Diameters in units and eighths, &c.

Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
1-64	.049087	.00019	3. 1/2	10.9956	9.6211	10 1/8	31.8086	80.516	19 1/4	60.4757	291.04
1-32	.098175	.00077	9-16	11.1919	9.9678	1/4	32.2013	82.516	3/8	60.8684	294.83
3-64	.147262	.00173	5/8	11.3883	10.321	3/8	32.5940	84.541	1/2	61.2611	298.65
1-16	.196350	.00307	11-16	11.5846	10.680	1/2	32.9867	86.590	5/8	61.6538	302.49
3-32	.294524	.00690	3/4	11.7810	11.045	5/8	33.3794	88.664	3/4	62.0465	306.35
1/8	.392699	.01227	13-16	11.9773	11.416	3/4	33.7721	90.763	7/8	62.4392	310.24
5-32	.490874	.01917	7/8	12.1737	11.793	7/8	34.1648	92.886	20.	62.8319	314.16
3-16	.589049	.02761	15-16	12.3700	12.177	11.	34.5575	95.033	1/8	63.2246	318.10
7-32	.687223	.03758	4.	12.5664	12.566	1/8	34.9502	97.205	1/4	63.6173	322.06
1/4	.785398	.04909	1-16	12.7627	12.962	1/4	35.3429	99.402	3/8	64.0100	326.05
9-32	.883573	.06213	1/8	12.9591	13.364	3/8	35.7356	101.62	1/2	64.4026	330.06
5-16	.981748	.07670	3-16	13.1554	13.772	1/2	36.1283	103.87	5/8	64.7953	334.10
11-32	1.07992	.09281	1/4	13.3518	14.186	5/8	36.5210	106.14	3/4	65.1880	338.16
3/8	1.17810	.11045	5-16	13.5481	14.607	3/4	36.9137	108.43	7/8	65.5807	342.25
13-32	1.27627	.12962	3/8	13.7445	15.033	7/8	37.3064	110.75	21.	65.9734	346.36
7-16	1.37445	.15033	7-16	13.9408	15.466	12.	37.6991	113.10	1/8	66.3661	350.50
15-32	1.47262	.17257	1/2	14.1372	15.904	1/8	38.0918	115.47	1/4	66.7588	354.66
1/2	1.57080	.19635	9-16	14.3335	16.349	1/4	38.4845	117.86	3/8	67.1515	358.84
17-32	1.66897	.22166	5/8	14.5299	16.800	3/8	38.8772	120.28	1/2	67.5442	363.05
9-16	1.76715	.24850	11-16	14.7262	17.257	1/2	39.2699	122.72	5/8	67.9369	367.28
19-32	1.86532	.27688	3/4	14.9226	17.721	5/8	39.6626	125.19	3/4	68.3296	371.54
5/8	1.96350	.30680	13-16	15.1189	18.190	3/4	40.0553	127.68	7/8	68.7223	375.83
21-32	2.06167	.33824	7/8	15.3153	18.665	7/8	40.4480	130.19	22	69.1150	380.13
11-16	2.15984	.37122	15-16	15.5116	19.147	13.	40.8407	132.73	1/8	69.5077	384.46
23-32	2.25802	.40574	5.	15.7080	19.635	1/8	41.2334	135.30	1/4	69.9004	388.82
3/4	2.35619	.44179	1-16	15.9043	20.129	1/4	41.6261	137.89	3/8	70.2931	393.20
25-32	2.45437	.47937	1/8	16.1007	20.629	3/8	42.0188	140.50	1/2	70.6858	397.61
13-16	2.55254	.51849	3-16	16.2970	21.135	1/2	42.4115	143.14	5/8	71.0785	402.04
27-32	2.65072	.55914	1/4	16.4934	21.648	5/8	42.8042	145.80	3/4	71.4712	406.49
7/8	2.74889	.60132	5-16	16.6897	22.166	3/4	43.1969	148.49	7/8	71.8639	410.97
29-32	2.84707	.64504	3/8	16.8861	22.691	7/8	43.5896	151.20	23	72.2566	415.48
15-16	2.94524	.69029	7-16	17.0824	23.221	14.	43.9823	153.94	1/8	72.6493	420.00
31-32	3.04342	.73708	1/2	17.2788	23.758	1/8	44.3750	156.70	1/4	73.0420	424.56
1.	3.14159	.78540	9-16	17.4751	24.301	1/4	44.7677	159.48	3/8	73.4347	429.13
1-16	3.33794	.88664	5/8	17.6715	24.850	3/8	45.1604	162.30	1/2	73.8274	433.74
1/8	3.53429	.99402	11-16	17.8678	25.406	1/2	45.5531	165.13	5/8	74.2201	438.36
3-16	3.73064	1.1075	3/4	18.0642	25.967	5/8	45.9458	167.99	3/4	74.6128	443.01
1/4	3.92699	1.2272	13-16	18.2605	26.535	3/4	46.3385	170.87	7/8	75.0055	447.69
5-16	4.12334	1.3530	7/8	18.4569	27.109	7/8	46.7312	173.78	24.	75.3982	452.39
3/8	4.31969	1.4849	15-16	18.6532	27.688	15.	47.1239	176.71	1/8	75.7909	457.11
7-16	4.51604	1.6230	6.	18.8496	28.274	1/8	47.5166	179.67	1/4	76.1836	461.86
1/2	4.71239	1.7671	1/8	19.2423	29.465	1/4	47.9093	182.65	3/8	76.5763	466.64
9-16	4.90874	1.9175	1/4	19.6350	30.680	3/8	48.3020	185.66	1/2	76.9690	471.44
5/8	5.10509	2.0739	3/8	20.0277	31.919	1/2	48.6947	188.69	5/8	77.3617	476.26
11-16	5.30144	2.2365	1/2	20.4204	33.183	5/8	49.0874	191.75	3/4	77.7544	481.11
3/4	5.49779	2.4053	5/8	20.8131	34.472	3/4	49.4801	194.83	7/8	78.1471	485.98
13-16	5.69414	2.5802	3/4	21.2058	35.785	7/8	49.8728	197.93	25	78.5398	490.87
7/8	5.89049	2.7612	7/8	21.5984	37.122	16.	50.2655	201.06	1/8	78.9325	495.79
15-16	6.08684	2.9483	7.	21.9911	38.485	1/8	50.6582	204.22	1/4	79.3252	500.74
2.	6.28319	3.1416	1/8	22.3838	39.871	1/4	51.0509	207.39	3/8	79.7179	505.71
1-16	6.47953	3.3410	1/4	22.7765	41.282	3/8	51.4436	210.60	1/2	80.1106	510.71
1/8	6.67588	3.5466	3/8	23.1692	42.718	1/2	51.8363	213.82	5/8	80.5033	515.72
3-16	6.87223	3.7583	1/2	23.5619	44.179	5/8	52.2290	217.08	3/4	80.8960	520.77
1/4	7.06858	3.9761	5/8	23.9546	45.664	3/4	52.6217	220.35	7/8	81.2887	525.84
5-16	7.26493	4.2000	3/4	24.3473	47.173	7/8	53.0144	223.65	26.	81.6814	530.93
3/8	7.46128	4.4301	7/8	24.7400	48.707	17.	53.4071	226.98	1/8	82.0741	536.05
7-16	7.65763	4.6664	8.	25.1327	50.265	1/8	53.7998	230.33	1/4	82.4668	541.19
1/2	7.85398	4.9087	1/8	25.5254	51.849	1/4	54.1925	233.71	3/8	82.8595	546.35
9-16	8.05033	5.1572	1/4	25.9181	53.456	3/8	54.5852	237.10	1/2	83.2522	551.55
5/8	8.24668	5.4119	3/8	26.3108	55.088	1/2	54.9779	240.53	5/8	83.6449	556.76
11-16	8.44303	5.6727	1/2	26.7035	56.745	5/8	55.3706	243.98	3/4	84.0376	562.00
3/4	8.63938	5.9396	5/8	27.0962	58.426	3/4	55.7633	247.45	7/8	84.4303	567.27
13-16	8.83573	6.2126	3/4	27.4889	60.132	7/8	56.1560	250.95	27.	84.8230	572.56
7/8	9.03208	6.4918	7/8	27.8816	61.862	18.	56.5487	254.47	1/8	85.2157	577.87
15-16	9.22843	6.7771	9.	28.2743	63.617	1/8	56.9414	258.02	1/4	85.6084	583.21
3.	9.42478	7.0686	1/8	28.6670	65.397	1/4	57.3341	261.59	3/8	86.0011	588.57
1-16	9.62113	7.3862	1/4	29.0597	67.201	3/8	57.7268	265.18	1/2	86.3938	593.96
1/8	9.81748	7.6699	3/8	29.4524	69.029	1/2	58.1195	268.80	5/8	86.7865	599.37
3-16	10.0138	7.9798	1/2	29.8451	70.882	5/8	58.5122	272.45	3/4	87.1792	604.81
1/4	10.2102	8.2958	5/8	30.2378	72.760	3/4	58.9049	276.12	7/8	87.5719	610.27
5-16	10.4065	8.6179	3/4	30.6305	74.662	7/8	59.2976	279.81	28.	87.9646	615.75
3/8	10.6029	8.9462	7/8	31.0232	76.589	19.	59.6903	283.53	1/8	88.3573	621.26
7-16	10.7992	9.2806	10.	31.4159	78.540	1/8	60.0830	287.27	1/4	88.7500	626.80

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam	Circumf.	Area.	Diam.	Circumf.	Area.
28⅜	89.1427	632.36	38.	119.381	1134.1	47⅝	149.618	1781.4	57¼	179.856	2574.1
½	89.5354	637.94	⅛	119.773	1141.6	¾	150.011	1790.8	⅜	180.249	2585.4
⅝	89.9281	643.55	¼	120.166	1149.1	⅞	150.404	1800.1	½	180.642	2596.7
¾	90.3208	649.18	⅜	120.559	1156.6	48	150.796	1809.6	⅝	181.034	2608.0
⅞	90.7135	654.84	½	120.951	1164.2	⅛	151.189	1819.0	¾	181.427	2619.4
29.	91.1062	660.52	⅝	121.344	1171.7	¼	151.582	1828.5	⅞	181.820	2630.7
⅛	91.4989	666.23	¾	121.737	1179.3	⅜	151.975	1837.9	58	182.212	2642.1
¼	91.8916	671.96	⅞	122.129	1186.9	½	152.367	1847.5	⅛	182.605	2653.5
⅜	92.2843	677.71	39.	122.522	1194.6	⅝	152.760	1857.0	¼	182.998	2664.9
½	92.6770	683.49	⅛	122.915	1202.3	¾	153.153	1866.5	⅜	183.390	2676.4
⅝	93.0697	689.30	¼	123.308	1210.0	⅞	153.545	1876.1	½	183.783	2687.8
¾	93.4624	695.13	⅜	123.700	1217.7	49.	153.938	1885.7	⅝	184.176	2699.3
⅞	93.8551	700.98	½	124.093	1225.4	⅛	154.331	1895.4	¾	184.569	2710.9
30.	94.2478	706.86	⅝	124.486	1233.2	¼	154.723	1905.0	⅞	184.961	2722.4
⅛	94.6405	712.76	¾	124.878	1241.0	⅜	155.116	1914.7	59	185.354	2734.0
¼	95.0332	718.69	⅞	125.271	1248.8	½	155.509	1924.4	⅛	185.747	2745.6
⅜	95.4259	724.64	40	125.664	1256.6	⅝	155.902	1934.2	¼	186.139	2757.2
½	95.8186	730.62	⅛	126.056	1264.5	¾	156.294	1943.9	⅜	186.532	2768.8
⅝	96.2113	736.62	¼	126.449	1272.4	⅞	156.687	1953.7	½	186.925	2780.5
¾	96.6040	742.64	⅜	126.842	1280.3	50.	157.080	1963.5	⅝	187.317	2792.2
⅞	96.9967	748.69	½	127.235	1288.2	⅛	157.472	1973.3	¾	187.710	2803.9
31.	97.3894	754.77	⅝	127.627	1296.2	¼	157.865	1983.2	⅞	188.103	2815.7
⅛	97.7821	760.87	¾	128.020	1304.2	⅜	158.258	1993.1	60	188.496	2827.4
¼	98.1748	766.99	⅞	128.413	1312.2	½	158.650	2003.0	⅛	188.888	2839.2
⅜	98.5675	773.14	41	128.805	1320.3	⅝	159.043	2012.9	¼	189.281	2851.0
½	98.9602	779.31	⅛	129.198	1328.3	¾	159.436	2022.8	⅜	189.674	2862.9
⅝	99.3529	785.51	¼	129.591	1336.4	⅞	159.829	2032.8	½	190.066	2874.8
¾	99.7456	791.73	⅜	129.983	1341.5	51.	160.221	2042.8	⅝	190.459	2886.6
⅞	100.138	797.98	½	130.376	1352.7	⅛	160.614	2052.8	¾	190.852	2898.6
32.	100.531	804.25	⅝	130.769	1360.8	¼	161.007	2062.9	⅞	191.244	2910.5
⅛	100.924	810.54	¾	131.161	1369.0	⅜	161.399	2073.0	61.	191.637	2922.5
¼	101.316	816.86	⅞	131.554	1377.2	½	161.792	2083.1	⅛	192.030	2934.5
⅜	101.709	823.21	42.	131.947	1385.4	⅝	162.185	2093.2	¼	192.423	2946.5
½	102.102	829.58	⅛	132.340	1393.7	¾	162.577	2103.3	⅜	192.815	2958.5
⅝	102.494	835.97	¼	132.732	1402.0	⅞	162.970	2113.5	½	193.208	2970.6
¾	102.887	842.39	⅜	133.125	1410.3	52.	163.363	2123.7	⅝	193.601	2982.7
⅞	103.280	848.83	½	133.518	1418.6	⅛	163.756	2133.9	¾	193.993	2994.8
33.	103.673	855.30	⅝	133.910	1427.0	¼	164.148	2144.2	⅞	194.386	3006.9
⅛	104.065	861.79	¾	134.303	1435.4	⅜	164.541	2154.5	62.	194.779	3019.1
¼	104.458	868.31	⅞	134.696	1443.8	½	164.934	2164.8	⅛	195.171	3031.3
⅜	104.851	874.85	43.	135.088	1452.2	⅝	165.326	2175.1	¼	195.564	3043.5
½	105.243	881.41	⅛	135.481	1460.7	¾	165.719	2185.4	⅜	195.957	3055.7
⅝	105.636	888.00	¼	135.874	1469.1	⅞	166.112	2195.8	½	196.350	3068.0
¾	106.029	894.62	⅜	136.267	1477.6	53.	166.504	2206.2	⅝	196.742	3080.3
⅞	106.421	901.26	½	136.659	1486.2	⅛	166.897	2216.6	¾	197.135	3092.6
34.	106.814	907.92	⅝	137.052	1494.7	¼	167.290	2227.0	⅞	197.528	3104.9
⅛	107.207	914.61	¾	137.445	1503.3	⅜	167.683	2237.5	63.	197.920	3117.2
¼	107.600	921.32	⅞	137.837	1511.9	½	168.075	2248.0	⅛	198.313	3129.6
⅜	107.992	928.06	44.	138.230	1520.5	⅝	168.468	2258.5	¼	198.706	3142.0
½	108.385	934.82	⅛	138.623	1529.2	¾	168.861	2269.1	⅜	199.098	3154.5
⅝	108.778	941.61	¼	139.015	1537.9	⅞	169.253	2279.6	½	199.491	3166.9
¾	109.170	948.42	⅜	139.408	1546.6	54.	169.646	2290.2	⅝	199.884	3179.4
⅞	109.563	955.25	½	139.801	1555.3	⅛	170.039	2300.8	¾	200.277	3191.9
35.	109.956	962.11	⅝	140.194	1564.0	¼	170.431	2311.5	⅞	200.669	3204.4
⅛	110.348	969.00	¾	140.586	1572.8	⅜	170.824	2322.1	64.	201.062	3217.0
¼	110.741	975.91	⅞	140.979	1581.6	½	171.217	2332.8	⅛	201.455	3229.6
⅜	111.134	982.84	45.	141.372	1590.4	⅝	171.609	2343.5	¼	201.847	3242.2
½	111.527	989.80	⅛	141.764	1599.3	¾	172.002	2354.3	⅜	202.240	3254.8
⅝	111.919	996.78	¼	142.157	1608.2	⅞	172.395	2365.0	½	202.633	3267.5
¾	112.312	1003.8	⅜	142.550	1617.0	55.	172.788	2375.8	⅝	203.025	3280.1
⅞	112.705	1010.8	½	142.942	1626.0	⅛	173.180	2386.6	¾	203.418	3292.8
36.	113.097	1017.9	⅝	143.335	1634.9	¼	173.573	2397.5	⅞	203.811	3305.6
⅛	113.490	1025.0	¾	143.728	1643.9	⅜	173.966	2408.3	65.	204.204	3318.3
¼	113.883	1032.1	⅞	144.121	1652.9	½	174.358	2419.2	⅛	204.596	3331.1
⅜	114.275	1039.2	46.	144.513	1661.9	⅝	174.751	2430.1	¼	204.989	3343.9
½	114.668	1046.3	⅛	144.906	1670.9	¾	175.144	2441.1	⅜	205.382	3356.7
⅝	115.061	1053.5	¼	145.299	1680.0	⅞	175.536	2452.0	½	205.774	3369.6
¾	115.454	1060.7	⅜	145.691	1689.1	56.	175.929	2463.0	⅝	206.167	3382.4
⅞	115.846	1068.0	½	146.084	1698.2	⅛	176.822	2474.0	¾	206.560	3395.3
37.	116.239	1075.2	⅝	146.477	1707.4	¼	176.715	2485.0	⅞	206.952	3408.2
⅛	116.632	1082.5	¾	146.869	1716.5	⅜	177.107	2496.1	66.	207.345	3421.2
¼	117.024	1089.8	⅞	147.262	1725.7	½	177.500	2507.2	⅛	207.738	3434.2
⅜	117.417	1097.1	47.	147.655	1734.9	⅝	177.893	2518.3	¼	208.131	3447.2
½	117.810	1104.5	⅛	148.048	1744.2	¾	178.285	2529.4	⅜	208.523	3460.2
⅝	118.202	1111.8	¼	148.440	1753.5	⅞	178.678	2540.6	½	208.916	3473.2
¾	118.596	1119.2	⅜	148.833	1762.7	57.	179.071	2551.8	⅝	209.309	3486.3
⅞	118.988	1126.7	½	149.226	1772.1	⅛	179.463	2563.0	¾	209.701	3499.4

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
18.7	[illegible]	274.6459	24.9	78.22566	486.9547	31.1	97.70353	759.6450
.8	[illegible]	277.5911	25.0	78.53982	490.8739	.2	98.01769	764.5380
.9	59.37610	280.5521	.1	78.85398	494.8087	.3	98.33185	769.4467
19.0	59.69026	283.5287	.2	79.16813	498.7592	.4	98.64601	774.3712
.1	[illegible]	286.5211	.3	79.48229	502.7255	.5	98.96017	779.3113
.2	[illegible]	289.5292	.4	79.79645	506.7075	.6	99.27433	784.2672
.3	60.63274	292.5530	.5	80.11061	510.7052	.7	99.58849	789.2388
	60.94690	295.5925	.6	80.42477	514.7185	.8	99.90265	794.2260
	61.26106	298.6477	.7	80.73893	518.7476	.9	100.2168	799.2290
	61.57522	301.7186	.8	81.05309	522.7924	32.0	100.5310	804.2477
.7	61.88938	304.8052	.9	81.36725	526.8529	.1	100.8451	809.2821
.8	62.20353	307.9075	26.0	81.68141	530.9292	.2	101.1593	814.3322
.9	62.51769	311.0255	.1	81.99557	535.0211	.3	101.4734	819.3980
20.0	62.83185	314.1593	.2	82.30973	539.1287	.4	101.7876	824.4796
.1	63.14601	317.3087	.3	82.62389	543.2521	.5	102.1018	829.5768
.2	63.46017	320.4739	.4	82.93805	547.3911	.6	102.4159	834.6898
.3	63.77433	323.6547	.5	83.25221	551.5459	.7	102.7301	839.8184
.4	64.08849	326.8513	.6	83.56636	555.7163	.8	103.0442	844.9628
.5	64.40265	330.0636	.7	83.88052	559.9025	.9	103.3584	850.1228
.6	64.71681	333.2916	.8	84.19468	564.1044	33.0	103.6726	855.2986
.7	65.03097	336.5353	.9	84.50884	568.3220	.1	103.9867	860.4901
.8	65.34513	339.7947	27.0	84.82300	572.5553	.2	104.3009	865.6973
.9	65.65929	343.0698	.1	85.13716	576.8043	.3	104.6150	870.9202
21.0	65.97345	346.3606	.2	85.45132	581.0690	.4	104.9292	876.1588
.1	66.28760	349.6671	.3	85.76548	585.3494	.5	105.2434	881.4131
.2	66.60176	352.9894	.4	86.07964	589.6455	.6	105.5575	886.6831
.3	66.91592	356.3273	.5	86.39380	593.9574	.7	105.8717	891.9688
.4	67.23008	359.6809	.6	86.70796	598.2849	.8	106.1858	897.2703
.5	67.54424	363.0503	.7	87.02212	602.6282	.9	106.5000	902.5874
.6	67.85840	366.4354	.8	87.33628	606.9871	34.0	106.8142	907.9203
.7	68.17256	369.8361	.9	87.65044	611.3618	.1	107.1283	913.2688
.8	68.48672	373.2526	28.0	87.96459	615.7522	.2	107.4425	918.6331
.9	68.80088	376.6848	.1	88.27875	620.1582	.3	107.7566	924.0131
22.0	69.11504	380.1327	.2	88.59291	624.5800	.4	108.0708	929.4088
.1	69.42920	383.5963	.3	88.90707	629.0175	.5	108.3849	934.8202
.2	69.74336	387.0756	.4	89.22123	633.4707	.6	108.6991	940.2473
.3	70.05752	390.5707	.5	89.53539	637.9397	.7	109.0133	945.6901
.4	70.37168	394.0814	.6	89.84955	642.4243	.8	109.3274	951.1486
.5	70.68583	397.6078	.7	90.16371	646.9246	.9	109.6416	956.6228
.6	70.99999	401.1500	.8	90.47787	651.4407	35.0	109.9557	962.1128
.7	71.31415	404.7078	.9	90.79203	655.9724	.1	110.2699	967.6184
.8	71.62831	408.2814	29.0	91.10619	660.5199	.2	110.5841	973.1397
.9	71.94247	411.8707	.1	91.42035	665.0830	.3	110.8982	978.6768
23.0	72.25663	415.4756	.2	91.73451	669.6619	.4	111.2124	984.2296
.1	72.57079	419.0963	.3	92.04866	674.2565	.5	111.5265	989.7980
.2	72.88495	422.7327	.4	92.36282	678.8668	.6	111.8407	995.3822
.3	73.19911	426.3848	.5	92.67698	683.4928	.7	112.1549	1000.9821
.4	73.51327	430.0526	.6	92.99114	688.1345	.8	112.4690	1006.5977
.5	73.82743	433.7361	.7	93.30530	692.7919	.9	112.7832	1012.2290
.6	74.14159	437.4354	.8	93.61946	697.4650	36.0	113.0973	1017.8760
.7	74.45575	441.1503	.9	93.93362	702.1538	.1	113.4115	1023.5387
.8	74.76991	444.8809	30.0	94.24778	706.8583	.2	113.7257	1029.2172
.9	75.08406	448.6273	.1	94.56194	711.5786	.3	114.0398	1034.9113
24.0	75.39822	452.3893	.2	94.87610	716.3145	.4	114.3540	1040.6212
.1	75.71238	456.1671	.3	95.19026	721.0662	.5	114.6681	1046.3467
.2	76.02654	459.9606	.4	95.50442	725.8336	.6	114.9823	1052.0880
.3	76.34070	463.7698	.5	95.81858	730.6166	.7	115.2965	1057.8449
.4	76.65486	467.5947	.6	96.13274	735.4154	.8	115.6106	1063.6176
.5	76.96902	471.4352	.7	96.44689	740.2299	.9	115.9248	1069.4060
.6	77.28318	475.2916	.8	96.76105	745.0601	37.0	116.2389	1075.2101
.7	77.59734	479.1636	.9	97.07521	749.9060	.1	116.5531	1081.0299
.8	77.91150	483.0513	31.0	97.38937	754.7676	.2	116.8672	1086.8654

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
37.3	117.1814	1092.7166	43.5	136.6593	1486.1697	49.7	156.1372	1940.0041
.4	117.4956	1098.5835	.6	136.9734	1493.0105	.8	156.4513	1947.8189
.5	117.8097	1104.4662	.7	137.2876	1499.8670	.9	156.7655	1955.6493
.6	118.1239	1110.3645	.8	137.6018	1506.7393	50.0	157.0796	1963.4954
.7	118.4380	1116.2786	.9	137.9159	1513.6272	.1	157.3938	1971.3572
.8	118.7522	1122.2083	44.0	138.2301	1520.5308	.2	157.7080	1979.2348
.9	119.0664	1128.1538	.1	138.5442	1527.4502	.3	158.0221	1987.1280
38.0	119.3805	1134.1149	.2	138.8584	1534.3853	.4	158.3363	1995.0370
.1	119.6947	1140.0918	.3	139.1726	1541.3360	.5	158.6504	2002.9617
.2	120.0088	1146.0844	.4	139.4867	1548.3025	.6	158.9646	2010.9020
.3	120.3230	1152.0927	.5	139.8009	1555.2847	.7	159.2787	2018.8581
.4	120.6372	1158.1167	.6	140.1150	1562.2826	.8	159.5929	2026.8299
.5	120.9513	1164.1564	.7	140.4292	1569.2962	.9	159.9071	2034.8174
.6	121.2655	1170.2118	.8	140.7434	1576.3255	51.0	160.2212	2042.8206
.7	121.5796	1176.2830	.9	141.0575	1583.3706	.1	160.5354	2050.8395
.8	121.8938	1182.3698	45.0	141.3717	1590.4313	.2	160.8495	2058.8742
.9	122.2080	1188.4724	.1	141.6858	1597.5077	.3	161.1637	2066.9245
39.0	122.5221	1194.5906	.2	142.0000	1604.5999	.4	161.4779	2074.9905
.1	122.8363	1200.7246	.3	142.3141	1611.7077	.5	161.7920	2083.0723
.2	123.1504	1206.8742	.4	142.6283	1618.8313	.6	162.1062	2091.1697
.3	123.4646	1213.0396	.5	142.9425	1625.9705	.7	162.4203	2099.2829
.4	123.7788	1219.2207	.6	143.2566	1633.1255	.8	162.7345	2107.4118
.5	124.0929	1225.4175	.7	143.5708	1640.2962	.9	163.0487	2115.5563
.6	124.4071	1231.6300	.8	143.8849	1647.4826	52.0	163.3628	2123.7166
.7	124.7212	1237.8582	.9	144.1991	1654.6847	.1	163.6770	2131.8926
.8	125.0354	1244.1021	46.0	144.5133	1661.9025	.2	163.9911	2140.0843
.9	125.3495	1250.3617	.1	144.8274	1669.1360	.3	164.3053	2148.2917
40.0	125.6637	1256.6371	.2	145.1416	1676.3853	.4	164.6195	2156.5149
.1	125.9779	1262.9281	.3	145.4557	1683.6502	.5	164.9336	2164.7537
.2	126.2920	1269.2348	.4	145.7699	1690.9308	.6	165.2478	2173.0082
.3	126.6062	1275.5573	.5	146.0841	1698.2272	.7	165.5619	2181.2785
.4	126.9203	1281.8955	.6	146.3982	1705.5392	.8	165.8761	2189.5644
.5	127.2345	1288.2493	.7	146.7124	1712.8670	.9	166.1903	2197.8661
.6	127.5487	1294.6189	.8	147.0265	1720.2105	53.0	166.5044	2206.1834
.7	127.8628	1301.0042	.9	147.3407	1727.5697	.1	166.8186	2214.5165
.8	128.1770	1307.4052	47.0	147.6549	1734.9445	.2	167.1327	2222.8653
.9	128.4911	1313.8219	.1	147.9690	1742.3351	.3	167.4469	2231.2298
41.0	128.8053	1320.2543	.2	148.2832	1749.7414	.4	167.7610	2239.6100
.1	129.1195	1326.7024	.3	148.5973	1757.1635	.5	168.0752	2248.0059
.2	129.4336	1333.1663	.4	148.9115	1764.6012	.6	168.3894	2256.4175
.3	129.7478	1339.6458	.5	149.2257	1772.0546	.7	168.7035	2264.8448
.4	130.0619	1346.1410	.6	149.5398	1779.5237	.8	169.0177	2273.2879
.5	130.3761	1352.6520	.7	149.8540	1787.0086	.9	169.3318	2281.7466
.6	130.6903	1359.1786	.8	150.1681	1794.5091	54.0	169.6460	2290.2210
.7	131.0044	1365.7210	.9	150.4823	1802.0254	.1	169.9602	2298.7112
.8	131.3186	1372.2791	48.0	150.7964	1809.5574	.2	170.2743	2307.2171
.9	131.6327	1378.8529	.1	151.1106	1817.1050	.3	170.5885	2315.7386
42.0	131.9469	1385.4424	.2	151.4248	1824.6684	.4	170.9026	2324.2759
.1	132.2611	1392.0476	.3	151.7389	1832.2475	.5	171.2168	2332.8289
.2	132.5752	1398.6685	.4	152.0531	1839.8423	.6	171.5310	2341.3976
.3	132.8894	1405.3051	.5	152.3672	1847.4528	.7	171.8451	2349.9820
.4	133.2035	1411.9574	.6	152.6814	1855.0790	.8	172.1593	2358.5821
.5	133.5177	1418.6254	.7	152.9956	1862.7210	.9	172.4734	2367.1979
.6	133.8318	1425.3092	.8	153.3097	1870.3786	55.0	172.7876	2375.8294
.7	134.1460	1432.0086	.9	153.6239	1878.0519	.1	173.1018	2384.4767
.8	134.4602	1438.7238	49.0	153.9380	1885.7410	.2	173.4159	2393.1396
.9	134.7743	1445.4546	.1	154.2522	1893.4457	.3	173.7301	2401.8183
43.0	135.0885	1452.2012	.2	154.5664	1901.1662	.4	174.0442	2410.5126
.1	135.4026	1458.9635	.3	154.8805	1908.9024	.5	174.3584	2419.2227
.2	135.7168	1465.7415	.4	155.1947	1916.6543	.6	174.6726	2427.9485
.3	136.0310	1472.5352	.5	155.5088	1924.4218	.7	174.9867	2436.6899
.4	136.3451	1479.3446	.6	155.8230	1932.2051	.8	175.3009	2445.4471

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
55.9	175.6150	2454.2200	62.1	195.0929	3028.8173	68.3	214.5708	3663.7960
56.0	175.9292	2463.0086	.2	195.4071	3038.5798	.4	214.8849	3674.5324
.1	176.2433	2471.8130	.3	195.7212	3048.3580	.5	215.1991	3685.2845
.2	176.5575	2480.6330	.4	196.0354	3058.1520	.6	215.5133	3696.0523
.3	176.8717	2489.4687	.5	196.3495	3067.9616	.7	215.8274	3706.8359
.4	177.1858	2498.3201	.6	196.6637	3077.7869	.8	216.1416	3717.6351
.5	177.5000	2507.1873	.7	196.9779	3087.6279	.9	216.4557	3728.4500
.6	177.8141	2516.0701	.8	197.2920	3097.4847	69.0	216.7699	3739.2807
.7	178.1283	2524.9687	.9	197.6062	3107.3571	.1	217.0841	3750.1270
.8	178.4425	2533.8830	63.0	197.9203	3117.2453	.2	217.3982	3760.9891
.9	178.7566	2542.8129	.1	198.2345	3127.1492	.3	217.7124	3771.8668
57.0	179.0708	2551.7586	.2	198.5487	3137.0688	.4	218.0265	3782.7603
.1	179.3849	2560.7200	.3	198.8628	3147.0040	.5	218.3407	3793.6695
.2	179.6991	2569.6971	.4	199.1770	3156.9550	.6	218.6548	3804.5944
.3	180.0133	2578.6899	.5	199.4911	3166.9217	.7	218.9690	3815.5350
.4	180.3274	2587.6985	.6	199.8053	3176.9042	.8	219.2832	3826.4913
.5	180.6416	2596.7227	.7	200.1195	3186.9023	.9	219.5973	3837.4633
.6	180.9557	2605.7626	.8	200.4336	3196.9161	70.0	219.9115	3848.4510
.7	181.2699	2614.8183	.9	200.7478	3206.9456	.1	220.2256	3859.4544
.8	181.5841	2623.8896	64.0	201.0619	3216.9909	.2	220.5398	3870.4736
9	181.8982	2632.9767	.1	201.3761	3227.0518	.3	220.8540	3881.5084
58.0	182.2124	2642.0794	.2	201.6902	3237.1285	.4	221.1681	3892.5590
.1	182.5265	2651.1979	.3	202.0044	3247.2209	.5	221.4823	3903.6252
.2	182.8407	2660.3321	.4	202.3186	3257.3289	.6	221.7964	3914.7072
.3	183.1549	2669.4820	.5	202.6327	3267.4527	.7	222.1106	3925.8049
.4	183.4690	2678.6476	.6	202.9469	3277.5922	.8	222.4248	3936.9182
5	183.7832	2687.8289	.7	203.2610	3287.7474	.9	222.7389	3948.0473
.6	184.0973	2697.0259	.8	203.5752	3297.9183	71.0	223.0531	3959.1921
.7	184.4115	2706.2386	.9	203.8894	3308.1049	.1	223.3672	3970.3526
.8	184.7256	2715.4670	65.0	204.2035	3318.3072	.2	223.6814	3981.5289
.9	185.0398	2724.7112	.1	204.5177	3328.5253	.3	223.9956	3992.7208
59.0	185.3540	2733.9710	.2	204.8318	3338.7590	.4	224.3097	4003.9284
.1	185.6681	2743.2466	.3	205.1460	3349.0085	.5	224.6239	4015.1518
.2	185.9823	2752.5378	.4	205.4602	3359.2736	.6	224.9380	4026.3908
.3	186.2964	2761.8448	.5	205.7743	3369.5545	.7	225.2522	4037.6456
.4	186.6106	2771.1675	.6	206.0885	3379.8510	.8	225.5664	4048.9160
.5	186.9248	2780.5058	.7	206.4026	3390.1633	.9	225.8805	4060.2022
.6	187.2389	2789.8599	.8	206.7168	3400.4913	72.0	226.1947	4071.5041
.7	187.5531	2799.2297	.9	207.0310	3410.8350	.1	226.5088	4082.8217
.8	187.8672	2808.6152	66.0	207.3451	3421.1944	.2	226.8230	4094.1550
.9	188.1814	2818.0165	.1	207.6593	3431.5695	.3	227.1871	4105.5040
60.0	188.4956	2827.4334	.2	207.9734	3441.9603	.4	227.4513	4116.8687
.1	188.8097	2836.8660	.3	208.2876	3452.3669	.5	227.7655	4128.2491
.2	189.1239	2846.3144	.4	208.6018	3462.7891	.6	228.0796	4139.6452
.3	189.4380	2855.7784	.5	208.9159	3473.2270	.7	228.3938	4151.0571
.4	189.7522	2865.2582	.6	209.2301	3483.6807	.8	228.7079	4162.4846
.5	190.0664	2874.7536	.7	209.5442	3494.1500	.9	229.0221	4173.9279
6	190.3805	2884.2648	.8	209.8584	3504.6351	73.0	229.3363	4185.3868
.7	190.6947	2893.7917	.9	210.1725	3515.1359	.1	229.6504	4196.8615
.8	191.0088	2903.3343	67.0	210.4867	3525.6524	.2	229.9646	4208.3519
.9	191.3230	2912.8026	.1	210.8009	3536.1845	.3	230.2787	4219.8579
61.0	191.6372	2922.4666	.2	211.1150	3546.7324	.4	230.5929	4231.3797
.1	191.9513	2932.0563	.3	211.4292	3557.2960	.5	230.9071	4242.9172
.2	192.2655	2941.6617	.4	211.7433	3567.8754	.6	231.2212	4254.4704
.3	192.5796	2951.2828	5	212.0575	3578.4704	.7	231.5354	4266.0394
.4	192.8938	2960.9197	6	212.3717	3589.0811	.8	231.8495	4277.6240
.5	193.2079	2970.5722	7	212.6858	3599.7075	.9	232.1637	4289.2243
.6	193.5221	2980.2405	8	213.0000	3610.3497	74.0	232.4779	4300.8403
.7	193.8363	2989.9244	.9	213.3141	3621.0075	.1	232.7920	4312.4721
.8	194.1504	2999.6241	68.0	213.6283	3631.6811	.2	233.1062	4324.1195
.9	194.4646	3009.3395	.1	213.9425	3642.3704	.3	233.4203	4335.7827
62.0	194.7787	3019.0705	.2	214.2566	3653.0754	.4	233.7345	4347.4616

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
74.5	234.0487	4359.1562	80.7	253.5265	5114.8977	86.9	273.0044	5931.0206
.6	234.3628	4370.8664	.8	253.8407	5127.5819	87.0	273.3186	5944.6787
.7	234 6770	4382.5924	.9	254.1548	5140.2818	.1	273.6327	5958.3525
.8	234.9911	4394.3341	81.0	254 4690	5152.9974	.2	273.9469	5972 0420
.9	235.3053	4406.0916	.1	254.7832	5165.7287	.3	274.2610	5985.7472
75.0	235.6194	4417.8647	.2	255.0973	5178.4757	.4	274.5752	5999.4681
.1	235.9336	4429.6535	.3	255.4115	5191.2384	.5	274.8894	6013.2047
.2	236.2478	4441.4580	.4	255.7256	5204.0168	.6	275.2035	6026 9570
.3	236.5619	4453.2783	.5	256.0398	5216.8110	.7	275.5177	6040.7250
.4	236.8761	4465.1142	.6	256.3540	5229 6208	.8	275.8318	6054.5088
.5	237.1902	4476.9659	.7	256.6681	5242.4463	.9	276.1460	6068.3082
.6	237.5044	4488.8332	.8	256.9823	5255.2876	88.0	276.4602	6082 1234
.7	237.8186	4500.7163	.9	257.2964	5268.1446	.1	276.7743	6095.9542
.8	238.1327	4512.6151	82.0	257.6106	5281.0173	.2	277.0885	6109 8008
.9	238.4469	4524.5296	.1	257.9248	5293.9056	.3	277.4026	6123.6631
76.0	238.7610	4536.4598	.2	258 2389	5306 8097	.4	277.7168	6137.5411
.1	239.0752	4548.4057	.3	258.5531	5319.7295	.5	278.0309	6151.4348
.2	239.3894	4560.3673	.4	258.8672	5332.6650	.6	278.3451	6165.3442
.3	239.7035	4572.3446	.5	259.1814	5345.6162	.7	278 6593	6179.2693
.4	240.0177	4584.3377	.6	259.4956	5358.5832	.8	278.9734	6193 2101
.5	240.3318	4596.3464	.7	259 8097	5371.5658	.9	279.2876	6207.1666
.6	240 6460	4608.3708	.8	260.1239	5384.5641	89.0	279 6017	6221.1389
.7	240.9602	4620.4110	.9	260.4380	5397.5782	.1	279 9159	6235.1268
.8	241.2743	4632.4669	83.0	260.7522	5410 6079	.2	280 2301	6249.1304
.9	241.5885	4644.5384	.1	261.0663	5423.6534	.3	280.5442	6263.1498
77.0	241.9026	4656.6257	.2	261.3805	5436 7146	.4	280.8584	6277.1849
.1	242.2168	4668.7287	.3	261.6947	5449.7915	.5	281.1725	6291 2356
.2	242.5310	4680.8474	.4	262.0088	5462.8840	.6	281.4867	6305.3021
.3	242.8451	4692.9818	.5	262.3230	5475.9923	.7	281.8009	6319 3843
.4	243.1593	4705.1319	.6	262.6371	5489.1163	.8	282.1150	6333.4822
.5	243.4734	4717.2977	.7	262.9513	5502.2561	.9	282.4292	6347.5958
.6	243.7876	4729.4792	.8	263.2655	5515.4115	90.0	282.7433	6361.7251
.7	244.1017	4741.6765	.9	263.5796	5528.5826	.1	283.0575	6375.8701
.8	244.4159	4753.8894	84.0	263.8938	5541.7694	.2	283.3717	6390.0309
.9	244.7301	4766.1181	.1	264.2079	5554.9720	.3	283.6858	6404.2073
78.0	245.0442	4778.3624	.2	264.5221	5568.1902	.4	284.0000	6418.3995
.1	245.3584	4790.6225	.3	264.8363	5581.4242	.5	284.3141	6432.6073
.2	245.6725	4802.8983	.4	265.1504	5594.6739	.6	284.6283	6446.8309
.3	245.9867	4815.1897	.5	265.4646	5607.9392	.7	284.9425	6461.0701
.4	246.3009	4827.4969	.6	265.7787	5621.2203	.8	285.2566	6475.3251
.5	246.6150	4839.8198	.7	266.0929	5634.5171	.9	285.5708	6489.5958
.6	246.9292	4852.1584	.8	266.4071	5647.8296	91.0	285.8849	6503.8822
.7	247.2433	4864.5128	.9	266.7212	5661.1578	.1	286.1991	6518.1843
.8	247.5575	4876.8828	85.0	267 0354	5674.5017	.2	286.5133	6532.5021
.9	247.8717	4889.2685	.1	267 3495	5687.8614	.3	286.8274	6546.8356
79.0	248.1858	4901.6699	.2	267 6637	5701.2367	.4	287.1416	6561.1848
.1	248.5000	4914.0871	.3	267 9779	5714.6277	.5	287.4557	6575.5498
.2	248.8141	4926.5199	.4	268.2920	5728.0345	.6	287.7699	6589.9804
.3	249.1283	4938.9685	.5	268.6062	5741.4569	.7	288.0840	6604.3268
.4	249.4425	4951.4328	.6	268.9203	5754.8951	.8	288.3982	6618.7388
.5	249.7566	4963.9127	.7	269.2345	5768.3490	.9	288.7124	6633.1666
.6	250.0708	4976.4084	.8	269.5486	5781.8185	92.0	289.0265	6647.6101
.7	250.3849	4988.9198	.9	269.8628	5795.3038	.1	289.3407	6662.0692
.8	250.6991	5001.4469	86.0	270.1770	5808.8048	.2	289.6548	6676.5441
.9	251.0133	5013.9897	.1	270.4911	5822.3215	.3	289.9690	6691.0347
80.0	251.3274	5026.5482	.2	270.8053	5835.8539	.4	290.2832	6705.5410
.1	251.6416	5039.1225	.3	271.1194	5849.4020	.5	290.5973	6720.0630
.2	251.9557	5051.7124	.4	271.4336	5862.9659	.6	290.9115	6734.6008
.3	252.2699	5064.3180	.5	271.7478	5876.5454	.7	291.2256	6749.1542
.4	252.5840	5076.9394	.6	272.0619	5890.1407	.8	291.5398	6763.7233
.5	252.8982	5089.5764	.7	272.3761	5903.7516	.9	291.8540	6778.3082
.6	253.2124	5102.2292	.8	272.6902	5917.3783	93.0	292.1681	6792.9087

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
93.1	292.4823	6807.5250	95.5	300.0221	7163.0276	97.8	307.2478	7512.2078
.2	292.7964	6822.1569	.6	300.3363	7178.0366	.9	307.5619	7527.5780
.3	293.1106	6836.8046	.7	300.6504	7193.0612	98.0	307.8761	7542.9640
.4	293.4248	6851.4680	.8	300.9646	7208.1016	.1	308.1902	7558.3656
.5	293.7389	6866.1471	.9	301.2787	7223.1577	.2	308.5044	7573.7830
.6	294.0531	6880.8419	96.0	301 5929	7238.2295	.3	308.8186	7589.2161
.7	294.3672	6895.5524	.1	301.9071	7253.3170	.4	309.1327	7604.6648
.8	294.6814	6910.2786	.2	302.2212	7268.4202	.5	309.4469	7620.1293
.9	294.9956	6925.0205	.3	302.5354	7283.5391	.6	309.7610	7635.6095
94.0	295.3097	6939.7782	.4	302.8495	7298.6737	.7	310.0752	7651.1054
.1	295.6239	6954.5515	.5	303.1637	7313.8240	.8	310.3894	7666.6170
.2	295.9380	6969.3406	.6	303 4779	7328.9901	.9	310.7035	7682.1444
.3	296.2522	6984 1453	.7	303.7920	7344.1718	99.0	311.0177	7697.6874
.4	296 5663	6998.9658	.8	304.1062	7359.3693	.1	311.3318	7713.2461
.5	296.8805	7013.8019	.9	304.4203	7374.5824	.2	311.6460	7728.8206
.6	297.1947	7028.6538	97 0	304.7345	7389.8113	.3	311.9602	7744.4107
.7	297.5088	7043.5214	.1	305.0486	7405.0559	.4	312.2743	7760.0166
.8	297 8230	7058.4047	.2	305.3628	7420.3162	.5	312.5885	7775.6382
.9	298.1371	7073.3037	.3	305.6770	7435.5922	.6	312.9026	7791.2754
95.0	298.4513	7088.2184	.4	305.9911	7450.8839	.7	313.2168	7806.9284
.1	298.7655	7103.1488	.5	306.3053	7466.1913	.8	313.5309	7822.5971
.2	299.0796	7118.0950	.6	306.6194	7481.5144	.9	313.8451	7838.2815
.3	299.3938	7133.0568	.7	306.9336	7496.8532	100.0	314.1593	7853.9816
.4	299.7079	7148.0343						

Circumferences when the diameter has more than one place of decimals.

Diam	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.
.1	.314159	.01	.031416	.001	.003142	.0001	.000314	.00001	.000031
.2	.628319	.02	.062832	.002	.006283	.0002	.000628	.00002	.000063
.3	.942478	.03	.094248	.003	.009425	.0003	.000942	.00003	.000094
.4	1.256637	.04	.125664	.004	.012566	.0004	.001257	.00004	.000126
.5	1 570796	.05	.157080	.005	.015708	.0005	.001571	.00005	.000157
.6	1.884956	.06	.188496	.006	.018850	.0006	.001885	.00006	.000188
.7	2.199115	.07	.219911	.007	.021991	.0007	.002199	.00007	.000220
.8	2 513274	.08	.251327	.008	.025133	.0008	.002513	.00008	.000251
.9	2.827433	.09	.282743	.009	.028274	.0009	.002827	.00009	.000283

Examples.

Diameter = 3.12699
Circumference = Sum of

Circ for dia of	3.1	=	9.738937
"	.02	=	.062832
"	.006	=	.018850
"	.0009	=	.002827
"	.00009	=	.000283
			9.823729

Circumfce = 9.823729
Diameter = Sum of

Dia for circ of	9.738937	=	3.1
	.084792		
"	.062832	=	.02
	.021960		
"	.018850	=	.006
	.003110		
"	.002827	=	.0009
	.000283		
"	.000283	=	.00009
			3.12699

TABLE 3 OF CIRCLES.

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.
			5 0	15.70796	19.63495	10 0	31.41593	78.53982
0 1	.261799	.005454	1	15.96976	20.29491	1	31.67773	79.85427
2	.523599	.021817	2	16.23156	20.96577	2	31.93953	81.17963
3	.785398	.049087	3	16.49336	21.64754	3	32.20132	82.51589
4	1.047198	.087266	4	16.75516	22.34021	4	32.46312	83.86307
5	1.308997	.136354	5	17.01696	23.04380	5	32.72492	85.22115
6	1.570796	.196350	6	17.27876	23.75829	6	32.98672	86.59015
7	1.832596	.267254	7	17.54056	24.48370	7	33.24852	87.97005
8	2.094395	.349066	8	17.80236	25.22001	8	33.51032	89.36086
9	2.356195	.441786	9	18.06416	25.96723	9	33.77212	90.76258
10	2.617994	.545415	10	18.32596	26.72535	10	34.03392	92.17520
11	2.879793	.659953	11	18.58776	27.49439	11	34.29572	93.59874
1 0	3.14159	.785398	6 0	18.84956	28.27433	11 0	34.55752	95.03318
1	3.40339	.921752	1	19.11136	29.06519	1	34.81932	96.47853
2	3.66519	1.06901	2	19.37315	29.86695	2	35.08112	97.93479
3	3.92699	1.22718	3	19.63495	30.67962	3	35.34292	99.40196
4	4.18879	1.39626	4	19.89675	31.50319	4	35.60472	100.8800
5	4.45059	1.57625	5	20.15855	32.33768	5	35.86652	102.3690
6	4.71239	1.76715	6	20.42035	33.18307	6	36.12832	103.8689
7	4.97419	1.96895	7	20.68215	34.03937	7	36.39011	105.3797
8	5.23599	2.18166	8	20.94395	34.90659	8	36.65191	106.9014
9	5.49779	2.40528	9	21.20575	35.78470	9	36.91371	108.4340
10	5.75959	2.63981	10	21.46755	36.67373	10	37.17551	109.9776
11	6.02139	2.88525	11	21.72935	37.57367	11	37.43731	111.5320
2 0	6.28319	3.14159	7 0	21.99115	38.48451	12 0	37.69911	113.0973
1	6.54498	3.40885	1	22.25295	39.40626	1	37.96091	114.6736
2	6.80678	3.68701	2	22.51475	40.33892	2	38.22271	116.2607
3	7.06858	3.97608	3	22.77655	41.28249	3	38.48451	117.8588
4	7.33038	4.27606	4	23.03835	42.23697	4	38.74631	119.4678
5	7.59218	4.58694	5	23.30015	43.20235	5	39.00811	121.0877
6	7.85398	4.90874	6	23.56194	44.17865	6	39.26991	122.7185
7	8.11578	5.24144	7	23.82374	45.16585	7	39.53171	124.3602
8	8.37758	5.58505	8	24.08554	46.16396	8	39.79351	126.0128
9	8.63938	5.93957	9	24.34734	47.17298	9	40.05531	127.6763
10	8.90118	6.30500	10	24.60914	48.19290	10	40.31711	129.3507
11	9.16298	6.68134	11	24.87094	49.22374	11	40.57891	131.0360
3 0	9.42478	7.06858	8 0	25.13274	50.26548	13 0	40.84070	132.7323
1	9.68658	7.46674	1	25.39454	51.31813	1	41.10250	134.4394
2	9.94838	7.87580	2	25.65634	52.38169	2	41.36430	136.1575
3	10.21018	8.29577	3	25.91814	53.45616	3	41.62610	137.8865
4	10.47198	8.72665	4	26.17994	54.54154	4	41.88790	139.6263
5	10.73377	9.16843	5	26.44174	55.63782	5	42.14970	141.3771
6	10.99557	9.62113	6	26.70354	56.74502	6	42.41150	143.1388
7	11.25737	10.08473	7	26.96534	57.86312	7	42.67330	144.9114
8	11.51917	10.55924	8	27.22714	58.99213	8	42.93510	146.6949
9	11.78097	11.04466	9	27.48894	60.13205	9	43.19690	148.4893
10	12.04277	11.54099	10	27.75074	61.28287	10	43.45870	150.2947
11	12.30457	12.04823	11	28.01253	62.44461	11	43.72050	152.1109
4 0	12.56637	12.56637	9 0	28.27433	63.61725	14 0	43.98230	153.9380
1	12.82817	13.09542	1	28.53613	64.80080	1	44.24410	155.7761
2	13.08997	13.63538	2	28.79793	65.99526	2	44.50590	157.6250
3	13.35177	14.18625	3	29.05973	67.20063	3	44.76770	159.4849
4	13.61357	14.74803	4	29.32153	68.41691	4	45.02949	161.3557
5	13.87537	15.32072	5	29.58333	69.64409	5	45.29129	163.2374
6	14.13717	15.90431	6	29.84513	70.88218	6	45.55309	165.1300
7	14.39897	16.49882	7	30.10693	72.13119	7	45.81489	167.0335
8	14.66077	17.10423	8	30.36873	73.39110	8	46.07669	168.9479
9	14.92257	17.72055	9	30.63053	74.66191	9	46.33849	170.8732
10	15.18436	18.34777	10	30.89233	75.94364	10	46.60029	172.8094
11	15.44616	18.98591	11	31.15413	77.23627	11	46.86209	174.7565

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
15 0	47.12389	176.7146	**20** 0	62.83185	314.1593	**25** 0	78.53982	490.8739
1	47.38569	178.6835	1	63.09365	316.7827	1	78.80162	494.1518
2	47 64749	180.6634	2	63.35545	319.4171	2	79.06342	497.4407
3	47.90929	182.6542	3	63.61725	322.0623	3	79.32521	500.7404
4	48.17109	184.6558	4	63.87905	324.7185	4	79.58701	504.0511
5	48.43289	186.6684	5	64.14085	327.3856	5	79.84881	507 3727
6	48 69469	188.6919	6	64.40265	330.0636	6	80.11061	510.7052
7	48 95649	190.7263	7	64.66445	332.7525	7	80.37241	514.0486
8	49 21828	192.7716	8	64.92625	335.4523	8	80.63421	517.4029
9	49.48008	194.8278	9	65.18805	338.1630	9	80.89601	520.7681
10	49.74188	196.8950	10	65.44985	340.8846	10	81.15781	524.1442
11	50.00368	198 9730	11	65.71165	343.6172	11	81.41961	527.5312
16 0	50.26548	201.0619	**21** 0	65.97345	346.3606	**26** 0	81.68141	530.9292
1	50.52728	203.1618	1	66.23525	349 1149	1	81.94321	534.3380
2	50.78908	205.2725	2	66.49704	351.8802	2	82.20501	537.7578
3	51.05088	207 3942	3	66.75884	354 6564	3	82.46681	541.1884
4	51.31268	209.5268	4	67.02064	357.4434	4	82.72861	544.6300
5	51.57448	211.6703	5	67.28244	360.2414	5	82.99041	548.0825
6	51.83628	213.8246	6	67.54424	363 0503	6	83.25221	551.5459
7	52.09808	215.9899	7	67.80604	365 8701	7	83.51400	555.0202
8	52.35988	218.1662	8	68.06784	368.7008	8	83.77580	558.5054
9	52.62168	220.3533	9	68.32964	371.5424	9	84.03760	562.0015
10	52.88348	222 5513	10	68.59144	374.3949	10	84 29940	565.5085
11	53.14528	224.7602	11	68 85324	377.2584	11	84 56120	569.0264
17 0	53.40708	226.9801	**22** 0	69 11504	380.1327	**27** 0	84 82300	572.5553
1	53.66887	229.2108	1	69 37684	383.0180	1	85.08480	576.0950
2	53.93067	231.4525	2	69 63864	385 9141	2	85.34660	579.6457
3	54.19247	233.7050	3	69.90044	388.8212	3	85.60840	583.2072
4	54.45427	235.9685	4	70.16224	391.7392	4	85.87020	586.7797
5	54.71607	238.2429	5	70.42404	394.6680	5	86.13200	590.3631
6	54.97787	240.5282	6	70 68583	397.6078	6	86 39380	593.9574
7	55.23967	242.8244	7	70.94763	400.5585	7	86.65560	597.5626
8	55.50147	245.1315	8	71.20943	403.5201	8	86 91740	601.1787
9	55.76327	247.4495	9	71.47123	406 4926	9	87.17920	604.8057
10	56.02507	249.7784	10	71.73303	409.4761	10	87.44100	608.4436
11	56 28687	252.1183	11	71.99483	412.4704	11	87.70279	612.0924
18 0	56.54867	254 4690	**23** 0	72 25663	415.4756	**28** 0	87.96459	615.7522
1	56.81047	256 8307	1	72.51843	418.4918	1	88.22639	619.4228
2	57.07227	259.2032	2	72.78023	421.5188	2	88.48819	623.1044
3	57.33407	261.5867	3	73.04203	424.5568	3	88.74999	626.7968
4	57.59587	263.9810	4	73.30383	427.6057	4	89.01179	630.5002
5	57.85766	266 3863	5	73.56563	430 6654	5	89.27359	634.2145
6	58.11946	268.8025	6	73.82743	433.7361	6	89.53539	637.9397
7	58.38126	271.2296	7	74.08923	436.8177	7	89.79719	641.6758
8	58.64306	273.6676	8	74.35103	439.9102	8	90.05899	645.4228
9	58.90486	276.1165	9	74.61283	443 0137	9	90.32079	649.1807
10	59.16666	278.5764	10	74.87462	446.1280	10	90.58259	652.9495
11	59.42846	281.0471	11	75.13642	449.2532	11	90.84439	656.7292
19 0	59.69026	283.5287	**24** 0	75.39822	452.3893	**29** 0	91.10619	660.5199
1	59.95206	286.0213	1	75.66002	455.5364	1	91.36799	664.3214
2	60.21386	288.5247	2	75.92182	458.6943	2	91.62979	668.1339
3	60.47566	291.0391	3	76.18362	461 8632	3	91.89159	671.9572
4	60.73746	293.5644	4	76.44542	465.0430	4	92.15338	675.7915
5	60.99926	296.1006	5	76.70722	468.2337	5	92 41518	679.6367
6	61.26106	298.6477	6	76.96902	471.4352	6	92.67698	683.4928
7	61.52286	301.2056	7	77.23082	474.6477	7	92.93878	687.3597
8	61.78466	303.7746	8	77.49262	477.8711	8	93.20058	691.2377
9	62.04645	306.3544	9	77.75442	481.1055	9	93 46238	695.1265
10	62.30825	308.9451	10	78.01622	484.3507	10	93.72418	699.0262
11	62.57005	311.5467	11	78.27802	487.6068	11	93.98598	702.9368

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft. In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
30 0	94.24778	706.8583	**35** 0	109.9557	962.1128	**40** 0	125.6637	1256.6371
1	94.50958	710.7908	1	110.2175	966.6997	1	125.9255	1261.8785
2	94.77138	714.7341	2	110.4793	971.2975	2	126.1873	1267.1309
3	95.03318	718.6884	3	110.7411	975.9063	3	126.4491	1272.3941
4	95.29498	722.6536	4	111.0029	980.5260	4	126.7109	1277.6683
5	95.55678	726.6297	5	111.2647	985.1566	5	126.9727	1282.9534
6	95.81858	730.6166	6	111.5265	989.7980	6	127.2345	1288.2493
7	96.08038	734.6145	7	111.7883	994.4504	7	127.4963	1293.5562
8	96.34217	738.6233	8	112.0501	999.1137	8	127.7581	1298.8740
9	96.60397	742.6431	9	112.3119	1003.7879	9	128.0199	1304.2027
10	96.86577	746.6737	10	112.5737	1008.4731	10	128.2817	1309.5424
11	97.12757	750.7152	11	112.8355	1013.1691	11	128.5435	1314.8929
31 0	97.38937	754.7676	**36** 0	113.0973	1017.8760	**41** 0	128.8053	1320.2543
1	97.65117	758.8310	1	113.3591	1022.5939	1	129.0671	1325.6267
2	97.91297	762.9052	2	113.6209	1027.3226	2	129.3289	1331.0099
3	98.17477	766.9904	3	113.8827	1032.0623	3	129.5907	1336.4041
4	98.43657	771.0865	4	114.1445	1036.8128	4	129.8525	1341.8091
5	98.69837	775.1934	5	114.4063	1041.5743	5	130.1143	1347.2251
6	98.96017	779.3113	6	114.6681	1046.3467	6	130.3761	1352.6520
7	99.22197	783.4401	7	114.9299	1051.1300	7	130.6379	1358.0898
8	99.48377	787.5798	8	115.1917	1055.9242	8	130.8997	1363.5385
9	99.74557	791.7304	9	115.4535	1060.7293	9	131.1615	1368.9981
10	100.0074	795.8920	10	115.7153	1065.5453	10	131.4233	1374.4686
11	100.2692	800.0644	11	115.9771	1070.3723	11	131.6851	1379.9500
32 0	100.5310	804.2477	**37** 0	116.2389	1075.2101	**42** 0	131.9469	1385.4424
1	100.7928	808.4420	1	116.5007	1080.0588	1	132.2087	1390.9456
2	101.0546	812.6471	2	116.7625	1084.9185	2	132.4705	1396.4598
3	101.3164	816.8632	3	117.0243	1089.7890	3	132.7323	1401.9848
4	101.5782	821.0901	4	117.2861	1094.6705	4	132.9941	1407.5208
5	101.8400	825.3280	5	117.5479	1099.5629	5	133.2559	1413.0676
6	102.1018	829.5768	6	117.8097	1104.4662	6	133.5177	1418.6254
7	102.3636	833.8365	7	118.0715	1109.3804	7	133.7795	1424.1941
8	102.6254	838.1071	8	118.3333	1114.3055	8	134.0413	1429.7737
9	102.8872	842.3886	9	118.5951	1119.2415	9	134.3031	1435.3642
10	103.1490	846.6810	10	118.8569	1124.1884	10	134.5649	1440.9656
11	103.4108	850.9844	11	119.1187	1129.1462	11	134.8267	1446.5780
33 0	103.6726	855.2986	**38** 0	119.3805	1134.1149	**43** 0	135.0885	1452.2012
1	103.9344	859.6237	1	119.6423	1139.0946	1	135.3503	1457.8353
2	104.1962	863.9598	2	119.9041	1144.0851	2	135.6121	1463.4804
3	104.4580	868.3068	3	120.1659	1149.0866	3	135.8739	1469.1364
4	104.7198	872.6646	4	120.4277	1154.0990	4	136.1357	1474.8032
5	104.9816	877.0334	5	120.6895	1159.1222	5	136.3975	1480.4810
6	105.2434	881.4131	6	120.9513	1164.1564	6	136.6593	1486.1697
7	105.5052	885.8037	7	121.2131	1169.2015	7	136.9211	1491.8693
8	105.7670	890.2052	8	121.4749	1174.2575	8	137.1829	1497.5798
9	106.0288	894.6176	9	121.7367	1179.3244	9	137.4447	1503.3012
10	106.2906	899.0409	10	121.9985	1184.4022	10	137.7065	1509.0335
11	106.5524	903.4751	11	122.2603	1189.4910	11	137.9683	1514.7767
34 0	106.8142	907.9203	**39** 0	122.5221	1194.5906	**44** 0	138.2301	1520.5308
1	107.0759	912.3763	1	122.7839	1199.7011	1	138.4919	1526.2959
2	107.3377	916.8433	2	123.0457	1204.8226	2	138.7537	1532.0718
3	107.5995	921.3211	3	123.3075	1209.9550	3	139.0155	1537.8587
4	107.8613	925.8099	4	123.5693	1215.0982	4	139.2773	1543.6565
5	108.1231	930.3096	5	123.8311	1220.2524	5	139.5391	1549.4651
6	108.3849	934.8202	6	124.0929	1225.4175	6	139.8009	1555.2847
7	108.6467	939.3417	7	124.3547	1230.5935	7	140.0627	1561.1152
8	108.9085	943.8741	8	124.6165	1235.7804	8	140.3245	1566.9566
9	109.1703	948.4174	9	124.8783	1240.9782	9	140.5863	1572.8089
10	109.4321	952.9716	10	125.1401	1246.1869	10	140.8481	1578.6721
11	109.6939	957.5367	11	125.4019	1251.4065	11	141.1099	1584.5462

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in **feet and inches.**

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.
45 0	141.3717	1590.4313	**50** 0	157.0796	1963.4954	**55** 0	172.7876	2375.8294
1	141.6335	1596.3272	1	157.3414	1970.0458	1	173.0494	2383.0344
2	141.8953	1602.2341	2	157.6032	1976.6072	2	173.3112	2390.2502
3	142.1571	1608.1518	3	157.8650	1983.1794	3	173 5730	2397.4770
4	142.4189	1614.0805	4	158.1268	1989.7626	4	173.8348	2404.7146
5	142 6807	1620.0201	5	158.3886	1996.3567	5	174.0966	2411.9632
6	142.9425	1625.9705	6	158.6504	2002.9617	6	174.3584	2419.2227
7	143.2043	1631.9319	7	158.9122	2009.5776	7	174.6202	2426.4931
8	143.4661	1637.9042	8	159.1740	2016.2044	8	174 8820	2433.7744
9	143.7279	1643.8874	9	159.4358	2022.8421	9	175 1438	2441.0666
10	143 9897	1649.8816	10	159.6976	2029.4907	10	175 4056	2448.3697
11	144.2515	1655 8866	11	159.9594	2036.1502	11	175 6674	2455.6837
46 0	144 5133	1661.9025	**51** 0	160 2212	2042.8206	**56** 0	175.9292	2463 0086
1	144.7751	1667 9294	1	160.4830	2049.5020	1	176.1910	2470.3445
2	145.0369	1673.9671	2	160.7448	2056.1942	2	176.4528	2477.6912
3	145.2987	1680.0158	3	161.0066	2062.8974	3	176.7146	2485.0489
4	145 5605	1686.0753	4	161.2684	2069.6114	4	176.9764	2492.4174
5	145.8223	1692 1458	5	161.5302	2076.3364	5	177.2382	2499.7969
6	146 0841	1698.2272	6	161.7920	2083.0723	6	177.5000	2507.1873
7	146.3459	1704.3195	7	162.0538	2089 8191	7	177.7618	2514.5886
8	146.6077	1710.4227	8	162.3156	2096.5768	8	178.0236	2522.0008
9	146.8695	1716 5368	9	162 5774	2103.3454	9	178.2854	2529.4239
10	147.1313	1722 6618	10	162.8392	2110.1249	10	178.5472	2536.8579
11	147.3931	1728.7977	11	163.1010	2116.9153	11	178.8090	2544.3028
47 0	147.6549	1734.9445	**52** 0	163.3628	2123.7166	**57** 0	179 0708	2551.7586
1	147.9167	1741.1023	1	163.6246	2130.5289	1	179.3326	2559.2254
2	148.1785	1747.2709	2	163.8864	2137.3520	2	179.5944	2566.7030
3	148.4403	1753.4505	3	164.1482	2144.1861	3	179.8562	2574.1916
4	148.7021	1759.6410	4	164.4100	2151.0310	4	180.1180	2581.6910
5	148.9639	1765.8423	5	164.6718	2157.8869	5	180.3798	2589.2014
6	149.2257	1772.0546	6	164.9336	2164.7537	6	180.6416	2596.7227
7	149 4875	1778.2778	7	165.1954	2171.6314	7	180.9034	2604.2549
8	149.7492	1784.5119	8	165.4572	2178.5200	8	181.1652	2611.7980
9	150.0110	1790.7569	9	165.7190	2185.4195	9	181.4270	2619.3520
10	150.2728	1797.0128	10	165.9808	2192.3299	10	181 6888	2626.9169
11	150.5346	1803.2796	11	166.2426	2199.2512	11	181.9506	2634.4927
48 0	150.7964	1809.5574	**53** 0	166.5044	2206.1834	**58** 0	182.2124	2642.0794
1	151.0582	1815 8460	1	166.7662	2213.1266	1	182.4742	2649.6771
2	151.3200	1822.1456	2	167.0280	2220.0806	2	182.7360	2657.2856
3	151.5818	1828.4560	3	167.2898	2227.0456	3	182.9978	2664.9051
4	151.8436	1834.7774	4	167.5516	2234.0214	4	183.2596	2672.5354
5	152.1054	1841.1096	5	167.8134	2241.0082	5	183.5214	2680.1767
6	152.3672	1847.4528	6	168.0752	2248.0059	6	183.7832	2687.8289
7	152.6290	1853.8069	7	168.3370	2255.0145	7	184.0450	2695.4920
8	152.8908	1860.1719	8	168.5988	2262.0340	8	184.3068	2703.1659
9	153.1526	1866 5478	9	168 8606	2269.0644	9	184.5686	2710.8508
10	153.4144	1872.9346	10	169.1224	2276.1057	10	184 8304	2718.5467
11	153.6762	1879.3324	11	169.3842	2283.1579	11	185 0922	2726.2534
49 0	153 9380	1885.7410	**54** 0	169.6460	2290 2210	**59** 0	185.3540	2733.9710
1	154.1998	1892 1605	1	169.9078	2297.2951	1	185.6158	2741.6995
2	154.4616	1898.5910	2	170.1696	2304 3800	2	185.8776	2749.4390
3	154.7234	1905.0323	3	170.4314	2311.4759	3	186.1394	2757.1893
4	154.9852	1911.4846	4	170.6932	2318.5826	4	186.4012	2764.9506
5	155 2470	1917.9478	5	170.9550	2325.7003	5	186.6630	2772.7228
6	155.5088	1924.4218	6	171.2168	2332 8289	6	186.9248	2780.5058
7	155.7706	1930.9068	7	171.4786	2339.9684	7	187.1866	2788.2998
8	156.0324	1937.4027	8	171.7404	2347.1188	8	187.4484	2796.1047
9	156.2942	1943.9095	9	172.0022	2354.2801	9	187.7102	2803.9205
10	156.5560	1950.4273	10	172.2640	2361.4523	10	187.9720	2811.7472
11	156.8178	1956.9559	11	172.5258	2368.6354	11	188.2338	2819.5849

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
60 0	188.4956	2827.4334	65 0	204.2035	3318.3072	70 0	219.9115	3848.4510
1	188.7574	2835.2928	1	204.4653	3326.8212	1	220.1733	3857.6194
2	189.0192	2843.1632	2	204.7271	3335.3460	2	220.4351	3866.7988
3	189.2810	2851.0414	3	204.9889	3343.8818	3	220.6969	3875.9890
4	189.5428	2858.9366	4	205.2507	3352.4284	4	220.9587	3885.1902
5	189.8046	2866.8397	5	205.5125	3360.9860	5	221.2205	3894.4022
6	190.0664	2874.7536	6	205.7743	3369.5545	6	221.4823	3903.6252
7	190.3282	2882.6785	7	206.0361	3378.1339	7	221.7441	3912.8591
8	190.5900	2890.6143	8	206.2979	3386.7241	8	222.0059	3922.1039
9	190.8518	2898.5610	9	206.5597	3395.3253	9	222.2677	3931.3596
10	191.1136	2906.5186	10	206.8215	3403.9375	10	222.5295	3940.6262
11	191.3754	2914.4871	11	207.0833	3412.5605	11	222.7913	3949.9037
61 0	191.6372	2922.4666	66 0	207.3451	3421.1944	71 0	223.0531	3959.1921
1	191.8990	2930.4569	1	207.6069	3429.8392	1	223.3149	3968.4915
2	192.1608	2938.4581	2	207.8687	3438.4950	2	223.5767	3977.8017
3	192.4226	2946.4703	3	208.1305	3447.1616	3	223.8385	3987.1229
4	192.6843	2954.4934	4	208.3923	3455.8392	4	224.1003	3996.4549
5	192.9461	2962.5273	5	208.6541	3464.5277	5	224.3621	4005.7979
6	193.2079	2970.5722	6	208.9159	3473.2270	6	224.6239	4015.1518
7	193.4697	2978.6280	7	209.1777	3481.9373	7	224.8857	4024.5165
8	193.7315	2986.6947	8	209.4395	3490.6585	8	225.1475	4033.8922
9	193.9933	2994.7723	9	209.7013	3499.3906	9	225.4093	4043.2788
10	194.2551	3002.8608	10	209.9631	3508.1336	10	225.6711	4052.6763
11	194.5169	3010.9602	11	210.2249	3516.8875	11	225.9329	4062.0848
62 0	194.7787	3019.0705	67 0	210.4867	3525.6524	72 0	226.1947	4071.5041
1	195.0405	3027.1918	1	210.7485	3534.4281	1	226.4565	4080.9343
2	195.3023	3035.3239	2	211.0103	3543.2147	2	226.7183	4090.3755
3	195.5641	3043.4670	3	211.2721	3552.0123	3	226.9801	4099.8275
4	195.8259	3051.6209	4	211.5339	3560.8207	4	227.2419	4109.2905
5	196.0877	3059.7858	5	211.7957	3569.6401	5	227.5037	4118.7643
6	196.3495	3067.9616	6	212.0575	3578.4704	6	227.7655	4128.2491
7	196.6113	3076.1483	7	212.3193	3587.3116	7	228.0273	4137.7448
8	196.8731	3084.3459	8	212.5811	3596.1637	8	228.2891	4147.2514
9	197.1349	3092.5544	9	212.8429	3605.0267	9	228.5509	4156.7689
10	197.3967	3100.7738	10	213.1047	3613.9006	10	228.8127	4166.2973
11	197.6585	3109.0041	11	213.3665	3622.7854	11	229.0745	4175.8366
63 0	197.9203	3117.2453	68 0	213.6283	3631.6811	73 0	229.3363	4185.3868
1	198.1821	3125.4974	1	213.8901	3640.5877	1	229.5981	4194.9479
2	198.4439	3133.7605	2	214.1519	3649.5053	2	229.8599	4204.5200
3	198.7057	3142.0344	3	214.4137	3658.4337	3	230.1217	4214.1029
4	198.9675	3150.3193	4	214.6755	3667.3731	4	230.3835	4223.6968
5	199.2293	3158.6151	5	214.9373	3676.3234	5	230.6453	4233.3016
6	199.4911	3166.9217	6	215.1991	3685.2845	6	230.9071	4242.9172
7	199.7529	3175.2393	7	215.4609	3694.2566	7	231.1689	4252.5438
8	200.0147	3183.5678	8	215.7227	3703.2396	8	231.4307	4262.1813
9	200.2765	3191.9072	9	215.9845	3712.2335	9	231.6925	4271.8297
10	200.5383	3200.2575	10	216.2463	3721.2383	10	231.9543	4281.4890
11	200.8001	3208.6188	11	216.5081	3730.2540	11	232.2161	4291.1592
64 0	201.0619	3216.9909	69 0	216.7699	3739.2807	74 0	232.4779	4300.8403
1	201.3237	3225.3739	1	217.0317	3748.3182	1	232.7397	4310.5824
2	201.5855	3233.7679	2	217.2935	3757.3666	2	233.0015	4320.2353
3	201.8473	3242.1727	3	217.5553	3766.4260	3	233.2633	4329.9492
4	202.1091	3250.5885	4	217.8171	3775.4962	4	233.5251	4339.6739
5	202.3709	3259.0151	5	218.0789	3784.5774	5	233.7869	4349.4096
6	202.6327	3267.4527	6	218.3407	3793.6695	6	234.0487	4359.1562
7	202.8945	3275.9012	7	218.6025	3802.7725	7	234.3105	4368.9186
8	203.1563	3284.3606	8	218.8643	3811.8864	8	234.5723	4378.6820
9	203.4181	3292.8309	9	219.1261	3821.0112	9	234.8341	4388.4613
10	203.6799	3301.3121	10	219.3879	3830.1469	10	235.0959	4398.2515
11	203.9417	3309.8042	11	219.6497	3839.2935	11	235.3576	4408.0526

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
75 0	235.6194	4417.8647	80 0	251.3274	5026.5482	85 0	267.0354	5674.5017
1	235.8812	4427.6876	1	251.5892	5037.0257	1	267.2972	5685.6337
2	236.1430	4437.5214	2	251.8510	5047.5140	2	267.5590	5696.7765
3	236.4048	4447.3662	3	252.1128	5058.0133	3	267.8208	5707.9302
4	236.6666	4457.2218	4	252.3746	5068.5234	4	268.0826	5719.0949
5	236.9284	4467.0884	5	252.6364	5079.0445	5	268.3444	5730.2705
6	237.1902	4476.9659	6	252.8982	5089.5764	6	268.6062	5741.4569
7	237.4520	4486.8543	7	253.1600	5100.1193	7	268.8680	5752.6543
8	237.7138	4496.7536	8	253.4218	5110.6731	8	269.1298	5763.8626
9	237.9756	4506.6637	9	253.6836	5121.2378	9	269.3916	5775.0818
10	238.2374	4516.5849	10	253.9454	5131.8134	10	269.6534	5786.3119
11	238.4992	4526.5169	11	254.2072	5142.3999	11	269.9152	5797.5529
76 0	238.7610	4536.4598	81 0	254.4690	5152.9974	86 0	270.1770	5808.8048
1	239.0228	4546.4136	1	254.7308	5163.6057	1	270.4388	5820.0676
2	239.2846	4556.3784	2	254.9926	5174.2249	2	270.7006	5831.3414
3	239.5464	4566.3540	3	255.2544	5184.8551	3	270.9624	5842.6260
4	239.8082	4576.3406	4	255.5162	5195.4961	4	271.2242	5853.9216
5	240.0700	4586.3380	5	255.7780	5206.1481	5	271.4860	5865.2280
6	240.3318	4596.3464	6	256.0398	5216.8110	6	271.7478	5876.5454
7	240.5936	4606.3657	7	256.3016	5227.4847	7	272.0096	5887.8737
8	240.8554	4616.3959	8	256.5634	5238.1694	8	272.2714	5899.2129
9	241.1172	4626.4370	9	256.8252	5248.8650	9	272.5332	5910.5630
10	241.3790	4636.4890	10	257.0870	5259.5715	10	272.7950	5921.9240
11	241.6408	4646.5519	11	257.3488	5270.2889	11	273.0568	5933.2959
77 0	241.9026	4656.6257	82 0	257.6106	5281.0178	87 0	273.3186	5944.6787
1	242.1644	4666.7104	1	257.8724	5291.7565	1	273.5804	5956.0724
2	242.4262	4676.8061	2	258.1342	5302.5066	2	273.8422	5967.4771
3	242.6880	4686.9126	3	258.3960	5313.2677	3	274.1040	5978.8926
4	242.9498	4697.0301	4	258.6578	5324.0396	4	274.3658	5990.3191
5	243.2116	4707.1584	5	258.9196	5334.8225	5	274.6276	6001.7564
6	243.4734	4717.2977	6	259.1814	5345.6162	6	274.8894	6013.2047
7	243.7352	4727.4479	7	259.4432	5356.4209	7	275.1512	6024.6639
8	243.9970	4737.6090	8	259.7050	5367.2365	8	275.4130	6036.1340
9	244.2588	4747.7810	9	259.9668	5378.0630	9	275.6748	6047.6149
10	244.5206	4757.9639	10	260.2286	5388.9004	10	275.9366	6059.1068
11	244.7824	4768.1577	11	260.4904	5399.7487	11	276.1984	6070.6097
78 0	245.0442	4778.3624	83 0	260.7522	5410.6079	88 0	276.4602	6082.1234
1	245.3060	4788.5781	1	261.0140	5421.4781	1	276.7220	6093.6480
2	245.5678	4798.8046	2	261.2758	5432.3591	2	276.9838	6105.1835
3	245.8296	4809.0420	3	261.5376	5443.2511	3	277.2456	6116.7300
4	246.0914	4819.2904	4	261.7994	5454.1539	4	277.5074	6128.2873
5	246.3532	4829.5497	5	262.0612	5465.0677	5	277.7692	6139.8556
6	246.6150	4839.8198	6	262.3230	5475.9923	6	278.0309	6151.4346
7	246.8768	4850.1009	7	262.5848	5486.9279	7	278.2927	6163.0248
8	247.1386	4860.3929	8	262.8466	5497.8744	8	278.5545	6174.6258
9	247.4004	4870.6958	9	263.1084	5508.8318	9	278.8163	6186.2377
10	247.6622	4881.0096	10	263.3702	5519.8001	10	279.0781	6197.8605
11	247.9240	4891.3343	11	263.6320	5530.7793	11	279.3399	6209.4942
79 0	248.1858	4901.6699	84 0	263.8938	5541.7694	89 0	279.6017	6221.1389
1	248.4476	4912.0165	1	264.1556	5552.7705	1	279.8635	6232.7944
2	248.7094	4922.3739	2	264.4174	5563.7824	2	280.1253	6244.4608
3	248.9712	4932.7423	3	264.6792	5574.8053	3	280.3871	6256.1382
4	249.2330	4943.1215	4	264.9410	5585.8390	4	280.6489	6267.8264
5	249.4948	4953.5117	5	265.2028	5596.8837	5	280.9107	6279.5256
6	249.7566	4963.9127	6	265.4646	5607.9392	6	281.1725	6291.2356
7	250.0184	4974.3247	7	265.7264	5619.0057	7	281.4343	6302.9566
8	250.2802	4984.7476	8	265.9882	5630.0831	8	281.6961	6314.6885
9	250.5420	4995.1814	9	266.2500	5641.1714	9	281.9579	6326.4313
10	250.8038	5005.6261	10	266.5118	5652.2706	10	282.2197	6338.1850
11	251.0656	5016.0817	11	266.7736	5663.3807	11	282.4815	6349.9496

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
90 0	282.7433	6361.7251	93 5	293.4771	6853.9134	96 9	303.9491	7351.7686
1	283.0051	6373.5116	6	293.7389	6866.1471	10	304.2109	7364.4386
2	283.2669	6385.3089	7	294.0007	6878.3917	11	304 4727	7377.1195
3	283 5287	6397.1171	8	294.2625	6890.6472	97 0	304.7345	7389.8113
4	283.7905	6408.9363	9	294.5243	6902.9135	1	304.9963	7402.5140
5	284.0523	6420 7663	10	294.7861	6915.1908	2	305.2581	7415.2277
6	284.3141	6432.6073	11	295.0479	6927.4791	3	305.5199	7427.9522
7	284.5759	6444.4592	94 0	295.3097	6939.7782	4	305.7817	7440.6877
8	284 8377	6456.3220	1	295.5715	6952.0882	5	306.0435	7453.4340
9	285.0995	6468.1957	2	295.8333	6964.4091	6	306.3053	7466.1913
10	285.3613	6480.0803	3	296.0951	6976.7410	7	306.5671	7478.9595
11	285.6231	6491.9758	4	296 3569	6989.0837	8	306.8289	7491.7385
91 0	285.8849	6503.8822	5	296.6187	7001.4374	9	307.0907	7504.5285
1	286.1467	6515.7995	6	296.8805	7013.8019	10	307.3525	7517.3294
2	286.4085	6527.7278	7	297.1423	7026.1774	11	307.6143	7530.1412
3	286.6703	6539.6669	8	297.4041	7038.5638	98 0	307.8761	7542.9640
4	286.9321	6551.6169	9	297.6659	7050.9611	1	308.1379	7555.7976
5	287.1939	6563.5779	10	297.9277	7063.3693	2	308.3997	7568.6421
6	287.4557	6575.5498	11	298.1895	7075.7884	3	308.6615	7581.4976
7	287.7175	6587.5325	95 0	298.4513	7088.2184	4	308.9233	7594.3639
8	287.9793	6599.5262	1	298.7131	7100.6593	5	309.1851	7607.2412
9	288.2411	6611.5308	2	298.9749	7113.1112	6	309.4469	7620.1293
10	288.5029	6623.5463	3	299.2367	7125.5739	7	309.7087	7633.0284
11	288.7647	6635.5727	4	299.4985	7138.0476	8	309.9705	7645.9384
92 0	289.0265	6647.6101	5	299.7603	7150.5321	9	310.2323	7658.8593
1	289.2883	6659.6583	6	300.0221	7163.0276	10	310.4941	7671.7911
2	289.5501	6671.7174	7	300.2839	7175.5340	11	310.7559	7684.7338
3	289.8119	6683.7875	8	300.5457	7188.0513	99 0	311.0177	7697.6874
4	290 0737	6695.8684	9	300.8075	7200.5794	1	311.2795	7710.6519
5	290.3355	6707.9603	10	301.0693	7213.1185	2	311.5413	7723.6274
6	290.5973	6720 0630	11	301.3311	7225.6686	3	311.8031	7736.6137
7	290.8591	6732.1767	96 0	301.5929	7238.2295	4	312.0649	7749.6109
8	291.1209	6744.3013	1	301.8547	7250.8013	5	312.3267	7762.6191
9	291.3827	6756.4368	2	302.1165	7263.3840	6	312.5885	7775.6382
10	291.6445	6768.5832	3	302.3783	7275.9777	7	312.8503	7788.6681
11	291.9063	6780.7405	4	302.6401	7288.5822	8	313.1121	7801 7090
93 0	292.1681	6792.9087	5	302.9019	7301.1977	9	313.3739	7814.7608
1	292.4299	6805.0878	6	303.1637	7313.8240	10	313.6357	7827.8235
2	292.6917	6817.2779	7	303.4255	7326 4613	11	313.8975	7840.8971
3	292.9535	6829.4788	8	303.6873	7339.1095	100 0	314.1593	7853.9816
4	293.2153	6841.6907						

Diam, inch.	Circumf, foot	Diam, inch.	Circumf, foot	Diam, inch	Circumf, foot.	Diam, inch.	Circumf, foot.	Diam, inch.	Circumf, foot.
1-64	.004091	7-32	.057269	27-64	.110447	5-8	.163625	53-64	.216803
1-32	.008181	15-64	.061359	7-16	.114537	41-64	.167715	27-32	.220893
3-64	.012272	¼	.065450	29-64	.118628	21-32	.171806	55-64	.224984
1-16	.016362	17-64	.069540	15-32	.122718	43-64	.175896	7-8	.229074
5-64	.020453	9-32	.073631	31-64	.126809	11-16	.179987	57-64	.233165
3-32	.024544	19-64	.077722	½	.130900	45-64	.184078	29-32	.237256
7-64	.028634	5-16	.081812	33-64	.134990	23-32	.188168	59-64	.241346
⅛	.032725	21-64	.085903	17-32	.139081	47-64	.192259	15-16	.245437
9-64	.036816	11-32	.089994	35-64	.143172	¾	.196350	61-64	.249528
5-32	.040906	23-64	.094084	9-16	.147262	49-64	.200440	31-32	.253618
11-64	.044997	⅜	.098175	37-64	.151353	25-32	.204531	63-64	.257709
3-16	.049087	25-64	.102265	19-32	.155443	51-64	.208621	1	.261799
13-64	.053178	13-32	.106356	39-64	.159534	13-16	.212712		

CIRCULAR ARCS.

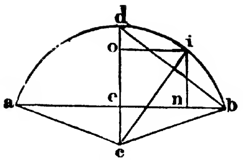


Fig.1

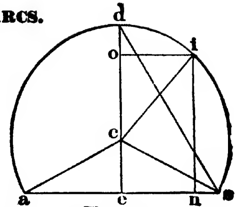


Fig. 2.

Rules for Fig. 1 apply to all arcs equal to, or less than, a semi-circle.
" " Fig. 2 " " " or greater than, a semi-circle.

Chord, *a b*, of whole arc, *a d b*,

$= 2 \times \sqrt{\text{radius}^2 - (\text{radius} - \text{rise})^2}$. Fig. 1.

$= 2 \times \sqrt{\text{radius}^2 - (\text{rise} - \text{radius})^2}$. Fig. 2.

$= 2 \times \sqrt{\text{rise} \times (2 \times \text{radius} - \text{rise})}$. Figs. 1 and 2.

$= 2 \times$ radius $\times$ sine of $\frac{1}{2}$ *a c b*. Figs. 1 and 2.

$= 2 \times \dfrac{\text{rise}}{\text{tangent of } a\,b\,d.*}$ Figs. 1 and 2.

$= 2 \times$ *d b* § $\times$ cosine of *a b d*.* Figs. 1 and 2.

$= 2 \times \sqrt{d\,b^2 - \text{rise}^2}$. Figs. 1 and 2.§

= approximately $8 \times$ *d b* § $- 3 \times$ length of arc *a d b* ¶. Fig. 1.

Length, *a d b*,

$= 2\,\pi$ radius $\times \dfrac{\text{arc } a\,d\,b \text{ in degrees}}{360}$. Figs. 1 and 2;

= .01745 $\times$ radius $\times$ arc *a d b* in degrees. Figs. 1 and 2.

= circumference of circle — length of *small* arc subtending angle *a c b*. Fig. 2.

= approximately $\dfrac{8 \times d\,b\,\S - \text{chord } a\,b.**}{3}$ Fig. 1.

---

* *a b d* is = $\frac{1}{4}$ of the angle *a c b*, subtended by the arc. In Fig. 2 the latter angle exceeds 180°.

§ *d b* = chord of *d i b*, or of half *a d b* $= \sqrt{\text{rise}^2 + (\frac{1}{2}\, a\, b)^2}$. Figs. 1 and 2.

¶ If rise =	multiply the result by	If rise =	multiply the result by
.5 chord,	1.036	.25 chord,	1.0044
.4 "	1.0196	.2 "	1.0021
.333 "	1.0114	.125 "	1.00036
.3 "	1.0083	.1 "	1.00015

** If rise =	multiply the result by	If rise =	multiply the result by
.5 chord	1.012	.25 chord	1.0015
.4 "	1.0065	.2 "	1.0007
.333 "	1.0038	.125 "	1.00012
.3 "	1.0028	.1 "	1.00005

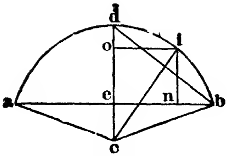


Fig.1

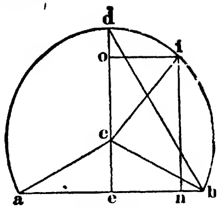


Fig. 2.

Continued from p. 179.

Rules for Fig. 1 apply to all arcs equal to or less than a semi-circle.
" " Fig. 2 " " " or greater than a semi-circle.

Radius, ca, cd, ci or cb,

$= \frac{(\frac{1}{2}\ ab)^2 + \text{rise}^2}{2 \times \text{rise}}$, Figs. 1 and 2.

$= \frac{db^2 §}{2 \times \text{rise}}$, Figs. 1 and 2.

$= \frac{\frac{1}{2}\ ab}{\text{sine of } \frac{1}{2}\ acb}$, Figs. 1 and 2. $= \frac{\frac{1}{2}\ db§}{\text{sine of } \frac{1}{2}\ bcd‖}$, Figs. 1 and 2.

$= \frac{\text{rise } de}{1 - \text{cosine of } \frac{1}{2}\ acb}$, Fig. 1. $= \frac{\text{rise } de}{1 + \text{cosine of } \frac{1}{2}\ acb†}$, Fig. 2.

Rise, or middle ordinate, de,

$= \text{radius} - \sqrt{\text{radius}^2 - (\frac{1}{2}\ ab)^2}$, Fig. 1.

$= \text{radius} + \sqrt{\text{radius}^2 - (\frac{1}{2}\ ab)^2}$, Fig. 2.

$= \text{radius} \times (1 - \text{cosine of } bcd‖)$, Fig. 1.

$= \text{radius} \times (1 + \text{cosine of } bcd‖)$,† Fig. 2.

$= \frac{db^2 §}{2 \times \text{radius}}$, Figs. 1 and 2.

$= \frac{1}{2}\ ab \times \text{tangent of } abd$,* Figs. 1 and 2.

$= \text{approximately } \frac{(\frac{1}{2}\ ab)^2}{2 \times \text{radius}}$, Fig 1.

When radius = chord ab, the result is 6.7 parts in 100 too short.
" " = 3 × chord ab, the result is 0.7 parts in 100 too short.

Side ordinate, as ni,

$= \sqrt{\text{radius}^2 - en^2} + \text{rise} - \text{radius}$, Figs. 1 and 2.

$= \text{approximately } \frac{an \times nb}{2 \times \text{radius}}$. Fig. 1.¶

* abd is = ¼ of the angle acb, subtended by the *arc*.

† Strictly, this should read 1 *minus* cosine; but the cosines of angles between 90° and 270° must then be regarded as *minus* or *negative*. Our rule, therefore, amounts to the same thing.

§ db = chord of dib, or of half adb, $= \sqrt{\text{rise}^2 + (\frac{1}{2}\ ab)^2}$. Figs. 1 and 2.

‖ bcd = half the angle acb subtended by the *arc*. In Fig. 2, the latter angle exceeds 180°.

¶ When radius = chord ab, this makes de 6.7 parts in 100 too short.
" " = 3 × chord ab, this makes de 0.7 parts in 100 too short.
The proportionate error is greater with the side ordinates.

Angle, $a\ c\ b$, subtended by arc, $a\ d\ b$.

Caution. The following sines, etc., are those of only *half* $a\ c\ b$.

Sine of $\frac{1}{2}\ a\ c\ b = \frac{\frac{1}{2}\ a\ b}{\text{radius}}$, Figs. 1 and 2.

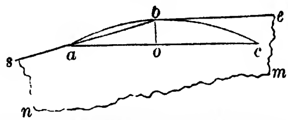
Cosine of $\frac{1}{2}\ a\ c\ b = \frac{\text{radius} - \text{rise}}{\text{radius}}$, Fig. 1; $= \frac{\text{rise} - \text{radius}}{\text{radius}}$, Fig. 2.

Tangent of $\frac{1}{2}\ a\ c\ b = \frac{\frac{1}{2}\ a\ b}{\text{radius} - \text{rise}}$, Fig. 1; $= \frac{\frac{1}{2}\ a\ b}{\text{rise} - \text{radius}}$, Fig. 2.

Versed sine of $\frac{1}{2}\ a\ c\ b = \frac{\text{rise}}{\text{radius}}$, Figs. 1 and 2.

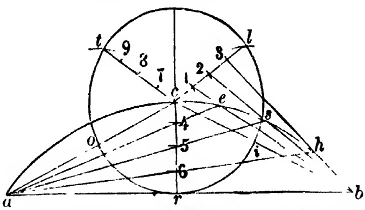
To describe the arc of a circle too large for the dividers.

1st Method. Let $a\ c$ be the chord, and $o\ b$ the height, of the required arc, as



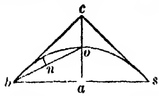
laid down *on the drawing.* On a separate strip of paper, $s\ e\ m\ n$, draw $a\ c$. $o\ b$. and $a\ b$ also $b\ e$, parallel to the chord $a\ c$. It is well to make $b\ s$ and $b\ e$, each a little longer than $a\ b$. Then cut off the paper carefully along the lines $s\ b$ and $b\ e$, so as to leave remaining only the strip $s\ a\ b\ e\ m\ n$. Now, if the straight sides $s\ b$ and $b\ e$ be applied to the drawing, so that any parts of them shall touch at the same time the points a and b, or b and c, the point b on the strip will be in the circumference of the arc, and may be pricked off. Thus, any number of points in the arc may be found, and afterward united to form the curve.

2d Method. Draw the span $a\ b$; the rise $r\ c$; and $a\ c$, $b\ c$. From c with radius



$c\ r$ describe a circle. Make each of the arcs $o\ t$ and $i\ l$ equal to $r\ o$ or $r\ i$; and draw $c\ t$, $c\ l$. Divide $c\ t$, $c\ l$, $c\ r$, each into half as many equal parts as the curve is to be divided into. Draw the lines $b\ 1$, $b\ 2$, $b\ 3$; and $a\ 4$, $a\ 5$, $a\ 6$, extended to meet the first ones at e, s, h. Then e, s, h, are points in one half the curve. Then for the other half, draw similar lines from a to 7, 8, 9; and others from b to meet them, as before. Trace the curve by hand.

Remark.—It may frequently be of use to remember, that in any arc *b o s*, not



exceeding 29°, or in other words, *whose chord bs is at least sixteen times its rise*, the **middle ordinate** *a o*, will be one-half of *a c*, quite near enough for many purposes; *b c* and *s c* being tangents to the arc.† And vice versa, if in such an arc we make *o c* equal *a o*, then will *c* be, very nearly, the point at which tangents from the ends of the arc will meet. Also the **middle ordinate** *n*, **of the half arc** *o b*, or *os*, will be approximately ¼ of *a o*, the middle ordinate of the whole arc. Indeed, this last observation will apply near enough for many approximate uses even if the arc be as great as 45°; for if in that case we take ¼ of *o a* for the ordinate *n*, *n* will then be but 1 part in 104 too small; and therefore the principle may often be used in drawings, for finding points in a curve of too great radius to be drawn by the dividers; for in the same manner, ¼ of *n* will be the middle ordinate for the arc *n b* or *n o*; and so on to any extent. Below will be found **a table by which the rise or middle ordinate of a half arc** can be obtained with greater accuracy when required for more exact drawings.

CIRCULAR ARCS IN FREQUENT USE.

The fifth column is of use for finding points for drawing arcs too large for the beam-compass, on the principle given above. In even the largest office drawings it will not be necessary to use more than the first three decimals of the fifth column; and after the arc is subdivided into parts smaller than about 35° each, the first two decimals .25 will generally suffice. Original.

Rise in parts of chord.	Degrees in whole arc.	For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by	Rise in parts of chord.	Degrees in whole arc.	For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by
	° ′					° ′			
1-50	9 9.75	313.	1.00107	.2501	⅛	56 8 70	8.5	1.04116	.2538
1-45	10 10.75	253.625	1.00132	.2501	1-7	63 46.90	6.625	1.05356	.2549
1-40	11 26.98	200.5	1.00167	.2502	.155	68 53.63	5.70291	1.06288	.2557
1-35	13 4.92	153.625	1.00219	.2502	1-6	73 44 39	5.	1.07250	.2566
1-30	15 15.38	113.	1.00296	.2503	.18	79 11.73	4.35803	1.08428	.2576
1-25	18 17.74	78.625	1.00426	.2504	1-5	87 12.34	3.625	1.10347	.2593
1-20	22 50.54	50.5	1.00665	.2506	.207107	90	3.41422	1 11072	.2599
1-19	24 2.16	45.625	1.00737	.2507	.225	96 54.66	2.96914	1.12996	.2615
1-18	25 21.65	41.	1.00821	.2508	¼	106 15.61	2.5	1.15912	.2639
1-17	26 50.36	36.625	1.00920	.2509	.275	115 14 59	2.15289	1.19082	.2665
1-16	28 30.00	32.5	1.01038	.2510	.3	123 51.30	1.88889	1.22495	.2692
1-15	30 22.71	28.625	1.01181	.2511	⅓	134 45.62	1.625	1.27401	.2729
1-14	32 31.22	25.	1.01355	2513	.365	144 31.07	1.43826	1.32413	.2766
1-13	34 59.08	21.625	1.01571	.2515	.4	154 38.35	1.28125	1.38322	.2808
1-12	37 50.96	18.5	1.01842	.2517	.425	161 27.49	1.19204	1.42764	.2838
1-11	41 13.16	15.625	1.02189	.2520	.45	167 56.93	1.11728	1.47377	.2868
1-10	45 14.38	13.	1.02646	.2525	.475	174 7.49	1.05401	1.52152	.2899
1-9	50 6.91	10.625	1.03260	.2530	.5	180	1.	1.57080	.2929

For **Rectification of circular arcs,** see p 184.

If radius = 1 mile, 1° = 92.1534 ft;
1′ = 1.53589 ft = 18.4307 ins,
1″ = 0.0256 ft = 0.3072 in.

† When arc *b o s* = 29°, half *a c* = 1.016450 × *a o*.

TABLE OF CIRCULAR ARCS. No errors.

Lengths of circular arcs. If arc exceeds a semicircle, see p 184.

Rise/Chord	Arc/Chord	Rise/Chord	Arc/Chord	Rise/Chord	Arc/Chord	Rise/Chord	Arc/Chord	Rise/Chord	Arc/Chord
.001	1.00002	.076	1.01533	.151	1.05973	.226	1.13108	.301	1.22636
.002	1.00002	.077	1.01573	.152	1.06051	.227	1.13219	.302	1.22778
.003	1.00003	.078	1.01614	.153	1.06130	.228	1.13331	.303	1.22920
.004	1.00004	.079	1.01656	.154	1.06209	.229	1.13444	.304	1.23063
.005	1.00007	.080	1.01698	.155	1.06288	.230	1.13557	.305	1.23206
.006	1.00010	.081	1.01741	.156	1.06368	.231	1.13671	.306	1.23349
.007	1.00013	.082	1.01784	.157	1.06449	.232	1.13785	.307	1.23492
.008	1.00017	.083	1.01828	.158	1.06530	.233	1.13900	.308	1.23636
.009	1.00022	.084	1.01872	.159	1.06611	.234	1.14015	.309	1.23781
.010	1.00027	.085	1.01916	.160	1.06693	.235	1.14131	.310	1.23926
.011	1.00032	.086	1.01961	.161	1.06775	.236	1.14247	.311	1.24070
.012	1.00038	.087	1.02006	.162	1.06858	.237	1.14363	.312	1.24216
.013	1.00045	.088	1.02052	.163	1.06941	.238	1.14480	.313	1.24361
.014	1.00053	.089	1.02098	.164	1.07025	.239	1.14597	.314	1.24507
.015	1.00061	.090	1.02146	.165	1.07109	.240	1.14714	.315	1.24654
.016	1.00069	.091	1.02192	.166	1.07194	.241	1.14832	.316	1.24801
.017	1.00078	.092	1.02240	.167	1.07279	.242	1.14951	.317	1.24948
.018	1.00087	.093	1.02289	.168	1.07365	.243	1.15070	.318	1.25095
.019	1.00097	.094	1.02339	.169	1.07451	.244	1.15189	.319	1.25243
.020	1.00107	.095	1.02389	.170	1.07537	.245	1.15308	.320	1.25391
.021	1.00117	.096	1.02440	.171	1.07624	.246	1.15428	.321	1.25540
.022	1.00128	.097	1.02491	.172	1.07711	.247	1.15549	.322	1.25689
.023	1.00140	.098	1.02542	.173	1.07799	.248	1.15670	.323	1.25838
.024	1.00153	.099	1.02593	.174	1.07888	.249	1.15791	.324	1.25988
.025	1.00167	.100	1.02646	.175	1.07977	.250	1.15912	.325	1.26138
.026	1.00182	.101	1.02698	.176	1.08066	.251	1.16034	.326	1.26288
.027	1.00196	.102	1.02752	.177	1.08156	.252	1.16156	.327	1.26437
.028	1.00210	.103	1.02806	.178	1.08246	.253	1.16279	.328	1.26588
.029	1.00225	.104	1.02860	.179	1.08337	.254	1.16402	.329	1.26740
.030	1.00240	.105	1.02914	.180	1.08428	.255	1.16526	.330	1.26892
.031	1.00256	.106	1.02970	.181	1.08519	.256	1.16650	.331	1.27044
.032	1.00272	.107	1.03026	.182	1.08611	.257	1.16774	.332	1.27196
.033	1.00289	.108	1.03082	.183	1.08704	.258	1.16899	.333	1.27349
.034	1.00307	.109	1.03139	.184	1.08797	.259	1.17024	.334	1.27502
.035	1.00327	.110	1.03196	.185	1.08890	.260	1.17150	.335	1.27656
.036	1.00345	.111	1.03254	.186	1.08984	.261	1.17276	.336	1.27810
.037	1.00364	.112	1.03312	.187	1.09079	.262	1.17403	.337	1.27964
.038	1.00384	.113	1.03371	.188	1.09174	.263	1.17530	.338	1.28118
.039	1.00405	.114	1.03430	.189	1.09269	.264	1.17657	.339	1.28273
.040	1.00426	.115	1.03490	.190	1.09365	.265	1.17784	.340	1.28428
.041	1.00447	.116	1.03551	.191	1.09461	.266	1.17912	.341	1.28583
.042	1.00469	.117	1.03611	.192	1.09557	.267	1.18040	.342	1.28739
.043	1.00492	.118	1.03672	.193	1.09654	.268	1.18169	.343	1.28895
.044	1.00515	.119	1.03734	.194	1.09752	.269	1.18299	.344	1.29052
.045	1.00539	.120	1.03797	.195	1.09850	.270	1.18429	.345	1.29209
.046	1.00563	.121	1.03860	.196	1.09949	.271	1.18559	.346	1.29366
.047	1.00587	.122	1.03923	.197	1.10048	.272	1.18689	.347	1.29523
.048	1.00612	.123	1.03987	.198	1.10147	.273	1.18820	.348	1.29681
.049	1.00638	.124	1.04051	.199	1.10247	.274	1.18951	.349	1.29839
.050	1.00665	.125	1.04116	.200	1.10347	.275	1.19082	.350	1.29997
.051	1.00692	.126	1.04181	.201	1.10447	.276	1.19214	.351	1.30156
.052	1.00720	.127	1.04247	.202	1.10548	.277	1.19346	.352	1.30315
.053	1.00748	.128	1.04313	.203	1.10650	.278	1.19479	.353	1.3047[illegible]
.054	1.00776	.129	1.04380	.204	1.10752	.279	1.19612	.354	1.30634
.055	1.00805	.130	1.04447	.205	1.10855	.280	1.19746	.355	1.30794
.056	1.00834	.131	1.04515	.206	1.10958	.281	1.19880	.356	1.30954
.057	1.00864	.132	1.04584	.207	1.11062	.282	1.20014	.357	1.31115
.058	1.00895	.133	1.04652	.208	1.11165	.283	1.20149	.358	1.31276
.059	1.00926	.134	1.04722	.209	1.11269	.284	1.20284	.359	1.31437
.060	1.00957	.135	1.04792	.210	1.11374	.285	1.20419	.360	1.31599
.061	1.00989	.136	1.04862	.211	1.11479	.286	1.20555	.361	1.31761
.062	1.01021	.137	1.04932	.212	1.11584	.287	1.20691	.362	1.31923
.063	1.01054	.138	1.05003	.213	1.11690	.288	1.20827	.363	1.32086
.064	1.01088	.139	1.05075	.214	1.11796	.289	1.20964	.364	1.32249
.065	1.01123	.140	1.05147	.215	1.11904	.290	1.21102	.365	1.32413
.066	1.01158	.141	1.05220	.216	1.12011	.291	1.21239	.366	1.32577
.067	1.01193	.142	1.05293	.217	1.12118	.292	1.21377	.367	1.32741
.068	1.01228	.143	1.05367	.218	1.12225	.293	1.21515	.368	1.32905
.069	1.01264	.144	1.05441	.219	1.12334	.294	1.21654	.369	1.33069
.070	1.01302	.145	1.05516	.220	1.12444	.295	1.21794	.370	1.33234
.071	1.01338	.146	1.05591	.221	1.12554	.296	1.21933	.371	1.33399
.072	1.01376	.147	1.05667	.222	1.12664	.297	1.22073	.372	1.33564
.073	1.01414	.148	1.05743	.223	1.12774	.298	1.22213	.373	1.33730
.074	1.01453	.149	1.05819	.224	1.12885	.299	1.22354	.374	1.33896
.075	1.01493	.150	1.05896	.225	1.12997	.300	1.22495	.375	1.34063

See p. 184.

TABLE OF CIRCULAR ARCS—(CONTINUED.)

Rise / Chord	Arc / Chord	Rise / Chord	Arc / Chord	Rise / Chord	Arc / Chord	Rise / Chord	Arc / Chord	Rise / Chord	Arc / Chord
.376	1.34229	.401	1.38496	.426	1.42945	.451	1.47565	.476	1.52346
.377	1.34396	.402	1.38671	.427	1.43127	.452	1.47753	.477	1.52541
.378	1.34563	.403	1.38846	.428	1.43309	.453	1.47942	.478	1.52736
.379	1.34731	.404	1.39021	.429	1.43491	.454	1.48131	.479	1.52931
.380	1.34899	.405	1.39196	.430	1.43673	.455	1.48320	.480	1.53126
.381	1.35068	.406	1.39372	.431	1.43856	.456	1.48509	.481	1.53322
.382	1.35237	.407	1.39548	.432	1.44039	.457	1.48699	.482	1.53518
.383	1.35406	.408	1.39724	.433	1.44222	.458	1.48889	.483	1.53714
.384	1.35575	.409	1.39900	.434	1.44406	.459	1.49079	.484	1.53910
.385	1.35744	.410	1.40077	.435	1.44589	.460	1.49269	.485	1.54106
.386	1.35914	.411	1.40254	.436	1.44773	.461	1.49460	.486	1.54302
.387	1.36084	.412	1.40432	.437	1.44957	.462	1.49651	.487	1.54499
.388	1.36254	.413	1.40610	.438	1.45142	.463	1.49842	.488	1.54696
.389	1.36425	.414	1.40788	.439	1.45327	.464	1.50033	.489	1.54893
.390	1.36596	.415	1.40966	.440	1.45512	.465	1.50224	.490	1.55091
.391	1.36767	.416	1.41145	.441	1.45697	.466	1.50416	.491	1.55289
.392	1.36939	.417	1.41324	.442	1.45883	.467	1.50608	.492	1.55487
.393	1.37111	.418	1.41503	.443	1.46069	.468	1.50800	.493	1.55685
.394	1.37283	.419	1.41682	.444	1.46255	.469	1.50992	.494	1.55884
.395	1.37455	.420	1.41861	.445	1.46441	.470	1.51185	.495	1.56083
.396	1.37628	.421	1.42041	.446	1.46628	.471	1.51378	.496	1.56282
.397	1.37801	.422	1.42221	.447	1.46815	.472	1.51571	.497	1.56481
.398	1.37974	.423	1.42402	.448	1.47002	.473	1.51764	.498	1.56681
.399	1.38148	.424	1.42583	.449	1.47189	.474	1.51958	.499	1.56881
.400	1.38322	.425	1.42764	.450	1.47377	.475	1.52152	.500	1.57080

If the arc, A, is greater than a semicircle, let H = its hight, a = circumf of circle — A, and h = hight of arc, a, = (half-chord)^2 ÷ H. Find a, by table, as above. Then A = circumf — a.

Rectification of Circular Arc.*

(a) Fig. A. **To find approximately* the length of an arc,** $G\ B\ C$. From one end, C, of the arc, draw a tangent, $C\ E$, to the arc (See p 162). Produce the chord, $G\ F\ C$, to D, and make $C\ D = F\ C = \frac{1}{2}\ G\ F\ C$. From D, with radius = $D\ G$, draw the arc $G\ E$. Then $C\ E$ = arc $C\ B\ G$, approximately.*

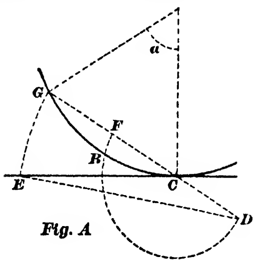


Fig. A

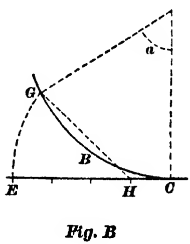


Fig. B

(b) Fig B. On a given circle, **to find an arc,** $G\ C$, approximately* **equal to a given straight line.** $C\ E$. From a point, C, on the circle, draw $C\ E$, tangent, of the given length. Make $C\ H = \frac{1}{4}\ C\ E$. From H, with radius = $H\ E$, draw arc $E\ G$. Then arc $C\ B\ G = C\ E$ approximately.*

(c). On a given circle, **to lay off an arc equal in length to a given arc** on another circle: first, by (a) find a straight line equal to the given arc; then, by (b) lay off the required arc equal to the line so found

* Prof. W. J. M. Rankine, Philosophical Mag., Vol. 34, Oct. and Nov., 1867 The error increases with the fourth power of the angle; but, in an arc of 60 degrees, the error is only about 4 minutes.

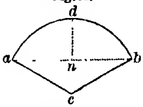
LENGTHS OF CIRCULAR ARCS TO RADIUS 1

= Arcs in radians. See p. 97, ¶ 2.

Deg.	Length	Deg.	Length	Min	Length.	Sec.	Length.
1	.0174533	61	1.0646508	1	.0002909	1	.0000048
2	.0349066	62	1.0821041	2	0005818	2	.0000097
3	.0523599	63	1.0995574	3	.0008727	3	.0000145
4	.0698132	64	1.1170107	4	.0011636	4	.0000194
5	.0872665	65	1.1344640	5	.0014544	5	.0000242
6	.1047198	66	1.1519173	6	.0017453	6	.0000291
7	.1221730	67	1.1693706	7	.0020362	7	.0000339
8	.1396263	68	1.1868239	8	.0023271	8	.0000388
9	.1570796	69	1.2042772	9	.0026180	9	.0000436
10	.1745329	70	1.2217305	10	.0029089	10	.0000485
11	.1919862	71	1.2391838	11	.0031998	11	.0000533
12	.2094395	72	1 2566371	12	.0034907	12	.0000582
13	.2268928	73	1.2740904	13	.0037815	13	.0000630
14	.2443461	74	1.2915436	14	.0040724	14	0000679
15	.2617994	75	1.3089969	15	0043633	15	.0000727
16	.2792527	76	1.3264502	16	.0046542	16	.0000776
17	.2967060	77	1 3439035	17	.0049451	17	.0000824
18	.3141593	78	1.3613568	18	.0052360	18	.0000873
19	.3316126	79	1.3788101	19	.0055269	19	.0000921
20	.3490659	80	1.3962634	20	.0058178	20	.0000970
21	.3665191	81	1.4137167	21	0061087	21	.0001018
22	.3839724	82	1.4311700	22	.0063995	22	.0001067
23	.4014257	83	1.4486233	23	.0066904	23	.0001115
24	.4188790	84	1 4660766	24	.0069813	24	.0001164
25	.4363323	85	1.4835299	25	.0072722	25	.0001212
26	.4537856	86	1 5009832	26	.0075631	26	.0001261
27	.4712389	87	1.5184364	27	0078540	27	.0001309
28	.4886922	88	1.5358897	28	.0081449	28	.0001357
29	5061455	89	1.5533430	29	.0084358	29	.0001406
30	.5235988	90	1 5707963	30	.0087266	30	.0001454
31	.5410521	91	1.5882496	31	.0090175	31	.0001503
32	.5585054	92	1.6057029	32	.0093084	32	.0001551
33	.5759587	93	1.6231562	33	.0095993	33	.0001600
34	.5934119	94	1 6406095	34	.0098902	34	.0001648
35	.6108652	95	1.6580628	35	.0101811	35	.0001697
36	.6283185	96	1.6755161	36	.0104720	36	.0001745
37	.6457718	97	1.6929694	37	0107629	37	.0001794
38	.6632251	98	1.7104227	38	.0110538	38	.0001842
39	.6806784	99	1.7278760	39	.0113446	39	.0001891
40	.6981317	100	1.7453293	40	.0116355	40	0001939
41	.7155850	101	1 7627825	41	.0119264	41	.0001988
42	.7330383	102	1.7802358	42	.0122173	42	.0002036
43	.7504916	103	1 7976891	43	0125082	43	.0002085
44	.7679449	104	1.8151424	44	.0127991	44	.0002133
45	.7853982	105	1.8325957	45	.0130900	45	.0002182
46	.8028515	106	1.8500490	46	.0133809	46	.0002230
47	.8203047	107	1 8675023	47	.0136717	47	.0002279
48	.8377580	108	1.8849556	48	.0139626	48	.0002327
49	.8552113	109	1.9024089	49	.0142535	49	.0002376
50	.8726646	110	1.9198622	50	0145444	50	.0002424
51	.8901179	111	1 9373155	51	.0148353	51	.0002473
52	.9075712	112	1.9547688	52	.0151262	52	.0002521
53	.9250245	113	1 9722221	53	.0154171	53	.0002570
54	.9424778	114	1.9896753	54	.0157080	54	.0002618
55	.9599311	115	2.0071286	55	.0159989	55	.0002666
56	.9773844	116	2.0245819	56	.0162897	56	.0002715
57	.9948377	117	2.0420352	57	.0165806	57	.0002763
58	1 0122910	118	2.0594885	58	.0168715	58	.0002812
59	1.0297443	119	2 0769418	59	.0171624	59	.0002860
60	1.0471976	120	2.0943951	60	.0174533	60	.0002909

CIRCULAR SECTORS, RINGS, SEGMENTS, ETC.

Fig. A.

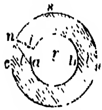


Area of a circular sector, $a\ d\ b\ c$, **Fig. A,**

$= \frac{\text{arc } a\,d\,b}{2} \times \text{radius } c\,a.$

$= \text{area of entire circle} \times \frac{\text{arc } a\,d\,b \text{ in degrees.}}{360}$

Fig. B.



Area of a circular ring, Fig. B,

= area of larger circle, $c\ d$, — area of smaller one, $a\ b$.

= .7854 $\times$ (sum of diams. $c\,d + a\,b$) $\times$ (diff. of diams. $c\,d - a\,b$.)

= 1.5708 $\times$ thickness $c\,a$ $\times$ sum of diameters $c\ d$ and $a\ b$.

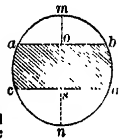
To find the radius of a circle which shall have the same area as a given circular ring $c\ s\ d\ a\ b$, Fig. B,

Draw any radius $n\ r$ of the outer circle; and from where said radius cuts the inner circle at t, draw $t\ s$ at right angles to it. Then will $t\ s$ be the required radius.

Breadth, $c\ a = b\ d$, **of a circular ring,** Fig. B,

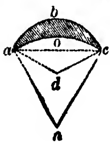
= ½ difference of diameters $c\ d$ and $a\ b$.

= ½ (diameter $c\ d - \sqrt{1.2732 \text{ area of circle } a\,b}$.)



Area of a circular zone $a\ b\ c\ d$,

= area of circle $m\ n$—areas of segments $a\,m\,b$ and $c\,n\,d$,
(for areas of segments, see below.)



A circular lune is a crescent-shaped figure, comprised between two arcs $a\,b\,c$ and $a\,o\,c$ of circles of different radii, $a\ d$ and $a\ n$.

Area of a circular lune $a\ b\ c\ o$

= area of segment $a\ b\ c$ — area of segment $a\ o\ c$,
(for areas of segments see below.)

Circular Segments.

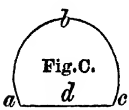
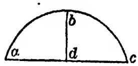


Fig. D.



Area of Segment $a\ d\ b\ n$, Fig. A (at top of page)
= Area of Sector $a\ d\ b\ c$ — Area of Triangle $a\ b\ c$.
= ½ (Arc $a\ d\ b$ $\times$ radius $a\ c$ — $c\ n$ $\times$ chord $a\ b$).

Table of Circular Segments, p 187. D = diam of circle.

Rise of seg = R D. **Area of seg** = D^2 A. **If the seg exceeds a semicircle,**

Area = area of circle — area of seg whose rise = (D — rise of given seg.)

For chord, etc. see pp 179–180.

Table of Areas of Circular Segments, Figs C, D, p 186.
D = diam of circle; **R** = Rise / D; **A** = Area / D^2.

R	A	R	A	R	A	R	A	R	A
.001	.000042	.064	.021168	.127	.057991	.190	.103900	.253	.156149
.002	.000119	.065	.021660	.128	.058658	.191	.104686	.254	.157019
.003	000219	.066	.022155	.129	.059328	.192	.105472	.255	.157891
.004	.000337	067	.022653	.130	.059999	.193	.106261	.256	.158763
.005	.000471	.068	.023155	.131	.060673	.194	.107051	.257	.159636
.006	.000619	.069	.023660	.132	.061349	.195	.107843	.258	.160511
.007	.000779	.070	.024168	.133	.062027	.196	.108636	.259	.161386
.008	.000952	.071	.024680	.134	.062707	.197	.109431	.260	.162263
.009	.001133	.072	025196	.135	.063389	.198	.110227	.261	.163141
.010	.001329	.073	.025714	.136	.064074	.199	.111025	.262	.164020
.011	.001533	.074	.026236	.137	.064761	.200	.111824	.263	.164900
.012	.001746	075	.026761	.138	.065449	.201	.112625	.264	.165781
.013	.001969	076	.027290	.139	.066140	.202	.113427	.265	.166663
.014	.002199	.077	.027821	.140	.066833	.203	.114231	.266	.167546
.015	.002438	078	.028356	.141	.067528	.204	.115036	.267	.168431
.016	.002685	079	.028894	.142	.068225	.205	.115842	.268	.169316
.017	.002940	.080	.029435	.143	.068924	.206	.116651	.269	.170202
.018	.003202	.081	.029979	.144	.069626	.207	.117460	.270	.171090
.019	.003472	082	.030526	.145	.070329	.208	.118271	.271	.171978
020	.003749	.083	031077	.146	.071034	.209	.119084	272	.172868
.021	.004032	.084	.031630	.147	.071741	.210	.119898	.273	.173758
.022	.004322	.085	032186	.148	.072450	.211	.120713	.274	.174650
.023	.004619	.086	.032746	.149	.073162	.212	.121530	.275	.175542
.024	.004922	.087	.033308	.150	.073875	.213	.122348	.276	.176436
.025	.005231	.088	.033873	.151	.074590	.214	.123167	.277	.177330
026	.005546	.089	.034441	.152	.075307	.215	.123988	.278	.178226
027	.005867	.090	.035012	.153	.076026	.216	.124811	.279	.179122
.028	.006194	.091	.035586	.154	.076747	.217	.125634	.280	.180020
.029	.006527	.092	.036162	155	.077470	.218	.126459	.281	.180918
.030	.006866	.093	.036742	.156	.078194	.219	.127286	.282	.181818
.031	007209	.094	.037324	.157	.078921	.220	.128114	.283	.182718
.032	.007559	.095	.037909	.158	.079650	.221	.128943	.284	.183619
.033	.007913	.096	.038497	.159	.080380	.222	.129773	.285	.184522
.034	.008273	.097	.039087	.160	.081112	.223	.130605	.286	.185425
.035	.008638	.098	.039681	.161	.081847	.224	.131438	.287	.186329
.036	.009008	.099	.040277	162	.082582	.225	.132273	.288	.187235
.037	.009383	.100	.040875	163	.083320	.226	.133109	.289	.188141
.038	.009764	.101	.041477	.164	.084060	.227	.133946	.290	.189048
039	.010148	.102	.042081	.165	.084801	.228	.134784	.291	.189956
.040	.010538	.103	.042687	.166	.085545	.229	.135624	.292	.190865
041	.010932	.104	.043296	.167	.086290	.230	.136465	.293	.191774
.042	.011331	.105	.043908	.168	.087037	.231	.137307	.294	.192685
.043	.011734	.106	.044523	.169	.087785	.232	.138151	.295	.193597
.044	.012142	.107	.045140	.170	.088536	.233	.138996	.296	.194509
.045	.012555	.108	.045759	.171	.089288	.234	.139842	.297	.195423
.046	.012971	.109	.046381	.172	.090042	.235	.140689	.298	.196337
.047	.013393	.110	.047006	.173	.090797	.236	.141538	.299	.197252
.048	.013818	.111	.047633	.174	.091555	.237	.142388	.300	.198168
.049	.014248	.112	.048262	.175	.092314	.238	.143239	.301	.199085
.050	.014681	.113	.048894	.176	.093074	.239	.144091	.302	.200003
.051	.015119	.114	.049529	.177	.093837	.240	.144945	.303	.200922
.052	.015561	.115	.050165	.178	.094601	241	.145800	.304	.201841
.053	.016008	.116	.050805	.179	.095367	.242	.146656	.305	.202762
.054	.016458	.117	.051446	.180	.096135	.243	.147513	.306	203683
.055	.016912	.118	.052090	.181	.096904	.244	.148371	.307	.204605
.056	.017369	.119	.052737	.182	.097675	.245	.149231	.308	.205528
.057	.017831	.120	.053385	.183	.098447	.246	.150091	.309	.206452
.058	.018297	.121	.054037	.184	.099221	.247	.150953	.310	.207376
.059	.018766	.122	.054690	.185	.099997	.248	.151816	.311	.208302
.060	.019239	.123	.055346	.186	.100774	.249	.152681	.312	.209228
.061	.019716	.124	.056004	.187	.101553	.250	.153546	.313	.210155
.062	.020197	.125	.056664	.188	.102334	.251	.154413	.314	.211083
.063	.020681	.126	.057327	.189	.103116	.252	.155281	.315	212011

Continued on next page.

Table of Areas of Circular Segments, Figs C, D, p 186.
Continued.

D = diam of circle; **R** = Rise / D; **A** = Area / D^2.

R	A	R	A	R	A	R	A	R	A
.316	.212941	.353	.247845	.390	.283593	.427	.319959	.464	.356730
.317	.213871	.354	.248801	.391	.284569	.428	.320949	.465	.357728
.318	.214802	.355	.249758	.392	.285545	.429	.321938	.466	.358725
.319	.215734	.356	.250715	.393	.286521	.430	.322928	.467	.359723
.320	.216666	.357	.251673	.394	.287499	.431	.323919	.468	.360721
.321	.217600	.358	.252632	.395	.288476	.432	.324909	.469	.361719
.322	.218534	.359	.253591	.396	.289454	.433	.325900	.470	362717
.323	.219469	.360	.254551	.397	.290432	.434	.326891	.471	.363716
.324	.220404	.361	.255511	.398	.291411	.435	.327883	.472	.364714
.325	.221341	.362	.256472	.399	.292390	436	.328874	.473	.365712
.326	.222278	.363	.257433	.400	.293370	.437	.329866	.474	.366711
.327	.223216	.364	.258395	.401	.294350	.438	.330858	.475	.367710
.328	.224154	.365	.259358	.402	.295330	.439	.331851	.476	.368708
.329	.225094	.366	.260321	.403	.296311	.440	.332843	.477	.369707
.330	.226034	367	.261285	.404	.297292	441	.333836	.478	.370706
.331	.226974	.368	.262249	.405	.298274	.442	.334829	.479	.371705
.332	.227916	.369	.263214	.406	.299256	443	.335823	.480	.372704
.333	.228858	.370	.264179	.407	.300238	444	.336816	.481	.373704
.334	.229801	371	.265145	.408	.301221	.445	.337810	.482	.374703
.335	.230745	.372	.266111	.409	.302204	.446	.338804	.483	.375702
.336	.231689	.373	.267078	.410	.303187	447	.339799	.484	.376702
.337	.232634	.374	.268046	.411	.304171	448	.340793	.485	.377701
.338	.233580	.375	.269014	.412	.305156	.449	.341788	.486	.378701
.339	.234526	.376	.269982	.413	.306140	.450	.342783	.487	.379701
.340	.235473	.377	.270951	.414	.307125	.451	.343778	.488	.380700
.341	.236421	.378	.271921	.415	.308110	.452	.344773	.489	.381700
.342	.237369	.379	.272891	416	.309096	.453	.345768	.490	.382700
.343	.238319	.380	.273861	.417	.310082	.454	.346764	.491	.383700
.344	.239268	.381	.274832	.418	.311068	.455	.347760	.492	384699
.345	.240219	.382	.275804	.419	.312055	456	.348756	.493	.385699
.346	.241170	383	.276776	.420	.313042	.457	.349752	.494	.386699
.347	.242122	.384	.277748	.421	.314029	458	.350749	.495	387699
.348	.243074	.385	.278721	.422	315017	.459	.351745	.496	.388699
.349	.244027	.386	.279695	.423	.316005	.460	352742	.497	.389699
.350	.244980	.387	.280669	.424	.316993	.461	.353739	.498	.390699
.351	.245935	.388	.281643	.425	.317981	.462	.354736	.499	.391699
.352	.246890	.389	.282618	.426	.318970	.463	.355733	.500	.392699

THE ELLIPSE

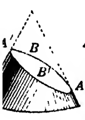


Fig. 1

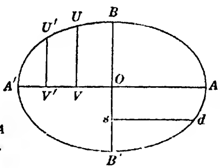


Fig. 2

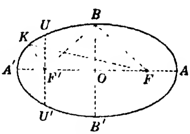


Fig. 3

The ellipse, Fig 1, *A B A' B'*, is the curve formed by the inter-
:tion of a plane with the curved surface of a cone or cylinder
ru which it passes completely and without cutting its base. The
me, "ellipse", is applied also to the plane *figure* enclosed by said
rve, which curve is then called the circumference or periphery of
e ellipse. When the cutting plane is perpendicular to the axis

of the solid ($e = 0$), the ellipse is a circle. When the plane touches the side of the solid ($e = 1$), the ellipse is a straight line.

A diameter, Figs 2 and 3, is any chord passing thru the center, O. Each diam is bisected at the center. An axis, $A A'$ or $B B'$, is a diam dividing the figure symmetrically. The major axis passes thru the foci, F, F', Fig 3. The vertices, A, A', B, B', are the extremities of the axes.

Figs 2-4.	O	=	the center of the ellipse;
	a	=	$O A$ = $O A'$ = the major or transverse semi-axis;
	b	=	$O B$ = $O B'$ = the minor or conjugate semi-axis;
Fig 2.	y, y'	=	$V U, V' U'$ = the ordinate over a point, V, V', in an axis;
	x	=	$O V$ = the distance of y from the center, O;
	x'	=	$A' V$ = $a - x$ = dist of y from the vertex, A';
Fig 3.	F, F'	=	the foci of the ellipse;
	c	=	$O F$ = $O F'$ = $a e$
		=	dist from center, O, to either focus;
	$2 c$	=	$F F'$ = the focal distance;
	e	=	c/a = $\sqrt{(a^2 - b^2)/a^2}$ = the eccentricity;
	Y	=	$F'U$ = the ordinate over either focus;
	p	=	$U U' = 2 Y$ = parameter or latus rectum;
	L	=	$A B A'B'A$ = the circumference or periphery;
Fig 4	r, r'	=	focal radii or radius vectors = dists, $F G$, $F'G$; or $F K$, $F'K$;
	θ, θ'	=	the angles, $G F A'$, $G F'A$; or $K F A'$, $KF'A$;
	S	=	the area of the ellipse;

Equations. Fig 2

(a) Origin of co-ordinates at cen, O : $x = O V$; $y = V U$.

$$(x^2/a^2) + (y^2/b^2) = 1; \text{ whence}$$

$$x = \pm (a/b) \sqrt{b^2 - y^2}; \quad y = \pm (b/a) \sqrt{a^2 - x^2} \quad \ldots\ldots (1)$$

$$y^2 = (a + x)(a - x) b^2/a^2 \quad \ldots\ldots (2)$$

(b) Origin taken at a vertex, A', of major axis, $A A'$; $x' = A'V$.

Here, in Eq 1, $x = a - x'$; $y = \pm (b/a) \sqrt{2ax' - x'^2}$

$$x' = a - x = a + (a/b) \sqrt{b^2 - y^2} \quad \ldots\ldots (3)$$

Ordinates. $VU^2 : AV.VA' = b^2 : a^2$;

$$VU^2 : V'U'^2 = AV.VA' : AV'.V'A' \quad \ldots\ldots (4)$$

Foci. Fig 4. The normal, $P X$, to a tangent, $T T$, thru any point, K, in an ellipse, bisects the angle, FKF', betw the focal radii, KF and KF', of the point K. Hence, a ray (as of light), emitted from either focus, is reflected back to the other. The bisectors, PX, give the positions of the **joints in an elliptic arch,** and enable us **to draw a tangent,** TT, (at right-angles to the bisector) at any point in the ellipse.

Fig 3 Let K and B be any two points in the ellipse. Then $KF + KF' = BF + BF' =$ major axis, $A A' = 2 a$ ..(5)

Or, the ellipse is the locus of a point, the sum of whose distances from two fixt points is constant and equal to the major axis.

Hence, **to locate the foci;** from either end, as B, of the minor axis, lay off BF and BF', each $= a$, to the major axis, AA'.

To draw an ellipse, Fig 4, having AA' and BB'; (a) Place a pin in each focus, located as above. Prepare a string, FKF' or FGF', with a loop at each end (length of string, from end to end of loops, $= 2 a$), and place a loop over each pin. Then a pencil, beginning as at K, and keeping the string constantly and equally stretched, will describe the ellipse, $K G B A B'A'K$. But the string may stretch unequally. Hence :—

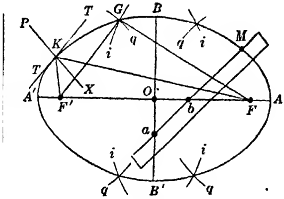


Fig. 4

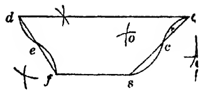


Fig. 7

(b) On a graduated straight-edge, M-a, take M-a = a, and M-b = b. Then, in whatever position the straight-edge is placed, keeping b on AA', and a on BB', M will be in the ellipse.

The ellipsograph (elliptic compasses) employs this principle.

(c) From F and from F', with any radius, R, less than AF', describe four short arcs, $q\ q\ q\ q$; and, with rad $= 2a - R$, four other arcs, $i\ i\ i\ i$. The intersections of these pairs of arcs are points in the ellipse. Find other such points by choosing other radii.

Eccentricity, $e = c/a = \sqrt{(a^2 - b^2)/a^2}$ ....(6)

Polar equation:— $r = \dfrac{a\,(1 - e^2)}{1 - e\cos\theta}$; $r' = \dfrac{a\,(1 - e^2)}{1 - e\cos\theta'}$.(7)

The directrix, EC, is a line such that, for any point, G, in the ellipse, distant g from EC, and with focal radius $r' = F'G$, we have $r'/g = e$. ....(8)

Parameter. Fig 3. For the ordinate, $Y = F'U$, over a focus, F', we have $x^2 = OF'^2 = F'B^2 - OB^2 = a^2 - b^2$; and (eq 1):—
$Y = b^2/a$ ....(9)

Parameter, $UU' = 2Y = 2b^2/a = 2de = 2a(1 - e^2)$ ..(10)
where d = distance from focus to directrix.

Periphery or Circumference. Let $(e^2)/4$ = "II";
$(3e^2/16) \times$ II = "III"; $(15e^2/36) \times$ III = "IV";
$(35e^2/64) \times$ IV = "V", etc. Then —
$L = 2\pi a$ (unity − II − III − IV − V − etc)......(11)
where (− II)/unity $= (-e^2/4)/(+1) = (-\tfrac{1}{4})e^2$;
(− III)/(− II) $= (+\,^3/_{16})e^2$; etc.

Note that, in any given ellipse, e^2 is constant, and that, in the fractions, $(-\,^1/_4)$, $^3/_{16}$, $^{15}/_{36}$, $^{35}/_{64}$, etc, we have:—

Numerators,	(35 − 15) − (15 − 3)	= (15 − 3) − (3 − −1) = 8
Denominators,	(64 − 36) − (36 − 16)	= (36 − 16) − (16 − 4)
		= (16 − 4) − (4 − 0) = 8
Denom − Num,	(64 − 35) − (36 − 15)	= (36 − 15) − (16 − 3)
		= (16 − 3) − (4 − −1) = 8

Or, eq (11) may be written thus:—
$L = 2\pi a\,(1 - Ae^2 - Be^4 - Ce^6 - De^8 - \text{etc})$ .....(12)

Where $A = \dfrac{1}{1}\left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$; $C = \dfrac{1}{5}\left(\dfrac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 = \dfrac{5}{256}$;

$B = \dfrac{1}{3}\left(\dfrac{1 \times 3}{2 \times 4}\right)^2 = \dfrac{3}{64}$; $D = \dfrac{1}{7}\left(\dfrac{1 \times 3 \times 5 \times 7}{2 \times 4 \times 6 \times 8}\right)^2 = \dfrac{175}{16384}$.

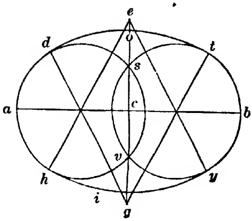


Fig. 5

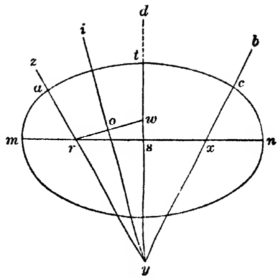


Fig. 6

Approximately :—(from Sir Thomas Muir, F R. S.)

$$L = \frac{\pi}{8}\left(21 \frac{a+b}{2} - 2\sqrt{ab} - \frac{6ab}{a+b}\right) \dots\dots\dots\dots (13)$$

For an ellipse where $a = 10$ and $b = 5$,
eq (11) gives $L = 48.4422$ after deducting term "XXIV";
eq (13) gives $L = 48.4426 = 1.000008 \times 48.4422$

Area of ellipse,

$S = \sqrt{\pi a^2 . \pi b^2} = \pi a b$
$= \pi a^2 (b/a) =$ (area of circle of rad a) $\times$ (b/a) ..(14)

Radius, R, of circle whose area = area, S, of ellipse,
$R = \sqrt{ab}$ ....(15)

Area, Ss, of an elliptic segment whose base is parallel to either axis Divide height of segment by that axis of which said height is a part. Find the quotient under R, in the table of *circular* segments, pp 187-8, and take out the corresponding A. Then
$Ss = 2a \times 2b \times A = 4abA$ ....(16)

The area of an elliptic segment included between two chords, each parallel to either axis, is the diff betw the areas of two segments, each found as above.

To draw an oval, or false ellipse. Fig 5. When only the long diam, *ab*, is given, and when span must not exceed about 3 × *co*. On *ab*, with any radius, *R*, describe two equal intersecting circles. Thru their intersections, *s* and *v*, draw *eg*. Make *sg* = *ve* = 2 *R*. Thru the cens of the circles draw *cy*, *ch*, *gd* and *gt*. From *c*, describe arc *hiy*; and, from *g*, describe *dot*.

Fig 6. When the span, *mn*, and the rise, *st*, are given. Lay off any *tw* and *mr*, equal and each less than *st*. Draw and bisect *rw*, and, thru its cen, *o*, draw the perpendicular, *yoi*. Draw *yrz*. Make *nx* = *mr*, and draw *yxb*. From *x* and from *r*, describe the arcs, *nc* and *ma*; and from *y* describe the arc *atc*. Making *sd* = *sy*, we find the cen, *d*, for the other half of the oval. The shape of the curve depends upon what portion of *st* is taken for *mr* and *tw*. Very flat ovals require more than four cens; and the finding of these cens is quite as troublesome as drawing a true ellipse.

Fig 7. On a given line, *as* or *df*, **to draw a "cyma" or "ogee"**, *acs* or *def*. The cyma recta, *acs*, is concave above; the cyma reversa, *def*, is concave below.

Bisect *as* in *c*. From *a*, *c* and *s*, with radius = $(as)/2$ = *ac* = *cs*, draw the four small arcs at *o* and *o*. The intersections, *o* and *o*, are the cens for drawing the cyma, with the same radius. For *def*, rad = $(df)/2$ = *de* = *cf*.

THE PARABOLA.

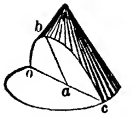


Fig. 1.

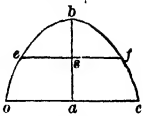


Fig. 2.

The common or conic parabola,

o b c, Fig 1, is a curve formed by cutting a cone in a direction *b a*, parallel to its side. The curved line *o b c* itself is called the *perimeter* of the parabola; the line *o c* is called its *base*; *b a* its height or *axis*; *b* its *apex* or *vertex*; any line *e s*, or *o a*, Fig 2, drawn from the curve, to, and at right angles to, the axis, is an *ordinate*; and the part *s b*, or *a b*, of the axis, between the ordinate and the apex *b*, is an *abscissa*. The *focus* of a parabola is that point in the axis, where the abscissa *b s*, is equal to one-half of the ord *e s*. The dist from apex to focus, called the *focal dist*, is found thus square any ord, as *o a*; div this square by the abscissa *b a* of that ord; div the quot by 4. The nature of the parabola is such that its abscissas, as *b s*, *b a*, &c, are to each other as, or in proportion to, the squares of their respective ords *e s*, *o a*, &c; that is, as $b\,s : b\,a :: e\,s^2 : o\,a^2$; or $b\,s : e\,s^2 \;\; b\,a : o\,a^2$. If the square of any ord be divided by its abscissa, the quot will be a constant quantity; that is, it will be equal to the square of any other ord divided by its abscissa. This quot or constant quantity is also equal to a certain quantity called the *parameter* of the parabola. Therefore the parameter may be found by squaring *e s*, or *o a*, (one-*half* of the base,) and dividing said square by the height *b s*, or *b a*, as the case may be If the square of any ord be divided by the parameter, the quot will be the abscissa of that ord

To find the length of a parabolic curve.

The approximate rule given by various pocket-books, is as follows·

$$\text{Length} = 2 \times \sqrt{(\tfrac{1}{2}\text{ base})^2 + 1\tfrac{1}{3}\text{ times the }(\text{Height}^2)}$$

Where the height does not exceed 1-10th of the base, this rule may, for practical purposes, be called exact. With ht = ¼ base, it gives about ⅙ per cent too much; ht = ½ base, about 3⅓ per cent; ht = base, about 8⅓ per cent; ht = twice the base, about 12⅙ per cent; ht = 10 × base, or more, about 15⅙ per cent

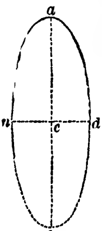


Fig. 4.

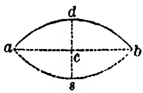


Fig. 3.

The following by the writer is correct within perhaps 1 part in 800 in all cases; and will therefore answer for many purposes.

Let *a d b*, Fig 3, or *n a d*, Fig 4, be the parabola, in which are given the base *a b* or *n d*; and the height *c d* or *c a*. Imagine the complete fig *a d b s*, or *n a d b*, to be drawn; and in *either* case, assume its *long* diam *a b* to be the chord or base, and one-half the short diam, or *c d*, to be the height, of a circular arc Find the length of this circular arc, by means of the rule and table given for that purpose. Then div the chord or base *a b*, or *n d* of the parabola, by its height *c d* or *c a* Look for the quot in the column of bases in the following table, and take from the table the corresponding multiplier. Mult the length of the circular arc by this; the prod will be the length of arc *a d b*, or *n a d*, as the case may be For bases of parabolas less than 05 of the height, or greater than 10 times the height, the multiplier is 1, and is very approximate; or in other words, the parabola will be of almost exactly the same length as the circular arc.

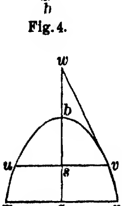


Fig. 5.

To find the area of a parabola *m a n b*.

Mult its base *m n*, Fig 5, by its height *a b*; and take ⅔ds of the prod The area of any segment, as *u b v*, whose base *u v* is parallel to *m n*, is found in the same way, using *u v* and *s b*, instead of *m n* and *a b*.

To find the area of a parabolic zone, or frustum, as *m n u v*.

RULE 1. First find by the preceding rule the area of the whole parabola *m b n*; then that of the segment *u b v*; and subtract the last from the first.

RULE 2. From the cube of *m n*, take the cube of *u v*; call the diff *c*. From the square of *m n*, take the square of *u v*; call the diff *s*. Div *c* by *s*. Mult the quot by ⅔ds of the height *a s*.

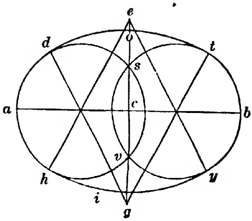


Fig. 5

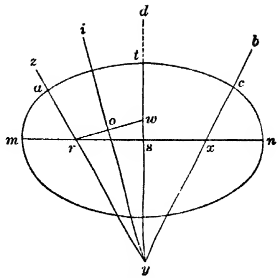


Fig. 6

Approximately :—(from Sir Thomas Muir, F R. S.)

$$L \quad = \quad \frac{\pi}{8} \left(21 \; \frac{a + b}{2} \; - \; 2 \; \sqrt{a\,b} \; - \; \frac{6\,a\,b}{a + b} \right) \ldots\ldots\ldots\ldots (13)$$

For an ellipse where $a \; = \; 10$ and $b \; = \; 5$,
eq (11) gives $L \; = \; 48.4422$ after deducting term "XXIV";
eq (13) gives $L \; = \; 48.4426 \; = \; 1.000008 \times 48.4422$

Area of ellipse,

$S \; = \; \sqrt{\pi\,a^2.\;\pi\,b^2} \; = \; \pi\,a\,b$
$\; = \; \pi\,a^2\,(b/a) \; = \;$ (area of circle of rad a) $\times$ (b/a) ..(14)

Radius, R, of circle whose area = area, S, of ellipse,
$R \; = \; \sqrt{a\,b}$ ....(15)

Area, Ss, of an elliptic segment whose base is parallel to either axis Divide height of segment by that axis of which said height is a part. Find the quotient under R, in the table of *circular* segments, pp 187-8, and take out the corresponding A. Then
$Ss \; = \; 2a \times 2b \times A \; = \; 4\,a\,b\,A$ ....(16)

The area of an elliptic segment included between two chords, each parallel to either axis, is the diff betw the areas of two segments, each found as above.

To draw an oval, or false ellipse. Fig 5. When only the long diam, *ab*, is given, and when span must not exceed about 3 $\times$ *co*. On *ab*, with any radius, R, describe two equal intersecting circles. Thru their intersections, *s* and *v*, draw *eg*. Make $sg = ve = 2\,R$. Thru the cens of the circles draw *cy*, *ch*, *gd* and *gt*. From *c*, describe arc *hiy*; and, from *g*, describe *dot*.

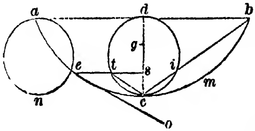
Fig 6. When the span, *mn*, and the rise, *st*, are given. Lay off any *tw* and *mr*, equal and each less than *st*. Draw and bisect *rw*, and, thru its cen, *o*, draw the perpendicular, *yot*. Draw *yrz*. Make $nx = mr$, and draw *yxb*. From *x* and from *r*, describe the arcs, *nc* and *ma*; and from *y* describe the arc *atc*. Making $sd = sy$, we find the cen, *d*, for the other half of the oval. The shape of the curve depends upon what portion of *st* is taken for *mr* and *tw*. Very flat ovals require more than four cens; and the finding of these cens is quite as troublesome as drawing a true ellipse.

Fig 7. On a given line, *as* or *df*, **to draw a "cyma" or "ogee"**, *acs* or *def*. The cyma recta, *acs*, is concave above; the cyma reversa, *def*, is concave below.

Bisect *as* in *c*. From *a*, *c* and *s*, with radius $= (as)/2 = ac = cs$, draw the four small arcs at *o* and *o*. The intersections, *o* and *o*, are the cens for drawing the cyma, with the same radius. For *def*, rad $= (df)/2 = de = ef$.

The Cycloid,

a c b, is the curve described by a point *a* in the circumference of a circle, *a n*, during one complete revolution of the circle, rolled along a straight line *a b*; which is called the base of the cycloid.



The vertex of the cycloid is at *c*.

Base, *a b*, = circumference of generating circle *a n* = diameter, *c d*, of generating circle $\times \pi = 3.1416cd$.

Axis, or height, $cd = an$.

Length, $acb, = 4\,cd$.

Area, $acbd = 3 \times$ area of generating circle, *a n*
$$= 3\frac{cd^2\pi}{4} = cd^2 \times \tfrac{3}{4}\pi = cd^2 \times 2.3562.$$

Center of gravity of *surface* at *g*. $cg = \frac{7}{12}\,cd$. Center of gravity of *cycloid* (curved line *a c b*) in axis *c d* at a point (as *s*) distant $\frac{1}{3}\,cd$ from *c*.

To draw a tangent, *e o*, from any point *e* in a cycloid; draw *e s* at right angles to the axis *c d*; on *c d* describe the generating circle *d c t*; join *t c*; from *e* draw *e o* parallel to *t c*. The cycloid is the **curve of quickest descent:** so that a body would fall from *b* to *c* along the curve *b m c*, in less time than along the inclined plane *b i c*, or any other line.

SOLIDS.

THE REGULAR BODIES.

A regular body, or regular polyhedron, is one which has all its sides, and its solid angles, respectively similar and equal to each other. There are but five such bodies, as follows:

Name.	Bounded by	Surface (= sum of surfaces of all the faces). Multiply the square of the length of one edge by	Volume. Multiply the cube of the length of one edge by
Tetrahedron	4 equilateral triangles.	1.7320	.1178
Hexahedron or cube	6 squares.	6.	1.
Octahedron	8 equilateral triangles.	3.4641	.4714
Dodecahedron	12 " pentagons.	20.6458	7.6631
Icosahedron	20 " triangles.	8.6602	2.1817

Guldinus' Theorem. To find the **volume of any body** (as the irregular mass *a b c m*, Fig A, or the ring *a b c m*, Fig B), generated by a complete or partial revolution of any figure (as *a b c a*) around one of its sides (as *a c*, Fig A), or around any other axis (as *x y*, Fig B).

Fig. A.

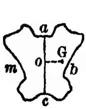
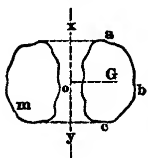


Fig. B.



Volume = surface *a b c a* $\times$ length of arc described by its center of gravity G.

If the revolution is complete, the arc described is = circumference = radius $oG^* \times 2\pi$ = radius $oG^* \times 6.283186$; and

Volume = surface *a b c a* $\times$ radius $oG^* \times 6.283186$.

If the revolution is incomplete,
$$\text{complete revolution} : \text{incomplete revolution} :: \text{circumference found as above} : \text{arc described}$$

* Measured perpendicularly to the axis of revolution.

PARALLELOPIPEDS.

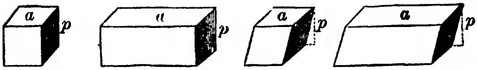


Fig. 1. Fig. 2. Fig. 3. Fig 4.

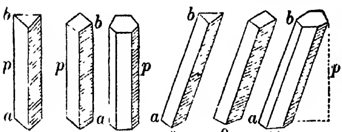
A parallelopiped is any solid contained within six sides, all of which are parallelograms; and those of each opposite pair, parallel to each other. We show but four of them; corresponding to the four parallelograms; namely, the *cube*, Fig 1, which has all its sides equal squares, and all its angles right angles; the *right rectangular prism*, Fig 2, has all its angles right angles, each pair of opposite faces equal, but not all of its faces equal; the *Rhombohedron*, Fig 3, which has all its sides equal rhombuses, and which, like the rhombus, p 157, is sometimes called "rhomb"; the *Rhombic* prism, Fig 4; its faces, rhombuses, or rhomboids, each pair of opposite faces equal, but not all its faces equal. All parallelopipeds are prisms.

Volume of any parallelopiped = area of any face, as a, × *perpendicular* distance, p, to the opposite face.

Volume of a cube = cube of length of one edge,
= 1.909859 × volume of inscribed sphere,
= 1.27324 × " " cylinder,
= 3.81972 × " " cone.

Diagonal of a cube = diameter of circumscribing sphere,
= 1.7320508 × length of one edge of cube.

PRISMS.



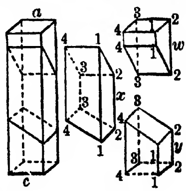
5 6 7 8 9 10

A prism is any solid whose two *ends* are parallel, similar, and equal; and whose *sides are parallelograms*, as Figs 5 to 10. Consequently the foregoing parallelopipeds are prisms. A *right* prism is one whose sides are perpendicular to its ends as 5, 6, 7; when not so, the prism is *oblique*, as 8, 9, 10. When all the sides of the figures which form the ends are equal, and the angles included between those sides are also equal, the prism is said to be *regular*: otherwise, *irregular*.

Volume of any prism (whether regular or irregular, right or oblique)
= area of one end × *perpendicular* distance, p, to the other end,
= area of cross section perpendicular to the sides × actual length, $a\,b$, Figs 5 to 10,
= 3 × volume of pyramid whose base and height are = those of the prism.

To find the volume of any frustum of any prism,

Whose cross section, perpendicular to its sides, is either any triangle; any parallelogram; a square, (as in Fig 10¼) or a *regular* polygon of any number of sides; no matter how the two ends of the frustum may be inclined with regard to each other; or whether one, or neither of them, is parallel to the base of the original prism.



Figs. 10¼.

$$\text{Volume of frustum} = \frac{\text{sum of lengths of } \textit{parallel} \text{ edges, } \overline{1\,1} + \overline{2\,2} + \overline{3\,3} + \overline{4\,4}}{\text{number of such edges (4 in Fig 10¼)}} \times \begin{array}{c}\text{area of cross section}\\ \textit{perpendicular}\\ \text{to such edges.}\end{array}$$

This rule may be used for ascertaining beforehand, the quantity of earth to be removed from a "borrow pit." The irregular surface of the ground is first staked out in squares; (the tape-line being stretched *horizontally*, when measuring off their sides). These squares should be of such a size that without material error each of them may be considered to be a plane surface, either horizontal or inclined. The depth of the horizontal bottom of the pit being determined on, and the levels being taken at every corner of the squares, we are thereby furnished with the lengths of the four parallel vertical edges of each of the resulting frustums of earth. In Figs 10¼, y may be supposed to represent one of these frustums.

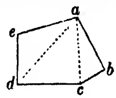


Fig. 10½.

If the frustum is that of an *irregular* 4-sided, or polygonal prism, first divide its cross section perpendicular to its sides, into triangles, by lines drawn from any one of its angles, as *a*, Fig 10½. Calculate the area of each of these triangles separately, then consider the entire frustum to be made up of so many triangular ones; calculate the volume of each of these by the preceding rule for triangular frustums; and add them together, for the volume of the entire frustum.

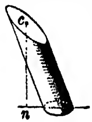


Fig. 10¾.

Volume of any frustum of any prism.

Or of a cylinder. Consider either end to be the base; and find its area. Also find the center of gravity *c* of the other end, and the *perpendicular* distance *n c*, from the base to said center of gravity. Then **Volume of frustum** = area of base × *n c*, Fig 10¾. The slant end, *c*, is an ellipse. Its area is greater than that of the circular end.

Surface of any prism, Figs 5 to 10, whether right or oblique, regular or irregular

$$= \left(\begin{array}{l}\text{circumference measured}\\ \text{perpendicular to the sides}\end{array}\right) \times \text{actual length, } a\,b \Big) + \begin{array}{l}\text{sum of the areas}\\ \text{of the two ends.}\end{array}$$

CYLINDERS.

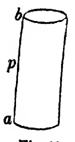


Fig. 11.

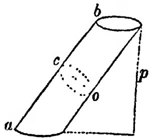


Fig. 12.

A cylinder is any solid whose ends are parallel, similar, and equal *curved* figures; and whose sections parallel to the ends are everywhere the same as the ends. Hence there are circular cylinders, elliptic cylinders (or cylindroids) and many others; but when not otherwise expressed, the circular one is understood. A *right* cylinder is one whose ends are perpendicular to its sides, as Fig 11; when otherwise, it is *oblique*, as Fig 12. If the ends of a right circular cylinder be cut so as to make it oblique, it becomes an elliptic one; because then both its ends, and all sections parallel to them, are ellipses. An oblique circular cylinder seldom occurs; it may be conceived of by imagining the two ends of Fig 12 to be circles, united by straight lines forming its curved sides.

A cylinder is a prism having an infinite number of sides.

Volume of any cylinder (whether circular or elliptic, &c, right or oblique

= area of one end × *perpendicular* distance, *p*, to the other end,

$$= \left\{\begin{array}{l}\text{area of cross section}\\ \text{measured perp to the sides}\end{array}\right. \times \text{actual length, } a\,b\text{, Figs 11 and 12,}$$

= 3 × volume of a cone whose base and height are = those of the cylinder.

Surface of any cylinder (whether circular or elliptic, &c, right or oblique)

$$= \left(\begin{array}{l}\text{circumference}\\ \text{measured perpendicularly}\\ \text{to the sides, as at } c\,o\text{, Fig 12,}\end{array} \times \text{actual length, } a\,b\right) + \begin{array}{l}\text{sum of the areas}\\ \text{of the two ends.}\end{array}$$

Right circular cylinder whose height = diameter.

Volume	= 1½ × volume of inscribed sphere.
Curved surface	= surface of inscribed sphere.
Area of one end	= ¼ surface of inscribed sphere = ¼ curved surface.
Entire surface	= 1½ × surface of inscribed sphere = 1½ × curved surface.

Contents for one foot in length, in Cub Ft, and in U. S. Gallons of 231 cub ins, or 7 4805 Galls to a Cub Ft. **A cub ft of water weighs** about 62½ lbs; and a gallon about 8⅓ lbs **Diams 2, 3, or 10 times as great,** give 4, 9, or 100 times the content.

Diam. in Ins	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft in length. Cub. Feet. Also area in sq. ft.	Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub Feet Also area in sq. ft.	Gallons of 231 Cub. Ins.
¼	0208	.0003	.0025	¾	.5625	.2485	1 859	19.	1.583	1.969	14.73
5-16	.0260	.0005	0040	7.	.5833	.2673	1 999	½	1.625	2.074	15.51
⅜	0313	.0008	.0057	¼	6042	.2867	2.145	20	1.667	2.182	16.32
7-16	.0365	0010	.0078	½	.6250	.3068	2.295	½	1.708	2 292	17.15
½	0417	.0014	0102	¾	.6458	.3276	2.450	21.	1.750	2 405	17.99
9-16	.0469	0017	0129	8	.6667	.3491	2.611	½	1.792	2.521	18.86
⅝	.0521	0021	.0159	¼	.6875	.3712	2.777	22	1.833	2.640	19 75
11-16	.0573	0026	0193	½	.7083	.3941	2.948	½	1.875	2.761	20.66
¾	0625	.0031	0230	¾	.7292	.4176	3 125	23.	1.917	2.885	21.58
13-16	.0677	.0036	0269	9	.7500	.4418	3.305	½	1.958	3 012	22.59
⅞	.0729	.0042	0312	¼	7708	.4667	3 491	24.	2.000	3.142	23.50
15-16	.0781	.0048	.0359	½	.7917	.4922	3.682	25.	2.083	3.409	25.50
1	0833	.0055	.0408	¾	.8125	.5185	3.879	26.	2.167	3.687	27.58
¼	.1042	.0085	.0638	10.	.8333	.5454	4 087	27.	2.250	3.976	29.74
½	.1250	.0123	.0918	¼	8542	.5730	4 286	28.	2.333	4 276	31.99
¾	1458	.0167	.1249	½	.8750	.6013	4.498	29.	2.417	4.587	34 31
2	.1667	.0218	1632	¾	8958	.6303	4.715	30	2.500	4.909	36 72
¼	.1875	0276	.2066	11.	.9167	.6600	4 937	31	2 583	5.241	39 21
½	2083	.0341	2550	¼	.9375	.6903	5 164	32.	2.667	5 585	41.78
¾	2292	0412	.3085	½	9583	.7213	5 396	33.	2.750	5.940	44 43
3.	.2500	.0491	.3672	¾	.9792	7530	5.633	34.	2 833	6.305	47.13
¼	2708	0576	.4309	12.	1 Foot.	7854	5.875	35	2.917	6.681	49.98
½	.2917	0668	.4998	½	1.042	.8522	6.375	36.	3 000	7.069	52.88
¾	3125	0767	.5738	13.	1 083	9218	6.895	37.	3.083	7.467	55.86
4.	.3333	0873	.6528	½	1.125	.9940	7.436	38.	3.167	7.876	58 92
¼	3542	0985	.7369	14.	1 167	1.069	7.997	39.	3 250	8 296	62.06
½	3750	1104	.8263	½	1 208	1.147	8 578	40.	3.333	8.727	65.28
¾	3958	1231	.9206	15.	1 250	1.227	9.180	41.	3.417	9.168	68.58
5.	.4167	.1364	1.020	½	1.292	1.310	9 801	42	3.500	9.621	71.97
¼	.4375	.1503	1.126	16.	1.333	1.396	10.44	43.	3 583	10.085	75.44
½	.4583	.1650	1 234	½	1.375	1 485	11 11	44.	3.667	10.559	78.99
¾	.4792	.1803	1.349	17.	1.417	1 576	11.79	45	3.750	11.045	82.62
6.	.5000	.1963	1.469	½	1.458	1 670	12 49	46.	3 833	11.541	86.33
¼	.5208	.2131	1.594	18.	1.500	1 767	13 22	47	3 917	12.048	90.13
½	.5417	.2304	1 724	½	1.542	1 867	13.96	48.	4.000	12.566	94.00

Table continued, but with the diams in feet.

Diam Feet.	Cub Feet.	U. S. Galls.	Diam. Feet.	Cub. Feet.	U. S. Galls.	Dia Feet	Cub. Feet.	U. S. Galls.	Dia. Feet	Cub. Feet.	U. S. Galls.
4	12 57	94.0	7	38.48	287.9	12	113.1	846.0	24	452.4	338
¼	14.19	106.1	¼	41.28	308.8	13	132.7	992.9	25	490.9	367
½	15.90	119.0	½	44.18	330.5	14	153.9	1152.	26	530.9	397
¾	17.72	132.6	¾	47.17	352.9	15	176.7	1322.	27	572.6	428
5	19.63	146.9	8	50.27	376.0	16	201.1	1504.	28	615.8	460
¼	21.65	161.9	½	56.75	424.5	17	227.0	1698.	29	660.5	494
½	23.76	177.7	9	63.62	475.9	18	254.5	1904.	30	706.9	528
¾	25.97	194.2	½	70.88	530.2	19	283.5	2121.	31	754.8	564
6	28.27	211.5	10	78.54	587.5	20	314.2	2350.	32	804.2	601
¼	30.68	229.5	½	86.59	647.7	21	346.4	2591.	33	855.3	639
½	33.18	248.2	11	95.03	710.9	22	380.1	2844.	34	907.9	679
¾	35.78	267.7	½	103.87	777.0	23	415.5	3108.	35	962.1	719

CONTENTS AND LININGS OF WELLS.

For diams twice as great as those in the table, for the cub yds of digging, take out those opposite *one half* of the greater diam; and mult them by 4. Thus, for the cub yds in each foot of depth of a well 31 feet in diam, first take out from the table those opposite the diam of 15½ feet; namely, 6.989. Then 6.989 × 4 = 27.956 cub yds reqd for the 31 ft diam. But for the stone lining or walling, bricks or plastering, mult the tabular quantity opposite *half* the greater diam, by 2. Thus, the perches of stone walling for each foot of depth of a well of 31 ft diam, will be 2.073 × 2 = 4.146. If the wall is more or less than one foot thick, within usual moderate limits, it will generally be near enough for practice to assume that the number of perches, or of bricks, will increase or decrease in the same proportion.

The size of the bricks is taken at 8¼ × 4 × 2 inches; and to be laid dry, or without mortar. In practice an addition of about 5 per cent should be made for waste. The brick lining is supposed to be 1 brick thick, or 8¼ ins.

CAUTION. — Be careful to observe that the diams to be used for the digging, are greater than those for the walling, bricks, or plastering. No errors.

Diam. in Feet.	For each foot of depth. For this col use the Diameter of the Digging.	For these three cols use the diam in clear of the lining.			Diam. in Feet.	For each foot of depth. For this col use the Diameter of the Digging.	For these three cols use the diam in clear of the lining.		
	Cub Yds. of Digging.	Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering		Cub Yds. of Digging.	Stone Lining 1 ft thick Perches of 25 Cub Ft	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.
1.	.0291	.2513	57	.3491	¼	5.107	1.791	750	4.625
¼	.0455	.2827	71	.4364	½	5.301	1.822	764	4.713
½	.0654	.3142	85	.5236	¾	5.500	1.854	778	4.800
¾	.0891	.3456	99	.6109	14.	5.701	1.885	792	4.887
2.	.1164	.3770	114	.6981	¼	5.907	1.916	806	4.974
¼	.1473	.4084	128	.7855	½	6.116	1.948	820	5.062
½	.1818	.4398	142	.8727	¾	6.329	1.979	834	5.149
¾	.2200	.4712	156	.9600	15.	6.545	2.011	849	5.236
3	.2618	.5027	170	1.047	¼	6.765	2.042	863	5.323
¼	.3078	.5341	184	1.135	½	6.989	2.073	877	5.411
½	.3563	.5655	198	1.222	¾	7.216	2.105	891	5.498
¾	.4091	.5969	212	1.309	16.	7.447	2.136	905	5.585
4.	.4654	.6283	227	1.396	¼	7.681	2.168	919	5.673
¼	.5254	.6597	241	1.484	½	7.919	2.199	933	5.760
½	.5890	.6912	255	1.571	¾	8.161	2.231	948	5.847
¾	.6563	.7226	269	1.658	17.	8.407	2.262	962	5.934
5.	.7272	.7540	283	1.745	¼	8.656	2.293	976	6.022
¼	.8018	.7854	297	1.833	½	8.908	2.325	990	6.109
½	.8799	.8168	311	1.920	¾	9.165	2.356	1004	6.196
¾	.9617	.8482	326	2.007	18.	9.425	2.388	1018	6.283
6.	1.047	.8796	340	2.095	¼	9.688	2.419	1032	6.371
¼	1.136	.9111	354	2.182	½	9.956	2.450	1046	6.458
½	1.229	.9425	368	2.269	¾	10.23	2.482	1061	6.545
¾	1.325	.9739	382	2.356	19.	10.50	2.513	1075	6.633
7.	1.425	1.005	396	2.444	¼	10.78	2.545	1089	6.720
¼	1.529	1.037	410	2.531	½	11.06	2.576	1103	6.807
½	1.636	1.068	425	2.618	¾	11.35	2.608	1117	6.894
¾	1.747	1.100	439	2.705	20.	11.64	2.639	1131	6.982
8.	1.862	1.131	453	2.793	¼	11.93	2.670	1145	7.069
¼	1.980	1.162	467	2.880	½	12.22	2.702	1160	7.156
½	2.102	1.194	481	2.967	¾	12.52	2.733	1174	7.243
¾	2.227	1.225	495	3.054	21.	12.83	2.765	1188	7.331
9.	2.356	1.257	509	3.142	¼	13.14	2.796	1202	7.418
¼	2.489	1.288	523	3.229	½	13.45	2.827	1216	7.505
½	2.625	1.319	538	3.316	¾	13.76	2.859	1230	7.593
¾	2.765	1.351	552	3.404	22.	14.08	2.890	1244	7.680
10.	2.909	1.382	566	3.491	¼	14.40	2.922	1259	7.767
¼	3.056	1.414	580	3.578	½	14.73	2.953	1273	7.854
½	3.207	1.445	594	3.665	¾	15.06	2.985	1287	7.942
¾	3.362	1.477	608	3.753	23.	15.39	3.016	1301	8.029
11.	3.520	1.508	622	3.840	¼	15.72	3.047	1315	8.116
¼	3.682	1.539	637	3.927	½	16.06	3.079	1329	8.203
½	3.847	1.571	651	4.014	¾	16.41	3.110	1343	8.291
¾	4.016	1.602	665	4.102	24.	16.76	3.142	1357	8.378
12.	4.189	1.634	679	4.189	¼	17.11	3.173	1372	8.465
¼	4.365	1.665	693	4.276	½	17.46	3.204	1386	8.552
½	4.545	1.696	707	4.364	¾	17.82	3.236	1400	8.640
¾	4.729	1.728	721	4.451	25.	18.18	3.267	1414	8.727
13.	4.916	1.759	736	4.538					

A cub yd = 202 U. S. galls.

CIRCULAR CYLINDRIC UNGULAS.

I. When the cutting plane does not cut the base. Figs 13, 14

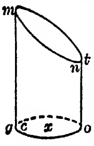


FIG. 13.

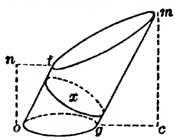


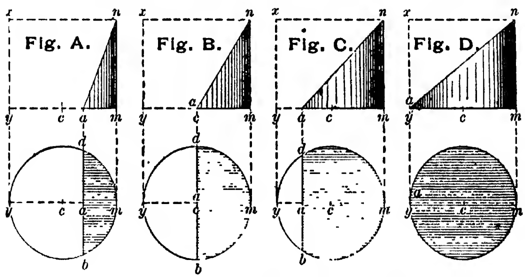
FIG. 14.

Volume of ungula = area of base $og \times \frac{1}{2}$ sum of greatest & least perp heights, on, cm,
= area of cross sec measd perp to sides, as x, $\times$ $\frac{1}{2}$ sum of greatest and least lengths, gm, ot, measd along the sides.

Area of *curved* surface = circumf measd perp to sides, as at x, $\times$ half sum of greatest and least lengths, gm, ot, measd along the sides.

Add areas of ends if required.
For areas of sections perpendicular to the sides, see Circles.
For areas of sections oblique to the sides, see The Ellipse.

II. When the cutting plane touches the base. Figs A to D.



Volume (whether right or oblique)

Fig **A** $= (\frac{2}{3} a b^2 - a c \times \text{area } a d m b \text{ of base}) \frac{mn}{am}$.

Fig **B** $= \frac{2}{3} c b^2 \times mn$.

Fig **C** $= (\frac{2}{3} a b^2 + a c \times \text{area } a d m b \text{ of base}) \frac{mn}{am}$.

mn meas'd per to base

Fig **D** $= \frac{1}{2}$ area of circle $ym \times mn$
$= \frac{1}{2}$ volume of cylinder $xymn$.

Curved surface (*right* ungula only)

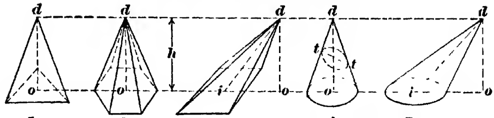
Fig **A** $= (ab \times my - ac \times \text{length of arc } dmb) \frac{mn}{am}$.

Fig **B** $= my \times mn$.

Fig **C** $= (ab \times my + ac \times \text{length of arc } dmb) \frac{mn}{am}$.

Fig **D** $= \frac{1}{2}$ circumference of base $my \times mn$
$= \frac{1}{2}$ curved surface of cylinder $xymn$.

PYRAMIDS AND CONES.



1 2 3 4 5

A pyramid, Figs. 1, 2, 3, is any solid which has, for its base, a plane figure of any number of sides, and, for its sides, plane triangles all terminating at one point d, called its *apex*, or top. When the base is a regular figure, the pyramid is *regular;* otherwise *irregular.*

A cone, Figs. 4 and 5, is a solid, of which the base is a curved figure; and which may be considered as made or generated by a line, of which one end is stationary at a certain point d, called the apex or top, while the line is being carried around the circumference of the base, which may be a circle, ellipse, or other curve. A cone may also be regarded as a pyramid with an infinite number of sides.

The axis of a pyramid or cone, is a straight line $d\,o$ in Figs. 1, 2, 4; and $d\,i$ in Figs. 3 and 5, from the apex d, to the center of gravity of the base. When the axis is perpendicular to the base, as in Figs. 1, 2, 4, the solid is said to be a *right* one; when otherwise, as Figs 3, 5, an *oblique* one. When the word cone is used alone, the right circular cone, Fig. 4, is understood. If such a cone be cut, as at $t\,t$, obliquely to its base, the new base $t\,t$ will be an ellipse; and the cone $d\,t\,t$ becomes an oblique elliptic one. Fig. 5 will represent either an oblique circular cone, or an oblique elliptic one, according as its base is a circle or an ellipse.

Volume of pyramid or cone, regular or irregular, right or oblique.

Volume $=$ ⅓ area of base $\times$ *perpendicular* height $d\,o$, or h, Figs 1 to 5.

$=$ ⅓ volume of prism or cylinder having same area of base and same perpendicular height.

$=$ ½ volume of hemisphere of same base and same height.

Or, a cone, hemisphere and cylinder, of the same base and same height, have volumes as 1, 2 and 3.

Area of surface of sides of right regular pyramid or right circular cone.

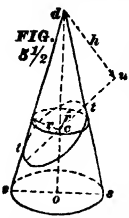
Area $=$ ½ circumference of base $\times$ slant height.*

In the *cone*, this becomes

$$\textbf{Area} = \frac{\text{area of base}}{\text{radius of base}} \times \text{slant height.}$$

Add area of base if required.

FIG. 5½



Area of surface of oblique elliptic cone, $d\,t\,t$, Fig. 5½, cut from a right circular cone, $d\,s\,s$. From the point c where the axis $d\,o$ of the right circular cone cuts the elliptic base $t\,t$, measure a perpendicular, r, in any direction, to the curved surface of the cone. Let v = the volume of oblique elliptic cone, $d\,t\,t$; let a = the area of its elliptic base $t\,t$, and let h = the height $d\,u$ measured perpendicularly to said base. Then

$$\textbf{Curved surface} = \frac{a\,h}{r} = \frac{3\,v}{r}.$$

Add area of base if required

No measurement has been devised for the surface of an **oblique circular cone.**

In an irregular pyramid, right or oblique, **surface area** $=$ sum of areas of sides, each calculated as a triangle, (p 148). Add area of base, if reqd.

* In the *pyramid*, this slant height must be measd along the middle of one of the *sides*, *not* along one of the *edges*.

FRUSTUMS OF PYRAMIDS AND OF CONES.

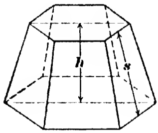


Fig. 6.

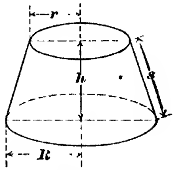


Fig. 7.

In Figs. 6 and 7.

a = area of top; c = circumference of top;
A = " " base; C = " " base;
M = " " section parallel to, and midway between, base and top;
h = height, perp to top and base; s = slant height.*

In Fig. 7.

r = radius of top; R = radius of base.

Volumes.

Frustum of pyramid (Fig. 6) **or of cone** (Fig. 7), regular or irregular, right or oblique, with base and top parallel.

$$\textbf{Volume} = \frac{h}{3}\left(a + A + \sqrt{a\,A}\right) = \frac{h}{6}\left(a + A + 4\,M\right).$$

Frustum of right or oblique circular cone. Fig. 7.

$$\textbf{Volume} = \frac{h}{3}\,\pi\left(r^2 + R^2 + r\,R\right); \quad \pi = 3.1416.$$

Surface area of sides. (Add top and base, if required.)

Frustum of right regular pyramid or cone, top and base parallel, Figs. 6 and 7.

$$\textbf{Area} = \frac{s}{2}\left(c + C\right).*$$

In the right circular conic frustum, this becomes:

$$\textbf{Area} = \pi\,s\,(r + R);* \quad \pi = 3.1416.$$

Frustum of irregular or oblique pyramid:

Area = sum of surfaces of sides.

Each side must be treated as a quadrilateral.
Add areas of top and base, if required.

---

* In the *pyramidal* frustum, Fig. 6, s must be measd along the *middle* of one of the *sides*, as shown, *not* along one of the *edges*.

PRISMOIDS.

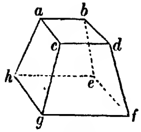


Fig. 1.

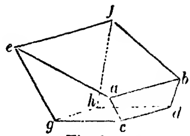


Fig. 2.

A prismoid is sometimes **defined as a solid having** for its ends two parallel plane figures, connected by other plane figures on which, and through every point of which, a straight line may be drawn from one of the two parallel ends to the other. These connecting planes may be parallelograms or not, and parallel to each other or not.

This definition would include the cube and all other parallelopipeds; the prism; the cylinder (considered as a prism having an infinite number of sides); the pyramid and cone (in which one of the two parallel ends, *i e* the one forming the apex, is considered to be infinitely small), and their frustums with top and base parallel; and the wedge.

But the use of the term **prismoid** is frequently **restricted** to six-sided solids, in which the two parallel ends are unequal quadrangles; and the connecting planes, trapezoids; as in Figs. 1 and 2; and, by some writers, to cases where the parallel quadrangular ends are *rectangles.*

The following **"prismoidal formula"** applies to all the foregoing solids, and to others, as noted below.

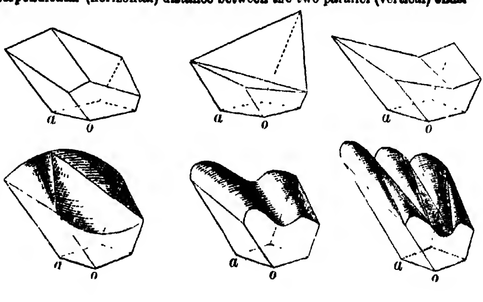
Let A = the area of one of the two parallel ends
a = " " the other of the two parallel ends.
M = " " a cross section midway between, and parallel to, the two parallel ends.
L = the perpendicular distance between the two parallel ends.

Then

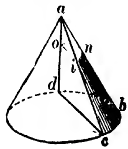
$$\textbf{Volume} = L \times \frac{A + a + 4M}{6}$$

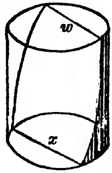
$$= L \times \text{mean area of cross section.}$$

The following six figures represent a few of the irregular solids which fall under the above broad definition of "prismoid," and to which the prismoidal formula applies. They may be regarded as one-chain lengths of railroad cuttings; *a o* being the length, or perpendicular (horizontal) distance between the two parallel (vertical) ends.



The prismoidal formula applies also to the sphere, hemisphere, and other spherical segments; also to any sections such as *a b c d*, and *o n i d b c*, of the





cone, in which the sides *a d*, *a c*, or *o d*, *i c*, are *straight*; as they are only when the cutting plane *a d c* passes *through the apex* or top *a*. **Also to the cylinder** when a plane *parallel to the sides* passes through both ends; but not if the plane *w x* is *oblique*, as in the fig., though never erring more than 1 in 142. **In this last case** we must imagine the plane to be extended until it cuts the side of the cylinder likewise extended; and then by page 199 find the solidity of the ungula thus formed. Then find the solidity of the small ungula above *w*, also thus formed, and subtract it from the large one.

This very extended applicability of the prismoidal formula was first discovered, and made known, by Ellwood Morris, C. E., of Philadelphia, in 1840.

WEDGES.

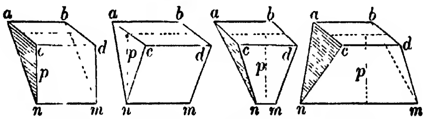


Fig. 8. Fig. 9. Fig. 10. Fig. 11.

A wedge

Is usually defined to be a solid, Figs. 8 and 9, generated by a plane triangle, *a n c*, moving parallel to itself in a straight line. This definition requires that the two triangular ends of the wedge should be parallel; but a wedge may be shaped as in Fig. 10 or 11. We would therefore propose the following definition, which embraces all the figs.; besides various modifications of them. A solid of five plane faces; one of which is a parallelogram *a b c d*, two opposite sides of which, as *a c* and *b d*, are united by means of two triangular faces *a c n*, and *b d m*, to an edge or line *n m*, parallel to the other opposite sides *a b* and *c d*. The parallelogram *a b c d* may be either rectangular, or not; the two triangular faces may be similar, or not; and the same with regard to the other two faces. The following rule applies equally to all:

$$\text{Volume of wedge} = \tfrac{1}{6} \times \begin{array}{c}\text{Sum of lengths}\\ \text{of the 3 edges}\\ ab + cd + nm\end{array} \times \begin{array}{c}\text{perp ht } p \text{ from}\\ \text{edge to back}\end{array} \times \begin{array}{c}\text{width of}\\ \text{back } (a\,b\,c\,d)\\ \text{meas'd perp to } a\,b\end{array}$$

SPHERES OR GLOBES.

A Sphere

Is a solid generated by the revolution of a semicircle around its diameter. Every point in the surface of a sphere is equidistant from a certain point called the center. Any line passing entirely through a sphere, and through its center, is called its *axis*, or diameter. Any circle described on the surface of a sphere, from the center of the sphere as the center of the circle, is called a *great circle* of that sphere, in other words any entire circumference of a sphere is a great circle. A sphere has a greater content or solidity than any other solid with the same amount of surface; so that if the shape of a sphere be any way changed, its content will be reduced. The intersection of a sphere with any plane is a circle.

Volume of sphere

$= \frac{4}{3}\,\pi$ radius^3 $\quad = 4.1888$ radius^3

$= \frac{1}{6}\,\pi$ diameter^3 $\quad = 0.5236$ diameter^3

$= \frac{1}{6}\,\frac{\text{circumference}^3}{\pi^2}$ $\quad = 0.01689$ circumference^3

$= \frac{1}{6}$ diameter $\times$ area of surface

$= \frac{2}{3}$ diameter $\times$ area of great circle

$= \frac{2}{3}$ volume of circumscribing cylinder

$= 0.5236$ volume of circumscribing cube.

Area of surface of sphere

$= 4\,\pi$ radius^2 $\quad = 12.5664$ radius^2

$= \pi$ diameter^2 $\quad = 3.1416$ diameter^2

$= \frac{\text{circumference}^2}{\pi}$ $\quad = 0.3183$ circumference^2

$=$ diameter $\times$ circumference

$= 4 \times$ area of great circle

$=$ area of circle whose diameter is equal to twice diameter of sphere

$=$ curved surface of circumscribing cylinder

$= \frac{6 \times \text{volume}}{\text{diameter.}}$

Radius of sphere

$$= \sqrt[3]{\frac{3}{4}\,\frac{\text{volume}}{\pi}} \quad = 0.62035\,\sqrt[3]{\text{volume}}$$

$$= \sqrt{\frac{\text{Area of surface}}{4\,\pi}} \quad = \sqrt{.07958 \times \text{area of surface}}$$

Circumference of sphere

$$= \sqrt[3]{6\,\pi^2\,\text{volume}} \quad = \sqrt[3]{59.2176\ \text{volume}}$$

$$= \sqrt{\pi\ \text{area of surface}} \quad = \sqrt{3.1416\ \text{area of surface}}$$

$$= \frac{\text{area of surface}}{\text{diameter.}}$$

SPHERES. (ORIGINAL.)

Some errors of 1 in the last figure only.

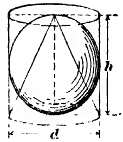
Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
1-64	.00077		13-32	18 190	7 2949	⅜	170.87	210 93	⅛	921 33	2629 6
1 32	00307	00002	7-16	18 666	7 5829	½	176.71	220 89	¼	934 83	2687 6
3-64	.00690	.00005	15-32	19.147	7 8783	⅝	182.66	232 13	⅜	948 43	2746.5
1 16	.01227	.00013	½	19 635	8.1813	¾	188.69	243 73	½	962.12	2806.2
3-32	02761	.00043	17-32	20.129	8.4919	⅞	194.83	255 72	⅝	975 91	2866.8
⅛	.04909	.00102	9 16	20 629	8 8103	8.	201.06	268 08	¾	989 80	2928 2
5 32	.07670	00200	19-32	21.135	9 1366	⅛	207 39	280 85	⅞	1003 8	2990 5
3-16	.11045	.00345	⅝	21 648	9 4708	¼	213 82	294 01	18.	1017.9	3053 6
7-32	.15033	00548	21 32	22 166	9 8131	⅜	220 36	307 58	⅛	1032 1	3117.7
¼	.19635	00818	11-16	22 691	10 164	½	226 98	321 56	¼	1046.4	3182 6
9-32	.24851	.01165	23 32	23 222	10 522	⅝	233 71	335 95	⅜	1060 8	3248.5
5-16	.30680	01598	¾	23 758	10 889	¾	240 53	350 77	½	1075 2	3315.3
11 32	37123	.02127	25 32	24 302	11 265	⅞	247 45	366 02	⅝	1089 8	3382 9
⅜	.44179	.02761	13 16	24 850	11 649	9.	254 47	381 70	¾	1104 5	3451 5
13 32	.51848	03511	27 32	25 405	12 041	⅛	261 59	397 83	⅞	1119 3	3521.0
7-16	.60132	.04385	⅞	25 967	12.443	¼	268 81	414 41	19.	1134 1	3591.4
15-32	.69028	.05393	29 32	26 535	12 853	⅜	276 12	431 44	⅛	1149.1	3662 8
½	.78540	·06545	15 16	27 109	13 272	½	283 53	448 92	¼	1164 2	3735.0
17-32	.88661	.07850	31 32	27 688	13 700	⅝	291 04	466 87	⅜	1179 3	3808.2
9-16	99403	09319	3.	28 274	14 137	¾	298 65	485 31	½	1194 6	3882 5
19 32	1.1075	10960	1-16	29 465	15 039	⅞	306 36	504 21	⅝	1210 0	3957.6
⅝	1.2272	12783	⅛	30 680	15 979	10.	314.16	523 60	¾	1225 4	4033 7
21-32	1 3530	14798	3 16	31 919	16 957	⅛	322 06	543 48	⅞	1241.0	4110 8
11-16	1 4849	.17014	¼	33 183	17 974	¼	330 06	563 86	20.	1256 7	4188 8
23-32	1 6230	.19442	5 16	34 472	19 031	⅜	338 16	584 74	⅛	1272 4	4267.5
¾	1.7671	22089	⅜	35 784	20 129	½	346 36	606 13	¼	1288 3	4347.8
25-32	1 9175	.24967	7 16	37 122	21 268	⅝	354 66	628 04	⅜	1304 2	4428.8
13-16	2 0739	28084	½	38 484	22 449	¾	363 05	650 46	½	1320 3	4510.9
27 32	2 2365	31451	9-16	39.872	23 674	⅞	371.54	673 42	⅝	1336 4	4593 9
⅞	2 4053	.35077	⅝	41 283	24 942	11.	380 13	696 91	¾	1352 7	4677 9
29 32	2 5802	.38971	11-16	42 719	26 254	⅛	388 83	720 95	⅞	1369 0	4763.0
15-16	2 7611	43143	¾	44 179	27 611	¼	397 61	745 51	21.	1385 5	4849.1
31-32	2.9483	.47603	13 16	45 664	29 016	⅜	406 49	770 64	⅛	1402 0	4936 2
1.	3 1416	52360	⅞	47 173	30 466	½	415 48	796 33	¼	1418 6	5024 3
1-32	3 3410	57424	15-16	48.708	31 965	⅝	424 56	822 58	⅜	1435 4	5113.5
1-16	3 5466	62804	4.	50 265	33 510	¾	433 73	849 40	½	1452 2	5203 7
3 32	3 7583	.68511	1 16	51 848	35 106	⅞	443 01	876 79	⅝	1469.2	5295.1
⅛	3 9761	74551	⅛	53 456	36.751	12.	452.39	904 78	¾	1486 2	5387 4
5-32	4.2000	.80939	3 16	55 089	38 448	⅛	461 87	933 34	⅞	1503 8	5480.8
3 16	4 4301	.87681	¼	56.745	40 195	¼	471 44	962 52	22.	1520 5	5575.3
7-32	4 6664	94786	5 16	58 427	41 994	⅜	481.11	992 28	⅛	1537 9	5670 8
¼	4 9088	1 0227	⅜	60 133	43 847	½	490 87	1022 7	¼	1555.5	5767.6
9 32	5 1573	1 1013	7 16	61 863	45 752	⅝	500 73	1053 6	⅜	1572 8	5865.2
5 16	5 4119	1,1839	½	63.617	47 713	¾	510.71	1085 3	½	1590 4	5964 1
11-32	5 6728	1 2704	9 16	65 397	49 729	⅞	520 77	1117 5	⅝	1608 2	6064.1
⅜	5 9396	1 3611	⅝	67 201	51 801	13.	530 93	1150 3	¾	1626 0	6165 2
13 32	6 2126	1.4561	11-16	69 030	53 929	⅛	541 19	1183 8	⅞	1643 9	6267.3
7-16	6 4919	1 5553	¾	70 883	56 116	¼	551 55	1218 0	23.	1661.9	6370 6
15 32	6 7771	1 6590	13-16	72 759	58 359	⅜	562 00	1252 7	⅛	1680 0	6475.0
½	7 0686	1 7671	⅞	74 663	60 663	½	572 55	1288 3	¼	1698 2	6580.6
17-32	7 3663	1 8799	15-16	76 589	63 026	⅝	583 20	1324 4	⅜	1716 5	6687.[illegible]
9 16	7.6699	1 9974	5.	78 540	65 450	¾	593 95	1361 2	½	1735 0	6795.2
19 32	7 9798	2 1196	1 16	80 516	67 935	⅞	604 80	1398 6	⅝	1753 5	6904 2
⅝	8 2957	2 2468	⅛	82 516	70.482	14	615 75	1436 8	¾	1772 1	7014 3
21 32	8 6180	2.3789	3 16	84 541	73 092	⅛	626 80	1475.6	⅞	1790.8	7125.6
11 16	8 9461	2 5161	¼	86 591	75 767	¼	637 95	1515 1	24	1809 6	7238.2
23 32	9 2805	2.6586	5-16	88 664	78.505	⅜	649 17	1555 3	⅛	1828 5	7351.9
¾	9 6211	2 8062	⅜	90 763	81.308	½	660 52	1596 3	¼	1847.5	7466.7
25-32	9 9678	2.9592	7 16	92 887	84.178	⅝	671 95	1637 9	⅜	1866 6	7583 0
13 16	10.321	3 1177	½	95 033	87 113	¾	683 49	1680 3	½	1885 8	7700.1
27 32	10 680	3 2818	9 16	97 205	90 118	⅞	695 13	1723 3	⅝	1905 1	7818 6
⅞	11 044	3 4514	⅝	99 401	93 189	15.	706 85	1767 2	¾	1924 4	7938.3
29-32	11 416	3 6270	11 16	101 62	96.331	⅛	718 69	1811 7	⅞	1943 9	8059.2
15 16	11 793	3 8083	¾	103 87	99 541	¼	730 63	1857 0	25	1963 5	8181.3
31 32	12 177	3 9956	13 16	106 14	102 82	⅜	742 65	1903 0	⅛	1983 2	8304 7
2.	12 566	4 1888	⅞	108.44	106 18	½	754 77	1949 8	¼	2002 9	8429.2
1-32	12 962	4.3882	15-16	110.75	109 60	⅝	767 00	1997 4	⅜	2022 9	8554.9
1 16	13 364	4 5939	6.	113 10	113 10	¾	779 32	2045 7	½	2042 8	8682.0
3 32	13 772	4 8060	⅛	117 87	120.31	⅞	791 73	2094 8	⅝	2062 9	8810.3
⅛	14 186	5 0243	¼	122 72	127 83	16.	804 25	2144 7	¾	2083 0	8939.9
5-32	14 607	5 2493	⅜	127 68	135 66	⅛	816 85	2195 3	⅞	2103.4	9070.6
3 16	15 033	5 4809	½	132 73	143 79	¼	829 57	2246 8	26.	2123 7	9202.8
7-32	15 466	5 7190	⅝	137 89	152 25	⅜	842.40	2299 1	⅛	2144.2	9336.2
¼	15 904	5 9641	¾	143 14	161.03	½	855 29	2352.1	¼	2164.7	9470.8
9-32	16.349	6 2161	⅞	148 49	170 14	⅝	868 31	2406 0	⅜	2185.5	9606.7
5-16	16.800	6 4751	7.	153 94	179 59	¾	881 42	2460 6	½	2206.2	9744.0
11-32	17.258	6 7412	⅛	159 49	189 39	⅞	894 62	2516 1	⅝	2227.1	9882.5
⅜	17.721	7.0144	¼	165.13	199.53	17.	907.93	2572.4	¾	2248.0	10022

SPHERES—(Continued.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
7/8	2269.1	10164	5/8	4214.1	25724	3/8	6756.5	52222	1/8	9896.0	92570
27.	2290.2	10306	3/4	4243.0	25988	1/2	6792.9	52645	1/4	9940.2	93190
1/8	2311.5	10450	7/8	4271.8	26254	5/8	6829.5	53071	3/8	9984.4	93812
1/4	2332.8	10595	37.	4300.9	26522	3/4	6866.1	53499	1/2	10029	94438
3/8	2354.3	10741	1/8	4330.0	26792	7/8	6902.9	53929	5/8	10073	95066
1/2	2375.8	10889	1/4	4359.2	27063	47.	6939.9	54362	3/4	10118	95697
5/8	2397.5	11038	3/8	4388.5	27337	1/8	6976.8	54797	7/8	10163	96330
3/4	2419.2	11189	1/2	4417.9	27612	1/4	7013.9	55231	57.	10207	96967
7/8	2441.1	11341	5/8	4447.5	27889	3/8	7050.9	55674	1/8	10252	97606
28.	2463.0	11494	3/4	4477.1	28168	1/2	7088.3	56115	1/4	10297	98248
1/8	2485.1	11649	7/8	4506.8	28449	5/8	7125.6	56559	3/8	10342	98893
1/4	2507.2	11805	38.	4536.5	28731	3/4	7163.1	57006	1/2	10387	99541
3/8	2529.5	11962	1/8	4566.5	29016	7/8	7200.7	57455	5/8	10432	100191
1/2	2551.8	12121	1/4	4596.4	29302	48.	7238.3	57906	3/4	10478	100845
5/8	2574.3	12281	3/8	4626.5	29590	1/8	7276.0	58360	7/8	10523	101501
3/4	2596.7	12443	1/2	4656.7	29880	1/4	7313.9	58815	58.	10568	102161
7/8	2619.4	12606	5/8	4686.9	30173	3/8	7351.9	59274	1/8	10614	102823
29.	2642.1	12770	3/4	4717.3	30466	1/2	7389.9	59734	1/4	10660	103488
1/8	2665.0	12936	7/8	4747.9	30762	5/8	7428.0	60197	3/8	10706	104155
1/4	2687.8	13103	39.	4778.4	31059	3/4	7466.3	60663	1/2	10751	104826
3/8	2710.9	13272	1/8	4809.0	31359	7/8	7504.5	61131	5/8	10798	105499
1/2	2734.0	13442	1/4	4839.9	31661	49.	7543.1	61601	3/4	10844	106175
5/8	2757.3	13614	3/8	4870.8	31964	1/8	7581.6	62074	7/8	10890	106854
3/4	2780.5	13787	1/2	4901.7	32270	1/4	7620.1	62549	59.	10936	107536
7/8	2804.0	13961	5/8	4932.7	32577	3/8	7658.9	63026	1/8	10983	108221
30.	2827.4	14137	3/4	4964.0	32886	1/2	7697.7	63506	1/4	11029	108909
1/8	2851.1	14315	7/8	4995.3	33197	5/8	7736.7	63989	3/8	11076	109600
1/4	2874.8	14494	40.	5026.5	33510	3/4	7775.7	64474	1/2	11122	110294
3/8	2898.7	14674	1/8	5058.1	33826	7/8	7814.8	64961	5/8	11169	110990
1/2	2922.5	14856	1/4	5089.6	34143	50.	7854.0	65450	3/4	11216	111690
5/8	2946.6	15039	3/8	5121.3	34462	1/8	7893.3	65941	7/8	11263	112392
3/4	2970.6	15224	1/2	5153.1	34783	1/4	7932.8	66436	60.	11310	113098
7/8	2994.9	15411	5/8	5184.9	35106	3/8	7972.2	66934	1/8	11357	113806
31.	3019.1	15599	3/4	5216.8	35431	1/2	8011.8	67433	1/4	11404	114518
1/8	3043.6	15788	7/8	5248.9	35758	5/8	8051.6	67933	3/8	11452	115232
1/4	3068.0	15979	41.	5281.1	36087	3/4	8091.4	68439	1/2	11499	115949
3/8	3092.7	16172	1/8	5313.3	36418	7/8	8131.3	68946	5/8	11547	116669
1/2	3117.3	16366	1/4	5345.6	36751	51.	8171.2	69456	3/4	11595	117392
5/8	3142.1	16561	3/8	5378.1	37086	1/8	8211.4	69967	7/8	11642	118118
3/4	3166.9	16758	1/2	5410.7	37423	1/4	8251.6	70482	61	11690	118847
7/8	3192.0	16957	5/8	5443.3	37763	3/8	8292.0	70999	1/8	11738	119579
32.	3217.0	17157	3/4	5476.0	38104	1/2	8332.8	71519	1/4	11786	120315
1/8	3242.2	17359	7/8	5508.9	38448	5/8	8372.8	72040	3/8	11834	121053
1/4	3267.4	17563	42.	5541.9	38792	3/4	8413.4	72565	1/2	11882	121794
3/8	3292.9	17768	1/8	5574.9	39140	7/8	8454.1	73092	5/8	11931	122538
1/2	3318.3	17974	1/4	5608.0	39490	52.	8494.8	73622	3/4	11980	123286
5/8	3343.9	18182	3/8	5641.3	39841	1/8	8535.8	74154	7/8	12028	124036
3/4	3369.6	18392	1/2	5674.5	40194	1/4	8576.8	74689	62	12076	124789
7/8	3395.4	18604	5/8	5708.0	40551	3/8	8617.8	75226	1/8	12126	125545
33.	3421.2	18817	3/4	5741.5	40908	1/2	8658.9	75767	1/4	12174	126305
1/8	3447.3	19032	7/8	5775.2	41268	5/8	8700.4	76309	3/8	12223	127067
1/4	3473.3	19248	43.	5808.8	41630	3/4	8741.7	76854	1/2	12272	127832
3/8	3499.5	19466	1/8	5842.7	41991	7/8	8783.2	77401	5/8	12322	128601
1/2	3525.7	19685	1/4	5876.5	42360	53.	8824.8	77952	3/4	12371	129373
5/8	3552.1	19907	3/8	5910.7	42729	1/8	8866.4	78505	7/8	12420	130147
3/4	3578.5	20129	1/2	5944.7	43099	1/4	8908.2	79060	63.	12469	130925
7/8	3605.1	20354	5/8	5978.9	43472	3/8	8950.1	79617	1/8	12519	131706
34.	3631.7	20580	3/4	6013.2	43846	1/2	8992.0	80178	1/4	12568	132490
1/8	3658.5	20808	7/8	6047.7	44224	5/8	9034.1	80741	3/8	12618	133277
1/4	3685.3	21037	44.	6082.1	44602	3/4	9076.4	81308	1/2	12668	134067
3/8	3712.3	21268	1/8	6116.8	44984	7/8	9118.5	81876	5/8	12718	134860
1/2	3739.3	21501	1/4	6151.5	45367	54.	9160.8	82448	3/4	12768	135657
5/8	3766.5	21736	3/8	6186.3	45753	1/8	9203.8	83021	7/8	12818	136456
3/4	3793.7	21972	1/2	6221.2	46141	1/4	9246.0	83598	64.	12868	137259
7/8	3821.1	22210	5/8	6256.1	46530	3/8	9288.5	84177	1/8	12918	138065
35.	3848.5	22449	3/4	6291.2	46922	1/2	9331.2	84760	1/4	12969	138874
1/8	3876.1	22691	7/8	6326.5	47317	5/8	9374.1	85344	3/8	13019	139686
1/4	3903.7	22934	45.	6361.7	47713	3/4	9417.2	85931	1/2	13070	140501
3/8	3931.5	23179	1/8	6397.2	48112	7/8	9460.2	86521	5/8	13121	141320
1/2	3959.2	23425	1/4	6432.7	48513	55.	9503.2	87114	3/4	13172	142142
5/8	3987.2	23674	3/8	6468.3	48916	1/8	9546.5	87709	7/8	13222	142966
3/4	4015.2	23924	1/2	6503.9	49321	1/4	9590.0	88307	65.	13273	143794
7/8	4043.3	24176	5/8	6539.7	49729	3/8	9633.8	88908	1/8	13324	144625
36.	4071.5	24429	3/4	6575.5	50139	1/2	9676.8	89511	1/4	13376	145460
1/8	4099.9	24685	7/8	6611.6	50551	5/8	9720.6	90117	3/8	13427	146297
1/4	4128.3	24942	46.	6647.6	50965	3/4	9764.4	90726	1/2	13478	147138
3/8	4156.9	25201	1/8	6683.7	51382	7/8	9808.1	91338	5/8	13530	[illegible]
1/2	4185.5	25461	1/4	6720.0	51801	56.	9852.0	91953	3/4	13582	[illegible]

SPHERES—(CONTINUED.)

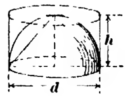
Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
7/8	13633	149680	1/2	17437	216505	1/8	21708	300743	3/4	26446	404406
66.	13685	150533	5/8	17496	217597	1/4	21773	302100	7/8	26518	406060
1/8	13737	151390	3/4	17554	218693	3/8	21839	303463	92.	26590	407721
1/4	13789	152251	7/8	17613	219792	1/2	21904	304831	1/8	26663	409384
3/8	13841	153114	75.	17672	220894	5/8	21976	306201	1/4	26735	411054
1/2	13893	153980	1/8	17731	222001	3/4	22036	307576	3/8	26808	412726
5/8	13946	154850	1/4	17790	223111	7/8	22102	308957	1/2	26880	414405
3/4	13998	155724	3/8	17849	224224	84.	22167	310340	5/8	26953	416086
7/8	14050	156600	1/2	17908	225341	1/8	22234	311728	3/4	27026	417774
67.	14103	157480	5/8	17968	226463	1/4	22300	313118	7/8	27099	419464
1/8	14156	158363	3/4	18027	227588	3/8	22366	314514	93.	27172	421161
1/4	14208	159250	7/8	18087	228716	1/2	22432	315915	1/8	27245	422862
3/8	14261	160139	76.	18146	229848	5/8	22499	317318	1/4	27318	424567
1/2	14314	161032	1/8	18206	230984	3/4	22565	318726	3/8	27391	426277
5/8	14367	161927	1/4	18266	232124	7/8	22632	320140	1/2	27464	427991
3/4	14420	162827	3/8	18326	233267	85.	22698	321556	5/8	27538	429710
7/8	14474	163731	1/2	18386	234414	1/8	22765	322977	3/4	27612	431433
68	14527	164637	5/8	18446	235566	1/4	22832	324402	7/8	27686	433160
1/8	14580	165547	3/4	18506	236719	3/8	22899	325831	94.	27759	434894
1/4	14634	166460	7/8	18566	237879	1/2	22966	327264	1/8	27833	436630
3/8	14688	167376	77	18626	239041	5/8	23034	328702	1/4	27907	438373
1/2	14741	168295	1/8	18687	240206	3/4	23101	330142	3/8	27981	440118
5/8	14795	169218	1/4	18748	241376	7/8	23168	331588	1/2	28055	441871
3/4	14849	170145	3/8	18809	242551	86.	23235	333039	5/8	28130	443625
7/8	14903	171074	1/2	18869	243728	1/8	23303	334492	3/4	28204	445387
69	14957	172007	5/8	18930	244908	1/4	23371	335951	7/8	28278	447151
1/8	15012	172944	3/4	18992	246093	3/8	23439	337414	95	28353	448920
1/4	15066	173888	7/8	19053	247283	1/2	23506	338882	1/8	28428	450695
3/8	15120	174828	78	19114	248475	5/8	23575	340352	1/4	28503	452475
1/2	15175	175774	1/8	19175	249672	3/4	23643	341829	3/8	28577	454259
5/8	15230	176728	1/4	19237	250873	7/8	23711	343307	1/2	28652	456047
3/4	15284	177677	3/8	19298	252077	87.	23779	344792	5/8	28727	457839
7/8	15339	178635	1/2	19360	253284	1/8	23847	346281	3/4	28802	459638
70	15394	179595	5/8	19422	254496	1/4	23916	347772	7/8	28878	461439
1/8	15449	180559	3/4	19483	255713	3/8	23984	349269	96	28953	463248
1/4	15504	181525	7/8	19545	256932	1/2	24053	350771	1/8	29028	465059
3/8	15560	182497	79.	19607	258155	5/8	24122	352277	1/4	29104	466875
1/2	15615	183471	1/8	19669	259383	3/4	24191	353785	3/8	29180	468697
5/8	15670	184449	1/4	19732	260613	7/8	24260	355301	1/2	29255	470524
3/4	15726	185430	3/8	19794	261848	88.	24328	356819	5/8	29331	472354
7/8	15782	186414	1/2	19856	263088	1/8	24398	358342	3/4	29407	474189
71.	15837	187402	5/8	19919	264330	1/4	24467	359869	7/8	29483	476029
1/8	15893	188394	3/4	19981	265577	3/8	24536	361400	97	29559	477874
1/4	15949	189389	7/8	20044	266829	1/2	24606	362935	1/8	29636	479725
3/8	16005	190387	80	20106	268083	5/8	24676	364476	1/4	29712	481579
1/2	16061	191389	1/8	20170	269342	3/4	24745	366019	3/8	29788	483438
5/8	16117	192395	1/4	20232	270604	7/8	24815	367568	1/2	29865	485302
3/4	16174	193404	3/8	20296	271871	89.	24885	369122	5/8	29942	487171
7/8	16230	194417	1/2	20358	273141	1/8	24955	370678	3/4	30018	489045
72.	16286	195433	5/8	20422	274416	1/4	25025	372240	7/8	30095	490924
1/8	16343	196453	3/4	20485	275694	3/8	25095	373806	98.	30172	492808
1/4	16400	197476	7/8	20549	276977	1/2	25165	375378	1/8	30249	494695
3/8	16456	198502	81.	20612	278263	5/8	25236	376954	1/4	30326	496588
1/2	16513	199532	1/8	20676	279553	3/4	25306	378531	3/8	30404	498486
5/8	16570	200566	1/4	20740	280847	7/8	25376	380115	1/2	30481	500388
3/4	16628	201604	3/8	20804	282145	90.	25447	381704	5/8	30558	502296
7/8	16685	202645	1/2	20867	283447	1/8	25518	383297	3/4	30636	504208
73	16742	203689	5/8	20932	284754	1/4	25589	384894	7/8	30713	506125
1/8	16799	204737	3/4	20996	286064	3/8	25660	386496	99	30791	508047
1/4	16857	205789	7/8	21060	287378	1/2	25730	388102	1/8	30869	509975
3/8	16914	206844	82.	21124	288696	5/8	25802	389711	1/4	30947	511906
1/2	16972	207903	1/8	21189	290019	3/4	25873	391327	3/8	31025	513843
5/8	17030	208966	1/4	21253	291345	7/8	25944	392945	1/2	31103	515785
3/4	17088	210032	3/8	21318	292674	91.	26016	394570	5/8	31181	517730
7/8	17146	211102	1/2	21382	294010	1/8	26087	396197	3/4	31259	519682
74.	17204	212175	5/8	21448	295347	1/4	26159	397831	7/8	31338	521638
1/8	17262	213252	3/4	21512	296691	3/8	26230	399468	100	31416	523598
1/4	17320	214333	7/8	21578	298036	1/2	26302	401109			
3/8	17379	215417	83	21642	299388	5/8	26374	402756			



Sphere, S: cone, C: and cylinder, Y: of equal diameters, d, and of equal heights, h ($d = h$).

$$\textbf{Volumes.}\quad Y = \frac{3}{2} S = 3\,C.$$

$$\textbf{Curved Surfaces.}\quad Y = S = \frac{2\,C}{\sqrt{1.25}}.$$



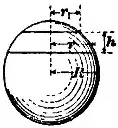
Hemisphere, H: cone, C: and cylinder, Y: of equal diameters, d, and of equal heights, h. ($d = 2\,h$).

$$\textbf{Volumes.}\quad Y = \frac{3}{2} H = 3\,C.$$

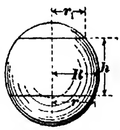
$$\textbf{Curved Surfaces.}\quad Y = H = C\sqrt{2}.$$

Spherical zones and segments.

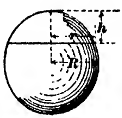
Let R = radius of sphere;
r = radius of base of segment
= radius of either end of zone;
r_1 = radius of other end of zone;
h = height of zone or segment,
$\pi = \frac{\text{circumference}}{\text{diameter}} = 3.14159$.
See p. 161.



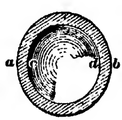
Zone



Zone



Segment



Shell

Then

$$\textbf{Volume of zone} = \frac{\pi}{2}\left(r^2 + r_1^2 + \frac{h^2}{3}\right) h$$

$$\textbf{Volume of segment} = \frac{\pi}{2}\left(r^2 + \frac{h^2}{3}\right) h = \pi\left(R - \frac{h}{3}\right) h^2.$$

$$\textbf{Curved surface of zone or segment} = \frac{h}{2\,R} \times \text{surface of sphere} = \frac{h}{2\,R} \times 4\,\pi\,R^2 = 2\,\pi\,R\,h$$

$$\text{In the segment, } 2\,R = \frac{r^2}{h} + h.$$

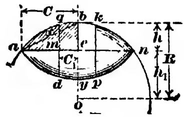
Spherical shell.

Volume = volume of sphere $a\,b$ — volume of sphere $c\,d$

Circular Spindle. Fig, p 209.

The circular spindle is the solid $a\,b\,n\,y\,a$, generated by the revolution of a circular segment, $a\,b\,n\,e\,a$, about its chord, $a\,n$, as an axis. Let

$C = a\,e$ = ½ chord of segment;
$h = e\,b$ = height of segment;
$R = o\,b$ = radius of circle $= \frac{C^2 + h^2}{2\,h}$,
$x = \sqrt{C^2 + h^2}$ = chord of half the segment;
L = length of arc, $a\,b\,n$;
S = area $a\,b\,e = \frac{L\,R}{4} - \frac{C\,h_1}{2}$;
$h_1 = o\,e = R - h$ = distance from cen, o, of circle, to cen, e, of chord.



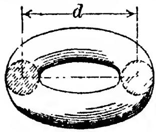
Then, **Volume** $= 4\pi\left(\frac{C^3}{3} - S\,h_1\right)$.

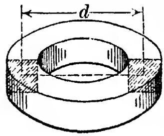
Surface $= 2\pi\,(2C\,R - L\,h_1)$

Middle Zone, $q\,d\,k\,p$; $(C_1 = m\,e.)$

Volume $= 2\pi\left[\left(C^2 - \frac{C_1^2}{3}\right) C_1 - h_1 S_1\right]$, where $S_1 = \frac{\text{area, } q\,d\,k\,p}{2}$.

Circular Ring, of Circular or Rectangular Cross-section.





Let

a = area of cross-section of bar of which ring is made;
c = circumference or periphery of cross-section of bar;
d = half-sum (or average) of inner and outer diameters.

Then,

Volume $= \pi\,a\,d$; **Surface** $= \pi\,c\,d$.

Ellipsoid.

The ellipsoid is the solid generated by the revolution of an ellipse about either axis The generating ellipse may be a variable It revolvs on a constant axis, and its vertices describe either another ellipse or a circle, whose center, in either case, coincides with the center of the generating ellipse, and whose plane is perpendicular to the axis of revolution. See "Spheroid," below.

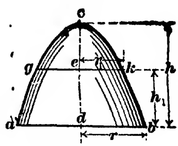
Spheroid.

The spheroid is an ellipsoid in which the generating ellipse is *constant* and in which its vertices describe a *circle*. When the generating ellipse revolvs about its *longer or transverse* axis, the **prolate** spheroid results; when about its *shorter or conjugate* axis, the **oblate** spheroid.

In either case, let f = the *fixt* diam; let m = the *moving* or *revolving* diam, of the ellipse Then: **Volume** $= \pi\,m^2 f/6$ **Surface** (approx) $= \pi\,m\sqrt{(m^2 + f^2)/2}$

The term "ellipsoid" is frequently defined as is "spheroid" above.

Paraboloid.



Paraboloid

The paraboloid is the solid generated by the revolution of a parabola, $a\,c\,b$, about its axis, $c\,d$. r = radius, $d\,b$, of base; h = height, $d\,c$

Volume $= \frac{\pi\,r^2 h}{2} = \frac{h \times \text{area of base}}{2}$;

Curved surface $= \left(\frac{2\,\pi\,r}{12\,h^2}\right)\left[(r^2 + 4h^2)^{\frac{3}{2}} - r^3\right]$

Frustum of paraboloid.

Ends, $g\,k$ and $a\,b$, perpendicular to axis, $d\,c$. r_1 = radius, $e\,k$, of base, $g\,k$. h_1 = height, $d\,e$, of frustum.

Volume = paraboloid, $a\,c\,b$ — paraboloid, $g\,c\,k$, $= \frac{\pi}{2}\,h_1\,(r^2 + r_1^2)$.

Curved surface = surface $a\,c\,b$ — surface $g\,c\,k$.

SPECIFIC GRAVITY.

1. The specific gravity, or relative density, D*, of a substance, is the ratio between the weight, W, of any given volume of that substance and the weight, A, of an equal volume of some substance adopted as a standard of comparison. Or: $D = \frac{W}{A}$.

2. For gaseous substances, **the standard substance** is air, at a temperature of 0° Cent. = 32° Fahr., with barometer at 760 millimeters = 29.922 inches.

3. For solids and liquids, the standard substance is distilled water, at its temperature (4° Cent. = 39.2° Fahr.) of maximum density.

4. For all ordinary purposes of civil engineering, **any clear fresh water, at any ordinary temperature,** may be used. Even with water at 30° Cent., = 86° Fahr., the result is only 4 parts in 1000 too great.

5. When a body is immersed in water, the upward force, or "**buoyancy,**" exerted upon it by the water, or the "**loss of weight**" of the body, due to its immersion, is equal to the weight of the water displaced by the immersion of the body †; or, if

W = the weight of the body in air,
w = its weight in water,
D = its relative density or specific gravity,
A = the weight of water displaced;

then $A = W - w$; and $D = \frac{W}{A} = \frac{W}{W - w}$.

6. Since the volume, V, of a body, of given weight, W, is inversely as its density, or specific gravity, D; the specific gravity is equal also to the ratio between the volume V_s of an equal weight of the standard substance, to the volume, V, of the body in question; or $D = \frac{V_s}{V}$.

7. The specific gravities of **substances heavier than water** are ordinarily determined by weighing a mass of the substance, first in air (obtaining its weight, W), and then when the mass is completely submerged in water (obtaining its diminished weight, w). Then $D = \frac{W}{W - w}$, as in ¶ 5.

8. If the body is lighter than water, it must be entirely immersed, and held down against its tendency to rise. Its weight, w, in water, or its upward tendency, is then a *negative* quantity, and means must be provided for measuring it, as by making it act upward against the scale pan. We then have, $A = W - (-w) = W + w$; or

Loss due to immersion = weight of body in air, *plus* its buoyancy.

9. Or, first allow the body to float upon the water, and note the resulting displacement, v, of water, as by the rise of its surface level in a prismatic vessel. Then immerse the body completely, and again note the displacement, V. Now v, the volume displaced by the body when floating, and V, the volume displaced by the body when completely immersed, are proportional respectively to the weight, W, of the body, and to the weight, $W - w$, of a mass of water of equal volume with the body. Hence $D = \frac{W}{W - w} = \frac{v}{V}$.

10. Or, attach to the light body, b, a heavier body, or sinker, S, of such density and mass that both bodies together will sink in water. Let W be the weight of the light body, b, in air; Q the weight of both bodies in air, and q their combined weight in water. Then $Q - q$ = the weight of a mass of water of equal volume with the two bodies, and $Q - W$ = the weight, S, of the sinker in air. By immersing the sinker alone, find the weight, k, of water equal in volume to the sinker alone, = loss of weight in sinker, due to immersion. Then, for the weight, A, of water of equal volume with the light body, b, or for

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*Strictly speaking, "specific gravity" refers to *weight*, and "relative density" to *mass* (see Mechanics, Art. 14 *a*); but, as specific gravity and density are numerically equal, they are often treated as identical.

† See Hydrostatics, Art. 18.

the loss of weight of b, due to immersion, we have $A - Q - q - k$; and, for the specific gravity, D, of the light body, b, we have $D = \frac{W}{Q - q - k} = \frac{W}{W - w}$ where w = the (unknown) buoyancy of b.

11. A granular body, as a mass of saw-dust, gravel, sand, cement, etc. or a porous body, as a mass of wood, cinder, concrete, sandstone, etc., is a composite body, consisting partly of solid matter and partly of air. Thus, a cubic foot of quartz sand weighs about 100 lbs., while a cubic foot of quartz weighs about 165 lbs.

12. The specific gravity of porous substances is usually taken as that of the composite mass of solid and air. Thus, a wood, weighing (with its contained air) 62.5 lbs. per cubic foot, or the same as water, is said to have a specific gravity of 1. The absorption of water, when such bodies are immersed for the purpose of determining their specific gravities, may be prevented by a thin coat of varnish.

13. The specific gravity of granular substances is sometimes taken as that of the solid part alone. Thus, Portland cements ordinarily weigh (in air) from 75 to 90 lbs per cubic foot, which would correspond to specific gravities of from 1.20 to 1.44, but the specific gravity of the solid portion ranges from 3.00 to 3.25; and the latter figures are usually taken as representing the specific gravities.

14. In determining the specific gravities of substances (such as cement) which are **soluble in water** or otherwise affected by it, the substances are weighed in some liquid (such as benzine, turpentine or alcohol) which will not affect them, instead of in water. The result, so obtained, must then be multiplied by the ratio between the density of the liquid and that of water.

15. The specific gravity of a liquid is most directly determined by weighing equal volumes of the liquid and of water.

16. Or weigh, in the liquid, some body, whose weight, W, in air, and whose specific gravity, d, are known. Let w' = its weight in the liquid. Then, for the specific gravity, D, of the liquid, we have

$$W : W - w' = d : D; \quad \text{or} \quad D = \frac{d\,(W - w')}{W}.$$

17. Or, let the body, in ¶ 16 (weighing W in air), weigh w in water, and (as before) w' in the liquid in question. Then, since specific gravity of water = 1, we have

$$W - w : W - w' = 1 : D; \quad \text{or} \quad D = \frac{W - w'}{W - w}.$$

18. The specific gravities of liquids are commonly obtained by observing the depth to which some standard instrument (called a **hydrometer**) sinks when allowed to float upon the surface of the liquid. The greater the depth, the less the specific gravity of the liquid. In **Beaumé's hydrometer** the depth of immersion is shown by a scale upon the instrument. The graduations of the scale are arbitrary. For liquids heavier than water, 0° corresponds to a specific gravity of 1, and 76° to a specific gravity of 2 For liquids lighter than water, 10° correspond to a specific gravity of 1, and 60° to a specific gravity of 0.745.

19. In Twaddell's hydrometer, used for liquids heavier than water,

$$\text{specific gravity} = \frac{5 \times \text{No. of degrees} + 1{,}000}{1{,}000}$$

Thus, if the reading be 90°,

$$\text{specific gravity} = \frac{5 \times 90 + 1{,}000}{1{,}000} = \frac{1{,}450}{1{,}000} = 1.45.$$

20. In Nicholson's hydrometer, largely used also for solids, the specific gravity is deduced from the weights required to produce a standard depth of immersion. It consists of a hollow metal float, from which rises a thin but stiff wire carrying a shallow dish, which always remains above water. From the float is suspended a loaded dish, which, like the float, is always submerged. On the wire supporting the upper dish is a standard mark, which, in observations, is always brought to the surface of the water. The specific gravity is then determined by means of the weights carried in the two dishes respectively.

21. The determination of the **specific gravities of gaseous substances** requires the skill of expert chemists.

Table of specific gravities, and weights.

In this table, the sp gr of air, and gases also, are compared with that of water instead of that of air; which last is usual.

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft Lbs.
Air, atmospheric; at 60° Fah, and under the pressure of one atmosphere or 14.7 lbs per sq inch, weighs $\frac{1}{815}$ part as much as water at 60°..........	.00123	0765
Alcohol, pure..........	.793	49 43
" of commerce..........	.834	62 1
" proof spirit..........	.916	57 2
Ash, perfectly dry. .... average..	.752	47.
1000 ft board measure weighs 1.748 tons.		
Ash, American white, dry.......... " ..	.61	38.
1000 ft board measure weighs 1.414 tons.		
Alabaster, falsely so called; but really Marbles..........	2 7	168
" real, a compact white plaster of Paris.......... average ..	2 31	144.
Aluminium	2 6	162.
Antimony, cast, 6.66 to 6.74.......... average .	6 70	418.
" native .... " ..	6 67	416.
Anthracite. See Coal, below.		
Asphaltum, 1 to 1.8.......... " ..	1.4	87 3
Basalt. See Limestones, quarried.......... " ..	2.9	181.
Bath Stone, Oolite.......... " ..	2.1	131.
Bismuth, cast. Also native.......... " ..	9.74	607.
Bitumen, solid. See Asphaltum		
Brass, (Copper and Zinc,) cast, 7 8 to 8.4.......... " ..	8 1	504.
" rolled .... " ..	8 4	524
Bronze. Copper 8 parts; Tin 1 (Gun metal) 8 4 to 8 6.......... " ..	8 5	529
Brick, best pressed	....	150.
" common hard	....	125
" soft, inferior	....	100.
Brickwork See Masonry.		
Boxwood, dry.......... " ..	.96	60
Calcite, transparent.......... " ..	2.722	169.9
Carbonic Acid Gas is 1½ times as heavy as air.......... " ..	.00187	
Cement. (See ¶ 13)		
" Portland, 3 00 to 3.25..........	3 12	75 to 90
" Natural, 2 75 to 3.00..........	2 87	50 to 56
Chalk, 2.2 to 2 8 See Limestones, quarried.	2.50	156.
Charcoal, of pines and oaks..........		15 to 30
Cherry, perfectly dry	0 67	42.
Chestnut, perfectly dry	0 68	41.
Coal. See also page 215.		
Anthracite, 1.4 to 1.7	1 50	81 to 106
" piled loose..........		47 to 58
Bituminous, 1 2 to 1.4	1.30	78 to 88
" piled loose		44 to 54
Coke..........	1 00	62.5
" piled loose		23 to 32
In coking, coals swell from 25 to 50 per cent.		
Copper, cast,..........8 6 to 8 8..........	8.7	542.
" rolled,..........8.8 to 9.0..........	8.9	555.
Crystal, pure Quartz. See Quartz.		
Cork	.25	15.6
Diamond, 3.44 to 3 55; usually 3.51 to 3.55	3.53	
Earth; common loam, perfectly dry, loose	....	72 to 80
" " " " " shaken	....	82 to 92
" " " " " moderately rammed..........	....	90 to 100
" " " slightly moist, loose..........	....	76 to 76
" " " more moist, "	....	66 to 68
" " " " shaken..........	....	75 to 90
" " " " moderately packed..........	....	90 to 100
" " " as a soft flowing mud..........	....	104 to 112
" " " as a soft mud, well pressed into a box..........	....	110 to 120
Æther	.716	44.6
Elm, perfectly dry. ....average..	.56	35.
1000 ft board measure weighs 1.302 tons.		
Ebony, dry .... " ..	1.22	76 1
Emerald, 2.63 to 2.76.......... " ..	2.7	
Fat .... " ..	.93	58.
Flint .... " ..	2.6	162.
Feldspar, 2.5 to 2.8.......... " ..	2.65	166.
Garnet, 3.5 to 4.3; Precious, 4.1 to 4.3.......... " ..	4.2	
Glass, 2.5 to 3 45.......... " ..	2.98	186.
" common window .... " ..	2.52	157.
" Millville, New Jersey. Thick flooring glass .... " ..	2.58	158.
Granite, 2.56 to 2.88. See Limestone, 160 to 180.......... " ..	2.72	170.

Table of specific gravities, and weights—(Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Gneiss, common, 2 62 to 2.76	2.69	168.
" in loose piles "	....	96.
" Hornblendic "	2 8	175.
" " quarried, in loose piles "	....	100.
Gypsum, Plaster of Paris, 2 24 to 2.30 "	2.27	141 6
" in irregular lumps "	....	82.
" ground, loose, per struck bushel, 70 "	....	56.
" " well shaken, " " 80 "	....	64.
" " Calcined, loose, per struck bush, 65 to 75 "	....	52 to 60
Greenstone, trap, 2.8 to 3 2 "	3.	187.
" " quarried, in loose piles "		107.
Gravel, about the same as sand, which see.		
Gold, cast, pure, or 24 carat "	19 258	1204.
" native, pure, 19.3 to 19 34 "	19.32	1206.
" " frequently containing silver, 15 6 to 19 3 "		
" pure, hammered, 19.4 to 19 6 "	19 5	1217.
Gutta Percha "	.98	61.1
Hornblende, black, 3.1 to 3 4 "	3.25	203.
Hydrogen Gas, is 14½ times lighter than air; and 16 times lighter than oxygen	....	
Hemlock, perfectly dry "	.4	25.
1000 feet board measure weighs 930 ton		
Hickory, perfectly dry "	.85	53.
1000 feet board measure weighs 1 971 tons		
Iron, and steel.		
" Pig and cast iron and cast steel	7 2	450
" Wrought iron and steel, and wire, 7.6 to 7.9	7.75	475 to 495
Ivory average	1 82	114.
Ice, .917 to .922 "	.92	57.4
India rubber "	.93	58
Lignum vitæ, dry "	1.33	83.
Lard "	.95	59.3
Lead, of commerce, 11.30 to 11 47; either rolled or cast "	11.38	709.6
Limestones and Marbles, 2.4 to 2 86, 150 to 178.8	2.6	164.4
" " " ordinarily about	2.7	168.
" " " quarried in irregular fragments, 1 cub yard solid, makes about 1 9 cub yds perfectly loose, or about 1¾ yds piled In this last case 571 of the pile is solid, and the remaining 429 part of it is voids piled	....	96.
Lime, quick, ground, loose, per struck bushel 62 to 70 lbs	....	53.
" " " well shaken, " " ....80 "	....	64.
" " " thoroughly shaken, " ....93¾ "	....	75.
Mahogany, Spanish, dry * average	.85	53.
" Honduras, dry "	.56	35.
Maple, dry * "	.79	49.
Marbles, see Limestones		
Masonry, of granite or limestones, well dressed throughout	....	165.
" " " well-scabbled mortar rubble. About $\frac{1}{5}$ of the mass will be mortar	....	154.
" " " well-scabbled dry rubble	....	138.
" " " roughly scabbled mortar rubble. About ¼ to ⅓ part will be mortar	....	150.
" " " roughly scabbled dry rubble	....	125.
At 155 lbs per cub ft, a cub yard weighs 1.868 tons; and 14.45 cub ft, 1 ton		
Masonry of sandstone; about ⅙ part less than the foregoing.		
" " brickwork, pressed brick, fine joints average	....	140.
" " " medium quality "	....	125.
" " " coarse, inferior soft bricks "	....	100.
At 125 lbs per cub ft, a cub yard weighs 1 507 tons; and 17.92 cub ft, 1 ton		
Mercury, at 32° Fah	13.62	849.
" 60° "	13.58	846.
" 212° "	13 38	836.
Mica, 2 75 to 3 1	2 93	183.
Mortar, hardened, 1.4 to 1.9	1.65	103.
Mud, dry, close	....	80 to 110
" wet, moderately pressed	....	110 to 130
" wet, fluid	....	104 to 120

* **Green timbers** usually weigh from one-fifth to nearly one-half more than dry; and ordinary building timbers when tolerably seasoned about one-sixth more than perfectly dry

Table of specific gravities, and weights—(Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Naphtha	.848	52.9
Nitrogen Gas is about $\frac{1}{35}$ part lighter than air	....	.0744
Oak, live, perfectly dry, .88 to 1.02 * average	.95	59.3
" white, " " .66 to .88 "	.77	48.
" red, black, &c* "		32 to 45
Oils, whale, olive "	.92	57.3
" of turpentine "	.87	54.3
Oolites, or Roestones, 1.9 to 2.5 "	2.2	137
Oxygen Gas, a little more than $\frac{1}{10}$ part heavier than air	.00136	.0846
Petroleum	.878	54.8
Peat, dry, unpressed	....	20 to 30
Pine, white, perfectly dry, .35 to .45* 1000 ft board measure weighs .930 ton.*	.40	25
" yellow, Northern, .48 to .62 1000 ft board measure weighs 1.276 tons *	.55	34.3
" " Southern, .64 to .80 1000 ft board measure weighs 1.674 tons *	.72	45
Pine, heart of long leafed Southern yellow, unseas. 1000 ft board measure weighs 2.418 tons.	1.04	65
Pitch	1.15	71.7
Plaster of Paris, see Gypsum.		
Powder, slightly shaken	1	62.3
Porphyry, 2.66 to 2.8	2.73	170.
Platinum 21 to 22	21.5	1342.
" native, in grains 16 to 19	17.5	
Quartz, common, pure 2.64 to 2.67	2.65	165
" " finely pulverized, loose		90
" " " " well shaken		105
" " " " well packed		112
" quarried, loose. One measure solid, makes full 1¾ broken and piled		94.
Ruby and Sapphire, 3.8 to 4.0	3.9	
Rosin	1.1	68.6
Salt		50 to 70
Sand, pure quartz, perfectly dry, loose		90 to 106
" " " " " slightly shaken		92 to 110
" " rammed, dry		100 to 120
Natural sand consists of grains of different sizes, and weighs more, per unit of volume, than a sand sifted from it and having grains of uniform size. Sharp sand with very large and very small grains may weigh as much as		117.
Sand is very retentive of moisture, and, when in large bulk, its natural moisture may diminish its weight from 5 to 10 per cent.		
" perfectly wet, voids full of water		118 to 120
Sandstones, fit for building, dry, 2.1 to 2.73, ...131 to 171	2.41	151.
" quarried, and piled, 1 measure solid, makes about 1¾ piled.	....	86.
Serpentines, good 2.5 to 2.65	2.6	162.
Snow, fresh fallen		5 to 12
" moistened, and compacted by rain		15 to 50
Sycamore, perfectly dry. 1000 ft board measure weighs 1.376 tons.	.59	37.
Shales, red or black 2.4 to 2.8 average	2.6	162
" quarried, in piles "	....	92
Slate 2.7 to 2.9 "	2.8	175
Silver "	10.5	655.
Soapstone, or Steatite 2.65 to 2.8 "	2.73	170.
Steel, 7.7 to 7.9. The heaviest contains least carbon "	7.85	490.
Steel is not heavier than the iron from which it is made; unless the iron had impurities which were expelled during its conversion into steel.		
Sulphur average	2.	125.
Spruce, perfectly dry. " 1000 ft board measure weighs .930 ton.	.4	25.
Spelter, or Zinc 6.8 to 7.2 "	7.00	437.5
Sapphire; and Ruby, 3.8 to 4 "	3.9	
Tallow "	.94	58.6
Tar "	1.	62.4
Trap, compact, 2.8 to 3.2 "	3.	187
" quarried; in piles "	....	107
Topaz, 3.45 to 3.65 "	3.55	

***Green timbers** usually weigh from one-fifth to nearly one-half more than dry; and ordinary building timbers when tolerably seasoned about one-sixth more than perfectly dry

Table of specific gravities, and weights — (Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft Lbs.
Tin, cast, 7 2 to 7 5 .... average..	7 35	459.
Turf, or Peat, dry, unpressed.	....	20 to 30
Water. See page 326.	. ..	62 417
Wax bees .... average..	.97	60.5
Wines, 993 to 1 04 .... " ..	.998	62.3
Walnut, black, perfectly dry .... " .. 1000 ft board measure weighs 1.414 tons.	.61	38.
Zinc, or Spelter, 6.8 to 7.2 .... " ..	7.00	437.5
Zircon, 4.0 to 4.9.......... " ..	4.45	

Space occupied by coal. In cubic feet per ton of 2240 pounds.

Pennsylvania Anthracite.

	Broken.	Egg.	Stove.	Nut.	Pea.	Buckwheat.	Average
Hard white ash *..........	39.4	38 6	39 2	39 8	40.5	41.1	39.8
		39.6	39 6	39 6	39.8	39.8	39 6
Free-burning white ash †..........	39.6	39.0	39 6	40.2	40.8	41.5	40.2
		39 6	39.6	41.2	41 9	42.4	40.7
Shamokin ‡..........	39.0	39.3	39.9	40.5	41 2	41.9	40.6
		39.9	42 6	45 7	46 5	47.7	43.6
Schuylkill white ash *		39.6	40.3	40.9	41 6	42.3	40.9
" red " *..........		40.0	40.5	41.1	41 7	42.3	41.1
Lykens Valley *..........	44.2	44.8	45.2	45 7	46.2	46.7	45.7
		44.3	44.3	45.0	46.1	46.5	45.1
Wyoming free-burning †..........		40 0	39.8	39.4			39.7
Lehigh †..........	39.4	38.8	38.5	38.4	42 1	41.4	40.0
Lehigh; Reading C. & I. Co. *...	38.5	38.8	40.1	40 3	40.3	40.5	39.7

Lehigh: † Lump, 40.5; cupola, 40 3; dust, 39.1.

Bituminous.

From Coxe Bros. & Co †	From Jour. U. S. Ass'n Charcoal Iron Workers. Vol. III, 1882.§	
Pittsburg.......... 48 2	Pittsburg.......... 47 1	Clover Hill, Va.......... 49 0
Erie.......... 46 6	Cumberland, max. .. 42 3	Richmond, Va. (Midlothian).......... 41.0
Hocking Valley.......... 45 4	" min 41 2	Cannelton, Ind.......... 47.0
Ohio Cannel.......... 45 5	Blossburg, Pa 42 2	Pictou, N. S.......... 45.0
Indiana Block.......... 51 1		Sydney, Cape Breton. 47.0
Illinois 47.4		

	Logarithm.
1 cubic foot per ton of 2240 pounds =	
0 89286 cubic foot per ton of 2000 pounds..........	$\bar{1}$.950 7820
2240 (exact) pounds per cubic foot..........	3 350 2480
1 cubic foot per ton of 2000 pounds =	
1.12 (exact) cubic feet per ton of 2240 pounds..........	0 049 2180
2000 (exact) pounds per cubic foot..........	3 301 0300
1 pound per cubic foot =	
2240 (exact) cubic feet per ton of 2240 pounds..........	3 350 2480
2000 " " " 2000 "	3.301 0300

* From Edwin F. Smith, Sup't & Eng'r, Canal Div., Phila. and Reading R. R

† From very careful weighings in the Chicago yards of Coxe Bros. & Co Note the irregular variation with size of anthracite in Coxe Bros.' figures.

§ Quoted from *The Mining Record*. On the authority of "many years' experience" of "a prominent retail dealer in Philadelphia," the Journal gives also figures requiring from 4 to 13 per cent. less volume per ton than those here quoted from the Journal and from other authorities.

WEIGHTS AND MEASURES.

United States and British measures of length and weight, of the same denomination, may, *for all ordinary purposes*, be considered as equal; but the **liquid and dry** measures of the same denomination differ widely in the two countries. **The standard measure of length** of both countries is theoretically that of a pendulum vibrating seconds at the level of the sea, in the latitude of London, in a vacuum, with Fahrenheit's thermometer at 62°. The length of such a pendulum is supposed to be divided into 39.1393 equal parts, called inches; and 36 of these inches were adopted as the standard yard of both countries. But the Parliamentary standard having been destroyed by fire, in 1834, it was found to be impossible to restore it by measurement of a pendulum. The present British Imperial yard, as determined, at a temperature of 62° Fahrenheit, by the standard preserved in the Houses of Parliament, is the standard of the United States Coast and Geodetic Survey, and is recognized as standard throughout the country and by the Departments of the Government, although not so declared by Act of Congress The yard between the 27th and 63d inches of a scale made for the U S Coast Survey by Troughton, of London, in 1814, is found to be of this standard length when at a temperature of 59°.62 Fahrenheit; but at 62° is too long by 0.00083 inch, or about 1 part in 43373, or 1 46 inch per mile, or 0.0277 inch in 100 feet.

The Coast Survey now uses, for purposes of comparison, two measures presented by the British Government in 1855, as copies of the Imperial standard, namely:

"Bronze standard, No 11," of standard length at 62° 25 Fahr.
"Malleable iron standard, No. 57," " " " 62°.10 "

See Appendix No. 12, Report of U. S. Coast and Geodetic Survey for 1877.

The legal standard of weight of the United States is the **Troy pound of the Mint** at Philadelphia. This standard, containing 5760 grains, is an exact copy of the **Imperial Troy pound of Great Britain.** The avoirdupois or commercial pound of the United States, containing 7000 grains, and derived from the standard Troy pound of the Mint, is found to agree within one thousandth of a grain with the British avoirdupois pound. The U. S. Coast Survey therefore declares the weights of the two countries identical.

The Ton. In Revised Statutes of the United States, 2d Edition, 1878, Title XXXIV, Collection of Duties upon Imports, Chapter Six, Appraisal, says:

"Sec 2951. Wherever the word 'ton' is used in this chapter, in reference to weight, it shall be construed as meaning twenty-hundredweight, each hundredweight being one hundred and twelve pounds avoirdupois."

This appears to be the only U. S. Government regulation on the subject.

The ton of 2240 lbs (often called **a gross ton** or **long ton**) is commonly used in buying and selling iron ore, pig iron, steel rails and other manufactured iron and steel. Coke and many other articles are bought and sold by the **net ton** or **short ton** of 2000 lbs. The bloom ton had 2464 lbs, = 2240 lbs + 2 hundredweight of 112 lbs each; and the pig iron ton had 2268 lbs, = 2240 lbs + a "sandage" of 28 lbs, or one "quarter," to allow for sand adhering to the pigs, but some furnace men allowed only 14 lbs. In electric traction work the ton means 2000 lbs.

As a measure, the ton, or tun, is defined as 252 gallons, as 40 cubic feet of round or rough timber or in ship measurement, or as 50 feet of hewn timber. 252 U. S. gallons of water weigh about 2100 lbs; 252 Imperial gallons about 2500 lbs; 50 cub ft yellow pine about 2500 lbs.

The metric system* **was legalized in the United States** in

* The metric system, as compared with the English, has much the same advantages and disadvantages that our American decimal coinage has in comparison with the English monetary system of pounds, shillings and pence. It will enormously facilitate all calculations, but, like all other improvements, it will necessarily cause some inconvenience while the change is being made. The metric system has also this further and very great advantage, that it bids fair to become universal among civilized nations.

1866, but has not been made obligatory. The government has since furnished very exact metric standards to the several States. The use of the metric system has been permitted in Great Britain, beginning with August 6, 1897, and in Russia, beginning with 1900. Its use is now at least permissive in most civilized nations.

The metric unit of length is the metre, or meter, which was intended to be one ten-millionth $\left(\frac{1}{10,000,000}\right)$ of the earth's quadrant, *i. e.*, of that portion of a meridian embraced between either pole and the equator This length was measured, and a set of metrical standards of weight and measure were prepared in accordance with the result, and deposited among the archives of France at Paris (Mètre des Archives. Kilogramme des Archives, etc.) It has since been discovered that errors occurred in the calculations for ascertaining the length of the quadrant; but the standards nevertheless remain as originally prepared

The metric measures of surface and of capacity are the squares and cubes of the meter and of its (decimal) fractions and multiples.

The metric unit of weight is the gramme or gram, which is the weight of a milliliter or cubic centimeter* of pure water at its temperature of maximum density, about 4° Centigrade or 39 2° Fahrenheit.

By the concurrent action of the principal governments of the world, an **International Bureau of Weights and Measures** has been established, with its seat near Paris It has prepared two ingots of pure platinum-iridium, from one of which a number of standard kilograms (1000 grams) have been made, and from the other a number of standard meter bars, both derived from the standards of the Archives of France. Of these copies, certain ones were selected as international standards, and the others were distributed to the different governments. Those sent to the United States are in the keeping of the U. S. Coast Survey.

The determination of the **equivalent of the meter in English measure** is a very difficult matter. The standard *meter* is measured *from end to end* of a *platinum* bar and at the *freezing* point; whereas the standard *yard* is measured *between two lines* drawn on a *silver* scale inlaid in a *bronze* bar, and *at* 62° *Fahrenheit* **The United States Coast Survey**† adopts, as the length of the meter at 62° Fahrenheit, the value determined by Capt. A. R. Clarke and Col. Sir Henry James, at the office of the British Ordnance Survey. In 1866, viz.: 39.370432 inches (= 3 2808666 + feet = 1.0936222 + yards); but the **lawful equivalent,** established by Congress, is 39 37 inches (= 3.28083 feet = 1.093611 yards). This value is as accurate as any that can be deduced from existing data.

The gram weighs, by Prof W. H. Miller's determination,‡ 15.43234874 grains An examination made at the International Bureau of Weights and Measures in 1884 makes it 15.43235639 grains. The **legal value** in the United States is 15.432 grains.

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* 1 centimeter = $\frac{1}{100}$ meter = 0.3937 inch. 1 milliliter ($\frac{1}{1000}$ liter) or cubic centimeter = 0.061 + cubic inches.

† Appendix No. 22 to report of 1876, page 6.

‡ Philosophical Transactions, 1856, pp. 893, etc.

Foreign Monetary Units and Coins. Values in U. S. Money.

From Report of Director of Mint, 1901, and Treasury Dept Circular, Jan. 1, 1911.

Africa. See Egypt, Liberia.
Argentine Republic. Peso = 100 centavos = $0.965. Argentine = $4.82.
Asia. See China, India, Japan, Persia, Siam, Straits Settlements.
Austria-Hungary. Crown = 100 hellers = $0.203. Ducat = $2.29.
Bavaria. See German Empire.
Belgium.* Franc = 100 centimes = $0.193.
Bolivia. Boliviano = 100 centavos = $0.389. 10 centimos = $0.0964.
Brazil. Milreis = 1000 reis = $0.546.
British Honduras Dollar = $1.00.
Canada. Dollar = 100 cents = $1.00.
Central America. See British Honduras, Costa Rica, Cuba, Haiti, Honduras, Guatemala, Nicaragua, Panama, Salvador, Santo Domingo.
Chile. Peso = 100 centavos = 0.05 condor = 0.1 doubloon = 0.2 escudo = $0.365.
China. Tael = 10 mace or tsien = 100 candareens or fun = 1000 cash or li = $0.604 to $0.673.
Colombia. Dollar = $1.00.
Costa Rica. Colon = 100 centimos = $0.465.
Cuba. Spanish quadruple (onza) = $15.74; doubloon Isabella = $5.02, Alphonse = 25 pesetas = $4.82.
Denmark.† Crown = 100 ore = $0.268.
Ecuador. Sucre = 100 cents = 5 pesetas = 10 reals = 20 medioreals = $0.487.
Egypt. Pound = 100 piasters = $4.943.
England. See Great Britain.
Esperantists. (From Am. Esperantist Co) Spesmilo = 10 spescentoj = 100 spesdekoj = 1000 spesoj = $0.4875.
Finland. Mark = $0.193.
France.* Franc = 100 centimes = $0.193.
German Empire. Mark = 100 pfennigs = $0.238; Crown = $2.382.
Great Britain. Pound sterling or sovereign = 20 shillings = 240 pence = 960 farthings = $4.8665. Guinea = 21 shillings, Crown = 5 shillings, Florin = 2 shillings.
Greece.* Drachma = 100 lepta = $0.193.
Guatemala. Peso = 10 dimes = $0.403. Onza or doubloon = $15.74.
Haiti. Gourde = 100 centimes = $0.965.
Honduras. Peso = 10 dimes = $0.403. Onza or doubloon = $15.74
India, British—. Rupee = $0.324 1/3 , anna = 4 pice = $0.02.
Italy.* Lira = 100 centesimi = $0.193.
Japan. Yen = 100 sen = $0.498.
Latin Union.* See France, Belgium, Italy, Switzerland, Greece.
Liberia. Dollar = $1.00.
Mexico. Peso = 100 centavos = $0.498.
Netherlands. Florin = 100 cents = $0.402. Rixdaler = 2.5 florins.
Newfoundland. Dollar = $1.014.
Nicaragua. Peso = 10 dimes = $0.403. Onza or doubloon = $15.74
North America. See Canada, Mexico, Newfoundland.
Norway.† Crown = 100 öre = $0.268.
Oceanica. See Philippine Islands.
Panama. Balboa = $1.00.
Persia. Kran = $0.1704; toman = 10 krans.
Peru. Libra = $4.8665. 1 sol = 2 British shillings.
Philippine Islands. Peso = $0.50.
Portugal. Milreis = 0.1 crown = 1000 reis = $1.08.
Prussia. See German Empire.
Roumania. Leu = $0.193.
Russia. Ruble = 100 copecks = $0.515.
Salvador. Peso = 10 dimes = $0.403. Onza or doubloon = $15.74
Santo Domingo. Dollar = $1.00.
Scandinavian Union.† See Denmark, Norway, Sweden.
Servia. Dinar = $0.193.
Siam. Tical = $0.3708.

*, †; See foot-notes, next page.

Foreign Monetary Units and Coins. *Continued.*

South America. See Argentine Republic, Bolivia, Brazil, Chile, Colombia, Ecuador, Peru, Uruguay, Venezuela.
Spain. Peseta = 100 centimes = $0.193.
Straits Settlements. Dollar = $0.421.
Sweden.† Crown = 100 öre = $0.268.
Switzerland.* Franc = $0.193.
Turkey. Piaster = 0.01 lira = $0.044.
Uruguay Peso = $1.034.
Venezuela. Bolivar = $0.193.

Standard Diameters and Weights of United States Coins.

	Value	Diameter.		Weight.	
	$	Inches.	Millimeters.	Grains.	Grams.
Gold, 10 per cent. alloy:					
Double Eagle	20	1 350	34 29	516.00	33.436
Eagle	10	1 060	26 92	258 00	16.718
Half Eagle	5	0 848	21.54	129.00	8.359
Quarter Eagle	2 50	0.700	17.78	64.50	4.180
Silver, 10 per cent alloy:					
Standard Dollar	1.00	1.500	38.10	412 50	26 729
Half Dollar	0.50	1.205	30.61	192 90	12.50
Quarter Dollar	0.25	0.955	24 26	96.45	6 25
Dime	0.10	0.705	17 91	38.58	2.50
Minor					
Five Cents, 75% copper, 25% nickel	0.05	0.835	21.20	77.16	5 00
One Cent, 95% copper, 5% tin and zinc	0 01	0.750	19.09	48.00	3.11

Perfectly pure gold is worth $1 per 23.22 grs = $20.67183 per troy oz = $18 81151 per avoir oz. **Standard** (U S coin) is worth $18 60465 per troy oz = $16.95736 per avoir oz It consists of 9 parts by weight of pure gold, to 1 par alloy Its value is that of the pure gold only; the cost of the alloy and of th coinage being borne by Government **A cubic foot of pure gold weigh** about 1204 avoir lbs; and is worth $362963. A cubic inch weighs about 11.14 avoir oz: and is worth $210.04.

Pure gold is called one, or 24 **carat** gold; and when alloyed, the alloy is sup posed to be divided into 24 parts by weight, and according as 10, 15, or 20, &c o these parts are pure gold, the alloy is said to be 10, 15, or 20, &c, carat.

The average fineness of California native gold, by some thou sands of assays at the U. S. Mint in Philada., is 88.5 parts gold, 11 5 silver. Som from Georgia, 99 per cent gold.

Pure silver fluctuates in value· thus, during 1878–1879 it ranged betwee $1 05 and $1 18 per troy oz, or $.957 and $1.076 per avoir. oz. A cubic inch weigh about 5.528 troy, or 6.065 avoir ounces

* Latin Union (France, Belgium, Italy, Switzerland, Greece). Fineness, gold and 5 franc silver, 0.9; minor silver coins, 0.835.

	Gold					Silver					
Francs	100	50	20	10	5	5	2	1	0.5	0.
Diameters, in millimeters	35	28	21	19	17	37	27	23	18	16
Weights, in grams	0.32258 per franc					25	10	5	2.5	1

† Scandinavian Union (Sweden, Norway, Denmark).

Troy Weight. U. S. and British.

24 grains	1 pennyweight, dwt.
20 pennyweights	1 ounce = 480 grains.
12 ounces	1 pound = 240 dwts. = 5760 grains.

Troy weight is used for gold and silver.

A carat of the jewellers, for precious stones is, in the U. S. = 3.2 grs.; in London, 3.17 grs., in Paris, 3.18 grains, divided into 4 jewellers' grs. **In troy, apothecaries' and avoirdupois, the grain is the same.**

Apothecaries' Weight. U. S. and British.

20 grains	1 scruple.
3 scruples	1 dram = 60 grains.
8 drams	1 ounce = 24 scruples = 480 grains
12 ounces	1 pound = 96 drams = 288 scruples = 5760 grains.

In troy and apothecaries' weights, the grain, ounce and pound are the same.

Avoirdupois or Commercial Weight. U. S. and British.

27.34375 grains	1 dram
16 drams	1 ounce = 437½ grains.
16 ounces	1 pound = 256 drams = 7000 grains.
28 pounds	1 quarter = 448 ounces.
4 quarters	1 hundredweight = 112 lbs.
20 hundredweights	1 ton = 80 quarters = 2240 lbs.

A stone = 14 pounds. **A quintal** = 100 pounds avoir.

The standard of the avoirdupois pound, which is the one in common commercial use, is the weight of 27.7015 cub ins of pure distilled water, at its maximum density at about 39°.2 Fahr, in latitude of London, at the level of the sea; barometer at 30 ins. But this involves an error of about 1 part in 1362, for the 1 lb of water = 27.68122 cub ins.

A troy lb = .82286 avoir lb. An avoir lb = 1.21528 troy lb, or apoth.

A troy oz. = 1.09714 avoir oz. An avoir oz = .911458 troy oz, or apoth.

Long Measure. U. S. and British.

12 inches	1 foot = .3047973 metre
3 feet	1 yard = 36 ins = .9143919 metre
5½ yards	1 rod, pole, or perch = 16½ feet = 198 ins.
40 rods	1 furlong = 220 yards = 660 feet
8 furlongs	1 statute, or land mile = 320 rods = 1760 yds = 5280 ft = 63360 ins.
3 miles	1 league = 24 furlongs = 960 rods = 5280 yds = 15840 ft

A point = $\frac{1}{72}$ inch. **A line** = 6 points = $\frac{1}{12}$ inch. **A palm** = 3 ins. **A hand** = 4 ins. **A span** = 9 ins. **A fathom** = 6 feet. **A cable's length** = 120 fathoms = 720 feet. **A Gunter's surveying chain** is 66 feet, or 4 rods long. It has 100 links, 7.92 inches long. 80 Gunter's chains = 1 mile.

A nautical mile, geographical mile, sea mile, or knot, is variously defined as being = the length of

		metres	feet	statute miles
1 min of longitude at the equator	=	1855.345	6087.15	1.15287
1 " latitude " "	=	1842.787	6045.95	1.14507
1 " " " pole	=	1861.655	6107.85	1.15679
1 " " at lat 45°	=	1852.181	6076.76	1.15090
1 " a great circle of a *true* sphere whose *surface area* is equal to that of the earth	=	value adopted by **U. S. Coast and Geodetic Survey** 1853.248	6080.27	1.15157
British **Admiralty knot**	=	1853.169	6080.00	1.15152

Navigators use **"knot"** to mean a *speed* of 1 knot *per hour*.

Minutes, in meters and in ft, as above, calculated from Clarke's spheroid. U. S. Coast & Geod Survey, Rept for 1881, App. 12.

Radius	Arc of 1 Deg	Arc of 1 Min	Arc of 1 Sec
100 ft	1.74533 ft	0.02909 ft	0.000485 ft
1 mile	92.1534 ft	1.5359 ft	0.0256 ft

Earth's great circle. Radius, equatorial, = 3963.27 miles; polar = 3949.83 miles. **Arc of 1°** = chord of 1° + 4.6356 feet.

Length of degree of latitude, in miles. At equator, 68.70; at lat 20°, 68.78; 40°, 69.00; 60°, 69.23; 80°, 69.39; 90°, 69.41.

Lengths of a Degree of Longitude in different Latitudes, and at the level of the Sea. These lengths are in common land or statute miles, of 5280 ft. Since the figure of the earth has never been *precisely* ascertained, these are but close approximations. Intermediate ones may be found correctly by simple proportion. 1° of longitude corresponds to 4 mins of civil or clock time; 1 min of longitude to 4 secs of time.

Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.
0	69.16	14	67.12	28	61.11	42	51.47	56	38.76	70	23.72
2	69.12	16	66.50	30	59.94	44	49.83	58	36.74	72	21.43
4	68.99	18	65.80	32	58.70	46	48.12	60	34.67	74	19.12
6	68.78	20	65.02	34	57.39	48	46.36	62	32.55	76	16.78
8	68.49	22	64.15	36	56.01	50	44.54	64	30.40	78	14.42
10	68.12	24	63.21	38	54.56	52	42.67	66	28.21	80	12.05
12	67.66	26	62.20	40	53.05	54	40.74	68	25.98	82	9.66

Inches reduced to Decimals of a Foot. No errors.

Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot	Ins.	Foot	Ins	Foot.
0	.0000	**2**	.1667	**4**	.3333	**6**	.5000	**8**	.6667	**10**	.8333
1-32	.0026		.1693		.3359		.5026		.6693		.8359
1-16	.0052		.1719		.3385		.5052		.6719		.8385
3-32	.0078		.1745		.3411		.5078		.6745		.8411
⅛	.0104	⅛	.1771	⅛	.3438	⅛	.5104	⅛	.6771	⅛	.8438
5-32	.0130		.1797		.3464		.5130		.6797		.8464
3-16	.0156		.1823		.3490		.5156		.6823		.8490
7-32	.0182		.1849		.3516		.5182		.6849		.8516
¼	.0208	¼	.1875	¼	.3542	¼	.5208	¼	.6875	¼	.8542
9-32	.0234		.1901		.3568		.5234		.6901		.8568
5-16	.0260		.1927		.3594		.5260		.6927		.8594
11-32	.0286		.1953		.3620		.5286		.6953		.8620
⅜	.0313	⅜	.1979	⅜	.3646	⅜	.5313	⅜	.6979	⅜	.8646
13-32	.0339		.2005		.3672		.5339		.7005		.8672
7-16	.0365		.2031		.3698		.5365		.7031		.8698
15-32	.0391		.2057		.3724		.5391		.7057		.8724
½	.0417	½	.2083	½	.3750	½	.5417	½	.7083	½	.8750
17-32	.0443		.2109		.3776		.5443		.7109		.8776
9-16	.0469		.2135		.3802		.5469		.7135		.8802
19-32	.0495		.2161		.3828		.5495		.7161		.8828
⅝	.0521	⅝	.2188	⅝	.3854	⅝	.5521	⅝	.7188	⅝	.8854
21-32	.0547		.2214		.3880		.5547		.7214		.8880
11-16	.0573		.2240		.3906		.5573		.7240		.8906
23-32	.0599		.2266		.3932		.5599		.7266		.8932
¾	.0625	¾	.2292	¾	.3958	¾	.5625	¾	.7292	¾	.8958
25-32	.0651		.2318		.3984		.5651		.7318		.8984
13-16	.0677		.2344		.4010		.5677		.7344		.9010
27-32	.0703		.2370		.4036		.5703		.7370		.9036
⅞	.0729	⅞	.2396	⅞	.4063	⅞	.5729	⅞	.7396	⅞	.9063
29-32	.0755		.2422		.4089		.5755		.7422		.9089
15-16	.0781		.2448		.4115		.5781		.7448		.9115
31-32	.0807		.2474		.4141		.5807		.7474		.9141
1	.0833	**3**	.2500	**5**	.4167	**7**	.5833	**9**	.7500	**11**	.9167
1-32	.0859		.2526		.4193		.5859		.7526		.9193
1-16	.0885		.2552		.4219		.5885		.7552		.9219
3-32	.0911		.2578		.4245		.5911		.7578		.9245
⅛	.0938	⅛	.2604	⅛	.4271	⅛	.5938	⅛	.7604	⅛	.9271
5-32	.0964		.2630		.4297		.5964		.7630		.9297
3-16	.0990		.2656		.4323		.5990		.7656		.9323
7-32	.1016		.2682		.4349		.6016		.7682		.9349
¼	.1042	¼	.2708	¼	.4375	¼	.6042	¼	.7708	¼	.9375
9-32	.1068		.2734		.4401		.6068		.7734		.9401
5-16	.1094		.2760		.4427		.6094		.7760		.9427
11-32	.1120		.2786		.4453		.6120		.7786		.9453
⅜	.1146	⅜	.2813	⅜	.4479	⅜	.6146	⅜	.7813	⅜	.9479
13-32	.1172		.2839		.4505		.6172		.7839		.9505
7-16	.1198		.2865		.4531		.6198		.7865		.9531
15-32	.1224		.2891		.4557		.6224		.7891		.9557
½	.1250	½	.2917	½	.4583	½	.6250	½	.7917	½	.9583
17-32	.1276		.2943		.4609		.6276		.7943		.9609
9-16	.1302		.2969		.4635		.6302		.7969		.9635
19-32	.1328		.2995		.4661		.6328		.7995		.9661
⅝	.1354	⅝	.3021	⅝	.4688	⅝	.6354	⅝	.8021	⅝	.9688
21-32	.1380		.3047		.4714		.6380		.8047		.9714
11-16	.1406		.3073		.4740		.6406		.8073		.9740
23-32	.1432		.3099		.4766		.6432		.8099		.9766
¾	.1458	¾	.3125	¾	.4792	¾	.6458	¾	.8125	¾	.9792
25-32	.1484		.3151		.4818		.6484		.8151		.9818
13-16	.1510		.3177		.4844		.6510		.8177		.9844
27-32	.1536		.3203		.4870		.6536		.8203		.9870
⅞	.1563	⅞	.3229	⅞	.4896	⅞	.6563	⅞	.8229	⅞	.9896
29-32	.1589		.3255		.4922		.6589		.8255		.9922
15-16	.1615		.3281		.4948		.6615		.8281		.9948
31-32	.1641		.3307		.4974		.6641		.8307		.9974

Square or Land Measure. U. S. and British.

144	square inches	=	1 square foot. (100 sq ft = **1 square**);
9	square feet	=	1 square yard = 1296 square inches;
30.25	square yards	=	1 square rod = 272.25 square feet;
40	square rods	=	1 rood = 1210 square yards;
4	roods	=	1 acre = 160 rods = 4840 sq yds = 43,560 sq ft;
640	acres	=	1 square mile.

U. S. Public Lands are divided into townships, sections and quarter-sections, bounded by meridians and parallels. Nominally a **township** is 6 miles square and contains 36 **sections**, each 1 mile square and containing 4 **quarter-sections**, 0.5 mile square. By law, errors, including those due to **convergence of the meridians**, are shifted to the northern and western quarter-sections of the township.

1 Circular Inch = circle, 1 inch in diameter, =

			Logarithm.
0.785398	square inch	n	1.895 0899

1 Square Inch =

1.27324	circular inches		0.104 9101

See also "Surface," pp 233, 234.

Cubic or Solid Measure. U. S. and British.

1728 cubic inches = 1 cubic foot. 27 cubic feet = 1 cubic yard.

1 British Rod of Bricklaying = 16.5 ft. sq. of 14 inch wall.

1 British Rod, engineering, = 306 cubic feet.

1 Toise = 261.5 cubic feet; **1 Chaldron** = 58.64 cubic feet.

See also "Volume," pp 234, 235.

1 Cubic Foot =

			Logarithm.
3300.24	spherical inches		3.518 5451
1.90986	spherical feet		0.281 0014
0.803564	U. S. bushel	n	1.905 0204
0.267855	flour bbl. of 3 struck bushels	n	1.427 8991
0.237477	U. S. liq. bbl. of 31½ gallons	n	1.375 6211

1 Cubic Inch =

1.90986	spherical inches		0.281 0014

1 Cubic Yard =

201.974	U. S. gallons		2.305 2955
7.23207	flour bbls. of 3 struck bushels		0.859 2629
21.6962	U. S. struck bushels		1.336 3842

1 Spherical Foot = sphere, 1 foot in diameter, =

0.523599	cubic foot	n	1.718 9986
904.779	cubic inches		2.956 5423
14.8268	liters		1.171 0461

1 Spherical Inch = sphere, 1 inch in diameter, =

0.523599	cubic inch	n	1.718 9986
8.58030	cubic centimeters		0.933 5024

1 Cylinder, 1 foot diameter, 1 foot long =

0.0290888	cubic yard	n	2.463 7261
0.785398	cubic foot	n	1.895 0899
1357.17	cubic inches		3.132 6336
5.87519	U. S. liquid gallons		0.769 0216
4.89468	British imperial gallons		0.689 7244
22.2401	liters		1.347 1374

1 Cylinder, 1 inch diameter, 1 foot long =

9.42478	cubic inches		0.974 2711
0.326399	U. S. liquid pint	n	1.513 7491
0.271927	British imperial pint	n	1.434 4519
0.154445	liter	n	1.188 7749

n Negative characteristic.

Liquid Measure. U. S. only.
See also "Volume," pp 234–5.

Based upon the old British wine gallon of 231 cubic inches. A cylinder 7 ins diameter, 6 ins high, contains 230.907 cubic inches. A cube, of 6.1358 ins, = 231.0008 cub ins.

4 gills = 1 pint		63 gallons	= 1 hogshead
2 pints = 1 quart	= 8 gills	2 hogsheads	= 1 pipe or butt
4 quarts = 1 gallon	= 8 pints = 32 gills	2 pipes	= 1 tun.

In the U S. and Great Britain, **a barrel** of wine or brandy = 31.5 gallons = 4.211 cub ft = cube of 1.6149 ft = cube of 19.3789 ins; in Pennsylvania, a half-barrel = 16 gallons; a double barrel = 64 gals. **A puncheon** = 84 gals. **A tierce** = 42 gals.

Contents of cylinders.

Pint, 3.5 ins diam, 3 001 ins high		2 gallons, 7 ins diam, 12.005 ins high	
Quart, 3 5 " " 6 002 " "		8 " 14 " " 12.005 " "	
Gallon, 7.0 " " 6.002 " "		10 " 14 " " 15.006 " "	

Apothecaries' or Wine Measure.

Measure.	Symbol.	Pints	Fluid ounces	Fluid drachms.	Minims.	Cubic inches.	Weight of water.‡	
							Pounds, av.	Grains.
1 Gallon	Cong *	8	128	1024	61440	231	8 345	58415
1 Pint...........	O †	1	16	128	7680	28.875	1 043	7301.9
							Ounces, av.	
1 Fluid ounce ...	f ℥	....	1	8	480	1.8047	1.043	456 4
1 Fluid drachm..	f ʒ	..	....	1	60	0 2256	....	57 05
1 Minim	♏	.. .	....	...	1	0 0038	....	0.95

Dry Measure. U. S. only.

Based upon the old British Winchester struck bushel of 2150.42 cubic inches. Dimensions, 18.5 ins inner diam; 19.5 ins outer diam; 8 ins deep. When heaped, cone not less than 6 ins high.

2 pints = 1 quart	2 gallons = 1 peck = 8 qts = 16 pints
4 quarts = 1 gallon = 8 pints	4 pecks = 1 bushel = 8 gals = 32 qts = 64 pints

Cement barrels, approximate dimensions, etc.

Portland. Height, between heads, 2 to 2.2 ft. Capacity, between heads, 3.1 to 3.5 cu ft. Cement in barrel, net 370 to 387 lbs; packed, 3 to 3.5 cu ft; loose, 3.7 to 4.2 cu ft. Weight per cu ft: packed, 114 to 123 lbs; loose, 89 to 100 lbs.§

Natural. Capacity, 3.4 to 3.8 cu ft. Net weight, Western states, 265 lbs; Eastern states, 300 lbs; making weight per cu ft, packed, 78 to 79 lbs.

Cement bags. Am Soc for Testing Materials¶ specifies, Nov 14, 1904, that a bag shall contain 94 lbs net. Portland, 4 bags to a barrel; natural, 3 bags.

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* Abbreviation of Latin, Congius. † Abbreviation of Latin, Octarius.

‡ At its maximum density, 62.425 lbs per cub ft, corresponding to a temp of 4° C = 39.2° F.

§ Sanford E. Thompson, Eng. News, Oct. 4, 1900.

¶ Proceedings, 1904, Vol IV, p. 107.

Bushels and Barrels.

A struck bushel = 1.24446 cubic feet;
A cubic foot = 0 80356 struck bushel;
A flour barrel = 3.75 cubic feet = 3 struck bushels.
In ordering by the barrel, specify its contents, as in pounds or in cubic inches.

British Imperial Measure.

Act of Parliament of 1897. Order of Council issued 1898, May 19.

			Logarithm
1 Imperial gallon =	4.545 963 1	liters	0 657 6259
=	277 420	cubic inches	2 443 1378
=	1.200 952	U S gallons	0 079 5258

To obtain the **sizes of commercial measures** by means of the weight of water

See Conversion Tables (14) Weights of Volumes of Water, p 241.
(15) Volumes of Weights of Water, p 242.

Metric Measures of Length.

By U. S. and British Standard.

	Ins.	Ft.	Yds.	**Miles.**
Millimetre*	.039370	.003281	....	....
Centimetre†	.39370428	.032809	....	....
Decimetre	3.9370428	.3280869	.1093623	....
Metre‡	39.370428	3.280869	1.093625	....
Decametre	393.70428	32.80869	10.93623	....
Hectometre	Road	328.0869	109.3623	**.0621375**
Kilometre	measures.	3280.869	1093.623	**.6213750**
Myriametre		32808.69	10936.23	**6.213750**

* Nearly the $\frac{1}{25}$ part of an inch. † Full $\frac{3}{8}$ inch.

‡ Very nearly 3 ft, $3\frac{3}{8}$ ins, which is too long by only 1 part in 8616.

Metric Square Measure.

By U. S. and British Standard.

	Sq. Ins.	Sq. Feet.	Sq. Yds.	Acres.
Sq Millimetre	.001550	.00001076	.0000012	....
Sq Centimetre	.155003	.00107641	.0001196	....
Sq Decimetre	15.5003	.10764101	.0119601	....
Sq Metre, or Centiare	1550.03	10.764101	1.19601	.000247
Sq Decametre, or Are	155003	1076.4101	119.6011	.024711
Decare (not used)	....	10764.101	1196.011	.247110
Hectare	....	107641.01	11960.11	2.47110
Sq Kilometre	.3861090 **sq miles.**	10764101	1196011.	247.110
Sq Myriametre	38.61090 "	....	....	24711.0

Metric Cubic or Solid Measure.

According to U. S. Standard.

Only those marked "Brit" are British.

	Cub Ins.	
Millilitre, or cub Centimetre	.0610254	Liquid. .0084537 gill. " .0070428 Brit gill. Dry. .0018162 dry pint.
Centilitre	.610254	Liquid. .084537 gill. " .070428 Brit gill. Dry. .018162 dry pint.
Decilitre	6.10254	Liquid. .84537 gill = .21134 pint. " .70428 Brit gill = .17607 Brit pint. Dry. .18162 dry pint.
Litre, or cubic Decimetre	61.0254	Liquid. 1.05671 quart = 2.1134 pints. " .88036 Brit quart = 1.7607 Brit pints. Dry. .11351 peck = .9081 dry qt = 1.8162 dry pt.
Decalitre, or Centistere	610.254 **Cub Ft.** .353156	Liquid. 2.64179 U. S. liquid gal. " 2.20090 Brit gal. Dry. .283783 bush = 1.1351 peck = 9.081 dry qts.
Hectolitre, or Decistere	3.53156	Liquid. 26.4179 U. S. liquid gal. " 22.0090 Brit gal. Dry. 2.83783 bush.
Kilolitre, or Cubic Metre, or Stere	35.3156	Liquid. 264.179 U. S. liquid gal. " 220.090 Brit gal. Dry. 28.3783 bush. } Cub yds, 1.3080.
Myriolitre, or Decastere	353.156	Liquid. 2641.79 U. S. liquid gal. Dry. 283.783 bush. } Cub yds, 13.080.

Metric Weights, reduced to common Commercial or Avoir Weight, of 1 pound = 16 ounces, or 7000 grains.

	Grains.
Milligramme	015432
Centigramme	.15432
Decigramme	1.5432
Gramme	15 432
	Pounds av.
Decagramme	022046
Hectogramme	22046
Kilogramme	2 2046
Myriogramme	22.046
Quintal*	220 46
Tonneau; Millier; or Tonne	2204 6

The gramme is the basis of French weights; and is the weight of a cub centimetre of distilled water at its max density, at sea level, in lat of Paris; barom 29.922 ins.

French Measures of the "Systeme Usuel."

This system was in use from about 1812 to 1840, when it was forbidden by law to use even its names. This was done in order to expedite the general use of the tables which we have before given. But as the Systeme Usuel appears in books published during the above interval, we add a table of some of its values.

Measures of Length.

	Yards.	Feet.	Inches.
Ligne usuel, or line	....	....	.09113
Pouce usuel, or inch, = 12 lignes	....	.09113	1.09362
Pied usuel, or foot, = 12 pouces	.36454	1 09362	13.12344
Aune usuel, or ell	1 31236	3 93708	47.245
Toise usuel, = 6 pieds	2 18727	6.56181	78.74172

Weights, Usuel.		Cubic, or Solid, Usuel.	
Grain usuel	8375 grains.	Litron usuel, or 1 litre	= 1.7608 British pint.
Gros usuel	60 297 "		
Once usuel	1 10258 avoir oz.		
Marc usuel	.55129 avoir lb.	Boisseau usuel	2.7512 British gals.
Livre usuel, or pound,	1.10258 avoir lb.		

Before 1812, or before the "Systeme usuel," the Old System, "Systeme Ancien," was in use.

French Measures of the "Systeme Ancien."

Lineal.	Square.			Cubic.		
Point ancien, .0148 ins	Sq ins.	Sq ft.	Sq. yds	C ins	C. ft.	C. yds.
Ligne ancien, .0888 ins	.00789	.....	....	.0007		
Pouce ancien, 1.06577 ins = .0888 ft	1 1359	.....	....	1.2106		
Pied ancien, 12 7892 ins = 1.06577 ft	....	1 1359	....	......	1.2106	
Aune ancien, 46.8939 ins = 3.90782 ft = 1.30261 yds						
Toise ancien, = 6.3946 ft = 2.1315 yds	....	40 8908	4.5434	......	261 482	9.6845
League = 2282 toises = 2.7637 miles						

There is, however, much confusion about these old measures. Different measures had the same name in different provinces.

* The *avoirdupois* quintal is 100 avoirdupois pounds.

Russian.

Foot; same as U. S. or British foot. **Sachine** = 7 feet. **Verst** = 500 sachine = 3500 feet = 1166⅔ yards = .6629 mile. **Pood** = 36.114 ℔s avoirdupois

Spanish.

The castellano of Spain and New Granada, for weighing gold, is variously estimated, from 71.07 to 71.04 grains. At 71.055 grains, (the mean between the two,) an avoirdupois, or common commercial ounce contains 6.1572 castellano; and a ℔ avoirdupois contains 98.515. Also a troy ounce = 6.7553 castellano; and a troy ℔ = 81.064 castellano. Three U. S. gold dollars weigh about 1.1 castellano.

The Spanish mark, or marco, for precious metals, in South America, may be taken in practice, as .5065 of a ℔ avoirdupois. In Spain, .5076 ℔. In other parts of Europe, it has a great number of values; most of them, however, being between .5 and .54 of a pound avoirdupois. The .5065 of a ℔ = 3545½ grains; and .5076 ℔ = 3553.2 grains. 1 marco = 50 castellanos = 400 tomine = 4800 *Spanish gold*-grains.

The arroba has various values in different parts of Spain. That of Castile or Madrid, is 25.4025 ℔s avoirdupois; **the tonelada** of Castile = 2032.2 ℔s avoirdupois; **the quintal** = 101.61 ℔s avoirdupois; **the libra** = 1.0161 ℔s avoirdupois; **the cantara** of wine, &c, of Castile = 4.263 U. S. gallons; that of Havana = 4.1 gallons.

The vara of Castile = 32.8748 inches, or almost precisely 32⅞ inches; or 2 feet 8⅞ inches. **The fanegada** of land since 1801 = 1.5871 acres = 69134.08 square feet. **The fanega** of corn, &c = 1.59914 U. S. struck bushels. In California, **the vara by law** = 33.372 U. S. inches; and **the legua** = 5000 varas; or 2.6335 U. S. miles.

CONVERSION TABLES OF UNITS OF MEASURES, WEIGHTS, Etc., Etc., Introduction.

In the following tables **repeating decimals** are indicated by a *. Thus: 1.2*01 = 1.2010101 . . . ; 333.*5657 = 333.56575657 . . .

Exact values are indicated by a †. Thus: 1 ft = 12† ins; 1 meter = 0.001† kilometer.

Negative characteristics are indicated by the letter n. Thus: n 1.285 5573 = $\bar{1}$ 285 5573 = 0.285 5573 — 1 = 9.285 5573 — 10. For the use of logarithms, see pp. 70, etc.

Gunter's and Engineers' **chains and links** are distinguished by the letters G and E respectively. Thus: 1 chain G = 66† ft; 1 link E = 1† ft.

Table A contains the equations taken as the basis of Tables 1 to 36 inclusive.

In Table C we give equivalents and numbers in common use, in order to facilitate the transformation of equations, as explained below.

Each table has been calculated independently by at least two persons, and their results compared and corrected. One of these results was used as copy for the compositor, and his proofs were compared with the other.

The calculations comprised many times the number of equations here given. To have published any considerable proportion of those calculated would have made far too bulky a table for an Engineer's Pocket Book. With a little practice, however, the reader can easily find other equations by means of those given. In doing this, the logarithms given in Table C will be found useful.

Thus, suppose we desire the length of the meter, in rods. In Table 1 (Length) or in Table C (Equivalents in Common Use), we find

1 meter = 3.28083 feet,	Log	0.515 9842

and in Table 1 we find

1 foot = 0.*0606 rod,	Log	n 2.782 5161

Adding logs, we find

1 meter = 0.19884 rod,	Log	n 1.298 5003

Again: 1 meter per second = ? miles per day.

Table 18 (Length per Time, or Velocity) gives

1 meter per second = 2.23693 miles per hour.	Log	0.349 6528

and Table 5 (Time) or Table C gives

1 day = 24 hours	Log	1.380 2112

Adding logs, we obtain

1 meter per second = 53.686 miles per day		1.729 8640

Again:

1 cu ft per sq mile per sec = ? U. S. gals per acre per day.

$$1 \text{ cu ft per sq mile per sec} = \frac{1{,}728 \times 86{,}400}{231 \times 640} \text{ gals per acre per day.}$$

Table C gives

	Log	1,728	3.237 5437
Add	Log	86,400	4 936 5137

1 cu ft per acre per sec = x cu ins per acre per sec 8.174 0574

Subtract sum of	Log 231	2.363 6120	
and	Log 640	2 806 1800	5 169 7920

1 cu ft per sq mile per sec = 1009.87 gals per acre per day 3.004 2654

In a few cases, of frequent occurrence, we give two tables, each containing the reciprocals of the values given in the other. Thus, we give Weights of Volumes (as 1 cu ft = ? lbs) and Volumes of Weights (as 1 lb = ? cu ft) of Water; Volume per Surface to Length (as 1 gal per acre = ? ins.) and Length to Volume per Surface (as 1 inch = ? gals per acre), etc., etc. Where only one of two such reciprocal values is given, the other may easily be found from it. Thus, (But see CAUTION, next page.)

	Log 1	0.000 0000
1 ft per sec = 720 ins per minute.	Log	2.857 3325
1 sec per ft = 0 0013*888 min per inch.	Log	n 3.142 6675

* Repeating decimals. † Exact values. n Negative characteristic. E. Engineer's. G. Gunter's.

CONVERSION TABLES, Introduction (Continued).

Caution. It will be noticed that, although a body moving 1 foot per second (or 720 inches per minute) requires 1 sec to travel 1 ft (or 0.0013*888 min to travel 1 inch), and although 1 ft per sec and 1 sec per ft are also *numerically* equal, they are not really identical, but reciprocal. Thus, also,

1 dime per dollar = 1 : 10
but 1 dollar per dime = 10 : 1.

In selecting equations for publication, we have in general given preference to those which cannot easily be found by means of others in the same table, or by means of simpler tables. Thus, having given, in Table 12 (Weight per Surface), the equivalent of 1 kilogram per sq centimeter, in pounds per sq *inch*, we omit the equivalent in pounds per sq *foot*, which is readily found, by using Table 2 (Surface), from the given equivalent in pounds per sq *inch*.

Again, the equivalents of a pressure of 1 pound per sq inch, in ounces per sq inch, and in pounds per sq foot, are not given in Table 12 (Weight per Surface); because they are easily deduced from 1 pound per sq inch, by means of Table 4 (Weight) and Table 2 (Surface), respectively.

The ratios between the values of the coins of different nations being subject to change, the equivalents in Tables 25 to 30 inclusive (Values) can at best be only rudely approximate. Their logarithms are therefore given to only three places of decimals, although the calculations were made by means of seven-place logarithms. Conversions of such approximate equivalents may be conveniently made by means of our two-page table of 4-place logarithms, pp. 78 and 79, omitting the last decimal place. Indeed, the two-page table, which obviates the necessity of turning over pages, may often be used to advantage in dealing with quantities more accurately known.

List of Tables.

* Repeating decimals. † Exact values. n, Negative characteristic. E. Engineer's. G, Gunter's.

CONVERSION TABLES, Introduction (Continued).

Table A.

Fundamental Equivalents.

(*See Notes below.*)

			Logarithm
1 meter =	39 37	inches (a)	1 595 1654
1 gram =	15 43235	grains (a)	1.188 4320
1 cu centimeter water weighs	1	gram (a)	0.000 0000
1 cu meter air weighs	1 293052	kilograms (b)	0.111 6159
1 inch mercury =	13 5956	inches water (c)	1.133 4112
1 atmosphere =	760	mm of mercury	2 880 8136
1 pound Fahr (B. T. U) =	778	foot-pounds (d)	2.890 9796
g = acceleration of gravity =	9 81	meters per second per second (e)	0.991 6690
£ 1 =	$4 86	(f)	0 686 6363
1 franc =	$0.193	(f)	n 1 285 5573
1 mark =	$0.238	(f)	n 1.376 5770

(a) See p 217.
(b) See Density of air, p 320.
(c) 1 inch mercury = 13 5956 inches water. Value adopted by International Bureau of Weights and Measures. Lat. 45°. Sea level. Temperature of mercury 0° C. = 32° Fahr.
(d) 1 pound Fahr = 778 foot-pounds. According to very careful determinations by Prof Henry A. Rowland (Proceedings, Am. Acad. Arts and Sciences for 1880) and generally accepted as standard.
(e) g = 9 81 meters per sec per sec Generally accepted average. At latitude 45° and at sea level, g = 9 806056 m. For other latitudes, L, and elevations h, in centimeters;
g in centimeters per sec per sec = approx 980 6056 — 2.5028 cos 2 L — 0.000003 h.
(f) According to circular of U. S. Treasury Department, Bureau of the Mint, Jan'y 1, 1887, £1 = $4 8665 gold; 1 franc = $0.193; 1 mark = $0 238.

Table B.

Abbreviations.

a	acre	hl	hectoliter or decistere
bu	bushel	hm	hectometer
C	Centigrade	H.P.	horse-power
c	centime	Imp	Imperial
cc	cubic centimeter	in	inch
cg	centigram	J	joule
ch	chain	kg	kilogram
cl	centiliter	kgm	kilogrammeter
cm	centimeter	kl	kiloliter
cs	centistere	km	kilometer
ct	cent	kw	kilowatt
cu	cubic	£	pound (money)
d	day	li	liter
d	penny or pence	℔	pound (weight)
dcg	decigram	liq	liquid
dcl	deciliter	m	meter
dcm	decimeter	mi	mile
dcs	decistere	min	minute
dkl	dekaliter	mlg	milligram
dkm	dekameter	mll	milliliter
dks	dekastere	mm	millimeter
dwt	pennyweight	myg	myriagram
F	Fahrenheit	pf	pfennig
ft	foot or feet	ql	quintal
fur	furlong	s	shilling
g	acceleration of gravity	sec	second
g	gramme	sect	section
gal	gallon	sq	square
gr	grains	st	stere
hec	hectare	w	Watt
h	hour	yd	yard
hg	hectogram	$	dollar

19

CONVERSION TABLES, Introduction (Continued).

Table C.

Equivalents and Numbers in Common Use.

(Arranged numerically.)

			Logarithm
1 cu cm of water weighs	1	gram *	0 000 0000
1 meter =	1 093611	yards	0 038 8629
	2		0 301 0300
1 kilogram =	2 20462	pounds	0.343 3340
1 yard =	3	feet	0.477 1213
1 circumference =	π	diameters	0.497 1499
1 meter =	3.280833	feet	0 515 9842
1 gallon =	4	quarts	0 602 0600
1 link G =	7.92	inches	0 898 7252
1 sq yard =	9	sq feet	0.954 2425
g = acceleration of gravity =	9.81	meters per sec per sec	0 991 6690
1 meter =	10	decimeters	1.000 0000
1 foot =	12	inches	1.079 1812
1 gram =	15 43235	grains†	1.188 4320
1 pound =	16	ounces	1 204 1200
1 rod =	16.5	feet	1 217 4839
1 day =	24	hours	1 380 2112
1 cu yard =	27	cu feet	1 431 3638
1 yard =	36	inches	1 556 3025
1 meter =	39 37	inches†	1.595 1654
1 hour =	60	minutes	1 778 1513
1 chain G =	66	feet	1 819 5439
1 cwt =	112	pounds	2.049 2180
1 sq foot =	144	sq inches	2.158 3625
1 gallon U. S. =	231	cu inches	2 363 6120
1 ounce =	437 5	grains	2 640 9781
1 horse-power	550	foot-pounds per sec.	2 740 3627
1 sq mile =	640	acres	2 806 1800
1 sq yard =	1 296	sq inches	3 112 6050
1 day =	1 440	minutes	3 158 3625
1 cu foot =	1,728	cu inches	3.237 5437
1 mile	1.760	yards	3 245 5127
1 ton =	2,240	pounds	3.350 2480
1 hour =	3,600	seconds	3 556 3025
1 acre =	4 840	sq yards	3 684 8454
1 mile =	5 280	feet	3 722 6339
1 pound =	7 000	grains	3 845 0980
1 horse-power =	33,000	foot-pounds per min	4 518 5139
1 acre =	43 560	sq feet	4.639 0879
1 cu yard =	46.656	cu inches	4.668 9075
1 mile =	63,360	inches	4 801 8152
1 day =	86,400	seconds	4 936 5137

Table D.

Prefixes used in the Metric System.

1 Myria-meter, gram, etc.,	=	10,000	meters, grams, etc.	4 000 0000
1 Kilo- " " "	=	1,000	" " "	3 000 0000
1 Hecto- " " "	=	100	" " "	2.000 0000
1 Deka- " " "	=	10	" " "	1.000 0000
1 ..... meter, gram, etc.	=	1	meter, gram, etc.	0.000 0000
1 Deci- " " "	=	0 1	" " "	n 1.000 0000
1 Centi- " " "	=	0.01	" " "	n 2.000 0000
1 Milli- " " "	=	0.001	" " "	n 3.000 0000

* At maximum density, = 4° Centigrade, = 39.2° Fahr.

† See page 217.

CONVERSION TABLES.

(1) LENGTH.

1 Chain E =

			Logarithm.
100 †	feet		2.000 0000
6.*0606	rods		0.782 5161
1.*5151	chains G		0.180 4561
0.018*93	mile	n	2.277 3661
30.4801	meters		1.484 0158

1 Chain G =

66 †	feet		1.819 5439
4 †	rods		0 602 0600
0.66†	chain E	n	1 819 5439
0.0125†	mile	n	2.096 9100
20.1168	meters		1.303 5597

1 Fathom =

6 †	feet		0.778 1513

1 Foot =

12 †	inches		1 079 1812
0.*333	yard	n	1 522 8787
0.*0606	rod	n	2.782 5161
0.0*1515	chain G	n	2.180 4561
0.00018*93	mile	n	4.277 3661
0.30480	meter	n	1.484 0158

1 Furlong =

0.125†	mile	n	1 096 9100

1 Inch =

1,000 †	mils		3 000 0000
0 08*33	foot	n	2.920 8188
0.02*777	yard	n	2.443 6975
2.540005	centimeters		0.404 8346

(1) LENGTH.—(Continued.)

1 Link G =

			Logarithm.
7.92†	inches		0 898 7252
0 66†	foot	n	1 819 5439

1 Mil =

0 001†	inch	n	3 000 0000
0.02540	millimeter	n	2.404 8346

1 Mile =

63,360 †	inches		4.801 8152
8,000 †	links G		3.903 0900
5,280 †	feet		3.722 6339
1,760 †	yards		3 245 5127
320 †	rods		2.505 1500
80 †	chains G		1.903 0900
52 8†	chains E		1 722 6339
8 †	furlongs		0.903 0900
1 60935	kilometers		0.206 6497

1 Rod =

25 †	links G		1.397 9400
16.5†	feet		1.217 4839
0 25†	chain G	n	1.397 9400
0 003125†	mile	n	3.494 8500

1 Yard =

36 †	inches		1 556 3025
3 †	feet		0.477 1213
0 00056*81	mile	n	4.754 4873
0.914402	meter	n	1.961 1371

1 Mikron =

0.001†	millimeter	n	3.000 0000

* Repeating decimals. † Exact values. n. Negative characteristic. E. Engineer's. G. Gunter's.

(1) LENGTH.—(Continued.)

		Logarithm.
1 Millimeter =		
39.37	mils	1.595 1654
0.03937	inch	n 2.595 1654
1,000 †	mikrons	3.000 0000
0.001†	meter	n 3.000 0000
1 Centimeter =		
0.3937	inch	n 1.595 1654
0.0328083	foot	n 2.515 9842
10 †	millimeters	1.000 0000
0.01†	meter	n 2.000 0000
1 Decimeter =		
3.937	inches	0.595 1654
0.1†	meter	n 1.000 0000
1 Meter =		
39.37	inches (see p 230)	1.595 1654
3.28083	feet	0.515 9842
1.09361	yards	0.038 8629
1,000 †	millimeters	3.000 0000
100 †	centimeters	2.000 0000
10 †	decimeters	1.000 0000
0.001†	kilometer	n 3.000 0000
1 Dekameter =		
10 †	meters	1.000 0000
1 Hectometer =		
100 †	meters	2.000 0000
1 Kilometer =		
3,280.83	feet	3.515 9842
1,093.61	yards	3.038 8629
0.62137	mile	n 1.793 3503
1,000 †	meters	3.000 0000

(2) SURFACE.

1 Acre = a square measuring 208.710 ft, or 3.16228 chains G, on a side.

		Logarithm.
100,000 †	sq links G	5.000 0000
43,560 †	sq feet	4.639 0879
4,840 †	sq yards	3.684 8454
160 †	sq rods	2.204 1200
10 †	sq chains G	1.000 0000
0.0015625†	sq mile	n 3.193 8200
4.046.87	sq meters	3.607 1196
0.404687	hectare	n 1.607 1196
1 Sq Foot =		
144 †	sq inches	2.158 3625
0.*111	sq yard	n 1.045 7575
0.0929034	sq meter	n 2.968 0316
1 Sq Inch =		
0.0069*44	sq foot	n 3.841 6375
6.45163	sq centimeters	0.809 6692
1,273,240	circular mils	6.104 9101
1 Sq Mil =		
0.000001†	sq inch	n 6.000 0000
0.000645163	sq millimeter	n 4.809 6692
1 Sq Mile =		
27,878,400 †	sq feet	7.445 2678
3,097,600 †	sq yards	6.491 0253
640 †	acres	2.806 1800
1	section	0.000 0000
259.000	hectares	2.413 2996
2.59000	sq kilometers	0.413 2996
1 Section =		
1	sq mile. (See sq mile.)	
1 Half-section =		
3,200 †	sq chains G	3.505 1500
1.29500	sq kilometers	0.112 2696

* Repeating decimals. † Exact values. n. Negative characteristic. E. Engineer's. G. Gunter's.

(2) SURFACE.—(Continued.)

1 Sq Yard =		Logarithm.
1,296 †	sq inches	3.112 6050
9 †	sq feet	0.954 2425
0.836131	sq meter	n 1.922 2742

1 Sq Millimeter =		
1,550.0	sq mils	3.190 3308
0.0015500	sq inch	n 3.190 3308
0.01†	sq centimeter	n 2.000 0000

1 Sq Centimeter =		
0.15500	sq inch	n 1.190 3308
0.0001†	sq meter	n 4.000 0000

1 Sq Meter =		
1,550.0	sq inches	3.190 3308
10.7639	sq feet	1.031 9683
1.19598	sq yard	0.077 7258
10,000 †	sq centimeters	4.000 0000

1 Sq Hectometer =
1 .... hectare. (See 1 hectare.)

1 Sq Kilometer =		
247.104	acres	2.392 8805
0.386101	sq mile	n 1.586 7005
1,000,000 †	sq meters	6.000 0000
100 †	hectares	2.000 0000

1 Hectare = a square measuring 100 meters on a side

107,639	sq feet	5.031 9683
2.47104	acres	0.392 8804
0.003861	sq mile	n 3.586 7005
10,000 †	sq meters	4.000 0000
0.01†	sq kilometer	n 2.000 0000

(3) VOLUME.

1 Bushel U S =		Logarithm.
1.24446	cu feet	0.094 9796

1 Cord of Wood =		
4 × 4 × 8 †	feet	
128 †	cu feet	2.107 2100

1 Cu Foot =		
1,728 †	cu inches	3.237 5437
$\frac{1728}{231}$ = 7.48052	gallons U S liquid	0.873 9317
0.*037	cu yard	n 2.568 6362
28.317	liters	1.452 0475
0.028317	cu meter	n 2.452 0475

1 Cu Inch =		
16.3872	cu centimeters	1.214 5038

1 Cu Yard =		
46,656 †	cu inches	4.668 9075
27 †	cu feet	1.431 3638
0.00061983	acre-foot	n 4.792 2759
0.76456	cu meter	n 1.883 4113

1 U S Gallon =		
231 †	cu inches	2.363 6120
8 †	U S pints liquid	0.903 0900
0.832673	Imp gallon	n 1.920 4742
$\frac{231}{1728}$ = 0.133681	cu foot	n 1.126 0683
3.78543	liters	0.578 1158
0.00378543	cu meter or stere	n 3.578 1158

1 Imp Gallon =		
277.420	cu inches	2.443 1378
1.20095	U S gallons liquid	0.079 5258
0.160544	cu foot	n 1.205 5941
4.5459631	liters	0.657 6259

* Repeating decimals; thus, 0.0*1515 = 0.0151515. . † Exact values. n Negative characteristic.

(3) VOLUME.—(Continued.)

		Logarithm.
1 Ounce, Fluid =		
1.8047	cu inches	0 256 4050
29.5739	cu centimeters	1.470 9088
1 Perch =		
24 75†	cu feet	1 393 5752
16.5 × 1 5 × 1† feet		
0 916667	cu yard	n 1.962 2114
0 700846	cu meter	n 1 845 6227
1 Cu Millimeter =		
0 000061023	cu inch	n 5 785 4962
0.001†	cu centimeter	n 3.000 0000
1 Cu Centimeter or Milliliter =		
0 0610234	cu inch	n 2.785 4962
1,000 †	cu millimeters	3 000 0000
0 001†	liter	n 3 000 0000
1 Centiliter =		
10 †	milliliters	1 000 0000
0 01†	liter	n 2 000 0000
1 Deciliter =		
0.1†	liter	n 1.000 0000
1 Liter or Cu Decimeter =		
61.0234	cu inches	1.785 4962
0 26417	U S gallon liquid	n 1 421 8842
0.219975	Imp gallon	n 1 342 3741
0.0353145	cu foot	n 2.547 9525
1,000 †	cu cm or milliliters	3.000 0000
0.001 †	cu meter or stere	n 3.000 0000
1 Dekaliter or Centistere =		
10 †	liters	1 000 0000
0.01 †	cu meter	n 2.000 0000

(3) VOLUME.—(Continued.)

		Logarithm.
1 Hectoliter or Decistere =		
100 †	liters	2.000 0000
1 Cu Meter, Stere, or Kiloliter =		
61,023 4	cu inches	4.785 4962
264 170	gallons U S liquid	2 421 8842
35 3145	cu feet	1.547 9525
1 30794	cu yards	0 116 5887
0 000810708	acre-foot	n 4 908 8646
1,000 †	liters	3.000 0000
1 Acre-foot =		
325,851	gallons U S liquid	5 513 0196
43,560 †	cu feet	4 639 0879
1,613 *333	cu yards	3 207 7241
1,233 49	cu meters	3.091 1354

(4) WEIGHT (AND FORCE).

Avoirdupois and Metric

1 Grain =		
0.0647989	gram	n 2 811 5680
1 Ounce =		
437 5†	grains	2.640 9781
0 0625†	pound	n 2 795 8800
28 3496	grams	1.452 5461
1 Pound =		
7,000 †	grains	3 845 0980
16 †	ounces	1 204 1200
453 593	grams	2.656 6660
0 453593	kilogram	n 1.656 6660

* Repeating decimals: thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(4) WEIGHT (AND FORCE).—(Continued.)

Avoirdupois and Metric.

			Logarithm.
1 Ton (2,240 Pounds) =			
2,240 †	pounds		3.350 2480
1,016.05	kilograms		3.006 9141
1.01605	metric tons		0.006 9141
1 Milligram =			
0.001†	gram	n	3.000 0000
1 Centigram =			
0.01†	gram	n	2 000 0000
1 Decigram =			
0.1†	gram	n	1.000 0000
1 Gram =			
15 43235	grains (see p. 230)		1.188 4320
0.001†	kilogram	n	3.000 0000
1 Dekagram =			
10 †	grams		1 000 0000
1 Hectogram =			
100 †	grams		2.000 0000
1 Kilogram =			
2.20462	pounds		0.343 3340
1,000 †	grams		3.000 0000
1 Myriagram =			
10 †	kilograms		1 000 0000
1 Quintal (Metric) =			
100 †	kilograms		2 000 0000
1 Tonne or Metric Ton =			
2,204.62	pounds		3 343 3340
1.10231	tons (2,000 lbs)		0.042 3040
0 984206	ton (2,240 lbs)	n	1.993 0860
1,000 †	kilograms		3.000 0000

(5) TIME.

1 year = 365† days. 1 leap year = 366† days.

			Logarithm.
1 Leap Year =			
8,784 †	hours		3 943 6923
366 †	days		2 563 4811
1 Year =			
8,760 †	hours		3 942 5041
365 †	days		2 562 2929
1 Day =			
86,400 †	seconds		4 936 5137
1,440 †	minutes		3 158 3625
24 †	hours		1 380 2112
0 142857	week	n	1.154 9020
0 00273973	year	n	3 437 7071
0 00273224	leap year	n	3.436 5189
1 Hour =			
3,600 †	seconds		3.556 3025
60 †	minutes		1.778 1513
0 041*666	day	n	2.619 7888
0.000114155	year	n	4.057 4959
1 Minute =			
60 †	seconds		1.778 1513
0 01*66	hour	n	2.221 8487
0 00069*44	day	n	4 841 6375
1 Second =			
0 01*66	minute	n	2 221 8487
0 0002*77	hour	n	4 443 6975
0 000011574	day	n	5 063 4863
1 Week =			
168 †	hours		2.225 3093
7 †	days		0.845 0980

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(6) WORK AND HEAT.

For Power, see Table (22).

		Logarithm.
1 Foot-pound =		
0.00128535	pound °F	n 3 109 0204
13,825 5	gram cm.	4 140 6818
1.3562844	joules	0 132 3508
0 138255	kilogrammeter	n 1 140 6818
0 00184340	metric H P second	n 3 265 6205
0 000323902	kilogram °C	n 4 510 4139
0 000376746	watt-hour	n 4 576 0483
1 H P (Horse-power) Hour =		
1,980,000 †	foot-pounds	6 296 6652
2,544 99	pound °F	3 405 6856
2,685,443	joules	6 429 0160
273,746	kilogrammeters	5 437 3470
641 326	kilogram °C	2 807 0791
1 Inch-pound =		
1,152 13	gram cm.	3 061 5006

1 B. T. U. (British Thermal Unit). See Pound Degree Fahrenheit.

1 Pound Degree Fahrenheit =		
1 †	British Thermal Unit	0 000 0000
778	foot-pounds	2 890 9796
107 563	kilogrammeters	2 031 6614
0 251996	kilogram °C	n 1 401 3935
1 Joule =		
0.737308	foot-pound	n 1.867 6492
0 101937	kilogrammeter	n 1 008 3310
1 Kilogrammeter =		
7.23300	foot-pounds	0 859 3182
0 0131509	H P second	n 2 118 9555
0 00929691	pound °F	n 3 968 3386
9 81	joules	0 991 6690
0 01*33	metric H P second	n 2 124 9387
0 00234278	kilogram °C	n 3 369 7321
0.002725	watt-hour	n 3.435 3665

(6) WORK AND HEAT.—(Continued.)

1 Calorie. See Kilogram Degree Centigrade.

		Logarithm.
1 Kilogram Degree Centigrade =		
3,087 35	foot-pounds	3 489 5861
3 96832	pound °F	0 598 6065
426 84?	kilogrammeters	2 630 2679
1 Watt-hour =		
2,654 31	foot-pounds	3 423 9517
3 41171	pound °F	0 532 9721
3,600 †	joules	3 556 3025
366 972	kilogrammeters	2 564 6335
0 859737	kilogram °C	n 1 934 3656
0 00135916	metric H P hour	n 3 133 2697
1 Metric H P (Horse-power) Hour =		
1,952,910	foot-pounds	6 290 6820
2,648,700	joules	6 423 0328
270,000 †	kilogrammeters	5 431 3638
632 551	kilogram °C	2 801 0959

(7) LENGTH PER LENGTH. (GRADES, ETC.)

1 Foot per Mile = 1:5280 † =		
0 00018*93		n 4 277 3661
0 002*27	inch per foot	n 3.356 5473
1 Inch per Foot = 1:12 =		
0 08*33 (tan 4° 46′ = 0.083386)		n 2 920 8188
0 0069*44	foot per inch	n 3 841 6376
440 †	feet per mile	2.643 4527
1 Inch per Mile =		
0.00157828	foot per 100 feet	n 3.198 1849
1 Mile per Inch =		
63,360 †		4 801 8152
21,120 †	yards per foot	4.324 6939

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(8) SURFACE PER LENGTH. (WIDTHS.)

1 Acre per Chain E =

		Logarithm.
435 6†	feet	2.639 0879
0 0825†	sq mile per mile	n 2.916 4539
0.132771	sq km per km	n 1.123 1038

1 Acre per Chain G =

660 †	feet	2.819 5439
66.000 †	sq feet per 100 feet	4 819 5439
0 125†	sq mile per mile	n 1.096 9100
0 201168	sq km per km	n 1.303 5599

1 Acre per Foot =

43.560 †	feet	4 639 0879
4,356,000 †	sq feet per 100 feet	6.639 0879
8 25†	sq miles per mile	0.916 4539
13.2771	sq km per km	1.123 1038

1 Acre per Mile =

8 25†	feet	0.916 4539
825 †	sq feet per 100 feet	2.916 4539
1,188 †	sq inches per foot	3.074 8165
0 251461	hectare per km	n 1.400 4699

1 Sq Foot per Mile =

0 002*27	inch = $\frac{1}{440}$† inch	n 3.356 5473
0 00018*93	foot = $\frac{1}{5280}$† foot	n 4.277 3661
0 0577274	sq meter per km	n 2.761 3819

1 Sq Inch per Foot =

36 *666	sq feet per mile	1 564 2714
2 11667	sq meters per km	0 325 6534

1 Sq Inch per Inch =

0 08*33	sq foot per foot	n 2 920 8187
0 0069*44	sq foot per inch	n 3 841 6375
0.*0101	acre per mile	n 2.004 3648
25 4001	sq m per km	1.404 8346

1 Sq Mile per Mile =

0.*1212	acre per foot	n 1.083 5461
1.60935	sq km per km	0.206 6499

(8) SURFACE PER LENGTH. (WIDTHS.) —(Continued.)

1 Sq Mile per Chain E =

		Logarithm.
440 †	sq feet per inch	2.643 4527

1 Sq Meter per Kilometer =

17 3228	sq feet per mile	1.238 6180
0.47244	sq inch per foot	n 1.674 3466

1 Hectare per Kilometer =

10 †	meters	1 000 0000
3.97677	acres per mile	0 599 5301

(9) VOLUME PER LENGTH. (SURFACES.)

1 Cu Foot per Foot =

1 †	sq foot	0 000 0000
195.*55	cu yards per mile	2.291 2701
92 9(34	liters per meter	1.968 0317

1 Cu Foot per Inch =

12 †	sq feet	1.079 1812
0 *444	cu yard per foot	n 1.647 8174
1,114 84	liters per meter	3.047 2129

1 Cu Foot per Mile =

0 00018*93	sq ft = $\frac{1}{5280}$† sq ft	n 4.277 3661
0 0*2727	sq in = $\frac{144}{5280}$† sq in	n 2.435 7286
0 3*2727	cu inch per foot	n 1.514 9098
0 0175953	liter per meter	n 2.245 3978

1 Cu Inch per Foot =

0 08*3	sq inch = $\frac{1}{12}$† sq inch	n 2.920 8188
3 0*55	cu feet per mile	0.485 0902
0 0537637	liter per meter	n 2.730 4880

1 Cu Inch per Inch =

1 †	sq inch	0 000 0000
36.*66	cu feet per mile	1.564 2715
0.645163	liter per meter	n 1.809 6692

* Repeating decimals: thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(9) VOLUME PER LENGTH. (SURFACES.)—(Continued.)

		Logarithm.
1 Cu Yard per Foot =		
3 †	sq yards	0.477 1213
2 25†	cu feet per inch	0.352 1826
2508.39	liters per meter	3 399 3955
1 Cu Yard per Inch =		
46,656 †	sq inches	4.668 9075
1 Cu Yard per Mile =		
0.00056818	sq yd = 1/1760† sq yd	n 4.754 4873
8.8*3636	cu inches per foot	0.946 2736
0.47507	liter per meter	n 1.676 7616
1 U S Gallon per Foot =		
19.25†	sq inches	1 284 4307
705.833	cu feet per mile	2 848 7022
12.4194	liters per meter	1.094 1000
1 U S Gallon per Inch =		
231 †	sq inches	2 363 6120
1 60417	cu feet per foot	0 205 2495
149.033	liters per meter	2.173 2812
1 Liter per Meter =		
0.001†	sq meter	n 3 000 0000
18.6	cu inches per foot	1.269 5120
56.8332	cu feet per mile	1.754 6022

(10) WEIGHT PER LENGTH.

1 Pound per Foot =		
1.48816	kg per meter	0.172 6502
1 Pound per Inch =		
17.8579	kg per meter	1.251 8314
1 Pound per Mile =		
0.281849	gram per meter	n 1.450 0163

(10) WEIGHT PER LENGTH.—(Continued.)

		Logarithm.
1 Gram per Meter =		
3.5480	pounds per mile	0.549 9837
1 Kilogram per Meter =		
0 67197	pound per foot	n 1.827 3498
1 Kilogram per Kilometer =		
3 548	pounds per mile	0 549 9837

(11) VOLUME PER SURFACE. (DEPTHS.)

1 Cu Foot per Acre =		
0.000275482	inch	n 4.440 0938
23.7037	cu yards per sq mile	1.374 8162
0 0699726	cu meter per hectare	n 2.844 9279
1 Cu Foot per Sq Foot =		
1 †	foot	0.000 0000
1,613.*333	cu yards per acre	3.207 7241
0.05195	U S gal per sq inch	2.715 5692
1 Cu Foot per Sq Mile =		
2.7†	cu inches per acre	0.431 3638
0.0109332	cu meter per sq km	n 2.038 7479
1 Cu Inch per Sq Foot =		
25.2083	cu feet per acre	1.401 5442
597.531	cu yards per sq mile	2.776 3603
0.176389	liter per sq meter	n 1.246 4722
1 Cu Inch per Sq Inch =		
1 †	inch	0.000 0000
3,630 †	cu feet per acre	3.559 9067
0.02540	cu meter per sq meter	n 2.404 8346

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(11) VOLUME PER SURFACE. (DEPTHS.) —(Continued.)

1 Cu Yard per Acre =

		Logarithm.
1.07107	cu inches per sq foot	0.029 8196
0.188926	liter per sq meter	n 1.276 2917

1 Cu Yard per Sq Foot =

		Logarithm.
27 †	feet	1.431 3638
8.22962	cu meters per sq meter	0.915 3797

1 Cu Yard per Sq Mile =

		Logarithm.
72.9†	cu inches per acre	1.862 7275
0.295197	cu meter per sq km	n 1.470 1117

1 U S Gallon per Acre =

		Logarithm.
85.*555	cu feet per sq mile	1.932 2483
0.935397	cu meter per sq km	n 1.970 9962

1 U S Gallon per Sq Foot =

		Logarithm.
5,823.13	cu feet per acre	3.765 1562
40.7459	liters per sq meter	1.610 0842

1 Liter per Sq Meter =

		Logarithm.
0.001†	meter	n 3.000 0000
142.913	cu feet per acre	2.155 0721
0.0245423	U S gal per sq foot	n 2.389 9159

1 Liter per Hectare =

		Logarithm.
9.14644	cu feet per sq mile	0.961 2521
0.106906	U S gal per acre	n 1.029 0038

1 Cu Meter per Sq Meter =

		Logarithm.
1 †	meter	0 000 0000
24.5423	U S gal per sq foot	1.389 9158

1 Cu Meter per Sq Kilometer =

		Logarithm.
0.001†	mm	n 3 000 0000
91.4644	cu feet per sq mile	1 961 2521
1.06906	U S gal per acre	0.029 0038

(12) WEIGHT PER SURFACE. (UNIT PRESSURES.)

Avoirdupois and Metric.

1 Atmosphere =

		Logarithm.
760	mm of mercury	2 880 8136
29.9212	inches of mercury	1.475 9790
33.9007	feet of water	1.530 2090
14.697	pounds per sq inch	1.167 2280
1.033296	kg per sq cm	0 014 2248
1 033296	metric atmospheres	0.014 2248

1 Metric Atmospher =

		Logarithm.
1	kg per sq cm	0 000 0000
0.967777	atmosphere	n 1.985 7752

1 Pound per Sq Inch =

		Logarithm.
2 30665	feet of water	0.362 9810
0 0680412	atmosphere	n 2 832 7720
0 0703067	kg per sq cm	n 2.846 9968
2.03589	inches of mercury	0.308 7510

1 Pound per Sq Foot =

		Logarithm.
0.0141380	inch of mercury	n 2.150 3885
0.0160184	foot of water	n 2.204 6185

1 Kilogram per Sq Centimeter =

		Logarithm.
1	metric atmosphere	0 000 0000
14 2234	pounds per sq inch	1.153 0032
0 967777	atmosphere	n 1.985 7752
32 8083	feet of water	1.515 9342
28.9570	inches of mercury	1.461 7542

1 Foot of Water =

		Logarithm.
0 0294979	atmosphere	n 2 469 7910
0 882612	inch of mercury	n 1 945 7700
0 433530	pound per sq inch	n 1.637 0190
30.4801	grams per sq cm	1.484 0158

* Repeating decimals; thus 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(12) WEIGHT PER SURFACE.

(UNIT PRESSURES.)—(Continued.)

Avoirdupois and Metric

		Logarithm
1 Foot of Mercury =		
0.401053	atmosphere	n 1 603 2022
0.414407	kg per sq cm	n 1 617 4270
1 Inch of Water =		
2 54001	grams per sq cm	0 404 8346
1 Inch of Mercury =		
0.03342	atmosphere	n 2 524 0210
1.133	feet of water	0 054 2299
0 491189	pound per sq inch	n 1 691 2490
34 5339	grams per sq cm.	1 538 2458

(13) WEIGHT PER VOLUME.

(SPECIFIC GRAVITY.)

Avoirdupois and Metric

1 Pound per Cu Inch =		
27.6797	grams per cu cm	1 442 1621
1 Pound per U S Gallon =		
0.119826	kg per liter	n 1 078 5501
1 Pound per Cu Foot =		
16.0184	kg per cu meter	1 204 6184
1 Pound per Cu Yard =		
0.593273	kg per cu meter	n 1 773 2546
1 Kilogram per Cu Meter =		
0.0624283	pound per cu foot	n 2.795 3816
1 Gram per Cu Centimeter =		
62.4283	pound per cu foot	1 795 3816

(14) WEIGHTS OF VOLUMES OF WATER.

Avoirdupois and Metric.

Water at maximum density = 1 gram per cu centimeter.

		Logarithm.
1 Cu Inch Weighs		
0 578040	ounce	n 1.761 9578
0 0361275	pound	n 2 557 8378
16 3872	grams	1.214 5038
1 Gallon U S Liquid Weighs		
8 34545	pounds	0 921 4498
3 78543	kilograms	0 578 1158
1 Gallon Imp Weighs		
10.022	pounds	1.000 9600
4 546	kilograms	0 657 6300
1 Cu Foot Weighs		
62 4283	pounds	1 795 3815
28 3170	kilograms	1.452 0475
1 Cu Yard Weighs		
1,685 56	pounds	3 226 7453
764 559	kilograms	2.883 4113
1 Cu Centimeter or Milliliter Weighs		
0 0352739	ounce	n 2 547 4540
1 †	gram	0 000 0000
1 Liter or Cu Decimeter Weighs		
2 20462	pounds	0.343 3340
1 †	kilogram	0.000 0000
1 Cu Meter, Stere, or Kiloliter Weighs		
2,204.62	pounds	3.343 3340
1,000 †	kilograms	3.000 0000

* Repeating decimals; thus, 0 0*1515 = 0 0151515 .. † Exact values n. Negative characteristic.

(15) VOLUMES OF WEIGHTS OF WATER.

Avoirdupois and Metric.

Water at maximum density = 1 gram per cu centimeter.

			Logarithm
1 Pound Measures			
27.6798	cu inches		1.442 1621
0.119825	gallon U S	n	1 078 5501
0.099779	gallon Imp.	n	2 999 0401
0.0160184	cu foot	n	2 204 6184
0.453592	liter	n	1.656 6660
1 Gram Measures			
0.0610233	cu inch	n	2.785 4962
1 †	cu centimeter		0.000 0000
1 Kilogram Measures			
61.0234	cu inches		1.785 4962
0.264170	gallon U S	n	1.421 8842
0.219975	gallon Imp.	n	1 342 3741
0 0353144	cu foot	n	2 547 9525
1 †	liter		0.000 0000

(16) WEIGHTS OF VOLUMES OF AIR.

Avoirdupois and Metric. (See p. 320.)

			Logarithm
1 Cu Inch Weighs			
0.327003	grain	n	1 514 5518
21.1895	milligram		1 326 1198
1 Cu Foot Weighs			
565.061	grains		2 752 0955
0.0807230	pound	n	2.906 9975
36.6154	grams		1.563 6635
1 Cu Yard Weighs			
2 17952	pounds		0 338 3613
988.615	grams		2 995 0273
1 Cu Meter Weighs			
2.85069	pounds		0.454 9500
1.293052	kilograms (see p. 230)		0.111 6159

(17) VOLUMES OF WEIGHTS OF AIR.

Avoirdupois and Metric. (See p. 320.)

			Logarithm.
1 Grain Measures			
3 05808	cu inches		0.485 4482
0.0501132	liter	n	2.699 9520
1 Pound Measures			
21,406.5	cu inches		4 330 5462
12.3880	cu feet		1.093 0025
0 458816	cu yard	n	1.661 6387
0.350792	cu meter	n	1.545 0500
1 Kilogram Measures			
47,193.3	cu inches		4 673 8802
27.3109	cu feet		1.436 3365
1 01152	cu yards		0 004 9727
0.773364	cu meter	n	1.888 3840

(18) LENGTH PER TIME. (VELOCITY.)

1 Foot per Second =			
0 6*8181	mile per hour	n	1.833 6686
1 09728	km per hour		0.040 3183
1 Inch per Second =			
300 †	feet per hour		2.477 1213
152.400	cm per minute		2.182 9859
1 Mile per Hour =			
1 4*66	feet per second		0.166 3314
0.447041	meter per second	n	1.650 3472
1 Mile per Minute =			
88 †	feet per second		1 944 4826
96.5608	km per hour		1.984 8010
1 Mile per Second =			
96 5608	km per minute		1.984 8010
1 Meter per Second =			
196 85	feet per minute		2 294 1354
2.23693	miles per hour		0.349 6528

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(18) LENGTH PER TIME. (VELOCITY.) —(Continued.)

		Logarithm.
1 Kilometer per Day =		
0 0379726	foot per second	n 2 579 4705
0 0258904	mile per hour	n 2.413 1391
1 Kilometer per Hour =		
0 911343	foot per second	n 1 959 6817
14 9129	miles per day	1 173 5615
1 Centimeter per Second =		
1.96850	feet per minute	0 294 1355

(19) SURFACE PER TIME.

1 Acre per Day =		
1,815 †	sq feet per hour	3 258 8766
1 Acre per Hour =		
12.1†	sq feet per second	1 082 7854
0 03750†	sq mile per day	n 2 574 0313
1 12413	sq meters per second	0.050 8170
1 Acre per Second =		
5 625†	sq miles per hour	0 750 1225
14 5687	sq km per hour	1.163 4222
1 Sq Foot per Hour =		
2 22968	sq meters per day	0 348 2429
1 Sq Foot per Second =		
0 0826446	acre per hour	n 2.917 2147
0 00309917	sq mile per day	n 3 491 2460
334.452	sq meters per hour	2 524 3341
1 Sq Inch per Second =		
25 †	sq feet per hour	1 397 9400
0.0137741	acre per day	n 2 139 0633
2.32259	sq meters per hour	0.365 9718

(19) SURFACE PER TIME.—(Continued.)

		Logarithm.
1 Sq Mile per Day =		
322.*666	sq feet per second	2 508 7541
26 *666	acres per hour	1 425 9688
0 107917	sq km per hour	n 1 033 0883
1 Sq Mile pe. Hour =		
7.744 †	sq feet per second	3 888 9653
15,360 †	acres per day	4 186 3912
719 444	sq meters per second	2 856 9970
1 Sq Meter per Second =		
21 3498	acres per day	1 329 3942
1 Sq Kilometer per Day =		
124 582	sq feet per second	2 095 4545
10 2960	acres per hour	1 012 6692
1 Sq Kilometer per Second =		
1,389 96	sq miles per hour	3 143 0030
1 Sq Centimeter per Second =		
3 87499	sq feet per hour	0 588 2708

(20) VOLUME PER TIME. (DISCHARGES.)

1 Cu Foot per Day =		
0 311688	U S gal per hour	n 1 493 7205
1.17987	liters per hour	0 071 8363
1 Cu Foot per Hour =		
179 532	U S gals per day	2 254 1429
0 679608	cu meter per day	n 1 832 2587
1 Cu Foot per Minute =		
448 831	U S gals per hour	2 652 0830
1 69902	cu meters per hour	0 230 1988
1 Cu Foot per Second =		
646,317	U S gals per day	5.810 4453
448.831	U S gals per minute	2 652 0828
1.98347	acre-feet per day	0.297 4258
101.941	cu meters per hour	2.008 3500
about 40	miner's inches	1.602 0600

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(20) VOLUME PER TIME. (DISCHARGES.)
—(Continued.)

		Logarithm.
1 Cu Inch per Second =		
50 †	cu feet per day	1 698 9700
15.5844	U S gal per hour	1.192 6905
58.9938	liters per hour	1.770 8063
1 Cu Yard per Hour =		
18 3494	cu meters per day	1 263 6225
1 Cu Yard per Second =		
53 5537	acre-feet per day	1.728 7396
1,000,000 U S Gallons per Day =		
1.54723	cu feet per second	0.189 5546
43 813	liters per second	1.641 6021
1 U S Gallon per Hour =		
3.208*33	cu feet per day	0 506 2795
0 0908504	cu meter per day	n 2 958 3270
1 U S Gallon per Minute =		
8.02084	cu feet per hour	0.904 2196
0 002228	cu foot per second	n 3.347 9171
0.00441919	acre-foot per day	n 3 645 3429
1 U S Gallon per Second =		
481 25†	cu feet per hour	2 682 3708
0 26*5151	acre-foot per day	n 1 423 4941
13 6276	cu meters per hour	1 134 4183
1 Miner's Inch =		
about 0.025	cu ft per sec	n 2 397 9400
1 Cu Centimeter per Second =		
3.05117	cu feet per day	0 484 4662
1 Liter per Second =		
127.132	cu feet per hour	2.104 2550
1 Cu Meter per Day =		
1.47144	cu feet per hour	0.167 7413

(20) VOLUME PER TIME. (DISCHARGES.)
—(Continued.)

		Logarithm.
1 Cu Meter per Hour =		
6,340 09	U S gal per day	3 802 0954
0.00980957	cu foot per second	n 3 991 6500
1 Cu Meter per Second =		
2 91855	acre-feet per hour	0 465 1671
1 Acre-foot per Day =		
1,815 †	cu feet per hour	3 258 8767
3.77143	U S gal per second	0 576 5059
0 0142765	cu meter per second	n 2 154 6217
1 Acre-foot per Hour =		
726 †	cu feet per minute	2 860 9366
90 5143	U S gal per second	1 956 7171
0.342636	cu meter per second	n 1 534 8329

(21) WEIGHT PER TIME.
Avoirdupois and Metric.

1 Pound per Second =		
27.2156	kg per minute	1 434 8173
1 Kilogram per Hour =		
52 9109	pounds per day	1 723 5452
1 Kilogram per Second =		
7,936.6	pounds per hour	3 899 6365

(22) WORK PER TIME. (POWER.)
For Work, see Table (6).

1 H P =		
5,694,120	foot-gallons per day	6 755 4265
33,000 †	foot-lbs per minute	4.518 5139
550 †	foot-lbs per second	2 740 3627
745 956	joules per second	2 872 7134
0 706941	pound F per second	n 1 849 3830
1 01387	metric H P	0 005 9832
0.178146	kg C per second	n 1 250 7764

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(22) WORK PER TIME. (POWER.)—(Continued.)

Equivalent		Logarithm.
1 Foot-pound per Second =		
0.00*1818	H P	n 3.259 6373
0.00184340	metric H P	n 3.265 6205
1 Million Foot-gallons Water per Day =		
0.175620	H P	n 1.244 5734
1 Watt =		
1 †	joule per second	0.000 0000
0.737308	foot-lb per second	n 1.867 6492
0 00134056	H P	n 3.127 2865
0 00135916	metric H P	n 3.133 2697
1 Kilowatt =		
1,000 †	watts. (See watt.)	
1 Metric H P =		
735 75	watts	2.866 7303
542 475	foot-lbs per second	2.734 3795
75 †	kgm per second	1 875 0613
0.986318	H P	n 1.994 0168
1 Joule per Second =		
0.00134056	H P	n 3.127 2865
0.00135916	metric H P	n 3.133 2697
366.973	kgm per hour	2.564 6336
1 Kilogrammeter per Second =		
0.0131509	H P	n 2.118 9555
0.01*33	metric H P	n 2.124 9387

(23) WORK PER VOLUME.

Equivalent		Logarithm.
1 Foot-pound per Cu Foot =		
0 000171826	pound F per gal U S	n 4.235 0886
4.88241	kgm per cu meter	0.688 6343
1 Foot-pound per Cu Inch =		
8.4368	kgm per liter	0.926 1780

(23) WORK PER VOLUME.—(Continued.)

Equivalent		Logarithm.
1 Foot-pound per Gallon U S =		
36.5230	kgm per cu meter	1.562 5660
1 H P Hour per Cu Foot =		
340 215	pounds F per gal U S	2.531 7538
22.6481	kg C per liter	1.355 0316
1 H P Hour per Cu Yard =		
73,333 *333	foot-lbs per cu foot	4.865 3014
12 6006	pounds F per gal U S	1 100 3901
358 044	kgm per liter	2.553 9358
1 H P Hour per Gallon U S =		
0.267835	metric H P h per liter	n 1.427 8674
1 Pound F per Cu Foot =		
0 0106091	H P hour per cu yard	n 2 025 6782
104 003	foot-lbs per gal U S	2 017 0478
8.89909	kg C per cu meter	0.949 3459
1 Pound F per Gallon U S =		
0 0793615	H P hour per cu yard	n 2 899 6099
66.5699	kg C per cu meter	1.823 2776
1 Kilogram Centigrade per Liter =		
50 5929	foot-lbs per cu inch	1 704 0899
0.0441538	H P hour per cu foot	n 2.644 9684
1 Kilogram Centigrade per Cu Meter =		
11.6870	foot-lbs per gal U S	1.067 7019
1 Kilogrammeter per Liter =		
204 817	foot-lbs per cu foot	2.311 3657
0 00279296	H P hour per cu yard	n 3 446 0643
0.0351929	pound F per gal U S	n 2.546 4544
1 Kilogrammeter per Cu Meter =		
0.027380	foot-lb per gal U S	n 2.437 4340

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(24) WORK PER WEIGHT.

		Logarithm.
1 Kilogrammeter per Gram =		
5.96515	H P seconds per pound	0.775 6215
1 Foot-pound per Pound =		
0.304801	kgm per kg	n 1.484 0158
1 Pound F per Pound =		
0.237135	kgm per gram	n 1.374 9954

(25) VALUE PER LENGTH.

1 Dollar per Foot =		
12.35	shillings per yard	1 092
0.1379	mark per cm	n 1.139
0.170	franc per cm	n 1.230
1 Penny per Foot =		
0.0607	dollar per yard	n 2 783
106.92	dollars per mile	2 029
440 †	shillings per mile	2 643
0.344	franc per meter	n 1.537
0.279	mark per meter	n 1.446
1 Shilling per Foot =		
0.729	dollar per yard	n 1.863
4.13	centimes per cm	0.616
3.35	marks per meter	0.525
1 Cent per Inch =		
0.12†	dollar per foot	n 1.079
0.494	shilling per foot	n 1.694
2.04	francs per meter	0 310
1.654	marks per meter	0.219
1 Penny per Inch =		
0.7290	dollar per yard	n 1.863
4.13	centimes per cm	0.616
3.35	pfennigs per cm	0.525

(25) VALUE PER LENGTH.—(Continued.)

		Logarithm.
1 Cent per Yard =		
17.60†	dollars per mile	1.246
72.43	shillings per mile	1.860
0.0566	franc per meter	n 2.753
0.0459	mark per meter	n 2.662
1 Penny per Yard =		
35.64	dollars per mile	1.552
7.*333	pounds per mile	0.865
0.1148	franc per meter	n 1.060
0.09305	mark per meter	n 2.969
1 Shilling per Yard =		
8 1	cents per foot	0.908
1.377	francs per meter	0.139
1.117	marks per meter	0.048
1 Dollar per Mile =		
0.018*93	cent per foot	n 2 277
0.009353	penny per foot	n 3.971
0.0007795	shilling per foot	n 4 892
3.22	francs per km	0.508
2.61	marks per km	0.417
1 Shilling per Mile =		
0.0046	cent per foot	n 3 663
0 002*2727	penny per foot	n 3.357
0 7823	franc per km	n 1 893
0.634	mark per km	n 1.802
1 Franc per Meter =		
0 0588	dollar per foot	n 2 770
2.905	pence per foot	0.463
0 242	shilling per foot	n 1.384
310.60	dollars per mile	2.492
1,278	shillings per mile	3.107

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(25) VALUE PER LENGTH.—(Continued.)

1 Mark per Meter =

		Logarithm.
0.0726	dollar per foot	n 2 861
3.58	pence per foot	0.554
0.2985	shilling per foot	n 1 475
383	dollars per mile	2 583
1,576	shillings per mile	3.198

1 Franc per Kilometer =

0.3106	dollar per mile	n 1.492
15.34	pence per mile	1.186
1.28	shillings per mile	0.107

1 Mark per Kilometer =

0.383	dollar per mile	n 1 583
18.92	pence per mile	1 277
1.576	shillings per mile	0.198

(26) VALUE PER SURFACE.

1 Dollar per Acre =

0.002295	cent per sq foot	n 3.361
0.00113	penny per sq foot	n 3 054
0.000850	shilling per sq yard	n 4.930
12.8	francs per hectare	1.107
10.38	marks per hectare	1.016

1 Cent per Sq Foot =

0.09†	dollar per sq yard	n 2.954
4.44	pence per sq yard	0 648
0.557	franc per sq meter	n 1 746
0.452	mark per sq meter	n 1 655

1 Shilling per Sq Foot =

10,585	dollars per acre	4.025
108 †	pence per sq yard	2 033
13.55	francs per sq meter	1.132
11.0	marks per sq meter	1.041

(26) VALUE PER SURFACE.—(Continued.)

1 Cent per Sq Inch =

		Logarithm.
12.96†	dollars per sq yard	1.113
71.1	pence per sq foot	1.852
53.3	shillings per sq yard	1.727
80.3	francs per sq meter	1 905
65.1	marks per sq meter	1.814

1 Dollar per Sq Yard =

0.457	shilling per sq foot	n 1.660
996	pounds per acre	2.998
6 20	francs per sq meter	0.792
5 02	marks per sq meter	0.701

1 Penny per Sq Yard =

98 01	dollars per acre	1.991
0.125	franc per sq meter	n 1.099
0.102	mark per sq meter	n 1.008

1 Shilling per Sq Yard =

1,176	dollars per acre	3.070
1.506	francs per sq meter	0.178
1.22	marks per sq meter	0 087

1 Franc per Sq Meter =

1.79	cents per sq foot	0 254
0 885	penny per sq foot	n 1 947
0.664	shilling per sq yard	n 1.822

1 Mark per Sq Meter =

2.21	cents per sq foot	0.345
1 09	pence per sq foot	0.038
0 819	shilling per sq yard	n 1.913

1 Franc per Hectare =

0 0781	dollar per acre	n 2.893
0 321	shilling per acre	n 1.507

1 Mark per Hectare =

0.0963	dollar per acre	n 2.984
0.396	shilling per acre	n 1.598

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(27) VALUE PER VOLUME.

		Logarithm.
1 Dollar per Cu Foot =		
111.1	shillings per cu yard	2.046
13.36	cents per U S gal	1.126
0.183	franc per liter	n 1.262
0.1483	mark per liter	n 1.171
1 Shilling per Cu Foot =		
6.56	dollars per cu yard	0.817
324 †	pence per cu yard	2.511
0.0445	franc per liter	n 2.648
0.03605	mark per liter	n 2.557
1 Cent per Cu Inch =		
17.28†	dollars per cu foot	1.238
71.11	shillings per cu foot	1.852
0.316	centime per cc	n 1.500
0.2565	pfennig per cc	n 1.409
1 Penny per Cu Inch =		
944.78	dollars per cu yard	2.975
3.888 †	shillings per cu yard	3.590
6.40	francs per liter	0.806
5.19	marks per liter	0.715
1 Shilling per Cu Inch =		
56.13	dollars per gal U S	1.749
0.07683	franc per cc	n 2.886
0.0624	mark per cc	n 2.795
1 Dollar per Cu Yard =		
3.*703	cents per cu foot	0.569
0.1523	shilling per cu foot	n 1.183
6.777	francs per cu meter	0.831
5.495	marks per cu meter	0.740
1 Shilling per Cu Yard =		
0.9000	cent per cu foot	n 1.954
0.*444	penny per cu foot	n 1.648
0.165	centime per liter	n 1.217
0.1336	pfennig per liter	n 1.126

(27) VALUE PER VOLUME.—(Continued.)

		Logarithm.
1 Penny per Gallon Imp =		
1.686	cents per gal U S	0.227
2.308	centimes per liter	0.363
1.87	pfennigs per liter	0.272
1 Shilling per Gallon Imp =		
0.2023	dollar per gal U S	n 1.306
0.277	franc per liter	n 1.442
0.225	mark per liter	n 1.351
1 Dollar per Gallon U S =		
4.94	shillings per gal Imp	0.694
1.369	francs per liter	0.136
1.11	marks per liter	0.045
1 Franc per Cu Meter =		
0.1477	dollar per cu yard	n 1.169
7.287	pence per cu yard	0.863
0.607	shilling per cu yard	n 1.783
1 Mark per Cu Meter =		
0.182	dollar per cu yard	n 1.260
8.985	pence per cu yard	0.954
0.7488	shilling per cu yard	n 1.874

(28) VALUE PER WEIGHT.

Avoirdupois and Metric

1 Dollar per Pound =		
11.43	francs per kg	1.058
9.263	marks per kg	0.967
1 Penny per Pound =		
0.231	franc per kg	n 1.364
0.1875	mark per kg	n 1.273
1 Shilling per Pound =		
2.775	francs per kg	0.443
2.25	marks per kg	0.352

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . † Exact values n. Negative characteristic.

(28) VALUE PER WEIGHT.—(Continued.)

Avoirdupois and Metric.

		Logarithm
1 Franc per Kilogram =		
0.08754	dollar per pound	n 2.942
0.3603	shilling per pound	n 1.557
1 Mark per Kilogram =		
0.108	dollar per pound	n 1.033
0.444	shilling per pound	n 1.648

(29) VALUE PER TIME.

1 Dollar per Day =		
4.1*66	cents per hour	0.620
2.058	pence per hour	0.313
0.216	franc per hour	n 1.334
0.175	mark per hour	n 1.243
1 Penny per Day =		
0.08437	cent per hour	n 2.926
0.437	centime per hour	n 1.641
0.355	pfennig per hour	n 1.550
1 Shilling per Day =		
1.012	cents per hour	0.005
0.5†	penny per hour	n 1.699
0.0525	franc per hour	n 2.720
0.0426	mark per hour	n 2.629
1 Franc per Day =		
0.0338	mark per hour	n 2.529
0.8042	cent per hour	n 1.905
0.397	penny per hour	n 1.599
1 Mark per Day =		
0.0514	franc per hour	n 2.711
0.9917	cent per hour	n 1.996
0.49	penny per hour	n 1.690

(29) VALUE PER TIME.—(Continued.)

		Logarithm.
1 Cent per Hour =		
0.9877	shilling per day	n 1.995
1.244	francs per day	0.095
1.008	marks per day	0.004
1 Dollar per Hour =		
0.0137	penny per second	n 2.137
0.144	centime per second	n 1.158
0.1168	pfennig per second	n 1.067
1 Penny per Hour =		
0.486	dollar per day	n 1.687
2†	shillings per day	0.301
2.52	francs per day	0.401
2.04	marks per day	0.310
1 Shilling per Hour =		
0.035	centime per second	n 2.544
0.0284	pfennig per second	n 2.453
5.83	dollars per day	0.766
1 Franc per Hour =		
19.46	marks per day	1.289
4.63	dollars per day	0.666
19.06	shillings per day	1.280
1 Mark per Hour =		
29.6	francs per day	1.471
5.712	dollars per day	0.757
23.5	shillings per day	1.371
1 Cent per Second =		
36†	dollars per hour	1.556
1,778	pence per hour	3.250
148.2	shillings per hour	2.171
186.5	francs per hour	2.271
151.3	marks per hour	2.180

* Repeating decimals, thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(29) VALUE PER TIME.—(Continued.)

1 Penny per Second =

		Logarithm.
72.90	dollars per hour	1.863
300 †	shillings per hour	2.477
378	francs per hour	2.577
306	marks per hour	2.486

1 Centime per Second =

36 †	francs per hour	1 556
29.2	marks per hour	1 465
6.95	dollars per hour	0 842
28.6	shillings per hour	1 456

1 Pfennig per Second =

44.4	francs per hour	1 647
36 †	marks per hour	1 556
8.568	dollars per hour	0.933
35.26	shillings per hour	1.547

(30) VALUE PER WORK.

1 Cent per Foot-pound =

0 375	franc per kgm	n 1 574
0.304	mark per kgm	n 1.483

1 Penny per Foot-pound =

0.7589	franc per kgm	n 1.880
0.615	mark per kgm	n 1 789

1 Shilling per Foot-pound =

9.107	francs per kgm	0 959
7.38	marks per kgm	0 868

1 Franc per Kilogrammeter =

2.668	cents per foot-lb	0 426
1.318	pence per foot-lb	0 120

1 Mark per Kilogrammeter =

3.29	cents per foot-lb	0 517
1.625	pence per foot-lb	0 211

(31) LENGTH PER TIME PER TIME.
(ACCELERATION.)

Acceleration of Gravity = g =

		Logarithm.
32 1850	ft per sec per sec	1 507 6532
9.81	m per sec per sec	0.991 6690

1 Foot per Second per Second =

0.0310704	g in ft per sec per sec	n 2.492 3468

1 Meter per Second per Second =

0.101937	g in cm per sec per sec	n 1 008 3310

(32) LENGTH TO VOLUME PER SURFACE.

1 Inch in Depth =

27,154 3	U S gal per acre	4.433 8384
3,630 †	cu feet per acre	3 559 9067
0 623376	U S gal per sq foot	n 1 794 7505
254.0005	cu meters per hectare	2.404 8346

1 Foot in Depth =

43,560 †	cu feet per acre	4 639 0879
67.3247	U S gal per sq yard	1 828 1742
3,048.006	cu meters per hectare	3.484 0158

1 Centimeter in Depth =

10 †	liters per sq meter	1 000 0000
1,429 13	cu feet per acre	3 155 0721
0.245423	U S gal per sq foot	n 1.389 9159

1 Meter in Depth =

1,000 †	liters per sq meter	3 000 0000
142.913	cu feet per acre	5.155 0721
24 5423	U S gal per sq foot	1.389 9159

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(33) VOLUME PER SURFACE TO LENGTH.

		Logarithm.
1 Cu Foot per Acre =		
0.000275482	inch in depth	n 4 440 0933
0.000699726	cm in depth	n 4.844 9279
1 Cu Inch per Sq Foot =		
0.0069*44	inch in depth	n 3 841 6375
0.0176389	cm in depth	n 2.246 4721
1 Cu Yard per Acre =		
0 00743802	inch in depth	n 3 871 4571
0 0188926	cm in depth	n 2 276 2917
1 Gallon U S per Sq Foot =		
1 60417	inches in depth	0.205 2495
4.07459	cm in depth	0.610 0841
1 Liter per Sq Meter =		
1 †	mm in depth	0 000 0000
0 03937	inch in depth	n 2 595 1654

(34) VOLUME PER SURFACE PER TIME.

1 Cu Foot per Acre per Day =		
0 00292	cu m per hec per h	n 3 464 7167
1 Cu Foot per Acre per Hour =		
1 32987	gal U S per sq mi per sec	0 123 8092
1.67934	cu m per hec per d	0.225 1391
1 Cu Foot per Acre per Second =		
0 618225	gal U S per sq ft per h	n 1 791 1463
251.901	cu m per hec per h	2.401 2304
1 Cu Foot per Sq Mile per Day =		
0.000487013	gal U S per a per h	n 4 687 5405
0 00455551	li per hec per h	n 3 658 5367
1 Cu Foot per Sq Mile per Hour =		
0.280519	gal U S per a per d	n 1 447 9629
0.262397	cu m per sq km per d	n 1.418 9591

(34) VOLUME PER SURFACE PER TIME.
—(Continued.)

		Logarithm.
1 Cu Foot per Sq Mile per Second =		
42.0779	gal U S per a per h	1 624 0542
944 63	cu m per sq km per d	2.975 2616
1 Gallon U S per Acre per Day =		
3 56482	cu ft per sq mi per h	0 552 0371
0.0389749	cu m per sq km per h	n 2.590 7850
1 Gallon U S per Acre per Hour =		
0 0237654	cu ft per sq mi per sec	n 2.375 9458
22 4495	cu m per sq km per d	1.351 2074
1 Gallon U S per Acre per Second =		
0 0110480	cu ft per sq ft per h	n 2 043 2829
56 1239	cu m per sq km per min	1.749 1475
1 Gallon U S per Sq Foot per Day =		
242 630	cu ft per a per h	2 384 9450
1.69775	li per sq m per h	0 229 8730
1 Gallon U S per Sq Foot per Hour =		
1 61754	cu ft per a per sec	0 208 8537
0 0113183	li per sq m per sec	n 2.053 7817
1 Gallon U S per Sq Foot per Second =		
146 685	cu m per sq m per h	2 166 3867
1 Gallon U S per Sq Inch per Day =		
0 802084	cu ft per sq ft per h	n 1 904 2196
0 244476	cu m per sq m per h	n 1.388 2354
1 Gallon U S per Sq Inch per Hour =		
462 †	cu ft per sq ft per d	2 664 6420
140 818	cu m per sq m per d	2 148 6578
1 Liter per Sq Meter per Second =		
283 464	cu ft per sq ft per d	2 452 4978
88.3524	gal U S per sq ft per h	1 946 2183

* Repeating decimals: thus. 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(34) VOLUME PER SURFACE PER TIME.—(Continued.)

		LOGARITHM.
1 Cu Meter per Sq Meter per Day =		
0 000284055	gal U S per sq ft per sec.	n 4.453 4021
1 Cu Meter per Hectare per Day =		
0 595471	cu ft per a per h	n 1.774 8609
1 Cu Meter per Hectare per Hour =		
342 991	cu ft per a per d	2 535 2833
1 Cu Meter per Hectare per Second =		
51,448.7	cu ft per a per h	4 711 3746
1 Cu Meter per Sq Kilometer per Day =		
3 81102	cu ft per sq mi per h	0 581 0409
1 Cu Meter per Sq Kilometer per Hour =		
2,195 15	cu ft per sq mi per d	3 341 4635
25 6575	gal U S per a per d	1.409 2150
1 Cu Meter per Sq Kilometer per Second =		
329,272	cu ft per sq mi per h	5 517 5546
64.1439	gal U S per a per min	1.807 1551

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.

		LOGARITHM.
1,000,000 Cu Feet per Acre per Day =		
0 000265704	foot per second	n 4 424 3984
0 00809868	cm per second	n 3 908 4142
1,000,000 Cu Feet per Acre per Hour =		
0 0063769	foot per second	n 3 804 6096
0.194368	cm per second	n 1 288 6254
1 Cu Foot per Acre per Second =		
1 98347	feet per day	0 297 4258
0.604563	meter per day	n 1.781 4416

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.—(Continued.)

		LOGARITHM.
1,000,000 Cu Feet per Sq Mile per Second =		
129 132	feet per hour	2 111 0346
944 63	meters per day	2.975 2616
1 Cu Inch per Sq Foot per Second =		
25 †	inches per hour	1 397 9400
50 †	feet per day	1 698 9700
15 240	meters per day	1.182 9858
1,000,000 Gallons U S per Sq Mile per Day =		
0.00239757	inch per hour	n 3 379 7704
1,000,000 Gallons U S per Sq Mile per Hour =		
0 115083	foot per day	n 1 061 0116
0.0350774	meter per day	n 2 545 0274
1,000,000 Gallons U S per Sq Mile per Second =		
5 26161	meters per hour	0 721 1187
1,000,000 Gallons U S per Acre per Day =		
0 000035519	foot per second	n 5 550 4667
0 00108264	cm per second	n 3 034 4825
1,000,000 Gallons U S per Acre per Hour =		
0 000852468	foot per second	n 4 930 6779
0.0259833	cm per second	n 2 414 6937
1 Gallon U S per Acre per Second =		
0 132576	inch per hour	n 1 122 4641
0 265152	foot per day	n 1 423 4941
8 08183	meters per day	0 907 5099
1,000,000 Gallons U S per Sq Foot per Day =		
1 54723	feet per second	0 189 5546
47.1596	cm per second	1 673 5705

* Repeating decimals; thus. 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.—(Continued.)

		Logarithm.
1 Gallon U S per Sq Foot per Hour =		
38.5†	inches per day	1 585 4607
0.977902	meter per day	n 1.990 2954
1 Gallon U S per Sq Foot per Second =		
146.685	meters per hour	2.166 3867
1 Cu Meter per Sq Meter per Second =		
11,811	feet per hour	4 072 2867
1 Cu Meter per Hectare per Second =		
340.157	inches per day	2.531 6791

(36) VELOCITY TO VOLUME PER SURFACE PER TIME.

1 Inch per Second =		
2,244 16	U S gal per sq ft per h	3 351 0531
91.4402	cu m per sq m per h	1.961 1372

(36) VELOCITY TO VOLUME PER SURFACE PER TIME.—(Continued.)

		Logarithm.
1 Inch per Hour =		
87,120†	cu ft per acre per d	4 940 1179
645 *333	cu ft per sq mi per sec	2 809 7841
14 9610	U S gal per sq ft per d	1.174 9617
7 54286	U S gal per a per sec	0.877 5358
6,096.01	cu m per hec per d	3.785 0458
1 Foot per Second =		
26,929 9	U S gal per sq ft per h	4 430 2343
1,097 28	cu m per sq m per h	3.040 3184
1 Foot per Hour =		
7,744 0†	cu ft per sq mi per sec	3 888 9653
179 532	U S gal per sq ft per d	2 254 1429
90 5143	U S gal per a per sec	1 956 7171
12 10†	cu ft per a per sec	1 082 7854
0 84667	cu m per hec per sec	n 1 927 7133
1 Centimeter per Second =		
118.11	cu ft per sq ft per h	2 072 2867
1 Meter per Hour =		
39 6981	cu ft per a per sec	1 598 7695
589 016	U S gal per sq ft per d	2.770 1271

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

TABLE OF ACRES REQUIRED per mile, and per 100 feet, for different widths.

Width Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft	Width. Feet.	Acres per Mile.	Acres per 100 Ft.
1	.121	.002	26	3.15	.060	52	6.30	.119	78	9.45	.179
2	.242	.005	27	3.27	.062	53	6.42	.122	79	9.58	.181
3	.364	.007	28	3.39	.064	54	6.55	.124	80	9.70	.184
4	.485	.009	29	3.52	.067	55	6.67	.126	81	9.82	.186
5	.606	.011	30	3.64	.069	56	6.79	.129	82	9.94	.188
6	.727	.014	31	3.76	.071	57	6.91	.131	½	10.	.189
7	.848	.016	32	3.88	.073	¾	7.	.133	83	10.1	.190
8	.970	.018	33	4.00	.076	58	7.03	.133	84	10.2	.193
¼	1.	.019	34	4.12	.078	59	7.15	.135	85	10.3	.195
9	1.09	.021	35	4.24	.080	60	7.27	.138	86	10.4	.197
10	1.21	.023	36	4.36	.083	61	7.39	.140	87	10.5	.200
11	1.33	.025	37	4.48	.085	62	7.52	.142	88	10.7	.202
12	1.46	.028	38	4.61	.087	63	7.64	.145	89	10.8	.204
13	1.58	.030	39	4.73	.090	64	7.76	.147	90	10.9	.207
14	1.70	.032	40	4.85	.092	65	7.88	.149	¾	11.	.209
15	1.82	.034	41	4.97	.094	66	8.	.151	91	11.0	.209
16	1.94	.037	¼	5.	.094	67	8.12	.154	92	11.2	.211
½	2.	.038	42	5.09	.096	68	8.24	.156	93	11.3	.213
17	2.06	.039	43	5.21	.099	69	8.36	.158	94	11.4	.216
18	2.18	.041	44	5.33	.101	70	8.48	.161	95	11.5	.218
19	2.30	.044	45	5.45	.103	71	8.61	.163	96	11.6	.220
20	2.42	.046	46	5.58	.106	72	8.73	.165	97	11.8	.223
21	2.55	.048	47	5.70	.108	73	8.85	.168	98	11.9	.225
22	2.67	.051	48	5.82	.110	74	8.97	.170	99	12.	.227
23	2.79	.053	49	5.94	.112	¼	9.	.170	100	12.1	.230
24	2.91	.055	½	6.	.114	75	9.09	.172			
¾	3.	.057	50	6.06	.115	76	9.21	.174			
25	3.03	.057	51	6.18	.117	77	9.33	.177			

Functions of Grades.

Tables of Grades. pages 255 to 257 See Figs, p. 255.

Rise $= v$; **Slope** $= s$; **horizontal distance** $= h$; $\sin A = v/s$; $\cos A = h/s$; $\tan A = v/h$; $\text{ctn}\ A = h/v$; $\sec A = s/h$; $\csc A = s/v$.

	In small angles, approximately
$v = s\,.\sin A = h\,.\tan A = h \div \text{ctn}\ A = s \div \csc A$	$v = 0.01745\ s\ A°$
$s = v \div \sin A = h \div \cos A = h\,.\sec A = v\,.\csc A$	$s = 57.30\ v \div A°$
$h = s\,.\cos A = v \div \tan A = v\,.\text{ctn}\ A = s \div \sec A$	$h = 57.29\ v \div A°$
In feet per mile, $v \div s = 5280 \sin A$	$v - s = 92.149\ A°$
$v \div h = 5280 \tan A$	$v \div h = 92.163\ A°$

The ratios for an angle of 1° are shown in the three figures opposite, in which the angle is necessarily exaggerated. **In small angles** the rise, in a given distance, may be taken as varying directly, and the sloping and horizontal distances, for a given rise, inversely, as the angle itself, expressed in degrees, as indicated above in the column headed "In small angles." Thus, in clinometer work, with small angles, 57.3 feet $\div A° =$ distance (horizontal or sloping) in feet per foot of rise. Hence, for small angles;

Sloping or horizontal distance, s or h, in feet $= 57.3\ v \div A°$.

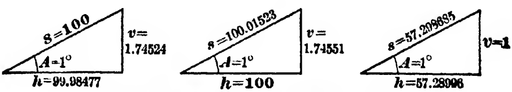
Table of grades per mile, and per 100 feet measured horizontally, and corresponding to different angles of inclination.

Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.
0	1	1.536	.0291	0	45	69.11	1.3090	1	58	181.3	3.4341	3	26	316.8	5.9994
	2	3.072	.0582		46	70.64	1.3381	2	0	184.4	3.4924		28	319.8	6.0579
	3	4.608	.0873		47	72.18	1.3672		2	187.5	3.5506		30	322.9	6.1163
	4	6.144	.1164		48	73.72	1.3963		4	190.6	3.6087		32	326.0	6.1747
	5	7.680	.1455		49	75.26	1.4254		6	193.6	3.6669		34	329.1	6.2330
	6	9.216	.1746		50	76.80	1.4545		8	196.7	3.7250		36	332.2	6.2914
	7	10.75	.2037		51	78.33	1.4837		10	199.8	3.7833		38	335.3	6.3498
	8	12.29	.2328		52	79.87	1.5128		12	202.8	3.8416		40	338.4	6.4083
	9	13.82	.2619		53	81.40	1.5419		14	205.9	3.8999		42	341.4	6.4664
	10	15.36	.2909		54	82.94	1.5710		16	208.9	3.9581		44	344.5	6.5246
	11	16.90	.3200		55	84.47	1.6000		18	212.0	4.0163		46	347.6	6.5832
	12	18.43	.3491		56	86.01	1.6291		20	215.1	4.0746		48	350.7	6.6418
	13	19.96	.3782		57	87.54	1.6583		22	218.1	4.1329		50	353.8	6.7004
	14	21.50	.4073		58	89.08	1.6873		24	221.2	4.1911		52	356.8	6.7583
	15	23.04	.4364		59	90.62	1.7164		26	224.3	4.2494		54	359.9	6.8163
	16	24.58	.4655	1		92.16	1.7455		28	227.4	4.3076		56	363.0	6.8751
	17	26.11	.4946		2	95.23	1.8038		30	230.5	4.3659		58	366.1	6.9339
	18	27.64	.5237		4	98.30	1.8620		32	233.5	4.4242	4		369.2	6.9926
	19	29.17	.5528		6	101.4	1.9202		34	236.6	4.4826		5	376.9	7.1384
	20	30.72	.5818		8	104.5	1.9784		36	239.7	4.5409		10	384.6	7.2842
	21	32.26	.6109		10	107.5	2.0366		38	242.8	4.5993		15	392.3	7.4300
	22	33.80	.6400		12	110.6	2.0948		40	245.9	4.6576		20	400.1	7.5767
	23	35.33	.6691		14	113.6	2.1530		42	248.9	4.7159		25	407.8	7.7234
	24	36.86	.6982		16	116.7	2.2112		44	252.0	4.7742		30	415.5	7.8701
	25	38.40	.7273		18	119.8	2.2694		46	255.1	4.8325		35	423.2	8.0163
	26	39.94	.7564		20	122.9	2.3277		48	258.2	4.8908		40	431.0	8.1625
	27	41.47	.7855		22	126.0	2.3859		50	261.3	4.9492		45	438.7	8.3087
	28	43.01	.8146		24	129.1	2.4441		52	264.3	5.0075		50	446.5	8.4554
	29	44.54	.8436		26	132.1	2.5023		54	267.4	5.0658		55	454.2	8.6021
	30	46.08	.8727		28	135.2	2.5604		56	270.5	5.1241	5		461.9	8.7489
	31	47.62	.9018		30	138.3	2.6186		58	273.6	5.1824		5	469.6	8.8951
	32	49.16	.9309		32	141.3	2.6768	3		276.7	5.2407		10	477.4	9.0413
	33	50.69	.9600		34	144.4	2.7350		2	279.7	5.2990		15	485.1	9.1875
	34	52.23	.9891		36	147.4	2.7932		4	282.8	5.3573		20	492.9	9.3347
	35	53.76	1.0182		38	150.5	2.8514		6	285.9	5.4158		25	500.6	9.4819
	36	55.30	1.0472		40	153.6	2.9097		8	289.0	5.4742		30	508.4	9.6292
	37	56.83	1.0763		42	156.6	2.9679		10	292.1	5.5326		35	516.1	9.7755
	38	58.37	1.1054		44	159.7	3.0262		12	295.1	5.5909		40	523.9	9.9218
	39	59.90	1.1345		46	162.8	3.0844		14	298.2	5.6493		45	531.6	10.068
	40	61.44	1.1636		48	165.9	3.1427		16	301.3	5.7077		50	539.4	10.215
	41	62.97	1.1927		50	169.0	3.2010		18	304.4	5.7660		55	547.2	10.362
	42	64.51	1.2218		52	172.0	3.2592		20	307.5	5.8244	6		555.	10.510
	43	66.04	1.2509		54	175.1	3.3175		22	310.5	5.8827				
	44	67.57	1.2800		56	178.2	3.3758		24	313.6	5.9410				

On a turnpike road 1° 38′, or about 1 in 35, or 151 feet per mile, is the greatest slope that will allow horses to trot down rapidly with safety. In crossing mountains, this is often increased to 3°, or even to 5°. It should never exceed 2½°, except when absolutely necessary.

Any hor dist is = sloping dist × cosine ang of slope.
" sloping dist is = hor dist ÷ cosine " " "
" vert height is = hor dist × tangent " " "
or = sloping dist × sine " " "

A grade of n feet rise per 100 feet horizontal is usually called a grade of **n per cent.**



Functions of grade of 1 degree. See p 254.

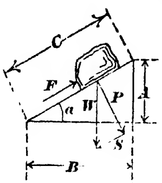
SLOPES IN FEET PER 100 FT. HORIZONTAL.

The fractions of minutes are given only to 34 feet in 100.

A clinometer graduated by the 3d column and numbered by the first one will give at sight the slopes in feet per 100 feet. No errors Original

Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.	
	Feet	Deg.	Min.		Feet	Deg.	Min.		Feet.	Deg.	Min
1	100.005	0	34.4	35	105.948	19	17	69	121.495	34	36
2	100.020	1	8.7	36	106.283	19	48	70	122.066	35	0
3	100.045	1	43.1	37	106.626	20	18	71	122.642	35	23
4	100.080	2	17.5	38	106.977	20	48	72	123.223	35	46
5	100.125	2	51.8	39	107.336	21	18	73	123.810	36	8
6	100.180	3	26.0	40	107.703	21	48	74	124.403	36	30
7	100.245	4	0.3	41	108.079	22	18	75	125.000	36	52
8	100.319	4	34.4	42	108.462	22	47	76	125.603	37	14
9	100.404	5	8.6	43	108.853	23	16	77	126.210	37	36
10	100.499	5	42.6	44	109.252	23	45	78	126.828	37	57
11	100.603	6	16.6	45	109.659	24	14	79	127.440	38	19
12	100.717	6	50.6	46	110.073	24	42	80	128.062	38	40
13	100.841	7	24.4	47	110.494	25	10	81	128.690	39	1
14	100.975	7	58.2	48	110.923	25	38	82	129.321	39	21
15	101.119	8	31.9	49	111.359	26	6	83	129.958	39	42
16	101.272	9	5.4	50	111.803	26	34	84	130.599	40	2
17	101.435	9	38.9	51	112.254	27	1	85	131.244	40	22
18	101.607	10	12.2	52	112.712	27	28	86	131.894	40	42
19	101.789	10	45.5	53	113.177	27	55	87	132.548	41	1
20	101.980	11	18.6	54	113.649	28	22	88	133.207	41	21
21	102.181	11	51.6	55	114.127	28	49	89	133.869	41	40
22	102.391	12	24.5	56	114.612	29	15	90	134.536	41	59
23	102.611	12	57.2	57	115.104	29	41	91	135.207	42	18
24	102.840	13	29.8	58	115.603	30	7	92	135.882	42	37
25	103.078	14	2.2	59	116.108	30	32	93	136.561	42	55
26	103.325	14	34.5	60	116.619	30	58	94	137.244	43	14
27	103.581	15	6.6	61	117.137	31	23	95	137.931	43	32
28	103.846	15	38.5	62	117.661	31	48	96	138.622	43	50
29	104.120	16	10.3	63	118.191	32	13	97	139.316	44	8
30	104.403	16	42.0	64	118.727	32	37	98	140.014	44	25
31	104.695	17	13.4	65	119.269	33	1	99	140.716	44	43
32	104.99[illegible]	17	44.7	66	119.817	33	25	100	141.421	45	00
33	105.304	18	15.8	67	120.370	33	49	101	142.130	45	17
34	105.622	18	46.7	68	120.930	34	13	102	142.843	45	34

In describing **railroad grades**, it is usual, as in our tables, to refer the rise, A, to the corresponding *horizontal* length, B. We then have $\frac{\text{rise}}{\text{length}} = \frac{A}{B}$ the *tangent* of the angle, a, between the plane and the horizontal. If the rise, A, be referred to the *sloping* length, C, we have $\frac{\text{rise}}{\text{length}} = \frac{A}{C} =$ *sine* A; and this fraction is proportional to the component, S, of the weight, W, in the direction of the slope. Thus, on a grade where rise, A, = 0.1 × sloping length, C, we have $\sin a = 0.1$, and S = 0.1 W. The *tangent* of a is only *approximately* proportional to S; but the steepest grades, surmounted by traction only, even on electric railways, rarely, if ever, exceed from 13 to 15 per cent; and, on these, the error, due to using $\tan a$ instead of $\sin a$, is less than a difference of 0.2 per cent in the grade, and about – 1 per cent of the true value of S. For steeper grades, such as those of rack railways, it should always be specified whether the rise refers to the horizontal or to the sloping measurement.



Transverse slopes, such as those of earthwork, are sometimes, like railroad grades, stated in $\frac{\text{ft. vertical}}{\text{ft. horizontal}}, \frac{A}{B}$, but usually in $\frac{\text{ft. horizontal}}{\text{ft. vertical}}$, as $\frac{B}{A}$. Here $\frac{B}{A}$ is the cotangent of the angle, a, with the horizontal, or the tangent of the angle ($90° - a$) with the vertical. Thus stated, a slope of 2 to 1 means a slope of 2 horizontal to 1 vertical.

Table of grades per mile; or per 100 feet measured horizontally.

Grade in ft. per mile	Grade in ft. per 100 ft	Grade in ft. per mile.	Grade in ft. per 100 ft	Grade in ft. per mile.	Grade in ft. per 100 ft	Grade in ft. per mile	Grade in ft. per 100 ft.
1	.01894	39	.73864	77	1.45833	115	2.17803
2	.03788	40	.75758	78	1.47727	116	2.19697
3	.05682	41	.77652	79	1.49621	117	2.21591
4	07576	42	.79545	80	1.51515	118	2.23485
5	.09470	43	.81439	81	1.53409	119	2.25379
6	.11364	44	.83333	82	1.55303	120	2.27273
7	.13258	45	.85227	83	1.57197	121	2.29167
8	.15152	46	.87121	84	1.59091	122	2.31061
9	.17045	47	.89015	85	1.60985	123	2.32955
10	.18939	48	.90909	86	1.62879	124	2.34848
11	.20833	49	.92803	87	1.64773	125	2.36742
12	.22727	50	.94697	88	1.66666	126	2.38636
13	.24621	51	.96591	89	1.68561	127	2.40530
14	.26515	52	.98485	90	1.70455	128	2.42424
15	.28409	53	1.00379	91	1.72348	129	2.44318
16	.30303	54	1.02273	92	1.74242	130	2.46212
17	32197	55	1.04167	93	1.76136	131	2.48106
18	.34091	56	1.06061	94	1.78030	132	2.50000
19	.35985	57	1.07955	95	1.79924	133	2.51894
20	.37879	58	1.09848	96	1.81818	134	2.53788
21	.39773	59	1.11742	97	1.83712	135	2.55682
22	.41667	60	1.13636	98	1.85606	136	2.57576
23	.43561	61	1.15530	99	1.87500	137	2.59470
24	.45455	62	1.17424	100	1.89394	138	2.61364
25	.47348	63	1.19318	101	1.91288	139	2.63258
26	.49242	64	1.21212	102	1.93182	140	2.65152
27	.51136	65	1.23106	103	1.95076	141	2.67045
28	.53030	66	1.25000	104	1.96970	142	2.68939
29	.54924	67	1.26894	105	1.98864	143	2.70833
30	.56818	68	1.28788	106	2.00758	144	2.72727
31	.58712	69	1.30682	107	2.02652	145	2.74621
32	.60606	70	1.32576	108	2.04545	146	2.76515
33	.62500	71	1.34470	109	2.06439	147	2.78409
34	64394	72	1.36364	110	2.08333	148	2.80303
35	.66288	73	1.38258	111	2.10227	149	2.82197
36	.68182	74	1.40152	112	2.12121	150	2.84091
37	.70076	75	1.42045	113	2.14015	151	2.85985
38	.71970	76	1.43939	114	2.15909	152	2.87879

If the grade per mile should consist of feet *and tenths*, add to the grade per 100 feet in the foregoing table, that corresponding to the number of tenths taken from the table below; thus, for a grade of 43.7 feet per mile, we have .81439 + .01326 = .82765 feet per 100 feet.

Ft per Mile.	Per 100 Feet.	Ft. per Mile	Per 100 Feet	Ft per Mile.	Per 100 Feet.
.05	.00094	.4	.00758	.7	.01326
.1	.00189	.45	.00852	.75	.01420
.15	.00283	.5	.00947	.8	.01515
.2	.00379	.55	.01041	.85	.01609
.25	.00473	.6	.01136	.9	.01705
.3	.00568	.65	.01230	.95	.01799
.35	.00662				

TABLE OF HEADS OF WATER CORRESPONDING TO GIVEN PRESSURES.

Water at maximum density, 62.425 lbs. per cubic foot = 1 gram per cubic centimeter; corresponding to a temperature of 4° Centigrade = 39 2° Fahrenheit.

Head in feet = 2.306768 × pressure in lbs. per square inch.
" " = 0.0160192 × pressure in lbs. per square foot

Heads corresponding to pressures not given in the table can be found by these formulæ, or taken from the table by simple proportion.

Pressure.		Head.	Pressure.		Head.	Pressure.		Head.
lbs. per sq in.	lbs per sq. ft.	Feet.	lbs per sq in	lbs. per sq ft.	Feet.	lbs. per sq. in.	lbs per sq. ft.	Feet.
1	144	2.3068	51	7344	117.645	101	14544	232.984
2	288	4.6135	52	7488	119.952	102	14688	235 290
3	432	6.9203	53	7632	122 259	103	14832	237.597
4	576	9.2271	54	7776	124.565	104	14976	239 904
5	720	11.5338	55	7920	126.872	105	15120	242.211
6	864	13.8406	56	8064	129.179	106	15264	244.517
7	1008	16.1474	57	8208	131.486	107	15408	246.824
8	1152	18.4541	58	8352	133 793	108	15552	249.131
9	1296	20 7609	59	8496	136.099	109	15696	251.438
10	1440	23.0677	60	8640	138.406	110	15840	253.744
11	1584	25.3744	61	8784	140.713	111	15984	256.051
12	1728	27.6812	62	8928	143.020	112	16128	258 358
13	1872	29.9880	63	9072	145.326	113	16272	260.665
14	2016	32.2948	64	9216	147.633	114	16416	262 972
15	2160	34.6015	65	9360	149.940	115	16560	265.278
16	2304	36.9083	66	9504	152.247	116	16704	267.585
17	2448	39.2151	67	9648	154.553	117	16848	269.892
18	2592	41.5218	68	9792	156.860	118	16992	272.199
19	2736	43.8286	69	9936	159.167	119	17136	274.505
20	2880	46.1354	70	10080	161.474	120	17280	276.812
21	3024	48.4421	71	10224	163.781	121	17424	279.119
22	3168	50 7489	72	10368	166.087	122	17568	281.426
23	3312	53.0557	73	10512	168.394	123	17712	283.732
24	3456	55.3624	74	10656	170.701	124	17856	286.039
25	3600	57.6692	75	10800	173.008	125	18000	288.346
26	3744	59.9760	76	10944	175.314	126	18144	290.653
27	3888	62.2827	77	11088	177.621	127	18288	292.960
28	4032	64.5895	78	11232	179.928	128	18432	295.266
29	4176	66.8963	79	11376	182.235	129	18576	297.573
30	4320	69.2030	80	11520	184.541	130	18720	299.880
31	4464	71.5098	81	11664	186 848	131	18864	302.187
32	4608	73.8166	82	11808	189.155	132	19008	304.493
33	4752	76.1233	83	11952	191.462	133	19152	306.800
34	4896	78.4301	84	12096	193.769	134	19296	309.107
35	5040	80.7369	85	12240	196.075	135	19440	311.414
36	5184	83.0436	86	12384	198.382	136	19584	313 720
37	5328	85.3504	87	12528	200.689	137	19728	316.027
38	5472	87.6572	88	12672	202.996	138	19872	318.334
39	5616	89.9640	89	12816	205.302	139	20016	320.641
40	5760	92.2707	90	12960	207.609	140	20160	322.948
41	5904	94.5775	91	13104	209.916	141	20304	325.254
42	6048	96 8843	92	13248	212.223	142	20448	327.561
43	6192	99.1910	93	13392	214.529	143	20592	329.868
44	6336	101.4978	94	13536	216.836	144	20736	332.175
45	6480	103.8046	95	13680	219.143	145	20880	334.481
46	6624	106.1113	96	13824	221.450	146	21024	336.788
47	6768	108.4181	97	13968	223.756	147	21168	339.095
48	6912	110.7249	98	14112	226.063	148	21312	341.402
49	7056	113.0316	99	14256	228.370	149	21456	343.708
50	7200	115.3384	100	14400	230.677	150	21600	346.015

Table of grades per mile; or per 100 feet measured horizontally.

Grade in ft. per mile	Grade in ft. per 100 ft	Grade in ft. per mile.	Grade in ft. per 100 ft	Grade in ft. per mile.	Grade in ft. per 100 ft	Grade in ft. per mile	Grade in ft. per 100 ft.
1	.01894	39	.73864	77	1.45833	115	2.17803
2	.03788	40	.75758	78	1.47727	116	2.19697
3	.05682	41	.77652	79	1.49621	117	2.21591
4	07576	42	.79545	80	1.51515	118	2.23485
5	.09470	43	.81439	81	1.53409	119	2.25379
6	.11364	44	.83333	82	1.55303	120	2.27273
7	.13258	45	.85227	83	1.57197	121	2.29167
8	.15152	46	.87121	84	1.59091	122	2.31061
9	.17045	47	.89015	85	1.60985	123	2.32955
10	.18939	48	.90909	86	1.62879	124	2.34848
11	.20833	49	.92803	87	1.64773	125	2.36742
12	.22727	50	.94697	88	1.66666	126	2.38636
13	.24621	51	.96591	89	1.68561	127	2.40530
14	.26515	52	.98485	90	1.70455	128	2.42424
15	.28409	53	1.00379	91	1.72348	129	2.44318
16	.30303	54	1.02273	92	1.74242	130	2.46212
17	32197	55	1.04167	93	1.76136	131	2.48106
18	.34091	56	1.06061	94	1.78030	132	2.50000
19	.35985	57	1.07955	95	1.79924	133	2.51894
20	.37879	58	1.09848	96	1.81818	134	2.53788
21	.39773	59	1.11742	97	1.83712	135	2.55682
22	.41667	60	1.13636	98	1.85606	136	2.57576
23	.43561	61	1.15530	99	1.87500	137	2.59470
24	.45455	62	1.17424	100	1.89394	138	2.61364
25	.47348	63	1.19318	101	1.91258	139	2.63258
26	.49242	64	1.21212	102	1.93182	140	2.65152
27	.51136	65	1.23106	103	1.95076	141	2.67045
28	.53030	66	1.25000	104	1.96970	142	2.68939
29	.54924	67	1.26894	105	1.98864	143	2.70833
30	.56818	68	1.28788	106	2.00758	144	2.72727
31	.58712	69	1.30682	107	2.02652	145	2.74621
32	.60606	70	1.32576	108	2.04545	146	2.76515
33	.62500	71	1.34470	109	2.06439	147	2.78409
34	64394	72	1.36364	110	2.08333	148	2.80303
35	.66288	73	1.38258	111	2.10227	149	2.82197
36	.68182	74	1.40152	112	2.12121	150	2.84091
37	.70076	75	1.42045	113	2.14015	151	2.85985
38	.71970	76	1.43939	114	2.15909	152	2.87879

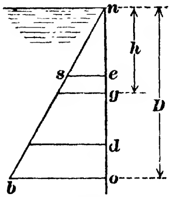
If the grade per mile should consist of feet *and tenths*, add to the grade per 100 feet in the foregoing table, that corresponding to the number of tenths taken from the table below; thus, for a grade of 43.7 feet per mile, we have .81439 + .01326 = .82765 feet per 100 feet.

Ft per Mile.	Per 100 Feet.	Ft. per Mile	Per 100 Feet	Ft per Mile.	Per 100 Feet.
.05	.00094	.4	.00758	.7	.01326
.1	.00189	.45	.00852	.75	.01420
.15	.00283	.5	.00947	.8	.01515
.2	.00379	.55	.01041	.85	.01609
.25	.00473	.6	.01136	.9	.01705
.3	.00568	.65	.01230	.95	.01799
.35	.00662				

TABLE OF PRESSURES (Continued).

Head.	Pressure.		Head.	Pressure.		Head.	Pressure.	
Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq in.	lbs. per sq. ft.
112	48 5528	6991.600	144	62.4250	8989.200	176	76.2972	10986.800
113	48.9863	7054.025	145	62.8585	9051 625	177	76.7307	11049 225
114	49.4198	7116.450	146	63 2920	9114.050	178	77.1642	11111.650
115	49.8533	7178.875	147	63.7255	9176.475	179	77 5978	11174 075
116	50.2868	7241.300	148	64 1590	9238 900	180	78 0313	11236 500
117	50 7203	7303.725	149	64 5925	9301.325	181	78.4648	11298.925
118	51.1538	7366.150	150	65.0260	9363.750	182	78.8983	11361.350
119	51.5873	7428.575	151	65.4596	9426.175	183	79 3318	11423.775
120	52.0208	7491.000	152	65 8931	9488.600	184	79.7653	11486.200
121	52 4543	7553.425	153	66 3266	9551.025	185	80.1988	11548.625
122	52.8879	7615.850	154	66 7601	9613.450	186	80.6323	11611 050
123	53.3214	7678 275	155	67.1936	9675.875	187	81 0658	11673 475
124	53.7549	7740.700	156	67.6271	9738.300	188	81.4993	11735.900
125	54.1884	7803 125	157	68.0606	9800.725	189	81.9328	11798.325
126	54 6219	7865.550	158	68 4941	9863 150	190	82.3663	11860.750
127	55.0554	7927.975	159	68 9276	9925.575	191	82.7998	11923 175
128	55.4889	7990 400	160	69 3611	9988.000	192	83 2333	11985.600
129	55.9224	8052 825	161	69.7946	10050 425	193	83 6669	12048 025
130	56.3555	8115.250	162	70.2281	10112 850	194	84 1004	12110.450
131	56.7894	8177.675	163	70 6616	10175 275	195	84.5339	12172 875
132	57.2229	8240.100	164	71.0951	10237.700	196	84 9674	12235.300
133	57.6564	8302.525	165	71 5287	10300.125	197	85.4009	12297.725
134	58 0899	8364 950	166	71.9622	10362.550	198	85.8344	12360 150
135	58.5234	8427.375	167	72.3957	10424 975	199	86.2679	12422 575
136	58.9570	8489.800	168	72.8292	10487.400	200	86.7014	12485 000
137	59 3905	8552.225	169	73 2627	10549 825	201	87 1349	12547.425
138	59.8240	8614.650	170	73 6962	10612.250	202	87.5684	12609 850
139	60.2575	8677.075	171	74.1297	10674.675	203	88 0019	12672.275
140	60.6910	8739 500	172	74.5632	10737.100	204	88.4354	12734.700
141	61.1245	8801.925	173	74 9967	10799 525	205	88 8689	12797 125
142	61.5580	8864 350	174	75.4302	10861.950	206	89.3024	12859 550
143	61.9915	8926 775	175	75.8637	10924.375	207	89 7359	12921.975

Total pressure, P, against a vert plane, $n\,o$, of unit width, y, perp to the paper, and of depth, $D = n\,o$, beginning at water surf, n. Area of plane = a $D = 2\,h$; h = depth from surf, n, to cen of grav, g, of plane, $n\,o$. Water at its max density, or w = 1 gram per cu cm = 62.425 lbs per cu ft, corresponding to a temp of 4° C = 39.2° F. Let $b\,o$ = $w\,D$ = unit pres at depth, D. Then $P = \frac{w\,D}{2}\,D = w\,h\,a$. Hence, P is represented by area of triangle, $n\,b\,o$.



With D in meters, and y = 1 meter, P, in kilograms, = $500\,D^2$.
With D in feet, and y = 1 foot, P, in pounds, = $31.2125\,D^2$.
With D in inches, and y = 1 foot, P, in pounds, = $0.21675\,D^2$.

Total pressure, P, in lbs, on vertical plane, 1 ft wide, extending from water surf to depth, D.

D. ins.	P. lbs.	D. ins.	P. lbs.	D. ins.	P. lbs.	D. ins.	P. lbs.
1	0.2168	7	10.621	15	48.770	30	195.08
2	0.8670	8	13.872	18	70.229	36	280.91
3	1.9508	9	17.557	21	95.589	42	382.36
4	3.4681	10	21.675	24	124.85	48	499.40
5	5.4189	11	26.227				
6	7.8032	12	31.213				

D. ft.	P. lbs.	D. ft.	P. lbs.	D. ft.	P. lbs.	D. ft.	P. lbs.
1	31.213	26	21,100	51	81,184	76	180,283
2	124.85	27	22,754	52	84,399	77	185,059
3	280.91	28	24,471	53	87,676	78	189,897
4	499.40	29	26,250	54	91,016	79	194,797
5	780.31	30	28,091	55	94,418	80	199,760
6	1,123.7	31	29,995	56	97,882	81	204,785
7	1,529.4	32	31,962	57	101,409	82	209,873
8	1,997.6	33	33,990	58	104,999	83	215,023
9	2,528.2	34	36,082	59	108,651	84	220,235
10	3,121.3	35	38,235	60	112,365	85	225,510
11	3,776.7	36	40,451	61	116,142	86	230,848
12	4,494.6	37	42,730	62	119,981	87	236,247
13	5,274.9	38	45,071	63	123,882	88	241,710
14	6,117.7	39	47,474	64	127,846	89	247,234
15	7,022.8	40	49,940	65	131,873	90	252,821
16	7,990.4	41	52,468	66	135,962	91	258,471
17	9,020.4	42	55,059	67	140,113	92	264,183
18	10,113	43	57,712	68	144,327	93	269,957
19	11,268	44	60,427	69	148,603	94	275,794
20	12,485	45	63,205	70	152,941	95	281,693
21	13,765	46	66,046	71	157,342	96	287,654
22	15,107	47	68,948	72	161,806	97	293,678
23	16,511	48	71,914	73	166,331	98	299,765
24	17,978	49	74,941	74	170,920	99	305,914
25	19,508	50	78,031	75	175,570	100	312,125

For depths not found in the table, we have, for any depth, D':

total pressure, P, = $\dfrac{\text{total pressure for } n\,D'}{n^2}$, where n = any number.

Thus, for a depth, D', of 8.5 feet, we may take $n = 2$. Then:

total pressure, P, = $\dfrac{\text{total pressure for } 2 \times 8.5 \text{ ft}}{4} = \dfrac{9020.4}{4} = 2255.10$ lbs

For a portion, $e\,d$, of the plane, we have, for total pressure on $e\,d$ represented by trapezoid, $d\,s$:
total pressure on $e\,d$ = total pressure on $n\,d$ — total pressure on $n\,e$.

TABLE OF DISCHARGES IN CUBIC FEET PER SECOND CORRESPONDING TO GIVEN DISCHARGES IN U. S. GALLONS PER 24 HOURS.

U. S. gallon = 231 cubic inches.
Discharge in cubic feet per second = 1.54723 × discharge in *millions* of U. S. gallons per 24 hours.

Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals per 24 hrs.	Cubic feet per second
.010	.0154723	13	20.1140	43	66.5308	72	111.400
.020	.0309446	14	21.6612	44	68.0781	73	112.948
.030	.0464169	15	23.2084	45	69.6253	74	114.495
.040	.0618891	16	24.7557	46	71.1725	75	116.042
.050	.0773614	17	26.3029	47	72.7197	76	117.589
.060	.0928337	18	27.8501	48	74.2670	77	119.137
.070	.108306	19	29.3973	49	75.8142	78	120.684
.080	.123778	20	30.9446	50	77.3614	79	122.231
.090	.139251	21	32.4918	51	78.9087	80	123.778
.100	.154723	22	34.0390	52	80.4559	81	125.326
.200	.309446	23	35.5863	53	82.0031	82	126.873
.300	.464169	24	37.1335	54	83.5503	83	128.420
.400	.618891	25	38.6807	55	85.0976	84	129.967
.500	.773614	26	40.2279	56	86.6448	85	131.514
.600	.928337	27	41.7752	57	88.1920	86	133.062
.700	1.08306	28	43.3224	58	89.7393	87	134.609
.800	1.23778	29	44.8696	59	91.2865	88	136.156
.900	1.39251	30	46.4169	60	92.8337	89	137.703
1	1.54723	31	47.9641	61	94.3809	90	139.251
2	3.09446	32	49.5113	62	95.9282	91	140.798
3	4.64169	33	51.0585	63	97.4754	92	142.345
4	6.18891	34	52.6058	64	99.0226	93	143.892
5	7.73614	35	54.1530	65	100.570	94	145.439
6	9.28337	36	55.7002	66	102.117	95	146.987
7	10.8306	37	57.2475	67	103.664	96	148.534
8	12.3778	38	58.7947	68	105.212	97	150.081
9	13.9251	39	60.3419	69	106.759	98	151.628
10	15.4723	40	61.8891	70	108.306	99	153.176
11	17.0195	41	63.4364	71	109.853	100	154.723
12	18.5667	42	64.9836				

TABLE OF DISCHARGES IN CUBIC FEET PER SECOND CORRESPONDING TO GIVEN DISCHARGES IN IMPERIAL GALLONS PER 24 HOURS.

Imperial gallon = 277.274 cubic inches.
Discharge in cubic feet per second = 1.85717 × discharge in Imperial gallons per 24 hours

Millions of Imp. gals. per 24 hrs	Cubic feet per second	Millions of Imp gals per 24 hrs.	Cubic feet per second.	Millions of Imp. gals per 24 hrs	Cubic feet per second.	Millions of Imp gals. per 24 hrs.	Cubic feet per second
.010	.0185717	13	24.1432	43	79.8583	72	133.7162
.020	.0371434	14	26.0004	44	81.7155	73	135.5734
.030	.0557151	15	27.8576	45	83.5727	74	137.4306
.040	.0742868	16	29.7117	46	85.4298	75	139.2878
.050	.0928585	17	31.5719	47	87.2870	76	141.1449
.060	.111430	18	33.4291	48	89.1442	77	143.0021
.070	.130002	19	35.2862	49	91.0013	78	144.8593
.080	.148574	20	37.1434	50	92.8585	79	146.7164
.090	.167145	21	39.0006	51	94.7157	80	148.5736
.100	.185717	22	40.8577	52	96.5728	81	150.4308
.200	.371434	23	42.7149	53	98.4300	82	152.2879
.300	.557151	24	44.5721	54	100.2872	83	154.1451
.400	.742868	25	46.4293	55	102.1444	84	156.0023
.500	.928585	26	48.2864	56	104.0015	85	157.8595
.600	1.11430	27	50.1436	57	105.8587	86	159.7166
.700	1.30002	28	52.0008	58	107.7159	87	161.5738
.800	1.48574	29	53.8579	59	109.5730	88	163.4310
.900	1.67145	30	55.7151	60	111.1302	89	165.2881
1	1.85717	31	57.5723	61	113.2874	90	167.1453
2	3.71434	32	59.4294	62	115.1445	91	169.0025
3	5.57151	33	61.2866	63	117.0017	92	170.8596
4	7.42868	34	63.1438	64	118.8589	93	172.7168
5	9.28585	35	65.0010	65	120.7160	94	174.5740
6	11.1430	36	66.8581	66	122.5732	95	176.4312
7	13.0002	37	68.7153	67	124.4301	96	178.2883
8	14.8574	38	70.5725	68	126.2876	97	180.1455
9	16.7145	39	72.4296	69	128.1447	98	182.0027
10	18.5717	40	74.2868	70	130.0019	99	183.8598
11	20.4289	41	76.1440	71	131.8591	100	185.7170
12	22.2860	42	78.0011				

TABLE OF DISCHARGES IN GALLONS PER 24 HOURS CORRESPONDING TO GIVEN DISCHARGES IN CUBIC FEET PER SECOND.

U. S gallon = 231 cubic inches. Imperial gallon = 277.274 cubic inches
Discharge in U. S. gallons per 24 hours = 646317 × discharge in cubic feet per second.
Discharge in Imperial gallons per 24 hours = 538454 × discharge in cubic feet per second.

Cub. ft. per sec.	Millions of U. S gallons per 24 hours.	Millions of Imperial gallons per 24 hours.	Cub. ft. per sec.	Millions of U. S gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
1	0.646317	0.538454	53	34.254795	28.538044
2	1.292634	1.076907	54	34.901112	29.076498
3	1.938951	1.615361	55	35.547428	29.614951
4	2.585268	2.153815	56	36.193745	30.153405
5	3.231584	2.692268	57	36.840062	30.691859
6	3.877901	3.230722	58	37.486379	31.230312
7	4.524218	3.769176	59	38.132696	31.768766
8	5.170535	4.307629	60	38.779013	32.307220
9	5.816852	4.846083	61	39.425330	32.845673
10	6.463169	5.384537	62	40.071647	33.384127
11	7.109486	5.922990	63	40.717963	33.922581
12	7.755803	6.461444	64	41.364280	34.461034
13	8.402119	6.999898	65	42.010597	34.999488
14	9.048436	7.538351	66	42.656914	35.537942
15	9.694753	8.076805	67	43.303231	36.076395
16	10.341070	8.615259	68	43.949548	36.614849
17	10.987387	9.153712	69	44.595865	37.153303
18	11.633704	9.692166	70	45.242182	37.691756
19	12.280021	10.230620	71	45.888498	38.230210
20	12.926338	10.769073	72	46.534815	38.768664
21	13.572654	11.307527	73	47.181132	39.307117
22	14.218971	11.845981	74	47.827449	39.845571
23	14.865288	12.384434	75	48.473766	40.384025
24	15.511605	12.922888	76	49.120083	40.922478
25	16.157922	13.461342	77	49.766400	41.460932
26	16.804239	13.999795	78	50.412717	41.999385
27	17.450556	14.538249	79	51.059034	42.537839
28	18.096873	15.076702	80	51.705350	43.076293
29	18.743190	15.615156	81	52.351667	43.614746
30	19.389506	16.153610	82	52.997984	44.153200
31	20.035823	16.692063	83	53.644301	44.691654
32	20.682140	17.230517	84	54.290618	45.230107
33	21.328457	17.768971	85	54.936935	45.768561
34	21.974774	18.307424	86	55.583252	46.307015
35	22.621091	18.845878	87	56.229569	46.845468
36	23.267408	19.384332	88	56.875885	47.383922
37	23.913725	19.922785	89	57.522202	47.922376
38	24.560041	20.461239	90	58.168519	48.460829
39	25.206358	20.999693	91	58.814836	48.999283
40	25.852675	21.538146	92	59.461153	49.537737
41	26.498992	22.076600	93	60.107470	50.076190
42	27.145309	22.615054	94	60.753787	50.614644
43	27.791626	23.153507	95	61.400104	51.153098
44	28.437943	23.691961	96	62.046420	51.691551
45	29.084260	24.230415	97	62.692737	52.230005
46	29.730576	24.768868	98	63.339054	52.768459
47	30.376893	25.307322	99	63.985371	53.306912
48	31.023210	25.845776	100	64.631688	53.845366
49	31.669527	26.384229	101	65.278005	54.383820
50	32.315844	26.922683	102	65.924322	54.922273
51	32.962161	27.461137	103	66.570639	55.460727
52	33.608478	27.999590	104	67.216956	55.999181

TABLE OF DISCHARGES (Continued).

Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.	Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
105	67.863272	56.537634	167	107.934919	89 921761
106	68 509589	57.076088	168	108.581236	90.460215
107	69.155906	57.614542	169	109.227553	90.998669
108	69.802223	58.152995	170	109 873870	91.537122
109	70.448540	58 691449	171	110.520186	92 075576
110	71.094857	59.229903	172	111.166503	92 614030
111	71.741174	59.768356	173	111.812820	93 152483
112	72.387491	60.306810	174	112.459137	93 690937
113	73.033807	60.845264	175	113.105454	94.229391
114	73.680124	61.383717	176	113.751771	94 767844
115	74.326441	61.922171	177	114.398088	95.306298
116	74 972758	62.460625	178	115 044405	95.844751
117	75.619075	62.999078	179	115.690722	96 383205
118	76.265392	63.537532	180	116.337038	96.921659
119	76 911709	64.075986	181	116 983355	97 460112
120	77.558026	64.614439	182	117.629672	97 998566
121	78 204342	65.152893	183	118 275989	98 537020
122	78.850659	65.691347	184	118.922306	99.075473
123	79.496976	66.229800	185	119 568623	99 613927
124	80.143293	66 768254	186	120 214940	100.152381
125	80.789610	67 306708	187	120.861257	100.690834
126	81 435927	67.845161	188	121.507573	101 229288
127	82.082244	68.383615	189	122.153890	101.767742
128	82.728561	68.922068	190	122.800207	102 306195
129	83.374878	69 460522	191	123.446524	102.844649
130	84.021194	69.998976	192	124.092841	103.383103
131	84.667511	70.537429	193	124.739158	103 921556
132	85.313828	71 075883	194	125.385475	104 460010
133	85 960145	71.614337	195	126.031792	105.098464
134	86.606462	72 152790	196	126.678108	105.536917
135	87 252779	72 691244	197	127.324425	106 075371
136	87.899096	73.229698	198	127.970742	106.613825
137	88.545413	73.768151	199	128.617059	107 152278
138	89.191729	74.306605	200	129 263376	107.690732
139	89.838046	74 845059	201	129.909693	108.229186
140	90.484363	75.383512	202	130.556010	108.767639
141	91.130680	75 921966	203	131 202327	109 306093
142	91.776997	76.460420	204	131 848644	109.844547
143	92.423314	76.998873	205	132.494960	110 383000
144	93 069631	77.537327	206	133.141277	110.921454
145	93.715948	78.075781	207	133 787594	111.459908
146	94.362264	78 614234	208	134.433911	111.998361
147	95 008581	79.152688	209	135.080228	112.536815
148	95.654898	79.691142	210	135.726545	113 075269
149	96 301215	80.229595	211	136.372862	113.613722
150	96.947532	80.768049	212	137.019179	114.152176
151	97.593849	81.306503	213	137.665495	114 690630
152	98.240166	81.844956	214	138 311812	115.229083
153	98.886483	82.383410	215	138.958129	115.767537
154	99.532800	82.921864	216	139.604446	116.305991
155	100.179116	83.460317	217	140.250763	116.844444
156	100.825433	83 998771	218	140.897080	117.382898
157	101.471750	84.537225	219	141.543397	117.921352
158	102.118067	85.075678	220	142.189714	118.459805
159	102.764384	85.614132	221	142.836030	118.998259
160	103.410701	86.152586	222	143.482347	119.536718
161	104 057018	86.691039	223	144.128664	120.075166
162	104.703335	87.229493	224	144.774981	120 613620
163	105 349651	87.767947	225	145.421298	121.152074
164	105 995968	88.306400	226	146.067615	121.690527
165	106.642285	88.844854	227	146.713932	122.228981
166	107.288602	89.383308	228	147.360249	122.767434

TABLE OF DISCHARGES (Continued).

Cub. ft per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours	Cub. ft. per sec.	Millions of U. S gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
229	148 006566	123.305888	240	155.116051	129 228878
230	148 652882	123.844342	241	155.762368	129.767332
231	149.299199	124.382795	242	156.408685	130.305786
232	149 945516	124.921249	243	157.055002	130.844239
233	150 591833	125.459703	244	157.701319	131.382693
234	151 238150	125.998156	245	158 347636	131.921147
235	151 884467	126 536610	246	158 993952	132 459600
236	152.530784	127 075064	247	159 640269	132.998054
237	153.177101	127.613517	248	160.286586	133.536508
238	153 823417	128 151971	249	160.932903	134 074961
239	154.469734	128 690425	250	161 579220	134.613415

TIME.

60 seconds,*†	marked s,	= 1 minute			
60 minutes,†	" m,	1 hour	= 3600 seconds		
24 hours,	" h,	= 1 day	= 1440 minutes	= 86400 seconds	
7 days,	" d,	= 1 week	= 168 hours	= 10080 minutes	

Arc	Time
1°	= 4 minutes
1′	= 4 seconds
1″	= 0.066 . second

Time	Arc
24 hours	= 360°
1 hour	= 15°
1 minute	= 0° 15′
1 second	= 0° 0′ 15″

Methods of reckoning time. Astronomers distinguish between mean solar time, true or apparent solar time, and sidereal time.

At a standard meridian (see page 267) **mean solar time** is the same as ordinary clock time. At any point not on a standard meridian, *standard* time is the local mean solar time of the meridian adopted as standard for such point; and *local* time is = time at a standard meridian *plus* correction for longitude from that meridian if the place is *east* of the meridian, and *vice versa*. For the amount of such correction, see second table above. **A true or apparent solar day** is the interval of time between two successive culminations of the sun, *i.e.*, between two successive transits or passages of the sun across the meridian of the same point on the earth; but, since these intervals are unequal, they do not correspond with the uniform movement of clock time. A fictitious or imaginary sun, called the "mean sun,' is therefore supposed to move along the equator in such a way that the interval between its culminations is constant This interval is called a day, or mean solar day, and is the average of the lengths of all the *apparent* solar days in a year. Apparent and mean time agree at four points in the year, viz, about the middle of April and of June, September 1 and December 24. The sun is sometimes behind and sometimes in advance of the mean sun, and is called "slow" or "fast" accordingly. The sun is "slow" in winter, the maximum being about February 11, when it passes any standard meridian, or "souths" (making *apparent* noon), about 14m, 28s, *after* noon by a correct clock. The sun is "fast," or in advance of the clock, in May and in the fall, with a maximum, about November 2, of about 16m, 20s.

The difference between apparent and mean time is called the **equation of time.** It can be obtained from the Nautical Almanac, or, approximately, by taking the mean between the times of sunrise and sunset, as given in ordinary almanacs.

As *solar* time is measured by the apparent daily motion of the *sun*, so *sidereal* time is measured by that of the fixed *stars*, or, more strictly speaking, by the motion of the vernal equinox which is the point where the sun crosses the equator in the spring.

* The second was formerly divided into 60 equal parts called thirds (marked ‴); but it is now divided decimally.

† The old and confusing practice of designating minutes, seconds and thirds of time (see footnote *) as ′, ″ and ‴, is no longer in vogue. Days, hours, minutes and seconds are now designated by d, h, m, and s, respectively, thus: 2d, 20h, 48m, 55.43 s., and the symbols ′ and ″ designate minutes and seconds of *arc*.

A sidereal day is the interval of time between two successive passages of the vernal equinox (or, practically, of any star) past the meridian of a given point on the earth It is, practically, the time required for one complete revolution of the earth on its axis, relatively to the stars

The length of the sidereal day is 23 h, 56 m, 4 09 s, of mean solar time or 3 m, 55 91 s of mean solar time less than the mean solar day of 24 hours. In other words, a star will, on any night, appear to set 3 m, 55 91 s earlier by a correct clock than it did on the preceding night Hence, substantially, the number of sidereal days in a year is greater by 1 than the number of solar days.

The sidereal day, like the solar day, is divided into 24 hours These hours are, of course, shorter than those of the solar day in the same proportion as the sidereal day is shorter than the solar day They are counted from 0 to 24, commencing with sidereal noon, or the instant when the vernal equinox passes the upper meridian

The civil day (= 24 hours of clock or mean solar time) commences at midnight; and **the astronomical solar day** at noon on the civil day of the same date. Thus, on a standard meridian, Thursday, May 9, 2 A M. civil time, is Wednesday, May 8, 14 h, astronomical time, but Thursday, May 9, 2 P M, civil time is Thursday, May 9, 2 h. astronomical time.

The civil month is the ordinary and arbitrary month of the calendar, varying in length from 28 to 31 mean solar days

A sidereal month is the time required for the moon to perform an entire revolution with reference to the stars Its mean length, in mean solar time, is about 27 d, 7 h, 43 m, 12 s

A lunation, or synodic month is the time from new moon to new moon. Its mean length is about 29 d, 12 h, 44 m, 3 s

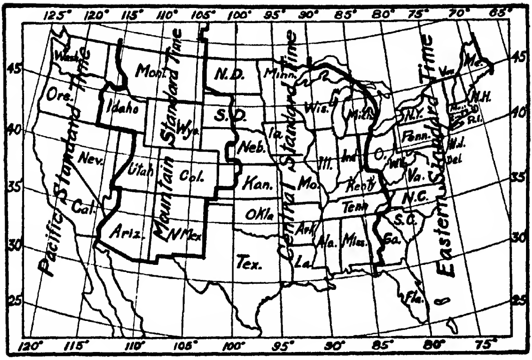
The tropical or natural year is the time during which the earth describes the circuit from either equinox to the same again Its mean length, in mean solar time, is now about 365 d, 5 h, 48 m 49 s

The sidereal year is the time during which the earth describes its orbit with reference to the stars Its mean length, in mean solar time, is about 365 d, 6 h, 9 m, 10 s

The civil year is that arbitrary or conventional and variable division of time comprised between the 1st of January and the 31st of the following December, both inclusive It contains ordinarily 365 mean solar days of 24 hours, but each year whose number is divisible by 4 contains 366 days, and is called a **leap year**, except that those years whose numbers end in 00 and are not multiples of 400 are not leap years.

To regulate a watch by the stars. The author, after having regulated his chronometer for a year by this method only differed but a few seconds from the actual time as deduced from careful solar observations Select a window facing west if possible, and commanding a view of a roof-crest or other fixed horizontal line, preferably about 40° above the horizon, in order to avoid disturbance due to refraction, and distant say 50 feet or more Note the time when any bright fixed star (not a planet) passes the range formed between the roof, etc., and any fixed horizontal line about the window frame as a pin fixed in either jamb The sight in the window, and the watch, must be illuminated The star will pass the range 3 m 55.91 s earlier on each succeeding evening Those stars which are nearest the equator appear to move the fastest, and are therefore best suited to the purpose. If the first observation of a given star be made as late as midnight, that same star will answer for about three months until at last it will begin to pass the range in daylight Before this happens, transfer the time to another star which sets later By thus tabulating, throughout the year, about half a dozen stars which follow each other at nearly equal intervals of time, we may provide a standard by means of which correct clock time may be ascertained on any clear night Experimenting in this way with two of the best **chronometers**, the author found that their *rates* varied, at times, as much as from three to eight seconds per day.

An average man takes **two steps** (one right, one left) **per second.** Hence, march music usually takes one second per measure (or "bar") Modern **watches** usually **tick** five times, and **clocks** either one, two, or four times, per second.



STANDARD TIME ZONES
of the United States

as establisht by the Interstate Commerce Commission in 1918, and as publisht in Circular C406 of the National Bureau of Standards in 1935; reckond from 75th, 90th, 105th and 120th meridians West of Greenwich.

The following table gives the times for each of 12 hours in the several zones of the U S, and in a number of cities of the world as compared with 12 noon, Eastern Standard Time:—Light-face, a.m., including 12 noon; bold-face, p.m. hours, including 12 midnight.

Eastern Standard Time,	12	1	2	3	4	5	6	7	8	9	10	11
Central Standard Time,	11	12	1	2	3	4	5	6	7	8	9	10
Montain Standard Time,	10	11	12	1	2	3	4	5	6	7	8	9
Pacific Standard Time,	9	10	11	12	1	2	3	4	5	6	7	8
Berlin, **Germany,**	6	7	8	9	10	11	12	1	2	3	4	5
Brussels, **Belgium,**	5	6	7	8	9	10	11	12	1	2	3	4
Cape Town, **So Africa,**	7	8	9	10	11	12	1	2	3	4	5	6
Geneva, **Switzerland,**	6	7	8	9	10	11	12	1	2	3	4	5
Halifax, **Nova Scotia,**	1	2	3	4	5	6	7	8	9	10	11	12
Havana, **Cuba,**	12	1	2	3	4	5	6	7	8	9	10	11
Hong Kong, **China,**	1*	2*	3*	4*	5*	6*	7*	8*	9*	10*	11*	12*
London, **England,**	5	6	7	8	9	10	11	12	1	2	3	4
Manila, **P I,**	1*	2*	3*	4*	5*	6*	7*	8*	9*	10*	11*	12*
Paris, **France,**	5	6	7	8	9	10	11	12	1	2	3	4
Rome, **Italy,**	6	7	8	9	10	11	12	1	2	3	4	5
Tokyo, **Japan,**	2*	3*	4*	5*	6*	7*	8*	9*	10*	11*	12*	1*
Vancouver, **Brit Col,**	9	10	11	12	1	2	3	4	5	6	7	8

Selected **RADIO TIME SIGNALS,** (Oct 1934).

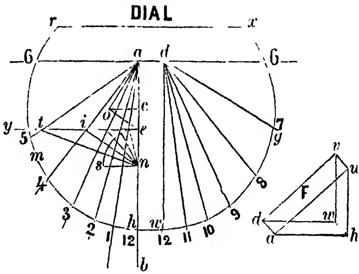
Station	Call Letters	Frequency Kilocycles	Time of Transmission at the Standard Time of the Station
Arlington, Va,	NAA	113	Each hr, except 9 & 11, a m & p m
		9,050	3, **4, 7, 10.**
		16,820	12.
Darien, C Z,	NBA	46	3. **12, 10.**
San Francisco,	NPG	42.8	9, **7, 12.**
		108	9, **7, 12.**
		12,855	9, **7.**

* Next day.

DIALLING.

To make a horizontal Sun-dial,

DRAW a line $a\ b$; and at right angles to it, draw 66. From any convenient point, as c, in $a\ b$, draw the perp $c\ o$. Make the angle $c\ a\ o$ equal to the lat of the place; also the angle $c\ o\ e$ equal to the same; join $o\ e$. Make $e\ n$ equal to $o\ e$; and from n as a center, with the rad $e\ n$, describe a quadrant $e\ s$; and div it into 6 equal parts. Draw $e\ y$, parallel to 6, 6; and from n, through the 6 points on the quadrant, draw lines $n\ t$, $n\ i$, &c, terminating in $e\ y$. From a draw lines a 5, a 4, &c, passing through t, i, &c. From any convenient point, as c, describe an arc $r\ m\ h$, as a kind of finish or border to half the dial. All the lines may now be effaced, except the hour lines a 6, a 5, a 4, &c, to a 12, or $a\ h$; unless, as is generally the case, the dial is to be divided to quarters of an hour at least. In this case each of the divisions on the quadrant $e\ s$, must be subdivided into 4 equal parts; and lines drawn from n, through the points of subdivision, terminating in $e\ y$. The quarter-hour lines must be drawn from a, as were the hour lines. Subdivisions of 5 min may be made in the same way; but these, as well as single min, may usually be laid off around the border, by eye. About 8 or 10 times the size of our Fig will be a convenient one for an ordinary dial. To draw the other half of the Fig, make $a\ d$ equal to the intended thickness of the gnomon, or style, of the dial; and draw d 12, parallel, and equal to a 12; and draw the arc $x\ g\ w$, precisely similar to the arc $r\ m\ h$. Between x and w, on the arc $x\ g\ w$ space off divisions equal to those on the arc $r\ m\ h$; and number them for the hours as in the Fig. The style F, of metal or stone, (wood is too liable to warp,) will be triangular; its thickness must throughout be equal to $a\ d$ or $h\ w$; its base must cover the space $a\ d\ h\ w$; its point will be at $a\ d$; and its perp height $h\ u$, over $h\ w$ must be such that lines $v\ d$, $u\ a$, drawn from its top, down to a and d, will make the angles $u\ a\ h$, $v\ d\ w$, each equal to the lat of the place. Its thickness, if of metal, may conveniently be from $\frac{1}{8}$ to $\frac{1}{4}$ inch; or if of stone, an inch or two, or more, according to the size of the dial. Usually, for neatness of appearance, the back $h\ u\ v\ w$ of the style is hollowed inward. The upper edges, $u\ a$, $v\ d$, which cast the shadows, must be sharp and straight. The dial must be fixed in place hor, or perfectly level; $a\ h$ and $d\ w$ must be placed truly north and south; $a\ d$ being south, and $h\ w$ north. The dial gives only sun or solar time; but clock time can be found by means of the "fast or slow of the sun," as given by all almanacs. If by the almanac the sun is 5 min &c, fast, the dial will be the same; and the clock or watch, to be correct, must be 5 min slower than it; and vice versa.



To make a Vertical Sun-Dial.

Proceed as directed above, except that the angles $c\,a\,o$ and $c\,o\,e$ on the drawing, and the angle $u\,a\,h$ or $v\,d\,w$ of the style, must be equal to the co latitude (= difference between the latitude and 90°) of the place, and the hours must be numbered the opposite way from those in the above figure; *i e.* from h to y number 12, 11, 10, 9, 8, 7; and from w to g number 12, 1, 2, 3, 4, 5. The dial plate must be placed vertically, in the position shown in the figure, facing exactly south, and with $a\,h$ and $d\,w$ vertical.

BOARD MEASURE.

Remark on following table. The table extends to 12 ins by 24 ins, but it is easy to find for greater sizes; thus, for example, the board measure in a piece of 19 by 22, will be twice that of a piece of 19 by 11, or 17 42 × 2 = 34 84 ft board meas., or that of 19¼ by 22, will be that of 10¼ by 22 added to that of 9 by 22, or 18.79 + 16.50 = 35.29. A foot of board meas is equal to 1 foot square and 1 inch thick, or to 144 cub ins. Hence 1 cub ft = 12 ft board meas.

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.) 1000 ft board measure = 83⅓ cub ft. THICKNESS IN INCHES.									Width in Inches.
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	
	Ft. Bd M	Ft. Bd M	Ft Bd M	Ft Bd M	Ft Bd M	Ft Bd M	Ft Bd M	Ft. Bd M.	Ft. Bd M	
¼	.0208	.0260	.0313	.0365	.0417	.0469	.0521	.0573	.0625	¼
½	.0417	.0521	.0625	.0729	.0833	.0938	.1042	.1146	.1250	½
¾	.0625	.0781	.0938	.1094	.1250	.1406	.1563	.1719	.1875	¾
1.	.0833	.1042	.1250	.1458	.1667	.1875	.2083	2292	.2500	1.
¼	.1042	1302	.1563	.1823	.2083	2344	.2604	.2865	.3125	¼
½	.1250	.1563	.1875	.2188	.2500	.2813	.3125	.3438	.3750	½
¾	.1458	.1823	.2187	.2552	.2917	.3281	.3646	.4010	.4375	¾
	1667	.2083	.2500	.2917	.3333	.3750	.4166	.4583	.5000	2.
¼	.1875	2344	.2813	.3281	.3750	.4219	.4688	.5156	.5625	¼
½	.2083	.2604	.3125	.3646	.4167	.4688	.5208	.5729	.6250	½
¾	.2292	.2865	.3438	.4010	.4583	.5156	.5729	.6302	.6875	¾
3.	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	3.
¼	.2708	.3385	.4063	.4739	.5416	.6094	.6771	.7448	.8125	¼
½	.2917	3646	.4375	.5104	.5833	.6563	.7292	.8021	.8750	½
¾	.3125	.3906	.4689	.5469	.6250	.7031	.7813	.8594	.9375	¾
4.	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1 000	4.
¼	.3542	.4427	.5312	.6198	.7083	.7969	.8854	.9740	1.063	¼
½	.3750	.4688	.5625	.6563	.7500	.8438	.9375	1 031	1 125	½
¾	.3958	.4948	.5938	.6927	.7917	.8906	.9896	1.086	1 188	¾
5.	.4167	.5208	.6250	.7292	.8333	.9375	1.042	1 146	1 250	5.
¼	.4375	.5469	.6563	.7656	8750	.9844	1.094	1 208	1 313	¼
½	.4583	.5729	.6875	.8020	9167	1 031	1 146	1 260	1.375	½
¾	.4792	.5990	.7188	.8385	9583	1 078	1 198	1 318	1.438	¾
6.	.5000	.6250	.7500	.8750	1 000	1 125	1.250	1 375	1.500	6.
¼	.5208	.6510	.7813	.9115	1 042	1 172	1.302	1.432	1.563	¼
½	.5417	.6771	.8125	.9479	1 083	1 219	1.354	1.490	1.625	½
¾	.5625	.7031	.8438	.9844	1 125	1.266	1 406	1 547	1.688	¾
7.	.5833	.7292	.8750	1.021	1 167	1.312	1 458	1 604	1.750	7.
¼	.6042	.7552	.9063	1.057	1 208	1.359	1 510	1 661	1.813	¼
½	.6250	.7813	.9375	1 094	1 250	1 406	1 563	1.719	1 875	½
¾	.6458	.8073	.9688	1 130	1 292	1.453	1 615	1.776	1.938	¾
8.	.6667	.8333	1 000	1.167	1 333	1.500	1.667	1 833	2 000	8.
¼	.6875	.8594	1.031	1 203	1 375	1 547	1.719	1 891	2 063	¼
½	.7083	.8854	1 063	1 240	1.417	1 594	1.771	1.948	2 125	½
¾	.7292	.9114	1.094	1.276	1 458	1 641	1 823	2 005	2 188	¾
9.	.7500	.9375	1.125	1 313	1.500	1 688	1.875	2.062	2 250	9.
¼	.7708	.9635	1 156	1 349	1 542	1 734	1 927	2 120	2 313	¼
½	.7917	.9895	1.188	1.385	1.583	1.781	1.979	2.177	2 375	½
¾	.8125	1.016	1.219	1.422	1.625	1 828	2 031	2 234	2 438	¾
10.	8333	1 042	1.250	1 458	1.667	1 875	2 083	2 292	2 500	10.
¼	8542	1 068	1.281	1.495	1 708	1.922	2 135	2.349	2 563	¼
½	8750	1 094	1.313	1 531	1.750	1.969	2 188	2.406	2 625	½
¾	.8958	1.120	1 344	1 568	1.792	2.016	2.240	2.468	2 688	¾
11.	.9167	1 146	1.375	1 604	1.833	2.063	2.292	2.521	2 750	11.
¼	.9375	1 172	1.406	1.641	1.875	2 109	2 344	2 578	2.813	¼
½	.9583	1 198	1 438	1.677	1 917	2.156	2.396	2 635	2 875	½
¾	.9792	1 224	1 469	1.714	1 958	2 203	2.448	2 693	2 938	¾
12.	1 000	1 250	1.500	1 750	2 000	2 250	2 500	2.750	3 000	12.
½	1 042	1 302	1 563	1 823	2 083	2 344	2 604	2 865	3.125	½
13.	1 083	1.354	1 625	1 896	2.167	2.438	2 708	2 979	3.250	13.
½	1 125	1 406	1 688	1 969	2 250	2.531	2.813	3 094	3.375	½
14.	1.167	1 458	1.750	2 042	2 333	2 625	2.917	3 208	3.500	14.
½	1 208	1.510	1.813	2 115	2 417	2 719	3 021	3 322	3.625	½
15.	1.250	1.563	1.875	2.188	2 500	2 813	3 125	3 438	3.750	15.
½	1 292	1.615	1 938	2.260	2 583	2 906	3.229	3 552	3 875	½
16.	1.333	1 667	2.000	2.333	2.667	3 000	3.333	3 667	4 000	16.
½	1.375	1 719	2.063	2 406	2 750	3 094	3.438	3 781	4 125	½
17.	1 417	1 771	2 125	2 479	2.833	3 188	3 542	3.896	4.250	17.
½	1.458	1.823	2 187	2.552	2 917	3.281	3 646	4.010	4 375	½
18.	1 500	1.875	2.250	2.625	3.000	3 375	3 750	4.125	4 500	18.
19.	1.583	1.979	2 375	2.771	3.167	3 563	3 958	4 354	4 750	19.
20.	1 667	2.083	2 500	2 917	3.333	3 750	4.167	4.583	5.000	20.
21.	1.750	2.188	2 625	3.063	3.500	3.938	4.375	4 812	5 250	21.
22.	1.833	2.292	2.750	3 208	3 667	4 125	4.583	5.042	5.500	22.
23.	1.917	2.396	2 875	3.354	3.833	4.313	4.792	5.270	5.750	23.
24.	2 000	2.500	3.000	3.500	4.000	4 500	5.000	5.500	6.000	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.) THICKNESS IN INCHES. 3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	Width in Inches.
	Ft Bd M	Ft Bd M	Ft.Bd.M	Ft.Bd.M.	Ft.Bd M	Ft.Bd M.	Ft Bd M	Ft Bd M	Ft Bd M	
¼	.0677	.0729	.0781	.0833	.0885	.0938	.0990	.1042	.1094	¼
½	.1354	.1457	.1562	.1667	.1770	.1875	.1979	.2083	.2188	½
¾	.2031	.2187	.2344	.2500	.2656	.2813	.2969	.3125	.3281	¾
1.	.2708	.2917	.3125	.3333	.3542	.3750	.3958	.4167	.4375	1
¼	.3385	.3646	.3906	.4167	.4427	.4688	.4948	.5208	.5469	¼
½	.4063	.4375	.4688	.5000	.5313	.5625	.5938	.6250	.6563	½
¾	.4740	.5104	.5469	.5833	.6198	.6563	.6927	.7292	.7656	¾
2	.5417	.5833	.6250	.6667	.7083	.7500	.7917	.8333	.8750	2
¼	.6094	.6563	.7031	.7500	.7969	.8438	.8906	.9375	.9844	¼
½	.6771	.7292	.7813	.8333	.8854	.9375	.9896	1.042	1.094	½
¾	.7448	.8021	.8594	.9167	.9740	1.031	1.089	1.146	1.203	¾
3.	.8125	.8750	.9375	1.000	1.062	1.125	1.188	1.250	1.313	3.
¼	.8802	.9179	1.016	1.083	1.151	1.219	1.286	1.354	1.422	¼
½	.9479	1.021	1.094	1.167	1.240	1.313	1.385	1.458	1.531	½
¾	1.016	1.094	1.172	1.250	1.327	1.406	1.484	1.563	1.641	¾
4.	1.083	1.167	1.250	1.333	1.416	1.500	1.583	1.667	1.750	4
¼	1.151	1.240	1.328	1.417	1.504	1.594	1.682	1.771	1.859	¼
½	1.219	1.313	1.406	1.500	1.593	1.688	1.781	1.875	1.969	½
¾	1.286	1.384	1.484	1.583	1.681	1.781	1.880	1.979	2.078	¾
5.	1.354	1.457	1.566	1.666	1.770	1.875	1.979	2.083	2.188	5
¼	1.422	1.530	1.644	1.750	1.858	1.969	2.078	2.188	2.297	¼
½	1.490	1.603	1.722	1.833	1.947	2.063	2.177	2.292	2.406	½
¾	1.557	1.676	1.800	1.917	2.035	2.156	2.276	2.396	2.516	¾
6	1.625	1.750	1.875	2.000	2.125	2.250	2.375	2.500	2.625	6
¼	1.693	1.823	1.953	2.083	2.214	2.344	2.474	2.604	2.734	¼
½	1.760	1.896	2.031	2.167	2.302	2.438	2.573	2.708	2.843	½
¾	1.828	1.969	2.109	2.250	2.391	2.531	2.672	2.813	2.953	¾
7	1.896	2.042	2.188	2.333	2.479	2.625	2.771	2.917	3.063	7
¼	1.964	2.115	2.266	2.416	2.568	2.719	2.870	3.021	3.172	¼
½	2.031	2.187	2.344	2.500	2.656	2.813	2.969	3.125	3.281	½
¾	2.099	2.260	2.422	2.583	2.745	2.906	3.068	3.229	3.391	¾
8	2.167	2.333	2.500	2.667	2.833	3.000	3.167	3.333	3.500	8
¼	2.234	2.406	2.578	2.750	2.922	3.094	3.266	3.438	3.609	¼
½	2.302	2.479	2.656	2.833	3.010	3.188	3.365	3.542	3.718	½
¾	2.370	2.552	2.734	2.916	3.099	3.281	3.464	3.646	3.828	¾
9.	2.438	2.625	2.813	3.000	3.187	3.375	3.563	3.750	3.938	9
¼	2.505	2.698	2.891	3.083	3.276	3.469	3.661	3.854	4.047	¼
½	2.573	2.771	2.969	3.167	3.365	3.564	3.760	3.958	4.156	½
¾	2.641	2.844	3.047	3.250	3.453	3.656	3.859	4.063	4.266	¾
10	2.708	2.917	3.125	3.333	3.542	3.750	3.958	4.167	4.375	10
¼	2.776	2.990	3.203	3.416	3.630	3.844	4.057	4.271	4.484	¼
½	2.844	3.063	3.281	3.500	3.719	3.938	4.156	4.375	4.594	½
¾	2.911	3.135	3.359	3.583	3.807	4.031	4.255	4.479	4.703	¾
11	2.979	3.208	3.438	3.666	3.896	4.125	4.354	4.583	4.813	11
¼	3.047	3.281	3.516	3.750	3.984	4.219	4.453	4.688	4.922	¼
½	3.115	3.354	3.594	3.833	4.073	4.313	4.552	4.792	5.031	½
¾	3.182	3.427	3.672	3.916	4.161	4.406	4.651	4.896	5.141	¾
12.	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000	5.250	12
½	3.385	3.646	3.906	4.167	4.427	4.688	4.948	5.208	5.469	½
13	3.521	3.792	4.063	4.333	4.604	4.875	5.146	5.417	5.688	13
½	3.656	3.938	4.219	4.500	4.781	5.063	5.344	5.625	5.906	½
14.	3.792	4.083	4.375	4.667	4.958	5.250	5.542	5.833	6.125	14
½	3.927	4.229	4.531	4.833	5.135	5.438	5.740	6.042	6.344	½
15.	4.063	4.375	4.688	5.000	5.313	5.625	5.938	6.250	6.563	15
½	4.198	4.521	4.844	5.166	5.490	5.813	6.135	6.458	6.781	½
16	4.333	4.667	5.000	5.333	5.667	6.000	6.333	6.667	7.000	16
½	4.469	4.813	5.156	5.500	5.844	6.188	6.531	6.875	7.219	½
17	4.604	4.958	5.313	5.667	6.021	6.375	6.729	7.083	7.438	17
½	4.740	5.104	5.469	5.833	6.198	6.563	6.927	7.292	7.656	½
18.	4.875	5.250	5.625	6.000	6.375	6.750	7.125	7.500	7.875	18
19	5.146	5.542	5.938	6.333	6.729	7.125	7.521	7.917	8.313	19
20.	5.417	5.833	6.250	6.667	7.083	7.500	7.917	8.333	8.750	20
21.	5.688	6.125	6.563	7.000	7.438	7.875	8.313	8.750	9.188	21.
22.	5.958	6.417	6.875	7.333	7.792	8.250	8.708	9.167	9.625	22
23.	6.229	6.708	7.188	7.667	8.145	8.625	9.104	9.583	10.06	23
24.	6.500	7.000	7.500	8.000	8.500	9.000	9.500	10.00	10.50	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)									Width in Inches.
	THICKNESS IN INCHES.									
	5½	5¾	6	6¼	6½	6¾	7	7¼	7½	
	Ft Bd M	Ft Bd M	Ft Bd.M	Ft Bd M	Ft.Bd M	Ft.Bd M	Ft.Bd M	Ft Bd M	Ft Bd M	
¼	.1146	.1198	.1250	.1302	.1354	.1406	.1458	.1510	.1563	¼
½	.2292	.2396	.2500	.2604	.2708	.2813	.2917	.3021	.3125	½
¾	.3438	.3594	.3750	.3906	.4063	.4219	.4375	.4531	.4688	¾
1	.4583	.4792	.5000	.5208	.5417	.5625	.5833	.6042	.6250	1.
¼	.5729	.5990	.6250	.6510	.6771	.7031	.7292	.7552	.7813	¼
½	.6875	.7188	.7500	.7812	.8125	.8438	.8750	.9062	.9375	½
¾	.8021	.8385	.8750	.9115	.9479	.9844	1.020	1.057	1.094	¾
2	.9167	.9583	1.000	1.042	1.083	1.125	1.167	1.208	1.250	2.
¼	1.031	1.078	1.125	1.172	1.219	1.266	1.313	1.359	1.406	¼
½	1.146	1.198	1.250	1.302	1.354	1.406	1.458	1.510	1.563	½
¾	1.260	1.318	1.375	1.432	1.490	1.547	1.604	1.661	1.719	¾
3	1.375	1.438	1.500	1.562	1.625	1.688	1.750	1.813	1.875	3.
¼	1.490	1.557	1.625	1.693	1.760	1.828	1.896	1.964	2.031	¼
½	1.604	1.677	1.750	1.823	1.896	1.969	2.042	2.115	2.188	½
¾	1.719	1.797	1.875	1.953	2.031	2.109	2.188	2.266	2.344	¾
4.	1.833	1.917	2.000	2.083	2.167	2.250	2.333	2.417	2.500	4.
¼	1.948	2.036	2.125	2.214	2.302	2.391	2.479	2.568	2.656	¼
½	2.063	2.156	2.250	2.344	2.438	2.531	2.625	2.719	2.813	½
¾	2.177	2.276	2.375	2.474	2.573	2.672	2.771	2.870	2.969	¾
5.	2.292	2.396	2.500	2.604	2.708	2.813	2.917	3.021	3.125	5.
¼	2.406	2.516	2.625	2.734	2.844	2.953	3.063	3.172	3.281	¼
½	2.521	2.635	2.750	2.865	2.979	3.094	3.208	3.323	3.438	½
¾	2.635	2.755	2.875	2.995	3.115	3.234	3.354	3.474	3.594	¾
6	2.750	2.875	3.000	3.125	3.250	3.375	3.500	3.625	3.750	6.
¼	2.865	2.995	3.125	3.255	3.385	3.516	3.646	3.776	3.906	¼
½	2.979	3.115	3.250	3.385	3.521	3.656	3.792	3.927	4.063	½
¾	3.094	3.234	3.375	3.516	3.656	3.797	3.938	4.078	4.219	¾
7.	3.208	3.354	3.500	3.646	3.792	3.938	4.083	4.229	4.375	7.
¼	3.323	3.474	3.625	3.776	3.927	4.078	4.229	4.380	4.531	¼
½	3.438	3.594	3.750	3.906	4.063	4.219	4.375	4.531	4.688	½
¾	3.552	3.714	3.875	4.036	4.198	4.359	4.521	4.682	4.844	¾
8	3.667	3.833	4.000	4.167	4.333	4.500	4.667	4.833	5.000	8.
¼	3.781	3.953	4.125	4.297	4.469	4.641	4.813	4.984	5.156	¼
½	3.896	4.073	4.250	4.427	4.604	4.781	4.957	5.135	5.313	½
¾	4.010	4.193	4.375	4.557	4.740	4.922	5.103	5.286	5.469	¾
9	4.125	4.313	4.500	4.687	4.875	5.063	5.249	5.438	5.625	9.
¼	4.240	4.432	4.625	4.818	5.010	5.203	5.395	5.589	5.781	¼
½	4.354	4.552	4.750	4.948	5.146	5.344	5.541	5.740	5.938	½
¾	4.469	4.672	4.875	5.078	5.281	5.484	5.687	5.891	6.094	¾
10.	4.583	4.792	5.000	5.208	5.417	5.625	5.833	6.042	6.250	10.
¼	4.698	4.911	5.125	5.339	5.552	5.766	5.979	6.193	6.406	¼
½	4.813	5.031	5.250	5.469	5.688	5.906	6.125	6.344	6.563	½
¾	4.927	5.151	5.375	5.599	5.823	6.047	6.271	6.495	6.719	¾
11.	5.042	5.271	5.500	5.729	5.958	6.188	6.417	6.646	6.876	11.
¼	5.156	5.391	5.625	5.859	6.094	6.328	6.563	6.797	7.031	¼
½	5.271	5.510	5.750	5.990	6.229	6.469	6.708	6.948	7.188	½
¾	5.385	5.630	5.875	6.120	6.365	6.609	6.854	7.099	7.344	¾
12.	5.500	5.750	6.000	6.250	6.500	6.750	7.000	7.250	7.500	12.
½	5.729	5.990	6.250	6.510	6.771	7.031	7.292	7.552	7.813	½
13.	5.958	6.229	6.500	6.771	7.042	7.313	7.583	7.854	8.125	13.
½	6.188	6.469	6.750	7.031	7.313	7.594	7.875	8.156	8.438	½
14.	6.417	6.708	7.000	7.292	7.583	7.875	8.167	8.458	8.750	14.
½	6.646	6.948	7.250	7.552	7.854	8.156	8.458	8.760	9.063	½
15.	6.875	7.188	7.500	7.812	8.125	8.438	8.750	9.063	9.375	15.
½	7.104	7.427	7.750	8.073	8.396	8.719	9.042	9.365	9.688	½
16.	7.333	7.667	8.000	8.333	8.667	9.000	9.333	9.667	10.00	16.
½	7.563	7.906	8.250	8.594	8.938	9.281	9.625	9.969	10.31	½
17	7.792	8.146	8.500	8.854	9.208	9.563	9.917	10.27	10.63	17.
½	8.021	8.385	8.750	9.115	9.479	9.844	10.21	10.57	10.94	½
18	8.250	8.625	9.000	9.375	9.750	10.13	10.50	10.88	11.25	18
19	8.708	9.104	9.500	9.896	10.29	10.69	11.08	11.48	11.88	19.
20.	9.167	9.583	10.00	10.42	10.83	11.25	11.67	12.08	12.50	20.
21	9.625	10.06	10.50	10.94	11.38	11.81	12.25	12.69	13.13	21.
22.	10.08	10.54	11.00	11.46	11.92	12.38	12.83	13.29	13.75	22.
23.	10.54	11.02	11.50	11.98	12.46	12.94	13.42	13.90	14.38	23.
24.	11.00	11.50	12.00	12.50	13.00	13.50	14.00	14.50	15.00	24.

Table of Board Measure — (Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions (Original.)

Width in Inches.	THICKNESS IN INCHES. 7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	Width in Inches.
	Ft Bd.M	Ft Bd M	Ft.Bd M	Ft Bd M	Ft Bd M.	Ft Bd M	Ft Bd M	Ft Bd M	Ft Bd M	
¼	.1615	.1667	.1719	.1771	.1823	.1875	.1927	.1979	.2031	¼
½	.3229	.3333	.3438	.3542	.3646	.3750	.3854	.3958	.4063	½
¾	.4844	.5000	.5156	.5313	.5467	.5625	.5781	.5938	.6094	¾
1.	.6458	.6667	.6875	.7083	.7292	.7500	.7708	.7917	.8125	1.
¼	.8073	.8333	.8594	.8854	.9115	.9375	.9635	.9896	1.016	¼
½	.9688	1.000	1.031	1.063	1.094	1.125	1.156	1.188	1.219	½
¾	1.130	1.167	1.203	1.240	1.276	1.313	1.349	1.385	1.122	¾
2.	1.292	1.333	1.375	1.417	1.458	1.500	1.542	1.583	1.625	2.
¼	1.453	1.500	1.547	1.594	1.641	1.688	1.734	1.781	1.828	¼
½	1.615	1.667	1.719	1.771	1.822	1.875	1.927	1.979	2.031	½
¾	1.776	1.833	1.891	1.948	2.005	2.063	2.120	2.177	2.234	¾
3.	1.938	2.000	2.063	2.125	2.188	2.250	2.313	2.375	2.438	3.
¼	2.099	2.167	2.234	2.302	2.370	2.438	2.505	2.573	2.641	¼
½	2.230	2.333	2.406	2.479	2.552	2.625	2.698	2.771	2.844	½
¾	2.422	2.500	2.578	2.656	2.734	2.813	2.891	2.969	3.047	¾
4.	2.583	2.667	2.750	2.833	2.917	3.000	3.083	3.167	3.250	4
¼	2.745	2.833	2.922	3.010	3.099	3.188	3.276	3.365	3.453	¼
½	2.906	3.000	3.094	3.188	3.281	3.375	3.469	3.563	3.656	½
¾	3.068	3.167	3.266	3.365	3.464	3.563	3.661	3.760	3.859	¾
5.	3.229	3.333	3.438	3.542	3.646	3.750	3.854	3.958	4.063	5.
¼	3.391	3.500	3.609	3.719	3.828	3.938	4.047	4.156	4.266	¼
½	3.552	3.667	3.781	3.896	4.010	4.125	4.240	4.354	4.469	½
¾	3.714	3.833	3.953	4.073	4.193	4.313	4.432	4.552	4.672	¾
6.	3.875	4.000	4.125	4.250	4.375	4.500	4.625	4.750	4.875	6
¼	4.036	4.167	4.297	4.427	4.557	4.688	4.818	4.948	5.078	¼
½	4.198	4.333	4.469	4.604	4.740	4.875	5.010	5.146	5.281	½
¾	4.359	4.500	4.641	4.781	4.922	5.063	5.203	5.344	5.484	¾
7.	4.521	4.667	4.813	4.958	5.104	5.250	5.360	5.542	5.688	7
¼	4.682	4.833	4.984	5.135	5.286	5.438	5.590	5.740	5.891	¼
½	4.844	5.000	5.156	5.313	5.469	5.625	5.782	5.938	6.094	½
¾	5.005	5.167	5.328	5.490	5.651	5.813	5.975	6.135	6.297	¾
8.	5.167	5.333	5.500	5.667	5.833	6.000	6.167	6.333	6.500	8
¼	5.328	5.500	5.672	5.844	6.016	6.188	6.359	6.531	6.703	¼
½	5.490	5.667	5.844	6.021	6.198	6.375	6.552	6.729	6.906	½
¾	5.651	5.833	6.016	6.198	6.380	6.563	6.745	6.927	7.109	¾
9.	5.813	6.000	6.188	6.375	6.563	6.750	6.938	7.125	7.313	9.
¼	5.974	6.167	6.359	6.552	6.745	6.938	7.130	7.323	7.516	¼
½	6.135	6.333	6.531	6.729	6.927	7.125	7.323	7.521	7.719	½
¾	6.297	6.500	6.703	6.906	7.109	7.313	7.516	7.719	7.922	¾
10.	6.458	6.667	6.875	7.083	7.292	7.500	7.708	7.917	8.125	10.
¼	6.620	6.833	7.047	7.260	7.474	7.688	7.901	8.115	8.328	¼
½	6.781	7.000	7.219	7.438	7.656	7.875	8.094	8.313	8.531	½
¾	6.943	7.167	7.391	7.615	7.839	8.063	8.286	8.510	8.734	¾
11.	7.104	7.333	7.563	7.792	8.021	8.250	8.479	8.708	8.938	11.
¼	7.266	7.500	7.735	7.969	8.203	8.438	8.672	8.906	9.141	¼
½	7.427	7.667	7.906	8.146	8.386	8.625	8.865	9.104	9.344	½
¾	7.589	7.833	8.078	8.323	8.568	8.813	9.057	9.302	9.547	¾
12.	7.750	8.000	8.250	8.500	8.750	9.000	9.250	9.500	9.750	12.
½	8.073	8.333	8.594	8.854	9.115	9.375	9.635	9.896	10.16	½
13.	8.396	8.666	8.938	9.208	9.479	9.750	10.02	10.29	10.56	13
½	8.719	9.000	9.281	9.563	9.844	10.13	10.41	10.69	10.97	½
14.	9.042	9.333	9.625	9.917	10.21	10.50	10.79	11.08	11.38	14
½	9.365	9.666	9.969	10.27	10.57	10.88	11.18	11.48	11.78	½
15.	9.688	10.000	10.31	10.63	10.94	11.25	11.56	11.88	12.19	15
½	10.01	10.33	10.66	10.98	11.30	11.63	11.95	12.27	12.59	½
16.	10.33	10.67	11.00	11.33	11.67	12.00	12.33	12.67	13.00	16.
½	10.66	11.00	11.34	11.69	12.03	12.38	12.72	13.06	13.41	½
17.	10.98	11.33	11.69	12.04	12.40	12.75	13.10	13.46	13.81	17.
½	11.30	11.66	12.03	12.40	12.76	13.13	13.49	13.85	14.22	½
18.	11.63	12.00	12.38	12.75	13.13	13.50	13.88	14.25	14.63	18.
19.	12.27	12.67	13.06	13.46	13.85	14.25	14.65	15.04	15.44	19.
20.	12.92	13.33	13.75	14.17	14.58	15.00	15.42	15.83	16.25	20.
21.	13.56	14.00	14.44	14.88	15.31	15.75	16.19	16.63	17.06	21.
22.	14.21	14.66	15.13	15.58	16.04	16.50	16.96	17.42	17.88	22.
23	14.85	15.33	15.81	16.29	16.77	17.25	17.73	18.21	18.69	23.
24	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	24.

Table of Board Measure—(Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	10	10¼	10½	10¾	11	11¼	11½	11¾	12	Width in Inches.
	THICKNESS IN INCHES.									
	Ft Bd M	Ft. Bd M	Ft Bd.M.	Ft Bd M	Ft. Bd M	Ft. Bd.M	Ft. Bd M	Ft Bd M	Ft. Bd M	
¼	.2083	.2135	.2188	.2240	.2292	.2344	.2396	.2448	.2500	¼
½	.4167	.4271	.4375	.4479	.4583	.4688	.4792	.4896	.5000	½
¾	.6250	.6406	.6563	.6719	.6875	.7031	.7188	.7344	.7500	¾
1.	.8333	.8542	.8750	.8958	.9167	.9375	.9583	.9792	1 000	1.
¼	1 042	1 068	1 094	1 120	1.146	1 172	1 198	1.224	1 250	¼
½	1 250	1 321	1.313	1.344	1.375	1 406	1.438	1.469	1 500	½
¾	1 458	1 495	1.531	1 568	1.604	1 641	1.677	1.714	1.750	¾
2.	1.667	1 708	1.750	1.792	1.833	1 875	1.917	1.958	2 000	2.
¼	1 875	1.922	1.969	2 016	2.063	2 109	2.156	2.203	2 250	¼
½	2.083	2,135	2.188	2.240	2.292	2 344	2.396	2.448	2.500	½
¾	2 292	2 349	2 406	2.464	2.521	2.578	2.635	2 693	2 750	¾
3.	2 500	2 563	2.625	2 688	2 750	2.813	2 875	2.938	3.000	3.
¼	2.708	2.776	2 844	2 911	2 979	3 047	3 115	3.182	3.250	¼
½	2 917	2 990	3 063	3 135	3 208	3.281	3 354	3.427	3 500	½
¾	3 125	3.203	3 281	3 359	3 438	3.516	3 594	3.672	3.750	¾
4.	3.333	3 417	3 500	3 583	3 667	3 750	3 833	3.917	4 000	4.
¼	3 542	3 630	3.719	3.807	3.896	3.984	4.073	4.161	4 250	¼
½	3 750	3 844	3 938	4.031	4 125	4 219	4 313	4 406	4.500	½
¾	3 958	4 057	4 156	4 255	4 354	4.453	4 552	4 651	4.750	¾
5.	4 167	4.271	4 375	4 479	4 583	4.688	4.791	4 896	5 000	5.
¼	4 375	4 484	4 594	4 703	4.813	4 922	5.031	5 141	5 250	¼
½	4 583	4 698	4 813	4.927	5 042	5 156	5.270	5.385	5 500	½
¾	4.792	4 911	5.031	5.151	5 271	5 391	5.510	5 630	5 750	¾
6.	5 000	5.125	5 250	5.375	5.500	5.625	5 750	5.875	6 000	6.
¼	5 208	5.339	5 469	5 599	5 729	5.859	5 990	6 120	6 250	¼
½	5 417	5.552	5 688	5 823	5.958	6.094	6 229	6 365	6 500	½
¾	5.625	5 766	5.906	6.047	6.188	6 328	6.469	6.609	6 750	¾
7.	5.833	5.979	6 125	6.271	6.417	6.563	6.708	6 854	7.000	7.
¼	6 042	6.193	6.344	6.495	6 646	6.797	6.948	7 099	7 250	¼
½	6.250	6.406	6 563	6 719	6.875	7.031	7.188	7.344	7 500	½
¾	6.458	6 620	6 781	6.943	7.104	7.266	7 427	7.589	7.750	¾
8.	6 667	6.833	7.000	7.167	7 333	7 500	7.667	7.833	8 000	8.
¼	6.875	7.047	7 219	7.391	7.563	7 734	7.906	8.078	8 250	¼
½	7 083	7.260	7.438	7 615	7.792	7 969	8 146	8 323	8.500	½
¾	7 292	7.474	7.656	7 839	8.021	8 203	8 385	8 568	8.750	¾
9.	7.500	7.688	7 875	8.063	8.250	8 438	8.625	8 813	9.000	9.
¼	7 708	7 901	8 094	8 286	8 479	8.672	8.865	9 057	9.250	¼
½	7.917	8.115	8 313	8.510	8 709	8 906	9 104	9.302	9.500	½
¾	8.125	8 323	8 531	8.734	8 939	9 141	9.344	9 547	9.750	¾
10.	8.333	8 542	8 750	8 958	9 167	9 375	9.583	9 792	10 00	10.
¼	8 542	8 755	8.969	9.182	9.396	9.609	9 823	10 04	10 25	¼
½	8 750	8 969	9 188	9.406	9 625	9 844	10.06	10.28	10 50	½
¾	8 958	9 182	9.406	9,630	9.854	10.08	10 30	10 53	10.75	¾
11.	9.167	9.396	9 625	9.854	10.08	10 31	10 54	10.77	11.00	11.
¼	9 375	9.609	9 844	10 08	10 31	10 55	10.78	11 02	11 25	¼
½	9 583	9 823	10 06	10 30	10.54	10.78	11.02	11.26	11 50	½
¾	9 792	10.04	10 28	10.53	10 77	11.02	11.26	11 51	11 75	¾
12.	10 00	10.25	10.50	10.75	11 00	11.25	11.50	11 75	12 00	12.
½	10 42	10 68	10 94	11.20	11.46	11.72	11.98	12 24	12 50	½
13.	10.83	11.10	11 38	11.65	11.92	12.19	12 46	12.73	13 00	13.
½	11.25	11.53	11.81	12 09	12 38	12.66	12.94	13 22	13 50	½
14.	11.67	11.96	12 25	12.54	12.83	13 13	13.42	13 71	14 00	14.
½	12 08	12 39	12 69	12 99	13.29	13 59	13.90	14 20	14.50	½
15.	12 50	12 81	13 13	13 44	13.75	14 06	14 38	14.69	15.00	15.
½	12 92	13 24	13,56	13 89	14 21	14 53	14 85	15.18	15 50	½
16	13 33	13 67	14 00	14 33	14 67	15 00	15 33	15 67	16 00	16.
½	13 75	14 09	14 44	14 78	15 13	15 47	15 81	16 16	16 50	½
17.	14 17	14 52	14.88	15 23	15.58	15.94	16 29	16.65	17.00	17.
½	14 58	14 95	15 31	15 77	16 04	16 41	16 77	17 14	17.50	½
18.	15 00	15 38	15 75	16.13	16 50	16 88	17 25	17 63	18 00	18.
19.	15 83	16.23	16 63	17 02	17 42	17 81	18 21	18 60	19 00	19.
20.	16 67	17.08	17.50	17 92	18 33	18 75	19.17	19 58	20 00	20.
21.	17 50	17 94	18 38	18 81	19 25	19 69	20 13	20 56	21 00	21.
22.	18 33	18 79	19 25	19 71	20.17	20 63	21 08	21 54	22.00	22.
23.	19.17	19 65	20.13	20 60	21 08	21.56	22 08	22 52	23 00	23.
24.	20.00	20.50	21.00	21 50	22.00	22 50	23.00	23.50	24.00	24.

LAND SURVEYING.

In surveying a tract of ground, the sides which compose its outline are designated by numbers in the order in which they occur. That end of each side which first presents itself in the course of the survey, may be called its *near* end; and the other its *far* end. The number of each side is placed at its far end. Thus, in Fig. 1, the survey being supposed to commence at the corner 6, and to follow the direction of the arrows, the first side is 6, 1, and its number is placed at its far end at 1; and so of the rest. Let N S be a meridian line, that is, a north and south line; and E W an east and west line. Then in any side which runs northwardly,

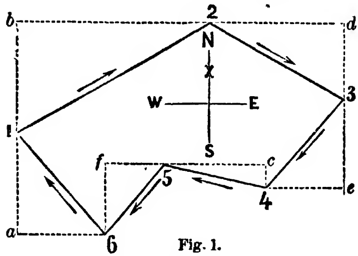


Fig. 1.

whether northeast, as side 2; or northwest, as sides 5 and 1; or due north; the distance in a due north direction between its near end and its far end, is called its *northing;* thus, *a* 1 is the northing of side 1; 1 *b* the northing of side 2; 4 *c* of side 5. In like manner, if any side runs in a southwardly direction, whether southeastwardly, as side 3; or southwestwardly, as sides 4 and 6; or due south; the corresponding distance in a due south direction between its near end and its far end, is called its *southing;* thus, *d* 3 is the southing of side 3: 3 *e* of side 4; *f* 6 of side 6. Both northings and southings are included in the general term *Difference of Latitude* of a side; or, more commonly but erroneously, its *latitude.* The distance due east, or due west, between the near and the far end of any side, is in like manner called the *easting,* or *westing,* of that side, as the case may be; thus, 6 *a* is the westing of side 1; 5 *f* of side 6; *c* 5 of side 5; *e* 4 of side 4; and *b* 2 is the easting of side 2; 2 *d* of side 3. Both eastings and westings are included in the general term *Departure* of a side; implying that the side *departs* so far from a north or south direction. We may say that a side norths, wests, southeasts, &c. We shall call the northings, southings, &c. the Ns, Ss, Es, and Ws; the latitudes, lats; and the departures, deps.

Perfect accuracy is unattainable in any operation involving the measurements of angles and distances.* That work is accurate enough, which cannot be made more so without an expenditure more than commensurate with the object to be gained. There is no great difficulty in confining the uncertainty within about one-half per cent. of the content, and this probably never prevents a transfer in farm transactions. But errors always become apparent when we come to *work out* the field notes; and since the map or plot of the survey, and the calculations for ascertaining the content, should be consistent within themselves, we do what is usually called *correcting* the errors, but what in fact is simply *humoring them in,* no matter how scientific the process may appear. We distribute them all around the survey. Two methods are used for this purpose, both based upon precisely the same principle; one by means of drawing; the other, more exact but much more troublesome, by calculation. The graphic method, in the hands of a correct draftsman, is sufficiently exact for all ordinary purposes. Add all the sides in feet together; and divide the sum by their number, for the average length. Divide this average by 8; the quotient will be the proper scale in feet per inch. In other words, take about 8 ins. to represent an average side. We shall take it for granted that an engineer does not consider it accurate work to

* A 100 ft. chain may vary its length 5 feet per mile, between winter and summer, by mere change of temperature; and this alone will make a difference of about 1 acre in 533. The student should practise plotting from perfectly accurate data; as from the example in table. p. 281, or

measure his angles to the nearest quarter of a degree, which is the usual practice among land-surveyors. They can, by means of the engineer's transit, now in universal use on our public works, be readily measured within a minute or two, and being thus much more accurate than the compass courses, (which cannot be read off so closely, and which are moreover subject to many sources of error,) they serve to correct the latter in the office. The noting of the courses, however, should not be confined to the nearest quarters of a degree, but should be read as closely as the observer can guess at the minutes. The back courses also should be taken at every corner, as an additional check, and for the detection of local attraction It is well in taking the compass bearings, to adopt as a rule, always to point the *north* of the compass-box toward the object whose bearing is to be taken, and to read off from the *north* end of the needle. A person who uses indifferently the N and the S of the box, and of the needle, will be very liable to make mistakes It is best to measure the *least* angle (shown by dotted arcs, Fig 2,) at the corners; whether it be exterior, as that at corner 5; or interior, as all the others; because it is always less than 180°, so that there is less danger of reading it off incorrectly, than if it exceeded 180°, taking it for granted that the transit instrument is graduated from the same zero to 180° each way; if it is graduated from zero to 360° the precaution is useless. When the small angle is exterior, subtract it from 360° for the interior one

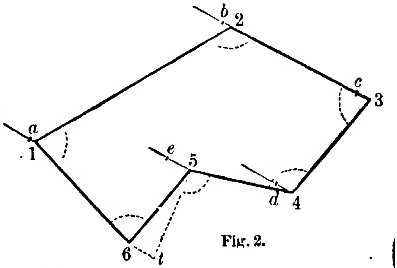


Fig. 2.

Supposing the field work to be finished, and that we require a plot from which the contents may be obtained mechanically, by dividing it into triangles, (the bases and heights of which may be measured by scale, and their areas calculated one by one,) a protraction of it may be made at once from the field notes, either by using the *angles*, or by first correcting the *bearings* by means of the angles, and then using them The last is the best, because in the first the protractor must be moved to each angle, whereas in the last it will remain stationary while all the bearings are being pricked off. Every movement of it increases the liability to errors. The manner of correcting the bearings is explained on the next page.

In either case the protracted plot will certainly not close precisely; not only in consequence of errors in the field work, but also in the protracting itself Thus the last side, No 6, Fig 2, instead of closing in at corner 6, will end somewhere else, say, for instance, at *t*; the dist *t* 6 being the *closing error*, which, however, as represented in Fig 2, is more than ten times as great, proportionally to the size of the survey, as would be allowable in practice. Now to humor-in this error, rule through every corner a short line parallel to *t* 6, and, in all cases, in the direction *from t* (wherever it may be) *to* the starting point 6 Add all the sides together, and measure *t* 6 by the scale of the plot. Then beginning at corner 1, at the far end of side 1, say, as the

Sum of all the sides	:	Total closing error *t* 6	::	Side 1	:	Error for side 1.

Lay off this error from 1 to *a*. Then at corner 2, say, as the

Sum of all the sides	:	Total closing error *t* 6	::	Sum of sides 1 and 2	:	Error for side 2.

Which error lay off from 2 to *b*; and so at each of the corners; always using, as the third term, the sum of the sides between the starting point and the given corner. Finally, join the points *a*, *b*, *c*, *d*, *e*, 6; and the plot is finished.

The correction has evidently changed the length of every side; lengthening some and shortening others It has also changed the angles. The new lengths and angles may with tolerable accuracy be found by means of the scale and protractor; and be marked on the plot instead of the old ones.

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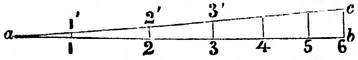
from those to be found in books on surveying. This is the only way in which he can learn what is meant by accurate work His semicircular protractor should be about 9 to 12 ins in diam and graduated to 10 min. His straight edge and triangle should be of metal; we prefer German silver, which does not rust as steel does; and they should be made with *scrupulous accuracy* by a skilful instrument maker. A very fine needle, with a sealing-wax head, should be used for pricking off dists and angles; it must be held vertically; and the eye of the draftsman must be directly over it. The lead pencil should be hard (Faber's No. 4 is good for protracting), and must be kept to a sharp point by rubbing on a fine file, after using a knife for removing the wood. The scale should be at least as long as the longest side of the plot, and should be made at the edge of a strip of the same paper as the plot is drawn on. This will obviate to a considerable extent, errors arising from contraction and expansion. Unfortunately, a sheet of paper does not contract and expand in the same proportion lengthwise and crosswise, thus preventing the paper scale from being a perfect corrective. In plots of common farm surveys, &c, however, the errors from this source may be neglected. For such plots as may be protracted, divided, and computed within a time too short to admit of appreciable change, the ordinary scales of wood, ivory or metal may be used; but satisfactory accuracy cannot be obtained with them on plots requiring several days, if the air be meanwhile alternately moist and dry, or subject to considerable variations in temperature. What is called *parchment* paper is worse in this respect than good ordinary drawing paper.

With the foregoing precautions we may work from a drawing, with as much accuracy as is usually attained in the field work.

When the plot has many sides, this calculating the error for each of them becomes tedious; and since, in a well-performed survey and protraction, the entire error will be but a very small quantity it should not *exceed* about $\frac{1}{400}$ part of the periphery,) it may usually be divided among the sides by merely placing about ¼, ½, and ¾ of it at corners about ¼, ½, and ¾ way around the plot, and at intermediate corners propor tion it by eye. Or calculation may be avoided entirely by drawing a line *a b* of a length equal to the united lengths of all the sides; dividing it into distances *a*, 1, 1, 2 &c equal to the respective sides. Make *b c* equal to the entire closing error join *a c*, and draw 1 1', 2, 2', &c, which will give the error at each corner.



When the plot is thus completed, it may be divided by fine pencil lines into triangles, whose bases and heights may be measured by the scale, in order to compute the contents. With care in both the survey and the drawing, the error should not *exceed* about $\frac{1}{200}$ part of the true area. At least two distinct sets of triangles should be drawn and computed as a guard against mistakes; and if the two sets differ in calculated contents more than about $\frac{1}{200}$ part, they have not been as carefully prepared as they should have been. The closing error due to imperfect field work, may be accurately calculated, as we shall show and laid down on the paper before beginning the plot, thus furnishing a perfect test of the accuracy of the protraction work, which, if correctly done, will not close at the point of beginning, but at the point which indicates the error. But this *calculation* of the error, by a little additional trouble, furnishes data also for dividing it by calculation among the diff sides besides the means of drawing the plot *correctly* at once, without the use of a protractor thus enabling us to make the subsequent measurements and computations of the triangles with more certainty

We shall now describe this process, but would recommend that even when it is employed, and especially in complicated surveys, a rough plot should first be made and corrected, by the first of the two mechanical methods already alluded to. It will prove to be of great service in using the method by calculation, inasmuch as it furnishes an eye check to vexatious mistakes which are otherwise apt to occur; for, although the principles involved are extremely simple, and easily remembered when once understood, yet the continual changes in the directions of the sides will, without great care cause us to use Ns instead of Ss, Es instead of Ws, &c

We suppose, then, that such a rough plot has been prepared, and that the angles, bearings, and distances, as taken from the field book, are figured upon it in *lead pencil*

Add together the *interior* angles formed at all the corners call their sum *a*. Mult the number of sides by 180°, from the prod subtract 360°. if the remainder is equal to the sum *a*, it is a proof that the angles have been correctly measured * This, however, will rarely if ever occur, there will always be some discrepancy, but if the field work has been performed with moderate care, this will not *exceed* about two min for each angle. In this case div it *in equal parts* among all the angles adding or subtracting, as the case may be, unless it amounts to less than a min to each angle, when it may be entirely disregarded in common farm surveys. The corrected angles may then be marked on the plot in ink, and the pencilled figures erased. We will suppose the corrected ones to be as shown in Fig 3.

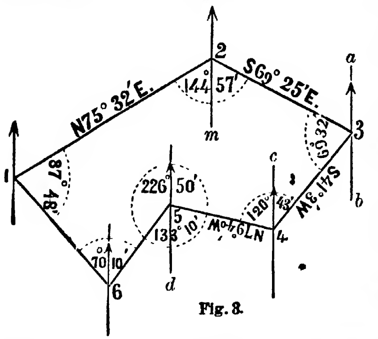


Fig. 8.

Next, by means of these corrected angles, correct the bearings also, thus, Fig 3 Select some side (the longer the better) from the two ends of which the bearing and the reverse bearing agreed; thus showing that that bearing was probably not influenced by local attraction Let side 2 be the one so selected; assume its bearing, N 75° 32' E as taken on the ground, to be correct; through either end of it, as at its far end 2, draw the short meridian line; parallel to which draw others through every corner. Now having the bearing of side 2 N 75° 32' E, and requiring that of side 3, it is plain that the reverse bearing from corner 2 is S 75° 32 W; and that therefore the angle 1, 2 *m*, is 75° 32. Therefore, if we take 75° 32 from the entire corrected angle 1, 2 3, or 144° 57', the rem 69° 25' will be the angle *m* 23; consequently the bearing of side 3 *must* be S 69° 25' E. For finding the bearing of side 4, we now have the angle 23 *a* of the reverse bearing of side 3, also equal to 69° 25'; and if we add this to the entire corrected angle 234, or to 69° 32', we have the angle *a* 34 = 69° 25' + 69° 32 = 138° 57'; which taken from 180°, leaves the angle *b* 34 = 41° 3'; consequently the bearing of side 4 *must* be S 41° 3' W. For the bearing of side 5 we now have the angle 34 *c* = 41° 3', which taken from the corrected angle 345, or 120° 43', leaves the angle *c* 45 = 79° 40'; consequently the bearing of side 5 *must* be N 79° 40' W. At corner 5, for the bearing of side 6 we have the angle 45 *d* = 79° 40', which taken from 133° 10', leaves the angle *d* 56 = 53° 30'; consequently the bearing of side 6 *must* be S 53° 30' W. And so with each of the sides, nothing be

* Because in every straight-lined figure the sum of all its interior angles is equal to twice as many right angles as the figure has sides, minus 4 right angles, or 360°.

careful observation is necessary to see how the several angles are to be employed at each corner. Rules are sometimes given for this purpose, but unless frequently used, they are soon forgotten The plot mechanically prepared obviates the necessity for such rules, inasmuch as the principle of proceeding thereby becomes merely a matter of sight, and tends greatly to prevent error from using the wrong bearings; while the protractor will at once detect any serious mistakes as to the angles, and thus prevent their being carried farther along After having obtained all the corrected bearings, they may be figured on the plot instead of those taken in the field. They will, however, require a still further correction after a while, since they will be affected by the adjustment of the closing error.

We now proceed to calculate the closing error *t* 6 of Fig 2, which is done on the principle that in a correct survey the northings will be equal to the southings, and the eastings to the westings. Prepare a table of 7 columns, as below, and in the first 3 cols place the numbers of the sides, and their corrected courses; also the dists or lengths of the sides, as measured on the rough plot, if such a one has been prepared; but if not, then as measured on the ground. Let them be as follows:

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015 5			304
2	N 75° 32′ E	1202	300.3		1163 9	
3	S 69° 25′ E	1110		390.2	1039.2	
4	S 41° 3′ W	850		641.		558.2
5	N 79° 40′ W	802	143.9			789.
6	S 53° 30′ W	705		419.3		566.7
			1459.7	1450.5	2203.1	2217.9
			1450 5			2203.1
			9.2	Error in Lat.	Error in Dep.	14.8

Now, by means of the Table of Sines, etc , find the N, S, E, W. of the several sides, and place them in the corresponding four columns Thus, for side 1, which is 1060 feet long, with bearing N 16° 40′ W ; cos 16° 40′ = 0 9580, sin 16° 40′ = 0.2868.

Here N = 1060 × 0 9580 = 1015 5; and W = 1060 × 0 2868 = 304. Proceed thus with all Add up the four cols; find the diff between the N and S cols; and also between the E and W ones. In this instance we find that the Ns are 9 2 feet greater than the Ss; and that the Ws are 14 8 ft greater than the Es; in other words, there is a closing error which would cause a correct protraction of our first three cols, to terminate 9 2 feet too far north of the starting point, and 14.8 feet too far west of it So that by placing this error upon the paper before beginning to protract, we should have a test for the accuracy of the protracting work, but as before remarked, a little more trouble will now enable us to div the error proportionally among all the Ns, Ss, Es, and Ws, and thereby give as data for drawing the plot correctly at once, without using a protractor at all

To divide the errors, prepare a table precisely the same as the foregoing, except that the hor spaces are farther apart, and that the addings-up of the *old* N, S, E, W columns are omitted. The additions here noticed are made subsequently.

The new table is on the next page.

REMARK. **The bearing and the reverse bearing** from the two ends of a line will not read precisely the same angle; and the difference varies with the latitude and with the length of the line, but not in the same proportion with either. It is, however, generally too small to be detected by the needle, being, according to Gummere, only three quarters of a minute in a line one mile long in lat 40°. In higher lats it is more, and in lower ones less. It is caused by the fact that meridians or north and south lines are not truly parallel to each other; but would if extended meet at the poles.

Hence the only bearing that can be run in a straight line, with strict accuracy, is a true N and S one, except on the very equator, where alone a due E and W one will also be straight But **a true curved E and W line** may be found anywhere with sufficient accuracy for the surveyor's purposes thus Having first by means of the N star p 284, or otherwise got a true N and S bearing at the starting point, lay off from it 90°, for a true E and W bearing *at that point* This E and W bearing will be tangent to the true E and W curve. Run this tangent carefully, and at intervals (say at the end of each mile) lay off from it (towards the N if in N lat, or vice versa) an offset whose length in *feet* is equal to the proper one from the following table, multiplied by the *square* of the distance in *miles* from the starting point These offsets will mark points in the true E and W curve.

Latitude N or S.												
5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
Offsets in ft one mile from starting point.												
.058	.118	.179	.243	.311	.385	.467	.559	.667	.795	.952	1 15	1.43

Or, any offset in ft = .6666 × Total Dist in miles^2 × Nat Tang of Lat.

A rhumb line is any one that crosses a meridian obliquely, that is, is neither due N and S, nor E and W.

Side.	Bearing.	Dist. Ft.	Latitudes. N	Latitudes. S.	Departures. E.	Departures. W.
1	N 16° 40′ W	1060	1015.5 1.7 1013.8			304.0 2.7 301.3
2	N 75° 32′ E	1202	300.3 1.9 298.4		1163.9 3.1 1167.0	
3	S 69° 25′ E	1110		390.2 1.8 392	1039.2 2.9 1042.1	
4	S 41° 3′ W	850		641.0 1.3 642.3		558.2 2.2 556.0
5	N 79° 40′ W	802	143.9 1.3 142.6			789.0 2.1 786.9
6	S 53° 30′ W	705		419.3 1.1 420.4		566.7 1.8 564.9
		5729 Sum of Sides.	1454.8 Cor'd Ns	1454.7 Cor'd Ss	2209.1 Cor'd Es	2209.1 Cor'd Ws.

Now we have already found by the old table that the Ns and the Ws are too long; consequently they must be shortened; while the Ss, and Es, must be lengthened, all in the following proportions: As the

Sum of all the sides : Any given side :: Total err of lat or dep : Err of lat or dep. of given side.

Thus, commencing with the lat of side 1, we have, as

Sum of all the sides. 5729 : Side 1. 1060 :: Total lat err. 9.2 : Lat err of side 1. 1.7

Now as the lat of side 1 is north, it must be shortened; hence it becomes = 1015.5 — 1.7 = 1013.8, as figured out in the new table. Again we have for the departure of side 1,

Sum of all the sides. 5729 : Side 1. 1060 :: Total dep err. 14.8 : Dep err of side 1. 2.7

Now as the dep of side 1 is west, it must be shortened; hence it becomes 304 — 2.7 = 301.3, as figured out in the new table.

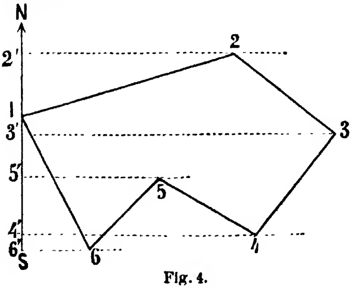


Fig. 4.

Proceeding thus with each side, we obtain all the corrected lats and deps as shown in the new table; where they are connected with their respective sides by dotted lines; but in practice it is better to cross out the original ones when the calculation is finished and proved. If we now add up the 4 cols of corrected N, S, E, W, we find that the Ns = the Ss; and the Es = the Ws, thus proving that the work is right. There is, it is true, a discrepancy of .1 of a ft between the Ns, and the Ss; but this is owing to our carrying out the corrections to only one decimal place; and is too small to be regarded. Discrepancies of 3 or 4 tenths of a foot will sometimes occur from this cause; but may be neglected. The corrected lats and deps must evidently change the bearing and distance of every side; but without knowing either of these, we can now plot the survey by means of the corrected

lats and deps alone. The principal is self-evident, explaining itself. First draw a meridian line N S, Fig. 4; and upon it fix on a point 1, to represent the *extreme west* corner* of the survey.

Then from the point 1, prick off by scale, northward, the dist 1, 2′=the corrected northing 298.4 of side 2, taken from the last table; from 2′ southward prick off the dist 2′, 3′, the corrected southing 392 of side 3, from 3′ southward prick off 3′, 4′,=southing 642 3 of side 4; from 4′ northward prick off 4′, 5′=northing 142.6 of side 5, from 5′ prick off southward 5′, 6′=southing of side 6†. Then from the points 2′, 3′, 4′, 5′, 6′, draw indefinite lines due eastward, or at right angles to the meridian line. Make by scale, 2′, 2=corrected departure of side 2; and join 1, 2. Make 3′, 3=dep of side 2+dep of side 3; and join 2, 3; make 4′, 4=3′, 3—dep of side 4, and join 3, 4; make 5′, 5=4′, 4—dep of side 5; and join 4, 5; make 6′, 6=5′, 5—dep of side 6; and join 5, 6 ‡ Finally join 6, 1; and the plot is complete If scrupulous accuracy is not required, the contents may be found by the mechanical method of triangles; the bearings, by the protractor; and the lengths of the sides, by the scale; all with an approximation sufficient for ordinary purposes; and perhaps quite as close as by the method by calculation, when, as is customary, the bearings are taken only to the nearest quarter of a degree. We have already said that with a scale of feet per inch $=\frac{\text{average length of sides}}{8}$, the error of area need not *exceed* the $\frac{1}{200}$th part.

But if it is required to calculate the area of the corrected survey with rigorous exactness, it may be done on the following principle, (see Fig 5) If a meridian line N S be supposed to be drawn through the extreme west corner 1 of a survey, and lines (called *middle distances*) drawn (as the dotted ones in the Fig) at right angles to said meridian, from the *center* of each side of the survey; then if each of the middle dists of such sides as have northings, be mult by the corrected northing of its corresponding side, and if each of the middle dists of such sides as have southings, be mult by the corrected southing of its corresponding side; if we add all the north prods into one sum; and all the south prods into another sum; and subtract the least of these sums from the greatest, the rem will be the area

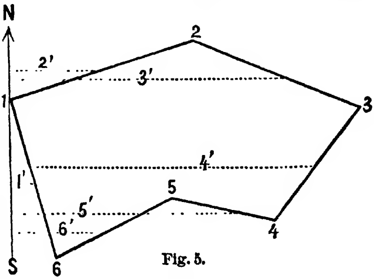


Fig. 5.

* The extreme east corner would answer as well, with a slight change in the subsequent operations, as will become evident.

† Instead of pricking off these northings and southings in succession, *from each other*, it will be more correct in practice to prepare first a table showing how far each of the points 2′,3′, &c, is north or south from 1. This being done, the points can be pricked off north or south from 1, without *moving the scale each time;* and of course with greater accuracy. Such a table is readily formed. Rule it as below, and in the first three columns place the numbers of the sides (starting with side 2 from point 1,) and their respective corrected northings and southings The formation of the 4th and 5th cols by means of the 3d and 4th ones, explains itself. Its accuracy is proved by the final result being 0.

Side.	N. lat.	S. lat.	Dist N or S from Point 1. N.	S.
2	298.4		298.4	
3		392.		93.6
4		642.3		735.9
5	142.6			593.3
6		420.4		1013.7
1	1013.8		000.0	000.0

‡ A similar table should be prepared beforehand for the dists of the points 2, 3, 4, &c, east from the meridian line. It is done in the same manner, but requires one col less, as all the dists are on the same side of the mer line. Thus, starting from point 1, with side 2.

Side.	E dep.	W. dep.	Dist east from meridian line.
2	1167.0		1167.0
3	1042.1		2209.1
4		556.0	1653.1
5		786.9	866.2
6		564.9	301.3
1		301.3	000.0

This work likewise proves itself by the final result being 0.

of the survey.* The corrected northings and southings we have already found; as also the eastings and westings. The middle dists are found by means of the latter, by employing their *halves*; adding half eastings, and subtracting half westings. Thus it is evident that the middle dist 2' of side 2, is equal to half the easting of side 2. To this add the other half easting of side 2, and a half easting of side 3, and the sum is plainly equal to the middle dist 3 of side 3. To this add the other half easting of side 3, and subtract a half westing of side 4, for the middle dist 4 of side 4. From this subtract the other half westing of side 4, and a half westing of side 5, for the middle dist 5' of side 5, and so on. The actual calculation may be made thus.

Half easting of side 2 = $\frac{1167}{2}$ = 583 5 E = mid dist of side 2.
583 5 E
————
Half easting of side 3 = $\frac{1042 1}{2}$ = 1167 0 E
521 0 E
————
1688 0 E = mid dist of side 3.
521 0 E
————
Half westing of side 4 = $\frac{556}{2}$ = 2209 0 E
278 0 W
————
1931 0 E = mid dist of side 4
278 0 W
————
Half westing of side 5 = $\frac{786 9}{2}$ = 1653 0 E
393 5 W
————
1259 5 E = mid dist of side 5
393 5 W
————
Half westing of side 6 = $\frac{564 9}{2}$ = 866 0 E
282 4 W
————
583 6 E = mid dist of side 6.
282 4 W
————
Half westing of side 1 = $\frac{301.3}{2}$ = 301 2 E
150 6 W
————
150.6 E = mid dist of side 1.

The work always proves itself by the last two results being equal.

Next make a table like the following, in the first 4 cols of which place the numbers of the sides, the middle dists, the northings, and southings. Mult each middle dist by its corresponding northing or southing, and place the products in their proper col. Add up each col, subtract the least from the

Side.	Middle dist.	Northing.	Southing.	North prod.	South prod.
1	150.6	1013.8		152678	
2	583.5	298.4		174116	
3	1688		392		661696
4	1931		642.3		1240281
5	1259.5	142.6		179605	
6	583 6		420.4		245345
				506399	2147322
					506399
					43560)1640923(37.67 Acres.

* *Proof.* To illustrate the principle upon which this rule is based, let *a b*, *b c*, and *c a*, Fig 6, represent in order the 3 sides of the triangular plot of a survey, with a meridian line *d f* drawn through the extreme west corner, *a*. Let lines *b d* and *c f* be drawn from each corner, perp to the meridian line; also from the middle of each side draw lines *w e*, *m n*, *s o*, also perp to meridian; and representing the middle dists of the sides. Then since the sides are regarded in the order *a b*, *b c*, *c a*, it is plain that *a d* represents the northing of the side *a b*; *f a* the northing of *c a*; and *d f* the southing of *b c*. Now if we mult the *northing a d* of the side *a b*, by its mid dist *e w*, the prod is the area of the triangle *a b d*. In like manner the *northing f a* of the side *c a*, mult by its mid dist *s o*, gives the area of the triangle *a c f*. Again, the *southing d f* of the side *b c*, mult by its mid dist *m n*, gives the area of the entire fig *d b c f d*. If from this area we subtract the areas of the two triangles *a b d*, and *a c f*, the rem is evidently the area of the plot *a b c*. So with any other plot, however complicated.

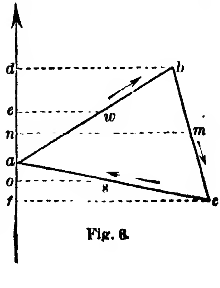


Fig. 6.

greatest. The rem will be the area of the survey in sq ft; which, div by 43560, (the number of sq ft in an acre,) will be the area in acres, in this instance, 37 67 ac.

It now remains only to calculate the corrected bearings and lengths of the sides of the survey, all of which are necessarily changed by the adoption of the corrected lats and deps. To find the bearing of any side, div its departure (E or W) by its lat (N or S), in the table of nat tang, find the quot; the angle opposite it is the reqd angle of bearing. Thus, for the course of side 1, we have $\frac{301.3 \text{ W}}{1013.8 \text{ N}}$ = .2972 = nat tang, opposite which in the table is the reqd angle, 16° 33′; the bearing, therefore, is N 16° 33′ W.

Again: for the dist or length of any side, from the table of nat cosines take the cos opposite to the angle of the corrected bearing, divide the corrected lat (N or S) of the side by the cos. Thus for the dist of side 1, we find opposite 16° 33′, the cos .9586 And

$$\overset{\text{Lat.}}{1013.8} \div \overset{\text{Cos.}}{.9586} = 1057.6 \text{ the reqd dist.}$$

The following table contains all the corrections of the foregoing survey; consequently, if the bear-

Side.	Bearing.	Dist. Ft.
1	N 16° 33′ W	1057.6
2	N 75° 39′ E	1204.0
3	S 69° 23′ E	1113.3
4	S 40° 53′ W	849.6
5	N 79° 44′ W	800.1
6	S 53° 21′ W	704.3

ings and dists are correctly plotted they will close perfectly. The young assistant is advised to practise doing this, as well as dividing the plot into triangles, and computing the content. In this manner he will soon learn what degree of care is necessary to insure accurate results

The following **hints** may often be of service.

1st. Avoid taking bearings and dists along a circuitous boundary line like *a b c*, Fig 7, but run the straight line *a c*; and at right angles to it, measure offsets to the crooked line. **2d.** Wishing to survey a straight line from *a* to *c*, but being unable to direct the instrument precisely toward *c*, on account of intervening woods, or other obstacles; first run a trial line, as *a m*, as nearly in the proper direction as can be guessed at.

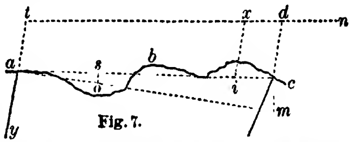


Fig. 7.

Measure *m c*, and say, as *a m* is to *m c*, so is 100 ft to ? Lay off *a o* equal to 100 ft, and *o s* equal to ?; and run the final line *a s c*. Or, if *m c* is quite small, calculate offsets like *o s* for every 100 ft along *a m*, and thus avoid the necessity for running a second line. **3d.** When *c* is visible from *a*, but the intervening ground difficult to measure along, on account of marshes, &c, extend the side *y a* to good ground at *t*: then making the angle *y t d* equal to *y a c*, run the line *t n* to that point *d* at which the angle *n d c* is found by trial to be equal to the angle *a t d*. It will rarely be necessary to make more than one trial for this point *d*, for, suppose it to be made at *x*, see where it strikes *a c* at *i*, measure *i c*, and continue from *x*, making *x d* = *i c*. **4th.** In case of a very irregular piece of land, or a lake, Fig 8, surround it by straight lines. Survey these, and at right angles to them, measure offsets to the crooked boundary


Fig. 8.

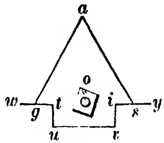


Fig. 9.

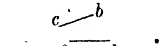


Fig. 10.

5th. Fig 9 Surveying a straight line from *w* toward *y*, an **obstacle**, *o*, is met. To pass it, lay off a right angle *w t u*, measure any *t u*, make *t u v* = 90°, measure *u v*; make *u v i* = 90°, make *v i* = *t u*, make *v i y* = 90°. Then is *t i* = *u v*, and *i y* is in the straight line. Or, with less trouble, at *g* make *t g a* = 60°, measure any *g a*; make *g a s* = 60°; and *a s* = *g a*, make *a s i* = 60°. Then is *g s* = *g a* or *a s*; and *i s*, continued toward *y*, is in the straight line. **6th** Fig 10. Being between two objects, *m* and *n*, and wishing to place myself in range with them, I lay a straight rod *c b* on the ground, and point it to one of the objects *m*; then going to the end *c*, I find that it does not point to the other object, *n*. By successive trials, I find the position *e d* in which it points to both objects, and consequently is in range with them.

CHAINING.

Chains. Engineers have abandoned the Gunter's chain of 66 ft, divided into 100 links of 7.92 ins each. They now use a chain of 100 ft, with 100 links of 1 ft each, and calculate areas in sq ft, the number of which, divided by 43,560, reduces to acres and decimals, instead of to acres, roods, and perches. Gunter's chain is used on U. S. Government land surveys.

Chains are commonly made of iron or steel wire. Each link is bent at each of its ends, to form an eye, by which it is connected with the adjacent links, either directly, as in the Grumman patent chain, or, more commonly, by from 1 to 3 small wire links. The wear of these links is a fruitful source of inaccuracy, inasmuch as even a very slight wear of each link considerably increases the length of the chain. Hence, chains should be compared with some standard, such as a target rod, every few days while in use. For transportation, the lengths are folded on each other, making a compact and sheaf-like bundle.

Tapes. With improved facilities for the manufacture of steel tape, the chain is going out of use. The tape, being much lighter, requires much less pull, and, as there are no links to wear, its length is much more nearly constant than that of the chain. It is replacing, to some extent, the base-measuring rod for accurate geodetic work. Steel tapes are made in continuous lengths up to 500, 600, and even 1000 ft, but those of 100 ft are the most commonly used. Very long tapes are liable to breakage in handling. Even the shorter lengths, unless handled carefully, are apt to kink and break. Breaks are difficult to mend, and the repaired joint is seldom satisfactory; whereas a kink in a wire chain seldom involves more than a temporary change of length. Being run over by a car or wagon will often kink steel tapes very badly, if it does not break them.* However, the lightness, neatness, and reliability of the tape offset these disadvantages, which, indeed, the surveyor soon learns to overcome.

Tapes for general field work are usually narrow (from 0.10 to 0.25 in) and thick (from 0.013 to 0.025 in),† and are graduated by means of small brass and copper rivets, spaced, in general, 5 ft apart, 1 ft apart in the 10 ft at each end, and 0.1 ft apart in the ft at each end. They are usually mounted on reels.

Tapes for city work are wider (from 0.25 to 0.5 in) and thinner (from 0.007 to 0.010 in)† and are graduated (usually to 0.01 ft) throughout their length by means of lines and numerals etched on the steel.

Pins are ordinarily of wire, pointed at the lower end, and bent to a ring at the upper end. They can be forced into almost any ground that is not exceedingly stony. A steel ring, like a large key ring, is often used for carrying the pins. Each pin should have a strip of bright red flannel tied to its top, in order that it may be readily found, among the grass, etc., by the rear chainman.

Corrections for Sag and Stretch. The following diagram‡ (see p. 283) gives the correction for a steel tape weighing 0.75 lb per 100 ft.†

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*The Nichols Engineering & Contracting Co., Chicago, guarantees that its tapes will not be injured by being run over by wagons.

†The sizes of tapes, as made by different manufacturers, vary greatly. In applying the corrections, therefore, the width and thickness of the tape to be used should be carefully measured, and its weight per ft computed.

‡Deduced from diagrams constructed by Mr. J. O. Clarke, Proceedings Engineers' Club of Philadelphia, April, 1901, Vol. XVIII, No. 2. from the formula:

$$\text{Stretch, in feet} = \frac{PS}{EA}$$

where

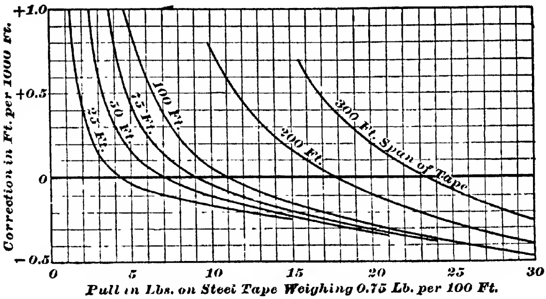
P = pull on tape, in lbs.
S = span of tape, in feet.
E = modulus of elasticity for steel = 27,500,000 lbs per sq in.
A = area of cross-section of tape weighing 0.75 lb per 100 ft.
= 0.0022 square ins,

and from the equation of the parabola, according to which

$$\text{shortening by sag, in feet} = \frac{W^2\ S^3}{24\ P^2}$$

where W = weight of tape, in pounds per foot.

Except for very light pulls, this last formula gives practically the same results as the equation of the catenary, which is absolutely correct, but much more cumbersome.



The diagram shows that a span of 100 ft of tape weighing 0 75 lb per 100 ft, requires a pull of 11 2 lbs to reduce the correction to zero; a span of 50 ft, 7.3 lbs, etc. At these tensions (which are called normal pulls), the opposite effects of sag and of stretch are equal. At higher tensions, the lengthening due to stretch exceeds the shortening due to sag, and vice versa.

Tapes of other weights require pulls proportional to their weights or to their areas of cross-section. Thus, a tape, of any length, weighing 1 lb per 100 ft, would require, for any given correction, a pull of $x = \frac{100}{75} y = 1\frac{1}{3} y$, where y = the pull for the same correction on the standard tape, weighing 0.75 lb per 100 ft.

Conversely; given a pull of 10 lbs on a 50 ft span of a tape weighing 0 6 lb per 100 ft; required the correction. To produce the same error in the tape weighing 0.75 lb per 100 ft would require a pull of $y = 10 \times \frac{75}{60} = 12.5$ lbs. Referring to the diagram at 12.5 lbs on the curve for a 50 ft span, we find correction = −0.16. This is the proper correction for either the heavier tape with 12.5 lbs or for the lighter tape with 10 lbs pull.

Corrections for temperature. Tapes are usually graduated so as to be of standard length at 62° Fahr. For ordinary steel tape, the correction for temperature is about 0.0000065 ft per ft per degree Fahr.

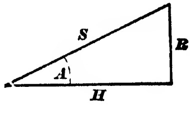
Corrections for temperature are uncertain, since the temperature of the tape cannot be determined with any accuracy. Measurements requiring great accuracy should therefore be made in cloudy weather, or at night, and the tape and the thermometer should be kept off the ground.

When measuring over **sloping ground**, in ordinary work, the chain or tape should be held as nearly horizontal as possible, transferring the position of the raised end to the ground by means of a plumb line. Where the ground is steep, it becomes necessary to use a short length of tape, as the down-hill chainman could not otherwise hold his end high enough; or the tape may be held parallel with the slope, and the distance corrected by the following formulas:

$\frac{H}{S} = \cos A$; $H = S. \cos A$; $S = \frac{H}{\cos A} = H. \sec A$;

$\frac{R}{H} = \tan A$; $R = H. \tan A$;

$\frac{R}{S} = \sin A$; $R = S. \sin A$.



LOCATION OF THE MERIDIAN.

By means of circumpolar stars.

(1) Seen from a point O (Figs. 1 and 2) on the earth, a circumpolar star (star near the pole *P*) appears to describe daily* and counterclockwise a small circle, *e u w l*, about the pole. The angle P O *e*, P O *u*, etc., subtended by the radius P *e*, P *u*, etc., of this circle, or the apparent distance of the star from the pole, is called its **polar distance.** The polar distances of stars vary slightly from year to year. See Table 3. They vary slightly also *during* each year. In the case of Polaris this latter variation amounts to about 50 seconds of arc.

(2) The **altitude** of the pole is the angle N O P of the pole's elevation above the horizon N E S W, and is = the **latitude** of the point of obser-

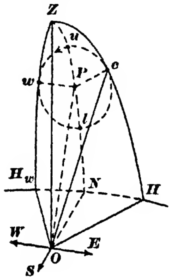


FIG. 1.

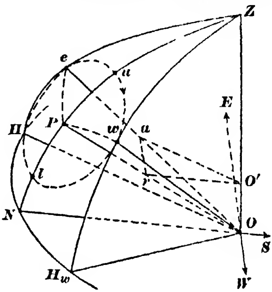


FIG. 2.

vation. **Declination** = angular distance north or south from the celestial equator. Thus, declination of pole = 90°. Declination of any star = 90°—its polar distance.

(3) Let Z *e* H be an arc of a vertical circle† passing through a circumpolar star, *e*, and let H be the point where this arc meets the horizon N E S W. Then the angle N Z H at the zenith Z, or N O H at the point O of observation, between the plane N Z O of the meridian and the plane H Z O of the star's vertical circle (or the arc N H), is called the **azimuth**‡ of the star. If this angle N O H be laid off from O H, on the ground, the line O N will be in the plane of the **meridian** N Z S, or will be a **north-and-south line.**‖

(4) When a star is on the meridian Z N of the observer, above or below the pole P, as at *u* or *l*, it is said to be at its **upper or lower culmination,** respectively. Its azimuth is then = 0, the line O H coinciding with the meridian line O N.

(5) When the star has reached its greatest distance east or west from the pole, as at *e* or *w*, it is said to be at its **eastern** or **western elongation.**§

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* In 23 h. 56.1 m.

† A great circle is that section of the surface of a sphere which is formed by a plane passing through the center of the sphere. A vertical circle is a great circle passing through the zenith Z.

‡ Astronomers usually reckon azimuth from the south point around through the west, north, and east points, to south again; but for our purpose it is evidently much more convenient to reckon it from the north point, and either to the east or to the west, as the case may be.

‖ The point N, on the horizon, is called the **north point,** and must not be confounded with the north *pole* P.

§ As seen from the equator, a star, at either elongation, is, like the pole itself, on the horizon; and the two lines P *e*, P *w*, joining it with the pole, form a single straight line perpendicular to the meridian, and lying in the

(6) The **hour angle** of any star, at any given moment, is the time which has elapsed since it was in upper culmination.*

(7) Evidently the azimuth of a star is continually changing. In circumpolar stars it varies from 0° to maximum (at elongation) and back to 0° twice daily, as the star appears to revolve about the pole; but when the star is near either elongation the change in azimuth takes place so slowly that, for some minutes, it is scarcely perceptible, the star appearing to travel vertically.

(8) For any star, whose declination (= 90° — its polar distance) exceeds the latitude of the point of observation, we have: †

$$\left.\begin{array}{l}\text{Sine of azimuth of star}\\ \text{at elongation}\end{array}\right\} = \frac{\text{sine of polar distance of star}}{\text{cosine of latitude of point of observation}}$$

or see (11) and Table 3. When lat > decl, sine az > 1. Hence this formula does not then apply.

(9) The following circumpolar stars are of service in connection with observations for determining the meridian. See Fig. 3.

Constellation	Letter	Called
Ursa minor (Little bear)	α (alpha)	Polaris
Ursa major (Great bear)	ε (epsilon)	Alioth
" " (" ")	ζ (zeta)	Mizar
Cassiopeia	δ (delta)	Delta ‡

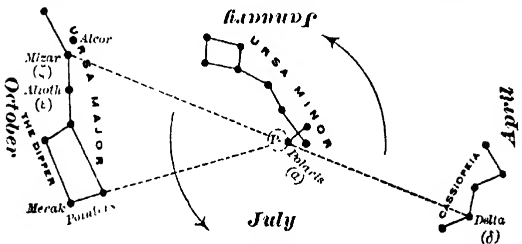

FIG. 3.

(10) **Polaris**, or the **north star**, is fortunately placed for the determination of the meridian, its polar distance being only about 1 °. See Table 3. Fig. 3 shows the circumpolar stars as they appear about midnight in July; inverted, as in January; with the left side uppermost, as in April; and, with the right side uppermost, as in October.‖

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horizon. The azimuth of the star is then = its polar distance. But in other latitudes P e and P w form acute angles with the meridian, as shown, and these angles decrease, and the azimuth of the star at elongation increases, as the latitude increases.

* In lat. 40° N., the hour angle, Z P e = Z P w, of Polaris, at elongation, is = 5 h 55 m. of solar time. **Caution.** It will be noticed that, except for an observer at the equator, the elongations do *not* occur at 90° from the meridian.

† In the spherical triangle Z P e, we have:

$$\frac{\sin e\,Z\,P}{\sin Z\,e\,P} = \frac{\sin P\,e}{\sin P\,Z}$$

But, since Z e P = 90°, sin Z e P = 1. Also, sin P Z = cos (90 − P Z), and e Z P = azimuth of e.

$$\text{Hence, sin azimuth of } e = \frac{\sin P\,e}{\sin P\,Z} = \frac{\text{sin polar distance } P\,O\,e}{\text{cos latitude}}$$

‡ δ Cassiopeia is here called Delta, for brevity.

‖ **Polaris is easily found** by means of the two well-known stars called the "**pointers**" in "the dipper," Fig. 3, which forms the hinder

(11) Table 3 gives the mean polar dists of Polaris and their log sines for Jan 1st in each second year from 1936 to 1956 incl, the log cosines of each fifth degree of lat from 25° to 50°, and the corresponding azimuths of Polaris at elongation. Intermediate values may be taken by interpolation.

(12) **By Observation of Polaris at Elongation.** This method has the convenience, that at and near elongation the star appears to travel vert'y for some minutes, its azimuth, during that time, remaining practically constant; but during certain parts of the year (see Table 1), the elongations of Polaris take place in daylight; so that this method cannot then be used (See note p. 290). See (18), (19), (22). Nor can it be used at any time in places south of about 4° N lat, because there Polaris is not visible.

(13) The approx times of elongation of Polaris for the first of each month in 1937 are given in Table 1, with instructions for finding the times for other dates. Or, watch Polaris in connection with any of those stars which are nearly in line with it and the pole, as Delta, Mizar, and Alioth. See Fig 3. The time of elongation is approximated, with sufficient closeness for the determination of the azimuth, by the cessation of apparent hor motion during the observation.

(14) From 15 to 30 minutes before the time of elongation, have the transit, see (21), set up and carefully centered over a stake previously driven and marked with a center point. The transit must be in adjustment, espec'y in regard to the second adjustment, p 294, or that of the hor axis, by which the line of collimation is made to describe a vert plane when the transit is leveled and the telescope is swung upward or downward.

(15) Means must be provided for making the stake and cross-hairs of the transit visible. This may be done quite satisfactorily by holding a sheet of paper behind the stake, and then illuminating the paper by a light held either behind the paper or in front of it, but not so that the light is in the field of the instrument. In this way, the stake and pencil point or knife-blade held on it, or the nail or tack in its top, and also the cross-hairs, are all seen in silhouet against the illuminated paper. Care should be taken, however, not to use the wrong cross-hair.

(16) Bring the vert hair to coincide with Polaris, and, by means of the tangent screw, follow the star as it appears to move, to the *right* if approaching *eastern* elongation, or *vice versa*, keeping the hair upon the star, as nearly as may be. As elongation is approacht, the star will appear to move more and more slowly in azimuth. When it appears to travel vert'y along the hair, it has reacht elongation, and the vert plane of the transit is in the plane of the star's vert circle at elongation. Depress the telescope, and fix a point in the line of sight, preferably 300′ or more distant from the transit. Immediately reverse the transit, (swinging it hor'y thru an arc of 180°), sight the star again, again depress, and, if the line of sight then coincides perfectly with the mark first set, both are in the plane of the star's vert circle. If not, note where the line of sight does strike, and make a third mark midway betw the two. The line of sight, when directed to this third mark, is in the req'd plane, from which the azimuth, found as in (8), has yet to be laid off to the meridian, to the *left* from *eastern* elongation, and *vice versa*.

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portion of the "great bear" (Ursa major), a line drawn through these two stars passing near Polaris. As the stars in the handle of the dipper form the tail of the great bear, as shown on celestial maps, so Polaris and the stars near it form the tail of the little bear (Ursa minor.) Polaris is also nearly midway and in line between Delta and Mizar. Polaris forms, with three other and less brilliant stars, a quite symmetrical cross, with Polaris at the end of the right arm. In Fig. 3 this cross is inverted. Its height is about 5°, or = the distance between the pointers.

(17) To avoid driving the distant stake and marking it during the night, a fixt target at any convenient point may be used, and the hor angle formed betw the line of sight to the star and that to the target merely noted, for use in ascertaining and laying off the azimuth of the target.

(18) **By observations of Polaris at Culmination.** Fig 3. See parag 9. The times of Polaris at culmination may be determined by the table below* in which are given the number of minutes (and tenths) to the culmination of Polaris *after* its being in the same vert plane with Delta Cassiopeiae or Mizar†.

	Jn 1	Fb 1	Mr 1	Ap 1	My 1	Jn 1	Jl 1	Ag 1	Ag 1	Sp 1	Oc 1	Nv 1	Dc 1	Jn 1
Lat	Delta below pole; L C of Polaris;								Mizar bel pole; U C of Polaris;					
40°	Time interval in mins								Time interval in mins					
1937	20.6	20.0	19.5	19.3	19.3	19.7	20.3	20.9	19.5	20.0	20.4	20.5	20.3	19.9
1942	23.3	22.7	22.2	22 0	22 0	22.4	23 0	23.6	22 2	22 7	23 1	23.2	23.0	22.6
Lat	Mizar above pole; L C of Polaris								Delta abv pole; U C of Polaris;					
30°	Time interval in mins								Time interval in mins					
1937	19.7	19.1	18.6	18.3	18.4	18.8	19.3	20.0	20.4	20.9	21.3	21.4	21.2	20.8
1942	22.5	21.9	21 4	21.1	21 2	21.6	22 1	22.8	23.1	23.6	24 0	24 1	23.9	23.5

In lat 50°, the time interval is 0.1 min greater than in lat 40°.
In lat 15°, the time interval is 0.1 minute less than in lat 30°.
The mean annual increase after 1942 is 33 secs, or 0.55 minute.

(19) **By observation of Polaris at any point in its path.** Table 1 gives the mean solar times of upper culmination of Polaris on the 1st of each month in 1937, and directions for ascertaining the times on other dates; and Table 2 gives the azimuths of Polaris corresponding to different values of its hour angle in civil or mean solar time, for different latitudes from 30° to 50°, and for Jan 1, 1938 and 1948. For hour angles, latitudes and dates intermediate of those in the table, the azimuths may be taken by interpolation. See caution and formula, p 290.

(20) The local‡ time of observation must be accurately known, and the time of the nearest upper culmination subtracted from it. The difference is the hour angle to be used in Table 2; the resulting azimuth is to the west of north if the hour angle is positive, and to the east of north if negative. Find the time of the nearest upper culmination from Table 1. For the months where only lower culminations are given, add or subtract 11 hours, 58 minutes.

(21) Where great accuracy is not req'd, Polaris may be observed by means of a plumb-line and sight. A brick, stone or other heavy object will answer perfectly as a plumb-bob. It should hang in a pail of water. A compass sight, or any other device with an accurate straight slit about $\frac{1}{16}$″ wide may be used. The sight must always remain perfectly vert, but must be adjustable hor'y for a few feet east and west. The plumb-line and sight should be at least 15 feet apart, and so placed that the star and plumb-line can be seen together thru the sight, thruout the observation. The plumb-line must be illuminated. It is well to arrange these matters on an evening preceding that of the observation. When the star reaches elongation, the sight must be fastened in range with the plumb-line and star. From the line so obtained, lay off the azimuth; to the *west* for *east* elongation, and *vice versa*.

* Taken from p 767 of the American Ephemeris and Nautical Almanac of 1937, by interpolation.

† Mizar may be recognized by the small star Alcor close to it.

‡ **Local time** agrees with standard time (p 267) on the standard meridians only. For other points *add* to standard time 4 minutes for each degree of longitude *east* of a standard meridian, and *vice versa*.

(22) **By any star at equal altitudes.** This method, applicable to south as well as to north lats, consists in observing a star when it is at any of two equal altitudes, E and W of the meridian, thus locating, on the horizon, two points of equal and opposite azimuth. The meridian is midway between them.

(23) **By equal shadows from the sun.** Approximate. Hang a plum-bob, as from a tripod, in the cord of which a knot has been tied, over a convenient spot on a level surface, where the sun will shine on it for several hours before and after noon. Or a card or metal plate with a hole in it may be suspended in place of the knot. The resulting shadow of the knot or image of the sun will trace a flat curve on the level surface. From the spot under the plum-bob, draw on the level surface one or more circles with a radius of a few feet. As the shadow of the knot or the image of the sun travels over the surf, note where it intersects the circle or circles. A point midway between the extreme intersections of each circle will be due north of the point under the plum-bob at the solstices (about June 21 and Dec 21). At the vernal equinox (Mar. 21) the line thus located will then be west, and at the autumnal equinox (Sep. 21) east, of the meridian, by less than 2½ minutes of arc. For intermediate dates the error is nearly proportional to the time elapst.

Table 1

Approx local times of elongation and culmination of Polaris in lat 40° N, long 90° W from Greenwich, on first of each month 1937.

P. M. times (from noon until midnight) are printed in **bold-face.**

Elongations. (E, eastern; W, western). **1937.**

Jan 1, W	Feb. 1, W	Mar 1, W	Apr 1, W	May 1, E	Jun 1, E
12:56 a.m.	**10:50 p.m.**	**8:59 p.m.**	**6:57 p.m.**	5:08 a.m.	3:06 a.m.
Jul 1, E	Aug 1, E	Sep 1, E	Oct 1, E	Nov 1, W	Dec 1, W
1:09 a.m.	**11:04 p.m.**	**9:02 p.m.**	**7:05 p.m.**	4:55 a.m.	3:00 a.m.

Culminations. (U, upper; L, lower). **1937.**

Jan 1, U	Feb 1, L	Mar 1, L	Apr 1, L	May 1, L	Jun 1, L
6:56 p.m.	4:56 a.m.	3:05 a.m.	1:03 a.m.	**11:01 p.m.**	**9:00 p.m.**
Jul 1, L	Aug 1, U	Sep 1, U	Oct 1, U	Nov 1, U	Dec 1, U
7:02 p.m.	5:03 a.m.	3:02 a.m.	1:04 a.m.	**10:59 p.m.**	**9:00 p.m.**

This table serves, with Table 3, for observations of Polaris at elongation (parag 16); and with Table 2 for any point in its path (parag 19).

In lat 25°, W elongations occur later and E earlier by 1½ mins.

In lat 50°, W elongations occur earlier and E later by 1½ mins.

In long 60°, elong'ns and culm'ns occur later by ⅓ minute.

In long 120°, elong'ns and culm'ns occur earlier by ⅓ minute.

For other days of the month, deduct 3.94 mins for each succeeding day, or add 3.94 mins for each earlier day.

Each Feb 29 makes each succeeding date occur one day later than it otherwise would, for which day 3.94 mins must be deducted.

Wherefore, for later years, add 1.6 min each year, and deduct 3.94 min for each Feb 29, when it occurs. More exactly, for 1941, four years later, including a Feb 29, the correction to add is 2.37 mins; for 1945, 4.82 mins; for 1949, 7.37 mins, etc. The corrections to be added are as follows, to the nearest minute:—

Year	1938	'39	1940 Jn-Fb	1940 Mr-Dc	1941	'42	'43	1944 Jn-Fb	1944 Mr-Dc	1945	'46	'47
Mins	2	3	5	1	2	4	6	7	3	5	6	8

At culmination, where azimuth change is most rapid, a time error of 1 min in observing Polaris involves an azimuth error of 0.3′ in lat 25° to 0.4′ in lat 50°.

Table 2.

AZIMUTHS OF POLARIS

Hr-angle Mean solar time		Azimuth for Latitude					Hr-angle Mean solar time		Azimuth for Latitude				
1937	1947	30°	35°	40°	45°	50°	1937	1947	30°	35°	40°	45°	50°
h m	h m	° ′	° ′	° ′	° ′	° ′	h m	h m	° ′	° ′	° ′	° ′	° ′
0 6	0 6	0 2	0 2	0 2	0 2	0 3	5 55		1 12	1 16	1 21	1 28	1 36
0 11	0 11	0 3	0 4	0 4	0 4	0 5	6 26		1 11	1 15	1 20	1 27	1 35
0 17	0 17	0 5	0 6	0 6	0 7	0 7	6 43		1 10	1 14	1 19	1 26	1 34
0 22	0 23	0 7	0 8	0 8	0 9	0 9	6 56		1 9	1 13	1 18	1 24	1 33
0 28	0 29	0 9	0 9	0 10	0 11	0 12	7 05	5 55	1 8	1 12	1 17	1 23	1 32
0 34	0 35	0 11	0 11	0 12	0 13	0 14	7 14	6 25	1 7	1 11	1 16	1 22	1 30
0 39	0 41	0 12	0 13	0 14	0 15	0 17	7 22	6 42	1 7	1 11	1 15	1 21	1 29
0 45	0 47	0 14	0 15	0 16	0 17	0 19	7 30	6 56	1 6	1 10	1 14	1 20	1 28
0 51	0 54	0 16	0 17	0 18	0 20	0 22	7 37	7 7	1 5	1 9	1 13	1 19	1 27
0 57	1 0	0 18	0 19	0 20	0 22	0 24	7 43	7 17	1 4	1 8	1 12	1 18	1 26
1 3	1 6	0 20	0 21	0 22	0 24	0 27	7 49	7 26	1 3	1 7	1 11	1 17	1 24
1 8	1 12	0 21	0 22	0 24	0 26	0 29	7 55	7 35	1 2	1 6	1 10	1 16	1 23
1 14	1 18	0 23	0 24	0 26	0 28	0 31	8 6	7 49	1 0	1 4	1 8	1 14	1 21
1 20	1 25	0 25	0 26	0 28	0 30	0 34	8 16	8 1	0 58	1 2	1 6	1 11	1 18
1 26	1 31	0 26	0 28	0 30	0 33	0 36	8 26	8 12	0 57	1 0	1 4	1 9	1 16
1 32	1 37	0 28	0 30	0 32	0 35	0 38	8 35	8 22	0 55	0 58	1 2	1 7	1 13
1 38	1 44	0 30	0 32	0 34	0 37	0 41	8 44	8 31	0 53	0 56	1 0	1 5	1 11
1 44	1 51	0 32	0 34	0 36	0 39	0 43	8 52	8 40	0 51	0 54	0 58	1 3	1 9
1 51	1 57	0 34	0 36	0 38	0 41	0 46	9 0	8 49	0 50	0 53	0 56	1 1	1 6
1 57	2 4	0 35	0 37	0 40	0 43	0 48	9 8	8 57	0 48	0 51	0 54	0 58	1 4
2 4	2 11	0 37	0 39	0 42	0 46	0 50	9 16	9 5	0 46	0 49	0 52	0 56	1 2
2 10	2 18	0 39	0 41	0 44	0 48	0 52	9 23	9 13	0 44	0 47	0 50	0 54	0 59
2 17	2 26	0 41	0 43	0 46	0 50	0 55	9 30	9 21	0 43	0 45	0 48	0 52	0 57
2 24	2 33	0 42	0 45	0 48	0 52	0 58	9 37	9 28	0 41	0 43	0 46	0 50	0 54
2 31	2 41	0 44	0 47	0 50	0 54	1 0	9 44	9 36	0 39	0 41	0 44	0 48	0 52
2 38	2 49	0 46	0 49	0 52	0 56	1 2	9 51	9 43	0 37	0 39	0 42	0 46	0 50
2 45	2 57	0 48	0 51	0 54	0 59	1 5	9 57	9 50	0 36	0 38	0 40	0 43	0 47
2 53	3 6	0 49	0 52	0 56	1 1	1 7	10 4	9 57	0 34	0 36	0 38	0 41	0 45
3 1	3 15	0 51	0 54	0 58	1 3	1 9	10 11	10 4	0 32	0 34	0 36	0 39	0 42
3 9	3 24	0 53	0 56	1 0	1 5	1 12	10 17	10 11	0 30	0 32	0 34	0 37	0 40
3 18	3 34	0 55	0 58	1 2	1 7	1 14	10 23	10 18	0 28	0 30	0 32	0 35	0 38
3 27	3 44	0 56	1 0	1 4	1 9	1 17	10 30	10 25	0 27	0 28	0 30	0 32	0 35
3 36	3 55	0 58	1 2	1 6	1 12	1 19	10 36	10 32	0 25	0 26	0 28	0 30	0 33
3 46	4 07	1 0	1 4	1 8	1 14	1 21	10 42	10 39	0 23	0 24	0 26	0 28	0 31
3 57	4 21	1 2	1 6	1 10	1 16	1 24	10 48	10 45	0 21	0 22	0 24	0 26	0 28
4 3	4 29	1 3	1 7	1 11	1 17	1 25	10 54	10 51	0 19	0 20	0 22	0 24	0 26
4 9	4 37	1 4	1 7	1 12	1 18	1 26	11 0	10 57	0 18	0 19	0 20	0 22	0 24
4 15	4 46	1 4	1 8	1 13	1 19	1 27	11 6	11 3	0 16	0 17	0 18	0 19	0 21
4 22	4 56	1 5	1 9	1 14	1 20	1 28	11 12	11 10	0 14	0 15	0 16	0 17	0 19
4 29	5 9	1 6	1 10	1 15	1 22	1 30	11 17	11 16	0 12	0 13	0 14	0 15	0 17
4 37	5 27	1 7	1 11	1 16	1 23	1 31	11 23	11 22	0 11	0 11	0 12	0 13	0 14
4 46	5 55	1 8	1 12	1 17	1 24	1 32	11 29	11 28	0 9	0 9	0 10	0 11	0 12
4 56		1 9	1 13	1 18	1 25	1 33	11 35	11 35	0 7	0 8	0 8	0 8	0 9
5 8		1 10	1 14	1 19	1 26	1 34	11 41	11 41	0 5	0 6	0 6	0 6	0 7
5 25		1 11	1 15	1 20	1 27	1 35	11 47	11 47	0 3	0 4	0 4	0 4	0 5
5 55		1 12	1 16	1 21	1 28	1 36	11 52	11 52	0 2	0 2	0 2	0 2	0 2
							11 58	11 58	0 0	0 0	0 0	0 0	0 0

At elonga'n, a time error of	20	10	5	1 mins,
causes an azimuth error of	16 to 21	4 to 5	1	0.4 secs.

In all latitudes, each lower culmination follows an upper culmination and precedes the next one by 11 hrs 58 mins, mean solar time. Each E elongation precedes, and each W elongation follows, the corresponding upper culmination by from 5 hrs 57 mins in lat 25° to 5 hrs 54 mins in lat 50°.

Table 3. See parag (11), p 286.

POLAR DISTANCES, AND AZIMUTHS AT ELONGATION.

Year	Mean Polar Dist. of Polaris Jan. 1	Log sin polar dist.	Azimuth at Elongation, in Latitude 25°	30°	35°	40°	45°	50°
	° ′ ″		° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1936	1 2 28	8.25937	1 8 56	1 12 8	1 16 16	1 21 33	1 28 21	1 37 12
1938	1 1 52	8.25514	1 8 16	1 11 26	1 15 32	1 20 46	1 27 30	1 36 15
1940	1 1 16	8.25088	1 7 36	1 10 44	1 14 47	1 19 58	1 26 38	1 35 19
1942	1 0 39	8.24659	1 6 56	1 10 3	1 14 3	1 19 11	1 25 47	1 34 22
1944	1 0 3	8.24226	1 6 16	1 9 21	1 13 19	1 18 24	1 24 56	1 33 26
1946	0 59 27	8.23790	1 5 36	1 8 39	1 12 35	1 17 37	1 24 5	1 32 30
1948	0 58 51	8.23350	1 4 56	1 7 58	1 11 51	1 16 50	1 23 14	1 31 34
1950	0 58 16	8.22906	1 4 17	1 7 16	1 11 7	1 16 3	1 22 23	1 30 38
1952	0 57 40	8.22459	1 3 37	1 6 35	1 10 24	1 15 16	1 21 33	1 29 43
1954	0 57 4	8.22008	1 2 58	1 5 54	1 9 40	1 14 30	1 20 42	1 28 47
1956	0 56 28	8.21554	1 2 19	1 5 13	1 8 56	1 13 43	1 19 52	1 27 52
Log cos latitude....			9.95728	9.93753	9.91336	9.88425	9.84949	9.80807

Owing to variations in the place of Polaris during the year, the azimuths given in table 3 above may be in error by as much as half a minute in Lat 40°, and more for higher latitudes.

Corrections:—The azimuths given in the table are correct for April and October For January, add 24″ in Lat 25°, 30″ in Lat 40°, and 36″ in Lat 50°; for July, subtract the same amounts; obtaining the correction for any intervening month by interpolation.

Having the north polar distance p of Polaris, and the latitude L of the point of observation, the **azimuth** A of Polaris, corresponding to any **siderial-time hour angle,** H, **may be found** by the following formulas:—

$$\tan m = \tan p \cdot \cos H, \text{ then } \tan A = \frac{\sin p \cdot \sin H}{\cos (L + m)}.$$

Sidereal hour angles = 366/355 of the mean-time hour angles of Table 2.

For greater accuracy, instead of Tables 2 or 3, use the above formulas, taking the declination of Polaris (complement of its polar distance) from the American Ephemeris and Nautical Almanac, which gives them to a fraction of a second for each day of the year.

Caution. For greater assurance against error, where great accuracy is desired, it is well to use more than one method and compare the results. For example, observe Polaris both E and W of the meridian, and a star at equal altitudes south of the zenith.

NOTE.—If Polaris be found during twilight, in the morning or evening, observations of it may be made without artificial illumination of the cross-hairs. For times of elongation, see Table 1.

Conversion of Longitude Arc into Time, and vice versa.

Arc	Time	Time	Arc
1°	= 4 minutes	24 hours	= 360°
1′	= 4 seconds	1 hour	= 15°
1″	= 0.066.. second	1 minute	= 0° 15′
		1 second	= 0° 0′ 15″

For relation between time error and corresponding azimuth error, for Polaris, see "At culmination" and "At elongation," p. 288.

THE ENGINEER'S TRANSIT.

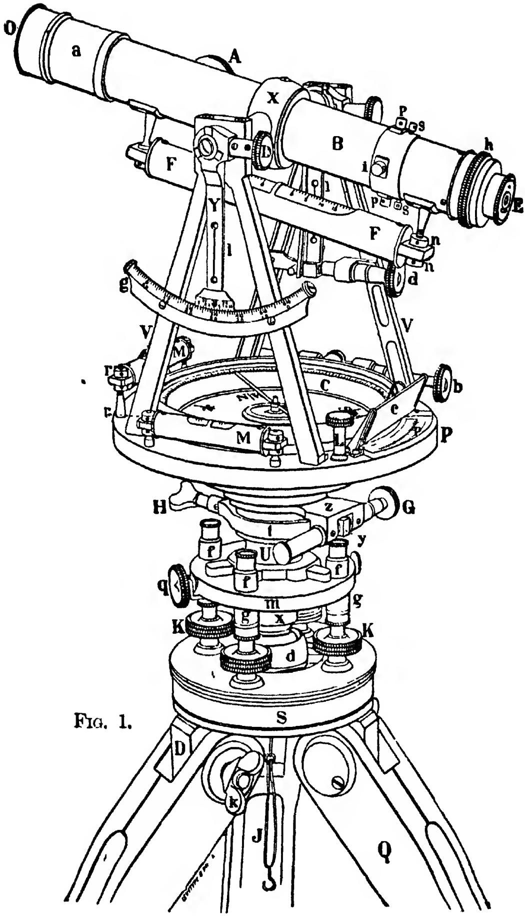


Fig. 1.

THE details of the transit, like those of the level, are differently arranged by diff makers, and to suit particular purposes. Without the **long bubble-tube** F F, Fig 1, under the telescope, and the **graduated arc** *g*, it is the **plain transit.** With these appendages, or rather with a graduated *circle* in place of the *arc*, it becomes virtually a **Complete Theodolite.**

B D D, Fig 2, is the **tripod-head.** The screw-threads at *v* receive the screw of a wooden tripod-head-cover when the instrument is out of use. S S A is the **lower parallel plate.** After the transit has been set very nearly over the center of a stake, the **shifting-plate,** *d d c c*, enables us, by slightly loosening the **levelling-screws** K, to shift the upper parts horizontally a trifle, and thus bring the plumb-bob exactly over the center with less trouble than by the older method of pushing one or two of the legs further into the ground, or spreading them more or less. The screws, K, are then tightened, thereby pushing upward the **upper parallel plate** *m m m x x*, and with it the half-ball *b*, thus pressing *c c* tightly up against the under side of S. The plumb-line passes

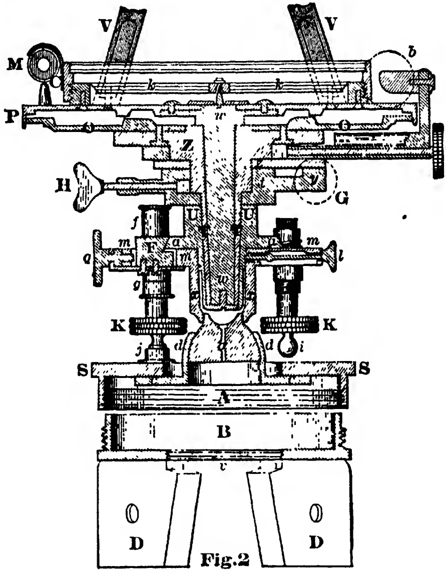


Fig.2

through the vert hole in *b*. Screw-caps, *f, g*, protect the levelling-screws from dust &c. The feet, *i*, of the screws, work in loose sockets, *j*, made flat at bottom to preserve S from being indented.

To set the upper parts upon the parallel plates. Place the lower end of U U in *x x*, holding the instrument so that the three blocks on *m m* (of which the one shown at F is movable) may enter the three corresponding

recesses in *a*, thus allowing *a* to bear fully on *m*, upon which the upper parts then rest. (The inner end of the spring-catch, *l*, in the meantime enters a groove around U, just below *a*, and prevents the upper parts from falling off, if the instrument is now carried over the shoulder.) Revolve the upper parts horizontally a trifle, in either direction, until they are stopped by the striking of a small lug on *a* against one of the blocks F. The recesses in *a* are now clear of the blocks. Tighten *q*, thereby pushing inward the movable block F, which clamps the bevelled flange *a* between it and the two fixed blocks on *m m*, and confines the spindle U to the fixed parallel plates. It remains so clamped while the instrument is being used.

To remove the upper parts from the parallel plates. Loosen *q*, bring the recesses in *a* opposite the blocks F. Hold back *l*, and lift the upper parts, which are then held together by the broad head of the screw inserted into the foot of the spindle *w*.

T T is the **outer revolving spindle,** cast in one with the **supporting-plate** Z Z, to which is fastened the **graduated limb** O O. The limb extends beyond the compass-box, and thus admits of larger graduations than would otherwise be obtainable. *w w* is the **inner revolving spindle.** At its top it has a broad flange, to which is fastened the vernier plate P. To the latter are fastened the compass-box C, the two bubble-tubes M M, the standards V V, supporting the telescope, &c. Each bubble-tube is supported and adjusted by four capstan head nuts, two at each end The bent strip, curving over the tube, protects the glass from accidental blows in swinging the telescope.

Control of motions of graduated limb O O **and vernier plate** P.—The tangent-screw G and a spiral spring (not shown) opposite to it are fixed to the graduated limb O O, and hold between them a projection *y* from the loose collar *t*, which is thus confined to the limb and made to travel with it. The clamp-screw H passes through the collar *t* and presses against the small lug shown at its inner end When H is tightened, this lug is pressed against the fixed spindle U U, to which the graduated limb is thus made fast. A slow motion may, however, still be given to the limb by means of the tangent-screw G

The motion of the vernier plate P over the graduated limb O O is similarly governed by the tangent-screw *b* and its spiral spring (not shown), fixed to the vernier plate P, and the clamp-screw *e*, which passes through the collar *z*, and presses against the small lug shown at its inner end.

There are two verniers One is shown at *p*, Fig 1. Both may be read, and their mean taken, when great accuracy is required. Ivory reflectors, *c*, facilitate their reading. Before the instrument is moved from one place to another, the **compass-needle,** *k*, Fig 2, should always be pressed up against the glass cover of the compass-box by means of the upright milled-head screw seen on the vernier-plate in Fig 1, just to the right of the nearest standard. The pivot-point is thus protected from injury.

R, Fig 1, is a ring with a clamp (the latter not shown) for holding the telescope in any required position. It is best to let the *eye*-end, E, of the telescope revolve *downward*, as otherwise the shade on O, if in use, may fall off. The tangent-screw, *d* moves a vert arm attached to R, and is thus used for slightly changing the elevation of the telescope. In the arm is a slit like that seen in the vernier-arm *l*. By means of the screw D, the movable vernier-arm Y may be clamped at any desired point on the vertical limb *g* When 0° of the vernier is placed at 90° on the arc *g*, and the index of the opposite arm is placed over a small notch on the horizontal brace (not seen in our figs) of the standards, the two slits will be opposite each other, and may be used for laying off offsets, &c, at right-angles to the line of sight.

One end, R, of the telescope axis rests in a movable box, under which is a screw By means of the screw, the box may be raised or lowered, and the axis thus adjusted for very slight derangements of the standards. For E, B, O, and A, see *Level*, p 306. *a* is a dust-guard for the object-slide.

Stadia Hairs. Immediately behind the capstan-screw, *p*, Fig 1, is seen a smaller one. This and a similar one on the opposite side of the telescope, work in a ring inside the telescope, and hold the ring in position. Across the ring are stretched two additional horizontal hairs, called stadia hairs, placed at such a distance apart, vertically, that they will subtend say 10 divisions of a graduated rod placed 100 ft from the instrument, 15 divisions at 150 ft, &c. They are thus used for measuring hor and sloping distances.

The long bubble-tube, F F, Fig 1, enables us to use the transit as a level although it is not so well adapted as the latter to this purpose.

To adjust a plain Transit.

When either a level or a transit is purchased, it is a good precaution (but one which the writer has never seen alluded to) to first screw the object-glass firmly home to its place; and then make a short continuous scratch upon the ring of the glass, and upon its slide; so as to be able to see at any time when at work, that the glass is always in the same position with regard to the slide. For if, after all the adjustments are completed, the position of the glass should become changed, (as it is apt to be if unscrewed, and afterward not screwed up to the same precise spot,) the adjustments may thereby become materially deranged; especially if the object-glass is eccentric, or not truly ground, which is often the case. Such scratches should be prepared by the maker. In making adjustments, as well as when using a transit or level, be careful that the eye-glass and object-glass are so drawn out that there shall be no parallax. The eye-glass must first be drawn out so as to obtain perfect distinctness of the cross-hairs; it must not be disturbed afterward; but the object-glass must be moved for different distances.

First, to ascertain that the bubble-tubes, M M, are placed parallel to the vernier-plate, *and that therefore when both bubbles are in the centers of their tubes the axis of the inst is vert.* By means of the four levelling-screws, K, bring both bubbles to the centers of their tubes in one position of the inst; then turn the upper parts of the inst *half-way* round. If the bubbles do not remain in the center, correct *half* the error by means of the two capstan-nuts *r r;* and the other half by the levelling-screws K. Repeat the trial until both bubbles remain in the center while the inst is being turned entirely around on its spindle.

Second, to see that the standards have suffered no derangement; that is, that they are of equal height and perpendicular to the vernier-plate, as they always are when they leave the maker's hands. Level the inst perfectly; then direct the intersection of the hairs to some point of a high object (as the top of a steeple) near by; clamp the inst by means of screws H and *e*, and lower the telescope until the intersection strikes some point of a low object. (If there is none such drive a stake or chain-pin, &c, in the line.) Then unclamp either H or *e*, and turn the upper parts of the inst half-way round; fix the intersection again upon the high point; clamp; lower the telescope to the low point. If the intersection still strikes the low point, the standards are in order. If not, correct *one-half* of the difference by means of the adjusting-block and screw at the end, R, of the telescope axis, Fig. 1, and repeat the trial *de novo*, resetting the stake or chain-pin at each trial. If the inst has no adjusting-block for the axis, it should be returned to the maker for correction of any derangement of the standards.

A transit may be used *for running straight lines*, even if the standards become slightly bent, by the process described at the end of the fourth adjustment.

Third, to see that the cross-hairs are truly vert and hor when the inst is level. When the telescope *inverts*, the cross-hairs are nearer the eye-end than when it shows objects erect. The maker takes care to place the cross-hairs at right-angles to each other in their ring, or diaphragm; and generally he so places the ring in the telescope, that when levelled, they shall be vert and hor. Sometimes, however, this is neglected; or the ring may by accident become turned a little. To be certain that one hair is vert, (in which case the other must, by construction, be hor,) after having adjusted the bubble-tubes, level the instrument carefully, and take sight with the telescope at a plumb-line, or other vert straight edge. If the vert hair coincides with this object, it is, *so far*, in adjustment; but if not, then loosen *slightly* only *two adjacent* screws of the four, *p p t t*, Fig 1; and with a knife, key, or other small instrument, tap very gently against the screw-heads, so as to turn the ring a little in the telescope; persevering until the hair becomes truly vertical. When this is done, tighten the screws. In the absence of a plumb-line, or vert straight edge, sight the cross-hair at a very small distinct point; and see if the hair still cuts that point, when the telescope is raised or lowered by revolving it on its axis

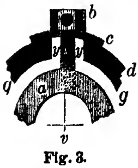


Fig. 3.

The mode of performing the foregoing will be readily understood from this Fig, which represents a section across the top part of the telescope, and at the cross-hairs. The hair-ring, or diaphragm, *a;* vert hair, *v;* telescope tube, *g;* ring outside of telescope tube, *d;* *b* is one of the four capstan-headed screws which hold the hair-ring, *a*, in its place, and also serve to adjust it. The lower ends of these screws work in the thickness of the hair-ring; so that when they are loosened somewhat, they do not lose their hold on the ring. Small loose

washers, c, are placed under the heads b of the screws. A space $y\,y$ is left around each screw where it passes through the telescope tube, to allow the screws and ring together to be moved a little sideways when the screws b are slightly loosened.

Fourth, to see that the vertical hair is in the line of collimation. Plant the tripod firmly upon the ground, as at a Level the inst; clamp it; and direct the vert hair by means of tangent-screw G (figs 1 and 2) upon some convenient object b, or if there is none such, drive a thin stake, or a chain-pin. Then revolving the telescope vert on its axis, observe some object, as c, where the vert hair now strikes, or if there is none, place a second pin. Unclamp the instrument by the clamp-screw H; and turn the whole upper part of it around until the vert hair again strikes b Clamp again; and again revolve the telescope vert on its axis. If the vert hair now strikes c, as it did before, it shows that c is really at o; and that b, a, c, *are in the same straight line*, and therefore this adjustment is in order If not, observe where it does strike, say at m, (the dist $a\,m$ being taken equal to $a\,c$,) and place a pin there also Measure $m\,c$; and place a pin at v, in the line $m\,c$, making $m\,v$ = one-fourth of $m\,c$. Also put a pin at o, half-way between m and c, or in range with a and b. By means of the two hor screws that move the ring carrying the cross-hairs, adjust the vert hair until it cuts v Now repeat the *entire* operation, and persevere until the telescope, after being directed to b, shall strike the same object o, *both times*, when revolved on its axis See whether the movement of the ring in this 4th adjustment has disturbed the verticality of the hair. If it has, repeat the 3d adjustment Then repeat the 4th, if necessary, and so on until both adjustments are found to be right at the same time Thus a straight line may be run, even if the hairs are out of adjustment; but with somewhat more trouble For at each station, as at a, two back-sights, and two fore-sights, $a\,c$ and $o\,m$, may be taken, as when making the adjustment, and the point o, half-way between c and m, will be in the straight line. The inst may then be moved to o, and the two back-sights be taken to a; and so on.

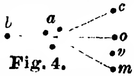
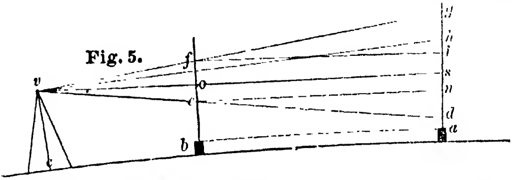


Fig. 4.

Angles measured by the transit, whether vert or hor, will evidently not be affected by the hairs being out of adjustment, provided either that the vert hair is truly vert, or that we use the *intersection* of the hairs when measuring.

The foregoing are all the adjustments needed, unless the transit is required for levelling, in which case the following one must be attended to:

Fig. 5.



To adjust the long bubble-tube, F F Fig 1, we first place the line of sight of the telescope hor, and then make the bubble-tube hor, so that the two are parallel Drive two pegs, a and b Fig 5, with their tops at precisely the same level (see Rem. p. 296) and at least about 100 ft apart; 300 or more will be better Plant the inst firmly, in range with them, as at c, making $b\,c$ an aliquot part of $a\,b$, and as short as will permit focusing on a rod at b. The inst need not be leveled Suppose the line of sight to cut e and d. Take the readings $b\,e$ and $a\,d$. Their diff is $b\,e - a\,d = a\,n - a\,d = d\,n$; and $a\,b : a\,c :: d\,n : d\,s$; s being the height of the target at a when the readings ($a\,s$, $b\,o$) on the two stakes are equal. $a\,s = a\,d + d\,s = a\,d + \frac{d\,n \times a\,c}{a\,b}$. If the reading on a *exceeds* that on b (as when the line of sight is $v\,f\,g$) the diff of readings is $= a\,g - b\,f = a\,g - a\,i = g\,i$; and $a\,s = a\,g - g\,s = a\,g - \frac{g\,i \times a\,c}{a\,b}$. Sight to s, bring the bubble to the cen of its tube by means of the two small nuts $n\,n$ at one end of the tube, Fig. 1, and assume that the telescope and tube are parallel.* The zeros of

* To correct for earth curvature and for atmosferic refraction (see p 153), make $s\,h$, ft, = 0 000 000 020 5 × ($a\,c$, ft)², and set target at h. (When $a\,c$ = 1,000 ft, $s\,h$ = 0.0205 ft.)

the vert circle, and of its vernier, may now be adjusted, if they require it, by loosening the vernier screws and then moving the vernier until the two coincide.

Rem. If no level is at hand for levelling the two pegs *a* and *b*, it may be done by the transit itself, thus. Carefully level the two short bubbles, by means of the levelling-screws K. Drive a peg *m*, from 100 to 300 feet from the instrument *o*. Then placing a target-rod on *m*, clamp the target tight at whatever height, as *e v*, the hor hair happens to cut it; it being of no importance whether the telescope is level or not; although it might as well be as nearly so as can conveniently be guessed at. Clamp the telescope in its position by the clamp-ring R, Fig. 1. Revolve the inst a considerable way round; say nearly or quite half way. Place another peg *n* *at precisely the same dist* from the instrument that *m* is; and continue to drive it until the hor hair cuts the target placed on it, and still kept clamped to the rod, at the same height as when it was on *m*. When this is done, the tops of the two pegs are on a level with each other, and are ready to be used as before directed.

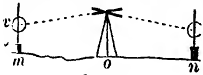
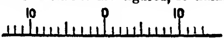


Fig. 6.

When a transit is intended to be used for surveying farms, &c, or for retracing lines of old surveys, it is very useful to set the compass so as to allow for the "variation" during the interval between the two surveys. For this purpose a **"variation-vernier"** is added to such transits; and also to the compass.

When the graduations of a transit are figured, or numbered, so as to read both ways from zero, thus, 

the vernier also is made double; that is, it also is graduated and numbered from its zero both ways. In this case, if the angle is measured from zero toward the right hand, the reading must be made from the right hand half of the vernier, and vice versa. If the figuring is single, or only in one direction, from zero to 360°, then only the single vernier is necessary, as the angles are then measured only in the direction that the figuring counts. Engineers differ in their preferences for various manners of figuring the graduations. The writer prefers from zero each way to 180°, with two double verniers.

To replace cross-hairs in a level, or transit. Take out the tube from the eye end of the telescope. Looking in, notice which side of the cross-hair diaphragm is turned toward the eye end. Then loosen the four screws which hold the diaphragm, so as to let the latter fall out of the telescope. Fasten on new hairs with beeswax, varnish, glue, or gum-arabic water, &c. This requires care. Then, to return the diaphragm to its place, press firmly into one of the screw-holes on the circumf of the diaphragm itself, the end of a piece of stick, long enough to reach easily into the telescope as far as to where the diaphragm belongs. By this stick, as a handle, insert the diaphragm edgewise to its place in the telescope and hold it there until two *opposite* screws are put in place and screwed. Then draw the stick out of the hole in the diaphragm; and with it turn the diaphragm until the same side presents itself toward the eye end as before; then put in the other two screws.

The so-called cross hairs are actually spider-web, so fine as to be barely visible to the naked eye.

To replace a spirit-level, or bubble-glass. Detach the level from the instrument; draw off its sliding ends; push out the broken glass vial, and the cement which held it; insert the new one, with the proper side up (the upper side is always marked with a file by the maker), wrapping some paper around its ends, if it fits loosely. Finally, put a little putty, or melted beeswax over the ends of the vial, to secure it against moving in its tube.

In purchasing instruments, especially when they are to be used far from a maker, it is advisable to provide extras of such parts as may be easily broken or lost; such as glass compass-covers, and needles; adjusting pins; level vials; magnifiers, &c.

Theodolite adjustments are performed like those of the level and transit.

1st. That of the cross-hairs; the same as in the level.

2d. The long bubble-tube of the telescope; also as in the level.

3d. The two short bubble-tubes; as in the transit.

4th. The vernier of the vert limb; as in the transit with a vert circle.

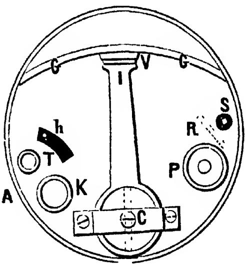
5th. To see that the vert hair travels vertically; as in the fourth adjustment of the transit. In some theodolites, no adjustment is provided for this; but in large ones it is provided for by screws under the feet of the standards.

Sometimes a second telescope is added; it is placed below the hor limb, and is

called a *watcher*. It has its own clamp, and tangent-screw. Its use is to ascertain whether the zero of that limb has moved during the measurement of hor angles. When, previously to beginning the measurement, the zero and upper telescope are directed toward the first object, point the lower telescope to any small distant object, and then clamp it. During the subsequent measurement, look through it, from time to time, to be sure that it still strikes that object; thus proving that no slipping has occurred.

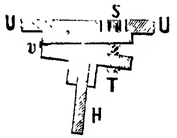
THE BOX OR POCKET SEXTANT.

THE portability of the pocket sextant, and the fact that it reads to single minutes, render it at times very useful to the engineer. By it, angles can be measured while in a boat, or on horseback; and in many situations which preclude the use of a transit. It is useful for obtaining latitudes, by aid of an artificial horizon. When closed, it resembles a cylindrical brass box, about 3 inches in diameter, and 1¼ inches deep. This box is in two parts: by unscrewing which, then inverting one part, and then screwing them together again, the lower part becomes a handle for holding the instrument. Looking down upon its top when thus arranged, we see, as in this figure, a movable arm I C, called the **index**, which turns on a center at C, and carries the vernier V at its other end. G G is the graduated arc or **limb.** It actually subtends about 73°, but is divided into about 146°. Its zero is at one end. Its graduations are not shown in the Fig.



Attached to the index is a small movable lens, (not shown in the figure,) likewise revolving around C, for reading the fine divisions of the limb. When measuring an angle, the index is moved by turning the milled-head P of a pinion, which works in a rack placed within the box. The eye is applied to a circular hole at the side of the box, near A. A small telescope, about 3 inches long, accompanies the instrument: but may generally be dispensed with. When so, the eye-hole at A should be partially closed by a slide which has a very small eye-hole in it; and which is moved by the pin *h*, moving in the curved slot. Another slide, at the side of the box, carries a dark glass for covering the eye-hole when observing the sun. When the telescope is used, it is fastened on by the milled-head screw T. The top part shown in our figure, can be separated from the cylindrical part, by removing 3 or 4 small screws around its edge; and the interior can then be examined, and cleaned if necessary. Like nautical, and other sextants, this one has two principal glasses, both of them mirrors. One, the **index-glass**, is attached to the underside of the index, at C; its upper edge being indicated by the two dotted lines. The other, the **horizon-glass**, (because, when measuring the vert angles of celestial bodies, it is directed toward the horizon,) is also within the box; the position of its upper edge being shown by the dotted lines at R. The horizon-glass is silvered only half-way down; so that one of the observed objects may be seen directly through its lower half, while the image of the other object is seen in the upper half, reflected from the index-glass. That the instrument may be in adjustment, ready for use, these two glasses must be at right angles to the plane of the instrument, that is, to the under side of the top of the box, to which they are attached; and must also be parallel to each other, when the zeros of the vernier and of the limb coincide. The index-glass is already permanently fixed by the maker, and requires no other adjustment. But the horizon-glass has two adjustments, which are made by a key like that of a watch, and having a milled-head K. It is screwed into the top of the box, so as to be always at hand for use. When needed, it is unscrewed. This key fits upon two small square-heads, (like that for

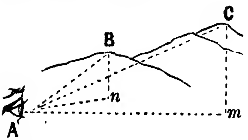
winding a watch;) one of which is shown at S; while the other is near it, but on the SIDE of the box. These squares are the heads of two small screws. If the horizon glass H should, as in this sketch, (where it is shown endwise,) not be at right angles to the top U U of the box, it is brought right by turning the square-head S of the screw S T; and if, after being so far rectified, it still is not parallel to the index-glass when the zeros coincide, it is moved a little backward or forward by the square head at the side.



To adjust a box sextant, bring the two zeros to coincide precisely; then look through the eye-hole, and the lower or unsilvered part of the horizon-glass, at some distant object. If the instrument is in adjustment, the object thus seen directly, will coincide precisely with its reflected image, seen at the same time, at the same spot. But if it is not in adjustment, the two will appear separated either hor or vert, or both, thus, * *; in which case apply the key K to the square-head S; and by turning it slightly in whichever direction may be necessary, *still looking at the object and its image*, bring the two into a hor position, or on a level with each other, thus, * *. Then apply the key to the square-head in the side of the box; and by turning it slightly, bring the two to coincide perfectly The instrument is then adjusted.

In some instruments, the hor glass has a *hinge* at *v*, to allow it play while being adjusted by the single screw S T; but others dispense with this hinge, and use *two* screws like S on top of the box, in addition to the one in the side.

If a sextant is used for measuring vert angles by means of an **artificial horizon**, the actual altitude will be but one-half of that read off on the limb; because we then read at once both the actual and the reflected angle. The great objection to the sextant for engineering purposes, is that it does not measure angles horizontally, as the transit does; unless when the observer, and the two objects happen to be in the same hor plane. Thus an observer with a sextant at A, if measuring the angle subtended by the mountain-peaks B and C, must hold the graduated plane of the sextant in the plane of A B C; and must actually measure the angle B A C: whereas what he wants is the hor angle *n* A *m*. This is greater than B A C, because the dists A *n* and A *m* are shorter than A B and A C. The transit gives the hor angle *n* A *m*, because its graduated plane is first fixed hor by the levelling-screws, and the subsequent measurement of the angle is not affected by his directing merely the line of sight upward, to any extent, in order to fix it upon B and C. For more on this subject; and for a method of partially obviating this objection to the sextant, see the note to Example 2, Case 4, of "Trigonometry."



The nautical sextant, used on ships, is constructed on the same principle as the box sextant; and its adjustments are very similar. In it, also, the index-glass is permanently fixed by the maker; and the horizon-glass has the two adjustments of the box sextant. It also has its dark glasses for looking at the sun; and a small sight-hole, to be used when the telescope is dispensed with.

THE COMPASS.

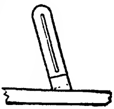
To adjust a Compass.

The first adjustment is that of the bubbles. Plant firmly; and level the instrument, in any position; that is, bring the bubbles to the centers of their tubes. Then turn the instrument half-way round If the bubbles then remain at the centers, they are in adjustment; but if not, correct *one-half* the diff in each bubble, by means of the adjusting-screws of the tubes. Level the instrument again; turn it half round; and if the bubbles still do not remain at the center, the adjusting-screws must be again moved a little, so as to rectify half the remaining diff. Gener

ally several trials must be thus made, until the bubbles will remain at the center while the compass is being turned entirely around.

Second adjustment. Level the compass, and then see that the needle is hor; and if not, make it so by means of the small piece of wire which is wrapped around it; sliding the wire toward the high end. A needle thus horizontally adjusted at one place, will not remain so if removed far north or south from that place. If carried to the north, the north end will dip down; and if to the south, the south end will do so. The sliding wire is intended to counteract this.

Third adjustment. This is always fixed right at first by the maker; that is, the sights, or slits for sighting through, are placed at right angles to the compass plate; so that when the latter is levelled by the bubbles, the sights are vert To test whether they are so, hang up a plumb-line; and having levelled the compass, take sight at the line, and see if the slits coincide with it. If one or both slits should prove to be out of plumb, as shown to an exaggerated extent in this sketch, it should be unscrewed from the compass, and a portion of its foot on the high side be filed or ground off, as per the dotted line; or as a temporary expedient, a small wedge may be placed under the low side, so as to raise it.

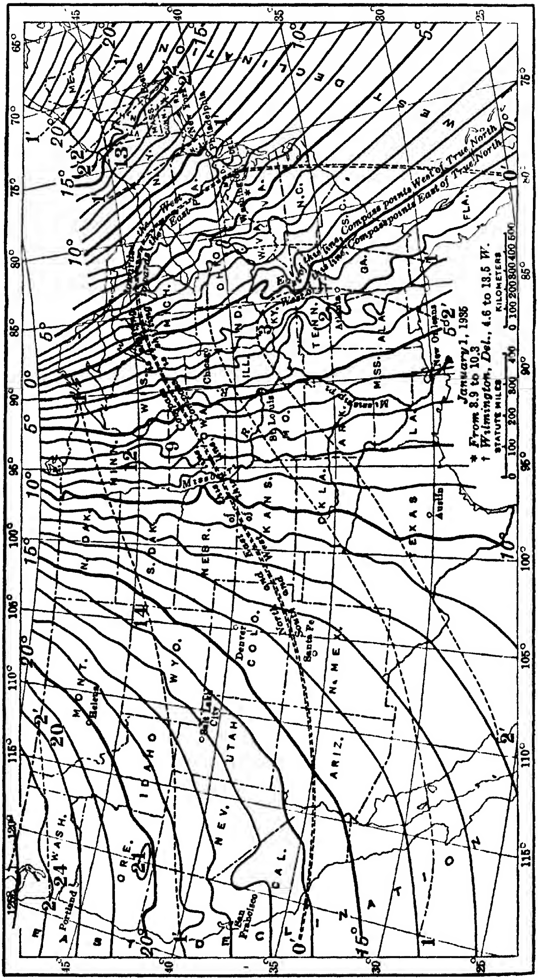


Fourth adjustment, to straighten the needle, if it should become bent The compass being levelled, and the needle hor, and loose on its pivot, see whether its two ends continue to point to exactly opposite graduations, (that is, graduations 180° apart,) while the compass is turned completely around. If it does, the needle is straight; and its pin is in the center of the graduated circle; but if it does not then one or both of these require adjusting. First level the compass. Then turn it until some graduation (say 90°) comes precisely to the north end of the needle. If the south end does not then point precisely to the opposite 90° division, lift off the needle, and bend the *pivot-point* until it does, remembering that every time said point is bent, the compass must be turned a hairsbreadth so as to keep the *north* end of the needle at its 90° mark. Then turn the compass half-way round, or until the *opposite* 90° mark comes precisely to the north end of the needle. Make a fine pencil mark where the *south* end of the needle now points. Then take off the needle, and bend it until its south end points *half-way between* its 90° mark and the pencil mark, while its north end is kept at 90° by moving the compass round a hairsbreadth. The needle will then be straight, and must not be altered in making the following adjustment, although it will not yet cut opposite degrees.

Fifth adjustment, of the pivot-pin. After being certain that the needle is straight, turn the compass around until a part is arrived at where the two ends of the needle happen to cut opposite degrees Then turn the compass *quarter way* around, or through 90°. If the needle then cuts opposite degrees, the pivot-point is already in adjustment, but if the needle does not so cut, bend the *pivot-point* until it does. Repeat, if necessary, until the needle cuts opposite degrees while being turned entirely around

Care and nicety of observation are necessary in making these adjustments properly; because the entire error to be rectified is, in itself, a minute quantity; and the novice is very apt to increase his trouble by not knowing how to use his **magnifier,** when looking at the end of the needle and the corresponding graduations. The magnifier must always be held with its *center directly over* the point to be examined; and it must be held parallel to the graduated circle. Otherwise annoying errors of several minutes will be made in a single observation; and the accumulation of two or three such errors, arising from a cause unknown to him, may compel him to abandon the adjustments in despair This suggestion applies also to the reading of angles taken by the transit, &c; although the errors are not then likely to be so great as in the case of the compass. In purchasing a magnifier for a compass, see that no part of it, as hinges, or rivets, are made of iron; for such would change the direction of the needle.

If the sight-slits of a compass are not fixed by the maker in line with the two opposite zeros, the engineer cannot remedy the defect. This can be ascertained by passing a piece of fine thread through the slits, and observing whether it stands precisely over the zeros.


West of this line, Compass points West of True North
East of this line, Compass points East of True North
January 1, 1935
* From 8.9 to 10.3
† Wilmington, Del., 4.6 to 13.5 W.
STATUTE MILES
KILOMETERS
Philadelphia
Boston
Chicago
St. Louis
New Orleans
Austin
Denver
Santa Fe
Salt Lake City
Helena
Portland
San Francisco

MAGNETIC DECLINATION AND VARIATION

The Isogonic Chart opposit is reduced from a large chart for 1935 (latest up to 1937), publisht by the U S Coast & Geodetic Survey.

Ordinarily, the compass needle does not lie exactly in the meridian, *i e*, not in a true N & S line. Its angle of deviation from that line is called the **magnetic declination.** This is different in different places, and, at any given place, it varies from time to time.

Variations with Place

Apart from local attractions causd by iron or steel in buildings, track, machines, etc, in the soil or in mines, in ferruginus gravel, trap rock, etc, the average declination varies with markt regularity from place to place, starting from the "**agonic line**", or line of no declination, markt 0°, entering the country in Michigan, and leaving along the South Carolina-Georgia boundary. Along this agonic line, the needle normally points approx N & S; but at other places the north end of the needle tends away from the true north, toward that line. At places NE of the agonic line, the declination is west, and *vice versa;* and this declination increases with the dist from the agonic line, reaching at present 22° W in Maine, and 24° E in the state of Washington. **The heavy solid lines** on the chart, join points of equal declination, and are called **isogonic lines.** The figures scatterd over the chart (not along the margins or boundaries) give the approx local declinations, in degrees, where the changes of declination are so abrupt that it has been impracticable to indicate them by isogonic lines. See also notes at foot of chart.

Variation with Time

Secular Variation. At any given point, there may be a gradual change in the average declination. The **Annual Variation** is the amount of such secular variation each year, and is shown by **lines of equal annual change** which are dotted, markt 1′, 2′, etc. The **line of no annual change** (shown double and dotted, and markt 0′) appears in southern California, extends E and NE to the Great Lakes, and thence down the Atlantic Coast to the tip of Florida.

South of the line of no annual change, the average position of the compass needle is gradually shifting toward the east (*i e*, the E declination is increasing); and *vice versa.* It is impossible to predict with accuracy what the secular motion will be, for even a few years in advance.

Diurnal Variation. At any given place, the needle swings back and forth once daily, thru an arc sometimes as great as 12′ or 15′, reaching its extreme eastern position generally between 8 and 8:30 A M; its extreme western position generally betw 1:15 and 1:30 P M; and its mean position, or normal declination, generally betw 10 and 11:30 A M and betw 7 and 8 P M. The eastern extreme is reacht about half an hour earlier in summer and half an hour later in winter, and the western extreme about a quarter hour earlier in summer, and a quarter hour later in winter.

The total arc described between the eastern and western positions, or betw morning and afternoon, is called the **diurnal range.** At Philadelphia, this range varies from 5′ in midwinter to 12′ in midsummer; at Key West from 3′ to 8′; and at Los Angeles from 4′ to 9′ respectivly. In years of least sun-spot activity, the range is about 20% less, and in years of greatest sun-spot activity about 30% greater than the av range; but the diurnal variation is very irregular, especially in seasons of small diurnal range.

Other Variations. The average position of the needle varies slightly each month and also each year. Magnetic storms frequently cause temporary deflections of 15′ or more from the normal position of the needle. Machines moving nearby may cause variations of many degrees. Indeed, the compass should hardly be depended upon within close range of the works of civilization, tho it may be of value as a rough check on angle mesurements with the transit.

Electricity, either atmospheric, or excited by rubbing the glass cover of the compass box, sometimes gives trouble. It may be removed by touching the glass with the moist tongue or finger.

DEMAGNETIZATION.

The needle, if of *soft* metal, sometimes loses part of its magnetism, and consequently does not work well. It may be restored by simply drawing the north pole of a common magnet (either straight or horseshoe) about a dozen times, from the center to the end of the south half of the needle; and the south pole, in the same way, along the north half, pressing the magnet gently upon the needle. After each stroke, remove the magnet several inches from the needle, while bringing it back to the center for making another stroke. Each half of the needle in turn, while being thus operated on, should be held flat upon a smooth hard surface. Sluggish action of the needle is, however, more generally produced by the dulling or other injury of the point of the pivot Remagnetizing will throw the needle out of balance, which must be counteracted by the sliding wire

In order to prevent mistakes by reading sometimes from one end, and sometimes from the other end of the needle, it is best *always* to point the N of the compass-box toward the object whose bearing is to be taken; and to read off from the north end of the needle. This is also more accurate.

CONTOUR LINES.

A CONTOUR LINE is a curved hor one, every point in which represents the same level; thus each of the contour lines 88*c*, 91*c*, 94*c*, &c, Fig 1, indicates that every point in the ground through which it is traced is at the same level, and that that level or height is everywhere 88, 91, or 94 ft above a certain other level or height called *datum;* to which all others are referred.

Frequently the level of the starting point of a survey is taken as being 0, or zero, or datum; and if we are sure of meeting with no points lower than it, this answers every purpose. But if there is a probability of many lower points, it is better to assume the starting point to be so far above a certain supposed datum, that none of these lower points shall become minus quantities, or *below* said supposed datum or zero. The only object in this is to avoid the liability to error which arises when some of the levels are +, or plus; and some −, or minus. Hence we may assume the level of the starting point to be 10, 100, 1000, &c, ft above datum, according to circumstances.

The vert dists between each two contour lines are supposed to be equal; and in railroad surveys through well-known districts, where the engineer knows that his actual line of survey will not require to be much changed, the dist may be 1 or 2 ft only; and the lines need not be laid down for widths greater than 100 or 200 ft on each side of his center-stakes. But in regions of which the topography is comparatively unknown; and where consequently unexpected obstacles may occur which require the line to be materially changed for a considerable dist back, the observations should extend to greater widths; and for expedition the vertical dists apart may be increased to 3, 5, or even 10 ft, depending on the character of the country, &c. Also, when a survey is made for a topographical map of a State, or of a county, vert dists of 5 or 10 ft will generally suffice.

Let the line A B, Fig 1, starting from 0, represent three stations (S 1, S 2, S 3,) of the center line of a railroad survey; and let the numbers 100, 103, 101, 104, along that line denote the heights at the stakes above datum, as determined by levelling. Then the use of the contour lines is to show in the office what would be the effect of changing the surveyed center line A B, by moving any part of it

to the right or left hand.* Thus, if it should be moved 100 ft to the left, the starting point 0 would be on ground about 6 ft higher than at present; inasmuch as its level would then be about 106 ft above datum, instead of 100. Station 1 would be about 7 ft higher, or 110 ft instead of 103. Station 2 would be about 7 ft higher, or 108 ft instead of 101. If the line be thrown to the right, it will plainly be on lower ground.

The field observations for contour lines are sometimes made with the spirit-level; but more frequently by a slope-man, with a straight 12-ft graduated rod, and a slope instrument, or clinometer. At each station he lays his rod upon the ground, as

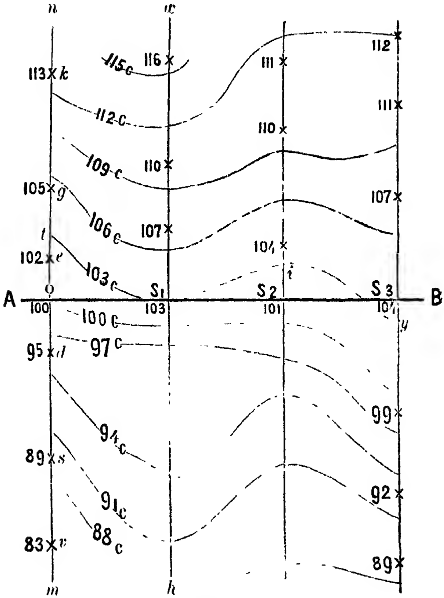


Fig. 1.

nearly at right angles to the center line A B as he can judge by eye; and placing the slope instrument upon it, he takes the angle of the slope of the ground to the nearest ¼ of a degree. He also observes how far beyond the rod the slope continues the same; and with the rod he measures the dist Then laying down the rod at that point also, he takes the next slope, and measures its length; and so on as far as may be judged necessary. His notes are entered in his field book as shown in Fig 2; the angles of the slopes being written above the lines, and their lengths below; and should be accompanied by such remarks as the locality suggests; such as woods rocks, marsh, sand, field, garden, across small run, &c, &c.

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* In thus using the words right and left we are supposed to have our backs turned to the starting point of the survey. **In a river, the right bank** or shore is that which is on the right hand as we **descend** it, that is, in speaking of its right or left bank, we are supposed to have our backs turned towards its head, or origin; and so with a survey

It is not absolutely necessary to represent the slopes roughly in the field-book, as in Fig 2; for by using the sign + to signify "up;" —"down;" and = "level," the slopes may be written in a straight line, as in Fig 2½.

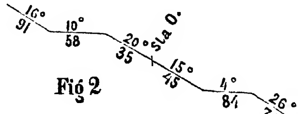


Fig 2

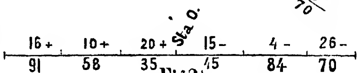


Fig 2½

The notes having been taken, the preparation of the contour lines by means of them, is of course office-work; and is usually done at the same time as the drawing of the map, &c. The field observations at each station are then separately drawn by protractor and scale as shown in Fig 3 for the starting point O. The scale should not be less than about $\frac{1}{12}$ inch to a ft, if anything like accuracy is aimed at. Suppose that at said station the slopes to the right, taken in their order, are, as in Fig 2, 15°, 4°, and 26°; and those to the left, 20°, 10°, and 16°; and their lengths as in the same Fig. Draw a hor line *h o*, Fig 3; and consider the center of it to be the station-stake. From this point as a center, lay off these angles with a protractor, as shown on the arcs in Fig 3. Then beginning say on the right hand, with a parallel ruler draw the first dist *a c*, at its proper slope of 15°; and of its proper length, 45 ft, by scale. Then the same with *c y* and *y t*. Do the same with those on the left hand. We then have a *cross-section* of the ground at Sta O Then on the map, as in Fig 1, draw a line as *m n*, or *h w*, at right angles to the line of road, and passing through the station-stake. On this line lay down the *hor* dists *a d*, *d s*, *s v*, *a e*, *e g*, *g k*, marking them with a small star, as is done and lettered in Fig 1, at Sta O.

When extreme accuracy is pretended to, these hor dists must be found by measure on Fig 3; but as a general rule it will be near enough, when the slopes do not exceed 10°, to assume them to be the same as the sloping dists measured in the field. Next ascertain how high each of the points *c y t l n i* is above datum Thus, measure by scale the vert dist *d c* Suppose it is found to be 5 ft; or in other words, that *c* is 5 ft *below* station-stake O. Then since the level at stake O is 100 ft above datum, that at *c* must be 5 ft *less*, or 100 — 5 = 95 ft above datum; which may be marked in light lead-pencil figures on the map, as at *d*, Fig 1. Next for the point *y*, suppose we find *s y* to be 11 ft, or *y* to be 11 ft *below* stake O; then its height above datum must be 100 — 11 = 89: which also write in pencil, as at *s* Proceed in the same way with *t*. Next going to the left hand of the station-stake, we find *e l* to be say 2 ft; but *l* is *above* the level of the station-stake, therefore its height above datum is

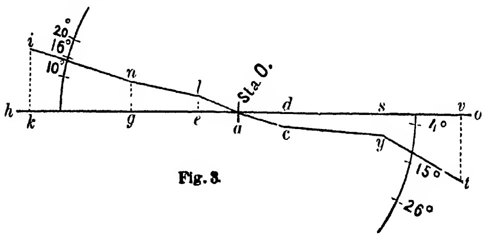


Fig. 3.

100 + 2 = 102 ft, as figured at *e* on the map. Let *n g* be 5 ft; then is *n*, 100 + 5 = 105 ft above datum, as marked at *g*; and so on at each station. When this has been done at several stations, we may draw in the contour lines of that portion by hand thus: Suppose they are to represent vert heights of 3 ft. Beginning at Station O (of which the height above datum is 100 ft) to lay down a contour line 103 ft above datum, we see at once that the height of 103 ft must be at *l*, or at ⅓ the dist from *e* to *g*. Make a light lead-pencil dot at *t*; and then go to the next Station 1. Here we see that the height of 103 ft coincides with the station-stake itself; place a dot there, and go to Sta 2. The level at this stake is 101; therefore the contour for 103

it must evidently be 2 ft higher, or at *i*, ⅔ of the dist from Sta 2 to +104; therefore make a dot at *i*. Then go to Sta 3. Here the level being 104 above datum, the contour of 103 must be at *y*, or $\frac{1}{5}$ of the dist from Sta 3 to +99; put a dot at *y*. Finally draw by hand a curving line through *t*, S1, *i*, and *y*; and the contour line of 103 ft is done. All the others are prepared in the same way, one by one. The level of each must be figured upon it at short intervals along the map, as at 103 *c*, 106 *c*, &c.

Or, instead of first placing the + points on the map, to denote the slope dists actually measured upon the ground, we may at once, and with less trouble, find and show those only which represent the points *t*, S 1, *i*, *y*, &c, of the contours themselves. Thus, say that at any given station-stake, Fig 4, the level is 104; that the cross-section *c s* of the ground has been prepared as before; and that we want the hor dists from the stake, to contour lines for 94, 97, 100 ft, &c, 3 ft apart vert.

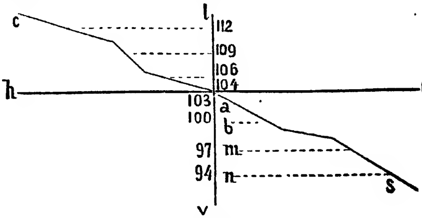


Fig. 4.

Draw a vert line *v l*, through the station-stake, and on it by scale mark levels of 94, 97, 100, &c. ft. This is readily done, inasmuch as we have the level 104 of the stake already given. Through these levels draw the hor lines *a*, *b*, *m*, *n*, &c. to the ground-slopes. Then these lines, measured by the scale, plainly give the required dists.

When the ground is very irregular transversely, the cross-sections must be taken in the field nearer together than 100 ft. The preparation of contour lines will be greatly facilitated by the use of paper ruled into small squares of not less than about $\frac{1}{12}$ inch to a side, for drawing the cross-sections upon.

When the ground is very steep, it is usual to shade such portions of the map to represent hill-side. The closer together the contours come, the steeper of course is the ground between them; and the shading should be proportionally darker at such portions. But for *working* maps it is best to omit the shading.

In surveys of wide districts, the transit instrument with a graduated vertical circle or arc, *g*, p 291, is used for measuring the angles of slope, instead of the common slope-instrument.

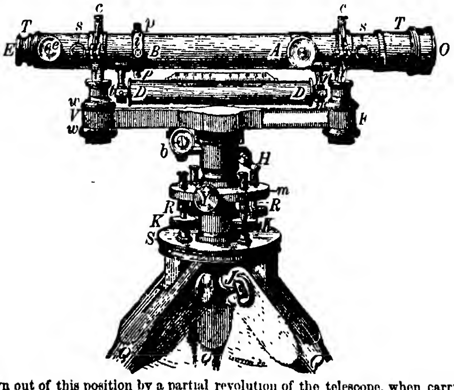
In many cases, notes similar to the following will serve the purpose of contour lines on railroad surveys.

Sta 60.. .	....— 3 1 R	+ 2 1 L.
61... ..	.... + 2. 2 R.	— 1 3 L.
62	..... = 1. R.	+ 4. 2 L.
63	"	"

Which means that at station 60, the slope of the ground on the right, as nearly as he can judge by eye, or by his hand-level, is about 3 ft downward for 1 chain, or 100 ft; and on the left, about 2 ft upward in 1 chain. At 61, 2 ft up, in 2 chains to the right; and 1 ft down in 3 chains to the left. At 62, level for 1 chain to the right, and ascending 4 ft in 2 chains to the left. At 63, the same as at 62. At some spots it will be well to add a sketch of a cross-section, like Fig 2; only, instead of the angles, use ft of rise or fall, to indicate the slopes, as judged by eye, or by a hand-level. By this method, the result at every station will be somewhat in error but these small errors will balance each other so nearly that the total may be regarded as sufficiently correct for all the purposes of a preliminary estimate of the cost of a road. When the final stakes for guiding the workmen are placed the slopes should be carefully taken, in order to calculate the quantity of excavation accurately for payment.

THE LEVEL.

ALTHOUGH the levels of different makers vary somewhat in their details, still their principal parts will be understood from the following figure. The telescope T T rests upon two supports YY, called Ys, out of which it can be lifted, first removing the pins *s s* which confine the semicircular *clips c c*, and then opening the clips. The pins should be tied to the Ys, by pieces of string, to prevent their being lost. The *slide* of the *object-glass* O, is moved backward or forward by a rack and pinion, by means of the milled head A. The slide of the *eye-glass* E, is moved in the same way by the milled head *e*. A cylindrical tube of brass, called a *shade*, is usually furnished with each level. It is intended to be slid on to the object-end O of the telescope, to prevent the glare of the sun upon the object-glass, when the sun is low. At B is an outer ring encircling the telescope, and carrying 4 small capstan-headed screws; two of which, *p p*, are at top and bottom; while the other two, of which *i* is one, are at the sides, and at right angles to *p p*. Inside of this outer ring is another, inside of the telescope, and which has stretched across it two spider-webs, usually called the CROSS-HAIRS. These are much finer than they appear to be, being considerably magnified. They are at right angles to each other; and, in levelling, one is kept vert, and the other hor. They are liable at times to be



thrown out of this position by a partial revolution of the telescope, when carrying the level, or when setting the tripod down suddenly upon the ground; but since, in levelling, the *intersection* of the hairs is directed to the target-rod, this derangement does not affect the accuracy of the work. Still it is well to keep them nearly vert and hor, by keeping the BUBBLE-TUBE D D as nearly directly over the bar V F as can be judged by eye. This enables the leveller to see that the rod-man holds his rod nearly vert, which is absolutely essential for correct levelling. If perfect verticality is desired, as is sometimes the case, when staking out work, it may be obtained (*if the instrument is in perfect adjustment, and levelled*) by sighting at a plumb-line, or other vert object, and then turning the telescope a little in its Ys, so as to bring the hair to correspond. When this is done, a short continuous scratch may be made on the telescope and Y, to save that trouble in future.

The small holes around the heads of the 4 small capstan-screws *p*, *i*, just referred to, are for admitting the end of a small steel pin, or lever, for turning them. If first the upper screw *p* be loosened and then the lower one tightened, the interior ring will be lowered, and the horizontal hair with it. But on looking through the tele-

scope they will *appear* to be raised. If first the lower one be loosened, and the upper one tightened, the hor hair will be actually raised, but apparently lowered. This is because the glasses in the eye-piece E reverse the apparent position of objects *inside* of the telescope; which effect is obviated, as regards *exterior* objects, by means of the object-glass O. This must be remembered when adjusting the cross-hairs; for if a hair appears to strike too high, it must be raised still higher; if it appears to be already too far to the right or left, it must be actually moved still more in the same direction.

This remark, however, does not apply to telescopes which make objects appear inverted.

There is no danger of injuring the hairs by these motions, inasmuch as the four screws act against the ring only, and do not come in contact with the hairs themselves

Under the telescope is the BUBBLE-TUBE D D One end of this tube can be raised or lowered slightly by means of the two capstan-headed nuts *n n*, one of which must be loosened before the other is tightened. On top of the bubble-tube are scratches for showing when the bubble is central in the tube. Frequently these scratches, or marks, are made on a strip of brass placed above the tube, as in our fig There are several of them, to allow for the lengthening or shortening of the bubble by changes of temperature. At the other end of the bubble-tube are two small capstan-screws, placed on opposite sides horizontally The circular head of one of them is shown near *t*. By means of these two screws, that end of the tube can be slightly moved hor, or to right or left Under the bubble-tube is the BAR V F; at one end of which, as at V, are two large capstan-nuts *w w*, which operate upon a stout interior screw which forms a prolongation of the Y. The holes in these nuts are larger than the others, as they require a larger lever for turning them. If the lower nut is loosened and the upper one tightened, the Y above is raised; and that end of the telescope becomes farther removed from the bar; and vice versa. Some makers place a similar screw and nuts under both Ys; while others dispense with the nuts entirely, and substitute beneath one end of the bar a large circular milled head, to be turned by the fingers This, however, is exposed to accidental alteration, which should be avoided.

When the portions above *m* are put upon *m*, and fastened by the screw **Y**, all the upper part may be swung round hor, in either direction, by loosening the **clamp-screw** H; or such motion may be prevented by tightening that screw. It frequently happens, after the telescope has been sighted very nearly upon an object, and then clamped by H, that we wish to bring the cross-hairs to coincide more precisely with the object than we can readily do by turning the telescope *by hand;* and in this case we use the **tangent-screw** *b*, by means of which a slight but steady motion may be given after the instrument is clamped. For fuller remarks on the clamp and tangent-screws, see "Transit."

The **parallel plates** *m* and S are operated by four **levelling-screws;** three of which are seen in the figure, at K K. The screws work in sockets R; which, as well as the screws, extend above the upper plate. When the instrument is placed on the ground for levelling, it is well to set it so that the lower parallel plate S shall be as nearly horizontal as can be roughly judged by eye; in order to avoid much turning of the levelling screws K K in making the upper plate *m* hor. The lower plate S, and the brass parts below it, are together called the **tripod-head;** and, in connection with three wooden legs Q Q Q, constitute the tripod. In the figure are seen the heads of wing-nuts J which confine the legs to the tripod-head. Under the center of the tripod-head should always be placed a small ring, from which a plumb-bob may be suspended. This is not needed in ordinary levelling, but becomes useful when ranging center-stakes, &c.

To adjust a Level.

This is a quite simple operation, but requires a little patience. Be careful to avoid *straining* any of the screws. The large Y nuts *w w* sometimes require some force to *start* them; but it should be applied by pressure, and not by blows. Before beginning to adjust, attend to the object-glass, as directed in the first sentence under "To adjust a plain transit."

Three adjustments are necessary: *and must be made in the following order:*

First, that of the cross-hairs; to secure that their intersection shall continue to strike the same point of a distant object, while the telescope is being turned round a complete revolution in its Ys. This is called adjusting the **line of collimation,** or sometimes, the *line of sight;* but it is not strictly the line of sight until all the adjustments are finished; for until then, the line of collimation will not serve for taking levelling sights. **If cross-hairs break.** see p 296.

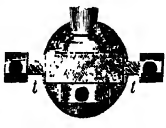
Second, that of the bubble-tube D D, to place it parallel to the line

of collimation. previously adjusted; so that when the bubble stands at the centre of its tube, indicating that it is level, we know that our sight through the telescope is hor. To replace **broken bubble tube.** see p 296

Third, that of the Ys, by which the telescope and bubble-tube are supported; so that the bubble-tube, and line of sight, shall be perp to the vert axis of the instrument; so as to *remain* hor while the telescope is pointed to objects in diff directions, as when taking back and fore sights.

To make the first adjustment, or that of the cross-hairs, plant the tripod *firmly* upon the ground. In this adjustment it is not necessary to level the instrument. Open the clips of the Ys; unclamp; draw out the eye-glass E, until the cross-hairs are seen *perfectly clear;* sight the telescope toward some clear distant point of an object; or still better, toward some straight line, whether vert or not. Move the object-glass O, by means of the milled head A, so that the object shall be clearly seen, **without parallax,** that is, without any apparent dancing about of the cross-hairs, if the eye is moved a little up or down or sideways. To secure this, the object-glass alone is moved to suit different distances; the eye-glass is not to be changed after it is once properly fixed upon the cross-hairs. The neglect of parallax is a source of frequent errors in levelling. Clamp; and, by means of the tangent-screw *b*, bring either one of the cross-hairs to coincide *precisely* with the object. Then gently, and without jarring, revolve the telescope half-way round in its Ys. When this is done, if the hair still coincides precisely with the object, it is in adjustment; and we proceed to try the other hair. But if it does not coincide, then by means of the 4 screws *p, t*, move the ring which carries the hairs, so as to rectify, as nearly as can be judged by eye, only *one-half* of the error; remembering that the ring must be moved in the direction opposite to what *appears* to be the right one; unless the telescope is an inverting one. Then turn the telescope back again to its former position; and again by the tangent-screw bring the cross-hair to coincide with the object. Then again turn the telescope half-way round as before. The hair will now be found to be more nearly in its right place, but, in all probability, not precisely so, inasmuch as it is difficult to estimate one-half the error accurately by eye. Therefore a little more alteration of the ring must be made; and it may be necessary to repeat the operation several times, before the adjustment is perfect. Afterward treat the other hair in precisely the same manner. When both are adjusted, their intersection will strike the same precise spot while the telescope is being turned *entirely* round in its Ys. This must be tried before the adjustment can be pronounced perfect; because at times the adjustment of the second hair slightly deranges that of the first one; especially if both were much *out* in the beginning.

To make the second adjustment, or to place the bubble-tube parallel to the line of collimation. This consists of two distinct adjustments, one vert. and one hor. The first of these is effected by means of the two nuts *n n* on the vert screw at one end of the tube; and the second by the two hor screws at the other end, *t*, of the tube. Looking at the bubble-tube endwise, from *t* in the foregoing Fig, its two hor adjusting-screws *t t* are seen as in this sketch. The larger capstan-headed nut *below*, has nothing to do with the adjustments; it merely holds the end of the tube in its place.



To make the vert adjustment of the bubble-tube, by means of the two nuts *n n*. Place the telescope over a diagonal pair of the levelling-screws K K, and clamp it there. Open the clips of the Ys; and by means of the levelling-screws bring the bubble to the center of its tube. Lift the telescope gently out of the Ys, turn it end for end, and put it back again in its reversed position. This being done, if the bubble still remains at the center of its tube, this adjustment is in order; but if it moves toward one end, that end is too high, and must be lowered; or else the other end must be raised. First, correct *half* the error by means of the levelling-screws K K, and then the remaining half by means of the two small capstan-headed nuts *n n*. To *raise* the end *n*, first loosen the upper nut and then tighten the lower one, to do which, turn each nut so that the *near* side moves toward your *right*. To *lower* it, first loosen the lower nut, then tighten the upper one, moving the *near* side of each nut toward your *left*. Having thus brought the bubble to the middle again, again lift the telescope out of its Ys; turn it end for end, and replace it. The bubble will now settle nearer the center than it did before, but will probably require still further adjustment. If so, correct *half* the remaining error by the levelling-screws, and half by the nuts, as before; and so continue to repeat the operation until the bubble remains at the center in both positions. For another method, see "To adjust the long bubble-tube," p 295.

Horizontal adjustment of bubble tube; to see that its axis is in the same plane with that of the telescope, as it usually is in new instruments. It is not easily de

ranged, except by blows. Have the bubble-tube, as nearly as may be, directly under the telescope, or over the center of the bar V F. Bring the telescope over two of the levelling-screws K K; clamp it there; center the bubble with said screws; turn the telescope in its Ys, say about ¼ inch, bringing the bubble-tube out from over the center of the bar, first on one side, then on the other. If the bubble stays centered while so swung out, this adjustment is correct. If it runs toward *opposite* ends of its tube when swung out on opposite sides of the center, move the end *t* of the tube by the two horizontal screws *t t* until the bubble stays centered when the tube is swung out on either side. If the bubble runs toward the *same* end of its tube *on both sides*, the tube is not truly cylindrical, but slightly conical,* so that if the telescope is turned in its Ys the bubble will leave the center, even when the horizontal adjustment is correct. It is known to be correct, in such tubes, if the bubble runs *the same distance* from the center when swung out the same distance on each side.

Having made the horizontal adjustment, turn the telescope back in its Ys until the bubble-tube is over the bar. Repeat the *vertical* adjustment (p 308), which may have become deranged in making this horizontal one. Persevere until both adjustments are found to be correct at the same time.

To make the third adjustment, or to adjust the heights of the Ys, so as to make the line of collimation parallel to the bar V F, or perp to the vert axis of the instrument. The other adjustments being made, fasten down the clips of the Ys. Make the instrument nearly level by means of all four of the levelling-screws K. Place the telescope over two of the levelling-screws which stand diagonally, and leave it there unclamped. Then bring the bubble to the center of its tube, by the two levelling-screws. Swing the upper part of the instrument half-way around, so that the telescope shall again stand over the same two screws; but end for end. This done, if the bubble leaves the center, bring it *half-way* back by the large capstan nuts *w*, *w*; and the other half by the two levelling-screws. Remember that to raise the Y, and the end of the bubble over *w*, *w*, the lower *w* must be loosened; and the upper one tightened; and vice versa. Now place the telescope over the other diagonal pair of levelling-screws; and repeat the whole operation with them. Having completed it, again try with the first pair; and so keep on until the bubble remains at the center of its tube, in every position of the telescope.

Correct levelling may be performed even if all the foregoing adjustments are out of order; provided each fore-sight be taken *at precisely the same distance from the instrument as the back-sight is*. But a good leveller will keep his instrument always in adjustment; and will test the adjustments at least once a day when at work. As much, however, depends upon the rodman, or target-man, as upon the leveller. A rodman who is careless about holding the rod vert, or about reading the sights correctly should be discharged without mercy.

The levelling-screws in many instruments become very hard to turn if dirty. Clean with water and a tooth-brush. Use no oil on field instruments.

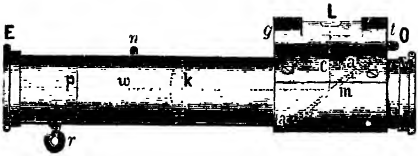
Forms for level note-books. When the distance is short, so as not to require two sets of books, the following is perhaps as good as any.

No. of Station.	Back sights	Fore sights.	Diff.	Level.	Grade.	Cut.	Fill.

But on public works generally the original field-books have only the first five cols After the grades have been determined by means of the profile drawn from these, the results are placed in another book, which has only the first col and the last four. In both cases, the right-hand page is reserved for memoranda. The writer considers it best, both with the level and with the transit, to consider the term "STATION" to apply to the whole dist between two consecutive stakes; and that its number shall be that written on the last stake. Thus, with the transit, Station 6 means the dist from stake 5 to stake 6; that it has a bearing or course of so and so; and its length is so and so. And with the level, Station 6 also means the dist from stake 5 to stake 6; the back-sight for that dist being taken at stake 5, and the fore-sight on stake 6; and that the level, grade, cut, or fill is that at stake 6. The starting-point of the survey, whether a stake, or any thing else, we call and mark simply 0.

* This defect can be remedied only by removing the tube and inserting a correctly-shaped one, and this is best done by an instrument-maker; but correct work can be done in spite of it, thus: Make all the adjustments as nearly correct as possible. Level the instrument. By turning the telescope in its Ys, make the vertical hair coincide with a plumb-line or other vertical line, and make a short continuous knife-scratch on the collar nearest the object-glass, and on the adjoining Y. Lift the telescope out of its Ys, turn it end for end, replace it in its Ys; again bring the upright hair vertical, and make on the other Y a scratch coinciding with that on the collar. Then, in levelling or in adjusting, always see that the scratch on the collar coincides with that on the adjoining Y when the bubble-tube is under the telescope.

THE HAND-LEVEL.



THIS very useful little instrument, as arranged by Professor Locke, of Cincinnati, is but about five or six inches long. Simply holding it in one hand, and looking through it in any direction, we can ascertain at once, approximately, what objects are at the same level with the eye. E is the eye end; and O the object end. L is a small level, enclosed in a kind of brass boxing t g, the bottom of which is open with a corresponding opening under it, through the top of the main tube E O. Immediately at the bottom of the small level L, is a cross-wire, stretched across said opening, and carried by a small plate, which, for adjusting the wire, can be pushed backward a trifle by tightening the screw t, or pushed forward by a small spring within the boxing, near g, when the screw t is loosened. At m is a small semicircular mirror a a, silvered on the back m. This is placed at an angle of 45°, and occupies one-half the width of the tube E O. Through the forementioned openings, the images of the cross-wire and of the level bubble are reflected down on the unsilvered face a a of the mirror, and thence to the eye, as shown by the single dotted lines c and w; and when the instrument is adjusted and held level, the wire will appear to be at the center of the bubble. At k is one half of a plano-convex lens, at the inner end of a short tube k p, which may be moved backward or forward by a pin n, projecting through a short slit in the main tube. By this means the image of the cross-wire is rendered distinct, and the half lens must be moved until, when viewing an object, the wire shall show no parallax; but appear steady against the object when the eye is slightly moved up or down. At each end of the tube E O is a circular piece of plain glass for excluding dust.

To adjust the hand-level, first fix two precisely level marks, say from 50 feet to 100 yards apart. This being done, rest the instrument against one of the level marks, and take sight at the other. If, then, the wire does not appear to be precisely at the center of the bubble, move it slightly backward or forward, as the case may be, by the screw t, until it does so appear. The two level marks may be fixed by means of the hand-level itself, even if it is entirely out of adjustment, thus. First, by the pin n arrange the half lens k, so as to show the wire distinctly and without parallax. Then holding the level steadily, at any selected object, as a, so that the wire appears to cut the center

of the bubble, see where it cuts any other convenient object, as b. Then go to b and from it, in like manner, sight back toward a. If the instrument is in adjustment, the wire will cut a; but if not, it will strike either above it or below it, as at c. In either case, make a mark m, half-way between c and a. Then b and m will be the two level marks required. With care, these adjustments, when once made, will remain in order for years. The instrument generally has a small ring r, for hanging it around the neck: it is not adapted to very accurate work, but admirably so for exploring a route. The height of a bare hill can be found by beginning at the foot, and sighting ahead at any little chance object which the cross-wire may strike, as a pebble, twig, &c; then going forward, stand at that object, and fix the wire on another one still farther on, and so to the top. At each observation we plainly rise a height equal to that of the eye, say 5¼ feet, or whatever it may be. Whether going up or down it, if the hill is covered with grass, bushes, &c. a target rod must be used for the fore-sights; and the constant height of the eye may be regarded as the back-sight at each station. An attachment may be made for screwing the level to a small ball and socket on top of a cane, or of a longer stick, for occasional use, when rather more accuracy is desired.

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LEVELLING BY THE BAROMETER.

1. Many circumstances combine to render the results of this kind of levelling unreliable where great accuracy is required. This fact was most conclusively proved by the observations made by Captain T. J. Cram, of the U. S. Coast Survey. See Report of U. S. C. S., vol. for 1854 It is difficult to read off from an aneroid (the kind of barom generally employed for engineering purposes) to within from two to five or six ft, depending on its size The moisture or dryness of the air affects the results; also winds, the vicinity of mountains, and the daily atmospheric tides, which cause incessant and irregular fluctuations in the barom. A barom hanging quietly in a room will often vary $\frac{1}{10}$ of an inch within a few hours, corresponding to a diff of elevation of nearly 100 ft No formula can possibly be devised that shall embrace these sources of error. The variations dependent upon temperature, latitude, &c, are in some measure provided for; so that with *very delicate* instruments, a *skilful* observer may measure the diff of altitude of two points close together, such as the bottom and top of a steeple, with a tolerable confidence that he is within two or three feet of the truth. But if as short an interval as even a few hours elapses between his two observations, such changes may occur in the condition of the atmosphere that he *may* make the top of the steeple to be lower than its bottom; or at least, cannot feel by any means certain that he is not ten or twenty ft in error; and this may occur without any *perceptible* change in the atmosphere Whenever practicable, therefore, there should be a person at each station, to observe at both points at the same time. Single observations at points many miles apart and made on different days, and in different states of the atmosphere, are of little value In such cases the mean of many observations, extending over several days, weeks, or months, and made when the air is apparently undisturbed, will give tolerable approximations to the truth. In the tropics the range of the atmospheric pres is much less than in other regions, seldom exceeding ½ inch at any one spot, also more regular in time, and, therefore, less productive of error. Still, the barometer, especially either the aneroid, or Bourdon's metallic, may be rendered highly useful to the civil engineer, in cases where great accuracy is not demanded. By hurrying from point to point, and especially by repeating, he can form a judgment as to which of two summits is the lowest Or a careful observer, keeping some miles ahead of a surveying party, may materially lessen their labors, especially in a rough country, by selecting the general route for them in advance. The accounts of the agreement within a few inches, in the measurements of high mountains, by diff observers, at diff periods; and those of ascertaining accurately the grades of a railroad, by means of an aneroid, while riding in a car, will be believed by those only who are ignorant of the subject. Such results can happen only by chance.

When possible, the observations at different places should be taken at the same time of day, as some check upon the effects of the daily atmospheric tides; and in very important cases, a memorandum should be made of the year, month, day, and hour, as well as of the state of the weather, direction of the wind, latitude of the place, &c, to be referred to an expert, if necessary.

The effects of latitude are not included in any of our formulas. When reqd they may be found in the table page 314. Several other corrections must be made when great accuracy is aimed at; but they require extensive tables.

In rapid railroad exploring, however, such refinements may be neglected, inasmuch as no approach to such accuracy is to be expected; but on the contrary, errors of from 1 to 10 or more feet in 100 of height, will frequently occur.

As a very rough average we may assume that the barometer falls $\frac{1}{10}$ inch for every 90 feet that we ascend above the level of the sea, up to 1000 ft. But in fact its rate of fall decreases continually as we rise; so that at one mile high it falls $\frac{1}{10}$ inch for about 106 ft rise. Table 2 shows the true rate.

To ascertain the diff of height between two points.

RULE 1. Take readings of the barom and therm (Fah) **in the shade** at both stations. Add together the two readings of the barom, and div their sum by 2, for their mean; which call b. Do the same with the two readings of the thermom, and call the mean t. Subtract the least reading of the barom from the greatest; and call the diff d. Then mult together this diff d; the number from the next Table No. 1, opposite t; and the constant number 30. Div the prod by b. Or

$$\text{Height in feet} = \frac{\text{Diff } (d) \text{ of barom} \times \text{Tabular number opposite mean } (t) \text{ of thermom} \times \text{Constant } 30.}{\text{mean } (b) \text{ of barom}}$$

EXAMPLE. Reading of the barom at lower station, 26.64 ins; and at the upper sta 20.82 ins. Thermom at lowest sta, 70°; at upper sta, 40°. What is the diff in height of the two stations? Here,

Barom, 26.64		Therm, 70°
" 20.82	Also,	" 40°
2)47.46		2)110
23.73 mean of bar, or b.		55° mean of therm, or t.

The tabular number opposite 55°, is 917.2.

Again, 26.64 — 20.82 = 5.82, diff of bar; or d. Hence,

$$\text{Height in feet} = \frac{\overset{d,}{5.82} \times \overset{\text{Tab No.}}{917.2} \times \overset{\text{Con.}}{30}}{23.73 \text{ (or } b)} = \frac{160143.12}{23.73} = 6748.5 \text{ ft; answer.}$$

Then correct for latitude, if more accuracy is reqd, by rule on next page.

The screw at the back of an aneroid is for adjusting the index by a standard barom. After this has been done it must by no means be meddled with. In some instruments specially made to order with that intention, this screw may be used also for turning the index back, after having risen to an elevation so great that the index has reached the extreme limit of the graduated arc. After thus turning it back, the indications of the index at greater heights must be added to that attained when it was turned back.

TABLE 1. For Rule 1.

Mean of Ther	No.	Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.
0°	801.1	30°	864.4	60°	927.7	90°	991.0
1	803.2	31	866.5	61	929.8	91	993.1
2	805.3	32	868.6	62	931.9	92	995.2
3	807.4	33	870.7	63	934.0	93	997.3
4	809.5	34	872.8	64	936.1	94	999.4
5	811.7	35	874.9	65	938.2	95	1001.6
6	813.8	36	877.0	66	940.3	96	1003.7
7	815.9	37	879.2	67	942.4	97	1005.8
8	818.0	38	881.3	68	944.5	98	1007.9
9	820.1	39	883.4	69	946.7	99	1010.0
10	822.2	40	885.4	70	948.8	100	1012.1
11	824.3	41	887.5	71	950.9	101	1014.2
12	826.4	42	889.6	72	953.0	102	1016.3
13	828.5	43	891.7	73	955.1	103	1018.4
14	830.6	44	893.8	74	957.2	104	1020.5
15	832.8	45	896.0	75	959.3	105	1022.7
16	834.9	46	898.1	76	961.4	106	1024.8
17	837.0	47	900.2	77	963.5	107	1026.9
18	839.1	48	902.3	78	965.6	108	1029.0
19	841.2	49	904.5	79	967.7	109	1031.1
20	843.3	50	906.6	80	969.9	110	1033.2
21	845.4	51	908.7	81	972.0	111	1035.3
22	847.5	52	910.8	82	974.1	112	1037.4
23	849.6	53	913.0	83	976.2	113	1039.5
24	851.8	54	915.1	84	978.3	114	1041.6
25	853.9	55	917.2	85	980.4	115	1043.8
26	856.0	56	919.3	86	982.6	116	1045.9
27	858.1	57	921.4	87	984.7	117	1048.0
28	860.2	58	923.5	88	986.8	118	1050.1
29	862.3	59	925.6	89	988.9	119	1052.2

RULE 2. **Belville's short approx rule** is the one best adapted to rapid field use, namely, add together the two readings of the barom only. Also find the diff between said two readings; then, **as the sum of the two readings is to their diff, so is 55000 feet to the reqd altitude.**

Correction for latitude is usually omitted where great accuracy is not required. To apply it, first find the altitude by the rule, as before. Then divide it by the number in the following table opposite the latitude of the place (If the two places are in different latitudes, use their mean.) *Add* the quotient to the altitude if the latitude is *less* than 45°. *Subtract* it if the latitude is *more* than 45°. No correction required for latitude 45°.

Table of corrections for latitude.

Lat.		Lat.		Lat		Lat.		Lat		Lat	
0°	352	14°	399	28°	630	42°	3367	54°	1140	68°	490
2	354	16	416	30	705	44	10101	56	941	70	460
4	356	18	436	32	804	45	∞	58	804	72	436
6	360	20	460	34	941	46	10101	60	705	74	416
8	367	22	490	36	1140	48	3367	62	630	76	399
10	375	24	527	38	1458	50	2028	64	572	78	386
12	386	26	572	40	2028	52	1458	66	527	80	375

Levelling by Barometer; or by the boiling point.

RULE 3. The following table, No. 2, enables us to measure heights either by means of boiling water, or by the barom The third column shows the approximate altitude above sea-level corresponding to diff heights, or readings of the barom; and to the diff degrees of Fahrenheit's thermom, at which water boils in the open air Thus when the barom, under undisturbed conditions of the atmosphere, stands at 24.08 inches, or when pure rain or distilled water boils at the temp of 201° Fah; the place is about 5764 ft above the level of the sea, as shown by the table. It is therefore very easy to find the *diff* of altitude of two places Thus: take out from table No 2, the altitudes opposite to the two boiling temperatures; or to the two barom readings Subtract the one opposite the lower reading, from that opposite the upper reading The rem will be the reqd height, as a rough approximation To correct this, add together the two therm readings; and div the sum by 2, for their mean. From table for temperature, p 316, take out the number opposite this mean. Mult the approximate height just found, by this tabular number. Then correct for lat if reqd.

EX. The same as preceding; namely, barom at lower sta, 26.64; and at upper sta, 20.82. Thermom at lower sta, 70° Fah; and at the upper one, 40°. What is the diff of height of the two stations?

	Alt.
Here the tabular altitudes are, for 20.82........ ...	9579
and for 26.64......	3115
	6464 ft, approx height.

To correct this, we have $\frac{70° + 40°}{2} = \frac{110°}{2} = 55°$ mean; and in table p 316, opp to 55°, we find 1.048. Therefore $6464 \times 1.048 = 6774$ ft, the reqd height.

This is about 26 ft more than by Rule 1; or nearly .4 of a ft in each 100 ft.

At 70° Fah, pure water will boil at 1° less of temp, for an average of about 550 ft of elevation above sea-level, up to a height of ½ a mile. At the height of 1 mile, 1° of boiling temp will correspond to about 560 ft of elevation. In table p 315 the mean of the temps at the two stations is assumed to be 32° Fah; at which no correction for temp is necessary in using the table; hence the tabular number opposite 32°, in table p 316, is 1.

This diff produced in the temp of the *boiling point*, by change of elevation, must not be confounded with that of the *atmosphere*, due to the same cause. The air becomes cooler as we ascend above sea-level, at the rate (very roughly) of about 1° Fah for every 200 ft near sea-level, to 350 ft at the height of 1 mile.

The following table, No. 2, (so far as it relates to the barom) was deduced by the writer from the standard work on the barom by Lieut.-Col. R. S. Williamson, U. S. army.*

* Published by permission of Government in 1868 by Van Nostrand, N. Y

TABLE 2.

Levelling by Barometer; or by the boiling point.

Assumed temp in the shade 32° Fah. If not 32°, mult barom alt as per Table, p 316

Boil point in deg Fah.	Barom Ins.	Altitude above sea level Feet.	Boil point in deg Fah	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet	Boil point in deg Fah	Barom Ins.	Altitude above sea level Feet.
184°	16 79	15221	.3	19 66	11083	.6	22 93	7018	.9	26 59	3164
.1	16 83	15159	4	19 70	11029	.7	22.98	6991	206	26 64	3115
.2	16 86	15112	.5	19 74	10976	.8	23.02	6945	.1	26 69	3066
3	16 90	15050	6	19 78	10923	9	23.07	6888	.2	26.75	3007
.4	16 93	15003	.7	19.82	10870	199	23.11	6843	.3	26 80	2958
.5	16 97	14941	.8	19.87	10804	.1	23 16	6786	4	26 86	2899
.6	17 00	14895	.9	19 92	10738	.2	23.21	6729	.5	26 91	2850
.7	17 04	14833	192	19 96	10685	.3	23.26	6673	6	26 97	2792
.8	17 08	14772	1	20 00	10633	.4	23.31	6617	.7	27 02	2743
.9	17 12	14710	.2	20 05	10567	.5	23 36	6560	8	27 08	2685
185	17 16	14649	.3	20 10	10502	.6	23 40	6516	9	27 13	2637
.1	17 20	14588	4	20 14	10450	.7	23 45	6460	207	27 18	2589
.2	17 23	14543	5	20 18	10398	.8	23 49	6415	1	27 23	2540
.3	17 27	14482	.6	20 22	10346	.9	23 54	6359	2	27 29	2483
.4	17.31	14421	7	20 27	10281	200	23 59	6304	.3	27.34	2435
.5	17 35	14361	8	20 31	10230	.1	23 64	6248	4	27.40	2377
.6	17.38	14315	.9	20 35	10178	.2	23 69	6193	5	27.45	2329
.7	17 42	14255	193	20 39	10127	.3	23 74	6157	6	27 51	2272
.8	17 46	14195	1	20 43	10075	.4	23 79	6082	7	27 56	2224
.9	17 50	14135	2	20 48	10011	.5	23 84	6027	8	27 62	2167
186	17 54	14075	3	20 53	9947	.6	23 89	5972	9	27 67	2120
.1	17 58	14015	4	20 57	9896	.7	23 94	5917	208	27.73	2063
.2	17 62	13956	5	20 61	9845	.8	23 98	5874	1	27 78	2016
.3	17 66	13896	6	20 65	9794	.9	24 03	5819	.2	27 84	1959
4	17 70	13837	7	20 69	9743	201	24 08	5764	.3	27 89	1912
.5	17.74	13778	8	20 73	9693	.1	24 13	5710	4	27 95	1856
.6	17.78	13718	.9	20 77	9642	.2	24 18	5656	.5	28 00	1809
.7	17 82	13660	194	20 82	9579	3	24 23	5602	.6	28 06	1753
.8	17 86	13601	.1	20 87	9516	.4	24 28	5547	7	28 11	1706
.9	17 90	13542	2	20 91	9466	.5	24 33	5494	8	28 17	1650
187	17 93	13498	3	20 96	9403	6	24 38	5440	.9	28.23	1595
.1	17 97	13440	.4	21.00	9353	7	24 43	5386	209	28 29	1539
.2	18.00	13396	.5	21 05	9291	.8	24 48	5332	.1	28 35	1483
.3	18.04	13338	6	21.09	9241	9	24 53	5279	.2	28.40	1437
.4	18 08	13280	.7	21.14	9179	202	24 58	5225	.3	28.45	1391
.5	18 12	13222	.8	21.18	9130	.1	24 63	5172	.4	28.51	1336
.6	18 16	13164	.9	21 22	9080	2	24 68	5119	.5	28.56	1290
.7	18.20	13106	195	21 26	9031	.3	24 73	5066	.6	28.62	1235
.8	18.24	13019	.1	21 31	8969	4	24.78	5013	.7	28 67	1189
.9	18 28	12991	2	21.35	8920	5	24 83	4960	.8	28 73	1134
188	18.32	12934	.3	21.40	8859	.6	24 88	4907	9	28.79	1079
.1	18 36	12877	4	21.44	8810	.7	24 93	4855	210	28 85	1025
.2	18 40	12820	.5	21 49	8749	8	24 98	4802	.1	28.91	970
.3	18.44	12763	.6	21 53	8700	.9	25.03	4750	.2	28.97	916
.4	18.48	12706	.7	21.58	8639	203	25 08	4697	.3	29.03	862
.5	18.52	12649	.8	21.62	8590	1	25 13	4645	.4	29.09	808
.6	18 56	12593	.9	21.67	8530	2	25 18	4593	5	29.15	754
.7	18 60	12536	196	21.71	8481	.3	25 23	4541	.6	29.20	709
.8	18 64	12480	.1	21 76	8421	.4	25 28	4489	.7	29 25	664
.9	18.68	12424	.2	21.81	8361	.5	25 33	4437	.8	29.31	610
189	18 72	12367	3	21 86	8301	.6	25 38	4386	.9	29 36	565
.1	18.76	12311	.4	21 90	8253	.7	25 43	4334	211	29.42	512
.2	18 80	12256	.5	21.95	8193	.8	25 49	4272	.1	29 48	458
.3	18 84	12200	.6	21 99	8145	.9	25 54	4221	.2	29.54	405
.4	18 88	12144	.7	22 04	8086	204	25 59	4169	.3	29.60	352
.5	18.92	12089	.8	22.08	8038	.1	25 64	4118	.4	29.65	308
.6	18 96	12033	.9	22 13	7979	.2	25 70	4057	.5	29.71	255
.7	19 00	11978	197	22 17	7932	.3	25 76	3996	6	29 77	202
.8	19.04	11923	1	22 22	7873	4	25 81	3945	.7	29 83	149
.9	19 08	11868	2	22 27	7814	.5	25 86	3894	.8	29.88	105
190	19 13	11799	3	22 32	7755	.6	25 91	3844	.9	29 94	52
.1	19 17	11745	.4	22 36	7708	.7	25.96	3793	212	30.00	sea lev
.2	19 21	11690	.5	22 41	7649	8	26 01	3742		Below sea level.	
.3	19 25	11635	.6	22 45	7602	.9	26 06	3692	.1	30.06	— 5
.4	19 29	11581	.7	22.50	7544	205	26.11	3642	.2	30 12	—1[illegible]
.5	19 33	11527	.8	22 54	7498	.1	26 17	3582	.3	30 18	—1[illegible]
.6	19.37	11472	.9	22.59	7439	.2	26.22	3592	.4	30.24	—2[illegible]
.7	19.41	11418	198	22 64	7381	.3	26 28	3472	.5	30.30	—2[illegible]
.8	19 45	11364	.1	22 69	7324	4	26 33	3422	.6	30 35	—3[illegible]
.9	19 49	11310	.2	22.74	7266	5	26 38	3372	.7	30.41	—3[illegible]
191	19.54	11243	.3	22.79	7208	.6	26.43	3322	.8	30.47	—4[illegible]
.1	19 58	11190	.4	22.84	7151	.7	26.48	3273	.9	30 53	—4[illegible]
.2	19.62	11136	.5	22.89	7093	8	26.54	3213	213	30.59	—5

Corrections for temperature; to be used in connection with Rule 3, when greater accuracy is necessary. Also in connection with Table 2 when the temp is not 32°.

Mean temp in the shade.	Mult by	Mean temp in the shade	Mult by	Mean temp in the shade	Mult by	Mean temp in the shade	Mult by
Zero	.933	28°	.992	56°	1.050	84°	1.108
2°	.937	30	.996	58	1.054	86	1.112
4	.942	32	1.000	60	1.058	88	1.117
6	.946	34	1.004	62	1.062	90	1.121
8	.950	36	1.008	64	1.066	92	1.125
10	.954	38	1.012	66	1.071	94	1.129
12	.958	40	1.016	68	1.075	96	1.133
14	.962	42	1.020	70	1.079	98	1.138
16	.967	44	1.024	72	1.083	100	1.142
18	.971	46	1.028	74	1.087	102	1.146
20	.975	48	1.032	76	1.091	104	1.150
22	.979	50	1.036	78	1.096	106	1.154
24	.983	52	1.041	80	1.100	108	1.158
26	.987	54	1.046	82	1.104	110	1.163

SOUND.

The velocity of sound in quiet open air, has been experimentally determined to be very approximately 1090 feet per second, when the temperature is at freezing point, or 32° Fahrenheit. For every degree Fahrenheit of increase of temperature, the velocity increases by from ½ foot to 1¼ feet per second, according to different authorities. Taking the increase at 1 foot per second for each degree (which agrees closely with theoretical calculations), we have

at	−	30°	Fahr	1030	feet	per sec	= 0.1951	mile	per sec	= 1	mile	in	5.13	seconds.
"	−	20°	"	1040	"	"	= 0.1970	"	"	= 1	"		5.08	"
"	−	10°	"	1050	"	"	= 0.1989	"	"	= 1	"		5.03	"
"		0	"	1060	"	"	= 0.2008	"	"	= 1	"		4.98	"
"		10°	"	1070	"	"	= 0.2027	"	"	= 1	"		4.93	"
"		20°	"	1080	"	"	= 0.2045	"	"	= 1	"		4.88	"
"		32°	"	1092	"	"	= 0.2068	"	"	= 1	"		4.83	"
"		40°	"	1100	"	"	= 0.2083	"	"	= 1	"		4.80	"
"		50°	"	1110	"	"	= 0.2102	"	"	= 1	"		4.78	"
"		60°	"	1120	"	"	= 0.2121	"	"	= 1	"		4.73	"
"		70°	"	1130	"	"	= 0.2140	"	"	= 1	"		4.68	"
"		80°	"	1140	"	"	= 0.2159	"	"	= 1	"		4.63	"
"		90°	"	1150	"	"	= 0.2178	"	"	= 1	"		4.59	"
"		100°	"	1160	"	"	= 0.2197	"	"	= 1	"		4.55	"
"		110°	"	1170	"	"	= 0.2216	"	"	= 1	"		4.51	"
"		120°	"	1180	"	"	= 0.2235	"	"	= 1	"		4.47	"

If the air is calm, fog or rain does not appreciably affect the result; but winds do. Very loud sounds appear to travel somewhat faster than low ones. The watchword of sentinels has been heard across still water, on a calm night, 10½ miles; and a cannon 20 miles. Separate sounds, at intervals of $\frac{1}{16}$ of a second, cannot be distinguished, but appear to be connected. The distances at which a speaker can be understood, in front, on one side, and behind him are about as 4, 3, and 1.

Dr. Charles M Cresson informs the writer that, by repeated trials, he found that in a Philadelphia gas main 20 inches diameter and 16000 feet long, laid and covered in the earth, but empty of gas, and having one horizontal bend of 90°, and of 40 feet radius the sound of a pistol-shot travelled 16000 feet in precisely 16 seconds, or 1000 feet per second. The arrival of the sound was barely audible; but was rendered very apparent to the eye by its blowing off a diaphragm of tissue-paper placed over the end of the main.

Two boats anchored some distance apart may serve as a base line for triangulating objects along the coast; the distance between them being first found by firing guns on board one of them.

In water the velocity is about 4708 feet per second, or about 4 times that in air. **In woods,** it is from 10 to 16 times; and **in metals,** from 4 to 16 times greater than in air, according to some authorities.

Approximate expansion of solids by heat; and their melting points by Fahrenheit's thermometer.‡

	For 1 degree.		For 180 degrees.*		Melting point in Deg.‡
	1 part in	⅛ inch in	1 part in	⅛ inch in	
Fire brick	365220	3804 ft	2029	21.14 ft	
Granite { from	187560	1954	1042	10.85	
Granite { to	228060	2375	1267	13.26	
Glass rod	221400	2306	1230	12.81	
Glass tube	214200	2231	1190	12.40	
" crown	211500	2203	1175	12.24	
" plate	209700	2184	1165	12.13	
Platina	208800	2175	1160	12.08	4593
Marble granular, white, dry	173000	1802	961	10.00	
" " " moist.	128000	1333	711	7.41	
" " black, compact	405000	4219	2250	23 44	
Antimony	166500	1722	925	9.63	955
Cast iron	162000	1688	900	9.38	1920 to
Slate	173000	1802	961	10.00	2800
Steel	151200	1575	840	8 75	2370 to
" blistered	159840	1665	888	9.25	2550
" untempered	167400	1744	930	9 69	
" tempered yellow	131400	1369	730	7.60	
" hardened	146880	1530	816	8.50	
" annealed	147600	1537	820	8 54	
Iron, rolled	149940	1562	833	8 68	3000 to
" soft, forged	147420	1536	819	8 53	3500
" wire	146340	1524	818	8 55	
Bismuth	129600	1350	720	7.50	506
Gold, annealed	123120	1282	684	7 12	2016
Copper average	104400	1088	580	6.04	2000
Sandstone†	103320	1076	574	5 98	
Brass average	97740	1018	543	5.66	1873
" wire	94140	981	523	5 45	
Silver	95040	990	528	5 50	1861
Tin average	87840	915	488	5 08	444
Lead average	63180	658	351	3 66	612
Pewter	78840	821	438	4.56	
Zinc (most of all metals)	61920	645	344	3 58	680 to 773
White pine	440530	4588	2447	25.49	

Let a_c or a_f = the **linear expansion coefficient** of a material = that fraction of its original length which a prismatic bar, of that material, gains or loses, for each degree, Cent. or Fahr. respectively, of change in its temperature. Then: $a_c = \frac{9}{5} a_f$; $a_f = \frac{5}{9} a_c$.

The coefficient is practically constant at ordinary temperatures.

The force. exerted longitudinally by such a bar, in expanding or contracting, is $P = a\,t\,E\,F$, where a = coefficient, as above; t = change in temperature, in degrees; E = elastic modulus (see p 456); F = area of cross section. **The work,** done by this force, in expanding or contracting the bar, of original length, L, through the length, l, is $W = P\,l = P\,L\,a\,t = a^2\,t^2\,E\,F\,L$.

The superficial expansion coefficient (ratio of change of area of a surface to its original area) = about $2\,a$; **voluminar coefficient** = about $3\,a$; assuming that the linear coefficient is the same in all directions.

Heat of a common wood fire variously estimated at from 800° to 1140° Fahr.; charcoal fire, about 2200° F.; coal fire, about 2400° F.

* By adding $\frac{1}{10}$ part to the lengths in the two cols under 180°, we get the lengths corresponding to a number of degrees $\frac{1}{11}$ less than 180°; or to 163°.63 deg which may be taken as about the *extremes* of temp in the colder portions of the United States. In the Middle States the extremes rarely reach 135°, or ¼ part less than 180°.

No dependence whatever is to be placed on results obtained by Wedgewood's pyrometer.

† The table shows that the contraction and **expansion of stone** will cause open joints in winter; and crushing of the mortar in summer, at the ends of long coping-stones.

‡ **The melting points are quite uncertain.** We give the mean of the best authorities. Assuming that with a change of temp of about 163°, wrought iron will alter its length 1 part in 916; this in a mile amounts to 5 764 ft, or about 5 ft 9¼ ins; and in 100 ft to .109 of a foot; or 1⅜ ins; so that a diff of 5 ft, or more, can readily result from measuring a mile in winter and in summer with the same chain; and a 25 ft rail will change its length full ⅜ of an inch.

THERMOMETERS.

Let C, R, F = the given reading, in degrees Celsius (Centigrade), Réaumur, ahrenheit, respectively. Then (See tables 1, 2, 3, below):

$$C = \frac{5}{4} R = \frac{5}{9} (F - 32);$$

$$R = \frac{4}{5} C = \frac{4}{9} (F - 32);$$

$$F = \frac{9}{5} C + 32 = \frac{9}{4} R + 32.$$

hus, let $F = -40$. Then $C = \frac{5}{9}(-40 - 32) = -40$. For **expan-ion coefficients**, see p 317.

Below about —37° C (= —30° R = —35° F), the mercurial thermometer nd barometer become irregular. Mercury begins to freeze at about —40° C —32° R = —40° F. Below this temperature alcohol is used.

TABLE 1. Fahrenheit compared with Centigrade and Réaumur. In *this* table the Cent and Réau readings are given to the nearest decimal.

F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
212	100	80.0	158	70.0	56.0	104	40.0	32.0	50	10.0	8.0	—3	—19.4	—15.6
211	99.4	79.6	157	69.4	55.6	103	39.4	31.6	49	9.4	7.6	—4	—20.0	—16.0
210	98.9	79.1	156	68.9	55.1	102	38.9	31.1	48	8.9	7.1	—5	—20.6	—16.4
209	98.3	78.7	155	68.3	54.7	101	38.3	30.7	47	8.3	6.7	—6	—21.1	—16.9
208	97.8	78.2	154	67.8	54.2	100	37.8	30.2	46	7.8	6.2	—7	—21.7	—17.3
207	97.2	77.8	153	67.2	53.8	99	37.2	29.8	45	7.2	5.8	—8	—22.2	—17.8
206	96.7	77.3	152	66.7	53.3	98	36.7	29.3	44	6.7	5.3	—9	—22.8	—18.2
205	96.1	76.9	151	66.1	52.9	97	36.1	28.9	43	6.1	4.9	—10	—23.3	—18.7
204	95.6	76.4	150	65.6	52.4	96	35.6	28.4	42	5.6	4.4	—11	—23.9	—19.1
203	95.0	76.0	149	65.0	52.0	95	35.0	28.0	41	5.0	4.0	—12	—24.4	—19.6
202	94.4	75.6	148	64.4	51.6	94	34.4	27.6	40	4.4	3.6	—13	—25.0	—20.0
201	93.9	75.1	147	63.9	51.1	93	33.9	27.1	39	3.9	3.1	—14	—25.6	—20.4
200	93.3	74.7	146	63.3	50.7	92	33.3	26.7	38	3.3	2.7	—15	—26.1	—20.9
199	92.8	74.2	145	62.8	50.2	91	32.8	26.2	37	2.8	2.2	—16	—26.7	—21.3
198	92.2	73.8	144	62.2	49.8	90	32.2	25.8	36	2.2	1.8	—17	—27.2	—21.8
197	91.7	73.3	143	61.7	49.3	89	31.7	25.3	35	1.7	1.3	—18	—27.8	—22.2
196	91.1	72.9	142	61.1	48.9	88	31.1	24.9	34	1.1	0.9	—19	—28.3	—22.7
195	90.6	72.4	141	60.6	48.4	87	30.6	24.4	33	0.6	0.4	—20	—28.9	—23.1
194	90.0	72.0	140	60.0	48.0	86	30.0	24.0	32	0.0	0.0	—21	—29.4	—23.6
193	89.4	71.6	139	59.4	47.6	85	29.4	23.6	31	—0.6	—0.4	—22	—30.0	—24.0
192	88.9	71.1	138	58.9	47.1	84	28.9	23.1	30	—1.1	—0.9	—23	—30.6	—24.4
191	88.3	70.7	137	58.3	46.7	83	28.3	22.7	29	—1.7	—1.3	—24	—31.1	—24.9
190	87.8	70.2	136	57.8	46.2	82	27.8	22.2	28	—2.2	—1.8	—25	—31.7	—25.3
189	87.2	69.8	135	57.2	45.8	81	27.2	21.8	27	—2.8	—2.2	—26	—32.2	—25.8
188	86.7	69.3	134	56.7	45.3	80	26.7	21.3	26	—3.3	—2.7	—27	—32.8	—26.2
187	86.1	68.9	133	56.1	44.9	79	26.1	20.9	25	—3.9	—3.1	—28	—33.3	—26.7
186	85.6	68.4	132	55.6	44.4	78	25.6	20.4	24	—4.4	—3.6	—29	—33.9	—27.1
185	85.0	68.0	131	55.0	44.0	77	25.0	20.0	23	—5.0	—4.0	—30	—34.4	—27.6
184	84.4	67.6	130	54.4	43.6	76	24.4	19.6	22	—5.6	—4.4	—31	—35.0	—28.0
183	83.9	67.1	129	53.9	43.1	75	23.9	19.1	21	—6.1	—4.9	—32	—35.6	—28.4
182	83.3	66.7	128	53.3	42.7	74	23.3	18.7	20	—6.7	—5.3	—33	—36.1	—28.9
181	82.8	66.2	127	52.8	42.2	73	22.8	18.2	19	—7.2	—5.8	—34	—36.7	—29.3
180	82.2	65.8	126	52.2	41.8	72	22.2	17.8	18	—7.8	—6.2	—35	—37.2	—29.8
179	81.7	65.3	125	51.7	41.3	71	21.7	17.3	17	—8.3	—6.7	—36	—37.8	—30.2
178	81.1	64.9	124	51.1	40.9	70	21.1	16.9	16	—8.9	—7.1	—37	—38.3	—30.7
177	80.6	64.4	123	50.6	40.4	69	20.6	16.4	15	—9.4	—7.6	—38	—38.9	—31.1
176	80.0	64.0	122	50.0	40.0	68	20.0	16.0	14	—10.0	—8.0	—39	—39.4	—31.6
175	79.4	63.6	121	49.4	39.6	67	19.4	15.6	13	—10.6	—8.4	—40	—40.0	—32.0
174	78.9	63.1	120	48.9	39.1	66	18.9	15.1	12	—11.1	—8.9	—41	—40.6	—32.4
173	78.3	62.7	119	48.3	38.7	65	18.3	14.7	11	—11.7	—9.3	—42	—41.1	—32.9
172	77.8	62.2	118	47.8	38.2	64	17.8	14.2	10	—12.2	—9.8	—43	—41.7	—33.3
171	77.2	61.8	117	47.2	37.8	63	17.2	13.8	9	—12.8	—10.2	—44	—42.2	—33.8
170	76.7	61.3	116	46.7	37.3	62	16.7	13.3	8	—13.3	—10.7	—45	—42.8	—34.2
169	76.1	60.9	115	46.1	36.9	61	16.1	12.9	7	—13.9	—11.1	—46	—43.3	—34.7
168	75.6	60.4	114	45.6	36.4	60	15.6	12.4	6	—14.4	—11.6	—47	—43.9	—35.1
167	75.0	60.0	113	45.0	36.0	59	15.0	12.0	5	—15.0	—12.0	—48	—44.4	—35.6
166	74.4	59.6	112	44.4	35.6	58	14.4	11.6	4	—15.6	—12.4	—49	—45.0	—36.0
165	73.9	59.1	111	43.9	35.1	57	13.9	11.1	3	—16.1	—12.9	—50	—45.6	—36.4
164	73.3	58.7	110	43.3	34.7	56	13.3	10.7	2	—16.7	—13.3	—51	—46.1	—36.9
163	72.8	58.2	109	42.8	34.2	55	12.8	10.2	1	—17.2	—13.8	—52	—46.7	—37.3
162	72.2	57.8	108	42.2	33.8	54	12.2	9.8	0	—17.8	—14.2	—53	—47.2	—37.8
161	71.7	57.3	107	41.7	33.3	53	11.7	9.3	—1	—18.3	—14.7	—54	—47.8	—38.2
160	71.1	56.9	106	41.1	32.9	52	11.1	8.9	—2	—18.9	—15.1	—55	—48.3	—38.7
159	70.6	56.4	105	40.6	32.4	51	10.6	8.4						

TABLE 2. Centigrade compared with Fahrenheit and Réaumur.

C.	F.	R.	C.	F.	R.	C.	F.	R.	C.	F.	R.
°	°	°	°	°	°	°	°	°	°	°	°
	Exact.	Exact.		Exact.	Exact.		Exact.	Exact.		Exact.	Exact.
100	212 0	80 0	62	143.6	49 6	24	75.2	19.2	−14	6 8	−11.2
99	210.2	79 2	61	141 8	48 8	23	73 4	18.4	−15	5.0	−12.0
98	208 4	78 4	60	140 0	48 0	22	71 6	17.6	−16	3 2	−12.8
97	206 6	77 6	59	138 2	47.2	21	69 8	16 8	−17	1 4	−13.6
96	204 8	76 8	58	136 4	46 4	20	68.0	16.0	−18	−0.4	−14.4
95	203 0	76.0	57	134 6	45 6	19	66 2	15.2	−19	−2.2	−15.2
94	201 2	75 2	56	132 8	44 8	18	64 4	14 4	−20	−4 0	−16 0
93	199 4	74 4	55	131 0	44 0	17	62 6	13 6	−21	−5 8	−16.8
92	197 6	73 6	54	129 2	43 2	16	60 8	12.8	−22	−7 6	−17.6
91	195 8	72.8	53	127 4	42.4	15	59 0	12 0	−23	−9 4	−18 4
90	194 0	72 0	52	125 6	41 6	14	57.2	11.2	−24	−11.2	−19 2
89	192 2	71 2	51	123 8	40 8	13	55.4	10 4	−25	−13 0	−20.0
88	190 4	70 4	50	122 0	40 0	12	53 6	9 6	−26	−14.8	−20 8
87	188 6	69 6	49	120 2	39.2	11	51 8	8 8	−27	−16 6	−21.6
86	186 8	68 8	48	118 4	38 4	10	50.0	8.0	−28	−18 4	−22.4
85	185 0	68 0	47	116 6	37 6	9	48 2	7.2	−29	−20 2	−23.2
84	183 2	67 2	46	114 8	36 8	8	46 4	6.4	−30	−22 0	−24.0
83	181 4	66 4	45	113 0	36 0	7	44 6	5.6	−31	−23 8	−24.8
82	179 6	65 6	44	111 2	35 2	6	42 8	4.8	−32	−25.6	−25 6
81	177 8	64 8	43	109 4	34 4	5	41 0	4.0	−33	−27.4	−26 4
80	176 0	64 0	42	107 6	33 6	4	39 2	3.2	−34	−29 2	−27.2
79	174 2	63 2	41	105 8	32 8	3	37 4	2 4	−35	−31 0	−28.0
78	172 4	62 4	40	104 0	32 0	2	35 6	1 6	−36	−32.8	−28 8
77	170 6	61 6	39	102 2	31 2	1	33.8	0 8	−37	−34 6	−29.6
76	168 8	60 8	38	100 4	30 4	0	32 0	0.0	−38	−36.4	−30.4
75	167 0	60 0	37	98 6	29 6	−1	30.2	−0.8	−39	−38.2	−31 2
74	165 2	59 2	36	96 8	28 8	−2	28 4	−1 6	−40	−40.0	−32 0
73	163 4	58 4	35	95 0	28 0	−3	26 6	−2.4	−41	−41 8	−32 8
72	161 6	57 6	34	93 2	27 2	−4	24 8	−3 2	−42	−43 6	−33.6
71	159 8	56 8	33	91 4	26 4	−5	23 0	−4 0	−43	−45 4	−34.4
70	158.0	56 0	32	89.6	25 6	−6	21.2	−4 8	−44	−47 2	−35.2
69	156 2	55 2	31	87 8	24 8	−7	19 4	−5 6	−45	−49 0	−36 0
68	154 4	54 4	30	86 0	24 0	−8	17 6	−6.4	−46	−50 8	−36 8
67	152 6	53 6	29	84.2	23 2	−9	15 8	−7 2	−47	−52 6	−37.6
66	150 8	52 8	28	82 4	22 4	−10	14 0	−8 0	−48	−54 4	−38.4
65	149 0	52 0	27	80 6	21 6	−11	12.2	−8 8	−49	−56.2	−39.2
64	147 2	51 2	26	78 8	20 8	−12	10 4	−9 6	−50	−58 0	−40 0
63	145 4	50 4	25	77.0	20.0	−13	8 6	−10.4			

TABLE 3. Réaumur compared with Fahrenheit and Centigrade.

R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.
°	°	°	°	°	°	°	°	°	°	°	°
	Exact	Exact		Exact.	Exact		Exact	Exact.		Exact	Exact
80	212 00	100 00	49	142 25	61 25	19	74 75	23 75	−11	7.25	−13.75
79	209 75	98 75	48	140 00	60 00	18	72.50	22.50	−12	5.00	−15.00
78	207 50	97 50	47	137 75	58 75	17	70 25	21 25	−13	2.75	−16 25
77	205 25	96 25	46	135 50	57.50	16	68 00	20 00	−14	0.50	−17.50
76	203 00	95 00	45	133 25	56 25	15	65 75	18.75	−15	−1.75	−18.75
75	200 75	93.75	44	131.00	55 00	14	63 50	17.50	−16	−4 00	−20.00
74	198 50	92 50	43	128 75	53.75	13	61 25	16 25	−17	−6 25	−21.25
73	196 25	91.25	42	126 50	52 50	12	59 00	15.00	−18	−8.50	−22.50
72	194 00	90 00	41	124 25	51 25	11	56 75	13 75	−19	−10 75	−23.75
71	191 75	88 75	40	122.00	50 00	10	54 50	12.50	−20	−13.00	−25 00
70	189 50	87 50	39	119 75	48 75	9	52 25	11.25	−21	−15.25	−26 25
69	187 25	86 25	38	117 50	47 50	8	50 00	10 00	−22	−17 50	−27 50
68	185 00	85 00	37	115 25	46 25	7	47.75	8 75	−23	−19 75	−28.75
67	182 75	83 75	36	113 00	45 00	6	45 50	7.50	−24	−22.00	−30.00
66	180 50	82 50	35	110.75	43.75	5	43 25	6 25	−25	−24.25	−31.25
65	178 25	81 25	34	108 50	42 50	4	41.00	5 00	−26	−26.50	−32.50
64	176 00	80 00	33	106.25	41 25	3	38.75	3 75	−27	−28.75	−33.75
63	173 75	78 75	32	104 00	40 00	2	36.50	2.50	−28	−31.00	−35 00
62	171 50	77.50	31	101.75	38 75	1	34.25	1.25	−29	−33 25	−36.25
61	169 25	76 25	30	99.50	37 50	0	32 00	0 00	−30	−35.50	−37.50
60	167.00	75.00	29	97.25	36 25	−1	29.75	−1.25	−31	−37.75	−38 75
59	164 75	73 75	28	95.00	35 00	−2	27.50	−2.50	−32	−40 00	−40 00
58	162 50	72.50	27	92.75	33 75	−3	25.25	−3 75	−33	−42 25	−41.25
57	160 25	71 25	26	90.50	32 50	−4	23 00	−5.00	−34	−44.50	−42.50
56	158 00	70.00	25	88 25	31 25	−5	20 75	−6.25	−35	−46.75	−43.75
55	155 75	68 75	24	86.00	30 00	−6	18 50	−7.50	−36	−49 00	−45 00
54	153 50	67 50	23	83.75	28 75	−7	16.25	−8 75	−37	−51 25	−46.25
53	151 25	66.25	22	81.50	27 50	−8	14.00	−10.00	−38	−53 50	−47.50
52	149 00	65 00	21	79.25	26 25	−9	11.75	−11.25	−39	−55.75	−48.75
51	146.75	63.75	20	77.00	25.00	−10	9.50	−12 50	−40	−58.00	−50 00
50	144 50	62.50									

AIR.—ATMOSPHERE.

The atmosphere is known to extend to at least 45 miles above the earth. It is a mixture of about 79 measures of nitrogen gas and 21 of oxygen gas; or about 77 nitrogen, 23 oxygen, by weight. It generally contains, however, a trace of water, and of carbonic acid and carburetted hydrogen gases, and still less ammonia.

Density of air. Under "normal" or "standard" conditions (sea level, lat 45°, barometer 760 mm = 29.922 ins, temperature 0° C = 32° F) **dry air weighs** 1.292673 kilograms per cubic meter * = 2.17888 lbs avoir per cubic yard. For other lats and elevations—

$$\text{Density, in kg per cu m,} = 1.292673 \times \frac{R}{R + 2h} \times (1 - 0.002837 \cos 2 \text{ lat})^*$$

where R = earth's mean radius = 6,366,198 meters; h = elevation above sea level, in meters. For other temperatures, see below.

Under normal conditions, but with 0.04 parts carbonic acid (CO_2) in 100 parts of air, density = 1.293052 kg per cu m.† = 2.17952 lbs avoir per cu yd.‡

The atmospheric pressure, at any given place, may vary 2 inches or more from day to day. **The average pressure,** at sea level § varies from about 745 to 770 millimeters of mercury according to the latitude and locality. 760 millimeters* is generally accepted as the mean atmospheric pressure, and called **an atmosphere.** The "**metric atmosphere,**" taken arbitrarily at 1 kilogram per square centimeter, is in general use in Continental Europe. The pressure diminishes as the altitude increases.† Therefore, a pump in a high region will not lift water to as great a height as in a low one. The pressure of air, like that of water, is, at any given point, equal in all directions.

It is often stated that the **temperature of the atmosphere** lowers at the rate of 1° Fah for each 300 feet of ascent **above the earth's surface;** but this is liable to many exceptions, and varies much with local causes. Actual observation in balloons seems to show that, up to the first 1000 feet, 1° in about 200 feet is nearer the truth; at 2000 feet, 1° in 250 feet; at 4000 feet, 1° in 300 feet; and, at a mile, 1° in 350 feet.

In breathing, a grown person at rest requires from 0.25 to 0.35 of a cubic foot of air per minute; which, when breathed, vitiates from 3.5 to 5 cubic feet. When walking, or hard at work, he breathes and vitiates two or three times as much. About 5 cubic feet of fresh air per person per minute are required for the perfect ventilation of rooms in winter; 8 in summer. Hospitals 40 to 80.

Beneath the general level of the surface of the earth, in temperate regions, a tolerably uniform **temperature** of about 50° to 60° Fah exists at the depth of about 50 to 60 feet; and increases about 1° for each additional 50 to 60 feet; all subject, however, to considerable deviations owing to many local causes. In the Rose Bridge Colliery, England, at the depth of 2424 feet, the temperature of the coal is 93.5° Fah; and at the bottom of a boring 4169 feet deep, near Berlin, the temperature is 119°.

The air is a very slow conductor of heat; hence hollow walls serve to retain the heat in dwellings; besides keeping them dry. **It rushes into a vacuum** near sea level with a velocity of about 1157 feet per second; or 13.3 miles per minute; or about as fast as sound ordinarily travels through quiet air. See Sound.

Like all other elastic fluids, air expands equally with equal increases of temperature. Every increase of 5° Fah, expands the bulk of any of them slightly more than 1 per cent of that which it has at 0° Fah; or 500° about doubles its bulk at zero. The bulk of any of them diminishes inversely in proportion to the total **pressure** to which it is subjected.

This holds good with air at least up to pressures of about 750 lbs per square inch, or 50 times its natural pressure; the air in this case occupying one-fiftieth of its natural bulk. In like manner the bulk will increase as the total pressure is diminished. Substances which follow these laws, are said to be **perfectly**

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* H. V. Regnault, Mémoires de l'Académie Royale des Sciences de l'Institut de France, Tome XXI, 1847. Translation in abstract, Journal of Franklin Institute, Phila., June, 1848.

† Travaux et Mémoires du Bureau International des Poids et Mesures, Tome 1, page A 54. Smithsonian Meteorological Tables, 1893, published in Smithsonian Miscellaneous Collections, Vol. XXXV, 1897.

‡ See Conversion Tables.

§ See Leveling by the Barometer.

elastic. Under a pressure of about 5½ tons per square inch, air would become as dense as water. Since the air at the surface of the earth is pressed 14¾ lbs per square inch by the atmosphere above it, and since this is equal to the weight of a column of water 1 inch square and 34 feet high, it follows that at the depths of 34, 68, 102 feet, &c, **below water,** air will be compressed into ½, ⅓, ¼, &c, of its bulk at the surface.

In a diving-bell, men, after some experience, can readily work for several hours at a depth of 51 feet, or under a pressure of 2½ atmospheres; or 37½ lbs per square inch. But at 90 feet deep, or under 3.64 atmospheres, or nearly 55 lbs per square inch, they can work for but about an hour, without serious suffering from paralysis, or even danger of death. Still, at the St. Louis bridge, work was done at a depth of 110½ feet; pressure 63.7 lbs per square inch.

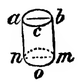
The dew point is that temp (varying) at which the air deposits its vapor.

The greatest heat of the air in the sun probably never exceeds 145° Fah, nor the greatest cold —74° at night. About 130° above, and 40° below zero, are the extremes in the U. S east of the Mississippi; and 65° below in the N. W.; all at common ground level. It is stated, however, that —81° has been observed in N. E. Siberia; and +101° Fah in the shade in Paris; and +153° in the sun at Greenwich Observatory, both in July, 1881. It has frequently exceeded +100° Fah in the shade in Philadelphia during recent years.

WIND.

The relation between the velocity of wind, and its pressure against an obstacle placed either at right angles to its course, or inclined to it, has not been well determined; and still less so its pressure against curved surfaces The pressure against a large surface is probably proportionally greater than against a small one. It is generally supposed to vary nearly as the squares of the velocities; and when the obstacle is at right angles to its direction, the pressure in lbs per square foot of exposed surface is considered to be equal to the square of the velocity in miles per hour, divided by 200. On this basis, which is probably quite defective, the following table, as given by Smeaton, is prepared. See also pp 710, 711, 713, 714, 758, 759, 764.

Vel. in Miles per Hour	Vel in Ft. per Sec.	Pres. in Lbs. per Sq Ft.	Remarks
1	1 467	005	Hardly perceptible.
2	2 933	020	Pleasant.
3	4 400	045	
4	5 867	080	
5	7 33	125	
10	14 67	.5	
12½	18.33	781	Fresh breeze.
15	22.	1 125	
20	29 33	2	
25	36 67	3 125	Brisk wind
30	44	4 5	Strong wind.
40	58 67	8	High wind.
50	73 33	12 5	Storm
60	88	18.	Violent storm.
80	117 3	32	Hurricane
100	146 7	50.	Violent hurricane, uprooting large trees.



The pres against a semicylindrical surface *a c b n o m* is about half that against the flat surf *a b n m*.

Tredgold recommends to allow 40 lbs per sq ft of roof for the pres of wind against it; but as roofs are constructed with a slope, and consequently do not receive the full force of the wind, this is plainly too much * Moreover, only one-half of a roof is usually exposed, even thus partially, to the wind. Probably the force in such cases varies approximately as the sines of the angles of slopes According to observations in Liverpool, in 1860, a wind of 38 miles per hour, produced a pres of 14 lbs per sq ft against an object perp to it: and one of 70 miles per hour, (the severest gale on record at that city,) 42 lbs per sq foot. These would make the pres per sq ft, more nearly equal to the square of the vel in miles per hour, div by 100, or nearly twice as great as given in Smeaton's table. We should ourselves give the preference to the Liverpool observations. A very violent gale in Scotland, registered by an excellent anemometer, or wind-gauge, 45 lbs per sq ft. It is stated that as high as 55 lbs has been observed at Glasgow. High winds often *lift* roofs.

The gauge at Girard College, Philada, broke under a strain of 42 lbs per sq ft; a tornado passing at the moment, within ¼ mile

By inversion of Smeaton's rule, if the force in lbs per sq ft, be mult by 200, the sq rt of the prod will give the vel in miles per hour

* The writer thinks 8 lbs per sq foot of *ordinary double-sloping roofs*, or 16 lbs for *shed-roofs*, sufficient allowance for pres of wind.

RAIN AND SNOW.

The annual precipitation* at any given place varies greatly from year to year, the ratio between maximum and minimum being frequently greater than 2 : 1. **Beware of averages.** In estimating *floods*, take the *maximum* falls, and in estimating *water supply*, the *minimum*, not only per annum, but for short periods. In estimating water supply, make deductions for evaporation and leakage.

Maxima and minima deduced from observations covering only 4 or 5 years are apt to be misleading. Data covering even 10 or more years may just miss including a very severe flood or drought. Records of from 15 to 20 years may usually be accepted as sufficient.

Table 1. Average Precipitation * † in the United States, in ins. (From Bulletin C of U. S. Department of Agriculture, compiled to end of 1891.)

State.	Spr.	Sum.	Aut.	Win.	Ann'l	State.	Spr.	Sum.	Aut.	Win.	Ann'l
Alabama	14.9	13.8	10.0	14.9	53.6	Montana	4.2	4.9	2.6	2.3	14.0
Arizona	1.3	4.3	2.2	3.1	10.9	Nebraska	8.9	10.9	4.9	2.2	26.9
Arkansas	14.3	12.5	11.0	12.8	50.6	Nevada	2.3	0.8	1.3	3.2	7.6
California	6.2	0.3	3.5	11.9	21.9	N. Hampshire	9.8	12.2	11.4	10.7	44.1
Colorado	4.2	5.5	2.8	2.3	14.8	New Jersey	11.7	13.3	11.2	11.1	47.3
Connecticut	11.1	12.5	11.7	11.5	46.8	New Mexico	1.4	5.8	3.5	2.0	12.7
Delaware	10.2	11.0	10.0	9.6	40.8	New York	8.5	10.4	9.7	7.9	36.5
Dist. Columbia	11.0	12.4	9.4	9.0	41.8	N. Carolina	12.9	16.6	12.0	12.2	53.7
Florida	10.2	21.4	14.2	9.1	54.9	N. Dakota	4.6	8.0	2.8	1.7	17.1
Georgia	12.4	15.6	10.7	12.7	51.4	Ohio	10.0	11.9	9.0	9.1	40.0
Idaho	4.4	2.1	3.6	7.0	17.1	Oregon	9.8	2.7	10.5	21.0	44.0
Illinois	10.2	11.2	9.0	7.7	38.1	Pennsylvania	10.3	12.7	10.0	9.5	42.5
Indiana	11.0	11.7	9.7	10.3	42.7	Rhode Island	11.9	10.7	11.7	12.4	46.7
Indian T'y	10.6	11.0	8.9	5.7	36.2	S. Carolina	9.8	16.2	9.7	9.7	45.4
Iowa	8.3	12.4	8.1	4.1	32.9	S. Dakota	7.2	9.7	3.5	2.5	22.9
Kansas	8.9	11.9	6.7	3.5	31.0	Tennessee	13.5	12.5	10.2	14.5	50.7
Kentucky	12.4	12.5	9.7	11.8	46.4	Texas	8.1	8.6	7.6	6.0	30.3
Louisiana	13.7	15.0	10.8	14.4	53.9	Utah	3.4	1.5	2.2	3.5	10.6
Maine	11.1	10.5	12.3	11.1	45.0	Vermont	9.2	12.2	11.4	9.3	42.1
Maryland	11.4	12.4	10.7	9.5	41.0	Virginia	10.9	12.5	9.5	9.7	42.6
Massachusetts	11.6	11.4	11.9	11.7	46.6	Washington	8.6	3.9	10.5	16.8	39.8
Michigan	7.9	9.7	9.2	7.0	33.8	W. Virginia	10.9	12.9	9.0	10.0	42.8
Minnesota	6.5	10.8	5.8	3.1	26.2	Wisconsin	7.8	11.6	7.8	5.2	32.5
Mississippi	14.9	12.6	10.1	15.4	53.0	Wyoming	4.3	3.5	2.2	1.6	11.6
Missouri	10.0	12.4	9.1	6.5	38.0	United States	9.2	10.3	8.3	8.6	36.3

At Philadelphia, in 1869, during which occurred the greatest drought known there for at least 50 years, 43.21 inches fell; August 13, 1873, 7.3 inches in 1 day; August, 1867, 15.8 inches in 1 month; July, 1842, 6 inches in 2 hours; 9 inches per month not more than 7 or 8 times in 25 years. From 1825 to 1893, greatest in one year, 61 inches, in 1867; least, 30 inches, in 1825 and 1880. At Norristown, Pennsylvania, in 1865, the writer saw evidence that at least 9 inches fell within 5 hours. At Genoa, Italy, on one occasion, 32 inches fell in 24 hours; at Geneva, Switzerland, 6 inches in 3 hours; at Marseilles, France, 13 inches in 14 hours; in Chicago, Sept., 1878, .97 inch in 7 minutes.

Near London, England, the mean total fall for many years is 23 inches. On one occasion, 6 inches fell in 1½ hours! In the mountain districts of the English lakes, the fall is enormous; reaching in some years to 180 or 240 inches; or from 15 to 20 feet! while, in the adjacent neighborhood, it is but 40 to 60 inches. At Liverpool, the average is 34 inches; at Edinburgh, 30; Glasgow, 22; Ireland, 36; Madras, 47; Calcutta, 60; maximum for 16 years, 82; Delhi, 21; Gibraltar, 30; Adelaide, Australia, 23; West Indies, 36 to 96; Rome, 39. On the Khassya hills north of Calcutta, 500 inches, or 41 feet 8 inches, have fallen in the 6 rainy months! In other mountainous districts of India, annual falls of 10 to 20 feet are common.

A moderate steady rain, continuing 24 hours, will yield a depth of about an inch.

As a general rule, more rain falls in warm than in cold countries; and more in elevated regions than in low ones. Local peculiar-

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* Precipitation includes snow, hail, and sleet, melted.

† Unmelted snow is here estimated at 10 inches snow = 1 inch rain. But see "Rainfall equivalent of snow," p 324.

ities, however, sometimes reverse this; and also cause great differences in the amounts in places quite near each other; as in the English lake districts just alluded to. It is sometimes difficult to account for these variations. In some lagoons in New Granada, South America, the writer has known three or four heavy rains to occur weekly for some months, during which not a drop fell on hills about 1000 feet high, within ten miles' distance, and within full sight. At another locality, almost a dead-level plain, fully three-quarters of the rains that fell for two years, at a spot two miles from his residence, occurred in the morning; while those which fell about three miles from it, in an opposite direction, were in the afternoon.

The relation between precipitation and stream-flow is greatly affected by the existence of forests or crops, by the slope and character of ground on the water-shed, especially as to rate of absorption, by the season of the year, the frost in the ground, etc. The stream-flow may ordinarily be taken as varying between 0 2 and 0 8 of the rainfall. Streams in limestone regions frequently lose a very large proportion of their flow through subterranean caverns.

Assuming a fall of 2 feet in 1 year (= 76,379 cubic feet per square mile per day), that half the rainfall is available for water supply, and that a per capita consumption of 4 cubic feet (= 30 gallons) per day is sufficient, one square mile will supply 19,095 persons; or a square of 38.25 feet on a side will supply one person.

An inch of rain amounts to 3630 cubic feet; or 27155 U. S. gallons; or 101.3 tons per acre; or to 2323200 cubic feet; or 17378743 U. S. gallons; or 64821 tons per square mile at 62½ lbs per cubic foot.

The most destructive rains are usually those which fall upon snow, under which the ground is frozen, so as not to absorb water.

Table 2. Maximum intensity of rainfall for periods of 5, 10, and 60 minutes at Weather Bureau stations equipped with self-registering gauges, compiled from all available records to the end of 1896.

(From Bulletin D of U. S. Department of Agriculture.)

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	5 min.	10 mins.	60 mins.		5 min.	10 mins.	60 mins.
	Ins.	Inches.	Inches.		Ins.	Inches.	Inches.
Bismarck	9.00	6.00	2.00	Chicago	6.60	5.92	1.60
St. Paul	8.40	6.00	1.30	Galveston	6.48	5.58	2.55
New Orleans	8.16	4.86	2.18	Omaha	6.00	4.80	1.55
Milwaukee	7.80	4 20	1 25	Dodge City	6.00	4.20	1.34
Kansas City	7.80	6 60	2.40	Norfolk	5.76	5.46	1.55
Washington	7.50	5.10	1.78	Cleveland	5.64	3.66	1.12
Jacksonville	7.44	7.08	2.20	Atlanta	5.46	5.46	1.50
Detroit	7.20	6 00	2.15	Key West	5 40	4.80	2.25
New York City.	7.20	4.92	1.60	Philadelphia	5.40	4.02	1.50
Boston	6 72	4.98	1 68	St. Louis	4.80	3.84	2.25
Savannah	6.60	6.00	2.21	Cincinnati	4.56	4.20	1.70
Indianapolis	6.60	3.90	1 60	Denver	3.60	3.30	1.18
Memphis	6.60	4.80	1.86	Duluth	3.60	2.40	1.35

The weight of freshly fallen snow, as measured by the author, varies from about 5 to 12 lbs per cubic foot; apparently depending chiefly upon the degree of humidity of the air through which it had passed. On one occasion, when mingled snow and hail had fallen to the depth of 6 inches, he found its weight to be 31 lbs per cubic foot. It was very dry and incoherent. A cubic foot of heavy snow may, by a gentle sprinkling of water, be converted into about half a cubic foot of slush, weighing 20 lbs.; which will not **slide or run off** from a shingled roof sloping 30°, if the weather is cold. A cubic block of snow saturated with water until it weighed 45 lbs per cubic foot, just slid on a rough board inclined at 45°; on a smoothly planed one at 30°; and on slate at 18°; all approximate. A prism of snow, saturated to 52 lbs per cubic foot, one inch square, and 4 inches high, **bore a weight** of 7 lbs; which at first compressed it about one-quarter part of its length. European engineers consider 6 lbs per square foot of roof to be sufficient **allowance for the weight of snow**

and 8 lbs for the pressure of wind; total, 14 lbs. The writer thinks that in the U. S. the allowance for snow should not be taken at *less* than 12 lbs; or the total for snow and wind, at 20 lbs. There is no danger that snow on a roof will become saturated to the extent just alluded to; because a rain that would supply the necessary quantity of water would also by its violence wash away the snow; but we entertain no doubt whatever that the united pressures from snow and wind, in our Northern States, do actually at times reach, and even surpass, 20 lbs per square foot of roof. **The limit of perpetual snow** at the equator is at the height of about 16000 feet, or say 3 miles above sea-level; in lat 45° north or south, it is about half that height; while near the poles it is about at sea-level.

Rain Gauges. Plain cylindrical vessels are ill adapted to service as rain gauges; because moderate rains, even though sufficient to yield a large run-off from a moderate area, are not of sufficient depth to be satisfactorily measured unless the depth be exaggerated. The inaccuracy of measurement, always considerable, is too great relatively to the depth.

In its simplest and most usual form, the gauge (see Fig.) consists essentially of a funnel, A, which receives the rain and leads it into a measuring tube, B, of smaller cross-section. The funnel should have a vertical and fairly sharp edge, and, in order to minimize the loss through evaporation, it should fit closely over the tube, and its lower end should be of small diameter.

The depth of water in the tube is ascertained by inserting, to the bottom of the tube, a measuring stick of some unpolished wood which will readily show to what depth it has been wet. The stick may be permanently graduated, or it may be compared with an ordinary scale at each observation. The tube is usually of such diameter that the area of its cross-section, minus that of the stick, is one-tenth of the area of the funnel mouth. The depth of rainfall is then one-tenth of the depth as measured by the stick.

DIMENSIONS OF STANDARD U. S. WEATHER BUREAU RAIN GAUGE.				Ins.
A.	Receiver or funnel.		Diameter	8
B.	Measuring tube.	Height 20 ins.	"	2.53
C C.	Overflow attachment and snow gauge.		"	8

Such gauges, with the tubes carefully made from seamless drawn brass tubing, cost about $5.00 each; but an intelligent and careful tinsmith, given the dimensions accurately, can construct, of galvanized iron, for about $1.00 a gauge that will answer every purpose of the engineer.

The exposure has a very marked effect upon the results obtained. The funnel should be elevated about 3 ft, in order to prevent rain from splashing back into it from the ground or roof. If on a roof, the latter should be flat, and preferably 50 ft wide or wider, and the gauge should be placed as far as possible from the edges; else the air currents, produced by the wind striking the side of the building, will carry some of the rain over the gauge. No objects much higher than the gauge should be near it, as they produce variable air currents which may seriously affect its indications.

An overflow tank, C, should be provided, for cases of overfilling the tube.

Water, freezing in the gauge, may burst it, or force the bottom off, or at least so deform the gauge as to destroy its accuracy.

To measure snow, the funnel is removed, and the snow is collected in the overflow attachment or other cylindrical vessel deep enough to prevent the snow from being blown out, and the cross-sectional area of which is accurately known. The snow is then melted, either by allowing it to stand in a warm place, or, with less loss through evaporation, by adding an accurately known quantity of luke-warm water. In the latter case, the volume of the added water must of course be deducted from the measurement.

Rainfall equivalent of snow. Ten inches of snow are usually taken as equivalent to 1 in of rain; but, according to various authorities, the equivalent may vary between 2½ and 34; *i. e.*, between 25 and 1.84 lbs. per cubic foot.

Self-recording gauges, of which several forms are on the market, are quite expensive, and, even when purchased from regular makers, seldom perfectly reliable. Gauges using a small tipping bucket register inaccurately in heavy rains; those using a float are limited as to the total depth which they can register; while those which weigh the rain, if exposed, are affected by wind.

ities, however, sometimes reverse this; and also cause great differences in the amounts in places quite near each other; as in the English lake districts just alluded to. It is sometimes difficult to account for these variations. In some lagoons in New Granada, South America, the writer has known three or four heavy rains to occur weekly for some months, during which not a drop fell on hills about 1000 feet high, within ten miles' distance, and within full sight. At another locality, almost a dead-level plain, fully three-quarters of the rains that fell for two years, at a spot two miles from his residence, occurred in the morning; while those which fell about three miles from it, in an opposite direction, were in the afternoon.

The relation between precipitation and stream-flow is greatly affected by the existence of forests or crops, by the slope and character of ground on the water-shed, especially as to rate of absorption, by the season of the year, the frost in the ground, etc. The stream-flow may ordinarily be taken as varying between 0.2 and 0.8 of the rainfall. Streams in limestone regions frequently lose a very large proportion of their flow through subterranean caverns.

Assuming a fall of 2 feet in 1 year (= 76,379 cubic feet per square mile per day), that half the rainfall is available for water supply, and that a per capita consumption of 4 cubic feet (= 30 gallons) per day is sufficient, one square mile will supply 19,095 persons; or a square of 38.25 feet on a side will supply one person.

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New Orleans	8.16	4.86	2.18	Omaha	6.00	4.80	1.55
Milwaukee	7.80	4.20	1.25	Dodge City	6.00	4.20	1.34
Kansas City	7.80	6.60	2.40	Norfolk	5.76	5.46	1.55
Washington	7.50	5.10	1.78	Cleveland	5.64	3.66	1.12
Jacksonville	7.44	7.08	2.20	Atlanta	5.46	5.46	1.50
Detroit	7.20	6.00	2.15	Key West	5.40	4.80	2.25
New York City	7.20	4.92	1.60	Philadelphia	5.40	4.02	1.50
Boston	6.72	4.98	1.68	St. Louis	4.80	3.84	2.25
Savannah	6.60	6.00	2.21	Cincinnati	4.56	4.20	1.70
Indianapolis	6.60	3.90	1.60	Denver	3.60	3.30	1.18
Memphis	6.60	4.80	1.86	Duluth	3.60	2.40	1.35

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WATER.

PURE water, as boiled and distilled, **is composed of** the two gases, hydrogen and oxygen; in the proportions of 2 measures hydrogen to 1 of oxygen; or 1 weight of hydrogen to 8 of oxygen. Ordinarily, however, it contains several foreign ingredients, as carbonic and other acids; and soluble mineral, or organic substances. When it contains much lime, it is said to be *hard*; and will not make a good lather with soap. **The air** in its ordinary state **contains** about 4 grains of water per cubic foot.

The average **pressure of the air at sea level, will balance a column of water** 34 feet high; or about 30 inches of mercury.

Weight of unit volume, as affected by temperature. barometer at 30 ins:

Temp, Fahr,	32°	39.2°	40°	50°	60°	70°	80°	90°	212°
Temp, Cent,	0°	4°	4.4°	10°	15.6°	21.1°	26 7°	32 2°	100°
lbs/cu ft,	62.417	62.428	62.423	62 409	62.367	62.302	62 218	62.119	59.700
grams/cu cm,	0.9998	1.0000	0.9999	0.9997	0 9990	0.9980	0.9966	0 9950	0.9563

Maximum density at 4° C = 39.2° F; 1 gram/cu cm = 62.428 lbs/cu ft. From this temp, it expands, either by heat or by cold.

At 62° F (16.7° C), barom, 30 ins; weight of water = about **815 × weight of air.**

Sea water. Weight of unit volume; 64 to 64 27 lbs per cu ft = 1.026 to 1.029 grams per cu cm. See also p 328.

Ice. Weight of unit volume; 57 2 lbs/cu ft = 0.916 gram/cu cm. (L. Dufour)

Hence, as ice, it has expanded one-twelfth of its original bulk as water; and the **sudden expansive force** exerted at the moment of freezing, is sufficiently great to split iron water-pipes; being probably not less than 30,000 lbs per square inch. Instances have occurred of its splitting cast tubular posts of iron bridges, and of ordinary buildings, when full of rain water from exposure. It also loosens and throws down masses of rock, through the joints of which rain or spring water has found its way. Retaining-walls also are sometimes overthrown, or at least bulged, by the freezing of water which has settled between their backs and the earth filling which they sustain; and walls which are not founded at a sufficient depth, are often lifted upward by the same process.

It is said that **in a glass tube ¼ inch in diameter,** water will not freeze until the temperature is reduced to 23°; and in tubes of less than $\frac{1}{20}$ inch, to 3° or 4°. Neither will it freeze until considerably colder than 32° in rapid running streams. **Anchor ice,** sometimes found at depths as great as 25 feet, consists of an aggregation of small crystals or needles of ice frozen at the surface of rapid open water; and probably carried below by the force of the stream. It does not form under frozen water.

Since ice floats in water: and a floating body displaces a weight of the liquid equal to its own weight, it follows that a cubic foot of floating ice weighing 57 2 lbs, must displace 57.2 lbs of water. But 57.2 lbs of water, one foot square, is 11 inches deep: therefore, floating ice of a cubical or parallelopipedal shape, will have $\frac{11}{12}$ of its volume under water; and only $\frac{1}{12}$ above; and a square foot of ice of any thickness, will require a weight equal to $\frac{1}{11}$ of its own weight to sink it to the surface of the water. In practice, however, this must be regarded merely as a close approximation, since the weight of ice is somewhat affected by enclosed air-bubbles.

Pure water is usually assumed to **boil** at 212° Fah in the open air, at the level of the sea; the barometer being at 30 inches; and at about 1° less for every 520 feet above sea level, for heights within 1 mile. In fact, its boiling point varies like its freezing point, with its purity, the density of the air, the material of the vessel, &c. In a metallic vessel, it may boil at 210°; and in a glass one, at from 212° to 220°; and it is stated that if all air be previously extracted, it requires 275°.

It evaporates at all temperatures; **dissolves** more substances than any other agent: and has a greater capacity for heat than any other known substance.

Compression, per atmosphere (14.7 lbs/sq in) about 1 in 21,740. The volume is restored when the pres is removed.

Effect on metals. The lime contained in many waters, forms deposits in metallic water-pipes, and in channels of earthenware, or of masonry, especially if the current be slow. Some other substances do the same; obstructing the flow of the water to such an extent, that it is always expedient to use pipes of diameters larger than would otherwise be necessary. The lime also forms very hard **incrustations at the bottoms of boilers**; very much impairing their efficiency; and rendering them more liable to burst. Such water is unfit for locomotives. We have seen it stated that the Southwestern R R Co, England, prevent this lime deposit, along their limestone sections, by dissolving 1 ounce of sal-ammoniac to 90 gallons of water. The salt of sea water forms similar deposits in boilers; as also does mud, and other impurities.

Water, either when very pure, as rain water; or when it contains carbonic acid, (as most water does,) **produces carbonate of lead in lead pipes**; and as this is an active poison, such pipes should not be used for such waters. Tinned lead pipes may be substituted for them. If, however, sulphate of lime also be present, as is very frequently the case, this effect is not always produced; and several other substances usually found in spring and river water, also diminish it to a greater or less degree. **Fresh water corrodes wrought iron more rapidly than cast**; but the reverse appears to be the case with **sea water**; although it also affects wrought iron very quickly; so that thick flakes may be detached from it with ease. The corrosion of iron or steel by sea water increases with the carbon. Cast-iron cannons from a vessel which had been sunk in the fresh water of the Delaware River for more than 40 years, were perfectly free from rust. Gen. Pasley, who had examined the metals found in the ships Royal George, and Edgar, the first of which had remained sunk in the sea for 62 years, and the last for 133 years, "stated that the cast iron had generally become quite soft; and in some cases resembled plumbago. Some of the shot when exposed to the air became hot; and burst into many pieces. The wrought iron was not so much injured, *except when in contact with copper, or brass gun-metal.* Neither of these last was much affected, except when in contact with iron. Some of the wrought iron was reworked by a blacksmith, and pronounced superior to modern iron." "Mr. Cottam stated that some of the guns had been carefully removed in their soft state, to the Tower of London; and in time (within 4 years) *resumed their original hardness.* Brass cannons from the Mary Rose, which had been sunk in the sea for 292 years, were considerably honeycombed in spots only; (perhaps where iron had been in contact with them) The old cannons, of wrought-iron bars hooped together, were corroded about ¼ inch deep; but had probably been protected by mud. The cast-iron shot became redhot on exposure to the air; and fell to pieces like dry clay!"

"Unprotected parts of cast-iron sluice-valves on the sea gates of the Caledonian canal were converted into a soft plumbaginous substance, to a depth of ¾ of an inch, within 4 years; but where they had been coated with common Swedish tar, they were entirely uninjured This softening effect on cast iron appears to be as rapid even when the water is but slightly brackish; and that only at intervals. It also takes place on cast iron imbedded in salt earth. Some water pipes thus laid near the Liverpool docks, at the expiration of 20 years were soft enough to be cut by a knife; while the same kind, on higher ground beyond the influence of the sea water, were as good as new at the end of 50 years."

Observation has, however, shown that **the rapidity of this action depends much on the quality of the iron**; that which is dark-colored and contains much carbon mechanically combined with it, corrodes most rapidly: while hard white, or light-gray castings remain secure for a long time. Some cast-iron sea-piles of this character, showed no deterioration in 40 years.

Contact with brass or copper is said to induce a galvanic action which greatly hastens decay in either fresh or salt water. Some muskets were recovered from a wreck which had been submerged in sea water for 70 years near New York. The brass parts were in perfect condition; but the iron parts had entirely disappeared. **Galvanizing** (coating with zinc) acts as a preservative to the iron, but at the expense of the zinc, which soon disappears. The iron then corrodes. If iron be well heated, and then coated with **hot coal-tar**, it will resist the action of either salt or fresh water for many years. It is very important that the tar be perfectly *purified.* Such a coating, or one of paint, will not prevent **barnacles and other shells** from attaching themselves to the iron. Asphaltum, if pure, answers as well as coal-tar.

Copper and bronze are very little affected by sea water.

No galvanic action has been detected where brass ferules are inserted into the water-pipes in Philadelphia.

The most prejudicial exposure for iron, as well as for wood, is that to alternate wet and dry. At some dangerous spots in Long Island Sound, it has been the practice to drive round bars of rolled iron about 4 inches diameter, for supporting signals. These wear away most rapidly between high and low water; at the rate of about an inch in depth in 20 years; in which time the 4-inch bar becomes reduced to a 2-inch one, along that portion of it. Under fresh water especially or under ground, a thin coating of coal-pitch varnish, carefully applied, will protect iron, such as water-pipes, &c., for a long time. See page 655. The sulphuric acid contained in the water from coal mines corrodes iron pipes rapidly. **In the fresh water of canals,** iron boats have continued in service from 20 to 40 years. **Wood remains sound** for centuries under either fresh or salt water, if not exposed to be worn away by the action of currents; or to be destroyed by marine insects.

Sea water weighs from 64 to 64.27 lbs per cubic foot, or say from 1.6 to 1.9 lbs per cubic foot more than fresh water, varying with the locality, and not appreciably with the depth. The excess, over the weight of fresh water, is chiefly common salt. At 64 lbs per cubic foot, 35 cubic feet weigh 2240 lbs. **Sea water freezes** at about 27° Fahr. The ice is fresh; but (especially at low temperatures) brine may be entrapped in the ice.

A teaspoonful of powdered alum, well stirred into a bucket of **dirty water,** will generally purify it sufficiently within a few hours to be drinkable. If a hole 3 or 4 feet deep be dug in the sand of the sea-shore, the infiltrating water will usually be sufficiently fresh for washing with soap; or even for drinking. It is also stated that water may be preserved sweet for many years by placing in the containing vessel 1 ounce of black oxide of manganese for each gallon of water.

It is said that water kept in zinc tanks; or flowing through iron tubes galvanized inside, rapidly becomes poisoned by soluble salts of zinc formed thereby; and it is recommended to coat zinc surfaces with asphalt varnish to prevent this. Yet, in the city of Hartford, Conn, service pipes of iron, galvanized inside and out, were adopted in 1855, at the recommendation of the water commissioners; and have been in use ever since. They are likewise used in Philadelphia and other cities to a considerable extent. In many hotels and other buildings in Boston, the "Seamless Drawn Brass Tube" of the American Tube Works at Boston, has for many years been in use for service pipe; and has given great satisfaction. It is stated that the softest water may be kept in brass vessels for years without any deleterious result.

The action of lead upon some waters (even pure ones) is highly poisonous. The subject, however, is a complicated one. An injurious ingredient may be attended by another which neutralizes its action. Organic matter, whether vegetable or animal, is injurious. Carbonic acid, when not in excess, is harmless.

Ice may be so impure that its water is dangerous to drink.

The popular notion that hot water freezes more quickly than cold, with air at the same temperature, is erroneous.

TIDES.

The tides are those well-known rises and falls of the surface of the sea and of some rivers, caused by the attraction of the sun and moon. There are two rises, floods, or high tides; and two falls, ebbs, or low tides, every 24 hours and 50 minutes (**a lunar day**); making the average of 6 hours 12½ minutes between high and low water. These intervals are, however, **subject to great variations;** as are also the heights of the tides; and this not only at different places, but at the same place. These irregularities are owing to the shape of the coast line, the depth of water, winds, and other causes. *Usually* at new and full moon, or rather a day or two after, (or twice in each lunar month, at intervals of two weeks,) the tides rise higher, and fall lower than at other times; and these are called **spring tides.** Also, one or two days after the moon is in her *quarters*, twice in a lunar month, they both rise and fall less than at other times; and are then called **neap tides.** From neap to spring they rise and fall more daily; and vice versa. The **time of high water** at any place, is generally two or three hours after the moon has passed over either the upper or lower meridian; and is called the **establishment** of that place; because, when this time is established, the time of high water on any other day may be found from it in most cases. The total height of spring tides is *generally* from 1½ to 2 times as great as that of neaps. The great **tidal wave** is merely an *undulation*, unattended by any current, or progressive motion of the particles of water. Each successive high tide occurs about 24 minutes later than the preceding one; and so with the low tides

EVAPORATION AND LEAKAGE.

The amount of evaporation from surfaces of water exposed to the natural effects of the open air, is of course greater in summer than in winter, although it is quite perceptible in even the coldest weather. It is greater in shallow water than in deep, inasmuch as the bottom also becomes heated by the sun. It is greater in running, than in standing water, on much the same principle that it is greater during winds than calms. It is probable that the average daily loss from a reservoir of moderate depth, from evaporation alone, throughout the 3 warmer months of the year, (June, July, August,) rarely exceeds about $\frac{3}{10}$ inch, in any part of the United States. Or $\frac{1}{10}$ inch during the 9 colder months, except in the Southern States. These two averages would give a daily one of .15 inch, or a total annual loss of 55 ins, or 4 ft 7 ins. It probably is 3.5 to 4 ft.

By some trials by the writer, in the tropics, ponds of pure water 8 ft deep, in a stiff retentive clay, and fully exposed to a very hot sun all day, lost during the dry season, precisely 2 ins in 16 days, or ⅛ inch per day, while the evaporation from a glass tumbler was ¼ inch per day. The air in that region is highly charged with moisture, and the dews are heavy. Every day during the trial the thermometer reached from 115° to 125° in the sun.

The total annual evaporation in several parts of England and Scotland is stated to average from 22 to 38 ins, at Paris, 31, Boston, Mass, 32, many places in the U. S. 30 to 36 ins. This last would give a daily average of $\frac{1}{10}$ inch for the whole year. Such statements, however, are of very little value, unless accompanied by memoranda of the circumstances of the case, such as the depth, exposure, size and nature of the vessel, pond, &c, which contains the water, &c. Sometimes the total annual evaporation from a district of country exceeds the rain fall, and vice versa.

On canals, reservoirs, &c, it is usual to combine the loss by evaporation, with that by filtration. The last is that which soaks into the earth; and of which some portion passes entirely through the banks, (when in embankt,) and if in very small quantity, may be dried up by the sun and air as fast as it reaches the outside, so as not to exhibit itself as water, but if in greater quantity, it becomes apparent, as *leakage*.

E. H. Gill, C E. states the average evaporation and filtration on the Sandy and Beaver canal, Ohio, (38 ft wide at water surface, 26 ft at bottom, and 4 ft deep,) to be but 13 cub ft per mile per minute, *in a dry season*. Here the exposed water surf in one mile is 200640 sq ft, and in order, with this surf, to lose 13 cub ft per min, or 18720 cub ft per day of 24 hours, the quantity lost must be $\frac{18720}{200640}$ = .0933 ft, = 1⅛ inch in depth per day. Moreover, one mile of the canal contains 675840 cub ft; therefore the number of days reqd for the combined evaporation and filtration to amount to as much as all the water in the canal, is $\frac{675840}{18720}$ = 36 days. Observations in warm weather on a 22 mile reach of the Chenango canal, N York, (40, 28, and 4 ft,) gave 65⅓ cub ft per mile per min; or 5 times as much as in the preceding case. This rate would empty the canal in about 8 days. Besides this there was an excessive leakage at the gates of a lock, (of only 5⅓ ft lift,) of 479 cub ft per min = 22 cub ft per mile per min; and at aqueducts, and waste weirs, others amounting to 19 cub ft per mile per min. The leakage at other locks with lifts of 8 ft, or less, did not *exceed* about 350 cub ft per min at each. On other canals, it has been found to be from 50, to 500 ft per min. On the Chesapeake and Ohio canal, (where 50, 32, and 6 ft,) Mr. Fisk, C E estimated the loss by evap and filtration in 2 weeks of warm weather, to be equal to all the water in the canal. **Professor Rankine assumes 2 ins per day, for leakage of canal bed, and evaporation, on English canals.** J B Jervis, C E, estimated the loss from evap, filtration, and leakage through lock-gates, on the original Erie canal, (40, 28, and 4 ft,) at 100 cub ft per mile per min, or 144000 cub ft per day. The water surf in a mile is 211200 sq ft; therefore, the daily loss would be equal to a depth of $\frac{144000}{211200}$ = .682 ft, = say 8⅛ ins.

On the Delaware division of the Pennsylvania canals, when the supply is temporarily shut off from any long reach, the water falls from 4 to 8 ins per day. The filtration will of course be much greater on embankts, than in cuts. In some of our canals, the depth at high embankts becomes quite considerable, the earth, from motives of economy not being filled in level under the bottom of the canal, but merely left to form its own natural slopes. At one spot at least, on the Ches and Ohio canal, where one side is a natural face of vertical rock, this depth is 40 ft. Such depths increase the leakage very greatly; especially when, as is frequently the case the embankts are not puddled, and the practice is not to be commended, for other reasons also.

The total average loss from reservoirs of moderate depths, in case the earthen dams be constructed with proper care, and well settled by time, will not exceed about from ½ to 1 inch per day, but in new ones, it will usually be considerably greater.

The loss from ditches, or channels of small area, is much greater than that from navigable canals, so that long canal feeders usually deliver but a small proportion of the water which enters them at their heads.

MECHANICS. FORCE IN RIGID BODIES.

In the following pages we endeavor to make clear a few elementary principles of Mechanics The opening articles are devoted chiefly to the subject of matter *in motion;* for, while an acquaintance with this is perhaps not absolutely required in obtaining a *working* knowledge of those principles of Statics which enter so largely into the computations of the civil engineer, yet it must be an important aid to their intelligent appreciation.

Art. 1 (a). Mechanics may be defined as that branch of science which treats of the effects of force upon matter.

This broad definition of the word "Mechanics" includes hydrostatics, hydraulics, pneumatics etc., if not also electricity, optics, acoustics, and indeed all branches of physics; but we shall here confine ourselves chiefly to the consideration of the action of extraneous forces upon bodies supposed to be *rigid* or incapable of change of shape

(b) Mechanics is divided into two branches, namely:

Kinematics; or the study of the *motions* of bodies, without reference to the *causes* of motion; and

Dynamics, or the study of force and its effects.

The latter is sub-divided into

Kinetics; which treats of the relations between force and *motion;* and

Statics; which considers those special, but very numerous cases, where *equal and opposite* forces counteract each other and thus destroy each other's motions.

Art 2 (a). Matter, or substance, may be defined as whatever occupies space; as meta , stone, wood, water, air, steam, gas, etc.

(b) A body is any portion of matter which is either more or less completely separated in fact from all other matter, or which we take into consideration by itself and as if it were so separated. Thus, a stone is a body, whether it be falling through the air or lying detached upon the ground, or built up into a wall. Also, the wall is a body; or, if we wish, we may consider any portion of the wall as any particular cubic foot or inch in it, as a body. The earth and the other planets are bodies and their smallest atoms are bodies.

A train of cars may be regarded as a body; as may also each car, each wheel or axle or other part of the car, each passenger, etc., etc.

Similarly, the ocean is a body, or we may take as a body any portion of it at pleasure, such as a cubic foot, a certain bay, a drop, etc

(c) But in what follows we shall (as already stated) consider chiefly *rigid* bodies; *i. e.* bodies which undergo no change in *shape*, such as by being crushed or stretched or pulled apart, or penetrated by another body. All actual bodies are of course more or less subject to some such changes of shape; *i. e*, no body is *in fact* absolutely rigid; but we may properly, for convenience, suppose such bodies to exist because many bodies are so nearly rigid that under ordinary circumstances they undergo little or no change of shape, and because such change as does occur may be considered under the distinct head of Strength of Materials

(d) But while bodies are thus to be regarded as incapable of change of *form*, it is equally important that we regard them as *susceptible* to change of *position* as *wholes*. Thus, they may be upset or turned around horizontally or in any other direction, or moved along in any straight or curved line, with or without turning around a point within themselves. In short they are capable of *motion*, as *wholes*.

Art. 3 (a). Motion of a body is change of its position in relation to another body or to some real or imaginary point, which (for convenience) we *regard* as fixed, or at rest. Thus, while a stone falls from a roof to the ground, its position, relatively to the roof is constantly changing, as is also that relatively to the ground and that relatively to any given point in the wall; and we say that the stone is *in motion relatively to either of those bodies*, or to any point in them. But if two stones, A and B, fall from the roof at the same instant and reach the ground at the same (subsequent) instant, we say that although each moves, relatively to roof and ground, yet they have *no motion relatively to each other*; or, they are *at rest* relatively to each other; for their position in regard to each other does not change; *i. e.*, in whatever direction and at whatever distance stone A may be from stone B at the time of starting, it remains in that same direction, and at that same distance from B during the whole time of the fall. Similarly, the roof, the wall and the ground are at rest relatively to each other, yet they are in motion relatively to a falling stone. They are also in motion relatively to the sun owing to the earth's daily rotation about its axis, and its annual movement around the sun.

(**b**) If a train-man walks toward the rear along the top of a freight train just as fast as the train moves forward he is in *motion* relatively to the *train*; but, as a whole, he is at *rest* relatively to *buildings*, etc. near by; for a spectator, standing at a little distance from the track, sees him continually opposite the same part of such building etc. If the man on the train now stops walking, he comes to *rest* relatively to the *train*, but at the same time comes into *motion* relatively to the surrounding *buildings*, etc., for the spectator sees him begin to move along with the train.

(**c**) Since we know of no absolutely fixed point in space we cannot say, of any body, what its *absolute* motion is. Consequently we do not know of such a thing as absolute *rest*, and are safe in saying that *all* bodies are in motion.

Art. 4 (a). The **velocity** of a moving body is its *rate* of motion. A body (as a railroad train) is said to move with **uniform velocity**, or **constant velocity**, when the *distances* moved over in *equal times* are *equal to each other*, no matter how small those times may be taken.

(**b**) The **velocity is expressed** by stating the *distance* passed over *during some given time*, or which *would* be passed over during that time if the uniform motion continued so long. Thus, if a railroad train, moving with constant velocity, passes over 10 miles in half an hour, we may say that its velocity, during that time, is (*i. e.*, that it moves *at the rate* of) 20 miles per hour or 105,600 feet per hour, or 1760 feet per minute, or 29$\frac{1}{3}$ feet per second. Or, we may, if desirable, say that it moves at the rate of 10 miles in *half* an hour or 88 feet in *three* seconds, etc.; but it is generally more convenient to state the distance passed over in a *unit* of time, as in *one* day *one* hour, *one* second, etc.

(**c**) If, of two trains, A and B, moving with constant velocity,

A moves 10 miles in half an hour,
B moves 10 miles in quarter of an hour,

then the velocities are,

A, 20 miles per hour,
B, 40 miles per hour.

In other words, the velocity of a body (which may be defined as the *distance* passed over in a given *time*) is *inversely* as the *time* required to pass over a given *distance*.

(**d**) By **unit velocity** is meant that velocity which, by common consent is taken as equal to *unity* or *one*. Where English measures are used, the unit velocity generally adopted in the study of Mechanics is 1 foot per second.

(**e**) When we say that a body has a velocity of 20 miles per hour, or 10 feet per second, etc. we do not imply that it will necessarily travel 20 miles, or 10 feet, etc.; for it may not have sufficient time for that. We mean merely that it is traveling at the *rate* of 20 miles per hour, or 10 feet per second, etc.; so that *if it continued* to move at that same rate for an hour, or a second, etc., it would travel 20 miles, or 10 feet etc.

(**f**) When velocity *increases* it is said to be **accelerated.** When it *decreases*, it is said to be **retarded.** If the acceleration or retardation is in exact proportion to the time; that is, when during any and every equal interval of time, the same degree of change takes place, it is *uniformly* accelerated, or retarded. When otherwise, the words *variable* and *variably* are used.

(**g**) A body may have at the same time, **two or more independent velocities** requiring to be considered. For instance, a ball fired vertically upward from a

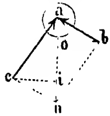
gun, and then falling again to the earth, has, during the whole time of its rise and fall, (1st) the *uniform upward* velocity with which it leaves the muzzle, and (2nd) the continually *accelerated downward* velocity given to it by gravity, which acts upon it during the whole time. Its *resultant* (or apparent) velocity at any moment is the *difference* between these two.

Thus, immediately after leaving the gun, the downward velocity given by gravity is very small, and the resultant velocity is therefore upward and very nearly equal to the whole upward velocity due to the powder. But after awhile the downward velocity (by constantly increasing) becomes equal to the upward velocity; *i. e.*, their difference, or the resultant velocity, becomes *nothing;* the ball at that instant stands still; but its downward velocity continues to increase, and immediately becomes a little greater than the upward velocity, then greater and greater, until the ball strikes the ground. At that instant its resultant velocity is

$$= \left\{ \begin{array}{c} \text{the downward velocity which it would} \\ \text{have acquired by falling } \textit{during the} \\ \textit{whole time of its rise and fall.} \end{array} \right\} - \left\{ \begin{array}{c} \text{the uniform upward} \\ \text{velocity given by the} \\ \text{powder.} \end{array} \right.$$

We have here neglected the resistance of the air, which of course retards both the ascent and the descent of the ball.

(**h**) As a further illustration, regard *a b n c* as a raft drifting in the direction *c a* or *n b*. A man on the raft walks with uniform velocity from corner *n* to corner *c* while the raft drifts (with a uniform velocity a little greater than that of the man) through the distance *n b*. Therefore, when the man reaches corner *c*, that corner has moved to the point which, when he started, was occupied by *a*. The man's resultant motion, relatively to the bed of the river or to a point on shore, has therefore been *n a*. His motion at right angles to *n a*, due to his walking, is *i c*, but that due to the drifting of the raft is *o b*. These two are equal and opposite. Hence his *resultant* motion *at right angles* to *n a* is nothing; he does not move from the line *n a*. His walking moves him through a distance equal to *n i*, in the direction *n a:* and the drifting through a distance equal to *i a*, and the sum of these two is *n a*.



(**i**) All the motions which we see given to bodies are but *changes* in their unknown *absolute* motions. For convenience, we may confine our attention to some one or more of these changes, neglecting others.

Thus, in the case of the ball fired upward from a gun (see (*g*) above) we may neglect its uniform upward motion and consider only its constantly accelerated downward motion under the action of gravity, or, as is more usual we may consider only the *resultant* or *apparent* motion, which is first upward and then downward. In both cases we neglect the motions of the ball caused by the several motions of the earth in space.

Art. 5 (a). Force, the cause of change of motion. Suppose a perfectly smooth ball resting upon a perfectly hard, frictionless and level surface, and suppose the resistance of the air to be removed. In order to merely *move* the ball horizontally (*i. e.*, to set it in motion—to change its state of motion) some *force* must act upon it. Or, if such a ball were already in motion, we could not retard, or hasten it, or turn it from its path without exerting force upon it. For, as stated in **Newton's first law of motion, "every body continues in its state** of rest or of motion in a straight line, except in so far as it may be compelled by impressed forces to change that state." On the other hand, if a force acts upon a body, the motion of the body must undergo change.

(**b**) **Force is an action between two bodies, tending either to separate them or to bring them closer together.** For instance, when a stone falls to the ground, we explain the fact by saying that a force (the attraction of gravitation) tends to draw the earth and the stone together.

Magnetic and electric attraction, and the cohesive force between the particles of a body, are other instances of *attractive* force.

(**c**) **Force applied by contact.** In practice we apply force to a body (B) by causing contact between it and another body (A) which has a tendency to motion toward B. A *repulsive* force is thus called into action between the two bodies (in some way which we cannot understand), and this force pushes B forward (or in the

direction of A's tendency to move) and pushes A backward, thus *diminishing* its forward tendency *

If, for instance, a stone be laid upon the ground, it tends to move downward, but does not do so, because a repulsive force pushes it and the earth apart just as hard as the force of gravity tends to draw them together.

Similarly, when we attempt to lift a moderate weight with our hand, we do so by giving the hand a tendency to move upward. If the hand slips from the weight, this tendency moves the hand rapidly upward before our will force can check it. But otherwise, the repulsive force, generated by contact between the hand (tending upward) and the weight, moves the latter upward in spite of the force of gravity, and pushes the hand downward, depriving it of much of the upward velocity which it would otherwise have. It is perhaps chiefly from the **effort,** of which we are conscious in such cases, that we derive our notions of **"force."**

When a moving billiard ball, A, strikes another one, B, at rest, the tendency of A to continue moving forward is resisted by a repulsive force acting between it and B. This force pushes B forward, and A backward, retarding its former velocity. As explained in Art. 23 (*a*), the repulsive force does not exist in either body until the two meet.

(d) The repulsive force thus generated by contact between two bodies, continues to act only so long as they remain in contact, and only so long as they tend (from some extraneous cause) to come closer together.

(e) Force acts either as a pull or as a push. Thus, when a weight is suspended by a hook at the end of a rope gravity *pulls* the weight downward, the weight *pushes* the hook, and the hook *pulls* the rope, each of these actions being accompanied, of course, by its corresponding and opposite "reaction." When two bodies collide, each pushes the other, generally for a very short time.

(f) Equality of action and reaction. A force always exerts itself *equally* upon the two bodies between which it acts. Thus, the force (or attraction) of gravitation, acting between the earth and a stone, draws the earth upward just as hard as it draws the stone downward; and the repulsive force, acting between a table and a stone resting upon it, pushes the table and the earth downward just as hard as it pushes the stone upward This is the fact expressed by **Newton's third law of motion,** that "to every action there is always an equal and contrary reaction" For measures of force, see Arts. 11, 12, 13.

If a cannon ball in its flight cuts a leaf from a tree, we say that the *leaf* has reacted against the *ball* with precisely the same force with which the ball acted against the leaf. That degree of force was sufficient to cut off a leaf, but not to arrest the ball. A ship of war, in running against a canoe, or the fist of a pugilist striking his opponent in the face, receives as violent a blow as it gives; but the same blow that will upset or sink a canoe, will not *appreciably* affect the motion of a ship, and the blow which may seriously damage a nose, mouth, or eyes, may have no such effect upon hard knuckles.

The resistance which an abutment opposes to the pressure of an arch; or a retaining-wall to the pressure of the earth behind it, is no greater than those pressures themselves; but the abutment and the wall are, for the sake of safety, made *capable* of sustaining much greater pressures, in case accidental circumstances should produce such.

(g) In most practical cases **we have to consider only one** of the two bodies between which a force acts. Hence, for convenience, we commonly speak as if the force were divided into *two* equal and opposite forces one for each of the two bodies, and confine our attention to *one* of the bodies and the force acting upon it, neglecting the other. Thus we may speak of the force of steam in an engine as acting upon the *piston*, and neglect its equal and opposite pressure against the head of the cylinder

(h) That point of a body to which, theoretically, a force is applied, is called the **point of application.** In practice we cannot apply force to a *point*, according to the scientific meaning of that word; but have to apply it distributed over an appreciable *area* (sometimes very large) of the surface of the body.

*** We ordinarily express all this by saying simply that A pushes B forward, and this is sufficiently exact for practical purposes; but it is well to recognize that it is merely a convenient expression and does not fully state the facts, and that every force *necessarily* consists of *two* equal and opposite pulls or pushes exerted between two bodies.**

For the present we shall assume that the line of action of the force passes through the center of gravity of the body and forms a right angle with the surface at the point of application.

Art. 7 (a). Acceleration. When an unresisted force, acting upon a body, sets it in motion (*i. e.*, gives it velocity) in the direction of the force, this velocity *increases* as the force continues to act; each equal interval of time (if the force remains constant) bringing its own equal increase of velocity.

Thus, if a stone be let fall, the force of gravity gives to it, in the first inconceivably short interval of time, a small velocity downward. In the next equal interval of time, it adds a second equal velocity, and so on, so that at the end of the second interval the velocity of the stone is twice as great, at the end of the third interval three times as great, as at the end of the first one, and so on. We may divide the time into as small equal intervals as we please. In each such interval the constant* force of gravity gives to the stone an equal increase of velocity.

Such increase of velocity is called acceleration.† When a body is thrown vertically *upward*, the downward acceleration of gravity appears as a *retardation* of the upward motion. When a force thus acts *against* the motion under consideration, its acceleration is called *negative*.

Art. 8 (a). The rate of acceleration† is the acceleration which takes place *in a given time*, as one second.

(b) The **unit rate of acceleration** is that which adds unit of velocity in a unit of time; or, where English measures are used, one foot per second *per second*.

(c) For a *given rate* of acceleration, the *total* accelerations are of course proportional to the *times* during which the velocity increases at that rate.

Art 9 (a). Laws of acceleration. Suppose two blocks of iron, one (which we will call A) twice as large as the other (*a*), placed each upon a perfectly frictionless and horizontal plane, so that in moving them horizontally we are opposed by no force tending to hold them still. Now apply to each block, through a spring balance, a pull such as will keep the pointer of each balance always at the same mark, as, for instance, constantly at 2 in both balances. We thus have equal forces acting upon unequal masses.‡ Here the rate of acceleration of *a* is double that of A; for **when the forces are equal the rates of acceleration are inversely as the masses.**

In other words, in one second (or in any other given time) the small block of iron, *a*, will acquire twice the increase of velocity that A (twice as large) will acquire so that if both blocks start at the same time from a state of rest, the smaller one, *a*, will have, at the end of any given time, twice the *velocity* of A, which has twice its mass.

(b) Again, let the two masses A and *a*, be equal, but let the *force exerted* upon *a* be twice that exerted upon A. Then the rate of acceleration of *a* will (as before) be twice that of A; for, **when the masses are equal, the rates of acceleration are directly as the forces.**

(c) We thus arrive at the principle that, in any case, **the rate of acceleration is directly proportional to the force and inversely proportional to the mass.**

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*We here speak of the force of gravity, exerted in a given place, as constant, because it is so for all practical purposes. Strictly speaking, it increases a very little as the stone approaches the earth.

†Since the *rate* of acceleration is generally of greater consequence, in Mechanics, than the *total* acceleration, or the "acceleration" proper, scientific writers (for the sake of brevity) use the term "acceleration" to denote that *rate*, and the term "*total* acceleration" to denote the total increase or decrease of velocity occurring during any given time. Thus, the *rate* of acceleration of gravity (about 32.2 ft. per second per second) is called, simply the "acceleration of gravity." As we shall not have to use either expression very frequently we shall generally to avoid misapprehension, give to each idea its *full* name; thus, "total acceleration" for the *whole change of velocity* in a given case, and "rate of acceleration" for the *rate* of that change.

‡The mass of a body is the quantity of matter that it contains.

(d) Hence, if we make the two forces proportional to the two masses, the rates of acceleration will be equal; or, **for a given rate of acceleration, the forces must be directly as the masses.**

(e) Hence, also, a greater force is required to impart a given velocity to a given body in a short time than to impart the same velocity in a longer time. For instance, the forward coupling links of a long train of cars would snap instantly under a pull sufficient to give to the train in two seconds a velocity of twenty miles per hour, supposing a sufficiently powerful locomotive to exist. In many such cases, therefore, we have to be contented with a slow, instead of a rapid acceleration.

A string may safely sustain a weight of one pound suspended from our hand. If we wish to impart a great upward velocity to the weight *in a very short time*, we evidently can do so only by exerting upon it a great force; in other words, by jerking the string violently upward. But if the string has not tensile strength sufficient to transmit this force from our hand to the weight, it will break. We might safely give to the weight the desired velocity by applying a *less force* during a *longer time*.

(f) When a stone falls, the *force* pulling the earth upward is (as remarked above) equal to that which pulls the stone downward, but the *mass* of the earth is so vastly greater than that of the stone that its motion is totally imperceptible to us, and would still be so, even if it were not counteracted by motions in other directions in other parts of the earth. Hence we are *practically*, though *not absolutely*, right when we say that the earth remains at rest while the stone falls.

(g) But in the case of the two billiard balls (Art. 5 *c* p. 333), we can clearly see the result of the action of the force upon each of the two bodies; for the second ball, B, which was at rest, now moves forward, while the forward velocity of the first one, A, is diminished or destroyed, its backward motion thus appearing as a *retardation* of its *forward* motion. And, (since the same force acts upon both balls)

$$\begin{array}{c}\text{mass}\\\text{of A}\end{array} : \begin{array}{c}\text{mass}\\\text{of B}\end{array} :: \begin{array}{c}\text{rate of acceleration}\\\text{of B}\end{array} : \begin{array}{c}\text{rate of negative acceleration}\\\text{of A}\end{array}$$

or (since the force acts for the same time upon both balls)

$$\begin{array}{c}\text{mass}\\\text{of A}\end{array} : \begin{array}{c}\text{mass}\\\text{of B}\end{array} :: \begin{array}{c}\text{forward velocity}\\\text{of B}\end{array} : \begin{array}{c}\text{loss of forward velocity}\\\text{of A}\end{array}$$

(h) Remark. A man cannot *lift* a weight of 20 tons; but if it be placed upon proper friction rollers, he can *move* it horizontally, as we see in some drawbridges, turntables, &c.; and if friction and the resistance of the air could be entirely removed, he could move it by a single breath; and it would continue to move forever after the force of the breath had ceased to act upon it. It would, however, move very slowly, because the force of the single breath would have to diffuse itself among 20 tons of matter. He can move it, if it be placed in a suitable vessel in water, or if suspended from a long rope. A powerful locomotive that may move 2000 tons, cannot lift 10 tons vertically.

If we imagine two bodies, each as large and heavy as the earth, to be precisely balanced in a pair of scales without friction, a single grain of sand added to either scale-pan, would give motion to both bodies.

Art. 10 (a). The constant force of gravity is a uniformly accelerating force when it acts upon a body falling freely; for it then increases the velocity at the uniform rate of .322 of a foot per second during every hundredth part of a second, or 32.2 feet per second in every second. Also when it acts upon a body moving down an inclined plane; although in this case the increase is not so rapid, because it is caused by only a part of the gravity, while another part presses the body to the plane, and a third part overcomes the friction. It is a uniformly retarding force, upon a body thrown vertically upward; for no matter what may be the velocity of the body when projected upward, it will be diminished .322 of a foot per second in each hundredth part of a second during its rise, or 32.2 feet per second during each entire second. At least, such would be the case were it not for the varying resistance of the air at different velocities. It is a uniformly straining force when it causes a body at rest, to press upon another body; or to pull upon a string by which it is suspended. The foregoing expressions, like those of momentum, strain, push, pull, lift, work, &c., do not indicate different *kinds* of force; but merely different kinds of *effects* produced by the one grand principle, force.

(b) The above 32.2 ft per second *per second* is called the **acceleration of gravity**, and customarily denoted by a small **g**; or, more correctly speaking, since the acceleration is not precisely the same at all parts of the earth, g denotes the acceleration, whatever it may be, at any particular place.

Art. 11 (a). Relation between force and mass. The mass of a body is the quantity of matter which it contains. *One* cubic foot of water has *twice* as great a mass as *half* a cubic foot of water, but a *less* mass than *one* cubic foot of iron. Thus, the *size* of a body is a measure of mass between bodies of the *same* material, but not between bodies of *different* materials.

(b) When bodies are allowed to fall freely in a vacuum at a given place, they are found to acquire equal velocities in any given time, of whatever different materials they may be composed. From this we know (Art. 9 (*d*), p. 335), that the *forces* moving them downward, viz.: their respective *weights* at that place, must be proportional to their *masses*.

Thus, *in any given place*, the *weight* of a body is a perfect measure of its *mass*. But the *weight* of a given body *changes* when the body is moved from one level above the sea to another, or from one latitude to another; while the *mass* of the body of course remains *the same* in all places. Thus, a piece of iron which weighs a pound at the level of the sea, will weigh *less* than a pound by a spring balance, upon the top of a mountain close by, because the attraction between the earth and a given mass diminishes when the latter recedes from the earth's center. Or if the piece of iron weighs one pound near the North or South Pole, it will, for the same reason, weigh *less* than a pound by a spring balance if weighed nearer to the equator and at the same level above the sea.

The difference in the weight of a body in different localities is so slight as to be of no account in questions of ordinary practical Mechanics;* but scientific exactness requires a measure of mass which will give the same expression for the quantity of matter in a given body, wherever it may be; and, since weighing is a very *convenient* way of arriving at the quantity of matter in a body, it is desirable that we should still be able to express the mass in terms of the weight. Now, when a given body is carried to a higher level, or to a lower latitude, its loss of weight is simply a decrease in the *force* with which gravity draws it downward, and this same decrease also causes a decrease of the *velocity* which the body acquires in falling during any given time. The change in velocity, by Art. 9 (*b*), p. 334, is necessarily proportional to the change in weight.

Therefore, if the weight of a body at any place be divided by the velocity which gravity imparts in one second at the same place (and called **g**, or the *acceleration of gravity* for that place), the *quotient* will be the same at all places, and therefore serves as an invariable measure of the mass.

(c) By common consent, the **unit of mass**, in scientific Mechanics, is said to be that quantity of matter to which a *unit* of force can give unit rate of acceleration. This unit rate, in countries where English measures are used, is one foot per second, per second. It remains then to adjust the units of *force* and of *mass*. Two methods (an old and a new one) are in use for doing this. We shall refer to them here as methods A and B respectively.

(d) In method A, still generally used in questions of *statics*, the **unit of force** is fixed as that force which is equal to the *weight of one pound* in a certain place; *i. e.*, the force with which the earth at that place attracts a certain standard piece of platinum called a pound; and the unit of *mass* is not this standard piece of metal, but, as stated in (c), that mass to which this unit force of one pound gives, in one second, a velocity of one foot per second. Now the one pound attraction of the earth upon a mass of *one pound* will (Art. 1, p. 330) in one second give to that mass a velocity = g or about 32 feet per second; and (Art. 9 (*a*), p. 334), for a given force the masses are inversely as the velocities imparted in a given time. Therefore, to give in one second a velocity of only *one* foot per second (instead of g or about 32) the one pound unit of force would have to act upon a mass g times (or about 32 times) that which weighs one pound.

This could be accomplished, with an Atwood's machine, Art. 16 (*c*), p. 339, by making the two equal weights each = 15½ lbs. and the third weight = 1 lb.

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*The greatest discrepancy that can occur at various heights and latitudes, by adopting weight as the measure of quantity, would not be likely to *exceed* 1 in 300; or, under ordinary circumstances, 1 in 1000.

By method A, therefore, the *unit* of mass is g times (or about 32 times) the mass of the standard piece of metal called a pound; *i. e.*, a body containing one such unit of mass weighs g lbs. or about 32 lbs.; or, **by method A,**

$$\text{the } \textit{weight} \text{ of any given body in lbs.} = g \times \text{the } \textit{mass} \text{ of the body, in units of mass.}$$

Or,

$$\text{the } \textit{mass} \text{ of a body, in units of mass} = \frac{\text{the weight of the body, in pounds}}{g}$$

For instance:

in a body weighing	the mass is about
½ pound	$\frac{1}{64}$ unit of mass
1 "	$\frac{1}{32}$ " "
2 "	$\frac{1}{16}$ " "
32 "	1 " "
64 "	2 " "

It has been suggested to call this unit of mass a "Matt."

(e) **In method B,** the *mass* of the standard pound piece of platinum is taken as the **unit of mass** and is called a **pound;** and the force which will give to it in one second a velocity of one foot per second is taken as the unit of *force*. This small unit of force is called a **poundal.** In order that it may in one second give to the mass of one pound a velocity of only one foot per second, it must (by Art. 9 b), be $\frac{1}{g}$ (or about $\frac{1}{32}$) of the *weight* of said pound mass.

Hence, **by method B,**

$$\text{the } \textit{mass} \text{ of any given body, in } \textit{pounds} = \frac{\text{the } \textit{weight} \text{ of the body in } \textit{poundals}}{g}$$

and

$$\text{the } \textit{weight} \text{ of a body, in } \textit{poundals} = g \times \text{the } \textit{mass} \text{ of the body in } \textit{pounds}.$$

For instance:

in a body weighing	the mass of the body is about
½ poundal = $\frac{1}{64}$ pound	$\frac{1}{64}$ pound
1 " = $\frac{1}{32}$ "	$\frac{1}{32}$ "
2 " = $\frac{1}{16}$ "	$\frac{1}{16}$ "
32 " = 1 "	1 "
64 " = 2 "	2 "

(f) In the **C. G. S. (centimeter-gram-second) system,** a force of **1 dyne,** acting, for **1 second,** upon a mass of **1 gram,** gives it a velocity of **1 centimeter per second.** See also Art. 17 (b), p 341.

Art. 12 (a). The product, force × time, is called **impulse.** The product, mass × velocity, is called **momentum.**

According to **Newton's second law of motion,**

$$\left.\begin{matrix}\text{Force} \times \text{time} = \text{mass} \times \text{velocity} \\ \text{or impulse} = \text{momentum*}\end{matrix}\right\} \text{ or } f\,t = m\,v \quad \ldots\ldots\ldots (1)$$

$$\text{Force} = \frac{\text{impulse}}{\text{time}} = \frac{\text{mass} \times \text{velocity}}{\text{time}} = \frac{\text{momentum}}{\text{time}}; \text{ or } f = \frac{m\,v}{t} \quad \ldots (2)$$

$$\text{Time} = \frac{\text{impulse}}{\text{force}} = \frac{\text{mass} \times \text{velocity}}{\text{force}} = \frac{\text{momentum}}{\text{force}}; \text{ or } t = \frac{m\,v}{f} \quad \ldots (3)$$

$$\text{Mass} = \frac{\text{momentum}}{\text{velocity}} = \frac{\text{force} \times \text{time}}{\text{velocity}} = \frac{\text{impulse}}{\text{velocity}}; \text{ or } m = \frac{f\,t}{v} \quad \ldots (4)$$

$$\text{Velocity} = \frac{\text{momentum}}{\text{mass}} = \frac{\text{force} \times \text{time}}{\text{mass}} = \frac{\text{impulse}}{\text{mass}}, \text{ or } v = \frac{f\,t}{m} \quad \ldots (5)$$

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*Strictly, impulse = *change* of momentum; but we here assume that the change is from or to a state of *rest*, so that momentum = change of momentum.

Let a force, *f*, of 320 poundals (= say 10 pounds *) act, for 3 seconds, upon a mass, *m*, of 5 lbs. (= say $\frac{5}{32}$ matt). Here the force, *f*, is twice the weight of the mass, *m*. Hence the acceleration will be twice that of gravity, *i. e.*, say 64 ft per sec per sec; and the velocity, generated in 3 secs, will be 3 × 64 = 192 ft per sec; and we have, from Eq (1):

$$\underbrace{f \cdot t}_{\text{Force} \times \text{time} = \text{impulse}} = \underbrace{m \cdot v}_{\text{mass} \times \text{velocity} = \text{momentum}}$$

Force	×	time	=	impulse	=	mass	×	velocity	=	momentum
320 pdl	×	3 sec	=	960 pdl-sec	=	5 lbs.	×	192 ft/sec	=	960 ft-lb/sec
10 pd	×	3 sec	=	30 pd-sec	=	$\frac{5}{32}$ matt	×	192 ft/sec	=	30 ft-matt/sec

From Eq (2) we see that **force = time rate of change of momentum.** Hence the momentum of a body, moving with a given velocity, is numerically equal to that *force* which, *in unit time*, can produce or destroy that velocity in that body. Thus, forces are proportional to the momentums which they can produce (or destroy) in a given time; or, in a given time, equal forces produce (or destroy) equal momentums. Therefore a force must always produce equal and opposite changes of momentum in the two bodies between which it acts. See Art. 5 (b).

If a force, *f*, whose *amount* is 10 poundals, act upon a mass, *m*, of 5 lbs,* its **intensity** is $\frac{10}{5}$ = 2 poundals per lb.* A force of given *intensity* will always produce its proper rate of *acceleration* †, or time rate of change of velocity. Thus:

a force intensity of 1 pdl per lb* produces an accel., *a*, of 1 ft/sec/sec.
" " " " 2 " " " " " " " " 2 "
" " " " g " " " " " " " " g "

The intensity (or acceleration) of gravity, or the acceleration produced, in any mass, by its *weight*, is called **g**, and at sea level is about 32.2 feet, or 981 centimeters, per second per second. See p 348.

Art. 13 (a). A force is commonly measured by ascertaining the amount of some other force which it can counteract. Thus, a spring balance gives, by its scale, the amount of the tension, in the spring, which just counterbalances the weight of the suspended mass. Thus, **forces are conveniently expressed by weights.**

A force may be constant, as that of gravity between the earth and a stone resting upon it. **It may vary regularly,** like the pressure of air compressed by a piston moving with constant velocity, or **it may vary irregularly,** as where the motion of such a piston is irregular. We shall deal only with forces supposed to be *constant.*

Art. 14 (a). Density. The *densities* of materials are proportional to the *masses* contained in a given *volume*, as a cubic inch; or *inversely* as the *volume* required to contain a given *mass*. Or, since the weights at a given place are proportional to the masses, the densities are proportional to the weights per unit of volume (or "specific gravities") of the materials Thus, a body weighing 100 lbs. per cubic foot is twice as dense as one weighing only 50 lbs. per cubic foot at the same place.

Art. 15 (a). Inertia. The inability of matter to set itself in motion, or to change the rate or direction of its motion, is called its inertia, or inertness. When we say that a certain body has twice the inertia (inertness) of another one, we mean that twice the *force* is required to give it an equal rate of acceleration; and that, since all force (Art. 5 *f*) acts equally in both directions, we experience twice as great a *reaction* (or so-called "resistance") from the larger body as from the smaller one. The "inertia" of a body is therefore a measure of the *force* required to produce in it a given rate of acceleration; or, which is the same thing, it is a measure of the *mass* of the body. We may therefore consider "inertia" and "mass" as identical.

(b) What is called the "resistance of inertia" of a body, is simply the reaction (*i. e.*, one of the two equal and opposite actions) of whatever force we apply to the body. Hence, its amount depends not only upon the mass of the body, but also upon the rate of acceleration which we choose to

* In order to distinguish between the pound of *mass* and the pound of *force* (*weight* of a pound of mass at sea level), we here use "pound" or "pnd" or "pd" for the pound of *force*, and "lb" for the pound of *mass*, "pdl" for poundal.

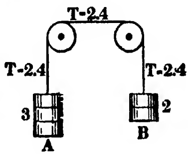
† See foot-note †, p 334.

give to it. Therefore we cannot tell, from the mass or weight of a body alone, what its "resistance of inertia" in any given case will be.

Art. 16 (a). Forces in opposite directions. When two equal and opposite forces act upon a body at the same time, and in the same straight line, we say that they destroy each other's tendencies to move the body, and it remains at rest. If two *unequal* forces thus act in opposition, the smaller force and an equal portion of the greater one are said to counteract each other in the same way, but the remainder of the greater force, acting as an unbalanced or unresisted force, moves the body in its own direction, as it would do if it were the only force acting upon it.

Thus, when we move bodies, in practice, we encounter not only the "resistance of inertia" (*i. e.*, we not only have to exert force in order to move inert matter), but we are also opposed by other *forces*, acting against us, as friction, the resistance of the air, and, often, all or a part of the *weight* of the body. By "resistances," in the following, we mean such resisting *forces*, and do not include in the term the "resistance of *inertia*."

(**b**) If separated, the two bodies, A and B, of 3 lbs and 2 lbs respectively, would fall with equal accelerations = g; each unit, $\frac{W}{g}$, of mass being acted upon by its own weight, W. But, connected as they are, A will move downward, and B upward, with an acceleration = only $\frac{g}{5}$; for now an unbalanced force of only 3 — 2 = 1 lb must give acceleration to a mass of $\frac{3+2}{g} = \frac{5}{g}$. But, to give to a mass, B, of $\frac{2}{g}$, an accel of $\frac{g}{5}$, requires a force of $\frac{2}{g} \cdot \frac{g}{5} = \frac{2}{5}$ lb = 0.4 lb. This, plus 2 lbs (required to balance the *weight* of B) is the tension, 2.4 lbs existing throughout the cord. Exerted at A, this tension balances 2.4 of the 3 lbs weight of A. The remainder (3 — 2.4 = 0.6 lb) of the weight, acting downward upon the mass, $\frac{3}{g}$, of A, gives to it the required acceleration of $\frac{g}{5}$; for here $\frac{\text{force}}{\text{mass}} = 0.6 \div \frac{3}{g} = \frac{0.6\,g}{3} = 0.2\,g = \frac{g}{5}$.



Or we may regard the total tension, 2.4 lbs, in the cord at A, as acting upon A and giving to it a *negative* or *upward* acceleration of $2.4 \div \frac{3}{g} = 0.8$ g, which, deducted from g (the acceleration which A would otherwise have) leaves

$$\text{Acceleration} = g - 0.8\,g = 0.2\,g = \frac{g}{5}.$$

Let W = weight of A
w = weight of B
F = net force available for acceleration = W — w
M = combined mass of both bodies = $\frac{W + w}{g}$
m = mass of B = $\frac{w}{g}$
a = acceleration
T = tension in cord.

$$\text{Then:}\quad a = \frac{F}{M} = (W - w) \div \frac{W + w}{g} = \frac{g\,(W - w)}{W + w}$$

$$T = w + m\,a = w + \frac{w}{g}a = w + \frac{w}{g} \cdot \frac{g\,(W - w)}{W + w} = w\left(1 + \frac{W - w}{W + w}\right).$$

(**c**) An "**Atwood's Machine**" consists essentially of a pulley, a flexible cord passing over the pulley, two equal weights (one suspended at each end of the cord), and a third weight, generally much lighter than either of the other two. The two equal weights balance each other by means of the pulley and cord. The third weight is laid upon one of the other two weights. The force of gravity, acting upon the third weight, then sets the masses of the three weights in motion at a small but constantly increasing velocity. In order to do this it must also overcome the friction of the pulley and cord, and the rigidity

of the latter; but, as these are made as slight as possible, they are, for convenience, neglected. The machine is used for illustrating the acceleration given to inert matter by unbalanced force, and forms an excellent example of the two distinct duties which a moving force generally has to perform, viz: (1st) the balancing of resistance, and (2nd) acceleration.

(d) In the case of **a locomotive, drawing a train on a level**, friction and the resistance of the air are the only resistances to be balanced; for the weight of the train here opposes no resistance. Unless the force of the steam is *more* than sufficient to balance the resistances, it cannot move the train. If it exceeds the resistances, the excess, however slight, gives motion to the inert matter of the train. If, at any moment while the train is moving, the force of the steam becomes *just equal to the resistances* (whether by an increase of the latter or by diminishing the force) the train will move on at a *uniform* velocity equal to that which it had at the moment when the force and resistance were equalized; and, if these could always be kept equal, it would so move on forever.

But so long as the excess of steam pressure over the resistances continues to act, the velocity is increased at each instant; for during each such instant the excess of force gives a small velocity *in addition* to that already existing.

On a level railroad, let

P = the total tractive force of the locomotive = say 13 tons
W = weight of locomotive = 50 tons
w = weight of train = 336 tons
R = resistance of locomotive (including internal friction, etc.) = 3 tons
r = resistance of train = 1 ton
F = net force available for acceleration = P — R — r = 9 tons

$$M = \text{mass of engine and train} = \frac{W + w}{g} = \frac{50 + 336}{32.2} = 12$$

$$m = \text{mass of train} = \frac{w}{g} = \frac{336}{32.2} = 10.44$$

a = acceleration
T = tension on draw-bar.

(Here the unit of mass is 2240 matts. See Art. 11 d, e.)

$$\text{Then: Acceleration} = a = \frac{F}{M} = \frac{9}{12} = 0.75 \text{ ft per second per second.}$$

The tension T on the draw-bar = resistance of train + force causing acceleration a, or $T = r + m\,a = 1 + 10.44 \times 0.75 = 1 + 7.83 = 8.83$ tons.

This tension, T, pulling backward against the locomotive, causes there a *retardation*, or *negative* acceleration, of $\frac{T}{\text{mass of locomotive}} = \frac{8.83\,g}{50} = 5.69$ ft per sec per sec, and thus reduces, by that amount, the acceleration which the locomotive would otherwise have, and which would be $= \frac{(P - R)\,g}{50} = \frac{10 \times 32.2}{50} = 6.44$. This, less 5.69, = 0.75 ft per sec per sec = acceleration of train.

(e) If the tractive force of a locomotive exceeds the resistances, due to friction, grades, and air, the velocity will be accelerated; but it then becomes more difficult to maintain the excess of force, for the pistons must travel faster through the cylinders, and the boiler can no longer supply steam fast enough to maintain the original cylinder pressure. Besides, some of the resistances increase with increase of velocity. We thus reach a speed at which the engine, although exerting its utmost force, can do no more than balance the resistances. The train then moves with a uniform velocity equal to that which it had when this condition was reached.

When it becomes necessary to stop at a station some distance ahead, steam is shut off, so that the steam force of the engine shall no longer counterbalance or destroy the resisting forces; and the number of the resistances themselves is increased by adding to them the friction of the brakes. The resistances, thus increased, are now the only forces acting upon the train, and their acceleration is *negative*, or a *retardation*. Hence, the train moves more and more slowly, and must eventually stop.

(f) **Caution.** When two opposite forces are in equilibrium, an addition to one of the forces does not always form an unbalanced force; for in many cases the *other force increases equally*, up to a certain point. For instance, when we attempt to lift a weight, W, its downward *resistance*, R, remains constantly just equal to our upward pull, P, however P may vary, until P *exceeds* W. Thus, R can never *exceed* W, but may be much less than it. Indeed, when we stop pulling, R ceases, although W (the attraction between the earth and the weight) of

course remains unchanged throughout. Such variation of resisting force, to meet varying demands, occurs in all those innumerable cases where structures sustain varying loads within their ultimate strength.

Art. 17 (a). Work. Force, when it moves a body,* is said to do "work" upon it. The *whole* work done by the force in moving the body through any distance is measured by multiplying the *force* by the distance; or, Work = Force × distance. If the *force* is taken in *pounds*, and the *distance* in *feet*, the product (or the *work* done) will be in *foot-pounds;* if the force is in tons and the distance in inches, the product will be in inch-tons; and so on.†

Thus, if a force of	moves a body through	we have work =
1 pound	10,000 feet	10,000 foot-pounds
100 pounds	100 "	10,000 "
10,000 "	1 foot	10,000 "

or, in any case, if a force of f pounds move a body through s feet, it does $f\,s$ foot-pounds of work. Thus **force is the linear rate of work.** ($f = f\,s \div s$.)

(b) With English measures, **the ordinary unit of work** is the **foot-pound. The metric unit of work** is the **kilogram-meter.** See Conversion Tables, p. 237.

In the C. G. S. system (Art. 11 *f*), 1 dyne, acting through 1 centimeter, does **1 erg (1 dyne-centimeter)** of work. 1 joule (p. 237) = 10,000,000 ergs = 0.7373 foot-pound. 1 foot-pound = about 13,563,000 ergs.

(c) In most cases, a portion at least of the **work** done by a force is **expended in overcoming resistances.** Thus, when a locomotive begins to move a train, a portion of its force works against, and balances, the resistances of friction or of an up-grade, while the remainder, acting as unbalanced force upon the inert mass of the train, increases its velocity.

An upward pull of exactly one pound will not raise a one pound weight, but will merely balance the downward force of gravity. If we increase the upward pull from one pound (= 16 ounces) to 17 ounces, the ounce so added, being unbalanced force, will give motion to the mass, and will accelerate its upward velocity as long as it continues to act. If we now reduce the upward pull to 1 pound, thus making it just equal to the downward pull of gravity, the body will move on upward with a uniform velocity; but if we reduce the upward force to 15 ounces (= $\frac{15}{16}$ pound), then there will be an unbalanced *downward* force of 1 ounce acting upon the body, and this downward force will generate in the body a downward or negative acceleration or *retardation*, and will *destroy* the upward velocity in the same time as the *upward* excess of 1 ounce required to *produce* it.

During any time, while the 17 ounces upward "force" were acting against the 16 ounces downward "resistance," the product of *total* upward force × distance must be *greater* than that of *resistance* × distance. The excess is the work done in accelerating the velocity, by virtue of which the body has acquired kinetic energy or capacity for doing work in coming to rest.

On the other hand, while the upward velocity was being *retarded*, the product of total upward force × dist was *less* than that of *resistance* × dist, the difference being the work done by the kinetic energy against the resistance of gravity.

In practice, the term "work" is usually restricted to that *portion* of the work which a force performs in balancing the *resistances* which act against it; in other words, to the work done by so much of the force as is equal to the resistance.

With this restriction, we have work = *force* × dist, = *resistance* × dist.

Thus, if the resistance be a friction of 4 lbs., overcome at every point along a distance of 3 feet; or if it be a weight of 4 lbs., lifted 3 feet high, then the work done amounts to 4 × 3 = 12 foot-lbs, provided the initial and the final velocities are equal.

(d) In cases **where the velocity is uniform,** as in a steadily running machine, the force is necessarily equal to the resistance; and where the velocities at the beginning and end of any work are equal (as where the machine starts from rest and comes to rest again) the *mean* force is equal to the *mean* resistance. In such cases, therefore, the two products, mean force × distance, and mean resistance × distance, are equal, and we have, as before,

Work = *force* × dist = *resistance* × dist.

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* A man who is standing still is not considered to be working, any more than is a post or a rope when sustaining a heavy load; although he may be supporting an oppressive burden, or holding a car-brake with all his strength; for his force *moves nothing* in either case.

† These products must not be confounded with *moments*, = force × leverage.

(**f**) In calculating the work done by machinery, etc., allowance must be made for his expenditure of a portion of the work in overcoming resistances. Thus, in pumping water, part of the applied force is required to balance the friction of the different parts of the pump; so that a steam or water "power," exerting a force of 100 lbs., and moving 6 feet per second, cannot raise 100 lbs. of water to a height of 6 feet per second. Therefore machines, so far from *gaining power*, according to the popular idea, actually lose it in one sense of the word. In *starting* a piece of machinery, the forces employed have (1st) to balance, react against, or destroy the resisting force of friction and the cohesive forces of the material which is to be operated on; and (2d) to give motion to the unresisting matter of the machine and of the material operated on, after the resisting forces which had acted upon them have thus been rendered ineffective. But after the desired velocity has been established, the forces have *merely to balance the resistances* in order that the velocity may continue uniform.

(**g**) That portion of the work of a machine, etc., which is expended against friction is sometimes called "**lost work**" or "**prejudicial work,**" while only that portion is called "**useful work**" which renders visible and tangible service in the shape of output, etc. Thus, in pumping water, the work done in overcoming the friction of the pump and of the water is said to be lost or prejudicial, while the useful work would be represented by the product, weight of water delivered × height to which it is lifted.

The distinction, although artificial, and somewhat arbitrary, is often a very convenient one; but the work is of course not actually "lost," and still less is it "prejudicial;" for the water could not be delivered without first overcoming the resistances. A merchant might as well call that portion of his money lost which he expends for clerk-hire, etc.

(**h**) For a given force and *distance*, **the work done is independent of the time;** for the product, force × distance, then remains the same, whatever the time may be. But the distance through which a given *force* will work at a given velocity is of course proportional to the time during which it is allowed to work. Thus, in order to lift 50 pounds 100 feet, a man must do the same work, (= 5000 foot-pounds) whether he do it in one hour or in ten; but, if he exerts constantly *the same force*, he will lift 50 lbs. ten times as high in ten hours as in one, and thus will do ten times the work. Thus, for a given *force*, **the work is proportional to the time.**

Art. 18 (a), Power. The quantity of any work may evidently be considered without regard to the *time* required to perform it; but we often require to know the *rate* at which work can be done; that is, how much can be done within a certain *time*.

The rate at which a machine, etc. can work is called its *power*. Thus, in selecting a steam-engine, it is important to know how much it can do *per minute, hour, or day*. We therefore stipulate that it shall be of so many *horse-powers*; which means nothing more than that it shall be capable of overcoming resisting forces at the rate of so many times 33,000 foot-pounds per minute when running at a uniform velocity, *i. e.*, when force × distance = resistance × distance.

(**b**) The **horse-power**, 33,000 foot-pounds per minute, or 550 foot pounds per second, is the **unit of power, or of rate of work,** commonly used in connection with engines. The **metric horse-power**, called "force de cheval," "cheval-vapeur," or (German) "Pferdekraft," is 75 kilogram-meters per second = 542.48 ft.-lbs per sec. = 32,549 ft.-lbs. per minute = 0.9863 horse-power. 1 horse-power = 1.0138 "force de cheval." In theoretical Mechanics the **foot-pound per second** is used in English measure; and the **kilogram-meter per second** in metric measure,

1 foot-pound per second = 0.13825 kilogram-meter per second.
1 kilogram-meter per second = 7.2331 foot-pounds per second.

(**c**) Up to the time when the velocity becomes uniform, the **power**, or **rate of work,** of the train, in Art 16 (*d*), is *variable*, being gradually *accelerated*. For in each second it overcomes its resistances (and moves its point of application) *through a greater distance* than during the preceding second. Also, after the steam is shut off, the rate of work is variable, being gradually *retarded*. When the force of the steam just balances the resistances, the rate of work is uniform.

(**d**) **Power = force × velocity.** Since the *rate* of work is equal to the work done *in a given time*, as so many *foot-pounds per second*, we may find it by dividing the work in foot-pounds done during any given time by the number of seconds in that time. Thus

$$\text{Power} = \text{rate of work} = \frac{\text{force in pounds} \times \text{distance in feet}}{\text{time in seconds}}$$

But this is equivalent to

$$\text{Power} = \text{rate of work} = \text{force in pounds} \times \frac{\text{distance in feet}}{\text{time in seconds}}$$

$$= \text{force in pounds} \times \text{velocity in feet per second.}$$

Or, if we treat only of the work of *that force which overcomes resistances:* or in cases where the velocity is either uniform throughout or the same at the beginning and end of the work;

$$\begin{array}{c}\text{Power,} \\ \text{in ft-lbs. per sec.}\end{array} = \begin{array}{c}\text{rate of work} \\ \text{in ft-lbs. per sec.}\end{array} = \begin{array}{c}\text{resistance,} \\ \text{in lbs.}\end{array} \times \begin{array}{c}\text{velocity,} \\ \text{in ft per sec.}\end{array}$$

Thus, if the resistance is 3300 lbs. and is overcome through a distance of 10 feet in every minute; or if the resistance is 33 lbs. and is overcome through a distance of 1000 feet per minute, the rate of the work is in each case the same, namely, 33,000 foot-pounds per minute, or one horse-power; for

$$\overset{\text{lbs.}}{3300} \times \overset{\text{vel.}}{10} = \overset{\text{lbs.}}{33} \times \overset{\text{vel.}}{1000} = 33{,}000 \text{ foot-pounds per minute.}$$

(c) The same "power" which will overcome a given resistance through a given distance, in a given time, will also overcome any other resistance through any other distance, in that same time, provided the resistance and distance when multiplied together give the same amount as in the first case. Thus, the power that will lift 50 pounds through 10 feet in a second, will in a second lift 500 pounds, 1 foot; or 25 pounds, 20 feet; or 5000 pounds $\frac{1}{10}$ of a foot. In practice, the adjustment of the speed to suit different resistances, is usually effected by the medium of **cog-wheels, belts, or levers.** By means of these the engine, water-wheel, horse, or other motive power, exerting a given force and running at a given velocity, may be made to overcome small resistances rapidly, or great ones slowly, as desired.

Art. 19 (a). The work which a body can do by virtue of its motion; or (which is the same thing) **the work required to bring the body to rest. Kinetic energy, vis viva, or "living force."** As already remarked, a force equal to the weight of any body, at any place, will, in one second, give to the mass or matter of the body a velocity $= g$, or (on the earth's surface) about 32.2 feet per second. Or if a body be thrown upward with a velocity $= g$, its weight will stop it in one second.

Since, in the latter case, the velocity at the beginning and at the end of the second are, respectively, $= g$ feet per second, and $= 0$, the *mean* velocity of the body is $\frac{g}{2}$ feet per second. Therefore, during the second it will rise $\frac{g}{2}$ feet, or about 16 feet. In other words, the work which any body can do, by virtue of being thrown vertically upward with an initial velocity (velocity at the *start*) of g feet per second, is equal to the product of its weight multiplied by $\frac{g}{2}$ feet. Or,

$$\text{work in foot-pounds} = \text{weight} \times \frac{g}{2}$$

Notice that in this case (since the initial velocity v is equal to g), $\frac{v}{g} = 1$.

Suppose now that the same body be thrown upward with *double* the former velocity; *i. e.*, with an initial velocity equal to $2\,g$ (or about 64 feet per second). Since gravity requires (Art. 8 *c*), *two* seconds to impart or destroy this velocity, the body will now move upward during *two* seconds, or twice as long a *time* as before. But its mean *velocity* now is g, or twice as great as before. Therefore, moving for double the time and with double the velocity, it will travel *four times* as *far*, overcoming the same resistance as before (viz.: its own weight) through *four* times the distance.

Thus, by making its initial velocity $v = 2\,g$, *i. e.*, by *doubling* its $\frac{v}{g}$, making it $= 2$, we have enabled the body to do *four* times the work which it could do when its $\frac{v}{g}$ was 1; so that the work in the second case is equal to the

product of that in the first case multiplied by the *square* of $\frac{v}{g}$; or,

$$\text{Work in ft-lbs.} = \text{weight} \times \frac{g}{2} \times \left(\frac{v}{g}\right)^2$$
$$= \text{weight} \times \frac{g}{2} \times \frac{v^2}{g^2}$$
$$= \text{weight} \times \frac{v^2}{2g}$$

And it is plain that this would be the case for any *other* velocity. Now the total amount of the work which the body can do, is independent of the amount of the resistance against which it is done; for if we increase the resistance we diminish the distance in the same proportion, so that their product, or the amount of work, remains the same. The above formula, therefore, applies to all cases; *i. e.*, the **total amount of work,** in foot pounds, which any body will do, against any resistance, by virtue of its motion alone, in coming to rest, is

$$\textbf{Work} = \text{weight of moving body, in lbs.} \times \frac{\text{square of its velocity in ft per sec'd}}{2g}$$
$$= \text{weight of moving body, in lbs.} \times \text{fall in ft required to give the velocity}$$
$$= \frac{\text{weight of moving body, in lbs.}}{g} \times \frac{\text{square of its velocity in ft per second}}{2}$$

In these equations, the *weight* is that which the body has in any given place, and g is the acceleration of gravity at that same place.

(b) Since the $\frac{\text{weight of a body}}{g}$ is its mass (Art. 11, p. 336), the last formula becomes, by "method A," Art. 11 (*d*),

$$\frac{\textbf{work}}{\text{in foot-}\textit{pounds}} = \text{mass of moving body in ``}\textit{matts}\text{''} \times \frac{\text{square of its velocity in ft per second}}{2}$$

and by "method B," Art. 11 (*e*),

$$\frac{\textbf{work}}{\text{in foot-}\textit{poundals}} = \text{mass of moving body in }\textit{pounds} \times \frac{\text{square of its velocity in ft per second}}{2}$$

(c) In the above equations the *left* hand side represents the *work* (or resistance overcome through a distance) in any given case, while the *right* hand side represents the **kinetic energy** of the body, by which it is enabled to do that work. Some writers call this energy **"vis viva,"** or "living force" a name formerly given (for convenience) to a quantity just *double* the energy, or = mass × velocity^2.

(d) As an illustration of the foregoing, take a train weighing 1,120,000 pounds, and moving at the rate of 22 feet per second. The kinetic energy of such a train is

$$\text{energy} = \text{weight} \times \frac{\text{velocity}^2}{2g}; \quad \text{or,}$$

$$1{,}120{,}000 \text{ lbs.} \times \frac{22^2}{64.4} = 8{,}400{,}000 \text{ ft.-lbs.}$$

That is, if steam be shut off, the train will perform a work of 8,400,000 ft.-lbs. in coming to rest. Thus, if the sum of all the resistances (of friction, air, grades, curves, etc.) remained constantly = 5000 lbs.,* the train would travel

$$\frac{8{,}400{,}000 \text{ ft.-lbs.}}{5000 \text{ lbs.}} = 1680 \text{ ft.}$$

(e) We thus see that the total quantity of work which a body can do by virtue of its motion alone, and without assistance from extraneous forces, is in proportion to the weight of the body and to the *square* of its velocity when it begins to do the work. For example, suppose that a train, at the moment when steam is shut off, has a velocity of 10 miles an hour and that the kinetic energy, which that velocity gives it, will by itself carry the train against the

* In practice, this would not be the case.

resistances of the road, etc., for a distance of *one* quarter of a mile before it stops. Then, if steam be shut off while the train is moving at 5, 20, 30 or 40 miles per hour (*i. e.* with ½, 2, 3 or 4 times 10 miles per hour) the train will travel 1/16, 1, 2¼ or 4 miles (or ¼, 4, 9 or 16 times ¼ mile) before coming to rest. The resistances are here supposed to be uniform.

But the *time rate* of work done is proportional simply to the resistance and the *velocity* (Art. 18 *d*). Therefore, the locomotive whose steam is shut off at 20, 30 or 40 miles per hour, will require, for running its 4, 9 or 16 quarters of a mile, but 2, 3 or 4 times as many seconds as it required at 10 miles per hour.

The same principle applies to all cases of acceleration or of retardation.

For instance, in the case of a falling body, the *distance* through which it must fall, in order to acquire any given velocity, is as the *square* of that velocity, but the *time* required is simply as the *velocity*. Also, if a body is thrown vertically upward with any given velocity, the *height* to which it will rise, by the time gravity destroys that velocity, will be as the *square* of the velocity, but the *time* will be simply as the velocity.

Art. 20 (a). When a body, starting from rest, and moving, under the action of a constant force, during a time, t, has acquired a velocity, v, its *mean* velocity, during the time, t, is ½ v; and the distance traversed is $s = \frac{1}{2} v\, t$. Since $m\, v = f\, t$ (Art. 12) we have:

$$\frac{m\ v^2}{2} = m\ v \cdot \frac{v}{2} = f\ t \cdot \frac{v}{2} = f\ s;$$

or, **the kinetic energy**, $K = \frac{1}{2} m\, v^2$, of a moving body, **is equal to the work, f s, done** by a force, f, in producing or destroying the momentum, m v, while traversing the space, s.

$$\text{Since } f\ s = K = \frac{m\ v^2}{2},$$

we have

$$\frac{f\ s}{t} = \frac{K}{t} = \frac{m\ v^2}{2\ t} = \frac{m\ v}{2} \cdot \frac{v}{t} = \frac{m\ v}{2} \cdot a = \frac{m\ v}{2} \cdot \frac{f}{m};$$

or: **time rate of work**

= mean momentum $\times$ acceleration
= mean momentum $\times$ force intensity;

or

$$\frac{\text{ft-lbs}}{\text{sec}} \cdot \frac{\text{poundals}}{\text{lb}} = \frac{\text{ft-poundals}}{\text{second}}.$$

In other words, if a mass, of m lbs, be moving with velocity, v ft per sec, it has a momentum of m v ft-lbs per second and an energy, K, of $\frac{m\ v^2}{2}$ ft-lbs; and $\frac{m\ v^2}{2}$ ft-lbs, = f s ft-poundals, of work must have been done upon it, in order to give it the velocity, v; and an equal amount of work must be done in order to bring it to rest.

Thus, if m = 4 lbs, and v = initial velocity = 12 ft per sec; we have $m\ v = 4 \times 12 = 48$ ft-lbs per sec, and $K = f\ s = \frac{m\ v^2}{2} = m\ v \cdot \frac{v}{2} = 48 \times 6 = 288$ ft-lbs.

By selecting different forces, f, to bring this mass to rest, we obtain different accelerations, different time rates of work, etc.

The momentum, m v, at any moment, is necessarily double the *mean* momentum, $\frac{m\ v}{2}$, which the mass has during the time, $t = \frac{v}{a}$, in which it acquires or loses the velocity, v, under the action of any constant force, f.

Let the weight of a falling body = W, and the acceleration of gravity = g. Then $W \cdot \frac{v}{2} \cdot \frac{v}{g}$ = weight $\times$ mean velocity $\times$ time of fall = weight $\times$ distance fallen = $W \cdot \frac{v^2}{2\ g}$ = the work done = $\frac{W}{g} \cdot \frac{v^2}{2} = \frac{m\ v^2}{2}$ = the kinetic energy acquired.

Art. 21 (a). Energy is indestructible. Energy, expended in work, is not destroyed. It is either transferred to other bodies, or else stored up in the body itself; or part may be thus transferred, and the rest thus stored. But, although energy cannot be destroyed, it may be rendered useless to us. Thus, a moving train, in coming to rest on a level track, transfers its kinetic energy into other kinetic energy; namely, the useless heat due to friction at the rails, brakes and journals; and this heat, although none of it is *destroyed*, is dissipated in the earth and air so as to be practically beyond our recovery.

Art. 22 (a). Potential energy, or possible energy, may be defined as stored-up energy. We lift a one-pound body one foot by expending upon it one foot-pound of energy. But this foot-pound is stored up in the "system" (composed of the earth and the body) as an addition to its stock of potential energy. For, while the stone falls through one foot,* the system will acquire a kinetic energy of one foot-pound, and will part with one foot-pound of its potential energy.

(**b**) The *potential* energy of a "system" of bodies (such as the earth and a weight raised above it, or the atoms of a mass of powder, or those of a bent spring) depends upon the relative *positions* of those bodies, and upon their *tendencies to change* those positions. The *kinetic* energy of a system (such as the earth and a moving train of cars) depends upon the *masses* of its bodies and upon their *motion* relatively to each other.

Familiar instances of potential energy are—the weight or spring of a clock when fully or partly wound up, and whether moving or not; the pent-up water in a reservoir; the steam pressure in a boiler; and the explosive energy of powder. We have mechanical energy in the case of the weight or springs or water; heat energy in the case of the steam; and chemical energy in that of the powder.

(**c**) In many cases we may conveniently estimate the *total* potential energy of a system. Thus (neglecting the resistance of the air) the explosive energy of a pound of powder is = the weight of any given cannon ball × the height to which the force of that powder could throw it, = the weight of the ball × (the square of the initial velocity given to it by the explosion) ÷ $2g$. But in other cases we care to find only a certain definite *portion* of the total potential energy. Thus, the *total* potential energy of a clock-weight* would not be exhausted until the weight reached the center of the earth; but we generally deal only with that portion which was stored in it by winding-up, and which it will give out again as kinetic energy in running down. This portion is = the weight × the height which it has to run down = the weight × (the square of the velocity which it would acquire in falling *freely* through that height) ÷ $2g$.

(**d**) There are many cases of energy in which we may hesitate as to whether the term "kinetic" or "potential" is the more appropriate. Thus, the pressure of steam in a boiler is believed to be due to the violent *motion* of the particles of steam, which bombard the inner surface of the boiler-shell; so that, from *this* point of view, we should call the energy of steam *kinetic*. But, on the other hand, the shell itself remains stationary; and, until the steam is permitted to escape from the boiler, there is no outward evidence of energy in the shape of work. The energy remains stored up in the boiler ready for use. From *this* point of view, we may call the energy of steam *potential* energy.

(**e**) It seems reasonable to suppose that further knowledge, as to the nature of other forms of energy, apparently potential (as is that of steam), might reveal the fact that all energy is ultimately kinetic.

Art. 23 (a). There is much **confusion of ideas** in regard to those actions to which, in Mechanics, we give the names, "**force**," "**energy**," "**power**," etc. This arises from the fact that, in every-day language, these terms are used indiscriminately to express the same ideas.

Thus, we commonly speak of the "force" of a cannon-ball flying through the air, meaning, however, the repulsive force which *would be* exerted between the ball and a building, etc. with which it might come into contact. This force would tend to move a part of the building along in the direction of the flight of the ball, and would move the ball backward; (*i. e.*, would retard its forward motion). But this great repulsive "*force*" does not exist until the ball strikes the building. Indeed, we cannot even tell, from the velocity and weight of the ball, what the *amount* of the force will be, for this depends upon the strength, etc., of the *building*. If the building is of glass, the force may be so slight as scarcely to retard the motion of the ball perceptibly, while, if the building is an

*For convenience we may thus speak of the energy of a *system* of bodies (the earth and the clock-weight) as residing in only *one* of the bodies.

earth embankment, the force will be much greater, and may retard the motion of the ball so rapidly as to entirely stop it before it has gone a foot farther.

The moving ball has great (kinetic) *energy;* but the only *force* that it exerts during its flight is the comparatively very slight one required to push aside the particles of air.

The energy of the ball, and therefore the total work which it can do, are independent of the nature of the obstruction which it meets; but since the work is the product of the resistance offered and the distance through which it can be overcome, the distance must be inversely as the resistance offered; or (which is the same thing) inversely as the force required of, and exerted by, the ball in balancing that resistance.

Since work, in ft.-lbs. = force, in lbs, $\times$ distance traversed, in feet, we have

$$\text{force, in lbs.} = \frac{\text{work, in ft.-lbs.}}{\text{distance traversed, in feet}} = \frac{\text{rate of work,}}{\text{in ft.-lbs. } per\ foot.}$$

Art. 24 (a). An impact, blow, stroke or collision takes place when a moving body encounters another body. The peculiarity of such cases is that the *time of action* of the repulsive force due to the collision is so *short* that generally it is impossible to measure it, and we therefore cannot calculate the force from the momentum produced by it in either of the two bodies: but since both bodies undergo a great change of velocity (*i. e.*, a great acceleration) during this short time, we know that the repulsive force acting between them must be very great.

We shall consider only cases of **direct impact**, or impact where the centers of gravity of the two bodies approach each other in one straight line, and where the nature of the surfaces of contact is such that the repulsive force caused by the impact also acts through those centers and in their line of approach

(**b**) This force, acting equally upon the two bodies (Art. 5 *f*), for the same length of time (namely, the time during which they are in contact), necessarily produces equal and opposite changes in their momentums (Art. 12, p 338). Hence, the total momentum (or product, mass $\times$ velocity) of the *two* bodies is always the same after impact as it was before

(**c**) But the relative behavior of the two bodies, after collision, depends upon their elasticity. If they could be perfectly inelastic, their velocities, after impact, would be equal. In other words, they would move on together. If they could be perfectly *elastic*, they would separate from each other, after collision, with the same velocity with which they approached each other before collision.

(**d**) Between these two extremes, neither of which is ever perfectly realized in practice, there are all possible degrees of elasticity, with corresponding differences in the behavior of the bodies The subject, especially that of *indirect* impact, is a very complex one, but seldom comes up in practical civil engineering.

(**e**) "In some careful experiments made at Portsmouth dock-yard, England, a man of medium strength, and striking with a maul weighing 18 lbs., the handle of which was 44 inches long, barely *started* a bolt about ⅛ of an inch at each blow; and it required a quiet pressure of 107 tons to press the bolt down the same quantity; but a small additional weight pressed it completely home."

GRAVITY, FALLING BODIES.

Bodies falling vertically. A body, falling freely in vacuo from a state of rest, acquires, by the end of the first second, a velocity of about 32.2 feet per second; and, in each succeeding second, an *addition* of velocity, or acceleration, of about 32.2 feet per second. In other words, the velocity receives in each second an acceleration of about 32.2 feet per second, or is accelerated at the *rate* of about 32.2 feet per second, *per second.* This rate is generally called (for brevity, see foot-note,† p. 334), simply the **acceleration of gravity** (but see * below), and is denoted by **g.** It increases from about 32.1 feet per second, per second, at the equator, to about 32.5 at the poles. In the latitude of London it is 32.19. These are its values at sea-level; but at a height of 5 miles above that level it is diminished by only about 1 part in 400. For most practical purposes it may be taken at 32.2.

Caution. Owing to the resistance of the air none of the following rules give perfectly accurate results in practice, especially at great vels. The greater the specific gravity of the body the better will be the result. The air resists both **rising** and **falling** bodies.

If a body be thrown vertically upwards with a given vel, it will rise to the same height from which it must have fallen in order to acquire said vel; and its vel will be retarded in each second 32.2 ft per sec. Its average ascending velocity will be half of that with which it started; as in all other cases of uniformly retarded vel. In falling it will acquire the same vel that it started up with, and in the same time. See above Caution.

Acceleration acquired*

in a given time $= g \times$ time

in a given fall from rest $= \sqrt{2\,g \times \text{fall}}$.

in a given fall from rest and given time $= \dfrac{\text{twice the fall}}{\text{time}}$

Time required

for a given acceleration $= \dfrac{\text{acceleration}}{g}$

for a given fall from rest $= \sqrt{\dfrac{\text{fall}}{\frac{1}{2}\,g}} = \dfrac{\text{fall}}{\frac{1}{2}\text{ final velocity}}$

for a given fall from rest or otherwise $= \dfrac{\text{fall}}{\text{mean vel}} = \dfrac{\text{fall}}{\frac{1}{2}(\text{initial vel} + \text{final vel})}$

Fall

in a given time (starting from rest) $=$ time $\times \frac{1}{2}$ final vel $=$ time$^2 \times \frac{1}{2} g$

in a given time (starting from rest or otherwise) $=$ time $\times$ mean vel $=$ time $\times \dfrac{\text{initial vel} + \text{final vel}}{2}$

reqd for a given acceleration (starting from rest) $= \dfrac{\text{acceleration}^2}{2\,g}$.

during any one given second (counting from rest)

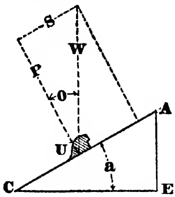
$$= g \times \left(\text{number of the second (1st, 2d, \&c)} - \tfrac{1}{2}\right)$$

during any equal consecutive times (starting from rest) $\propto$ 1, 3, 5, 7, 9, &c.

Calling $g = 32.2$ we have	At the end of the 1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	10th seconds
Velocity; ft per sec.	32.2	64.4	96.6	128 8	161.0	193.2	225 4	257.6	289.8	322.0
Dist fallen since end of preceding sec; ft.	16.1	48.3	80.5	112.7	144.9	177.1	209.3	241.5	273.7	305.9
Total dist fallen; ft.	16.1	64.4	144 9	257.6	402.5	579.6	788 9	1030 4	1304 1	1610.0

* By "acceleration," *in this article*, we mean the *total* acceleration; *i. e.*, the whole change of velocity occurring in the given time or fall. For the *rate* of acceleration

Descent on inclined planes. When a body, U, is placed upon an inclined plane, A C, its whole weight W is not employed in giving it velocity (as in the case of bodies falling vertically) but a portion, P, of it (= W × cosine of o = W × cosine of a*) is expended in perpendicular pressure against the plane; while only S, (= W × sine of o = W × sine of a,*) acts upon U in a direction parallel to the surface A C of the plane, and tends to slide it down that surf



The acceleration, generated in a given body in a given time, is proportional to the force acting upon the body in the direction of the acceleration

Hence if we make W to represent by scale the acceleration g (say 32.2 ft per sec per sec) which grav would give to U in a sec if falling freely, then S will give, by the same scale, the acceleration in ft per sec which the actual sliding force S would give to U in one sec if there were no friction between U and the plane. We have therefore

theoretical acceleration down the plane = g × sine of a.

Therefore we have only to substitute "g sin a" in place of "g;" and the *sloping* distance or "slide" A C in place of the corresponding *vertical* distance or "fall" A E in the equations, in order to obtain the accelerations etc as follows:

on an inclined plane without friction.

Acceleration† of sliding velocity

$$\text{in a given time} = \left.\begin{matrix}\text{vert accel acquired in falling}\\ \text{vert during the same time}\end{matrix}\right\} \times \sin a$$

$$= g.\ \sin a \times \text{time}$$

$$\left.\begin{matrix}\text{in a given slide, as A C,}\\ \text{from rest}\end{matrix}\right\} = \frac{\text{slide}}{\tfrac{1}{2}\ \text{time}}$$

$$= \left\{\begin{matrix}\text{vert accel acquired in falling}\\ \text{freely thro the corresponding}\\ \text{vert ht A E}\end{matrix}\right\} = \sqrt{2\,g.\ \text{A E}}$$

$$= \sqrt{2\ g.\ \sin a \times \text{slide}}$$

Time required

$$\text{for a given sliding acceleration} = \frac{\text{sliding acceleration}}{g.\ \sin a}$$

$$\text{for a given slide, as A C, from rest} = \frac{\text{slide}}{\tfrac{1}{2}\ \text{final sliding velocity}} = \sqrt{\frac{\text{slide}}{\tfrac{1}{2}\,g.\ \sin a}}$$

$$= \frac{\text{time reqd to fall freely thro the corresponding vert ht A E}}{\sin a}$$

$$\left.\begin{matrix}\text{for a given slide, from}\\ \text{rest or otherwise}\end{matrix}\right\} = \frac{\text{slide}}{\text{mean sliding vel}} = \frac{\text{slide}}{\tfrac{1}{2}\,(\text{initial} + \text{final sliding vels})}$$

$$\textbf{Cosine a} = \frac{\text{base E C}}{\text{length A C}} = \frac{\text{horizontal stretch, as E C, of any length, as A C}}{\text{that length}} = \frac{\sqrt{\text{A C}^2 - \text{A E}^2}}{\text{A C}}$$

$$\textbf{Sine a} = \frac{\text{height A E}}{\text{length A C}} = \frac{\text{fall, A E, in any given length, A C}}{\text{that length}} = \frac{\sqrt{\text{A C}^2 - \text{E C}^2}}{\text{A C}}$$

* Because o and a are equal.

† By acceleration, *in this article*, we mean the total acceleration, *i. e.*, the whole change in velocity occurring in the given time or slide. For the *rate* of acceleration we use simply the letter g.

Slide, as A C

in a given time, starting from rest = time × ½ final sliding vel
= time² × ½ g. sin a.

in a given time, starting from rest or otherwise = time × mean sliding vel
= time × ½ (initial + final, sliding vels)

required for a given sliding acceleration (starting from rest) $= \dfrac{\text{sliding acceleration}^2}{2\, g.\ \sin a}$

But in practice the sliding on the plane is always opposed by friction. To include the effect of friction, we have only to substitute

"$g \times \left[\sin a - (\cos a.\ \text{coeff fric})\right]$" in place of "$g.\ \sin a$" in the above equations. Because

Friction = Perpendicular pressure P × coefficient of friction
= weight W × cosine a × coefficient of friction

and

retardation of friction = g × cosine a × coefficient of friction.

Resultant sliding acceleration

= theoretical sliding accel (due to the sliding force, S) — retardation of fric
= (g. sin a) — (g cosine a. coeff fric)
= $g \times \left[\sin a - (\text{cosine } a.\ \text{coeff fric})\right]$

If the retardation of friction (= g. cos a × coeff fric) is *not less* than the total or theoretical accel ("g sin a") the body cannot slide down the plane.

PENDULUMS.

THE numbers of vibrations which diff pendulums will make in any given place in a given time, are *inversely* as the square roots of their lengths; thus, if one of them is 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; but its number of vibrations will be but ½, ⅓, or ¼ as great. The times in which diff pendulums will make a vibration, are *directly* as the sq rts of their lengths. Thus, if one be 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; and so also will be the time occupied in one of its vibrations.

The length of a pendulum vibrating seconds at the level of the sea, in a vacuum, in the lat of London (51½° North) is 39.1393 ins; and in the lat of N. York (40¾° North) 39.1013 ins [illegible]uator about $\frac{1}{10}$ inch shorter; and at the poles, about $\frac{1}{10}$ inch longer. Approximately enough for experiments which occupy but a few sec, we may at any place call the length of a seconds pendulum in the open air, 39 ins; half sec, 9¾ ins; and may assume that long and short vibrations of the same pendulum are made in the same time; which they actually are, *very* nearly. For measuring depths, or dists by sound, a sufficiently good sec pendulum may be made of a pebble (a small piece of metal is better) and a piece of thread suspended from a common pin The length of 39 ins should be measured from the centre of the pebble

In starting the vibrations, the pebble, or *bob*, must not be *thrown* into motion, but merely *let drop*, after extending the string at the proper height.

To find the length of a pendulum reqd to make a given number of vibrations in a min, divide 375 by said reqd number. The square of the quot will be the length in ins, near enough for such temporary purposes as the foregoing. Thus, for a pendulum to make 100 vibrations per min, we have $\frac{375}{100} = 3.75$; and the square of $3.75 = 14.06$ ins, the reqd length.

To find the number of vibrations per min for a pendulum of given length, in ins, take the sq rt of said length, and div 375 by said sq rt. Thus, for a pendulum 14.06 ins long, the sq rt is 3.75; and $\frac{375}{3.75} = 100$, the reqd number.

REM 1. By practising before the sec pendulum of a clock, or one prepared as just stated, a person will soon learn to count 5 in a sec, for a few sec in succession; and will thus be able to divide a sec into 5 equal parts; and this may at times be useful for very rough estimating when he has no pendulum.

Centre of Oscillation and Percussion.

REM. 2. When a pendulum, or any other suspended body, is vibrating or oscillating backward and forward, it is plain that those particles of it which are far from the point of suspension move faster than those which are near it But there is always a certain point in the body, such that if *all* the particles were concentrated at it, so that all should move with the same actual vel, neither the number of oscillations, nor their angular vel, would be changed. This point is called the *center of oscillation*. It is not the same as the cen of grav, and is always farther than it from the point of suspension. It is also the *centre of percussion* of the suspended vibrating body. The dist of this point from the point of susp is found thus: Suppose the body to be divided into many (the more the better) small parts; the smaller the better. Find the weight of each part. Also find the cen of grav of each part; also the dist from each such cen of grav to the point of susp. Square each of these dists, and mult each square by the wt of the corresponding small part of the body. Add the products together, and call their sum *p*. Next mult the weight of the entire body by the dist of its cen of grav from the point of susp. Call the prod *g*. Divide *p* by *g*. This *p* is the **moment of inertia** of the body, and if divided by the wt of the body the sq rt of the quotient will be the **Radius of Gyration.**

Angular Velocity.

When a body revolves around any axis, the parts which are farther from that axis move faster than those nearer to it. Therefore we cannot assign a stated linear velocity in feet per second, or miles per hour etc, that shall apply to *every part* of it. But every part of the body revolves around an entire circle, or through an angle of 360°, in the same time. Hence, all the parts have the same velocity in *degrees* per second, or in *revolutions* per second. This is called the angular velocity. Scientific writers measure it by the length of the arc described by any point in the body in a given time, as a second, the length of the arc being measured by the number of times the length *of its own radius* is contained in it. When so measured,

$$\begin{array}{c}\text{Angular velocity}\\ \text{in radii per second}\end{array} = \frac{\text{linear velocity (in feet etc) per sec}}{\text{length of radius (in feet etc)}}$$

Here, as before, the angular velocity is the same for all the points in the body, because the velocities of the several points are directly as their radii or distances from the axis of revolution.

In each revolution, each point describes the circumference of the circle in which it revolves $= 2\pi r$ ($\pi = 3.1416$ etc; r = radius of said circle). Consequently, if the body makes n revolutions per second, the length of the arc described by each point in one second is $2\pi r n$; and the angular velocity of the body, or *linear* velocity of any point *measured in its own radii*, is

$$a = \frac{2\pi r n}{r} = 2\pi n = \text{say } 6.2832 \times \text{revs per } \textit{second} = \text{say } .1047 \times \text{revs per } \textit{minute}.$$

Moment of Inertia.

Suppose a body revolving around an axis, as a grindstone; or oscillating, like a pendulum. Suppose that the distance from the axis of revolution (which, in the pendulum, is the point of suspension) to each individual particle of the body, has been measured; and that the square of each such distance has been multiplied by the weight of that particle to which said distance was measured.

Slide, as A C

in a given time, starting from rest $= \text{time} \times \frac{1}{2}$ final sliding vel

$= \text{time}^2 \times \frac{1}{2} g. \sin a.$

in a given time, starting from rest or otherwise $=$ time $\times$ mean sliding vel

$= \text{time} \times \frac{1}{2}$ (initial + final, sliding vels)

required for a given sliding acceleration (starting from rest) $= \dfrac{\text{sliding acceleration}^2}{2\, g. \sin a}$

But in practice the sliding on the plane is always opposed by friction. To include the effect of friction, we have only to substitute

"$g \times \left[\sin a - (\cos a.\ \text{coeff fric})\right]$" in place of "$g. \sin a$" in the above equations. Because

Friction = Perpendicular pressure P $\times$ coefficient of friction

= weight W $\times$ cosine a $\times$ coefficient of friction

and

retardation of friction = g $\times$ cosine a $\times$ coefficient of friction.

Resultant sliding acceleration

= theoretical sliding accel (due to the sliding force, S) — retardation of fric

$= (g. \sin a) - (g\ \text{cosine}\ a.\ \text{coeff fric})$

$= g \times \left[\sin a - (\text{cosine}\ a.\ \text{coeff fric})\right]$

If the retardation of friction ($= g. \cos a \times$ coeft fric) is *not less* than the total or theoretical accel ("$g \sin a$") the body cannot slide down the plane.

PENDULUMS.

The numbers of vibrations which diff pendulums will make in any given place in a given time, are *inversely* as the square roots of their lengths; thus, if one of them is 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; but its number of vibrations will be but ½, ⅓, or ¼ as great. The times in which diff pendulums will make a vibration, are *directly* as the sq rts of their lengths. Thus, if one be 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; and so also will be the time occupied in one of its vibrations.

The length of a pendulum vibrating seconds at the level of the sea, in a vacuum, in the lat of London (51½° North) is 39.1393 ins; and in the lat of N. York (40¾° North) 39.1013 ins quator about $\frac{1}{10}$ inch shorter; and at the poles, about $\frac{1}{10}$ inch longer. Approximately enough for experiments which occupy but a few sec, we may at any place call the length of a seconds pendulum in the open air, 39 ins; half sec, 9¾ ins; and may assume that long and short vibrations of the same pendulum are made in the same time; which they actually are, *very* nearly. For measuring depths, or dists by sound, a sufficiently good sec pendulum may be made of a pebble (a small piece of metal is better) and a piece of thread suspended from a common pin The length of 39 ins should be measured from the centre of the pebble

Table of Radii of Gyration.—CONTINUED.

Body	Revolving around	Radius of Gyration
Hollow sphere of any thickness	a diameter	$\sqrt{\frac{2\,(\text{outer rad}^5 - \text{inner rad}^5)}{5\,(\text{outer rad}^3 - \text{inner rad}^3)}}$
ditto, thin	ditto	approx (outer rad + inner rad) $\times$ .4085
ditto, infinitely thin (spherical surface)	ditto	radius of sphere $\times \sqrt{\frac{2}{3}}$ = radius of sphere $\times$ about .8165
Straight line, *a b*	any point, *x*, in its length	$\sqrt{\frac{a\,x^3 + x\,b^3}{3\,a\,b}}$
a *x* *c* *b*	either end, *a* or *b*	length *a b* $\times \sqrt{\frac{1}{3}}$ = length *a b* $\times$ about .5775
	its center, *c*	*a c* $\times \sqrt{\frac{1}{3}}$ = length *a b* $\times$ about .2887
Solid cone	its axis	radius of base of cone $\times \sqrt{.3}$ = radius of base of cone $\times$ .5477
Circular plate, of rectangular cross section	See Solid cylinder	For the *thickness* of plate or ring, measured perpendicularly to the plane of the circumference, take the *length* of the cylinder.
Circular ring, of rectangular cross section	See Hollow cylinder	
Square, rectangle and other surfaces	For *least* radius of gyration, or that around the *longest* axis, see pp 353 *a* and *b*.	

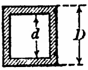
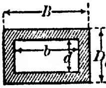
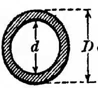
Relations of least radius of gyration, r, and least side, D.

In any cross section, let

I = least moment of inertia;
a = area;
r = $\sqrt{I/a}$ = least rad of gyration;
D = least external diam or side;
d = least internal diam or side;

B = greatest external diam or side;
b = greatest internal diam or side;
t = thickness of wall;
c = D/t; $1/12$ = 0 0833
m = B/D; $\sqrt{12}$ = 3.4641

We then have the following relations:

Cross Section	r	r^2	$\frac{D}{r}$	$\left(\frac{D}{r}\right)^2$
 Solid square	$\frac{D}{\sqrt{12}}$	$\frac{D^2}{12}$	$\sqrt{12}$	12
 Hollow square, of uniform thickness *	$\sqrt{\frac{D^2+d^2}{12}}$	$\frac{D^2+d^2}{12}$	$\sqrt{\frac{12\ D^2}{D^2+d^2}}$ *	$\frac{12\ D^2}{D^2+d^2}$ *
 Solid rectangle,	$\frac{D}{\sqrt{12}}$	$\frac{D^2}{12}$	$\sqrt{12}$	12
 Hollow rectangle, of uniform thickness †	$\sqrt{\frac{D^3B-d^3b}{12(DB-db)}}$	$\frac{D^3B-d^3b}{12(DB-db)}$	$\sqrt{12D^2\frac{DB-db}{D^3B-d^3b}}$ †	$12D^2\ \frac{DB-db}{D^3B-d^3b}$
 Solid circle	$\frac{D}{4}$	$\frac{D^2}{16}$	4	16
 Hollow circle, of uniform thickness **	$\sqrt{\frac{D^2+d^2}{16}}$	$\frac{D^2+d^2}{16}$	$4D\sqrt{\frac{1}{D^2+d^2}}$ **	$\frac{16\ D^2}{D^2+d^2}$ *

* Hollow square; $c = D/t$.

When c =	5	10	20,
$(D/r)^2$ =	8.82	7.32	6.63.

† Hollow rectangle of uniform thickness; $c = D/t$; $m = B/D$.

When c =	5		10		20,	
and m =	1.5	2	1.5	2	1.5	2,
$(D/r)^2$ =	7.98	7.54	6.62	6.23	6.01	5.66.

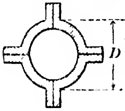
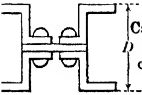
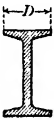
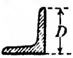
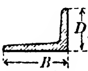
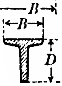
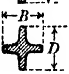
** Hollow circle of uniform thickness; $c = D/t$.

When c =	5	10	20,
$(D/r)^2$ =	11.76	9.76	8.84.

†† Angle with unequal legs; $m = B/D$. See p 353 *b*.

When m =	1.25	1.50	1.75	2.00	2.25,
$(D/r)^2$ =	21.60	18.78	17.00	15.70	14.80.

The equations below are approximate only.

Cross Section	r	r^2	$\frac{D}{r}$	$\left(\frac{D}{r}\right)^2$
 Phoenix column	$0.3636\,D$	$0.1322\,D^2$	$\frac{1}{0.3636} = 2.7527$	$\frac{1}{0.1322} = 7.5643$
 Carnegie Z-bar column	$0.295\,D$	$0.087\,D^2$	$\frac{1}{0.295} = 3.3898$	$\frac{1}{0.087} = 11.4943$
 I-beam	$\frac{D}{4.58}$	$\frac{D^2}{21}$	4.58	21
 Channel	$\frac{D}{3.54}$	$\frac{D^2}{12.5}$	3.54	12.50
 Deck beam	$\frac{D}{6}$	$\frac{D^2}{36.5}$	6	36.5
 Angle, equal legs	$\frac{D}{5}$	$\frac{D^2}{25}$	5	25
 Angle, unequal legs ††	$\frac{BD}{2.6(B+D)}$	$\frac{B^2D^2}{13(B^2+D^2)}$	$\sqrt{13\left(\frac{D^2}{B^2}+1\right)}$ ††	$13\left(\frac{D^2}{B^2}+1\right)$ ††
 T, with $D=B$  Cross, with $D=B$	$\frac{D}{4.74}$	$\frac{D^2}{22.5}$	4.74	22.5

CENTRIFUGAL FORCE.

When a body *a*, Fig. 1, moves in a circular path *abd*, it tends, at each point, as *a* or *b*, to move in a tangent *at* or *bt'* to the circle at that point. But at each point, as *a*, etc., in the path, it is *deflected* from the tangent by a force acting toward the center, *c*, of the circle. This force may be the tension of a string, *ca*, or the attraction between a planet at *c* and its moon *a*, or the inward pressure of the rails, *a b*, on a curve, etc., etc. Like all force, it is an action between *two* bodies, tending either to separate them or to draw them closer together, and acting equally upon both. (See Art 5 (*b*), p 332) In the case of the string, it *pulls* the body *a*, *toward* the center, *c*, and the nail or hand, etc., at *c*, toward the body at *a* or *b*, etc.; *i. e.*, *from* the center. In the case of a car on a curve it *pushes* the car *toward* the center, and the rails *from* the center. The pull or push on the *revolving* body *toward* the center is called the **centripetal force;** while the pull or push tending to move the *deflecting* body *from* the center is called the **centrifugal force.** These two "forces," being merely the two "sides" (as it were) of the same stress, are necessarily equal and opposite, and can only exist together. The moment the stress or tension exceeds the strength (or inherent cohesive force) of the string, etc., the latter breaks. The centripetal and centrifugal forces therefore instantly cease, and the body, no longer disturbed by a deflecting force, moves on, at a uniform velocity,* in a tangent, *at* or *bt'*, etc., to its circular path; *i. e.*, at right angles to the direction which the centrifugal force had at the moment it ceased.

(*a*). **A single revolving body**, *a*, Fig. 1. Let

f = the centrifugal or centripetal force, in pounds.
W = the weight of the body *a*, in pounds,
R = the radius *ca* of the path of the *center of gravity* of the body *a*, in feet,
v = the uniform velocity of the body *a* in its circular path *abd*, in feet per second,
n = the number of revolutions per minute,
g = the acceleration of gravity = say 32.2 feet per second per second,
$900\,g$ = about 28980.
π = circumference ÷ diameter = say 3.1416. π^2 = about 9.8696.

Then, for the **centrifugal force**, f:

If we have the velocity v in feet per second: $$f = W\frac{v^2}{R\,g}\dagger \quad \ldots \quad (1)$$

If we have the number n of revolutions per minute: $$f = W\frac{\pi^2 R n^2}{900\,g}\ddagger \quad \ldots \quad (2)$$

$$f = \text{about } .0003406\ W R n^2 \S \quad \ldots \quad (3).$$

---

* Neglecting friction, gravity, the resistance of the air, etc.

† For let *at*, Fig. 1, represent the amount and direction of the velocity v of the body at *a* in feet per second. Then at the end of one second the body will have reached the point *b* (the arc *ab* being made = *at*), and the amount and direction of its velocity at *b* will then be represented by the line *bt'* = *at* in length, but differing in direction. Drawing *cu* and *cu'* at the center, equal and parallel respectively to *at* and *bt'*, we find that the *change in the direction* of the motion (*i. e.*, the acceleration toward the center) during the second is represented by the arc *uu'*; and, since angle *acb* = angle *ucu'*, we have the proportion, radius R or *ac* : *ab* or *at* : : *cu* or *at* : arc *uu'*. In other words, the acceleration *uu'* in one second, or rate of acceleration, is = $\frac{at^2}{R} = \frac{v^2}{R}$; and, for the force causing that acceleration, we have

$$f = \text{mass of body} \times \text{rate of acceleration} = \text{mass of body} \times \frac{v^2}{R} = W\frac{v^2}{R\,g}.$$

‡ By formula (1), $f = W\frac{v^2}{R\,g}$. But $v = \frac{2\,\pi\,R\,n}{60}$; and $v^2 = \frac{4\pi^2 R^2 n^2}{3600} = \frac{\pi^2 R^2 n^2}{900}$.

Hence, $f = W\frac{\pi^2 R^2 n^2}{900\,R\,g} = W\frac{\pi^2 R n^2}{900\,g}$.

§ Formula (3) is obtained from (2) by substituting the values 9.8696 and 28980 for π^2 and $900\,g$ respectively.

(b) Wheels and discs. Suppose the rim of a wheel to be cut into very short slices, as shown (much exaggerated) at a, Fig. 2. Then for each slice, as a, by formula (1): $f =$ weight W of slice $\times \frac{v^2}{R\,g}$;* and if each slice were connected

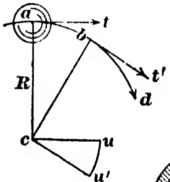


Fig. 1

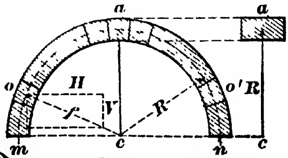


Fig. 2

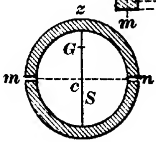


Fig. 3

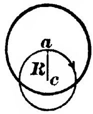


Fig. 4

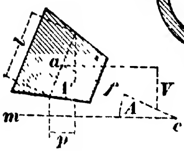


Fig. 5

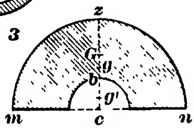


Fig. 6

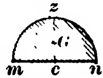


Fig. 7

with the center by a separate string, the sum of the stresses in all the strings (neglecting friction between adjacent slices) would be ·

$F =$ sum of centrifugal forces of all the slices † = weight of *rim* $\times \frac{v^2}{R\,g}$ . . . (4).

But the stress with which we are usually concerned in such cases (viz: **the tension in the rim itself** in the direction of a tangent to its own circumference) is *much less* than the theoretical quantity F obtained from formula (4), being in fact only $\frac{1}{6.2832}$ of it. For suppose first that the same thin rim is cut only at two opposite points m and n, Fig. 3, and that its two halves are held together only by the string S.

---

* If the rim is very thin in proportion to its diameter mn, we may take the center of gravity of each slice as being in a circle mn midway between the inner and outer edges of the rim, so that $R = \frac{\text{inner radius} + \text{outer radius}}{2}$. In a rim of appreciable thickness, this is not the case, because each slice is a little thicker at its outer than at its inner end. See Fig. 5. Hence its center of gravity is a little outside of the curved line mn, Fig. 2.

† In a perfectly balanced rim (*i. e.*, a rim whose center of gravity coincides with its center of rotation, as in Fig. 3) the centrifugal forces of the particles on one side of c counterbalance those on the opposite side. Here, too, $R = 0$. Hence, *as a whole*, such a rim has *no centrifugal force*; *i. e.*, no tendency to leave the center in any one direction by virtue of its rotation. But if the two centers do *not* coincide (Fig. 4), then the rim is a single revolving body, and its centrifugal force is: $f =$ weight of entire rim $\times \frac{v^2}{R\,g}$; where R is the distance between the two centers, and v the velocity of the center of gravity a. The force f acts in the line joining the two centers.

Then:*

semi-circumference mzn : diameter mn :: $\frac{F}{2}$: pull on the string S;

so that

$$\textbf{pull on string } S = \frac{\text{half weight}}{\text{of rim}} \times \frac{v^2}{R\,g} \times \frac{2}{\pi} = \frac{\text{weight}}{\text{of rim}} \times \frac{v^2}{R\,g\,\pi} = \frac{F}{\pi} = \frac{F}{3.1416} \quad \ldots (5)$$

and if the rim is now made complete by joining the ends at m and n, and if the string S is removed, then the pull on the string by formula (5) will be equally divided between m and n. Hence each cross-section, as m or n, of the rim, will sustain a tensile stress equal to half the pull on the string; or

$$\textbf{tension in rim} = \frac{F}{2\pi} = \frac{F}{6.2832} = \frac{\text{weight of rim} \times v^2}{6.2832\ R\,g} \quad \ldots\ldots\ldots \quad (6).$$

The centripetal force, f, Fig. 2, holding any part o of the rim to its circular path, is the resultant of the two equal tensions at the ends of that part.

For the *stress per square inch* of cross-section of rim, we have:

$$\textbf{unit stress} = \frac{\text{tension in rim}}{\text{area A of cross-section of rim, in square inches}}$$

$$= \frac{F}{6.2832\ A} = \frac{\text{weight of rim} \times v^2}{6.2832\ A\ R\,g} \quad \ldots\ldots \quad (7).$$

We shall arrive at the same result if we reflect that the pull in the string S, or the *sum* of the two tensions at m and n, is equal to the centrifugal force f of either *half* of the rim, revolving, as a single body, about the center c. Find the center of gravity G of the half rim, and then, in formula (1), use the velocity of that point, and the radius cG instead of velocity at z and radius cz respectively; thus:

$$\text{pull in string} = f = \frac{\text{centrifugal force}}{\text{of half-rim}} = \text{weight of half-rim} \times \frac{(\text{velocity at G})^2}{cG \times g},$$

and half of this is the tension in each cross-section of the rim.†

If the rim were infinitely thin, cG, Fig. 3, would be 0.6366 cz.

If its thickness must be taken into consideration, and if it is of rectangular cross-section find the centers of gravity g and g', Fig 6, of the whole semicircular segment cz and of the small segment cb respectively (cg = 0.4244 cz, and cg' = 0.4244 cb. Then

$$g'G = gg' \times \frac{\text{area of entire segment } cz}{\text{area of half rim}}.$$

For rims of other than rectangular cross-section, use formulæ (4), (5) and (6)

In a disc, such as a grindstone, the tension in each full cross-section mn, Fig. 7, is equal to the centrifugal force f of *half* the disc. Let W = weight of half disc. The distance cG from the center c to the center of gravity G of the half disc, is cG = 0.4244 cz; and the

---

* In Fig. 2, let the centrifugal force of any slice, o, be represented by the diagonal, f, of a rectangle, whose sides, H and V, are respectively parallel and perpendicular to the given diameter mn. Then H and V represent the components of f in those two directions. The equal and opposite horizontal components H, of o and of the corresponding slice o', being parallel to mn, have no tendency to pull the rim apart at m or n. Hence, the pull on a string S, Fig. 3, perpendicular to mn, is the sum of the components V of all the slices For each very thin slice, Fig. 5 (greatly exaggerated) we have (since angle A = angle A'):

Length l of slice : its horizontal projection, p :: centrifugal force f, of slice : its vertical component V.

Hence, for the entire half-rim mn, Fig. 3 (made up of such slices), we have:

Length mzn of half-rim : its horizontal projection mn :: the sum of the centrifugal forces of all the slices of the half-rim, : the sum of the vertical components V for all those slices;

which is identical with the proportion at top of page.

† The *rims* of revolving wheels are usually made strong enough to resist the tension due to the centrifugal force, without aid from the *spokes*, which thus have merely to support the *weight* of the wheel. But if the rim *breaks*, the centrifugal forces of its fragments come entirely upon the spokes; and, since the breakage is always irregular, some of the spokes will always receive more than their share.

$$\textbf{tension in } mn = \mathbf{f} = W\frac{(\text{vel. at G})^2}{\text{rad. }cG\times g} = W\frac{0.4244^2\,(\text{vel. at }z)^2}{0.4244\;cz\times g}$$

$$= W\frac{0.4244\,(\text{vel. at }z)^2}{cz\times g}\quad\ldots\ldots(8).$$

$$= W\frac{0.4244\,\pi^2\;n^2\;cz}{900\,g}\quad\ldots\ldots\ldots(9).$$

The stress *per square inch* in any full section mn is

$$\textbf{unit stress} = \frac{\text{tension in }mn}{\text{area of cross-section in square inches}},$$

$$= W\frac{0.4244\,(\text{velocity at }z)^2}{\text{diam. }mn\text{, ins.}\times\text{thickness, ins.}\times cz\times g}\quad\ldots(10).$$

$$= W\frac{0.4244\,\pi^2\,n^2\,cz}{\text{diam. }mn\text{, ins.}\times\text{thickness, ins.}\times 900\,g}\quad\ldots(11).$$

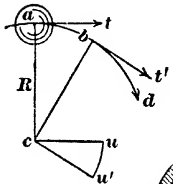


Fig. 1

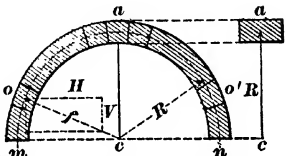


Fig. 2

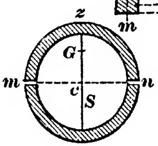


Fig. 3

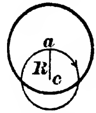


Fig. 4

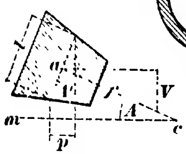


Fig. 5

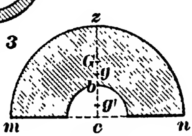


Fig. 6

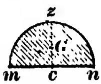


Fig. 7

f = the centripetal force, in pounds acting upon a single revolving body, a, Figs. 1, 2, 4 and 5, or upon the half-rim or half-disc, Figs. 3, 6 and 7 the centrifugal force exerted by such body

F = the sum of the centrifugal forces f, of all the particles of a rim, Fig. 3.

W = the weight of the body, in pounds.

R = the radius ca, Figs 1, 4 and 5, of the path of the *center of gravity* of the body.

v = the uniform velocity of the body in its circular path, in feet per second.

n = the number of revolutions per minute.

g = the acceleration of gravity = say 32.2 feet per second. 900 g = about 28980.

$\pi = \frac{\text{circumference}}{\text{diameter}}$ = say 3.1416 π^2 = about 9.8696

In a rolling wheel, each point in the rim, during the moment when it touches the ground, is stationary *with respect to the earth;* but each particle has the same velocity *about the center* as if the latter were stationary, and hence the centrifugal force has no effect upon the weight.

STATICS.

FORCES.

1. Statics Defined. The science of statics, or of equilibrium of forces, takes account of those very numerous cases where the forces under consideration are in equilibrium, or balanced. It embraces, therefore, all cases of bodies which are said to be "at rest."*

2. In the problems usually presented in civil engineering, a certain given force, or certain given forces, applied to a stationary * body (as a bridge or building) tend to produce motion, either in the structure as a whole or in one or more of its members; and it is required to find and to apply another force or other forces which will balance the tendency to motion, and thus permit the structure and its members to remain at rest. See ¶ 33, below.

3. Equilibrium. Suppose a body to be acted upon by certain forces. Then those forces are said to be in equilibrium, when, as a whole, they produce no change in the body's state of rest or of motion, either as regards its motion as a whole along any particular line (motion of translation), or as regards its rotation about any point, either within or without the body. In such cases the body also is said to be in equilibrium. See ¶ 84, below.

4. A body may be in equilibrium as regards the forces under consideration, even though not in equilibrium as regards other forces. Thus, a stone, held between the thumb and finger, is in equilibrium as regards their two equal pressures, even though it may be lifted upward by the excess of the muscular force of the arm over the attraction between the earth and the stone. Similarly, on a level railroad, a car is in equilibrium as regards gravity and the upward resistance of the rails, although the horizontal pull of the locomotive may exceed the resistance to traction.

5. Molecular Action. Any force, applied to a body, is in fact made up of a system of forces, often parallel or nearly so, applied to the several particles of the body. Thus, the attraction exerted by the earth upon a grain of sand or upon the moon is, strictly speaking, a cluster of nearly parallel forces exerted upon the several particles of those bodies; but, for convenience, and so far only as concerns their tendency to move the body as a whole, we conceive of such forces as replaced by a single force, equal to their sum and acting in one line. In thus considering the forces, we assume that the bodies are absolutely rigid, so that each of them acts as a single "**material particle**" or "**material point.**"

6. Transmission of Force. The upward pressure of the ground, upon a stone resting upon it, acts directly only upon those particles which are nearest to the ground. These, in turn, exert a (practically) equal upward force upon those immediately above them, and so on; and the force is thus transmitted throughout the stone.

7. Rigid Bodies. In treating of bodies as rigid, we assume that the intermolecular forces hold the several particles absolutely in their original relative positions.

It is not the *material* that resists being broken, but the *forces* which hold its particles in their places. Thus, a cake of ice may sustain a great pressure; but its particles yield readily when its cohesive forces are destroyed by a melting temperature.

8. Force Units. The force units generally used in statics are those of weight, as the pound and the kilogram. See Conversion Tables, p 235.

In statics we have no occasion to consider the masses of bodies (except

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* Strictly speaking, *absolute* rest is scarcely conceivable, since all bodies are actually in motion (see Art. 3, p. 331), so that unbalanced forces produce merely *changes in the states of motion* of bodies. Yet, for a body to be at rest, *relative to other bodies*, is a very common condition, and, in practical statics, we usually regard the body under consideration as being at rest relatively to the earth or to some other large body, so that the change of state of motion, due to the action of unbalanced force upon it, consists in a change from relative rest to relative motion. See ¶ 33, below.

in so far as these determine their weights, or the force of gravity exerted upon them), bodies being regarded merely as the media upon and through which the forces under consideration are exerted. Hence we require, in statics, no units of mass; and, as the bodies are regarded as being "at rest," no units of time, velocity, acceleration, momentum, or energy.

9. Forces, how Determined. A force is fully determined when we know (1) its amount (as in pounds, or in some other weight unit), (2) its direction, (3) its sense (see ¶ 10), and (4) its position or its point of application.

10. When a force is represented by a line, the length of the line may be made to represent by scale the amount of the force, and its direction and position may often be made to indicate those of the force, while the sense of the force may be shown by arrows or letters affixed to the lines, or by the signs, + and —.

Thus, the *directions* of the forces represented by lines *a* and *b*, Fig. 1, are vertical, and those of *c* and *d* are horizontal. The *sense* of *a* is upward, of *b* downward, of *c* right-handed, of *d* left-handed. Thus, *a* and *b* are of like direction, but of opposite sense; and so with *c* and *d*. In treating of vertical or horizontal forces, we usually call upward or right-handed forces **positive,** and downward or left-handed forces **negative,** as indicated by the signs, + and —, in Fig. 1. When a force is designated by two letters, attached to the line representing it, one at each end of the line, the sense of the force may be indicated by the order in which the letters are taken. Thus, in Fig. 1, having regard to the directions of the arrows, we have forces, *e f*, *h g*, *k l*, and *n m*.

11. Line of Action, etc. The point (see ¶ 5) at which a force P, Fig 2, is supposed to be applied, as *a*, is called its point of application. But the force is transmitted, by the particles, throughout the body (see ¶ 6), and

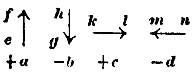

Fig. 1.

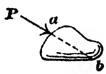

Fig. 2.

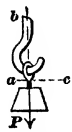

Fig. 3.

the effect of the force, as regards the body as a whole, is not changed if it be regarded as acting at any other point, as *b*, in its line of action. We may therefore regard any point in that line as a point of application of the force. For instance, the tendency to move the stone, Fig. 2, as a whole, will not be changed if, instead of *pushing* it, at *a*, we apply a *pull* (in the same direction and in the same sense) at *b*; and if a weight, P, be laid upon the top of the hook, at *b*, Fig. 3, it will have the same tendency, to move the hook as a whole, as it has when suspended from the hook as in the Fig.

A force cannot actually be applied to a body at a point outside of the substance of the body, as between the upper and lower portions of the hook in Fig. 3, yet this portion also of the line *a b* is a part of the line of action of the force. The vertical force, exerted by the weight, P, is transmitted to *b* by means of bending moments in the bent portion of the hook.

12. Stress. (See Art. 1, Strength of Materials, p. 454.) Opposing forces, applied to a body by contact (see Art. 5 c, p. 332), cause stress, or the exertion of intermolecular force, within it, or between its particles, tending to pull them apart (tension) or to press them closer together (compression). **The stress, due to two equal opposing forces,** is equal to *one* of them.

Tension and Compression. Ties, Struts, etc. If the action of the forces tends to pull farther apart the particles of the body upon which they act, the stress is called a tension or pull, or a tensile stress. If it tends to press them closer together, the stress is called a pressure, compression or push, or a compressive stress. A long slender piece sustaining tension is called a tie. One sustaining compression is called a strut or post. One capable of sustaining either tension or compression is called a tie-strut or strut-tie

MOMENTS.

13. Moments. If, from any point, o, or o', Fig. 4, a line, $o\,c$ or $o'\,s$, be drawn normally to the line of action, $n\,m$, of a force, P_1, whether the point, o or o', be within or outside of the body upon which the force, P_1, is acting, said line, $o\,c$ or $o'\,s$, is called the **arm** or **leverage** of the force about such point; and if the amount of the force, in lbs., etc., be multiplied by the length of the arm, in ft., etc., then the product, in ft.-lbs., etc., is called the **moment** of the force about that point.* The moment represents the total tendency of the force to produce rotation about the given point. A force has evidently no moment about any point in its line of action.

14. Sense of Moments. Since the moment of P_1 about o, Fig. 4, tends to cause rotation (about that point) in the direction of the motion of the hands of a clock, as we look at the clock and at the figure, or from left to right, as indicated by the arrow on the circle around o, it is called a **clockwise** or **right-hand** moment; but the moment of the same force about o' tends to produce rotation from right to left. Hence it is called a **counter-clockwise** or **left-hand** moment, as is also that of P_2 about o. Right-hand or clockwise moments are conventionally considered as **positive**, or +, and left-hand or counter-clockwise moments as **negative**, or —.

15. The plane of a moment is that plane in which lie both the line of action and the arm of the force.

16. The resultant or combined tendency of two or more moments in the same plane is equal to the algebraic sum of the several moments. Thus, Fig. 4, if the forces, P_1, P_2, and P_3, are respectively 6, 5, and 3 lbs., and if the arms, $o\,c$, $o\,y$, and $o\,e$, of their moments about o are respectively 7, 6, and 3 ft., we have

$$\begin{aligned} & P_1 . o\,c - P_2 . o\,y + P_3 . o\,e \\ &= 6 \times 7 - 5 \times 6 + 3 \times 3 \\ &= 42 - 30 + 9 = 21 \text{ ft-lbs.} \end{aligned}$$

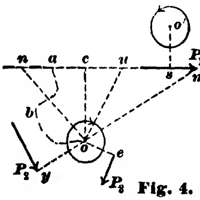

Fig. 4.

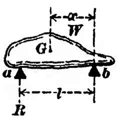

Fig. 5.

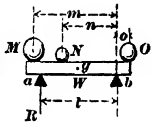

Fig. 6.

17. If the algebraic sum of the moments is zero, they are in **equilibrium** and tend to cause no rotation of the body about the given point.

Thus, in Fig. 5, where W is the weight, and G the center of gravity of the body, and R the upward reaction of the left support, a, taking moments about the right support, b, we have $R\,l - W\,x =$ zero; or $R\,l = W\,x$. Hence, having W, x and l, to find R, we have $R = \frac{W\,x}{l}$.

Similarly, in Fig. 6, where W = weight of beam alone, and g, the center of gravity of W, is at the center of the span l, so that the leverage $b\,g$ of the weight of the beam about b, is $= \frac{l}{2}$, we take moments about b, thus:

$$R\,l + O\,o - W\frac{l}{2} - M\,m - N\,n = \text{zero; or}$$

$$R = \frac{M\,m + N\,n + W\frac{l}{2} - O\,o}{l}.$$

*Note that a very small force may have a great moment about a point, while a much greater force, passing nearer to the same point, may have a smaller moment about it; or, passing through the point, no moment at all.

In Fig. 7, where W is the weight of the beam itself, and w its leverage, taking moments about b, we have

$$+ R\,l + O\,o - N\,n - W\,w + M\,m = 0;$$

Hence, Reaction at a = $R = \dfrac{W\,w + N\,n - M\,m - O\,o}{l}$.

In any case, if W be the combined weight and G the common center of gravity, of the beam and its several loads, and x the horizontal distance of that center from the right support, b; and if l be the span, R the reaction of the left support, a, and R′ that of the right support, b, we have

$$R = \frac{W\,x}{l}; \text{ and } R' = W - R.$$

If $x = \frac{l}{2}$, R is $= \frac{W}{2} = R'$.

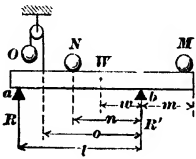

Fig. 7.

Note that the moments of two or more forces, about a given point, may be in equilibrium, while the forces themselves are not in equilibrium. See ¶ 84, below.

18. Center of Moments. So far as concerns equilibrium of moments, it is immaterial what point is selected as a center of moments; but it is generally convenient to take the center of moments in the line of action of one (or more, if there be concurrent forces, see ¶ 19) of the unknown forces, for we thus eliminate that force or those forces from the equation.

CLASSIFICATION OF FORCES.

19. Classification of Forces. Concurrent, Colinear, Coplanar, and Parallel Forces. Forces are called concurrent when their lines of

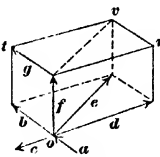

Fig. 8.

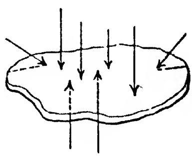

Fig. 9.

action meet at one point, as a, b, c, d, e and f, or f and g, Fig. 8; non-concurrent when they do not so meet, as c and g; colinear when their lines of action coincide, as a and b, or c and d; non-colinear when they do not coincide, as b and f; coplanar when their lines of action lie in one plane,* as a, b, c, d and e, or b, f and g, etc.; non-coplanar, as c and g, or b, f and d, when they do not lie in one plane; parallel when their lines of action are parallel, as b and g; non-parallel when those lines are not parallel, as b and f.

*Acting *upon* a plane, as in Fig. 9, must not be confounded with acting *in* that plane, as in Figs. 70, &c.

Any two parallel forces must be coplanar Three or more parallel forces may or may not be coplanar Any two concurrent forces must be coplanar. Three or more concurrent forces may or may not be coplanar Any two coplanar forces must be either parallel or concurrent

COMPOSITION AND RESOLUTION OF FORCES.

20. Resultant. A single force, which can produce, upon a body considered as a whole, the same effect as two or more given forces combined, is called the resultant of those forces Thus, in Fig 10 (*b*), a downward pressure, G, = *w* + W, is the resultant of the downward pressures *w* and W; and, in Fig. 11 (*b*), a downward pressure, = W — *w*, is the resultant of the downward pressure W and the upward pull *w* of the left-hand string *

21. Component. Any two or more forces which, together, produce, upon a body considered as a whole, the same effect as one given force, are called the components of that force, which thus becomes their resultant Thus, in Fig. 10 (*b*), *w* and W are the components of the total force, G, = *w* + W. In Fig. 11 (*b*), + W (= 5) and *w* (= — 3) are the components of G.*

22. If we take into account the resultant of any given forces, those forces (components) themselves must of course be left out of account, as regards their action upon the body as a whole; although we may still have to consider their effect upon its particles Vice versa, if the forces (components) are considered, their resultant must be neglected.

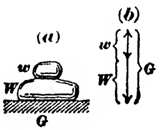

Fig. 10.

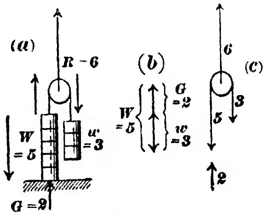

Fig. 11.

23. Anti-resultant. The anti-resultant of one or more forces is a single force which, acting upon any body or system of bodies considered as a whole, produces an effect equal, but opposite, to that of their resultant. In other words, tne anti-resultant is the force required to hold the given force or forces in equilibrium. Thus, in Fig. 10 (*b*), the upward reaction, G, of the ground, is the anti-resultant of the two downward forces, *w* and W; and the downward resultant, W + *w*, of W and *w*, is the anti-resultant of G. In Fig. 11 (*b*), G (upward) is the anti-resultant of W (downward) and *w* (acting upward through the left-hand string). Similarly, this upward pull of *w* is the anti-resultant of W and G.

24. In any system of balanced forces (forces in equilibrium), any one of the forces is the anti-resultant of all the rest; and any two or more of them have, for their resultant, the anti-resultant of all the rest. In such a system, the resultant (and the anti-resultant) of all the (balanced) forces is zero.

25. Anti-component. The anti-components of a given force, or of a given system of forces, are any two or more forces whose resultant is the anti-resultant of the given force or of the given system of forces.

26. Composition and Resolution of Forces. The operation of finding the resultant of any given system of forces is called the composition of forces; while that of finding any desired components of a given force is called the resolution of the force.

* For convenience, we here reverse the convention of ¶ 10.

Colinear Forces.

27. Let the vertical line, w, Fig. 10 (b), represent, by any convenient scale, the weight of the upper stone in Fig. 10 (a), and W that of the lower stone. Then, w + W, = G, = the combined length of the two lines, gives, by the same scale, the combined weight of the two stones, and a vertical line G, coincident with them, equal to their sum, and pointing upward, would represent their anti-resultant, or the reaction of the ground.

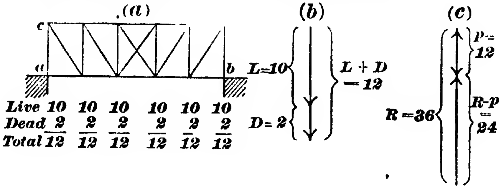

Fig. 12.

28. Similarly, if, at each panel point of the lower chord in the bridge truss in Fig. 12 (a). we have 2 tons dead load (weight of bridge and floor, etc.*) and 10 tons live load (train, vehicles, cattle, passengers, etc.), the combined length of the two lines in Fig. 12 (b), L = 10, and D = 2, gives the tota panel load of 12 tons.

29. In Fig. 11 the pressure, 5 lbs., of W upon the ground, is diminished by the 3 lbs. upward pull of the cord, transmitted from the smaller weight w, leaving 2 lbs. upward pressure to be exerted by the ground in order to maintain equilibrium. The upward reaction, R, of the pulley is = w + W — G = 3 + 5 — 2 = 6. This is represented graphically in Fig. 11 (c).

30. In the truss shown in Fig. 12 (a), the total dead and live load is = 6 × 12 = 72 tons, and half this total load, or 36 tons, rests upon each abutment. Hence, to preserve equilibrium, each abutment must exert an upward reaction of 36 tons; but, in order to ascertain how much of these 36 tons is *transmitted through the end-post, a c*, we must deduct from it the 12 tons which we assume to be originally concentrated, as dead and live load, at the panel point a; for this portion is evidently not transmitted through a c. Accordingly, in Fig. 12 (c), we draw R upward, and equal by scale to 36 tons; and, from its upper end, draw p downward and = 12 tons. The remainder of R, = R — p = 36 — 12 = 24 tons, is then the pressure transmitted through a c.

31. Colinear forces are called **similar** when they are of like sense, and **opposite** when of opposite sense. The same distinction applies to resultants.

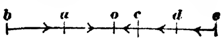

Fig. 13.

32. For equilibrium, under the action of colinear forces, it is, of course, necessary that the sum of the forces acting in one sense be equal to the sum of those acting in the opposite sense, or, in other words, that the algebraic sum of all the forces be zero. Thus, in Fig. 13, if the forces are in equilibrium, the sum, b a + a o, of the two right-handed forces must be equal to the sum, e d + d c + c o, of the three left-handed forces. Or, considering the right-handed forces, b a and a o, as positive, and the left-handed forces, e d, d c and c o, as negative, as in ¶ 10, we have, as the condition of equilibrium of colinear forces:

$$b\,a + a\,o - o\,c - c\,d - d\,e = 0.$$

---

* The dead load is, of course, never actually concentrated upon one chord as here indicated; but it is often assumed, for convenience, that it is so concentrated.

In other words, the algebraic sum of all the forces must be zero; or, more briefly,

$$\Sigma \text{ forces} = 0,$$

where the Greek letter Σ (sigma), or sign of summation, is to be read "The sum of—."

33. Two equal and opposite forces, acting upon a body, are commonly said to keep it at rest; but, strictly speaking, they merely *prevent each other* from moving the body, and thus *permit* it to remain at rest, so far as they are concerned; for they cannot keep it at rest against the action of any third force, however slight and in whatever direction it may act; and the body itself has no tendency to move.

34. Unequal Opposite Forces. If two opposite forces, acting upon a body, are unequal, the smaller one, and an equal portion of the greater one, act against each other, producing no effect upon the body as a whole; while the remainder, the resultant, moves the body in its own direction.

Concurrent Coplanar Forces. The Force Parallelogram.

35. Composition. Let the two lines, $a\,o$, $b\,o$, in any of the diagrams of Fig. 14, represent, in magnitude, direction and sense, concurrent forces whose lines of action meet at the point o. Then, in the parallelogram, $a\,c\,b\,o$, formed upon the lines $a\,o$, $b\,o$, the resultant of those two forces is represented, in magnitude and in direction, by that diagonal, R, which passes through the point, o, of concurrence. The parallelogram, $a\,c\,b\,o$, is called a **force parallelogram.**

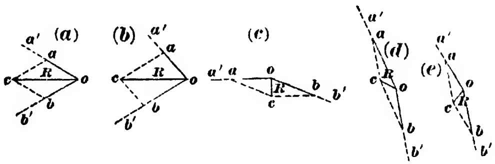

Fig. 14.

36. Resolution. Conversely, to find the components of a given force, $o\,c$, Fig. 14, when it is resolved in any two given directions, $o\,a$, $o\,b$, draw the lines, $o\,a'$, $o\,b'$, in those directions and of indefinite length, and upon these lines, with the diagonal $R = o\,c$, construct the force parallelogram $a\,c\,b\,o$. The sides, $o\,a$, $o\,b$, of the parallelogram then represent the required components in amount and in direction.

37. Caution. The two forces, $a\,o$ and $b\,o$, Fig. 14, may act either toward or from the point o; or, in other words, they may act either as pulls or as pushes; but the lines representing them in the parallelogram, and meeting at the point, o, must be drawn, either both as pushes or both as pulls; and the resultant, R, as represented by the diagonal of the parallelogram, will be a pull or a push, according as the two forces are represented as pulls or as pushes.

38. Thus, in Fig. 15 (a), the inclined end-post of the truss pushes obliquely downward toward o, with a force represented by $a'\,o$, while the lower chord pulls away from o, toward the right, with a force represented by $o\,b'$. If, now, we were to construct, in Fig. 15 (a), the parallelogram $o\,a'\,c'\,b'$, we should obtain the diagonal $o\,c'$ or $c'\,o$, which does not represent the true resultant. In fact, as one of the two forces acts toward, and the other from, the point, o, we could not tell (even if R′ were the *direction* of the resultant) in which *sense* its arrow should point.

We must first either suppose the push, $a'\,o$, in the end-post, toward o, to be carried on beyond o, so as to act as a pull, $o\,a$, Fig. 15 (b) (of course, in the same direction and sense as before), thus treating both forces as pulls; or

else we must similarly suppose the pull, *o b'*, in the chord, to be transformed into the push, *b o*, of Fig. 15 (*c*), thus treating both forces as pushes. In either case we obtain the true resultant, R (= *a' b'*, Fig. 15 *a*), which, in this case, represents the vertical downward pressure of the end of the truss upon the abutment.

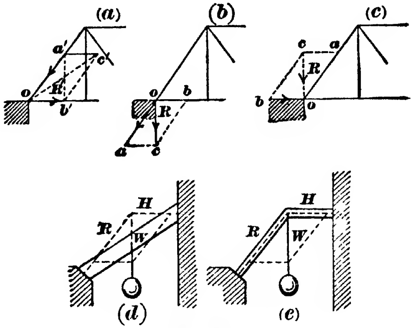

Fig. 15.

Caution. The tensile force, exerted at the end of a flexible tie, necessarily acts in the line of the tie; but, in general, the pressure, exerted at the end of a strut, acts in the line of the axis of the strut only when all the forces producing it are applied at the other end of the strut. Thus, in Fig. 15 (*d*), the components, R and H, of the weight, W, do not coincide with the axis of the beam which supports the load; but in Fig. 15 (*e*), where the weight acts at the intersection of the two struts, its components, R and H, do coincide with the axes of the struts. See also Figs. 143 and 145 (*b*).

39. Demonstration. The rational demonstration of the principle of the force parallelogram is given in treatises on Mechanics (See Bibliography.) It may be established experimentally as indicated in Fig. 16, where *c o* represents by scale the pull shown by the spring balance C, while *o a* and *o b* represent those shown by A and B respectively

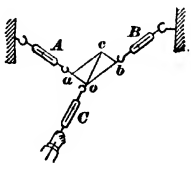

Fig. 16.

40. Equations for Components and Resultant. Given the amounts of the forces, *a* and *c*, or of the resultant, R, and the angles formed between them, Fig. 17 (*a*), we have*:

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* See dotted lines, Fig. 17 (*a*), noting that $c' = c$; $c. \sin (x + y) = R. \sin x$, and $a. \sin (x + y) = R. \sin y$.

$$R = c\,\frac{\sin (x + y)}{\sin x} = a\,\frac{\sin (x + y)}{\sin y} = \sqrt{a^2 + c^2 + 2\,a\,c\,\cos (x + y)}$$

$$c = R\,\frac{\sin x}{\sin (x + y)};\quad a = R\,\frac{\sin y}{\sin (x + y)}.$$

If the angle between the two forces is 90°, Fig. 17 (*b*), these formulas become:

$$R = \frac{c}{\cos y} = \frac{a}{\cos x}$$

$$c = R \cos y;\quad a = R \cos x.$$

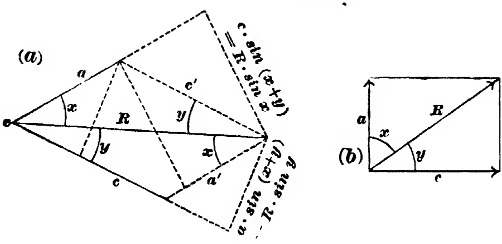


Fig. 17.

41. Position and Sense of Resultant. Figs. 18 If the lines representing the components be drawn in accordance with ¶¶ 37 and 38, and if a straight line, *m n* or *m' n'*, be drawn through the point, *o*, of concurrence, in such a way that both forces are on one side of that line, then the line representing the resultant will be found upon the same side of that line with the components, and between them; and it will act toward the line, *m n* or *m' n'*, if the components act toward it, and vice versa The resultant is necessarily in the same plane with its two components

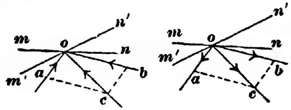


Fig. 18.

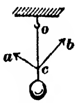


Fig. 19.

42. If one of the components is colinear with the force, it is the force itself, and the other component is zero. In other words, a force cannot be resolved into two non-colinear components, one of which is in the line of action of the force. Thus the rope, *o c*, Fig. 19, may receive assistance from *two* additional ropes, pulling in the directions *a c*, and *c b*; for the resultant of their pulls may coincide with *o c*; but, so long as *o c* remains vertical, no *single* force, as *c a* or *c b*, can relieve it, unless acting in its own direction *c o*.

43. In Fig 20, the load, P, placed at C, is suspended entirely by the vertical member B C, and exerts directly no pull along the horizontal member, C E. Neither does a pull in the latter exert any effect upon the force acting in B C, so long as B C remains vertical. But the tension in B C, acting at B, does exert a thrust *o a* along B D, although that member is at right angles to B C; for B C meets there also the inclined member A B; and the tension *o d* is thus resolved into *o a* and *o b*, along B D and B A respectively. The horizontal thrust, *o a*, in B D, is really the anti-resultant of the horizontal component, *d b*, of the oblique thrust in the end-post B A at its head. B, which thrust is = the pull in A E, due to P.

44. In Fig. 21, the tension, *o c*, in the inclined tie, D G, is resolved, at D into *o a* and *o b*, acting at right angles to each other along D F and D E respectively.

45. A resultant may be either greater or less than either one of its two oblique components, but it is always less than their sum. If the components are equal, and if the angle between them = 120°, the resultant is equal to one of them. Therefore the same weight which would break a single vertical rope or post, would break two such ropes or posts, each inclined 60° to the vertical

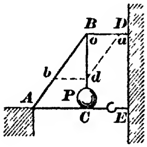


Fig. 20.

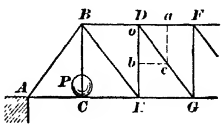


Fig. 21.

The Force Triangle.

46. The Force Triangle. Inasmuch as the two triangles, into which a parallelogram is divided by its diagonal, are similar and equal, it is sufficient to draw either one of these *triangles*, *a o c* or *b o c*, Figs. 14, 16, 18, instead of the entire parallelogram

47. If three concurrent coplanar forces are in equilibrium, the lines representing them form a triangle; and the arrows, indicating their senses follow each other around the triangle Thus, in Fig. 22 (*a*), we have, acting at *o* and balancing each other there, three forces: viz., (1) the vertical downward force *o c* of the weight, acting as a pull through the rope *o c*, (2) the horizontal thrust *a o* through the beam *a o*, and (3) the upward inclined thrust *b o* of the strut *o b*, all acting in the senses (*o c*, *a o*, *b o*) in which the letters are taken, and as indicated by the arrows.

48. Each of the forces in Fig. 22 (*b*) and (*c*) is the anti-resultant of the other two in the same triangle; and, if its sense be reversed, it becomes their resultant. Thus, *o c*, Fig. 22 (*b*), is the anti-resultant, and *c o* the resultant of *c a* and *a o*; and *o c*, Fig. 22 (*c*), is the anti-resultant, and *c o* the resultant of *c b* and *b o*, *c b* being parallel to *a o*, Fig (*b*), and representing the thrust exerted by the horizontal beam against the joint *o*, Fig. (*a*).*

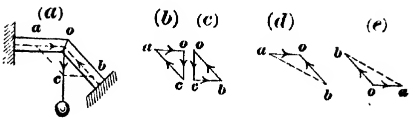


Fig. 22.

*Fig. 22 (*d*) and (*e*), representing the same two forces, *a o*, *b o*, of Fig 22 (*a*), show the erroneous resultant (*a b*) obtained if the lines are drawn with their arrows pointing both toward or both from the meeting-point of the lines. See ¶¶ 37, 38. A comparison of any force parallelogram, as that in Fig. 18, with either of the two force triangles composing it, will show that this, while apparently contradicting ¶¶ 37 and 38, is merely another statement of the same fact The apparent contradiction is due to the fact that, in the force triangle, the lines representing the forces do not meet at the point, *o*, of concurrence of the forces.

49. Conversely, if the three sides of a triangle be taken as representing, in direction and in amount, three concurrent forces whose senses are such that arrows, representing them and affixed to their respective sides in the triangle, follow each other around it, then those forces are in equilibrium.

50. The three forces, Fig. 23, are proportional, respectively, to the sines of their opposite angles. Thus:

Force a : force b : force c
= Sin A : sin B : sin C.

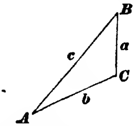


Fig. 23.

51. Example. In Fig. 24, the half arch and its spandrel, acting as a single rigid body, are assumed to be held in equilibrium by their combined weight, W, the horizontal pressure h at the crown, and the reaction R of the skewback, which is assumed to act through the center of the skewback. In the force triangle $c\ s\ t$, $c\ s$, acting through the center of gravity of the half arch and spandrel, represents the known weight W, and $s\ t$ is drawn horizontal, or parallel to h. From c, where h, produced, meets the line of action of W, draw $c\ t$ through the center of the skewback. Then $s\ t$ and $c\ t$ give us the amounts of h and R respectively.

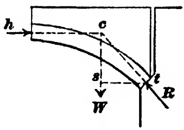


Fig. 24.

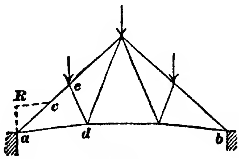


Fig. 25.

52. Example. Let Fig. 25 represent a roof truss, resting upon its abutments and carrying three loads, as shown by the arrows. Draw a R vertically, to represent the proportion of the loads carried by the left abutment, a, or, which is the same thing, the vertical upward reaction of that abutment. Then, drawing R c, parallel to the chord member, $a\ d$, to intersect $a\ e$ in c, we have, for the stresses in $a\ e$ and $a\ d$, due to the three loads:

Stress in $a\ e = a\ c$
" " $a\ d =$ R c

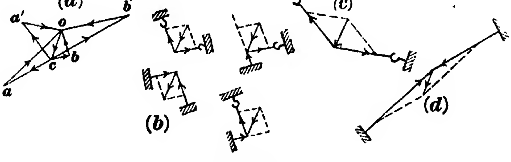


Fig. 26.

53. While any two or more given forces, as $o\ b$ and $b\ c$, Fig. 26 (a) (arrows reversed), or $o\ b'$ and $b'\ c$, or $o\ a$ and $a\ c$, or $o\ a'$ and $a'\ c$, can have but one resultant $o\ c$; a single force, as $o\ c$, may be resolved into two or more concurrent components in any desired directions. In other words, there is an infinite number of possible systems of concurrent forces which have $o\ c$ for their resultant.

Rectangular Components.

54. Resolutes, or Rectangular Components. A very common case of resolution of forces is that where a force, as the pressure, *c n*, of the post, Fig. 27, is to be resolved into components at right angles to each other, as are the vertical and horizontal components *c t* and *t n* in Fig. 27 (*a*). Two such components, taken together, are called the resolutes or rectangular components of the force. The joint, *o d*, in Fig. 27 (*a*), is properly placed at right angles to *c n*; but the joint *c i b*, Fig. 27 (*b*), provides also against accidental changes in the direction of *c n*. In Fig. 27 (*b*), the surfaces, *c i* and *i b*, are preferably proportioned as the components, *c t* and *t n*, Fig. 27 (*a*), respectively, by similarity of triangles, *c i b*, *c t n*.

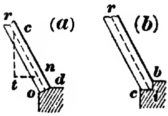


Fig. 27.

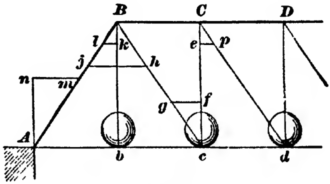


Fig. 28.

55. Example. In bridge and roof trusses it is often required to find the vertical and horizontal resolutes of the stress in an inclined member, or to find the stress brought upon an inclined member by a given vertical or horizontal stress applied at one of its ends, in conjunction with another stress (whose amount may or may not be given) at right angles to it.

Thus, in Fig. 28, the tension C *p* in the diagonal C *d* is resolved into a compression *e p* along the upper chord member C D* and a compression C *e* in the post C *c*.* Adding to C *e* the load at *c*, and representing their sum by *f c*, we have tension *f g* in chord member *c d*, and tension *c g* in the diagonal B *c*. Making B *h* = *c g*, we have *j h*, compression in B C, and B *j*, compression in the end-post or batter post B A. But the load at *b* also sends to B, through the hip vertical B *b*, a load (tension) equal to itself Representing this by B *k*, we have *l k* as its component along the chord member B C, and B *l* as its component along the end-post B A. Now, making A *m* = the sum of B *j* and B *l*, we find the vertical resolute A *n* = so much of the vertical reaction of the abutment as is due to the three loads only, and the horizontal resolute *m n* = the corresponding stress in the chord member, A *c*.

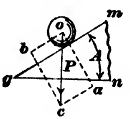


Fig. 29.

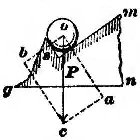


Fig. 30.

56. Example. Inclined Plane. Again, in Fig. 29, let it be required to find the two resolutes of P (the weight of the ball) respectively parallel and perpendicular to the inclined plane. The former is the tendency of the ball to move down the plane, and is called the **tangential component.** The

*The stress thus found is not necessarily the *total* stress in the member. The compression in C *c* (neglecting its own weight and that of the top chord) is due entirely to the tension C *p* in C *d*, acting at its top, and hence C *e* represents the total compression in C *c*; but *e p* is only a *portion* of the compression sustained by C D; for B C also contributes its share toward this.

latter is the pressure of the ball against the plane, and is called the **normal component.**

Here we have only to draw the triangle of forces *o a c*,* drawing *o c* = P to represent the weight of the ball, and *o a* and *a c* in the required directions. Then *o a* and *a c* give respectively the normal and the tangential components of the force, P.†

57. If we suppose the inclined plane *g m*, Fig 29, to be frictionless, and if the body *o* is to be prevented from sliding down the plane, by means of a force applied in a direction parallel to the plane, that force must be = *c a*.
Thus, in Fig. 30, supposing the plane *o m* to be frictionless, we have *a c* = pressure against the stop, *s*.

58. Table of normal and tangential components for different angles of inclination:

Inclination or Slope of the Plane. The sloping length is = vertical height / sine, col 6.						Pres on Plane, in parts of the wt. Or, nat cos of angle of Plane.	Pres on Plane, in lbs per ton.	Tendency down the Plane, in parts of the wt Or, nat sine of angle of Plane.	Tendency down the Plane, in lbs per ton.
Vert.		Hor.	Ft per mile	Deg	Min.				
1	in	3	1760 00	18	26	9487	2125	.3162	708
1	in	4.	1320 00	14	2	9702	2173	2425	543.
1	in	5.	1056 00	11	19	9806	2196	1962	439.
1	in	6.	880 00	9	28	9864	2210	.1645	368.
1	in	8	660 00	7	8	.9923	2223	.1242	278
1	in	9.	588 66	6	20	9939	2226	.1108	247.
1	in	10	528 00	5	43	.9950	2229	.0996	223.
1	in	11 4	461 94	5	00	.9962	2231	.0872	195.
1	in	12	440.00	4	46	.9965	2232	0831	186.
1	in	14 3	369 23	4	00	.9976	2232	.0698	156.
1	in	15	352 00	3	49	.9978	2233	.0666	149.
1	in	19 1	276 73	3	00	9986	2237	.0523	117.
1	in	20	264 00	2	52	9987	"	.0500	112
1	in	23 1	229 04	2	30	9990	"	.0436	97.7
1	in	25.	211.20	2	17	9992	2238	.0398	89.2
1	in	28 6	184 36	2	00	.9994	"	0349	78 2
1	in	30	176 00	1	55	"	"	.0334	74 8
1	in	32 7	161 47	1	45	.9995	2239	0.305	68 4
1	in	35	150 86	1	38	.9996	"	.0285	63 8
1	in	38 2	138 22	1	30	.9997	2240	.0262	58 6
1	in	40.	132 00	1	26	"	"	.0250	56 0
1	in	45 8	115.29	1	15	"	"	.0218	48 8
1	in	50.	105.60	1	9	.9998	"	0201	45 0
1	in	57 3	92.16	1	0	"	"	.0175	39 1
1	in	60	88 00	0	57½	.9999	"	.0167	37 4
1	in	70	75 43	0	49	"	"	.0143	32 0
1	in	76 4	69 12	0	45	"	"	.0131	29.3
1	in	80.	66.00	0	43	"	"	0125	28 0
1	in	90.	58 67	0	38	"	"	.0111	24 9
1	in	100.	52 80	0	34	1 0000	"	.0100	22 4
1	in	114.6	46 07	0	30	"	"	.0087	19 6
1	in	125	42 24	0	27½	"	"	0080	17 9
1	in	150.	35 20	0	23	"	"	0067	15 0
1	in	175.	30 17	0	19⅗	"	"	0057	12 8
1	in	200.	26 40	0	17	"	"	.0050	11.2
1	in	229 2	23 04	0	15	"	"	.0044	9.77
1	in	250.	21 12	0	14	"	"	0041	9.18
1	in	300	17 60	0	11½	"	"	0033	7 39
1	in	343 9	15 35	0	10	"	"	.0029	6.52
1	in	400	13 20	0	8⅗	"	"	.0025	5 60
1	in	500.	10 56	0	7	"	"	.0020	4 48
1	in	600.	8 80	0	6	"	"	0017	3 81
1	in	800	6 60	0	4⅓	"	"	0013	2.91
1	in	1000.	5 28	0	3¾	"	"	0016	2.24
1	in	3437.	1 54	0	1	"	"	.0003	0.65
Level.			0 00	0	0	"	"	0000	0.00

* Or *o b c*. If both triangles are drawn, we have the force parallelogram, *o a c b*.

†The line *a c* (or *c a*) is called the **projection** of *o c* upon the inclined plane; and *o a* (or *a o*) is the projection of *o c* upon a *normal* to the inclined plane

59. Equations. In Fig. 29,

$$o\ a = \mathrm{P} \,.\, \cos\ c\ o\ a$$
$$a\ c = \mathrm{P} \,.\, \sin\ c\ o\ a$$

and, since the angle $c\ o\ a$ between the vertical $o\ c$ and the normal component $o\ a$ is equal to the angle A of inclination between the plane $g\ m$ and the horizontal $g\ n$, we have.

$$\text{Normal component,} \quad o\ a = \mathrm{P} \,.\, \cos \mathrm{A}.$$
$$\text{Tangential component,} \quad a\ c = \mathrm{P} \,.\, \sin \mathrm{A}.$$

60. When a force is resolved into *rectangular* components, as in Figs. 29 and 30, each of these components represents the *total effort or tendency* which *that force alone* can exert *in that direction.*

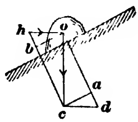


Fig. 31.

Thus, in Fig 31, the utmost force which the weight $o\ c$ alone can exert *perpendicularly against the plane* is that represented by the component $o\ a$. True, if, in order to prevent the body from sliding down the plane, we apply a force in some other direction, such as the horizontal one, $h\ o$, instead of the tangential one $b\ o$, and find the components of $o\ c$ in the directions $h\ o$ and $o\ a$, we shall find the normal component $o\ d$ greater than before; but the increase $a\ d$ is due entirely to the normal component, $h\ b$, of the horizontal force $h\ o$. Thus, the only effect upon the body o, and upon the plane, of substituting $h\ o$ for $b\ o$, is to add the normal component, $h\ b$, of the former, to that $(o\ a)$ of $o\ c$.

Stress Components.

61. Stress Components. In Fig. 32, let $a\ o$ and $b\ o$ be any two forces, and $c\ o$ their resultant. From a and b draw $a\ a'$ and $b\ b'$ at right angles to the diagonal $o\ c$ of the force parallelogram $a\ o\ b\ c$, and construct the sub-parallelograms (rectangles), $o\ a'\ a\ a''$ and $o\ b'\ b\ b''$. Each of the original components, $o\ a$, $o\ b$, is thus resolved into two sub-components, perpendicular to each other, one of which is perpendicular also to the resultant, $o\ c$, while the other coincides with $o\ c$ in position and in sense. Now, perpendiculars, let fall from the opposite angles of a parallelogram upon its diagonal, are equal,

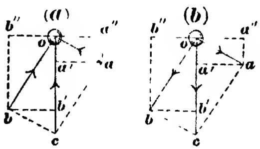


Fig. 32.

Hence the two colinear forces, $o\ a''$ and $o\ b''$, acting upon the body at o, are equal and opposite (although the lines, $a'\ a$ and $b'\ b$, representing them, are not opposite). Hence also they are in equilibrium, and their only effect upon the body is a stress of compression in Fig. 32 (a), and of tension in Fig 32 (b). They may therefore be called the stress components. The other two sub-components ($o\ a'$ of $o\ a$, and $o\ b'$ of $o\ b$) combine to form the resultant $o\ c$, which is equal to their sum, and which tends to move the body in its own direction.

62. The two great forces, $o\,a$, $o\,b$, in Fig. 33 (b) have the same resultant, $o\,c$, $=$ $o\,c'$, as the two small forces, $o\,a'$ $o\,b'$, in Fig. 33 (a), although their stress components, $a''\,a$, $=$ $b''\,b$, are much greater.

63. It often happens that one of the components is itself normal to the resultant. Thus, in Fig. 22, where $o\,c$ is vertical, its component, $o\,a$, is horizontal, and the perpendicular, let fall from a upon $o\,c$, represents its horizontal anti-component, $a\,o$. Here the horizontal and the inclined beam sustain equal horizontal pressures; but the vertical pressure, $o\,c$, $=$ the weight, W, is borne entirely by the inclined beam.

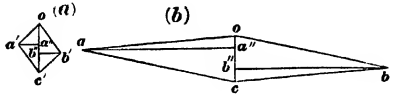


Fig. 33.

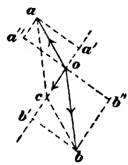


Fig. 34.

64. When, as in Fig. 34, the resultant, $o\,c$, forms, with one of the original components, $o\,a$ and $o\,b$, an angle, $a\,o\,c$, greater than 90°, the perpendiculars, $a\,a'$, $b\,b'$, from a and b, must be let fall upon the line of the resultant *produced*. Here, however, as before, the two equal and opposite sub-components, $o\,a''$ and $o\,b''$, are in equilibrium at o, while the other two sub-components, $o\,b'$ and $o\,a'$, go to make up the resultant $o\,c$; which, however (since $o\,b'$ and $o\,a'$ here act in *opposite* senses) is equal to their *difference*, and not to their sum, as in Fig. 32.

Fig. 34 shows that a *downward* force, $o\,c$, may be so resolved that one of its components is an *upward* force, $o\,a$, *greater* than the original *downward* force, and that the pressure, $o\,b$, has a component, $o\,b'$ or $b''\,b$, parallel to $o\,c$, and greater than $o\,c$ itself; for $b''\,b$ $=$ $o\,b'$ $=$ $o\,c$ $+$ $c\,b'$.

Applied and Imparted Forces.

65. Applied and Imparted Forces. In Fig. 29, the ball is free to roll down the inclined plane. Hence, although the entire weight P of the ball is *applied* to the body $g\,m\,n$, only the *normal component $o\,a$* is *imparted* to it or exerts any pressure upon it, and this pressure is in the direction $o\,a$.

But in Fig. 30, the body $g\,m\,n$ receives and resists not only the normal component $o\,a$, but also (by means of the stop s) the *tangential* component $o\,b$; and the entire force P, or $o\,c$, is thus *imparted* to the body $g\,m\,n$, pressing it in the direction $o\,c$.

Composition and Resolution of Concurrent Forces by Means of Co-ordinates.

66. In Fig. 35 (a) let the three coplanar forces E, F and G act through the point x. Draw two lines, H H, and V V, Fig. 35 (b), crossing each other at right angles, as at o.* These lines are called rectangular co-ordinates. From o, draw lines E o, F o, G o, parallel to E x, F x, G x, Fig. 35 (a), and equal respectively to the forces E, F, and G by any convenient scale. Resolve each of these forces, Fig. 35 (b), into two components, parallel to H H and V V respectively. Thus, E o is resolved into $t\,o$ and $n\,o$, F o into $u\,o$ and $e\,o$, G o into $i\,o$ and $m\,o$. Then, summing up the resolutes, we have:

Sum of horizontal resolutes $= u\,o - i\,o - t\,o = -\,s\,o$, and

Sum of vertical resolutes $= n\,o + e\,o - m\,o = a\,o$;

---

*It is only for convenience that the co-ordinates are usually drawn (as in Fig. 35) at right angles. They may be drawn at any other angle (see Fig. 36); but, in any case, the forces must of course be resolved into components *parallel to the co-ordinates*, whatever the directions of those co-ordinates may be.

and — *s o* and *a o* are the resolutes of the resultant, R, of the three forces, E, F and G.

67. When a system of (concurrent) forces is in equilibrium, the algebraic* sum of the components of all the forces, along either of the two co-ordinates, is zero. Thus, in Fig. 35 (*b*) or 36, if the sense of R be such that it shall act as the anti-resultant of the other three forces E, F and G, its component, *o s* or *o a*, along either co-ordinate, will be found to balance those of the other forces along the same co-ordinate.

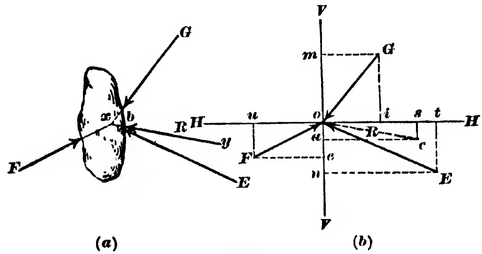


Fig. 35.

Hence we have the very important proposition that: When a system of concurrent coplanar forces is in equilibrium, the algebraic sums of their components, in any two directions, are each equal to zero.

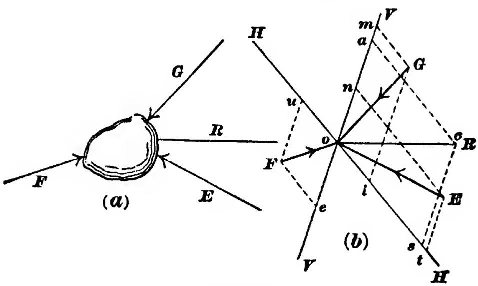


Fig. 36.

68. Conversely, in a system of concurrent forces, if the algebraic sums of the components in any two directions are each equal to zero, the forces are in equilibrium.

If the sum of the components in one of any two directions is not equal to zero, the forces cannot be in equilibrium. Thus, in Fig. 35 (*b*) or 36 (*b*), the sum of the components, along either one (as VV) of the two co-ordinates, may be zero; and yet, if the sum of those along the other co-ordinate is not zero, their resultant, or algebraic sum, will move the body, on which they act, in the direction of that resultant.

---

*The components being taken as + or —, according to the sense of each.

69. With vertical and horizontal co-ordinates, the condition of equilibrium* becomes:

The sum of the horizontal resolutes must be equal to zero;
The sum of the vertical resolutes must be equal to zero;

or, more briefly:

$$\Sigma \text{ horizontal resolutes} = 0$$
$$\Sigma \text{ vertical resolutes} = 0$$

Conversely, if these conditions are fulfilled, the forces are in equilibrium.

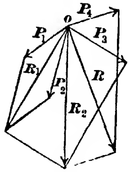


Fig. 37.

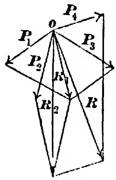


Fig. 38.

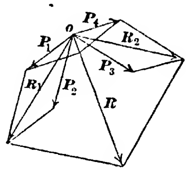


Fig. 39.

70. Resultant of More than Two Coplanar Forces. Where it is required to find the resultant of more than two concurrent and coplanar forces, as in Fig. 37, we may first find the resultant R_1 of any two of them, as of P_1 and P_2; then the resultant, R_2, of R_1 and a third force, as P_3; and so on, until we finally obtain the resultant R of all the forces. This resultant is evidently concurrent and coplanar with the given forces.

71. It is quite immaterial **in what order the forces are taken.** Thus, we may, as in Fig. 38, first combine P_1 and P_3; then their resultant R_1 with P_2, obtaining R_2; and, finally, R_2 with P_4, obtaining R; or, as in Fig 39, we may first combine any two of the forces, as P_1 and P_2, obtaining their resultant R_1; then proceed to any other two forces, as P_3 and P_4, and obtain their resultant R_2; and finally combine the two resultants, R_1 and R_2, obtaining the resultant R.

The Force Polygon.

72. The Force Polygon. Comparing Figs. 37 and 38 with Figs 40 and 41, respectively, we see that we may arrive at the same resultant R by simply drawing, as in Fig. 41, lines representing the several forces in any order, but following each other according to their senses. It will be noticed that this is merely an abbreviation of the process of drawing the several force parallelograms.

73. Resultant and Anti-resultant. The line,—R, required to complete the polygon, represents the *anti*-resultant of the other forces if its sense is such that it *follows* them around the polygon, as in Fig. 40. If its sense is *opposed* to theirs, as in Fig. 41, it is their *resultant*, R.

74. In other words, if any number of concurrent forces, as P_1, P_2, P_3, P_4 and R, Figs. 37 and 38,† are in equilibrium, the lines representing them, if drawn in any order, but so that their senses follow each other, will form a closed polygon, as in Fig. 40 (or in Fig 41 if the sense of R be reversed).

75. Conversely, if the lines representing any system of concurrent coplanar forces, when drawn with their senses following each other, form a closed polygon, as in Fig. 40, those forces are in equilibrium.

---

*With non-concurrent forces, another condition must be satisfied. See ¶ 83.

†R is here regarded as tending *upward*, so as to form the *anti*-resultant of the other forces.

It will be noticed that the force triangle, and the straight line representing a system of colinear forces, Figs. 10 and 11, ¶¶ 20, etc, or a system of parallel forces, Figs. 55, etc, ¶¶ 111, etc., are merely special cases of the force polygon.

76. In a force polygon, Fig. 42, any one of the forces is the anti-resultant of all the rest. Any two or more of the forces balance all the rest; or, their resultant is the anti-resultant of all the rest.

If a line *a c* or *b d*, Fig. 42, be drawn, connecting any two corners of a force

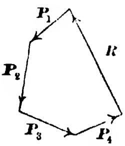


Fig. 40.

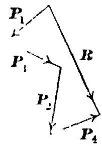


Fig. 41.

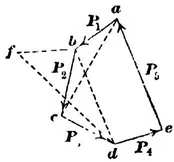


Fig. 42.

polygon, that line represents the resultant, or the anti-resultant (according as its arrow is drawn) of all the forces on either side of it. Thus:

a c	is the resultant of P_1 P_2	and the anti-resultant of	P_3 P_4 P_5
c a	" " " P_3 P_4 P_5	" "	" P_1 P_2
b d	" " " P_2 P_3	" "	" P_4 P_5 P_1
d b	" " " P_4 P_5 P_1	" "	" P_2 P_3

77. Knowing the *directions* of *all* the forces of a system, as $P_1 \ldots\ldots P_5$, Fig. 42, and the *amounts* of *all but two* of them, as P_2 and P_3, we may find the amounts of those two by first drawing the others, P_4, P_5 and P_1, as in the figure. Then two lines *b c* and *c d*, drawn in the directions of the other two and closing the polygon, will necessarily give their amounts

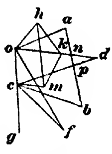


Fig. 43.

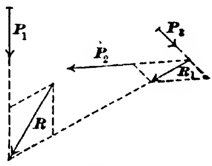


Fig. 44.

78. If any two points, as *o* and *c*, Fig 43, be taken, then the force or forces represented by any line or system of lines joining those two points will be equivalent to *o c*. Thus: *o c* = *o a b c* = *o d c* = *o n p c* = *o h k m c* = *o h m c* = *o f c* − *o g c*, etc, etc.

Similarly, in Fig 42, the force polygon *a b c d e a* is equivalent to the force polygon *a b f d e a*, and to the force triangle, *a b c a*, each being = zero.

Non-concurrent Coplanar Forces.

79. Non-concurrent Coplanar Forces. Fig 44. The process of finding the resultant of three or more coplanar but non-concurrent forces is the same as if they were concurrent. Thus, let P_1, P_2 and P_3 represent three such forces.* We may first find the resultant R_1 of any two of them, as P_2

*Any *two* coplanar non-parallel forces, as P_1 and P_2, or P_2 and P_3 are necessarily concurrent (see ¶ 19); but there is no single point in which the *three* forces meet.

and P_3; and then, by combining R_1 with the remaining force P_1, we find the resultant R of the three forces. Here the line R represents the resultant, not only in amount and in direction, but also in position. That is, the line of action of the resultant coincides with R.

80. The resultant R is the same, in amount and in direction, as if the forces were concurrent, and its position is the same as it would have been if their point of concurrence were in the line of R. If there are more than three forces, we proceed in the same way.

81. Conversely, the resultant R, or any other force, may be resolved into a system of any number of concurrent or nonconcurrent coplanar forces, in any directions, at pleasure. Thus, we may first resolve R into P_1 and R_1; then either of these into two other forces, as R_1 into P_2 and P_3, and so on.

82. If a system of non-concurrent coplanar forces is in equilibrium, the forces will still be in equilibrium if they are so placed as to be concurrent; provided, of course, that their directions, senses and amounts remain unchanged; but it does not follow that a system of forces, which is in equilibrium when concurrent, will remain in equilibrium when so placed as to be non-concurrent.

Thus, the five forces, $P_1 \ldots\ldots P_5$, Fig. 45 (*a*), may be so placed, as in Fig. 45 (*b*), that the resultant *a c*, of P_1 and P_2, does not coincide with the resultant *c a* of P_3, P_4 and P_5, but is parallel to it. These two resultants then form a couple. (See ¶¶ 155, etc.)

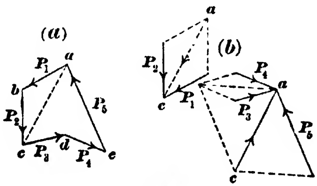


Fig. 45.

83. Third Condition of Equilibrium. Hence, the conditions of equilibrium for concurrent forces, stated in ¶ 69,

$$\Sigma \text{ vertical components} = 0$$
$$\Sigma \text{ horizontal components} = 0$$

do not suffice for non-concurrent forces, and a third condition must be added, viz.:—

$$\Sigma \text{ moments} = 0;$$

i. e., the moments of the forces, taken about any point, must be in equilibrium.

A system of forces in equilibrium has no resultant; hence it has no moment about any point. In other words, the moments of the forces, as well as the forces themselves, are in equilibrium.

84. The resultant of a system of unbalanced non-concurrent forces, acting upon a body, may be either

(1) a single force, acting through the center of gravity of the body; or

(2) a couple; *i. e.*, two equal and parallel forces of opposite sense (see ¶¶ 155, etc.); or

(3) either (a) a single force, acting through the center of gravity of the body, and a couple; or (b) a single force acting elsewhere than through the center of gravity of the body.

In Case (3), the two alternative resultants are interchangeable; *i. e.*, a single force, acting elsewhere than through the center of gravity of the body, may always be replaced by an equivalent combination consisting of an equal

parallel force, acting through the center of gravity of the body, and a couple, and vice versa. See ¶¶ 161, etc.

The resultant gives to the body, in Case (1), motion of translation in a straight line, without rotation; in Case (2), rotation without translation; and in Case (3), both translation and rotation. See foot-note (*), ¶ 1.

85. The force polygon, ¶ 72, Figs. 40, etc., and the method by coordinates, ¶ 66, Fig. 35, therefore, give us only the amount, direction and sense of the resultant of non-concurrent forces, and *not* its *position*. To find the position of the resultant of non-concurrent forces, we may have recourse to a figure, like Fig. 44, where the forces are represented in their actual positions, or to the cord polygon, ¶¶ 86, etc., Fig. 46.

The Cord Polygon.

86. In the force triangle any two of the three lines may be regarded as representing, by their directions, the positions of two members (two struts or two ties, or one strut and one tie) of indefinite length, resisting the third force; while their lengths give the amounts of the forces which those members must exert in order to maintain equilibrium.

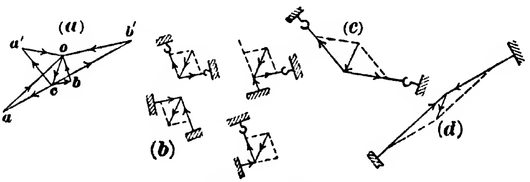


Fig. 26 (repeated).

87. Thus, in Fig. 26 (*b*), are shown four different systems, of two members each, inclined respectively like the forces *c b* and *b o* in Fig. 26 (*a*) and balancing the third force *o c*. The stresses in these two members are given by the lengths of the lines *c b* and *b o* in Fig. 26 (*a*).

The members acting as struts are represented, in Fig. 26 (*b*), as abutting against flat surfaces, while those acting as ties are represented as attached to hooks, against which they pull.

In Fig. 26 (*c*) and (*d*) are indicated systems of members, inclined like the forces *c a'* and *a' o*, *c a* and *a o*, respectively, of Fig 26 (*a*), by which the third force *o c* might be supported.

88. In the force polygon *a b c d e a*, Fig. 46 (*b*), representing the four forces, P_1, P_2, P_3, P_4, of Fig 46 (*a*), if we select, at pleasure, any point *o* (called the **pole**) and draw from it a series of straight lines *o a*, *o b*, etc. (called **rays**), radiating to the ends, *a*, *b*, *c*, etc., of the lines P_1, P_2, etc., representing the forces, we shall form a series of force triangles, *a o b*, *b o c*, etc.

Thus, in the triangle *a b o* we have the force P_1, or *a b*, balanced by the two forces *o a* and *b o;* in the triangle *b c o*, the force P_2, or *b c*, balanced by the two forces *o b* and *c o;* and so on.

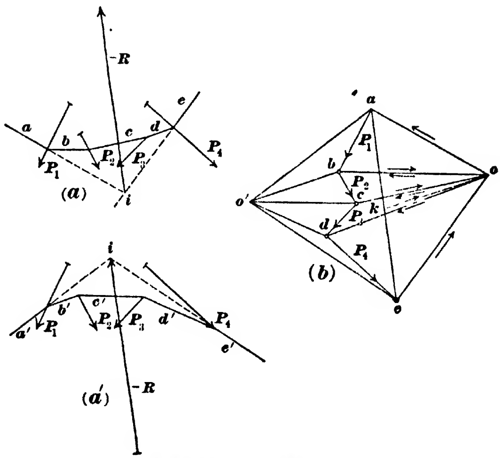
89. The Cord Polygon. If, now, in Fig. 46 (*a*), we draw the lines *a* and *b*, parallel respectively to the rays *o a* and *o b* of Fig 46 (*b*) and meeting in the line representing the force P_1, they will represent the positions of two tension members of indefinite length, which will balance the force P_1 by exerting forces represented, in amount as well as in direction, by the rays *o a* and *b o*, Fig. 46 (*b*). Again, taking pole *o'*, Fig. 46 (*b*), instead of *o*, we have *a'* and *b'*, Fig. 46 (*a'*), parallel respectively to the rays, *o' a* and *o' b*, and representing a pair of struts performing the same duty.

90. Similarly, the lines *b* and *c*, Fig. 46 (*a*), parallel respectively to rays *o b* and *o c*, represent two tension members, which, with stresses equal respectively to *o b* and *c o*, Fig. 46 (*b*), balance the force P_2.

91. We thus obtain, finally, a system of five tension members, *a b c d e*, Fig. 46 (*a*), which, if properly fastened at the ends *a* and *e* respectively, will, by exerting forces represented respectively by the rays, *o a*, *o b*, *o c*, etc., Fig. 46 (*b*), balance the four given forces P_1, P_2, P_3 and P_4.

92. The figure *a b c d e*, Fig. 46 (*a*), is called a cord polygon, funicular polygon, or equilibrium polygon.

93. Resultant, Anti-resultant. Amount and Direction. In the force polygon, Fig. 46 (*b*) or (*d*), the line *e a*, joining the end of the last force-line *d e* with the beginning of the first one *a b*, represents the anti-resultant of the given system of four forces, and *a e* their resultant. Evidently, therefore, the rays *a o* and *o e*, which represent two components of *a e*, represent also, in direction and in amount, two forces which would balance *e a*, or which would be equivalent to the given system of (four) forces.



Figs. 46 (*a*), (*a′*) and (*b*).

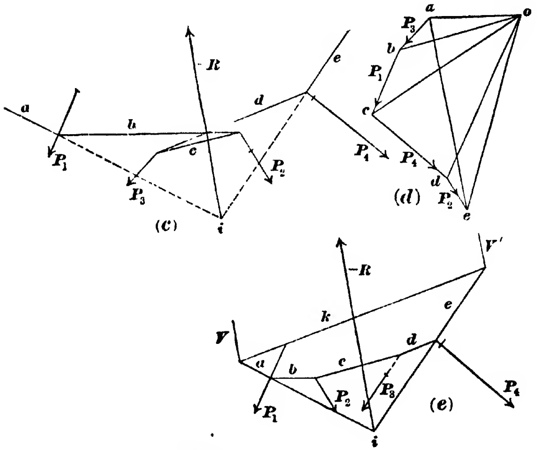
94. Position of Resultant. Hence, in the cord polygon, Fig. 46 (*a*), the intersection, *i*, of the cords *a* and *e*, parallel respectively to the rays *o a* and *e o*, is a point in the line of action of the resultant R; and, if we imagine *a i* and *e i* to be rigid rods, and apply, at *i*, a force, — R, equal and parallel to *a e*, but of opposite sense, that force will be the anti-resultant of the (four) given forces, and we shall have a frame work *b c d i* of cords and rods, kept in equilibrium by the action of the five forces, P_1, P_2, P_3, P_4 and — R.

95. The choice of position of the pole, *o*, in the force polygon, Fig. 46 (*b*), does not affect the resultant, R; but it does affect the shape of the cord polygon, Fig. 46 (*a*) or (*a′*). Thus, with the forces drawn in the order shown in (*b*), and with the pole, *o*, on the *right*, we obtain, as in Fig. (*a*), a series of supposed *ties*, *a*, *b*, *c*, etc.; but, with the pole, *o′*, on the *left* (forces drawn as before) we obtain a series of *struts*, *a′*, *b′*, *c′*, etc., Fig. (*a′*)

96. In constructing the cord polygon, Fig. 46 (*a*), (*a′*), (*c*), and (*e*), care must be taken to draw the cords in their proper places; and for this it is necessary to remember, simply, that the two rays pertaining to any particular force line in the force polygon, Fig. 46 (*b*), represent those members which, in the cord polygon, Fig. 46 (*a*), take the components of that force.

Thus, $o\ a$ and $b\ o$, Fig. 46 (b), pertain to the force P_1; $o\ b$ and $c\ o$ to the force P_2. Hence, in Fig 46 (a) or (c) we draw a and b (parallel respectively to $o\ a$ and $b\ o$) meeting in the line of action of P_1; b and c (parallel respectively to $o\ b$ and $c\ o$) meeting in the line of action of P_2, etc., etc.

97. Each ray in the force polygon, Fig. 46 (b), including the outside ones, is thus seen to pertain to two forces, and each force has two rays. The two cords, parallel respectively to the two rays of any force, must be drawn to meet in the line of action of that force; and each cord must join the lines of action of the two forces to which its parallel ray pertains. The lines, a, b, c, etc, in the *cord* polygon, Fig 46 (a) and (c), give merely the *inclinations* of members which, as there arranged, would sustain the given forces. The lengths of these lines have nothing to do with the amounts of the *stresses*. These are given by the lengths of the corresponding *rays* in the *force* polygon, Fig. 46 (b).



Figs. 46 (c), (d) and (e).

98. If the anti-resultant force, — R, is not applied, the cords a and e may be supposed fastened to firm supports, against which they exert stresses represented, in amount and in direction, by the rays $a\ o$ and $o\ e$, respectively. But the resistances of those two supports are plainly equal and opposite to those stresses, or equal to $o\ a$ and $e\ o$ respectively. Hence, their resultant is the anti-resultant, — R, of the four original forces.

99. If, Fig 46 (e), the two end members a and e were attached merely to two ties, V and V′, parallel to the anti-resultant, —R, they would evidently draw the ends of those ties inward toward each other. To prevent this, let the strut k be inserted, making it of such length that the ties V and V′ may remain parallel to — R, and draw $o\ k$, Fig. 46 (b), parallel to k. Then $a\ k$ and $k\ e$ give the stresses in V and V′ respectively.

100. If the anti-resultant, — R, found by means of the force polygon, be applied in a line passing through the intersection of the outer (initial and final) members in the cord polygon, all the forces, including of course the anti-resultant, will be in equilibrium. In other words, coplanar forces are in equilibrium if they may be so drawn as to form a closed force polygon, and if a closed cord polygon may be drawn between them. But if the anti-resultant be applied elsewhere, we shall have a couple, composed of the anti-resultant, — R, and the resultant R of the forces.

Concurrent Non-coplanar Forces.

101. Any two of the concurrent forces, as *o a* and *o c*, Fig. 47 (*a*) or (*b*), are necessarily coplanar. Find their resultant, *o r*, which must be coplanar with them and with a third force *o b* Then the resultant, R, of *o r* and *o b* is the resultant of the three forces. If there are other forces, proceed in the same way.

102. No three non-coplanar forces, whether concurrent or not, can be in equilibrium.

103. Force Parallelopiped. The resultant of any three concurrent non-coplanar forces, *o a*, *o b*, *o c*, Figs. 47, will be represented by the diagonal *o* R, of a parallelopiped, of which three converging edges represent the three forces.

104. Methods by Models. (a) For three forces. Construct a box, Fig. 47 (*a*) or (*b*), with three convergent edges representing the three forces in position and amount. Then a string *o* R, joining the proper corners, will represent the resultant.

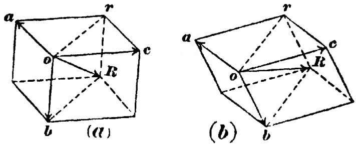
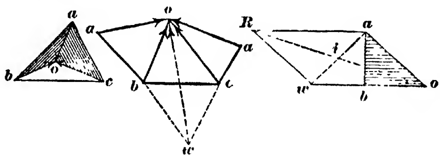


Fig. 47.

Or, let *a o*, *b o*, *c o*, Fig. 48 (*a*), be three forces, meeting at *o*. Draw on pasteboard the three forces *a o*, *b o*, *c o*, as in Fig. 48 (*b*), with their *actual* angles *a o b*, *b o c*, *c o a*, and find the resultant *w o* of the middle pair, *b o* and *c o*. Cut out neatly the whole figure, *a o a c w b a*. Make deep knife-scratches along *o b*, *o c*, so that the two outer triangles may be more readily turned at angles to the middle one. Turn them until the two edges *o a*, *o a* meet, and then paste a piece of thin paper along the meeting joint to keep



(*a*) (*b*) (*c*)

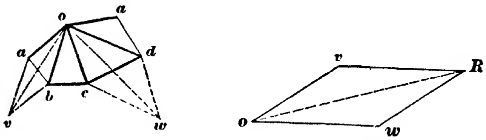
Fig. 48.

them in place. Stand the model upon its side *o b w c* as a base, and we shall have the *slipper* shape *a o b w*, Fig. 48 (*c*); *o w* being the sole, and *a o b* the hollow foot. In the model, the force *a o* and the resultant *w o* of the other two forces, are now in their actual relative positions To find their resultant, cut out a separate piece of pasteboard, R *a o w*, with R *a* and R *w* parallel respectively to *w o* and *a o*. Draw upon each side of it the diagonal R *o*. Paste this piece inside the model, with its lower edge *w o* on the line *w o*, Fig. 48 (*b*), and its edge *a o* in the corner *a o* This done, R *o* represents the resultant of *a o*, *b o*, *c o*, Fig 48 (*a*), in its actual position relative to them.

105. (b) For four forces, as *a o*, *b o*, *c o*, *d o*, in Fig. 49. Draw them as in Fig. 49 (*a*), with their angles *a o b*, *b o c*, etc. Draw also the resultants *v o*, of *c o* and *b o*; and *w o*, of *c o* and *d o*. Then cut out the entire figure, as before, and paste together the two edges *a o*, *a o*. Hold the model in such a way that two of its *planes* (as *a o b* and *b o c*) form the same angle with each other

as do the two corresponding planes between the forces. Then we have the two resultants $v\,o$, $w\,o$, Fig. 49 (*b*), in their *actual relative positions*. Cut out a separate piece of pasteboard R $v\,o\,w$, Fig. 49 (*b*), draw the diagonal R o on each side of it, and paste it inside the model, with $o\,v$ and $o\,w$ on the corresponding lines of the model. Then R o will represent the resultant of the four forces, $a\,o$, $b\,o$, $c\,o$, $d\,o$, in its actual position relative to them.

The model may be made of wood, the triangles $a\,o\,b$, $b\,o\,c$, etc., being cut out separately, the joining edges bevelled, and then glued together.



(*a*) **Fig. 49.** (*b*)

Non-concurrent Non-coplanar Forces.

106. Non-concurrent Non-coplanar Forces. Fig 50 (*a*). (For parallel non-coplanar forces, see ¶¶ 110, etc) Resolve each force into two rectangular components, one normal to an assumed plane, the other coinciding with the plane.* Find the resultant of the (coplanar) components coinciding with the plane, by methods already given, and that of the normal (parallel) components, by ¶¶ 110, etc. If these two resultants are coplanar, they are also concurrent, and their resultant (which is the resultant of the system) is readily found

107. If not, let V, Fig 50 (*b*), be the resultant normal to the plane, and H the resultant lying in the plane By ¶ 162, substitute, for H, the equal and parallel force H′, meeting V at O, and the couple H . O a, and find the resultant, R′, of V and H′ The system of forces is thus reduced to the single force R′ and the couple H . O a For Couples, see ¶ 155

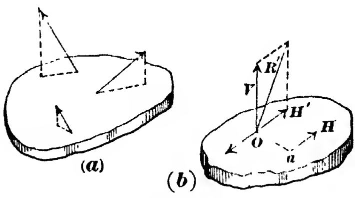


Fig. 50.

108. Moments of Non-coplanar Forces. The action of the weight W of the wall, Fig 51 (*a*), and of the non-coplanar forces P_1 and P_2, may be represented as in Fig. 51 (*b*), where the axle $a'\,c'$ represents the edge $a\,c$ about which the wall tends to turn, while the bars or levers represent the leverages of the forces. So far as regards the overturning stability of the wall, regarded as a rigid body and as capable of turning only about the edge $a\,c$, it is immaterial whether an extraneous force, as P_1, is applied at p or at q; but it is plainly not immaterial as regards a tendency to swing the wall around horizontally, or to fracture it; or as regards pressures (and consequent friction) between the axle $a'\,c'$ and its bearings. For equilibrium, $P_1\,m = P_2\,h + W.\,\frac{b}{2}$. Here a torsional or twisting stress is exerted in the axle,

*Wires, stuck in a board representing the plane, will facilitate this.

and the pressures of its ends in the bearings are more or less modified; but, so far as merely the equilibrium of the moments is concerned, we may suppose all of the forces and their moments to be shifted into one and the same plane, as in Fig. 51 (*c*).

109. In cases like that represented in Fig. 51, it is usual, for convenience, to restrict ourselves to a supposed vertical *slice*, *s*, 1 foot thick, and to the forces acting upon such slice; supposing the weight of the slice to be concentrated at its center of gravity, and the extraneous forces to be applied in the same vertical plane with gravity. In effect, we are then dealing with a slice indefinitely thin, but having the weight of the 1-ft. slice.

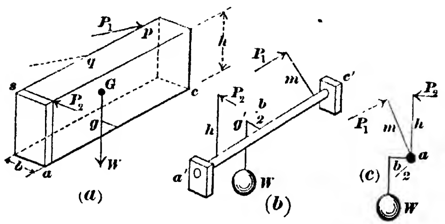


Fig. 51.

PARALLEL FORCES.

110. The resultant of any number of parallel forces, whether they are in the same plane or not, and whether in the same direction or not, is parallel to them and = their algebraic sum.

Coplanar Parallel Forces.

111. The resultant of any number of coplanar parallel forces is in the same plane with them, whether the forces are of the same or of opposite sense; and the leverages, or arms, of such forces, and of their resultant, about any given point in the same plane, are in one straight line. Thus, in Fig 56 (*a*), where the five forces, *a*, *b*, *c*, *d* and *e* are in one plane, their resultant, R, is in that same plane; and the leverages of the forces, and of R, about any point, as *b* or *v*, in the same plane, are in the straight line R *v*.

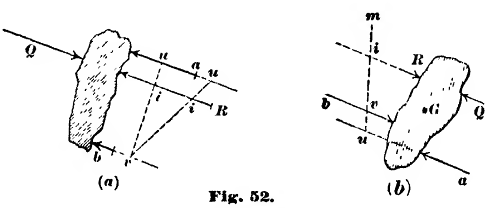


Fig. 52.

112. The resultant, R, or anti-resultant, Q, Fig. 52, **of two parallel forces,** *a* and *b*, intersects any straight line, *u v*, joining the directions of the two forces. Hence, if three parallel forces are in equilibrium, they are in the same plane. In Fig. 52 (*a*), the two forces, *a* and *b*, are of like sense. R is then between *a* and *b*, and R = *b* + *a*. In Fig. 52 (*b*), *a* and *b* are of opposite sense. R is then not between *a* and *b*, and R = *b* — *a*.

113. To find the position of the resultant, draw and measure any straight line, $u\,v$, joining the lines of action of the forces. It is immaterial whether $u\,v$ is perpendicular to said directions, or not. The line representing the resultant cuts $u\,v$, and its position is found thus:

$$u\,i = u\,v \times \frac{b}{\mathrm{R}}; \quad \text{and } v\,i = u\,v \times \frac{a}{\mathrm{R}}.$$

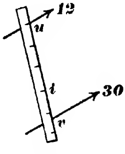


Fig. 53.

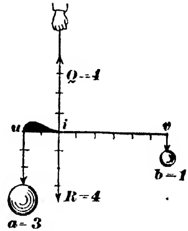
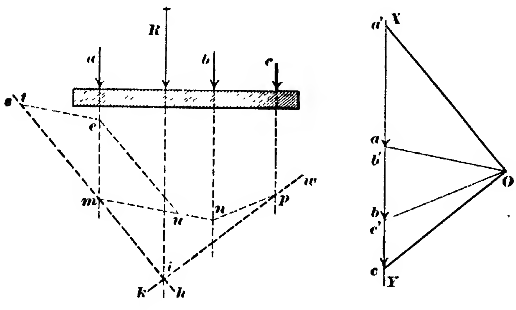


Fig. 54.

114. This may be conveniently done by making $u\,v$ equal, by any convenient scale, to the sum of the forces, as in Fig 53, where $u\,v = 42$. Then make $u\,i$ *equal*, by the same scale, to the force at v, or $v\,i$ equal to the force at u. Then a line, R, Fig 52 (a), drawn through i parallel to a and b, gives the position and direction of their resultant; and its *amount* is equal to the *sum* of a and b; or $\mathrm{R} = a + b$. In other words, if a force, Q, parallel to a and b, and equal to their sum, but of opposite sense, be applied to the body anywhere in a line passing through i, it will balance a and b, or will be their antiresultant.



(a) Fig. 55. (b)

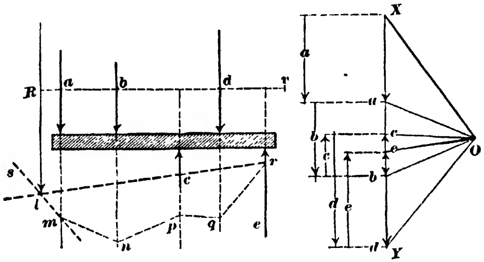
115. The position of the resultant, so found, satisfies the condition of equilibrium of moments: thus, $b.vi - a\,ui$ = zero.

If the two forces are equal, their resultant R is evidently midway between them.

116. In the common steelyard, Fig. 54, the two forces a and b, of Fig. 52 (a), are represented by the two weights, a = 3 pounds at u, and b = 1 pound at v, with leverages ui and vi respectively, as 2 : 6, or as 1 : 3.

It will be noticed that in Fig. 56 (*a*) the resultant, R, owing to the positions and amounts of the several forces, falls outside of the system of given forces.

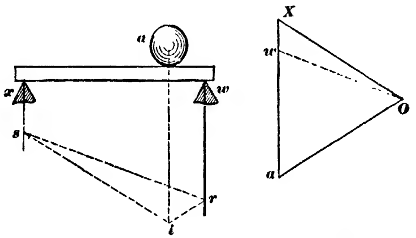
117. Figs. 55 to 58 illustrate the application of the cord polygon (¶¶ 86 to 100) to coplanar parallel forces. Here the force polygon is necessarily a straight line.



(*a*) (*b*)

Fig. 56.

118. Resolution. Let Fig. 57 (*a*) represent **a beam bearing a single concentrated load,** *a*, elsewhere than at its center; and let it be required to find the pressure on each of the two supports, *w* and *x*.



(*a*) (*b*)

Fig. 57.

Draw X *a*, Fig. 57 (*b*), to represent the load *a* by scale, and rays X O, *a* O, to any point O not in the line X *a*. In Fig. (*a*), from any point. *i*, in the vertical through the point, *a*, where the load is applied, draw *i s* and *i r*, parallel respectively to O X and O *a*. Join *r s*, and in Fig. (*b*) draw O *w* parallel to *r s*. Then the two segments, *w a* and X *w*, of X *a*, give by scale the pressures upon the two supports, *w* and *x* respectively. The greater pressure will of course be upon the support nearest to the load; but we may be guided also by remembering that the segment X *w*, adjoining the radial line O X in Fig. (*b*) represents the pressure on that support, *x*, Fig. (*a*), which pertains to the line *i s* parallel to O X; and vice versa.

119. Fig. 58 represents a case where there are several loads on the beam. Here the intersection, *i*, of the lines *h s* and *k r*, Fig. (*a*), drawn parallel respectively to O X and *c* O, Fig. (*b*) shows the position of the resultant of the three loads. Here, as in Fig. 57, we join *r s*, Fig. (*a*),

and draw O w, Fig. (b), parallel to $r\,s$. Then X w, Fig. (b), gives the pressure upon x, and $w\,c$ that upon w.

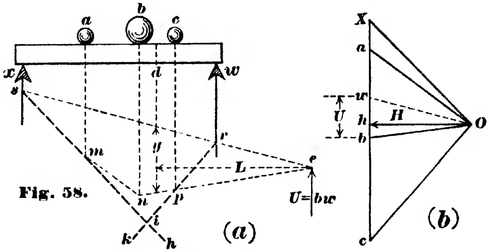


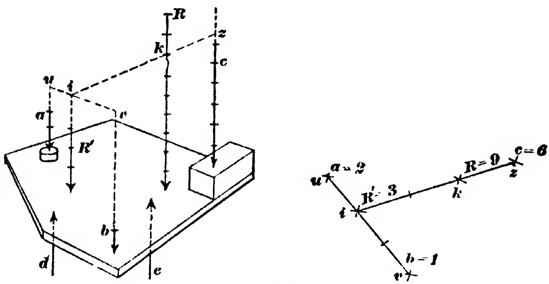
Fig. 58.

Non-coplanar Parallel Forces.

120. Non-coplanar Parallel Forces. Fig. 59 (a). Between the lines of action of any two of the forces, as a and b, draw any straight line, $u\,v$, and make

$$u\,i = u\,v \times \frac{b}{a+b}; \quad \text{or} \quad v\,i = u\,v \times \frac{a}{a+b}.$$

Through i draw R′, parallel to a and b, and equal to their sum. Then is R′ the resultant of a and b. Then, from any point, i, in the line of action of R′, draw $i\,z$ to any point, z, in the line of action of c, and make $i\,k = i\,z \times \frac{c}{c + R'}$; or $z\,k = i\,z \times \frac{R'}{c + R'}$. Through k draw R parallel to a, b and c, and equal by scale to their sum. Then is R the resultant of the three forces, a, b and c. If there are other forces, proceed in the same way with them.



(a) **Fig. 59.** (b)

121. In Fig. 59 (a) we have shown the forces, a and c, acting upon surfaces raised above the general plane, merely in order to illustrate the fact that it is not at all necessary that the forces be supposed to act upon or against a plane surface.

122. Although Fig. 59 (a) illustrates the method of finding the resultant of non-coplanar parallel forces, yet it plainly does not give the actual relative positions of the forces and their resultant; because it is necessarily drawn in a kind of perspective, and therefore all the parts cannot be measured by a scale. The true relative positions may of course be represented in plan, as by the five stars, a, b, c, i and k, Fig. 59 (b), corresponding to the points where

the forces and resultants intersect some one chosen plane. But it is now impossible to represent the forces themselves by lines. They must therefore be stated in figures, as is here done. It is then easy to find the positions of the resultants, as before

123. If there are also **forces acting in the opposite direction,** as d and e, Fig. 59 (a), find their resultant separately. We thus obtain, finally, two resultants of opposite sense. These resultants may be equal or unequal, and colinear or non-colinear. If they are non-colinear, see ¶ 84, and Couples, ¶¶ 155, etc.

124. Method by projections, Fig. 60. First find the projections, a', b' and c' of the forces, a, b and c, upon any plane, as $x\,y$, parallel to them; and then their projections, a'', b'', and c'', upon a second plane, $x\,v$, parallel to them and normal to the first. Find the position, R′, of the resultant of a', b' and c', in plane $x\,y$, and that R″, of a'', b'' and c'', in plane $x\,v$. Now, as the lines, a', b', c', and a'', b'', c'', are projections of the forces, a, b and c, so R′, R″, are projections of the resultant, R, of the forces. The position of R is therefore at the intersection of two planes, R R′ and R R″, perpendicular to the planes, $x\,y$ and $x\,v$, and standing upon the projections R′ and R″, of the resultant, R. R = $a + b + c$.

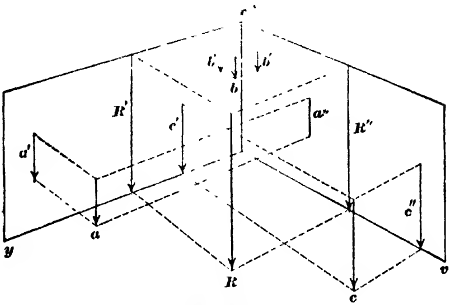


Fig. 60.

CENTER OF GRAVITY.

125. If a body, Fig. 1,* or a system of bodies, Fig. 2, be held successively in different positions, (a), (b), etc., the resultant of the parallel forces of gravity, acting upon its particles and indicated by the arrows in the figures, will occupy different positions, relatively to the figure of the body or system. That point, where all these positions, or lines of gravity, meet, is called the center of gravity of the body or system.' Thus, if a homogeneous cylinder be stood vertically upon either end, the line of gravity will coincide with the axis of the cylinder; but if the cylinder be then laid upon its side, the line of gravity will intersect the axis at right angles and will bisect it. Hence, in the cylinder, the center of gravity is at the center of the axis.

126. About the center of gravity the moments of all the forces of gravity are in equilibrium, in whatever position the body or system may be Hence, the body, or system, if suspended by this point, and acted upon by gravity alone, will balance itself; *i. e.*, if at rest it will remain at rest; or, if set in motion revolving about its center of gravity, and then left to itself, it will continue to revolve about that center indefinitely and with uniform angular velocity. Or, if suspended freely from any point, it will oscillate until the center of gravity comes to rest vertically under such point.

---

* Figs. 1 to 45, relating to Center of Gravity, are numbered independently of the rest of the series of figures relating to Statics.

127. In some bodies, such as the cube, or other parallelopiped, the sphere, etc., the center of *gravity* is also the center of the *weight* of the body; but very frequently this is not the case. Thus, in a body *a b*, Fig. 2, with its center of gravity at G, there is more weight on the side *a* G, than on the side G *b*.

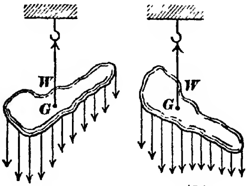


Fig. 1.

Stable, Unstable, and Indifferent Equilibrium.

128. A body is said to be in stable equilibrium when, as in the pendulum, it is so suspended that, if swung a little to either side, it tends to oscillate until it comes to rest again, with its center of gravity vertically under the point of suspension.

129. It is said to be in unstable equilibrium when, as in the case of an egg, stood upon its point, it is so supported that, if swung a little to either side, and left to itself, it swings farther out from the vertical and eventually falls

130. It is said to be in indifferent equilibrium when, as in the case of a grindstone, supported by its horizontal axis, or of a sphere resting upon a horizontal table, it is so suspended or supported that, if made to rotate about its center of gravity and then left to itself, it will continue in that state of rest or of angular motion in which it is left.

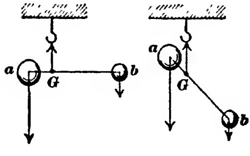


Fig. 2.

General Rules.

131. The following general rules (1) to (6), form the basis of the special rules, (7) to (39)

In speaking of the center of gravity of one or more **bodies**, we shall assume, for simplicity, that they are *homogeneous* (*i. e.*, of uniform density throughout) and of the same density with each other. The center of *gravity* is then the same as the center of *volume*, and we may use the *volumes* of the bodies (as in *cubic feet*, etc.) in the rules, instead of their *weights* (as in *pounds*, etc).

In applying these general rules to **surfaces**, use the *areas* of the surfaces, and in applying them to **lines**, use the *lengths* of the lines, in place of the weights or volumes of the bodies.

In all of the rules and figures, pp. 388 to 398, **G represents** the center of gravity, except where otherwise stated.

(1). Any two bodies, Fig. 3. Having found the center of gravity, g, g', of each body, by means of the rules given below: then **G** is in the line joining g and g'; and

$$g\,\mathbf{G} = g g' \times \frac{\text{weight of } g'}{\text{sum of weights of } g \text{ and } g'}$$

$$g'\,\mathbf{G} = g g' \times \frac{\text{weight of } g}{\text{sum of weights of } g \text{ and } g'}$$

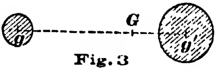


Fig. 3

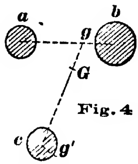


Fig. 4

(2). Any number of bodies, as a, b and c, Fig. 4, whether their **centers** of gravity are in the same plane or not.

First, by means of rule (1) find the center of gravity, g, of any two of the bodies, as a and b. Then the center of gravity, **G,** of the three bodies, a, b and c, is in the line $g g'$ joining g with the center of gravity, g' of c; and

$$g\,\mathbf{G} = g g' \times \frac{\text{weight of } c}{\text{sum of weights of } a,\, b \text{ and } c};$$

$$g'\,\mathbf{G} = g g' \times \frac{\text{sum of weights of } a \text{ and } b}{\text{sum of weights of } a,\, b \text{ and } c};$$

and so on, if there are other bodies.

(3). In many cases, **a single complex body** may be supposed to be divided into parts whose several centers of gravity can be readily found. Then the center of gravity of the whole may be found by the foregoing and following rules. Thus, in Fig. 5, we may find separately the centers of gravity of the

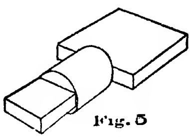


Fig. 5

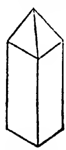


Fig. 6

two parallelopipeds and of the cylinder between them (each in the center of its respective portion of the whole solid); and in Fig. 6 the centers of gravity of the square prism and the square pyramid (the latter by rule (36), and then, knowing in either case the weights of the several parts, find their common center of gravity as directed in rules (1) and (2).

(4). **Any hollow body,** or body containing one or more openings, Fig. 7 Find the common center of gravity, g', of the openings by rule (1) or (2), and

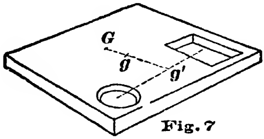


Fig. 7

the center of gravity, g, of the entire figure, as though it had no openings. Then G is in the line $g\,g'$, extended, and

$$g\,G = g\,g' \times \frac{\text{sum of volumes of openings}}{\text{volume of entire body} - \text{volumes of openings}}$$

$$g'\,G = g\,g' \times \frac{\text{volume of entire body}}{\text{volume of entire body} - \text{volumes of openings}}$$

REMARK. For convenience, we have shown the several centers of gravity, g, g', G, upon the *surface* of the figure. In the real solid (supposed to be of uniform thickness) they would of course be in the middle of its thickness and immediately under the positions shown in the figure.

(5). In any line, figure or body, or in any system of lines, figures or bodies, any plane passing through the center of gravity is called a **"plane of gravity"** for said line, etc., or system of lines, etc. The intersection of two such planes of gravity is called a **"line of gravity."** The center of gravity is (1st) the intersection of two lines of gravity; (2nd) the intersection of three planes of gravity, or (3rd) the intersection of a plane of gravity with a line of gravity not lying in said plane.

If a figure or body has an axis or plane of **symmetry** (*i. e.*, a line or plane dividing it into two equal and similar portions) said axis or plane is a line or plane of gravity. If a figure or body has a central point, said point is the center of gravity.

In Fig. 1, the string represents a line of gravity; and any plane with which the string coincides is a plane of gravity. Thus G may often be conveniently found, especially in the case of a flat body, by allowing it to hang freely from a string attached alternately at different corners of it, or by balancing it in two or more positions over a knife-edge, etc., and finding G in either case by the intersection of the lines or planes of gravity thus found.

(6). **The graphic method** of finding the resultant of parallel forces may often be advantageously used for finding the center of gravity of a compound body or figure, or of a system of bodies or figures, when the centers of gravity of the several parts are known.

Thus, in Fig. 8, let a, b and c represent three figures or bodies whose centers of gravity are in one plane. Draw vertical lines through said centers, and construct the polygon of forces, $x\,a\,b\,c$, Fig. 9, making the lines $x\,a$, $a\,b$, etc., proportional to the weights of a, b and c; and from any convenient point O draw radial lines O x, O a, etc. In Fig. 8, draw $m\,h$, $m\,n$, $n\,p$, and $p\,k$, parallel respectively to O x, O a, O b, O c. Then a vertical line, i G, drawn through the intersection, i, of $m\,h$ and $p\,k$, is a line of gravity of the system or figure. If the body or figure is *symmetrical*, as in the cross section of a T rail, I beam or deck beam, etc., the axis of symmetry, dividing the figure, etc. into two similar and equal parts, is also a line of gravity, and its intersection with the line i G already found is the required center of gravity G. In such cases it is generally most convenient to draw the lines through the several centers of gravity *perpendicular* to the axis of symmetry, so that the line of gravity found will also be perpendicular to it.

But if, as in Fig. 8, the body or figure, etc., is not symmetrical, we must find a second line of gravity, the intersection of which with the first will give the center of gravity, G. To do this, repeat the process, drawing another set of parallel lines through the several centers of gravity, Fig. 8. It will be most convenient to draw them horizontally, or *at right angles* to those already drawn, and in the following instructions we suppose this to be done.

Then draw a second funicular polygon, $m'\,n'\,p'\,i'$, Fig. 8, making the lines $m'\,n'$, etc., *perpendicular* (instead of parallel) to the radial lines O x, etc., Fig. 9; and draw the second line of gravity, i' G, through i', perpendicular to the first. Then G is at the intersection of the two lines of gravity.

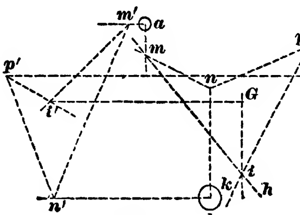


Fig. 8

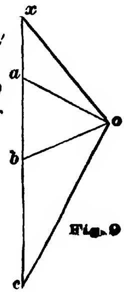


Fig. 9

The drawing of the second funicular polygon is often less simple than that of the first, because in the second the parallel lines through the several centers of gravity do not necessarily follow each other in the same order as in the first. Bear in mind that the two lines (as $n'\,p'$, $n'\,m'$) meeting in the parallel line (as $b\,n'$) pertaining to any given part, b, of the figure, must be perpendicular respectively to those radial lines (O a, O b) which meet the ends of the line, $a\,b$, that represents that same part.

Figs. 10 and 11 show the application of the same process to an irregular figure composed of three rectangles, a, b and c. The lettering is the same as in Figs. 8 and 9; but in Fig. 10 it happens that i' and p' of the second funicular polygon fall upon the same point.

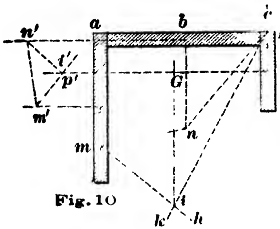


Fig. 10

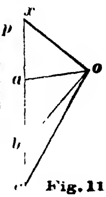


Fig. 11

If the centers of gravity of the several bodies, or of the several parts of the body, etc., are in more than one plane, we must find their projections upon certain planes, and apply the process to those projections.

Special Rules.

132. Special Rules, derived from the general rules, (1) to (6).

Lines.

(7). Straight line. **G** is in the line, and at the middle of its length.

(8). Circular arc,* $a\,o\,b$, Figs. 12 and 13 (center of circle at c). **G** is in the line $c\,o$ joining the center of the circle with the middle of the arc, and

$$c\,G = \text{radius } a\,c \times \frac{\text{chord } a\,b}{\text{length of arc } a\,o\,b}.$$

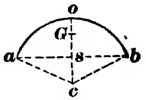


Fig. 12

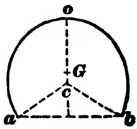


Fig. 13

(8a). If the arc is a semi-circle,*

$$c\,G = \text{radius } a\,c \times \frac{2}{\pi} = \text{radius } a\,c \times 0.6366.$$

(8b). Approximate rules for distance $s\,G$, Fig. 12, from *chord* to center of gravity.

If rise $s\,o$ = .01 chord $a\,b$;	$s\,G$ = .666 $s\,o$	If rise $s\,o$ = .30 chord $a\,b$;	$s\,G$ = .653 $s\,o$
" " " = .10 " " ;	" = .665 $s\,o$	" " " = .35 " " ;	" = .649 $s\,o$
" " " = .15 " " ;	" = .663 $s\,o$	" " " = .40 " " ;	" = .645 $s\,o$
" " " = .20 " " ;	" = .660 $s\,o$	" " " = .45 " " ;	" = .641 $s\,o$
" " " = .25 " " ;	" = .657 $s\,o$	" " " = .50 " " ;	" = .637 $s\,o$

(9). Triangle, $a\,b\,c$, Fig. 14. The center of gravity, **G**, of its three sides* is the center of the circle inscribed by a triangle, $d\,e\,f$, whose corners are in the centers of the sides of the given triangle.

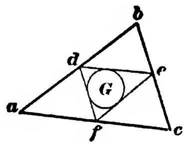


Fig. 14

(10). Parallelogram (square, rectangle, rhombus or rhomboid). The center of gravity of the four sides* is at the intersection of the diagonals.

(11). Circle, ellipse, or regular polygon. The center of gravity of the outline or circumference* is the center of the figure.

(12). Regular prism, right or oblique, and **right regular pyramid, or frustum.** The center of gravity of the edges* is the center of the axis. In the *prism*, the position of G is not affected by either including or excluding the sides of *both* of the polygons forming the ends.

(12a). Cycloid.* See p. 194.

Surfaces.

A. Plane surfaces.

We now treat of the centers of gravity of plane *surfaces*, which may be regarded as infinitely thin flat bodies. The rules for surfaces may be used also for actual flat bodies, in which, however, the center of gravity is in the middle of the thickness, immediately under the points found by the rules.

(13) Parallelogram (square, rectangle, rhombus or rhomboid), **circle, ellipse** or **regular polygon.** **G** is the center of the figure; or the intersection of any two diameters, or the middle of any diameter. In a **Parallelogram, G** is the intersection of the two diagonals.

(14). Triangle, Fig. 15. **G** is at the intersection of lines (as $a\,e$ and $c\,d$ drawn from any two angles, a and c, to the centers, e and d, of the sides, b

* We are now treating of *lines* only; not of the surfaces bounded by them. For surfaces, see rules (13), etc., etc.

and ab, respectively opposite to said angles. Such lines are called "medial lines."
$e\,\mathbf{G} = \frac{1}{3}\,a\,e;\quad d\,\mathbf{G} = \frac{1}{3}\,c\,d;\quad f\,\mathbf{G} = \frac{1}{3}\,b\,f$ (f being the middle of $a\,c$).

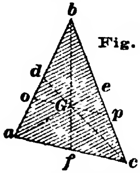


Fig. 15

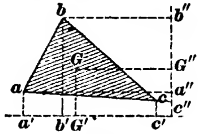


Fig. 16

(**14a**). Fig. 15. Or, on either one of the sides (as $a\,b$), meeting at any angle, a, make $a\,o = \frac{1}{3}\,ab$. Draw $o\,p$ parallel to the other side, $a\,c$. Then $o\,\mathbf{G} = \frac{1}{2}\,o\,p$, and $\mathbf{G}$ is at the intersection of $o\,p$ with any medial line, as $a\,e$, etc.

(**14b**). Fig. 16. If aa', bb', cc' and GG' are the distances of the three corners and of $\mathbf{G}$ from any straight line or plane $a'c'$; then

$$\mathbf{G}\,G' = \tfrac{1}{3}\,(a\,a' + b\,b' + c\,c').$$

This gives us the position of the line of gravity G G″. In the same way we find the distance G G″ of G from any second line or plane, $b''\,c''$ This gives us the position of a second line of gravity G G′. $\mathbf{G}$ is at the intersection of G G′ and G G″.

(**14c**). Fig. 17. The distance $\mathbf{G}\,n$ of $\mathbf{G}$ in any direction from any side, as $a\,c$ (extended if necessary) is $= \frac{1}{3}$ the distance $n'\,b$ measured in a parallel direction from the same side to the opposite angle, b.

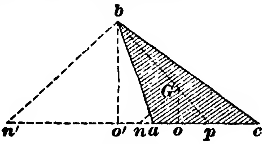


Fig. 17

It follows from this that the *shortest* distance, $\mathbf{G}\,o$, of $\mathbf{G}$ from any side (as $a\,c$) is $= \frac{1}{3}$ the shortest distance, $o'\,b$, from the same side to its opposite angle b. It follows also that $p\,G = \frac{1}{3}\,p\,b$, as in Rule (14)

(**15**). **Trapezium or trapezoid,** Fig. 18. For trapezoids, see also Rule (16). Draw the two diagonals, $a\,c$ and $b\,d$. Divide either of them, as $a\,c$, into two equal parts, $a\,m$ and $c\,m$. From b, on $b\,d$, lay off $b\,n = d\,s$ (or from d lay off $d\,n = b\,s$). Join $m\,n$. $\mathbf{G}$ is in $m\,n$, and

$$m\,\mathbf{G} = \tfrac{1}{3}\,m\,n.$$

($\mathbf{G}$ is the center of gravity of the triangle $a\,c\,n$).

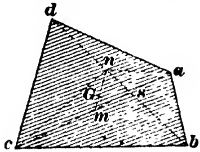


Fig. 18

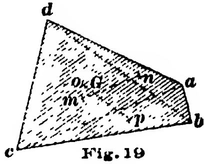


Fig. 19

(**15a**). Or, Fig. 19, find first the centers of gravity, m and n, of the two triangles, $c\,b\,d$ and $a\,b\,d$, into which the trapezium is divided by one of its diagonals, $b\,d$. Join $m\,n$ Then find the centers of gravity, o and p, of the two triangles, $d\,a\,c$ and $b\,a\,c$, into which the trapezium is divided by its *other* diagonal, $a\,c$. Join $o\,p$. Then $\mathbf{G}$ is the intersection of $m\,n$ and $o\,p$.

(**16**). **Trapezoid only,** Fig 20. **G** is in the line $e\,f$ joining the centers, e and f, of the two parallel sides, $a\,b$ and $c\,d$. To find its position in said line, prolong either parallel side, as $a\,b$, in either direction, say toward i; and make $b\,i$ equal to the opposite side, $c\,d$. Then prolong said opposite side, $c\,d$, in the opposite direction, making $d\,h = a\,b$. Join $h\,i$. Then **G** is the intersection of $h\,i$ and $e\,f$. Or

$$f\,G = \frac{e\,f}{3} \times \frac{2\,a\,b + c\,d}{a\,b + c\,d}; \quad \text{or } o\,G = \frac{e\,n}{3} \times \frac{2\,a\,b + c\,d}{a\,b + c\,d}$$

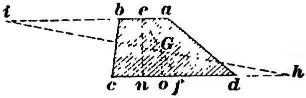


Fig. 20

(**17**). **Regular polygon.** **G** is the center of the figure.

(**17a**). **Irregular polygon.** If the polygon be divided into any two portions, as by any diagonal, **G** must be in the line (of gravity) joining the centers of gravity of those two portions. If we again divide the whole polygon into two *other* parts by *another* diagonal, and join the centers of gravity of those two parts, **G** is the intersection of the two lines of gravity.

(**17b**). Or we may divide the polygon into triangles, find the center of gravity of each triangle, by Rules (14), etc., and then find **G** by general Rule (1), (2) or (6).

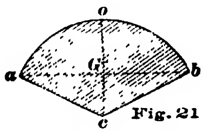


Fig. 21

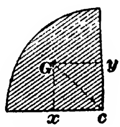


Fig. 22

(**18**). **Circular sector,** $a\,o\,b\,c$, **Fig.** 21. (Center of circle at c).

$$c\,G = \frac{2}{3}\ \text{radius}\ a\,c \times \frac{\text{chord}\ a\,b}{\text{arc}\ a\,o\,b} = \frac{\text{radius}^2 \times \text{chord}}{3 \times \text{area}}$$

For length of arc, see pp 179, etc.

(**18a**). If the sector is a **sextant,**

$$c\,G = \text{radius} \times \frac{2}{\pi} = \text{radius} \times 0.6366.$$

(**18b**). If the sector is a **quadrant,** Fig. 22,

$$c\,G = \frac{4}{3}\,\text{radius} \times \frac{\sqrt{2}}{\pi} = \text{radius} \times 0.6002.$$

$$c\,x = x\,G = \frac{4}{3}\ \text{radius} \times \frac{1}{\pi}.$$

(**18c**). If the sector is a **semi-circle,**

$$c\,G = \frac{4}{3}\ \text{radius} \times \frac{1}{\pi} = \text{radius} \times 0.4244.$$

$$= \text{(approximately) radius} \times \frac{14}{33}.$$

(19). **Circular segment,** $a\,o\,b\,s$, Fig. 23. (Center of circle at c).

$$c\,G = \frac{\text{cube of chord } a\,b}{12 \times \text{area of segment}}.$$

(19 a). If the segment is a **semi-circle,**

$$c\,G = \frac{4}{3}\,\text{radius} \times \frac{1}{\pi} = \text{radius} \times 0.4244$$

$$= \text{(approximately) radius} \times \frac{14}{33}.$$

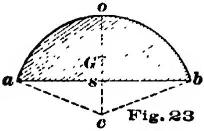


Fig. 23

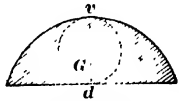


Fig. 24

(20). **Cycloid,** Fig. 24. (Vertex at v).

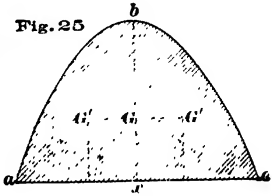
$$v\,G = \frac{7}{12}\,v\,d.$$

(21). **Parabola,** $a\,b\,c$, Fig. 25. $a\,c$ is the base; $a\,x$ and $c\,x$, ordinates; and the height or axis, $b\,x$, an abscissa. Center of gravity at **G,** in the axis $x\,b$, and

$$x\,G = \frac{2}{5}\,x\,b.$$

(21 a). **Semi-parabola,** $a\,b\,x$ or $c\,b\,x$. Center of gravity at G′, and

Fig. 25



$$x\,G = \frac{2}{5}\,x\,b; \qquad G\,G' = \frac{3}{8}\,a\,x = \frac{3}{8}\,c\,x.$$

(22). **Ellipse,** $m\,n\,o\,p$, Fig. 26. The center of gravity, c, of the **whole** ellipse is at the center of the figure.

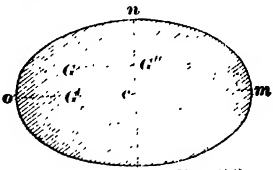


Fig. 26

G is the center of gravity of the quarter ellipse, $o\,n\,c$.
G′ " " " " " half " $n\,o\,p$.
G″ " " " " " " " $m\,n\,o$.

$$c\,G' = \frac{4}{3}\,o\,c \times \frac{1}{\pi} = 0.4244\,o\,c = \text{(approximately)}\ \frac{14}{33}\,o\,c.$$

$$c\,G'' = G'\,G = \frac{4}{3}\,c\,n \times \frac{1}{\pi} = 0.4244\,c\,n = \text{(approximately)}\ \frac{14}{33}\,c\,n.$$

(23). **Any plane figure.** Draw the figure to scale on stout card-board. Cut it out and balance it in two or more positions over the edge of a table or on a knife-edge; and mark on it the several positions of the supporting edge. Where these intersect is the center of gravity. Considerable care is of course necessary to obtain very close results by this method. Before balancing the card, its upper edges should be marked off into small equal spaces. Otherwise it will be difficult to locate the positions of the supporting edge. The paper on which the figure is prepared must of course be so stiff that the figure will not bend when balanced on the knife-edge. See Rule (5).

B. Surfaces of Solids.*

(24). Curved surface* of **sphere** or **spheroid (ellipsoid).** G is the center of the figure.

(25). Curved surface* of any **spherical zone,** as a **spherical segment, hemisphere,** etc., Figs. 27. G is the center of the axis or height, *a o*.†

In the **hemisphere,** $o\,G = \frac{1}{2}$ radius.†

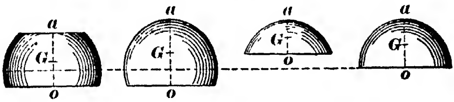


Fig. 27

(26). Right or oblique **prism,** whose ends are either regular figures or parallelograms (this includes the **cube** and other **parallelopipeds**); and right or oblique **cylinder** (circular or elliptic). *Surface** (either including both or excluding both of the two parallel ends). G is the center of the **axis,** or line joining the centers of the two parallel ends.

(27). Curved surface*† of right **cone,** Fig. 28 (circular or elliptic), or slanting surfaces*† of right regular **pyramid,** Fig. 29. G is in the **axis** *o a* (the line joining the apex and the center of the base); and

$$o\,G = \frac{1}{3}\,o\,a.$$

In an *oblique* cone or pyramid, the perpendicular distance of G* from the base is one-third of the perpendicular height, as in the right cone and pyramid; *but does not lie in the axis.*

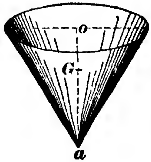


Fig. 28

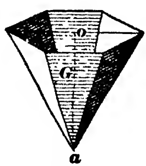


Fig. 29

(28). **Frustums** with top and base parallel, Figs 30 and 31. Curved surface*† of frustum of right cone (circular or elliptic); or slanting surfaces*† of frustum of right regular pyramid. G is in the **axis** *o a* (the line joining the centers of the two parallel ends); and

$$o\,G = \frac{1}{3}\,o\,a \times \frac{\text{circumference of } o + 2\,\text{circumference of } a.}{\text{circumference of } o + \text{circumference of } a.}$$

*We treat now of the *surfaces* of solids, not of their contents or volumes or weights. For these, see Rules (29), etc.

†If the **top** or **base** is to be included, see Rules (1) and (2).

In the **conic frustum,** Fig. 30, we may use the *radii* of the two ends; **and** in the **frustum of a regular pyramid,** Fig. 31, any *side* of each end (**as** bc and de) instead of the circumferences.

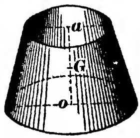


Fig. 30

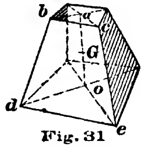


Fig. 31

Solids.

In the following rules for center of gravity of solids, the solid is supposed to be *homogeneous*; *i. e.*, of uniform density throughout; so that the center of *gravity* is the center of *magnitude* or of *volume*.

(29). Sphere and **spheroid (ellipsoid).** G is the center of the body.

(30). Hemisphere, Fig. 32. (Center of sphere at c). Height cT = radius cb. G is in the axis, cT, and

$$c\mathrm{G} = \frac{3}{8}\,c\mathrm{T} = \frac{3}{8}\text{ radius } cb.$$

(31). Spherical sector, Fig. 33. (Center of sphere at c).

$$c\mathrm{G} = \frac{3}{4}\left(\text{radius } cb - \frac{h}{2}\right).$$

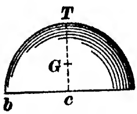


Fig. 32

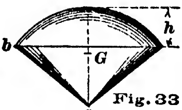


Fig. 33

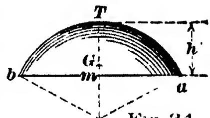


Fig. 34

(32). Spherical segment, ambT, Fig. 34. Center of *sphere* at c. Center of *base* at m. Rise or height of segment = mT = h. G is in the axis mT; and

$$c\mathrm{G} = \frac{3}{4} \times \frac{(2\text{ radius } cb\text{ of } sphere - \text{height } h)^2}{3\text{ radius } cb\text{ of } sphere - \text{height } h}$$

$$m\mathrm{G} = \frac{\text{height}, h}{2} \times \frac{2\,(\text{radius } mb\text{ of } base)^2 + (\text{height}, h)^2}{3\,(\text{radius } mb\text{ of } base)^2 + (\text{height}, h)^2}$$

$$= \frac{\text{height}, h}{4} \times \frac{4 \times \text{radius } cb\text{ of } sphere - \text{height}, h}{3 \times \text{radius } cb\text{ of } sphere - \text{height}, h}.$$

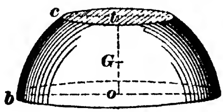


Fig. 35

(33). Spherical zone, Fig. 35.

$$o\mathrm{G} = \frac{ot}{2} \times \frac{2\,(\text{radius } ob\text{ of base})^2 + 4\,(\text{radius } tc\text{ of top})^2 + (\text{height } ot)^2}{3\,(\text{radius } ob\text{ of base})^2 + 3\,(\text{radius } tc\text{ of top})^2 + (\text{height } ot)^2}.$$

(34). Prism, regular or irregular, right or oblique (including the **cube** and other **parallelopipeds**), and **cylinder,** circular or elliptic, etc., regular or irregular, right or oblique. G is the center of the **axis** joining the **centers of gravity** of the two ends.

(34 *a*). A flat body, such as an iron plate, etc, may be treated as a very short cylinder or prism. See (34)

(35). Ungula of a cylinder, circular, or elliptic (provided one of the axes of the ellipse coincides with the oblique cutting plane); right or oblique. Figs. 36 and 37.

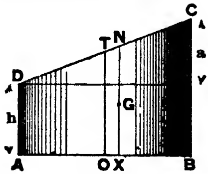


Fig. 36

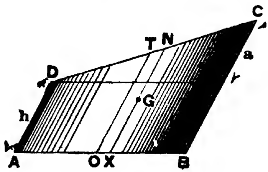


Fig. 37

Let O T be the axis (joining the centers of gravity of the ends), and X N a line drawn parallel to the axis, in the plane, A B C D, passing through the axis and through the uppermost and lowermost points C and D of the oblique cutting plane. Then the position of G in the plane A B C D, is found thus:

$$OX = \frac{OB}{4} \times \frac{a}{2h + a};$$

$$XG = \frac{XN}{2} = \frac{1}{4}\left(2h + a + \frac{1}{4}\,\frac{a^2}{2h + a}\right).$$

(35 *a*), Figs 38 and 39. If the oblique plane C D *meets* the base, A B, at A, so that $h = 0$, while C D remains a complete ellipse or circle, this becomes

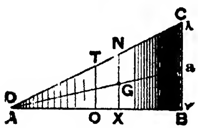


Fig. 38

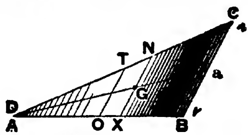


Fig. 39

$$OX = \frac{OB}{4};$$

$$XG = \frac{XN}{2} = \frac{5}{16}a$$

(36). Cone, Figs. 40 and 41, circular, elliptic, etc., right or oblique; or **pyramid,** regular or irregular, right or oblique. The center of gravity G is in the axis O T, drawn from the apex, or top, T, to the center of gravity O of the base; and

$$OG = \frac{OT}{4}.$$

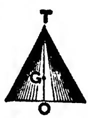


Fig. 40

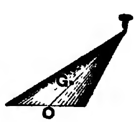


Fig. 41

(37). Frustum of a cone, Figs. 42 and 43, circular or elliptic, right or oblique; or **of a pyramid,** regular or irregular, right or oblique; provided the two ends A B and C D are parallel.

Call the area of the large end A, and that of the small end *a*; and let *h* be the height O Z of the frustum, *measured along its axis.* Then

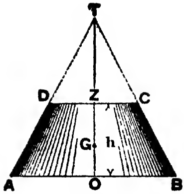


Fig. 42

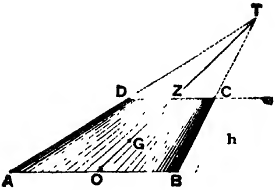


Fig. 43

G is in the axis O Z, which joins the centers of gravity O and Z of the two ends; and its distance from the base, A B, *measured along the axis*, is

$$O\,G = \frac{h}{4} \times \frac{A + 2\sqrt{A\,a} + 3\,a}{A + \sqrt{A\,a} + a}.$$

(37*a*). **In a frustum of a circular cone,** right or oblique, with **parallel ends,** this becomes

$$O\,G = \frac{h}{4} \times \frac{R^2 + 2\,R\,r + 3\,r^2}{R^2 + R\,r + r^2},$$

where R and *r* are the radii of the large and small ends of the frustum respectively.

(38). Figs. 44 and 45. **Frustum,** A B C D, **of a cone,** circular, elliptic etc., right or oblique; or **of a pyramid,** regular or irregular, right or oblique; whether the ends are parallel or not. By rule (36) find the center of gravity N of the *entire* pyramid (or cone, as the case may be) A B T, of which the frustum forms the lower part; and the center of gravity S of the smaller pyramid or cone D C T (= entire pyramid or cone, minus the frustum). Also find the *volume* of each: thus,

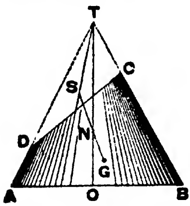


Fig. 44

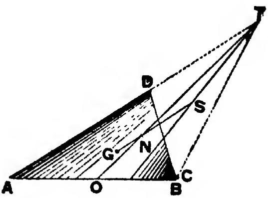


Fig. 45

$$\text{Volume of pyramid or cone} = \frac{\text{area of base} \times \text{perpendicular height}}{3},$$

and

Volume of the frustum A B C D	=	volume of entire pyramid or cone, A B T	−	volume of smaller one, D C T

Then the center of gravity G of the frustum A B C D is in the extension of the line S N; and

$$N\,G = S\,N \times \frac{\text{volume of smaller pyramid or cone, D C T}}{\text{volume of frustum, A B C D}}.$$

(39). **Paraboloid.** G is in the axis, and at one-third of its length from the base

LINE OF PRESSURE.
CENTER OF FORCE OR OF PRESSURE.

Position of Resultant.

133. In ¶¶ 133 to 154 we discuss the position of the resultant, or line of pressure, of a system of parallel forces acting against a surface. For the changes in that position within a structure, due to the action of non-parallel forces, see Arches, Dams, etc., ¶¶ 251, etc.

134. In a system of parallel forces, acting against a surface, the line of pressure, or pressure line, is the position of the resultant of the forces; and the center of force or center of pressure is the point where the pressure line meets that surface against which the forces act.

135. If the lengths of the lines which represent the forces be taken as representing weights, to scale, then the position of the pressure line is the line of gravity (see (5), ¶ 131) corresponding to those weights.

136. Thus, in Fig. 55 (*a*), ¶ 117, if the three forces, *a*, *b* and *c*, be taken as weights, represented to scale by the arrows, *a*, *b* and *c*, respectively, then the resultant R of the three forces occupies the position of the line of gravity of the three weights.

137. Again, in a mass of sand, Fig 61,* with an irregular surface, we may

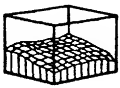


Fig. 61.

suppose the mass to consist of innumerable vertical columns of sand, of different heights, and exerting pressures proportional to those heights. Here, also, the pressure line is the vertical line of gravity of the mass, and the center of pressure against the base of the containing box is the point where said pressure line meets that base.

138. Although we are usually concerned with forces acting against *surfaces*, so that the lines representing the forces form a solid and not merely a surface, yet, in a majority of the cases which occur in civil engineering, we may, for convenience, regard the forces as concentrated in a single plane, and therefore as acting against a mere line.

139. Thus, in the case of an arch, pressing against its skewback, the pressure is ordinarily distributed over all or a considerable part of the bearing surface of the skewback; but we may, for convenience, regard it as concentrated in a single plane, midway between, and parallel to, the two faces of the arch.

140. Similarly, in the case of the water pressure against the back of a dam (or against a small strip of the back, extending from the water surface to the bottom, or to any other depth), the water, of course, presses upon the entire surface of such strip; but we may, for convenience, regard the pressure as concentrated in a vertical plane normal to the back of the dam and meeting it in the vertical axis of the assumed strip.

141. We have just seen (¶¶ 138 to 140) that, when a system of parallel pressures acts against a surface, they may often be assumed to act, in one plane, against a single line—viz., the intersection of that plane with the surface. It also frequently happens that such forces are so distributed along that line that the lines representing the forces are either of equal length or of lengths increasing uniformly from one end of the line to the other.

---

*Following Fig. 60, of Parallel Forces, ¶ 124. Figs. 1 to 45, illustrating Center of Gravity, are numbered independently of the rest of the series of figures relating to Statics.

142. Thus, in the case of water resting upon a horizontal surface, Fig. 62, *the pressure is uniformly distributed, and the diagram*, Fig. (*b*), representing the pressures, is a rectangle bounded by a horizontal line, and its center of gravity, G, is at the center of the figure. Hence, the center of pressure, *c*, is at the center of the line *a b*, or *l*.

Here the unit pressure, *p*, is uniform, and $R = p\,l$.

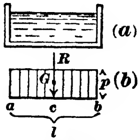


Fig. 62.

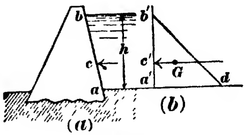


Fig. 63.

143. But when the water presses horizontally against a vertical or inclined surface, *a b*, Fig. 63, the unit pressure increases uniformly from zero, at the water surface, *b*, to a max at the bottom, *a*; and the hor pressures are represented, in Fig (*b*), by the ordinates of the triangle *b' a' d*. Since the resultant passes through the center of gravity, G, of the triangle, the center of pressure, *c*, is at such a depth that $c\,a = \frac{1}{3}\,a\,b$, and $c'\,a' = \frac{1}{3}\,h$. See Rule (**14** *c*) under Center of Gravity.

Here the mean horizontal unit pressure, *p*, is half the maximum horizontal pressure at *a*, and the total horizontal pressure is $= p\,h$.

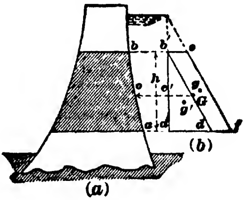


Fig. 64.

144. Again, if we consider only the water pressures against a certain *part*, *a b*, Fig. 64, of the depth of the back of a dam, the diagram, Fig. (*b*), representing the horizontal unit pressures, becomes a trapezoid, composed of a parallelogram *b' f*, and a triangle *b' a' d*, with their centers of gravity at *g* and *g'* respectively; and the center of pressure, *c*, on *a b*, is opposite their common center of gravity (center of gravity of trapezoid), G. If *h* be the vertical depth of the portion considered, then

$$a'\,c' = \frac{h}{3} \times \frac{2\,b'\,e + a'\,f}{b'\,e + a'\,f}.$$

See Rule (**16**) under Center of Gravity.* See also Center of Pressure, under Hydrostatics.

Distribution of Pressure.

145. Conversely, if two surfaces, as those of a masonry joint, are in such contact that the pressure is, or may be regarded as, regularly distributed, and if the position of the resultant is known, the rectilinear figure, representing the distribution of pressure, may be drawn by means of the principles just stated.

146. In Figs. 65 to 68 inclusive, let

o = the center of the joint $a\,b$ between the two surfaces;
R = the total pressure = resultant of all the pressures;
c = point of application of resultant, R;
l = $a\,b$ = the length of the joint;
x = $o\,c$ = the distance of the center of pressure from the center of the joint;
y = $\frac{l}{2} - x = a\,c$ = distance of center of pressure from nearest end of joint;
p = the mean unit pressure = $\frac{R}{l}$;
p_a = the maximum unit pressure;
p_b = the minimum unit pressure.

¶¶ 147 to 154 apply equally whether the surface is horizontal, vertical or inclined, and whether the forces are normal or inclined to it.

147. If x is not greater than $\frac{l}{6}$, or, in any case, if the joint is capable of sustaining tension, as well as compression, we have:

$$\text{Maximum unit pressure} = p_a = p\,(1 + \frac{6\,x}{l});$$

$$\text{Minimum unit pressure} = p_b = p\,(1 - \frac{6\,x}{l}).$$

If x exceeds $\frac{l}{6}$, and if the joint is incapable of resisting tension, see ¶¶ 151, 152, 154.

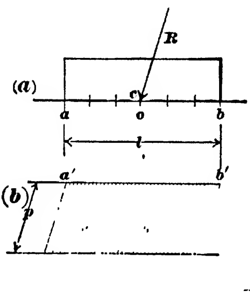


Fig. 65.

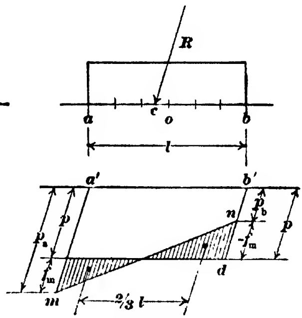


Fig. 66.

148. Demonstration. In Fig. 66, where the parallelogram $a'\,d$ represents the total pressure R as it would be if uniformly distributed along l, we see that the moment of R, about o, which changes the parallelogram $a'\,d$ into the trapezoid $a'\,b'\,n\,m$, is equivalent to a couple (see Couples, ¶ 155, etc) composed of two forces—viz., a pressure, f (not shown) distributed over $o\,a$ and represented by the shaded triangle on the left, and a tension, $-f$, or diminution of pressure, distributed over $o\,b$ and represented by the triangle on the right. The forces, f and $-f$, act through the centers of gravity of these two triangles respectively; and the distance of each of these centers of gravity from the center, o, of the joint, measured parallel to the joint, is $= \frac{2}{3} \cdot \frac{l}{2}$. Hence the distance between the two centers of gravity, meas-

ured parallel to the joint, is $= \frac{2l}{3}$. Let x be the eccentricity, $c\,o$, of R, measured along the joint, and let A_R and A_C (not shown) be the lever arms of R and of the couple, respectively, about the center, o, of the joint. Then, since R is parallel to f and $-f$, A_R to A_C, and x to l, we have:

$$A_R : A_C = x : \frac{2l}{3}.$$

If R is normal to the joint, we have: $A_R = x$; and $A_C = \frac{2l}{3}$.

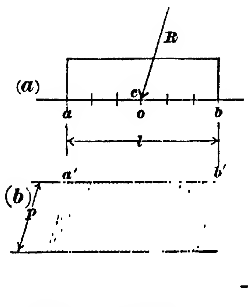


Fig. 65 (repeated).

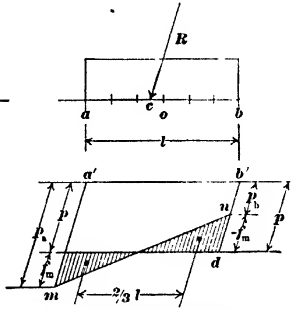


Fig. 66 (repeated).

$$\text{Now} \qquad f = \frac{\text{moment of R}}{\text{arm of couple}} = \frac{R\ A_R}{A_C}.$$

$$\text{Hence,} \qquad f = \frac{R\ x}{\frac{2l}{3}} = \frac{R}{l} \cdot \frac{3x}{2} = p\,\frac{3x}{2}.$$

The *mean* additional pressure on $o\,a$ (or mean tension on $o\,b$) is $= \frac{f}{\frac{1}{2}l}$ and the corresponding maximum additional pressure is

$$f_m = 2\,\frac{f}{\frac{1}{2}l} = \frac{4}{l}\,f = \frac{4}{l}\,p\,\frac{3x}{2} = p\,\frac{6x}{l}.$$

$$\text{Now } p_a = p + f_m = p + p\,\frac{6x}{l} = p\left(1 + \frac{6x}{l}\right)$$

$$\text{and } p_b = p - f_m = p - p\,\frac{6x}{l} = p\left(1 - \frac{6x}{l}\right).$$

149. If, as in Fig. 65, the center of pressure, c, is at the center, o, of the surface, we have $x = o\,c =$ zero, and the pressure, R, is uniformly distributed over the surface.

150. "The Middle Third." If, as in Fig 67, $x = \frac{l}{6}$, — i e, if the resultant, R, of all the forces, meets the surface at the edge of the middle third of that surface, then $p_a = 2\,p$; and $p_b = 0$ See ¶¶ 143 and 148.

151. When, as in Fig. 68 (a), x exceeds $\frac{l}{6}$, — i e, when the center of pressure, c, falls beyond the middle third of the surface of pressure, a portion, $s\,b$, of the surface, is in tension, the maximum tension, p_b, Fig. 68 (b), being $= p\left(1 - \frac{6x}{l}\right)$ as above; maximum pressure $= p\left(1 + \frac{6x}{l}\right)$, and total pressure on $a\,s = \frac{p_a \cdot a\,s}{2} =$ R *plus* the tension in $s\,b$; but if, as usually

happens in masonry, the surfaces are incapable of resisting tension, the total pressure, R, is simply concentrated upon a portion $a\,v$, Fig. 68 (a), of the surface, $a\,v$ being $= 3\,y$.

Then, mean unit pressure on $a\,v = \frac{R}{a\,v} = \frac{R}{3\,y}$;

$$p_a = 2\,\frac{R}{3\,y} = 2\,\frac{p\,l}{3\,y}.$$

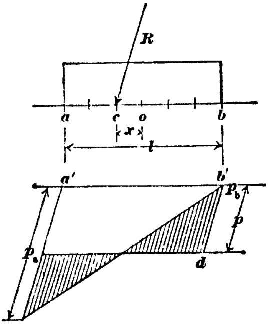


Fig. 67.

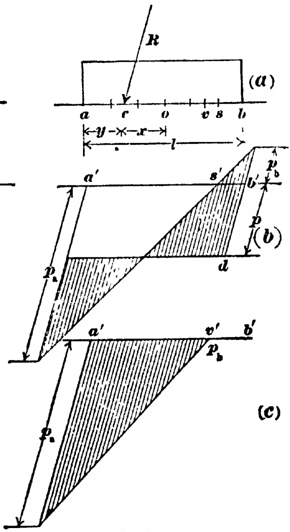


Fig. 68.

152. Hence, in a joint incapable of sustaining tension, Fig. 68 (c), if $p_a = 2\,\frac{R}{3\,y}$ is the maximum permissible unit stress, the distance $a\,c$, from the center of pressure, c, to the nearest end of the joint, must not be less than $y = \frac{2\,R}{3\,p_a}$.

153. If the joint can resist tension, Fig. 68 (b), we substitute, in the equation, $p_a = p\,(1 + \frac{6\,x}{l})$, the value of $x = \frac{l}{2} - y$, and, solving for y, we have

$$y = \frac{2}{3}\,l - \frac{p_a\,l}{6\,p}.$$

154. The influence diagrams, Fig. 69 (see ¶¶ 339, etc., and Trusses, ¶¶ 79, etc.), show the changes in the maximum and minimum unit pressures, p_a and p_b, as the center of pressure, c, recedes from the center, o, of the joint. The diagrams are constructed for a mean unit pressure, p, of 1. If the surfaces of the joint are capable of sustaining tension, every part of the joint always sustains either pressure or tension; and (see dotted lines,

Fig. 69) the maximum unit pressure, $p_a = p\,(1 + \frac{6\,x}{l})$, see ¶ 146, increases proportionally with x; becoming $= 4\,p = \frac{4\,R}{l}$ when c reaches the end, a, of the joint, and when $\frac{x}{l} = \frac{1}{2}$. The max tension, p_b, is then $= 2\,p = \frac{2\,R}{l}$. But if the surfaces are incapable of sustaining tension (see solid lines, Fig. 69), the increase of p_a is proportional to x only so long as $x < \frac{l}{6}$;—*i. e.*, so long as the resultant of all the pressures falls within the middle third of the base $a\,b$. When that limit is exceeded, the maximum unit pressure, p_a, begins to increase more rapidly than does the distance, x, of c, from the center, o, of the joint, the diagram becoming a rectangular hyperbola; so that, if the resultant could be actually applied at the very edge of the joint, the unit pressure there would become infinite.

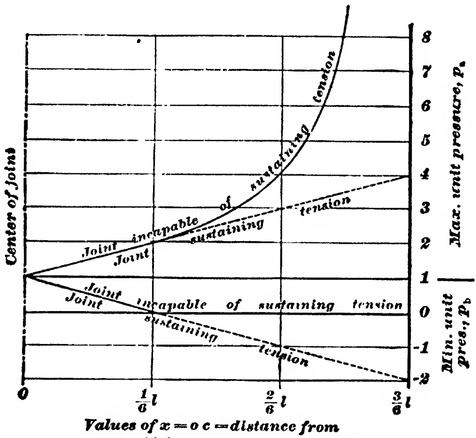


Fig. 69.

COUPLES.

155. Couples. Two equal parallel forces, p and q, or p' and q', Fig. 70,* of opposite sense, are called a couple. A couple has no tendency to move the body † as a whole in any straight line. In other words, the two forces, forming a couple, can have no resultant. Their only tendency is to make the body revolve about its center of gravity, G, and in the **plane of the couple**—*i. e.*, the plane in which the two forces lie. A body with a fixed axis can revolve only in a plane normal to that axis. The actual plane of rotation of a free body depends upon the distribution of mass in the body, and is not necessarily the plane of the couple.

---

* Figs. 70 to 75 are supposed to be seen in perspective, and the forces are supposed to act in the planes shown.

† See foot-note (*), ¶ 1.

156. The moment of a couple is equal to the product of one of the two forces, p or q, into the perpendicular distance, d, between the two forces. Or, in our figures,

$$\text{moment of couple} = p \cdot d = q \cdot d.$$

157. Graphic Representation of Couples. A couple, M or N, Fig. 70, is indicated, in amount, in direction and in sense, by a line, L or L', normal to the plane of the couple, so placed that, looking along it toward that plane, the couple appears positive or right-handed, and of such length as to represent, by scale, the moment of the couple. In Fig. 70, the two couples M and N are of opposite sense. Hence the lines L and L', representing them, project in opposite directions from their respective planes.

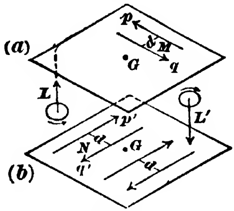


Fig. 70.

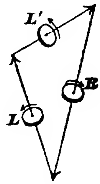


Fig. 71.

158. Composition of Couples. If the lines, L and L', Fig. 71, represent two couples, in accordance with ¶ 157, then the line R, completing the triangle, will, in the same way, represent their resultant or anti-resultant. As drawn, with its arrow *following* those of the other two sides, it represents their *anti*-resultant. For their resultant, the arrow on R, and that indicating the direction of rotation, must be reversed.

159. Equality of Couples. Two couples, M and N, in the same plane, Fig. 72 or Fig. 73, or in parallel planes, Fig. 70, are equal if their moments are equal, whether or not the forces of one of the couples be equal or parallel to those of the other. In Fig. 73, the two couples, M and N, are of like sense; in Figs. 70 and 72, of opposite sense.

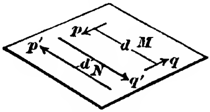


Fig. 72.

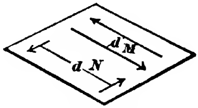


Fig. 73.

160. Since a couple has no resultant (¶ 155), it can have no anti-resultant; *i. e.*, no single force can balance a couple and thus preserve equilibrium. (But see ¶ 168.) To do this requires an equal and opposite couple. Thus, in Fig. 72 the couple M is balanced by the equal and opposite couple N. If, as in Fig. 72, the two couples are in the same plane, and if we find first the resultant of either pair of non-parallel forces, as p and p', and then those of the other pair, q and q', we shall find these resultants equal and opposite, maintaining equilibrium.

161. Any couple, as M, Fig. 73, **may be replaced** by any other equal couple, N, in the same plane or in a parallel plane, and of like sense.

162. If, to a force, P, Fig. 74 (a), **we add a couple,** M, Fig. 74 (b), in the same plane with the force, we may replace the couple, M, by an equal and like couple, N, Fig. (c), composed of the forces, —P and P', each = P, placing —P opposite P, as shown. Then P and —P counteract each other, and we have left only P', equal and parallel to P; and, since Pd = M, we have

$d = \frac{M}{p}$. In other words, the effect of the addition of the couple, M, Fig. (*b*), to the force, P, is simply to shift the line of action of P, parallel with itself, through the distance, *d*. If the couple M is left-handed, as in the figure, P will be shifted to the right (looking in its own direction), and vice versa.

163. Conversely, the force, P′, Fig. (*c*), is equivalent to the combination of force P and couple M, Fig. (*b*).

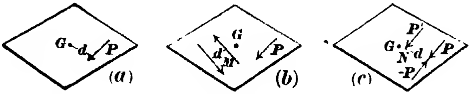


Fig. 74.

164. Again, having only the force P″, Fig. (*c*), if we apply, at a distance, *d*, from P′, the two opposite forces, P and —P, each equal and parallel to P″, we shall thus substitute, for P″, the equal and parallel force, P, and a couple = P*d* = M.

165. Hence, also, the combination of the force P and the couple M, Fig (*b*), is equivalent to the combination of the force P and the couple N, Fig. (*c*).

166. If the moment of the couple, M, Fig. (*b*), or N, Fig. (*c*), be equal and opposite to the moment of the force P about the center of gravity, G, of a body, we have $d = \frac{M}{P}$ = distance from P to G. In other words, the effect of such a couple is to shift the force, P, parallel with itself, to a line passing through the center of gravity, G.

167. Hence, the effect of a force, P, Fig. (*a*), applied to a body at a distance, *d*, from its center of gravity, G, is equivalent to the combined effect of an equal and parallel force, P′, Fig. (*c*), applied at the center of gravity, and a couple (as M, Fig *b*) = P*d*, and of like sense, applied to any part of the body in a plane parallel to P and P′.

168. It will be seen that, although (¶ 160) no single force can balance a couple and establish equilibrium, yet, if a force, P, be so applied that its moment, P*d*, about the center of gravity, G, of the body, is equal and opposite to the moment of the couple, it will counteract the tendency to rotation, due to the couple, and substitute for it a motion of translation only.

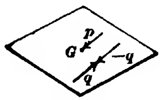


Fig. 75.

169. Thus, in Fig. 75, where the force, *p*, acts through the center of gravity, G, of the body, let a force, —*q*, equal and opposite to *q*, be applied in the same line with it. Then rotation will be prevented, and the body will move * under the action of *p* (= the resultant of the three forces), which acts through the center of gravity, G, of the body. The rotation will similarly be prevented if a force *less* than *q* be applied *farther* from G than *q* is; or if a force *greater* than *q* be applied *nearer* G than *q* is; provided always that the moment of said third force, about G, be equal and opposite to that of the couple *p q*. But in the first case the resultant of the three forces (being always equal to the third force) will be less, and in the second case greater, than *p*.

170. If, to a couple, be added a third force, colinear with one of the forces of the couple, we have the case of two unequal parallel forces of opposite sense. See ¶ 112, under Parallel Forces.

* See foot-note (*) ¶ 1.

170 a. Moments of Couples. The two equal and parallel forces, P and P', Fig A, in opposite directions, form a clockwise couple $= PL = P'L$, where L = the distance between the lines of action of the forces. Taking the moments of the forces about any point, as A, B, C or D, and considering *clockwise* moments as *positive*, we have

$$Pm - P'n = P(m - n) = P's - Pr = P(s - r)$$
$$= Pt + P'u = P(t + u) = P \times 0 + P'L = PL = P'L.$$

In other words, while the moments, Pm, Pr, etc, of the *forces* are *different* for different points, A, B, etc, the moment, $PL = P'L$, of the *couple* is the *same* for *all* points.

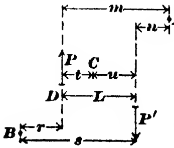


Fig. A.

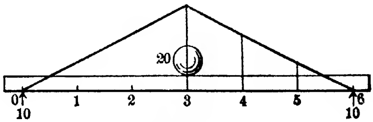


Fig. B.

Thus, suppose the beam, Fig B, to be divided by a vertical section at 4. The right segment is acted upon, at Section 4, by a downward force = load — left reaction = 20 — 10 = 10, and, at 6, by an upward force = right reaction = 10. These forces form a left-hand couple, with moment = 2 × 10 = 20. The left segment is acted upon, at 0, by the left reaction = 10; at 3, by the load = 20; and, at Section 4, by the right reaction = 10. Combining the two reactions, we find their resultant, = 10 + 10 = 20, at 2 (midway betw 0 and 4), forming, with the load, 20, a right-hand couple, with moment = 1 × 20 = 20.

At Section 5, by a similar process, we find couples with moments = 10, and, at Section 3, couples with moments = 30. Thus, altho the moment of a couple is the same, about whatever point moments be taken, yet, in a beam, under a given set of forces arranged in a given manner, the moment varies from point to point; because, in that case, the *couples* are different.

As the tendency to rotation, at any section, is thus seen to be caused by two equal and opposite couples, acting upon the two segments respectively; so the tendency to rotation is resisted by two equal and opposite couples, due to the internal stresses of the material, and acting, in the section, upon the two segments respectively

FRICTION.*

171. When one rough body rests upon another, the projections and depressions, forming **the roughnesses** of their surfaces of contact, **interlock** to a greater or less extent; and, in order to slide one over the other, we must expend a portion of the sliding force, either in *separating* the bodies (as by lifting the upper one) sufficiently to clear the projections, or in breaking off some of the projections and clearing the others.

172. Even the most highly polished flat surface, as *x y*, Fig. 76, is *not* (as it appears to the eye) a *plane*, but is, in fact, a more or less jagged surface, as would appear under a sufficiently powerful microscope; so that the force, *a b*, instead of forming the *apparent* angle, *a b x*, with *one* smooth surface, *x y*, of application, really becomes a series of parallel forces, as *c*, *d* and *e*, which form *other* angles with a number of surfaces, *m m*, *n n*, etc., of application, *inclined* (often in different directions) to the general surface, *x y*, as shown. Among these surfaces may be some, as *m m*, at right angles to the applied force; and, the force *c* will be imparted to them in its original direction, although applied *obliquely* to the *apparent* surface, *x y* In the case of the two forces, *d* and *e*, applied to the surfaces, *n n* and *s s*, if the sliding tendencies along the two surfaces are *equal* and act in *opposition* to each other, the *combined* resistance of the two surfaces, *n n* and *s s*, is directly opposite to the forces, as would be that of a single surface at right angles to those forces.

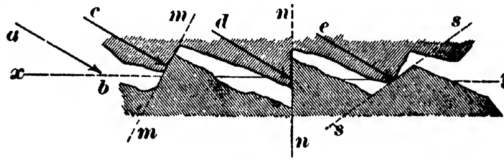


Fig. 76.

173. It is of course entirely out of the question to ascertain the exact resistance of each such microscopic projection in any given case. Instead of this, we find by experiment the *combined* resistance which *all* of the projections, in a given case, offer to the sliding force, and give to this resistance the name of **friction.**

174. Friction always tends to *prevent relative motion of the two bodies between which it acts; i e*, motion of one of the bodies relatively to the other. In doing so, however, it tends equally to *cause* relative motion † between each of those two and *a third, or outside body* Thus, the fric between a belt and the pulley *driven by it* tends to prevent slipping between *them;* but thus tends to make the belt slip on the *driving* pulley, and sets the *driven* pulley and its shaft in motion *relatively to the bearing* in which the shaft revolves. *This* motion is resisted by the fric *between journal and bearing*; and *this* fric, in turn, tends equally to make the *bearing* revolve with the journal, and to make the belt slip on the driven pulley.

175. The fric between two bodies *at rest* relatively to each other is called **static friction,** or fric of rest. That between two bodies *in relative motion* is called **kinetic friction** or fric of motion.

176. The ultimate or maximum static fric between two bodies, as U and L, Fig. 77 (or the greatest fric resistance which they are capable of opposing to any sliding force when at rest), is equal to a force (as that of

---

* "Friction" (meaning rubbing) is a misnomer in so far as it implies that rubbing must take place in order to produce the resistance. For we meet this resistance, not only during rubbing, but also before motion (or rubbing) takes place. "Resistance of roughness" would better express its nature.

† See foot-note (*), ¶ 1.

the wt F) which is just upon the point of making U begin to slide upon L.* Thus fric, like other forces, may be expressed in *weights*, as in lbs.

177. A resistce cannot exceed the force which it resists.† Therefore if F is less than the ult static fric between U and L, the *frictional resistce actually exerted* by them is also less. When F is = the ult fric (and U is therefore on the point of sliding) the actual resistce is = the ult stat fric. If F exceeds the ult stat fric, the excess gives motion to U.

178. If, when a body is in motion, all extraneous forces and resistces are removed or kept in equilib, it moves at a uniform vel. Hence, if the force, F, Fig. 77, is just = the ultimate kinetic fric between U and L, their vel is uniform. If F exceeds this, the excess *accelerates* the vel. If the ult kinetic fric exceeds F, the excess *retards* the vel. Thus **the actual frictional resistce** exerted by two bodies *in relative motion* is = their **ult kinetic fric** = that force (as F) which can just maintain their relative vel uniform.

179. Hence, if the hor surf S upon which L rests, could be made perfectly frictionless, the pres of L against the lug *m* (which would then always be = the *actual* fric resistce between U and L) would also be = their *ult* fric so long as U continued in motion over L, and might therefore be greater or less than or = F; but when U was at rest the pres against *m* would be = F, and less than (or at most just =) the ult fric.

Coefficient of Friction.

180. Since no surface can be made *absolutely* smooth, some separation of the two bodies must in all cases take place in order to clear such projections as exist. Hence the fric is always more or less affected by the amount of the perp pres which tends to keep them together.

181. The ratio of the ult fric, in a given case, to the perp pres, is called **the coefficient of friction** for that case. Or,

$$\text{Coefficient of friction} = \frac{\text{ultimate friction}}{\text{perpendicular pressure}}$$

and

$$\text{Ultimate friction} = \text{perp pres} \times \text{coeff of fric}$$

Thus, if a force F, Fig 77, of 10 lbs, just balances the ult fric between U and L, and if the wt of U (the perp pres in this case since the surf between U and L is *hor*) is 50 lbs, then the coeff of fric between U and L is $= \frac{10 \text{ lbs}}{50 \text{ lbs}}$ = 0.2.

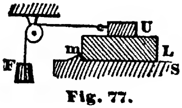


Fig. 77.

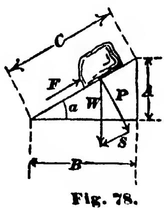


Fig. 78.

182. The coeff is usually expressed decimally, or by a common fraction; but sometimes, as in the case of railroad cars and engines, in lbs (of fric) per ton (of perp pres). Or by the "angle of fric" in degs and mins.

* We here neglect the fric of the string and pulley, and assume that *all* the force of the wt F is transmitted by the string to U.

† If a resisting *force* exceeds the force resisted, the excess is not resistce, but motive force

183. Angle of Friction. In Fig. 78, let W = the weight of the body; P = its pressure normal to the plane, and S = the component tending to slide the body down the plane.

When the angle a is such that the body is just on the point of sliding down the plane, it is called the angle of friction, or angle of repose. The friction F and the sliding force S are then equal.

But $\frac{S}{P} = \frac{F}{P} = \frac{A}{B}$ = coefficient of friction = tan a. Hence F = P tan a = W cosin a . tan a.

184. Frictional Stability. Let R, Fig. 79, be the resultant of all the forces pressing a body against a plane, and N a normal to the plane. If the angle i between R and N exceeds the angle of friction (a, Fig. 78) between the two surfaces in contact, the body will slide on the plane, but not otherwise. If i does not exceed the angle of friction, the entire resultant R will be imparted to the plane and in its own direction, and not merely its normal component V, as would be the case if the surfaces were frictionless.

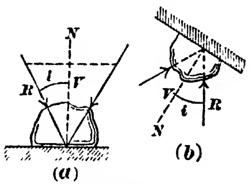


Fig. 79.

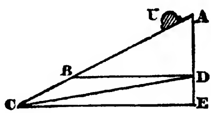


Fig. 80.

185. To find the coeff of kinetic fric, allow one of the bodies, U, Fig. 80, to slide down an inclined plane A C formed of the other one and having any convenient known steepness A C E greater than the angle of fric (¶ 183). Note the *vert* dist A E through which U descends in sliding any dist as A C (A E = A C × sine of A C E); also its actual *sliding* vel in ft per sec on reaching C. Calculate the *vert* dist A D through which it would have to descend *along the plane* (from A to B) to acquire that vel *if there were no fric.*

$$\left(\text{A D} = \frac{\text{velocity}^2 \text{ in ft per sec}}{\text{twice the accel } g \text{ of grav *}}\right).$$

Find D E (= A E — A D), and the hor dist E C corresponding to A C (E C = A C × cosine A C E = $\sqrt{\text{A C}^2 - \text{A E}^2}$). Then

$$\text{Coeff of the average fric in sliding from A to C} = \frac{\text{D E}}{\text{E C}},$$

because, if we let A E represent the *total* sliding force expended (in acceleration and in overcoming the fric), then A D represents the portion of A E expended on *vel*, and D E that expended on *fric*, and, since C E represents the perpendicular pressure (¶ 183),

$$\frac{\text{D E}}{\text{E C}} = \frac{\text{friction}}{\text{prep pres}} = \text{coeff.}$$

186. Or, find sine and tangent of A C E; and the dist A C (= time² in secs × ½ g * × sine of A C E) through which U *would* slide in a given time *if there were no fric.* Measure the dist A B through which it *actually slides* in that time; and find B C = A C — A B. Then

$$\left.\begin{array}{l}\text{coeff of the average}\\ \text{fric in sliding from A to B}\end{array}\right\} = \tan \text{D C E} = \tan \text{A C E} \times \frac{\text{B C}}{\text{A C}}$$

because

* g = about 32.2 ft per second per second.

(1st) A C : A B : B C :: A E : A D : D E

:: the theoretical velocity due to total sliding force : the actual velocity : the frictional retardation

:: the total sliding force : sliding force employed in giving the actual velocity : the friction, or the sliding force required to balance the friction.

And, if A E is = the total sliding force, then E C is = the perpendicular pressure, and

$$\frac{DE}{EC} = \text{the coefficient of friction} = \text{tangent of } DCE.$$

(2nd) Owing to the similarity of the two triangles, A B D and A C E, we have A C : B C :: A E : D E :: $\frac{AE}{EC} : \frac{DE}{EC}$ :: tangent A C E : tangent D C E.

187. In 1831 to 1834, **Gen'l Arthur Morin*** experimented with pressures not exceeding about 30 lbs per sq in; and arrived at the following conclusions in regard to sliding fric where the perp pres is considerably less than would be necessary to abrade the surfs appreciably. These were for a long time generally regarded as constituting the **three fundamental laws of fric.**

1st. The ult fric between two bodies is proportional to the total perp force which presses them together; *i e*, the **coeff is independent of the perp pres** and of its *intensity* (pres *per unit of surf*). Hence

2d. For any given *total* perp pres, **the coeff is independent of the area of surf in contact.**

If upon a hor support we lay a brick, measuring 8 × 4 × 2 ins, first upon its long edge (8 × 2 ins) and then upon its side (8 × 4 ins), we double the area of contact, while the total pres (the wt of the brick) remains the same, and *thus reduce the pres per sq in* by one-half. Consequently (the coeff remaining practically the same) we have only half the *fric per sq in*. But we have twice as many sq ins of contact, and therefore the same *total* fric.

But if we can increase or diminish the area of contact *without affecting the pres per sq in*, the *total* pres will of course *vary as the area*, and the total fric will vary in the same proportion, for the coeff remains the same Thus, if we place two similar sheets of paper between the leaves of a book (taking care not to place both sheets between the same two leaves) and then squeeze the book in a letter-copying press, it will require about twice as much force to pull out both sheets as to pull out only one of them.

3d. Although the coeff of *static* fric between two bodies is often much greater than their coeff of kinetic fric; yet **the coeff of kinetic fric is independent of the vel.**

This applies also (approx) to the *fric*, and hence to the *work* (in *foot*-pounds etc) of overcoming fric through a given *dist*; for then the work (= resistce × dist) is independent of the vel. But in a given *time*, the dist (and consequently the work also) of course varies as the vel.

188. **(a)** Some kinds of surfaces appear to interlock their projections much more perfectly when at rest relatively to each other, than when in even very slow motion; and in some cases the degree of interlocking seems to increase with time of contact. Hence there is often a great diff in amount between fric of rest and fric of motion. Thus, Gen'l Morin found that with oak upon oak, fibres of the two pieces at right angles, the resistce to sliding while still at rest, and after being for "some time in contact," was about one eighth greater than when the pieces had a relative vel of from 1 to 5 ft per sec.

(b) But experience shows that even very slight jarring suffices to remove this diff; and since all structures, even the heaviest, are subject to occasional jarring, (as a bridge, or a neighboring building, or even a hill, during the passage of a train; or a large factory by the motion of its machinery; or in numberless cases, by the action of the wind) it is expedient, in construction, not to rely on fric for *stability* any further than the coeff for *moving* fric will justify. When it is to be regarded as a resistce, which we must provide force for overcoming, it should be taken at considerably *more* than our tabular statement.

* See his "Fundamental Ideas of Mechanics", translated by Jos. Bennett; D. Appleton & Co., New York. 1860.

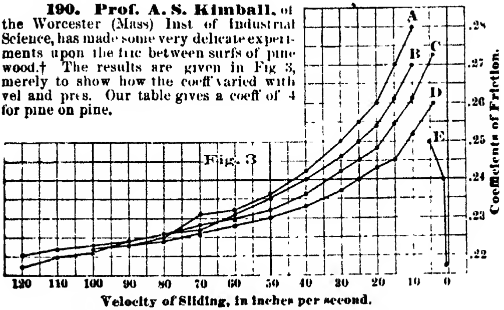
Table of moving friction, of perfectly smooth, clean, and dry, plane surfaces, chiefly from Morin.

Materials Experimented with.	Coeff of Fric; or Proportion of Fric to the Pres.	Angle of Fric. Deg.	Min
Oak on oak; all the fibers parallel to the motion	.48	25	38
" " moving fibres at right angles to the others; and to the motion	.32	17	45
" " all the fibres at right angles to the motion	.34	18	47
" " moving fibres on end, resting fibres parallel to the motion	.19	10	46
" cast iron, fibres at right angles to motion	.37	20	19
Elm on oak, fibres all parallel to motion	.43	23	17
Oak on elm, " " "	.25	14	3
Elm on oak, moving fibres at right angles to the others, and to motion	.45	24	16
Ash on oak, fibres all parallel to motion	.40	21	49
Fir on oak, " " " "	.36	19	48
Beech on oak " " " "	.36	19	48
Wrought iron on oak, fibres parallel to motion	.62	31	47
Wrought iron on elm, " " " "	.25	14	3
Wrought iron on cast iron, fibres parallel to motion	.19	10	46
" " on wrought iron, fibres all parallel to motion	.14	7	58
Wrought iron on brass	.17	9	39
Wrought iron on soft limestone, well dressed	.49	26	6
" " " hard " " "	.24	13	30
" " " " " " " wet	.30	16	42
" " or steel on hard marble, sawed. By the writer ... about	.17	9	39
" " " " " smoothly planed, and rubbed mahogany, fibres parallel to motion	18	10	12
" " " " " smoothly planed wh pine	.16	9	6
Cast iron on oak, fibres parallel to motion	.49	26	6
" " " elm, " " " "	20	11	19
" " " cast iron	.15	8	3[illegible]
" " " brass	15	8	32
Steel on cast iron	20	11	19
Steel on steel By the writer	.14	7	59
Steel on brass	.15	8	32
Steel on polished glass By the writer ... about	.11	6	17
" quite smooth but not polished, on perfectly dry planed wh pine, fibres parallel to motion ... about	.16	9	6
" quite smooth, but not polished, on perfectly dry planed and smoothed mahogany, fibres parallel to motion ... about	.18	10	12
Yellow copper on cast iron	.19	10	46
" " on oak	62	31	48
Brass on cast iron	22	12	25
" on wrought iron, fibres parallel to motion	.16	9	6
" on brass	.20	11	19
" on perfectly dry planed wh pine, fibres parallel to motion ... about	.19	10	46
" " " " " and smoothed mahogany, fibres parallel to motion ... about	.24	13	30
Polished marble on polished marble By the writer ... average	.16	9	6
" " on common brick ... "	.44	23	4[illegible]
Common brick on common brick ... "	.64	32	[illegible]
Soft limestone well dressed, on the same	.64	32	38
Common brick, on well-dressed soft limestone	.65	33	2
" " " " " hard "	.60	31	00
Oak across the grain, on soft limestone, well dressed	.38	20	48
" " " " " hard " " "	.38	20	48
Hard limestone on hard limestone, both " "	.38	20	48
" " " soft " " " "	.67	33	50
Soft " " hard " " " "	.65	33	2
Wood on metal, generally, 2 to .62 ... mean	.41	22	18
Wood, *very smooth*, on the same, generally, .25 to .5 ... "	.38	20	48
Wood, " " on metal, " .2 to .62 ... "	.41	22	18
Metal on metal, very smooth, dry " .15 to .22 ... "	.18	10	12
Masonry and brickwork, dry " .6 to 7 ... "	.65	33	2
" " " with wet mortar ... about	.47	25	30
" " " " slightly damp mortar ... "	.74	36	30
" on dry clay ... "	.51	27	00
" " moist " ... "	.33	18	15
Marble, sawed, on the same; both dry. By the writer.* ... average "	4	21	49
" " " " " both damp ... " ..* ... " "	.55	28	49
" " on perfectly dry planed wh pine. " ..* ... " "	.45	24	14
" " on damp planed wh pine ... " ..* ... "	.6	31	00
" polished, on perfectly dry planed wh pine " ... "	.26	14	35
White pine, perfectly dry; planed; on the same; all the fibres parallel to motion ... about	.4	21	48
" " damp, planed; on the same ... "	.6	31	00

* But after a few trials the surfaces become so much smoother as to reduce the angles as much as 8 m 2° to 5°; the sliding blocks weighing about 30 lbs each.

189. Recent experiments, with much greater variations of pres and of vel, and with more delicate apparatus for detecting slight changes in the coeff, although giving conflicting results,* show that **the three laws** in ¶ 187 are far from correct for surfs moving at high vels, and under great pres; and that they **are only approximately correct** for ordinary vels and pressures; for the coeff is found to vary both with the intensity of the pres and with the vel, as also with the temperature.* But in the cases with which the civil engineer has mostly to deal, slight diffs in the character of the surfs, or even in the dampness of the air, will often cause much greater changes of coeff than those due to any probable changes of pres, vel and temp: so that, within the limits of abrasion, we may generally take Morin's rules as sufficiently correct for such cases.

190. Prof. A. S. Kimball, of the Worcester (Mass) Inst of Industrial Science, has made some very delicate experiments upon the fric between surfs of pine wood.† The results are given in Fig 3, merely to show how the coeff varied with vel and pres. Our table gives a coeff of .4 for pine on pine.



Line A	shows coeffs at diff vels under a pres of			1.58	lbs per sq in.		
" B	"	"	"	1.59	"	"	
" C	"	"	"	1.60	"	"	
" D	"	"	"	1.61	"	"	
" E	"	"	"	4.17	"	"	

It will be seen that at low vels the coeff decreased when the pres per sq in was almost imperceptibly increased; but this diff disappeared as the vel increased. At vels from 4 to 120 ins per sec, the coeff generally decreased as the vel increased; rapidly at first, but more slowly as the vel became greater. This agrees with other recent expts. But at very low vels (.08 to 5 ins per sec) Prof. Kimball found the coeff (line E) *increasing very rapidly with the vel.*

We have made the scale of coeffs large in order to show their variations, which are so slight that they would otherwise be scarcely perceptible. Less delicate expts would have failed to show them at all.

191. (a) In 1878 **Capt. Douglas Galton** and **Mr. George Westinghouse, Jr.,** made careful experiments in England to ascertain the effect of friction in connection **with railway brakes.**‡ The friction and pressure were

* This is not surprising in view of the extent to which the coeff is affected by the nature of the surf. If the shape of the minute projections is such that they fit into each other as perfectly under small pressures as under great ones, and if they are too strong to be broken by the pressures applied, the coeff, as stated in the 1st law, should be independent of the pres. But if high pres wedges the projections of one body more closely between those of the other, the coeff should increase under such pres. On the other hand, if the higher pres breaks down the projections while the lower ones are unable to do so, the coeff should decrease under the higher pres. The particles thus broken off may either act as a lubricant and thus still further *reduce* the fric and its coeff, or (if angular and hard) may *increase* it. Change of area of contact, under a given total pres, may, by affecting the *intensity* of the pres, make changes in the coeff similar to those just mentioned.

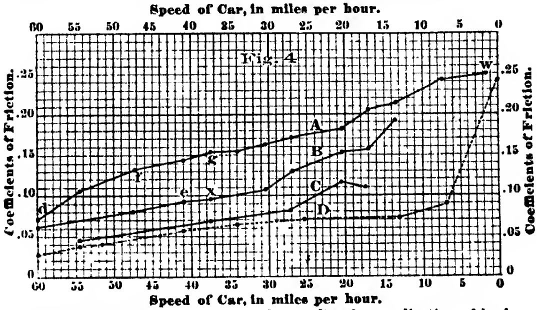
At high vels the roughnesses have not time to interlock as perfectly as at low vels. Hence we should expect a less coeff at high vels. But high vel generally increases the number of projections broken away; and these may either increase or diminish the coeff, as explained above. High vel often *indirectly* affects it by increasing the *temperature.*

† Silliman's Journal (American Journal of Science) March 1876 and May 1877.

‡ See Proc. Instn of Mechl Engrs, London, June and Oct 1878 and April 1879; and "Engineering" London, 1878; vol. 25, pp 432, 469, 490, vol. 26, pp 153, 386, 395.

automatically recorded by means of hydraulic gauges. With cast iron brake-blocks and steel-tired wooden wheels, 43⅝ inches in diameter, they found coefficients about as shown in Fig. 4.

The points in lines A, B and C show the average *brake* coeffs, or coeffs of sliding fric between the tread of a *rolling* wheel and the *brake-block*.



Line A shows brake coeffs obtained immed'y after application of brake
" B " " 5 secs " "
" C " " 15 " " "
" D shows *rail* coeffs or coeffs of sliding fric between the tread of a *sliding* or "*skidding*" wheel (held fast by the brake) and the *rail*.

(**b**) From lines A B and C it appears that the **brake coeff** obtained at a given length of time after the application of the brake was generally **greater at low than at high vels.** But where the vel was maintained uniform the **brake coeff diminished as block and wheel remained longer in contact.** Thus, lines A and B show that at 37½ miles per hour the brake coeff was .154 when the brake was first applied (point *g*), but fell to .096 in 5 secs (*x*). Line A (immed'y after application) shows a higher brake coeff (.132 at *f*) at 47½ miles than line B (5 secs after application) shows at 37½ miles (.096 at *x*).

The diminution of the *rail* coeff with length of time of application of brake was scarcely noticeable.

(**c**) When the brake fric (owing to the reduction of vel and consequent increase of coeff) becomes = the "adhesion" or static fric between the rail and the tire of the rolling wheel, the vel of rotation rapidly falls below that due to the vel of the car; *i e*, **the wheel begins to "skid"** or slide along the rail and in from .75 to 3 secs the rotation of the wheel ceases entirely.

(**d**) **The rail coeff**, line D, **is generally much less than the brake coeff**, lines A, B and C. The pres on the rail (= the wt on a wheel) was about 5000 lbs per sq in, or greatly in excess of the limit of abrasion. That at the brake was about 200 lbs per sq in. A few expts were made with brake blocks having but ⅓ of the usual area of contact, and therefore 3 times the pres per sq in under a given total pres. They failed to show conclusively that this caused any marked change in the coeff.

(**e**) **The rail coeff**, line D, like the brake coeff, **increases as the vel diminishes;** slowly at first, but much more rapidly as the speed becomes less; until, at the moment of stopping, it is generally even greater than the brake coeff just before skidding. With steel tires on *iron* rails at high vels it was somewhat greater than on steel rails, but this diff disappeared as the vel diminished.

(**f**) **Locomotives** overcome resistces = from ⅕ to ⅓ or more of the wt on all the drivers; *i e*, they have a coeff of .33 or more, although the experimental coeff for steel on steel in motion at low pres, is only about .15. But the cases are so diff that a similarity in their coeffs could hardly be expected. The great wt, say from 2 to 6 or even 7 tons, on a driver, is concentrated on a surf (where the wheel touches the rail) about 2 ins long × about ½ inch wide, or = say 1 sq in. The pres per sq in thus greatly exceeds not only that upon which the tables are based, but also the limit of abrasion. Besides, any point in the tread, during

the instant when it is acting as the fulcrum for the steam pres in the cyl, is *stationary* upon the rail. Its fric (miscalled "adhesion") is therefore *static*.

Capt. Galton found that **the coeff of "adhesion"** was independent of the vel, and depended only on the character of the surfs in contact. With a four-wheeled car having about 5000 lbs load on each wheel, it was generally over .20 on dry rails; in some cases .25 or even higher. On wet or greasy rails, without sand, it fell as low as .15 in one case, but averaged about .18. **With sand** on wet rails it was over .20. Sand applied to dry rails before starting gave .35 and even over .40 at the start, and an average of about .28 during motion; but sand applied to dry rails while the car was in motion was apt to be blown away by the movement of the car and wheels.

(g) Owing to the constancy of the coeff of "adhesion" under given conditions of tire and rail, the brake fric necessary to "skid" the wheels in any case was also practically constant for all vels. But at high vels, owing to the *lower* brake *coeff*, a *higher* brake *pres* was reqd to produce this fixed amount of brake *fric*. The skidding also reqd a longer *time* than at low speeds.

192. If the pres is sufficient to produce **abrasion** (indeed, while it is much less) the fric often varies greatly, but no precise law has yet been discovered for estimating it. Rennie gives the following **table of coeffs of fric of dry surfaces, under pressures gradually increased up to the limits of abrasion.** It will be noticed that **in this table the coeff generally increases** with the *intensity* of the pres:

Coeffs of friction of dry surfaces, under pressures gradually increased up to the limits of abrasion. (By G. Rennie, C E.)

Pres in Lbs. per Square Inch	Wrought Iron on Wrought Iron	Wrought Iron on Cast Iron	Steel on Cast Iron	Brass on Cast Iron
32 5	140	.174	166	157
186	.250	.275	.300	225
224	271	292	333	219
336	.312	333	347	215
448	.376	365	354	.208
560	.409	367	.358	[illegible]
672	......	376	403	[illegible]
709	......	.434	......	254
784	......	......	......	.232
821	......	......	......	273

193. (a) Rolling friction, or that between the circumf of a rolling body and the surf upon which it rolls, is somewhat similar to that of a pinion rolling upon a rack. In disengaging the interlocking projections, or in lifting the wheel over an obstacle *o*, Figs 5 and 6, the motive force F, instead of *dragging* one over the other, as in Fig 76, p. 407, acts at the end of a bent lever F R W Figs 5 and 6, the other end W of which acts in a direction *perp* to the contact surf; and in practical cases of rolling fric proper the leverage R W of the resisting wt of the wheel and its load is very much less, in proportion to that (F R) of the force F, than in our exaggerated figs. Hence the force F reqd to *roll* a wheel etc is usually very much less than would be necessary to *slide* it.

(b) There are usually **two ways of applying the force** in overcoming rolling fric: 1st (Fig 5) at the *axis* of the rolling body; as the force of a horse is applied at the axle of a wagon-wheel; or that of a man at the axle of a wheel-barrow: 2d (Fig 6) at the *circumf;* as when workmen push along a heavy timber laid on top of two or more rollers; or as the ends of an iron bridge-truss play backward and forward by contraction and expansion, on top of metallic rollers or balls (p725). In Fig 5 we have, in addition to the rolling fric of the circumf of the wheel on its support, the sliding fric of the axle in its bearing. In Fig 6 we have only rolling fric. but at both top and bottom of the wheel.

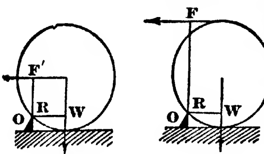


Fig. 5 Fig. 6

(c) When the obstacles *o* are very small, as in the case of cart-wheels on smooth hard roads, or of car-wheels on iron or steel rails, the leverage

(F R) of F becomes, practically, in Fig 5 the *radius*, and in Fig 6 the *diam*, of the wheel; while that (R W) of the resistce is very small. Hence, neglecting axle fric in Fig 5, the force F reqd to overcome rolling fric in such cases is directly as the wt W of and on the wheel, and inversely as the diam of the wheel.

The few expts that have been made upon the coeffs of rolling fric, apart from axle fric, are too incomplete to serve as a basis for practical rules.

(d) The fric (or "**adhesion**") between wheel and rail, which enables a locomotive to move itself and train, or which tends to make a car-wheel revolve notwithstanding the pres of the brake, is a resistce to the *sliding* of the wheel on the rail; and is therefore not rolling but *sliding* fric; *static* when the wheels either stand still or roll perfectly on the rails; and *kinetic* when they slip or "skid".

194. The friction of liquids moving in contact with solid bodies is independent of the pressure, because the "lifting" of the particles of the fluid over the projections on the surf of the solid body, is aided by the pres of the surrounding particles of the liquid, which tend to occupy the places of those lifted. Hence we have, for liquids, no coeff of fric corresponding with that (= resistce ÷ pres) of solids. The resistce is believed to be directly as the area of surf of contact. Recent researches indicate that Resistce = a coeff × area of surf × vel^n, in which both n and the coeff depend upon the vel and upon the character of the surf; and that at low vels $n = 1$, but that at a certain "critical" vel (which varies with the circumstances) n suddenly becomes = 2, owing to the breaking up of the stream into marked counter currents or eddies. The resistance of fluid fric arises principally from the counter currents thus set in motion, and which must be brought into compliance with the direction of the force which is urging the stream forward.

195. Table of coefficients of moving friction of smooth plane surfaces, when kept perfectly lubricated. (Morin.)

Substances.	Dry Soap.	Olive Oil.	Tallow.	Lard.	Lard & Plumbago.
Oak on oak, fibres parallel to motion	.164	....	.075	.067	
" " " fibres perpendicular to motion	....	....	.083	.072	
" on elm, fibres parallel to motion	.136	....	.073	.066	
" on cast iron, fibres parallel to motion	....	....	.080		
" on wrought iron, " " "	....	....	.098		
Beech on oak, fibres " " "	....	....	.055		
Elm on oak, " " " "	.137	....	.070	.060	
" on elm, " " " "	.139				
" cast iron, " " " "	....	....	.066		
Wrought iron on oak, fibres parallel, greased and wet, .256.					
" " " " fibres parallel to motion	.214	....	.085		
" " on elm, " " " "	....	.055	.078	.076	
" " on cast iron, " " "	....	.066	.103	.076	
" " on wrought iron, " " "	....	.070	.082	.081	
" " on brass, fibres " " "	....	.078	.103	.075	
Cast iron on oak, fibres parallel to motion	189				
" " " " " " " " greased and wet, .218	..	.075	.078	.075	
" " on elm, " " " "	....	.061	.077	....	.091
" " on cast iron, with water, .314	.197	.064	.100	.070	.055
" " on brass	....	.078	.103	.075	
Copper on oak, fibres parallel to motion	....	....	.069		
Yellow copper on cast iron	...	.066	.072	.068	
Brass on cast iron	....	.077	.086		
" on wrought iron	....	.072	.081	....	.089
" on brass	...	.058			
Steel on cast iron	....	.079	.105	.081	
" on wrought iron	....	..	.093	.076	
" on brass	...	.053	.056	....	.067
Tanned oxhide on cast iron, greased and very wet, .365	....	.133	.159		
" " on brass	...	.191	.241		
" " on oak, with water, .29					

The launching friction of the wooden frigate Princeton was found by a committee of the Franklin Institute in 1844, to average about .067 or one-fifteenth of the pressure during the first .75 of a second and .022 or one forty-fifth for the next 4 seconds of her motion. The slope of the ways was 1 in 13, or 4 degrees 24 minutes. They were heavily coated with tallow. Pressure on them = 15.84 lbs. per square inch, or 2280 lbs. per square foot. In the first .75 of a second the vessel slid 2.5 inches; in the next 4 seconds 15 feet 6.5 inches; total for 4.75 seconds 15.75 feet.

196. The friction of lubricated surfaces varies greatly with the character of the surfs and with that of the lubricant and the manner of its application. If the lubricant is of poor quality, and scantily and unevenly applied under great pres, it may wear away in places and leave portions of the dry surfs in contact. The conditions then approximate to those of unlubricated surfaces. But if the best lubricants for the purpose are used, and supplied regularly and in proper quantity, so as to keep the surfs always perfectly separated, the case becomes practically one of liquid friction, and the resistce is very small. Between these two extremes there is a wide range of variations (see table, ¶ 197 (*d*)), the coeff being affected by the smallest change in the conditions. Where any degree of accuracy is reqd, we would refer the reader to the experimental results given in Prof. Thurston's very exhaustive work,* devoted exclusively to this intricate subject.

197. (a) Expts by Mr. Arthur M. Wellington upon the **fric of lubricated journals** † gave a gradual and continuous increase of coeff as the vel of revolution diminished from 18 ft per sec (= a car speed of 12 miles per hour) to a stop. This increase was very slight at high vels, but much more rapid at low ones; as in Figs 3 and 4. At vels from 2 to 18 ft per sec the coeff was much less under high pressures than under low ones; but at starting there was little diff in this respect. The coeff increased rapidly as the **temperature** rose from 100° to 120° and 150° Fahr.

(b) Prof. Thurston, also experimenting with **lubricated journals**, ‡ found that at starting, the coeff increased with increase of pres, as it did also when in motion, if the pres greatly exceeded the max (say 500 to 600 lbs per sq in) allowable in machinery. He also found that at high vels the coeff *increased* very slowly (instead of continuing to decrease) as the vel increased.

(c) Prof. Thurston gives the following **approx formulæ for journal friction** at ordinary temperatures, pressures and speeds, with journal and bearing in good condition and well lubricated:

$$\textbf{Coeff for starting} = (.015 \text{ to } .02) \times \sqrt[3]{\text{pres in lbs per sq in.}}$$

$$\textbf{Coeff} \text{ when the shaft is } \textbf{revolving} = (.02 \text{ to } .03) \times \frac{\sqrt[5]{\text{vel in ft per } min}}{\sqrt[2]{\text{pres in lbs per sq in.}}}$$

At pressures of about 200 lbs per sq in:

$$\textbf{Temperature of minimum fric}\text{; in Fahr degs} = 15 \times \sqrt[3]{\text{vel in ft per min}}$$

Caution. The leverage, with which journal fric resists motion, **increases with the diam** of the journal.

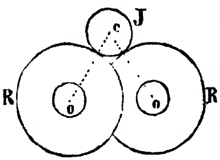
(d) The following figures, selected from a table of experimental results given by Prof. Thurston, merely show **the extent to which the coeff of journal fric is affected by pres, vel and temperature:** and hence the risk incurred in rigidly applying general rules to such cases. In these expts the character of journal and bearing, the lubricant and its method of application, remained the same throughout. Where these vary, still further, and much greater, variations in the coeff may occur.

Steel journal in bronze bearing, lubricated with standard sperm oil.

Temperature. Fahrenheit.	Speed of revolution: 30 feet per *minute*			100 feet per *minute*			500 ft per *min*		1200 ft per *min*	
	Pressures. 200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	200 lbs per sq in	100 lbs per sq in
	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff
130°	.0160	.0044	.125	.0087	.0019	.0630	.0053	.0037	.0065	.0075
90°	.0056	.0031	.094	.0040	.0019	.0630	.0075	.0061	.0100	.0150

* Friction and Lost Work in Machinery and Mill Work. John Wiley & Sons, New York.
† Trans Amer Soc of Civil Engrs New York Dec. 1884.
‡ [illegible]

(e) **Where the force is applied first on one side of the journal and then on the opposite side,** as in crank pins, the fric is less than where the resultant pres is always upon one side, as in fly-wheel shafts because in the former case the oil has time to spread itself alternately upon both sides of the journal.



(f) **Friction rollers.** If a journal J, instead of revolving on ordinary bearings, be supported on friction rollers R, R, the force required to make J revolve will be reduced in nearly the same proportion that the diam of the axle *o* or *o* of the rollers, is less than the diam of the rollers themselves.

Mr. Wellington experimented with a patent bearing on this principle, invented by Mr. A. Higley. Diam of rollers R R, 8 ins; of their axles *o o* 1¾ ins; of the journal *c*, 3½ ins. Here. theoretically,

$$\text{fric of patent journal} = \text{fric of } 3\tfrac{1}{2} \text{ in journal} \times \frac{\text{diam of axles } oo}{\text{diam of rollers RR}} = \frac{1\tfrac{3}{4} \text{ ins}}{8 \text{ ins}}$$

or as 1 to 4.6. Under a load of 279 lbs per sq in, Mr. Wellington found it about as 1 to 4 when starting from rest; and about as 1 to 2 at a car speed of 10 miles per hour.

198. (a) **Resistance of railroad rolling stock.** This consists of rolling fric between the treads of the wheels and the rails (the treads also sometimes *slide* on the rails, as in going around curves); of sliding fric between the journals and their bearings, and between the wheel flanges and the rail heads; of the resistce of the air; and of oscillations and concussions, which consume motive power by their lateral and vert motions, and also increase the wheel and journal frics.

Its amount depends greatly upon the condition of the road-bed and rails (as to ballast, alignment, surf, spaces at the joints, dryness etc); upon that of the rolling stock (as to wt carried, kind of springs used, kind and quantity of lubricant, condition and dimensions of wheels and axles etc); upon grades and curvature; upon the direction and force of the wind; and upon many minor considerations. Experiments give very conflicting results.

(b) During the summer of 1878, **Mr. Wellington** experimented with loaded and empty box and flat freight cars, passenger and sleeping cars, and at speeds varying from 0 to 35 miles per hour. The cars were started rolling (by grav) down a nearly uniform grade of .7 foot per 100 feet, or 36.5 feet per mile, and 6400 ft long. Their resistces were calculated as in ¶ 185. "The rails were of iron, 60 lbs per yd, and the track was well ballasted and in good line and surf, but not strictly first class." The following approx figures are deduced from Mr. Wellington's expts upon cars fitted with ordinary journals:*

Car Resistance in pounds per ton (2240 lbs) of weight of train, on straight and level track in good condition.

Speed of train in miles per hour	Empty cars				Loaded cars			
	Axle, tire and flange	Oscillation and concuss'n	Air	Total	Axle, tire and flange	Oscillation and concuss'n	Air	Total
0	14	0	0	14	18	0	0	18
10	6	.6	.4	7	4	.6	.4	5
20	6	2.7	1.3	10	4	2.	1.	7
30	6	5.3	2.7	14	4	4.7	2.2	11

(c) With the Higley patent anti-fric roller journal, the resistce to *starting* was but about 4 lbs per ton.

(d) About midway in the track experimented upon, was a **curve of 1° deflection angle** (5730 ft rad) 8000 ft long, with its outer rail elevated 3 to 4 ins

* Transactions, American Society of Civil Engineers, Feb 1879.

above the inner one. The rise of the outer rail was begun on the tangent, about 500 ft before reaching the curve. In the first 500 ft of the curve the resistce was greater than that encountered just before reaching the curve, by from .6 to 2.1 (average 1.1) lbs per ton. In the last 500 ft of the curve this excess had diminished to from .2 to .9 (average .6) lbs per ton. Owing to the continuance of the down grade on the curve, the vel increased as the train traversed the curve; but it does not clearly appear whether the decrease in curve resistce was due to the increase in vel, or to the fact that the oscillations caused by entering the curve gradually ceased as the train went on.

(e) **Mr. P. H. Dudley,** experimenting with his **"dynagraph"*** obtained results from which the following are deduced:

Train Resistance in pounds per ton (2240 lbs) of weight of train, including grades.

Description of train			Trip	Average speed. Miles per hour	Average resistance.
Loaded cars	Empty cars	Weight tons (2240 lbs)			
29	2	526	Toledo to Cleveland. 95 miles	20	8.34
37	0	633	Cleveland to Erie 95.5 miles	20	7.67
25	2	458	Erie to Buffalo. 88 miles	20	8.89

"With the long and heavy trains of the L. S. & M. S. Ry, of 600 to 650 tons, it reqd less fuel with the same engine to run trains at 18 to 20 miles per hour than it did at 10 to 12 miles per hour", owing to the fact that at the higher speeds steam was used expansively to a greater extent, and hence more economically.

199. The work, in ft-lbs, reqd to overcome fric through any dist, is = the fric in lbs × the dist in ft. In order that a body, started sliding or rolling freely on a hor plane and then left to itself, may do this work; i e, may slide or roll through the given dist, its kinetic energy (= its wt in lbs × its vel² in ft per sec ÷ 2*g*†) must = the first-named prod. Conversely, **the dist in ft** through which such a body will slide or roll on a hor plane, is

$$= \frac{\text{its kinetic energy in ft-lbs, at start}}{\text{fric in lbs}}$$

$$= \frac{\text{wt of body in lbs} \times \text{initial vel}^2 \text{ in ft per sec}}{\text{wt of body in lbs} \times \text{coeff of fric} \times 2g\dagger} = \frac{\text{initial vel}^2 \text{ in ft per sec}}{\text{coeff of fric} \times 2g}$$

The time reqd, in secs, is $= \frac{\text{dist in ft, so found}}{\text{mean vel, in ft per sec}} = \frac{\text{dist in ft}}{\frac{1}{2}\text{ initial vel in ft per sec}}$

Suppose two similar locomotives, A and B, each drawing a train on a level straight track; A at 10 miles, and B at 20 miles, per hour. The total resistce of each eng and train (which, for convenience, we suppose to be independent of vel) is 1000 lbs. Hence the force, or total steam pres in the two cyls reqd to balance the fric and thus maintain the vel, is the same in each eng. In traveling *ten miles* this force does the same amount of work (1000 lbs × 10 miles = 10000 pound-miles) in each eng, and with the same expenditure of steam in each; although B must supply steam to its cyls *twice as fast* as A, in order to *maintain* in them the same pres. In *one hour* the force in A does 10000 lb-miles as before, but that in B does (1000 lbs × 20 miles =) 20000 lb-miles, and with twice A's expenditure of steam.

But in fact the resistce of a given train is much greater at higher vels. See table, ¶ 198 (*b*) And even if we still assumed the resistce to be the same at both vels, B must exert more force than A in order to *acquire* a vel of 20 miles per hour while A is acquiring 10 miles per hour.

* An inst for measuring the strain on the draw-bar of a locomotive, or the force which the latter exerts upon the train.
† g = acceleration of gravity = say 32.2 : $2g$ = say 64.4.

200. Natural Slope. When granular materials, as sand, earth, grain, etc., are deposited loosely, as when they are shoveled from a cutting or dumped from a cart, the angle, formed between a level plane and the sloping surface of the pile of material, is called the natural slope. This angle depends upon the friction and adhesion between the separate particles of the material, and often varies, in one and the same material, from time to time, with changes in weather conditions, etc., especially with dampness.

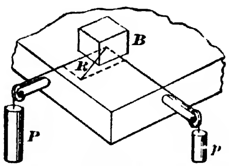


Fig. 85.

201. Any force, p, Fig. 85, acting upon a body, B, will suffice to move the body (see foot-note (*), ¶ 1), provided it exceeds the sum, S, of all resistances, including friction between B and the surface upon which B rests, or if it forms, with any other force or forces, P, a resultant, R, greater than S.

If, before the application of p, the body is already in uniform motion, P is = S; and any force, p, however small, will suffice to change the direction of motion. This accounts for the ease with which a *revolving* shaft may be slid longitudinally in its bearings, and for the fact that a cork may be more easily drawn if we first give it a twisting motion in the neck of the bottle.

LEVERS.

202. Classes of Levers. Figs 86. Levers are classed according to the relative positions of "power," * "weight" * and fulcrum, as follows:

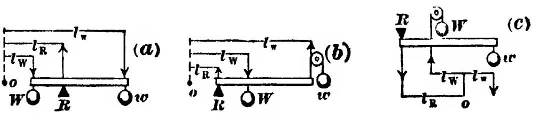


Fig. 86.

Fig. (*a*), Class 1. Fulcrum R between power w and weight W;
" (*b*), " 2. Weight W between power w and fulcrum R;
" (*c*), " 3. Power W between weight w and fulcrum R.

In class 2, the leverage of the power is necessarily greater than that of the weight. In class 3, vice versa.

203. In Fig. 86, taking the moments of the forces about any point at pleasure, as o, we have, for equilibrium:

$$\text{Fig. } (a),\ W \cdot l_W - R\, l_R + w \cdot l_w = 0;$$
$$\text{Fig. } (b),\ W \cdot l_W - R\, l_R - w \cdot l_w = 0;$$
$$\text{Fig. } (c),\ W \cdot l_W - R\, l_R + w \cdot l_w = 0.$$

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* When levers are used for lifting weights or for overcoming other resistances, the force applied is called the "power," and the resistance to be overcome is called the "weight."

204. Compound levers, Fig. 87, may be used where there is not room for the arms of a single lever of sufficient length. In a compound lever, neglecting friction,

$$\frac{\text{weight}}{\text{power}} = \frac{\text{product of lengths of power arms}}{\text{product of lengths of weight arms}} = \frac{8 \times 10 \times 2}{2 \times 1 \times 4} = \frac{160}{8} = 20$$

The three levers of Fig. 87, taken separately and beginning at the power end, give:

$$\frac{\text{weight}}{\text{power}} = \frac{8}{2} = 4, \quad \frac{10}{1} = 10, \quad \frac{2}{4} = \frac{1}{2},$$

and $4 \times 10 \times \frac{1}{2} = 20$, as before.

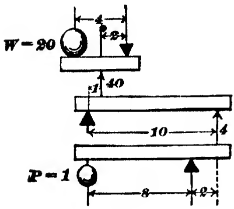

Fig. 87.

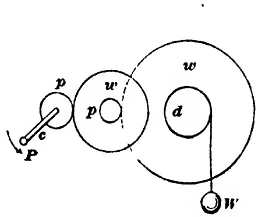

Fig. 88.

205. Toothed or Cog Gearing. Wheels and Pinions. Fig. 88. These are a series of continuous compound levers. The power is usually applied to a crank, *c*, and the weight is attached to a drum, *d*. The larger wheel, *w*, on a given shaft, is called the wheel; the smaller one, *p*, the pinion.

Let c = the radius of the crank, d = that of the drum, m = the product of the radii of the pinions, and n = the product of the radii of the wheels. Then, neglecting friction,

$$\frac{\text{weight}}{\text{power}} = \frac{c \, . \, n}{m \, . \, d}.$$

Instead of the several radii, we may of course use the corresponding diameters or circumferences; and, as the teeth are necessarily of equal "pitch" (length, measured along the circumference), the number of teeth on a wheel or pinion is usually taken instead of the radius.

When the ratio, $\frac{\text{weight}}{\text{power}}$, is great, the system is said to be of **high gear.** When that ratio is small, we have **low gear.**

Compound levers and gearing are used for converting low into high velocity, as well as for lifting great weights by means of small powers. When used for increasing the velocity, the positions of power and of weight are the reverse of those shown in Figs. 87 and 88.

206. Whenever the power and the weight balance each other, either in a single lever, or in a connected system of levers or leverages, of any kind whatever, then if we suppose them to be put into motion about the fulcrum, their respective velocities will be in the same proportion or ratio as their leverages; that is, if the leverage of the power is 2, 5, or 50 times as great as that of the weight, the power will move 2, 5, or 50 times as fast as the weight. Therefore, by observing these velocities, we may determine the ratio of the leverages. The weight and the power are to each other, therefore, *inversely* as their velocities, as well as inversely as their leverages.

207. No mechanical advantage is gained by merely increasing the *length* of a lever, as by curving it, as at *abo*, Fig. 4, ¶ 13, or by giving it an inclination to the line of action of the power, P, as at *o m*, *o u* or *o n*.

208. Thus, in Fig. 89, representing a bent lever, $a\,f\,b$, the *length* of the *lever*, or of any of its members, as $f\,b$, must not be confounded with the *arm* or *leverage* of the *force* acting upon the lever. These may or may not be equal. Thus, the member $f\,b$ is much longer than the member $f\,a$; yet, if the arms, $f\,a$ and $f\,c$, of the forces or weights are equal, the weights n and m must also be equal in order to insure equilibrium

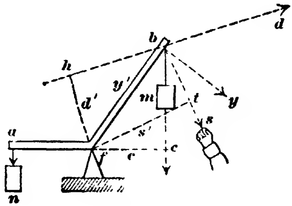

Fig. 89.

209. If the weight m be removed, a force c, or s, or y, or d, with leverage $= c', s', y', d'$, respectively, may be applied at any point, as b, to balance the moment of n. In any case this force must be such that

$$\text{force} \times \text{its leverage} = n\ a\ f.$$

Hence,

$$\text{force} = \frac{n\ a\ f}{\text{leverage of force}}.$$

210. Hence also the force required is *least* when, as at y, it is *perpendicular* to the length of the member $f\,b$; for the leverage (which evidently *cannot exceed* $f\,b$) is then greatest. The force required increases as it deviates in either direction from the line $b\,y$ (perpendicular to $f\,b$) and approaches more nearly to the direction of $f\,b$ itself; for its leverage then constantly decreases. No force, however great, could balance the moment of n about f, if applied in the direction $f\,b$, or $b\,f$; for such a force would have no moment about f.

211. Similarly, in Fig. 90, the moment, about a, of a load W, placed at b is $=$ W . $a\,c$, or the same as if it were placed at c, and not $=$ W . $a\,b$.

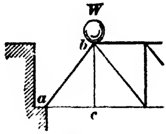

Fig. 90.

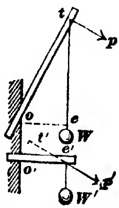

Fig. 91.

212. In Figs. 91, also, the moments W. $o\,e$ and W'. $o'\,e'$, of the equal weights, W and W', are equal. But if forces, p and p', be applied in directions perpendicular to the longer beam, $o\,t$, the leverage $o\,t$ of p becomes about 6 times that ($o'\,t'$) of p'. Hence a force, p, applied at t, has about the same bending moment as a parallel force $= 6\,p$, applied at e'.

STABILITY.

213. Stability. Figs. 92. If the resultant, R, Fig. (*a*), of the force P and weight W, falls beyond the base, as shown, then the overturning moment of P, Fig. 92 (*b*), about the toe *n*, will exceed the moment of stability of the weight W about the same point, and the body will overturn about *n*. If not, it will stand.

214. Assuming stability against overturning, the body will slide if the horizontal component, *h*, of R, Fig. 92 (*a*), exceeds the frictional and other resistances.

215. In practice, the toe, *n*, or the ground beneath it, might yield if the stone revolved upon it, or if R fell near *n* (see ¶¶ 145, etc.); but this is a question of strength of materials. Cement, clamps, etc., between the base and the ground, would add a third force, and thus change the problem.

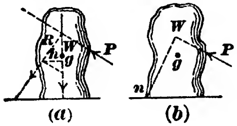

Fig. 92.

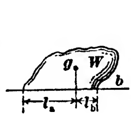

Fig. 93.

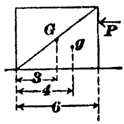

Fig. 94.

216. Owing to the greater leverage, l_a, Fig. 93, of W about *a*, the moment of stability is much greater about *a* than about *b*.

217. In Fig. 94, let G = 2 lbs.; *g* = 1 lb.; leverages = 3, 4 and 6 ft., as shown. Then the moment of stability of the rectangular body, G, against a horizontal force, P, is = 3 G = 3 × 2 = 6 ft.-lbs.; and the moment of the lower triangular body, *g*, is = 4 *g* = 4 × 1 = 4 ft.-lbs.; so that, although the larger body weighs twice as much as the smaller one, yet its moment of stability is only 1.5 times as great.

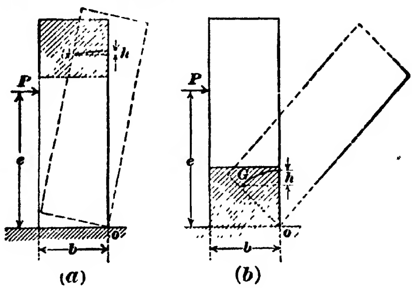

Fig. 95.

218. Work of Overturning. In Figs. 95 (*a*) and (*b*), let the shaded portion of each figure be of lead, and the remainder of wood, and let the center of gravity of the entire body, in each case, be at G. Then, since the weight, W, is the same in both cases, as is also its leverage of stability, about o, $= \frac{b}{2}$, the moment of stability, $= \frac{1}{2} b.W$, is the same in both cases, as is also the force, P, required to balance that moment when applied at a given elevation, *e*. As overturning proceeds, the weight, W, remaining unchanged, the leverage and moment of stability, and the overturning moment required, decrease, becoming = 0 when the bodies reach the positions

shown by the dotted lines. If the elevation, e, remains constant, the force, P, required for overturning, decreases in the same proportion as the leverage, etc.

219. But in order that the bodies may be overturned by the force of gravity alone, they must be brought into the positions shown by the dotted lines. This requires that the weights of the bodies be lifted through a height = the distance, h, through which their centers of gravity, G, are raised. Hence

$$\text{work of overturning} = W . h.$$

Since h is greater in Fig (*b*), the work of overturning is greater in that case.

In civil engineering we are generally concerned with the amount of the *force* which will *begin* overturning, rather than with the amount of *work* required to *complete* the overthrow

220. Stability against overturning is of course affected, and may be increased, by forces other than the weight of the body itself. Thus, the stability of a bridge pier is ordinarily increased by the weight of the bridge itself if this be brought upon the pier symmetrically. Otherwise the weight of the bridge may either increase or diminish the stability of the pier, according to circumstances.

221. The coefficient of stability, in any given case, is the ratio of the moment of stability to the overturning moment. Or,

$$\text{Coefficient of stability} = \frac{\text{moment of stability}}{\text{overturning moment}}.$$

222. Let the weight, W, of the stone in Fig 96 be 10 lbs., G its center of gravity, and $o\,g$ = 2 feet. Then the moment of the weight about o, or the moment of stability about o, is 10 × 2 = 20 ft.-lbs.; and, if $o\,n$ = 5 feet, a force $P = \frac{20}{5} = 4$ lbs., will just hold in equilibrium the moment of the weight, so that, except at the corner, o, no pressure will be exerted upon the base $o\,m$, although the stone remains in contact with the base. If the force P exceeds 4 lbs, the stone will begin to turn about o. If P is less than 4 lbs., the stone will exert a pressure upon the base $o\,m$.

Let the stone be supported at o and at m only. The leverage of the supporting force R, at m, is = the length $o\,m$ of the base, = l. Let P = 1 and base $o\,m$ = 4.5 ft. Then, for equilibrium,

$$W . o\,g - P . o\,n - R . o\,m = 0;$$

or,

$$20 \text{ ft.-lbs} - 1 \times 5 = 1.5 \times R;$$

or,

$$R = \frac{20 - 5}{4.5} = 3.33 \ldots \text{ lbs.}$$

In other words, a vertical upward force, R, of 3.33 . . . lbs., at m, will maintain equilibrium.

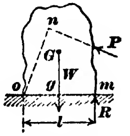

Fig. 96.

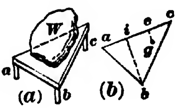

Fig. 97.

223. In Fig. 97 (*b*), let g be the center of gravity of the load W and the table, combined, Fig 97 (*a*). Then, upward reaction of $b = \frac{W . e\,g}{i\,b}$. Those of a and c may be similarly found.

224. In Fig. 98, let h be the horizontal force exerted at the crown by the left-hand half of the arch, against the half-arch shown, and e its leverage about o. Let W be the weight of the half-arch with its spandrel, acting as a single rigid body, and l its leverage about o. Then, for equilibrium, we have

$$h \cdot e = W \cdot l; \text{ or } h = \frac{W \cdot l}{e}.$$

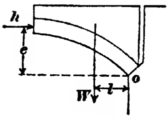

Fig. 98.

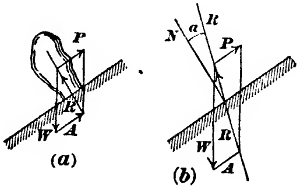

Fig. 99.

Stability on Inclined Planes.

225. Stability on Inclined Planes. Fig. 99. Here, as in ¶ 213, if the resultant, R, of the force P and weight W, falls beyond the base,—*i. e.*, if the overturning moment exceeds the moment of stability,—the body will overturn. If not, it will stand.

The force, P, in any given direction, required to prevent overturning, is = the anti-resultant, A, of weight W and reaction R; and reaction R = anti-resultant of force P and weight W.

226. Neglecting friction, as in Fig. 99 (a), R will be normal to the plane. Taking friction into account, Fig. 99 (b), R may form, with a normal, N, to the plane, an angle, a, not exceeding the angle of friction between the body and the plane. R may be either uphill or downhill from N.

227. In Fig. 100, the body B has less stability against overturning about its toe, a, than has the similar body, A, when the force, n, tends to upset it downhill; but a greater stability than A against overturning about c under the action of a force tending to upset it uphill.

228. The body C, which would upset if upon a horizontal base, would be stable against overturning if placed upon an inclined plane, as at D. Assuming $a\,o = t\,c$, a given upward vertical force would have the same overturning

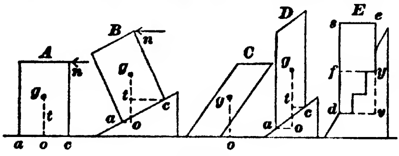

Fig. 100.

moment, whether applied at a or at c. But a given horizontal force, applied at any given height, as at g, has a greater leverage, $g\,o$, when pushing downhill than when pushing uphill. In the latter case its leverage is only $g\,t$.

229. Structures built upon slopes are liable to slide. This may be obviated by cutting the slope into horizontal steps, as at $d\,y$, Fig. E; but the vertical faces of such steps break the bond of the masonry; and, moreover, the joints being more numerous, and the mortar therefore in greater quantity, on the deeper side, $s\,d$, than on the shallower uphill side, $e\,y$, the structure is liable to unequal settlement, the downhill side settling most and tending to split away from the uphill portion, as might be the case with a founda-

tion firm in some parts and compressible in others. Hence, when circumstances permit, it is preferable to level off the foundation, as at $d\,v$; or, if the structure has to withstand downhillward pressures, v should be lower than d, and the courses of masonry laid with a corresponding inclination.

THE CORD.

230. The Cord. Figs 101 (*a*) and (*b*) and 102 (*a*) and (*b*). In ¶¶ 230 to 239 we deal with cords supposed to be perfectly flexible, inextensible, frictionless, weightless and infinitely thin.

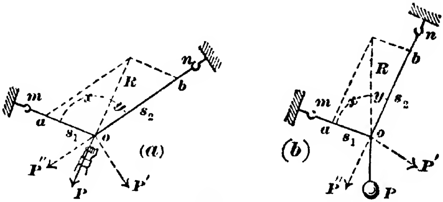

Fig. 101.

231. Let P be the external force applied to the cord at the knot or pin, o, and let R be the resultant of the stresses, s_1 and s_2, or $o\,a$ and $o\,b$, in the two segments, $o\,m$ and $o\,n$, of the cord. Then, for equilibrium, R must be equal to and colinear with P.

232. Knowing the amount of P (= R), the tensions s_1 and s_2 may be found by means of ¶ 36; and, vice versa, given s_1 and s_2, we may find R (= P) by ¶ 35. Or see ¶ 40.

233. If, as in Figs. 101 (*a*) and (*b*), the force P be applied to the cord, at o, by means of a fixed knot, incapable of sliding along the cord, so that the segments, $o\,m$ and $o\,n$, of the cord, are of fixed lengths, and the angle, $x + y$, between them, of fixed magnitude, then the force may be applied in any direction, as P or P′, passing between the two segments of the cord; and the components, s_1 and s_2, will be equal only when R (P produced) forms equal angles, x and y, with the two segments of the cord. If the direction of the force, as P″, coincides with either segment, as $o\,n$, of the cord, that segment transmits the entire force, P″, and the other segment none.

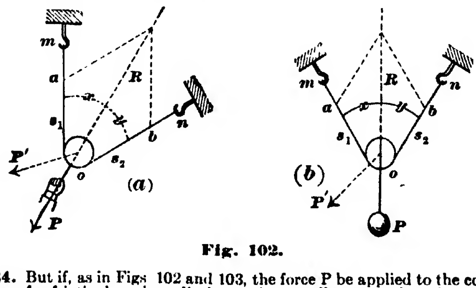

Fig. 102.

234. But if, as in Figs 102 and 103, the force P be applied to the cord by means of a frictionless ring, slip-knot, pin or pulley, etc., then, for equilibrium, the two stresses, s_1 and s_2, must be equal, as must also the two angles, x and y; and, if we suppose the direction of the force P to be changed, as to P′, the pin and the cord will readjust themselves, as indicated by the dotted lines in Fig. 103, until the pin finally comes to rest at that point, o', where the angles, x' and y', are equal, and also the stresses, s_1' and s_2'.

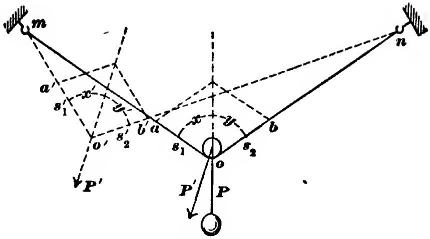

Fig. 103.

235. Even though the pin or pulley be rigidly fixed to some external object, as at o, Fig. 104, yet, if there is no friction at its axle, or between it and the cord, the components, s_1 and s_2, will still be equal, and their resultant, R, will bisect the angle, $x + y$, between them. In other words, the angles, x and y, will be equal.

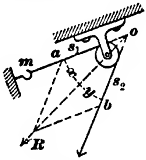

Fig. 104.

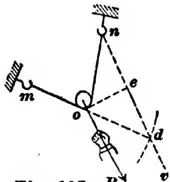

Fig. 105. P

236. When the pin is movable, Figs. 102 and 103, to find the position, o, Fig. 105, which it will assume. From the end, n, of one of the segments, $o\ n$, of the cord, draw $n\ v$ parallel to P. From the end, m, of the other segment, with radius $= m\ o + o\ n$, $=$ length of cord, describe an arc, cutting $n\ v$ in d. Bisect $n\ d$ in e. Draw $e\ o$ normal to $n\ v$, intersecting $m\ d$ in o. Then o is the required point.

237. Whether o be a fixed knot or a movable pin or pulley, it is always in the circumference of an ellipse whose foci are at the ends, m and n, of the cord.

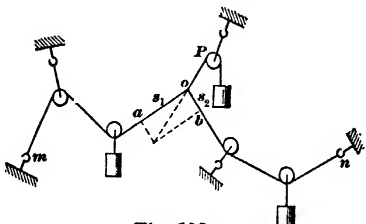

Fig. 106.

238. From the foregoing it follows that, if o, Fig. 106, be a fixed knot, and if the other pins or pulleys, etc., are frictionless, the stress $a\ o$, or s_1, will be transmitted uniformly throughout the left segment of the cord, from o to its end at m; and $b\ o$, or s_2, throughout the right segment, from o to n.

239. Caution. Note that, in Fig. 107 (*b*), the stresses in all the cords are twice as great as the stresses in the corresponding cords in Fig. 107 (*a*), although each Fig. shows a load = 4 suspended from the pulley. Thus, if the weight be that of a man, hanging by the rope, and if the rope, in Fig. (*a*), be just sufficiently strong to hold, it will break if he gives one end of the rope to another man to hold, or makes it fast, as in Fig (*b*).

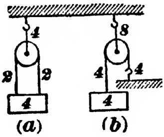

Fig. 107.

The Funicular Machine.

240. When the angles, x and y, Figs. 101, etc., are very great, a very small force, P, will balance a very great stress, s_1 or s_2, in the cord. When $x = y = 90°$, we have $\cos x = \cos y = 0$, and $s_1 = s_2$ = infinity, however small P may be If a line, $m\ n$, joining the ends of the cord, is horizontal or inclined, the weight of the cord itself acts as a force P. Hence

"There is no force, however great, can stretch a cord, however fine, into a horizontal line that shall be absolutely straight."

241. The funicular machine takes advantage of the fact that, when the total angle, $x + y$, between the two segments of the cord, approaches 180°, a small force, P, may balance great stresses, s_1 and s_2 Thus, in Fig 108, let W represent a heavy boat (seen in plan) which is to be hauled ashore. One end of a rope being made fast to the bow of the boat, the rope is passed around one smooth post, n, to another, m, around which it is given one or more whole turns; and a man stands at the end, e, to take in the slack; while others, taking hold of the rope between m and n, pull it, in the direction of P, into a position $m\ o\ n$. If the two angles, x and y, are equal, the component in the segment $o\ n$ exceeds P, so long as the angle x exceeds 60°, and a pull, equal to this component (except in so far as it is reduced by the rigidity of the rope and by its friction against the post n), is exerted upon the boat at W, drawing it a short distance up the beach. The rope is then straightened again, from m to n, by taking in the slack at e, and the operation is repeated as often as may be necessary.

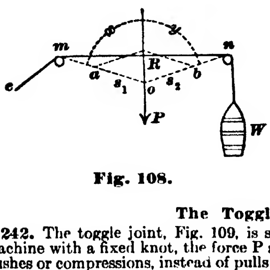

Fig. 108.

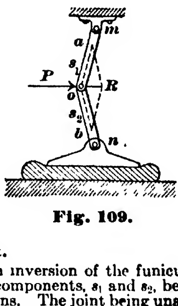

Fig. 109.

The Toggle Joint.

242. The toggle joint, Fig. 109, is simply an inversion of the funicular machine with a fixed knot, the force P and the components, s_1 and s_2, being pushes or compressions, instead of pulls or tensions. The joint being unable to move along the arms, the force P may be applied in any direction at pleasure, but it is usually exerted in a directio' forming approximately equal angles with the two arms.

The Pulley.

243. Figs. 110 show the relations of stresses and weights in several arrangements of fixed and movable pulleys. Thus, in (*a*), 1 lb. balances 1 lb., in (*b*) 2 lbs., in (*c*) and in (*d*) 4 lbs. In each case, if the bodies or weights be set in motion, their velocities are inversely as their masses. See ¶ 206.

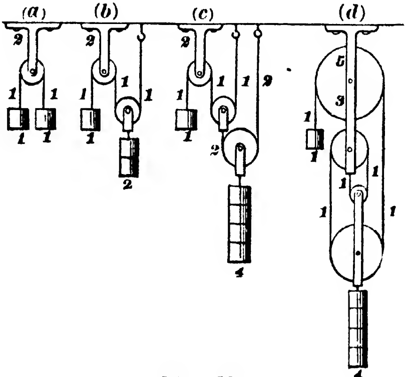

Fig. 110.

244. The simple pulley, Fig. 110 (*a*), is used simply for convenience of changing direction of stress, for the forces at the two ends of the cord are equal; but in the compound pulley, Figs 110 (*b*), (*c*), (*d*), a small force (the "power"), moving rapidly, at one part of the rope, balances a greater force (the "weight"), moving slowly, at another part Hence, the compound pulley is used for the purpose of overcoming great resistances slowly, by means of small forces, moving rapidly.

245. To set such a system in motion * (*i e*. to raise the "weight") requires that the equilibrium be disturbed by making the "power" exceed the stress in the cord due to the "weight" But the motion, once generated, will continue indefinitely if the "power" is made sufficiently greater than the "weight" to balance the resistances of friction, etc.

The Loaded Cord or Chain.

246. In Figs. 111 the principle of the cord polygon, ¶¶ 86, etc., is applied to the case of a flexible cord or chain, sustaining four loads, $p_1 \ldots p_4$, at fixed points, and exerting a horizontal † pull, H, at its lower end, and an inclined pull, R, at its upper end. The loads, $p_1 \ldots p_4$, are represented by the vertical line, 0–4, Fig. 111 (*a*); the horizontal pull, H, by 0–*c*; the amount and direction of the inclined pull, R, at the upper end of the cord, by 4–*c*, and the tensions in the segments, 1–2, 2–3 and 3–4, by the rays, 1–*c*, 2–*c* and 3–*c*, respectively.

247. The horizontal tension, H (= the horizontal component of the tension in each segment), is uniform throughout the cord; but the vertical component of the tension in any segment is equal to the sum of the loads between that segment and the pulley, *m*. Thus, the vertical component

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* See foot-note (*), ¶ 1.

† See foot-note (*), ¶ 249.

(0–2, Fig. a) of the tension, c–2, in segment 2–3, is $= p_1 + p_2$; that in segment 3–4 is $0–3 = p_1 + p_2 + p_3$, etc.

248. If all the loads (including W) be increased in the same proportion, as indicated by the dotted lines in Fig. 111 (a), or diminished in the same proportion, the new triangles, c' 4′ 0, etc., Fig (a), will be similar to the old, and the profile of the cord, Fig. (b), will remain unchanged, although the stresses in its segments will of course be increased or diminished in the same proportion.

249. In Fig. 111 we make the weight, W, which is necessarily equal to the horizontal* pull, H (see The Cord, ¶¶ 230, etc.), equal also to the

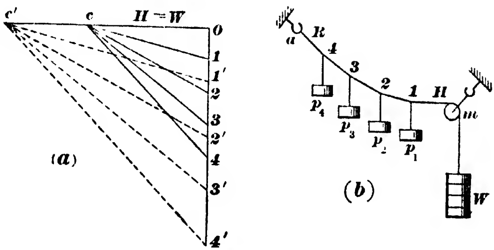

Fig. 111.

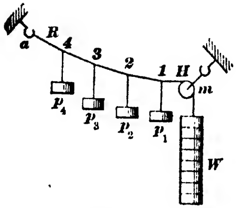

Fig. 112.

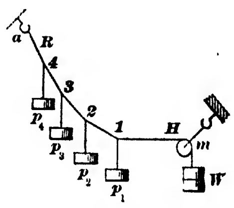

Fig. 113.

sum of the loads, $p_1 \ldots p_4$. When this is the case, the cord segment, a–4, next to the support, a, and the corresponding line, c–4, Fig. (a), will be inclined 45° to the vertical.

250. But if, while the loads, $p_1 \ldots p_4$, remain unchanged, we raise the pulley m, so as to keep H horizontal,* we shall obtain a flatter curve, as in Fig 112; and, for equilibrium, H (= W) must be made *greater* than the sum of $p_1 \ldots p_4$. On the other hand, if we place the pulley, m, lower than in Fig. 111 (still keeping H horizontal), we obtain a deeper curve, as in Fig 113; and H (= W) must be made *less* than the sum of $p_1 \ldots p_4$.

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* In Figs. 111, 112 and 113 we suppose the weight, W, and the position of the pulley, m, to be so adjusted, relatively to the support, a, that the pull, H, shall remain horizontal.

ARCHES, DAMS, ETC. THRUST AND RESISTANCE LINES.

The Arch.

In ¶¶ 251 to 257 are given the elements of the commonly accepted theory of the arch. For practical considerations, see ¶¶ 258 to 266, and Stone Bridges.

251. If Figs. 111, 112 and 113 be inverted, the cord segments will represent struts, sustaining compression, as do the stones of a masonry arch, Fig. 114.

Thrust Line. Resistance Line.

252. In the case of an arch, Fig 114, assuming * that the horizontal thrust H, at the crown, *m*, and the reaction, R, of the skewback, *a*, act at the center (or at some other definite point) of crown and of skewback, respectively, their amounts, and the direction of the reaction, R, may be found by means of the Force Triangle, ¶ 51, or by Moments, ¶ 224. (See ¶ 257.) We then suppose the half-arch and its spandrel to be divided, by vertical planes,* Fig.

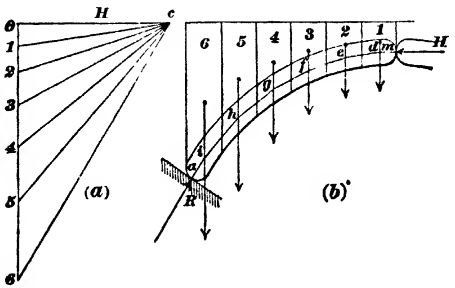

Fig. 114.

114 (*b*), into a number of segments, as shown; and, finding the weight and the center of gravity of each such segment (see ¶¶ 257 and 266), we treat these segments as we treated the loads, $p_1 \ldots p_4$, of Figs. 111 to 113, laying them off from 0 to 6, Fig. 114 (*a*), and laying off 0–*c* horizontal and = H. The rays, *c*–1, *c*–2, etc., then give, theoretically,* the directions and amounts of the pressures exerted by the segments, 1, 2, etc., respectively.

The broken line, *m d e f . . . . a*, Fig. 114 (*b*), thus formed, is called the **thrust line,** or line of resultants. It corresponds with the cord polygons of Figs. 111 (*b*), etc *

253. The resistance line is a broken line joining the points where the several resultants, forming the thrust line, cut the respective joints between the arch stones.

254. When the planes, by which the arch is supposed to be divided into segments, are vertical,* as in Fig. 114 (*b*), and, indeed, in most actual arches, the thrust and resistance lines, Fig 115, practically coincide; but if these planes are far from vertical, as in Fig. 115, the two lines separate, the resistance line being always the outer one.

Thus, in Fig. 115 (where the thrust line is shown solid, and the resistance line dotted), noticing where resultant *a* cuts joint A, where resultant *b* cuts joint B, etc., it will be seen that the two lines practically coincide as far as to joint C, where they begin to diverge.

* See Practical Considerations, ¶¶ 258, etc.

255. In ¶ 252 we assumed that the arch and its spandrel are divided into vertical segments, incapable (except in the arch ring) of exerting other than vertical pressures. The theoretical resistance line, thus obtained, may, especially in deep arches, pass from the thickness of the arch ring in places; so that, if no other forces were acting, the arch would open at such places; on the intrados when the resistance line cuts the extrados, and vice versa;

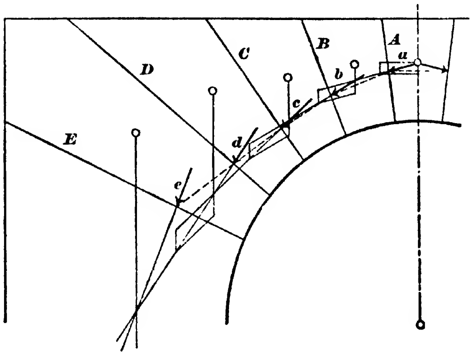

Fig. 115.

but such opening is usually prevented by other forces, such as the horizontal or inclined pressures of the spandrels. The actual resistance line is thus confined within the thickness of the arch ring. In general, the actual resistance line, Fig. 116, approaches the extrados at the crown, and the intrados at the haunches, so that the arch tends to sink at the crown (opening there on the intrados), and to rise at the haunches (opening there on the extrados), as shown.

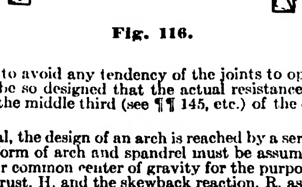

Fig. 116.

256. In order to avoid any tendency of the joints to open at either side, the arch should be so designed that the actual resistance line shall everywhere be within the middle third (see ¶¶ 145, etc.) of the depth of the arch ring.

257. In general, the design of an arch is reached by a series of approximations. Thus, a form of arch and spandrel must be assumed in advance, in order to find their common center of gravity for the purpose of determining the horizontal thrust, H, and the skewback reaction, R, as in ¶ 252; and, if it is afterward found necessary to modify the form first assumed, in order to satisfy the requirements of ¶ 256, or for other reasons, we may have to recompute H and R, again modifying the design, and so on.

Practical Considerations.

258. While the theoretical thrust and resistance lines, based upon the foregoing assumptions, are easily found, much uncertainty exists as to the positions of the actual thrust and resistance lines in a masonry arch.

259. *In the first place, we do not know through what points in the crown and skewback, respectively, the resultants, H and R, pass.*

260. Again, we have assumed that the loads on the arch, like those on the cord, Figs. 111 to 113, are incapable of acting otherwise than vertically; whereas the spandrel walls and filling, which form a large portion of the load on a masonry arch, may offer resistances acting in other directions. If the loading were a liquid, like water, its pressures upon the arch ring would be radial, like those of the particles of steam, in a boiler, upon the boiler tubes; and this condition is probably more or less closely approximated in the case of a loading of clean dry sand; and, less closely, in the case of earth filling. Hence, although the determination of the theoretical thrust and resistance lines in an arch is facilitated by the assumption that the arch is correctly represented by the inverted cord, the distinction between the two cases must be borne in mind when drawing practical conclusions from the lines so found.

261. Thus, in many cases, the theoretical thrust and resistance lines cut the intrados or the extrados in places, thus passing entirely out of the arch ring; so that this would inevitably fall (see ¶ 255), were it not for horizontal or inclined resistances exerted by the upper parts of the abutments through the spandrel walls and filling

262. Hence, in order to determine the actual resistance line, we should not only have to know through what points, in crown and in skewback respectively, the resultants, H and R, pass, but we should also have to ascertain and take into account the possible horizontal and inclined resistances of the spandrel walls and filling. But, as this is ordinarily impracticable, we content ourselves either with determining the theoretical thrust and resistance lines, as directed above, and then estimating, as well as may be, the resistances of the spandrels, or with reasoning by analogy from the behavior of actual structures. See Stone Bridges

263. If the inverted cord correctly represented the actual thrust line in a masonry arch, the arch stones, in elliptic or in deep segmental arches, would have to be made inordinately deep, in order that the resistance line should nowhere leave the middle third of their depth (see ¶¶ 145, etc.); and it might therefore appear rational to make the profile of the arch correspond approximately with the thrust line, which usually approaches a parabola. But, owing to the spandrel resistances, the actual thrust line, even in semicircular arches, probably seldom greatly oversteps the middle third.

264. With a wall or a deep continuous filling, over an arch, if the arch were to settle, or were to be removed, the wall and the filling above it would form an arch, as indicated by the broken lines in Fig 117; and only that portion

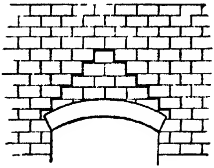

Fig. 117.

below this arch would fall out. Hence, only this portion can properly be regarded as pressing upon the arch.

265. Neglecting the strength of the mortar, the inclination of each joint between two arch stones must of course be such that the angle, between the thrust, at any joint, and a normal to that joint, shall be less than the angle of friction See ¶¶ 183, 184.

266. It is often the case that the spandrels or the spandrel filling are of less specific gravity than the arch ring. In such cases, in order to facilitate the finding of the lines of gravity of the segments, we may, before dividing the half-arch and its spandrels into vertical segments (¶ 252), consider the lighter structure of the spandrels as being reduced to an equivalent depth of material having equal specific gravity with the arch. The areas of the several segments, as seen in profile, and as thus reduced, may then be taken as representing their weights. Thus, in Fig. 118, where *t t t* represents the

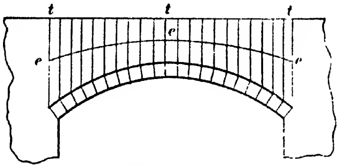

Fig. 118.

top of the spandrels, the curved line *e e e* represents the top of a filling of equal weight per foot run with the spandrels, but of equal specific gravity with the arch ring. When, as in Fig. 119, the spandrels consist of a series of transverse arches, we may assume that the main arch carries a series of loads concentrated at the piers of these transverse arches.

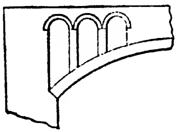

Fig. 119.

The Masonry Dam.

267. A dam must be secure against sliding, on its base or on any plane within the body of the dam, against overturning, and against crushing of the material at any point and consequent opening of a seam at either face of the dam.

268. The dam will be secure against sliding if the resultant of all the pressures, upon any surface, forms, with a normal to that surface, an angle less than the angle of friction of the surface. See ¶¶ 183, etc. In practice, the base of the dam is let well down into the rock foundation, as indicated in Fig. 122 (*a*), and continuity of joints is avoided by making all the stones break joints. The angle of friction thus becomes, in effect, 90°, and sliding cannot occur without shearing the stones themselves.

269. If the material is sufficiently strong to resist crushing, under the maximum unit stresses brought upon it, and if the resultant of all the forces acting upon any section falls within the body of the dam, the dam will be secure against overturning. But see ¶ 270.

270. For a given total pressure upon any section, the maximum unit pressure in the section would be least when the resultant cut the middle point of the section. See Center of Pressure, ¶¶ 133, etc. It is generally impracticable to secure this; but the dam must be so designed that, under the maximum unit pressure, the given material shall not be taxed beyond its safe crushing strength. If this is done, and if, under all conditions, the center of pressure is kept within the middle third (see ¶ 150) of each horizontal section throughout the dam, there will be no tendency to open on either face of the dam.

271. Let Fig. 120 represent a stone block, resting upon a solid foundation and intended to sustain the pressure, p, of quiet water on one side. Through the center of gravity, g, of the block, draw o'N vertically, to represent the weight, W, of the block. Then the point, s, where o' N meets the foundation, is the center of pressure for the block alone, *i e*, when the water is removed.

272. Let h be the depth of water back of the block, and let the block be one foot in length, measured normally to the paper. Then the amount, in pounds, of the water pressure, against the vertical back, $a\,b$, is $p = 62.5\,h \times \frac{1}{2}\,h$, and its center of pressure is at a depth, $d = \frac{2}{3}\,h$, below the water surface.

273. Combining p with W (¶ 35) we obtain R as their resultant, and r as the center of pressure upon the foundation when the block is sustaining the water pressure.

274. Let Fig. 121 represent several such blocks superposed Let

g_1 = cen of grav, p_1 = cen of water pres, for block 1;
g_2 = " " " p_2 = " " " " " blocks 1 and 2 combined;
g_3 = " " " p_3 = " " " " " " 1, 2, and 3 combined,
etc

Then, finding r_1 and s_1, r_2 and s_2, r_3 and s_3, etc , for joints 1–2, 2–3, 3–4, etc , as before, we have the points r_1, r_2, r_3, etc , in the resistance line for full dam, and the points s_1, s_2, s_3, etc , in the resistance line for empty dam.

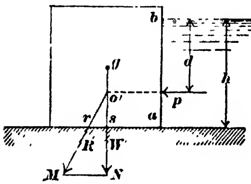

Fig. 120.

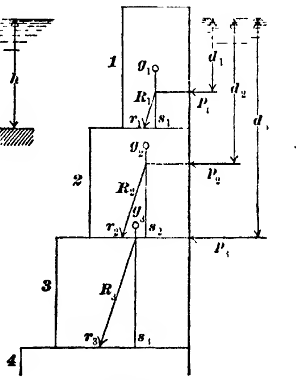

Fig. 121.

275. In Fig. 122 (a) the curves, $u\,u$ and $d\,d$, indicate the up-stream and down-stream limits, respectively, of the middle third of the plane separating each two blocks, assuming the profile with vertical back; and the points $r_1 \ldots r_7$ and $s_1 \ldots s_7$ are points in the corresponding resistance lines for full dam and for empty dam respectively.

276. While theory would require the cross-section of the dam to terminate in a sharp angle at the top, it is, of course, always made heavier in practice, as indicated in Fig. 122 (a).

277. For joint 2–3 we may suppose that block 2 had at first been designed rectangular, as shown by dotted line $c\,a$ (Fig. 122 (c), showing blocks 1 and 2 enlarged); but this makes the center of pressure, r', for the full dam, fall beyond the middle third of the narrow base, $a\,b$. We therefore try the trapezoidal shape $c\,i\,b$, with its wider base, $i\,b$, and find that, with this, the center of pressure, r_2, although further down-stream than before, falls within the middle third of said wider base. The remainder of the profile is determined by similar trials.

278. Graphic Method. Suppose the cross-section to be divided, by horizontal sections, into numerous blocks, 1, 2, 3, 4, etc., of a depth approximately = the top width of the dam. In Fig. 122 (*b*), draw 0–1, 1–2, 2–3, 3–4, etc., vertically, to represent, by scale, the weights of the several blocks, respectively, and 0–1′, 1′–2′, 2′–3′, 3′–4′, etc., horizontally, to represent the water pressures against said blocks respectively; and draw 1′–1, 2′–2, 3′–3, 4′–4, etc., representing, in amount and in direction, the total inclined pressures upon joints 1–2, 2–3, 3–4, etc., and upon the base, respectively. Thus, 2′–2 represents the resultant of the water pressure (see Hydrostatics) upon blocks 1 and 2, and the combined weight of those blocks. From the points, o_1, o_2, etc., Fig. 122 (*c*), where these two forces meet in each case, draw $o_1\ r_1$, $o_2\ r_2$, etc., parallel respectively to 1′–1, 2′–2, etc., Fig. 122 (*b*), to the corresponding joint. We thus obtain points, $r_1 \ldots r_6$, in the resistance line for the case where the dam is filled to the assumed depth.

The foregoing refers to the diagram for a dam already completed or designed. In designing *de novo*, we of course begin at the top, and lay off the lines 0–1, 0–1′ and 1′–1 in Fig. (*b*) for the first block; then lines 1–2, 1′–2′ and 2′–2, for the second block, and so on; making necessary changes, as in ¶ 277.

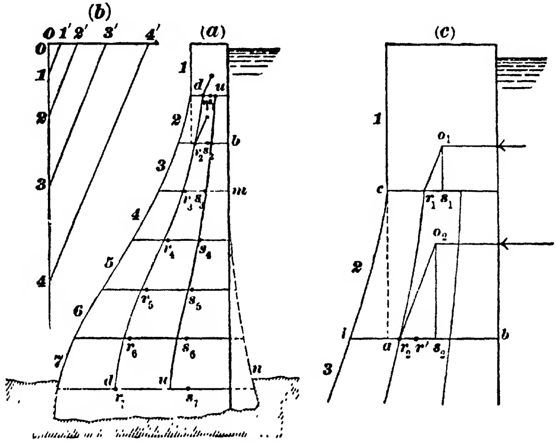

Fig. 122.

279. In order that the resistance line, $r_1 \ldots r_6$, for the full dam, may be brought well within the middle third, it may sometimes be necessary to adopt a somewhat unwieldy cross-section; but, in view of the imminent danger involved in the smallest opening on the up-stream side of the dam, (see ¶ 281), it is well here to err on the safe side.

280. As this process is carried further down, the angle formed between the down-stream face and the vertical becomes considerable; and the middle third, in each of the lower joints, is thus brought further down-stream. The centers of pressure, $s_1 \ldots s_6$, for empty dam, may then fall beyond the middle third, on the up-stream side, as indicated at joints 5–6 and 6–7. To obviate this, the up-stream face is sometimes given a curved profile, as at $m\ n$.

Practical Considerations.

281. The assumption of ideal conditions is particularly dangerous in the case of masonry dams. Thus, any compression of the material at the down-stream face may open seams on the up-stream face; and water, entering these seams, will exert a wedge-like action, shifting the resistance line further down-stream, thus still further increasing the tendency to crushing on the down-stream face and to opening on the up-stream face. Again, if any relatively smooth joints have been left, the water, thus penetrating into or under the dam, increases the tendency to slide, not only by diminishing the effective weight of the upper portions, but also by acting as a lubricant upon the seam where it penetrates.

It has been suggested that failures of dams may have been occasioned, in part at least, by vacuum, formed in front of the down-stream face, by the action of the sheet of water falling in front of that face.

282. Theoretically, the deflections of arches, dams and other structures composed of blocks, may be found by means of the formulas in ¶¶ 162–167 of Trusses; but, owing to uncertainty as to the values of the moduli of elasticity, E, of building stones and of mortar, and to the relative inaccuracy of finish in masonry work, the formulas are of but little practical value in such cases.

THE SCREW.

283. The screw is a spiral inclined plane. The force (or "power") describes a spiral, at the end of a lever arm, while the resistance (or "weight") moves along the axis of the screw. During the time in which the force makes one revolution, the resistance traverses the "pitch," or distance between the centers of two adjacent threads.

284. Hence, if P = power, w = weight, d = pitch, l = lever arm, v = rectilinear velocity of weight, and V = linear (circular) velocity of power, we have, theoretically:*

$$\frac{w}{P} = \frac{V}{v} = \frac{2\,\pi\,l}{d}.$$

* Neglecting friction, which, however, very greatly modifies the result.

FORCES ACTING UPON BEAMS AND TRUSSES.

Conditions of Equilibrium.

285. In beams and trusses, for equilibrium, it is necessary and sufficient that the resisting forces, exerted by the material of the structure, and the moments of those forces, shall balance the external or destructive forces and their moments. We here discuss chiefly the destructive forces. For the resisting forces, see Stresses, under Trusses, and Beams or Transverse Strength, under Strength of Materials.

286. The destructive forces are (1) the loads upon the structure, including its own weight, "live" or moving loads, wind, etc., and (2) the reactions of the supports. We shall here discuss the action of vertical loads only, including (*a*) the dead load, or the weight of the structure itself, together with the roadway, etc., and (*b*) the live, moving or extraneous load of vehicles, trains, persons, etc. The action of horizontal loads (wind, centrifugal force, etc.) is governed by similar laws, and is discussed under Stresses, in Trusses.

287. Let Fig. 123 (*a*) represent a cantilever, resting upon a support, *b*, and bearing a load, W, at its outer end, *a*. The cantilever is prevented from turning about *b*, by the tension, T, of a horizontal chain, and by the compression, C, in a horizontal strut.* Neglecting the weight of the cantilever

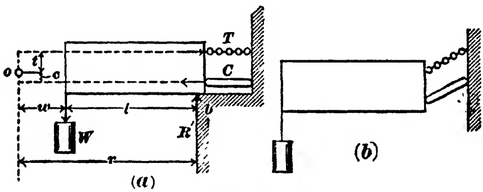

Fig. 123.

itself, the cantilever is acted upon by four external forces, forming two couples; one couple consisting of two vertical forces—viz., the load, W, and the reaction, R', of the support; the other couple consisting of two horizontal forces—viz., the tension, T, near the top, and the compression, C, near the bottom. Were it not for the reaction, R', of the support, *b*, the load, W, would pull the cantilever downward, as indicated in Fig. 123 (*b*).

288. In Fig. 123 (*a*) we have:

Algebraic sum of vertical forces $= R' - W = 0$;
" " " horizontal " $= T - C = 0$;
" " " moments, about any point, as *o*,

$$W.w - R'.r + T.t + C.c = 0.$$

---

* In Figs. 123 to 127, inclusive, and Figs. 132 and 133, showing cantilevers, beams and part beams, acted upon by loads, by reactions, by pulls of chains and by pushes of struts, the arrows denote forces acting *upon the cantilever or beam or upon its segments*, and not forces acting upon the load, the supports, or the connecting chains or struts. Thus, the tension in a chain tends to *draw together* the two bodies which it connects. Hence, in these cases, the corresponding arrows point *toward each other*. On the other hand, the compression in a strut tends to *separate* the two bodies between which it acts. Hence its two arrows point *away from each other*.

289. If, as in Fig. 124, the horizontal forces are exerted at the end farthest from the support, and at the same distance apart as before, their amounts and senses must remain respectively the same as before; but we now have compression, C, at the top, and tension, T, below. Or, if Fig. 123 be inverted, R′ acts as the load, and W as the upward reaction; and we have, as in Fig. 124, compression, C, at top, and tension, T, below. Thus, Fig. 124 is practically Fig. 123 inverted.

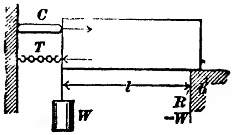

Fig. 124.

290. The condition described in ¶ 289. Fig. 124, represents also the condition in each segment, A, B, of a beam, Figs. 125 (*a*) and (*b*) or Figs. 126 (*a*) and (*b*), supported at both ends and bearing a concentrated load, W + W, Fig. 125, or W + *w*, Fig. 126.

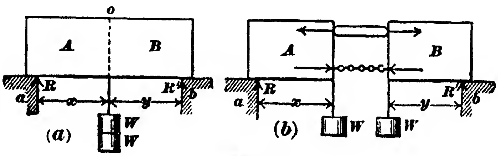

Fig. 125.

291. Suppose the beam, Fig. 125 (*a*) or Fig. 126 (*a*), to be divided into two cantilevers, or part beams, as in Fig. 125 (*b*) or Fig. 126 (*b*); each part sustaining, at its end, a part of the original load. (See ¶ 292.) The stresses in the strut and chain, Figs. (*b*), take the place of stresses in the material (situated in the dotted line) of the truss or beam, Figs. (*a*). In a truss, these forces are exerted by the chords: in a beam, by the particles or fibers throughout the section.

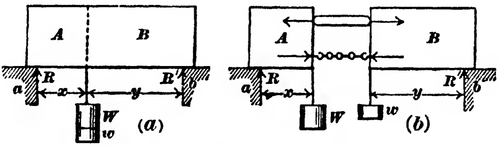

Fig. 126.

292. If, as in Fig. 125 (*a*), the load is at the center of the span, the spans, x and y, of the cantilevers, Fig. 125 (*b*), are equal, as are also the loads, W = W, carried by them. But if, as in Fig. 126 (*a*), the load, W + w, on the beam, is not at the center of the span, the partial loads, W and w, supposed to be supported at the ends of the two cantilevers, or part beams, respectively, Fig. 126 (*b*), are unequal, and inversely proportional to their leverages about their respective supports. Hence, the moments of the two opposite couples are equal. The reaction of each support is equal to the weight carried by the cantilever resting upon it.

End Reactions.

293. In a cantilever, Fig. 127, there is but one vertical support; the reaction, R′, of that support, is = the sum of all the loads, including the weight of the cantilever itself; and the reaction due to each partial load is = such partial load. Thus, if B = weight of cantilever,

$$R' = W + w + B.$$

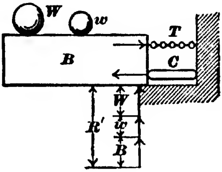

Fig. 127.

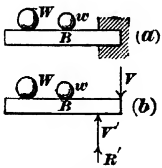

Fig. 128.

294. The reaction, R′, must not be confounded with other vertical forces. Thus, a cantilever is often supported as in Fig. 128 (*a*). The couple, composed of two horizontal forces, T and C, Fig. 127, is then replaced by a couple composed of two vertical forces, V and V′, Fig. 128 (*b*); one of which, V′, coincides with the reaction, R′. Here, R′ + V′, acting upward, is the anti-resultant of W, *w*, B and V, acting downward.

295. In a beam, Fig. 129, the sum of the two end reactions is = the sum of all the loads, including the weight of the beam itself.

296. The reaction, R, of the left support, *a*, Fig. 129, due to the load, W, alone, is $R = W \cdot \frac{y}{l}$ (see ¶ 17), and the reaction, R′, of the right support, *b*, is $= W - R = W \cdot \frac{x}{l}$. If the load is central, $\frac{x}{l} = \frac{y}{l} = \frac{1}{2}$, and $R = R' = \frac{W}{2}$.

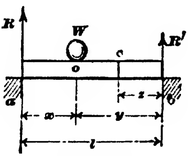

Fig. 129.

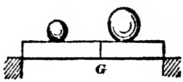

Fig. 130.

297. Graphically, Fig. 156, suppose a concentrated load, W (not shown), to be placed on the beam at any point, as *c*. Draw *a′ a″* and *b′ b″*, vertical and each = W. Join *a″ b′*; also join *a′ b″*, and draw *g h* vertically through *c′*. Then the ordinate, *c′ g*, to the upper line, *a″ b′*, and the ordinate, *c′ h*, to the lower line, *a′ b″*, give the left and the right end reactions, R and R′, respectively.

298. Where, as in Fig. 130, there are two or more loads (in which the weight of the beam may or may not be included), the reactions due to each load may be separately obtained, the sums of these reactions giving the total reactions; or, the common center of gravity, G, of all the loads may first be found (see ¶¶ 125, etc.), and then the reactions found as for a single load, W, Fig. 129; the combined weight of the loads, whose center of gravity is at G, being supposed concentrated there.

299. In a beam, Fig. 131, under a load, W, uniformly distributed over any part of the span, let G be the center of gravity of the load, and let x and y be the segments of the span, l, to the left and right of G respectively. Then, neglecting the weight of the beam, $R = W\frac{y}{l}$; and $R' = W - R = W\frac{x}{l}$.

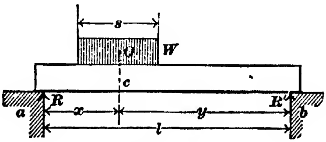

Fig. 131.

300. If the load is uniformly distributed over the entire span, its center of gravity is at the center of the span, and we have:

$$\frac{x}{l} = \frac{y}{l} = \frac{1}{2}; \text{ and } R = R' = \frac{W}{2}.$$

Moments and Shears.

301. In order to determine what internal stresses are required, at any point in the span, to maintain equilibrium, we may suppose the cantilever or beam to be cut in two by a section, *c c*, Fig. 132 or Fig. 133, at such point, and inquire what forces must be applied, in the section, in order to maintain equilibrium and hold in position the two segments, E and F, into which the section, *c c*, divides the span, Fig. 132, or that part of the span between the load and a support, Fig. 133. The forces, so ascertained, are evidently equivalent to those actually exerted, for the same purpose, by the material of the beam itself.

302. In Figs. 132 and 133, moments of loads and of reactions, or *external* or bending moments, are indicated by arrows *below* the cantilever and beam respectively; while the resisting moments of the *internal* forces are indicated by arrows *within* the body of the cantilever or beam respectively.

303. In the cantilever, Fig. 132, the load, w, = 4 lbs., distant 6 ft. from the section, *c c*, produces there a left-hand or negative moment of $6\,w = 6 \times 4 = 24$ ft.-lbs. Hence, for equilibrium, the horizontal strut and chain, at *c c*, must exert a right-hand or positive resisting moment of 24 ft.-lbs; and, being 2 ft. apart, they must exert a tension, T, and compression, C, of $\frac{24}{2} = 12$ lbs. each. At the support, moment of load $= 9\,w = 9 \times 4 = 36$ ft.-lbs.; and $T' = C' = \frac{36}{2} = 18$ lbs.

304. But, considering only the forces thus far discussed, we should find the right segment, F, acted upon, at *c c*, by a left-hand couple, $= d \times T = d \times C = 2 \times 12 = 24$ ft.-lbs.; and, at the support, by a right-hand couple, $= d \times T' = d \times C' = 2 \times 18 = 36$ ft.-lbs. In other words, there would be an unbalanced excess of right-hand moment, $= 36 - 24 = 3\,R' = 3 \times 4 = 12$ ft.-lbs., acting upon F. F also receives, at the support, the upward reaction, R', = 4 lbs., of that support. Similarly, the couple, $d \times T = d \times C$, at *c c*, exerts, upon the left segment, E, an apparently unbalanced right-hand moment of $2 \times 12 = 6\,w = 6 \times 4 = 24$ ft.-lbs., and E receives, from the load, w, a downward pull = 4 lbs.

305. For equilibrium, therefore, the vertical chain at *c c* must exert a tension $= S = w = R' = 4$ lbs., pulling F downward, and E upward. The downward tension, $-S$, acting on F at *c c*, forms, with the reaction, R', of the support, a left-hand couple $= 3\,R' = 3 \times 4 = 12$ ft.-lbs., balancing the excess of right-hand moment acting upon F; while the upward tension, $+S$, acting on E at *c c*, forms, with the weight, w, a left-hand couple, $= 6\,w = 6 \times 4 = 24$ ft.-lbs., balancing the excess of right-hand mom. acting on E

306. Similarly, if we suppose the cantilever cut through by a section at any other point, we shall find that a vertical force, = S = w = R′, acting upward upon the left segment and downward upon the right segment, is required in order to maintain equilibrium and to transmit the load, w, to the support, so that the two segments may act unitedly as a single cantilever. This force, S, is called a shear. See ¶¶ 325, etc. Without it, section E would fall, as in Fig. 123 (b).

307. In the beam, Fig. 133, the total load is 16 lbs.; and, its distances, 3 ft. and 9 ft., from the left and from the right support respectively, being as 1 to 3, the end reactions (¶¶ 293, etc.) are as 3 to 1; or R = 16 × ¾ = 12 lbs.; R′ = 16 × ¼ = 4 lbs. We therefore regard the beam as being cut by a section at the load (as well as at c c), and the total load of 16 lbs. as divided into two portions; one, W = R = 12 lbs., attached to the end segment, M; and the other, w = R′ = 4 lbs., supported by the middle segment, E. Here, as in Fig. 132, segments E and F together form a cantilever, 9 ft. long, loaded with a weight, w, of 4 lbs., at its end; but,

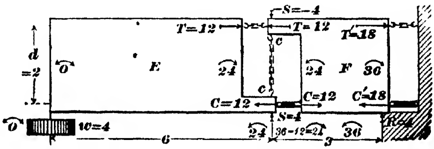

Fig. 132.

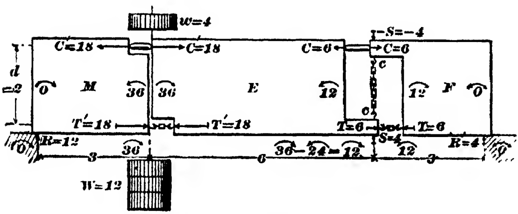

Fig. 133.

in Fig. 133, the horizontal resisting forces, T′ and C′, by which the entire cantilever (E + F) is upheld, are exerted, not at the support, as in Fig. 132, but at the end farthest from the support.

308. We have, therefore, in Fig. 133, positive (clockwise) and negative (counterclockwise) moments, as follows, acting upon E, about the lower end of c c:

$$d\,C' - 6\,w - d\,C = 2 \times 18 - 6 \times 4 - 2 \times 6 = 0.$$

Here, $d\,C' - 6\,w$, = 36 − 24 = 12, is bending moment, and $-\,d\,C$, = − 12, is resisting moment.

We have, also, in c c, shear, S = w = R′ = 4 lbs.

309. In Fig. 132 or in Fig. 133, considering the segment extending from the load to either support (in Fig. 132 there is but one such segment), it will be seen that, at the free end of any such segment, the horizontal stresses are zero, and that they increase uniformly to a maximum at the other end of the segment. Thus, in Fig. 132, they increase uniformly from 0, at the loaded or free end, to 18 lbs., at the support; while, in Fig. 133, they increase uniformly from 0, at each support, or free end, to 18 lbs., at the load.

Moments in Cantilevers. See also pp 437 &c.

310. In a cantilever, Fig. 134, each load exerts, about any point between itself and the support, a moment = its weight × the horizontal distance of its center of gravity from such point; and the total moment, at any point is the sum of the moments of the several loads about that point. Thus neglecting the weight of the cantilever itself, we have:

about b, mom = $A\ x + B\ y$; about d, mom = $A\ z$;
about c, mom = $A\ m + B\ n$; about A, or any point beyond A, mom = 0

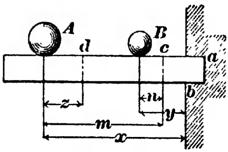

Fig. 134.

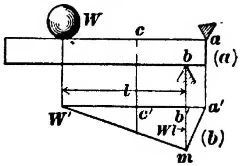

Fig. 135.

311. In a cantilever, Fig 135, the maximum leverage of any load, W, is evidently its distance, l, from the support, b. Hence, the maximum bending moment of any load upon a cantilever is at the support, and is = $W\ l$. From this max, the mom diminishes uniformly to 0, at the load and at a.

In Fig. 134, the end, $b\ a$, is uniformly loaded, and the diminution of mom, betw b and a, follows the ordinates of a parabola, as in $m\ k'$, Fig. 137.

311 a. In cantilevers and beams, Figs 129, 134, *clockwise* moments of forces, to the *left* of a given point, c, about that point, are considered *positive*, and vice versa. *Positive* moments *extend* the *lower* and *compress* the *upper* fibers, and vice versa.

312. In Fig. 135 (b), (cantilever with a single load, W,) draw $b'\ m$, = max mom by scale, and join $m\ W'$ and $m\ a'$. Then, for any point, c,

moment = ordinate at c' to line $m\ W'$.

313. In a cantilever, Fig. 136 (a), with two or more concentrated loads, W and w, let

$b'\ m$, Fig. 136 (b), = moment of W, at the support;
$b'\ m'$, Fig. 136 (b), = moment of w, at the support.

Then, for both loads, W and w, neglecting the weight of the beam,
at d, moment = moment of W alone, = ordinate at d';
at c, moment = sum of moments of W and w,
= sum of two ordinates, $c'\ n$ and $c'\ n'$, at c'.

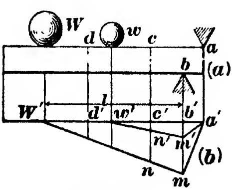

Fig. 136.

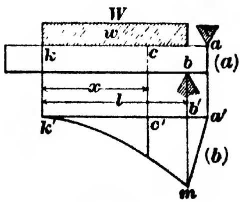

Fig. 137.

314. In a cantilever, Fig. 137 (a), under a load, W, uniformly distributed over a length, l, beginning at the support, b, the maximum moment, at the support, b, is = $W\ l/2$.

In Fig. 137 (*b*), make $b'\,m$ = said max moment, and draw a semi-parabola $m\,k'$, with apex at k'. Then, at any other section, c, the moment is represented by the ordinate, c', of said parabola, and is $= w \cdot \frac{x}{2}$, where w = the weight of that portion of W beyond c, and x = the length of that portion.

At k, or at any point beyond k, moment = 0.

315. In Fig. 138, neglecting the weight of the cantilever itself, let W represent the weight of the whole load, and w, that of the shaded portion, concentrated at their respective centers of gravity, G and g. Then,

about b, moment $= W \cdot x$;
" c, " $= W \cdot y$,
" d, " $= w \cdot v$;
" k, or any point beyond k, moment = 0.

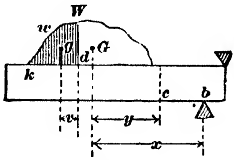

Fig. 138.

Moments in Beams.

316. In a beam, Fig. 129, the upward reaction of each abutment exerts, about any point, a moment = reaction × distance of support from such point; but any load, between such point and the support, exerts a contrary moment = load × distance of load from such point. Thus,

about c, moment $= R'\ z = R\,(l - z) - W\,(y - z)$.

At each support, the moment is 0

317. In a beam, Fig. 129, carrying a single concentrated load, W, the moment, $R'.z$, at any point, c, is $= R'z = W\frac{x}{l} \cdot z - R\,(l - z) - W\,(y - z)$ $= W \cdot \frac{y}{l}\,(l - z) - W\,(y - z)$. At a point, as c, not under the load, the moment, $R'z$, is evidently less than the moment, $R'.y$, about the point, o, under the load. In other words, the maximum moment is at the point, o, under the load.

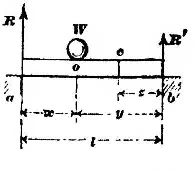

Fig. 129 (repeated).

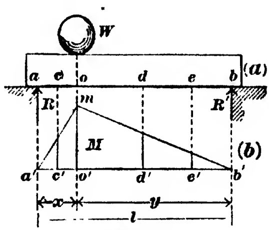

Fig. 139.

318. From the point, o', Fig. 139 (*b*), corresponding to the point, o, Fig. (*a*), where the load is applied, erect an ordinate, $o'\,m$, equal by scale to the (maximum) moment, $= R' \cdot y = R \cdot x$, at that point. Join $a'm\,b'$. Then the ordinate to $a'm$, or to $m\,b'$, at any point, c', d', e', etc., represents by scale the moment at the corresponding point, c, d, e, etc., in the span.

319. When the load, W, is at the center of the span, l, Fig. 140, each end reaction is $= \frac{W}{2}$. Hence, the moment s', at any point, s, distant y from a support, as b, is

$$\text{moment} = \frac{W}{2} \cdot y.$$

At the center of the span (*i. e.*, at the point under the central load, W) we have:

$$\text{maximum moment, M,} = \frac{W}{2} \cdot \frac{l}{2} = \frac{W.l}{4}.$$

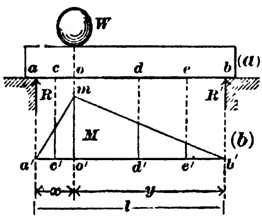

Fig. 139 (repeated).

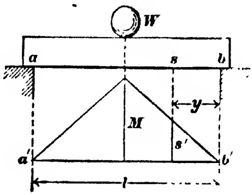

Fig. 140.

In order that the maximum moment (at o, Fig. 139) due to an eccentric load, W, may be equal to the maximum moment (at center of span, l) due to a given center load, C, we must have

$$W \frac{x}{l} . y = C . \frac{l}{4}; \text{ or } W = C\frac{l}{4} . \frac{l}{xy} = C\frac{\left(\frac{l}{2}\right)^2}{xy}$$

320. When there are two or more concentrated loads, c, d, e, Fig 141, treat each load as in Fig 139, making each short ordinate, m, m', m'', represent the maximum moment of its single load, c, d or e, alone. Make the long ordinates, M, M′ and M″ = the sums of the separate moments, as measured at c', at d', and at e', respectively. Then the ordinate to a' M M′ M″ b', at any point, represents the total moment at that point, due to the several loads combined.

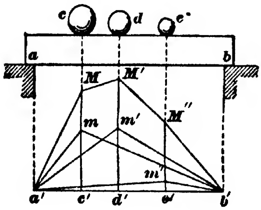

Fig. 141.

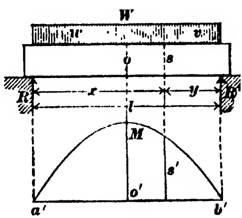

Fig. 142.

321. In a beam, Fig. 142, under a uniform load, W, covering the span, l, the maximum moment is at the center of the span, and is

$$\text{moment} = R.\frac{l}{2} - \frac{W}{2}.\frac{l}{4} = \frac{W}{2}.\frac{l}{2} - \frac{W}{2}.\frac{l}{4} = \frac{W}{2}.\frac{l}{4} = \frac{W.l}{8}.$$

Make $o'M$, by scale, = the maximum moment; and draw the parabola, a' M b', with vertex at M. Then the moment, at any section, as s, is represented by the corresponding ordinate, s', to that parabola.

Let w and v = the portions of W to the left and right of s, respectively. Then, moment at $s = \frac{w}{2} y * = \frac{v}{2} x * = \frac{1}{2}$ moment due to whole load, W, concentrated at s.*

At either support, moment = 0.

In Fig. 131, at a point, c, under the center of gravity of a load, W, uniformly distributed over a portion, s, of the span, neglecting the weight of the beam,

$$\text{moment} = R.x - \frac{W}{2} \cdot \frac{s}{4} = R.x - \frac{W.s}{8} = R'.y - \frac{W.s}{8}.$$

322. Let W = the total load, whether concentrated or uniform, and let l = the span. Then the maximum moment, M, is as given below:

Cantilever	Load, W, at end	M at support	$M = W\,l$;
"	" " uniform.	" " "	$M = \frac{W\,l}{2}$;
Supported beam †	" " at center.	" at center	$M = \frac{W\,l}{4}$;
" "	" " uniform.	" " "	$M = \frac{W\,l}{8}$;
Fixed beam.‡	" " at center.	" " " or support	$M = \frac{W\,l}{8}$,
" "	" " uniform.	" " support	$M = \frac{W\,l}{12}$.

323. In the inclined beam, Fig 143, the inclined distances may be used, instead of the horizontal distances, in finding the reactions. Thus,

$$\text{reaction } R' = W \cdot \frac{x}{l} = W \cdot \frac{x'}{l'}.$$

But, in finding moments of vertical forces, we must of course use the horizontal, not the inclined, distances. Thus, at c, moment $R'c$; not $R'c'$.

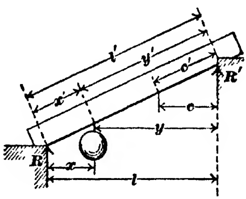

Fig. 143.

* Moment at $s = R'\,y - v\,\frac{y}{2} = \frac{W}{2}\,y - \frac{v}{2}\,y = \frac{W - v}{2}\,y = \frac{w}{2}\,y$
$= R\,x - w\,\frac{x}{2} = \frac{W}{2}\,x - \frac{w}{2}\,x = \frac{W - w}{2}\,x = \frac{v}{2}\,x.$

With W concentrated at s,

$$\text{moment at } s = W\,\frac{x}{l}\,y = w\,y = W\,\frac{y}{l}\,x = v\,x.$$

† Beam supported at each end, but not fixed.

‡ Beam fixed at each end.

324. In curved beams, the same principles apply as in straight beams. Thus, Fig. 144, at *s*, moment = W . *l*. Again, in Fig. 145 (*a*), reaction R' $= \frac{W . x}{l}$, and at *s*, moment = R' . *y*. Or, as in Fig. 145 (*b*), from *o*, where the load is applied, draw *o a* and *o b*, to the two supports respectively, and, by means of the force parallelogram, find the components, *p* and *q*, of W. Then, at *s*,

moment = *p* . *n*.

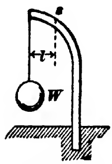

Fig. 144.

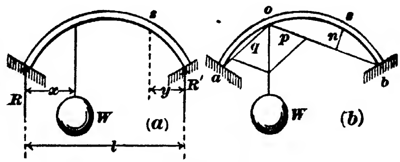

Fig. 145.

Shear.

325. In the beam, *a b*, Fig 146 (*a*), consider the segments, *a c* and *c b*, to the left and to the right respectively of the plane *n n*. Besides the horizontal forces acting across the plane *n n*, we have seen (¶ 305) that we require also, for equilibrium, a vertical force, = the left end reaction, R, acting downward upon the left segment, *a c*, and forming a couple with R; and, at the same time, acting upward on the right segment, *c b*, being = the load, W, minus the right end reaction, R'. This force is called the shear, S, in the section *n n*. It may be regarded as the transmission of the vertical forces from loads to supports or vice versa.

326. The two segments, *a c* and *c b*, thus tend to slide vertically past each other, the right segment, *c b*, tending downward, owing to the preponderance of the load, W, over the right end reaction, R'; and this tendency is resisted by the shear, S, which is = the left end reaction, R. The same tendency exists uniformly between W and *a*, and is resisted throughout by a shear = S = R

327. Between the load, W, and the right support, *b*, also, a uniform shear exists; but here the shear, S', is = the right end reaction, R', = R — W; and, whereas the shear, S, to the *left* of the load was *right*-handed or *clockwise* (the portion to the *right* of any section, *n n*, receiving the *downward* force), and is called *positive*, or +, the shear on the *right* of the load is *left*-handed or *counterclockwise* (the portion to the *left* of any section receiving the *downward* force), and is called *negative*, or —.

328. The shears, S and S', to the left and to the right of the load, W, are represented by the diagrams in Fig. 146 (*b*); that, S, on the left of the load being drawn *above* the zero line, *a' b'*, to indicate a *positive* shear, and vice versa.

329. Comparing Figs 146, 147 and 148, notice that, between the left support, *a*, and the load, W, Fig. 146, we have positive shears, S = 90, Fig. 146, and *s* = 15, Fig 147; so that, in Fig. 148, where *both* loads, W and *w*, are placed upon the same beam, we have, between *a* and W, a total positive shear of S + *s* = 90 + 15 = 105. Between the right support, *b*, and the load, *w*, Fig. 147, we have negative shears, S' = — 30, Fig. 146, and *s'* = — 45, Fig 147; so that, in Fig 148, between *b* and *w*, we have a total negative shear = S' + *s'* = — 30 — 45 = — 75. But, between the points of application of W and of *w*, we have S' = — 30, Fig. 146, and *s* = + 15, Fig. 147; leaving, between W and *w*, Fig. 148, *s* + S' = 15 — 30 = — 15. If the total right end reaction, R' + *r'*, exceeds *w*, as we here suppose, the shear, at any point between the two loads, W and *w*, Fig. 148, is negative, as indicated; and vice versa.

330. In any section, the shear is = the reaction at either end, minus any loads between that end and the given section.

331. If, as in Fig. 149, the right end reaction, $R' + r'$, is = the load, w, then the left end reaction, $R + r$, is = the load, W; and there is no shear at any point between the two loads. In other words, if the beam be cut by a section at any point between W and w, horizontal forces alone will preserve equilibrium, no vertical forces being required, since the two segments have no tendency to slide vertically past each other.

332. A similar condition exists in any section where the sign of the shear changes from + to — or vice versa. Thus, if the beam be cut by a section immediately under W, Fig. 146 or 148, or under w, Fig. 147, horizontal forces equivalent to the fiber stresses in the beam, will suffice to preserve equilibrium, without a vertical force, or shear; there being no tendency of the two segments to slide past each other. Also, when, as in Fig. 149, under W and under w, the shear changes, in amount, from any value, on one side of a section, to 0, on the other side, the shear in the section itself is = 0.

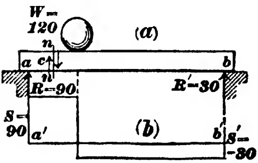

Fig. 146.

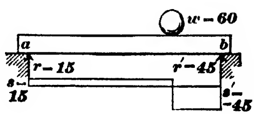

Fig. 147.

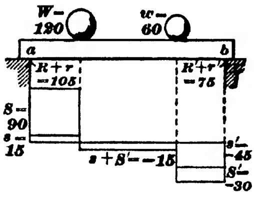

Fig. 148.

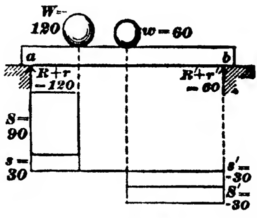

Fig. 149.

333. But in the section under w, Fig. 148, where the shear changes in amount, although not changing sign from + to — or vice versa, there is a shear = the *less* of the two shears on the opposite sides of the section, for this is the amount of the shear transferred through the section, or is the tendency of either segment to slide past the other.

334. With any number of loads, if that portion of the total load to the left of any section be called X, and that portion to the right of the same section be called Y, it will be found that the shear in the section is equal to the difference between that part of X which goes to the right support, b, and that part of Y which goes to the left support, a.

335. With a load, W, Fig. 150 (a), uniformly distributed over the entire span, the maximum shear, $= R = R' = \frac{W}{2}$, is at each support, a and b. The minimum shear, = 0, is at the center, c, of the span, which is also the point of maximum bending moment, see ¶ 321 and Fig. 142. At any point,

d, the shear is given by the corresponding ordinate, d', Fig 150 (b). See Relation between Moment and Shear, ¶¶ 359, etc.

336. With a load, W, uniformly distributed over any part y, of the span, Fig. 151 (a), find the end reactions, R and R', as in ¶ 299. Then

$$\begin{array}{lll} \text{between } a \text{ and } d, & \text{shear} & = S = R; \\ \quad '' \quad e \text{ and } b, & \quad '' & = S' = R'; \\ \text{at } c, & \quad '' & = 0. \end{array}$$

$$x = d\,c = y\cdot\frac{R}{W};\quad z = y - x = c\,e = y\cdot\frac{R'}{W}.$$

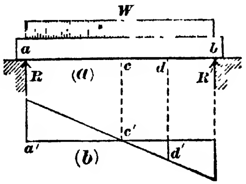

Fig. 150.

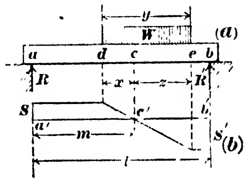

Fig. 151.

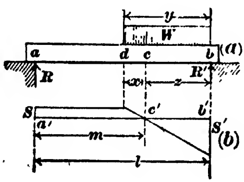

Fig. 152.

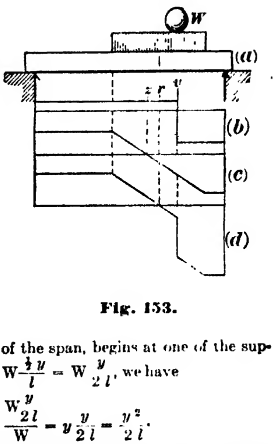

Fig. 153.

337. When the loaded portion, y, of the span, begins at one of the supports, b, Fig. 152 (a), then since $R = W\frac{\frac{1}{2}y}{l} = W\frac{y}{2\,l}$, we have

$$x = d\,c = y\cdot\frac{R}{W} = y\,\frac{W\frac{y}{2\,l}}{W} = y\,\frac{y}{2\,l} = \frac{y^2}{2\,l}.$$

338. When a concentrated load, W, Fig. 153, is added to a load uniformly distributed over the entire span, or over a part of it, each load produces the same shears as if it alone were upon the span. Those due to W are represented in Fig. 153 (b), while those due to the uniform load are represented in Fig. 153 (c). The resultant shear, due to both loads combined, is represented in Fig. 153 (d). Note that, between v and r, the addition of W, with its *positive* shear, *reduces* the *negative* shear due to the uniform load, and that, between r and z, the addition of W reverses the negative shear; also that it shifts the zero point from z to r.

For Continuous Beams, see Beams, under Strength of Materials.

Influence Diagrams.

339. The end reactions, due to a given load, and consequently the moments, shears and stresses, produced, at any given point in a span, by such load, vary as the position of the load, relatively to the supports, is changed. A diagram, Figs. 154 (*b*), 155 (*b*), 156 (*b*), showing the changes thus produced at any given point, is called an influence diagram, or influence line.*

Influence Diagram for Moments.

340. Thus, in Fig 154 (*b*), *a'Mb'* is the moment influence diagram for the point, *c*, under a single concentrated load, W.†

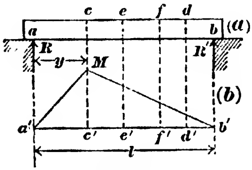

Fig. 154.

341. In Fig. 154, let l be the span, x the variable distance of the load, W,† from the right support, *b*, and y the constant distance, *a c*, of a given point, *c*, from the left support, *a*. Then, for any position of W, the left end reaction, R, is $= W \cdot \frac{x}{l}$, and the moment of that reaction about c, $= R \cdot y$, $= W \cdot \frac{x}{l} \cdot y$. The right end reaction is $R' = W \frac{l - x}{l}$, and its moment, about c, is $R' \, (l - y) = W \frac{l - x}{l} \, (l - y)$.

So long as W is between *b* and *c*, the moment at *c* is $= R \cdot y = W \cdot \frac{x}{l} \cdot y$.

342. Since W, y and l are constant, the moment, at *c*, while W is between *b* and *c*, is proportional to the variable distance, x, of the load from *b*. It therefore increases uniformly, from 0, when W is at *b*, to its maximum value, M, when W is at *c*. See ¶ 317. Hence, if the ordinate, *c'* M, be made equal, by scale, to the maximum moment, M, then the moment, at *c*, for any position, *d*, *e* or *f*, of W, between *c* and *b*, is given by the corresponding ordinate, *d'*, *e'* or *f'*, to the line *b'* M. Similarly, the moments, at *c*, for any positions of W between *c* and *a*, are given by the ordinates to the line *a'* M.

343. For the moment, at *c*, for any number of loads, in any positions, find the moment, at *c*, for each load separately, as above, and take their sum.

344. It is customary to construct the moment influence diagram for the moments of a load, W, = unity (1 ton, 1 pound, 1 thousand kilograms, etc.). Each ordinate must then be multiplied by its corresponding load, measured in the corresponding unit, in order to obtain the required moment.

345. When W is at the point, *c*, we have $x = l - y$. Hence, ordinate, c' M, = maximum moment, $= W \cdot \frac{l - y}{l} \cdot y$; or, if W = 1, $c' M = \frac{l - y}{l} \cdot y$. The area of the diagram, *a'Mb'*, is $= \frac{l}{2} \cdot c' M = \frac{l}{2} \cdot \frac{l - y}{l} \cdot y = \frac{(l - y)\, y}{2}$.

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* See "Calculation of the Stresses in Bridges for Actual Concentrated Loads," by Prof. Geo. F. Swain, "Trans. Am. Soc. C. E.," vol. XVII, July, 1887.

† Inasmuch as the load, in this discussion, occupies different positions at different times, it is not shown in Fig. 154.

346. If a load, = 1, be distributed over a length, = 1, at c, Fig. 155 (a), the resulting moment, at c, may be represented by the area of the rectangle standing on c', Fig. (b), the height of said rectangle being the ordinate, c' M, and its length = 1. Similarly, the moment, at c, due to a uniformly distributed load, $e\,f$, of 1 per unit length, Fig. (a), may be represented by the sum of the areas of the rectangles between e' and f', Fig. (b); and, if we suppose the load, $e\,f$, Fig. (a), of 1 per unit length, to be divided into a very large number of very narrow vertical strips, the resulting moment, at c, may be taken as represented by the area of the shaded trapezoid over $e'\,f'$, Fig. 155 (b). The moment, at c, due to a load of p (lbs., tons, etc.) per unit length, and occupying the same length, $e\,f$, is = p × area of trapezoid over $e'\,f'$, Fig. 155 (b).

347. Hence, the maximum moment, at c, due to a uniform load of p (lbs., tons, etc.) per unit of length, occurs when that load covers the entire span This maximum moment is = p × area a'Mb', $= p\,\dfrac{(l-y)\,y}{2}$. See ¶ 345.

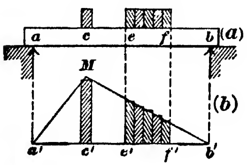

Fig. 155.

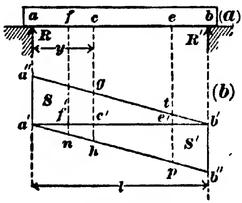

Fig. 156.

Influence Diagram for Shear.

348. Under a single concentrated load, the shear, at any point between the load and either support, is = the reaction of that support. See ¶¶ 326 and 327.

349. In the shear influence diagram, Fig. 156, as in the moment influence diagram, Fig. 154, let l be the span; x the variable distance of the load, W, from the right support, b, and y the constant distance, $a\,c$, of a given point, c, from the left support, a. Then, for any position of W, the left end reaction, R, or the shear, S, at any point between the load and the left support, is $= \mathrm{W} \,.\, \dfrac{x}{l}$; and the right reaction, R′, or the shear, S′, at any point between the load and the right support, is $= \mathrm{W} \,.\, \dfrac{l-x}{l}$.

350. The influence line for shear, like that for moments, ¶ 344, is usually constructed for a load = unity, so that $\mathrm{S} = \mathrm{R} = \dfrac{x}{l}$; and $\mathrm{S}' = \mathrm{R}' = \dfrac{l-x}{l}$. Each ordinate of the shear diagram must then be multiplied by W, in order to obtain the required shear.

351. Since W (= 1) and l are constant, R and S vary directly (and R′ and S′ inversely) with x. Thus, when W (= 1) is at b, we have $x = 0$; S = R = 0, and S′ = R′ = W = 1. When W (= 1) is at a, we have $x = l$, S = R = 1, and S′ = R′ = 0. Draw $a'\,a''$ and $b'\,b''$, each = W (= 1), and join $a''\,b'$ and $a'\,b''$. Then, with W at c, the (positive) shear, S, at each point, as f, between c and a, is given by the ordinate, $c'\,g$, to the line $a''\,b'$; while the ordinate, $c'\,h$, to the line, $a'\,b''$, gives the (negative) shear at each point, as e, between c and b.

352. Similarly, with W at e, the ordinate, $e'\,t$, gives the (positive) shear at each point, as c, between e and a; while $e'\,p$ gives the (negative) shear at each point between e and b.

353. It will be noticed that, as the load, W, passes from one side to the other of any point, as *c*, the shear at that point is reversed, the total change in shear being $= h\,c' + c'\,g = h\,g =$ the load, W.

354. With a load, W ($= 1$), at *b*, the shear at *c* is $= 0$. See ¶ 351. As the load advances from *b* toward *a*, the positive shear, $S = R = \frac{x}{l}$, at *c*, increases in proportion to the ordinates to the line $b'\,g$, becoming $= c'\,g = \frac{l - y}{l}$, when W is just to the right of *c*. With W just to the left of *c*, we have, negative shear at $c = S' = R' = c'\,h = \frac{y}{l}$. But as W proceeds from *c* to *a*, this negative shear, at *c*, decreases in proportion to the ordinates to the line $h\,a'$, becoming 0 when W reaches *a*. Thus, $a'hgb'$ is the shear influence diagram for the point, *c*. Similarly, $a'ptb'$ is the shear influence diagram for the point, *e*, etc.

355. If a series of nearly uniform and equidistant concentrated loads, such as the wheel loads of a locomotive and train, come upon the span, at the support, *b*, and advance toward *a*, the shear at *c* evidently increases until the first load reaches *c*. It is then suddenly diminished, by an amount = the first load, as that load passes *c*. It then continues to diminish, as each wheel passes over *c*, but more slowly, until the first load reaches *a*. See ¶ 358.

356. With a *uniformly distributed* load, of unity per unit length, moving as in ¶ 355, the shears at *c* (see ¶ 346) are represented by the *areas* of those portions of the diagram, $a'hgb'$, successively covered by the load, portions of the diagram below the zero line, $a'\,b'$, being taken as negative. Thus, when the head of the load reaches *e*, the (positive) shear at *c* is given by the area of the triangle, $b'\,e'\,t$. With head of load at *c*, the shear at *c* reaches its maximum, and is given by the area of triangle, $b'\,c'\,g$. With head of load at *f*, the shear at *c* is = area $b'\,c'\,g$ — area $f'\,c'\,h\,n$.

357. Similarly, the shears at *e* are given by the areas of portions of the diagram, $a'\,p\,t\,b'$.

358. Fig. 157* shows the influence diagram, 0 *d z* 14 16, for the shears at

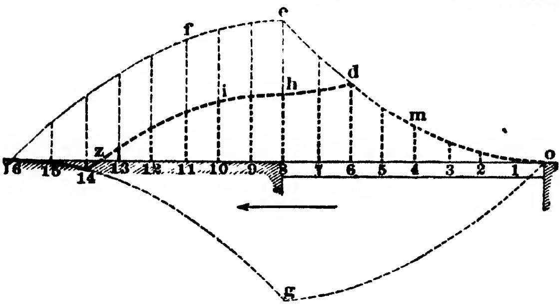

Fig. 157.

point 6, for a given uniformly distributed load, at least as long as the span, coming upon the span at point 0, passing across it, and leaving it at point 8; also the corresponding diagram, 0 *e* 16, for the left support, 8; and that, 0 *g* 16, for the right support, 0.

For the action of internal resisting forces in beams and trusses, see Transverse Strength, under Strength of Materials, and Stresses, under Trusses.

* First published in our 9th Edition, 1885.

Relation between Moment and Shear.

359. The shear, at any point in the span, is simply the **rate at which the bending moment is changing** at that point.

360. Thus, in Fig. 158, the moment, M, Fig. (*b*), at the support, *b*, due to the concentrated load, W, of 6 lbs., is = W l = 6 × 4 = 24 ft.-lbs.; but, between the support and the load, the moment is decreasing at the *uniform rate* of 6 ft.-lbs. for each foot of *x*, or 6 ft.-lbs. per foot = 6 lbs.; and this 6 lbs. is the *uniform shear*, V, Fig. (*c*), throughout the beam. Hence the shear diagram, Fig. (*c*), is a *horizontal* line; *i. e.*, its ordinates are of *equal* length.

361. Again, in Fig. 159, the shear diagram ordinates between *a*″ and *o*″, Fig. (*c*), are *positive*, showing the (algebraic) *increase* of the bending moment, M, Fig. (*b*), as we proceed from the left support, *a*, toward the center, *o*, of the span; while the *negative* shear diagram ordinates, between *o*″ and *b*″, show the (algebraic) *decrease* of the bending moment as we proceed from the center, *o*, to the right support, *b*. At the center, *o*, the rate of change of bending moment is zero, as is also the vertical shear.

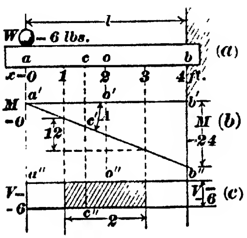

Fig. 158.

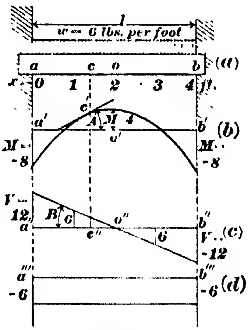

Fig. 159.

362. Both in Figs. 158 and 159, the bending moment, M, is constantly changing; but in Fig. 158 its *rate of change* (= 6 ft.-lbs. per ft. of span) is *constant*. Hence, the *moment* diagram is a *straight inclined* line, and the *shear* diagram is a *horizontal* line; whereas in Fig. 159 the rate of change of bending moment is constantly *varying*, being = 12 ft.-lbs. per foot of span (shear = 12 lbs.) at the support, and diminishing to zero at the center, *o*, of the span. Hence, in Fig. 159, the *moment* diagram, Fig. (*b*), is no longer straight, but *curved;* and the *shear* diagram, Fig. (*c*), is no longer horizontal, but *inclined*.

363. But, in Fig. 159 (*c*), the shear, V, or the *rate* of change of the bending moment (although no longer *constant*, as it was in Fig. 158 (*c*)), nevertheless *diminishes uniformly*, as we proceed from *a* toward *b*. Thus, at the point, 1, Fig. 159, midway between *a* and *o*, the bending moment is changing at the rate of 6 ft.-lbs. per foot, or *half* as fast as at *a*. Hence, the shear diagram, although no longer a *horizontal* line, is still a *straight* line; and the *uniform* decrease (= 6 ft.-lbs. per foot per foot) in the rate of change of the bending moment, or the *uniform* decrease (= 6 lbs. per foot) in shear, is indicated by the *horizontal* diagram in Fig. 159 (*d*).

364. In either Fig., let a straight line be drawn, tangential to the moment diagram (*b*), at any point, *c*′, and forming, with the horizontal zero line, *a*′ *b*′, an angle, A. (In Fig. 158, this line coincides with the moment diagram.) Then the tangent of A is given by the shear diagram ordinate, *c*″, corresponding to the point, *c;* or, for any point, V = tan A.

365. In Fig. 158, where this angle, A, Fig. (*b*), is *constant*, the shear ordinates, Fig. (*c*), are of *constant* length. In other words, the shear diagram, Fig. 158, is a *horizontal* line.

366. Since the shear diagram ordinates represent *forces* (as in lbs., etc.) and the abscissas represent *distances* (as in ft., etc.) the product of the distance between any two shear ordinates, multiplied by the mean of those ordinates, is an *area* representing a *moment* (in ft.-lbs., etc.). This moment is = the *difference* between the two moments represented by the corresponding ordinates in the moment diagram.

367. Thus, in Fig. 158 (*b*), the increase in (negative) bending moment, between points 1 and 3, is = 18 — 6 = 12 ft.-lbs.; and, in Fig. 158 (*c*), the moment represented by the (shaded) area, between the same two points, is = 2 ft. × 6 lbs. = 12 ft.-lbs. In Fig. 159, the (algebraic) increase in bending moment, Fig. (*b*), between the left support, *a*, and the center, *o*, of the span, is = 8 + 4 = 12 ft.-lbs.; and the moment, represented by the shear diagram area (triangle) between the same two points, Fig. (*c*), is

$$= \frac{1}{2} V \frac{l}{2} = \frac{12 \times 2}{2} = 12 \text{ ft.-lbs.}$$

368. Again, in Fig. 159, at any two points equally distant from the center, *o*, of the span, the moments are equal; or difference of moments = zero; and, since shear ordinates *below* the zero line, $a'' \, b''$, Fig. (*c*), are considered as *negative*, the algebraic sum of the two corresponding shear triangles, Fig (*c*), is also = zero.

Similarly, areas in Fig. 159 (*d*) correspond to differences of ordinates in Fig. 159 (*c*).

Diagram for Reactions, Shears and Moments.

369. In Fig. 58 (*b*), p. 385, *c w* and *w X* give respectively the **reactions** of the right and left supports, *w* and *x*, Fig. 58 (a).

The shear, pp. 446, &c., is constant for all the sections between any two loads. For the shear in any such section, as *d*, find, in the equilibrium polygon, Fig. 58 (*b*) the lines, *c w* and *b c*, representing the forces, *w* and *c*, to the right, or those, *w X*, *X a* and *a b*, representing the forces *x*, *a* and *b*, to the left, of the section. Then the line, *b w*, = *c w* — *b c*, or *w b* = *w X* *X a* *a b*, represents the resultant of either set of forces, and thus represents the shear in the section, *d*.

To find the bending moment in any section, as *d*, Fig. 58 (*a*); from *d* draw a vertical, cutting the lines *s r* and *n p* of the cord polygon, and let *y* = the length of the ordinate, intercepted on the vertical, between these lines.

Produce *s r* and *n p* to intersect, as at *e*. Then *e* is the point of application of the resultant, *U* = *b w*, Fig. 58 (*b*), of the two forces to the right of *d*, viz: of the load *c*, represented, in Fig. 58 (*b*) by *b c*, and the reaction at *w*, represented by *c w*; for, if the load, *c*, and the reaction at *w* were removed, we should require, for equilibrium, a vertical reaction, *b w* = *c w* *b c*; and the corresponding lines, *b O* and *w O*, in the force diagram, are respectively parallel to *n p* and *s r* in the cord polygon.

In other words, by eliminating the load, *c*, and the reaction at *w*, and substituting their resultant, *U* = *b w*, we substitute also a new cord polygon, *s m n e s*, Fig. 58 (*a*), with *U* (= *b w*) applied at *e*.

In section *d*, let *L* be the lever arm of this resultant, *U*, = the horizontal distance from *e* to *y*. In Fig. 58 (*b*), draw *O h* horizontal, representing the horizontal component of each of the stresses, *X O*, *a O*, etc. Let *H* = the length of *O h*. Then the bending moment, in section *d*, is *M* = *H y*; for *M* = *U L*; and, by similar triangles, *L* : *y* = *H* : *U*; or *U L* = *H y*.

Scale of moments. Since *M* = *H y*, we can, by choosing the position of *O** at the proper distance from *X c*, obtain any desired scale of moments by which to measure moments directly, upon *y*. Thus, suppose that a scale of 1 inch = 2 feet has been used in Fig. 58 (*a*), and that it is desired to have a moment scale of 1 inch = 50 ft-lbs. Then we need only so choose the position of *O** that its distance, *H*, from *X c*, shall represent 50 ft-lbs 2 ft = 25 lbs, by the scale of Fig. 58 (*b*). Then 1 inch, in length of *y*, will correspond to 2 ft × 25 lbs. = 50 ft-lbs.

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* See p. 378, ¶ 95.

STRENGTH OF MATERIALS.

GENERAL PRINCIPLES.

Stress.

1. Stress occurs when forces act upon a body in such a way that its particles tend to move simultaneously with different velocities or in different directions; to do which, the particles must change their relative positions. This occurs, for instance, when a body is so placed as to oppose the relative motion of two other bodies, as when a block is placed between a weight and a hor table. Here the two bodies (the wt and the table) tend to come closer together; but they cannot do so without distortion of the intervening block; and such distortion is resisted by **internal forces,** acting betw the particles of the block and tending to keep those particles in their original relative positions. The action of these internal forces is called **stress.***

2. Similarly, if a body be suspended by a long chord, and if we push or pull the body to one side, the particles, on the side acted upon, will first tend to move, and the transmission of this tendency to the remaining particles causes stress within the body.

3. For internal equilibrium, **the internal stresses must balance the external forces.** Hence, it is not unusual to apply the term, "stress," indifferently to either.

4. Let the two forces, a and b, Figs A, B, acting upon the body, o, meet at an angle, $a\ o\ b$. Then the two equal and opposite **components,** $a''\ o$ and $b''\ o$, cause compressive or tensile stress in the body, o, as in ¶ 1; while the other two components, $a'\ o$ and $b'\ o$, unite to form the resultant, $c\ o$, which, unless balanced by other forces, moves the body, o, in its own direction, causing, as in ¶ 2, another comp stress, Fig A, or tensile stress, Fig B

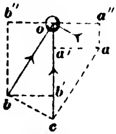
Fig. A.

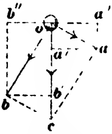
Fig. B.

5. Upon any plane within a body, a force may act **(1) normally, (2) tangentially, or (3) obliquely.** If it act obliquely, it may be resolved into two components (see Statics, ¶ 65, p 372), one acting normally and the other tangentially, upon the plane.

6. Consider the two portions into which the body is divided by such a plane Then **(1)** forces, acting **normally** upon the plane, produce **tension** (or **compression**) in the plane, tending to separate the two portions (or to push them closer together); and **(2)** forces, acting **tangentially** upon the plane, produce **shear** (or **torsion**) in the plane, tending to slide the two portions one past the other in a straight line (or with a twisting motion). **Torsion occurs** in planes betw and parallel to **two contrary couples,** as in cross sections of a hand-brake axle when the brake is applied.

7. Thus, if an iron bar be pulled (or pushed) lengthwise, its cross sections sustain normal tension (or compression). If it be sheared across (or twisted), the cross sections, between and parallel to the two shearing (or twisting) forces, sustain shearing (or torsional) stress.

8. At any point, in the circular path of a torsional stress, we may consider the tangents to the path as representing shearing forces. **Torsion** is

* In every-day language, and often in the writings of engineers, this action of the internal forces, or the external force causing it, is called "strain", but scientists apply the word "**strain**" to the *deformation* occurring under stress. See "**stretch,**" ¶¶ 11 etc.

therefore merely a **shearing** stress in which the direction changes at each point.

9. Transverse stress. In Fig 124, p 438, the two equal and parallel forces, W and R, in opposite directions, cause a tangential or *shearing* stress, $= W = R$, in the vertical planes lying between their lines of action; but W and R, as a *couple*, have a *moment*, which, for equilibrium, must be resisted by the equal and opposite moment of another couple, as C and T; and the opposition of these two couples causes normal (comp and tensile) stresses in the same vert planes parallel to and betw W and R.

10. The ultimate tendency of any opposing external forces is to fracture the body by *increasing* the distances between its particles. Even under *compressive* stress, rupture can occur only by *separation* of particles.

Stretch.

11. When the internal stresses and the external forces are in equilibrium, no distortion takes place, but, at the instant when opposing external forces are first applied to a body, the internal stresses are not yet developed, and **distortion** begins, under the unopposed action of the external forces. See ¶¶ 35 etc. But the stresses are brought into action by the distortion, and they increase with it, and, if the external force is not increased beyond the elastic limit (¶ 26) the stresses finally equal the external forces, and prevent further distortion.

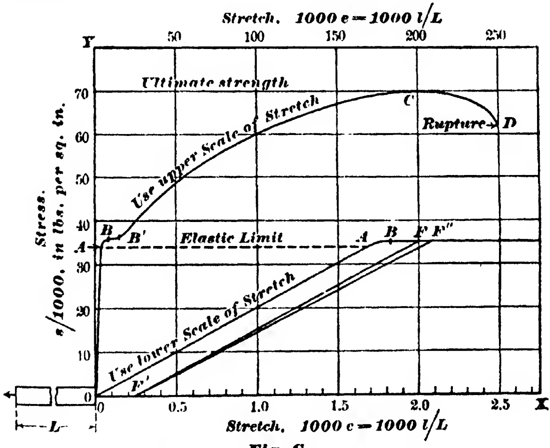

Fig. C.

Behavior under Normal Stresses.

12. Fig C represents the **behavior of a typical material** (mild steel) **under tension.** From 0 to A, i.e., under stresses up to the **elastic limit** (¶ 26), say 34,000 lbs per sq inch, the stretch progresses proportionally with the stress, as indicated by the straight line, 0 A. (The earlier portions of the process are represented, in the *lower* diagram, to a scale of stretch 100 times as great as that of the upper diagram.) After passing the point A, the stretch increases faster than the stress; and, betw B and B', the stretch (in iron and steel) increases with little or no increase of stress, or even under a slightly diminishing stress.* B is called the **yield point.** See ¶ 31. The scale of the lower diagram does not extend to B'. Beyond B' (upper diagram), the stretch increases much less rapidly than betw B

* See ¶¶ 16, 17

and B', and remains, for a time, nearly proportional to the stress* (though much greater, relatively to stretch, than in $0\ A$); but the stretch now proceeds faster and faster, and in increasing ratio with the stress, until the stress reaches its maximum or **ultimate** value (say 70,000 lbs per sq inch) at C. At C, the stretch is increasing without increase of stress (diagram horizontal), and, beyond C, the stretch continues increasing altho the stress is diminishing, until, finally, at D, **rupture** occurs.

13. If, after passing the elastic limit, the bar is relieved from stress, as at F, Fig C, lower diagram, its recovery is incomplete, the length remaining somewhat greater than in its original unstressed condition. The permanent increase, $0\ F'$, is called the **permanent set.** or simply the **set.** The line $F\ F'$ is, in general, approx parallel to the line, $0\ A$, of elastic stretch. When the same stress is again applied, the stretch is greater than before, by a small amount represented by $F'\ F''$.

14. When the stress is **within the elastic limit** (¶ 26), **the recovery**, upon release from stress, is so nearly complete that the permanent set cannot be indicated in our Figs. (¶ 28.)

15. Under *tension*, the **sec area** is *diminished*, and, under *compression increased*. In ductile materials, under tension, the reduction of sec area is very marked, especially along a relatively short portion of the length, usually near the middle of said length; and fracture occurs normally at the point of maximum reduction.

16. In Fig C, both diagrams, and, in Fig D, the *solid* curves, represent the **nominal unit stresses**, or those usually stated. These are found by dividing the total stresses, respectively, by the *original* section area, as in ¶ 18.

17. The *dotted* curves, Fig D, represent the **actual unit stresses**, found by dividing the total stresses, respectively, by the *actual* section area, as diminished or increased by stress. Under *tension*, the actual unit stresses are of course *greater*, and, under *comp*, *less* than the corresponding nominal unit stresses.

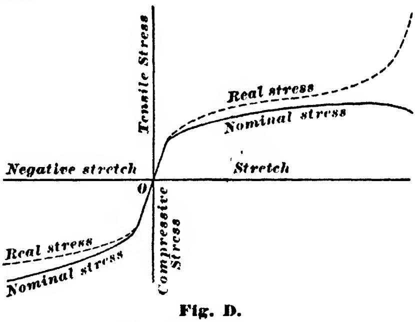

Fig. D.

Elastic Modulus. Fig. C.

18. Let P = the **load** (one of the two equal and opposite external forces) acting at one end of a bar and in line with the axis of the bar; and let a = the original* cross-section area, or **section area**, of the bar, normal to its axis. Then, s, = $P\ /\ a$, is the normal stress *per unit of area*, or stress *intensity*, or normal **unit stress**, in the bar. We assume that, so long as the external force acts axially, P is *uniformly* distributed over a, altho this is seldom strictly the case in practice.

* See ¶¶ 16, 17

19. Let L = the original length of the bar, or of some designated portion of that length, and l = the **stretch** * which takes place, in the length, l, under the action of a given unit stress, s. Then, e, = l / L, is the stretch *per unit of length*, or **unit stretch.*** corresponding to the unit stress, s.

20. In many materials, the unit stress, s, and the unit stretch, e, at first increase *proportionally*, the ratio, s/e, or unit stress ÷ unit stretch, remaining practically constant. This ratio is called the **elastic modulus**, and is designated by E, or

Elastic modulus = E = s/e = unit stress ÷ unit stretch.

20 a. The elastic modulus is thus **proportional to the tangent** of the angle, X 0 1, Fig C, the proportion depending upon the scales adopted.

20 b. The elastic modulus, E, increases with the unit stress reqd to produce a given unit stretch. Hence E is a measure of the *stiffness* of a body, i.e., of its ability to resist change of shape. "**Stiffness modulus**" would have been a better name.

20 c. If equal additions of stress could indefinitely continue producing equal additional stretches in a bar, beyond as well as within the elastic limit (¶ 26), then a stress, equal to the elastic modulus, would *double the length* of a bar when applied to it in *tension*, or would *shorten it to zero* in *compression*.

20 d. For example, within the elastic limit, a one-inch square bar of rolled steel will stretch or shorten, on an average, about $\frac{1}{30,000}$ of its length under each additional load of 1000 lbs. If it could stretch or shorten indefinitely at the rate of $\frac{1}{30,000}$ of its *original* length for each 1000 lbs. of added load, then 30,000 times 1000 lbs., or 30,000,000 lbs., (which is about the average modulus of elasticity for such bars) could either stretch the bar to double its length or reduce it to zero.

20 e. If equal infinitesimal stresses, applied to a bar, could indefinitely produce stretches, each bearing a *constant ratio to the increased length of the bar*, if in *tension*; or to the *diminished* length, if in *compression*; then the same load which would *double* the *original* length of the bar, if applied in *tension*, would reduce it to *half* its *original* length, if applied in *compression*.

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* We regard *shortening*, under *compression*, as *negative* stretch.

21. In a prismatic bar, under longitudinal tension or compression, let

W = the total load;
a = the cross section area;
$s = \frac{W}{a}$ = the unit stress = the stress per unit of area;
L = the original length;
l = the stretch*;
$e = l/L$ = the unit stretch* = the stretch* per unit of original length;
E = the elastic modulus of the material;
$r = E\,a$ = a measure of the resistance of the bar

Then

Elastic modulus $= E = \frac{W}{a} \cdot \frac{L}{l} = s/e$ .... (1)

Total load $= W = E\,a\,\frac{l}{L} = r\,e$ .... (2)

Unit stress $= s = \frac{W}{a} = E\,e$ .... (3)

Total stretch* $= l = \frac{W}{a} \cdot \frac{L}{E}$ .... (4)

$= s \cdot \frac{L}{E}$ .... (5)

Unit stretch* $= e = \frac{l}{L} = \frac{W}{a\,E} = \frac{s}{E}$ .... (6)

22. In a beam, supported at both ends and loaded at the center, let

L = length of clear span of beam;
w = weight " " " " ;
$\triangle$ = deflection " " " " ;
b = breadth of cross section of beam;
d = depth " " " " ;
I = moment of inertia " " " "

Then

$$E = \frac{(W + 5/8\,w)\,L^3}{48\,\triangle\,I} \quad \text{............} \quad (7)$$

If the beam is rectangular, $I = \frac{b\,d^3}{12}$ (p 469), and

$$E = \frac{12\,(W + 5/8\,w)\,L^3}{48\,\triangle\,b\,d^3} = \frac{(W + 5/8\,w)\,L^3}{4\,\triangle\,b\,d^3} \quad \text{............} \quad (8)$$

For beams, see also pp 480–481.

23. Reciprocal of elastic modulus. The elastic modulus, $\frac{\text{unit stress}}{\text{unit stretch}}$, indicates the *stress* required to produce a certain *distortion*

Its reciprocal, $= \frac{\text{unit stretch}}{\text{unit stress}}$, shows to what extent a bar etc of a given material must be *distorted* in order to produce a given *stress*. This may be of great importance, especially in the design of structures of timber, the elastic modulus of which is low, relatively to that of steel, and in which, therefore, a relatively great distortion must take place before a given fiber stress (such as the maximum safe fiber stress) can be brought into action Thus, in the case of a wharf, supported by long timber piles, the piles may submit to so great a lateral deflection as to give the load, resting upon them, a dangerously great horizontal leverage, and thus a dangerous overturning moment.

* *Compression* is regarded as *negative* stretch.

24. Variable elastic modulus. Fig 11, Concrete experiments 81*a* p 1340, shows an example (in both tension and compression) of a material in which the elastic modulus, *E*, is constantly changing; the stretches, from the first, increasing faster than the stresses.

25. Even in the case of ductile materials, the stretches, produced by stresses within the elastic limit (¶ 26), are so small and so irregular that a satisfactory average value of the elastic modulus can be arrived at only by comparing the results of many experiments. In the case of brittle materials, where scarcely any perceptible stretch takes place before rupture, the determination of the elastic modulus is very uncertain.

Elastic Limit.

26. The stress, 0 *A*, Fig C, beyond which the stretches in any body increase perceptibly faster than the stresses, is called its **elastic limit**, or limit of elasticity. Owing to the irregularity in the behavior of different specimens of the same material, and to the extreme smallness of the distortions caused in most materials by moderate loads, and because we often cannot decide just when the stretch begins to increase faster than the load, the elastic limit is seldom, if ever, determinable with exactness and certainty.* But by means of a large number of experiments upon a given material we may obtain useful average or minimum values for it, and should in all cases of practice keep the stresses well within such values, since, if the elastic limit be exceeded (through miscalculation, or through subsequent increase in the stress or decrease in the strength of the material) the structure rapidly fails. The table, p 460, gives approximate average elastic limits for a few materials. The elastic limit, as here defined, is sometimes called **the "true" elastic limit.** Compare ¶ 31.

27. Brittle materials, such as stones, cements, bricks, etc., can scarcely be said to have an elastic limit; or, if they have, it is almost impossible to determine it; since rupture, in such bodies, takes place before any stretch can be satisfactorily measured.

28. A small **permanent "set"** (stretch) probably takes place in all cases of stress even under very moderate loads; but ordinarily it first becomes noticeable at about the time when the elastic limit is exceeded. **The elastic limit is sometimes defined** as that stress at which the first marked permanent set appears.

29. The elastic ratio of a material is the quotient, $\frac{\text{elastic limit}}{\text{ultimate strength}}$. It is usually expressed as a decimal fraction.

The permissible working load of a material should be determined by its elastic limit rather than by its ultimate strength. Hence, other things being equal, a high elastic ratio is in general a desirable qualification; but, on the other hand, it is possible, by modifying the process of manufacture, to obtain material of high elastic ratio, but deficient in "body" or in resilience—*i. e.*, in capacity to resist the effect of blows or shocks, or of sudden application or fluctuation of stress. See ¶ 34; also ¶¶ 35 etc.

In the manufacture of steel, the elastic ratio is increased by increasing the reduction of area in hammering or rolling, and the rate of increase of elastic ratio with reduction of area increases rapidly as the reduction becomes very great. Kirkaldy found†

for steel plates	1 inch thick, mean elastic ratio	=	0.53
" " "	¾ " " " " "	=	0.53
" " "	½ " " " " "	=	0.54
" " "	¼ " " " " "	=	0.61

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* The U. S. Board appointed to test Iron, Steel, &c., found a variation of nearly 4000 lbs. per square inch in the elastic limit of bars of one make of rolled iron, prepared with great care and having very uniform tensile strength; and, in another very carefully made iron, a difference of over 30 per cent. between two bars of the same size. Report, 1881, Vol. 1, p. 31.

† Annual Report of the Secretary of the Navy, Washington, 1885, Vol. I, p. 499; and Merchant Shipping Experiments on Steel, Parliamentary Paper, C. 2897, London, 1885.

30. Elastic Moduli and Elastic Limits. Approximate averages.†

E = **elastic modulus,** in millions of pounds per square inch;

l = **stretch or compression,** in ins, in a length of 10 feet, under a load of 1000 pounds per square inch.

$= (10 \times 12 \times 1{,}000) \div (1{,}000{,}000\ E)$;

s_e = **stress at elastic limit,** in thousands of pounds per square inch.

	E	l	s_e
Metals.			
Iron, cast	10 to 30	0.012 to 0.004	4 to 8
" " ordinarily	12 to 15	0.010 to 0.008	6 to 7
" wrought *	27 to 31	0.004	20 to 40
Steel, structural*	" to "	"	34 to 38
Brass, cast	8 to 10	0.015 to 0.012	5 to 7
" wire	12 to 16	0.010 to 0.007	14 to 18
Copper, cast	10 to 14	0.012 to 0.009	6 to 7
" wire	10 to 14	0.012 to 0.009	8 to 12
Lead	0.8 to 1.0	0.150 to 0.120	1 to 1.2
Tin, cast	6 to 7	0.020 to 0.017	1.4 to 1.6
Bronzes	13 to 15	0.009 to 0.008	14 to 15
Stones, etc.†	4 to 8	0.030 to 0.015	1 to 2
Masonry †	0.5 to 2	0.240 to 0.060	Art. 4 (b)
Wood ‡	1.5 to 2	0.080 to 0.060	5 to 7

31. Yield point. Commercial, Relative or Apparent Elastic Limit. In testing specimens of iron and steel, it is commonly found that, at a stress slightly exceeding the true elastic limit (¶ 26), the stretch begins to increase without further increase of load. This point is usually called "the yield point," or "the elastic limit" in commercial testing. The French Commission on Methods of Testing the Materials of Construction called it the "apparent elastic limit." The late Prof. J. B. Johnson ("The Materials of Construction," New York, John Wiley & Sons, 1906, p. 19) applied the term, "relative or apparent elastic limit" to that point on the stress diagram at which the rate of deformation is 50 per cent. greater than at points below the true elastic limit.

Resilience.

32. The resilience of a bar, under a stress, s, is the work done, upon the bar, in producing that stress, or, theoretically, the work which the bar will do, in regaining its original shape, when relieved from stress. Usually we are concerned with the **elastic resilience,** or that corresponding to the stress, s_e *at the elastic limit.*

33. Let

s_e = the unit stress at the elastic limit;

a = the section area of the bar,

$P_e = a\ s_e$ = the load corresponding to s_e;

L = the original length of the bar;

l = its stretch, at the elastic limit;

E = the elastic modulus.

*In rolled iron and steel, the elastic modulus is remarkably constant for all grades. In wrought iron, the elastic limit depends chiefly upon the degree of reduction of cross section in rolling; the smaller sizes having the higher elastic limit. In steel, this effect is less marked.

† See ¶¶ 25, 26.

‡ In wood, "the extreme fiber stress at the true elastic limit (¶ 26) of a beam is practically identical with the compressive stress endwise of the material," table, p. 1138. See discussion by S. T. Neely, in "Timber Physics," 1889 to 1898, by Filibert Roth, House Document No. 181, 55th Congress, 3d Session, Washington. 1899, p. 374.

The work has been done by the *mean* load, $P_e/2 = a\,s_e/2$, acting thru the dist, $l = L\,s_e/E$. Hence,

Resilience $= K = P_e\,l/2 = a\,s_e\,L\,s_e\,/2\,E = (s_e^2/2\,E)\,a\,L$.

34. Here $s_e^2/2\,E$ is the

resilience modulus = resilience of a bar of unit section area and unit lgth.

The resilience modulus of a material is a measure of its capacity for resisting shocks or blows.

Suddenly applied loads.

35. Let a body, of weight, W, be suspended by a string, and let it just touch the scale-pan of a spring balance, without depressing it. Now let the string be cut with a pair of scissors.

36. At the moment of cutting, the spring has not been stretched; its resisting stress, S, is therefore zero, and the net or resultant downward force, acting upon the body, is $F = W - S = W - 0 = W$.

37. Under the action of this force, the spring stretches, and S increases proportionally with the stretch. Hence (W remaining constant) the resultant downward or accelerating force, F, acting upon the body, decreases until $S = W$, when $F = W - S = W - W = 0$.

38. The body, having thus far been constantly accelerated, (by a diminishing force, F), has constantly increased its velocity. Let h = the height thru which it has now fallen, and let x be the point reached, at the end of h.

39. Beyond x (W remaining constant, while S continues to increase), the moving body is acted upon by a constantly increasing, retarding upward force, $-F = W - S$, which brings it to rest at a second point, z, at the end of a second distance = h. Its **total fall** is therefore $2\,h$.

40. Let S max = the max value of S, or that at the end, z, of the fall, $2\,h$. Then, since S has increased proportionally with h, its *mean* value, during the fall, $2\,h$, was S max$/2$; and the work done, during the entire fall, $2\,h$, was $2\,W\,h$ = (S max/2) $2\,h$ = S max $\times\,h$. Hence,

S max = 2 W.

41. At the end, z, of the fall, $2\,h$, the body, having come to rest, is acted upon by an upward force, $-F = W - S$ max $= W - 2W = -W$; and (neglecting friction) the same performance is now repeated, but in the upward direction, and so on indefinitely.

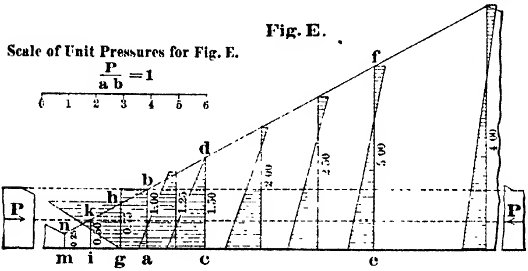
42. But losses of energy, due to air resistance and to internal friction, render each oscillation less than its theoretical value; and the body therefore finally comes to rest at the point, x, midway of the fall, $2\,h$.

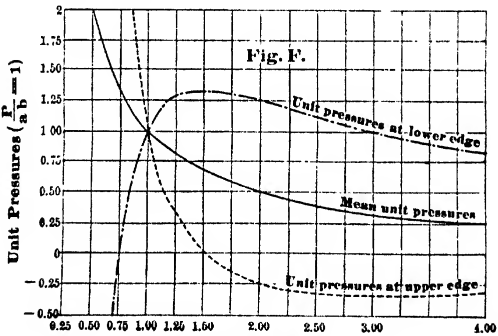
43. Thus (¶ 40), within the elastic limit, **a load, suddenly applied** (tho without shock) **produces temporarily a stretch** nearly equal to **twice that which it could produce if applied gradually:** *i.e.*, twice that which it can maintain after it comes to rest; and develops temporarily, in the stretched body, **a resisting stress = twice the load.**

44. If the load be added in small instalments, each applied suddenly, then each instalment produces a small temporary stretch, and afterward maintains a stretch half as great. Under the last small instalment of load, the spring stretches temporarily to a length greater than that which the total load can maintain, by an amount equal to half the small temporary stretch produced by the sudden application of the last small instalment.

A section may be weakened by increasing its width. On pp 400, etc., we considered the case where the width of the base is fixed and where the point of application of the resultant of the forces acting upon it is shifted to different positions along the base We will now notice the case where the resultant is applied at a constant distance from one end of the base, but where the base is of varying width, so that this constant distance may be equal to, or greater or less than, the half width of the base.

Let Fig. E represent a side view of a bar of uniform thickness = 1,* but (as





shown) of varying width, and subjected to pressures, the resultant P of which is = 1,* and passes through the center of that section $a\,b$ whose width is 1 * †

* We adopt the value 1 for the pressure P, the width $a\,b$, and the thickness merely in order to facilitate the explanation. It is not essential to the application of the principle.

† We here suppose ourselves dealing with a perfectly rigid and homogeneous material. In practice, these values would be more or less modified by yielding of the particles under stress, by unevennesses in the surfaces of the supposed cross sections, etc. Nevertheless, the general principles here laid down hold good.

The pressure per unit of area of cross section (or "unit stress") in the section ab is then $\frac{P}{ab \times 1} = \frac{P}{1} = P = 1$*, and may be assumed to be uniformly distributed over it.

But at other sections of the bar the resultant is nearer to one edge than to the other, and the unit stress can then no longer be assumed to be uniformly distributed over the cross section, but, as explained on pp 400 to 403 is a maximum at the edge nearest to the resultant, and diminishes gradually and uniformly to a minimum at the farther edge.

This is indicated by the shaded triangles, etc., in Fig. E and by the curves in Fig. F, which show, for the several sections in Fig. E, the mean unit stress* and the unit stresses at the upper and lower edges respectively, calculated by the rules on pp 401–404.

These stresses are also given in the following table:

Unit Stresses in Fig. E; the unit stress $\frac{P}{ab}$ in section ab being taken as 1.†

Section.	Width.	Stress per unit of area of cross section.		
		Mean.	At lower edge *m e*.	At upper edge *n f*.
	4.00	0.25	0.8125	− 0.3125 ‡
e f	3.00	⅓	1.00	− ⅓
	2.50	0.40	1.12	− 0.32
	2.00	0.50	1.25	− 0.25
c d	1.50	⅔	1⅓	0
	1.25	0.80	1.28	0.32
a b	1.00	1.00	1.00	1.00
g h	0.75	1⅓	0	2⅔
i k ¶	0.50	2.00	− 4 ‡	8.00
m n ¶	0.25	4.00	− 32 ‡	40.00

It is important to notice that for a given force P, and for widths less than 3 ab, the strongest section of this bar is *not the widest one*, but that (ab) at which the resultant P passes through the center of the section. In other words, **a bar may be weakened by additions to its cross section** if those additions are such as to cause the resultant of the pressures to pass elsewhere than through the center of any cross section. This fact is entirely independent of the *weight* of the added portion.

Among the sections wider than ab, the weakest is that (cd) whose width is = 1.5 ab. At that section the lower edge me has its maximum unit stress $\left(= 1\frac{1}{3} \times \frac{P}{ab}\right)$ while at d in the upper edge there is no pressure. Beyond cd the upper edge nf is in *tension* ‡ and the unit pressure along me decreases, becoming again $= \frac{P}{ab}$ at ef, where the width ef is $= 3\,ab$, and decreasing still further with further increase in width.

---

* In the case discussed on pp 400 to 403, the mean pressure $un = \frac{P}{uv}$, remained constant so long as the entire surface uv was called into play. Here, on the contrary, the area of the section varies. Hence the mean unit pressure varies also, and inversely as the area

† See foot-note *, p. 462.

‡ In the present discussion, as well as in that on pp. 400 to 403, we have assumed cases of *compression* for illustration, but the principle involved applies equally to cases where the force applied is *tensile*. In such cases, however, the terms "pressure" and "tension" are of course reversed

¶ The unit stresses at the edges in section ik are too great to be shown conveniently in either figure; while those in section mn (as the table shows) far exceed the limits of the figures. The pressure at k would be $\frac{P}{0} = \infty$ (infinity) were it not for the tensions in the lower part of the section.

When the width becomes *less* than that at $a\,b$, as at $g\,h$, etc., the *upper* edge of the bar comes nearer to the resultant than the lower edge, and hence receives the maximum pressure.

When the width $= \frac{3}{4}\,a\,b$, as at $g\,h$, the distance of the resultant from the upper edge is $\frac{1}{3}$ the width of the section. The pressure at the lower edge is then $= 0$; the mean pressure in $g\,h$ is $\frac{P}{a\,b} \times \frac{1}{0.75} = 1\frac{1}{3} \times \frac{P}{a\,b}$, and the pressure at the upper edge is twice the mean pressure in $g\,h$, or $2\frac{2}{3} \times \frac{P}{a\,b}$.

When the width becomes less than $\frac{3}{4}\,a\,b$, as at $i\,k$ and $m\,n$, the pressure at the lower edge $m\,e$ becomes negative or tensile.* Thus, when, as at $i\,k$, the width is $= \frac{1}{2}\,a\,b$, and the resultant passes through the upper edge, the unit pressure at that edge is $= 8 \times \frac{P}{a\,b}$, while the lower edge sustains a *tension* of $4 \times \frac{P}{a\,b}$; and, as the section is further reduced, these stresses are still further and very rapidly increased.

The condition of those sections (such as $m\,n$) where the line of the resultant passes outside the section, is similar to that of the section $m\,n$ of a bent hook sustaining a load, as in Fig. G.

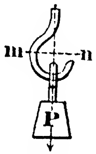


Fig. G.

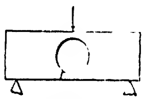


Fig. H.

Messrs. William Sellers & Co., of Philadelphia, had occasion to test a number of cast-iron beams, each having a large circular opening, as in the annexed figure. These beams broke, not at the smallest section directly under the center of the opening, but a little to one side, where the section was deeper, as indicated in Fig. H.

Fatigue of Materials. In the following articles on Strength of Materials, the ultimate or breaking load is that which will, during its first application, rupture the given piece within a *short* time. But Wohler's and Spangenberg's experiments show that a piece may be ruptured by **repeated applications** of a load much *less* than this; and that the oftener the load is applied the less it needs to be in order to produce rupture. Thus, wrought iron which required a tension of 53000 lbs per sq inch to break it in 800 applications, broke with 35000 lbs per sq inch applied about 10 million times; the stress, after each application, returning to zero in both cases.

The *diff* between the maximum and minimum tension in a piece subjected to tension only, or between the max and min compression in a piece subjected to comp only; or the *sum* of the max tension and max comp in a piece subjected alternately to tension and comp; is called the **range of stress** in the piece. Stresses alternating between 0 and any point within the elastic limit may be repeated many million times without producing rupture.*

For a given number of applications, the load required for rupture is least when the range of stress is greatest. If the stress is alternately comp and tension, rupture takes place more readily than if it is always comp or always tension. That is, it takes place with a less range of stress applied a given number of times, or with a less number of applications of a given range of stress. For a given range of stress and given number of applications, the most unfavorable condition is where the tension and comp are equal.

The fatigue of materials is taken into consideration in designing members of important structures subject to moving loads. See Truss specifications, pp 760, 761.

Experiments show that materials may fail under a **long continued stress** of much less intensity than that produced by the ult or bkg load.

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* This does not always hold in cases where the elastic limit has been artificially raised by process of manufacture, etc. Oft-repeated alternations between tension and compression below such a limit reduce it to the natural one. A slight flaw may cause rupture under comparatively few applications of a range of stress but little greater, or even less, than the elastic limit. Rest between stresses increases the resisting power of a piece. In many cases, stresses a little beyond the elastic limit, even if oft-repeated, raise that limit and the strength but render the piece brittle and thus more liable to rupture from shocks; and a little further increase of stress rapidly lessens, or may entirely destroy, the elasticity. A *tensile* stress above the elastic limit greatly lowers, or may even destroy, the *compressive* elasticity, and vice versa. If a tensile stress, by stretching a piece, reduces its resisting area, it may thus reduce its *total* strength, even though the strength *per sq in* has increased. Mr B Baker finds that hard steel fatigues much faster under repeated loads than soft steel or iron.

TRANSVERSE STRENGTH.

1. In Statics, ¶¶ 285, etc., we discuss the action of external or destructive forces upon cantilevers, beams and trusses. We here discuss the reaction of the internal or resisting forces (stresses) in solid cantilevers and beams, in order to determine their loads. See also ¶¶ 104, etc.

2. Unless otherwise stated or apparent, we assume that the stresses in all parts of the cantilever or beam are within the elastic limit.

Conditions of Equilibrium.

3. For equilibrium, the internal forces, and their moments, must balance the external forces and their moments. In other words, if the cantilever or beam be supposed cut by a section at any point, we must have

(1) Σ vertical forces = 0
(2) Σ horizontal forces = 0
(3) Σ moments = 0

Or:

(1) Algebraic sum of the internal vertical *stresses* = algebraic sum of the external vertical *forces* on either side of the section;

(2) Sum of horizontal *tensile* stresses = sum of horizontal *compressive* stresses; and

(3) Algebraic sum of the moments of the internal *stresses* = algebraic sum of moments of external *forces* on either side of the section.

4. Cantilevers and beams of uniform cross-section have usually a superabundance of strength against shearing, and fail (if at all) where the bending moment is greatest. Hence the discussion of their resistance turns principally upon equilibrium of *moments*. For their resistance to vertical shear, see Statics, ¶¶ 325, etc., and p. 499. See also Horizontal Shear, ¶¶ 51 to 53, below.

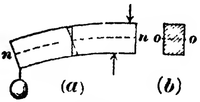


Fig. 1.

5. For equilibrium, therefore, the resisting moment, R (= the sum of the resisting moments, r, of all the particles in any cross-section of the cantilever or beam, Fig. 1 or 2), must be equal to the bending moment, M, or algebraic sum of the moments of all the external forces on either side of the section.

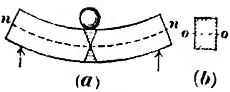


Fig. 2.

Reactions of Fibers.

6. In a *truss* or *framed* beam (see Trusses) the resistance of each of its two chords is regarded as acting in a line passing through the centers of gravity of the cross-sections of the chord; but, in a solid cantilever, Fig. 1, or beam, Fig. 2, the total resisting moment is the sum of the separate resisting moments of the several fibers throughout the cross-section.

Neutral Surface. Neutral Axis.

7. When a cantilever (or beam) bends, the fibers in the upper (or lower) part of each cross-section are extended, while those in the lower (or upper)

part are compressed (see Figs. 1 and 2); the extension and compression being greatest at the top and bottom of the section, and thence decreasing uniformly inward toward a surface, *n n*, Figs. (*a*), near the center of the cross-section. In this surface, which is called the **neutral surface**, the fibers are neither extended nor compressed. The line, *o o*, Figs. (*b*), formed by the intersection of the neutral surface with any cross-section of the cantilever or beam, is called the **neutral axis** of that section.

8. In order that the algebraic sum of all the horizontal stresses in the cross-section may be zero, as required for equilibrium, the neutral axis must pass through the center of gravity of the section. Hence, the neutral surface passes through the centers of gravity of all the cross-sections.

9. The neutral axis may be found by balancing the section (cut out of cardboard) over a knife-edge. Or see Center of Gravity, under Statics, ¶¶ 125, etc. Every section has an indefinite number of neutral axes, all passing through its center of gravity in as many different directions. The axis required, in any given case, is that one which is normal to the plane of the bending moment under consideration.

In the following discussion, we assume that the neutral axis of the section is normal to the line of action (usually vertical) of the load, as it generally is. For other cases, as, for instance, the case of roof purlins, see "The Determination of Unit Stresses in the General Case of Flexure," by Prof. L. J. Johnson, Boston Soc. of Civil Engineers, in Jour. Ass'n of Eng'ng Socs., vol. XXVIII, No. 5, May, 1902.

Resisting Moment. Unit Stress.

10. It is assumed that the extension or compression of each fiber, and therefore the resisting force actually exerted by it, is proportional to its vertical distance, *t*, above or below the neutral axis.

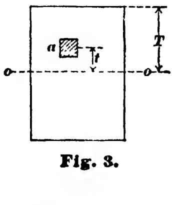


Fig. 3.

In Fig. 3, let

T = the distance from the neutral axis, *o o*, to the fiber farthest from that axis, either above or below the axis;
S = the unit stress in said farthest fiber;
t = the distance from the neutral axis to any given fiber;
s = the unit stress in said given fiber;
a = the area of said given fiber;
F = the total stress in said given fiber;
r = the resisting moment of said given fiber about the neutral axis;
M = bending moment at the cross-section under consideration;
R = the resisting moment of the entire cross-section;
= Σr = the sum of the resisting moments of all the fibers;
I = the moment of inertia of the cross-section. See ¶¶ 14, etc.;
= $\Sigma t^2 a$ = the sum, for all the fibers, of $t^2 a$;
X = the section modulus, $= \frac{I}{T} = \frac{R}{S}$. See ¶¶ 25, etc.

Then the unit stress, in any given fiber, is $= s = S \frac{t}{T}$; its total stress, F, is $= a s = S a \frac{t}{T}$; and its resisting moment, *r*, is $= F t = S a \frac{t^2}{T}$. Hence, the resisting moment, R, of the entire section, is

$$R = M = \Sigma r = \Sigma S a \frac{t^2}{T} = \frac{S}{T} \Sigma t^2 a = \frac{S}{T} \cdot I.$$

Hence also, $S = MT/I$; $I = MT/S = TX$; $T = SI/M$.
Since $S/s = T/t$, we have $S/T = s/t$, and

$$R = M = SI/T = sI/t = SX.$$

In beams of rectangular section (p 469), of breadth, B, and depth, D, we have $I = BD^3/12$, and $T = D/2$. Hence,

$$S = 12\,MT/BD^3 = 6\,M/BD^2; \text{ and}$$
$$R = M = SBD^3/12\,T = SBD^2/6$$

When beams are tested to *destruction*, the value attained by S is called **the Rupture modulus,** or modulus of rupture.

11. It will be noticed that the strengths of similar beams of any shape, and those of rectangular beams, whether similar or not, are directly proportional to the product, width $\times$ square of depth. See ¶ 63.

12. When the stress, S, upon the extreme fibers, is = the elastic limit of the material, failure is imminent. The permissible unit stress is usually taken as not more than half the elastic limit, and the safe load is that under which S does not exceed the permissible unit stress.

13. The same quantity of material that composes a solid beam, Fig. 2, would present greater resistance to bending or breaking if it were cut in two lengthwise along the neutral surface, *n n*, and converted into top and bottom chords of a truss; because, first, the *leverage* with which the resistance acts is thus greatly increased; and, second, the depths of the chords are so small, compared with their distances from the neutral axis, that their fibers may be assumed to act *unitedly* and *equally*. Hence, practically, *all* the fibers in the upper chord must be crushed, or *all* those in the lower pulled apart, *at the same instant*, before the truss can give way; whereas, in the solid beam, the extreme upper or lower fibers yield first; then those next to them, and so on, one after the other.

Moment of Inertia.

14. Unlike the moment of a force, which is the product of a force and a distance, the moment of inertia, being the sum of the products of areas of fibers by the squares of their distances from the neutral axis, is a purely *geometrical* quantity. Thus, the moment of inertia of a given section depends solely upon the dimensions and shape of that section, and is independent of the material and the span of the beam and of the manner in which it is supported or loaded.

Unit of Moment of Inertia. The moment of inertia of a figure being the product of an area by the square of a distance, its unit is the fourth power of a unit of length. Thus, in a rectangle 3 ins. wide and 4 ins. deep, $I = \frac{b\,d^3}{12} = \frac{3 \times 64}{12} = \frac{192}{12} = 16$ biquadratic inches = 16 inch^4.

In a rectangle 1 inch wide and 6 ins. deep, $I = \frac{1 \times 6^3}{12} = 18$ inch^4.

15. Comparing similar sections of any shape, their moments of inertia are proportional to the product, breadth $\times$ cube of depth. Compare ¶ 11.

16. The following illustrated table, pp. 469–471, gives, for several figures of frequent occurrence,

(1) I = the moment of inertia $= \Sigma\, t^2 a$;

(2) T = the distance from the neutral axis to the farthest fiber;

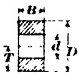
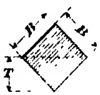
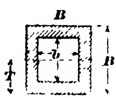
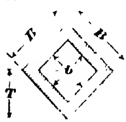
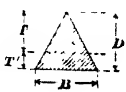
(3) X = the section modulus $= \frac{I}{T} = \frac{\Sigma\, t^2 a}{T} = \frac{R}{S} = \frac{M}{S}$;

(4) A = the area of the cross-section.

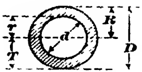
17. In sections where the distance from the neutral axis to the lowermost fiber, and the corresponding section modulus, differ from those (T and X) pertaining to the uppermost fiber, those corresponding to the lowermost fiber are distinguished as T′ and X′ respectively.

18. In each figure the neutral axis is indicated by a horizontal line crossing the section.

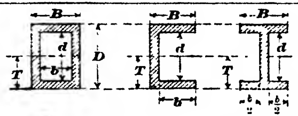
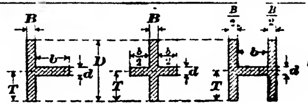
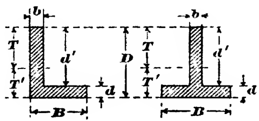
Moments of Inertia, etc.

		I Moment of inertia	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus	A Area of Section.
1		$\frac{BD^3}{12}$	$\frac{D}{2}$	$\frac{BD^2}{6}$	BD
2		$\frac{B(D^3 - d^3)}{12}$	$\frac{D}{2}$	$\frac{B(D^3 - d^3)}{6D}$	B (D - d)
3		$\frac{B^4}{12}$	$\frac{B}{2}$	$\frac{B^3}{6}$	B^2
4		$\frac{B^4}{12}$	$\frac{B}{\sqrt{2}}$	$\frac{\sqrt{2}}{12} B^3$ $= 0.118 B^3$	B^2
5		$\frac{B^4 - b^4}{12}$	$\frac{B}{2}$	$\frac{B^4 - b^4}{6B}$	$B^2 - b^2$
6		$\frac{B^4 - b^4}{12}$	$\frac{B}{\sqrt{2}}$	$\frac{\sqrt{2}}{12} \frac{B^4 - b^4}{B}$ $= 0.118 \frac{B^4 - b^4}{B}$	$B^2 - b^2$
7		$\frac{BD^3}{36}$	$T = \frac{2}{3} D$ $T' = \frac{1}{3} D$	$X = \frac{BD^2}{24}$ $X' = \frac{BD^2}{12}$	$\frac{BD}{2}$

Moments of Inertia, etc. continued. ($\pi = 3.14159$)

		I Moment of inertia	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus	A Area of Section
8	 Circle	$\frac{\pi R^4}{4} = 0.7854\ R^4$ $\frac{\pi D^4}{64} = 0.0491\ D^4$	$R = \frac{D}{2}$	$\frac{\pi R^3}{4} = \frac{\pi D^3}{32}$	$\pi R^2 = \frac{\pi D^2}{4}$
9	 Hollow Circle	$\frac{\pi(R^4 - r^4)}{4} = \frac{\pi(D^4 - d^4)}{64}$	$R = \frac{D}{2}$	$\frac{\pi(R^4 - r^4)}{4R} = \frac{\pi(D^4 - d^4)}{32D}$	$\pi(R^2 - r^2) = \frac{\pi(D^2 - d^2)}{4}$
10	 Semicircle	$\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4 = 0.1098\ R^4$	$T = 0.5756\ R$ $T' = 0.4244\ R$	$X = 0.1908\ R^3$ $X' = 0.2587\ R^3$	$\frac{\pi R^2}{2} = \frac{\pi D^2}{8}$
11	 Ellipse	$\frac{\pi B D^3}{64}$	$\frac{D}{2}$	$\frac{\pi B D^2}{32}$	$\frac{\pi B D}{4}$

Moments of Inertia, etc. continued.

		I Moment of inertia	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus	A Area of Section
12		$\frac{BD^3 - bd^3}{12}$	$\frac{D}{2}$	$\frac{BD^3 - bd^3}{6D}$	$BD - bd$
13		$\frac{BD^3 + bd^3}{12}$	$\frac{D}{2}$	$\frac{BD^3 + bd^3}{6D}$	$BD + bd$
14		$I = \frac{B\left[T'^3 - (T'-d)^3\right] + b\left[T^3 + (T'-d)^3\right]}{3}$ $T' = \frac{Bd^2 + bd'(D+d)}{2[BD - (B-b)d']}$, $X' = \frac{I}{T'}$, $T = D - T'$, $X = \frac{I}{T}$,			$Bd + bd'$

19. The moment of inertia of any figure, about its neutral axis, is the sum of the moments of inertia of its several parts, about that same axis.

20. Let I = the moment of inertia of the entire figure about its neutral axis, *o o*;

i = the moment of inertia of any part, about the neutral axis, *o o*, of the entire figure;

m = the moment of inertia of that part, about its own neutral axis;

a = the area of that part;

t = the distance of its center of gravity from the neutral axis, *o o*, of the entire figure.

Then $I = \Sigma i$; and $i = m + a t^2$.

21. Thus, in Fig. 4,

$$i_1 = \frac{B D^3}{12} + B D t_1^2;$$

$$i_2 = \frac{b d^3}{12} + b d t_2^2;$$

and $I = i_1 + i_2$.

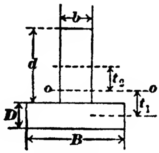


Fig. 4.

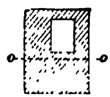


Fig. 5.

22. Hence, in any hollow section, as in the hollow rectangle, Fig. 5, let I' = the moment of inertia of the whole figure (including both the shaded and the unshaded rectangles), i = that of the missing or unshaded rectangle, and I = that of the shaded portion; all referred to the neutral axis, *o o*, of the *shaded portion*. Then $I = I' - i$.

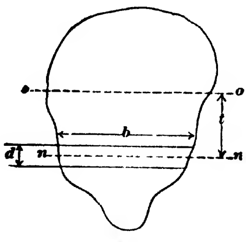


Fig. 6.

23. In the case of an irregular section, as Fig. 6, let the section be divided into numerous strips, parallel to the neutral axis and narrow enough to be considered as rectangular; and proceed as in ¶¶ 19 to 21.

24. The narrower the strips are taken, the less m becomes. If the strips be taken so narrow (relatively to the depth of the section) that m may be neglected, then $I = \Sigma\, t^2\, a$, as in ¶ 10. The strips need not be of uniform width.

The Section Modulus.

25. Definition. If the resisting moment, $R = \frac{S}{T} \cdot \Sigma\, t^2\, a$, be divided by the unit stress, S, in the extreme fibers, the quotient, $X = \frac{R}{S} = \frac{\Sigma\, t^2\, a}{T} = \frac{I}{T}$, is called the Section Modulus. This, like the moment of inertia, ¶¶ 14, etc., is a purely *geometrical* quantity, depending solely upon the dimensions and shape of the section, and being independent of the material, of the span, and of the manner of loading.

26. Having the section modulus, X, we have only to multiply it by the unit stress, S, in the extreme fibers, in order to obtain the resisting moment, R, or $R = S\, X$.

27. Multiplying the section modulus, X, by the distance, T, from the neutral axis to the farthest fibers, we obtain the moment of inertia, I; or, $I = T\, X$.

28. The section modulus is usually given in tables of rolled beams, channels and shapes. See tables of Carnegie Beams, etc.

Loading. Strength.

29. The following illustrated table gives (1) the max moment, M, corresponding to a given load, W; and (2) the load, W,* corresponding to a given unit stress, S, for different conditions of support and of loading. In this table,

M = maximum bending moment;
R = M = resisting moment of cross section;
W = the total extraneous load* on the beam, whether concentrated at one point (as shown) or uniformly distributed over the span;
l = the span;
S = the unit stress, in the fibers farthest from the neutral axis, due to the extraneous load, W, *
T = the distance from the neutral axis to the farthest fibers;
I = the moment of inertia.

In rectangular beams,

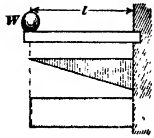
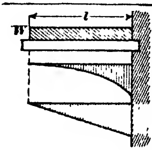
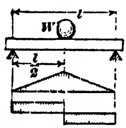
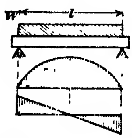
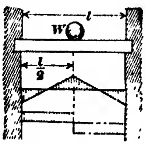
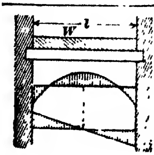
b = breadth;
d = depth;
I = moment of inertia $= \frac{b\, d^3}{12}$;
$n = \frac{W\, l}{S\, b\, d^2}$.

Of the two diagrams under each loading, the first represents the moments, and the second the shears, in the several parts of the span.

30. If S = the permissible unit fiber stress, then, in the foregoing formulas, W = the permissible extraneous load.*

31. It will be noticed that the strengths of similar beams are proportional to their values of $\frac{b\, d^2}{l}$; *i. e.*, the strengths of beams of similar cross-sections are directly proportional to their breadths and to the squares of their depths, and inversely proportional to their spans.

* The beam is here supposed to be without weight. See ¶¶ 42 etc.

For symbols, see opposite page.	$M = R$ Max. Bending moment	Load, W, and Stress, S. $S = MT/I = RT/I$ General	In rectangular beams	n
	At Support $M = Wl$	$W = \frac{M}{l} = S \cdot \frac{I}{Tl}$	$W = \frac{S}{12} \cdot \frac{bd^3}{Tl} = \frac{S}{6} \cdot \frac{bd^2}{l}$	$\frac{1}{6}$
		$S = \frac{WTl}{I}$	$S = \frac{12\,WTl}{bd^3} = \frac{6\,Wl}{bd^2}$	
	At Support. $M = \frac{Wl}{2}$	$W = 2\frac{M}{l} = 2S \cdot \frac{I}{Tl}$	$W = \frac{S}{6} \cdot \frac{bd^3}{Tl} = \frac{S}{3} \cdot \frac{bd^2}{l}$	$\frac{1}{3}$
		$S = \frac{WTl}{2I}$	$S = \frac{6\,WTl}{bd^3} = \frac{3\,Wl}{bd^2}$	
	At Center $M = \frac{Wl}{4}$	$W = 4\frac{M}{l} = 4S \cdot \frac{I}{Tl}$	$W = \frac{S}{3} \cdot \frac{bd^3}{Tl} = \frac{2S}{3} \cdot \frac{bd^2}{l}$	$\frac{2}{3}$
		$S = \frac{WTl}{4I}$	$S = \frac{3\,WTl}{bd^3} = \frac{3\,Wl}{2bd^2}$	
	At Center $M = \frac{Wl}{8}$	$W = 8\frac{M}{l} = 8S \cdot \frac{I}{Tl}$	$W = \frac{2S}{3} \cdot \frac{bd^3}{Tl} = \frac{4S}{3} \cdot \frac{bd^2}{l}$	$\frac{4}{3}$
	At Center and at Support $M = \frac{Wl}{8}$	$S = \frac{WTl}{8I}$	$S = \frac{3\,WTl}{2bd^3} = \frac{3\,Wl}{4bd^2}$	$\frac{4}{3}$
	At Support. $M = \frac{Wl}{12}$	$W = 12\frac{M}{l} = 12S \cdot \frac{I}{Tl}$	$W = S \cdot \frac{bd^3}{Tl} = 2S \cdot \frac{bd^2}{l}$	2
		$S = \frac{WTl}{12I}$	$S = \frac{WTl}{bd^3} = \frac{Wl}{2bd^2}$	

Symbols in Table Opposite.

M = maximum bending moment;
R = M = resisting moment of cross section;
S = unit stress, due to W, in the extreme fibers;
T = distance from the neutral axis to the extreme fibers;
I = moment of inertia;
W = load; l = span.
In rectangular beams, b = breadth; d = depth; $n = \frac{W\,l}{S\,b\,d^2}$; $W = n\,S\,\frac{b\,d^2}{l}$.

Beam 1 Inch Square, 1 Foot Span.

32. In a beam, 1 inch square and 1 foot (12 inches) span, supported at both ends, we have, for the extraneous center load:*

$$W' = S \cdot \frac{2\,b\,d^2}{3\,l} = \frac{n\,S\,b\,d^2}{l} = S \cdot \frac{2}{3 \times 12} = \frac{S}{18}; \text{ and } S = 18\,W'.$$

33. For any other rectangular beam, let L = the span in feet. Then the extraneous center load,* W, required to produce the same unit stress, S, in the extreme fibers, is

$$W = W' \frac{b\,d^2}{L}.$$

34. Thus, for yellow pine, let S = the permissible unit stress = 1620 lbs. per sq. in. Then, for a beam 1 inch square, 1 foot span, supported at both ends and loaded at center, the permissible load,* W', is

$$W' = \frac{n\,S}{12} = \frac{S}{18} = \frac{1620}{18} = 90 \text{ lbs.},$$

and, for a joist, 3 × 12 ins., 20 ft. span; the permissible extraneous * center load is

$$W = W' \frac{b\,d^2}{L} = 90 \times \frac{3 \times 144}{20} = 1944 \text{ lbs.};$$

and the permissible extraneous uniform load is = 2 W = 2 × 1944 = 3888 lbs.

35. If the load, W the span, L, and the coefficient, W', are given, we have $b\,d^2 = \frac{W\,L}{W'}$. Thus, in the case of the yellow pine joist, mentioned in ¶ 34, of 20 ft span, with a uniform extraneous* load, where 2 W = 2 × 1944 = 3888 lbs., we have $b\,d^2 = \frac{W\,L}{W'} = \frac{1944 \times 20}{90} = 432$.

36. Then, if either b or d is given, the other is easily found. If not, assign to either of them, an arbitrary value. Thus, if $b = 6$, we have $d^2 = \frac{432}{6} = 72$; and $d = \sqrt{72}$ = say $8\frac{1}{2}$. With $b = 3$, $d^2 = 144$, and $d = 12$.

37. With the slide rule, in the foregoing example, place the runner at 432, and, assuming $b = 6$, place 1 (or 10) on the slide, opposite 6 on the rule Then, in the scale of square roots, on the slide, opposite 432 on the rule, will be found 848 and 268. The former of these represents the desired root, and we take 8.5 as a sufficient approximation.

38. If the relation, S = 18 W, held beyond the elastic limit, and if W' = the center *breaking* load, in lbs., on a beam of any material, 1 inch square, 1 ft. span, supported at each end; then, for any other beam of the same material, and of breadth b ins., depth d ins., and span L ft., the center *breaking* load would be $W = W' \frac{b\,d^2}{L}$.

39. Notwithstanding the defective basis of this method, as applied to loads beyond the elastic limit, its simplicity renders it very convenient, and it is much in use. See the following table of values of W', and example, ¶ 40.

---

* The beam is here supposed to be without weight. For the weight of the beam itself, see ¶¶ 42, etc.

Center Breaking Loads, W', in Pounds, for Beams 1 Inch Square, 1 Foot Span, Supported at Each End.

In wooden beams in practice deduct one-third part, to allow for knots, crooked grain, &c.

WOODS.	W'
Ash, English	650
" Amer White (Author).	650
" Swamp	400
" Black	600
Arbor Vitæ, Amer	250
Balsam, Canada	350
Beech,	
" Amer	850
Birch, Amer Black	550
" Amer Yellow	850
Cedar, Bermuda	400
" Guadaloupe	600
" Amer White, or Arbor Vitæ	250
Chestnut	450
Elm, Amer White	650
" Rock, Canada	800
Hemlock	500
Hickory, Amer	800
" " Bitter nut	800
Iron Wood, Canada	600
Locust	700
Lignum Vitæ	650
Larch	400
Mahogany	750
Mangrove, White	650
" Black	550
Maple, Black	750
" Soft	750
Oak, English	550
" Amer White (by Author).	600
" " Red, Black, Basket	850
" Live	600
Pine, Amer White (by Author)	450
" " Yellow " "	500
" " Pitch " "	550
" Georgia	850
Poplar	550
Poon	700
Spruce (by Author).	450
" Black	550
Sycamore	500
Tamarack	400
Teak	750
Walnut	550
Willow	350
METALS.	
Brass	850
Iron, cast, 1500 to 2700 ...average	2100
" " common pig	2000
" " castings from pig	2300
" " employed in our tables	2025
" " for castings 2½ or 3 ins thick	1800?
Iron, wrought, 1900 to 2600 ....av	2250
Wrought iron does not *break*; but at about the average of 2250 lbs its elas limit is reached.	
Steel, hammered or rolled; elas destroyed by 3000 to 7000	5000
Under heavy loads hard steel snaps like cast iron, and soft steel bends like wrought iron.	
STONES, ETC.	
Blue stone flagging, Hudson River	125
Brick, common, 10 to 30 average	20
" good Amer pressed, 30 to 50 .....average	40
Caen Stone	25
Concrete, see article on Concrete.	
Granite, 50 to 150 ....average	100
" Quincy	100
Glass, Millville, N Jersey, thick flooring ..(by Author).	170
Mortar, of lime alone, 60 days old	10
" 1 measure of slacked lime in powder, 1 sand	8
" 1 measure of slacked lime in powder, 2 sand	7
Marble, Italian, White (Author)	116
" Manchester, Vt, " "	95
" East Dorset, Vt, " "	111
" Lee, Mass, " "	86
" Montg'y Co, Pa, Gray "	103
" " " Clouded "	142
" Rutland, Vt, Gray "	70
" Glenn's Falls, N.Y. Black "	155
" Baltimore, Md, white, coarse "	102
Oolites, 20 to 50	35
Sandstones, 20 to 70 .....average	45
" Red of Connecticut and New Jersey	45
Slate, laid on its bed, 200 to 450, av	325

40. Example. In the yellow pine joist of ¶¶ 34 and 36, 3 × 12 ins., 20 ft. span, we have, from the table, W′ = say 500 lbs. Hence

Center breaking load * $W = W' \frac{bd^2}{L} = \frac{500 \times 3 \times 144}{20} = 10{,}800$ lbs., or about 5.5 times the permissible load, found, by means of the permissible unit stress, in ¶ 34.

Dimensions.

41. Since $W = n\,S\,\frac{bd^2}{l} = W'\,\frac{bd^2}{L}$, and $W' = \frac{n\,S}{12}$, we have *

$$\textbf{Breadth} = b = \frac{W\,l}{n\,S\,d^2} = \frac{W\,L}{W'\,d^2};$$

$$\textbf{Depth} = d = \sqrt{\frac{W\,l}{n\,S\,b}} = \sqrt{\frac{W\,L}{W'\,b}};$$

where W = extraneous load * required;
W′ = extraneous load on beam 1 inch square, 1 ft. span;
n = coefficient from last column of table = $\frac{W\,l}{S\,b\,d^2}$;
S = unit fiber stress;
l = span in inches;
L = span in feet.

Weight of Beam Considered as Load.

42. For simplicity we have hitherto regarded our cantilevers and beams as having no weight of their own; and, in beams of the moderate dimensions usually employed in buildings, their own weight, w, is so small, in comparison with their loads, W, that it may often be safely neglected; but in larger beams it must generally be taken into account. The loads, found as above, with S = greatest permissible unit stress, must then be regarded as including not only the extraneous load, W, but also the weight, w, of the beam itself, for a length = span.

43. If the beam is prismatic,—*i. e.*, of uniform cross-section,—its weight, w, acts as a uniformly distributed load, and we have, for the extraneous load, W, in the case of a concentrated center load on a beam, or of a concentrated load at the end of the span, l, in cantilevers,

$$W = \text{whole load} - \frac{w}{2};$$

in the case of a uniform load,

$$W = \text{whole load} - w.$$

44. In finding the breadth or the depth of a rectangular beam, required to carry a given load with a given span and given unit stress, we may provide for the weight by successive approximations. Thus,

45. To find the breadth, b, required for a beam of given depth, d. Neglecting the weight, w, of the beam, find the first approximate breadth, b, by the formulas in ¶ 41, for the extraneous load, W. Next, calculate the weight, w, of a beam with width, b, treat said weight as a uniform load; and, by the same formulas, find the additional breadth, b', required to carry this additional load, w. Then $b + b'$ = a second approximate breadth. If necessary, find the weight, w', of a beam of breadth, b', and, from this, a second additional breadth, b'', required to carry it. Then $b + b' + b''$ = a third approximate breadth, and so on.

46. To find the depth, d, required for a beam of given breadth, b, find a first approximate depth, d, by the formula, ¶ 41, for the extraneous load, W. Find the weight, w, of a beam of that depth; and again apply the formula, using (in place of W) W + w if W is a uniform load, or W + $\frac{w}{2}$ if W is a concentrated load. The depth, d', so found, is a second approximation. We may again apply the formula, as before, using the weight, w', of beam of depth d'; or, more simply, increase the breadth, as in ¶ 45.

---

* The beam is here supposed to be without weight. See ¶¶ 42, etc.

47. In practice, beams of rectangular section are almost always of timber and such beams are economically obtainable only in certain commercial sizes. Hence, the second approximation will usually be all that is required.

Strengths and Weights of Similar Beams of Different Dimensions. Comparison between Models and Actual Structures.

48. In any given beam, let W_1 = the load causing any given unit stress, S. Then, $W_1 = \frac{n\,S\,b\,d^2}{l}$ (for n, see table, p. 474); and, in any similar beam, of a times the breadth, depth and span, the corresponding load, $W = \frac{n\,S\,a\,b\,a^2\,d^2}{a\,l}$. Hence, the ratio of their *loads* is $\frac{W}{W_1} = a^2$; or $W = a^2\,W_1$; but the ratio of their *weights* is $\frac{w}{w_1} = \frac{ab\;ad\;al}{b\;d\;l} = a^3$; or $w = a^3\,w_1$.

49. In other words, comparing one beam with another, of a times its breadth, depth and span, their *strengths* are as the *squares* of their respective dimensions; but their *weights* are as the *cubes* of those dimensions.

50. Hence, if a model of a beam will just break under a uniform load (including its own weight, w) = 2, 3 or 4, etc., times its own weight, then a beam of similar cross-section, but of 2, 3 or 4, etc., times its breadth, depth and span, will just break under its own weight alone.

Horizontal Shear. See also ¶¶ 119–122.

51. When (Figs. 7 and 8) deflection occurs in a cantilever or beam composed of separate horizontal layers, like a pile of loose boards, the several layers slide upon each other; but, if they are firmly joined together, or otherwise prevented from sliding, they exert, upon each other, a horizontal shearing force. In any section, this force diminishes from a maximum, at the neutral surface, $n\,n$, to zero, at the top and bottom of the section.

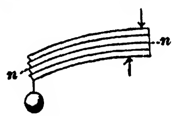


Fig. 7.

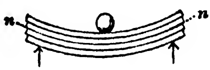


Fig. 8.

52. In any section of a rectangular beam, the maximum horizontal shear, per unit of neutral surface, is

$$H = \frac{3\,V}{2\,b\,d},$$

where V = the vertical shear in the section, and b and d = the breadth and the depth of the section, respectively.

In words, the unit horizontal shear, at any point, is directly proportional to the vertical shear at that point. Hence, the horizontal shear diagram is similar, in character, to the vertical shear diagram; but is opposite in sense, positive vertical shear corresponding to negative horizontal shear.

53. If the horizontal shear is resisted by a fastening applied at only one point, said fastening must be made sufficiently strong to resist the *sum of all* the horizontal shears between such point and that where the shear is = 0.

54. In Fig. 9, diagrams (*b*) and (*c*) show respectively the moments and the vertical shears due to concentrated and distributed loads on a beam as shown; and, in Fig. (*d*), each ordinate represents the force which must be applied, at the corresponding point, in order to resist the sum of all the horizontal shears between that point and the point of zero shear. Ordinates above a zero line indicate positive moments or shears, and vice versa. In *positive* moments, the segment to the *left* of a

section tends to turn *clockwise*. In *positive* shears, the *left*-hand segment tends to slide *upward* or the *upper* segment to slide toward the *right*. Between a and c, between c and d, between g and h, and between h and b, all the diagrams are straight lines, Figs. (b) and (d) being inclined, and Fig. (c) horizontal. At c and at h, Figs. (b) and (d) change their inclination, and Fig. (c) shifts its position. Between d and f, and between f and g (*i. e.*, under the distributed load), Figs. (b) and (d) are parabolic curves, and Fig. (c) shows inclined straight lines. At f, Figs. (b) and (d) change curvature, and Fig. (c) shifts its position. At e, the point of maximum moment, Figs. (c) and (d) change signs. See Relation between Moment and Shear, Statics, ¶¶ 359 to 368.

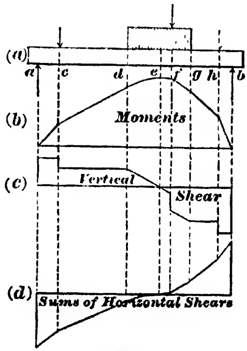


Fig. 9.

55. Inasmuch as the horizontal shear is a resistance to bending, its neglect, in the common theory of beams, as heretofore explained, is in general on the side of safety. But, in beams composed of horizontal layers, means must be provided for its transmission from one layer to the next.

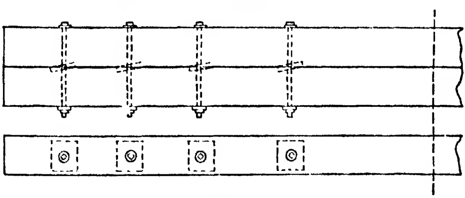
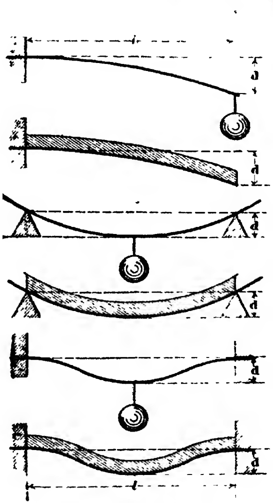


Fig. 10.

56. Thus, deep wooden beams, Fig. 10, are frequently built up of two or more timbers, one above the other. In order to prevent deflection, due to the sliding of these timbers upon each other, blocks are inserted between them at intervals, as shown, or the adjacent sides of the timbers are so notched as to interlock. In either case, the timbers are tightly bound together. The blocks or notches then serve to transmit the horizontal shear from one timber to the other. In Fig. 10 the blocks are more numerous near the ends of the span, as required by the diagram of horizontal shear, Fig. 9 (d).

DEFLECTIONS OF BEAMS OF UNIFORM CROSS SECTION THROUGHOUT. (For signification of letters, see opposite.)



If the beam is	Deflection d, in inches, caused by extraneous load.	Deflection d, in inches, caused by weight of beam	Deflection d, in inches, caused by weight of beam and load	Extraneous load for a given deflection (weight of beam neglected.)
Fixed at one end, loaded at the other	$\frac{1}{3} \frac{l^3 W}{E I}$	$\frac{1}{8} \frac{l^3 w}{E I}$	$\frac{1}{3} \frac{l^3 (W + \frac{3}{8} w)}{E I}$	$3 \frac{d E I}{l^3}$
" " " uniformly	$\frac{1}{8}$ "	$\frac{1}{8}$ "	$\frac{1}{8} \frac{l^3 (W + w)}{E I}$	8 "
Supported at both ends, loaded at the center	$\frac{1}{48}$ "	$\frac{5}{384}$ "	$\frac{1}{48} \frac{l^3 (W + \frac{5}{8} w)}{E I}$	48 "
" " " uniformly	$\frac{5}{384}$ "	$\frac{5}{384}$ "	$\frac{5}{384} \frac{l^3 (W + w)}{E I}$	$\frac{384}{5}$ "
Fixed " " at the center	$\frac{1}{192}$ "	$\frac{1}{384}$ "	$\frac{1}{192} \frac{l^3 (W + \frac{1}{2} w)}{E I}$	192 "
" " " uniformly	$\frac{1}{384}$ "	$\frac{1}{384}$ "	$\frac{1}{384} \frac{l^3 (W + w)}{E I}$	384 "

Deflections.

57. The opposite table gives the **deflections** within the elastic limit of any *prismatic* beam (beam of uniform cross section throughout) **under different arrangements of support and of load;** also (in the last column) the *extraneous* **load which will produce a given deflection,** without assistance from the weight of the beam itself. All the formulæ are based upon the assumption that the increase of deflection is proportional to increase of load.

The letters signify as follows:

d = deflection of beam, in inches (see Figs).
W = weight of extraneous load in pounds
w = " " clear span of beam, in pounds.
l = clear span of beam, in inches (see Figs).
E = modulus of elasticity of the material of the beam, in lbs. per sq. inch.
I = moment of inertia of the cross section of the beam, in biquadratic inches. See "Unit of Moment of Inertia," p. 468.

From the principles embodied in the opposite table, we find that in beams of similar cross section and of the same material, and within the elastic limit, the loads and deflections (neglecting the weight of the beam itself) are as follows:

With the same	The deflections under a given extraneous load are
span	inversely as the breadths and as the cubes of the depths
" and breadth	" " " " "
" " depth	" " breadths
breadth " "	directly " cubes of the spans

With the same	The extraneous loads for a given deflection are
span	directly as the breadths and as the cubes of the depths
" and breadth	" " " " "
" " depth	" " breadths
breadth " "	inversely " cubes of the spans

Deflection in Terms of Extreme Fiber Stress. In table, p. 474, the load $W = k\,S\,\frac{I}{T\,l}$; where k = a coefficient, as below; S = unit stress in extreme fibers; I = moment of inertia; T = distance from neutral axis to extreme fiber, and l = span. From the table opposite, we have: $W = m\,\frac{d\,E\,I}{l^3}$; where m = a coefficient, as below; d = deflection, and E = modulus of elasticity. Hence, $k\,S\,\frac{I}{T\,l} = m\,\frac{d\,E\,I}{l^3}$; and $d = \frac{k}{m}\cdot\frac{l^2\,S}{E\,T} = \frac{l^2\,S}{E\,T\,c}$; where $c = \frac{m}{k}$.

In a cantilever, loaded at end,	m = 3;	k = 1;	c = 3.
" " " uniformly,	m = 8;	k = 2;	c = 4.
In a beam, supported, and loaded at center,	m = 48;	k = 4;	c = 12.
" " " " " uniformly,	m = 76.8;	k = 8;	c = 9.6
" " fixed, " " at center,	m = 192;	k = 8;	c = 24.
" " " " " uniformly,	m = 384;	k = 12;	c = 32.

Elastic Limit.

58. Under moderate loads, the deflections are practically proportional to the load. When they begin to increase perceptibly faster than the load, the latter is said to have reached the elastic limit, or limit of elasticity. It is generally at this point that the "**permanent set**" first becomes noticeable; i. e., after removal of the load, the beam fails to return to its original unstrained condition, and remains more or less bent. The deflections then also begin to increase *irregularly*; and to continue indefinitely without further increase of load. In short, the beam is in danger. Hence, the actual load *must never exceed* the elastic limit; and should not exceed from one-third to two-thirds of it, according to circumstances.

The limit of elasticity of a beam of any particular form, or material, is **determined by experiment** with a similar beam, as in the case of constants for breaking loads, &c. Thus, load a beam at the center, by the careful gradual addition of small equal loads; carefully note down the deflection that takes place within some minutes (the more the better) after each load has been applied; in order to ascertain when the deflections begin to increase more rapidly than the loads; for when this takes place, the load for elastic limit has been reached.*

It is not the deflections of the *whole* beam that are to be noted, but those of its clear span only. Several beams should be tried, in order to get an *average* constant, for even in rolled iron beams of the same pattern, and same iron, there is a very appreciable difference of strengths and deflections.

Then, to get the constant, using the *total load* applied during the equal deflections, including half the weight of the beam itself,

$$\textbf{Constant for elastic limit} = \frac{\text{Span in feet} \times \text{Total load in lbs.}}{\text{Breadth in inches} \times \text{Square of depth in inches}}$$

The constant, for wooden beams, may be had, near enough for common practice, by taking *one third* of the *breaking* constants in the table, page 476.

Said constant, thus calculated, is the elastic limit of a beam of the given shape and material, 1 inch broad, 1 inch deep, and of 1 foot span, supported at both ends and loaded at the center. To obtain from it the elastic limit of any other beam of the same design ‡ and the same material, similarly supported and loaded, but of other dimensions,

$$\textbf{Elastic limit} = \text{constant} \times \frac{\text{breadth in inches} \times \text{square of depth in inches}}{\text{span in feet}}.$$

If the beam is			Multiply the result by
supported	at both ends	and loaded at center,	1
"	" "	" " uniformly,	2
fixed †	" "	" " at center,	2
"	" "	" " uniformly,	3
"	" one end	" " at other end,	¼
"	" " "	" " uniformly,	½

* Of course, in practice, it is frequently difficult to ascertain with precision, when, or under what load, the deflections actually do begin to increase more rapidly than the successive loads. For although *by theory* the deflections are practically equal for equal loads, until the elastic limit is reached, yet in *fact* they are subject to more or less irregularity; for no material composing a beam is perfectly uniform throughout in texture and strength. Hence, instead of regular increase of deflection, we shall have an alternation of larger and smaller ones. Therefore, some judgment is required to determine the final point; in doing which, it is better, in case of doubt, to lean to the side of *safety*. It is assumed always that the load is not subject to jars or vibrations. These would increase the deflections.

† A beam is said to be "fixed" at either end when the tangent to the longitudinal axis of the deflected beam at that end remains always horizontal.

‡ The *shapes* of the two beams need not be *similar*. For instance, the constant deduced from experiments upon any rectangular beam is applicable to any other rectangular beam, whether square or oblong.

The Elastic Curve.

59. When a cantilever, Fig. 1, or a beam, Fig. 2, supported or fixed in any manner, bends, under the action of any load, the neutral surface, $n\ n$, forms a curve, such that, at any section,

$$R = \frac{E\,I}{M} = \frac{I\,S}{M\,k};$$

where

- R = the radius of curvature, at the section;
- M = the bending moment, at the section;
- I = the moment of inertia of the section;
- E = the elasticity coefficient of the material, $= \frac{S}{k}$;
- S = any unit stress within the elastic limit;
- k = the unit "stretch" (elongation or compression) produced by S in the given material.

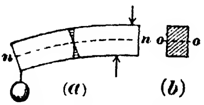


Fig. 1 (repeated).

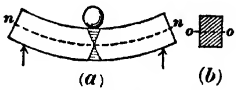


Fig. 2 (repeated).

The Deflection Coefficient.

60. Definition. The deflection coefficient, for any given material, is the deflection, in inches, of a beam, of that material, 1 inch square and of 1 foot span, supported at each end, and carrying, at its center, an extraneous load of 1 lb. — ⅝ w', where w' = weight of clear span of beam alone, in lbs.

61. Let y = the deflection coefficient for any given material. Then, in any rectangular beam, of the same material, with center load or uniform load, let

- b = the breadth, in inches;
- d = the depth, in inches;
- L = the span, in feet;
- w = the weight, in lbs., of the clear span of the beam itself;
- W = center load + ⅝ w;
  = ⅝ (uniform load + w).

Then, in the given beam,

Deflection $= Y = y\,\frac{W\,.\,L^3}{b\,.\,d^3}$; **Breadth*** $= b = W\,.\,\frac{L^3\,.\,y}{d^3\,.\,Y}$;

Load $= W = Y\,.\,\frac{b\,.\,d^3}{L^3\,.\,y}$. **Depth*** $= d = L\,\sqrt[3]{\frac{W\,y}{b\,Y}}$.

62. The deflection coefficient, y, for any given material, is obtained by experiment, thus: At the center of any rectangular beam, of the given material, placed horizontally upon two supports, at any convenient and known distance apart, place any load that is within the elastic limit, and measure the resulting deflection, Y. Let W = the extraneous center load + ⅝ w, where w = the weight, in lbs., of the clear span of the beam itself. Then the deflection coefficient is

$$y = Y\,.\,\frac{b\,.\,d^3}{W\,.\,L^3};$$

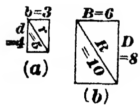
where b and d = the breadth and depth, in inches, and L = the span, in feet, of the experimental beam.

---

* In calculating the breadth or the depth, if it is necessary to provide for the weight of the beam itself, we first let W = the extraneous load only, and then proceed by successive approximations, as in ¶¶ 45 and 46, remembering, however, that in the case of deflections, 5-8 of the weight of each additional section is to be taken as equivalent center load, and not 1-2 as in the case of strengths.

63. The ratio between any two homologous lines, in any two similar figures, is constant. Hence, in determining or using coefficients, whether for strength or for deflection, by comparing beams of similar sections but of different sizes, we may use any two homologous lines in place of the two breadths, or in place of the two depths, or the same line may be taken in place of both breadth and depth.

Thus, in Figs. 11, $B/b = D/d = R/r = 2$.



Figs. 11.

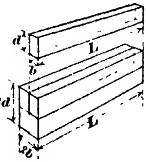


Fig. X.

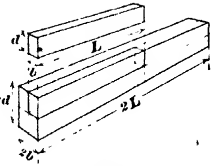


Fig. Y.

Hence,

$$\frac{B\,D^2}{b\,d^2} = \frac{384}{48} = 8 = 2^3 = \frac{R\,R^2}{r\,r^2} = \frac{1000}{125} = 8 = 2^3;$$

Also,

$$\frac{B\,R^2}{b\,r^2} = \frac{6 \times 100}{3 \times 25} = \frac{600}{75} = 8 = 2^3.$$

Also,

$$\frac{B^3\,D}{b^3\,d} = \frac{1728}{108} = 16 = 2^4 = \frac{R^3\,R}{r^3\,r} = \frac{10{,}000}{625} = 16 = 2^4$$

63 a. From the foregoing and from ¶ 31, p. 473, it follows that, with *similar cross sections* but *equal spans*, Fig. X, the strengths are proportional to the *cubes* of any homologous lines in the two sections, but, with beams similar *in all respects, including span*, Fig. Y, the strengths are as the *squares* of any homologous lines in the two sections; *i. e.*, as the *areas* of the two sections.

64. Deflection coefficient for beams of rectangular cross section. See ¶¶ 60, 61. Beam 1 inch square, 12 ins span. W (= cen load + ⅝ × weight of clear span) = 1 pound. From p 480, we have,

$$\text{Defl coeff} = \text{defl, } y, \text{ in ins, at cen,} = \frac{L^3\,W}{48\,E\,I} = \frac{12^3 \times 12}{48\,E} = \frac{432}{E}.$$

65. Caution. The deflections of timber of the same kind vary greatly with the degree of seasoning, the age of the tree, the part from which the beam is cut, etc. In our own experiments on good pieces, well seasoned, on which the loads were allowed to remain for months, less than 2 per cent. of the breaking load produced a permanent set in a few months. Several of the sticks bore their breaking loads for months before actually giving way. The vibrations and jars, to which all structures are exposed, in time increase the deflections.

66. Eccentric Concentrated Loads. Let Y, Fig. 12 (*a*), be the deflection, at the center of the span (*i. e.*, at the point of application of the load) of a beam supported at each end, due to a load, W, within the elastic limit, at the center of the span. Then, if the same load, W, be placed eccentrically upon the same beam, as in Fig. 12 (*b*), the deflection, Y′, at the point, *c*, of application of the load, and due to the load, W, is

$$Y' = Y\,\frac{16\,m^2\,n^2}{l^4};$$

where

l = the span;

m and n = the segments into which the load divides the span.

67. Uniform Loads. Let Y be the deflection, due to any central extraneous load (within the elastic limit), on a beam supported at each end. Then the deflection, Y′, of the same beam, due to the same load uniformly distributed over the span, is

$$Y' = \frac{5}{8}\,Y.$$

68. Inclined Beams. If the beam is inclined, use the horizontal projection of its span, in place of the span, l, in determining its deflections.

69. Cylindrical Beams. Let Y be the deflection of a square beam under any given load. Then, for a cylindrical beam whose diameter = side the square, the deflection, under the same load, is = 1.698 Y.

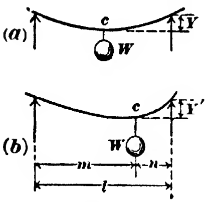


Fig. 12.

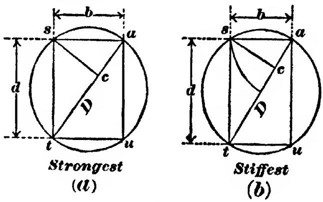


Fig. 13.

70. Figs. 13 (*a*) and (*b*) show, respectively, the strongest and the stiffest rectangular sections which can be cut from a given cylindrical log, of diameter, D. In the strongest section Fig. (*a*), $a\,c = \frac{D}{3}$, and $b = \sqrt{\frac{1}{3}\,D^2}$. In the stiffest section Fig (*b*), $a\,c = \frac{D}{4}$, and $d = \sqrt{\frac{3}{4}\,D^2}$.

71. Maximum Permissible Deflection. Under even a perfectly safe load, a beam may bend too much for certain purposes. Thus, to prevent the cracking of the plaster of ceilings, it is usual to limit the deflection of beams to $\frac{\text{span}}{360} = \frac{1}{30}$ inch per foot of span = 3⅓ ins. per 100 ft. In long lines of shafting, for machinery, the deflection is usually limited to $\frac{\text{span}}{1200}$ = 1 inch per 100 feet of span; in highway bridges to $\frac{\text{span}}{640} = \frac{3}{16}$ inch in 10 ft.; in railroad bridges to $\frac{\text{span}}{1600} = \frac{3}{4}$ inch per 100 feet.

72. Let Y = the maximum permissible deflection, in inches per foot of span, in any given case;
y = the deflection coefficient, ¶¶ 60, etc.
L = the clear span of the beam, in feet;
w = the weight of the clear span of the beam, in lbs.;
W = the center load + ⅝ w;
= ⅝ (uniform load + w).

Then, Y L = the deflection, in inches, for the whole span, L, and we have, for the permissible load, W, and the required breadth, b, and depth, d, for a rectangular beam (see ¶ 61):

$$\textbf{Load} = W = \frac{Y\,L\,b\,d^3}{L^3\,y} = \frac{Y\,b\,d^3}{L^2\,y};$$

$$\textbf{Breadth}^{*} = b = W \cdot \frac{L^2\,y}{d^3\,Y};$$

$$\textbf{Depth}^{*} = d = \sqrt[3]{W \cdot \frac{L^2\,y}{b\,Y}}.$$

* See foot-note to ¶ 61.

Suddenly Applied Loads.

73. Suppose a load to be applied to a flexible beam suddenly, though without falling or jarring; as, for instance, if it be supported by a cord which allows it just to touch the beam without bearing upon it, and the cord be then suddenly cut in two. The deflection of the beam, in such a case, is theoretically twice as great as when the same load is applied gradually, as by very slowly relaxing the cord, or by dividing the weight into small fragments and applying them at intervals, one by one. See ¶¶ 35 etc, under Strength of Materials. Hence the strength of the beam (within the elastic limit) is much more severely taxed in the former than in the latter case. A heavy train, coming very rapidly upon a bridge, presents a condition intermediate between the two.

Cantilevers and Beams of Uniform Strength.

74. For equilibrium, the resisting moment, R, of any section, must balance the bending moment, M, at that section. Or,

$$\frac{S}{T} \cdot I = M; \text{ or, } S = M \cdot \frac{T}{I};$$

where S = unit stress in extreme fibers;
T = distance from neutral axis to extreme fibers;
I = moment of inertia of section.

75. In a beam of uniform cross-section, therefore, since T and I are uniform throughout the span, the unit stress, S, on the extreme fibers, varies with the bending moment, M. For uniform strength against bending moments, the cross-section must so vary that $\frac{T}{I}$ shall be inversely proportional to M, in order that S may remain constant.

76. The following table shows, in elevation and in plan, the theoretical shapes of rectangular cantilevers and beams of uniform strength against bending moments, under concentrated and uniform loads. In practice, some of these shapes would of course have to be made stronger near their ends, in order to provide a sufficient section to resist shear.

77. Notwithstanding the reduction in material which would be effected, by using beams of uniform strength, their use is seldom economical, except in the case of cast iron. In timber, the material removed would not be saved; and, in steel, the saving in material would often be offset by the cost of additional labor.

Moreover, it will be noticed that the **deflections** of beams of uniform strength, under a given loading, are considerably greater than those of beams of uniform cross-section.

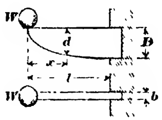
In the table,

W = concentrated load;
w = uniform load per unit of span;
l = span;
x = distance from a support to any given section;
d = depth of beam at that section;
b = breadth of beam at that section;
D = maximum depth of beam;
B = maximum breadth of beam;
S = unit stress in extreme fibers;
E = elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;
Y′ = deflection, due to extraneous load, in beam of uniform strength;
Y = deflection, due to extraneous load, in beam of uniform cross-section = maximum cross-section of beam of uniform strength

Cantilevers of Rectangular Cross-section and of Uniform Strength. Profiles, Plans and Deflections.

For symbols, see ¶ 77.

Concentrated Load, = *W*, at end.

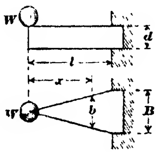


Breadth, b, constant

Profile, parabola, with vertex at load.

$$d = \sqrt{\frac{6\,W\,x}{S\,b}}, \quad Y' = \frac{8\,W\,l^3}{E\,b\,D^3} = 2\,Y,$$

D = Maximum depth.

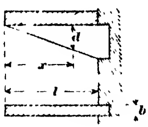


Depth, d, constant.

Plan, triangle.

$$b = \frac{6\,W\,x}{S\,d^2}; \qquad Y' = \frac{6\,W\,l^3}{E\,B\,d^3} = \frac{3}{2}\,Y.$$

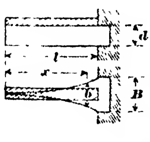
Uniform Load, = *w* per unit of span.



Breadth, b, constant

Profile, triangle

$$d = x\sqrt{\frac{3\,w}{S\,b}}$$



Depth, d, constant

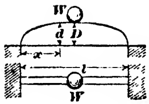
Plan, two parabolic curves, with vertices at free end.

$$b = \frac{3\,w\,x^2}{S\,d^2}, \qquad Y' = \frac{3\,W\,l^3}{E\,B\,d^3} = 2\,Y.$$

Beams of Rectangular Cross-section and of Uniform Strength. Profiles, Plans and Deflections.

For symbols, see opposite page.

Concentrated Load, = *W*, at center.

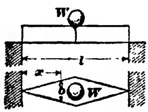


Breadth, *b*, constant.

Profile, two parabolic curves, with vertices at supports.

$d = \sqrt{\frac{3\,W\,x}{S\,b}}$ D at center of span

$Y' = \frac{W\,l^3}{2\,E\,b\,D^3} = 2\,Y$

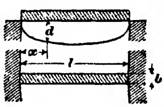


Depth, *d*, constant B = maximum width

Plan, two triangles

$b = \frac{3\,W\,x}{S\,d^2}$, $Y' = \frac{3\,W\,l^3}{8\,E\,B\,d^3} = \frac{3}{2}\,Y$

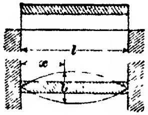
Uniform Load, = *w* per unit of span.



Breadth, *b*, constant

Profile, ellipse or semi-ellipse

$d = \sqrt{\frac{3\,w}{S\,b}\,(\,l\,x - x^2\,)}$



Depth, *d*, constant

Plan, parabolas with vertices at center of span

$b = \frac{3\,w}{S\,d^2}\,(\,l\,x - x^2)$

Symbols in table opposite:

W = concentrated load; w = uniform load per unit of span;
l = span; x = dist from a support to given sec;
d = depth of beam at that sec; b = breadth of beam at that sec;
D = maximum depth of beam; B = maximum breadth of beam;
S = unit stress in extreme fibers;
E = elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;
Y′ = deflection due to extraneous load in beam of uniform strength;
Y = deflection, due to extraneous load, in beam of uniform cross-section = max cross-section of beam of uniform strength.

Continuous Beams. See also ¶¶ 134–6.

78. A continuous beam is one which rests upon more than two supports.

79. The resistances and deflections of continuous beams, like those of beams with fixed ends, are determined by means of the elastic curve, using the calculus. The more important facts, thus deduced, are indicated in Fig. 14 and illustrated table, ¶ 89.

80. Fig. 14 represents the general character of the deflections, and the variations of the moments and of the shears, in uniformly loaded continuous beams.

81. Moments. Fig 14 (*b*). Ordinates drawn *above* the zero line, *a′ b′*, represent *positive* moments, or those where the segment of the beam, to the left of any section, tends to revolve *clockwise;* and vice versa.

82. At each end of the beam, at one point, *i* (called the inflection point, or point of contrary flexure) in each end span, and at two such points in each remaining span, the moment is zero.

83. At another point, *m*, in each span, the positive moment reaches a maximum for that span; while the negative moments reach their maxima at the supports. Both the positive and the negative moments vary in the different spans; but, if the spans are equal, then the moments, at any two points equidistant from the center of the whole beam, are equal.

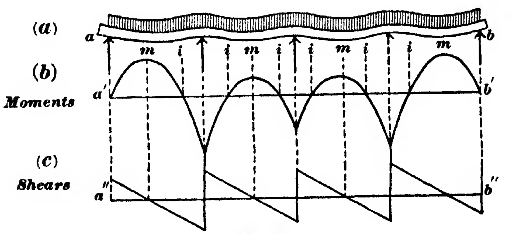


Fig. 14.

84. The moment diagram, between each support and the point, *m*, of maximum positive moment on either side of it, is a semi-parabola, with its apex at *m*.

85. Shears. Fig. 14 (*c*). Ordinates drawn *above* the zero line, *a″ b″*, represent *positive* shears, or those in which the *left*-hand segment, at any section, tends to slide *upward* past the *right*-hand segment; and vice versa.

86. At the point, *m*, of maximum moment, in each span, the shear is zero. Between each such point and the next support on the left, the shear is positive, and vice versa.

87. At each support the shear suddenly changes, by an amount = the reaction of the support.

88. The shear diagram is a series of straight lines.

Continuous Beams.

$m\,w\,l^2$ — Moment.
$v\,w\,l$ — Shear.
$a\,l$ — Distance from left support of any span to point of maximum positive moment in that span.
$x\,l$ — Distance from any left support to inflection point.
w — Load per unit of span.
l — Span.

Number of Spans	Number of Supports	m =	a =	x =	v =
2	3	0; + 0.0703; − 0.125	0.375	0.75	0; + 0.375; − 0.625; + 0.625
3	4	0; + 0.08; − 0.10; + 0.025	0.4; 0.5	0.8; {0.276, 0.724}	0; + 0.4; − 0.6; + 0.5
4	5	0; + 0.0772; − 0.1071; + 0.0364; − 0.0714	0.3930; 0.5357	0.7860; {0.2659, 0.8055}	0; + 0.3929; − 0.6071; + 0.5357; − 0.4643; + 0.4643
5	6	0; + 0.0779; − 0.1053; + 0.0332; − 0.07895; + 0.0461	0.3947; 0.5264; 0.5	0.7894; {0.2680, 0.7830}; {0.1964, 0.8036}	0; + 0.3947; − 0.6053; + 0.5263; − 0.4737; + 0.5000

89. The illustrated table opposite represents the conditions theoretically existing in uniformly loaded continuous beams of from two to five equal spans. Only the left half of each such beam is shown, the right half being symmetrical with it.

90. The Figs. show the amount of the maximum positive moment in each span, that of the negative moment at each support, and the shear on each side of each support.

91. The Figs. show, also, the coefficient, a, for the distance, $a\,l$, from the left support of each span to the point of maximum moment in that span; and the coefficient, x, for the distance or distances, $x\,l$, from the same support to the inflection point or points in that span. In each central span, the sum of the two values of x is = 1. In each end span, $x = 2\,a$.

92. In each central span, the point of maximum positive moment is at the center of the span. In other words, the deflection in that span is symmetrical, or $a = 0.5$.

93. The numerical sum of the two shears, one on each side of a support, is = the reaction of that support. At each central support, the shears, on its two sides, are equal.

In the Figs.,

w = load per unit of span;
l = span;
m = the coefficient for moment;
$m\,w\,l^2$ = moment;
v = the coefficient for shear;
$v\,w\,l$ = shear;
a = the coefficient for distance to point of maximum moment;
$a\,l$ = distance from left support of any span to point of maximum positive moment in that span;
x = the coefficient for distance to inflection point;
$x\,l$ = distance from left support of any span to either inflection point in that span.

94. Fig. 15 shows the values of m and of v in a uniformly loaded non-continuous beam. Comparing these with the corresponding values in continuous beams, as shown in the illustrated table, opposite, we see that the continuous beam has considerable theoretical advantage. But see ¶ 95.

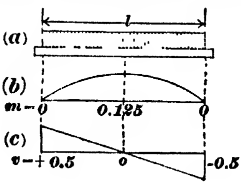


Fig. 15.

95. Certain practical considerations, however, materially reduce these advantages in many cases. Thus, in a continuous railroad bridge of 100 ft. spans, so designed that the maximum deflection shall not exceed $\frac{3}{4}$ inch, a settlement of $\frac{3}{4}$ inch, in an intermediate pier, would deprive the bridge of the support of such pier, and thus practically throw two adjacent spans into one, bringing upon their members stresses far in excess of those for which they were designed. Again, with moving loads, the theoretical advantage may at times be much less than that due to a stationary load and indicated in the illustrated table.

Cross-shaped Beam.*

96. In a cross-shaped beam, Fig. 16, of homogeneous material, loaded at center, let

W = the load;

E = the elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;

Y = the deflection at center;

L, l = the spans of
D, d = the depths of
T, t = the half depths of
I, i = the moments of inertia of
S, s = the unit stresses in the extreme fibers of
P, p = the portions of W borne by
} the two branches respectively.

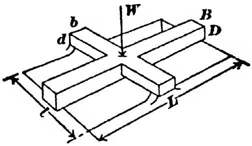


Fig. 16.

Then (see illustrated table, p 480), since the deflection is necessarily the same for both branches,

$$\frac{L^3 P}{E I} = \frac{l^3 p}{E i}; \text{ or}$$

$$\frac{P}{p} = \frac{I \cdot l^3}{i \cdot L^3};$$

and, since $P = 4 \cdot \frac{S}{T} \cdot \frac{I}{L}$, and $p = 4 \frac{s}{t} \cdot \frac{i}{l}$ (see table, p. 474), we have

$$\frac{S}{s} = \frac{D}{d}\left(\frac{l}{L}\right)^2.$$

97. In other words, in order that both branches may be equally strong, their depths (independently of their breadths) must be directly as the squares of their spans, or their spans directly as the square roots of their depths.

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* F. Reuleaux, "Der Konstrukteur," Braunschweig, 1889. "The Constructor," translated by H. H. Suplee, Philadelphia, 1893.

Transverse Resistance of Flat Plates. For Buckled Plates, see p 1167.

98 The laws governing the resistance of plates, to pressures normal to their surfaces, are but imperfectly understood; and formulas expressing them must be used with caution and as probable approximations.

99. In the following table, the moments are those given, as sufficiently approximate, by Rankine, Civil Engineering, p 544, for **supported edges;** and the stresses are deduced from these moments by means of the formula, $S = M \frac{6}{L\, d^2}$. See symbols, ¶ 101. See formulas for load, in rectangular beams, p 474. For plates with *fixed* edges, see ¶ 100.

A homogeneous oblong plate tends to split along its longer axis.

100. Fixed edges. If M be the moment, and S the maximum fiber stress, for a plate with *supported* edges, and if M_f and S_f be the corresponding values for the same load on the same plate with *fixed* edges, we may assume —

for central load, $M_f = \frac{1}{2} M$; $S_f = \frac{1}{2} S$;
for uniform load, $M_f = \frac{2}{3} M$; $S_f = \frac{2}{3} S$.

See rules for ordinary beams, p 474.

101. In a plate with *supported* edges, let

d = thickness of plate,
w = load per unit of surface;
W = total load,
M = max bending moment;
S = max fiber stress.

In an oblong plate, let
L = longer span;
b = shorter span;
in a square or circular plate, $L = b$;
in a circular plate, r = radius.

Maximum bending moment, M, and maximum fiber stress, S, in plates *supported* at edges. For fixed edges, see ¶ 100.

Central Load.

Plate	Moment, M.	Stress, S.
Oblong $L \not< 1.19\, b$	$\frac{1}{4} W b$	$\frac{3}{2} \frac{b}{L} \frac{W}{d^2}$
$L < 1.19\, b$	$\frac{3}{8} W \frac{L^4 b}{L^4 + b^4}$	$\frac{9}{4} \cdot \frac{L^3 b}{L^4 + b^4} \cdot \frac{W}{d^2}$
Square	$\frac{3}{16} W L$	$\frac{9}{8} \frac{W}{d^2}$*
Circular	$W \frac{r}{\pi}$	$\frac{3}{\pi} \frac{W}{d^2}$*

Uniform Load.

Plate	Moment, M.		Stress, S.	
Oblong	$\frac{1}{8} W \frac{L^4 b}{L^4 + b^4}$	$= \frac{1}{8} w \frac{L^5 b^2}{L^4 + b^4}$	$\frac{3}{4} \cdot \frac{L^3 b}{L^4 + b^4} \cdot \frac{W}{d^2}$	$= \frac{3}{4} \cdot \frac{L^4 b^2}{L^4 + b^4} \cdot \frac{w}{d^2}$
Square	$\frac{1}{16} W L$	$= \frac{1}{16} w L^3$	$\frac{3}{8} \frac{W}{d^2}$*	$= \frac{3}{8} \cdot L^2 \cdot \frac{w}{d^2}$
Circular	$\frac{1}{3} W \frac{r}{\pi}$	$= \frac{1}{3} w r^3$	$\frac{1}{\pi} \cdot \frac{W}{d^2}$*	$= r^2 \frac{w}{d^2}$

102. In a circular plate, uniformly loaded, **deflection** at center $= c \frac{w\, r^4}{E\, d^3}$ where †$c = \frac{2}{3}$ for plates *supported* at edges; $c = \frac{1}{6}$ for plates *fixed* at edges; E = elastic modulus.

---

*Since the stress ($S = M \frac{6}{L\, d^2}$) is proportional to $\frac{W\, L}{L\, d^2}$, the stress, S, under a given *total* load, W, in *square or circular* plates, and in rectangular plates where L/b is constant, is independent of the surface dimensions.

†Compare F. Grashof, Theorie der Elasticitat und Festigkeit, Berlin, 1878, pp 336, 337.

TRANSVERSE AND LONGITUDINAL STRESS COMBINED.

103. Although the combination of longitudinal and transverse stress in the same piece is objectionable, it is often unavoidable. Thus, in a timber roof, the rafters generally act both as columns and as beams.

In such cases, the total unit stress, S, in the extreme fibers, is the sum of the uniform stress, S_c, due to direct compression or tension, and the extreme fiber stress, S_b, due to bending moments only, under the action of the transverse and longitudinal loads combined. Or $S = S_c + S_b$.

Let M_b = the bending moment due to the transverse load; M_c = the bending moment due to longitudinal load, P; and M = the total resultant bending moment, $= M_b - M_c$ when the longitudinal load is tensile; $= M_b + M_c$ when the longitudinal load is compressive.

But $M_c = P\,d$, where P = the longitudinal load, and d = its leverage, = the deflection of the beam, due to all causes; and (see ¶ 57) $d = \frac{l^2 S_b}{E\,T\,c}$; where l = span, S_b = unit stress in extreme fibers, due to bending; E = modulus of elasticity; T = distance from neutral axis to extreme fibers, and c = a coefficient, whose values, for different cases, are given in ¶ 57.

Hence, $M_c = P\frac{l^2 S_b}{E\,T\,c}$; and resultant moment $M = M_b + P\frac{l^2 S_b}{E\,T\,c}$. The resisting moment, R (see ¶ 10), is $= S_b\frac{I}{T}$; and, for equilibrium, R = M.

Hence, $S_b.\frac{I}{T} = M_b + P\frac{l^2 S_b}{E\,T\,c}$; whence we derive, for the extreme fiber stress, S_b, due to bending only, under the action of the transverse and longitudinal loads combined,

$$S_b = \frac{M_b\,T}{I + \frac{P\,l^2}{E\,c}} \quad \text{where the longitudinal stress is tensile}$$

$$S_b = \frac{M_b\,T}{I - \frac{P\,l^2}{E\,c}} \quad \text{where the longitudinal stress is compressive}$$

Besides this we have the unit stress, S_c, due directly to the longitudinal load, P, and $= \frac{P}{A}$, where A is the area of cross-section of the beam. Hence, for the total unit stress, S, in the extreme fibers, we have

$$S = S_c + S_b = \frac{P}{A} + \frac{M_b\,T}{I \pm \frac{P\,l^2}{E\,c}}.$$

When the deflection, d, is negligible, $M_c = 0$; $M = M_b = S_b\frac{I}{T}$; and $S_b = \frac{M_b\,T}{I} = \frac{M\,T}{I}$, as in ¶ 10; and $S = \frac{P}{A} + \frac{M\,T}{I}$. In practice, d is frequently neglected, and this formula used.

DIAGONAL STRESSES IN BEAMS.
Maximum Unit Stresses.

104. When a body (as a bolt) is **under tensile (or comp) stress only**, the tendency of the body, *as regards sections normal to the stress*, is to pull apart (or crush together) in the direction of the stress, or normally to the section, and the entire stress acts normally upon the section; but, *on planes oblique to the stress*, the stress is resolved into two components, one (n) of tension (or comp) normal to the plane, and one (t) tangential to the plane (shearing stress).

105. Under shearing stress alone, the effect, upon a plane *parallel* to & betw the 2 shearing forces, is pure shear; but, upon planes *oblique* to the forces, the shearing forces are resolved into (t) tangential or shearing stresses, and (n) normal (tensile or comp) stresses.

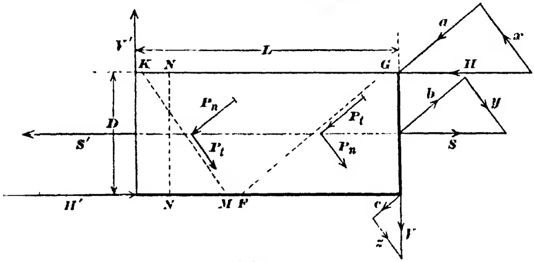

Fig. 17.

106. Thus, Fig 17, let a bar, of length, L, and depth, D, be subjected to a tension, $S = S'$, in line with its hor axis, and to two pairs of forces, $V = V'$ and $H = H'$, as shown; V and V' constituting a right-hand vert shear, while H and H' constitute a left-hand hor shear.

Suppose the bar divided by a section, as $N\ N$, $F\ G$ or $K\ M$, and consider the forces acting, in either case, upon the right-hand segment of the bar as thus divided.

Upon the *normal* section, $N\ N$, the tension, S, and the hor shear, H, act normally (S as tension, H as compression), and the vert shear, V, tangentially (as shear), but, for an *oblique* section, $F\ G$ or $K\ M$, we first resolve each force, S, V and H, into two components, b and y, c and z, a and x, respectively normal and parallel to the section, as shown by the force-triangles on the right.* Then, summing these comps, algebraically, we obtain the resultant forces, P_n (normal) and P_t (tangential or shearing), acting upon the section in question. With the forces, S, V and H, as shown in Fig 17, we have.

On sec $F\ G$, P_n, tension, $= y + z - x$;

P_t, right-hand shear, $= a + c - b$;

On sec $K\ M$, P_n, compression, $= a + c - b$;

P_t, right-hand shear, $= y + z - x$.

107. If, now, we examine **all possible planes** cutting the body at a given point, we shall find (1) one such plane upon which the resultant unit *tensile* stress reaches its max; (2) another, normal to (1), upon which the resultant unit *comp* stress reaches its max; and (3) two planes, normal to each other & bisecting the right angles betw planes (1) & (2). Upon the two planes last named, (3), the resultant unit *shearing* stresses reach their max.

---

*In order that, for either force, S, V or H, the two force-triangles (for the two sections, $F\ G$ and $K\ M$) may be identical, and thus simplify the figure, we take the two sections, $F\ G$ and $K\ M$, normal to each other.

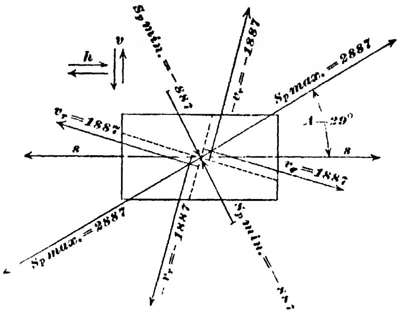

Fig. 18.

108. Let Fig. 18 represent **a small element** in a bar under tensile & shearing stresses; and let it be required to determine the **positions of these planes and the corresponding max stresses.** Let

s = the original normal (tensile or comp) unit stress;
v = " " vertical (shearing) unit stress;
= h = " " horizontal (shearing) unit stress;
s_p = " max or min resultant normal unit stress;
v_r = " max resultant shearing unit stress,
A = " angle betw s and s_p.

Then $\tan 2A = \dfrac{v}{s/2}$ . . . . (1)

$v_r = \sqrt{(s/2)^2 + v^2}$ . . . . (2)

$s_p \max = s/2 + v_r = s/2 + \sqrt{(s/2)^2 + v^2}$ . . . . . (3)

$s_p \min = s/2 - v_r = s/2 - \sqrt{(s/2)^2 + v^2}$ . . . . . (4)

If s is tension { + sign gives max tension; − " " " comp = min tension }
If s is comp { + " " " comp; − " " " tension = min comp. }

109. Example. Let

s = 2000 lbs/sq inch, tension (not drawn to scale);
v = h = 1600 " / " " , shear (" " " ").

Here v is left-handed, h right-handed. If this be reversed, the angle, A, betw the resulting tension, s_p, & the hor, will be *below* the neut axis.

110. Then $\tan 2A = \dfrac{v}{s/2} = \dfrac{1600}{1000} = 1.6$; $2A = 58°$; $A = 29°$;

$v_r = \sqrt{(s/2)^2 + v^2} = \sqrt{1000^2 + 1600^2} = 1887$;
$s_p \max = s/2 + v_r = 1000 + 1887 = 2887$ (tension)
$s_p \min = s/2 - v_r = 1000 - 1887 = -887$ (comp)

111. In other words, **we have, as resultants,** (1) a max unit tension, s_p max = 2887 lbs/sq in, forming an angle, A = 29°, with the axis of the bar or with the direction of s; (2) a min unit tension or max comp, s_p min = − 887 lbs/sq in, normal to s_p max; (3) a right-hand unit shear, v_r = 1887 lbs/sq in; and a left-hand unit shear, − v_r = − 1887 lbs/sq inch; the

directions of the shearing stresses bisecting the right angles betw the max normal stresses.

112. The max tension and compression, at any point, are called the "**principal stresses**" for that point.

Horizontal and Vertical Shear in Beams.

See also pp 440 &c, 446 &c, 450 to 453, 478-9.

113. Let Fig. 19 represent the left half of a **homogeneous beam**, of rectangular section; breadth, b, = 1 inch; depth, d, = 10 ins; span, L, = 100 ins; with cen load, W,* of 200 lbs; left reaction, $R = W/2 = 100$ lbs. Weight of beam neglected. The bendg mom, at cen of span, is $M = RL/2 = WL/4$* = 5000 inch-lbs; and the mom decreases uniformly,* from its max, at cen of span, to zero at the supports. In the extreme upper & lower fibers, the longitudinal unit stress, (¶ 10, p 468) s, = MT/I, where $T = d/2$ = dist from neut axis to extreme fibers = 5 ins; I = inertia mom of cross section = $bd^3/12$ = 1000/12. Hence, in Fig 19, $s = 12 \times 5\ M/1000 = 0.06\ M$. Now s, being thus proportional to M, also decreases uniformly,* from its max, at cen of span, to zero at the supports. Values of M and of s, for the sections 0, a, b, c, d, e, are figured on the diagram.

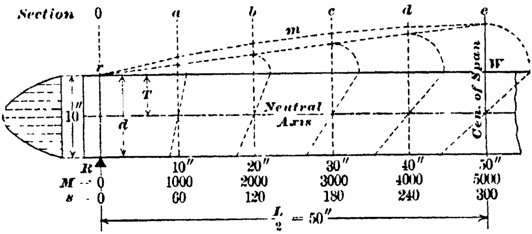

Fig. 19.

* Under a *uniformly distributed* load, the bendg mom, at cen of span, is $WL/8$; and the bendg moms, M, and the resulting longitudinal unit stresses, s, vary as the ordinates of a parabola, as indicated by the dotted parabola, $r\ m\ e$, at top of Fig 19, which corresponds to a uniform load = 400 lbs = 2 W The unit shears, v, in a given hor section, then decrease uniformly, from a max, at the supports, to zero at the cen of the span. Compare 3d and 4th figures, p 474.

114. The unit hor **tensile and comp stresses,** *s*, at the several points in any vert section, are **proportional to the dists** of those points **from the neutral axis,** as indicated by the diagram at each vert section, Fig 19.

115. In Fig 20, let *n* and *g* be two vert sections of this beam, such that, at *n* and at *g*, the extreme unit fiber stresses are: $m\ n = 15$, and $u\ g = 25$, respectively. Then the **rectangular portion, n f, of the beam, betw sections n & g,** is acted upon by a series of net or resultant forces, ranging from compression, $e\ g = u\ g - m\ n = -25 - (-15) = -10$, at the top, to tension, $= +10$, at bottom, as indicated by the diagram, *e k*.

116. Suppose the piece *n f* to be divided into 10 hor strips of equal depth, = 1 inch. Then the **net unit stresses,** *s*, acting at the tops and bottoms of these strips, respectively, are those, (—10, —8, —6, ... 6, 8, 10) figured from *e* to *k*; and the mean stress, or (since depth of each strip = *b* = 1) **the force, acting upon each strip,** is that (—9, —7, —5, ... 5, 7, 9) figured betw *g* and *f*.

117. These forces are transmitted, from strip to strip, thru their surfs of contact; and, in determining the shearing force, acting in the hor plane betw any 2 strips, we regard the upper (or lower) strip as acted upon by its own push or pull plus (algebraically) those of all the strips above (or below) it.

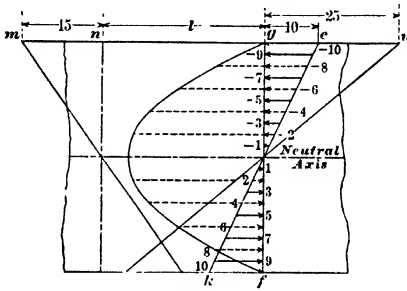

Fig. 20.

118. Thus, the 3d strip from the top is pushed to the left by a force of $-9 -7 -5 = -21$, while the 4th strip, just below it, is pulled to the right by a force of $9 + 7 + 5 + 3 + 1 - 1 - 3 = 21$. Hence the surf betw the 3d and 4th strips, sustains a counterclockwise **shear** of 21; which, divided by the area, $b\,l = l$, of that surface, gives the **unit shear** in the plane betw the 3d and 4th strips. With central load,* this unit shear is uniform from each support to cen of span, where it changes sense (from plus to minus, or vice versa) but is of the same intensity in the other half-span. See 3d Fig, p 474.

119. In any vert section of the beam, let

V = the total shear
= " reaction of either support, minus the sum of all loads betw that support and the section;
I = " inertia moment with respect to the neut axis;
b = " breadth; d = depth;
a = " area above (or below) any given point in the section;
c = " dist from neut axis to grav cen of a;
$M_s = a\,c$ = static mom of a, with respect to the neut axis;
v = the unit vert shear = unit hor shear at a given point.

120. Then

$$v = V \frac{M_s}{I} \div b = V \frac{a\,c}{I\,b}$$

* See foot-note p 494 *c*.

At the neut axis, M_s (= $a\,c$) = $\frac{d\,b}{2} \cdot \frac{d}{4} = \frac{d^2\,b}{8}$.

Hence, **at the neutral axis:**

$$v = V \frac{d^2}{8\,I} = V \frac{12\,d^2}{8\,b\,d^3} = \frac{3}{2} \cdot \frac{V}{b\,d}$$

$$= \frac{3}{2} \times \text{the mean vert shear in the cross section.}$$

See also ¶¶ 51 etc.

Since, under a center load, (¶ 113 and Fig 19) s increases *uniformly*, from zero (at support) to s_{max} (at span center), we have, for the increase of s, in any portion, as $n\,g = l$, Fig 20, of the span:

$$s_g - s_n = s_{max} \frac{l}{L/2} = 2\,s_{max} \frac{l}{L}.$$

121. At the left of Fig. 19 is a **diagram showing the unit shears** in the several hor sections.

122. Let Fig 21 represent a small element of a body, of unit thickness, normal to the paper, and acted upon by a right-hand vert shear, $V = v\,D$, (where v = the unit vert shear, and D = the depth of the element) and by a left-hand hor shear, $H = h\,L$ (where h = the unit hor shear, and L = the length of the element). For equilib of moments, we must have

$$V\,L = H\,D; \quad \text{or } v\,D\,L = h\,L\,D; \quad \text{or } v = h.$$

In other words,

unit vert shear = unit hor shear.

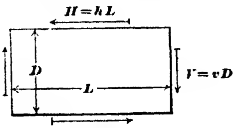

Fig. 21.

Maximum Unit Stresses in Beams.

123. The common theory of beams (pp 466 to 494, ¶ ¶ 1–103) considers only the **longitudinal tensile and compressive forces** and the vert and hor **shearing forces**, due directly to the load and to the upward reactions of the supports, and acting, at any point, upon *vert and hor planes* passing thru such point; but, except in certain limited portions of the beam, these stresses are **not the maximum stresses** acting at such point; for they combine to form resultant *diagonal* stresses, acting upon *diagonal* planes (passing thru the same point); and, upon some of these diag planes, the resulting normal and tangential stresses are greater than either of the original stresses.

124. The common theory is sufficiently well **adapted** to beams of many kinds, and especially to **steel beams**, where the longitudinal forces are resisted by the flanges, and the shears by the web; but in certain portions of deep and heavily loaded beams, especially those of **reinforced concrete**, the diagonal resultant or **maximum stresses** are the *ruling* stresses, and must not be neglected.

125. In a beam, at top and bottom, we have, respectively, hor tensile and comp stresses only, and, at the neut axis, shear (vert & hor) only; but, at all other points, we have **shear** (vert & hor) **acting conjointly with hor stresses**, either tensile or comp. At all points, these shearing and longitudinal stresses may be **resolved into components**, normal & tangl to any plane, at pleasure, as in the case of the bar or bolt, Fig 17.

126. Thus, each element of the beam, Figs 22, 23, 24, is acted upon by hor & vert forces (unit stresses), which, acting upon diagonal planes, are resolved into diagonal components, and these components may be alge-

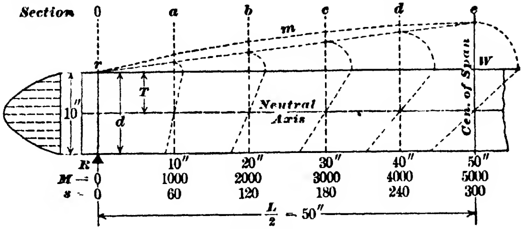

Fig. 22.

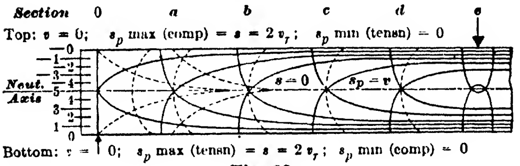

Fig. 23

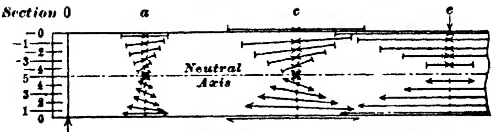

Fig. 24.

braically **summed into resultants;** but the original stresses vary in intensity, and the resultant stresses both in intensity and in direction, from point to point. For the **directions and values** of these resultant stresses at their maxima, we have, from Eqs 1-4, ¶ 108, p 494*b*:

$$\text{Tan } 2\,A = \frac{v}{s/2} \quad\ldots\ldots(1)$$

$$v_r = \sqrt{(s/2)^2 + v^2} \quad\ldots\ldots(2)$$

$$s_p = s/2 \pm v_r = s/2 \pm \sqrt{(s/2)^2 + v^2} \quad\ldots\ldots(3)(4)$$

where

s = original unit tensile or comp stress at the point;
v = original (vert or hor) unit shear at the point.

The max normal stresses, s_p, are called the **principal stresses.**

127. Applying these formulas at numerous points in the profile of the beam, Fig 22, we are enabled to construct **curves,** Fig 23, showing the **directions** of the stresses; and to plot, as in Fig 24, for given points, the **directions and intensities** of the stresses there acting. At any given point, Fig 24, we have resultant normal and shearing stresses analogous to those in Fig 18, p 494 *b*; but, in the present Fig 24, owing to want of space, only the *max principal* stress, s_p *max*, is shown for each point selected.

128. In Fig. 23, the directions of the **principal stresses**, s_p, are represented by the solid curves; those of the **resultant shears**, v_r, by dotted curves.

Of the solid curves (principal stresses)	concave	horizontal at cen of span	at 45° with	at 90° with
The tension curves are	upward	below neut axis	neut axis	top of beam
The compression curves are	downwd	above " "	" "	bot " "

The tensile and comp curves are normal to each other at their intersections.

129. Following any curve (concave upward) of normal tension,* we find that,

(1) for its point of **tangency with the hor** (viz: at cen of span) s_p max = tension = s, s_p min = comp = 0;

(2) for the point **where the curve crosses the neut axis** (at 45°) s_p max (tension) = s_p min (comp) = v_r = $\pm v$ (shear),

(3) **above the neut axis**, the tension becomes s_p min, and continues diminishing, as the direction approaches the vert, becoming zero at top, where A = 90°. Above the neut axis, for points in the same curve, the *compression* (*normal* to the curve) is now s_p max, and increases from s_p = v_r = $\pm v$, at the neut axis, to s_p max (comp) = s, at top.

130. Where v = zero (viz. at any point in the vert cross section at cen of span, and along the extreme upper and lower fibers), we have (¶ 126):

$$v_r = s/2$$
$$s_p \max = s/2 + v_r = s; \qquad \tan 2A = 0;$$
$$s_p \min = s/2 - v_r = 0; \qquad \tan 2A = 0.$$

131. The equation, $\tan 2A = 0$, gives either $2A = 0°$ or $2A = 180°$; *i. e.*, $A = 0°$, or $A = 90°$; but we know that, at cen of span and along the extreme upper and lower fibers, s_p max is hor, or $A = 0°$; and s_p min is vert, or $A = 90°$.

132. Where s = zero (as at the neut surf and where bending mom = zero), we have (¶ 126): $v_r = \pm v$; $s_p \max = s_p \min = \sqrt{v^2} = \pm v$; $\tan 2A = \infty$; $2A = 90°$; and $A = 45°$.

133. Of the (dotted) **shear curves**, Fig 23, those of one set are tangential to the neut axis and reach top & bottom of beam at angles of 45°, tending away from cen of span; while those of the other set are normal to these and to the neut axis at their intersections, reaching top and bottom of beam at 45°, tending toward cen of span.

MOMENTS IN CONTINUOUS BEAMS.

See also ¶¶ 78, etc.

134. Figs 25 and 26 show positive and negative **bending moments in two continuous beams**, Fig 25 of two equal spans, and Fig 26 of three equal spans, resting freely upon their supports. Each span = 1. Fig 26 (three spans) may be used, with sufficient approximation, for cases where the spans are more numerous.

* *Conversely* for curves (concave downward) of normal *compression*.

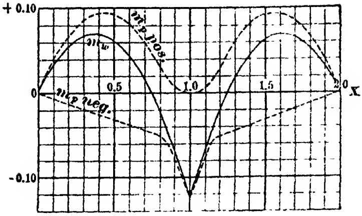

Fig. 25.

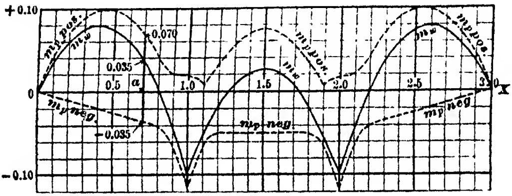

Fig. 26.

135. At any cross section, the ordinate, betw the axis, 0 X, of abscissas, and the curve, (1) m_w, (2) m_p *pos*, or (3) m_p *neg*, represents, respectively, by the scale of ordinates on the left, (1) the dead load moment, m_w, (2) the max positive live-load mom, m_p *pos*, or (3) the max negative live-load mom, m_p *neg*, at that section, the dead load (1 per unit of span) being uniformly distributed over the entire length (two or three spans, as shown) of the beam, and the live load (1 per unit of span) being uniformly distributed alternately over two portions of the length of the beam, said portions being, for each cross section, such that the uniformly distributed live load, placed upon said portions, will produce, alternately, the max pos and the max neg mom at that section.

136. In an actual beam, at any point, we have, for bending mom:

$$M = m_w \, w \, L^2 + m_p \, p \, L^2 ;$$

where

m_w = the ordinate, at the point, from 0 X to the curve m_w;
m_p = " " " " " " " " " " " " m_p *pos* or m_p *neg*;
w = uniform dead load per unit of span;
p = " live " " " " " " , placed as explained in ¶ 135.
L = the actual span.

Thus, at the point, a, Fig 26 (distant 0.7 L from 0), we have, by scale, m_w = 0.035; m_p *pos* = 0.070; m_p *neg* = − 0.035. Hence, at point a,

$$\text{max pos mom} = 0.035 \, w \, L^2 + 0.070 \, p \, L^2;$$
$$\text{max neg mom} = 0.035 \, w \, L^2 - 0.035 \, p \, L^2.$$

If, therefore, $p = w$, the max neg mom, at a, is zero, and there is no resultant neg mom to the left of a; but, if $p = 2\,w$, we have $w = p/2 = (w + p)/3$; and, at a, with $p = 2\,w$:

$$\begin{aligned} \text{max neg mom} &= 0.035 \, w \, L^2 - 0.035 \times 2 \, w \, L^2 \\ &= 0.035 \, w \, L^2 - 0.070 \, w \, L^2 = - 0.035 \, w \, L^2 \\ &= - 0.035 \, (w + p) \, L^2/3. \end{aligned}$$

COLUMNS IN GENERAL.

Strength of Columns, Pillars, Struts, of Uniform Cross Section.

(For iron and steel columns, see pp 1189, etc. For wooden columns, see pp 1143, etc.)

1. The axis of a column is a line passing through the centers of gravity of all its cross sections.

2. If the line of action of the load coincides with the axis of the column in the end sections, the column is said to be **axially loaded**: otherwise, **eccentrically loaded.**

3. Under an axial load, if the axis remains mathematically straight, it coincides with the line of pressure throughout the length of the column, and the stress is uniformly distributed over each cross section, as indicated by the parallelogram, *a b*, Fig 2, but eccentricity of loading, Fig 1 *a*, or lack of straightness in the axis, Fig 1 *b*, or of homogeneity in the material, will cause the axis to diverge from the line of pressure.

4. When the axis and the line of pressure fail to coincide, in any section, their divergence sets up, in that section, a "couple" (see ¶¶ 148, etc, pp 401, etc; ¶¶ 155, etc, pp 404, etc) represented by the two triangles, *b c d* and *e c f*, Fig 2, ¶ 6, below.

5. In all cases, therefore, **the diagram** of the stresses in the cross section consists of the **parallelogram**, *a b*, modified or not (as the case may be) by the **two triangles**, *b c d* and *e c f*. (In Fig 2, the extreme fiber stresses are: $s = af\ (= ae + ef)$ and $gd\ (= gb - bd.)$

6. In very short columns, axially loaded, the parallelogram is but little (if at all) affected by the triangles; i e, the pressure is nearly or quite uniformly distributed over the cross section; while, **in very slender columns**, on the contrary, the pressure is practically certain to be unequally distributed; and the disturbing effects, represented by the two triangles, may become the principal feature; so that the maximum unit stress, on the concave side, may greatly exceed the mean stress, *a e* or *g b*, Fig 2; while the unit stress, *g d*, on the convex side, may fall to zero or may even become tension.

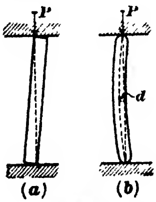

Fig. 1.

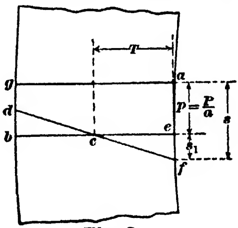

Fig. 2.

7. In very slender columns, under moderate deflections, the changes of length in, and therefore the stresses exerted by, the extreme fibers, are relatively small. Hence, the column may fail to support its load long before these stresses reach the ultimate strength, or even the elastic limit, of the material; as when a whale-bone is compressed endwise between thumb and finger, and thus sprung out laterally.

8. But, as the deflection, *d*, Fig 1 *b*, increases, the moment, *Pd*, of the load, *P*, increases also. The changes of length of the fibers, and therefore the stresses exerted by them, increase also, owing to the increase of curvature; so that, eventually, further movement of the load may be prevented, or, if the load is sufficient, the column may be broken or permanently crippled by it.

9. A very slight obliquity, Fig 1 *a*, or deflection, Fig 1 *b*, may reduce the strength as much as 50 per cent; and differences of 10 per cent or more, in the ultimate load, may occur between two pillars which, to all appearances,

are precisely similar, and tested under the same conditions. Hence, the behavior of columns is much less certain than that of pieces in tension, or even of beams; and **liberal safety factors should be employed** in using formulas or tables for columns.

10. This lack of certainty, in respect to the strengths of columns, is especially unfortunate in view of the necessary and extensive use of **columns as compression members in bridge and roof trusses.** See Quebec Bridge, p 1202 [under Iron & Steel Columns].

11. The neutral axis of any section is a line, in that section, passing through its center of gravity and perpendicular to the plane in which deflection takes place, or in which it is supposed to take place. A cross section may therefore have a number of neut axes, as the col may be supposed to deflect in any one of a number of planes. Usually, however, when a col, of homogeneous material, is left free to deflect in any plane, the plane of deflection is determined by the shape of its cross section, and is that of its least radius of gyration. Thus, if the section is circular, whether solid or hollow, deflection may equally well occur in any plane; but, in a square, deflection occurs in a plane parallel to either side; in a rectangle, in a plane parallel to the shorter sides, etc, etc.

12. When the plane of deflection is predetermined by some constraining feature, as by the pins in Figs 3 *c*, **economy of material** requires that the section be so disposed, with reference to the pin, etc, that the **least radius of gyration** (see pp 353 *a*, *b*) shall be perpendicular to the plane of deflection, as so predetermined. When the plane of deflection is not thus predetermined, economy of material requires that the section be so designed that the several radii of gyration be as nearly equal as may be.

13. Since a structure is in danger when any portion of it sustains a stress exceeding the **elastic limit,** that limit should be taken as constituting the ultimate stress.

14. Fatigue. In our following remarks on this subject, the pillars are supposed to sustain a *constant* load; and the ultimate or breaking load referred to is that one which would, during its first application, cripple or rupture the pillar in a short time. But struts in bridges etc often have to endure stresses which vary greatly in amount from time to time. Their ultimate load is then less. See p 465.

15. The resistance of a column is greatly affected by the arrangement of its ends, see Fig 3, below. Thus:

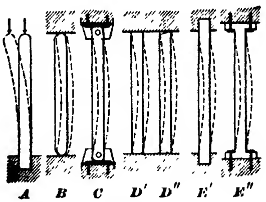

Fig. 3.

A. Free end. Column free to deflect, in any direction, about the other (fixed) end.

In the following cases, the ends are supposed to remain stationary, altho the axis is more or less free to bend.

B. Round ends. Axis free to deflect in any vertical plane.

C. Hinged or pinned ends. Deflection limited to one vertical plane.

D. Flat or square ends. Tangents to axis of column, at ends, fixed in position (see *D′*), until rotation, at ends, takes place, as in *D″*.

E. Fixed ends. Tangents to axis of column, at ends, fixed in position.

The two ends of a column may be differently arranged. Thus, one may be fixed and the other round; or one flat and the other pinned, etc.

Axial Loading.

16. The weight of the column itself, whether vertical or horizontal, is here neglected.

17. Let all dimensions be expressed in one and the same unit, as the inch, and all forces in one and the same unit, as the pound; and let

a = area of cross section;
r = its least radius of gyration;
I = ar^2 = its least moment of inertia;
T = distance from neutral axis of section to extreme fiber on concave side of bent col.
L = unsupported length of col (see ¶ 41).
K = $\frac{L}{r}$ = length ratio, or slenderness, of col;
b, m, c = coefficients, as explained below.
P* = load on column;
p* = P/a = av unit load on col;
s* = max unit stress in cross section,
s_e = elastic limit of material;
s_s = max unit stress in short col.;
E = elastic modulus of material;
e = eccentricity of load, P; Fig 5.
d = deflection under load P Fig 1 b = max dist from line of pres to axis of col.

18. Euler's formula. See Fig 4. Let the column be slightly bent by a lateral force. Then the average unit load p, which will just suffice to hold the column in equilibrium, in its bent condition, is

$$p = \frac{P}{a} = b \cdot \frac{\pi^2 E}{K^2}; \qquad K = \pi \sqrt{\frac{bE}{p}} \quad \ldots\ldots\ldots (1)$$

For round ends, b = 1; for fixed ends, b = 4. For one round and one fixed end, b is usually taken between 2 and $2\frac{1}{4}$.

Under any less load, or less K, the column will return to the straight condition. Under any greater load, or greater K, the bending will increase. See ¶ 8.

19. For very long columns, **Euler's formula**, ¶ 18, gives results agreeing well with experiment; but, for practical lengths, it gives excessive loads. See lines $E\,e$, $E'\,e'$, Fig 4. For practical lengths (if the column could remain straight and axially loaded), and down to short blocks, we should have, for the ultimate strength, $p = P/a = s_e$ = elastic limit, and the diagram would be $J\,b\,e$ or $J\,b'\,e'$, Fig 4. See ¶ 13.

20. But, owing to unavoidable imperfections, columns of usual lengths are liable to lateral deflection, and the diagram therefore falls below the line $J\,b\,b'$. See "Parabolic Formula," ¶ 28.

21. Rankine's formula† for axially loaded columns of practical lengths. Fig 2.

As in ¶ 17, let

p = mean unit load on column; **s** = max unit stress in cross sectn;
K = L/r = unsupported length ÷ least rad of gyration; **m** = a coefficient.

22. In an *axially* loaded column, (¶ 2) while unbent, we have

$$s = p = P/a.$$

23. But, when the column *bends*, an *additional* unit stress, s_1, is thereby thrown upon the material on its concave side (see ¶ 6), and we have, for the total unit fiber stress, s, on that side:

$$s = p + s_1 = (P/a) + s_1$$

24. The bending moment, due to the deflection, d, Fig 1 b, of the column, is $M = Pd$.

---

* Under any given conditions, P and p are the total load and the avge unit load, respectively, corresponding to the extreme fiber stress, s, existing under those same conditions. Thus, P, p and s may be those corresponding to ultimate or safe or any other loading.

† W. J. M. Rankine, "Civil Engineering," p 523. "Gordon's formula," attributed to Prof Lewis Gordon, of Glasgow, uses the least *diameter*, instead of the least *radius of gyration*, of the cross section.

25. As in beams, p 467, we have, for equilibrium between the bending moment, M, and the resisting moment, R,

$$M = R; \quad \text{or } Pd = \frac{s_1}{T} I.$$

$$\text{Hence } s_1 = \frac{M\,T}{I} = \frac{P\,d\,T}{a\,r^2} = p\,\frac{d\,T}{r^2}$$

$$\text{and } s = p + s_1 = p + p\,\frac{d\,T}{r^2} = p\left(1 + \frac{d\,T}{r^2}\right)$$

26. In a beam, p 481, for a given fiber stress, the deflection, d, is proportional to L^2/T. Assuming that this is true of columns also, we have:

$$d = m\,(L^2/T)$$

Hence,

$$s_1 = p \,.\, \frac{d\,T}{r^2} = p \,.\, m\,\frac{L^2}{T} \,.\, \frac{T}{r^2} = p\,m\,K^2; \quad \text{and}$$

$$s = p\,(1 + m\,K^2) \quad \ldots\ldots (2)$$

Hence, **Rankine's formula:**

$$p = \frac{P}{a} = \frac{s}{1 + m\,K^2} = s\,\frac{1}{1 + m\,K^2} \quad \ldots\ldots (3)$$

$$K = \sqrt{\frac{\frac{s}{p} - 1}{m}} \quad \ldots\ldots (3a)$$

27. Ritter* gives $m = s_e/(b\,\pi^2\,E)$; and Crehore† gives $m = s/(b\,\pi^2\,E)$. For values of b, corresponding to different arrangements of the ends of the column, see ¶ 18.

For values of s and of $1/m$, commonly used in practice, and for diagrams of values of $p/s = 1/(1 + m\,K^2)$, in columns of metal and wood, see Iron and Steel Columns, pp 1189, etc, and Wooden Columns, pp 1143, etc.

28. Parabolic formula. J. B. Johnson.‡

s_e = elastic limit of the material; $\quad$ q = a coefficient.

$$\left.\begin{array}{l}\text{Ultimate strength}\\ \text{lbs per sq inch,}\end{array}\right\} = p = \frac{P}{a} = s_e - \frac{s_e^2}{q\,\pi^2\,E}\,K^2 = s_e - c_p\,K^2 \ldots (4)$$

$$K = \sqrt{\frac{(s_e - p)\,\pi^2\,E\,q}{s_e^2}} \quad \ldots\ldots (4a)$$

For values of q, see Iron and Steel Columns, ¶ 20, p 1197. See also Wooden Columns, ¶ 6, p 1146.

29. Straight-line formula. For the sake of simplicity, with columns of practical lengths, we may use the straight-line formula:

$$\text{Ultimate strength, lbs per sq inch} = p = P/a = s_s - c\,K \quad \ldots\ldots (5)$$

$$K = (s_s - p)/c. \quad \ldots\ldots (5a)$$

For values of s_s and c, assigned by Thos. H. Johnson and others, see Steel and Iron Columns, pp 1189, etc, and Wooden Columns, pp 1143, etc.

The error, involved in the use of this formula, is probably much less than those inseparable from the nature of columns.

30. Fig 4 shows a **comparison of results by formulas for, and experiments upon, round-end mild steel columns**, as follows:—**For Euler's formula**, p is that unit load, in lbs per sq inch, which will just hold, in slightly bent condition, a column of the given E and K. **For the other formulas** and for Tetmajer's experiments, p is the unit *crippling* load, in lbs per sq inch.

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* Dach- und Brücken-Constructionen, 1873.
† Van Nostrand's Magazine, 1879.
‡ Modern Framed Structures, New York, John Wiley & Sons, 1893. p 150.
§ Trans Am Soc Civ Engrs, 1882, Vol viii, pp 97, 113, 115; 1886, Vol xv, p 517.

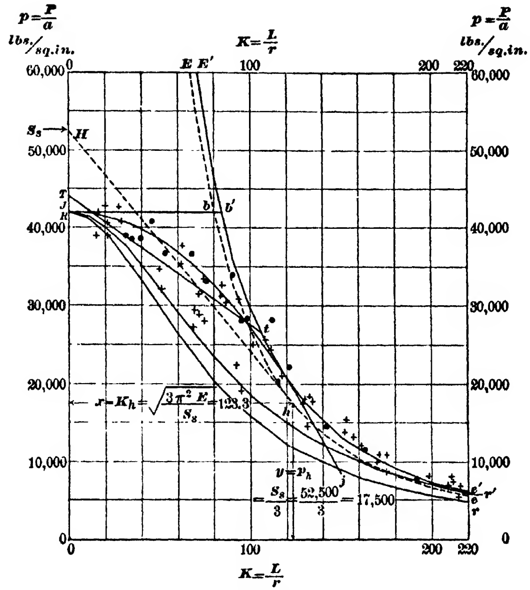

Fig. 4.

Formulas.

E e. Euler, $p = \pi^2 E/K^2$; $E = 27,000,000$ lbs per sq inch;
E' e'. Euler, $p = \pi^2 E/K^2$; $E = 30,000,000$ lbs per sq inch;
H h. straight-line, $p = 52,500 - 284\ K$;
T t. Tetmajer, straight-line, $p = 44,000 - 162\ K$;*
J j J. B. Johnson, parabolic, $p = 42,000 - 1.489\ K^2$;†
R r. Rankine, $s = 42,000$; $1/m = 6000$;
R' r'. Rankine, $s = 42,000$; $1/m = 8000$.

Experiments.

From Tetmajer *: (dimensions approximate)

● 16 round bars, from ¾ inch to 1¾ inches diameter.

✚ 102 experiments, covering angles, 4 × 4 ins, T-shapes 5 × 2½ ins, Channels, 5½ × 2¼ ins, I-beams, 7 × 3¼ ins; riveted columns as follows:— 2 angles, 3¼ ins; 4 angles, 2½ ins; 2 T-shapes, 3½ × 1¾ ins; 2 channels, 3¼ × 1½ ins. Each point represents the average of two experiments.

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* Mitteilungen der Materialprufungsanstalt am schweizerischen Polytechnikum in Zürich. Heft VIII, 1896.

† $1.489 = s_c^2/\pi^2 b E = 42,000^2/(9.87 \times 30,000,000\ b)$. For round ends, $b = 4$. Materials of Construction, p 363. Compare Iron and Steel Columns, ¶ 20, p 1197.

31. T. H. Johnson finds that, **for all tangents to Euler's curve,** the co-ordinates of the point (as h, Fig 4) of tangency, are:

Abscissa, $x = K_h = \sqrt{3\,z\,E/s_s}$; Ordinate, $y = p_h = s_s/3$;

where $z = \pi^2 = 9.87$, for round ends; $z = (5/3)\,\pi^2 = 16.45$, for hinged ends, and $z = (5/2)\,\pi^2 = 24.67$, for flat ends; and s_s = the ordinate of that point, H, where the tangent meets the axis of ordinates.

Mr. Johnson takes $E = 27{,}000{,}000$ lbs per sq in, and $s_s = 52{,}500$. Hence, in Fig 4, $x = 123.3$, and $y = 17{,}500$.

32. Tetmajer gives the following values, x', for the abscissa of the point where his straight line meets Euler's curve

	E	x'
Wood	1,400,000	100
Cast iron	14,000,000	80
Wrought iron	28,000,000	112
Structural steel, soft	30,000,000	105
Structural steel, medium	32,000,000	105

Hence, in Fig 4, his straight line, $T'\ t$, meets Euler's curve, $E'\ e'$ (not tangentially) at t, where $K = 105$.

33. J. B. Johnson takes $E = 30{,}000{,}000$ lbs per sq in; and his curve, $J\ j$, is tangent to Euler's curve, $E'\ e'$, at the point where $K = 120$. For mild steel columns, with pin ends, he limits K to a max value of 150

34. Stepped formula. J. R. Worcester. In view of the wide divergence in the strengths of columns, as found by the several formulas and coefficients in use, and of the similarly wide variation in the results of experiments, Mr. J. R. Worcester * considers it idle to assume a different value of p for each value of K; and proposes a *fixed* value of p for a certain *range* of values of K. Thus, for compression members of structural steel in bridges he proposes ($K/12 = L$, in *feet*, ÷ r, in *inches*)

for $K/12$ from 2 to 4, $p = 13{,}000$ lbs per sq inch,
for $K/12$ from 4 to 6, $p = 12{,}000$ lbs per sq inch;

and so on, with a decrease, in p, of 1000 lbs per square inch for each succeeding increase of 2 in the value of $K/12$. For steel columns, he gives $(K/12) \not> 16$. Mr. Worcester's formula gives a "stepped" diagram, composed of horizontal and vertical lines.

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* Trans Am Soc Civ Engrs, Vol 54 p 417, June, 1905.

Eccentric Loading. Fig 5.

35. Hitherto we have assumed that the column is axially loaded (see ¶ 2) and, in considering inequality of distribution of stress in a cross section, Fig 2, with its resulting excess load on the extreme fibers of the concave side, we have taken into account only that excess, s_1, which is due to the *deflection* of an *axially* loaded column.

36. But it very commonly happens that columns are (accidentally or intentionally) eccentrically loaded; and this eccentricity, e, brings, upon the extreme fibers on the side nearest the line of pressure, an *additional* excess unit stress, s_2, Fig 5. If the column remained perfectly straight, we should have (see pp 467–8) $s_2 = TM/I = TPe/ar^2 = (P/a)(Te/r^2) = pTe/r^2$; and hence, in a bent column, (see Eq 2, where $s_1 = p\,m\,K^2$),

$$s = p + s_1 + s_2 = p\left(1 + mK^2 + \frac{Te}{r^2}\right) \qquad (6)$$

and

$$p = \frac{P}{a} = \frac{s}{1 + mK^2 + \dfrac{Te}{r^2}} \qquad (7)$$

$$K = \sqrt{\frac{\dfrac{s}{p} - 1 - \dfrac{Te}{r^2}}{m}} \qquad (7a)$$

where, as in ¶ 17:

- **p** = mean unit load on column; **e** = eccentricity, Fig 5;
- **s** = maximum unit stress in cross section;
- **K** = L/r = unsupported length ÷ least radius of gyration;
- **T** = dist from neut ax of sectn to extreme fiber on concave side.

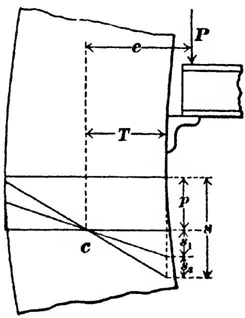

Fig. 5.

37. As a matter of fact, the deflection of the column, provided for in the term, mK^2 (Eqs 6 and 7), slightly increases the eccentricity, e, of the load, (see Fig 5) and this is neglected in the term Te/r^2. Hence, the fiber stress, s, given by equation (6) is a little too small, and the load, p, given by equation (7), is a little too great; but the discrepancy is ordinarily regarded as negligible in practice.

38. For round end columns, **J. M. Moncrieff*** gives:

$$p = \frac{P}{a} = \frac{s}{1 + \frac{Te}{r^2}\left(\frac{48\,E + p\,K^2}{48\,E - 5\,p\,K^2}\right)} \quad\ldots\ldots\ldots\ldots(8)$$

$$K = \sqrt{\frac{48\,E}{5\,s + p\,(Te/r^2 - 5)}\left(\frac{s}{p} - 1 - \frac{T\,e}{r^2}\right)} \quad\ldots\ldots\ldots\ldots(8a)$$

39. Let

M = $P\,e$, Fig 5, = moment of an eccentric load, P, about c;
I = moment of inertia of cross section;
T = distance from neutral axis to furthest fiber;
X = I/T = section modulus of cross section in direction of bending;
s_2 = maximum fiber stress due to eccentricity only.

Then

$$s_2 = M\,T/I = M/X \quad\ldots\ldots\ldots\ldots(9)$$

40. With columns supposed to be axially loaded, the loading is, in fact, very generally more or less eccentric, and to an unknown degree; but where a load is applied at a point at or beyond the edge of the column, as in Fig 5, the eccentricity is so great that unavoidable uncertainties, as to its exact amount, become relatively negligible, and Eqs (7) and (7*a*) may be used with some confidence.

41. If a very long col be so **braced at intervals** as to prevent its bending at those points, then its length becomes virtually diminished, and its strength increased. Thus, if a column 100 ft long be sufficiently braced at intervals of 20 ft, then the load sustained may be that due to a column only 20 ft long.

Cautions.

42. Cast iron columns are subject to blow holes and to other hidden defects, and are easily broken by side blows. Columns of all kinds are subject to jars and vibrations from moving loads. It very rarely happens that the pressure is equally distributed over the whole area of the pillar; or that the top and bottom ends have perfect bearing at every part.

43. In column tests, exceptionally high results may occur where the conditions happen to approach the ideal.

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* Trans Am Soc Civ Engrs, Vol 45, p 349, June, 1901.

SHEARING STRENGTH.

Shearing or detrusion occurs when a body is acted upon by two opposite forces in parallel and closely adjacent planes, tending to slide some of the particles over the others. In Fig 1, the two forces are (1) the downward pressure of the weight, W, and (2) the upward reaction of the support, A.

In **single shear**, Fig 1, the shearing area, a, = the section $g\,g$. In **double shear**, Fig 2, $a = g\,g + o\,o = 2 \times g\,g$. In Fig 3, $a = 6 \times$ cross section of piece. In Fig 4 (single shear), a = section $c\,c$. In punching rivet holes, a = circumference of hole $\times$ thickness of plate

In any case, if S = the ultimate unit shearing stress, Shearing strength = S a.

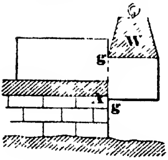

Fig. 1

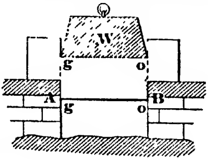

Fig. 2

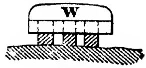

Fig. 3

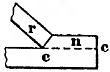

Fig. 4

Ultimate unit shearing stress, S, in lbs per sq inch. The following figures indicate the range of values of S in metals and in timber.

Metals. Wrought iron, 35,000 to 55,000; cast iron, 20,000 to 30,000; steel, 45,000 to 75,000; copper, 33,000.

		With the grain, Fig 4.	Across the grain.
Timber: From our experiments:	Spruce	250 to 500	3,250
	White pine	"	2,500
	Hemlock	"	"
	Yellow pine	......	4,300 to 5,600
	Oak	400 to 700	....
	White oak	......	4,400

TORSIONAL STRENGTH.

Torsion occurs when a body is acted upon by two couples or moments of contrary sense and in different planes. Thus, torsion takes place in a brake axle when we try to turn it while its lower end is held fast by the brake chain; and in shafting, when it transmits the motive power of an engine to tools. Suppose such a body to be divided, by cross sections, into layers. Then each layer tends to shear across from those next to it. Hence, in order to maintain equilibrium, each two adjacent layers must exert, in the cross section between them, an internal resisting moment equal to one of the two external and contrary torsional moments.

Resisting moment in a circular cross section of a cylindrical shaft. Let

P = the torsional force of one of the two external moments..... ....in pounds;
l = its leverage, = its distance from the axis of the shaft..................in inches;
M = P l = external or torsional moment..................................in inch-pounds;
T = distance from axis to farthest fibers, = radius of shaft.................in inches;
D = diameter of shaft, = 2 T...in inches;
S = unit shearing stress in farthest fibers.......................in pounds per sq inch;
t = distance from axis to any given fiber...in inches
s = unit stress in said given fiber ....in pounds per square inch;
a = area of said given fiber..in square inches;
F = total stress in said fiber ....in pounds;
r = resisting moment of said fiber about the axis.....................in inch-pounds;
R = internal resisting moment of the entire cross section
= Σr = sum of resisting moments of all the fibers..................in inch-pounds;
I_p = polar moment of inertia* of the cross section
= moment of inertia of cross section about the axis of the shaft
= $\Sigma t^2 a$, = the sum, for all the fibers, of $t^2 a$.................................in inches.

*In any figure, the polar moment of inertia, I_p, is = the sum of the greatest and the least moments of inertia of the same figure, about two axes lying in the figure and intersecting in its center. In a solid circle, each of these is a moment of inertia about a diameter, and is $= \pi T^4 \div 4$. Hence, in such a circle, $I_p = \pi T^4 \div 2$.

Then the unit stress, in any given fiber, is $s = S\,l \div T$; its total stress, F, is $= a\,s = S\,a\,l \div T$; and its resisting moment, r, is $= F\,l = S\,a\,l^2 \div T$. For equilibrium, the internal resisting moment, R, of the entire section, must be = the external torsional moment, M.

Hence, for the **Internal Resisting moment,** R, we have:

$R = M = \Sigma\, r = \Sigma\, S\,a\,l^2 \div T = (S \div T)\,\Sigma\, l^2\,a = (S \div T)\,I_p.$

Hence, also, $S = M\,T \div I_p$; $M = S\,I_p \div T$; and $P = M \div l = S\,I_p \div (T\,l)$.

In a solid circle, $I_p = \pi\,T^4 \div 2$. Hence, $S = 2\,M\,T \div (\pi\,T^4) = 2\,M \div (\pi\,T^3)$; $M = S\,\pi\,T^3 \div 2$; $P = S\,\pi\,T^3 \div (2\,l)$; and

$$\textbf{Diameter,}\ D, = 2\,T = 2 \times \sqrt[3]{\frac{2\,P\,l}{\pi\,S}} = \sqrt[3]{\frac{5.1\ M}{S}} = 1.72\,\sqrt[3]{\frac{M}{S}}.$$

For approximate ultimate values of S, for torsion, use the values for shearing, p 499, with **safety factors** from 5 to 10.

Horse power of shafting. In one revolution, the force, P lbs, describing a circle with radius l ins, does a work $2\,\pi\,l\,P$ inch-lbs, and, in n revolutions, work $= 2\,\pi\,l\,P\,n$ inch-lbs. If n be the number of revolutions *per minute*, the horse power is:

$H = 2\,\pi\,l\,P\,n \div (12 \times 33{,}000) = 2\,\pi\,M\,n \div (12 \times 33{,}000)$;

or, since $P\,l = M = R = S\,I_p \div T$, we have:

$H = S\,\pi\,n\,I_p \div (12 \times 16{,}500\,T)$; and $S = 12 \times 16{,}500\,T\,H \div (\pi\,n\,I_p)$.

In a solid cylindrical shaft, $I_p = \pi\,T^4 \div 2$. Hence,

$H = S\,\pi\,n\,\pi\,T^4 \div (12 \times 33{,}000\,T) = S\,\pi^2\,n\,T^3 \div (12 \times 33{,}000)$;

$S = 12 \times 33{,}000\,H \div (\pi^2\,n\,T^3) = 12 \times 3{,}343\,H \div (n\,T^3)$;

$n = 321{,}000\,H \div (D^3\,S)$; and

$$\textbf{Diameter,}\ D = 2\,T = 2 \times \sqrt[3]{\frac{12 \times 3{,}343\,H}{S\,n}} = \sqrt[3]{\frac{321{,}000\,H}{S\,n}} = 68\,\sqrt[3]{\frac{H}{S\,n}}.$$

The higher the speed, the less is the force, and hence the less is the strength of shaft, required in order to transmit a *given horse-power*; but if the speed is increased by increasing the torsional force, the horse-power transmitted is thereby increased also.

Example. Given, a wrought iron shaft; let S = 6,000 lbs per sq inch; P = 7,500 lbs; l = 10 ins; M = 75,000 inch-lbs. Required the diameter, D. Here, $D = 1.72 \times \sqrt[3]{M \div S} = 1.72 \times \sqrt[3]{75{,}000 \div 6{,}000} = 1.72 \times \sqrt[3]{12.5} = 1.72 \times 2.32 = 4$ ins. Let the horse-power, H, = 25. Then $n = 321{,}000\,H \div (D^3\,S) = 321{,}000 \times 25 \div (4^3 \times 6{,}000) = 21$ revolutions per minute. Checking, $D = 68 \times \sqrt[3]{H \div (S\,n)} = 68 \times \sqrt[3]{25 \div (6{,}000 \times 21)} = 68 \times 0.058 =$ say 4 inches.

Rectangular Sections. The foregoing equations are based upon the assumption that the stress increases uniformly from the axis of the shaft outward. It has been shown (notably by St. Venant*) that this assumption is not applicable to square and rectangular sections. In a rectangle, let B = the longer, b = the shorter side, and $c = b \div B$. Then $S = M\,(3 + 1.8\,c) \div (B\,b^2)$; and $P = S\,B\,b^2 \div [(3 + 1.8\,c)\,l]$. In a square, with side $= b$, this becomes: $S = 4.8\,M \div b^3$; $M = S\,b^3 \div 4.8$. $P = S\,b^3 \div (4.8\,l)$.

The angle of torsion is that described by one of the external torsional moments, relatively to the other. Within the elastic limit, this angle is proportional to the torsional moment, M, and (assuming l constant) proportional to the force, P. Other things being equal, the angle is proportional to the distance between the planes of the two contrary external moments, and, in a solid cylindrical shaft, is inversely proportional to D^4. It is inadvisable to allow the angle of torsion to exceed 1° in a length = 20 diameters, in shafts revolving in one direction. In reciprocating shafts allow still less. See Fatigue, p 465.

Practical Considerations. In many cases the diameter of the shaft must be made greater than that required by the foregoing formulas; as in a long shaft, in order to keep the angle of torsion within permissible limits; in fly-wheel and other shafts, carrying considerable bending loads in addition to the torsion; and, in most cases, to allow for additional moments due to alternate acceleration and retardation.

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*See Treatise on Natural Philosophy, by Sir William Thomson and Peter Guthrie Tait, Part II, New Edition, Cambridge, 1890, pp 236, etc.

HYDROSTATICS.

Art. 1. Hydrostatics treats of the pressure of water and of other fluids at rest.

At any given point within a fluid **the pressure is equal in all directions;** and the pressure against any point of any surface, whether plane or curved, is **normal to the pressed surface** (or to a plane tangent to that surface) at that point.

The intensity of the pressure is proportional to the depth of the point below the water surface.

Pressure against any plane surface.

Let

a = the area of the pressed surface;
h = the vertical depth of the center of gravity of the pressed surface below the free surface of the fluid;
H = the total depth of the fluid;
w = the weight of a unit volume of the fluid;*
p = the mean unit pressure on the pressed surface;
P = the total pressure on the pressed surface;

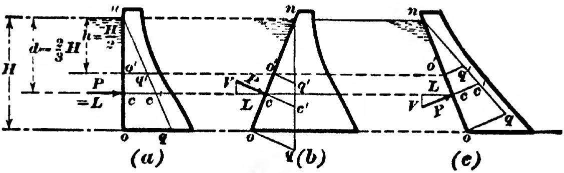


Fig. 1.

Then the **mean unit pressure,** p, is equal to the weight of a prism of the fluid, whose base is = 1, and whose length is = h; or

$$p = h\,w;$$

and the **total pressure,** P, is equal to the weight of a prism of the fluid, whose base is = a, and whose length is = h. Or

$$P = a\,h\,w = a\,p.$$

In the diagrams of Figs. 1 and 2, the *ordinates* (supposed to be drawn from, and normally to, the pressed surfaces respectively) represent the *unit* pressures (as in lbs. per sq. inch, kilograms per sq. centimeter, etc.), and the *area*, opposite any given surface, represents the *total* pressure on that surface. Thus, in Fig. 1 (a), the unit pressures at n, at o', at c and at o, are represented by n (= 0), by $o'\,q'$, by $c\,c'$ and by $o\,q$ respectively; and the total pressure on $n\,o$ is represented by the area $n\,o\,q$, that on $n\,o'$ by the area $n\,o'\,q'$, that on $o'\,o$ by the area $o\,q'$, and that on $o'\,c$ by the area $c\,q'$.

The center of pressure upon any surface is opposite the center of gravity of the area representing the total pressure upon that surface. Thus, in Figs. 1, the center of pressure on $n\,o$, is opposite the center of gravity of the triangle, $n\,o\,q$, or at a depth, d, = $\frac{2}{3}$ H, below the water surface. See The Center of Pressure, Arts 8, etc., also ¶¶ 133 etc., of Statics.

The hydrostatic paradox. For a given depth, h, both the mean unit pressure, p, and (for a given area, a) the total pressure, P, are independent of the quantity of water. Thus, in Figs. 1, the walls sustain as great a pressure from a vertical film of water only an inch thick, as if the water extended back

* For water, w = about 62.5 lbs. per cub. ft., = about 0.0362 lbs. per cub. inch.

for miles. In Fig. 2 (*b*), the excess of weight of water, over that in Fig. 2 (*a*), is carried by the lower sloping *wall* of the vessel. The total pressures, P, upon the equal bases, *a b* and *a' b'*, Figs. 2 (*c*) and (*d*), are equal. In Fig. 2 (*c*), the total pressure upon the base is greater, and in Fig. 2 (*d*) less, than the weight of the water; but, in either case, the algebraic sum of all the vertical pressures in the vessel (*upward* pressures taken as *negative*) is = the weight of all the water in the vessel; and the algebraic sum of all the horizontal pressures is = 0.

Thus, let the lower part of Fig. 2 (*c*) represent a cubical box, 8 feet on a side, and filled with water. Now let the tube, *n o*, 36 ft. high and of 0.287 inch bore, be filled with water. The water in the tube alone, although weighing only about 1 pound, will cause an additional bursting pressure of 2250 lbs. per square foot, or say 384 tons total, to be exerted upon the top, bottom and sides of the box.

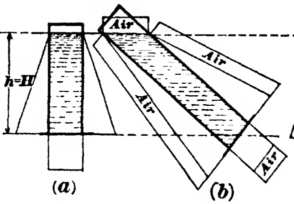


Fig. 2.

Air pressure on water surface. In addition to the pressure of the water itself, the free surface of any body of water sustains also the pressure of the air, = about 14.7 lbs. per sq. inch. This pressure (transmitted through the water to the walls of the vessel) is indicated by the diagrams (parallelograms) marked "air" in Fig. 2 (*b*). In most cases, the pressure against a surface, due to the air pressure on the water surface, is counter-balanced by an equal pressure of the air in the opposite direction, as against the outer sides of the walls of the vessels in Figs. 2, and against the down-stream sides of the dams in Figs. 1.

Strictly speaking, the air pressure, exerted directly against the walls, Fig. 2, or against the dams, Figs. 1, being exerted practically at the centers of gravity of those surfaces, and therefore at lower elevations than the opposite pressure due to the air pressure on the water surface, is a very little greater than the latter. In a dam 100 ft deep, this difference would amount to about 0.027 lb per sq inch, = 0.0018 atmosphere.

Braces for dams. The water pressure being normal to the pressed surface, the posts, Fig. 3, must also be normal to the surface, D, if they are to receive

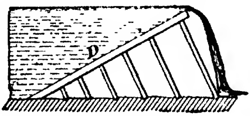


Fig. 3.

that pressure longitudinally and thus avoid bending moments; but other considerations may forbid their being so placed. Thus, if the face, D, of the dam, is nearly vertical, the posts would have to be made inordinately long; and their consequent weakness, as pillars (unless made inconveniently thick) might more than offset the advantage due to directness of pressure. Moreover, the feet of the down-stream posts would project beyond the crest of the dam, and would thus be liable to injury by ice or logs, etc., tumbling over the dam.

Inasmuch as the pressure increases uniformly from the water surface downward, the posts are usually placed closer together near the bottom than near the top, although the shortness of the lower ones renders them stronger as pillars. Similarly, the hoops on tanks, if of uniform strength, are placed closer together near the bottom.

Horizontal and vertical components. In Figs. 1 (*b*) and (*c*), the force triangle (Statics, ¶¶ 46, etc.) gives us the horizontal and vertical components, L and V, of the total normal pressure, P. Or, if *n o* be taken, in each case, as representing the total normal pressure, P, by scale, then H = the total horizontal pressure. In Fig 1 (*a*), with pressure against a vertical surface, L = P, and V = 0.

In Fig. 1 (*b*), the vertical component, V, presses the wall downward against its base, but in Fig. 1 (*c*) it tends to uplift and overturn it.

The depth, H, being the same in each of the three figures, Figs. 1 *a*, *b* and *c*, the *vertical* projections of the three submerged surfaces are equal, and hence the total *horizontal* pressures are equal in the three cases; but the *horizontal* projection, and consequently the total *vertical* pressure, vary with the inclination of the surface. Thus, in Fig. 1 (*a*), the horizontal projection and the vertical pressure are each = 0.

Pressures in cubical and other vessels, full of water. Let F = the weight of water contained in a prismatic vessel; and $f = \frac{F}{3}$ = the weight of that in a conical or pyramidal vessel of the same base and height.

In a cubical vessel, pressure on base = F; pressure on one side = $\frac{F}{2}$; pressure on base and four sides together = $F + 4\frac{F}{2} = 3\,F$.

In a conical or pyramidal vessel, pressure on base = $3\,f$ = F.

In a spherical vessel, total pressure = 3 × weight of water.

Art. 2. Unequal pressure in opposite directions. In Figs. 4, let *n o* represent the edge of a parallelogram, whose top coincides with the water surface, and whose bottom is parallel with that surface; let *a* = the area of that portion, *n' o*, which is subjected to pressure on both sides; and let H and

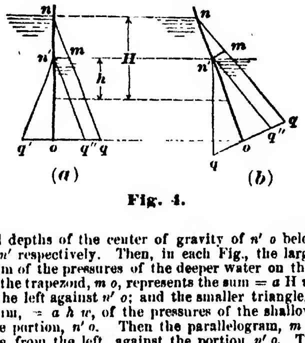


Fig. 4.

h = the vertical depths of the center of gravity of *n' o* below the two water surfaces, *n* and *n'* respectively. Then, in each Fig., the large triangle, *n o q*, represents the sum of the pressures of the deeper water on the left against the entire wall, *n o*; the trapezoid, *m o*, represents the sum = *a* H *w*, of the pressures of the water on the left against *n' o*; and the smaller triangle, *n' o q'*, = *n' o q''*, represents the sum, = *a h w*, of the pressures of the shallower water on the right against the portion, *n' o*. Then the parallelogram, *m q''* represents the *excess* of pressure, from the left, against the portion, *n' o*. This excess, due to the difference, H — *h*, between the two levels, is *uniformly* distributed over *n' o*, the uniform excess *unit* pressure being represented by the ordinate *n' m* = *q'' q*. The *total* excess pressure, represented by the parallelogram, *m q''* = *a* H *w* — *a h w* = (H — *h*) *a w*, is therefore proportional to H — *h*.

The pressure coming from the right against the portion *n' o*, and represented by the triangle *n' o q'*, is balanced by an equal portion (represented by the triangle *n' o q''*) of the total pressure, *m o*, from the left against the portion *n' o*; and the centers of these two pressures, each being at a depth = ⅔ *n' o* below *n'*, are opposite. Hence these two pressures are in equilibrium. But the center of the excess pressure, *n' q*, from the left, is opposite the center of gravity of the parallelogram, *n' q*, or at the center of gravity of *n' o*. Hence the portion *n' o*, considered independently of *n' n*, is acted upon by the unbalanced force *n' q*, coming from the left, acting through its center of gravity and therefore tending to move it bodily toward the right, without rotation.

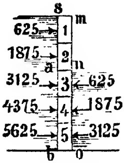


Fig 5

This will be understood by means of Fig 5 which may represent five planks, 1, 2, 3, 4, and 5, forming a dam, and seen endwise, each one 1 ft in depth, and say 20 ft long hor, making the area of each surf pressed, equal to 20 sq ft. The pres in lbs against each separate 20 sq ft of area, calculated by the rule in Art 1, is shown in the fig Now, the outward pres against the upper *immersed* 20 ft area, or that of plank 3, is 3125 lbs, while the counter-pres against it from the other side is 625 lbs, making the *excess* of outward pres equal to 3125 — 625 = 2500 lbs. Again, at the lowest plank, number 5, the outward pres exceeds the inward one by 5625 — 3125 = 2500 lbs, the same as in the upper one And so of any other equal area of surf, at any depth whatever; the excess depending upon the vert height of *m n*, will be equally distributed over *a b*. It only remains to show that the total excess of outward pres against *a b*, is equal in amount to the wt of a uniform column of water with a base equal in area to *a b*, and with a height equal to *m n* * Thus, we have seen that in the instance before us, the excess amounts to 3 times 2500 lbs, or to 7500 lbs. Now, the wt of the column of water will be 60 (or area of *a b*) × *m n* (or 2 ft; × 62.5 lbs = 7500 lbs; or the same as the *excess* pres on *a b*

The excess of pres against the entire side *s b*, over that against *n o*, is evidently the diff between those two pressures calculated respectively by the rule in Art 1.

Art. 3. Surfaces, vert, as *b m c s*, *a n o t*, Fig 6, or otherwise, of equal widths, *b m*, *a n*; commencing at the level, *b a n m*, of the water, but extending to diff depths, *m c*, *n o*, measured vert; and having the same inclination to the surf of the water; sustain total pressures proportional to the squares of those depths.

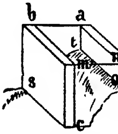


Fig 6

In Fig 6, let the two vert sides, *a n o t*, and *b m c s*, of a vessel, have the same width *a n*, and *b m*, then if the depth *m c*, be 2, 3, 4 5, &c times greater than the depth *n o*, the pres against the surf *b m c s* will be 4, 9, 16, 25, &c, times greater than that against *a n o t* This will be seen by referring to the pressures figured on the left side of Fig 5, where, as stated in Art 2, the surf of plank 1, exposed to the pres on the left side, is 20 sq ft, that of planks 1 and 2, 40 sq ft, that of planks 1, 2, and 3, 60 sq ft, &c All these surfs commence at the level of the water, and all of them being vert are of course at the same inclination with the water surf, but their depths are respectively 1, 2, and 3 ft The pres against the surf of 1, is 625 lbs; that against the surf of 1, 2, is 625 + 1875 = 2500, and that against the surf of 1, 2, 3, is 625 + 1875 + 3125 = 5625 But 2500 is *four* times 625, and 5625 is *nine* times 625. And the pres against the entire surf *s b*, (which is 5 times as deep as plank 1,) is 25 times as great as that against plank 1, or 625 × 25 = 15625 lbs = the sum of all the pressures marked on the left side of Fig 5

This follows, from the Rule in Art 1, for twice the area of surf, mult by twice the vert depth of the cen of grav below the surf, must give 4 times the pres, three times the area, by three times the depth, must give 9 times the pres, &c.

It follows, also, that at any particular *point*, or against any *given* area placed at various depths, the pres will increase simply as the vert depth thus, if there be three areas, each one sq ft, placed in the same positions, but with their centers of grav respectively 8, 16, and 24 ft below the surf, the pres against them will be respectively *as* 8, 16, and 24, or *as* 1, 2, and 3

Art. 4. The pressure of quiet water, in any one given direction, against any given plane surface, whether vertical, horizontal, or inclined, is equal to the weight of a prismatic column of water, the area of whose section, parallel to its base, is equal to the area of the projection of the given surface taken at right angles to the given direction, and whose height is equal to the vertical depth of the center of gravity of the given surface below the upper surface of the water. Hence the

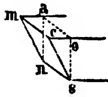


Fig 7

RULE **To find the pres in lbs,** mult together the area in sq ft of the projection taken at right angles to the given direction; the vert depth in ft of the cen of grav of the pressed surf below the upper surf of the water, and the constant 62.5 lbs wt of a cub ft of water.

Ex. Let *m c s n*, Fig 7, be an inclined surf, sustaining the pres of water which is level with its top *m c* Then the *total* pres against *m c s n*, and at right angles to it, as found by the rule in Art 1, is an illustration of the present rule; because the projection of *m c s n*, taken at right angles to the given direction, or parallel to *m c s n*, is in fact *m c s n* itself, or equal to it Hence the rule in Art 1 is merely a simple modification of the present one, applicable to the case of *total* pres against any surf.

But if it be reqd to find only the vert or downward pres against *m c s n*, in pounds, mult together the area of the hor projection *a o c m* in sq ft; the vert depth in ft of the cen of grav of *m c s n* below the surf, and 62 5. Or if only the *hor* pres against *m c s n* be sought, mult together the area of the vert projection *a o s n*; the vert depth of the cen of grav of *m c s n*, and 62 5.

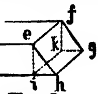


Fig 8

In Fig 8 also, the *total* pres against *e f g h* is found by rule in Art 1; while the *hor* and vert pressures against it are found as in Fig 7, by using the projections *e f k i*, and *k i g h*. In Fig 7 the vert pres is downward; while in Fig 8 it is upward, but this circumstance in no respect affects the rule.

REM. 1. At any given depth, the pres, perp to any given surf, is the same in all directions; but Figs 7 and 8 show that the total pres *oblique* to a given surf will be less than the perp one at the same depth; because an oblique projection of a surf must be less than the surf itself, which last is the projection when the pres is perp to it. Thus, in a reservoir, the total pres perp to a sloping side, as *m n s s*, Fig 7 is *greater* than either the vert or the hor pres upon it.

Again, let Fig 9 represent **a conical vessel full of water;** its base $b\ c$, 2 ft diam; its vert height $a\ n$, 3 ft, then the circumf of the base will be 6.2832 ft, the area of the base 3.1416 sq ft, the length of its slant side $a\ b$ or $a\ c$, 3.16 ft, the area of its curved slanting sides will be $\frac{6.2832 \times 3.16}{2} = 9.93$ sq ft; and the vert depth of the cen of grav of the slanting sides will be at two thirds of the vert height $a\ n$ from the apex a, or 2 ft.

Fig 9

Here, to find the total pres against the base, we have by rule in Art 1, 3.1416 × 3 × 62.5 = 589.05 lbs. For the total pres against the slant sides, by the same rule, 9.93 × 2 × 62.5 = 1241.25 lbs. For the vert pres upward against the entire area of the slant sides, we have given the area of the base (which is here the hor projection of the slant sides) = 3.1416, and the vert depth of the cen of grav of the slant sides, 2 ft. Therefore, 3.1416 × 2 × 62.5 = 392.7 lbs, the upward vert pres.

Finally, for the hor pres in any given direction against the slant sides of *one half* of the cone, we have the vert projection of that half, represented by the triangle $a\ b\ c$, with its base 2 ft, and its perp height 3 ft, and consequently, with an area of 3 sq ft. The depth of its cen of grav is 2 ft; therefore, 3 × 2 × 62.5 = 375 lbs, the reqd hor pres.*

In Fig 10, which represents a vessel full of water, the *total* pres against the semi cylindrical surf $a\ v\ e\ m\ d\ k$, and perp to it, must be also hor, because the surf is vert, but inasmuch as the surf is *curved*, this total pres, as found by rule in Art 1, acts against it in many directions, which might be represented by an infinite number of radii drawn from o as a center. But let it be reqd to find the hor pres in lbs, in one direction only, say parallel to $o\ e$, or perp to $a\ d$; which would be the force tending to tear the curved surf away from the flat sides $a\ b\ n\ i$, and $d\ c\ s\ k$, by producing fractures along the lines $a\ v$ and $d\ k$, or which would tend to burst a pipe or other cylinder. In this case, mult together the area of the vert projection $a\ d\ k\ v$ in sq ft, the depth of the cen of grav of the curved surf in ft, (which, in the semi cylinder would be half of $e\ m$, or of $o\ i$,) and 62.5. Since the resulting pres is resisted equally by the strength of the vessel along the two lines $a\ v$ and $d\ k$, it is plain that each single thickness along those lines need only be sufficient to resist safely *one half* of it. and so in the case of pipes, or other cylinders, such as hooped cisterns or tanks. See Art 17.

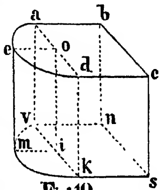


Fig 10

Should the pres against only one half of the curved surf, as $e\ d\ m\ k$ be sought, and in a direction parallel to $o\ d$, tending to produce fractures along the lines $e\ m$, and $d\ k$, then use the vert projection $o\ e\ m\ i$, with the same depth, and 62.5 as before.

It follows, that if the face of a metallic piston be made concave or convex, no more pres will be reqd to force the piston through any dist, than if it were flat, for the pres against the face of the piston, in the direction in which it moves, must be measured by the area of a projection of that face, taken at right angles to said direction, and the area of said projection will be the same in all three cases.

REM 2. **If a bridge pier, or other construction, Fig 10½, be founded on sand or gravel,** or on any kind of foundation through which water may find its way underneath, even in a very thin sheet, then the upward pres of the water will take effect upon the pier, and will tend to lift it, with a force equal to the wt of the water displaced by the pier; (Arts 18, 19.) In other words, the effective wt of the *submerged* portion of the pier, will be reduced 62½ lbs per cub ft; or nearly the half of the ordinary wt of masonry.

Fig 10½

But if the foundation be on rock, covered with a layer of cement to prevent the infiltration of water beneath the masonry, no such effect will be produced but on the contrary, the vert pres downward, afforded by the battering sides of the pier, and by its offsets, will tend to hold it down, and thus increase its stability; which, in *quiet* water, will then actually be greater than on land.

Art. 5. To divide a rectangular surf, whether vert as *a b c d*, or inclined as *m n o p*, Fig 11, whose top *a b* or *m n* is level with the surf of the water, by a hor line *x* 2, such that the total pres against the part above said hor line, shall equal that against the part below it.

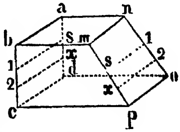


Fig 11

RULE. Mult one half of the length of $b\ c$, or $m\ p$, as the case may be, by the constant number 1.4142, the prod will be b 2, or $m\ x$.

Ex. Let $b\ c$ = 12 ft. Then 6 × 1.4142 = 8.4852 ft, or b 2. Let $m\ p$ = 16 ft. Then 8 × 1.4142 = 11.3136 ft, or $m\ x$.

REM. The line x 2, thus found, must not be confounded with the *cen of pres* which is entirely diff. See Art 8.

Art. 6. In a rectangular surf, whether vert as *a b c d*, or inclined as *m n o p*, Fig 11, whose top *a b* or *m n* coincides with the surf of the water, to find any number of points, as 1, 2, &c, through which if hor lines, as 1 *s*, 2 *x*, &c, be drawn, they will divide the given surf into smaller rectangles, all of which shall sustain equal pressures.

RULE. First fix on the number of small rectangles reqd. Then for point 1 from the top, mult the number 1, by this number of rectangles. Take the sq rt of the prod. Mult this sq rt by the entire length

* **In a sphere** filled with a fluid the total inside pres = 3 times wt of fluid.

$b\,c$ or $m\,p$, as the case may be. Div the prod by the number of rectangles. The quot will be the dist b 1, or n 1, as the case may be.

For the dist b 2, or n 2, proceed in precisely the same way, only instead of the number 1, use the number 2 to be mult by the number of rectangles· and so use successively the numbers 3, 4, 5, &c, if it be reqd to find that number of points

Ex. Let $b\,c = 10$ ft; and let it be reqd to find 2 points, 1 and 2, for dividing the rectangular surf $a\,b\,c\,d$ into 3 rectangular parts, which shall sustain equal pressures. Here we have for point 1,

$1 \times 3 = 3$ The sq rt of $3 = 1.732$. And 1.732×10 (or $b\,c$) $= 17.32$. And $\dfrac{17.32}{3 \text{ rectangles}} = 5.773$ ft $= b$ 1.

For point 2, we have

$2 \times 3 = 6$. The sq rt of $6 = 2.449$. And 2.449×10 (or $b\,c$) $= 24.49$. And $\dfrac{24.49}{3 \text{ rectangles}} = 8.163$ ft $= b$ 2.

And so for any number of points.

REM. 1. **This rule will be found useful in spacing the cross-bars of lock-gates; the hoops around cylindrical cisterns: and the props to a structure, like Fig 3.**

REM. 2. **For dividing any surf, as $o\,b\,c\,d$, Fig 12, which is not rectangular, in the same manner,** with an accuracy sufficient for most practical purposes, perhaps the following method is as convenient as any.

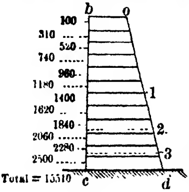


Fig. 12.

RULE. First div the surf, as in Fig 12, into several small hor parts, equal or not, at pleasure. Then by Rule in Art 1, find the pres on each part separately, as is supposed to be done in the numbers on the left hand of the fig The sum of these (in this case 15510) is the total pres against the entire surf $o\,b\,c\,d$. Now suppose we wish to div this surf in 4 parts bearing equal pres; first div 15510 by $4 = 3878$. Then beginning at the top, add together a number of the separate pressures sufficient to amount to 3878, by this means find point 1. Then proceed with the addition until the sum amounts to twice 3878, or 7756, which will indicate point 2, and in the same manner find point 3, by adding up to three times 3878, or 11634 Then the hor dotted lines ruled through points 1, 2, and 3, will give the reqd divisions approximately. In this manner the hoops of conical, and other shaped vessels, may be spaced nearly enough for practical purposes.

Art. 7. The transmission of pressure through water. Water, in common with other fluids, possesses the important property of transmitting pres equally in all directions. Thus, suppose the vessel, Fig 13, to be entirely closed, and filled with water; and suppose the transverse area of T, C, D, and E, to be each equal to one sq inch. Then, if by means of a piston, or otherwise, a pres of 1 ℔, 1 ton, or any other amount, be applied to the one sq inch of area of T, C, D, or E, every sq inch of the inner surf of the vessel, and of the pipe a, will instantly receive, at right angles to itself an equal pres of 1 ℔, or 1 ton, &c, in addition to the pres which it before sustained from the water itself, and this will occur if the vessel consist of parts even miles asunder; as, for instance, if T were miles distant from E; and united to it by a long series of tubes. If the vessel were a strong steam boiler full of water, a single pres of a few hundred pounds at T, C, &c, would burst it. See also Fig 2 (c) and paragraph above it.

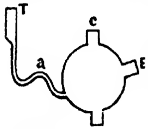


Fig. 13.

The hydrostatic press acts on this principle. Any body, within the vessel, would also receive an equal additional pres on each sq inch of its surf.

If the top of T be open, the air will press upon the sq inch of the exposed surf of water to the extent of nearly 15 ℔s; and the same degree of pres will also be transmitted to every sq inch of the interior surf of the vessel, and its connecting tubes; but no danger of bursting will result from this atmospheric pres, because the air also presses every sq inch of the outside of the vessel to the same extent.

Air, and other gaseous fluids, transmit pres equally in all directions, like liquids; but not as rapidly.

Art. 8. The center of pressure. Let Fig 14 represent a vessel full of water, and suppose the side P to be perfectly loose, so as to be thrown outward by the slightest pres of the water from within. Now, there is but one single point, P, in every surf so pressed, no matter what its shape may be to which if we apply a force equal to the pres of the water, and in a direction opposite to said pres, the side P will be thereby prevented from yielding Such point is called the *center of pressure.* It must not be understood by this that the actual amount of pres of the water against that part of the surface which is above the hor dotted line passing through P, is equal to that of the water below said line; but that the sum of the products of the several pressures above it, mult by their several leverages, or vert dists from P, is equal to the sum of the products of the pressures below, mult by their leverages; or, in other words, that the sum of the *moments* around the point P, of the pressures above the line, is equal to the sum of the *moments* of those below it; so that if a hor iron rod $b\,b$ were passed entirely through the side P, at the same level as the dotted line, as shown in the fig, so as to serve as a hinge for the side P to turn on, the side would have no tendency to turn.

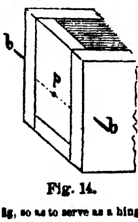


Fig. 14.

Art. 9. To find the cen of pres of a quiet fluid, against a plane surface. Fig 15.

1. The center of pressure of a quiet fluid against any plane surface whose width is uniform throughout its depth, whether said surface be vertical, as *e o*, or inclined, as *c a*, (or inclined in the opposite direction:) and whose top *c*, or *e*, coincides with the hor water surf, is distant *vert* below the water surf, two-thirds of the vert depth, *s x*, from said water surf to the bottom of the plane, as at *n* and *i*. Inasmuch as a hor line at ⅔ of the depth of *s x*, intersects both *c a* and *e o* at ⅔ of their lengths respectively, we might say at once that the center of pres against a plane parallelogram, with its top at the water surface, is at two-thirds of its length below the water surface.

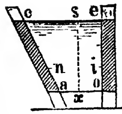


Fig. 15

Throughout Art 9 **any measure,** as yard, foot, or inch &c, may be used.

2. But if the hor top *a*, or *o*, Fig 16, of the rectangular plane *a g*, or *o h*, be covered to some depth with water, then the *vert* depth *s m*, of the cen of pres *d*, or *e*, below the surf of the water, will be equal to

$$\tfrac{2}{3} \text{ of } \frac{\text{cube of } sc - \text{cube of } sw}{\text{square of } sc - \text{square of } sw}$$

where *s c* is the vert depth of the bottom, and *s w* the vert depth of the top, of the pressed surf, below the water surf. Or, in words From the cube of *s c*, take the cube of *s w*, and call the rem *a*. Then, from the square of *s c*, take the square of *s w*; and call the rem *b*. Div *a* by *b*, and take two thirds of the quot for *s m*.

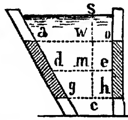


Fig. 16.

3. When a plane surf of any shape whatever, whether rectangular, triangular, or circular, &c, whether vert as *o p*, Fig 17, or inclined as *m n*, is *entirely immersed*, so as to be pressed over the entire area of *both* sides but by *diff depths* of water on its two sides, then the cen of pres coincides with the *cen of grav* of the pressed surf.

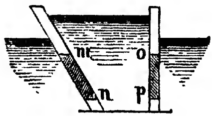


Fig. 17.

In the 3 foregoing figures the supposed surfaces are shown edgewise, so that their widths do not appear.

4. In any triangular plane surf, whether right-angled, or otherwise, as *a b c*, Fig 18, whether vert, or inclined; the *base* *a b* of which coincides with the hor surf of the water, the cen of pres *o*, will be in the center of the line *c r*, which bisects the base *a b*.

5 But if the triangle, as *a s c*, vert, or inclined, have its *apex*, *a*, at the surf of the water; and its base *s c*, *hor*; then the cen of pres *x*, will also be in the line *a m* which bisects the base; but *a x* will be ¾ of *a m*.

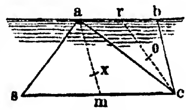


Fig. 18.

6 If any plane triangle *a b c*, Fig 19, base up, and hor, have its base *a b* covered to some depth *n d*, with water, then the cen of pres *o*, will be in the line *c s* which bisects the base; and *n o* will be equal to

$$\frac{mx^2 + (2mx \times ma) + 3ma^2}{(mx + 2ma) \times 2.}$$

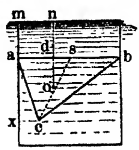


Fig. 19.

7. The center of pres against any plane *rectangular* surface, Fig 20, whether vert as *m n*, or inclined as *p o*, or *w x*, having its top coinciding with the surf of the water; and pressed by diff depths of water on its opposite sides, as shown in the fig; will be *vert* below the *upper* water surf, a dist equal to

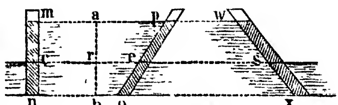


Fig. 20.

$$\frac{\left(\begin{array}{c}\textit{area of surf } \text{m n},\\ \textit{or } \text{p o}, \textit{or } \text{w x}\end{array} \times \frac{\textit{sq of vert depth } \text{a b}}{3}\right) - \left(\begin{array}{c}\textit{area of surf}\\ \text{c n}, \textit{or } \text{e o}, \textit{or } \text{s x}\end{array} \times \frac{\textit{sq of vert depth } \text{r b}}{3}\right) - \left(\begin{array}{c}\textit{vert depth}\\ \text{a r}\end{array} \times \begin{array}{c}\textit{area of surf } \text{c n},\\ \textit{or } \text{e o}, \textit{or } \text{s x}\end{array} \times \frac{\textit{vert depth } \text{r b}}{2}\right)}{\left(\begin{array}{c}\textit{area of surf}\\ \text{m n}, \textit{or } \text{p o}, \textit{or } \text{w x}\end{array} \times \textit{half of } \text{a b}\right) - \left(\begin{array}{c}\textit{area of surf}\\ \text{c n}, \textit{or } \text{e o}, \textit{or } \text{s x}\end{array} \times \textit{half of } \text{r b}\right).}$$

9. To find the center of pressure against either a circular or an elliptic surface, pressed on one side only; whether vertical or inclined, and having its top either coinciding with the surface of the water or below it.

Let h = the vertical depth of the center of pressure below the water surface.
r = the vertical or inclined *semi*-diameter of the surface.
d = the *vertical* distance of the *center* of the pressed surface, below the water surface.

Then

$$h = \frac{r^2}{4d} + d.$$

In a **vertical circle,** with top at water surface, $h = 1\frac{1}{4} \times$ radius.

Art. 10. Walls for resisting the pressure of quiet water. A study of our remarks on retaining-walls for earth, pp. 603, etc, will be of use in this connection. It is of course assumed that the water does not find its way under the wall; and that the wall cannot slide. In making calculations for walls to resist the pressure of either earth or water, it is convenient to assume the wall to be but one foot in *length* (not height, or thickness), for then the number of cubic feet contained in it, is equal to that of the square feet of area of its cross-section, or profile; so that these square feet, when multiplied by the weight of a cubic foot of the masonry, give the weight of the wall. In ordinary cases, it is well, for safety, to assume that the water extends down to the very bottom line of the wall.

Now, by Art 1, the total pressure of quiet water, against the rectilineal back of a wall, whether vertical or sloping, is found in lbs, by multiplying together the area in square feet of the part actually pressed, (or in contact with the water;) half the *vertical* depth of the water, in feet, (being the *vertical* depth of the center of gravity of a rectilineal back, below the surface); and the constant 62.5 lbs; and this total pressure is always *perpendicular to the pressed area.*

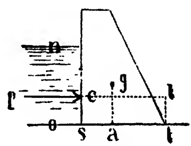


Fig 20½.

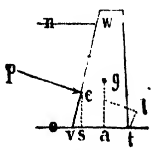


Fig 21.

When the back of the wall is vertical, as in Fig 20½, this pressure p is of course less than when it is battered; and is also *horizontal;* and it tends to overthrow the wall, by making it revolve around its outer toe, or edge t. The center of pressure is at c; $c\,s$ being ⅓ the *vertical* depth $o\,n$; in other words, the entire pressure of the water, so far as regards overthrowing the wall as one mass, may be considered as concentrated at the point c; where it acts with an overthrowing leverage $t\,l$. The pressure in lbs, multiplied by this leverage in feet, gives the *moment* in foot-lbs of the overturning force. The wall, on the other hand, resists in a vertical direction $g\,a$, with a moment equal to its weight (supposed to be concentrated at its center of gravity g), multiplied by the horizontal distance $a\,t$, which constitutes the leverage of the weight with respect to the point t as a fulcrum. If the moment of the water is greater than that of the wall, the latter will be overthrown; but if less, it will stand.

In Fig 21 the overturning moment of the water is equal to its calculated pressure $p \times$ its leverage $t\,l$; while the moment of stability of the wall is equal to its weight $\times$ its leverage $a\,t$. By aid of a drawing to a scale, we may on this principle ascertain whether any proposed wall will stand. For we have only to calculate the pressure p, then apply it at c, and at right angles to the back; prolong it to l; measure $t\,l$ by the same scale. Then calculate the weight of wall; find its center of gravity g; draw $g\,a$ vertical, and measure the leverage $a\,t$. We then have the data for calculating the two moments.

If the water, instead of being quiet, is liable to waves, the wall should be made thicker.

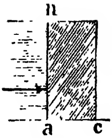


Fig. 22.

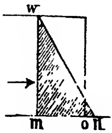


Fig. 23.

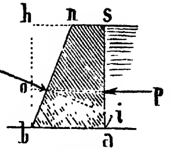


Fig. 24.

Art. 11. To find the thickness at base of a wall required to be safe against *overturning* under the pres of quiet water level with its top, and pressing against its entire vert back. **Caution.** See Art. 13.

(1st) Vertical wall, Fig 22.

$$\text{Thickness in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety *}}{3 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

(2d) Right angled triangular wall, Fig 23.

$$\text{Thickness at base in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety *}}{2 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

$$= \text{thickness, } m\,o\text{, of vertical wall} \times 1.225.$$

Notwithstanding their greater thickness at base, such triangular walls contain, as seen by the fig, not much more than half the quantity of masonry reqd for vert ones of equal stability. This is owing to the fact that their cent of grav is thrown farther back; thus increasing the leverage by which the wt of the wall resists overthrow.

(3d) Wall with vertical back and sloping face, Fig 24.

$$\text{Thickness at base in feet} = \sqrt{\frac{(\text{Ht}^2, \text{ft} \times \text{factor of safety *}) + (\text{batter } h\,n^2, \text{ft} \times \text{sp grav of wall})}{3 \times \text{specific gravity of wall}}}$$

$$= \text{Height in feet} \times \text{the proper decimal in the following table.}$$

Fig. 22.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.	Resist = 2 pres.	Resist = 3 pres.
Dressed Granite...	2.5	156	.447	.516	.633
Dressed Sandstone	2.2	137	.477	.550	.674
Mortar Rubble.....	2.	125	.500	.578	.707
Brickwork	1.8	112	.527	.609	.746
Fig. 23.					
Dressed Granite...	2.5	156	.548	.633	.775
Dressed Sandstone	2.2	137	.584	.675	.826
Mortar Rubble.....	2	125	.613	.707	.866
Brickwork	1.8	112	.646	.746	.913

Fig. 24.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.				Resist = 2 pres.			
			Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.	Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.
Dressed Granite..	2.5	156	.449	.458	.487	.532	.519	.526	.561	.593
Dressed Sandstone	2.2	137	.480	.488	.515	.558	.552	.560	.583	.622
Mortar Rubble.....	2.	125	.502	.510	.536	.578	.571	.586	.609	.646
Brickwork	1.8	112	.530	.539	.562	.602	.610	.618	.640	.674

$$\text{* Factor of safety} = \frac{\text{Required moment of stability of wall}}{\text{overturning moment of water}}.$$

Art. 12. Table showing how the stability of a wall sustaining water is affected by a change in the form of the wall; the quantity of masonry remaining the same. REM. When the base of a triangular wall, of sp grav 2, is less than ½ the ht, the stability is greatest when the water presses the vert side; but if the base exceeds ½ the ht, the stability is greatest with the water on the battered side. **Caution.** See Art. 13.

	All these walls contain precisely the same quantity of masonry. The masonry is supposed to be mortar rubble, weighing 125 lbs per cubic foot; or twice as much as water; or about the same as ordinary rough mortar rubble. If the sp gr of the masonry is actually greater or less than this, the safety also will be greater or less, in precisely the same proportion.	Base in parts of height	Approx resist of wall
1	Vertical wall	.5	1.
2	Face vertical; back batters one-tenth height	.55	1.
3	" " " " one-fifth "	.6	2.2
4	" " " " one-fourth "	.625	2.[illegible]
5	" " " " one-third "	.667	3.[illegible]
6	" " " " four-tenths "	.7	4.9
7	" " " " one half "	.75	14.0
8	Back vertical; face batters one-tenth height	.55	1.8
9	" " " " one fifth "	.6	2.1
10	" " " " one-fourth "	.625	2.2
11	" " " " one-third "	.667	2.4
12	" " " " four-tenths "	.7	2.6
13	" " " " one half "	.75	2.9
14	Back and face, each batter one tenth height	.6	2.2
15	" " " " one-fifth "	.7	3.4
16	" " " " one fourth "	.75	4.6
17	" " " " one third "	.833	9.0
18	" " " " four-tenths "	.9	36.0

Art. 13. Liability of wall or foundation to crush under unequal distribution of pressure. Arts 11 and 12 apply only to the *stability* of a *rigid* wall resting upon a *rigid* base, and therefore incapable of failure *except by overturning as a whole.* They show that the *stability* is greatest when the water presses against the *sloping* side. But in practice the point where the resultant of all the pressures on the base of the wall cuts the base, must not be so near to either toe as to endanger a *crushing* of wall or of foundation. This consideration often makes it best to let the water press against the *vert* back, notwithstanding the consequent loss in stability.

Art. 14. Fig. 25 shows, to scale, a dam wall at Poona, India, designed by Mr. Fife, C. E., of England. It is of mortar rubble, of 150 lbs per cub ft. Its total vert height is 100 ft; thickness *uv* at base, 60 ft 9 ins; at top, *rx*, 13 ft 9 ins. The front *ru* slopes 42 ft in 100 ft; and the back *xv*, 5 ft in 100 ft. Its foundation is 7 feet deep; but we here assume that the water presses against its *entire* back *xv*. Through the cen of grav G draw G*s* vert. From *c*, where the direction of the pres P of the water strikes G*s*, lay off *cn* by scale = 139.6 tons (of 2240 lbs) water pres against 1 ft in length of *xv*; and *ct* = 249.4 tons wt of 1 ft length of wall. Complete the parallelogram *cnmt* of forces. Its diag *cm* represents the resultant of all the pressures upon the base *uv*, and cuts the base at *a*, 20 ft back from the toe *u*. Doing the same with the 151.4 tons pres *p* against *ru*, we get the resultant *oy*, which is greater than *cm*, and cuts the base (at *i*) only 12.7 ft from the toe *v*, or 7.3 ft less than *a* is from *u*.

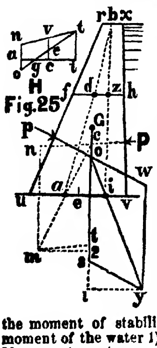


Fig. 25

Hence, when the water presses against *xv* the wall is less liable to fracture or crushing, and the earth foundation *uv* is more evenly loaded, and hence less liable to yield unequally so as to cause cracks in the wall. On this account *xv* is made the back of the wall, although the moment of stability of the wall is then only 2.2 (calling the overturning moment of the water 1), while if the water pressed against *ru* it would be 3, or 36 per cent greater.

Art. 15. The points a and i, Fig 25, are called **centers of pressure** upon the base, or **centers of resistance** of the base. If similar points, as d and z, be found in the same way for other lines, as fh, by treating a part (as $r\,x\,h\,f$) of the wall as if it were an entire wall; a slightly curved line joining these points is called the **line of pressure.** Thus, $b\,a$ is the line of pressure when the water presses against $x\,v$. Each point, as d, in $b\,a$, shows where any joint, as fh, drawn through that point, is cut by the resultant of all the forces acting upon said joint. $b\,i$ is the line of pres when the water presses against $r\,u$. These lines do not show the *direction* of the resultants Thus, at a, the latter is $c\,m$, not $b\,a$. The angle between the direction of the resultant and a line at right angles to the bed or joint, must be less than the angle of friction of the materials forming the joint.

If from the end m or y of the resultant of the pressures upon any joint, we draw $m\,2$ or $y\,l$ hor, then $c\,2$ or $o\,l$ (as the case may be) measures the entire *vert* pres on that joint: and $m\,2$ or $y\,l$ measures the *hor* pres against the back of the wall, which tends to cause sliding at the same joint If the direction of the resultant comes within the limit stated in the preceding paragraph, $m\,2$ or $y\,l$ will be less than the frictional resistance to sliding, which last is $= c\,2$ (or $o\,l$) $\times$ the coeff of friction for the surfaces forming the joint. Hence sliding cannot take place Sliding never occurs in the *masonry* of walls of ordinary forms. Good mortar well set aids to prevent sliding, but it is better not to rely upon it. But entire walls have slidden on slippery foundations.

Art. 16. In California, about 1860, were built dams of dry rough stone, founded on rock. Height 70 ft; base 90 ft; top 6 ft; hor spread of up-stream slope 70 ft; of outer slope 14 ft. Stone laid with some care by hand, except a core of about one-fifth of the mass, which was roughly thrown in. Water slope covered with 3-inch plank, laid hor on 12 × 12 inch stringers built into the stone work Cost about $3 per cubic yard. G. H. Mendell, Report on projects for San Francisco water supply, 1877, page 18.

Art. 17. To find the thickness of a cylinder to resist safely the pressure of water, steam, &c, against its interior. If riveted, see next page.

Where the thickness is less than one-thirtieth of the radius, as it is in most cases, the usual formula

$$\text{(1)}\qquad \underset{\text{in inches}}{\textbf{Thickness}} = \frac{\text{pressure}}{\text{safe strength}} \times \text{radius}^{*}$$

is employed. It regards the material as being subjected only to a direct tensile strain, which is sufficiently correct in such thin shells.

For somewhat greater pressures and thicknesses, Professor F. Reuleaux (Der Konstrukteur, p 52) gives

$$\text{(2)}\qquad \underset{\text{in inches}}{\textbf{Thickness}} = \frac{\text{pressure}}{\text{safe strength}}\left(1 + \frac{\text{pressure}}{2 \times \text{safe strength}}\right) \times \text{radius.}^{*}$$

For very great pressures and thicknesses, as in hydraulic presses, cannons, &c, Professor Reuleaux (Konstrukteur, p 53) gives Lamé's formula:

$$\text{(3)}\qquad \underset{\text{in inches}}{\textbf{Thickness}} = \left(\sqrt{\frac{\text{safe strength} + \text{pressure}}{\text{safe strength} - \text{pressure}}} - 1\right) \times \text{radius.}^{*}$$

The three formulæ give results as follows, pressures and strengths in lbs per square inch:

Diameter.	Radius.	Pressure.	Safe tensile strength.	Thickness, inches. Formula (1).	Formula (2).	Formula (3).
20 inches.	10 inches.	50	10000	**.05**	.050125	.05
"	"	500	"	.50	**.5125**	.513
"	"	5000	"	5.00	6 25	**7.32**

The thicknesses given by the formulæ appropriate to the several pressures are printed **in heavy type.** It will be seen that in these cases the results differ but slightly, except for *very* great pressures.

* In all three formulæ take the *radius in inches*, and the pressure and strength in *pounds per square inch.*

Rem. 2. Want of uniformity in the cooling of thick castings makes them proportionally weaker than thin ones, so that in order to reduce thickness in important cases we should use only best iron remelted 3 or 4 times, by which means an ult cohesion of about 30000 lbs per sq inch may be secured. But even with this precaution **no rule will apply safely** in practice to cast cylinders whose thickness exceeds either about 8 to 10 ins, or the inner rad however small

Under a pres of 8000 lbs per sq inch, **water will ooze through cast iron 8 or 10 ins** thick; and under but 250 lbs per sq inch, through .5 inch.

Table of thicknesses of single-riveted wrought iron pipes, tanks, standpipes, &c, by the above rule, to bear with a safety of 6 a quiet pressure of 1000 ft head of water, or 434 lbs per sq inch, the ult coh of fair quality plate iron being taken at 48000 lbs per sq inch, or at 8000 lbs for a safety of 6, which is farther reduced to 8000 × .56 = 4480 lbs, to allow for weakening by rivet holes; for **single-riveted** cyls have but about .56 of the strength of the solid sheet; and **double-riveted** ones about .7. With the above pres and other data, the rule here leads to thickness = .1016 × inner rad in ins.

Di. Ins.	Ths. Ins	Di Ins	Ths. Ins.	Di Ins.	Ths Ins	Di. Ins	Ths Ins.	Di Ins.	Ths Ins.	Di Ins	Di. Ft	Ths Ins.
.5	.025	5	.254	16	.813	30	1.52	60	3.05	120	10	6.09
1.0	.051	6	.305	18	.914	33	1.68	66	3.35	132	11	6.70
1.5	.076	8	.406	20	1.016	36	1.83	72	3.66	144	12	7.31
2.0	.102	10	.508	22	1.117	42	2.13	84	4.27	192	16	9.75
3.0	.152	12	.609	24	1.219	48	2.44	96	4.88	240	20	12.19
4.0	.203	14	.711	27	1.371	54	2.74	108	5.49	288	24	14.63

For a less head or pressure, or for any safety less than 6, it is safe and near enough in practice, to reduce the thickness of wrought iron cyls in the same proportion as said head, pres, or safety is less than the tabular one

Double-riveted cylinders, Fairbairn says, are about 1.25 times as strong as single-riveted. Hence they may be one-fifth part thinner **Lap-welded ones** are nearly 1.8 times as strong as single-riveted; and hence may be only .56 as thick.

Many continuous miles of **double-riveted pipes in California** have been in use for years with safetys of but 2 to 2.6. In one case the head is 1720 ft, with a pres of 746 lbs per sq inch, diam 11.5 ins; thickness, .34 inch.

Cast iron city water pipes must be thicker than required by formula (1), in order to endure rough handling and the effects of "water-ram" (due to sudden stoppage of flow, see second Rem, p 513), and to provide against irregularity of casting and the air bubbles or voids to which all castings are more or less liable. In the following table the ultimate tensile strength of cast iron is taken at 18,000 lbs per square inch. Column A gives thicknesses by Mr. J. T. Fanning's formula (Hydraulic Engineering, p 454).

$$\left.\begin{array}{l}\text{Thickness}\\ \text{in inches}\end{array}\right\} = \frac{(\text{pres, lbs per sq in} + 100) \times \text{bore, ins}}{.4 \times \text{ultimate tensile strength}} + .333\left(1 - \frac{\text{bore, ins}}{100}\right)$$

These correspond with average practice. The addition of 100 lbs to the pres is made in order to allow for water-ram. Column B gives thicknesses by formula (1), taking coefficient of safety = 8 (thus making safe tensile strain = 2250 lbs per square inch) and adding three-tenths of an inch to each thickness given by the formula:

Head in feet	50		100		200		300		500		1000	
Pressure, lbs per sq in.	21.7		43.4		86.8		130		217		434	
Bore, ins.	Thickness of pipe, in Inches.											
	A	B	A	B	A	B	A	B	A	B	A	B
2	.36	.31	.37	.32	.38	.34	.39	.36	.42	.40	.48	.51
3	.37	.31	.38	.33	.40	.35	.42	.40	.45	.45	.54	.60
4	.39	.32	.40	.34	.42	.38	.45	.42	.50	.50	.61	.72
6	.41	.33	.43	.36	.47	.42	.50	.48	.57	.60	.75	.94
8	.45	.34	.47	.38	.52	.47	.57	.55	.66	.70	.90	1.16
10	.47	.35	.50	.40	.56	.50	.62	.60	.74	.81	1.04	1.35
12	.49	.36	.53	.42	.60	.54	.67	.66	.82	.91	1.18	1.57
16	.55	.38	.60	.46	.70	.62	.79	.77	.98	1.10	1.46	2.00
18	.57	.39	.63	.48	.74	.65	.85	.84	1.06	1.21	1.60	2.20
20	.61	.40	.67	.50	.79	.68	.91	.90	1.15	1.31	1.75	2.50
24	.66	.42	.73	.53	.87	.77	1.02	1.01	1.30	1.51	2.03	2.84
30	.74	.45	.83	.59	1.01	.89	1.19	1.19	1.55	1.82	2.46	3.47
36	.82	.47	.93	.65	1.15	1.01	1.36	1.37	1.80	2.12	2.88	4.11
48	.98	.53	1.13	.77	1.42	1.24	1.70	1.73	2.28	2.73	3.73	5.38

Table of thickness of lead pipe to bear internal pressures with a safety of 6; taking the ultimate cohesion of lead at 1400 lbs per sq inch.

Rem. Although these thicknesses are safe against quiet pressures, they might not resist shocks caused by too sudden closing of stop cocks against running water.

Bore in Inches.	Heads in Feet. 100	200	300	400	500	Bore in Inches.	Heads in Feet. 100	200	300	400	500
	Pres in lbs per sq inch. 43.4	86.8	130	174	217		Pres in lbs per sq inch. 43.4	86.8	130	174	217
	Thickness in Inches.						Thickness in Inches.				
¼	.026	.055	.089	.128	.171	1	.102	.221	.357	.511	.682
⅜	.038	.083	.134	.192	.256	1¼	.127	.276	.447	.639	.853
½	.051	.111	.179	.256	.341	1½	.153	.332	.536	.767	1.02
⅝	.064	.138	.223	.320	.427	1¾	.178	.387	.626	.895	1.20
¾	.076	.166	.268	.383	.512	2	.204	.442	.714	1.02	1.36
⅞	.089	.193	.313	.447	.597						

Rem. The valves of water-pipes must be closed slowly, and the necessity for this precaution increases with their diams. Otherwise the sudden arresting of the momentum of the running water will create a great pressure against the pipes in all directions, and throughout their entire length behind the gate, even if it be many miles, thus endangering their bursting at any point. Hence stop gates are shut by screws.

Art. 18. Buoyancy. When a body is placed in a liquid, whether it float or sink, it evidently displaces a bulk of the liquid equal to the bulk of the immersed portion of the body; and the body, in both cases and at any depth, and in any position whatever, is buoyed up by the liquid with a force equal to the weight of the liquid so displaced. Thus, if we immerse entirely in water a piece of cork, *c*, *c*, Fig 26, or any body of less specific gravity than water, the cork will, by its weight, or force of gravity, tend to descend still deeper; but the upward buoyant force of the water, being greater than the downward force of gravity of the cork, will compel the latter to rise. In this case, the cork receives a total downward pressure equal to the weight of the vertical column of water above it, shown by the vertical lines in vessel 1; and a total upward pressure equal to the weight of the column shown in vessel 2. The difference between these two columns is evidently (from the figs) equal to the bulk of the cork itself; therefore the difference between their weights or pressures (or, in other words, the buoyancy of the water) is equal to the weight or pressure of the water which would have occupied the place of the cork; or, in other words, of the water which is displaced by the cork. This *difference*, or buoyancy, will plainly be very nearly the same at any depth whatever of *entire* immersion. It increases slightly with the depth, owing to increase in the density of the water; but, on the other hand it is diminished by compression of the cork. Now the cork, if left to itself, will continue to rise until a portion of it reaches above the surface, as in vessel 3. The downward pressing column then ceases to exist; and the cork is pressed downward only by its own weight. But, as it now remains stationary, the upward pressure of the water must be equal to the weight of the cork. But the upward pressure of the water arises only from the shaded column shown in vessel 3: and this column is (as in the case of total immersion) equal to the bulk of water displaced. Therefore, *in all cases*, the buoyancy is equal to the weight of water displaced; and when the body is in equilibrium, whether at or below water level, the buoyancy, or the weight of water displaced, is also equal to the weight of the body itself.

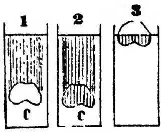


Fig. 26.

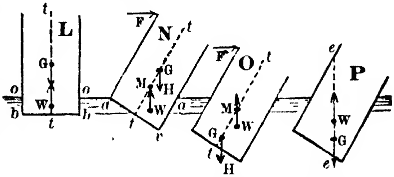
If the body be of a substance **heavier than water**, its weight is greater than the buoyancy of the displaced water, and the body therefore sinks, with a force equal to the difference between the two. Thus, a cubic foot of cast iron weighs 450 lbs., and a cubic foot of water 62.5 lbs., so that the iron sinks with a force of 450 — 62.5 = 387.5 lbs.

The same principle applies to **other fluids.** Thus, light bodies, such as smoke, a balloon, etc., in air, all tend, like a cork in water, to fall; but the air, being heavier, crowds them out of the lower positions which they tend to assume, and pushes them upward.

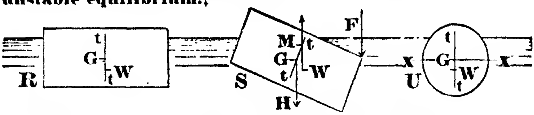
Although a pound of lead and a pound of feathers, weighed in the air, balance each other, yet in a vacuum the feathers will outweigh the lead, by as much as the air displaced by them outweighs that displaced by the lead.

The downwd force of grav may be regarded as concentrated at the cen of grav G of a floating body. The upwd pres, or buoyancy,† of the water may similarly be regarded as acting at the cen of gr W of the displaced water.* W is also called the **center of pressure,** or **of buoyancy,** of the water; and a vert line drawn through it is called the **axis,** or **vertical, of buoyancy,** or **of flotation.** Ordinarily,‡ W shifts its position with every change in that of the body Thus in L it is at the cen of gr of the rectangle *o o b b*; and in N at that of the triangle *a a r*

When a floating body, L, P or R, is at rest, and undisturbed by any third force, as F, it is said to be in **equilibrium,** and G and W are then in the same vert line *t t* Figs L and R, or *e e* Fig P; which line is called the **axis,** or **vertical, of equilibrium.**



When a third force,§ as F, in N and O, causes the line, joining G and W, to lean, as in Figs N, O and S, then if a vert line be drawn upwd from the cen W of buoy, the point M where said line cuts said axis, is called the **metacenter** of the body.‖ G and W are then no longer in the same vert line;‡ and the two opp and vert forces, grav and buoy, acting upon those points respectively, form a "couple" and, when the third force F is removed, they no longer hold the body in equilib, but cause it to rotate If (as in Figs O and S) the positions of G and W are then such that the metacenter M is *above* the cen of gr G, this rotation will tend to *restore the body to its former position*, and the body is said to have been (before the application of the third force F) in **stable equilibrium.**¶ But if (as in N) M is *below* G the direction of rotation is such as to *upset* the body, by causing it to *depart further from its former position*, and the body is said to have been in **unstable equilibrium.**‡



The tendency or moment in ft lbs of a floating body either to upset or to right itself, is,

= the wt of the body (or the equal upwd pres of the water) in lbs × the hor dist between W M and G H, Figs N, O and S, in ft.

The third force F may of course be so great as to overpower the tendency of the body to right itself. Thus, a ship may upset in a hurricane, although judiciously loaded and ballasted for ordinary winds. A hor section of a body at water-line is called its **plane of flotation.**

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* **The body is in fact acted upon by other forces,** such as the hor pressures of the water against its immersed portions but as all of these in any one given direction are balanced by equal ones in the opposite direction, they have no effect upon the forces G and W It is also acted upon by the air, which presses it downwards with a force of 14 75 lbs per sq inch, but this is balanced by an equal pres of the surrounding air upon the surface of the water

† **This buoyancy is made up** of the parallel upward pressures of the innumerable vert filaments of the displaced water as shown by Fig 26, and the **axis of flotation** is their resultant, as in the case of parallel forces.

‡ The shape of a body (as that of a sphere or cylinder U) may be such that the position of its cen of buoy W, relatively to that of its cen of gr G, is not changed by the rotation of the body about a given axis (as any axis of the sphere or the *longitudinal* axis of the cyl), but remains constantly in the same vert line with G, so that the body, in rotating, remains in equilib Such a body is said to be in **indifferent equilibrium** about said axis. But if a cyl U be made to rotate about its *transverse* axis *x x*, it plainly comes under the remarks on Figs R and S, and may (before rotating) be in either stable or unstable equilib about that axis according to the way in which its wt is distributed.

‖ This metacenter shifts its position on the line *t t* according to the inclination of the latter.

§ **Uneven loading,** instead of a third force, may cause a vessel at rest to lean as at P; and yet the vessel so leaning may be in equilib, for its axis *e e* of equilib may be vert, although not coinciding with the **axis of symmetry** of the vessel, as it does at *t t* in L.

¶ In *floating* bodies, this may sometimes (as in Figs R and S) be the case even when the cen of buoy W (*not the metacenter*) is *below* the cen of gr G; because, when the body is forced to lean, W moves to another point in it, and this point may be such as to bring M above G. W is always below G in bodies of uniform density, floating at rest, if any part of the body is above water. When such bodies are entirely submerged, W and G coincide.

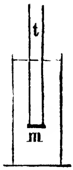


Fig 27

Art. 19. A body lighter than water, if placed at the bottom of a vessel containing water, will not rise unless the water can get under it, to buoy it, or press it upward, as the air presses a balloon or smoke upward. Thus, if one side of a block of light wood, *perfectly* flat and smooth be placed upon the similarly flat and smooth bottom of a vessel, and held there until the vessel is filled with water, the downward pres will keep it in its place, until water insinuates itself beneath through the pores of the wood. But if the wood be smoothly varnished, to exclude water from its pores, it will remain at the bottom.

On the other hand, a piece of metal may be prevented from sinking in water, by subjecting it to a sufficient *upward* pres only, while the downward pres is excluded. Thus, if the bottom of an open glass tube, *t*, Fig 27, and a plate of iron *m*, be made smooth enough to be water tight when placed as in the fig, and if in this position they be placed in a vessel of water to a depth greater than about 8 times the thickness of the iron, the upward pres of the water will hold the iron in its place, and prevent its sinking; because it is pressed upward by a column of water heavier than both the column of air, and its own weight, which press it downward. On this principle iron ships float.

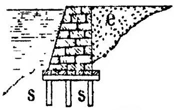


Fig 28

Rem 1. **A retaining-wall, as in Fig 28, founded on piles,** may be strong enough to resist the pres of the earth *e* behind it, in case water does not find its way underneath; and yet may be overthrown if it does; or even if the earth *s s* around the heads of the piles becomes saturated with water so as to form a fluid mud. In either case, the upward pres of the water against the bottom of the wall will virtually reduce the wt of all such parts as are below the water surf, to the extent of 62½ lbs per cub ft; or nearly one-half of the ordinary wt of rubble masonry in mortar.

Rem. 2. Although the piles under a wall, as in Fig 28, may be abundantly sufficient to sustain the wt of the wall; and the wall equally strong *in itself* to resist the pres of the backing *e*; yet if the soil *s s* around the piles be soft, both they and the wall may be pushed outward, and the latter overthrown by the pres of the backing *e*. From this cause the wing-walls of bridges, when built on piles in very soft soil, are frequently bulged outward and disfigured. In such cases, the piling, and the wooden platform on top of it, should extend over the whole space between the walls; or else some other remedy be applied.

Art. 20. Draught of vessels. Since a *floating* body displaces a wt of liquid equal to the wt of the body, we may determine the wt of a vessel and its cargo, by ascertaining how many cub ft of water they displace. The cub ft, mult by 62½, will give the reqd wt in lbs. Suppose, for instance, a flat boat, with vert sides, 60 ft long, 15 ft wide, and drawing unloaded 6 ins, or .5 of a ft. In this case it displaces 60 × 15 × .5 = 450 cub ft of water, which weighs 450 × 62½ = 28125 lbs, which consequently is the wt of the boat also. If the cargo then be put in, and found to sink the boat 2 ft *more*, we have for the wt of water displaced by the cargo alone, 60 × 15 × 2 × 62½ = 112500 lbs; which is also the wt of the cargo. So also, knowing beforehand the wt of the boat and cargo, and the dimensions of the boat, we can find what the draught will be. Thus, if the wt as before be 140625 lbs, and the boat 60 × 15, we have 60 × 15 × 62½ = 56250, and $\frac{140625}{56250} = 2.5$ ft the required draught. In vessels of more complex shapes, as in ordinary sailing vessels, the calculation of the amount of displacement becomes more tedious; but the principle remains the same.

HYDRAULICS.

Flow of Water through Pipes.

Much of the theory of hydraulics is still matter of dispute. This, and the unavoidable imperfections of actual work, render it advisable to use liberal safety factors in applying hydraulic formulas. Even new pipes are liable to tuberculations, which materially diminish the flow, and these sometimes greatly increase under the action of chemicals in the water. Air in the pipes also diminishes the flow.

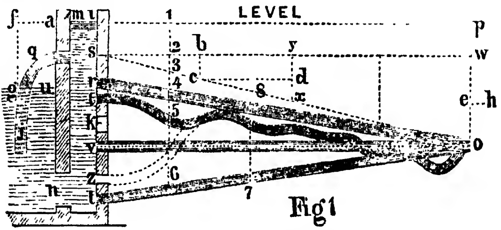


Fig 1

The term **HEAD** or **TOTAL HEAD** of water, as applied to the flowage of water through canals, pipes, or openings in reservoirs, &c, means the *vert* dist *i v* or *p o*, Fig 1, from the level surf, *m i*, of the water in the reservoir, or source of supply, to the center (or more properly to the cen of grav) *o*, of the orifice (whether the end of a pipe, *r o*, *t o*, *v o*, *z o*, *l o*; or any other kind of opening) through which the disch takes place freely, *into the air*; or the vert dist *a u*, or *f g*, from the same surf, *m i*, to the level *surf*, *g u*, of the water in the lower reservoir, when the disch takes place *under water*. Thus, in the case of disch *into the air*, the vert dist *i v* or *p o*, is the total head for either of the pipes *r o*, *t o*, *v o*, *z o*, or *l o*; and *i k* is the head for the orifice, *k*, in the side of the reservoir. And for disch *under water*, *a u*, or *f g*, is the head for either the pipe *j*, or the opening *n*; without any regard whatever to their depths below the surf of the lower water, which, according to the older authorities, do not at all affect their disch.

A portion of a pipe may have a head greater than the total head of the entire pipe. Thus the point 6 in the pipe *l o*, has a head 6 1; while the entire pipe has only the head *p o*.

Both in theory and in practice it is immaterial as regards the vel, and the quantity of water discharged, whether the pipe is inclined downward, as *r o*, Fig 1; or hor, as *v o*; or inclined upward, as *l o*: provided the total head *p o*, and also the length of the pipe, remain unchanged. If one pipe is longer than another, its sides will evidently present more friction against the water, and thus diminish the vel and the quantity of disch. The inclined pipes, *r o*, *l o*, being of course a little longer than the hor one *v o*, will therefore each disch a trifle less water, but if the hor one were extended slightly beyond *o*, so as to give it the same length as the others, then each of the three would disch the same quantity in the same time.

Art. 1 *a*. Divisions of the Total Head. In any pipe, as *s o*, *r o*, *t o*, *v o*, *z o*, or *l o*, Fig 1, the total head has three distinct duties to perform; 1st, to overcome the resistance to *entry* at *s*, *r*, *t*, *v*, *z*, or *l*; 2d, to overcome the resistances *within* the pipe; and, 3d, to give to the water, entering the pipe, the uniform *velocity* with which it actually flows

For convenience, we regard the total head as divided into three portions, corresponding to these duties; namely, 1st, the *entry* head; 2d, the *resistance*, or *friction*, head; and, 3d, the *velocity* head

Art. 1 *b*. The **velocity head** is the height through which a body must fall, in vacuo, to acquire the vel with which the water actually flows into the pipe. It is therefore = $\frac{v^2}{2g}$, in which v is the vel in ft per sec; and g is the acceleration of gravity, or 32 2

Art. 1 *c*. Experiment shows that, with the usual sharp-edged entry, the **entry head** is, near enough for practice, = half the vel head. If the entry is shaped like Fig 7, scarcely any entry head will be required But, in pipes longer than about 1000 diameters, the entry head bears so slight a proportion to the total head, that this advantage is of but little importance. It becomes more apparent in shorter pipes.

Art. 1 *d*. In Fig 1 we will assume that for any of the pipes, *i s* represents the sum of the vel and entry heads. Then the remainder *s v*, or *w o*, of the total head, is the **friction head**; or the head which is just sufficient to *balance* the friction and other resistances *within* the pipe; and, since the *entry* head balances the resistance at the *entrance* to the pipe, the *velocity* head has only to give velocity to the water in the vessel, causing it to enter

the pipe as rapidly as it flows through it, and thus keeping the pipe supplied. If, by shortening the pipe, or by smoothing its inner surf, we diminish the total friction, then a less friction head will be required, but the vel will, at the same time, be increased, and this will require a greater vel head, and entry head, so that the three together make up the total head, as before. Since the *friction is* equal to the force or head reqd to overcome it, it also is represented by *w o*.

Art. 1 e. The friction head may as in *v o*, *z o*, and *l o*, Fig 1, be all *above* the entrance to the pipe, and therefore *outside* of the pipe; or, as in a pipe laid from *s* to *o*, it may be all *below* the entrance, and *within* the pipe; or as in *r o* and *t o*, it may be partly above, and partly below, the entrance, and therefore partly within, and partly without, the pipe. The vel and disch, *after the pipe is filled*, are not affected by this difference in position of the entry end; but the *pressures* in the pipe, and the vels *while the water is filling an empty pipe*, are affected by it.

Art. 1 f. But it is necessary that the entry end of the pipe should be placed so far below the surf *m i*, that there shall be left, above the cen of grav of the entry end, at least a head, *i s*, sufficient to perform the duties of the entry and vel heads. If the entry end of any of the pipes be raised above *s*, a portion of the *vel* head will be *in* the pipe. In other words, the head *in* the pipe will be more than sufficient to overcome the resistances in the pipe; and the surplus will act as vel head, and will give greater vel to the water *in* the pipe. The *reduced* head thus left above the entry end will plainly be insufficient to maintain the supply for the *greater* vel, and the pipe will run only partly full.

In ordinary cases of pipes of considerable length, the sum of the entry and vel heads theoretically required, is but a small portion of the total head, and rarely exceeds a foot. Indeed, in a pipe of considerable diameter, the upper half of its cross section at the entry end may often be more than enough to provide sufficient entry and vel heads above the cen of grav of said cross section; so that the top of the entry end might, so far as these considerations alone are concerned, project above the surf of the water in the reservoir. But the end of the pipe should in practice always be entirely below the surf; otherwise air and floating impurities will be drawn into it, and cause obstructions. Moreover, the water surf of reservoirs is always liable to considerable changes of height; and the entry end of the pipe must be placed at such a depth that the water can flow into it with sufficient vel when at its *lowest* stages. As before stated, this will cause no diminution or increase of disch.

Art. 1 g. To find the friction head reqd for any part of a pipe; knowing the fric head reqd for the whole pipe. Since the friction, in a pipe of uniform diam, is (other things being equal) in proportion to its length; and since *w o*, Fig 1, represents the total friction, or reqd friction head, we have

Total length of the pipe : Length of the given portion : : *w o* : The friction head reqd for that portion

Or, having drawn *w o* by scale, *s w* hor, and *s o*;

Total length of the pipe : Length of the given portion : : *s o* : A dist, as *s c*, to be laid off from *s* on *s o*.

Or : : *s w* : A dist, as *s b*, to be laid off from *s* on *s w*

Then a vert line, as *b c*, drawn from *b* or *c*, and joining *s w* and *s o*, gives by scale the friction head reqd.

Art. 1 h. If the pipe is straight, as *r o*, *v o*, *l o*, the friction in any part *beginning at the reservoir*, as *l 6* in the pipe *l o*, may be found at once by drawing a line 6 1 vert upward from the axis of the pipe at 6. The line 2 8 will then give the friction in *l 6*. It also gives the friction in *r 4*, or in that part of *v o* which lies between *v* and the dotted line 1 6. It must be remembered that all the pipes in Fig 1 are supposed to be of the same actual length. They would thus end at different points *o*, and strictly a separate diagram must be drawn for each pipe. In a part of the pipe *not* beginning at the reservoir, as in *r o*, *v o*, or *l o*, between points vertically under *c* and *x*, the amount of friction is given by the line *d x*, for it is plainly $= y\,x - b\,c$

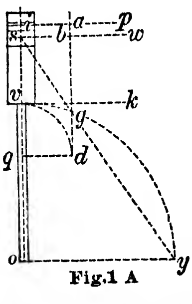


Fig.1 A

Art. 1 j. If the pipe is vert, as *v o*, Fig 1 A, let *v s* (on its axis *l o*) represent, as before, the sum of the vel and entry heads. From *s*, *v*, and *o*, respectively draw hor lines *s w*, *v k*, and *o y*, making *o y* = *v o*. Draw the oblique line *s y*. Then, to find the friction in any part, as *v q*, beginning at the reservoir, from *q* lay off *q d* hor, and equal to *v q*, and draw the vert line *a d*, crossing *s y* at *g*. Then *b g* will give the friction in *v q*.

Art. 1 k. If the pipe is curved, and if the curvature is uniformly distributed along its length, or so slight that it may be neglected; the friction heads reqd for the several portions of the pipe, may be found in the same way as for straight pipes, as in Art 1 H. Otherwise they must be found by proportion, as in Art 1 G.

Art. 1 l. While water is filling an empty pipe, the excess of the total head above the requirements of friction, &c, gives to the water a **greater vel** than it has after the pipe is filled; but this gradually decreases as the advancing water encounters the friction along the increased lengths of pipe filled; and finally becomes least when the water fills the whole length, and begins to flow from the disch end. *o*. But if only the vel and entry heads are left above the entry end, as in a pipe laid from *s* to *o*, there will plainly be no such excess of total head, and, consequently, no such change of vel during the filling of the pipe.

Art. 1 *m-r*.* Relation between discharge, area, velocity and pressure. In Fig 1 *B-D*, where the pipe, *b F*, running full, receives water from an unlimited reservoir, *R*, at *b*, and discharges through an orifice, *F*; the volume of water, passing any given cross section of *b F*, in a given time, is constant and equal to the rate of discharge at *F*. Thus;—if the rate of discharge, at *F*, be *Q* cubic feet per second, then *Q* cubic feet will pass each cross section of the pipe, *b F*, per second.

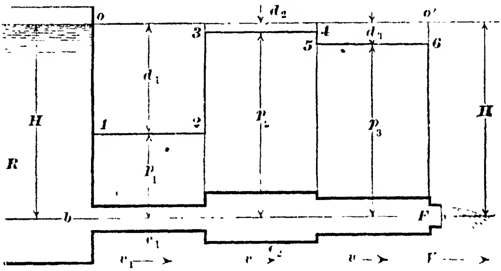


Fig. 1 B-D.

Let a = the area of cross section, and V = the velocity, of the stream issuing through the short pipe beyond *F*. V is called the velocity of efflux.

Let A_1, A_2, etc., be the different areas of cross section of *b F*, and let v_1, v_2, etc., be the velocities at those cross sections respectively. Then $Q = a\,V = A_1\,v_1 = A_2\,v_2$, etc.; or $V = \frac{Q}{a}$, $v_1 = \frac{Q}{A_1}$, $v_2 = \frac{Q}{A_2}$, etc. In other words, the velocities are inversely as the areas of cross section. Also, $a = \frac{Q}{V}$, $A_1 = \frac{Q}{v_1}$, $A_2 = \frac{Q}{v_2}$, etc.

The losses of pressure, due to the velocities, respectively, are $d_1 = \frac{v_1^2}{2g}$, $d_2 = \frac{v_2^2}{2g}$, etc.; as represented by the ordinates between the line $o\,o'$, of static pressure, and the diagram, *o* 1 2 3 4 5 6 *F*, of actual pressures. The difference, due to velocity, between the pres heads at any two points, as c_1 and c_2, where the velocities are v_1 and v_2 respectively, is $p_2 - p_1 = d_1 - d_2 = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = \frac{v_1^2 - v_2^2}{2g}$.*

The remaining pressure head, p_1, p_2, etc., at any point, is = static head in reservoir — velocity head at the point, = $H - d_1$, $H - d_2$, etc.

The loss of pressure head, at *F*, is $(6\ F) = p_3 = H - d_3$, and the pressure drops to zero, i.e., to the atmospheric pressure.

Art. 1 *s*. Open piezometers. If the lower ends of vertical or inclined tubes, open at both ends, be inserted into a pipe, *b F*, Fig. 1 *B D*, as at c_1, c_2, etc., the water surface, in these tubes, will stand at heights, p_1, p_2, etc., corresponding to the pressure heads at the points where the tubes are inserted. Such tubes are called open piezometers. In order that the water level may be observed, they are of glass, at least in those portions where that level is likely to be found. An obstruction, in the pipe, between c_2 and *F*, would *raise* the level in a piezometer at c_2; while an obstruction between *b* and c_2 would *lower* it.

*In Art. 1 *m-r*, for simplicity, we neglect all resistances, including those due to the abrupt enlargements and contractions of the pipe.

Art. 1 t. If we imagine any pipe, full of water, to be supplied with a number of piezometers, then a line, joining the tops of the columns of water in the several piezometers, is called the **hydraulic grade line.**

Art. 1 u. In a straight tube of uniform diam throughout, as *r o*, *r o*, or *l o*, Fig 1, running full and discharging freely into the air, the hyd grade line is a straight line drawn from its disch end *o* to a point *s* immediately over the entry end of the pipe, and at a depth below the surf equal to the sum of the vel and entry heads.

If the orifice at *o* be contracted, the hyd grade line must be drawn from *s* to some point, as *e*, immediately over *o*, and depending, for its height, upon the amount of contraction at *o*. But in this case the point *s* will also be higher than before, because the vel in the pipe is reduced by the contraction; and the sum *i s* of the vel and entry heads will be less.

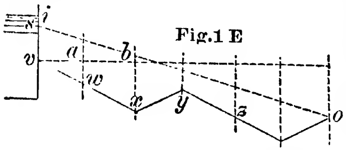


Fig.1 E

If the disch at *o* is under water, the effect upon the position of the grade line will be the same as that of a contraction of the orifice at *o*. The point *e* will be on the surf of the lower water, and immediately over *o*.

Art. 1 v. If the pipe, of uniform diam, (whether discharging freely or through a contracted opening at *o*, whether into the air or under water), **is bent or curved,** the hyd grade line will still be straight, **provided** the resistances are equal in each equal division of the *hor* length of the pipe, as in Fig 1 E, where equal divisions *v w*, *w x*, &c, of the *total* length, correspond with equal divisions *i a*, *a b*, &c, of the *hor* length.

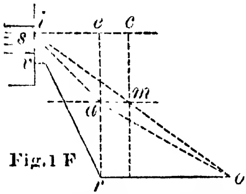


Fig.1 F

But in Fig 1 F, the hyd grade line will take the shape *s a o*. For if, in accordance with Art 1 G, we divide *s o* into two equal parts, *s m*, *m o*, corresponding with the two equal parts *v r*, *r o* of the length of the pipe, we obtain *m c* = *a e* for the head consumed in the resistances in *v r*, leaving only *r a* for the pres head at *r*.

Art 1 w. In a very large vessel, the total head upon any point at the level of the entrance *l* to a pipe *l o o'* Fig 1 G, is represented by *i l*, as already explained but of this total head a portion, as *i s*, is required to act as velocity head and entry head for the entrance at *l*, leaving only *s l* as the pressure head upon a point in the *pipe*, immediately to the right of *l*. Thus while the pressure, in pounds per square inch, in the *vessel* at *l*, is

$$p = i\,l \times 0.434\,,$$

that in the *pipe* at *l* is

$$p = s\,l \times 0.434.$$

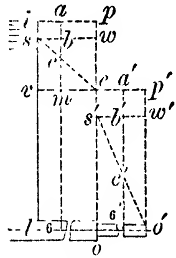


Fig.1 G

But now a portion, as *s r*, of *s l*, is expended in *l o* in balancing or "overcoming" the resistances throughout that portion of the pipe, and in doing this work, it gradually diminishes from *s r* (at *l*) to nothing (at *o*) as indicated by the dotted line *s e*. Thus, at the point 6, a portion *b e* has already been expended in overcoming the resistances in the pipe between *l* and 6, leaving *e* 6 as the pressure head at 6, of which *e m* must still be expended against resistances in the wide pipe between 6 and *o*, leaving *m* 6 = *r l* = *e o* as the pressure head for a point just to the left of the contraction at *o*. The pressure in *l o* is thus gradually diminished from *s l* (at *l*) to *e o* = *r l* (at *o*).

Now a portion *e s'* of *e o* is required to act as velocity and entry head for the entrance *o* to the narrower portion *o o'* of the pipe; because we need at *o* not only an additional *entry* head to overcome the resistance due to the square shoulder formed by the contraction, but also an additional *velocity* head to give the increase of velocity which must take place as the water passes from the wide pipe *l o* to the narrower one *o o'*; for, so long as a pipe runs *full* and the discharge remains constant, the *velocity* in each part of the pipe must be *inversely as the area* of cross section of that part; because in each second the same quantity of water passes each point; and this constant quantity is = area × velocity. Hence, as the area diminishes, the velocity increases.

There remains, therefore, *s' o* as the pressure head upon a point in the narrow part just to the right of *o*; and this in turn gradually diminishes to nothing at

the end o' of the pipe, as indicated by the dotted line $s\,o'$, being all expended in overcoming the resistances in $o\,o'$. We thus have, for the hydraulic gradient in Fig 1 G, the broken line $i\,s\,e\,s'\,o'$.

When the pressure is thus diminished by overcoming resistances, or by accelerating velocity, the diminution is called **loss of head.** Thus we say that $i\,s$ is *lost* at the entrance l, $s\,i$ as friction head in $l\,o$, $e\,s'$ at the contraction o, and $s'\,o$ as friction head in $o\,o'$,

At o', *all* the available head, $i\,l$, has been used up. The water flowing out at o', therefore exerts no lateral pressure, so that the stream flows out in parallel lines, and its capacity for forward pressure is due entirely to its kinetic energy (energy of motion) and to the pressure of the air upon the surface in the reservoir at i; but this last is of course balanced by the air pressure from without against the opening o'. Where a great reduction of cross sectional area in a pipe is followed (down-stream) by a re-enlargement, the increase of velocity may (under certain circumstances) consume not only the entire head of *water*, but also a portion or all of the *atmospheric* pressure on the surface in the reservoir, thus causing a partial or complete vacuum at the constriction. See the Venturi Meter.

The syphon, or siphon If one leg $a\,b$ of a bent tube or pipe $a\,b\,c$, Fig M, of any diam, filled with water, and with both its ends stopped, be placed in a reservoir of water, as in the fig, and if the stoppers be then removed, the water in the reservoir will begin to flow out at c, and will continue to do so until its level is reduced to t, which is the same as that of the *highest* end c of the pipe or syphon. The flow will then stop. The parts $a\,b$ and $b\,c$ are called the legs of the syphon, b being its highest point, and this is correct so far as relates to it merely as a piece of tube, but considering it purely with regard to its character as a hydraulic machine, the part $t\,a$ below the level of the highest end c, may be entirely neglected, for the water in the reservoir will not be drawn down below the level of the highest end, whether that be the inner or the outer one. Therefore, if the disch end be above the water in the reservoir, as, for instance, at w, no flow will take place. The *vert* height $b\,o$, from the highest part of the syphon, to the lowest level t, to which the reservoir is to be drawn down, must not, theoretically, exceed about 33 or 34 ft, or that at which the pres of the air will sustain a column of water. Practically it must be less, to allow for the friction of the flowing water, and for air which forces its way in. And still less at places far above sea level; for at such the reduced weight of the atmospheric column will not balance so great a height of water. In order readily to understand, or at any time to recall the principle on which the syphon acts, bear in mind that we may theoretically consider the end of the inner leg to be not actually immersed *below* the water surf, but only to be kept precisely *at* it, as the surf descends while the water is flowing out, but may regard the vert dist $b\,o$ as the length of the outer leg; and a varying dist, which at first is $b\,s$, and finally $b\,o$ (as the surf of the reservoir descends) as the length of the inner leg; and that the flow continues only while *this* outer leg is longer than *this* inner one The books are wrong in saying that the outer leg $b\,c$ *must* be longer than the inner one $b\,a$, in order that the water may run at all. The principle then is simply this: that both these legs $b\,c$, and $b\,i$, being first filled with water, (the part $i\,a$ being considered at first as a portion of the *reservoir*, and not of the syphon,) it follows that when the stoppers are removed from the ends c and a, the air presses equally against these ends; but the great vert head of water $b\,o$ in the outer leg $b\,c$, presses against the air at c, with more force than the small head of water $b\,s$ in the inner leg $b\,i$, does against the air at a or i.* Consequently, the water in $b\,c$ will tend to fall out more rapidly than that in $b\,i$; and as it commences to fall, would produce a vacuum at b, were it not that the pres of the air against the other end a or i, forces the water up $i\,b$, to supply the place of that which flows out at c. In this manner the flow continues until the surf of the water in the reservoir descends to t, on the same level as c. The pressures of the vert heads $b\,o$, $b\,o$, in the two legs $b\,c$, $b\,t$, being then equal, it ceases.

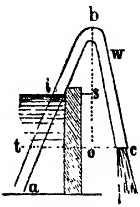


Fig. M.

The syphon principle may be employed for draining ponds into lower ground at a considerable dist, even though an elevation of several feet (in practice perhaps not *exceeding* about 28 ft above the level to which the pond is to be reduced) may intervene. In such a case an **escape** must be provided at the summit (or summits, if there are more than one) of the bends, for the disch of free air, which will inevitably enter, and soon stop the flow, unless this precaution be taken. The air-valve will not answer for this, because as soon as the valve v opens, the syphon becomes in effect two separate tubes open at top; and the water will fall in both. An orifice at the escape will be needed for filling the syphon at the start; and to prevent the water thus introduced, from running out, stopcocks must be provided at the ends, and kept closed until the filling is completed.

The greatest pains must be taken to make all the joints perfectly air-tight.

The motive power or head which causes the flow in a syphon, is the vert dist $s\,o$, from the surf of the reservoir, to the disch end c; or in other words, it is the diff, $s\,o$, between the theoretical lengths $b\,s$ and $b\,o$, of the two legs. Con-

* Said pressure of the air is of course not exerted directly at a or i; but is transmitted to a through the water in the vessel; and thence upward to i through the water in the syphon.

sequently, the farther *c* is below *s* the more rapid will be the flow; and it is plain that as the surf gradually sinks below *s*, the less rapid will the flow become. Having this head, the entire length *abc* of the syphon, and its diam, all in ft, the disch may be found approximately by either of the rules given in Art 2 for straight pipes. These rules give 55½ galls per min, instead of the 43½ galls actually dischd by Col Crozet's syphon, with a head of 20 ft. See below.

In a true syphon, *a g n y o* Fig 1½, *free from air* inside, and *running full*, **the total head** *p o* is measured vertically from the surface *m i* in the reservoir to the center of gravity of the outlet *o*, as in Fig 1, **the hydraulic gradient** (with the restriction named in Art 1 *i*) is, as before, a straight line *s r o* drawn from the foot *s* of the combined entry and velocity heads to the end *o*, and the velocity and discharge are the same as they would be if all parts of the pipe were brought below *s r o*. But see cautions 1 and 2, below.

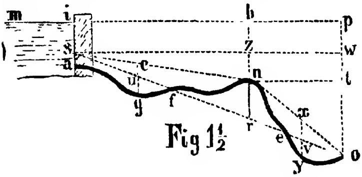


Fig 1½

The pressure at any point, *g*, *n* or *y*, is then given by a vertical line, *g u*, *n r* or *y v*, drawn from the point in question to *s r o* but for points, as *n*, situated *above s r o*, this pressure is *negative* or *inward*, while at points where *s r o* and the pipe are at the same level, as at *f* and *e*, there is neither pressure nor vacuum

Caution 1. But if the water be admitted to the empty pipe at *a*, while the end *o* is open, the pipe will not form a true syphon The part *a g n* will then run full, and will have *s c n* as its hydraulic gradient, but upon reaching at *n*, a portion *n o* of the pipe with a much steeper grade, the water will run off, in *n o*, with a velocity greater than that with which it arrives from *a n*. Hence the stream in *n o* will have a less area of cross section than in *a n*, and therefore cannot fill *n o*, but will run off in it as in an open gutter.

Caution 2. The tendency to vacuum at points above *s r o* causes an accumulation, at *n*, of particles of **air** that have been carried into the syphon by the water or have found their way in through imperfect joints, etc.; and these bring about a condition approaching that described in Caution 1; for their expansive force, by reducing the negative pressure or vacuum *n r* at *n*, diminishes the total head *h r* of the part *a g n*, while, by practically reducing the cross-section of the syphon at *n*, they require that a portion of the remaining head be used at *n*, as entry head to overcome the resistance caused by the contraction, and as velocity head to give the increase of velocity needed for passing the narrowed section at *n* Now since the friction head required for the part *a g n* remains about the same, the velocity head in the reservoir is considerably diminished, and the water arrives at *n* too slowly to keep *n o* filled. The accumulation of air at *n* thus retards the flow and disturbs the distribution of the pressures, so that these are no longer correctly indicated by vertical lines drawn to *s r o*.

At Blue Ridge Tunnel, Virginia, Col. C. Crozet constructed a drainage syphon 1792 ft long of cast iron faucet pipes 3 ins bore, 9 ft long. Its summit was 9 ft above the surface of the water to be drained; and its discharge end was 20 ft below said surface, thus giving it a head of 20 ft. At the summit 570 ft from the inlet, was an ordinary cast iron **air-vessel** with a chamber 3 ft high and 15 ins inner diam. In the stem connecting it with the syphon was a **cut-off stop-cock**; and at its top was an opening 6 ins diam, closed by an air tight screw lid. At each end of the syphon was a stopcock. **To start the flow** these end cocks are closed, and the entire syphon and air-vessel are filled with water through the opening at top of air-vessel This opening is then closed airtight, and the two end cocks afterwards opened; the cut-off cock remaining open. The flow then begins, and **theoretically** it should continue without diminution, except so far as the head diminishes by the lowering of the surface level of the pond. **But in practice** with very long syphons this is not the case, for air begins at once to disengage itself from the water, and to travel up the syphon to the summit, where it enters the air-vessel, and rising to the top of the chamber gradually drives out the water. If this is allowed to continue the air would first fill the entire chamber, and then the summit of the syphon itself, where it would act as a wad completely stopping the flow. **The water-level in the air chamber** can be detected by the sound made by tapping against the outside with a hammer

To prevent this stoppage, the cut-off at the foot of the chamber is closed before the water is all driven out; and the lid on top being removed the chamber is refilled with water, the lid replaced, and the cut-off again opened. The flow in the meantime continues uninterrupted, but still gradually diminishing notwithstanding the refilling of the chamber; and after a number of refillings it will cease altogether, and the whole operation must then be repeated by filling the whole syphon and air chamber with water as at the start.

At Col. Crozet's syphon at first owing to the porosity of the joint-caulking, which was nothing but oakum and pitch, air entered the pipes so rapidly as to drive all the water from the chamber and thus require it to be refilled every 5 or 10 minutes; but still in two hours the syphon would run dry. The joints were then thoroughly recaulked with lead, and protected by a covering of white and red lead made into a putty with Japan varnish and boiled linseed oil. But even then the chamber had to be refilled with water about every two hours; and after six hours the syphon ran dry, and the whole had to be refilled. In this way it continued to work.

Care in making the joints air-tight, and an outside and inside coating of the pipes and air-vessel with coal pitch varnish are important precautions.

Art. 2. Approximate formulæ for the velocity of water in straight, smooth, cylindrical iron pipes, as *r o*, *i o*, *l o*, Fig. 1. Having the total head *p o*, and the length and diameter of the pipe.

$$\left.\begin{array}{c}\textbf{Approx}\\ \textbf{mean vel}\\ \text{in ft per sec}\end{array}\right\} = \begin{array}{c}\text{coefficient}\\ \textbf{m}\\ \text{as below}\end{array} \times \sqrt{\frac{\text{diam in ft} \times \text{total head in ft}}{\text{total length in ft} + 54 \text{ diams in ft}}}$$

Table of coefficients "m".

Diam of pipe, feet	inches	**m**	**Diam** of pipe, feet	inches	**m**
0.1	1 2	23	1.5	18	53
0 2	2 4	30	2.0	24	57
0 3	3.6	34	2 5	30	60
0.4	4 8	37	3.0	36	62
0.5	6.0	39	3.5	42	64
0.6	7 2	42	4.0	48	66
0 7	8.4	44	5.0	60	68
0 8	9.6	46	6 0	72	70
0 9	10.8	47	7.0	84	72
1.0	12 0	48	10.0	120	77

For heads not less than 4 feet per mile, this formula gives results practically corresponding with those by Kutter's formula (p. 523) with coefficient n of roughness = 0.012. But slight differences, as to roughness, etc, may cause considerable variations of velocity, especially in small pipes; for, in such pipes, a given roughness of surface bears a greater proportion to the whole area of surface than in a pipe of large diameter. Extreme accuracy is not to be expected in such matters.

As in a river the velocity half way across it, and at the surface, is usually greater than at the bottom and sides, so in a pipe the velocity is greater at the center of its cross section than at its circumf. The **mean velocity** referred to in our rules is an assumed uniform one which would give the *same discharge* that the actual ununiform one does.

Hence

$$\begin{array}{c}\textbf{Discharge}\\ \text{in cub ft per sec}\end{array} = \begin{array}{c}\textbf{Mean velocity}\\ \text{in ft per sec}\end{array} \times \begin{array}{c}\textbf{Area of cross section}\\ \text{of pipe in sq ft.}\end{array}$$

1 cubic foot = 7.48052 U.S. gallons
1 U. S. gallon = .13368 cubic foot = 231 cubic inches.

* For intermediate diameters, etc, take intermediate coefficients from the table by simple proportion.

In the case of long pipes with low heads, the sum of the velocity and entry heads is frequently so small that it may be neglected. Where this is the case, or where their amount can be approximately ascertained, **Kutter's formula**, although designed for open channels, may be used. This formula is the joint production of two eminent Swiss engineers, E. Ganguillet and W. R. Kutter, but for convenience it is usually called by the name of the latter.*

It is, properly speaking, a formula for finding the coefficient c in the well known formula,

$$\textbf{Mean velocity} = c\sqrt{\text{mean radius} \times \text{slope}}$$

$$= c\sqrt{\frac{\text{diameter}}{4} \times \text{slope}}$$

According to Kutter,

For English measure. **For metric measure.**

$$\mathbf{c} = \frac{41.6 + \frac{.00281}{\text{slope}} + \frac{1.811}{n}}{1 + \frac{\left(41.6 + \frac{.00281}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } feet}}} = \frac{23 + \frac{.00155}{\text{slope}} + \frac{1}{n}}{1 + \frac{\left(23 + \frac{.00155}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } metres}}}$$

See also **tables of c**, pp 566 etc.

The mean radius is the quotient, in feet or in metres, obtained by dividing the area of wet cross section, in square feet or in square metres, by the wet perimeter (see below) in feet or in metres. In pipes running full, or exactly half full, and in semicircular open channels running full, it is equal to one-fourth of the inner diameter.

The wet perimeter is the sum, *a b c o* Fig C, p 528, of the lengths, *a b*, *b c*, *c o*, in feet or in metres, found by measuring (at right angles to the length of the channel) such parts of its sides and bottom as are in contact with the water. In pipes running full, it is of course equal to the inner circumference.

For the slope, we have

$$\textbf{Slope} = \frac{\textit{friction} \text{ head } w\ o \text{ Fig 1,}}{\text{length of pipe.}}$$

In open channels, this becomes

$$\textbf{slope} = \frac{\text{fall of water surface in any portion of the length of the channel}}{\text{length of that portion}}$$

$$= \text{fall of water surface per unit of length of channel}$$

$$= \text{sine of the angle formed between the sloping surface and the horizon.}$$

The number indicating the slope in any given case is plainly the same for English, metric and all other measures.

"n" is a "coefficient of roughness" of wet perimeter, and of course depends chiefly upon the character of the inner surface of the pipe as related to its size, a given diff in roughness resulting in a greater diff of n in large than in small pipes. For iron pipes in good order and from 1 inch to 4 feet diameter, n may be taken at from 0.010 to 0.012; the lower figures being used where the pipe is in exceptionally good condition. See pp. 564–5.

*See "Flow of Water," translated from Ganguillet and Kutter, by Rudolph Hering and John C. Trautwine, Jr., New York, John Wiley & Sons, 1889. $4.00.

Curves and bends do not greatly affect the discharge, so long as the total heads, and total actual lengths of the pipes remain the same; provided the tops of all the curves be kept below the hydraulic grade line; and provision be made for the escape of air accumulating at the tops of the curves.

Relation between area, velocity, and discharge.

Let q = rate of discharge (as in cubic feet per second),
v = mean velocity (as in feet per second),
a = area of cross section (as in square feet).

Then: $q = a\,v;\quad v = \dfrac{q}{a};\quad a = \dfrac{q}{v}$.

Relation of discharge to diameter* and slope. If we assume velocity = $c\sqrt{\text{mean radius} \times \text{slope}}$, or $v = c\sqrt{r\,s}$ (page 523); and if the pipe be of circular cross section, we have, for the rate, Q, of discharge through a pipe of diameter, d, and area, A, of cross section, running full:—

$$Q = A\,v = \frac{\pi\,d^2}{4} \cdot c\,\frac{d^{1/2}}{2}\,s^{1/2} = \frac{c\,\pi\,d^{5/2}\,s^{1/2}}{8};$$

or: Q is proportional to the $^5/_2$ power (square root of fifth power) of the diameter, and to the ½ power (square root) of the slope. For tables of fifth powers, and of square roots of fifth powers, see pp 67–69

Effect of resistances.

The pressure head of running water, upon any point in a pipe between the orifice and the reservoir, is:

$$= \left\{\begin{array}{l}\text{the total}\\ \text{head on}\\ \text{that point}\end{array}\right\} \text{minus} \left\{\begin{array}{l}\text{the head}\\ \text{due to the}\\ \text{vel at}\\ \text{that point}\end{array} + \begin{array}{c}\text{the}\\ \text{entry}\\ \text{head}\end{array} + \begin{array}{l}\text{the head consumed}\\ \text{in overcoming re-}\\ \text{sistances in the pipe}\\ \text{between the reservoir}\\ \text{and the point.}\end{array}\right\}$$

Thus, at the point 6, in the pipe, $l\,o$, Fig 1, the pressure head is $h = (3\,6) = (1\,6) - [(1\,2) + (2\,3)]$; where $(1\,2) = i\,s$ = the sum of the velocity and entry heads At 4, in the pipe $r\,o$, $h = (3\,4) = (1\,4) - [(1\,2) + (2\,3)]$.

In Fig 1, let the straight line, $s\,o$, represent the actual length of the pipe, whether straight, bent or curved, etc.; and $s\,v$ the sum of the resistances (supposed to be uniformly distributed) within the pipe Then, the angle, $s\,o\,v$, is called the hydraulic gradient, and sine $s\,o\,v = s\,v \div s\,o$

In the vertical pipe, $v\,o$, Fig 1 A, the pressure, at q, is = $g\,d$

---

* Diameter = 4 × mean radius, or $d = 4\,r$ (p 523).

TABLE OF WEIGHT OF WATER CONTAINED IN ONE FOOT LENGTH OF PIPES OF DIFFERENT BORES. *

(Original.)

Water at maximum density, 62.425 lbs per cubic foot = 1 gram per cubic centimeter, corresponding to a temperature of 4° Centigrade = 39.2° Fahrenheit.

Weight = 0.340475558 × square of bore in inches.

Bore.* Ins.	Water. Lbs.	Bore.* Ins.	Water. Lbs.	Bore.* Ins.	Water. Lbs.	Bore.* Ins.	Water. Lbs.
1/8	0.005320	6	12.25712	23	180.1116	62	1308.788
1/4	0.021280	6 1/4	13.29983	24	196.1139	63	1351.347
3/8	0.047879	6 1/2	14.38509	25	212.7972	64	1394.588
1/2	0.085119	6 3/4	15.51292	26	230.1615	65	1438.509
5/8	0.132998	7	16.68330	27	248.2067	66	1483.112
3/4	0.191518	7 1/4	17.89625	28	266.9328	67	1528.395
7/8	0.260677	7 1/2	19.15175	29	286.3399	68	1574.359
1	0.340476	7 3/4	20.44981	30	306.4280	69	1621.004
1 1/8	0.430914	8	21.79044	31	327.1970	70	1668.330
1 1/4	0.531993	8 1/4	23.17362	32	348.6470	71	1716.337
1 3/8	0.643712	8 1/2	24.59936	33	370.7779	72	1765.025
1 1/2	0.766070	8 3/4	26.06766	34	393.5897	73	1814.394
1 5/8	0.899068	9	27.57852	35	417.0826	74	1864.444
1 3/4	1.042706	9 1/4	29.13194	36	441.2563	75	1915.175
1 7/8	1.196984	9 1/2	30.72792	37	466.1110	76	1966.587
2	1.361902	9 3/4	32.36646	38	491.6467	77	2018.680
2 1/8	1.537460	10	34.04756	39	517.8633	78	2071.453
2 1/4	1.723658	10 1/2	37.53743	40	544.7609	79	2124.908
2 3/8	1.920495	11	41.19754	41	572.3394	80	2179.044
2 1/2	2.127972	11 1/2	45.02789	42	600.5989	81	2233.860
2 5/8	2.346089	12	49.02848	43	629.5393	82	2289.358
2 3/4	2.574846	12 1/2	53.19931	44	659.1607	83	2345.536
2 7/8	2.814243	13	57.54037	45	689.4630	84	2402.396
3	3.064280	13 1/2	62.05167	46	720.4463	85	2459.936
3 1/8	3.324957	14	66.73321	47	752.1105	86	2518.157
3 1/4	3.596273	14 1/2	71.58499	48	784.4557	87	2577.060
3 3/8	3.878229	15	76.60700	49	817.4818	88	2636.643
3 1/2	4.170826	15 1/2	81.79925	50	851.1889	89	2696.907
3 5/8	4.474062	16	87.16174	51	885.5769	90	2757.852
3 3/4	4.787938	16 1/2	92.69417	52	920.6459	91	2819.478
3 7/8	5.112453	17	98.39744	53	956.3958	92	2881.785
4	5.447609	17 1/2	104.27064	54	992.8267	93	2944.773
4 1/4	6.149840	18	110.31408	55	1029.9386	94	3008.442
4 1/2	6.894630	18 1/2	116.52776	56	1067.7314	95	3072.792
4 3/4	7.681980	19	122.91168	57	1106.2051	96	3137.823
5	8.511889	19 1/2	129.46583	58	1145.3598	97	3203.535
5 1/4	9.384358	20	136.19022	59	1185.1954	98	3269.927
5 1/2	10.299386	21	150.14972	60	1225.7120	99	3337.001
5 3/4	11.256973	22	164.79017	61	1266.9096	100	3404.756

The weight of water in a given length (as one foot) of any pipe or other circular cylinder **is in proportion to the square of the bore** or inner diameter. Hence the weight of water in 1 foot length of any cylinder of other diameter than those in the table can be found by multiplying that for a 1 inch pipe, 0.340475558, by the square of the inner diameter of the given cylinder in inches. Thus, for a cylinder 120 inches diameter: diameter^2 = 120^2 = 14400, and weight of water in 1 foot depth = 0.340475558 × 14400 = 4902.848 lbs. Or, weight for 120 ins. diam. = 100 × weight for 12 ins. diam. = 100 × 49.02848 = 4902.848 lbs. Similarly, $\left(\frac{7}{16}\right)^2 = \frac{49}{256}$ = 0.191406, and 0.340475558 × 0.191406 = 0.065169 lb. = weight in 1 foot of $\frac{7}{16}$ inch pipe. Here, also, $\frac{7}{16}$ = *half* of $\frac{7}{8}$; hence, weight for $\frac{7}{16}$ inch = one-*fourth* of weight for $\frac{7}{8}$ inch = one-fourth of 0.260677 = 0.065169.

Weight of one square inch of water 1 foot high, at 62.425 lbs. per cubic foot = 62.425 ÷ 144 = 0.433507 lb.

* Actual. See nominal and actual diameters, foot note, p 526.

Pipes*. Areas; Contents; Square roots of Diameters. Original.
D = diameter* in inches; **d** = diameter in feet.
A = cross section area in sq ft, = cub ft in 1 ft length.
For **d**, see p 221. Thus; let D = 17¾ ins = 12 ins + 5¾ ins.
Then (p 221) d = 1 ft + 0.4792 ft = 1.4792 ft. Or, square $\sqrt{d}$.

D	A	$\sqrt{d}$	D	A	$\sqrt{d}$	D	A	$\sqrt{d}$
1/4	0.0003409	0.144	4.	0.08727	0.577	15	1.227	1.118
5/16	0.0005327	0.161	1/8	0.09281	0.586	1/4	1.268	1.127
3/8	0.0007670	0.177	1/4	0.09851	0.595	1/2	1.310	1.137
7/16	0.001044	0.191	3/8	0.1044	0.604	3/4	1.353	1.146
1/2	0.001364	0.204	1/2	0.1105	0.612*	16	1.396	1.155
9/16	0.001726	0.217	5/8	0.1167	0.621	1/4	1.440	1.164
5/8	0.002131	0.228	3/4	0.1231	0.629	1/2	1.485	1.173
11/16	0.002578	0.239	7/8	0.1296	0.637	3/4	1.530	1.181
3/4	0.003068	0.250	5.	0.1364	0.645	17.	1.576	1.190
13/16	0.003601	0.260	1/8	0.1433	0.654	1/4	1.623	1.199
7/8	0.004176	0.270	1/4	0.1503	0.661	1/2	1.670	1.208
15/16	0.004794	0.280	3/8	0.1576	0.669	3/4	1.718	1.216
1	0.005454	0.289	1/2	0.1650	0.677	18	1.767	1.225
1/16	0.006157	0.298	5/8	0.1726	0.685	1/2	1.867	1.242
1/8	0.006903	0.306	3/4	0.1803	0.692	19	1.969	1.258
3/16	0.007691	0.315	7/8	0.1883	0.700	1/2	2.074	1.275
1/4	0.008522	0.323	6	0.1964	0.707	20.	2.182	1.291
5/16	0.009396	0.331	1/4	0.2131	0.722	1/2	2.292	1.307
3/8	0.01031	0.339	1/2	0.2304	0.736	21.	2.405	1.323
7/16	0.01127	0.346	3/4	0.2485	0.750	1/2	2.521	1.339
1/2	0.01227	0.354	7	0.2673	0.764	22	2.640	1.354
9/16	0.01332	0.361	1/4	0.2867	0.777	1/2	2.761	1.369
5/8	0.01440	0.368	1/2	0.3068	0.791	23.	2.885	1.384
11/16	0.01553	0.375	3/4	0.3276	0.804	1/2	3.012	1.399
3/4	0.01670	0.382	8.	0.3491	0.816	24	3.142	1.414
13/16	0.01792	0.389	1/4	0.3712	0.829	25	3.409	1.443
7/8	0.01918	0.395	1/2	0.3941	0.842	26	3.687	1.472
15/16	0.02047	0.402	3/4	0.4176	0.854	27	3.976	1.500
2.	0.02182	0.408	9.	0.4417	0.866	28	4.276	1.528
1/16	0.02320	0.415	1/4	0.4667	0.878	29	4.587	1.555
1/8	0.02463	0.421	1/2	0.4922	0.890	30.	4.909	1.581
3/16	0.02610	0.427	3/4	0.5185	0.901	31	5.242	1.607
1/4	0.02761	0.433	10	0.5454	0.913	32.	5.585	1.633
5/16	0.02917	0.439	1/4	0.5730	0.924	33.	5.940	1.658
3/8	0.03077	0.445	1/2	0.6013	0.935	34	6.305	1.683
7/16	0.03240	0.451	3/4	0.6303	0.946	35	6.681	1.708
1/2	0.03409	0.456	11.	0.6599	0.957	36	7.069	1.732
9/16	0.03581	0.462	1/4	0.6903	0.968	38	7.876	1.780
5/8	0.03759	0.468	1/2	0.7213	0.979	40	8.727	1.826
11/16	0.03939	0.473	3/4	0.7530	0.990	42.	9.621	1.871
3/4	0.04125	0.479	12	0.7854	1.000	44	10.56	1.915
13/16	0.04315	0.484	1/4	0.8185	1.010	48	12.57	2.000
7/8	0.04508	0.489	1/2	0.8522	1.021	54	15.90	2.121
15/16	0.04707	0.495	3/4	0.8866	1.031	60	19.64	2.236
3	0.04909	0.500	13	0.9218	1.041	66	23.76	2.345
1/8	0.05327	0.510	1/4	0.9575	1.051	72	28.27	2.449
1/4	0.05761	0.520	1/2	0.9940	1.061	78	33.18	2.550
3/8	0.06213	0.530	3/4	1.031	1.070	84	38.49	2.646
1/2	0.06681	0.540	14.	1.069	1.080	90.	44.18	2.739
5/8	0.07167	0.550	1/4	1.108	1.090	96	50.27	2.828
3/4	0.07670	0.559	1/2	1.147	1.099			
7/8	0.08190	0.568	3/4	1.187	1.109			

Caution.** In the tables, pp 525 and 526, the diameters or bores are the *actual* ones, as mesured in inches. Wrought iron and steel steam, gas and water pipes are commonly designated by fictitious or **"nominal"** diameters, which are mere arbitrary names. * In the smaller sizes especially, the discrepancies are serious. Thus, the pipe whose "nominal" inner diam is *one eighth* inch has an ***actual inner diam of *full quarter* inch. See p 1164.

Art. 3. Theory of flow in long pipes and channels.

Resistance to flow. When one solid body moves over another, most of its particles retain practically their original relative places; but when water flows, as in a pipe or channel, the roughnesses and irregularities, which exist in even the smoothest surfaces, disturb the current, the several particles describing spiral or other paths not parallel to the general direction of flow. Resistance to flow is believed to be due to such disturbances, rather than to friction proper, as we understand it in solids See p 407, and ¶ 194, p. 415

In the absence of complete knowledge respecting the nature of these disturbances, we are forced to rely upon experiment in determining the manner and extent of their influence upon the flow.

Mean velocity: rate of discharge. Figs A, B. In a pipe or

Fig. A.

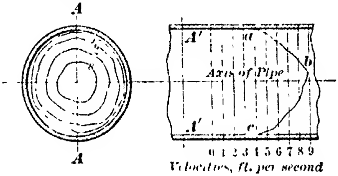
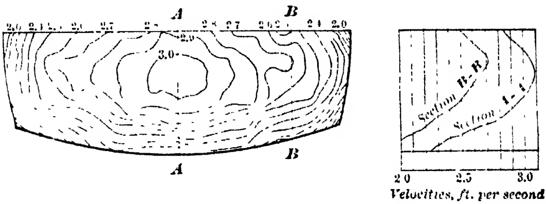


Fig. B.



channel, owing to the disturbing influence of contact with the sides, the particles of water move in tortuous paths The **mean velocity**, through the entire cross section at any point, is the quotient obtained by dividing the rate of discharge by the area of cross section **At any given point** in a cross section, **the velocity**, as recorded by any form of current meter, is the component, parallel to the axis of the pipe, of the actual velocity of the particles passing that point. We deal, at present, only with cases of **"steady flow,"** *i. e*, where the velocity, at each point, remains constant.

The velocities, as measured at different points in the cross section of a pipe or channel, are generally least near the sides of pipes, and near the sides and bottoms of channels

In Fig A, the longitudinal section shows one of many series of velocity measurements by Messrs Williams, Hubbell and Fenkell* on a cast iron pipe, 16 ins diameter The measurements were made, by means of the Pitot tube, on a vertical diameter, such as A A in the cross section The horizontal distances of the several points in the curve, *a b c*, from the vertical line, 0, represent, by the scale below the figure, the velocities at the several points in the diameter. In the cross section, the several curved lines are lines of equal vel.

Fig B shows the results of measurements of velocities in a cross section of the Sudbury Conduit, Mass.,† 9 feet wide, 3 feet deep. In the longitudinal section, the velocities, at different depths on the lines A A and B B of the cross section, are (approximately) indicated as in Fig A. In the cross section, the several curved lines are lines of equal velocity. The innermost one corresponds to a velocity of 3 ft. per second; the next, to 2.9 ft per second, and so on.

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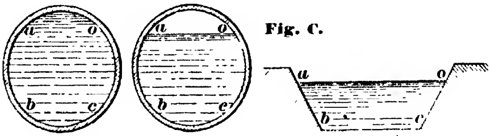
*Trans. Am. Soc. Civ. Engrs., Vol. XLVII, plate LV, p. 66.

†F. P. Stearns, Trans. Am. Soc. C. E., Aug. 1883, Vol. XII, p. 324.

Formulas for discharge, q, and mean velocity, v, in a given length, L, of a straight **pipe or channel** of uniform wet cross section. Referring to Fig. C, let

Fig. C.



$L =$ the given length ;
$p =$ the wet perimeter, $a\,b\,c\,o$;
$A = L\,p =$ the area of the wetted surface ;
$a =$ the cross section area of the liquid stream ;
$r = \frac{a}{p} =$ the mean radius of its cross section ;
$d =$ diam of pipe ; *
$q =$ the rate of discharge
$=$ volume passing a given cross section, a, in unit time ;
$v = \frac{q}{a} =$ the mean velocity ;
$h_v = \frac{v^2}{2\,g} =$ the velocity head ;
$m\ c,\ f,\ F =$ resistance factors, as explained below and at top of p 530.
$h_f =$ the resistance (or "friction") head in the length, L ;
$s = \frac{h_f}{L} =$ the slope.

It is generally held that the resistances to flow are directly proportional to A and to some power (usually taken as v^2 †) of v, and inversely proportional to a. Or:

friction head, $h_f = m\,\frac{A\,v^2}{a} = m\,\frac{L\,p\,v^2}{a} = m\,\frac{L\,v^2}{r}$; and $m = h_f\,\frac{r}{L\,v^2} = \frac{r\,s}{v^2}$;

whence

$v^2 = h_f\,\frac{a}{m\,L\,p} = \frac{r\,s}{m}$; and **velocity**, $v = \sqrt{\frac{r\,s}{m}} = \sqrt{\frac{1}{m}}\,\sqrt{r}\,\sqrt{s}$.

Let $c = \sqrt{\frac{1}{m}}$; $m = \frac{1}{c^2}$. Then we have the **Chezy formula**:—

velocity, $v = c\,\sqrt{r\,s} = c\,\sqrt{r}\,\sqrt{s} = c\,r^{0.5}\,s^{0.5}$.

For "Kutter's formula," giving values of c, see p 523, and pp 564 to 574. For tables of c, by Kutter's formula, see pp. 565 to 570

The resistance, or roughness factor, such as m, f, F or c, above, or c_H in the Hazen formula below, must be selected by judgment, or determined, as by Kutter's formula, pp. 523, 564, &c, according to the known or assumed condition of the wetted surface.

Weisbach's formula for friction head, $h_f = m\,L\,v^2/r$, in feet and seconds. ($g = 32.2$ ft per sec per sec.)

Friction head $h_f = [0.0144 + (0.017 / \sqrt{v})]\,L\,v^2 / 2\,g\,d$.

This formula is based upon early experiments with small pipes. For pipes above say 20″ diam, modern practice shows greater friction heads for given vels. Compare Fig D, pp 529 a, 529 b.

* In full semi-circular channels, and in full or half-full pipes, $d = 4\,r$.
† See "Exponential" formulas, p 529.

Art. 3 a. "Exponential" formulas.* Careful study of experiments upon large and small pipes and channels, under varying conditions, indicates that the mean velocity depends, not upon the *square* roots, or 0.5 powers, of r and of s, but generally upon somewhat higher powers; and that the inaccuracy, involved in taking 0.5 for the exponents in the Chezy formula, accounts for the wide variation of the Chezy c, with r and with s, for a given condition of inner surface of pipe or channel.

Gardner S. Williams and Allen Hazen adopt the formula:—

$$v = c_H \, r^{0.63} \, s^{0.54} \; 0.001^{-0.04};$$

the last factor, $0.001^{-0.04} = 0.001^{0.5 - 0.54} = 1000^{0.04} = 1.32$, being inserted in order to make their values of c_H, in ordinary cases, more nearly equal with the values of c in the Chezy formula.† To facilitate the application of their formula, they furnish a "hydraulic slide rule," by means of which mean vels, and the losses of head due to different vels in different pipes and channels, may be readily found. Their "Hydraulic Tables,"‡ have been calculated in this way.

Fig D, pp 529 *a* and *b*. **Diagram of discharges, velocities and head-losses,** in pipes and conduits, by Williams-Hazen formula, as above.

Example. Given a 24" cast iron pipe, 2000 ft long, to carry 2,000,000 gals/24 hrs, after 40 yrs use carrying av soft unfilterd water. Reqd, the vel and the head-loss.

Velocity. Starting at D, on lower scale (2,000,000 gals/24 hrs) follow the vert line until it meets, at E, the inclined *dotted* line for 24" pipe. The hor line, thru E, gives the reqd vel, 1 ft/sec.

Head-loss. On small diagram A (either page) find intersection, B, of 40-yr age line with curvd line for 24" pipe, giving $c_H = 80$. Find intersection, C, of line for $c_H = 80$ with curvd line of main diagram for 24" pipe. Follow the *direction* of the *solid* inclined lines from C to the intersection, F, with the hor line for vel = 1 ft/sec. From F, follow the vert line to G, on the lower scale, giving the reqd head-loss, 0.4 ft/1000 ft = 0.8 ft in the given length of 2000 ft. (And vice versa.)

In diagram A, the curv markt ∞ applies to cases where the diam is not reduced by service, and to large pipes in general.

Approximate values of c_H:

Pipes and conduits, running full;

3 to 60 inches diameter;		**Over 60 inches diameter;**	
Cast iron;	c_H	Iron, masonry, etc.	c_H
Very best, new, carefully coated and laid	140	Extremely smooth and straight	140
Good, new	130 to 120	Very smooth	130
In fair condition	100	Good masonry	120
Tuberculated	80 to 40	Brick sewers	100
Riveted steel;		Rough	90
New	110	Very rough	80
		Tile sewers:	
Ten years old	100	4 to 36 ins diam	110

Open channels. (For a given character of surface, and given wet perimeter, c_H, like c in the Chezy formula, is max for max area of cross section.)

Sudbury conduit, 8 ft wide,	c_H	Rough masonry, 4 ft wide,	c_H
1 to 5 ft deep	130 to 140	1 ft deep	65 to 75
Smooth wood	110 to 140	Gravel	50 to 80
Unplaned plank	100 to 120	Earth, very rough	65 to 75
Good Masonry	80 to 120	With mud, grass and weeds	35 to 70

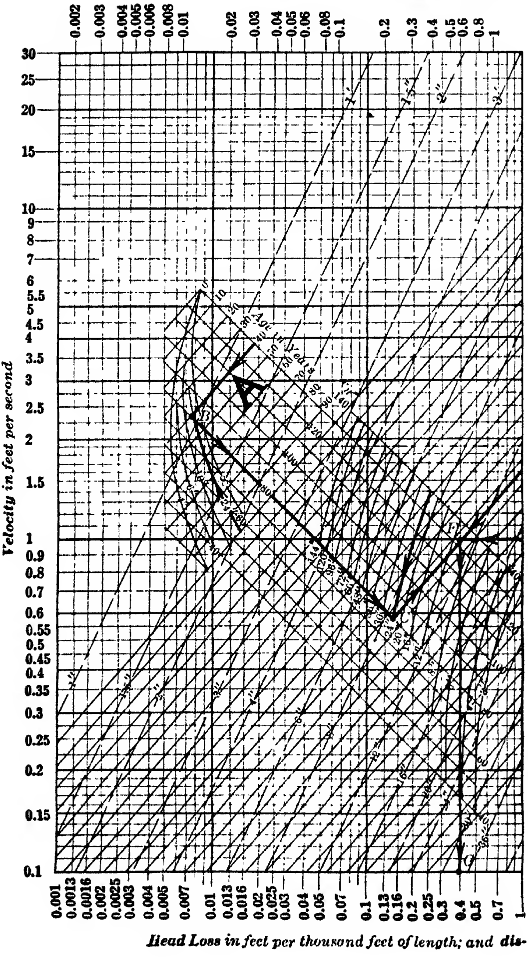
* Formulas in which the exponents are other than 0.5 have been called "exponential," in order to distinguish them from those where the exponent is 0.5; but these last are, of course, no less "exponential."

† The values of c, given by the Kutter formula and in our tables, pp. 566, etc., contain the correction necessitated by the fact that the velocity is not strictly proportionate to the square roots of r and of s.

‡ Hydraulic Tables. John Wiley & Sons, New York

Fig. D. *Discharges, Velocities and Head Losses in Pipes and Con-*

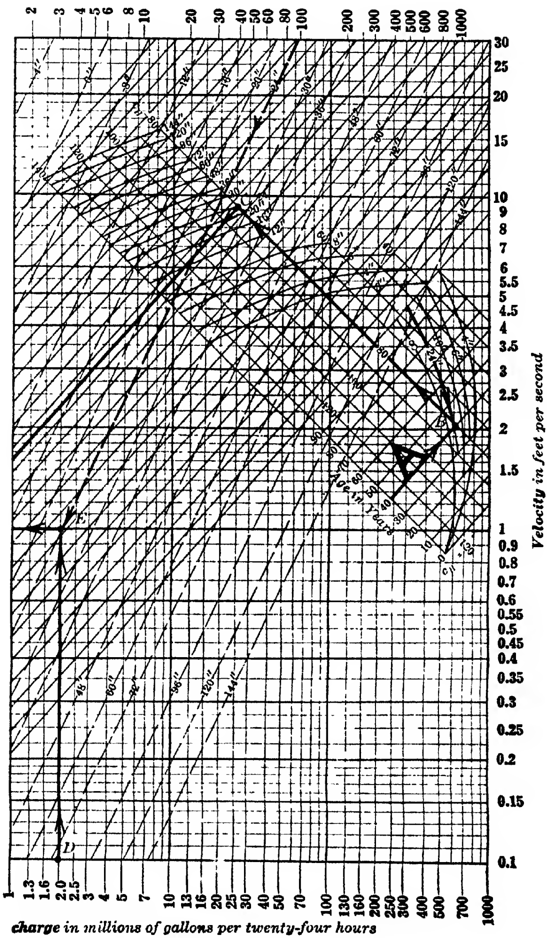
Discharge in Cub-



Head Loss in feet per thousand feet of length; and dis-

duits, by Williams-Hazen Formula; $V = C_H\, r^{0.63}\, s^{0.54}\, 0.001^{-0.04}$

ic feet per second



charge in millions of gallons per twenty-four hours

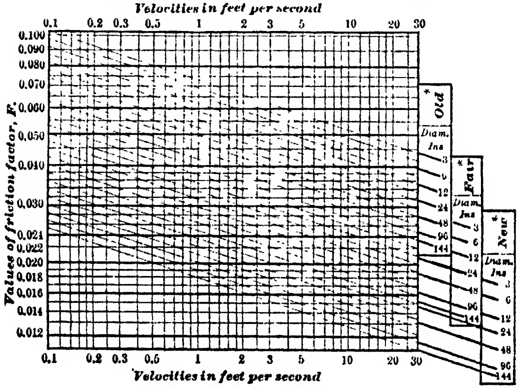
Friction factor. Let

$$f = 2\,g\,m = \frac{2\,g}{c^2} = 2\,g\,.\,h_f\,\frac{r}{\mathrm{L}\,v^2} = 2\,g\,.\,\frac{r\,s}{v^2}; \qquad \mathrm{F} = 4\,f^{*} = 2\,g\,.\,\frac{d\,s}{v^2}.$$

F or f is usually called **the friction factor.** It expresses the relation (determined by the condition of the wetted surface of the pipe or channel) between velocity and dimensions, on the one hand, and the resistance to flow on the other. Thus

$$h_f = f\,.\,\frac{\mathrm{L}}{r}\,.\,\frac{v^2}{2\,g}\dagger = 4\,f\,.\,\frac{\mathrm{L}}{4\,r}\,.\,\frac{v^2}{2\,g} = \mathrm{F}\,.\,\frac{\mathrm{L}}{d}\,.\,\frac{v^2}{2\,g}.\ddagger{}^{*}$$

Values of friction factor, *F*, for iron pipe. See page 531.



Example of use of diagram. Given a 6 inch pipe, in fair condition. In the column, on the right, headed "Fair," find diam, 6 ins. Following to the left the direction of the short inclined line, preferably by means of a rule or straight-edge of paper, we find that it coincides nearly with one of the inclined lines which cross the diagram. By means of the intersections of this line with the others, we find that, for the pipe in question, a velocity of 5 ft per sec corresponds approximately with F = 0.035; 0 5 ft per sec, F = 0.048, etc., etc.

* "Old," "fair," and "new" correspond approximately with values of Kutter's "n" (p. 564) as follows:

Diameter..	3 inch		6 inch		12 inch		60 inch		120 inch	
Slope, in ft per 1000 ft	10.0	1.0	10 0	1.0	10 00	0 4	10 00	0 05	1 000	0.025
	n	n	n	n	n	n	n	n	n	n
Old	0.012	0.013	0 014	0 014	0 015	0.016	0.018	0.020	0.019	0.021
Fair.........	0 011	0.011	0.012	0 012	0.013	0 014	0 014	0.015	0.015	0.017
New	0.010	0.010	0 010	0.010	0.011	0.012	0.012	0.013	0.013	0.014

* This applies where $d = 4\,r$. In pipes running full or half-full, and in semicircular channels running full, $d = 4\,r$.

† See Mechanics of Engineering, by I P Church, 1890, p. 714, Eq. (8).

‡ See Hydraulics, by Mansfield Merriman, 1905, p. 209. Eq. (86).

Art. 4. To find the discharge, q, through a long compound pipe, or pipe of varying diameter, Fig. 1 H.

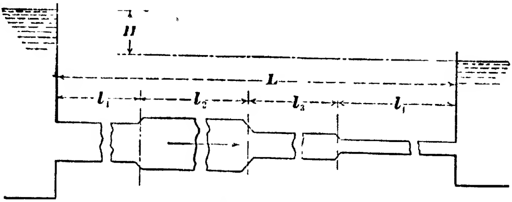


Fig. 1 H.

Let

l_1, l_2, l_3, etc. = the lengths of the several portions of the pipe;
d_1, d_2, d_3, etc = the corresponding diameters;
v_1, v_2, v_3, etc. = the corresponding velocities;
F_1, F_2, F_3, etc = the corresponding values of the resistance or "friction" factor. See p. 530.
$L = l_1 + l_2 + l_3 +$ etc. = the total length of the pipe;
H = the total head (p. 516);
q = rate of discharge $= \frac{1}{4} \pi d_1^2 v_1 = \frac{1}{4} \pi d_2^2 v_2 =$ etc.

In a long pipe, the velocity and entry heads are usually negligible, relatively to the friction head. Neglecting them, we have

H = total head = friction head.

In each portion of the pipe, the resistance, and the corresponding "friction" head, h_f, are believed to be proportional directly to the length, l, of such portion, and to the velocity head, $\frac{v^2}{2g}$, and inversely to the diameter, d; or

$$h_f = F \cdot \frac{l}{d} \cdot \frac{v^2}{2g}.$$

Hence,

$$H = F_1 \cdot \frac{l_1}{d_1} \cdot \frac{v_1^2}{2g} + F_2 \cdot \frac{l_2}{d_2} \cdot \frac{v_2^2}{2g} + F_3 \cdot \frac{l_3}{d_3} \cdot \frac{v_3^2}{2g} + \text{etc.};$$

and, since $v_1 = \frac{4q}{\pi d_1^2}$, $v_2 = \frac{4q}{\pi d_2^2}$, etc.,

we have, also,

$$2gH = F_1 \cdot \frac{l_1}{d_1} \cdot \frac{16 q^2}{\pi^2 d_1^4} + F_2 \cdot \frac{l_2}{d_2} \cdot \frac{16 q^2}{\pi^2 d_2^4} + \text{etc.}$$

$$= \frac{16 q^2}{\pi^2} \left(F_1 \cdot \frac{l_1}{d_1^5} + F_2 \cdot \frac{l_2}{d_2^5} + \text{etc.} \right)$$

whence

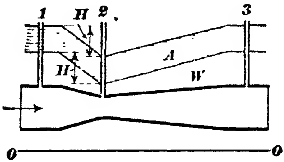
$$q = \frac{\pi}{4} \sqrt{\frac{2gH}{F_1 \frac{l_1}{d_1^5} + F_2 \frac{l_2}{d_2^5} + \text{etc.}}}$$

Art. 4 a.* **The Venturi Meter** is designed for the measurement of the flow of liquids in pipes of large dimensions, running full.

The meter proper, patented by Clemens Herschel, consists essentially of a mere constriction in the area of cross-section of the pipe, with openings in the pipe opposite its normal and its constricted diameters, for measuring, by piezometers or pressure-gauges, the pressures at those points, while **the register** is an elaborate mechanism, provided with clock-work and dials.

Theory.† Let Figs. 1 to 3 represent a Venturi meter tube, with three piezometers in place, viz: No 1, over the tube up-stream from the constriction; No 2, over the constriction itself, and No 3, over the tube down-stream from the constriction. Let the unshaded area *W* in Figs 1 to 3, represent the depths at which the water stands above any assumed horizontal datum plane *O O*, and let the shaded area *A* represent the uniform pressure of the atmosphere, which, for convenience, we may suppose to be converted into some liquid of the specific gravity of water, but distinguishable, by its appearance, from the water.

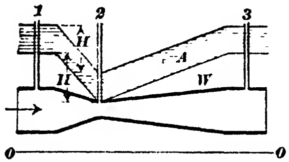
Fig 1



The vertical distance, between the upper boundary of this latter area and any given point in the tube, represents the combined pressure of air and water at such point.

The velocities in the meter tube at any instant, are of necessity inversely proportional to the areas of cross section, and, as the heads corresponding to the several velocities are proportional to the squares of those velocities, the remaining or pressure heads must vary also, the smallest or lowest pressure head standing over the throat, where the velocity is greatest.

Fig 2.



The increase of velocity, acquired by the fluid in passing from section 1 to section 2, is again given up in passing from section 2 to section 3, and, in the case of a perfect fluid the pressure lost between sections 1 and 2 would be perfectly restored in passing from section 2 to section 3. In practice, a small total loss occurs. This loss is greater with high than with low velocities.

For a given head in piezometer No 1 and given diameter of pipe at section 1, the expenditure of head in velocity between sections 1 and 2 increases as the area of the throat is diminished and as the throat velocity is thereby increased.‡ In Fig. 2 is shown the case where all of the water head above the top of the throat is required to maintain the velocity through the throat.

In Figs 1 and 2 the head, *H*, expended in the increase of velocity between sections 1 and 2 is represented by the difference in level between the tops of the two water columns 1 and 2, or between the tops of the two corresponding air columns. In Fig. 2 this difference is equal to the *total* vertical height of the water column at section 1 above the top of the throat at section 2.

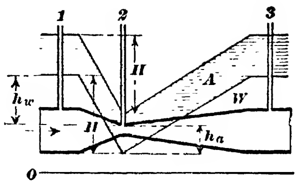
* Abridged from a description prepared by the writer as Chairman of a Committee of the Franklin Institute. *Journal of the Franklin Institute,* February, 1899.

† The Venturi meter, apart from its merits as a measuring device, embodies important hydraulic principles. Hence its theory is here stated more fully than would otherwise be necessary.

‡ In a given Venturi tube the pressure and velocity at the throat may be varied also by modifying those at sectons 1 and 3, as by regulating the openings of the valves of influx to and of efflux from the meter tube, by changing the total head on the system, etc.

If now (Fig. 3) the throat section be still further reduced (the other conditions remaining as before) the throat velocity will thereby be still further increased, for the *total* pressure available for increase of velocity between sections 1 and 2 consists not merely in the depth of *water* above the tube, but also in the *atmospheric pressure*, represented by the shaded area *A* above the water *W*.

Fig. 3.

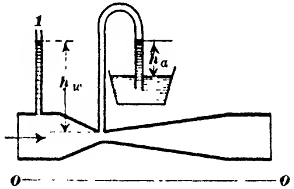


In Fig 3 all the water has disappeared from piezometer 2, and even a portion of the liquid representing the air has also disappeared, leaving only a portion of the latter to represent such pressure as now remains in the throat In other words, the pressure within the throat is now less than the atmospheric pressure

In Fig 3, the loss of head, due to increase of velocity between sections 1 and 2, is $H = h_w + h_a$ - the entire available head of water, h_w, plus a portion, h_a, of the atmospheric pressure The latter portion, h_a, is frequently called "the vacuum"

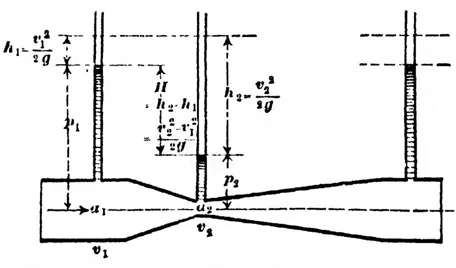
The top of the water column having now disappeared below the top of the throat, it is no longer feasible to ascertain the loss of head by taking the difference between the levels of the water surfaces in piezometers 1 and 2. The degree of "vacuum" may be found, as shown in Fig 4, by using, in place of the piezometers, a glass tube bent over and led *downward* into an open vessel containing water or mercury. The height to which the water (or the mercury, converted into feet of water) rises in this tube, shows the extent of the vacuum, or the portion, h_a, of the air pressure which has been called into service in producing the high velocity through the throat. By adding this to h_w, we obtain, as above, the total loss of head H between sections 1 and 2

Fig. 4.



When the reduction of area at the throat has proceeded so far that the entire available pressure of water and air at section 1 is required, in order to maintain the corresponding velocity through the throat (*i e.*, when the line representing the upper surface of the air falls to the level of the top of the throat),

Fig. 5



no further increase of throat velocity can be secured (with a given total head over section 1) by still further narrowing the throat. If the throat is further narrowed, the velocity through it will remain the same; and, the rate of discharge being thus diminished, the velocity through section 1 will be necessarily reduced. In other words, throttling begins

Let v_1 be the velocity in section 1, above the throat, and v_2 the "throat velocity," or velocity in the throat or section 2.

Neglecting resistances, Fig. 5, the velocity head at section 1, measured from an assumed datum represented by the upper horizontal lines, is

$$h_1 = \frac{v_1^2}{2g},$$

and that at section 2 is

$$h_2 = \frac{v_2^2}{2g}.$$

Neglecting resistances to flow, the loss of head, between sections 1 and 2, or "the head on the Venturi," is equal to the increase in the velocity head, or to the loss in pressure, between a_1, and a_2, or

$$H = h_2 - h_1 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{v_2^2 - v_1^2}{2g} = p_1 - p_2{}^*$$

Hence, $$h_2 = \frac{v_2^2}{2g} = H + h_1$$

and **throat velocity** $= v_2 = \sqrt{2g\,(H + h_1)} = \sqrt{2g\left(H + \frac{v_1^2}{2g}\right)}.$

In other words, the velocity at the throat is that corresponding to the "head H on the Venturi," plus the head corresponding to the velocity of approach v_1 in section 1.

But, since the velocities are inversely as the areas of cross-section a_1 and a_2,

$$v_1 = \frac{a_2}{a_1} v_2, \qquad \text{and } v_1^2 = \frac{a_2^2}{a_1^2} v_2^2,$$

$$H = \frac{v_2^2 - v_1^2}{2g} = \frac{v_2^2 - \frac{a_2^2}{a_1^2} v_2^2}{2g} = \frac{\left(1 - \frac{a_2^2}{a_1^2}\right) v_2^2}{2g} = \frac{\frac{a_1^2 - a_2^2}{a_1^2} v_2^2}{2g},$$

$$v_2^2 = \frac{2g\,H a_1^2}{a_1^2 - a_2^2},$$

and **throat velocity** $= v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2g\,H}.$

The ratio $$\frac{a_2}{a_1},$$

between the area a_2 of cross-section at the throat, and that, a_1, at the upper end of the up-stream cone, is called the **throat ratio.** For a ratio of 1 : 9 we have

$$\frac{a_1}{\sqrt{a_1^2 - a_2^2}} = \frac{9}{\sqrt{9^2 - 1^2}} = \frac{9}{\sqrt{80}} = \sqrt{\frac{81}{80}} = 1.0062.$$

or $$v_2 = 1.0062\sqrt{2g\,H}.$$

The Venturi tube, for pipes not over 60 inches in diameter, is formed of several short sections of cast iron pipe, having the required taper, and fur-

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* By Bernouilli's theorem, $p_1 + h_1 = p_2 + h_2$.

nished with flanges, by means of which the sections are bolted together to form the two truncated cones required.

In the smaller sizes, the shorter cone is generally in one section and the longer cone in two or more sections.

The throat section is generally made in a separate piece, and is either made of bronze or lined with that metal.

The ends of the Venturi tube are furnished with either bell, spigot, or flanged ends, according to the character of the pipe in which the tube is to be used.

For still larger streams, such as those in masonry conduits or riveted flumes, the Venturi tube may be made of wooden staves, sheet steel, cement concrete, brick or other suitable material, metal being used for the throat piece and where required by the pressure.

The throat piece is surrounded by an annular chamber called the **pressure chamber,** which communicates with the interior of the throat by means of several holes drilled radially through the walls of the latter at equal or nearly equal distances around the circumference.

A similar pressure chamber is provided at the larger end of the short cone for observing the pressure in the normal section up-stream from the throat; and, if it is desired to ascertain the final loss of head due to the passage of the water through the Venturi, a similar chamber must be provided at the larger end of the longer or down-stream cone.

In designating the size of the meter, the diameter of the pipe of which it forms a part is used, and not the throat diameter. Thus, a meter for use in a 6-inch pipe is called a 6-inch meter.

The register gives periodic registrations, usually every ten minutes, in which the head $H = h_2 - h_1$, existing at the instant of registry, is recorded in terms of the total discharge in cubic feet since the last registry and as an increase in the total number of cubic feet registered. In other words, the registry involves the assumption that the average velocity, during the period between two registrations, is equal to the velocity at the instant of the following registration.

The register may be placed at a considerable distance (not exceeding, say, 500 feet) from the Venturi tube. It must be placed at such a depth below the hydraulic grade line that the pressures existing in the Venturi tube shall at all times be transmitted to the register.

The pipe lines, connecting the Venturi with the register, must be covered, and a shelter from weather and frost must be provided for the register.

The size and cost of the register are independent of the size of the Venturi.

Behavior. From experiments by Mr. Herschel,* † ¶ by the Bureau of Water, Philadelphia,† and by others,† it appears that the Venturi meter may ordinarily be depended upon to give results within 3 per cent of the true discharge.

With a 48 inch Venturi, Mr Herschel ¶ found a total **loss of head,** due to the passage of the water through the Venturi tube, of about 10.6 per cent. of the head H on the Venturi. With two 54 inch Venturis, Professors Marx, Wing, and Hoskins § † found a loss of 14.9 per cent, part of which, no doubt, was due to the presence of a 42 inch gate valve in the down-stream cone. This last result would add about 1.12 feet to the head required in pumping 20,000,000 gallons daily through a 48 inch main and a Venturi having a throat ratio of 1 : 9.

The Venturi meter has been found to give perfectly satisfactory results in measuring the flow of brine and very hot water.

Venturi tubes are made with throat ratios ranging from 1 : 4½ (or 2 : 9) to 1 : 16. The former are adapted to high, and the latter to low velocities; for, where the velocity in the pipe is low, it is necessary to accelerate it greatly in the throat in order to obtain sufficient loss of pressure to secure reliable indications in the register. These cannot be obtained where the throat velocity is less than about 3 feet per second. With a throat ratio of 1 : 16, this would give a pipe velocity of $\frac{3}{16}$ foot per second. On the other hand, a meter with a high throat ratio, adapted to low velocities, would, with high velocities, exceed the upper limit of the register.

Owing to its unobstructed channel, free from moving parts, the Venturi meter is far less liable to clogging than the forms of meter in common use.

The **prices** of the principal sizes of the Venturi meter are as follows:—on board cars at Providence, R. I.

6 inch $600.00	24 inch $1,130.00	48 inch $3,060.00
12 inch 770 00	36 inch 1,680.00	60 inch 4,890 00

These prices include the register, which, in the smaller sizes, constitutes the principal item of cost. Discount, 1901, 10 per cent.

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* Trans. Am. Soc. Civil Engrs., Nov., 1887, Vol. XVII., page 228.

† Journal of the Franklin Institute. Feb., 1899.

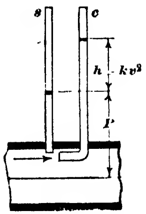
¶ Journal New England Waterworks Assn., Vol. VIII., No. 1, Sep., 1893.

§ Trans. Am. Soc. Civil Engrs., Vol XL., Dec., 1898, pp. 471 etc.

Art. 4 b. The Ferris-Pitot meter, invented and patented by Mr Walter Ferris, of Philadelphia, is designed to measure the flow of liquids in pipes running full. It consists of a device for the registration of the results obtained by the Pitot tube, described on pages 561 and 562, and of special devices to prevent the clogging of the tubes and to permit their examination while in use.

In Fig. 6 let P represent the level at which the water stands in the straight Pitot tube, s. Then $h = kv^2$, or the difference in level between the columns in the two tubes, is the head (theoretically $= \frac{v^2}{2g}$) due to the velocity of the water in the pipe as it impinges against the open up-stream end of the bent tube, c. For a given velocity, v, this difference, h, is constant, and is independent of the pressure represented by P.

Fig. 6.



The Ferris register, like that of the Venturi meter, records the velocity (existing at the instant of registration) in terms of the total discharge since the last registry and as an increase in the total number of cubic feet registered. The registry thus involves the assumption that the average velocity, during the period between registrations, is equal to the velocity at the end of that period. In the Ferris meter the registration is made every two minutes.

Evidently the instrument measures the velocity at only one point in the cross-section of the pipe, and it may thus be used to determine successively the velocities at any number of such points, but the velocity at such a point may or may not be equal to the mean velocity in the entire cross-section. The instrument is therefore usually calibrated by reference to some accepted standard, and the coefficient or coefficients thus obtained are used in subsequent observations.

The recording mechanism is operated by a small hydraulic motor, driven by means of the flow of the water in the pipe itself. For this purpose a second pair of Pitot tubes, is inserted into the pipe, and the current, flowing through these tubes, drives the motor without loss of water, the water used for power being returned to the pipe. If the velocity in the pipe is less than 3 feet per second it must be increased by means of a "reducer."

Experiments made by Mr. Ferris and by the Bureau of Water, Philadelphia, indicate that the Ferris-Pitot meter will ordinarily register within 3 per cent of the true discharge.

In general, the size and cost of the registering apparatus are independent of the size of the pipe.

Art. 5. Resistance of curves and bends in water pipes.

Much uncertainty exists respecting these matters. **Weisbach's formula,*** for the resistance due to a circular curve, Figs. 2 and 3, is

$$h = C\,\frac{A}{180}\cdot\frac{v^2}{2\,g} = \left[0.131 + 1.847\left(\frac{r}{R}\right)^{\frac{7}{2}}\right]\frac{A}{180}\cdot\frac{v^2}{2\,g}\text{, where}$$

h = head in feet required to overcome resistance due to curve or bend,
C = experimental coefficient,
A = angle of deflection, in degrees,
v = mean velocity of flow in pipe, in feet per second,
g = acceleration of gravity = 32.2 ft per sec per sec,
$v^2/2\,g$ = head theoretically due to velocity v,
D = inside diameter of pipe, in feet,
r = inside radius of pipe, in feet,
R = radius of axis of curve, in feet.

If $r \div R$ =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	**0.9**	**1.0**
then C =	0.131	0.138	0.158	0.206	0.294	0.440	0.661	0.977	**1.408**	**1.978**

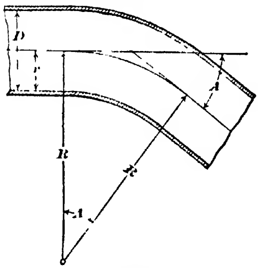


Fig. 2.

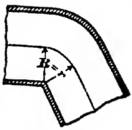


Fig. 3.

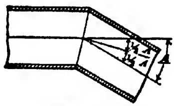


Fig. 4.
(See next page.)

According to this formula, the resistance due to curvature decreases rapidly as R increases from ½ D to 2 D; and but little further decrease occurs beyond R = 5 D; but, from very careful and elaborate experiments on city water mains, from 12 to 30 ins diameter, in Detroit, Mich.,† the investigators conclude that a line of pipe with a curve of *short* radius R (down to a limit of R = 2½ D) causes *less* resistance than does a line of equal length and equal total angle A, with a curve of longer radius R. Their results were approximately as follows, where

H = resistance due to a section of 80 diameters in length, with a curve of A = 90° at mid-length,
h = resistance in a tangent of length = 80 diameters.

If R ÷ D =	1	2	2.5	3	4	5	10	15	20	25
then H ÷ h =	1.35	1.14	1.13	1.14	1.18	1.24	1.50	1.66	1.80	1.93

They found also that the **loss of head,** due to a curve, occurs **not only in the curve itself,** but that head continues to be lost in the following tangent, for some distance **down stream** from the curve.

Their experiments led to the inference that even very **slight deflections, A,** in the line, **cause material losses of head,** and that care in securing a straight alignment is therefore highly advisable. For **bends,** see next page.

* Der Ingenieur, pp. 444, 445.
† Paper by Gardner S. Williams, Clarence W. Hubbell, and George H. Fenkell, Transactions, American Society of Civil Engineers, Vol. XLVII, April, 1902.

For abrupt angles, Fig. 4, Weisbach gives: Resistance, in feet of head =

$$h = c\,\frac{v^2}{2\,g} = (0.95 \sin^2 \tfrac{1}{2}\,A + 2.05 \sin^4 \tfrac{1}{2}\,A)\,\frac{v^2}{2\,g}$$

If $\frac{1}{2}$ A =	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
then c =	0.03	0.14	0.36	0.74	0.98	1.26	1.56	1.86	2.16	2.43

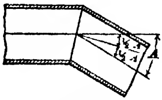


Fig. 4.

In addition to the resistance offered to flow, curves and bends involve additional labor and expense in manufacture and in laying; and vertical bends and curves lead to the formation of pockets of sediment at the feet of slopes, and of air cushions at their summits.

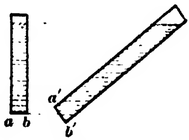


Fig. 5.

Art. 6. Although, in Fig. 5, the static pressures upon the equal bases, $a\,b$ and $a'\,b'$, of the two pipes are equal (see Hydrostatics, Art. 1); yet, in order to pump water through either pipe, at a given velocity, an *additional* force is required, in order to overcome resistances to flow, and these resistances and the additional force required in order to overcome them, will be greater in the longer than in the shorter pipe.

Art. 7. Flow through orifices. Theoretically the velocity, v, of a fluid, flowing through a small orifice in the side or bottom of a very large vessel, is equal to that acquired by a body falling freely in vacuo through a height equal to the head, h, or depth, measured vertically from the level surface of the fluid in the vessel, to the center of gravity of the orifice; or,

$$v = \sqrt{2\,g\,h} = \sqrt{64.4\,h} = 8.03\,\sqrt{h};$$

and

$$h = \frac{v^2}{2\,g} = \frac{v^2}{64.4} = 0.0155\,v^2.$$

This law applies equally to all fluids. Thus, theoretically, mercury, water, air, etc., all flow with equal velocities from a given orifice under a given head.

For deviations in practice from this theoretical law, see Art. 9, etc.

Table 10.

Velocities theoretically due to given heads.

Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.
Feet	Ft per sec	Feet	Ft per sec	Feet	Ft per sec	Feet	Ft per sec	Feet	Ft per sec	Feet	Ft per sec	Feet.	Ft per sec
.005	.57	.29	4.32	.77	7.04	1.50	9.83	7	21.2	28	42.5	76	69.9
.010	.80	.30	4.39	.78	7.09	1.52	9.90	.2	21.5	29	43.2	77	70.4
.015	.98	.31	4.47	.79	7.13	1.54	9.96	.4	21.8	30	43.9	78	70.9
.020	1.13	.32	4.54	.80	7.18	1.56	10.0	.6	22.1	31	44.7	79	71.3
.025	1.27	.33	4.61	.81	7.22	1.58	10.1	.8	22.4	32	45.4	80	71.8
.030	1.39	.34	4.68	.82	7.26	1.60	10.2	8.	22.7	33	46.1	81	72.2
.035	1.50	.35	4.75	.83	7.31	1.65	10.3	.2	23.0	34	46.7	82	72.6
.040	1.60	.36	4.81	.84	7.35	1.70	10.5	.4	23.3	35	47.4	83	73.1
.045	1.70	.37	4.87	.85	7.40	1.75	10.6	.6	23.5	36	48.1	84	73.5
.050	1.79	.38	4.94	.86	7.44	1.80	10.8	.8	23.8	37	48.8	85	74.0
.055	1.88	.39	5.01	.87	7.48	1.85	10.9	9	24.1	38	49.5	86	74.4
.060	1.97	.40	5.07	.88	7.53	1.90	11.1	.2	24.3	39	50.1	87	74.8
.065	2.04	.41	5.14	.89	7.57	1.95	11.2	.4	24.6	40	50.7	88	75.3
.070	2.12	.42	5.20	.90	7.61	2	11.4	.6	24.8	41	51.3	89	75.7
.075	2.20	.43	5.26	.91	7.65	2.1	11.7	.8	25.1	42	52.0	90	76.1
.080	2.27	.44	5.32	.92	7.70	2.2	11.9	10	25.4	43	52.6	91	76.5
.085	2.34	.45	5.38	.93	7.74	2.3	12.2	.5	26.0	44	53.2	92	76.9
.090	2.41	.46	5.44	.94	7.78	2.4	12.4	11	26.6	45	53.8	93	77.4
.095	2.47	.47	5.50	.95	7.82	2.5	12.6	.5	27.2	46	54.4	94	77.8
.100	2.54	.48	5.56	.96	7.86	2.6	12.9	12.	27.8	47	55.0	95	78.2
.105	2.60	.49	5.62	.97	7.90	2.7	13.2	.5	28.4	48	55.6	96	78.6
.110	2.66	.50	5.67	.98	7.94	2.8	13.4	13.	28.9	49	56.2	97	79.0
.115	2.72	.51	5.73	.99	7.98	2.9	13.7	.5	29.5	50	56.7	98	79.4
.120	2.78	.52	5.79	1 Ft.	8.03	3	13.9	14	30.0	51	57.3	99	79.8
.125	2.84	.53	5.85	1.02	8.10	3.1	14.1	.5	30.5	52	57.8	100	80.3
.130	2.89	.54	5.90	1.04	8.18	3.2	14.3	15.	31.1	53	58.4	125	89.7
.135	2.95	.55	5.95	1.06	8.26	3.3	14.5	.5	31.6	54	59.0	150	98.3
.140	3.00	.56	6.00	1.08	8.34	3.4	14.8	16.	32.1	55	59.5	175	106
.145	3.05	.57	6.06	1.10	8.41	3.5	15.	.5	32.6	56	60.0	200	114
.150	3.11	.58	6.11	1.12	8.49	3.6	15.2	17	33.1	57	60.6	225	120
.155	3.16	.59	6.17	1.14	8.57	3.7	15.4	.5	33.6	58	61.1	250	126
.160	3.21	.60	6.22	1.16	8.64	3.8	15.6	18	34.0	59	61.6	275	133
.165	3.26	.61	6.28	1.18	8.72	3.9	15.8	.5	34.5	60	62.1	300	139
.170	3.31	.62	6.32	1.20	8.79	4.	16.0	19	35.0	61	62.7	350	150
.175	3.36	.63	6.37	1.22	8.87	.2	16.4	.5	35.4	62	63.2	400	160
.180	3.40	.64	6.42	1.24	8.94	.4	16.8	20	35.9	63	63.7	450	170
.185	3.45	.65	6.47	1.26	9.01	.6	17.2	.5	36.3	64	64.2	500	179
.190	3.50	.66	6.52	1.28	9.08	.8	17.6	21.	36.8	65	64.7	550	188
.195	3.55	.67	6.57	1.30	9.15	5	17.9	.5	37.2	66	65.2	600	197
.200	3.59	.68	6.61	1.32	9.21	.2	18.3	22.	37.6	67	65.7	700	212
.21	3.68	.69	6.66	1.34	9.29	.4	18.7	.5	38.1	68	66.2	800	227
.22	3.76	.70	6.71	1.36	9.36	.6	19.	23.	38.5	69	66.7	900	241
.23	3.85	.71	6.76	1.38	9.43	.8	19.3	.5	38.9	70	67.1	1000	254
.24	3.93	.72	6.81	1.40	9.49	6	19.7	24.	39.3	71	67.6		
.25	4.01	.73	6.86	1.42	9.57	.2	20.0	.5	39.7	72	68.1		
.26	4.09	.74	6.91	1.44	9.63	.4	20.3	25	40.1	73	68.5		
.27	4.17	.75	6.95	1.46	9.70	.6	20.6	26	40.9	74	69.0		
.28	4.25	.76	6.99	1.48	9.77	.8	20.9	27	41.7	75	69.5		

Art 8. Flow, into air, thru short tubes, Fig. 6. Length, L, = $n'\,i$ or $n\,e$, of tube, $\nless$ 2.5 to 3 times its *least* transverse dimension, d. Head, h, constant and > half hight of tube.

Let v = mean velocity thru tube;
g = acceleration of gravity
= 32.2 ft per sec per sec; ($\sqrt{2\,g} = 8$);
= 9.81 meters per sec per sec; ($\sqrt{2\,g} = 4.43$).
h = head, $n\,s$, from grav center of tube cross section to water surface;
L = length, $n'\,i$ or $n\,e$, of tube;
d = least transverse dimension of tube;
a = tube cross section area,
q = discharge rate thru tube;
c = an experimental coefficient.

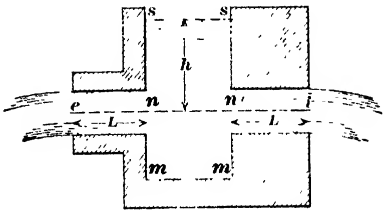


Fig. 6.

Then, if the flow fills the tube ("full flow" or "full bore"), we have.

$v = c\sqrt{2\,g\,h}$; $q = v\,a = c\,a\sqrt{2\,g\,h}$.

L/d	< 1.5	2.5 to 3	4	6	10	15	20	40	60	100
c, approx	0.61	0.81	0.80	0.76	0.74	0.71	0.69	0.62	0.57	0.48

If the outflowing stream fails to fill the tube cross section, the coeff, c, is greatly reduced. This may be obviated by temporarily closing the outlet from the tube, thus backing up the water and compelling it to fill the tube. Then, when the outlet is opened, the air pressure, on the outer end of the outflowing stream, may be expected to keep the stream in contact with the inner tube surface. The sides of the tube should be free from greasiness.

If $s\,m$ be a channel cross section, the vel and disch thru the short tube, $n'\,i$ or $n\,e$, are **not affected by the flow** of the water in the channel, $s\,m$, provided the head, $h = n\,s$, be maintained constant.

The flow may be increased nearly to the theoretical value ($v = \sqrt{2\,g\,h}$, or $c = 1$) by neatly **rounding off the edges** of the entrance end or mouth of the tube, as in Fig. 7, which represents, half-size, one with which Weisbach obtained $c = 0.975$ when $h = 10$ ft, and $c = 0.958$ when $h = 1$ ft

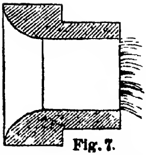


Fig. 7.

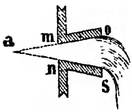


Fig. 8.

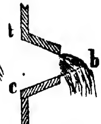


Fig. 9.

As much as .92 to .94 may be obtained by widening the opening, *m n*, toward its outer mouth, *o s*, Fig, 8, making the divergence, or angle *a*, about 5°, or by widening it toward its inner mouth, as at *t c*, Fig 9, but increasing the angle of divergence, at *b*, to from 11° to 16°. In all cases, we consider the small end as being the opening whose area must be multiplied by the vel to get the discharge.

In some experiments made with large pyramidal wooden troughs 9.5 ft long, with an inner mouth of 3.2 × 2.4 ft, and a discharging one of .62 × .44 ft; and under a head of 9⅓ feet, the discharge was .98 of the theoretical one, due to the smaller end.

Rem 5. With an adjutage shaped as in Fig 10, about 9 ins long; diams, *m n* = 1 inch, and *b c* = 1.8 ins, Venturi and Eytelwein obtained a discharge = $1.55\ a\ \sqrt{2\ g\ h}$; where *a* = the smaller cross section area at *m n*, and *h* = distance from to free water surface.

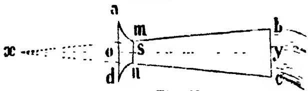


Fig. 10.

See the Venturi Meter, Art 4 *a*.

Art. 9. On the disch of water through openings in thin vert partitions, with plane or flat faces, *e e*, or *n n*, Fig 11.* If the face *e e*, or *n n*, instead of being plane, and vert, should be curved, or inclining in diff directions toward the opening then the disch will be altered When water flows from a reservoir, Fig 11, through a vert plane plate or partition *n n*, which is not thicker than about the least transverse dimension of the opening, whether that dimension be its breadth, or its height *o o*, † or when, if the partition *e e* itself is much thicker, we give the opening the shape shown at *b*, (which evidently amounts to the same thing) then the effluent stream will not pass out with a *full flow* as in Fig 6 but will assume the shape shown in Fig 11, forming, just outside of the opening, what is called the *vena contracta*, or *contracted vein*. In order that this contraction may take place to its fullest extent, or become *complete*, the inner sharp edges of the opening must not approach either the surf of the water, or the bottom or sides of the reservoir, nearer than about 1⅓ times the least transverse dimension of the opening. The contracted vein occurs at a dist of about half the smallest dimension of the orifice, from the orifice itself. In a circular orifice, at about half the diam dist: and ordinarily its area is about .62 or nearly ⅝ that of the orifice itself. At this point the actual mean *vel* of the stream is very nearly (about .97) the theoretical vel given by Table 10, and hence the actual *dischs* are but .62, or nearly ⅝ of the theoretical ones.

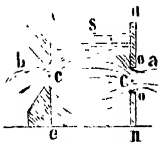


Fig. 11.

Case 1. **To find the actual disch into air,† through either a circular or rectilinear§ opening in a thin vert plane parti-**

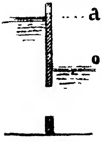


Fig. 12.

* We believe that these rules for thin plate are also sufficiently approximate for most practical purposes, if the opening be in the *bottom* of the reservoir; or in an *inclined*, instead of a *vert* side.

† When the side of a reservoir, or the edge of a plank, &c. over which water flows, has no greater thickness than this, the water is said to flow through, or over, **thin plate,** or **thin partition.**

‡ Should the disch take place *under water*, as in Fig 12, *both surf-levels remaining constant*, then the head to be used is the *vert* diff *a o*, of the two levels After making the calculation with this head, we should, according to Weisbach, deduct the $\frac{1}{75}$ part; inasmuch as he states that the disch is that much less when under water, than when it takes place freely into the air. Other experimenters, however, assert that it is precisely the same in both cases.

§ If the shape of the opening is oval, triangular, or irregular, the head must be measured vert from its cen of grav.

tion, when the contraction is complete; and when the surf. level, *s*, remains constantly at the same height: water being supplied to the reservoir as fast as it runs out at the opening.*

Rule 1. When the head, measured vert from the center (or rather from the cen of grav) c, of the opening, to the surf level *s* of the reservoir, is not less than 1 ft, nor more than 10 ft; and when the least transverse dimension of the opening is not less than an inch, mult the theoretical vel in ft per sec due to the head. (Table 10,) by the coefficient of disch .62. The prod will be the actual mean vel of the water through the opening. Mult this vel by the area of the opening in sq ft; the prod will be the disch in cub ft per sec, approximately.

When the head is greater than 10 ft, use .6, instead of .62

Rule 2. Find the sq rt of the head in ft. Mult this sq rt by 5, the prod will be the vel in ft per sec; which mult by the area as before for the disch

Ex. What will be the disch through an opening in complete contraction, whose dimensions are 6 ins, or .5 ft vert, and 4 ft hor; the vert head above the cen of grav of the opening being constantly 6 feet?

By Rule 1. The theoretical vel (Table 10, corresponding to 6 ft head, is 19.7 ft per sec And 19.7 × .62 = 12.214 ft, the reqd vel. Again, the area of the opening = .5 × 4 = 2 sq ft, and 12.214 × 2 = 24.428 cub ft per sec, the disch

By Rule 2. The sq rt of 6 = 2.45; and 2.45 × 5 = 12.25 ft per sec, the reqd vel; and 12.25 × 2 = 24.5 cub ft per sec, the disch.

Both very approx even if the orifice reaches to the surface of the issuing water.

Rem. 1. The coef .62 is a mean of results of many old experimenters In 1874 Genl. T. G. Ellis of Massachusetts conducted an elaborate series (Trans Am Soc C E, Feb 1876) on a large scale, the general results of which, within less than 1 per ct, are given in the following table. See also Rem 3. The sharp edged orifices were in iron plates .25 to .5 inch thick

Orifice.	Head above Center.	Coef.
2 ft sq.	2 to 3.5 ft	.60 to .61
2 " long, 1 ft high	1.8 to 11.5 "	.60 to .61
2 " long, .5 high	1.4 to 17.0 "	.61 to .60
2 " diam.	1.8 to 9.6	.59 to .61

Rem. 2. Extreme care is reqd to obtain correct results, but for many purposes of the engineer an error of 5 to 10 per ct is unimportant

It will rarely happen that greater accuracy is required than may be obtained by the foregoing rules; but when such does occur, aid may be derived from the following **table deduced from the experiments of Lesbros and Poncelet**, on openings 8 ins wide, of diff heights, and with diff heads Use that coeff in the table which applies to the case, instead of the .62 of Rule 1 In some of the cases in this table, the upper edge of the opening is nearer the surf level of the reservoir than 1½ times its least transverse dimension

TABLE 12. Coefficients for rectangular openings in thin vertical partitions in full contraction.*

Head above cen. of grav. of opening in Feet.	Head above cen. of grav. of opening in Inches.	The breadth in all the openings = 8 inches. HEIGHT OF OPENING Ins. 8	Ins. 6	Ins. 4	Ins 3	Ins 2	Ins. 1	Ins .4
.033	.4	......	......	......	......	......	......	.70
.0666	.8	......	......	......	......	......	.65	.69
.0833	1	......	......	......	......	......	.64	.68
.125	1½	......	......	......	......	.61	.64	.68
.1666	2	......	......	......	.60	.62	.64	.68
.2083	2½	......	......	.59	.61	.62	.64	.67
.250	3	......	......	.60	.61	.62	.64	.67
.2917	3½	......	.57	.60	.61	.62	.64	.66
.3333	4	......	.58	.60	.61	.63	.64	.66
.3750	4½	.56	.59	.60	.61	.63	.64	.66
.4167	5	.57	.59	.61	.62	.63	.64	.66
.6666	8	.59	.60	.61	.62	.63	.64	.65
1	12	.60	.60	.61	.62	.63	.63	.64
3	36	.60	.60	.61	.62	.62	.63	.63
5	60	.60	.60	.61	.61	.62	.62	.62
10	120	.60	.60	.60	.60	.60	.61	.61

Rem. 3. Careful experiments on openings 4¼ ft wide, and 18 ins high, under heads of from 6 to 15 ft, gave coeff = 0.62, correct within about 3 per cent, although the thickness of the partition varied, on its diff sides from 12 to 20 ins.

Rem. 4. When two or more contiguous openings discharge at the same time from the same reservoir, they appear to disch the same as, or a little less than, a single orifice of their combined size.

Case 2. **The discharge through thin vert partitions in complete contraction, when the surface-level, *m*, Fig 13, descends as the water flows out into the air.** In this case, if the reservoir is prismatic, that is, if its hor sections are everywhere equal; and if no water is flowing into the reservoir, to supply the place of that which flows out, then, to find the time reqd to disch the reservoir.

RULE. Inasmuch as the time in which such a reservoir entirely discharges itself, is twice that in which the same quantity would flow out under a constant head, as in Case 1, therefore, calculate the disch in cub ft per sec by Rule 1, Art 9, div the number of cub ft contained in the reservoir, above the level *g* of the bottom of the opening, Fig 13, by this disch, the quot will be the number of sec in which a volume equal to that in the reservoir, to the depth *g*, would run out in Case 1, of a *constant* head. And *twice* this number will be the seconds reqd to empty the reservoir in Case 2, of a *varying* head.

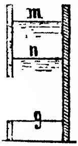


Fig. 13.

EX. If it should be reqd to find the time in which such a prismatic reservoir would *partly* empty itself, as, for instance, from *m* to *n*, Fig 13, first calculate, by the above rule, the secs necessary to empty it if it had only been filled to *n*; and afterward calculate as if it had been filled to *m*. The diff between the two times will evidently be the time reqd to empty it from *m* to *n*. If the opening is not in complete contraction, see Arts 11, &c.

If the disch is into a lower reservoir, whose surf-level remains constant, proceed in the same manner; only use the diff of level of the two surfs as the head, and afterward (according to Weisbach) increase the time $\frac{1}{75}$ part.

Art. 10. Disch from a reservoir R, Fig 14, the surf-level, *s*, of which remains constantly at the same height; through an opening, *o*, in thin vert partition; and in complete contraction; but entirely under water; and into a prismatic reservoir, *m*.

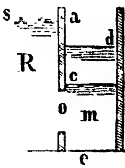


Fig. 14.

Seconds required to discharge a quantity *c d a*, *the level c remaining constant*:

$$= \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft}}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required *to raise level in m from c to a*:

$$= \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required *to raise level in m from c to any other level, d*:

$$= \frac{\left(\sqrt{a\,c} - \sqrt{a\,d} \text{ both in ft}\right) \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

REM. 1. **If it should be reqd to find the time of filling *m*, from its bottom *e*, up to *d*,** we may do so very approximately by calculating by the first rule in Art 9, the time reqd from *e* to the center of the opening *o*, as if all that portion of the disch took place into air; and afterward, from the center of the opening to *d*, by the rule just given. This case is similar to that of filling a lock from the canal reach above, in which the surf-level may be considered constant.

REM. 2. **If the bottom of the opening *o*, should coincide with the bottom of the reservoir,** then the coeff will become greater than .62. See Art 11, for obtaining coeffs for imperfect contraction.

REM. 3. **If the opening, instead of being in complete contraction,** is of any of the shapes Figs 6 to 9, then a reference to Art 8 will show what coeff must be substituted for .62.

Case 3. **Disch from one prismatic reservoir, Fig 15, W, into another, X, of any comparative sizes whatever, through an opening *o*, in a plane thin vert partition, and in complete contraction; when the water rises in X, while it falls in W.**

To find the time in which the water, flowing from W into X, through o, will fall through the dist a s, so as to stand at the same level s c, in both reservoirs.

In this case, the water reqd to fill X from *e* to *d*, (*d* being the bottom of the opening *o*,) flows out into the air; and the time necessary for it to do so, must be calculated separately from that reqd above *d*, which flows into water.

RULE. First from *e* to *d*. Find the hor area of each reservoir, in sq ft. Mult the hor area of X, by the vert depth *d e* in ft, for the cub ft contained in that portion. Div these cub ft by the hor area of W. The quot will be the dist *a m*, in feet, through which the water in W must descend, in order to fill X to *d*.

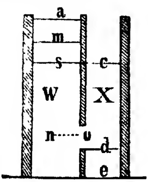


Fig. 15.

Seconds required to lower from *a* to *m*, and raise from *e* to *d*:

$$= \frac{\text{Twice the hor area of W in sq ft} \times \left(\sqrt{\text{head } a\,n \text{ in ft}} - \sqrt{\text{head } m\,n \text{ in ft}}\right)}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

$$\textbf{Seconds required}\ \text{to lower from } m \text{ to } s\text{, and raise from } d \text{ to } c\text{. (Very approx)} = \frac{\begin{matrix}\text{Hor area of}\\ \text{X in sq ft}\end{matrix} \times \begin{matrix}\text{twice the hor area}\\ \text{of W in sq ft}\end{matrix} \times \sqrt{\begin{matrix}\text{head } m\ s\\ \text{in ft}\end{matrix}}}{\begin{matrix}\text{Area of}\\ \text{opening}\\ o \text{ in sq ft}\end{matrix} \times \left(\begin{matrix}\text{hor area}\\ \text{of W}\\ \text{in sq ft}\end{matrix} + \begin{matrix}\text{hor area}\\ \text{of X}\\ \text{in sq ft}\end{matrix}\right) \times .62 \times 8.03}$$

Ex Let the hor area of W be 100 sq ft; and that of X, 60 sq ft Let *a n* be 20 ft, and *m n* 16 ft, and the area of the opening *o*, 3 sq ft. In what time will the water descend from *a* to *s*, and rise from *e* to *c*?

Inasmuch as the method of finding the time for filling from *e* to *d*, by the water falling from *a* to *m*, requires no further exemplification, we will confine ourselves to the *additional* time necessary for filling from *d* to *c*, by the water falling from *m* to *s*. To find this, we have, the sq rt of the head $m\ n = 4$ ft, and the sum of the 2 areas $= 100 + 60 = 160$. Hence, $\frac{4 \times 100 \times 60 \times 2}{160 \times 8.03 \times 3 \times .62} = \frac{48000}{2389.73} =$ 20 1 sec, the additional time reqd, very approximately.

NOTE 1. **If the opening, as *d*, Fig 16, reaches to the very bottom of the reservoirs,** we may consider all the water flowing from R into T, as flowing into water. Therefore, using the head *a m*, we at once calculate the time necessary for the water in the two reservoirs to arrive at the same level *s c*, by the last process of the preceding rule, or, in other words, by the process given in the preceding example. But in this case it must be borne in mind that the opening *o* is no longer in *complete* contraction, inasmuch as the contraction along its lower edge is suppressed.

The disch will consequently be somewhat increased, and a coeff greater than .62 becomes necessary. The method of finding this, is given in the following Case 4. A reference to Art 8 will give the coeff in case the opening is shaped as Figs 6 to 9.

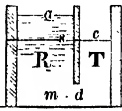


Fig 16.

Art. 11. *Case* 4. The discharge through openings in plane thin vert partitions; but in incomplete contraction.

The opening may be such that contraction will take place along one portion of its perimeter, or at the top of the opening *a*, Fig 17, while it is suppressed on another portion; as at the bottom and two ends of the opening *a*; where suppression is caused by the addition of *short* side and bottom pieces *c, c, c*. Or it may be caused by the bottom, or ends, or both, coinciding with the bottom and sides of the reservoir. In such cases the disch will be greater than in those of complete contraction; but less than in those of full flow, inasmuch as the opening now partakes somewhat of the character of the *short* tubes of Art 8; and the coeff will rise from .62, or that which *usually* pertains to openings in full contraction; and will approach .8, or that of full flow, in proportion to the extent of perimeter along which contraction is suppressed, or even to .9 or .98 by the use of such openings as are shown by Figs 7, 8, 9.

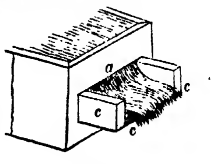


Fig. 17.

To find approximately a new coeff of disch; and the disch itself, in cases of incomplete contraction.

RULE. First find by the foregoing rules, what would be the disch in the particular case that may be under consideration, supposing the contraction to be complete. Then div that portion of the perimeter of the opening on which contraction is suppressed, by the entire perimeter. Mult the quot by the dec .152 if the opening is rectangular, or by .128 if circular To the prod add unity, or 1. Call the sum, *g*. Then say, as unity, or 1, is to *g*, so is the coeff for complete contraction in ordinary cases (usually .62) to the reqd new coeff Finally, repeat the original calculation, only substituting this new coeff in the place of .62.

According to this rule, we have the following coeff of discharge for rectangular openings within probably 3 or 4 per cent, when contraction is not suppressed on more than ¾ of the perimeter. The theoretical discharge multiplied by the corresponding coeff will give the actual discharge. When the contraction is carried farther, the coeff becomes extremely irregular, and is probably indeterminable.

For complete contraction (ordinarily)	.62
When contraction is suppressed on ¼ *the perimeter*	.64
" " " " " ½ " "	.67
" " " " " ¾ " "	.69
" " " " *entirely around the orifice*	.80

Intermediate ones can be estimated nearly enough, mentally

Art. 12. Fig. 18 **Contraction suppressed at the bottom and at both vert sides of the opening o, in consequence of their coinciding with the bottom and sides of the reservoir.** Front of reservoir sloped, as at *f*. Use Rule 1, Case 1, Art. 9; but, according to Poncelet, the coefficient, instead of 0.62, as in Rule 1, should be 0.80 if *f* slopes 45°, or 1 : 1; and coeff = 0.74 if *f* slopes 63° 30′, or 1 hor : 2 vert.

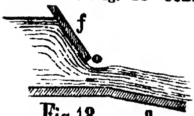


Fig 18

Art. 13. To find, approximately, the time reqd for the emptying of a pond, or any other reservoir, as Fig 19, which is not of a prismatic shape; through an opening, *n*, near the bottom.

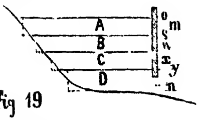


Fig 19

RULE. First ascertain the exact shape and dimensions of the reservoir. If large, and irregular, it must be carefully surveyed, and soundings taken, and figured upon a correct plan and cross-sections. Next, consider the entire body of water to be divided into a series of thin hor strata. A, B, C, D, the top line of the lower one being at least a few ins above the top of the opening *n*. It is not necessary that these strata should be of equal thickness; although the thinner they are, the more correct will the result be. The depth of the lower one, D, will vary to some extent with the height of the opening; those next above it should not exceed about a foot in thickness, until a depth of 6 or 8 feet is reached; then they may conveniently, and with sufficient accuracy, be increased to about 2 ft, for 6 or 8 ft more; and so on; becoming thicker as they approach the surf. By aid of the drawings, calculate the content of each stratum in cub ft. Now, since the strata are thin, we may, without serious error, assume each of them to be prismatic, as shown by the dotted lines; and may assume that the head under which each stratum (except the lowest) empties itself through *n*, is equal to the vert height from the center of the opening to the center of the stratum. Thus, *m n* will be the head of A; *w n*, the head of B; *x n*, the head of C. Then, for the stratum A, by Rule 1, Art 9, (only using *m n* as the head instead of *o n*,) and instead of the coeff .62 of that rule (which can only be used if *n* is in complete contraction) using .64, or whatever other coeff near the end of Art 11 applies to the case, calculate the disch in cub ft per sec. Div the content of the stratum A by this disch, and the quot will be the number of sec reqd for discharging A. Using the head *w n*, proceed in precisely the same way with the stratum B; and using the head *x n*, do the same with C. Finally, for the lower stratum D, find by Rule 1, Art 9, (with the same caution as before respecting the proper coeff.) in what time it would empty itself under a *constant* head equal to *y n*, measured from its *surf* to the center of the opening. *Double* this time will be that reqd to empty itself in the case before us, under its *varying* head. Finally, add together all these separate times; and their sum will be the entire time reqd to empty the pond, or reservoir, approximately enough for practical purposes.

(*f*) **The length L of the crest,** Figs. 21 to 22 *a*, should be at least three times the head H, in order to reduce the effect of friction of the sides *s s* and that of end contractions where such exist. **The height** *p*, Fig. 20, of the vertical back *a b* in contact with the water should be not less than twice the head H; for, in order to reduce the velocity of approach (see Art. 14 *u*), the cross-section of **the channel** leading to the weir should be large in proportion to that of the stream *a c*. The cross-section of the channel of approach should be as *regular* as possible.

(*g*) The weir should be stoutly built, as **vibrations** of the structure may seriously modify the discharge

(*h*) *Theoretically*, **the head** is the vertical distance H', Fig. 24, from the crest *a* to a point *o'* where the water is *perfectly still*, and the surface therefore *horizontal*. But in fact the head is usually measured from the crest *a* to a point *o* a few feet back from the weir where the water is only *comparatively* still, the velocity of approach being perceptible (See Art. 14, *u*.) The difference between the head H actually measured and the head H' to *still* water is usually very slight. It is greatly exaggerated in the figure.

The correct **measurement of the head** is a delicate matter, the discharge being increased or diminished about $1\frac{1}{2}$ per cent. by 1 per cent of increase or diminution of the head Waves or ripples and other disturbances of the surface, and capillary attraction, are the chief sources of error.

(*i*) To avoid the latter difficulty, the **hook-gauge** is used for measuring the height of the water surface in important cases. This consists of a long graduated rod, provided at its *foot* with an upturned hook or point, and sliding vertically (by means of a screw motion) in a fixed support, to which is attached a vernier indicating on the scale the height of the point. The sliding rod is first run down until the point is well below the surface, and then gradually raised by means of the screw until the point just reaches the surface, which is indicated by the first appearance of a "pimple" in the water surface immediately over the hook Under favorable circumstances a good hook-gauge may be read within from .0002 to .0005 foot

(*j*) To avoid inaccuracies due to the **disturbance of the surface** by the current, by wind, etc., the level is sometimes taken (with the hook-gauge or otherwise) in a side chamber which communicates with the main channel of approach. The surface in the chamber maintains the same level as that in the channel itself, but is comparatively free from disturbance Or a bucket communicating with the channel by means of a pipe can be made to serve in the same way Either may of course be sheltered from the wind **Caution.** Messrs Fteley and Stearns found that when the bucket or chamber communicated with the water *near the bottom and close behind the weir*, the head thus obtained was generally somewhat greater than that found by measurement near the surface and 6 feet back from the weir. But Mr. Francis found the difference scarcely perceptible.

(*k*) Great care is necessary in **adjusting the hook-gauge for the height of the crest;** for any error in this affects all the subsequent experiments. The hook is usually adjusted to the height of the surface when the latter just reaches the level of the crest; but this method is rendered inaccurate by capillary attraction at the crest A more accurate method is to have, in addition to the hook-gauge, a stout *fixed* hook, pointing upward, the level of which, relatively to that of the crest, may be ascertained by means of an engineer's level, holding the rod on the crest and also on the point of the fixed hook. The water surface is then allowed to fall slowly until a "pimple" just appears over the fixed hook. It is then kept at that level and the hook-gauge adjusted accordingly. Or if the gauge-hook is a stout one, the levelling rod may be set at once upon its point without having recourse to a fixed hook. It is better to adjust the hook-gauge so as to read *zero* for the crest level, which is thus made the datum; for the reading of the hook-gauge for the water surface then gives the head H at once, and without subtracting the height of the crest.

(1) Formulæ for weir discharge.
Let

Q = the actual discharge over the weir, in cubic feet per second; *
Q' = the theoretical discharge over the weir, in cubic feet per second,
H = **H'** † = the vertical distance or head *a m*, Fig. 24, p. 556, in feet,* measured from the crest *a* to the *horizontal* surface *o'* of *still* water up-stream from the weir;
L = the length *a a* of the weir, in feet,* Figs 21 to 22 *a*;
g = the acceleration of gravity = say 32.2 feet * per second per second;

$$\left.\begin{aligned} c &= \text{coefficient of discharge} = \frac{\text{actual discharge}}{\text{theoretical discharge}} = \frac{Q}{Q'}; \\ m &= \frac{2}{3}c, \\ x &= \frac{2}{3}c\sqrt{2g} = m\sqrt{2g} = \text{say } 5.36\,c = \text{say } 8.025\,m. \end{aligned}\right\}\ddagger$$

Then, for the **theoretical discharge,** we have

$$Q' = \frac{2}{3}\,L\,H\sqrt{2gH}; \S \ldots\ldots \quad (1)$$

and for the **actual discharge,**

$$\left.\begin{aligned} Q &= c\,Q' \\ &= \frac{2}{3}c\,L\,H\sqrt{2gH} \ldots\ldots\ldots\ldots \quad (2) \\ &= m\,L\,H\sqrt{2gH} \ldots\ldots\ldots\ldots \quad (3) \\ &= x\,L\,H\sqrt{H} = x\,L\sqrt{H^3} = x\,L\,H^{\frac{3}{2}} \ldots \quad (4) \end{aligned}\right\}\text{See foot-notes } *\dagger\ddagger$$

For the value of the coefficient (*c*, *m*, or *x* ‡) we have recourse to experiment, measuring the actual discharge and comparing it with the theoretical one, as in the following articles.

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* The formulæ apply equally to any system of measures, as the English, the metric, etc. It is requisite merely that the units, of length, of time, etc., used, be the same throughout. In metric measure, g = 9.81 meters per second per second.

† *For the present* we suppose the head to be measured to *still* water, so that H = H'. When this is not the case, see "Velocity of Approach," Art. 14 (*a*), etc.

‡ It will be noticed that the formulæ (2), (3) and (4), with their corresponding coefficients, *c*, *m*, and *x*, are really identical, differing only in form. The last is the most convenient in practice, but all are met with in works on hydraulics.

§ When water issues under a head H, from a *horizontal* orifice in the *bottom* of a vessel, the theoretical velocity (Art. 7, is $= \sqrt{2gH}$; and this may be regarded as true also for *vertical* orifices in the *sides* of vessels, provided the head H to the center of gravity of the orifice is at least two or three times the vertical dimension of the orifice; for in both cases the theoretical velocities through the several parts of the orifice may be taken as *equal*. But when a vertical orifice is nearer to the surface, or when it *reaches* to the surface as in the case of a weir, we must take into consideration the differences in the velocities with which the water issues from points at different depths.

Theoretically, the particles pass the oblique plane *a o'*, Fig. 23, in horizontal lines, with velocities ($= \sqrt{2gh} = 8.025\sqrt{h}$) proportional to the square roots of their several vertical depths *h* (not indicated in fig.) below still water surface at *o'*. Therefore if from *a m* we imagine horizontal lines *a a'*, *d d'*, *v v'*, *c c'*, etc., etc., to be drawn, representing all these velocities to any scale, then the outer ends *a'*, *d'*, *v'*, *c'*, etc., etc., of these lines will form, with *a m* and *a a'*, a parabolic segment *a m c' a'*, the area of which is:

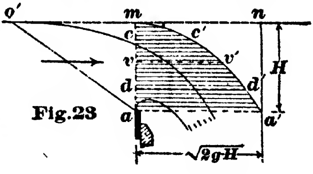

Fig.23

area $= \frac{2}{3}$ area of rectangle *a n* (see Parabola, p. 192) $= \frac{2}{3}\,a\,m \times a\,a' = \frac{2}{3}\,H\sqrt{2gH}$; and this area in square feet, multiplied by the thickness of the escaping sheet of

(*m*) **Mr. James B. Francis*** experimented at "the lower locks," Lowell, Mass., in 1852, with weirs 10 feet long, 5 feet and 2 feet high, under heads from 7 to 19 inches. To apply his results, the following conditions must exist:

The head H, Fig 20, must be between 6 and 24 inches. The height p of the vertical back of the weir above the bottom b of the channel must be at least twice the head H. The crest a must be "in thin partition" (foot-note † p 541), and its length L, Figs. 21 to 22 a, must be at least 3 times the head H. The ends $a\,h$, $a\,h$ must be vertical, and, when there is end contraction, "in thin partition."

When there is end contraction, Mr. Francis first deducts from the actual length L of the weir *one tenth* † *of the head* H for *each* end where contraction occurs. Thus, if n = the number of end contractions (two in Fig. 22),

$$Q = x\left(L - n\,\frac{H}{10}\right) H\,\sqrt{H} = x\left(L - n\,\frac{H}{10}\right) H^{\frac{3}{2}} \ddagger \quad \ldots \quad (5)$$

$$\text{In Fig. 22, } Q = x\left(L - \frac{H}{5}\right) H\,\sqrt{H}.\dagger = x\left(L - \frac{H}{5}\right) H^{\frac{3}{2}}$$

But within the limits specified above, the formula is very approximate *without* correction for end contraction, provided the length L of the weir is at least 10 times the head H; and within 6 per cent. of the truth when L is = 4 H. When there is no end contraction, of course no such correction is required, and the formula remains $Q = x\,L\,H\,\sqrt{H}\,\dagger = x\,L\,H^{\frac{3}{2}}$.

Mr. Francis gives x = 3.33 for *feet*; § or

$$\textbf{Discharge} = 3.33 \times \left(\text{length} - \frac{\text{number of}}{\text{end contractions}} \times \frac{\text{head H}}{10}\right) \times H^{\frac{3}{2}}, \ddagger$$

the mean of his 88 experiments being 3.3318. The least value of x obtained by him was 3.3002, or 1 in 112 less than 3.33; and the greatest was 3.3617, or 1 in 105 more than 3.33. Hence, with x = 3.33, the formula will give the discharge for each of his experiments within 1 per cent. In 67 out of the 88 experiments x ranged between 3.32 and 3.35, and in 53 between 3.32 and 3.34. When x is 3.33, **m** is = 0.415, and **c** is = 0.622

The height of the surface was measured six feet back from the weir by two hook-gauges, one on each side of the channel; and the mean of their readings was used in calculating the coefficient x.

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water, or length L of weir, in feet, gives the theoretical discharge in cubic feet per second. Or

$$Q' = L \times \text{area } a\,m\,c'\,a' = L \times \frac{2}{3} H\,\sqrt{2\,g\,H}.$$

Hence, area $a\,m\,c'\,a'$ in sq. ft. represents the theoretical disch. in cub. ft. per sec. over 1 ft. length of weir, under head H. The theoretical *mean vel.* through the section $a\,o'$ is

$$V = v\,v' = \frac{\text{theoretical discharge } Q'}{\text{area } a\,m} = \frac{2}{3}\,L\,H\,\sqrt{2\,g\,H} \div L\,H = \frac{2}{3}\sqrt{2\,g\,H},$$

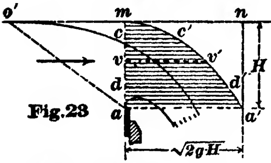


Fig.23

or two thirds of the theoretical horizontal velocity $a\,a'$ of the particles passing immediately over the weir. As in the case of orifices (Art. 9, p. 541), the actual vel. *at the smallest section* of the sheet after passing the weir (corresponding to the "vena contracta") is probably very nearly equal to this theoretical velocity.

* "Lowell Hydraulic Experiments," Van Nostrand, New York, 1883.

† In Messrs. Fteley and Stearns' experiments this figure was not constant at 0.10, but varied between 0.061 and 0.124, generally increasing as the head decreased.

‡ We here suppose the head to be measured to the surface of *still* water, so that H and H' (see Art. 14 *h*, p. 548) are the same. See Velocity of Approach, Art. 14 (*u*).

§ Since 1 meter = 3.2808 ft., the value of x for metric measure corresponding to Mr. Francis' 3.33, is = 3.33 ÷ $\sqrt{3.2808}$ = 3.33 ÷ 1.8113 = 1.838

Table 13.* **Discharge in cubic feet per second for each foot in length of weir** in thin plate and without end contraction, by the Francis formula: **Discharge,** Q/L, per foot of weir length, $= 3.33\ H^{\frac{3}{2}} = 3.33\ H\sqrt{H}$.

Very approximate also when there is end contraction, provided that L is at least = 10 H; and but about 6 per cent. in excess of the truth if L = 4 H. Mr. Francis limits the formula to heads H from 0.5 foot to 2.0 feet, but no serious error will result from using the table for any of the heads given. **For weirs of other lengths** than 1 foot, multiply the tabular discharge by the actual length in feet.

H = head, in ft; **Q/L** = disch in cu ft per sec per weir-lgth ft

H	Q/L	H	Q/L	H	Q/L	H	Q/L	H	Q/L
.01	0.003	.51	1.213	1.01	3.380	1.51	6.179	2.01	9.489
.02	0.009	.52	1.249	1.02	3.430	1.52	6.240	2.02	9.560
.03	0.017	.53	1.285	1.03	3.481	1.53	6.302	2.03	9.631
.04	0.027	.54	1.321	1.04	3.532	1.54	6.364	2.04	9.703
.05	0.037	.55	1.358	1.05	3.583	1.55	6.426	2.05	9.774
.06	0.049	.56	1.395	1.06	3.634	1.56	6.488	2.06	9.846
.07	0.062	.57	1.433	1.07	3.686	1.57	6.551	2.07	9.917
.08	0.075	.58	1.471	1.08	3.737	1.58	6.613	2.08	9.989
.09	0.090	.59	1.509	1.09	3.790	1.59	6.676	2.09	10.062
.10	0.105	.60	1.548	1.10	3.842	1.60	6.739	2.10	10.134
.11	0.121	.61	1.586	1.11	3.894	1.61	6.803	2.11	10.206
.12	0.138	.62	1.626	1.12	3.947	1.62	6.866	2.12	10.279
.13	0.156	.63	1.665	1.13	4.000	1.63	6.930	2.13	10.352
.14	0.174	.64	1.705	1.14	4.053	1.64	6.994	2.14	10.425
.15	0.193	.65	1.745	1.15	4.107	1.65	7.058	2.15	10.498
.16	0.213	.66	1.786	1.16	4.160	1.66	7.122	2.16	10.571
.17	0.233	.67	1.826	1.17	4.214	1.67	7.187	2.17	10.645
.18	0.254	.68	1.867	1.18	4.268	1.68	7.251	2.18	10.718
.19	0.276	.69	1.909	1.19	4.323	1.69	7.316	2.19	10.792
.20	0.298	.70	1.950	1.20	4.377	1.70	7.381	2.20	10.866
.21	0.320	.71	1.992	1.21	4.432	1.71	7.446	2.21	10.940
.22	0.344	.72	2.034	1.22	4.487	1.72	7.512	2.22	11.015
.23	0.367	.73	2.077	1.23	4.543	1.73	7.577	2.23	11.089
.24	0.392	.74	2.120	1.24	4.598	1.74	7.643	2.24	11.164
.25	0.416	.75	2.163	1.25	4.654	1.75	7.709	2.25	11.239
.26	0.441	.76	2.206	1.26	4.710	1.76	7.775	2.26	11.314
.27	0.467	.77	2.250	1.27	4.766	1.77	7.842	2.27	11.389
.28	0.493	.78	2.294	1.28	4.822	1.78	7.908	2.28	11.464
.29	0.520	.79	2.338	1.29	4.879	1.79	7.975	2.29	11.540
.30	0.547	.80	2.383	1.30	4.936	1.80	8.042	2.30	11.615
.31	0.575	.81	2.428	1.31	4.993	1.81	8.109	2.31	11.691
.32	0.603	.82	2.473	1.32	5.050	1.82	8.176	2.32	11.767
.33	0.631	.83	2.518	1.33	5.108	1.83	8.244	2.33	11.843
.34	0.660	.84	2.564	1.34	5.165	1.84	8.311	2.34	11.920
.35	0.690	.85	2.610	1.35	5.223	1.85	8.379	2.35	11.996
.36	0.719	.86	2.656	1.36	5.281	1.86	8.447	2.36	12.073
.37	0.749	.87	2.702	1.37	5.340	1.87	8.515	2.37	12.150
.38	0.780	.88	2.749	1.38	5.398	1.88	8.584	2.38	12.227
.39	0.811	.89	2.796	1.39	5.457	1.89	8.652	2.39	12.304
.40	0.842	.90	2.843	1.40	5.516	1.90	8.721	2.40	12.381
.41	0.874	.91	2.891	1.41	5.575	1.91	8.790	2.41	12.459
.42	0.906	.92	2.939	1.42	5.635	1.92	8.859	2.42	12.536
.43	0.939	.93	2.987	1.43	5.694	1.93	8.929	2.43	12.614
.44	0.972	.94	3.035	1.44	5.754	1.94	8.998	2.44	12.692
.45	1.005	.95	3.083	1.45	5.814	1.95	9.068	2.45	12.770
.46	1.039	.96	3.132	1.46	5.875	1.96	9.138	2.46	12.848
.47	1.073	.97	3.181	1.47	5.935	1.97	9.208	2.47	12.927
.48	1.107	.98	3.231	1.48	5.996	1.98	9.278	2.48	13.005
.49	1.142	.99	3.280	1.49	6.057	1.99	9.348	2.49	13.084
.50	1.177	1.00	3.330	1.50	6.118	2.00	9.419	2.50	13.163

* Table 13 is an extension of the "original" table published in our first edition, 1872. Most of the values now given are taken, by permission, from a table published by Messrs. A. W. Hunking and Frank S Hart, of Lowell, Mass., in May, 1884.

(*n*) **Messrs. A. Fteley and F. P. Stearns** * experimented at Boston, Mass., in 1877–79, upon weirs 5 feet and 19 feet long, 3 feet 2 inches and 6 feet 6¼ inches high, and under heads from 0.8 inch to 19 inches. For weirs in thin partition and without end contraction, with a rectangular and uniform channel of approach and under heads greater than 0.07 foot or 0.84 inch (other conditions as specified in (*b*) and (*d*)), their formula is:

$$\textbf{Discharge, } \left.\begin{aligned} Q &= 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L \\ &= 0.4125\ L\ H\ \sqrt{2\,g\,H} + 0.007\ L \end{aligned}\right\}\dagger \quad \ldots \ (6).$$

In their experiments, the heads were measured six feet back from the weir. The total variation in the values of the coefficients obtained was about 2½ per cent. Compare foot-note § below.

(*o*) **M. Bazin** ‡ experimented at Dijon, France, in 1886–88, with weirs from about 1½ to 6½ feet long, from about 9 inches to 3 feet 9 inches high, and under heads from 2½ to 21 inches. The top of the weir is shown in Fig. 23 *a*. The weirs were placed at different points in a rectangular and regular canal 700 feet long, smoothly lined with cement. Compare foot-note § below.

While Mr. Francis and Messrs. Fteley and Stearns provide for the effect of velocity of approach (see Art. 14 *u* and *v*) by modifying the measured *head* H, M. Bazin includes it in the *coefficient m* in the formula $Q = m\ L\ H\ \sqrt{2\,g\,H}$.

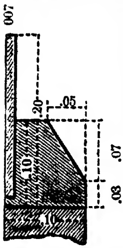


Fig. 23 a.
Measurements in meters.

Let M = the value of *m* for the case where velocity of approach = 0. Then, very approximately:

$$M = 0.405 + \frac{0.003}{H\ \text{meters}} = 0.405 + \frac{3.28 \times 0.003}{H\ \text{feet}} = 0.405 + \frac{0.00984}{H\ \text{feet}}$$

When velocity of approach is to be taken into account:

$$\mathbf{m} = M\left[1 + 0.55\left(\frac{H}{H + p}\right)^2\right] \ldots \quad \ldots (7);$$

where H is the head actually measured to running water, and *p* is the height *a b* of the weir, Fig. 20. H and *p* must of course both be measured in the same unit, as both in meters, or both in feet, etc.

M. Bazin believes that except in the case of very low weirs (which should be avoided) the values of *m* given by formula (7) and in Table 14 calculated from it, will be found within 1 per cent. of the truth for weirs in thin partition and without end contraction, if the conditions of his experiments are exactly reproduced, and provided especially that the sheet of water is not allowed to expand laterally after passing the crest (Art. 14 (*d*)) and that the air has free access to the space *w*, Fig. 20, behind the falling sheet of water.

For heads between 4 inches and 1 foot, M. Bazin gives, as sufficiently approximate,

when there is no velocity of approach, M = 0.425,§

and, to allow for velocity of approach, $\mathbf{m} = 0.425 + 0.21\left(\frac{H}{H + p}\right)^2$.

* Transactions, American Society of Civil Engineers, Jan., Feb. and March, 1883.

† See correction for Velocity of Approach, Art 14 (*u*).

‡ Expériences nouvelles sur l'écoulement en déversoir. Extrait des Annales des Ponts et Chaussées, Oct., 1888. Paris, Vve Ch. Dunod, 1888. Translation by A Marichal and John C. Trautwine, Jr., presented to Engineers' Club of Philadelphia, in 1889, for publication in its Proceedings.

§ This would make $x = 3.41$ (since $x = m\sqrt{2\,g} = 8.025\ m$); whereas Mr. Francis gives $x = 3.33$, which agrees very well with Messrs. Fteley and Stearns, within the limits of Art. 14 (*m*). Yet M. Bazin measured the head 16 feet back from the weir, while the other experimenters measured it only 6 feet back, and the slight increase of head thus obtained by M. Bazin would of itself have made his coefficient *lower* than theirs. Its excess may be largely due to the character of the channel of approach, which in his case was from 50 to 700 feet long, rectangular and regular in cross-section, and smoothly coated with cement. In the other experiments it was much less regular.

Table 14. Values of Bazin's m, in the formula:

$$Q = m\,L\,H\,\sqrt{2\,g\,H}.*$$

The coefficient, $m = \frac{2}{3}\,c = \frac{2}{3}\cdot\frac{Q}{Q'}$, p. 549, being a mere ratio, is independent of the unit of length adopted; but *Bazin's* M and *m* include correction for velocity of approach. They therefore depend upon the unit in which H is expressed. See p. 552.

Using Bazin's *m*, as given in the table below, we have, for the **discharge** per second:

Cubic *meters* = *m* × length L in met. × head H* in met. × $\sqrt{2 \times 9.81\ \text{H in met.}}$;

Cubic *feet* = *m* × length L in feet × head H* in feet × $\sqrt{2 \times 32.2\ \text{H in feet.}}$

It will be noticed that **below the heavy lines** the head H is greater than ½ height *p*, and thus exceeds the limit laid down in (**f**) and (**m**).

Head H, Fig. 24, p. 556. Meters.	approximate feet	approximate inches.	Height, *p*, Fig. 20, of crest of weir above bed of up-stream channel. meters 0.20 / approx. feet 0.656 / inches 7.87	0.30 / 0.984 / 11.81	0.40 / 1.312 / 15.75	0.50 / 1.640 / 19.69	0.60 / 1.969 / 23.62	0.80 / 2.624 / 31.50	1.00 / 3.280 / 39.38	1.50 / 4.920 / 59.07	2.00 / 6.560 / 78.76	Infinite.
			m	m	m	m	m	m	m	m	m	M†
.05	.164	1.97	.458	.454	.451	.450	.449	.449	.449	.448	.448	.4481
.06	.197	2.36	.456	.450	.447	.445	.445	.444	.443	.443	.443	.4427
.07	.230	2.76	.455	.448	.445	.443	.442	.441	.440	.440	.439	.4391
.08	.262	3.15	.456	.447	.443	.441	.440	.438	.438	.437	.437	.4362
.09	.295	3.54	.457	.447	.442	.440	.438	.436	.436	.435	.434	.4340
.10	.328	3.94	.459	.447	.442	.439	.437	.435	.434	.433	.433	.4322
.12	.394	4.72	.462	.448	.442	.438	.436	.433	.432	.430	.430	.4291
.14	.459	5.51	.466	.450	.443	.438	.435	.432	.430	.428	.428	.4267
.16	.525	6.30	.471	.453	.444	.438	.435	.431	.429	.427	.426	.4246
.18	.591	7.09	.475	.456	.445	.439	.435	.431	.428	.426	.425	.4229
.20	.656	7.87	.480	.459	.447	.440	.436	.431	.428	.425	.423	.4215
.22	.722	8.66	.484	.462	.449	.442	.437	.431	.428	.424	.423	.4204
.24	.787	9.45	.488	.465	.452	.444	.438	.432	.428	.424	.422	.4194
.26	.853	10.24	.492	.468	.455	.446	.440	.432	.429	.424	.422	.4187
.28	.919	11.02	.496	.472	.457	.448	.441	.433	.429	.424	.422	.4181
.30	.984	11.81	.500	.475	.460	.450	.443	.434	.430	.424	.421	.4174
.32	1.050	12.60	—	.478	.462	.452	.444	.436	.430	.424	.421	.4168
.34	1.116	13.39	—	.481	.464	.454	.446	.437	.431	.424	.421	.4162
.36	1.181	14.17	—	.483	.467	.456	.448	.438	.432	.424	.421	.4156
.38	1.247	14.96	—	.486	.469	.458	.449	.439	.432	.424	.421	.4150
.40	1.312	15.75	—	.489	.472	.459	.451	.440	.433	.424	.421	.4144
.42	1.378	16.54	—	.491	.474	.461	.452	.441	.434	.425	.421	.4139
.44	1.444	17.32	—	.494	.476	.463	.454	.442	.435	.425	.421	.4134
.46	1.509	18.21	—	.496	.478	.465	.456	.443	.435	.425	.421	.4128
.48	1.575	18.90	—	—	.480	.467	.457	.444	.436	.425	.421	.4122
.50	1.640	19.69	—	—	.482	.468	.459	.445	.437	.426	.421	.4118
.52	1.706	20.47	—	—	.483	.470	.460	.446	.438	.426	.421	.4112
.54	1.772	21.26	—	—	.485	.472	.461	.447	.438	.426	.421	.4107
.56	1.837	22.05	—	—	.487	.473	.463	.448	.439	.427	.421	.4101
.58	1.903	22.83	—	—	.489	.475	.464	.449	.440	.427	.421	.4096
.60	1.969	23.62	—	—	.490	.476	.466	.451	.441	.427	.421	.4092

Owing to the wide range of the head H and of the height *p* in these experiments, we find in them **a wider divergence** in the values of the coefficient than resulted from the earlier investigations Thus, the smallest value of *m* above the heavy lines is 0.4092, or about one nineteenth less than the mean, 0.4325; and the greatest is 0.459, or about one sixteenth more than 0.4325.

* In these experiments, the head H was measured at a point 5 meters (16.4 ft) back from the weir. The correction for velocity of approach is contained in the coefficient *m*.

† M is the value of *m* when there is no velocity of approach; *i. e.*, where the cross-section of the channel of approach is indefinitely great compared with that of the stream of water passing over the weir.

(*p*) From a comparison of a number of experimental data, **the author** deduced the following

Table 15. Approximate values of the coefficient m in the formula:

$$Q = m\,L\,H\,\sqrt{2\,g\,H},$$

for weirs of several different shapes and thicknesses. (Original)

Head, H. Feet.	Head, H. Inches.	Sharp Edge.*	2 Inches thick.	3 ft. thick; smooth; sloping outward and downward, from 1 in 12 to 1 in 18.	3 ft. thick; smooth; and level.
		m	m	m	m
.0833	1	.41	.37	.32	.27
.1666	2	.40	.38	.34	.30
.25	3	.40	.39	.34	.31
.3333	4	.40	.41	.35	.31
.4166	5	.40	.41	.35	.32
.5	6	.39	.41	.35	.33
.5833	7	.39	.41	.35	.32
.6666	8	.39	.41	.34	.31
.8333	10	.38	.40	.34	.31
1.	12	.38	.40	.33	.31
2.	24	.37	.39	.32	.30
3.	36	.37	.39	.32	.30

(*q*) **To find the head H, approximately;** having the discharge Q According to formulæ (3) and (4), Art. 14 (*l*).

$$Q = m\,L\,H\,\sqrt{2\,g\,H} = x\,L\,H\,\sqrt{H} = m\,L\,\sqrt{2\,g}\,\sqrt{H^3}, = x\,L\,\sqrt{H^3},$$

Hence

$$H = \sqrt[3]{\frac{Q^2}{m^2\,L^2\,2\,g}} = \sqrt[3]{\frac{Q^2}{x^2\,L^2}} \quad . \quad . \quad (8)$$

or

$$\textbf{Head, } H, \text{ approximately} = \sqrt[3]{\frac{\text{square of discharge of stream, in cub. ft. per sec.}}{m^2 \times \text{length}^2 \times 64.4}}$$

$$= \sqrt[3]{\frac{\text{sq. of discharge}}{x^2 \times \text{length}^2}}$$

The coefficient *m* or *x* itself varies somewhat with the head; but the formula may be usefully employed as an approximation by taking, for sharp-crested weirs, $m = 0.415$ ($m^2 = 0.172$) or $x = 3.33$ ($x^2 = 11$). For other shapes, see Table 15, above.

(*r*) **Submerged weirs,** Fig 23 *b*, are those in which the surface of the *down*-stream water at *h*, after the construction of the weir, is *higher* than the crest *a*.

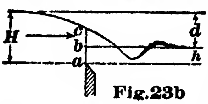


Fig.23b

In a weir discharging freely into the air, as in Fig. 20, Mr. Francis found that with a head of 1 foot the discharge was diminished only about one thousandth part by placing a solid horizontal floor about 6 inches below and in front of the crest of the weir for the water to fall upon. Also, when the head was 10 inches, and the water fell freely through the air into water of considerable depth (as in Fig 20), the quantity discharged was the same whether the surface of the down-stream water was about 3 inches or about 13 inches below the crest *a*.

In experiments by Mr. Francis and by Messrs Fteley and Stearns, with air freely admitted underneath the falling sheet of water just below the crest *a*, the discharge was not appreciably affected by a submergence of h = from 0.017 H to 0.023 H. When air was only partially admitted, the discharge was affected (*increased*) by less than one per cent. while *h* remained less than 0.15 H.

* These values are lower than those given in Art. 14 (*m*) and (*n*), and much lower than those in (*o*).

Dubuat's formula for submerged weirs. Let

H and h = the heads measured vertically from the crest a of the weir to the surface of still water* up-stream and down-stream from the weir, respectively.

$d = H - h$ = their difference = the difference in level between the up-stream and down-stream surfaces of still-water,*

c = coefficient of discharge = $\frac{\text{actual discharge}}{\text{theoretical discharge}}$.

Then

$$Q = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d};\dagger \quad \ldots\ldots\ldots \quad (9) \text{ or:}$$

$$\text{Actual \textbf{discharge} in cub. ft. per sec.} = c \times \frac{\text{length of}}{\text{weir in ft.}} \times 8.025\,\sqrt{d \text{ in ft.}} \times \left(h \text{ in ft.} + \frac{2}{3}\,d \text{ in ft.}\right).$$

(n) **Messrs. Fteley and Stearns**‡ experimented at Boston in 1877 with **submerged weirs** under up-stream heads H from about 4 to 10 inches; and **Mr. Francis**§ at Lowell in 1883 under heads from about 1 foot to 2 feet 4 inches.

From these experiments we deduce the following

Table 16, of approximate values of the **coefficient c** in the formula for discharge over **submerged weirs.**

$$Q = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d}.$$

Deduced from experiments by Fteley and Stearns and by J. B. Francis. In Mr. Francis' experiments, the value of c for a given value of $h \div H$ generally increased as H increased.

	Fteley and Stearns. (H = 0.325 to 0.815 feet.)	J. B. Francis. (H = 1 to 2.32 feet.)
$h \div H$	**c**	**c**
.05	......	.623 to .632
.10	.625 to .635	.620 to .630
.20	.618 to .628	.610 to .625
.30	.600 to .610	.598 to .615
.40	.590 to .600	.586 to .610
.50	.585 to .595	.585 to .607
.60	.583 to .593	.585 to .607
.70	.580 to .590	.585 to .607
.80	.581 to .591	.585 to .607
.90	.590 to .600	......
.95	.610 to .615	......

* For velocity of approach, see Art. 14 (u) etc.

† In deducing this formula, the water that passes over the weir between c and b is assumed to flow as over a weir with its crest at b, and with free discharge into the air, as over the crest a in Fig. 20; and for this portion, by formula (2) in Art. 14 (l), the discharge would be:

$$Q_b = c\,L\,\frac{2}{3}\,d\,\sqrt{2\,g\,d};$$

while the water that passes through the lower portion between b and a is regarded as flowing through a submerged vertical *orifice* whose height is $b\,a = h$, under a head $= d$. For this lower portion, therefore, the discharge would be:

$$Q_a = c\,L\,h\,\sqrt{2\,g\,d}.$$

It is assumed that the coefficient of discharge c is the same for the upper section $c\,b$ as for the lower one $a\,b$. Hence, adding these two discharges together, we obtain, for the entire disharge:

$$Q = Q_b + Q_a = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d}.$$

‡ Transactions, American Society of Civil Engineers, March, 1883, p. 101, etc.

§ Transactions, American Society of Civil Engineers, Sept., 1884, p. 295, etc.

(*t*) **Mr. Clemens Herschel,*** comparing these experiments with some earlier ones by Mr. Francis, gives the following:

Having ascertained the depths H and h of the crest below the still-water levels up-stream and down-stream respectively, divide h by H. Find the quotient, as nearly as may be, in the column headed $h \div H$ in Table 17. Take out the corresponding coefficient a, and multiply it by the up-stream head H.†

The product a H is the head which would cause the given weir to discharge the same quantity freely *into the air*, as in Fig. 20. Find the discharge into air over the given weir with the head a H; and this discharge will be approximately the same as that of the actual *submerged* weir under the up-stream head H and against the down-stream head h; or (H being the actual up-stream head on the *submerged* weir) the discharge is

$$Q = m\,L\,a\,H\,\sqrt{2\,g\,a\,H} = x\,L\,a\,H\,\sqrt{a\,H} \ldots\ldots \quad (10).$$

TABLE 17.

$h \div H$	a	$h \div H$	a	$h \div H$	a
.10	1.000 to 1.010	.45	0.894 to 0.930	.72	0.762 to 0.784
.20	0.975 to 0.995	.50	0.874 to 0.910	.74	0.747 to 0.769
.25	0.960 to 0.984	.55	0.853 to 0.889	.76	0.732 to 0.752
.30	0.945 to 0.973	.60	0.829 to 0.863	.78	0.713 to 0.733
.35	0.928 to 0.960	.65	0.803 to 0.833	.80	0.693 to 0.713
.40	0.912 to 0.946	.70	0.775 to 0.799		

(*u*) **Velocity of approach.** See Fig. 24. It is generally impracticable to measure the head H′ to perfectly still water o' up-stream. The head is usually measured at a point o, from 2 or 3 to 6 or 8 feet or more up-stream from the weir, according to the size of the latter. At such points the velocity is generally appreciable, and the surface therefore a little lower than at o'. Hence a formula using the smaller head H so measured, instead of H′, and coefficients based upon H′, will give too small a discharge. Mr. Francis found that a current of 1 foot per second, or nearly 0.7 mile per hour, at the point o to which the head was measured, increased the discharge but about 2 per cent. when the head was 3 inches; and a current of 6 inches per second increased the discharge about 1 per cent. when the head was 8 inches.

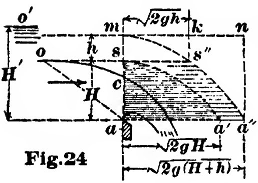


Fig.24

If, however, the velocity of approach is such as to require consideration, proceed as follows: For the approximate mean velocity of approach, we have:

$$v = \frac{\text{approximate discharge}}{\text{area of entire cross section of stream at } o} = \frac{3.33\ L\ H^{\frac{3}{2}}}{\text{area at } o};$$

and, for the head due to this velocity, $h = \dfrac{v^2}{2g}$

Then, for all practical purposes, we may say: $H' = H + h$; or

$$Q = m\,L\,(H + h)\,\sqrt{2\,g\,(H + h)} = x\,L\,(H + h)^{\frac{3}{2}} \ldots\ldots \quad (11)$$

Although, strictly speaking, the difference of level between o' and o is really (as shown in Fig. 24) somewhat *greater* than h, or than $v^2 \div 2\,g$, because some head is lost in friction between o' and o.

* Transactions, American Society of Civil Engineers, May, 1885, pp. 189, etc.

† Mr. Herschel's table, from which ours is condensed, gives a for every 0.01 foot of $h \div H$; but the values of a intermediate of those we have selected may be taken from our table almost exactly by simple proportion.

(*v*) **Messrs. Fteley and Stearns** make $H' = H + 1.5\,h$ for suppressed weirs, and $H' = H + 2.05\,h$ for weirs with complete end contractions, as averages; and Mr. Hamilton Smith, Jr.,* after comparing their experiments with others by Lesbros, Castel and Mr. Francis, gives $H' = H + 1\frac{1}{3}\,h$, and $H' = H + 1.4\,h$, for the two cases respectively.

(*w*) On the other hand, Mr. Francis' formula, as modified for velocity of approach,

$$Q = x\,L\dagger\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right) = m\,L\dagger\sqrt{2g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right)\ddagger \quad . \; . \; . \quad (12),$$

makes the effect of H' *less* than that of $H + h$.

(*x*) **Messrs. A. W. Hunking and Frank S. Hart,** Civil and Hydraulic Engineers, have substituted for the expression $(\sqrt{(H+h)^3} - \sqrt{h^3})$ in formula (12), the equivalent one $K\,\sqrt{H^3}$, in which K is a coefficient deduced from the former expression, and therefore depending upon the relation between H and *h*, or, ultimately, upon that between the cross-section *a s* Fig. 24 at the weir and the entire cross-section of the stream at *o*

Having found the area of cross section at *o*, divide it by $\left(L - n\,\frac{H}{10}\right)$, which is the length of the weir corrected for contraction. See Art. 14 (*m*). Call the quotient D.§ Divide the measured head H by D. Find this last quotient in the column $\frac{H}{D}$ of the table. Multiply the approximate discharge, $Q = 3.33 \left(L - n\,\frac{H}{10}\right) H^{\frac{3}{2}}$, by the corresponding coefficient K; or

$$\textbf{Actual Discharge}\ Q = 3.33\ K\left(L - n\,\frac{H}{10}\right) H^{\frac{3}{2}}. \quad . \; . \; . \; . \; . \quad (13)$$

Table 18. Coefficient K in formula (13).

$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K
.01	1.0000	.09	1.0020	.17	1.0072	.24	1.0143	.31	1.0239
.02	1.0001	.10	1.0025	.18	1.0081	.25	1.0155	.32	1.0254
.03	1.0002	.11	1.0030	.19	1.0090	.26	1.0168	.33	1.0271
.04	1.0004	.12	1.0036	.20	1.0100	.27	1.0181	.34	1.0287
.05	1.0006	.13	1.0042	.21	1.0110	.28	1.0195	.35	1.0305
.06	1.0009	.14	1.0049	.22	1.0121	.29	1.0209	.36	1.0322
.07	1.0012	.15	1.0056	.23	1.0132	.30	1.0224	.37	1.0341
.08	1.0016	.16	1.0064						

* "Hydraulics," John Wiley & Sons, New York, 1886.

† If there are end contractions, L here becomes $\left(L - n\,\frac{H}{10}\right)$. See Art. 14 (*m*).

‡ This formula is deduced as follows: Let the area of the parabolic segment *a s a'*, Fig. 24, represent the theoretical discharge over a weir one foot long (as explained in foot-note § p. 549) under the measured head $H = a\,s$, as though there were no current at *o*. Let $m\,s = h = v^2 \div 2\,g$. The theoretical velocities of the particles passing the oblique plane *o a* under their actual heads, will now be represented by horizontal lines *s s''*, *a a''*, etc., etc., drawn from every point in *s a* to the *outer* curve *s'' a''*; the line *s s''* representing v = velocity of approach $= \sqrt{2\,g\,h}$, and *a a''* representing $\sqrt{2\,g\,(H+h)}$. Then, area *s s'' a'' a*

$$= \text{area } m\,a''\,a - \text{area } m\,s''\,s = \frac{2}{3}\ \text{area of rectangle } a\,n - \frac{2}{3}\ \text{area of rectangle } s\,k$$

$$= \frac{2}{3}\left(H + h\right)\sqrt{2\,g\,(H+h)} - \frac{2}{3}\,h\,\sqrt{2\,g\,h}, = \frac{2}{3}\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right);$$

and the actual discharge is

$$Q = c \times \text{length of weir} \times \text{area } s\,s''\,a''\,a = c\,L\,\frac{2}{3}\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right)$$

$$= m\,L\,\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right) = x\,L\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right).$$

§ In a weir without end contraction, $D = H + p$.

K is very approximately $= 1 + \frac{1}{4}\left(\frac{H}{D}\right)^2$ Hence

$$Q = 3.33\left[1 + \frac{1}{4}\left(\frac{H}{D}\right)^2\right]\left(L - n\,\frac{H}{10}\right)H^{\frac{3}{2}}$$

$$= \left[3.33 + 0.83\left(\frac{H}{D}\right)^2\right]\left(L - n\,\frac{H}{10}\right)H^{\frac{3}{2}}. \quad . \quad . \quad . \quad (14)$$

See Journal of the Franklin Institute, Philadelphia, August, 1884, from which we condense the above table

(*y*) **M. Bazin**, see Art. 14 (*o*), provides for the velocity of approach by modifying the coefficient m instead of the head H, making $\dot{m} = 0.425 + 0.21\left(\frac{H}{D}\right)^2$; while by Messrs Hunking and Hart's method (based upon Mr. Francis' experiments) m becomes $= 0.415 + 0.10\left(\frac{H}{D}\right)^2$.

Art. 15. Inclined weirs. If the up-stream face of the weir, instead of being vertical, as in Fig. 25, is inclined up-stream, as in Fig. 25 *a*, or down-stream, as in Fig. 25 *b*, the character and amount of the discharge are modified With an up-stream inclination (Fig 25 *a*) the lower side of the sheet of water passing over the weir leaps higher, and tends more and more up-stream as the

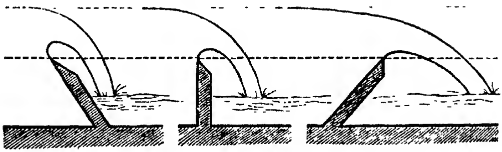


Fig. 25 *a*. **Inclined up-stream.** Fig. 25. **Vertical.** Fig 25 *b*. **Inclined down-stream.**

inclination is increased With a down-stream inclination (Fig 25 *b*), on the contrary, as the inclination increases the upward leap of the sheet decreases, its profile becomes more and more flattened, and the curve of the upper surface, due to the fall, extends farther up-stream from the crest of the weir.

An up-stream inclination (Fig 25 *a*) decreases, and a down-stream one (Fig. 25 *b*) increases, the discharge, as is indicated by the following coefficients obtained by M. H. Bazin:*

For the discharge over an inclined weir, having ascertained the discharge over a vertical weir of the same height and head and under similar conditions in other respects, multiply the discharge over the vertical weir by the following approximate coefficients·

	Inclination.		Angle		Coefficient.
	Horizontal.	Vertical.	with hor.	with vert.	
Weirs inclined up-stream, Fig. 25 *a*....	1	1	45°	45°	0.93
	$\frac{2}{3}$	1	56° 19′	33° 41′	0.94
	$\frac{1}{3}$	1	71° 34′	18° 26′	0 96
Vertical weirs, Fig 25...	0	1	90°	0°	1.00
Weirs inclined down-stream, Fig. 25 *b*....	$\frac{1}{3}$	1	71° 34′	18° 26′	1.04
	$\frac{2}{3}$	1	56° 19′	33° 41′	1.07
	1	1	45°	45°	1.10
	2	1	26° 34′	63° 26′	1.12

* "Expériences Nouvelles sur l'Écoulement en Déversoir," 2e Article; "Annales des Ponts et Chaussées," January, 1890, translated in *Proceedings, Engineers' Club of Philadelphia*, vol. ix., 1892.

The discharge will be increased also if the inner corner or edge of the crest be rounded off, instead of being left sharp; or if the sides of the reservoir converge more or less as they approach the weir, so as to form wings for guiding the water more directly to it; or if *a b*, Fig 20, be less than twice *a m*. Indeed, so many modifying circumstances exist to embarrass experiments on this and similar subjects that some of those which have been made with great care are rendered inapplicable as other than tolerable approximations, in consequence of the neglect to take into consideration some local peculiarity which was not at the time regarded as exerting an appreciable effect. Unless, therefore, circumstances admit of our combining all the conditions mentioned in Art. 14 (*d*), (*f*) and (*m*), pp. **547**, **548** and **550**, thereby securing *very* approximate results, we must either resort to an actual measurement of the discharge in a vessel of known capacity; or else be content with rules which may lead to errors of 5, 10, or more per cent in proportion as we deviate from these conditions. Frequently even 10 per cent. of error may be of little real importance.

REMARK 1. When the water, after passing over a weir, Fig. 26, instead of fall-

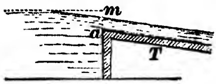


Fig. 26.

ing freely into the air, is carried away by a slightly inclined apron or trough, T, the floor of which coincides with the crest *a*, of the weir, then the discharge is not appreciably diminished thereby when the head *a m*, is 15 inches or more. But if the head *a m* is but 1 foot, then the calculated discharge must be reduced about one-tenth; if 6 inches, two-tenths; if 2½ inches, three-tenths; and if 1 inch, five-tenths, or one-half, as approximations.

REMARK 2. Professor Thomson, of Dublin, proposed the use of triangular notches, or weirs, for measuring the discharge, inasmuch as then the periphery always bears the same ratio to the area of the stream flowing over it; which is not the case with any other form. Experimenting with a right-angled triangular

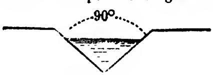


Fig. 26 A.

notch in thin sheet-iron, Fig. 26 A, with heads of from 2 to 7 inches, measured vertically from the bottom of the notch to *the level surface of the quiet water*, he found *discharge* in cubic feet per second = .0051 × $\sqrt{\text{fifth power of head in inches}}$ = 2.54 × $\sqrt{\text{fifth power of head in feet}}$* Or, in general, if m = coefficient of contraction (Art. 9, p 541) T = tangent of ½ the angle of the notch = ½ width of water-surface ÷ the depth in the notch, g = the acceleration of gravity = say 32 2 feet per second, and h = the head, measured as above; then

$$\text{Discharge} = \frac{8m}{15}\, T \sqrt{2 g h^5}\,^* = 4.28\, m\, T \sqrt{h^5}.^*$$

Remark 3. In constructing the irrigating canal, Canale Villoresi, near Milan, in 1881–4, the Italian engineer, Cesare Cippoletti,† adopted **a trapezoidal notch**, with its bottom edge horizontal and its ends sloping at $\frac{4 \text{ vertical}}{1 \text{ horizontal}}$, in order to avoid the necessity of either suppressing or allowing for end contractions. (See Art. 14 *c*, p **547**, and *m*, p **550**.) The contraction was found to affect only the triangular spaces over the sloping ends of the weir, and the effective length of the weir thus remained constant (and equal to the length of the bottom edge) for all heads. In using these weirs the contraction is complete along the bottom as well as at the ends.

* For such roots see p. 68.

† See his work, *Canale Villoresi; Modulo per la Dispensa della Acqua, etc.*, **Milan**, 1886; published by Societa Italiana per Condotte d'Acqua. **Results summarized by** L. G. Carpenter, in Bulletin No. 13, Agricultural Experiment Station, Fort Collins, Colorado, October, 1890.

ON THE FLOW OF WATER IN OPEN CHANNELS.

Art. 16. The mean velocity of flow is an imaginary uniform one, which, if given to the water at every point in the cross section, would give *the same discharge* that the actual ununiform one does. Or

$$\textbf{mean velocity} = \frac{\text{volume of discharge}}{\text{area of cross section}}$$

In channels of uniform cross section, **the maximum velocity** is found about midway between the two banks, and generally at some dist below the surface. This dist varies in diff streams; but, as an average, it seems to be about one third of the total depth. Where the total depth is great in proportion to the width, (say ½ the width or more), the max vel has been found as deep as midway between surf and bottom; while in small shallow streams it appears to approach the surf to within from .1 to .2 of the total depth. Many experiments upon shallow streams have indeed indicated that the max vel was *at* the surf.

The ratio between the velocities in different parts of the cross section varies greatly in diff streams; so that but little dependence can be placed upon rules for obtaining one from the other. With the same surf vel, wide and deep streams have greater mean and bottom vels than small shallow ones. In order **to approximate roughly to the mean vel** when the greatest surf vel is given, it is frequently assumed that the former is = ⅘ (or .8) of the latter. But Mr. Francis found, in his experiments at Lowell, that *surface* floats of wax, 2 ins diam, floating down the center of a rectangular flume 10 ft wide, and 8 ft deep, actually moved about 6 per cent *slower* than a tin tube 2 ins diam, reaching from a few ins above the surf, down to within 1½ ins of the bottom of the flume; and loaded at bottom with lead, to insure its maintaining a nearly vert position. While the wax *surf* float moved at the rate of 3.73 ft per sec, the rate of the tube (which was evidently very nearly the same as that of the center *vert* thread of water) was 3.98 ft per sec. Also, that in the same flume, with vels of the center tube varying from 1.55 to 4 ft per sec, the vel of the tube was *less* than that of the mean vel of the entire cross section of water in the flume, about as .96 to 1, for the lesser vel; and .93 to 1 for the greater vel. While, in another rectangular flume 20 ft wide and 8 ft deep, with vels varying from 1.16 to 1.84 ft per sec, that of the tubes was *greater* than that of the entire mass of water, about as 1.04 to 1. In a flume 29 ft wide, by 8.1 ft deep, with vels of about 3 ft per sec, it was as 1 to .9; and in a flume 36½ ft wide, by 8.4 ft deep, with vels of about 3½ ft per sec, as 1 to .97.

Charles Ellet, Jr, C E, found in the Mississippi "at diff points *on the river, in depths varying from 54 to 100 ft; and in currents varying from* 3 to 7 miles an hour that the speed of a float supporting a line 50 ft long, is almost always grea er than that of the surf float alone." The same results were obtained with lines 25 and 75 ft long; the excess of the speed of the line floats being about 2 per cent over that of the simple floats: and Mr. Ellet concludes, therefore, that the mean vel of the entire cross section of the Mississippi, instead *of being less, is absolutely greater by about 2 per cent, than the MEAN surf vel. He, however, employed .8 of the greatest surf vel as representing approximately,* in his opinion, the mean vel of the *entire* cross section of water. In *shallow* streams, he always found the surf float to travel more rapidly than a line float.

European trials of the *mean* vel of *separate single verticals*, in tolerably deep rivers, have resulted in from .85 to .96 of the surf vel at each vertical. The mean *of all may be taken at .9.*

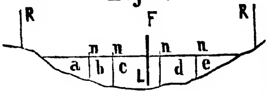
Bottom velocity. In streams of nearly uniform slope and cross section, there is a great reduction of vel near the bottom. As a very rough approximation, the deepest measurable vel, in streams of uniform slope etc, appears to be from ½ to ⅔ of the mean vel.

Art. 17. To measure the surface velocity, select a place where the stream is for some dist (the longer the better) of tolerably uniform cross section; and free from counter-currents, slackwater, eddies, rapids, etc. Observe, by a seconds-watch, or pendulum, how long a time a float (such as a small block of wood), placed in the *swiftest* part of the current, occupies in passing through some previously measured dist. From 50 feet for slow streams, to 150 ft for rapid ones, will answer very well. This dist in ft, or ins, div by the entire number of seconds reqd by the float to traverse it, will give the greatest surf vel in ft or ins per sec.

The surf vel should be measd in perfectly calm weather, so that the float may not be disturbed by wind; and, for the same reason, the float should not project much above the water. The measurement should be

repeated several times to insure accuracy. In very small streams, the banks and bed may be trimmed for a short dist, so as to present a uniform channel-way. The float should be placed in the water a little dist above the point for commencing the observation; so that it may acquire the full vel of the water, before reaching that point.

Fig 27



Art. 18. To gauge a stream by means of its velocity. Select a place where the cross-section remains, for a short distance, tolerably uniform, and free from counter-currents, eddies, still water, or other irregularities Prepare a careful cross-section, as Fig 27 By means of poles, or buoys, *n. n.* divide the stream into sections, *a. b. c.* &c Plant two range-poles, R R, at the upper end, and two others at the lower end, of the distance through which the floats are to pass: for observing, by a seconds watch or a pendulum, the time which they occupy in the passage Then measure the *mean* velocity of each section *a. b. c.* &c, separately, and directly, by means of long floats, as F L, reaching to near the bottom: and projecting a little above the surface. The floats may be tin tubes, or wooden rods; weighted in either case, at the lower end, until they will float nearly vertical. They must be of different lengths, to suit the depths of the different sections. For this purpose the float may be made in pieces, with screw-joints. The area of each separate section of the stream in square feet, being multiplied by the observed mean velocity of its water in feet per second, will give the discharge of that section in cubic feet per second And the discharges of all the separate sections, thus obtained, when added together, will give the total discharge of the stream. And this total discharge, divided by the entire area of cross-section of the stream in square feet, gives the mean velocity of *all* the water of the stream, in feet per second.

Rem. If the channel is in common earth, especially if sandy, the loss by soakage into the soil, and by evaporation, will frequently abstract so much water that the disch will gradually become less and less, the farther down stream it is measured. Long canal feeders thus generally deliver into the canal but a small proportion of the water that enters their upper ends.

The double float is used for ascertaining vels at diff depths. It consists of a float resting upon the surface of the water, and of a heavier body, or "lower float", *which is suspended from the upper float by means of a cord.* The depth of the lower float of course depends upon the length of the suspending cord (which may be increased or diminished at pleasure until the lower float is believed to be at that depth for which the vel is wanted), and upon its straightness, which is more or less affected by the current. Owing to this latter circumstance, it is difficult to know whether the lower float is really at the proper depth. *Moreover it is uncertain to what extent the two floats and the string* interfere with one another's motions. In deep water the string may oppose a greater area to the current than the lower float itself does. It thus becomes doubtful to what extent the vel of the upper float can be relied upon as indicating that of the water at the depth of the lower one.

Art. 19. Castelli's quadrant, or hydrometric pendulum, consisted of a metallic ball suspended by a thread from the center of a graduated arc. The instrument was placed in the current, with the arc parallel to the direction of flow; and the vel was then calculated from the angle formed between the thread and a vert line.

Gauthey's pressure plate was a sheet of metal suspended by one of its ends, about which it was left free to swing. The plate was immersed in the stream, with its face at right angles to the current. The vel was estimated by means of the weight required to make the plate hang vert in opposition to the force of the current.

Pitot's tube was originally a simple glass tube, Fig 27 A, open at both ends and bent in the shape of the letter L. One leg of the L was held horizontal under water, with its open end facing the current; and the velocity v at the point o where it was placed was measured by the vertical height h $\left(\text{theoretically} = \frac{v^2}{2g}\right)$ to which the water rose in the other leg above the surface of the stream.

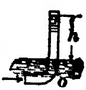


Fig. 27A

As developed by M. Darcy and by Prof. S. W. Robinson,* and rudely indicated in Fig. 27 B, Pitot's tube consists essentially of *two* horizontal glass or metal tubes *a* and *b*, of very small bore, placed side by side in the current and pointed up-stream. Tube *a* receives the current in its open up-stream end, while *b* is closed at its up-stream end, and has small *lateral* openings only. The other end of each tube communicates, by means of small metal or rubber piping, with one leg of an inverted U shaped glass gauge fixed in a boat or on shore. For convenience, the two flexible pipes may be joined together into one double pipe. By sucking through a stop-cock T at the top, water is drawn up to any convenient height in the two legs of the gauge. When measuring the flow in pipes under pressure, such suction is unnecessary, and a U-shaped manometer is commonly used, the U being filld with a liquid heavier than water. When there is no current, the two columns of course stand at the same level; but, when a current is flowing, they stand at diff levels, as in Fig 27B, and the velocity is given by the equation, $v = c\sqrt{2\,g\,h}$

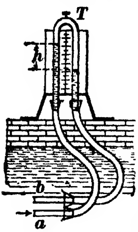


Fig. 27B

where h = the diff of level of the two column-tops, v = the vel of the water at the point where it impinges upon tube a, g = acceleration of gravity = 32 2 ft (9.81 meters) per sec per sec, and c = an experimental coefficient, found preferably by rating the apparatus in a current of known vel. When the hor end of tube b is drawn to a long, smooth, fine point, facing the current, and its (tangential) orifices are placed at some dist down-stream from said point and are made small, and their edges carefully smoothd, to avoid disturbance of flow, the coeff c approximates very closely to unity, so that, for practical purposes, $v = \sqrt{2\,g\,h}$. The instrument is remarkably simple and accurate, and can be used in very narrow and shallow streams of water or of gas. It measures velocities as low as 4 inches per second.

In practice, a and b are fixed together in one piece, and placed, when in use, in a metal frame which slides vertically, either upon a wire passing through it and provided with a plummet which rests upon the bottom and keeps the wire stretched, or (in streams shallower than about 20 feet) upon a vertical wooden rod, the lower end of which holds in the bed of the stream. In the former case, the frame is provided with a long vane for keeping the instrument headed upstream. In either case, means are provided for showing the depth to which the instrument is submerged

By making the gauge scale adjustable vertically, and placing it (at each change of depth of instrument) with its zero opposite the top of the lower column, we obviate the necessity of observing the height of both columns at each reading; for the reading of the upper column alone then gives the head h at once.

Art. 20. The wheel meter consists of a wheel which is turned by the current, and which communicates its motion, by means of its axle and gearing, to indices which record the number of revolutions. The instrument may be clamped to any part of a long pole reaching to the bottom of a stream, and thus may be used at any depth The observer, by means of a wire, rod or string reaching down to the instrument, throws the registering apparatus first into, and then out of, gear with the wheel (applying a brake to the former at the instant it is thrown out of gear), and carefully noting the times when he does so. The instrument is then raised, the number of revolutions in the measured time is read off from the indices, and from it the velocity is calculated But the meter is often made **self-registering**; the wheel, at each revolution, automatically breaking and re-establishing a galvanic current generated by a battery. The wire carrying this current is thus made to operate Morse telegraphic registering apparatus placed in a boat or on shore

A number of meters, so arranged, can be attached at different points on the same pole at the same time, and thus **simultaneous observations of velocities at different depths** may be made and registered.

Meters are usually so arranged as to swing freely about the long vertical pole to which they are clamped, and are provided each with a vane or tail similar to that of a windmill, for keeping the wheel in the proper position as regards the current. The wheels are generally made like those of a windmill; *i. e.*, with blades set at such an angle as to present a sloping surface to the current; and

*See Van Nostrand's Magazine, March, 1878, and August, 1886.

with the axis of the wheel parallel to the direction of flow. The axis runs in agate bearings. When desired, the rim of the wheel is furnished with an air-chamber, which just counterbalances the weight of the wheel, and thus removes journal friction due to it. Meters provided with electrical registering apparatus sometimes have the gearing and indices, etc., enclosed in a glass case, to prevent them from becoming clogged by weeds, sediment, etc.

A wheel meter is rated by moving it at a known velocity through still water, and noting the effect produced. In this way a coefficient is obtained for each meter, which, when multiplied by the number of revolutions recorded in any given case, gives the velocity for that case.

REM. 1. **Care must be taken that the bottom vel is not so great as to wear away the soil.** If there is any such danger artificial means must be applied to protect the channel-way; or it may be advisable to reduce the rate of fall, and increase the cross section of the channel; so as to secure the same disch, but with less vel. A liberal increase should also be made in the dimensions of such channels, to compensate for obstructions to the flow, arising from the growth of **aquatic plants,** or deposits of mud from rain-washes, etc; or even from very strong winds blowing against the current.

REM. 2. **Water running in a channel with a horizontal bed, or bottom, cannot have a uniform vel, or depth, throughout its course;** because the action of gravity due to the inclined plane of a sloping bottom, is wanting in this case; and the water can flow only by forming its *surface* into an inclined plane; which evidently involves a diminution of depth at every successive dist from the reservoir.

For theory of flow in long pipes and channels, mean velocity, distribution of velocity in cross section, Chezy formula, friction factor, exponential formulas, see pp 527, etc.

For Kutter's formula. see pp 523, 564, etc.

Bazin's Formula for Flow in Channels. Annales des Ponts et Chaussées, 1897, 4e trimestre, p 40.

In the Chezy formula, $v = c\sqrt{rs}$ where; (see p 528)

v = mean velocity;

$$r = \text{mean radius} = \frac{\text{cross section area}}{\text{wet perimeter}};$$

$$s = \text{slope} = \frac{\text{friction head}}{\text{length}}.$$

M. Bazin considers c independent of the slope, and gives:

For English measure.	**For metric measure,**
$c = \dfrac{87}{0.552 + \dfrac{\gamma}{\sqrt{r}}}$	$c = \dfrac{87}{1 + \dfrac{\gamma}{\sqrt{r}}}$

where γ has the following values;—

Very smooth surfaces, cement, planed wood, etc.,	$\gamma = 0.06$
Smooth surfaces, boards, brick, cut stone, etc.,	$\gamma = 0.16$
Ashlar Masonry,	$\gamma = 0.46$
Earth, very smooth, or paved with dry stone,	$\gamma = 0.85$
Earth, in ordinary condition,	$\gamma = 1.30$
Earth, very rough,	$\gamma = 1.75$

"In measuring the slope of a large river, the ordinary errors of the most careful leveling are a large proportion of the whole fall: the variation of level in the cross section of the surface is often as great as the slope for ten miles or more; the exact point where the level should be taken is often uncertain: the rise and fall of the water makes it extremely difficult to decide when the levels should be taken at the upper and lower points; waves of translation may affect the inclination to a great and uncertain degree, and may even make the surface slope the reverse way." Genl. T. G. Ellis, Trans Am Soc Civ Engrs., Aug 1877.

41

Kutter's Formula.

See also p 523. **For theory of flow** in long pipes and channels, see p 527.

Ganguillet and Kutter sought to establish a formula for the value of c in the Chezy formula:

$$v = c\sqrt{r\,s};$$

where v = mean velocity; r = mean radius $= \dfrac{\text{area}}{\text{wet perimeter}}$

s = slope $= \dfrac{\text{friction head}}{\text{length}}$.

The earlier hydraulicians gave (each according to the results of his investigations) *fixed* **values for the coeff c,** (generally about 95 to 100 for channels in earth or gravel, as in our early editions), making it, in other words, a *constant*, and independent of the shape, size, slope and roughness of the channel. But more recent investigators have shown that the coefficient c is affected by differences in any of these particulars.

According to the formula of Ganguillet and Kutter (generally called, for convenience, "**Kutter's formula**"*) the value of c is:

For English measure. **For metric measure.**

$$\mathbf{c} = \frac{41.6 + \dfrac{.00281}{\text{slope}} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{.00281}{\text{slope}}\right)\times n}{\sqrt{\text{mean rad in } \textit{feet}}}} = \frac{23 + \dfrac{.00155}{\text{slope}} + \dfrac{1}{n}}{1 + \dfrac{\left(23 + \dfrac{.00155}{\text{slope}}\right)\times n}{\sqrt{\text{mean rad in } \textit{metres}}}}$$

Tables giving values of c for diff grades, mean radii and degrees of roughness, and for English and metric measures, are given on pp 566, etc.

Here **n** is a "**coefficient of roughness**" of sides of channel as given below. These values of n were obtained from experience, by averaging a large number of experiments made under very different circumstances. They therefore embrace all the disturbing effects arising from obstructions existing upon the bottom and sides of the channel in the cases experimented upon. In small artificial channels of uniform cross section and slope, these obstructions may be said to consist entirely of the comparatively minute roughnesses of the *material* of which the bed of the channel consists. But in rivers and earth canals, even where the *general* direction, slope and cross section are tolerably uniform, (as they were in the cases upon which our list is based), there are still many considerable irregularities in the sides and bottom; and these exert a much greater retarding effect upon the mean vel than the mere *roughness* of the material of the banks. We therefore find larger values given for n in such cases than for small regular artificial channels, although the material of the sides etc was in many cases smooth mud; and we must not apply to such comparatively irregular channels the small values of n obtained by experiments with small and carefully made straight flumes of uniform section and slope, even if we suppose the bottom and sides of the former to be made as smooth as those of the latter.

No general formula is applicable to cases of **decided bends** in the course of a natural stream, or of **marked irregularities in the cross section.** Such cases would require still higher coefficients n than those here given for rivers and canals; but they would have to be ascertained by experiment for each case, and would be useless for other cases. For such streams we must therefore depend upon actual measurements of the velocity, either direct or by means of the disch.

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* See "Flow of Water," translated from Ganguillet and Kutter, by Rudolph Hering and John C. Trautwine, Jr., New York, John Wiley & Sons, 1889. $4.00.

There is much room for the exercise of judgment in the **selection of the proper coefficient n** for any given case, even where the condition of the channel is well known. It may frequently be necessary to use values of n intermediate between those given; for careless brickwork may be rougher than well finished rubble; side slopes in "very firm gravel" may have very diff degrees of roughness; etc etc. The engineer should make lists of values of n from his own experience, fully noting the peculiarities of each case, and calculating n from the tables.

A given diff in the deg n of roughness exerts a much greater effect upon the coefficient c, and thus upon the velocity, in small channels than in larger ones. It is therefore especially necessary in small channels that care be exercised in finding (by experiment if necessary) the proper value of n; and, where a large disch is desired, the sides of small channels should be made particularly smooth.

Table of n, or coefficient of roughness.

In any given case the value of **n is the same whether the mean radius is given in English, metric or any other measure.**

Artificial channels of uniform cross section.

Sides and bottom of channel lined with	n =
well planed timber	.009
neat cement (applies also to glazed pipes and very smooth iron pipes).	.010
plaster of 1 measure of sand to 3 of cement; (or smooth iron pipes).	.011
unplaned timber (applies also to ordinary iron pipes)	.012
ashlar or brickwork	.013
rubble	.017

Channels subject to irregularity of cross section.

Canals in very firm gravel	.020
Canals and rivers of tolerably uniform cross section, slope and direction, in moderately good order and regimen, and free from stones and weeds	.025
having stones and weeds occasionally	.030
in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	.035

Art. 22. The following tables give values of the coefficient c as obtained by Kutter's formula for diff slopes (S) mean radii (R) and degrees of roughness (n).*

Caution. Different values of c must be used with English and with metric measures. We give tables for both measures.

1st. Having the slope S, the mean rad R and the deg n of roughness; **to find the coeff c.** Turn to the division of the table corresponding to the given slope S. In the first column find the given mean rad, R. In the same line with this R, and under the given n, is the proper value of c.*

2d. Having the slope S, the mean rad R and either c or the actual or reqd vel v; **to find the actual, or the greatest permissible, deg n of roughness** of channel. If the vel is given, and not c, first find

$$c = \frac{\text{velocity}}{\sqrt{\text{slope} \times \text{mean radius}}}.$$

Turn to the division of the table corresponding to the given S, and in the first col find the given R. In the same line find the value given, or just obtained, for c; over which will be found the reqd n.*

3d. Having the slope S, the deg n of roughness, and the actual or required vel v; **to find the actual or necessary mean rad, R.** Assume a mean rad; and from the division of the table corresponding to the given S take out the value of c corresponding to the given n and the assumed R. Then say

$$v' = c \text{ so found} \times \sqrt{\text{assumed mean radius} \times \text{slope}}$$

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* It is often necessary to interpolate values of S, R, n and c intermediate of those in the tables; this may be done mentally by simple proportion.

If this v' is the same as the given vel, or near enough to it, take the assumed R as the proper one. Otherwise, repeat the whole process, assuming a new R, *greater* than the former one if v' is *less* than the given vel, and *vice versa*.*

4th. Having the dimensions of the wetted portion (*abco* Fig C, p 528,) of the channel, the deg *n* of roughness, and the actual or reqd vel; **to find the actual or necessary slope,** S:

$$\text{Find the mean rad, R} = \frac{\text{area of wet cross section}}{\text{length, } abco, \text{ of wet perimeter}}$$

Assume one of the slopes of the tables to be the proper one. From the corresponding division of the table take out the value of c corresponding to the given R and n.

If R is 3.28 feet, or 1 metre, the value of c thus found is the proper one (because then c, for any given n, remains the same for all slopes); and the slope, S, may be found at once, thus:

$$\text{Slope, S} = \left(\frac{\text{given velocity}}{c \times \sqrt{\text{mean radius}}}\right)^2$$

But if R is greater or less than 3.28 feet, or 1 metre, say

$$v' = c \text{ thus found} \times \sqrt{\text{mean radius} \times \text{assumed slope}}$$

If this v' is near enough to the given vel, take the assumed S as the proper one. Otherwise, assume a new S, *greater* than the former one if v' is *less* than the given vel, and *vice versa*; and repeat the whole process.*

* It is often necessary to interpolate values of S, R, n and c intermediate of those in the tables. This may be done mentally by simple proportion.

Tables of coefficient c,

for mean radii *in feet*, and for mean radii *in meters*.

Table 1: for mean radii in FEET.

(For table with mean radii in *meters*, see pp 569, 570.)

Slope S = .000025 per unit of length, = 1 in 40000, = .132 foot per mile.

Mean rad R FEET	.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R FEET
	c	c	c	c	c	c	c	c	c	c	c	c	
.1	65	57	50	44	40	33	28	23	17	14	12	10	.1
.2	87	75	67	59	53	45	38	31	24	19	16	14	.2
.4	111	97	87	78	70	59	51	42	32	26	22	19	.4
.6	127	112	100	90	81	69	60	49	38	31	26	22	.6
.8	138	122	109	99	90	77	66	55	43	35	30	25	.8
1	148	131	118	106	97	83	72	60	47	38	32	28	1
1.5	166	148	133	121	111	95	83	69	55	45	38	33	1.5
2	179	160	144	131	121	104	91	77	61	50	43	37	2
3	197	177	160	147	135	117	103	88	70	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	209	188	172	158	146	127	113	96	78	65	56	49	4
6	226	206	188	174	161	142	126	108	88	74	64	57	6
8	238	216	199	184	171	151	135	117	96	82	71	63	8
10	246	225	207	192	179	159	142	124	102	87	76	68	10
12	253	231	214	198	186	165	149	129	107	92	81	72	12
16	263	242	223	208	195	174	157	138	115	100	88	79	16
20	271	249	231	215	202	181	164	144	121	106	94	84	20
30	283	261	243	228	215	193	176	157	133	117	104	95	30
50	297	274	257	241	228	207	190	170	147	130	117	107	50
75	306	284	267	251	238	217	200	180	157	140	127	117	75
100	312	290	273	257	244	223	207	187	163	147	134	124	100

Table 1, of coefficient c, for mean radii in FEET.—CONT'D.

Mean rad R FEET	.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R FEET
	c	c	c	c	c	c	c	c	c	c	c	c	
Slope S = .00005 per unit of length, = 1 in 20000, = .264 foot per mile.													
.1	78	67	59	52	47	39	33	26	20	16	13	11	.1
.15	91	79	69	62	56	46	39	31	28	19	16	13	.15
.2	100	87	77	68	62	51	44	35	26	21	18	15	.2
.3	114	99	88	79	71	59	50	41	31	25	21	18	.3
.4	124	109	97	88	79	66	57	46	35	28	24	20	.4
.6	139	122	109	98	90	76	65	53	41	33	28	24	.6
.8	150	133	119	107	98	83	71	59	46	37	31	27	.8
1	158	140	126	114	104	89	77	64	49	40	34	29	1
1.5	173	154	139	126	116	99	87	72	57	47	40	34	1.5
2	184	164	148	135	124	107	94	79	62	51	44	38	2
3	198	178	161	148	136	118	104	88	71	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	207	187	170	156	145	126	111	95	77	64	56	49	4
6	220	199	182	168	156	137	122	105	85	72	68	56	6
8	228	206	189	175	163	144	129	111	91	78	68	61	8
10	234	212	195	181	169	149	134	116	96	82	72	64	10
12	238	217	200	185	173	153	138	120	99	86	75	68	12
16	245	223	206	191	180	160	144	126	106	91	81	73	16
20	250	228	211	196	184	165	149	131	110	96	85	77	20
30	257	236	219	204	192	172	157	139	118	103	92	84	30
50	266	245	228	213	201	181	165	148	127	112	191	98	50
75	272	250	233	218	207	187	171	153	133	119	108	99	75
100	275	254	237	222	210	190	175	158	137	123	112	104	100
Slope S = .0001 per unit of length, = 1 in 10000, = .528 foot per mile.													
.1	90	78	68	60	54	44	37	30	22	17	14	12	.1
.2	112	98	86	76	69	57	48	39	29	23	19	16	.2
.3	125	109	97	87	78	65	56	45	34	27	22	19	.3
.4	136	119	106	95	86	72	62	50	38	31	25	22	.4
.6	149	131	118	105	96	81	70	57	44	35	30	25	.6
.8	158	140	126	114	103	88	76	63	48	39	33	28	.8
1	166	147	132	120	109	93	81	67	52	42	35	31	1
1.5	178	159	144	130	120	103	89	75	59	48	41	35	1.5
2	187	168	151	138	127	109	96	81	64	53	45	39	2
3	198	178	162	149	137	119	104	89	71	59	51	45	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	206	186	169	155	143	125	111	94	76	64	55	49	4
6	215	195	178	164	152	134	119	102	84	71	61	54	6
8	221	201	184	170	158	139	124	107	88	75	66	59	8
10	226	205	188	174	162	143	128	111	92	78	69	62	10
15	233	212	195	181	169	150	135	118	98	85	75	68	15
20	237	216	200	185	173	154	139	122	102	89	79	71	20
30	243	222	206	191	179	160	145	123	108	95	84	77	30
50	249	227	211	197	185	166	151	134	114	100	91	83	50
100	255	234	218	204	191	172	158	140	121	108	98	91	100
Slope S = .0002 per unit of length, = 1 in 5000, = 1.056 feet per mile.													
.1	99	85	74	65	59	48	41	32	24	18	15	12	.1
.2	121	105	93	83	74	61	52	42	31	25	21	17	.2
.3	133	116	103	92	83	69	59	48	36	29	24	20	.3
.4	143	125	112	100	91	76	65	53	40	32	27	23	.4
.6	155	138	122	111	100	85	73	60	46	37	31	26	.6
.8	164	145	131	118	107	91	79	65	50	41	34	29	.8
1	170	151	136	123	113	96	88	69	54	44	37	32	1
1.5	181	162	146	133	122	105	91	77	60	49	42	36	1.5
2	188	170	154	140	129	111	97	82	64	54	45	40	2
3	200	179	163	149	137	119	105	89	72	59	51	45	3
4	205	185	168	155	143	125	111	94	73	63	55	48	4
6	213	193	176	162	150	132	117	100	82	69	60	53	6
8	218	198	181	167	155	137	122	105	87	73	64	57	8
10	222	201	185	170	158	140	125	108	89	76	67	60	10
15	228	207	190	176	164	145	131	113	95	82	72	65	15
20	231	210	194	180	168	149	134	117	98	85	76	68	20
30	235	215	198	184	172	154	139	122	103	89	80	73	30
50	240	220	203	189	177	158	143	126	108	94	85	78	50
100	245	224	208	194	182	163	148	131	113	99	90	83	100

Table 1, of coefficient c, for mean radii in FEET.—Concl'd.

	Mean rad R FEET	.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R FEET
		Coefficients n of roughness.												
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope S = .0004 per unit of length, = 1 in 2500, = 2.112 feet per mile.	.1	104	89	78	69	62	50	43	34	25	19	16	13	.1
	.15	116	101	90	80	71	59	50	40	29	23	19	16	.15
	.2	126	110	97	87	78	65	54	44	32	25	21	18	.2
	.3	138	120	107	96	87	73	62	50	37	30	24	21	.3
	.4	148	129	115	104	94	79	68	55	42	33	27	23	.4
	.6	157	140	126	113	103	87	75	62	47	38	31	27	.6
	.8	166	148	133	121	110	93	81	67	51	42	35	30	.8
	1	172	154	138	125	115	98	85	70	55	45	37	32	1
	1.5	183	164	148	135	124	106	93	78	61	50	42	37	1.5
	2	190	170	154	141	130	112	98	83	65	54	45	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	94	76	63	55	48	4
	6	211	191	175	161	149	130	116	99	81	69	60	53	6
	10	219	199	183	168	157	138	123	107	88	75	66	59	10
	20	227	207	190	176	164	146	131	115	96	83	73	66	20
	50	235	215	198	184	173	154	139	123	104	91	82	75	50
	100	239	219	203	189	177	158	148	127	108	96	87	80	100
Slope S = .0010 per unit of length, = 1 in 1000, = 5.28 ft. per m.	.1	110	94	83	73	65	54	45	36	27	21	17	14	.1
	.2	129	113	99	89	81	66	57	45	34	27	22	18	.2
	.3	141	124	109	98	89	74	63	51	39	30	25	21	.3
	.4	150	131	117	105	96	80	69	56	43	34	28	24	.4
	.6	161	142	127	115	104	88	76	63	48	39	32	27	.6
	.8	169	150	134	122	111	94	82	68	52	42	35	30	.8
	1	175	155	139	127	116	99	86	71	56	45	38	33	1
	1.5	184	165	149	136	124	108	93	78	62	50	43	37	1.5
	2	191	171	155	142	130	112	98	83	66	54	46	40	2
	3	199	179	163	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	93	75	63	54	48	4
	6	211	190	174	160	149	130	116	99	81	68	59	52	6
	10	218	197	181	167	155	136	122	105	87	74	65	58	10
	20	225	205	188	175	163	144	129	113	94	81	72	65	20
	50	232	212	196	182	170	151	137	120	101	89	79	72	50
	100	236	216	200	186	174	155	141	124	105	94	85	77	100
Slope S = .01 per unit of length, = 1 in 100, = 52.8 feet per mile.	.1	110	95	83	74	66	54	46	36	27	21	17	14	.1
	.15	122	105	93	83	75	62	52	42	31	24	20	17	.15
	.2	130	114	100	90	81	67	57	46	34	27	22	19	.2
	.3	143	125	111	100	90	76	64	52	39	31	25	22	.3
	.4	151	133	119	107	98	82	70	57	44	35	29	24	.4
	.6	162	143	129	116	106	90	77	64	49	39	33	28	.6
	.8	170	151	135	123	112	95	82	68	53	43	35	31	.8
	1	175	156	141	128	117	99	87	72	56	45	38	33	1
	1.5	185	165	149	136	125	107	94	79	62	51	43	37	1.5
	2	191	171	155	142	130	112	99	83	66	55	46	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
	4	204	184	167	154	142	123	109	93	76	63	55	48	4
	6	210	190	173	160	148	129	115	99	81	68	59	52	6
	10	217	196	180	166	154	136	121	105	86	74	65	58	10
	20	225	204	187	173	161	143	128	112	93	80	71	64	20
	50	231	210	194	181	168	150	135	119	100	87	78	71	50
	100	235	214	197	184	172	153	139	122	104	91	82	75	100

For slopes steeper than .01 per unit of length, = 1 in 100 = 52.8 feet per mile, *c* remains practically the same as at that slope. But the *velocity* (being = $c \times \sqrt{\text{mean radius} \times \textit{slope}}$) of course continues to increase as the slope becomes steeper.

Table above is for mean radii in **FEET.**

For mean radii in **METERS**, see table, **pp 569-570.**

Table 2, of coefficient c, for mean radii in METERS.

Slope	Mean rad R metres	.009 c	.010 c	.011 c	.012 c	.013 c	.015 c	.017 c	.020 c	.025 c	.030 c	.035 c	.040 c	Mean rad R metres
Slope = .000025 per unit of length, = 1 in 40000.	.025	34	29	25	22	20	17	14	11	9	7	6	5	.025
	.05	44	38	33	30	27	22	19	16	12	9	8	7	.05
	.1	58	50	44	40	36	30	26	21	16	13	11	9	.1
	.2	72	63	56	51	46	39	34	28	21	18	15	13	.2
	.3	82	72	64	58	53	45	39	33	25	21	17	15	.3
	.4	89	79	71	64	59	50	44	37	29	23	20	17	.4
	.6	99	88	80	72	67	57	50	42	33	28	23	20	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.50	121	109	100	92	85	74	66	57	46	38	33	29	1.50
	2	127	115	106	98	91	80	71	61	50	42	37	32	2
	3	136	124	114	106	99	87	78	68	56	48	42	37	3
	4	142	130	120	111	104	93	83	73	61	52	46	41	4
	6	149	137	127	119	111	100	90	80	67	58	51	46	6
	10	158	145	135	127	120	108	98	88	75	66	59	53	10
	15	164	151	141	133	126	114	104	94	81	72	64	59	15
	20	167	155	145	137	130	118	108	98	85	75	68	62	20
	30	172	160	150	142	135	128	113	103	90	81	74	68	30
Slope = .00005 per unit of length, = 1 in 20000.	.025	40	85	80	26	24	20	17	13	10	8	7	5	.025
	.05	52	44	39	34	81	26	22	18	13	11	9	7	.05
	.1	65	57	50	44	40	34	29	24	18	14	12	10	.1
	.2	79	69	62	55	51	43	87	30	23	19	16	13	.2
	.8	87	77	69	62	57	48	42	35	27	22	18	16	.8
	.4	93	83	74	67	62	53	46	38	80	25	21	18	.4
	.6	102	90	82	74	69	59	52	43	34	28	24	21	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	118	107	97	90	83	73	65	55	45	88	33	28	1.5
	2	123	111	102	94	87	77	68	59	48	41	35	31	2
	8	129	117	108	100	93	83	74	64	53	45	40	35	8
	4	133	121	112	104	97	86	77	68	56	49	48	38	4
	6	138	126	117	109	102	91	82	72	61	53	47	42	6
	10	143	131	122	114	107	96	87	78	66	58	52	47	10
	15	147	135	126	118	111	100	91	82	70	62	56	51	15
	20	150	137	128	120	113	103	94	84	72	64	58	53	20
	30	152	140	131	123	116	105	97	87	76	68	62	57	30
Slope = .0001 per unit of length, = 1 in 10000.	.025	47	40	35	31	28	22	19	15	11	9	7	6	.025
	.05	59	50	44	40	35	29	25	20	15	12	10	8	.05
	.1	72	62	55	50	45	37	32	26	19	16	13	11	.1
	.2	84	74	66	60	54	46	39	32	25	20	17	14	.2
	.3	91	81	73	66	60	51	44	37	28	23	19	17	.3
	.4	97	86	77	70	64	55	48	40	31	25	21	18	.4
	.6	104	92	83	76	70	60	53	45	35	29	25	21	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	117	105	96	88	82	72	64	54	44	37	32	28	1.5
	2	120	109	100	92	85	75	67	57	47	40	34	30	2
	4	128	116	107	99	92	82	73	64	53	46	40	36	4
	6	131	119	110	102	96	85	77	67	56	49	43	89	6
	10	135	123	114	106	100	89	81	71	60	53	47	43	10
	15	137	126	116	109	102	92	83	74	63	55	50	46	15
	30	141	129	120	112	106	95	87	78	67	59	54	50	80
Slope = .0002 unit of length, = 1 in 5000.	.025	52	45	40	35	31	25	21	17	12	9	8	6	.025
	.050	63	55	48	43	39	32	27	21	16	12	10	8	.050
	.1	75	66	59	53	48	40	34	27	21	16	13	11	.1
	.2	87	77	69	62	57	48	41	34	26	21	17	15	.2
	.4	99	88	80	72	66	57	49	41	32	26	22	19	.4
	.6	104	93	84	77	71	61	53	45	36	29	25	22	.6
	1	111	100	90	83	77	67	59	50	40	83	28	25	1
	2	118	107	98	90	84	74	65	56	46	39	34	30	2
	4	124	113	104	97	90	79	71	62	51	44	89	35	4
	10	130	119	110	102	96	85	77	67	57	50	45	40	10
	80	135	124	114	107	100	90	82	73	62	55	50	46	80

Table 2, of coeff c, for mean radii in METERS.—CONCL'D.

Slope	Mean rad R meters	.009	.010	.011	.012	.013	.015	.017	.020	025	.030	.035	.040	Mean rad R metres
		Coefficients n of roughness.												
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope = .0004 per unit of length, = 1 in 2500.	.025	55	47	41	37	83	27	22	17	13	10	8	7	.025
	.050	66	58	51	45	40	83	28	23	17	13	11	9	.050
	.1	78	68	61	55	50	42	35	28	21	17	14	12	.1
	.2	90	80	70	64	59	49	42	35	27	22	18	15	.2
	.3	95	85	76	70	63	54	47	39	30	24	21	17	.3
	.4	99	89	80	73	67	57	50	42	32	27	22	20	.4
	.6	105	94	85	78	72	62	54	45	36	30	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	117	106	97	89	83	73	65	56	45	38	34	30	2
	4	123	111	102	95	88	78	70	61	50	43	38	34	4
	6	125	114	105	97	91	81	72	63	58	46	40	36	6
	10	128	117	108	100	93	83	75	66	55	48	43	39	10
	30	132	121	112	104	98	87	79	70	60	52	48	43	30
Slope = .0010 per unit of length, = 1 in 1000.	.025	57	50	43	38	84	28	23	18	13	11	9	7	.025
	.050	69	59	52	47	42	84	29	23	17	13	11	9	.050
	.1	80	70	63	56	50	42	36	30	22	17	14	12	.1
	.2	90	80	72	65	60	50	43	85	27	22	18	16	.2
	.3	96	86	77	70	64	54	47	89	80	25	21	18	.3
	.4	100	89	81	74	67	58	50	42	88	27	23	19	.4
	.6	104	94	85	78	72	62	54	46	86	80	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	116	106	97	90	83	72	64	55	45	38	33	29	2
	4	121	111	102	94	87	77	69	60	50	42	37	33	4
	6	124	113	104	97	90	80	71	62	52	45	40	36	6
	10	127	115	106	99	92	82	73	64	54	47	42	38	10
	30	130	119	110	102	96	86	77	68	58	51	46	42	30
Slope = .01 per unit of length, = 1 in 100.	.025	59	50	44	39	85	28	24	19	14	10	9	7	.025
	.05	69	60	53	48	43	85	29	24	18	14	11	9	.05
	.1	81	71	63	57	51	43	36	30	22	18	15	12	.1
	.2	91	81	72	65	60	50	44	86	27	22	18	16	.2
	.3	97	86	77	71	65	55	48	40	81	25	21	18	.3
	.4	101	90	81	74	68	58	50	42	83	27	23	20	.4
	.6	106	95	86	78	72	62	54	46	86	80	25	22	.6
	1.	111	100	90	83	77	67	59	50	40	83	28	25	1.
	1.5	115	104	94	87	80	70	62	53	43	36	31	27	1.5
	2	117	105	96	89	83	72	64	55	45	38	33	29	2
	4	121	110	101	93	87	76	68	59	49	42	37	33	4
	10	126	114	105	98	91	81	78	64	53	46	41	37	10
	30	129	118	108	101	95	84	77	67	57	50	45	41	30

For slopes steeper than .01 per unit of length, = 1 in 100, the *coefficient c* remains practically the same as at that slope. The *velocity*, however, being $= c \times \sqrt{\text{mean radius} \times \textit{slope}}$, continues to increase as the slope becomes steeper.

To construct a diagram, fig 30 A, from which the **values given by Kutter's formula** may be taken by inspection.

Draw *zz* hor, and say from 2 to 4 ft long; and *oy* vert at any point *o* within say the middle third of *zz*. On *oy* lay off, as shown on the left, the values of *c* for which the diagram will probably be used. If a scale of .05 inch, or .082 metre, per unit of *c* be used, and be made to include *c* = 250 for English measure, or 150 for metric measure, *oy* will be about 1 ft long. For the sake of clearness we show only the larger divisions in this and in what follows.

On *oz* lay off, as shown on its upper side, the *square roots* of all the values of the mean rad R for which the diagram is to be used. One inch per ft, or .06 metre per metre, of sq rt, is a convenient scale. *Mark* the dividing points with the respective values of the *mean radii themselves*

Having decided upon the *flattest* slope to be embraced in the diagram, say

$$w = 41.6 + \frac{.0028}{\text{flattest slope per unit of length}} \quad \text{for English measure.}$$

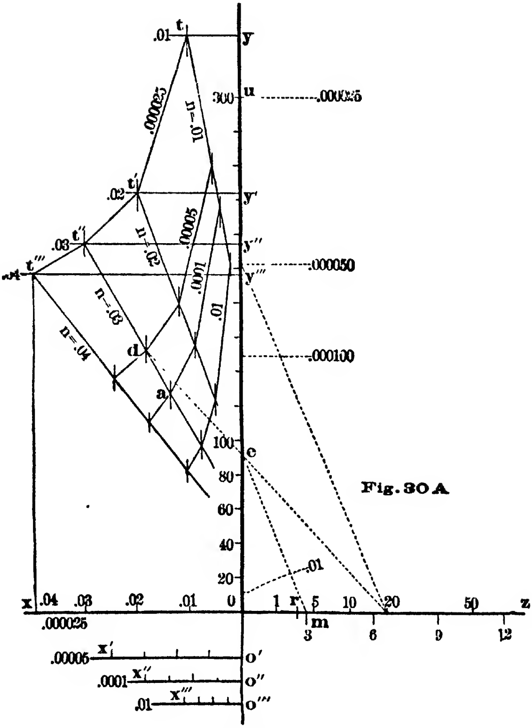

Fig. 30 A

$$w = 23 + \frac{.00155}{\text{flattest slope per unit of length}} \text{ for metric measure.}$$

For each value of n to be embraced in the diagram, say

$$y - w = \frac{1.811}{n} \text{ (English) or } y - w = \frac{1}{n} \text{ (metric).}$$

To each value of $y - w$, add w, thus obtaining values of y. We take .000025 per unit of length as the flattest slope, and .01, .02, .03 and .04 for n. Hence (using English measure)

$$w = 41.6 + \frac{.0028}{.000025} = 41.6 + 112 = 153.6.$$

$$y - w = \frac{1.811}{.01}, \frac{1.811}{.02}, \frac{1.811}{.03}, \frac{1.811}{.04}; = 181.1,\ 90.5,\ 60.4,\ 45.3 \text{ respectively,}$$

and $y = 181.1 + 153.6$, $90.5 + 153.6$, $60.4 + 153.6$ and $45.3 + 153.6$, or 334.7, 244.1, 214.0 and 198.9 respectively. Lay off these values of y on oy in pencil, as at y, y', y'', and y''', using the scale already laid off for c on oy.

From each point, y, y' etc, draw a hor pencil line yt, $y't'$ etc, and mark on it, in pencil, the value of n used in determining its height oy etc.

Next say $x = w \times$ *greatest* value of n. Make $ox = x$ by the scale of *sq rts* of R on oz. In our case $ox = 153.6 \times .04 = 6.144$ by the scale of *sq rts* of R, or $- - 6.144^2 = 37.75$ by the scale of R.

Divide ox into as many equal spaces (4 in our case) as .01 is contained in greatest n. Mark the dividing points with the values of n, as in our Fig.

From each dividing mark on ox erect a perpendicular, (xt''' etc) in pencil, to cut that hor line ($y'''t'''$ etc) which corresponds to the same value of n. The intersections are points in a hyperbola. Join them by straight lines $t'''t''$, $t''t'$, $t't$ etc.

From r in oz (corresponding to a mean rad of 3.28 ft, or 1 metre) draw radial lines, rt, rt', rt'' etc. Mark them "$n = .01$", "$n = .02$" etc, the same as their corresponding lines yt, $y't'$ etc.

For each slope (S) to be used in the diagram (except the flattest, for which this has already been done) say

$$x',\ x'' \text{ etc} = \left(41.6 + \frac{.0028}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for English measure.}$$

$$x',\ x'' \text{ etc} = \left(23 + \frac{.00155}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for metric measure.}$$

Thus, our slopes are = .000025, .00005, .0001 and .01 per unit of length. Hence,

$$x' = \left(41.6 + \frac{.0028}{.00005}\right) \times .04 = 3.904\,,\quad x'' = \left(41.6 + \frac{.0028}{.0001}\right) \times .04 = 2.784\,;$$

$$x''' = \left(41.6 + \frac{.0028}{.01}\right) \times .04 = 1.675.$$

Lay off each value of x', x'' etc from oy on a separate hor pencil line $o'x'$ etc, using the scale of sq rts of R as on oz.

Mark each line $o'x'$ etc in pencil with the slope used in fixing its length.

Divide each dist $o'x'$ etc into the same number of equal parts as ox. From the dividing points (which, like those of ox, represent the values of n) erect perps to cut the radial lines rt''', rt'' etc, each perp cutting that radial line which corresponds to the value of n represented by the point at the foot of the perp. The intersections corresponding to each line $o'x'$ etc form a hyperbolic curve. Mark each curve with the slope of its corresponding line, ox, $o'x'$ etc.

The drawing is now in the shape proposed by Mess Ganguillet and Kutter, and is ready for use in finding either c, n, R or S when the other three are given. Thus:

1st. Having R, S and n, **to find c.** For example let R = 20 ft, S = .00005, $n = .03$. From the intersection d of slope curve .00005 and radial line $n = .03$, draw* d–20 to the point (20) in oz corresponding to the given R. At e, where d–20 cuts oy, is the reqd c, = 96 in this case.

2d. Having R, S and c, **to find n.** For example let R = 20 ft, S = .00005, $c = 96$. Through the points R = 20 in oz, and $c = 96$ in oy, draw* d–20 to cut curve .00005. n (= .03) is found by means of the radial lines nearest to the intersection, d.

3d. Having S, n and c, **to find R.** For example let S = .00005, $n = .03$, $c = 96$. Find curve .00005 and radial line $n = .03$. From their intersection d draw d–20 through the point e showing $c = 96$. Its intersection with oz shows the reqd R, 20 in this case.

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* Instead of *drawing* these lines, we may use a fine black thread with a loop at one end. Drive a needle either into one of the points R or into one of the intersections, d etc. Slip the loop over the needle. The other end of the thread is held between the fingers and the thread is made to cut the other points as reqd. The diagram should lie perfectly flat, and the string be drawn tight at each observation, in order that friction between string and paper may not prevent the string from forming a straight line. Or the free end of the string may rest on a pamphlet or other object about ½ inch thick, to keep the string clear of the diagram. Special care must then be taken to have the eye perp over the point observed.

4th. Having R, c and n, **to find S.** For example let R = 20 ft, c = 96, n = .03. Through R = 20 and c = 96 draw d-20. S (.00005) is found by means of the curves nearest to the point d of intersection of d-20 with radial line n = .03.

The following addition to Kutter's diagram, proposed by Mr Rudolph Hering, Civil and Sanitary Engineer, New York,* enables us **to read the velocity from the diagram.**

Find the sq rt of the reciprocal of each slope to be embraced in the diagram $= \sqrt{\frac{1}{\text{slope per unit of length}}}$. Lay off these sq rts on the right of oy, using the scale of c already laid off on its left In our fig we have so proportioned the two scales that $\frac{c}{\sqrt{\text{recip of S}}} = \frac{1.5}{1}$. *Mark* the dividing points with the *slopes per unit of length.*

On oz lay off the vels to be embraced in the diagram, using the scale of sq rts of R already laid off on oz, and making $\frac{\text{vel}}{\sqrt{\text{R}}} = \frac{c}{\sqrt{\text{recip of S}}}$

1st. Having R, S and n; **to find v.** For example let R = 20 ft, S = .00005, n = .03. From R = 20 draw d-20 to the intersection d of curve .00005 with radial line n = .03. d-20 cuts oy at e, where c = 96. With a parallel ruler join R = 20 with S = .00005 on oy. Draw a parallel line through c = 96. It cuts oz at m, giving the reqd vel, 3.03 ft per sec.

2d. Having R, S and v; **to find n.** For example let R = 20 ft, S = .00005, v = 3.03 ft per sec. With a parallel ruler join R = 20 and slope .00005 on oy. Draw a parallel line through v = 3.03. It cuts oy at e, where c = 96. Through R = 20 and c = 96, draw d-20 to cut curve .00005. The point d of intersection, being on radial line n = .03, shows .03 to be the proper value of n.

Any line drawn to the curves from R = 3.28 ft or 1 metre, is one of the radial lines used in making the diagram. It therefore necessarily cuts all the slope curves at points showing the same value of n.

3d. Having S, n and v; **to find R.** For example, let S = .00005, n = .03, v = 3.03 ft per sec. *Assume* a value of R, say 10 ft. Find curve .00005 and radial line n = .03. Join their intersection d with R = 10 ft. The connecting line cuts oy at c = 82. With a parallel ruler join c = 82 with v = 3.03. Draw a parallel line through slope = .00005 on oy. It cuts oz at R = 27.3, showing that a new trial is necessary, and with an assumed R *greater* than 10 ft.

If R thus found is the same as the assumed one, the latter is correct. If they are nearly equal, their mean may be taken.

4th. Having R, n and v; **to find S.** For example, let R = 20 ft, n = .03, v = 3.03 ft per sec. *Assume* a slope (say .0001). Find its curve, and radial line n = .03. Join their intersection with R = 20, and note the value (89) of c where the connecting line cuts oy. With a parallel ruler join c = 89 with v = 3.03. Draw a parallel line through R = 20. It cuts oy at slope .000058, showing that a new trial is necessary, and with an assumed S *flatter* than .0001. If R is 3.28 ft, or 1 metre, the diagram gives the correct S at the first trial, no matter what S was assumed at starting. With any other R, if the diagram gives the same S as that assumed, the latter is correct. If the two differ but slightly, we may take their mean.

* Transactions of the American Society of Civil Engineers, January 1879.

Table of vels in Circular Brick Sewers when running full, by Kutter's formula, but taking **n** at .015 instead of his .013, in consideration of the rough character of sewer brickwork generally.

When running only half full the vel will be the same as when full but this is not the case at any other depth whether greater or less. At greater ones it increases until the depth equals very nearly .9 of the diam, when it is about 10 per cent greater than when either full or half full From depth of .9 of the diam the vel decreases whether the depth becomes greater or less At depth of .25 diam the vel is about .78 of that when full; and then diminishes much more rapidly for less depths. All this applies also to pipes.

The vel for any fall or diam intermediate of those in the table can be found by simple proportion. Original

Fall in ft per mile.	Diameters in feet. 2	3	4	6	8	12	16	20	Fall in ft per 100 ft.
	Velocities in feet per second.								
.1	.19	.27	.35	.50	.64	.89	1 10	1.34	.0019
.2	.30	.42	.53	.74	.93	1.26	1.56	1.84	0038
.4	.46	.65	. 80	1.08	1.39	1 81	2 20	2.60	.0076
.6	.59	.81	1 00	1.35	1.70	2 22	2 70	3.18	.0114
.8	.69	.95	1.17	1.57	1.94	2.56	3.08	3.60	.0151
1.0	.79	1 07	1 32	1.77	2.16	2 84	3 43	3.96	.0189
1.25	.89	1 21	1.49	1.98	2.42	3.17	3.8	4.5	.0237
1.50	.98	1 33	1.64	2.18	2 64	3 5	4.2	4.9	.0284
1.75	1.06	1 41	1.78	2.34	2.85	3.8	4.5	5.3	.0331
2.0	1 15	1.55	1 91	2.53	3.1	4 0	4.8	5.6	.0379
2.5	1.32	1.78	2.18	2.85	3 5	4 5	5 4	6 3	.0473
3 0	1 44	1.94	2 38	3.2	3 8	5 0	6 0	6 9	0568
3.5	1.58	2.10	2.58	3.4	4 1	5 3	6.5	7 4	.0662
4.	1.68	2.2	2.7	3.6	4 4	5 7	6 9	7.9	.0758
5.	1.90	2.5	3 1	4.1	4.9	6.3	7 6	8.7	.0947
6.	2.06	2.7	3.3	4.4	5 4	6 9	8 3	9.6	.1136
7.	2.2	3.0	3.6	4.8	5 8	7.5	9 0	10.4	.1325
8.	2.4	3.2	3 8	5.1	6 2	8 0	9 7	11.1	.1514
9.	2.5	3.4	4.1	5 4	6.6	8.5	10 3	11.8	.1703
10.	2.7	3.5	4.3	5.7	6.9	9 0	10.8	12.5	.1894
12.	2.9	3.9	4.8	6.3	7.6	9.9	11.9	13.6	.2273
15.	3.3	4.4	5.4	7.1	8.5	11.0	13.3	15.3	.2841
18.	3.6	4.8	5.9	7.7	9.3	12 1	14.5	16 7	.3409
21.	3.9	5.1	6.3	8.4	10.0	13 0	15.7	17.9	.3975
24.	4.2	5.5	6.8	8.9	10.8	13.9	16 8	19.2	.4546
27.	4.5	5.9	7.2	9.5	11 4	14.8	17.9	20.4	.5109
30.	4.7	6.2	7.5	9 9	12.0	15 6	18 8	21.5	.5682
35.	5.0	6.7	8.2	10 8	13 0	16.8	20 4	23.2	.6629
40.	5.4	7.1	8.7	11.5	13 9	18.0	21.7	24.8	.7576
45.	5.6	7.5	9.2	12.2	14.8	19 1	23.0	26.3	.8523
50.	5.9	8.0	9.7	12.8	15.5	20.1	24.2	27.7	.9470
60.	6.5	8.7	10.7	14.1	17 0	22 1	26.5	30.3	1.136
70.	7.0	9.4	11.5	15.2	18 4	23 9	28.5	32.8	1.326
80.	7.4	10 1	12.3	16.2	19.7	25.5	31.0	35.0	1.515
90.	7.9	10.7	13.1	17.2	20 9	27.0	32.3	37.1	1.705
100.	8.4	11.3	13.8	18 2	22.0	28.5	34.1	39.1	1.894

A vel of 10 ft per sec = 600 ft per minute = 36000 ft, or 6 818 miles per hour. About 5 ft per sec is as great as can be adopted in practice to prevent the lower parts of the sewers from wearing away too rapidly by the debris carried along by the water.

Art. 23. The rate at which rain water reaches a sewer or culvert, etc. **Burkli-Ziegler Formula.** See "European Sewerage Systems," by Rudolph Hering, C. E., in Trans. Am. Soc. C. E., Nov. 1881.

$$\text{Cub. ft. per second per acre, reaching sewer} = \text{A coef according to judgment} \times \text{Av. cub. ft. of rainfall per } \textbf{second} \text{ per } \textbf{acre,} \text{ during heaviest fall.} \times \sqrt[4]{\frac{\text{Av. slope of ground in feet per 1000 ft.}}{\text{No. of acres drained}}}$$

Coefficient, for paved streets, 0.75; for ordinary cases, 0.625; for suburbs with gardens, lawns, and macadamized streets, 0.31.

Note that 1 inch of rainfall per *hour* may be taken as equivalent to 1 cubic foot per second per acre. See Conversion Tables, pp. 235, etc.

Example. If an area of 3100 acres (nearly 5 square miles), with an average slope of 5 feet per 1000 feet, receives a maximum rainfall of 3 inches per hour, then, assuming a coefficient of 0.5, the rate at which the water would reach the mouth of a sewer at the lower end of the 3100 acres would be

$$0\,5 \times 3 \times \sqrt[4]{\frac{5}{3\,1\,0\,0}} = 0\,5 \times 3 \times 0\,2004 = 0.3006 \text{ cubic ft per second per acre;}$$

or 0 3006 $\times$ 3100 = 931.9 cubic feet per second, total.

Let the grade of the intended sewer be say 4 feet per mile; and, to avoid excessive wear of its brickwork by debris swept along by the water, let its velocity be limited to 6.3 feet per second, which may be permitted on occasions as rare as rains of 3 inches per hour, although, for tolerably constant flow, where liable to debris, it should not exceed about 5 feet per second.

Find, in table opposite, the diameter, 14 ft., corresponding, as nearly as may be, to a velocity of 6 3, and to a grade of 4 feet per mile. The area is 154 square feet. Hence, 154 $\times$ 6.3 = 970 cubic feet per second = capacity of sewer. To allow for deposits in the sewer, make the diameter say 14.5 or 15 feet.

Table of least velocities and grades for drain-pipes and sewers in cities, in order that they may under ordinary circumstances keep themselves clean, or free from deposits. (Wicksteed.)

Diam. in Inches	Vel. in ft per Min.	Grade, 1 in	Grade. Feet per Mile.	Diam. in Inches.	Vel. in ft per Min	Grade, 1 in	Grade. Feet per Mile.
4	240	36	146 7	18	180	294	18 0
6	220	65	81 2	21	180	343	15.4
7	220	76	69 5	24	180	392	13.5
8	220	87	60 7	30	180	490	10.8
9	220	98	53 9	36	180	588	9.0
10	210	119	44.4	42	180	686	7.7
11	200	145	36.7	48	180	784	6.8
12	190	175	30 2	54	180	882	6.0
15	180	244	21 6	60	180	980	5.4

Weight per foot run **of glazed terra cotta pipes** for drains, etc. **prices** per foot run adopted by the United Sewer Pipe Makers of the United States, March, 1887. For discounts, see Price List.

Drain pipe, with socket joint						Sewer pipe, with sleeve joint					
Bore	Wt	Price	Bore	Wt	Price	Bore	Wt	Price	Bore	Wt	Price
ins	lbs	$	ins	lbs	$	ins	lbs	$	ins	lbs	$
2	4	0 14	6	18	0.30	15	45	1.25	30	150	5.5
3	7	0 16	8	22	0 45	18	65	1.70	36	195	7.0
4	10	0 20	10	30	0 65	21	89	2.50	42	203	8.5
5	12	0.25	12	33	0 85	24	100	3 25	48	230	10.0

The joints are filled with cement mortar; or, when used for drainage only with clay. Drain pipes (3 to 12 ins bore) are about $\frac{3}{4}$ inch thick. A bend or branch costs about as much as from 3 to 5 feet of pipe. The 48-inch pipes are about 2 ins thick.

Art. 24. When the area of cross section of channel is reduced at any point, as by a dam (Fig 33, p. 576) or by narrowing it, either at its sides (Fig 32) or by placing in it a pier etc. Fig 34; a portion at least of the force of grav (which would otherwise be giving *vel* to the water up-stream from the point where the obstruction takes place), causes *pressure* against the dam etc. This pres maintains the up-stream water at a higher level than

would otherwise have. Said water is then practically in a reservoir; *i e*, it has less vel and greater pres than before. If the reservoir has no outlet, there is *no* vel; and *all* of the head, or force of grav, acting on the water is expended in *pres.*

But if there is an outlet, as over the dam, or between the piers etc, a portion *c o*, Figs 31, 33, 34, of this pres or head, is expended in giving *vel* (or an acceleration of vel) to the water escaping by that outlet; after which only so much head (in the shape of surface slope) is needed as will overcome the resistances of the channel down-stream from the obstruction, and so *maintain uniform* the vel given to the water by the head *c o*.

Where a large canal, such as those intended for navigation, is fed from a reservoir, the fall *c o* in feet is approximately

= mean velocity² in canal, in feet per second, × .017;

and in smaller canals, such as mill courses,

= mean velocity² in canal, in feet per second, × .02.

The abruptness of the fall may be diminished by rounding off or sloping the edges of the piers, or the corners at the sides of the channel (Fig 32) or the approach to the dam.

Fig 33 is a cross section of **Clegg's dam,** across Cape Fear River, N. C. It is from measurements made by Ellwood Morris, C E; by whom they were communicated to the writer. The dam is of wooden cribwork; and its level crest, 8 ft 5 ins wide, is covered with plank; along which the water glides in a smooth sheet, 6 ins deep, (at the time of measurement). At the upper end of this sheet, and in a dist of about 2 ft, a head *c o* of 9 ins forms itself, as in the fig.

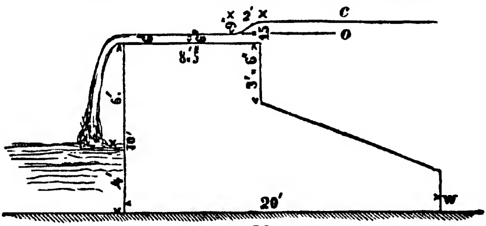

Fig 33

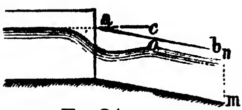

Fig 31

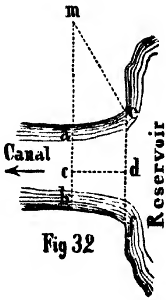

Fig 32

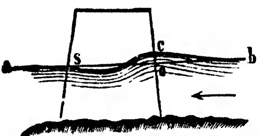

Fig 34

HOISTING, CONVEYING AND EXCAVATING MACHINERY

We here treat of the handling or transferring of objects or materials from place to place, as well as of excavation where that is involvd. (For Dredging, see p 581.) We endevor to outline the varius devices available that are of considerable use to the C E, together with their characteristics and general utility, without troubling too much about mechanical details.

0.1. Classification

We present first, in sections 1.0, 1.1, etc, the details of cranes and derricks and excavators in general, because to describe machines of all possible or even of all usual existing combinations would require many times the space here used. These details may be classified and are treated as follows:—

1.1, the grabbing, holding or digging device; such as the hook, sling, magnet, skip, scoop, bucket, scraper, skimmer, clam-shell, orange-peel, shovel, etc,

1.2, the means for connecting the source of power to the load; such as rope (hemp or wire);

1.3, the mast and the boom, jib, etc, for making possible the moving of the load closer to or farther from the pivot;

1.4, the means for rotating the load; such as a pivoted mast or post, bull-wheel, circular gear, etc;

1.5, the means, if any, for moving the machine over the ground; such as skids, rollers, wheels, caterpillars, etc;

1.6, the hoist, (gearing, drums, etc); and the power itself, such as animal, steam, gas, gasoline, oil, Diesel, comprest air, electric motor, or combinations of them.

0.2. Capacities and Utilities

of some of the machines as a whole, are given under 1.1 (to 1.19 incl); giving further descriptions of details, as cable, booms, power and movement over ground, in secs 1.2 to 1.6

0.3. Convertible Machines

in which different grabbing, holding or digging devices may be applied, are generally available, and in addition, machines may be made up of almost any combination of details from 1.1 to 1.6 inclusiv.

0.4. Other Complete Machines

such as elevating devices, cableways, conveyors, trenchers, back-fillers, etc, are treated in secs 2 to 9 inclusiv.

1.0. DETAILS

1.1. HOOKS, SLINGS, GRAPPLES, BUCKETS, SHOVELS, etc

Hooks, tho essentially very simple, should be scientifically designd, and should be made of material, such as soft steel forgings, that will yield before cracking, thus increasing the chance of detecting complete failure before it occurs. They may have many special forms for special purposes. Some hooks are provided with safety shackles or catches to prevent the load from slipping off; some with catches that may be releast for dropping the load suddenly, as in operating "skull-crackers" for breaking concrete or castings. Hooks are often

made double, reducing eccentricity of loading and affording more room for chains or slings. Two hooks may be framed with a pivot, like ice-tongs, or the two (or more) hooks may hang loosely from rods or chains, so that when separated the width or length of the load and attacht to it, they exert a hor component tending to prevent slipping. Hooks or chains may be attacht in varius ways to **platforms** or **skips**, on which the load (earth, rock, merchandize, packages, etc) may be placed. See also 1.18, scrapers. Hooks are depended upon for most of the work on building erection.

Slings are much used for hoisting. These are merely chains or rope with or without hooks or eyes, sockets or rings. The sling may consist of an endless loop of wire rope only wrapt around the object to be hoisted, tho more usually the ropes are fastend around eyes, Fig 1, one large enough to pass the other thru, or at one end they may be attacht to a hook. **Bridle slings** consist of a length of rope with a hook at each end and the middle past around a socket. When hoisting metal beams, etc, thin pieces of wood must be placed between the beam and the rope or hook. The wood crushes and increases the friction, reducing the danger of slipping. The two lengths should form an equilateral triangle with the beam or other object held hor'y. **Caution:** increasing the spread materially greatly increases the stress and danger of breaking. Several bridge companies have largely used chains of ½" to ¾" with rings at each end.

The first two items treated under 1.1 (hooks and lifting magnets) are not limited in their use to cranes or derricks, being often of use in other types of apparatus.

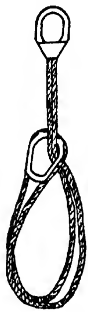

Fig. 1

1.12. Lifting Magnets

Electro-magnets have a number of advantages where direct current electricity is available, and where iron or steel are to be handled, especially scrap, or where many small or irregular pieces are to be handled. Iron or steel may be pickt up, and lowerd or releast, or dropt, all with fewer if any extra men for attaching and detaching. Magnets can handle and drop skull-crackers efficiently. They can lift iron or steel which is under snow, or when packt in boxes, or when hot. Of course, reliable current is essential for safety, tho automatic underhung hooks may reduce the danger. Such hooks may be regularly used for carrying loads of fairly uniform shape from place to place while current is shut off. A single hoisting line is sufficient where safety hooks are not used. Magnets are of course useless for anything but ferrous metals.

The **lifting capacity** depends largely upon the form of the iron or steel to be handled. A magnet 20" diam, will lift 3600 lbs of billets or slabs, and one of 65", 50,000 lbs. Skull-crackers of 2/3 of these weights may be lifted; but only about 1/20 to 1/30 of these wts of scrap iron. The great ease of handling irregular scrap, etc, compensates for the reduced attraction. The current consumption at 220 volts will range from 6 to 60 amperes. (From Material Handling Cyclopedia, Simmons-Boardman Publishing Co.)

1.12a. Vacuum Lifters have been used for seizing non-ferrous mat'ls. They are similar in operation to the pneumatic fingers used on printing presses for handling sheets of paper one at a time, and may be used on any material such as copper, zinc, wood, wall-board, etc, as well as iron or steel, having a smooth surface. The lifting capacity depends upon the area of the suction cups employd and may be so com-

puted. A valve controls the vacuum or air pressure for seizing or releasing the mat'l to be handled.

1.13. Buckets

The preceding devices, except perhaps platforms or skips, sec 1.1, cannot well handle granular mat'l, as earth, coal, ore, loose rock, etc, and for these buckets are of use. We first consider those that **can not dig**, and must have the mat'l deliverd into them, as by hand shoveling. Nearly all such buckets may be hoisted by a single hoisting line, to which they are attacht. The unloading is controld by a latch or trip operated by hand at the bucket, or by a light line puld from the hoisting engin or near by, or by a second line operating levers or doors.

1.131. Turn-over buckets, one type of which is ill'd in Fig 2, are so balanst that when not full they assume an upright position, but when loaded the tendency is to turn over. This is resisted by the latch or trip. Thus, after filling, the bucket is hoisted and swung and lowerd over the desired point, and the latch is tript. The bucket turns over, drops its load, and then automatically rights itself and relatches ready for the next loading. These buckets are made with either straight sides, curvd back or bottom, or as a vertical cylinder. For contractor's work they are made in sizes of 1½ cu yds cap down to ½ cu yd or less. It is difficult to deliver the contents of turn-over buckets exactly where wanted.

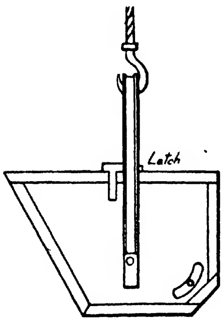

Fig. 2

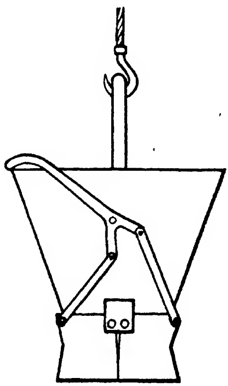

Fig. 3

1.132. Bottom-dump buckets, one of which is ill'd in Fig 3, are made in many forms and with many types of doors or hoppers operated with varius arrangements of levers too numerous to mention and too unimportant to describe in detail. In some types the load is dumpt all at once; in others the discharge is controllable, some having the hopper door designd especially for dumping (concrete, *e.g.*) accurately into narrow spaces. Capacities range from abt 2 to 3 cu yds down to 1 cu yd or less.

1.14. Grabs; Clam-Shells & Orange-Peels

Grabs may be described as buckets **capable of digging**, and those for land excavation are much the same as for dredging. See p 581*e*, &c, including paragrafs 45, 54 and 56.

1.141. Utility. Grabs may be used for either plain lifting or the handling of varius objects, or for digging loose or soft mat'l, as earth, coal, sand, etc, and to a certain extent both together, as where old timbers or debris are encountered in digging; but not hard mat'l. **Care** should be taken to obtain the machine best suited for the servis desired, especially a correct ratio of bucket-closing force to lifting force, which depends upon the number of sheavs and "parts" of hoisting cable (see sec 1.2) and leverage of arm (p 581*e*, paras 38, 39 and 40), shape of bucket bottom, teeth, etc. A bucket may cut so deeply that it becomes overloaded and the hoisting is retarded, or the cut may be so light that there is much waste motion. (See Engg-Contracting, '32 Aug, p 193). Clam-shells are obtainable with **electric motor** mounted on a bucket, to force the closing of the bucket. This has the advantage of a surer bite into the mat'l, and may be used with hoists having only one line, or on overhead tracks, tho the addition of the motor and wires increases complication.

1.142. Operation. As in dredging, the bucket is hoisted and lowerd, and opend and closed by means of two lines of cable. The rotation of the boom is accomplisht by the engin acting on a bull-wheel (p 581*k*, para 60), pivoted mast or circular gear (sec 1.4).

1.143. Capacities of land grabs usually employd range from ½ cu yd to 3 cu yds by ¼ cu yd diffs, and the corresponding wts are from abt 2000 lbs to 7000 lbs. Clam-shells of 16½ cu yds cap have been built, weighing 28,000 lbs

1.144. Dimensions. The following approx figures are taken from the catalog of a large mfr.

Bucket cap, cu yds	½	¾	1	1¼ to 1½
Boom length, ft	30	38	40	40 to 45
Max clear lift of bucket, ft	20	28	30	30 to 35
Max ht of boom, ft	35		43	47 to 52
Max ht of boom, lowerd, ft	12	12	12	15 15

1.145. Special forms, usually made up of bars or prongs, are used for handling **long materials** in quantity, as timbers, rods, pipes, etc.

1.146. Miniature or dwarf orange-peel buckets may be had small enough to dig holes only a foot in diam, for wells, test holes, etc. Owing to their small capacity, they may usually be operated by **hand**; but unless the mat'l is very soft, they require a heavy "hammer" (wt) mounted so as to slide on a vert rod above the bucket, to force the orange-peel points into the mat'l.

1.15. Dippers and Shovels

Shovels for land excavation, Fig 4, are much the same as shovels or dippers for dredging. **See p 581*j***, etc, paragrafs 58, 60-63 incl.

1.151. "Crowding" devices used to force the "stick," *T*, up or down thru the boom, *B*, may be operated either by a **separate engin** or motor on the boom, as usual in dredging, or the gearing may be operated by either **cable or chain**, *N*, driven from shafts from the main engin or motor in the cab. There are arguments in favor of and against each, but the differences seem to be slight

1.152. Capacities. The usual small shovels for the local contractor have a cap around 1 cu yd, the smallest holding ⅜ cu yd, and sizes up to 2 or 3 or more cu yds are available for larger operations. Special machines for strip mining used by the Electric Coal Cos at DuQuoin, Ill, had buckets holding 15 cu yds. (Eng News-Rec, '32 May 5).

1.153. Lips and Dipper Teeth. Manganese steel is largely used. Even so, dippers working in boulders in stream beds may wear out one set of teeth every 8 hours. Mr. George Sykes reports (Eng News-Rec, '33 Feb 16, p 215) that by welding hard surfacing to the bucket teeth with an alloy of cobalt, chromium and tungsten, their life can be increast more than ten times. Stelite is another mat'l used.

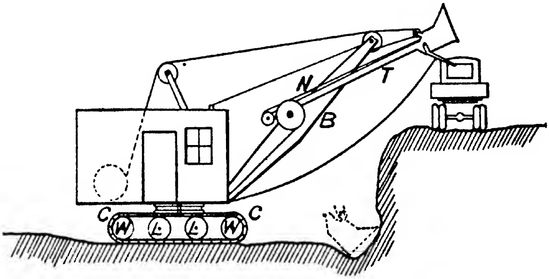

Fig. 4. Shovel

1.154. Utility. Shovels will handle much the same mat'ls as grabs (sec 1.14), and will dig much harder mat'l, even loose rock or ore They will easily handle mixtures of earth and debris, and are of great servis in digging ore or coal, tearing up road surfaces, old foundations, floors, walls, in excavation of basements, grading, sewer work, quarry excavation and gravel pits, irrigation and RR work. Special compact shovels for **underground work** are often satisfactorily used for excavating subways and tunnels.

1.155. Operating Speeds range as follows, the figures being in seconds for each operation of a complete cycle:—

	Loading	Swinging	Dumping	Returning	Total
Maximum*	16	7½	5¾	7¼	34
Average†	9.6	6.0	2.4	6.2	24.2
Minimum*	7	5¼	2	5½	21

the maxima being usual for poorly blasted rock, and the minima for good common mat'l.

1.156. Delays. The following are taken from figures given by Andrew P. Anderson, Roads & Sts, '34 July, p 255 *et seq*, obtaind from 51 power shovel highway jobs:—

Minor delays, per cent		Major delays, per cent	
Hauling equipment ..	9.9	Weather	14.9
Shovel, mvg, repairs.	9.1	Shovel, mvg, repairs..	9.5
Miscellaneus	14.2	Miscellaneus	6.0

* Eng News-Record, '29 Nov 28, p 860.
† Roads and Streets, '34 July.

1.157. Effect of Material on time req'd to load dipper; good earth, 5.7 secs; earth and some rock, cemented mat'l, 8.4 secs; poorly blasted rock, 10.3 secs; very poorly blasted hard granite, 16.7 secs.

1.158. Dimensions. The following figures are averages; the smaller sizes being taken from mfr's catalogs, while the larger sizes are those of RR power shovels from Mat'l Handling Cyclopedia by Simmons-Boardman Pub Co.

Capacity of dipper, cu yds	2/3 to 1	2½ to 6
Length of boom, B, ft	18 to 22	24 to 31½
Length of dipper handle, T, ft	14 to 17	17 to 20½
Dumping radius, max lift, ft	24 to 27	26½ to 32
Maximum height of cut, ft	19 to 21	
Maximum dumping height, ft	13 to 15	16 to 18½
Digging radius, ft	27 to 29	26 to 33
Height of boom at max angle, ft	24 to 30	
Max height, boom lowerd, ft	12 to 15	

In a 3/8 cu yd full revolving shovel,

with angle of boom	40°	60°	with angle of boom	40°	60°
max dumping ht, ft	12	17	max digging rad, ft	21½	19
max dumping rad, ft	18¼	14¾	depth of digging, ft	3½	2
max digging ht, ft	16½	21¾	swing of boom, ft	15¼	11

Some of the smaller machines may be transported on a heavy duty motor truck.

1.159. Miniature Shovels or Loaders are available, of somewhat different designs, but very compact, being but little if any taller than a man, some of them having no slewing device and depending upon their caterpillars for changing direction, yet they are mostly over ½ cu yd capacity.

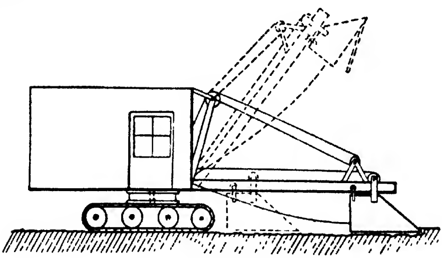

Fig. 5. Skimmer

1.16. Skimmers

A skimmer, skimmer scoop, or hor'l crowd shovel, Fig. 5, is akin to a shovel or dipper, but with more restricted movement. A dipper stick is not used, the skimmer bucket being arranged to be slid long'ly along the boom by cable from one of the engin drums. When digging, the boom is kept level or at grade desired, and the bucket is pulled out along the boom until full, when the boom is raisd and

turnd hor'y until over the truck or place where the mat'l is wanted. The load is dropt by either tilting the bucket down, or by unlatching a pivoted bottom.

Capacities from ½ to abt 2 cu yds. A ¾ cu yd machine can handle 400 to 500 cu yds or over of loose dirt per day.

Utility is limited chiefly to operations that might be called "skimming", *i e*, shallow excavating along the surface, tearing up old roads or highways of almost any mat'l. The machine is simpler than the dipper shovel with its boom and stick, and more easily levels off large surfaces with satisfactory accuracy.

Dimensions. In a ¾ cu yd machine; length of boom, 16'; lift, 16'; min ht of machine, 15'; max ht, boom raisd, 23½'; hor length of cut, 12½'.

1.17. Pull-Scoop,

also called Pull Shovel, Goose-Neck, Trench Hoe, Back Digger, or Back Hoe.

Pull-scoops, Fig 6, may be regarded somewhat as dipper shovels in reverse, except that the "stick", *T*, does not pass thru the boom, *B*, but is pivoted to the outer end of the boom, and the cutting bucket, *K*, is pivoted to the outer end of the stick, facing back toward the engin. It dumps by tilting. Pull-scoops are readily interchangeable with skimmers, sec 1.16.

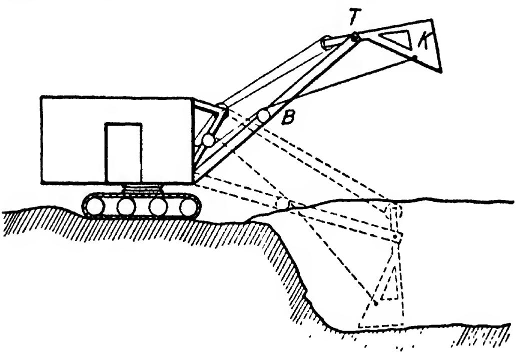

Fig. 6. Pull-Scoop

Capacities from abt ¾ to 2 cu yds. In hard digging in trench work, all day may be req'd for one 12' length of pipe; in easier ground, enough may be dug for 10 or 15 lengths.

Utility. It is useful for digging pipe, sewer or other trenches, as deep as 25'; but can be swung hor'y like a shovel or skimmer, so as to excavate areas as well. It can cut thru hard slate, shale, blasted rock or a foot of frozen dirt. See also 7.6, trenchers and ditchers. A hook on the bucket may be used for laying pipe.

1.18. Scrapers

1.180. Scrapers are of many types and are operated in a number of ways. They consist essentially of a strong box, usually open at one end and more or less on top, with a cutting edge at the open end of the lower side, and means for dragging along the ground. When the scraper reaches its destination, it is dumpt so as to spil out the mat'l. Scrapers may be puld by horses, tractors, cables or almost any other means.

Their **utility** lies in clay, soft shales, loam, sand, gumbo, gravel, etc. They become troublesome in wet mat'l, or mat'l containing large boulders or snags. Their size is varied according to the power used, and their form according to the kind of mat'l and the service to be renderd, such as gutter-work, back-filling, ditching, grading. In some types a **double bottom** is employd, the outer shell of which may be removed when worn thru, before wearing out the main body. In others **ribs** are attacht to the bottom, tho they seem to be of doubtful advantage.

Length of haul. As they are excavating as well as transporting devices, too large a percentage of their time should not be spent for hauling, so that it becomes better, if the haul is over 100′ or so, to resort to some other device or combination, such as power shovels and wagons or trucks, each of which may be kept working almost continuously at its own specialty. The economical limit of haul is variously set at anywhere from 100′ to 250′, tho some mfrs claim as high as 1500′.

1.181. Horse-drawn drag-scrapers or slip-scrapers, and wheel-scrapers. See pp 1029-1031...Art 12. (Note the necessity for pro-rating costs from the assumed $1 per 10-hour day). In mfr, the material of the scraper should have been bent or workt into shape when cold.

1.182. Tractor-drawn scrapers (on wheels or caterpillars) to be puld by tractors, may be very much larger than those puld by animals. Some are arranged to be drawn in trains of 3 or 4 units. Nearly every mfr has his own special form or device. One type is crescent-shaped; another is cylindrical in form (the axis of the cyl lying hor'y at rt angles to the line of travel) having an open side with a cutting edge in front, the dumping being accomplisht by permitting the cyl to rotate; another is bottomless, the earth being dragd along with it; while still other types, by means of a moveable lower lip, gather the mat'ls into a large bin, from which it may subsequently be dumpt or forst out. Some are reversible and may be dumpt by backing up. Some are controld from the tractor by cords or cables. Not a few are provided with **power control,** using oil or air to adjust and maintain the ht of the cutting edge, so as to control depth of cut, and either to dump all at once or distribute over a distance.

Capacities range from 1 or 2 cu yds up to 6, 8 or 12. The rate of earth handling varies too greatly with the size, length of haul, character of ground, speed of tractor, etc, to be stated even approx'y; but may ordinarily be estimated fairly closely by computation. Some mfrs have prepared tables or diagrams which include most of the factors. Most scrapers will drag along with them considerably more than their computed capacity.

Sizes of several makes of tractor-drawn scrapers; cutting width, 5′ to 10′; ht at back, 22″ to 48″; wts, 750 lbs to nearly 6000 lbs, and as high as 17,000 lbs in certain forms. Tractors to pull such scrapers must be capable of 10 to 30 HP or more; those of 12 cu yds cap, from 75 to 100 HP.

1.183. Derrick- or Crane-operated scrapers are usually called **Drag-Line Scrapers or Drag-Line Excavators.** They are much the same as where used for dredging; see p 581*b*, paragrafs 24, 25, 27 and 28. For this work, some buckets are constructed for rear or **back dumping** by being provided with a door or flap controld by a latch, as in dipper buckets, but they are usually dumpt by releasing the drag-line.

For **capacities,** see "Dimensions", below.

Utility. As an excavator, the drag-line is perhaps more generally useful than any other excavating machine. The bucket is lighter than other types, and this combined with the direct pull of the drag-line, makes possible a lighter and longer boom. In addition, the operator, by pulling the bucket toward the crane and then releasing, can throw the bucket beyond the end of the boom some 30% farther than the length of the boom. It can handle all loose earthy mat'ls, and do good work in blasted rock, or in handling boulders, stumps and roots. It has exceptional range of high and deep cuts. At Taylorsville (Miami Conservancy) machines excavated as deep as 63', and as high as 37'. It is accurate enough for effectiv loading of trucks and cars; and is largely used for mine stripping, drainage and canal excavating, levee building and trenching. It can build its own "trackway" ahead and clean up behind. Thus, in Miami Conservancy work, machines crost shallow rivers by maintaining "moving islands" for themselvs, which they kept adding to ahead as they crost, while they restored the river channel by digging out behind. About the only disadvantage is a tendency to cut irregularly, leaving an uneven surface. In the Miami work this was largely corrected by going over the ground afterwards with heavy drag platforms or frames. This resulted in an earth surface within 6" of that intended; a max variation of 1' in rock; and 2' under water.

Dimensions. The figures below are averages taken chiefly from the catalogs of well-known mfrs.

Capacity, cu yds....	½	¾	1	1¼	2½	5	10
Boom length, ft....	30	38	40	40	85	155	160
Dumping reach, ft, boom 25°.........	32	39	42	42	92	164	
Add'l throw beyond dumping reach...	13	13	13	13			40
Dumping, ht, ft, boom 40°.........	15	20	21	22	45	71	
Max ht, boom lowerd, ft...... ..	11½	12	12	15			
Depth of cut, ft....	16	18	19	21	58	75	
Pull on bucket, tons	6	8	10½	12½			
Drag-line speed, ft/min	110	100	116	110			
Rotating speed, rev/min	5¼	3½	3½	2¼			0.85

Machines up to and including 1¼ cu yd cap can usually be transported on flat-car without dismantling.

Operation. See p 581*c*, para 25. Drag-line scrapers can be used in many ways. A favorit method is to arrange them in "series", so that one can pass mat'l on to the next, and so on. For suggestions and accounts of plans adopted, see Eng News-Rec '31 Apr 16, p 636, etc, and '32 Jun 2, p 796, etc. **Speeds** range for the usual small machines from 200 cu yds/8 hrs for loading into wagons, 250 to 500 for excavation, 350 to 600 for ditch cleaning and RR drainage. **Delays** on the Miami Conservancy District: Englewood dam, 1920, from

1.7% to 18.1% in different months, but with the av for the season 7.7% for steam machines, and 7.8 for electric. On the Huffman dam, 1921, (electric), 21.6%. In all of these "waiting for cars" was responsible for half or more of the delay.

1.184. Cable-operated scrapers are treated under 4.1.

1.19. Pile Driver

attachments, consisting of vert hammer gides, may be attacht to the ends of the booms, and the hammer operated by the hoisting line.

1.2. CABLE

1.20. Hemp rope is seldom used for any prolonged hoisting or conveying operations, **wire** rope being used almost exclusively for such work. **See also** pp 1386-8 incl.

1.21*. Hoisting cable should be of a flexible type, hemp center, plow steel, known as 6 × 19, Lang lay. **Life** is about 50% greater than for drag-line (below).

1.22*. Drag-line cable as above, but with wire or wire-rope or wire-rope center. **Life;*** cable hauld 45,000 to 60,000 cu yds at a **cost** for cable of $2.06 to $1.22/1000 cu yds of excavation.

1.23. †Use and Care of wire rope are very important, the replacement cost being estimated as betw 50 and 80% of all maintenance. Bad handling may increase the cost 2 to 4 times, or even 10 times* the cost under careful handling Care should be taken to unreel a shipment of cable as a spool of thread is unwound, and to keep the unwound cable out straight and under control, so that it cannot slip out of hand and kink. It should be clampt before cutting to prevent untwisting. Operators are likely to be indifferent and to cause great damage to cable quite unnecessarily, especially by jerking or kinking The winding drum and all sheavs over which the cable passes should be of ample diam, from 400 to 500 times the diam of the individual wires, and kept in good alinement, the drum groovs should fit the cable, and if possible there should be not more than one layer of cable on the drum. Frequent lubrication is important, especially when the rope is to be idle for weeks or months, exposed to the weather A cable showing more than 6 broken wires in any foot of its length, or has 3 or more adjacent broken wires in the same strand, should not be used. (J. Feld, Civil Engg, '34 Dec, p 647). To minimize enforst idleness of machines, extra lengths of cable should be kept on hand ready for immediate use. There is probably no economy in obtaining any but the highest grade.

1.3. MASTS AND BOOMS, CRANES AND DERRICKS

Masts and booms, except where part of a mfd hoisting or excavating unit, are often specially built for the work in hand, and are usually of wood or steel. **Caution.** They must of course be carefully designed as long colums (but see recommended dimensions under 1.311 and 1.312, below)

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* Most of this information from "Construction Plant, Methods and Costs", by Chas. H. Paul, Miami Conservancy District, Dayton, Ohio, 1925.

† Chiefly from article by Herbert MacMillan, Eng & Contr'g, '31 Jul, p 175; and the Miami report above.

not neglecting ample allowance for overloading due to sudden stoppage of a descending load by the hoisting cable, accidental lateral impact, and, in the case of booms, for the wt of the boom acting on itself as a beam, making further allowance for side stresses due to sudden stoppage of the boom while being lowerd.

1.30. Materials

1.301. Wood is often used for masts and booms for moderate loads. Iron or steel **fittings** may be obtaind, such as foot blocks, mast steps and tops, boom seats, caps, stiff-leg straps, boom bands, back-leg bottom plates, sheavs in blocks ready for framing to the timber, etc, for the complete construction of derrick frames. See also 1.31.

1.302. Steel masts and booms are usually built-up structural members with latticing.

1.303. Aluminum alloy booms have been built as long as 175′ or more.

1.31. Derricks

Generally speaking, a derrick is a hoisting device in which the mast (carrying sheavs at or near the top over which pass ropes controlling the load and the end of the boom) is kept vert by means of guys or struts, and is not capable of movement over the ground without dismantling. (Towers used for oil wells and for pile driving are called "derricks" also, even tho they are usually stiff enough to require no lateral supports at their tops).

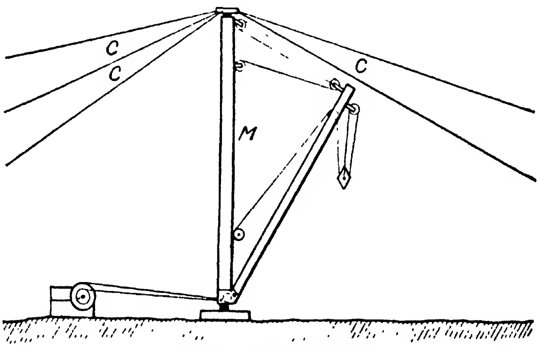

Fig. 7. Guy Derrick

1.311. In a **Guy** derrick, Fig 7, the mast, *M*, is kept vert by means of wires or cables or rods, *C*, *C*, *C*, in tension. Theoretically, three should be sufficient, but it is well to have five, so that in case one breaks, the derrick will not fall. The safe sizes for such guys may be computed, knowing their inclination, the wt and angle of the boom, and the loads to be handled, making ample allowance for jerking of

the load. The following dimensions for wooden guy derricks are recommended by Mat'l Handling Cyclopedia;—

Capacity Tons	Mast	Boom
1½	34′ × 8″ × 8″	25′ × 6″ × 6″
6	50′ × 12″ × 12″	40′ × 10″ × 10″
16	60′ × 16″ × 16″	50′ × 14″ × 14″

Probably 40-ton capacity derricks, and even smaller, are better made with steel masts and booms.

1.312. In a **Stiff-leg** derrick, Fig 8, the mast, *M*, is kept vert by two or more stiff members, *P*, *P*, inclined at abt 45°, spaced abt 90° in plan. For light or "home-made" derricks, these may be simple timbers, similar to the mast and boom, but are usually reinforst at their middles either by additional struts, *S*, *S*, to the ground, or by added thickness or "trussing", for they must be able to sustain their own wt as inclined beams, and to take compression, and are usually nec'y longer than the mast. The following dimensions for wooden stiff-leg derricks are recommended by Mat'l Handling Cyclopedia;—

Capacity Tons	Mast	Boom
1½	16′ × 8″ × 8″	25′ × 6″ × 6″
5	26′ × 12″ × 12″	40′ × 10″ × 10″
12	33′ × 16″ × 16″	50′ × 14″ × 14″
33	36′ × 22″ × 22″	55′ × 20″ × 20″

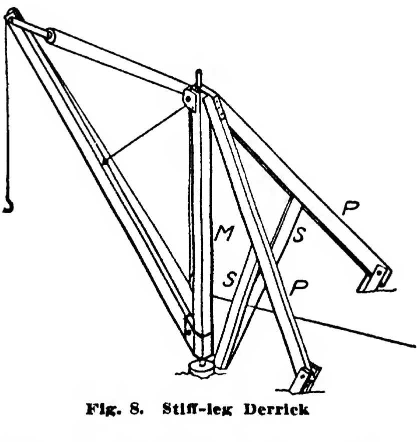

Fig. 8. Stiff-leg Derrick

The largest of three stiff-leg derricks used in the construction of the Boulder (formerly Hoover) dam, had a boom length of 180′; mast ht, 75′; legs of abt the same length, since they bore almost hor'y against the canyon walls; and had a capacity of 12 tons. (Eng News-Rec, '34 Jun 28, p 832). Others have capacities of 50 tons or more.

1.3121. A **Jinniwink** is a stiff-legd derrick in which the lower ends of the legs and of the mast are attacht to the ends of a hor T-shaped frame of timbers, which can be lasht to the floor of a building in course of construction, and then moved about from place to place as req'd. Capacities, 3 to 5 tons.

1.313. A **Gin Pole** consists of a single pole, slightly inclined and supported by four guys. The hoisting line passes over a sheav near the top. It can handle loads only vert'y under the upper end. It can be readily rigd up on the job, and so may frequently be of use, as may also

1.314. Shears, which consist of two poles mounted like the A-frame of a dredge, and similarly inclined, being held in position by a back-guy.

1.315. Tripods, consisting of three stiff legs, need no guys, but can handle loads only well within the space embraced by them. These are dangerus on a hard smooth surface unless the feet of the legs are properly tied to each other to prevent spreading.

1.316. Sulkies. It may be worth while in the case of small rigs (tripods, shears, etc) to mount a pair of wagon wheels on an axle against two of the members, the axle being horizontal. By tipping the outfit over and bringing the wheels to the ground, it may be more readily moved from place to place.

1.32. Cranes

In the predominating type of crane there is no tall mast or tower requiring guying. The hoisting and boom lines are attacht to the engin or motor or to the base or to an A-frame such as that in dredges, the device being kept upright by virtue of the width of its base and the wt of the engin or motor and cab (mounted well to the rear) and possible counterweights.

Utility. Probably no other type of machine for hoisting, conveying and excavating is more versatil and in more general use. By varying the boom and attachments, nearly all makes can be used, as simple hoists, as shovels (Fig 4), or to handle clam-shells or orange-peels, skimmers (Fig 5), drag-lines, backdiggers (Fig 6), or pile drivers. For their respectiv utilities (except hoists) **see secs 1.14 to 1.18 incl.** **Care** should be taken that the load does not tip the crane. Tipping depends upon the dist of the load from the center of the crane, as well as its wt, and possible jerking by the hoister.

Dimensions, etc. The following data are closely representativ of average practice, there being but small variations between different makes,—

Dipper of bucket, capacity, cu yds..	⅜	½	¾	1
Boom length				
for clamshell or dragline, ft......	25-35			35-40
for shovel, ft....................	16	18	18	18
Dipper stick, for shovel, ft.........	12	14	14	14
Travel speed, 15% grade, mi/hr....		1.6	1.5	1.5
" " 30% grade, mi/hr....		0.4	¾	¾
Overall length, ft..................	9½		11	13
Working weight, thousands of lbs, approx	20		50	60
Swing speed, rev/min..............		4.8	4.6	4.5
HP (gasoline).....................	42	47	70	80

In the cranes "Hercules" and "Ajax" used on the Panama canal (Eng News '15 May 13) there was a tower, from the top of which the boom projected, but it was not guyd, and obtaind its stability from the barge on which it was

mounted. A floating crane of 200 tons cap was used at the Port of Havre (Eng News-Rec, '34 June 14). Cranes of similar design, sometimes referd to as **"Portal Cranes"**, are largely used on docks.

1.321. Jib, Pillar or Foundry Cranes. While some are maintaind upright by means of a post or mast embedded in the floor, they are usually supported at their tops by framing to a wall or to the ceiling or roof. Either an inclined jib or boom is held up by connections to the top of the pillar, in which case a load can be handled only from the boom's end, or else the arm extends hor'y from the top of the pillar, and a hoist (sec 2 2) is mounted on a trolley moveable hor'y along the boom, in which case a load can be handled anywhere within the circle swept by the boom. Jib cranes are usually shop fixtures and have little use in eng'g construction.

1.322. Hammerhead cranes have a boom permanently hor which is mounted on a tower (without guying), the load traveling along it as in jib cranes. These too are generally used in permanent positions, as on docks, supply yards, etc. A notable exception was their use on the towers of the West Bay crossing of the San Francisco-Oakland Bay bridge (Construc'n Methods, '35 Aug, p 33).

1.4. ROTATION, SWINGING OR SLEWING

of the boom is accomplisht in different ways. **In derricks** mere pivots are provided at the tops and bottoms of the masts which may then be swung by hand or by power by means of a bull-wheel, (8' to 16' or more in diam). **In cranes** the boom is usually mounted on the platform or "base" on which is mounted the engin or motor. This platform acts as a turntable, riding on wheels or rollers arranged on the circumf of a circle, and resting on the floor beneath or upon the frame of the car or truck which is provided for motion over the ground. Rotation is effected by the engin or motor driving a vert shaft which extends down thru the platform, carrying a small gear which engages with a large gear mounted on the frame below. Thus there is usually no limit to the angular motion of such a crane, sometimes referd to as a **"whirley"** or **"whirler"**.

1.5. HORIZONTAL MOTION

Many methods are available for moving the hoisting, conveying or excavating machine as a whole, aside from such hor motion as can be obtaind by trolley on jib crane, by changing elevation of boom, etc, depending largely upon the kind of surface to be traverst.

1.51. via Water

Machines mounted on vessels may of course be moved wherever there is sufficient draft. While sometimes it may be practicable to have the boom reach over land, so as to serv for hoisting and conveying, the usual use is for dredging, or excavating under water. **See p 581, et seq.**

1.52. Skids

may be used where the machines are not very heavy, but in view of the many modern methods available, they need seldom be resorted to.

1.53. Rollers

rolling on prepared plankways constitute an improvement on skids in that they greatly reduce friction.

1.54. Flanged Wheels on Track Rails

are a further development and further reduce friction, but require more elaborate preparation. If the ground is hard, the preparation may be easy; but if soft, it may be advisable to construct a track almost as elaborate as for a RR. Indeed, for heavy machines, two standard gage tracks may be advisable or necessary. This is often used for derricks, ordinarily immovable without dismantling. For the building of dams, long buildings, stadiums, elevated RRs, etc, rails may often be laid along the structure. Where two rails may naturally come at different elevations, the framework under the derrick may have legs of correspondingly different lengths.

1.54a. A **Locomotive Crane or Shovel** consists of a crane mechanism (see sec 1.32) mounted on a railroad flat-car or frame, on one or preferably two four- or six-wheeld trucks (usually standard RR trucks), the wheels being driven by power transmitted down thru the central shaft or pivot, from the engin or motor.

Utility. Within reach of a RR track, its uses are approx those of other cranes, being largely used not only for wrecking, but also for digging ditches, taking care of embankments, driving piles, erecting bridges, etc.

Dimensions, etc.

Capacity, tons	15	20	25
Boom length, ft	40	45	50
Speed, mi/hr	2-4	2-4¼	4½
Max grade, %	8½	8½	
Min curv, rad, ft	55	55-60	60
Overall length, ft	25	25	
Weight, thousands of pounds	83	120	142
Swing speed, rev/min	4	4½	4-5
HP (gasoline)	80	97	140

1.55. Wheels on Ground

Metal tires or rims of great width with deep markings are sometimes used with satisfaction where the roadway is fairly good. But it is generally more satisfactory to use

Rubber tires, which are of four main types; (1) solid, (2) "cushion", with a hollow tunnel thruout its length, or other forms of hollowing, (3) "high pressure", using from 50 to 100 lbs/sq in, and (4) "balloon", using from abt 30 to as little as 5 lbs/sq inch in the super-balloon or "doughnut" tires. Where soft or wet ground may be encounterd, the larger tires are preferable, as they exert lower unit pressures on the ground. Apparently, suction greatly vitiates the law stated under "2nd" on p 410, and the grooving or other surface patterns used afford greatly exaggerated roughness, thus greatly increasing the traction of the larger tires over that of the smaller solid and cushion tires, so that the greater **care** req'd for maintenance is well worth while if actually put into effect. There should be careful application or attachment of tire to rim, correct wheel alinement, no overloading, proper air pressure, moderate acceleration and deceleration, no skidding, avoidance of sharp objects in the road, and no high speed with heavy loads or at sharp turns. Where even a small fleet of trucks is employd, there should be one or more men detaild to attend to tires, their inflation, cuts and punctures and loading. Bonuses to drivers for increast tire mileage have proved profitable. Much can be done by a skilful driver to **avoid slipping** (which reduces tractiv effort, increases danger of accident, and increases wear) by applying power gradually when starting, and stopping by applying brakes

carefully and leaving the engin in gear until almost stopt. At any given speed, the **impact** of a solid tire over a given obstacle or road roughness has been found to be abt double that of a balloon tire. **Dual** or double tires, in which two wheels with tires are used side by side in place of a single wheel and tire, are frequently used for rear wheels and for trailers, to carry heavier loads. Additional care is needed with duals to avoid unequal division of the load betw them, as by running the outer one off the edge of a road, or using unequal pressures. As high as 16 doughnut tires are sometimes used under trailers, in which cases equalizing devices are provided to distribute the load evenly.

1.56. Caterpillars, Crawlers

(We here treat only of the running gear that goes by these names. Tractors using caterpillar mechanism are also called "Caterpillars", but are treated in sec 6.44).

1.561. Construction. The caterpillar or crawler running gear consists of two endless bands, *CC*, Fig 4, one on each side of the vehicle, tractor, crane, excavator or other machine to which it may be applied. Each band passes around two large wheels, *WW*, with teeth or sprockets with which it engages, the two rear or front wheels doing the driving. There may be two or more additional wheels, *LL*, on each side, to keep the entire lower portion of the band prest down against the ground. Each band is several ft wide and the length on the ground is 5 ft or much more, giving a bearing surf of 25 sq ft or more, far in excess of what even a large number of balloon tired wheels can afford, thus greatly reducing the penetration of the driving mechanism into the ground and the depth of the resulting hole continually encounterd. Usually each band is made up of a series of plates of hardend steel, linkt together, often provided with transvers ridges, webs, grouters or grousers; but continuus rubber belts have been used as well. In some makes, the metal belts are readily detachable, when the machine may run on its wheels at higher speed. The device is, in effect, a locomotiv running on a cog ry or rack track which is automatically laid on the ground ahead of the wheels and taken up behind them and brought forward again. None of the wheels are arranged for **steering or turning**, and this is accomplisht by retarding the band on one side, and maintaining speed on the other. With one belt held fast, the machine may be turnd 180° in almost if not quite its own width.

1.562. Operation is effected almost invariably by a gasoline or oil engine, with clutch and brake for each band. **Speeds** may range from 2 to 10 mi/hr, but at higher speeds, the machine becomes cumbersome and noisy with much impact due to the flapping of successiv plates onto the ground.

1.563. Horse-powers and weights depend upon the type of machine to which the caterpillar is applied, such as tractor, excavator, etc, *q v*.

1.564. Utility of caterpillars is very great, and altho super-balloon tires have been encroaching on their field where the road or ground is not too difficult, yet caterpillars excell for use on rough and irregular ground of sand, muck or mud, with steep grades. They may be used under tractors, cranes, excavators, etc.

1.57. Walkers

are machines provided with a device on each side that acts together as legs and feet, tho both move simultaneusly, and the whole machine is advanst much as a man walks on crutches. Each leg is driven by suitable gearing, and to its lower end is attacht a hor member or foot whose lower surf is amply large to support the entire machine without sinking into soft ground. Normally, when digging (it is usually applied to drag-lines) the machine rests on its own base and the feet are raisd several feet from the ground. To "walk", the mechanism causes the lower ends of the feet to be thrust forward, then down against the ground until the base of the machine is lifted, and then, by a rearward thrust of the legs and feet, the machine is skidded forward. By repeating the steps, the machine may be advanst as desired. The direction of travel may be changed by merely rotating the machine, with its walking mechanism, on its base, as other cranes and shovels are rotated. As ordinarily mfd, "walkers" have booms 35′ to 175′ long, and can handle buckets up to 20 cu yds. Each "step" is about 7′, and walking speed about 18′/min.

1.58. Trolleys on Monorail

or overhead tracks are seldom practicable for engineering construction on account of the difficulty of constructing and shifting the tracks as the work progresses. They are used mostly in shops. The cableway, (sec 4) is usually far preferable for construction work. Trolleys are used on the hor jibs of cranes (sec 1.321) or on traveling cranes (sec 1.59), or on overhead tracks or monorail, or cableways. The trolley may be a mere car or truck carrying a hook hoist, propeld by pulling or pushing the hanging load, or the wheels may be geard for propulsion by pendant hand chains or other means, or by a motor on the trolley, operated either from the ground, from a distance, or from an operator's cage on the trolley. The trolley usually runs on four wheels bearing on the lower flanges of I-beams supported by their upper flanges. Such tracks may be provided with moveable sections acting as switches or turntables.

1.59. Traveling Cranes

In these, loads are handled from a trolley running on a "bridge", which in turn runs at rt angles on tracks on girders near the roof. They are usually permanent installations in shops, and not practicable for eng'g construction. But see sec 1.54.

1.591. Gantries or Bridge Cranes are similar to traveling cranes, except that the "bridge" is usually stay'y, or, if it is moveable, the colums or towers on which it is mounted run on tracks on the ground.

1.6. POWER MECHANISM AND POWER

1.61. Mechanism

For practically all of the hoisting and excavating machinery described in section 1.1 and its subdivisions (1.11, 1.12, etc), and also in 1.31 (derricks) and 1.32 (cranes), there is used or may be used one or another hoisting mechanism described in 2.22 and subdivisions, the number of gears or drums and lines depending upon what is to be controld, which may include hoisting (2 lines for "grabs"), "crowd", boom elevation, slewing, and movement over ground.

1.62. Power

The engin should be capable of exerting great force at starting, and at low speeds, especially where digging is involvd. The fuel should be cheap and such that it can readily be transported to the machine and stored, and it should not injure the engin. The engin should be of a type readily understood and operated.

1.621. Animals. A caution may be in order at least to consider the use of animals for certain purposes. Scrapers and light graders can be had especially designd to be pulled by **horses,** mules, etc. While the probable economy of horse *vs* mechanical power can be estimated for any given job, other considerations may enter, such as the availability of power machinery and men suitably skild in its use, as contrasted with that of horses and farmers or others (comparativly unskild with machinery) to handle them. See also pp 683-88 incl. **Elephants,** obtaind from circuses, sometimes available during the winter, have been used for miscellaneus handling of logs, poles or other objects, and even the pushing of vehicles as heavy as freight cars.

1.622. Electricity, where and when available, is especially good for the slow heavy starting pull, and has simple control. See also p 581 *t*, para 116.

1.623. Steam is especially good for the slow heavy starting pull, and has simple control, but the boiler's great weight is useful only as a counterweight, and the boiler may be troublesome on account of poor water, the fuel is cumbersome (even if oil is used), it is slower to start after shutting down, and is much less efficient than an internal combustion engin. By 1930 it had almost gone out of use except for locomotive cranes.

1.624. Gas (not gasoline) being generally not available, is seldom to be considerd.

1.625. Gasoline is still (1937) the most generally favord. Gears and clutches have been so developt that it can largely overcome slow heavy starting pulls. It is more efficient than steam, and the fuel is available almost anywhere.

1.6251. Gasoline-electric, in which a gasoline engin generates current for an electric motor or motors, has been resorted to and of course avoids many gears, and can exert great starting force, but it is not generally used. Initial cost, involving not only a gasoline engine and one or more electric motors, but also a dynamo, must be comparativly high.

1.626. Oil engins cost somewhat more than gasoline engins, but the operation is as simple and the fuel cheaper.

1.627. Diesel engins are being developt until they are now (1937) largely replacing gasoline engins. They are more capable than gasoline for slow heavy pulls. They cost about twice as much, but the fuel costs only about 25% as much as gasoline. The choice depends chiefly on whether the future use of the machine will effect suff saving to pay for the greater first cost and interest as compared with gasoline engins. They are more complicated and require special devices or engins for starting.

1.628. Comprest Air is largely used where it is available, as where rock-drilling is being done.

2.0. ELEVATING DEVICES

2.1. JACKS

Jacks are compact and readily portable devices capable of moving (especially lifting or lowering) heavy objects thru very short distances only, but can often be used for precise movements in very crampt places where other devices could not operate satisfactorily.

2.11. A Mechanical Jack, Figs 9, a, b and c, consists of a frame, *F*, which may be placed on the ground or other solid support, thru which slides a plunger, *G*, carrying a bearing plate, *B*, at its upper end, or a step (not shown) on one side, for supporting the load to be lifted, or for pushing laterally or otherwise. Between the frame and the plunger there is gearing which makes it possible to lift the load by means of a moderate force applied to the end of a lever. (Only the socket, *L*, into which the lever is fitted, is shown) In many cases this is merely a heavy **screw**-thread on the plunger, and the plunger is turnd by a stick or rod past thru holes in it, the top bearing plate being swiveld; or Fig 9a, the plunger, *G*, is turnd by means of a bevel gear turnd by a hor shaft with a square hole into which is fitted a square-ended rod (not shown) with a crank on the other end. Or, in the **rack-and-lever** jack, Fig 9b, the plunger carries a rack, *R*, which engages with gears or against which one or more pawls operate from a lever. Additional devices are provided either for reversing the action, so that the load may be lowerd step by step, or dropt at once. In some cases other kinds of gearing are employd, but these are the more usual The load can be held by the jack only at the stept levels determind by the ratchet—and not at any level as in the screw or hydraulic types.

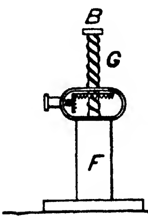

Fig. 9a
Screw
Jack

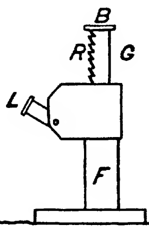

Fig. 9b
Rack-and-
Lever Jack

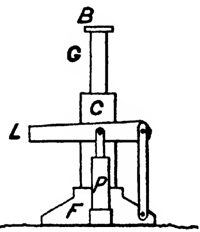

Fig. 9c
Hydraulic
Jack

2.12. A **Hydraulic** jack, Fig 9c, likewise consists of a frame or plunger, but the frame contains chambers filled with liquid. A lever actuates a pump, *P*, with a very small piston, which pumps the liquid from a storage chamber into a large cylinder, *C*, into which fits another piston carrying the plunger, *G*. Check and other valvs are provided for holding or lowering the load. Usually the pump is containd within the jack, tho sometimes it is separate and is connected with the jack proper by flexible metallic tubing.

2.13. Jacks on wheels are available for lifting one end or corner of a load, as an automobile, and moving it around on any smooth hard surface.

Commercial jacks may be obtaind in a great variety of sizes and capacities, from light one-ton capacity jacks for light autos, up to 50 and 75-ton cap for jacking up ry cars and locomotivs and structures generally. The ht may be anywhere from ½′ to 3′. The lift is usually a fraction of the ht, 1/3 to ½ or more, tho some, with double screws, have a lift as great as or greater than their own ht. For greater lifts, the load, when raisd as far as possible, must be blockt up, and the jack then shifted so as to raise the load again, and so on. Wts of jacks may be from abt 10 lbs to 400 lbs or more. Small trucks may be had for transporting heavy jacks.

2.15. Pneumatic or other power may be applied to some types of jacks.

2.16. Melting Ice Cakes were used to lower telefone conduit abt 3′ in 500′ sections weighing 60 tons. Every 6′ the earth under the conduit was removed and replaced by blocks of ice 21″ × 22″ × 11″. The remaining earth was then excavated gradually as the ice melted. The conduit sank abt 1/3″/hr with air temp around 55°. The operation was very satisfactory. (Construction Methods, 1935. July). Melting ice has been used in other cases.

2.17. Sand Jacks are sometimes used for the lowering of heavy loads such as arch centers, plate girders, etc, and may obviate much cribbing and shifting of mech'l jacks.

In lowering a 1000 ton drawbridge over the Passaic R at Newark, N J, dry sand was confined in a box open at the top, 4′ wide, 52′ long, abt 11′ high, built of 12″ × 12″ timbers. Four plungers fitted into the box with ½″ clearance, and settled without jerking, on releas of sand thru 2″ holes in the bottom. Pressure on sand, 2,815 lbs/sq in. Sand would not flow from holes in side.

In lowering two 142′ girders, wt 121 tons each, at Glenview, Ill, (Eng News-Rec, '36 Oct.8), the boxes were 3′ wide, 11′ long, 12′3″ high, of timbers 8″ × 16″. Heavy timbers rested on the sand, coverd by a bearing plate on which rested the girders. Lowering was accomplisht by men standing on top of the jacks, removing the sand with small shovels.

Sand jacks used in Switzerland (Eng News, '11 Jan 5) were metal cyls, diam 26.8″, ht 39.4″. Metal piston, fitting closely, was 31½″ high. Total lowering, 24″. Load 282 tons, or 1000 lbs/sq in on dry quartz sand from sand-blast work. Hole in bottom 2½″ diam. Sand flowd slowly and uniformly. Operation very satisfactory. A sheet of lead draped over top of jack acted as a hood to prevent entrance of water.

2.2. HOISTS

There is much ambiguity regarding the word "hoist", for the name is applied, usually without any qualifying word to distinguish, to (1) hoisting devices which hang from a support above, as described in sec 2.21, and (2) hoisting devices resting on the ground or other foundation, as in sec 2.22. To distinguish betw the two, we call the first "Hanging Hoists", the smaller sizes of which especially are called "Winches"; and the second "Drum Hoists". Hoists, unless mounted on trolleys, booms, or otherwise, move the load only vert'y.

2.21. A Hanging Hoist

consists of a frame, usually with a hook or other means at the top for attaching it to an overhead support, containing an assemblage of gears or other means for applying power, and a hook or other grabbing device hanging from the gears or sheavs, for attaching the load. It may be of use for light loads, where high efficiency and high speeds are not required.

2.211. A Hand Hoist is operated by a "hand chain" which hangs in a loop below the gearing. Pulling down on one side of the loop, the load is hoisted; pulling on the other side, the load is lowerd; while provision is made for holding the load in position when there is no pull either way on the hand-chain loop. The gearing may be either a worm gear, a train of one or more spur gears, a planetary gear, a differential gear, or any one of several other special types. The **worm gear** or **screw hoist**, can handle loads but slowly, tho a heavy load requires a comparativly easy pull on the hand-chain; while it requires no brake to hold the load in position. Owing to the friction of the worm, its efficiency is low, around 40%. Capacity from 500 lbs to 6000 lbs or more. **Spur** gears require special additional mechanism for holding the load when it is not being operated, but their efficiency is high, around 80% to 90%, and the ratio of load to pull may be almost anything, depending upon the number of gears in the train. In the **differential** hoist, only one chain is used for both hand and load. The chain is continuous and hangs in two loops, one loop hanging free for pulling by hand, and one under a sheav to which the hoisting hook is attacht. The two upper loops pass over a sheav having two groovs, one a little larger than the other. Thus, the larger sheav hauls up its length of chain a little faster than the smaller one lets down its length of chain; and these two lengths, passing under the hoisting sheav, thus move the load much more slowly than the chain itself moves. Lowering is accomplisht by reversing the motion. Owing to internal friction, this hoist requires no brake, but its eff'y runs from 28% to 38%. Cap, 500 lbs to 6000 lbs or more.

2.212. An **Air or Pneumatic Hoist.** The simplest of these is a mere **cylinder,** containing a piston or rod, to which is attacht a hook or other device for attaching the load. It may be operated by varius combinations of air pressure, as (1) pressure only below the piston, (2) variable pressures below and above, or (3) full air pressure below and variable above. (2) or (3) give better control than (1) by tending to prevent sticking of the piston. There may be in addition an oil control which affords even more definit movement. Cylinder hoists have the advantage of quick movement, but the amount of movement is limited by the length of the piston, the stroke being from 4′ to 8′. These may be had with cyls from 2″ to 24″ diam, operated by pressures from 60 to 100 lbs/sq in, with lifting capacities from 175 lbs to 40,000

lbs, using from 1 to 100 cu ft of free air for a 1' lift For greater movements, **air-motor** hoists may be used. These are two- or three-cyl air motors, usually employing oscillating or rotating cyls, with a max lift of 20', and capacities from 1,000 lbs to 10,000 lbs.

2.213. Electric Hoists, tho generally heavier than air-hoists, are very serviceable, where 110 to 550 volt, D C or A C is available. Motor, gearing and hoisting drum are all mounted together compactly. Control may be had from the floor by means of pendant cords, or, especially where the hoist is mounted as a trolley running on overhead track, (see sec 1.58) the operator may ride with the hoist, with or without a cab; or by "remote control" from a convenient point away from the hoist They are of course supplied with one device or another, usually automatic, for holding the load when it is not being operated. Capacities run from 500 lbs to 40,000 lbs, with lifts from 30' to 10', respectivly.

2.22. Winches and Drum Hoists

are usually mounted on the ground or engine- or crane-platform.

2.221. Hand Winches or Crabs. The simplest form of winch consists of a crank or cranks turning an axle, geard to turn a drum on which a rope is wound, all being held in place by a wooden or metal frame. Capacities, up to 1½ to 2½ tons. To hold the hoisted load, either a worm gear is used, or (with spur gearing) a pawl or brake is provided, the brake permitting lowering under good control. All winches and drum hoists are elaborations of this A double drum may be provided with special gearing for handling two lines. In a "double-purchase" winch, gearings are provided whereby either of two drum speeds may be obtaind from a given crank speed, with inverse changes in pulling force.

2.222. Horse Winches may be useful where power is seldom req'd. By means of bevel gearing, a vert shaft is provided, to which is attacht a long lever to which is hitcht a horse that walks around the winch. Provision must be made to lead the hoisting rope either well above or below the horse.

2.223. Power Hoists or **Power Winches** may be operated by any kind of power mentioned in 1.622, *et seq.* They are usually equipt up to 50 to 100 HP, tho occasionally as much as 300 to 400 HP or more for special hoisting and conveying. For excavating machinery, there are usually three drums. See sec 1 61. These drums usually receive their power thru appropriate gears and clutches, altho in electric excavators there is sometimes one motor for each drum, thus reducing gearing but increasing cost for motors. "The modern standard heavy duty hoist would include wide, capacius drums, brakes mounted on flanges opposit the fric flanges, and compact lever arrangement so that even three-drum and swinging gear hoists are operated from one position.......The entire outfit with levers at the side; hand friction, foot brake, or extension levers bankt in a rack located at the point of greatest operating convenience." (F D Hooper, Eng News-Rec, '30, Apr 3). Many of the heavier machines are now (1936) provided with electric, pneumatic or hydraulic control, greatly reducing the labor of the operator.

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*Materia' Handling Cyclopedia, Simmons-Boardman Pub Co.

2.224. Capstans (almost invariably power-operated) may be of servis for occasional pulling, as for short hauls of cars or ships, or assistance to trucks. The (hemp) rope is not fastend to the drum, but is merely wrapt around it three or four times, which makes it possible to apply a very strong pull to the object to be moved, by the strength of one man holding back on the free end and taking up the slack as it is wound in. The drum or **niggerhead** is conceaved to keep the rope from working its way off the end or jamming against the end flange. *Lowering* by this means is awkward and dangerus unless the capstan is reversible.

2.23. Material Hoist Towers or Elevators

are of great servis in building construction for lifting mat'ls to different floors, and for returning barrows, etc. The best location is usually 6 or 8 ft outside the building wall, and opp a vert row of windows, altho they are often placed within the building.

A mat'l hoist consists of a crude elevator with only a platform and side frame-work, to the top of which is attacht a hoistin rope. It runs in gides inside a rectangular tower, often of timber, of temporary construction, but also frequently of steel of stock mfr with bolted connections, which may be used repeatedly in different locations. The cable passes over a sheav or sheavs at the top of the tower, and thence to a drum-hoist (sec 2.223).

For ordinary loads, the tower uprights may be of 4″ × 4″ timbers if only a few stories high, and 6″ × 6″ or more if over 100′. For heavy loads, and especially where the lowering speeds are high and the retardation likely to be severe, the tower should be carefully designd, and the cable stresses well taken care of. The towers should be braced to the building at intervals or maintaind upright by frequent guys, and completely braced by diagonals to resist hor thrusts due to wind or irregularity of track, and to aid in its action as a colum, the bents being abt 6 or 7 ft high, or approx'y square. The size of the elevator platform will be governd by the mat'ls to be hoisted, or by the barrows and trucks loaded on it, say 6 or 7 ft square.

Danger in the operation of such a hoist is great. Excessiv speeds and sudden stops may shake the load out of place on the platform, and create very high stresses thruout the tower. The operator must be thoroly competent and careful, and there should be such safety devices as automatic gates and interlocking signals, as may be req'd by law. In interior instalations these may be as elaborate as for permanent passenger elevators, especially if men are to ride on the hoist platform.

2.24. Vertical Conveyors

of the bucket type, see sec 5.2.

2.25. Inclined Hoists

Where material is to be hoisted up a slope, a drum hoist may wind a cable to the lower end of which is attacht a car running on a track, as is prevailing practis in mines having a steeply inclined passage for haulage. There seems to be no standard design for such a rig, and it may often be improvised from idle equipment on hand. In mine work, there are often two cars running on two tracks (possibly using a gauntlet) thus doubling capacity and avoiding waste of power req'd to hoist one unbalanst car. The inclination of the cable requires the use of sheavs or rollers all along the slope, spaced suff'y close to keep the cable from dragging on the ground or ties.

Assistance to Trucks on heavy grades may be given by a winch or drum hoist, a cable from the drum being hookt to each truck to be handled on the grade which may be as high as 35% or 19°, without danger due to slippery surface. This may be useful in removing mat'l from a building excavation. It is an old device formerly used to assist horses with wagons or men with wheelbarrows up grades entirely too steep for them alone.

3.0. TROLLEYS, TRAVELERS

See section 1.58.

4.0. CABLEWAYS

Where there is need to transport mat'l further than can be done by "throwing the bucket" of a crane, the crane may still be used by stringing a stationary or **"standing cable"** or **"track-cable"** from the end of its boom to some distant anchorage. On this cable run the wheels of **a trolley** or **carriage,** much as a car runs on a track, tho the load hangs below. This is largely used for operating scrapers.

4.01. From this a great variety of forms has been developt; only the more usual ones being described below. There are **two main types,** (1), secs 4.1 and 4.2, that in which the supporting cable is stationary, and the load is carried by a carriage running on wheels on the cable, and is propeld by a second moving cable, and (2), sec 4.3, that in which the supporting cable moves continuusly, and to which the carriages are attacht. **Nomenclature** has become very vague and varied, and altho we mention the more customary names, we shall in general use those names which are the more naturally descriptiv.

4.02. Utility. For utilities of special types, see "Utility" under each type described below. In general, cableways are often valuable for the excavating, transporting and depositing of earth, coal, ore, concrete, or other mat'ls. Except where moveable towers are used, irregular topografy is no obstacle, it being perfectly practicable to cross deep valleys, canons, marshes, swamps or bodies of water (see Dredging, p 581c, para 28); and for handling mat'ls as timber, steel, concrete, etc, in building bridges, dams and other long structures. There need be no interference with highways, RRs, or water traffic. For moderate dists and loads, espec'y where a constant delivery of mat'ls is req'd over very rough country, the cableway compares very favorably with a RR and usually has many advantages over it. It requires a min rt of way (towers being placed on property lines where possible), and much less mat'l for "track", no earthwork, and no bridges (being itself a kind of suspension bridge of max simplicity*). Usually hor curvature can be avoided. Weather has practically no effect on transportation.

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* For example, a single-span cableway may be elaborated into a suspension bridge, carrying track and dump trains out on it for constructing an earth fill.

4.1. Tower Cableways

In Fig 10 the span is shown very much shorter than it usually would be. To enlarge the range of the rig outlined above (in 4.0), the crane or hoisting engin, *E*, may be built into a tower called the **"main tower"**, *M*, and the track cable, *C*, stretcht from it to a **"tail tower"**, *T*, which is usually much smaller, so as to give a grade to the cable, so that the carriage may run in one direction by gravity. Then, for transporting, only one **traction cable** is needed from the engin to the carriage. But where suff grade cannot be obtaind, the traction cable must consist not only of this **"inhaul"** cable, *I*, but also of an **"outhaul"** cable, *O*; thus forming a loop of cable betw the two towers, with the carriage, *K*, cut into the loop. This, too, is largely used for handling scrapers, when the inhaul cable may be called the **"drag-line"** and the outhaul cable the **"back-haul line"**.

4.11. Hoisting and lowering the load is sometimes accomplisht by pulling up or slacking off on the standing cable; and the device is then known as a **"slack-line cableway"**. But more usually, another line, called a **"hoisting line"**, *H*, is provided from the crane to the carriage, thru which it runs over a sheav down to the load (and usually under another sheav and back up again to the carriage).

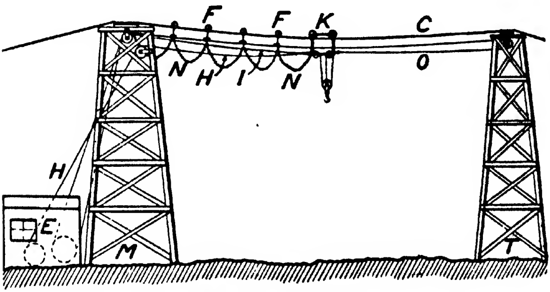

Fig. 10. Cableway

4.12. Where the grade is sufficient, and the load is to be taken on or dropt off at one point for some time, one cable may serv as a **combined traction and hoisting line**, by attaching a block to the standing cable, to stop the carriage over the required point. The block may be shifted from time to time as required. However, the use of separate cables affords a more flexible tho more expensiv installation.

4.13. Rope- or Cable-trolleys or Fall-rope Carriers, *F*, are often employd to support one or more of the cables and to prevent the several ropes from fouling each other. Each consists of a light frame, with a wheel at its top, which may run on the standing cable, and thru which slide the other ropes or cables. Sometimes they run on an additional line of their own, called a **"trolley rope"** or **"button rope,"** carrying stop-buttons of graded sizes for spacing the trolleys, of which there may be a string of 6 or more. In some cases they are connected with each other by chains, *N*, which sag between them, so that they tend to space themselvs equally.

The equipment described above (4.1 etc) can serv only a very narrow area, limited in width by the extent to which the hanging load may be deflected by one means or another.

4.14. To Serve a Small Area, varius devices are used. For coal piles and similar limited areas, the towers are sometimes mounted so as to **oscillate on rockers,** so that they may be tilted at right angles to the cables, thus serving an area perhaps 25′ or 50′ wide.

4.15. For Larger Areas the towers may be mounted either on wheels running on tracks at rt angles to the cables, or on caterpillars. The "width" and "length" of the area servd may then become interchanged, the cables stretching across the *width*, and the towers moving paralel with each other along the *length*. Where **tracks** are used and the area is great, it becomes economical to use a limited length of track and shift it as progress is made, either by hand or by small crane. Where **caterpillars**, sec 1.56, are used, and the installation is heavy and the ground soft, it may be necessary to provide corduroy roadways, and these then must be shifted. **Radial cableways** serv a sector, the tail tower being arranged to travers a portion of a circle on a permanent track.

4.17. Utility. See also sec 4.02 When the towers are fixt in position, they may serv only a very narrow area. When the ground is fairly level, and there is a large area to be servd, moveable cableway towers are coming into extensiv use, especially on the Mississippi R; but the equipment is too costly to be profitable where only a comparativly small area is to be servd.

4.18. Dimensions. Head tower hts range from 75′ or less to 125′ or more; **tail towers,** 25′ to 50′. **Spans** of a few hundred ft may be used for scraper outfits having no towers. With towers, spans of 1000′ to 1500′ are not unusual, a span of 2295′ having been used on the Hetch Hetchy project (Eng News-Rec, '29 May 16, p 793); and another of 2700′, carrying 20 tons, using four 2″ cables (Eng & Contr, '29 Sep, p 358). **Standing cable** diams should be computed from span, sag and load, but run from 1″ to 3″. **Other ropes** for traction, hoisting, etc, are usually about 1″ diam or less. Their size should be kept as small as consistent with hauling stress, to minimize wear over sheavs.

4.19. Capacities. Where **buckets** are used, their capacities may be from 5 to 10 cu yds or more. **Loads** of 15 tons are quite practicable. A cableway of 1200′ span at the Hoover (Boulder) dam could handle 150 tons (Eng News-Rec, '32 Oct 6, p 408). Traction **cable speeds** of 300 to 500 ft/min (3½ to 5½ mi/hr) are usual, tho some have run as high as 1500′/min. On levee work, tower excavators have handled from 150,000 to 200,000 cu yds per mo, during season.

4.2. Aerial or Cable Tramways

(with stationary supporting cable), are the next development where it is desired to transport mat'ls over greater dists than can be negotiated by one span. In this type, the standing cable is supported on intermediate towers as well as by the end towers, placed at suitable intervals of several hundred ft to around 3000′. In order to increas the capacity of the installation, a number of carriages or carriers is used, each running on the standing cable, and attacht either permanently or by means of grips to the traction cable. **Loading and unloading** may be accomplisht by any one of a number of devices too numerus to detail here. In some cases the carriers are attacht or detacht from the traction cable, and run off from, or on to the standing cable.

in others, each carrier remains permanently attacht to the traction cable, and is servd as it passes loading stations by an auxiliary carrier, which runs along in paralel a dist sufficient to load or unload, and which then backs up to serv the next carrier, being itself loaded or unloaded by any one of a number of means.

4.21. Such cable tramways must nec'y be **"double-track"**, having two standing cables, usually 6′ to 10′ apart, with a complete loop of traction cable, for the return of the carriers, and the ends of the standing cables must lead to and from mono-rails which gide the carriers around a half circle back to the other standing cable for the return trip. By such mechanism, hor angles in the alignment may be establisht, tho the speed should then not be over 400 or 500 ft/min*.

4.25. Utility is much the same as described in secs 4.02 and 4.1 relating to fixt towers, except that loading and unloading can take place only at *points* along the *line*, and *no area* whatever can be served. But *lengths* of five miles are entirely practicable, and there are many installations over 10 miles long, and up to 15 and more. When the length of a traction line is so great as to be cumbersome, other sections of traction line may be added almost indefinitly, the carriers being automatically detacht at the end of one traction section, then coasting a short dist on a monorail to the next section where it is transported by the next traction cable, each cable being driven by a separate motor. Grades up to 20 and 40% are usual, and 85% has been attaind, and a moderate amt of curvature is quite practicable.

4.26. Capacities in tons per hour are *independent of length* of cableway, but are approx proportional to speed of traction cable, number of carriers on the line, and the capacity of each carrier. **Speeds** range from 300 to 600′/min; the **spacing** of carriers from 150′ to 500′; and the **capacity of carriers** from 5 to 40 cu ft, or 600 lbs to 3500 lbs; and the resulting **load per hour** from abt 5 or 10 tons to as much as 250 or 200 tons.

4.27. Horse-Power required depends on diff of elevation, loads and speed, but is usually light compared with a RR, ranging from 5 or 10 to 100, and averaging around 50.

4.28. Crews required are comparativly small; 3 or 4 men for small installations, up to 10 or more for larger ones.

4.3. Moving or "Single" Ropeway

"Single" refers to a cableway in which *one* cable moves, and does both the carrying and the hauling. Like cable tramways, sec 4.21, they are "double-track", *not* "single". Their use is common in Europe, and rather rare in the U S. They are simpler, and cheaper to *instal* but they call for an exceptional rope, for as the size is increast to support the load, the bending stresses over the sheavs become serius, and such ropes wear out quickly, increasing *maintenance* costs. Hor angles are hardly practicable. There is increast difficulty keeping the line taut, and in designing clips to attach the carriers to the rope so that there will be no interference with the carrying sheavs, and this precludes the use of grades much over 30%. Economic **capacities** run from 50 to 100 tons/hr*, each carrier taking from 100 lbs to 800 lbs or more*. **Speeds** up to 450′/min.*

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* Aerial Tramways, by F C Carstarphen, A S C E paper No 1675.

5.0. CONVEYORS

In this section we treat only of "*continuus* conveyors" which deliver material continuusly, or practically so; and do not include cranes, hoists, cableways and vehicles, which are intermittent in their action.

Utility. The conveyors most useful to the C E are the belt and bucket conveyors. Their fields of application overlap somewhat, **belt conveyors** being of max use for hor or nearly hor movements, since mat'ls cannot be handled by them on very steep grades. **Bucket conveyors** also can carry mat'ls hor'y, but their chief usefulness lies in vert movements or those too steep for belt conveyors, or where it is desirable to transport, by the same machine, vert'y as well as hor'y. It is not possible to have hor angles in the line of movement of either type. See also under "Utility" under each type treated below in sections 5.1, 5.2, etc.

5.1. BELT CONVEYORS

See also "Belt Conveyors and Belt Elevators" by Frederick Hetzel, publisht by John Wiley & Sons, New York, N Y.

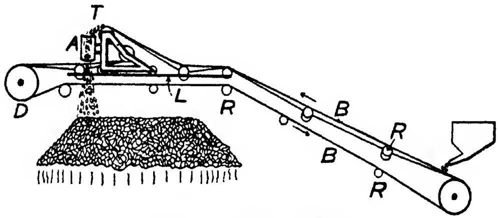

Fig. 11. Belt Conveyor

5.11. Construction

A belt conveyor, Fig 11, consists primarily of a continuus belt, *B*, hor or somewhat inclined, the two halves traveling in opp directions over rollers, *R*, or idlers spaced closely enough to prevent troublesome sag of the belt betw them, and driven by power applied to a drum, *D*, around which the belt passes at one end. The upper half of the belt carries the mat'l to be transported, and the lower half returns empty. The illustration shows a very much shorter conveyor than usual.

5.111. The **Belt,** *B*, is usually of cotton or canvas fabric, coated with rubber, the two being in different proportions according to the servis req'd. Plain cotton belts are of use only in dry locations and for light work. The function of the fabric is to prevent stretching, and that of the rubber to protect the fabric from dampness, chemicals, gases, abrasion, etc. Belts are not as durable as buckets, but generally much cheaper. They are made in varius thicknesses up to 10-ply, and in widths (varying by 2″) from 12″ to 60″, the more usual being 24″ and 36″, and are used in conveyors up to 1000′ in **length** (the belt being double that, counting the return half) altho the usual limit of conveyor length is more like 800′ or less, two or more belt conveyors being used in tandem where greater dists are to be coverd, the first delivering to the second, and so on.

5.112. Idlers or Rollers, *R*, used to support the belt, are usually of metal, cylindrical under the returning belt, but under the upper or delivery half, they may be concaved somewhat, (now practically obsolete) so that the belt forms a trough to prevent the mat'l from being shaken off. The ends of the idlers, being of larger diam than the center, travel faster, and cause rubbing against the belt, which hastens wear. To overcome this, the idlers are usually made up of from 3 to 5 or 6 short rollers, the central ones being hor, and the end ones being tilted 20° to 45° from the hor'l, so as to form the desired trough in the belt. The **bearings** of the idlers are very important, especially if abrasiv mat'ls are handled. They are preferably ball- or roller-bearing, self-oiling, and carefully designd to exclude mat'l falling from the belt. An idler which fails to rotate, causes rapid wear of the belt and of itself, wastes power.

5.113. Drive. It seems best to have the driving drum, *D*, at the unloading end, for then the max pull need not be transmitted thru the returning belt. It may be faced with rubber to minimize the pull nec'y to maintain suff friction. The tension may be controld by screw **take-ups** adjusted by hand from time to time, which may be suff for comparativly short machines. For long ones, a sliding take-up which maintains constant tension by means of a weight, seems preferable.

5.114. Loading. The belt may be quickly damaged by violent loading. The mat'l should be deliverd to it in its direction of movement, in small pieces and with min drop, and a steel plate under the belt at that point may be of help if heavy chunks are likely to be deliverd to it.

5.115. Unloading. No special device is needed for unloading, if it is to take place at the end of the run, the mat'l merely falling off as the belt curvs around the driving drum. However, **trippers,** *T*, are of great value where it is desired to distribute the mat'l over a long line. The tripper is a car running on rails, *L*, lengthwise of the conveyor, with two principal idlers or rollers, over which the delivery belt is drawn down and back under and then bent forward so as to resume its course. The mat'l falling off the belt passing over the first roller is caught by an inclined apron or trough, *A*, at rt angles to the belt and is thereby deliverd to one side of the conveyor. One of the rollers is geard to the wheels of the tripper, so that the tripper is causd to move slowly longitudinally, thus delivering the mat'l along the side of the conveyor as far as the tripper is arranged to run. At the end of each run, it encounters a stopping device, not shown, which causes it to reverse and return. A **Discharge Sweep,** consisting of a scraper placed diag'y over the belt, may be used in place of the tripper to unload mat'l before it reaches the end, but its position must be continually changed by hand unless the unloaded mat'l can be tolerated in piles instead of being evenly distributed.

5.116. Mounting. The idlers must of course be mounted on something constituting a frame. Often a metal truss is provided; sometimes, if the installation is fairly permanent, wooden stringers on posts or piles are used.

5.117. Stackers. To cover an *area*, the truss or frame becomes a boom, and is either pivoted at the lower or loading end, while the delivery end is arranged to swing hor'y (as well as being adjustable vert'y), or it is mounted on crawlers or tracks. An example of this was used on the intake canal of the Beauharnois power project, Quebec, in which a

36″ belt conveyor was carried on a truss boom 106′ long, pivoted from the base of a gantry 40′ long, running on track of 32.5′ gage, for the placing of rip-rap with a max size of 14″. (Eng-News Rec, '32 Sep 8). Booms range betw 50′ and 135′ in length. Smaller stackers designd primarily for mobility, are known as Portable Loaders, see sec 5.3.

5.118. "Elephant Trunk" Spouts, fed by funnels under the delivery ends of conveyors, made of flexible canvas or similar mat'l, are often convenient for the placing of concrete within a limited area without moving the entire machine.

5.12. Operation

The **speed** at which the belt is operated is usually betw 300 and 600′/min. Belt **running out of line** may be partially corrected by reducing the troughing, and also by setting the idlers a few degrees off from a rt angle with the belt. Other remedies are objectionable. (Hetzel, Belt Conveyors).

Lubrication of idlers is important, and if not self-lubricating, regular times should be establisht for lubrication

Remote Control may be employd where two or more conveyors are used in tandem, delivering from one to the next, preferably interlockt if there are many conveyors, so that no one conveyor will deliver to another which may not be running.

5.13. Capacity and HP

The following table is computed and averaged from figures given by Link Belt Co, Materials Handling Cyclopedia, and Marks's Mech Engr's Hand-Book, assuming speed of belt as 500′/min and wt of mat'l 50 lbs cu ft

Width of belt, inches	12	24	36	48
Capacity, cu yds/hr	85	340	790	1400
" tons/hr	115	465	1050	1850
HP per 100′, level	2.0*	8.4*	15.7*	25.5

If mat'l is raised, add'l HP is tons/hr × ht lifted in ft ÷ 1000.

5.14. Utility

Belt conveyors are of use chiefly for transporting granular mat'l hor'y or up moderate **grades**, 15° to 20°. This may be increast somewhat by adding cleats to the belt. In one case, aggregate was reported as carried up a 32° grade. The **materials** may be coal, coke, sand, gravel, ore, earth, cement (bulk or bags), concrete, and other mat'ls, such as grains, usually handled in fixt installations in bldgs. Hot or wet ashes will injure the belt. With concrete and other somewhat adhesiv mat'ls, it may be nec'y to employ scrapers or "discharge sweeps" placed diag'y over the surface of the belt at the discharge point

Outfits for handling **packages** are similar to those for granular mat'ls, with a few differences, but are usually permanent installations in bldgs.

5.15. Examples

In delivering concrete for the construction of a sewage treatment plant at Ward's Island, New York, four 24″ rubber belt conveyors were used, aggregating 1865′, three of them with trippers arranged to deliver conc at any pt thruout their length. (Eng News-Rec, '32 Nov 3).

In the construction of the Merchandise Mart in Chicago, 44 portable conveyors were employd, using cleated belts with effectiv lengths of 40′ to 60′, running 250′/min, each delivering abt 1 cu yd of conc per min. (Eng News-Rec, '29 Sep 12).

* F V Hetzel's "Belt Conveyors and Belt Elevators" gives HP as about half these values.

In the construction of the 100′ × 2200′ roof of a subway in St Louis, Mo, 8 portable belt conveyors were used, 26′ c to c, totaling over 200′, having 24″ belts run at 150′/min, handling abt 30 cu ft/min at angles up to 20°. 14 men in attendance. (Eng-Contr, '31 Aug).

5.2. BUCKET CONVEYORS

5.21. Construction

In the bucket conveyor, buckets are arranged in a continuus series, mounted on one or two chains, or on a belt. If the run is vert and short, they move without any gides, but if inclined, the part that is rising travels on gides or rails, and the part that is descending may hang freely, altho long runs of a story or more in ht usually need to be gided, both up and down, as they may get to swaying badly. Much or most of the run may be hor, when tracks are provided for both directions. The chains pass over sprockets at each end, and wherever there may be changes in direction.

5.211. The **Buckets** are usually malleable iron castings, or made up of sheet metal; usually V-shaped as seen from the side, with the upper end of the V open and up during carrying movements. They range in size from much less than 100 cu ins contents to 1000 or more, the open ends having areas from less than 25 sq ins to 100 or more.

5.212. **Chains.** Usually there is one linkt chain on each side of the row of buckets, and the buckets are attacht to them, as are also the **rollers** or **flanged wheels** which run on the gides or rails. The joints are the critical parts of the chain because of wear, and should be well designd and of hardend mat'l, with good provisions for oiling.

5.213. **Drive.** The chains of buckets may be driven by power applied to one of the sprockets at the end of the run.

5.214. **Loading** is accomplisht by throwing the mat'l to be transported into a pile or into a trough, "boot" or "hopper", or pit at the lower end of the run, the buckets digging into the mat'l and lifting it as it accumulates.

5.215. **Unloading** is accomplisht in either of two ways. (1) In the **"centrifugal"** method, speed is maintained suff to throw the mat'l from each bucket at the top of the run far enough out to prevent it from falling back onto any part of the conveyor, the mat'l being caught in an apron or trough, or falling directly onto the stock pile or embankment or other place where it may be wanted. (2) In the **"perfect"** or **"positiv"** discharge, the buckets are pivoted to the chains, and a tripping device, set at the place for unloading, causes them to tilt and drop their loads at that point. These trippers may be shifted to change the point of discharge as desired.

5.22. Operation

The **speed** of the chain of buckets is from 150 to 400′/min for "centrifugal" unloading, and from 75 to 150′/min for the "perfect" or "positiv" discharge, the speed of the latter being kept low on account of the impact of the tripper.

5.23. Capacity and HP

Authorities disagree so widely on these figures that it seems useless to say more than that on account of the lower speed and greater wt of moving parts, the capacities of bucket conveyors, are generally markedly less and the HPs higher per ton/hr, than those of belt conveyors.

5.24. Utility

Being a much more cumbersome machine than the belt conveyor, installations are seldom more than a few hundred ft long, and the rise is limited to about 150′. However, it is only where the grade is less than 15° or 20° that the belt conveyor can be used in place of the buckets, which are well adapted to handling coal, ashes, cement and water, and are largely used for such work in permanent installations. But see below, 5.3, Portable Loaders.

5.3. PORTABLE LOADERS OR CONVEYORS

5.30. Fig. 12. These units are simply self-containd continuus conveyors, belt or bucket type, of comparativly small dimensions, with sufficient lift to load mat'ls into trucks, *T*, and mounted so as to be moved readily from place to place, and so as to be fed against a pile of mat'l, *E*.

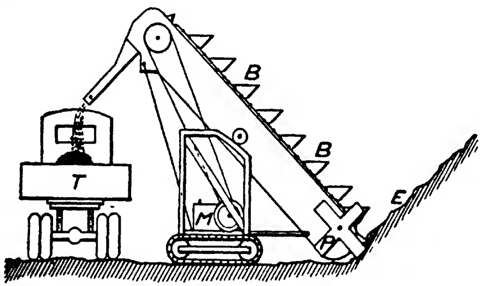

Fig. 12. Portable Loader

5.31. Construction

does not differ greatly from that of the conveyors described in secs 5.1 and 5.2.

5.313. The **Drive** is part of the machine and takes its power from the motor, *M*, provided.

5.314. Loading is accomplisht with the **belt** machines by loading onto the belt at its lower end, by shoveling or from hoppers, or from another belt or bucket conveyor. The buckets, *B*, of **bucket** conveyors, Fig 12, can receiv mat'l in the same ways, but have the advantage that the faces of the buckets can be gradually advanst against a pile of mat'l, *E*, or even against light earth in place. To gather together the sides of the pile, and not merely dig a channel thru it, **"paddles"**, *P*, are frequently provided at the two sides of the foot of the conveyor, on a shaft at rt angles to it, of propeller type, or as a ribbon in the form of a helix. Sometimes they carry cutting tools of varius kinds to assist in breaking up fairly hard mat'l.

5.315. Unloading is similar to that of larger conveyors, but always at the upper end only.

5.316. Mounting. Where **belt** conveyors are to be used in the same place for considerable periods of time, they are often mounted on a pair of wheels with wide treds. The **bucket** conveyors, however, are now (1937) almost always mounted on crawlers or caterpillars, which makes it possible not only to move them readily from job to job or from place to place, but also to feed the machine against the pile of mat'l to be handled.

5.32. Operation

The **Speed** of the **belts** in portable conveyors is from 150 to 250'/min, and of **buckets** from 100 to 135'/min.

The entire unit can usually be handled by one man, tho other men may be req'd to keep it properly fed.

5.33. Capacity

of portable **belt** conveyors may be from 10 to 50 cu ft/min, while portable **bucket** conveyors may handle from 20 to 80 cu ft/min. **Horse-power** required for **bucket** machines, from 2 to 20 and more; for **belts** somewhat less. The belt conveyors for loading trucks are at a disadvantage because the low angle necessitates a greater length and cumbersomeness to obtain the req'd height. The **weights** of belt machines run from 2000 lbs to 8000 lbs, and of bucket machines from 1000 lbs to 6000 lbs.

5.34. Utility

is much the same as for larger conveyors as regards mat'l handled. Recehtly wet concrete has been handled very successfully. As with the larger machines, the angle of the belt conveyor is limited to about 20°. Both belt and bucket portable conveyors have proven very useful on account of their mobility, and are very largely used in construction work.

5.4. APRON OR PAN CONVEYORS

5.41. Construction. Instead of a series of buckets, there is a series of steel plates 18" to 72" wide, carried by chains, and arranged so as to overlap while carrying, forming an almost continuus surface. The surface or apron thus formd often moves betw longitudinal stationary vert plates along each side, to increas capacity.

Loading is done much as in other conveyors, care being taken that large chunks do not fall heavily on the plates.

Unloading, on account of the ridges and side plates, can be done only at the end of the carry.

5.42. Operation Speeds range from 50 to 75'/min or more, tho sometimes very low speeds are used, as where coal is to be fed at a low rate for boilers.

5.43. Capacity ranges from 40 to 400 or 500 tons/hr for mat'ls of 50 lbs/cu ft, depending chiefly upon the width of the apron.

5.44. Utility is limited to mat'l not so finely divided that it may fall thru the joints betw the plates as they pass over sprockets, or betw the plates and the vert side pieces; nor must the mat'l be adhesiv. By providing a transvers ridge at the edge of each plate, mat'l may be carried up grades as high as 30°.

5.5. SCRAPER OR FLIGHT CONVEYORS

are similar in **construction** to the bucket and apron conveyors, except that the mat'l is pusht along by **"flights", scrapers** or **pushers,** spaced from 1½′ to 3′, which are merely vert plates of wood or steel, attacht to and pulled by a chain, that fit loosely into a **trough.** The flights may be either in contact with the trough or may be suspended slightly to reduce friction. The chain and flights run on gides or tracks suitably mounted. **Unloading** may be accomplisht at any point along the trough by providing a gate or trap-door in the trough, or at the end. They are built in **lengths** up to several hundred ft. The **speed** of the chain is usually around 100 or 150′/min. **Capacity,** approx number of tons per hour = two-thirds the area of a flight in sq inches. The **utility** is limited chiefly to non-abrasiv mat'ls such as coal and ashes at grades not over 45°.

5.6. SCREW, HELICOIDAL, OR "SPIRAL" CONVEYORS

Such a conveyor consists of a hor or inclined shaft, fixt in position, to which is attacht a metal helicoidal blade, all forced to revolv in a trough of the same **length,** 8′ to 12′ long. **Loading** is done by throwing the mat'l around the lower or intake end of the screw. **Unloading** may be done as in flight conveyors. **Capacities** as follows,—

Diam of screw, inches	7	9	12	16
Approx speed, rev/min..........	110	100	90	80
Cement, bbls/hr..................	57	132	300	670
Coal, tons/hr..................	6½	16	36½	80
Sand, cu ft/hr	180	420	930	2000
Approx HP req'd for sand.	2¼	5½	12½	25½

Utility. In addition to handling grains and the like, the screw conveyor can handle cement, coal, sand, gravel and ashes, provided lumps are not over 1″ across, on grades up to 15° to 20°. The handling of sticky mat'ls may be facilitated by the use of a helicoidal metal ribbon providing an open space near the shaft, while a great variety of forms of paddles, propellers, or ribbons, are used for **mixing** several mat'ls, while transporting. With abrasiv mat'ls, wear is excessiv.

5.7. PNEUMATIC OR "CURRENT" CONVEYORS,

in which finely divided materials are conveyed thru pipes by the movement of air currents, are of three types; (1) the pressure or blast system, (2) the vacuum or suction system, and (3) vacuum and pressure combined.

5.71. In the **pressure** system, the mat'l is introduced into the pipe by an injector by a blast of comprest air, and is blown out of the other end of the pipe into the bin or place in which it is wanted.

5.72. In the **vacuum** system, the mat'l is suckt in by means of a partial vacuum produced at the delivery end of the pipe. The mat'l is carried along the pipe and enters a large chamber which reduces the velocity sufficiently to permit it to fall to the bottom.

5.73. In the **vacuum and pressure** system, the mat'l is suckt in thru a nozzle and travels thru a pipe to one or two large chambers, where the mat'l settles. If only one chamber is used, a rotary or alternating air-trap is provided at the base, which feeds the mat'l from the chamber with partial vac to another chamber under pressure, from which the mat'l is blown thru the delivery pipe to the place where

it is wanted. Where two chambers are employd, they are causd to alternate their functions of receiving the mat'l in partial vac, and of blowing it thru the delivery pipe under pressure.

5.74. The **intake** end of the nozzle may be carried by a crane or derrick which can lower it into the bin or ship's hold from which the mat'l is to be taken, and move it around until all the mat'l has been suckt up, or the nozzle may be on a length of hose or flexible pipe which may be moved about by one man.

5.76. **Utility.** In general, any fine or powdery mat'l can be handled if not sticky or friable, such as grain, certain chemicals, fine coal, ashes, lime or cement. Pneumatic conveyors are distinctly inefficient as regards power consumption, but they are by far the easiest type of long haul conveyor to instal or shift from place to place, delivering as far as 500′ or more, requiring only pipes as conveyors, and one man at each intake nozzle. They serv very well in unloading box cars containing cement in bulk, effecting complete clean-up.

6.0. VEHICLES

6.02. **Classification.** We find a very great variety of combinations of machines and devices. Indeed, the practising engineer may often devise other new and useful combinations.

From 6.1 to 6.3 incl (wheelbarrows, carts and wagons), we treat of vehicles utilizing hand or animal power only; and from 6.4 to 6.7 incl, (automobiles, trucks, tractors, trailers, cars and locomotivs), those employing such power as steam, gasoline, oil, etc.

6.1. WHEELBARROWS

See also p 1027, Art 10, and p 1034, Art 15.

6.11. **Construction, dimensions,** etc. Wooden wheelbarrows weigh abt 50 lbs each, and are not very durable. Steel barrows, with trays of stampt steel, some with wooden handles and frame, weigh from 65 to 90 lbs each. Aluminum alloy barrows weigh abt 35 lbs, the saving in wt over steel barrows making poss the addition of from 30 to 55 lbs of load carried. Wheels are usually 15″ in diam, tho some are as large as 18″. The larger the wheel, the easier the rolling over rough ground, but the ht of the load becomes objectionable. Some barrows are equipt with pneumatic tires, and are claimd to require no plankways. The man wheeling a barrow may have to hold from 1/5 to 1/3 the load.

6.12. **Capacity.** Barrows are built to contain from 3 to 6 or even 8 cu ft of material, but in practis the amt carried depends chiefly upon the wt of the mat'l. Only 1½ to 1¾ cu ft of concrete can be carried, or 2 to 2½ cu ft of earthy mat'ls, equivalent to abt 180 lbs, tho some authorities allow 3 or even 4 cu ft.

6.13. **Utility** is confined to short hauls of a few hundred ft, and to small quantities, and to cases where more efficient equipment cannot be employd. Barrows must be expected to receive very hard treatment, so that cheap ones are very uneconomical.

6.14. **Operation.** It is usually nec'y or very desirable to provide plankways 2″ thick and as wide as obtainable. If practicable, it is best to arrange the work so that the bar-

row may be dumpt from the side. Time is saved if the barrow is not reverst and only the man turns around at the end of the run. On a heavy grade, a horse may sometimes be employd to assist by pulling with a rope.

6.2. CARTS

may be of use when power trucks or tractors and trailers are not available. Three types have predominated, as follows;—

6.21. Those referd to on pp 1024 *et seq* have wooden wheels 5 or 6 ft in diam, wide steel tires, a wooden box body with a cap of from 18 to 27 cu ft, pivoted at the middle of its base for dumping from the rear by withdrawing a locking-bar, the frame being provided with two shafts for a horse.

6.22. Push-Carts or Concrete Buggies now (1937) available for transporting concrete, coal, coke, earth, etc, for short dists, have steel wheels from 20″ to 40″ in diam (some pneumatic tired), with a steel body with capacity of 6 to 11 cu ft, and a handle for operation by hand, the body being so pivoted as to be readily tipt forward for dumping, with feet at the rear for keeping it level when standing.

6.23. The **pick-up cart** consists of two wheels from 45″ to 60″ in diam, 3″ tires, with an axle bent like an inverted "U", to the top of which is rigidly attacht one end of a long lever, with a hook on each end. It may be of use in picking up and transporting heavy objects (up to 5 tons) such as pipe. By raising the free end of the lever, the other end is lowerd. The object is then attacht by rope or chain, and lifted abt a ft from the ground by pulling down on the free end of the lever, when it may be moved about over fairly even ground

6.3. WAGONS

Like carts, wagons pulled by 2 or 3 horses may be of use where power vehicles are not available. They were used with satisfaction by the Miami Conservancy District (report of 1925, p 236) over loamy silt or light sand, where power trucks had trouble, especially when the ground was wet. **Speed,** abt 240′/min = 2¾ mi/hr.

Ordinary wagons are inefficient because the material hauld must be shoveld out. **Dump** wagons have been built of varius types; those with a floor of removable long'l wooden bars, called pole wagons (which must be stopt to unload); with hopper bottom, and with body tipping backward like a dump cart.

Capacities range from 1¼ to 2 cu yds.

See also section 6.6, Trailers.

6.4. AUTOMOBILES

6.41. Passenger

automobiles are generally too well known to warrant description here.

For **Tires,** see sec 1.55.

6.411. Cost of Operation, as determined by many who have made records of experiments and regular use, *appear* to vary widely betw 3 and 13 c/mi. Much of the variation is due to use of autos on different kinds of roads, and to diff speeds, but much is nec'y due to the combination of time costs (such as garage, interest, insurance and lisences)

with distance costs (as gasoline, oil, tires, maintenance and depreciation). Depreciation, taken as diff betw buying and selling costs divided by mileage coverd, may be from 1 to 3 c/mi. Taking depreciation as 2 c/mi, the total expense due to **distance** over **high-grade roads,** as given by varius writers, usually ranges within 4¼ to abt 5 c/mi, gasoline consumption being taken as from 1 to 1.6 c/mi. The expenses due to **time,** are best computed by the user, and when known they may be combined with the distance costs, resulting in a **total** cost of anywhere from 5½ to 10 c/mi or more. Note that if one drives comparativly little, say 2000 mi/yr (or 5.5 mi/day), and has time expenses of say $80/yr or 4 c/mi on an *average,* then the cost of a trip of several hundred miles covering only a day or so, will be primarily a distance cost of perhaps 5 c/mi, instead of 5 + 4 = 9 c/mi.

6.412. Weight of a passenger auto may be from 2500 lbs to 4500 lbs (1¼ to 2¼ tons) the more usual machines weighing around 3000 lbs (1½ tons).

6.413. Resistances. Different experiments conducted by different persons, using diff autos, naturally give rather divergent results. The following table gives approximate figures only, for passenger autos on level concrete or other high-grade road, assuming front projection of auto as 30 sq ft.

Speeds, mi/hr	10	25	40	60
Total resistance, lbs*	75	105	155	340
Air resistance only, lbs†	6	37½	96	216
" " (**"streamlined"**) lbs‡	2	12	31½	71
One **stop-and-start,** cost, cents¶	0.056	0.134	0.215	0.330

Road resistance. Gravel roads may be expected to increase cost of operation (over that on conc roads) from 20% to 40%, according to different authorities.

Grade increases resistance by wt of auto × sine of angle.

6.42. Trucks

are mfd in great variety of sizes, **capacities** varying by tons or half tons, four-wheeld trucks ranging from ½ to 10 tons, with chassis **weights** (without bodies) from 2200 lbs to 7000 lbs, 6-wheeld trucks, cap from 1½ to 12 tons or more, and chassis wts from 5000 lbs to 10,000 lbs. **HP** from 75 to 125 and higher.

Tires, see sec 1 55.

6.421. Speeds. Loaded trucks ordinarily make speeds about equal to passenger autos over the same kinds of roads, except where up-grades are encounterd, but seldom much over 50 mi/hr.

6.428. Utility. The ability of a truck to travel almost any road or fairly firm surface, without the need for construction of tracks, to transport widely different objects, and so minimize idle time, and to go usually to the exact spots where loading and unloading are to be done, without changing the load from one conveyance to another, makes it of great service to the C E. But its

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* Derived from data in Bull 119, Iowa Engg Expt Sta, by R G Paustian, '34 Aug 1.

† Derived from Kansas State Agr Coll Bull No 18, '27 Dec 15, by L E Conrad and E R Dawley. Air resistance varies very closely as the square of the speed.

‡ Deduced from U S Buro of Standards Research Paper RP591.

¶ Expts by T T Wiley, Civil Eng'g, '35 May, p 286.

6.429. Operation is very important. Where even small fleets of trucks are used, there should be men and equipment devoted solely to maintaining the trucks in good condition, and in **maintaining temporary roads** so that they may be used without trouble or undue delay even in bad weather. On certain extensiv dam building operations, fleets of scores of trucks have been used for conveying earth, traveling 40 mi/hr at intervals of only a few hundred feet. The selection of drivers is very important.

6.4291. Costs of operation. The following are taken from bulletins issued by the Economic Dept of the Coll of Agriculture, Cornell University, Ithaca, N Y, and relate to trucks used in central N Y state.

Averages of 43 trucks, averaging 6615 mi/yr each, 1935, showing effect of **size** of truck,—

Capacity of truck, tons.	½ & ¾	1	1½
Depreciation, fuel, oil, and repairs, c/mi	2.7	3.9	5.9
Other costs, c/mi	1.0	1.8	1.6
Totals, c/mi	3.7	5.7	5.9

Averages of 19 trucks, 1 ton capacity or less, 1935, showing effect of **mileage** per year,—

Miles per year	10,000	3,000
Cost per mile, cents	3.1	11.5

Average of 47 trucks, approx 6 c/mi.

6.43. Dump Trucks

are little more than ordinary trucks with provision for dumping the containd material to the ground. Almost every possible method of dumping is available, almost always by power taken from the engin. The front end may be raisd to as much as 70° from the hor, the tail-board releast, and the mat'l dumpt from the rear. This is in some cases assisted by a false bottom, known as a kick-out pan, lying on the bottom of the body, so hung by chains at the lifted end that it swings free to an almost vert position. Or the body is tilted to one side or the other, the lower side being releast. Some have hopper bottoms. Bodies are usually of prest steel.

6.431. Horse-power provided is usually betw 75 and 100. **Speeds** were formerly kept down by governors to abt 15 mi/hr, but are now generally unrestricted and are almost if not quite equal to those of ordinary trucks. **Weights** from 11,000 lbs to 14,500 lbs. **Capacities** usually range from 1½ to 4 or 5 tons, and as high as 10 or 15 tons; or 1½ to 4, 5 or 6 cu yds, and as high as 12 cu yds.

6.435. "Dumptors" or "Iron Mules" are a special type of dump truck, usually of very short wheel-base, in which the driver sits behind the body and faces forward over it, making possible quick sharp turns within 15′, and also indefinit revers running, as the driver has a completely unobstructed view to the rear. **Speeds** of 20 mi/hr are claimd for some, tho 10 to 13 mi/hr in high gear and 3½ to 4 in low, in *both forward and revers* gear, seem more usual. **Weights** from 13,000 lbs to 18,000 lbs. **Capacities**, 5½ cu yds and less.

6.44. Tractors

The use of caterpillars or crawlers with tractors is so usual that "caterpillar" is often used to denote "tractor."

6.441. Construction. The tractor is a unit of great compactness, combining an engin mounted on a frame carried usually by caterpillars, (see sec 1.56) abt 5 or 6 ft apart, (occasionally by wheels, as for farm work), with a coupling in the rear for pulling.

6.442. **Power** is usually gasoline or oil or Diesel engin, and may range from abt 25 to 95 HP, in some cases as high as 125 and even 160 HP, 50 to 70 HP being the most usual and generally practicable. The following figures are approximations from data supplied by one of the foremost mfrs;—

	1st gear	2nd gear	3rd gear
Draw-bar pull, in lbs, largest ..	22,000	15,000	13,000
" " " " smallest ..	4,000	3,000	2,000
Speed, mi/hr, average	2	2¾	4

6.443. **Weights** range from 5,000 lbs to over 30,000 lbs, 11,000 lbs being abt the av. **Unit wt on ground** is very close to 6 lbs/sq inch for all sizes, the caterpillar treads being so proportiond as to keep near that figure.

6.444. **Speeds** in mi/hr = approx 0.5 + the number of the gear. Thus, in 3d gear, speed = 0.5 + 3.0 = 3 5 mi/hr.

6.448. **Utility** is very great, becaus almost anything may be hitcht to a tractor and puld. The machine can travel over almost any surface, negotiating steep grades, as was done by the "tanks" in the World War. A tractor may be used for hauling disc plows, gang plows, scarifiers, scrapers, graders, ditchers, elevating graders, trailers (wagons or buggies), for logging, road grading, maintenance and construction, for almost any purpose requiring a strong slow hor pull, even such as pulling down old structures or moving buildings, and, by the addition of a blade in front, for bull-dozing and snow plowing.

6.449. **Costs** of operating tractors. The following figures are from varius bulletins of the Economic Dept of the Coll of Agriculture, Cornell University, Ithaca, N Y, reporting, 1935, on abt 70 machines, from 16 to 30 brake HP. Extremes were 25c and $1.12 per hr, avs from abt 50 to 70 c/hr.

6.45. Tractor Trucks

A tractor truck is a combination of a power truck and "**semi-trailer**" carrying the load.

6.451. The **power unit** consists of an automobile truck, comparable in wt and HP to trucks in which the body contains the load. The rear portion, however, is a low platform carrying the lower half of a vert pivot.

6.452. The **load-carrying unit,** or semi-trailer, has no running wheels under its front end, but instead the upper half of the pivot, resting on the rear of the truck. The pivot is arranged to permit the truck to turn 100° or more hor'y either way under the semi-trailer, thus affording great maneuverability in spite of size, and is also arranged to permit tilting of the units with respect to each other when traversing uneven ground. Just back of the pivot, the semi-trailer is provided with one or two small wheels or other supports that are off the ground when running, but which, when standing, are lowerd, and bear the wt of the front end, so that the truck may then be removed and used under another semi-trailer while the first is being loaded or unloaded. Thus one truck may serv two or more semi-trailers, depending upon the ratio betw loading and running times. These semi-trailers, having to provide no room for engin or cab, are made up to very large sizes with great capacities Since the semi-trailer delivers only half its wt onto the rear wheels of the truck (and half to its own rear wheels) it may have about double the ordinary capacity of such a truck. The added load nec'y reduces the speed and the grades that can be surmounted, tho air-brakes on all wheels afford almost as good control going down hill as the truck itself would have.

6.6. TRAILERS, "WAGONS" or "BUGGIES"

In many cases, trailers have been specially built for particular cases, such as the 194-ton trailer for Boulder (formerly Hoover) dam, (Roads & Sts, '34 Mar), and many others too numerus to list. These nec'y have many wheels under them to carry their loads, and in some of them the wheels are steerd by power. For carrying long girders, a trailer may consist of merely two trucks (of 2, 4 or more wheels each) connected to each other by the girder itself, which is lasht to them. Large bins for cement, aggregate, etc, have been mounted on wheels, that they may be towd from place to place (provided there is head-room).

The usual trailers, however, have become well standardized, tho of many types. They should be hung low, to minimize chance of overturning while being towd, and to make it possible for elevating graders to deliver into them. The use of wheels (up to 8 or 16) is about as usual as the use of caterpillars. **Capacities** may range from a few cu yds each to 24 or 30 cu yds in the larger sizes, capable of carrying loads of from 20 to 35 tons. **Trains** of two trailers or "buggies" (often on caterpillars) are frequently used, tho more than two increases difficulty and danger of maneuvering. They are often or usually provided with means for **dumping** in one manner or another.

6.7. INDUSTRIAL RAILWAYS

may be of use to the C E when there is much hauling to be done over a long fixt route, as in constructing RRs, highways, dams, tunnels, aqueducts, canals and sewers. In some cases it may be well to operate the cars by means of a cable, especially where grades are over 3%. Greater flexibility, however, is obtaind by the use of locomotivs or motor cars, tho grades of over a few per cent will seriusly limit the wt of cars that can be hauld. Electric motor cars have been operated by remote control over track divided electrically into separate sections.

6.71. Track

Gage is usually 18″, 24″ or 36″, some 42″. **Rails** may weigh almost any even figure from 8 to 20 lbs/yd, or any even ten pounds from 20 to 70 lbs/yd, and come in lengths of 20, 30 or 33 ft. Second-hand rails and ties (worn but not defectiv) will often serv well to construct such track, especially if it is not to be used over a long time. **Portability** is usually of great importance as the work progresses, and light track is to be had in which ties and rails come ready assembled in lengths of 15 to 20 or even 30 ft, that can be carried by 2 or 3 men, with curvd sections, switches and turntables; having joints that may redily be fastend and unfastend. Such track can be laid on firm ground with very little if any grading. **Ties** may be of wood, but steel is better for portability.

6.72. Motiv Power

6.721. Power. Animals may sometimes be used to advantage where mech'l power is not available. Steam has been largely displaced, as has "fireless" (steam storage), and comprest air power. Power now used (1937) is chiefly oil or Diesel, gasoline, gasoline-electric, electric storage battery, electric trolley or third-rail, depending upon facilities available. **HP** from 50 to 200. **Speeds** are not generally high, 2 to 12 mi/hr for gasoline.

6.722. Weights of locos available vary by very small differences from 2 to 35 tons, up to 50 or 60 tons.

6.723. Tractiv Effort. The power employd is usually suff to slip the wheels on dry rails when starting, so that the practical tractiv effort will be equal to the wt of the loco or motor multiplied by the coeff of fric. This coeff may be counted on in theory at least as 0.20 for dry rails, tho 0.35 may be achievd with sanded rails; while on wet or greasy rails (without sand) it may be as low as 0.15 or 0.10 or much less. The **draw-bar pull** = tractiv effort — force (= abt 8 lbs/ton of loco on well laid track) req'd to move loco. As speed is increast, the propulsiv force of almost any power diminishes, so that the hevier the train, the lower the max speed possible. What is said in sec 1.55 about **avoiding slipping** applies also to steel wheels and rails.

Caution. A loco **and train** will not be able to negotiate a grade equal to that which the loco alone can manage, but only that grade multiplied by the wt of loco alone, divided by the wt of loco and train. Thus, where a loco alone can operate on an 8% grade, it can handle a train of its own wt on grades of only half that, or 4%. Altho the operation of a loco may approach the theoretical coefs of fric, yet **in practis** a very little water or grease on a short length of rail may so reduce the coef there that the wheels may skid and the machine or train become unmanageable.

6.729. Operation, as in standard railroading, should be by careful and competent men only.

6.73. Cars

for use on industrial rys may have only a platform, a box body, or any of a great variety of forms for handling special objects as logs, brick, ore, concrete, etc. **Dump** cars may be bottom-, side- or end-dump, and of many different designs. **Sizes** vary from 4-wheeld hand-cars up to 8-wheeld (two-truck) cars of standard RR type. The bodies are usually made of steel, prest or welded.

6.8. TURNTABLES

Portable turntables have been available, with a short ramp on one end, onto which trucks or other vehicles may be run, and then turnd around in places too narrow for the vehicle to turn itself, such as that half of a roadway not under construction.

7.0. OTHER EXCAVATORS

in addition to those described in sections 1, 2 or 5.

7.1. PLOWS

7.11. Construction. A plow consists of a pointed "share" which divides the earth, and above it a "moldboard" so curvd as to throw the earth to one side. On the opposit side is the "landside" which prevents the thrust of the share and moldboard from deflecting the plow. These are attacht to a frame or "frog", to the rear of which handles are attacht for hand plowing, and to the front of which is fastend a "beam" to which horses or a tractor may be attacht, and to the under side of which a "gage wheel" is sometimes fixt to control the depth of cut.

7.12. Types. The **rooter plow** or **subsoil plow** has no moldboard, is much sharper, and is used to loosen particularly hard ground. A **gang plow,** mounted on wheels, carries three or more plows to cut as many furrows. In the **disc plow** a concave, edged, steel disc, mounted on edge

in the plane of motion, replaces the moldboard plow, and is especially adapted for sticky or very hard dry soils.

7.13. Furrows may be cut from 4″ to 12″ deep. In shaly rock 7″ or 8″ is about all that can be attempted. Width of cut, 12″ to 14″ for deep cuts, and less for shallower cuts.

7.14. Weights of farm plows are usually somewhat over 100 lbs; while contractor's plows weigh from 180 lbs to 430 lbs.

7.15. Power may consist of from 2 to 12 horses, while some plows are rigged to be attacht to a tractor.

See also 7.5, Elevating graders.

7.2 ROOTERS

A rooter is similar to the harrow used by farmers, but much stronger, to withstand the stresses imposed by being dragd by a tractor, and by deeper and harder digging to depths of abt 2 ft. It is mounted on a pair of wheels from 2 to 3 ft in diam The number of **teeth** is much smaller than in the harrow, being as high as 9, tho more usually 5 or 3, with provision for leaving only 2 or 1 for digging. Some are claimd to be so designd as to be self-sharpening until worn out. **Weights** may be anywhere from 2500 lbs to 8000 lbs.

Rooters seem to have their greatest **utility** in making ready ground for bulldozers (sec 7.3) and graders (7.4) where it is very hard or stony, consisting of boulders or decomposed rocks. They are of use also in dislodging stumps, or breaking up concrete. It is claimd they frequently obviate blasting.

7.21. Scarifiers

are similar to rooters. They consist of a series of vert teeth mounted in one line on one bar, which may be used in place of the blade of a grader (7.4) or may be attacht to the rear of a road roller. Their **utility** lies in their ability to break up hardend surfaces, and to break off high spots of uneven ground.

7.3. BULLDOZERS, TRAILBUILDERS

7.31. Construction. A bulldozer consists of a self-propelling unit, usually running on caterpillars (sec 1.56) tho sometimes on rubber-tired wheels, powerd by gasoline or Diesel engin; provided with a very strong blade.

7.311. The **Blade or Scraper** is of steel, oblong, mounted in front of the caterpillar or wheels, its longer axis hor. It is usually held at rt angles to the line of motion, but in some makes may be placed at any hor angle up to 30°, right or left, to throw dirt to one side, and it may also be tilted from the vert to better suit different kinds of earth. The lower side carries a cutting edge of specially hardend steel, while the main portion of the blade is usually more or less concaved to carry along as much mat'l as possible. The blade is arranged to be lifted as much as 3 or 3½ ft, and lowerd 5 or 6 ft below the level of the ground on which the machine may be operating.

7.312. Weights of such machines run from 3500 lbs to 6000 lbs. **Capacities** when loaded for shoving, 2 to 4 cu yds. **HP,** 30 to 70.

7.318. Operation frequently consists of an initial movement of 25 to 40 ft at abt 2 ft/sec, in which the blade is lowerd and the mat'l scraped up; and then a hauling or shoving movement which may be as long as 300′ at abt 3½ ft/sec, with the lower edge of the blade at ground level.

After depositing the mat'l where wanted, the return may be made with the blade raisd, at full speed, say 4 ft/sec. If the **material is hard,** it should first be gone over and loosend with a rooter. Experiment should determin the best speed for shoving, as it is usually possible to travel fast enough to lose much of the mat'l.

7.319. Utility. The bulldozer is of use for clearing, maintenance of roads, spreading mat'l, making fills, making paths for trucks, shovels, etc, but the **mat'l must be fairly loose**, without soft pockets, without rock, and the haul not too long. They are particularly effectiv scraping *down* slopes as steep as 35% to 45%. If the gear is right, they may be *backt* up the grade for the next operation.

7.4. GRADERS

7.41. A grader, also known as **blade grader, road shaper,** or **maintainer,** consists essentially of a blade or moldboard mounted on a frame on wheels, much as in a bulldozer except that the blade is betw the front and rear wheels instead of in front of all the wheels.

7.42. Power. Graders may be either powerless, to be puld by animals or a tractor requiring 15 to 80 HP; or they may be propeld by their own power, which is usually around 50 HP, using 4 steel wheels, or 4, 6 or 8 rubber-tired wheels, or caterpillars, for propelling.

7.43. The **Blade or Moldboard** is furnisht in almost any length from 7 to 16 ft, or less when horse-drawn, 12 ft being usual, with a ht of abt 18″. The blade is adjustable in ht, hor angle and tip.

7.45. Weights may be as low as 1400 lbs in graders to be towd, and as high as 17,000 lbs or more in power machines. **Speeds** of power machines are usually abt 2 mi/hr in low gear, and 10 mi/hr in high gear, with two intermediate gears, and reverse.

7.48. Utility. Graders are not essentially excavators, tho they may do a moderate amount of cutting. They are chiefly levelers or eveners, and are used largely for maintaining the surfaces of earth roads, and ditches and banks; taking displaced mat'l and shoving it along until it finds low spots to lodge in.

7.49. Leaning-Wheel Grader

In this the wheels may be leand to one side or the other as much as 25° or 30°. This is of value (1) in opposing the thrust of the blade when forming banks, and (2) to keep the wheel-treds hor when it is working on steeply sloping ground. The blade may be swung out to the side (its plane approx at rt angles to the line of motion) and set at different positions and vert angles for the forming of banks as high as 8 ft. Or it may be set low so as to partly cut and form small ditches.

7.5. ELEVATING GRADERS

are *not* a development of graders, but are a combination of plow and conveyor mounted on a frame carried by wheels or caterpillars.

7.51. The **Plow** may be either of the moldboard type, or both disc and moldboard in series, capable of cutting a furrow abt 1′ wide, and 6″ to 7″ deep. The moldboard delivers the broken-up mat'l onto the lower end of a small

7.52. Conveyor or **"Elevator"** of changeable inclination, (usually of the belt type, tho buckets have been used) from 14 to 25 ft long, the belt being usually 42" or 48" wide. It is mounted at rt angles to the direction of motion, extending outward to the right or left, and upward suff'y to deliver over the top of a wagon or truck moving alongside and keeping pace with it.

7.54. Power. Either horses (as many as a dozen) or a tractor (around 75 HP) may be used for traction. The conveyor may be operated either by a separate power unit on it, which makes belt speed control easier, or by a "power-take-off" from the wheels or tractor, which is simpler and lighter.

7.55. The **Weight** of such a machine may be around 15,000 lbs.

7.56. Capacity will vary greatly according to the kind of mat'l encounterd, but will usually be over 1000 cu yds per day, while some mfrs claim that about 3000 cu yds per day have been handled.

7.58. Utility. These machines can be very servisable on even level ground of uniform plowable loam, sandy soil, or gravel, for the forming of canals, irrigation and other ditches, and levees, and for RR and highway construction. They are said to be particularly helpless on wet ground, being unable to handle wet or sticky mat'l, and especially so among rocks or boulders or roots, in hard-pan or deeply frozen ground, or where there are abrupt grades.

7.59. Operation. Unless the conveyor is delivering mat'l directly to the tops of levees or other embankments, the servising of the machine by the optimum number of wagons or trucks is most important, as changes in hauling dist may call for more or fewer vehicles.

7.6. TRENCHERS AND DITCHERS

7.60. Definitions. Engineers seem to make a quite definit distinction between trenches and ditches (tho not perceptibly indicated in dictionaries), a trench being comparativly narrow with vert or nearly vert sides, and the ditch being much wider, with side slopes of say 1 on 1 or flatter.

7.61. Types. Almost any excavating device may be used and has been used for digging trenches and ditches. Years ago, cableways were much used, strung over the line of the trench, some handling a number of buckets which were loaded by men digging in the trench. These may be used effectivly if nothing else is available, and they can be used for deeper digging than the machines described below. In at least one case, a power shovel dug and deliverd to a belt conveyor that carried the mat'l along for back-filling. A power shovel mounted on a flat-car is frequently used by RRs for maintaining the ditches alongside their tracks. Two types of machines using **buckets for digging** as described below, have been specially developt for trenching and are largely used. But where these are not available, the machine most generally resorted to is the **pull-scoop** or back-digger (sec 1.17).

7.62. Bucket-Wheel Trenchers

7.621. Construction. Fig. 13. An engin, *E*, (formerly steam, but now generally gasoline or oil) is mounted on a frame carried by wheels, *W*, or caterpillars, *C*, or both. Extending from the rear is a large vert wheel, *L*, *L*, which carries a series of buckets, *B*, *B*, equipt for cutting. There is also a belt conveyor, *V*, as in elevating graders (sec 7 52). The engin is geard for driving the entire machine ahed at varying speeds, for rotating the bucket-wheel, and for operating the conveyor. The wheel, *L*, has no axle, but is a circular frame rotating on three or more rollers, *R*, *R*, and the buckets, on reaching the top, dump inside the circumference of the wheel thru an opening in a fixt ring (not shown) which the wheel embraces, onto the conveyor which extends in to abt its center. Where tile or pipe is to be laid, or other work done in the trench directly behind the machine, not only may the sides of the wheel be enclosed to prevent the sides of the trench from falling in, but there may be carried behind the wheel a large hollow box, *X*, in which the workmen may lay their pipe with a fair degree

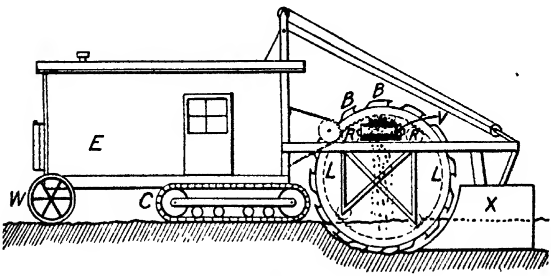

Fig. 13. Bucket-Wheel Trencher

of protection. In some machines the wheel (without the protecting box) has been arranged to be turnd right or left abt 20°, so as to excavate a ditch rather than a trench.

7.622. Depth of Trench which bucket-wheel machines can dig, is from 8 to 12 ft max. The **width** may be from 12″ to 28″ or as much as 42″, altho 12″ or even 15″ is too narrow for men to work in efficiently, so that 18″ or more is usual.

7.623. The **Conveyor**, *V*, which extends out at rt angles to the right or left, to deliver the excavated mat'l either in a long pile at the side, or to trucks, is usually of the belt type, (sec 5.1) of ordinary width, from 20″ to 30″.

7.624. HP, from 25 to 100. **Speed** or feed, in ordinary earth, from 1¼ to 4 ft of trench per min, in exceptional cases as high as 5′/min, while partly frozen ground may slow it down to ½′/min.

7.63. Bucket-Chain Trenchers or Ladder Excavators operate much like ladder dredges (p 581*l*, paragrafs 72, 73 & 74).

7.631. Construction is much the same as that of bucket-wheel trenchers, except that a chain carrying buckets takes the place of the wheel. The course of the chain of buckets has been different in different machines, having either a triangular or elliptical outline as seen from the side, while in some machines the forward buckets are moved downward in a vert line, forming a vert face to the trench. Some machines are so designd as to be able to cut a trench only a fraction of a foot from the foundation wall of a building.

Depth of trench which bucket-chain trenchers can dig, is from 12′ to 20′. **Width** of cut is about the same as in bucket-wheel machines. The **conveyor** is arranged as in bucket-wheel machines. **HP**, from 30 to 70. **Speed** or feed, from 1¼ to 5 ft/min.

7.64. Operation of bucket machines (either type) is usually accomplisht by two men with the machine. Others must of course be employd for laying the tile, pipe, cable, or whatever is to be placed in the trench, as well as for back-filling. One of the **main objects** in the design of these machines is to minimize the time that a trench must be open; particularly important on roads or streets.

7.65. Utility of bucket machines is likely to be limited if hard rocky ground is encounterd, advance blasting frequently being required. The figures above, sec 7.624, for speed, are for earth, gravel, tough clay or sand. Trenchers are particularly useful for laying water, sewer, gas or oil pipe, or telefone cable. Pull-scoops (or "back-diggers") (sec 1.17) are very largely used in place of bucket machines.

7.66. DITCHES (meaning wide trenches with sloping sides) may often be dug with other excavators, such as power shovels, or even clam-shells if the ground is soft enough, and especially by pull-scoops. Leaning wheel graders (sec 7.49) are largely used for shallow ditches, while shovels operating from RR track do much of the maint of RR ditches.

7.8. HYDRAULIC EXCAVATION or HYDRAULICKING originating chiefly as a means of surface mining, consists of squirting large quantities of water at high pressure thru a large nozzle, against the material to be excavated.

7.81. Water Supply. In some cases suff water may be obtaind by **gravity** from some not too distant source at a higher level. In other cases it will be necessary to **pump** the water, centrifugal pumps being usual and efficient, making possible the use of salt or gritty water. The **head** req'd will be several hundred feet. As little as 80 or 100 ft head has been used for washing mat'l away, but usually less than 200′ will not be satisfactory for cutting, while 300 to 400 and even 600 ft head are used where poss. The **quantity** of water used will be determind by the head and the size of the

7.82. Nozzle, "Hydraulic Giant" or "Monitor" used. According to Peele's Mining Engineer's Handbook, we have approx'y;—

Diam of nozzle, inches................	1	3	6	9
Flow under 200′ head, cu ft/min......	33	250	1500	2700

Diams of nozzles vary from ¾″ to 10″. Experience indicates that a number of smaller nozzles are more effectiv than a very large one, altho an 8″ nozzle, using 3600 cu ft/min, has excavated 800 cu yds/hr. The nozzle must be **fastend** to the ground in some firm manner, to resist the great reaction from the escaping jet. The base of the nozzle is attacht to the supply pipe by a swivel **joint** that will permit considerable changes of direction. A small nozzle may be directed by hand by a long handle extending back to the rear; but the larger ones will usually require a **deflector.** This may consist of a short tube, larger than the nozzle, surrounding the front of the nozzle and extending a short dist in front of it, and so mounted as to be turnd slightly by hand in different directions, so that the jet strikes its inside surface on one side or the other, the reaction being suff to turn the nozzle as desired. The deflector is then again turnd straight ahead. The **bore** of the nozzle should be very smooth and even, as slight roughnesses will break part of the jet into spray, losing much of its force.

7.83. **Sluices** used for carrying away the mixture of water and earth usually require a grade of 4%. Sometimes a mere gully, reinforst on the sides by planks, will be suff, but it is usually better to provide a wooden or metal flume, especially as it is easier to shift as the work progresses.

7.84. **Capacities.** Records of mat'l excavated in given times are often unsatisfactory because some factor is not given. It has been found, however, that the proportion of solids that the water will carry ranges betw 6 and 20%, with 11 or 12% as an average. From this it would seem that the amt of excavation may be conservativly estimated and computed as 10% of the water deliverd by the nozzle or "giant".

7.88. **Utility** depends upon the head and quantity of water available, character of mat'l to be excavated, and ability to dispose of the waste. Its efficiency will be greatly increast when the waste may be used for dam construction or for making fills. **Material** excavated may be sand, gravel, earth, and even loose rock. Glacial till, hard blue clay and boulders were excavated in large quantities on a number of the Miami Conservancy works, but with some difficulty. Sometimes advance blasting is necessary or very desirable. **Freezing**, if severe, may suspend operations entirely. See also "Operation", following.

7.89. **Operation** requires care and skill. Nozzles should be kept as close to the face to be excavated as possible, altho even more **care** should be exercized to keep far enough away to avoid danger from caving. 50 to 60 ft is further than desirable for efficient cutting, altho fair work has been done at 250′ with a small nozzle. The nozzle is best aimd at the base of the wall of mat'l to be cut away, striking at a small angle. It is frequently well to have a low pressure nozzle to assist by breaking up masses that may cave down and by keeping the mat'l excavated by the high pres nozzle moving on its way to the sluice. For efficiency it is important to watch the output closely. A reduction of **solids carried** to half the max poss, may otherwise pass unnoticed.

Hydraulic excavation was used most effectivly in very large grading operations in both Portland, Ore, and in Seattle, Wash, where a hill about a half mile across was reduced by 210′ in ht, street grades being reduced from betw 7 and 15% to 5% over a dist of 800′. Much of the mat'l was sluiced to tidal flats, forming useful land as well as almost eliminating a hill impracticable for habitation.

9.0. OPERATION

In the following paragrafs on "balance", waste motions, layout, choice and maintenance of machines, and personnel; nearly every point brought out may be regarded as a *factor;* for if the efficiency of any one of them be reduced by any percentage, the resultant output will be reduced by almost if not quite the same percentage. Hence all are about equally important and none should be slighted.

9.1. FUNDAMENTALS

9.11. Balance

9.111. Machines. Perhaps the most important and comprehensiv principle to observ for ideal efficiency in the operation of equipment, where more than one machine is involvd, is that all of the several machines working in connection with each other shall "team together" as nearly as possible, and that the work of any one shall fit in with that of others as regards capacity and reliability For example, bad balance obtains when too many trucks must wait for an excavator, or too few trucks keep the excavator waiting. One machine, unable to keep up with the rest, may keep them all down to its own low rate. Or, again, if the equipment is already well balanst, the purchase and introduction of any one machine capable of working twice as fast as the one displaced, will *not* speed up the work in the slightest, and will increase maintenance cost and interest on the investment. Usually there is one expensiv "key" machine, such as a power shovel, whose output it is not practicable to change materially; and it is to this that all the other co-operating machines should be timed. **Caution:**—But see "in practis" in sec 9.3.

9.112. Material and Labor also should be kept balanst. Unless the work is well supervised, machines may be kept waiting for insufficient or incompetent men; and conversely, expensiv men may be kept waiting for machines or equipment. Lack of pipe, wire, tools, etc, may add much to man-hours req'd to overcome the handicap.

9.113. Time and Materials should be balanst so that needed supplies, as gasoline, oil, cement, food, etc, arrive at as nearly the best time as possible. Not only may lateness hold back the work, but pre-delivery may do so by cluttering up places where other things belong or where free movements of men and mat'ls are important.

9.12. Waste Motions

should be reduced to a minimum. No fixt rules can here be set down for such matters. Many books have been written on this subject alone. See section **620.e** in Bibliography preceding Index. The management must be on the watch for needless **revers movements, re-handling, excessiv travel,** such as that of an excavating bucket or the movement of a truck. Only common sense and ingenuity can here be prescribed for most of such efficiency work. Wasteful **Repetitiv Operations** should not be lost sight of or ignored, even when apparently trivial. It may be said; "It is *not* the little things that count, but the large number of them". A loss of only 3 secs in one operation of a shovel making a cycle every 30 secs, is multiplied by almost a thousand in a day's work, and means the loss of over an hour in a 10-hr day.

* 9.13. Time Studies

of machines it is proposed to use, are usually of great help in planning the work, and if kept up as each job progresses, they frequently reveal unsuspected causes of delay and inefficiency, especially valuable if of repetitiv work. Record-

ing ammeters, truck movement indicators, and other recorders and meters are valuable if used and studied, and advantage is taken of their indications.

9.14. Scheduling

should be undertaken in almost every case, especially where many machines and operations are involvd. Even tho precise predicting may be impracticable, it is usually far better than forging ahead by hit-or-miss judgment. As an aid in scheduling, we suggest "Engineering Office Systems" by John P. Davies, McGraw-Hill Book Co, New York, N Y.

9.141. Diagramming is usually of great assistance in scheduling. Many methods are available, too numerus to outline here. For this, "Graphic Methods for Presenting Facts", by Willard C. Brinton, the Engineering Magazine Co, New York, N Y, may be of value.

9.2. PLANT LAYOUT

Conditions vary too widely to permit the giving of any detaild rules for plant layout. It will depend upon topografy, materials to be handled, machines available, locations of supplies of both men and equipment, meteorological conditions, time and money available, and probably other factors. For good results, it calls for detaild planning and the ingenuity and common sense of the C E. A review of engineering periodicals may often afford valuable suggestions. See Eng & Contr'g, '29 Nov, p 443.

9.21. Surveys

are essential for all but the simplest jobs. These should be plotted to a generus scale, should include all the territory involvd, and show all objects that may be encounterd; and of especial value are **contours** accurately plotted for every few feet of elevation.

9.22. Excavation, etc,

should be pland as closely as possible. In the case of power shovels at least, diagrams supplied by mfrs (or obtainable by measurement) showing the reach, clearances, ht of lift, etc (see secs 1.14, 1.15 and 1.18) together with capacity and speed of operation, make possible close planning of the cuts to be made and estimates of time req'd.

9.23. Roads

should be thoughtfully laid out so as to serv all machines as well as possible, and to minimize their reconstruction as the work progresses, using moderate grades and curvs that will utilize the high speeds now possible with trucks. See also 9.43, road maintenance.

9.3. CHOICE OF MACHINE

will depend upon many factors, among them the work to be done, the kind of mat'l to be handled, topografy, meteorological conditions, money available, and the abilities of the machines. For this last, **see paragrafs** entitled "Capacity", "Utility" and "Operation", in each article descriptiv of each machine in question, on preceding pages.

In practis, unless the machine will be worn out when the job is done, the ideal machine for max efficiency, or even that combination of machines that produces the best balance (secs 9.11) is not always the best. Often the cost of purchasing or hiring will be prohibitiv. Even with ample funds, it frequently may be cheaper in the end to use machines already in hand, even tho rather inefficient for the purpose, than to purchase the ideal machines which cannot be sold or hired out to advantage when no longer of use on the job for which they were obtaind, particularly when they will not be used over a long period of time. Usually a

moderate sized machine will be best, because more likely to be of use on subsequent jobs. Some machines are more **versatil** than others, such as the crane, sec 1.32, and the pull-scoop, sec 1.17. Some are extremely limited, such as the bucket conveyor and the bucket-wheel and bucket-chain excavators. See also sec 0.3.

The following articles may be found to be of value. Civil Engineering, '36 Mar, p 139, Eng & Contr'g, '29 Dec, p 491. For a comparison of **draglines** with **other** devices, see article on New Madrid (Mississippi R) Floodway Levee, Eng News-Rec, '31 Apr 16. See also Construction Plant, Methods and Cost, by Chas H Paul, Miami Conservancy District, Dayton, O.

So many kinds of machines, often of increasing size, are being continually put on the market, some to endure and develop, some to disappear, that when more knowledge is req'd, we suggest that catalogs of mfrs be sent for and studied. For addresses, see advts in eng'g periodicals.

9.4. MAINTENANCE AND SERVISING

Hardly too much emfacis can be placed on the importance of ample and effectiv means for servising, maintaining and repairing machines, especially where a half dozen or more of them are in use. As pointed out in sec 9.1, the failure of any one machine may mean the stoppage of a number of others as well. One man should be in sole charge of maintenance, with no other duties. With a dozen or more machines (trucks, excavators, etc), he should have a special **machine shop** with blacksmith shop and welding equipment, and all tools and devices usually accompanying it.

9.41. Spare Parts should be kept on hand for all parts judged likely to fail, especially parts of which there are many alike, even tho it is impossible to predict accurately what may be needed; and renewals should be ordered promptly for any parts found wearing out. For small devices, as jacks, carts, tools, etc, it is well to have complete duplicates on hand.

9.42. Oiling and other routine **servising** may well be done by schedule, at lunch-time, betw shifts, or at night, making thoro **inspection** of machinery, of chassis and tires of autos. Where only trucks are involvd, they may be required to report regularly to one or more gasoline stations having water, comprest air, oil, tires, etc. Where less mobile machines are in use, as power shovels, cableways, etc, trucks should be specially appointed to deliver these things to all machines on regular trips.

9.43. Road Maintenance is of equal importance now that trucks are very greatly depended upon. They should be kept well surfaced at all times.

9.5. OPERATION OF CRANES, ETC

The following points, likely to be overlookt, and regarded as especially important, are taken from the Code of Safety Standards for Cranes, prepared by the Am Soc Mech Engrs, and agree closely with rules by Assd Genl Contractors of America.

Proper provisions for strength shall be made for all parts subject to impact and rough usage. No cast iron.....except .....drums, bearings, brackets. Gears.....shall be provided with standard gards. Drums.....shall have a flange at each end. Not less than two full wraps of hoisting cable in the groovs when...at.....lowest position.

Cranes shall be operated only by the regular crane operators. Operator should not eat, smoke nor read when operating,...nor operate....when....physically unfit. Someone specially designated should lubricate all working parts.

Cranes shall be examind daily for loose parts or defects .....(and) kept clean. Avoid.....swinging loads over workmen. On electric machines, close no main switches until sure no one about, and that all controllers are off. When leaving machine, thru all switches off.

The operator shall not make side pulls with the crane ....lower carefully. When handling max loads....test the hoist brakes after the load has been lifted a few inches; if the brakes do not hold....lower....and....adjust or repair.

Operator shall recognize **signals** only from the one man who is supervising the lift. The following are among those recommended by the Am Ry Bridge & Bldg Assn;—

Hoist; forearm vert, making small hor circle with hand.

Lower; wave forearm downward.

Stop; hold hand level with hip.

Emergency stop; hand level with hip, move hand right and left.

Raise or lower boom; point up or down with thum.

Close or open clam; close or open hand.

9.6 PERSONNEL

Another factor of great importance in operating machines is the character and ability of the men employd. They must be both competent and careful—able and willing. Careless or unskild men will probably kink cables, batter buckets and shovels, jerk hoisting lines, thus introducing dangerus stresses or overturning machines, overload conveyors, abuse autos and trucks and their tires, cause unnecessary breakages and consequent inordinate delays; neglect lubrication, loose bolts and worn parts; leav ragged excavations or sides so steep they cave in; hinder an entire plant by lateness, or slow it down by indolent work, or injure men by careless handling, or cause dissatisfaction among the other men; all of this resulting also in increast "turnover" and its expense. Altho their practical experience may give them certain knowledge the C E may not have, yet when they feel that no one can know more or do better than they, they become dangerus. A careful and expert man will probably save very many times the increase in his pay; will be willing to work hard and do overtime if necessary, being content and even eager to remain on the work, acquiring an interest in it, and will respond favorably to decent treatment and respect, and to efforts to maintain his health and happiness.

9.7. DISASTERS

such as war, revolutions, strikes, tornados, shipwrecks, floods, long rains, rock-falls and land-slides, failures due to errors in computation or construction, fires, explosions of gasoline and dynamite, epidemics, "rackets" and financial "depressions", may add tremendusly to the cost of a project, and delay it indefinitly. Probably it is practicable to insure only against fires, tornados, shipwrecks and epidemics. Floods may usually be predicted in time by receiving prompt reports of heavy rains and snow meltings from rain-gage or river-gaging stations back on the water-shed. Rains which may delay work for weeks at a time cannot be predicted. Chance of error may be minimized by ample checking, while watchfulness and care should be maintaind against construction failures, explosions or epidemics. The engineer may often be on gard against depressions by a study of financial and business curvs of the past 50 or 100 years or more.

9.9. COSTS

9.911. In sec 6.411 we were able to give fairly definit costs of automobile operation by considering distance-costs and time-costs separately, as applied to level high-grade roads only; and from these the reader may obtain the tot cost for any particular case by merely adding the two costs, and then making allowances for other surfaces and grades.

9.912. When it comes to total costs of *earthwork*, (excavating and conveying), it becomes increasingly difficult to give figures that show any sign of agreement, because of the many more elements or factors in the more complex operations. There may be any one or more of many kinds and sizes of excavator used, the mat'l encounterd may be anything from mud to rock, the work may be in open country over wide areas or trenching in trecherus soil in city streets where traffic interferes, the dist the mat'l must be transported may be from a few feet to several miles, the difference in elev'n may be plus or minus a hundred ft or more, the depth of cut may be a few inches or beyond the reach of the machine, and any one or more of a number of kinds and sizes of conveying devices may be employd.

9.913. The C E with wide experience in excavating and conveying may usually be able to estimate unit costs with a tolerable degree of accuracy, by merely looking at the work to be done, and reviewing the machines available.

9.914. It seems best, however, in addition to referring to what figures we can in sec 9.92 with their wide discrepancies, to expect the less experienst C E to figure out each problem for himself as nearly as may be, from the figures we have given in the preceding sections, of capacities, weights, speeds, etc, of the machines available, subject to careful judgment based on what we have said regarding utility of each device. The computations usually involv nothing more than simple arithmetic, but it is essential that all factors or elements be considerd. To this end, we give in sec 9.94 a list of such items as *may* enter into any given problem, tho some or many of them may amount to zero in any particular case. See sec 9.95 for an example.

9.92. Cost Data

9.921. For suggestions in **this book**, see as follows;—

Pages 1024 *et seq*, for comparativly **primitiv** methods of excavation and transportation, noting that figures are based on labor at $1 per 10-hr day.

Pages 1105-7 incl, paragrafs 53-56 incl. These figures are for **railroad** work, practically all for 1914 and earlier.

Page 1408, para 2.11. From **Bidding Estimates.** Years of 1925-6.

Pages 581*w* and *x*, paragrafs 138-153 incl. Figures apply to **dredging** only, 1914 and earlier.

9.922. On the Welland Canal (1922-1931 only) **rock** excavation and disposal cost on an av $6.27/cu yd under water, and $2.12 on dry land.

9.923. A review of "Current Construction Unit Prices" as estimated by bidders, in recent issues of Eng News-Rec, may disclose figures that may be of servis, and that will require no adjustment for time, altho of course there is nothing to indicate how far the actual cost came from the amts bid.

9.93. Coefficients

Tables and charts frequently found in engineering periodicals may be of great use "*provided the conditions are those for which the chart* (or table) *is made*", or if proper coefficients may be applied to the result to adjust to the actual conditions. We offer the following suggestions.

9.931. Coeffs for different **Years.** In Eng & Contr'g, '28 Aug, p 416, is a table and diagram showing yearly av prices of **common excavation** on the Wabash RR from **1899 to 1915** incl, from which may be computed a coeff for changing the cost of any one of these years to any other.

From "Eng News-Rec Construction Costs", publisht annually by McGraw-Hill Pub Co, New York, N Y, may be obtaind comparativ costs of many factors entering into C E work, from **1913** in most cases, **to date.** At least, the curv for **common labor** may be of servis in adjusting data betw such dates.

9.932. Output Coefficients following are abstracted from "Output Factors for Excavation and Material Handling Equipment", by A. E. Holcolm, in Civil Eng'g, '30 Oct.

9.9321. The factor is 1.00 when the operating conditions are reasonably favorable, the dipper shovel or drag-line bucket has a capacity of 1 cu yd, an 8 ft cut is being made in ord'y earth, and the av swing of the boom is 90°. **Note** that these are *not cost* coeffs such as in sec 9.931, and that the two are not nec'y reciprocal of each other.

For **Materials,***

Hard shale and other rocky formations poorly blasted	0.40
Fairly well blasted rock or hard-pan, and tough rubbery clay	0.50
Clay and boulders	0.60
Heavy clay (not sticky)	0.70
Clay gravel	0.80
Wet sandy clay	0.90
Ordinary earth	1.00
Light dry loam or clay, loose sand and gravel, cinders and ashes	1.10
Light moist clay and loam	1.25

Size of Shovel Dipper or **Size of Drag-line Bucket;** is directly proportional to output. Thus, a half cu yd dipper will take out only half as much as a 1 cu yd dipper in a given time.

Output coeffs for **Depth of Cut** for a **Shovel** swinging 4 revs/min, and other conditions as above (9.9321), which should excavate about 100 cu yds/hr,

Depth of cut, ft	1	4	8	12	16
Coefficient	0.62	0.89	1.00	0.88	0.76

Output coeffs for **Depth** of end-cut excavation by **Drag-Line** with conditions as in sec 9.9321, which should excavate 80 cu yds/hr;

Depth of cut, ft	1	4	8	16	24	32	40
Coefficient	0.82	0.95	1.00	0.90	0.80	0.70	0.60

9.9322. Example. A ¾ cu yd shovel, working in clay gravel, with a cut of 12 ft, may be expected to produce abt

$$\underset{\textit{cu yds/hr}}{100} \times \underset{\textit{dipper coeff}}{0.75} \times \underset{\textit{mat'l coeff}}{0.80} \times \underset{\textit{cut coeff}}{0.88} = \underset{\textit{cu yds/hr}}{52.8}$$

* Cost coeffs establisht by A T Cushing, Eng News-Rec, '26 Sep 9, applying esp'y to **trench** work, agree fairly well with Mr Holcomb's figures, when the reciprocals are used.

9.94. Cost Elements

to be considerd when computing output or costs.

9.941. Major Elements. (a) Excavation, (b) Transportation, (c) Backfilling or Spreading.

9.942. Working Rates for computation of volume excavated and transported per time, should include time for (1) Loading, (2) Transporting, (3) Unloading, (4) Returning, and also for (5) Idleness due to weather, repairs and lack of co-ordination.

9.943. General. Camp Maint, Supt and Eng'g, Maint of Roads, Profit, Effect of Size of Job.

9.944. Machines. In any case, Power, Labor, Repairs, Maintenance (oil, etc). If rented, cost of Rental, if ownd, Interest and Depreciation, and Idle Time.

9.95. Example

To illustrate the way in which the above suggestions under sec 9.94 may be used, we give the following comparativly simple example, in which all quantities are entirely **fictitius**, and are not to be taken in any way as figures for estimating. Our computations are carried to not more than 3 or 4 significant figures, because it is impossible to compute such results to within a few percent.

Given:—20,000 cu yds of earth to move an av dist of 1000′, and that it can be dumpt on a spoil bank without spreading. Suppose we have a 2½ cu yd gasoline shovel capable of digging 300 cu yds/hr, and that we can hire all the 5 cu yd trucks we need, capable of 15 mi/hr loaded, and 20 mi/hr unloaded over the territory involvd.

At 300 cu yds/hr, our **shovel** should finish, theoretically, in 20,000/300 = 66 hrs, or, allowing 25% lost time, in 83 hrs. Using sec 9.944 as a reminder, we might have, $2/hr for gasoline, $1/hr for labor, $2.50/hr for repairs and maintenance, and $2.50/hr for int and depreciation, or a tot of $8.00/hr, which, for 83 hrs would cost $664. To this we should perhaps add interest for 15 days since the last job, say $50, and $50 for moving from it to the present location, making tot cost for shovel, **$764.**

Taking up the items in sec 9.942, as applied to the **trucks,** and considering loading first, we allow two shovel cycles of 30 secs each for loading, plus 15 secs to "spot" under the shovel and to pull out, or 75 secs. To travel 1000′ at 15 mi/hr (= 22 ft/sec), will require theoretically 45 secs, or say 50 secs, allowing 5 secs for accel and decel. For dumping we allow 10 secs. For the return, at 20 mi/hr, 33 + 5 = 38 secs. Tot, 173 secs for each truck load of 5 cu yds. As there are 20,000 cu yds to move, we shall need 4000 truck loads, or, at 173 secs/load, 692,000 secs. The shovel, however, needs only 83 hrs or 298,800 secs. We shall need then, 692,000/298,800 = 2.31 trucks, theoretically, or 3 actually. This excess of truckage should allow suff time for item (5) under sec 1.942, (idleness due to repairs) altho serving the shovel with only two trucks, for a time, may perhaps more than help to use up the 25% allowd for delays to it. (Delay due to weather we take up later). Considering sec 9.944, we assume for each truck $1/hr for gasoline, $1/hr for labor, 25c/hr for repairs and maintenance, totaling $2.25/hr. This, multiplied by 3 × 83 = 249 truck-hours, givs $560. In addition we must allow say $15/day for rental of each of three trucks for 83 hrs or 10½ days, plus 4 days allowance for bad weather, (tho it may

rain for weeks at a time or not at all) or 14½ days, totaling $652. This, with the $560 above for operating, totals **$1212.**

General expenses. Considering now sec 9.943, we allow 25c/hr for office overhead (rental, etc); superintendence, $1.25/hr; total $1.50/hr. Allowing for 100 hrs to include office work outside the machine time, this, with $25 for surveying, totals $175. We assume no special road maintenance, but allow for 3 helpers for 10½ days at 50c/hr each, totaling $126. For general expense, then, we allow $175 + $126 = **$301.**

Totals. Adding totals for shovel, trucks and general expenses, we have $2277. Allowing 10% for profit, this becomes **$2505**, the total for the job. Dividing by 20,000 cu yds, we obtain 12.52c/cu yd. Bear in mind that this example is entirely fictitius, for purely imaginary conditions, to illustrate the kind of method that may be used.

DREDGING

References. Those contributing most largely have been;— W L Saunders, XII International Congress of Navigation; A W Robinson, Am Soc C E, Vol LIV, Part C, and numerus others writing in Eng News, Eng Rec, etc, and F Lester Simon's book "Dredging Engineering".

1. Excavating, in general, is the work of loosening and moving materials, such as earth, rock, etc. It is called "dredging" when water overlies the mat'l to be moved. In many instances, the methods or devices employd may be the same or similar, whether the work goes on above or below water.

2. Usually the **object** of moving the material is to **increase the depth** of the waterway, to accommodate vessels of greater draft. Sometimes, however, the object is primarily to obtain gold or other **precius metals** from the dredged material, or to **raise the level of land**, by heaping dredged material on it. If it is desired both to dredge and to raise the land level at places near each other, the two operations may often be combined.

CONTROLLING FACTORS

3. Methods of dredging, and the devices used, depend chiefly upon the character of the material to be excavated, which may vary from silt to solid rock. However, there are often other considerations of much importance.

Materials

4. Mud, Silt and Sand are generally the easiest materials to excavate, but the means used to dislodge them, may so mingle them with the water, that they may be carried away by the water to some other place where they are equally unwelcome, or other mat'l may drift in and refill the excavations, necessitating repetition of the work. Or, it may become necessary to handle disproportionately large quantities of water with the mat'l, making subsequent separation very troublesome.

5. Loam, Clay. These, and the gravels, are ordinarily easy to dredge, altho sticky clays are likely to give trouble by adhering to the buckets after they have been excavated. When they are handled by hydr dredges, they may stick or jam in the suction orifices or pipe-line.

6. Indurated Clays or Hardpans may be so hard as to be quite difficult to dredge, and it may even be expedient or necessary to blast them.

7. Disintegrated Rock, Boulders, Rock. Solid rock, must, of course, be broken up before it can be removed. The fragments so obtaind, may still be too large to be handled by the machinery employd, and may have to be broken to a still smaller size. In disposing of the dredged rock, it is often desirable to provide additional apparatus for sorting out the varying sizes, which may be wanted for different purposes, or for separating the rock from soil that may be mixt with it.

8. Gold and Other Precius Metals are usually found mixt with gravel or small boulders. See ¶80 and immediately before.

9. Additional apparatus, tailings stackers, gold-saving tables, etc, not strictly dredging machinery, will be needed to separate the metal from the gravel, and where rock may be utilized, still more apparatus may be required for sorting, as in ¶7.

10. Debris may include almost anything except fairly uniform ground. It may be ground containing old stumps of trees, often making necessary more than one kind of dredging apparatus to complete the work. Under this head would come also wrecks of all kinds, from wreckt masonry (sea walls, bridges, etc) to wreckt ships. Dredging machinery may be useful also in removing piles or other structures in water

11. Coral formations are usually quite difficult to handle, and often require much previus blasting

Operating Space

12. The type of dredge to be used is determind largely by the character of the space in which it is to work. In the open, work may be delayd by **rough water or storms,** for few dredges can weather storms well, and the several units of a plant may become damaged by pounding against each other In channels, the rapidity of the **current** may make it impracticable to hold in place and operate certain kinds of dredges. In channels, anchoring becomes troublesome, as the cables, or the pipe-lines of hydraulic dredges, are likely to interfere with passing ships, or, in very narrow channels, the **passing ships** may disturb the dredge by creating suction on one side **Confined spaces** around docks, etc, may make it impracticable to employ certain types of dredges.

Character of Work

13. The Depth and Width of cut required may determin the type of dredge. Care must be taken in considering the proposed depth of dredging, to allow for the **"squatting"** of vessels, that is, the increast draft due to their motion, an item that is different for each vessel Tho squat becomes noticeable only in large craft moving at high speed, yet it has produced an increase in draft of as much as 4 ft. (Simon, "Dredging Engineering").

14. The Distance the dredged mat'l must be transported is a very important consideration, often calling for much auxiliary equipment, such as scows, and tugs to handle them, or long pipe lines. Sometimes the mat'l must be rehandled at some intermediate point.

15. The Permanence of the work also controls very largely **New work, or isolated** bits of work, with completion in sight, may justify the use of almost any dredge available that can do the work, even tho very poorly suited to it. For **maintenance** work, however, or where there is prospect of repetitions of the same kind of work nearby, the tendency should be to obtain machines closely adapted to the work in hand, and with high operating efficiencies.

16. Much may depend, also, upon whether **other kinds of work** are to be done that can be combined with the dredging, such as land filling, obtaining sand, road materials, etc, since one type of dredge might be much better adapted to such combination than another which might be ideal for one kind only.

17. Government or Public operation, more common in foren countries, makes possible and economical the employment of much larger and more elaborate dredges than are possible under private control, and has been the chief reason for certain diffs in types of dredge used here and abroad.

18. Time, also, may enter into the problem, for if there is not time to obtain the ideal dredge, or, if the work must be done very rapidly, it may be necessary to employ dredges that will do the work uneconomically.

Operating Facilities Available

19. The Kind of Power available will sometimes influence the selection of the type of machine to be used Conversely, the mech'l equipment of an available dredge, may determin the kind of power to be used. See ¶114.

20. The Crew available may also be a controlling factor, both as regards their number and experience on the particular type of dredge contemplated for the work

21. Supplies. The availability of fuel, repair facilities, new or duplicate parts, as well as food, shelter, etc, may all largely determin the choice of a dredge type, since one dredge, otherwise well suited to the work in hand, may be grossly lacking in such facilities, while another, by its superior equipment, may promis better, even tho not otherwise the appropriate machine for the work

DREDGES *

and Dredging Devices

Classification

22. Means for dredging may be classified broadly as follows;—

Training walls, reaction jetties, etc
Drag-line or scoop dredges
Cableway excavators
Bucket or grapple† dredges
  Clam-shell dredges
  Orange-peel dredges
Dipper dredges
Ladder, or continuus bucket dredges
Hydraulic dredges
Rock dredging
  Rock breakers
  Rock cutters
  Blasting
"Universal" and miscellaneus dredges.

Training Devices

23. Walls, dikes, partial dams, etc, may be used under certain favorable conditions, to create or maintain deep water, by causing currents to operate in such ways as to increas depth by scour; but their treatment is reservd for another place.

Drag-line or Scoop Dredges

24. General Features. Fig 1. The boiler and engin *E* used for the work, are mounted together on a frame, or platform, and may be housd in. From the front of the platform there extends a long boom *B*, held up by an adjustable guy or stay *G*, which bears on an A-frame *A*, and is fastend at the back of the platform Connected to the engin drum is a cable which extends to the end of the boom, and then downward over a pulley, as shown at *H*, to the rear of a bucket or **scraper** *S*. At the front of the scraper is a harness or bail, to which is fastend a second cable, or **drag-line** *D*, also manipulated by the engin. The entire outfit is mounted on a turn-table which is itself mounted on a truck of some kind. The turntable is usually able to swing thru an entire hor circle, in either direction. The truck is usually fitted with either (a) wheels which run on temporary tracks; (b) skids bearing on rollers, or (c) a "caterpillar" gear.

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* Called "dredgers" in Great Britain.
† This word is used by some to denote a bucket specially designd for picking up bulky objects, as stones, logs, etc.

25. Operation. The scraper S is lowerd by means of the cable H, to the bottom. The cable D is then tightend, and the lip of the scraper thus forst into the mat'l, which it scrapes up and collects within itself. When the scraper is full, the cables are so adjusted that it is raisd from the bottom still maintaining its nearly level position Then, if the mat'l cannot be dumpt betw the digging position and the machine itself, the entire apparatus is swung to a position such as that shown by the dotted lines, so that the mat'l may be dumpt where it will be least in the way. D is then relaxt, and the scraper drops, open end down, thus dumping its contents The machine is then swung back for another cut.

26. Sizes and Capacities. The following figs are approx av extremes of regular construction

Boom, 40′ to 150′ long
Bucket capacity, 1 to 5 cu yds.
Digging depth, side cut, 15′ to 50′, end cut, 20′ to 70′.
Weight, 75,000 lbs to 550,000 lbs.

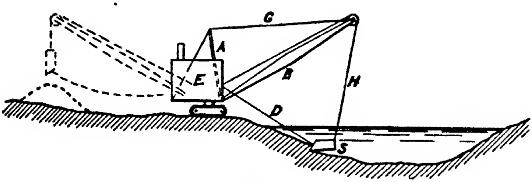


Fig. 1.

27. Utility. The use of drag-line scoops under water is rather recent, but seems to have been very satisfactory so far as tried The scoop, not being mounted on a vessel, must do all its work by reaching out from land, and the dist beyond shore to which it can excavate is necessarily limited by its reach, whereas dredges bilt as parts of vessels can go as far from shore as their seaworthiness will permit. The chief use of the scoop as a dredge, therefore, is for deepening comparativly narrow channels, such as canals and irrigation ditches. Owing to the almost direct hor pull, and negligible vert pull on the end of the boom, when digging, and the resulting small overturning moment, the machine is capable of distinctly difficult work, such as rooting out snags, boulders, stumps, etc.

28. Cableways, or Tower Excavators are used sometimes insted of derricks, for handling scoops, and also for handling grapple buckets But if a revolving derrick is available, even if it must be shifted across the stream to complete its work, it will usually be found preferable to the cableway with its two cumbersome towers If the derrick cannot reach far enough, either a cableway or a dredge becomes necessary, and of course there are limits to the economical span of a cableway.

Grapple* Dredges

The Dredge.

29. The dredge itself (see also Scows, etc, ¶111), Figs 2(a) and (b), consists of a vessel V, which carries the necessary machinery DD' are two hoisting drums operated by steam or gasoline engins or electric motors. Over these pass two wire hoisting cables, known as the "closing" and "opening" lines, H and H', which are gided by sheavs CC' on an A-frame A, and thence pass over sheavs on the end of the boom B, and down to the bucket K. Other apparatus, not here shown, and to be described later (¶46, etc) must be provided for holding the dredge in place, and for holding to the side of the dredge the scow which is to receiv the dredged mat'l

* The word "grapple" is sometimes confined to special forms of grab buckets designd for handling bulky objects, as stone, logs, etc.

30. The Vessel or Hull *V* is usually rectangular, since such a form affords a max of stability to resist the moments set up by digging and by swinging the boom over the scow. There is seldom need for a hull designd for easy navigation. It must be very substantially constructed, and braced by suitable trusses, bulkheds and knees. There is bilt on the vessel, also, such **housing,** *E*, for machinery and men, as may seem desirable.

31. The Boiler and Engin, or motors, are of the usual types, Scotch marine, or other boiler: 2-cyl hor engins, driving standard hoisting machinery. (See Mechanical Enginer's Hand-Books). The boiler (or power-line, if electric) should be of sufficient capacity to operate all the aux'y engins simultaneusly if need be, and the hoisting engin should be of ample size to withstand hard usage See under "Power", ¶114, etc

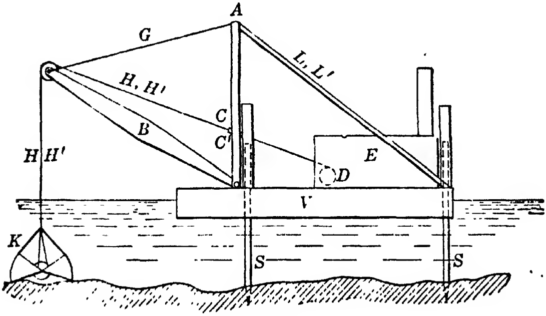


Fig. 2 (a)

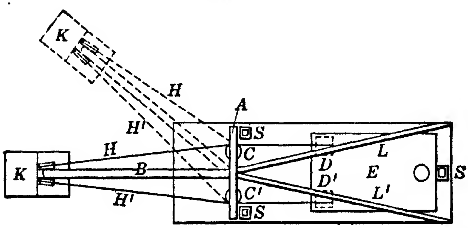


Fig. 2 (b)

32. The A-frame, as its name implies (tho not obvius from the figs, since it is there viewd edgewise) is shaped like a letter "A". The element corresponding to the hor portion of the letter is generally referd to as **"the table".** At or near the table are mounted two sheavs *CC'* which hold apart the hoisting wires. The A-frame is held in position at its top by **back-guys or back-legs,** or both, *LL'*.

33. The Boom, *B*, is usually maintaind at a fixt vert angle by means of a guy-wire or "topping-lift" *G*, but is arranged to rotate on a pivot at its lower end thru a considerable hor angle. The upper end of the boom carries the two main sheavs over which pass the bucket-hoisting lines.

34. The Bucket.

35. The Grab, Grapple or Bucket is mfd in two types, the **"clam-shell"** or **"crab"**, Fig 7, having two segments or shells, and the **"orange-peel"**, Fig 8, having three or four segments.

36. The Principle on which the two kinds of buckets operate is similar, and is perhaps best explaind by first assuming a hypothetical clam-shell bucket, Figs 3 (*a*) and (*b*) and then explaining the modifications necessary Each shell, *S* or *S'*, consists of a curvd plate, forming about a quarter of a circumf, and of two side sectors or flat cheek plates. Thus, Fig 3 (*a*), the two shells, when closed, form a semi-cylindrical bucket, open at the top The shells are pivoted together at *C* by pins or hinges, and if supported by a sufficiently large force at *C*, the bucket would retain material *M* within it. But provision is made, thru the rods *OP* and *OP'*, for applying upward forces at *P* and *P'*, so that on reducing *C* practically to zero, the bucket must collaps at the middle, as shown in Fig 3 (*b*), forming a wide opening under it, thru which the mat'l falls If now the bucket is lowerd upon loose mat'l, and the forces again be reverst, the two shells will not only reshape themselvs into a bucket, but will gather up a certain amount of any loose mat'l on which they rest.

37. At *O* is attacht one of two wire ropes, known as the **"opening line"**, while at *C* is attacht a second rope or cable, known as the **"closing line"**. It is thus possible, with only two lines or ropes, not only to raise and lower the bucket as a whole by hauling up or lowering on both lines simultaneusly, but also to open or close the bucket, by paying out or reeling in one line with relation to the other

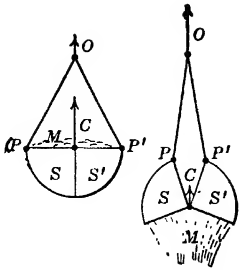
38. Actually, the bucket shown in Fig 3 will gather up but little of any hard material, since not enough of the lifting force *C* can exert closing or biting forces.

39. Practical Design. To overcome this, the bucket closing forces are increast. In one make of bucket, a simple **lever** *L*, Fig 4, is made a part of one of the shells *S* The closing line passes down around a pulley on the end of the lever, and returns to a point near *O*, where it is made fast The force *C* must therefore exert a closing force greater in relation to the lifting force, than in the case assumed above in Fig 3 Other pulleys may be provided near *O* and at *C*, and the line led back and forth until suff closing force, for a given lifting force, is secured

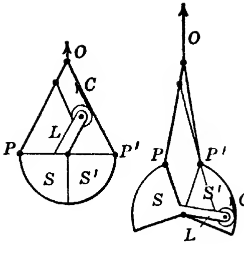
40. A "Drum and Shaft" arrangement, however, is more generally provided, as in Fig 5. The "drum" indicated by the large circle at the center is little more than a large pulley. Around it is wrapt and attacht the end of the "**closing** line" *C* (or *H* Fig 7). The "shaft", indicated by the smaller circle, **consists** in reality of a pair of small drums, one on each side of the large pulley, and around them are wrapt and fastend two branches of chain *Z*, the other ends of which are attacht to the frame at *O* The force *C* thus sets up in *Z* a much greater force than itself, and an increast closing couple is set up by *Z*, pulling up at the shaft, and exerting at *O* a downward thrust equal to *Z*, carried out to *P* and *P'*; this without increasing the lifting force (approx $=C$) on the bucket as a whole

41. Yet such design would mean serius fouling of the shaft and drum by the contents of the bucket. In practis, therefore, the shaft and drum are mounted much higher, somewhat as shown in Fig 6 (*a*), the bucket shells having heavy arms or extensions. See also Figs 7 and 8.

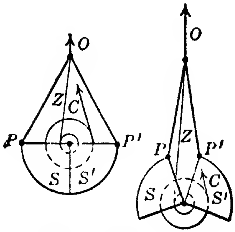
42. Bucket Details. Figs 7 and 8 are outline sketches of the clam-shel and orange-peel buckets respectivly, shown open ready for grabbing a load Since all dredging devices must work in dirt which necessarily increases the wear, all **pivots and bearings** should be made with bushings than can redily be removed and renewd. "The **cutting edge or lip** of each shell is generally strengthend by the addition of a steel casting, or, preferably manganese steel, the better to resist the severe abrasion". (Simon, Dredging Engineering). **Teeth** may be attacht, but they are often used mistakenly,



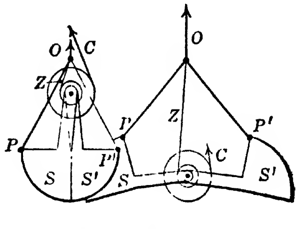
(a) Fig. 3 (b)



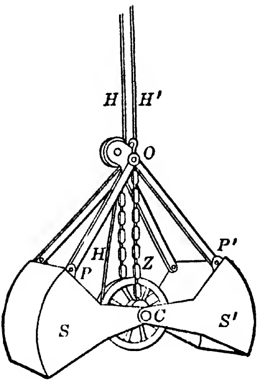
(a) Fig. 4 (b)



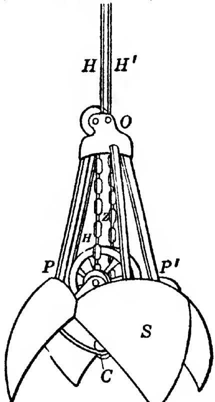
(a) Fig. 5 (b)



(a) Fig. 6 (b)



"Clam-Shell"
Fig. 7



"Orange Peel"
Fig. 8

and are seldom of use on large buckets "Teeth are of no advantage in handling coal, sand, gravel, crusht stone, or other bulk material of a granular lumpy kind" (Blaw Bucket Manual).

43. On account of the twist of the strands of the hoisting cables, and because there is practically no force to prevent the rotation of the bucket about a vert axis (which would make the hoisting wires bind on each other), it is usally necessary to provide some means to prevent such twisting A **"dorsey wire"** may be used, which is simply a light line, one end of which is attacht to the side of the bucket nearest the dredge, while the other is past over a pulley or sheav about midlength of the boom, with a weight attacht, just heavy enough to prevent such twisting, but in no way interfering wth the other motions. Another device consists of two long **uprights** attacht rigidly to the two sides of the bucket, which extend up thru gides provided at the end of the boom, thru which they slide vert'y These uprights have the additional advantage of holding the bucket vert in case it is lowerd on an uneven bottom.

44 In France for dredging in clay, the bucket is sometimes lined with wood, to which the clay less readily adheres,

45. Other Types of buckets have been bilt, one like a pair of tongs, and another in which the closing force is supplied by comprest air **Single-line buckets** are bilt, requiring only one hoisting line They usually depend upon some arrangement of latches or catches Where only a single hoisting line is available, such an arrangement is essential; but none of those mfd seem to be sufficiently satisfactory in their operation to take the place of the two-wire arrangement where that can be had.

Moorings.

46. Perhaps the simplest and cheapest method of fixing the position of the dredge and of holding to it the scow which is to receiv the dredged mat'l, is that of using **wire ropes,** which are controld by aux'y engins thru capstans or drums on the dredge, and which are made fast to nearby objects, or to anchors or especially driven piles On at least one dredging operation (in Galops Rapids, St. Lawrence R, Report by W. L Saunders, XIIth International Congress of Navigation), where the current was very violent, **chains** were used insted of wire ropes See ¶104. The use of lines for holding the dredge is generally unsatisfactory when many other vessels must pass the dredge.

47. Spuds *SS*, Fig 2, are therefore often employd Many dredges are equipt both with spuds and mooring lines, thus affording greater flexibility of operation. The spuds are simply large uprights, of timber or steel, arranged to move vert'y in wells in the dredge or in pockets on the sides. Usually there are three or four spuds. Two of them are rigged near the front of the dredge, and are maintaind upright by means of a gallows frame spanning the entire beam of the vessel, (usually near the A-frame, and sometimes combined with it); while the rear spud or spuds are held alongside rigid or guyd posts, or else pass vert'y thru other framing. Usually the spuds merely rest upon the bottom, being prevented from floating up by hevy metal pointed shoes, which also aid in penetrating the bottom, and in protecting the ends of the spuds. They are raisd or lowerd, usually by means of cables, sometimes by racks and gears, operated by special engins on the dredge The spud cables are sometimes past over sheavs at the tops of the spuds, or other means are used for forcing the spuds down, sometimes to the extent of lifting the dredge slightly from its normal floating level, thus making it quite rigid; but more usually the vessel is free to ride up and down on the spuds, which serve merely to maintain the desired lateral position.

48. To Shift the Position of the dredge when spuds are used, the bucket may be gript into the bottom, and all the spuds raisd but one of the forward two. Then, by a pull on the proper bucket line, the dredge may be rotated about the one spud as a pivot. Then the two front spuds are made to exchang functions, and the dredge is advanst still further by pulling on the other line. The process is well referd to as **"walking on the spuds".** See also "walking", "fleeting" or "trailing spuds," ¶64.

49. Sometimes the spuds are utilized to keep the mooring cables below the keels of passing vessels. In this case, sheavs are provided at the bottoms of the spuds, around and under which the mooring cables are past, the spuds acting only as brackets, and the cables doing the work of maintaining the position of the dredge. (Patented by Osgood Dredge Co)

50. Dredges bilt for drainage ditches sometimes employ **"bank spuds"**, which reach out at a vert angle from the dredge, and penetrate the bank or rest in grillages set in the bank. These afford great lateral stability.

51. In some cases (where the bank cannot be reacht) **side floats** have been provided, which greatly increas the lat'l stability These may be essential if the dredge is of narrow beam.

52. Where the dredge can do all of its work from the bank, it may be mounted like the drag-line scrapers, (¶24), the **caterpillar** mounting being now (1925) generally preferd.

Operation.

53. After the dredge is made fast, a scow (¶111) is brought up along one side, and made fast If on the right side, as is usual, then the "closing line" is the right-hand or starbord line H, Fig 12 (b), while the "opening line" is the left or port line H' The "table" sheavs CC keep these lines so deflected that as the tensions in them are altered, th boom swings hor'y. Thus, after the bucket has been lowerd to the bottom, tension is put on the closing line, which therefore (1) closes the bucket upon its load, (2) hoists the load, and, at the same time (3) swings the boom over to the right. When the boom comes over the scow, the tensions in the lines are reverst, with the result that (4) the bucket is opend and dumps its load into the scow, and (5) the boom is swung back to its digging position Both lines are then slackt off, and the bucket so lowerd The position of the boom can be nicely controld by an experienst operator, by reversing and adjusting the stresses in the two wires at the proper instants.

54. The curv of the bucket shells should not be such as to fit well into their own bite, since it is likely that a suction may be establisht in soft mat'l which may be very hard to break Attempts to use a light bucket not designd for hard biting, will soon result in injury to it. If teeth are added, they may only bend the lips of the shels, and if the operator tries to make a light bucket bite by dropping it violently on to the mat'l, the bucket will soon be injured The remedy is either a hevier bucket, or means for exerting greater closing forces, or both.

Sizes and Capacities.

55. Some very large dredges have been bilt, one recently (1921) advertised having a bucket cap of 30 cu yds, the bucket alone weighing 40,000 lbs. The bucket capacities more usually range from a fraction of a cu yd up to 7 or 8 or 10 cu yds, with wts from 4000 to 26,000 lbs. On machines in quantity for the market, the booms are generally under 50 ft long, but machines are constructed with booms from 100 to 200 ft long or longer.

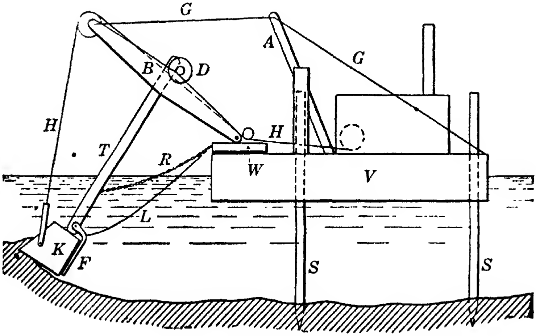
Utility.

56. In suitable mat'l (as sand, clay or mud), the grapple dredge is very efficient The **depth** to which it is able to dig is theoretically limited only by the length of cable which can be wound on the hoisting drums; but with increast depth, the bucket cannot be counted on to settle on just that part of the bottom desired The bottom made by a grapple dredge is usually uneven, with holes and high spots A grapple dredge is generally not good for very hard digging, unless the masses to be grappled with are broken. The dredge is useful around docks and in narrow channels, tho it cannot well be used where there is no room for the scow to be maneuverd alongside. The **clam-shell** is most useful in soft mat'l, stiff mud and sand (when not too fine), gravel, coal, ore, and small disconnected masses of rock of from ½ to 4 or 5 tons. The **orange-peel** seems especially adapted, in spite of considerable uncertainty of grasping when far under water, to the dredging of boulders, blasted ledge rock, pig iron, stumps and the like—the roots of stumps being cut thru on all sides by the 3 or 4 shells with their 6 or 8 cutting edges; also for excavating within caissons in foundation work.

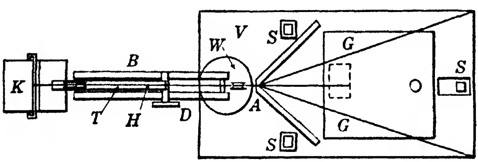
Dipper Dredges

The Dredge.

57. The dipper dredge, Figs 9(*a*) and (*b*), consists of a vessel *V*, much the same as that of the grapple dredge, ¶30, tho even more strongly constructed, and carrying the necessary machinery. The vessel is almost invariably equipt with spuds *SS*, the forward ones being sometimes held by a frame (not shown) spanning the width of the vessel There is also an "A"-frame *A*, tho not necessarily with the hor "table", and this frame is often inclined, as shown Over it pass guy wires *GG* which hold up the end of the boom *B* and maintain it at a fixt angle. The foot of the boom, however, is placed at the bow and is so arranged that the boom may be swung hor'y The boom is so constructed that another member, called the "stick" or "dipper handle" *T* can pass thru it To the lower end of the stick is attacht the bucket *K*, which has a flap-bottom door *F* engaged by a latch (not shown).



(*a*)



(*b*)

Fig. 9.

58. **Boilers and Engins,** or electric or gasoline motors, are provided for the varius operations described below and also for handling spuds and other auxiliaries. The **hoisting line, main line or bucket wire** *H* passes from the drum of the main engin under a sheav at the foot of the boom, thence to a sheav on the outer end of the boom, and from there down to the bucket bale. Usually the line passes under another sheav on the bucket bale and

back up to the end of the boom, where it is made fast. There may be also a **backing chain** *R* with its engin or motor (not shown). In many cases there is a **"crowding engin"** or motor *D* (shown only diagrammatically), mounted on the boom where the stick passes thru, by means of which the stick may be forst to rise or fall.

59. It is well to have the hoisting line wound on a **"differential" drum,** conically shaped, and having helical groovs, so that during the first part of the pull, when the resistance due to separating the load from the bottom is greatest, the line is wound on the small part of the drum, affording the engin a max of leverage, whereas, when more line is wound on the drum, it lies on portions of larger diam, and so makes possible higher hoisting speed when the resistance is less.

60. The Boom *B* of a dipper dredge, must be exceptionally strong and well designd It consists usually of girders, deepest at the center, the better to resist the bending moments It is usually made in two halvs, with a space betw, thru which the stick (see ¶61) may operate, tho in some cases the stick is divided and straddles the boom In operation, and especially where a crowding engin is used, the direct bending stresses set up at the middle of the boom may be quite large In addition, the rotation of the boom may be effected by an engin driving a **bull-wheel** *W* at its heel, and this will set up twisting stresses which may be quite serius in case the lateral movement of the dipper is arrested abruptly by striking the bottom or other object

61. The Stick *T*, as stated, passes thru the boom *B*, and is handled by the lines *H* and *R* Usually it carries a rack which engages with gears and wheels at *D* Not only can the stick move longit'y, but it can rotate about the pinion at *D* In some cases these wheels are provided with brakes only, and in others they are part of the "crowding engin", which can aid in the movements by forcing the stick up or down In gen'l, the motion produced is similar to that of a man digging with a hand shovel.

62. The Bucket *K* carries a cutting lip at its outer end, usually bearing individual detachable steel spikes or **"teeth"** where hard mat'l is encounterd. It is rigidly attacht to the end of the boom Its bottom consists entirely of a **flap or trap-door** *F* hinged at the rear. A latch is provided for holding the flap closed, and a line *L* passes from the latch back to the enginman who may thus open the flap The flap is so counter-weighted as to caus it to hang at about 45°, and the impact of striking the water, in its descent, is counted on for closing it.

63. The Digging Operation, then, consists in releasing the hoisting line *H* and the clutches or crowding engin at *D*, thus permitting the bucket to descend (the flap being closed by the water surface) while the backing chain *R* is pulled so as to draw the bucket to the rear. When the bucket reaches the bottom, the clutches at *D* are closed, and the hoisting engin is operated, with the result that the bucket makes a cut along the bottom. The hoisting is continued (clutches releast) and the bucket is hauld out of the water. The boom is swung hor'y either by two operating lines as is customary with grapple dredges, or by means of a bull-wheel *W* at the base of the boom, until the bucket is over a scow which has been brought up alongside, or over a bank or other available point at which it may be desired to deposit the mat'l. The latch is then drawn, the flap opens, and the contents drop out If the mat'l sticks badly in the bucket, the operator may use his crowding engin to jerk the stick back and forth (while in nearly hor position) thus slapping the flap violently against the bottom edges of the sides, and helping to jar the mat'l loose. The boom is then swung back, and another cut is started, as above.

Moorings.

64. For dipper dredges, moorings may be substantially the same as for grapple dredges, except that mooring lines are rarely if ever used, and the **forward spuds are especially strong** to resist the high and erratic digging stresses. The rear spud stands in a slotted well to provide motion fore and aft, and is called the **"walking", "fleeting" or "trailing spud".**

65. To Shift the Position of the dredge, the bucket is lowerd on to the bottom, the forward spuds are releast, and then the backing chain is pulled; the dredge moving ahead and the rear spud tilting in its slotted well. The forward spuds are then set down again, and the rear spud raisd and advanst for the next "step".

66. Scows may be Handled by the dipper, the spuds remaining down, and the dipper resting on the scow and moving it about by means of the bull-wheel and crowding engin, or hoisting and backing lines.

Sizes, Capacities, etc.

67. The bucket capacities generally used are from 1 to 5 cu yds Larger sizes are not unusual, while the famus dipper dredges of the Panama Canal had a cap of 15 cu yds in mud or 10 in rock The following figures will afford a fair idea of the extremes.

Hull		
Length, ft	144	40
Width, ft	44	18
Depth, ft	13½	4½
Draft, ft	8	
Digging depth, ft	50	9
Dipper capacity, cu yds	15	¾

68. Speed. In medium sized machines, the dipper can make one cut in from about ½ to 1 min during steady working (not allowing for waits for scows, repairs, etc)

69. Utility. On account of its ability to maneuver itself and its scows without lines, the dipper dredge is especially useful in confined spaces, around docks, and in narrow channels By its ability to force the bucket into the bottom, it is especially capable in hard mat'ls, for the removal of boulders or rock that has been broken up (as by blasting), and for tearing out miscellaneus obstructions, as piles, old cribs, wrecks or foundations. It can "cut its own flotation" ahed of it, thus digging a new channel where none existed. On account of the poor leverage, however, it is not available for as great depths as the grapple or hydr dredges but it can level off the bottom very much more accurately than the grapple dredge.

Continuus Bucket, Elevator or Ladder Dredges

70. Three Types of "ladder" or continuus bucket dredge are in gen'l use, referd to as (1) the **"Stationary"**, (2) the **"Barge Loading"**, and (3) **"Sea-going Hopper"** dredge.

71. The Vessel or Hull is different for each of these The "stationary" dredge has a hull similar to those of the grapple and dipper dredges; but the other two (2) and (3), are bilt with molded hulls to minimize resistance, and are equipt with propelling engins The "barge-loading" dredge (2) depends upon barges or scows to receiv the mat'l it excavates, while the "sea-going" dredge, (3), Fig 10, is bilt with self-containd hoppers *HH*, since it is intended for use in open waters, where there would be serius pounding of any aux'y craft alongside.

71½. A notch or elongated well *W* is formd down thru the bow or near the middle of the vessel to receiv the "ladder" *L* and the chain of buckets *BBB*, tho in some "stationary" dredges, two ladders are employd, one on each side, outside the hull.

72. "The Ladder" *L* is a long frame, pivoted at its upper end *P*, while the lower end is held at the desired elev'n by a line *N* operated by a winch *C*. At each end of the ladder is a large tumbler or drum, and rollers are provided along the upper side of the ladder.

73. The Buckets *BBB* are carried on an endless chain over the drums and rollers, hanging freely from the under side. The chain of buckets is operated by a special engin or motor (only the main wheel *E* of which is shown) engaging with the shaft of the upper drum. Each bucket is provided with a reinforst cutting lip, or with teeth.

74. Operation. As may be seen, each bucket, in turn, cuts some mat'l from the bottom. Then, as the bucket revolvs, the mat'l is caught up, and is then carried to the top of the ladder, where it is dumpt out as the bucket turns over The bucket then descends (inverted) back along the lower side of the ladder As the cutting progresses, the dredge is advanst by its own engins, or is swung as in ¶77.

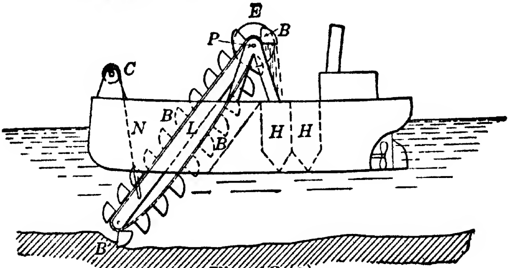


Fig. 10 (a)

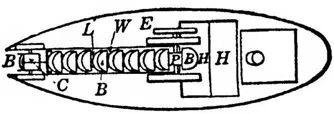


Fig. 10 (b)

75. The Disposal of the Material is different in diff types of dredges, and again, with diff kinds of mat'ls. In the sta'y barge-loading dredge, suitable chutes are provided to pass the mat'l over the edge of the vessel into a scow made fast at the side (Where ladders are placed on each side of the vessel, the mat'l may fall directly into the scows). In the hopper dredge, Fig 10, the mat'l is dropt into the hoppers *H*, and the dredge travels to a suitable dumping ground, where the bottoms of the hoppers are opend and the mat'l is deposited. When desirable to separate diff kinds of mat'l, as in digging for gold, or where the bottom consists of rock and finer mat'l to be sorted, rotary screens or separating devices of one kind or another are introduced betw the top of the ladder and the chutes or hoppers. These, not being strictly dredging devices, are not treated here. In some cases, the mat'l can be conveyd hydraulically thru long pipes, as in hydr dredging (see ¶89, etc) insted of depending upon scows. (Simon, "Dredging Engineering").

76. The Engins or Motors for such dredges may be of any suitable type, but on account of the complete balancing of the moving parts, and the relativly stedy cutting forces, except when encountering markt irregularities of the bottom, and the nearly uniform loads to be lifted, steam engins may be operated expansivly if the bottom is soft and uniform, while these conditions invite also the use of electric and possibly gasoline motors.

77. Moorings. The so-called "stationary" dredges are usually held in place by means of a half dozen lines, operated by winches on the vessel, several lines or a rear spud controlling the advance of the vessel, while several other lines cause the dredge to swing from side to side with a radial movement as the cutting progresses.

Sizes, Capacities, etc.

78. The following figures are intended to give approx values of the usual extremes or averages.

Hull, length, ft	130* to 275*;
width, ft	36 to 56,
depth, ft,	15 to 20+.
Draft, ft,	up to 12
Digging depth, ft,	30 to 60.
Buckets,	
cap each, cu ft,	5 to 50; number on ladder, 42 to 70;
speed, ft/min,	45 to 80; number per min, 10 to 30

Hopper capacity (sea-going), tons, 500* to 2200*
Digging capacity, cu yds per day, 8000 to 28,000.
Speed of vessel (sea-going), mi/hr, 6 to 8 or 10.

Utility.

79. If **Sand and Gravel** are not too fine, the continuus bucket dredge can handle them well; also rather hard mat'ls, as indurated **clays, shales,** and even **soft or broken rock** and hard-pan, especially where the depth is too great for a dipper It can cut its own flotation It is especially valuable where the disturbance of the bottom would be serius On this account, perhaps, its greatest use in this country is in:

80. Gold Dredging. Neither grapple nor dipper dredges can clean the bed rocks next which gold is usually found, and either will permit the dredged mat'l to leak serusly thru the cracks in the bottom, while hydr dredges cannot create suff suction to gather up the heavy gold along with the sand. The continuus delivery of the "ladder" dredge lends itself also to the smooth and continuus operation of separating devices Gold digging operations are usually sufficiently extensiv to warrant the large investment needed.

Hydraulic Dredges

81. Two main Types of hydr or suction dredge may be recognized, (**A**) Figs 11(*a*) and (*b*), that in which the dredged mat'l is disposed of by means of a hydr pipe-line, constituting a dredge of the "stationary" type, usually designated as a "pipe-line" dredge, and sometimes as a dredge of the "river type" (Simon's "Dredging Engineering"), or (**B**), ¶93, the dredge may be of the "sea-going hopper" type

(A). "Stationary", Pipe-line or River type Dredge.

82. The vessel or hull *V* Figs 11, is of the common rectangular form, (housing not shown), but it need not be so strong as other kinds of dredges, for there are practically no digging stresses, except those produced by the cutter, ¶86.

83. The dredging machinery consists of a suction pipe *S*, Figs 11, mounted on a "ladder"* At the lower end of the suction pipe and ladder is usually a "cutter" *C*, ¶96, which servs to break up the mat'l to be dredged. The dredged mat'l and accompanying water are drawn up thru the suction pipe by means of a centrifugal pump *P*, which creates the suction and sends the accumulation thru a delivery or discharge pipe-line, *D*.

84. The "ladder"† The upper end of the ladder is pivoted at the front of the dredge, while the lower end is held at the desired elevation by means of a line *L* passing over an inclined A-frame *A*, which is back-guyd by other frames and wires *W*.

85. The suction pipe *S* usually lies within the ladder, and at its upper end, at *F*, is provided with ball-and-socket joint, which connects it with the pipe leading to the pump *P*, thus permitting vert, and in some cases, hor movement.

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* From Simon's "Dredging Engineering."

† Tho such ladders are unlike those familiarly known, yet the name "ladder," for this arm or boom, seems to be entirely usual. Care should be taken, also, not to confuse this with the so-called "ladder" dredges, which we have usually referd to as "continuus bucket" dredges.

86. The cutter *C*, Fig 11(*c*), is often used on "stationary" dredges It is generally of approx barrel, conical or paraboloidal form, and consists of an aggregation of cutting edges or blades. In clayey ground, the mat'l is likely to stick and clog the cutter unless the openings betw the blades are ample The cutter is mounted on a shaft coincident with its axis, and the shaft is usually just above the opening of the suction pipe *S*, and extends back along the ladder *L* to the gears which connect it with its engin (see end of this paragraf). The diam of the cutter is about twice that of the pipe. As the cutter is rotated, its lower half, extending somewhat below the end

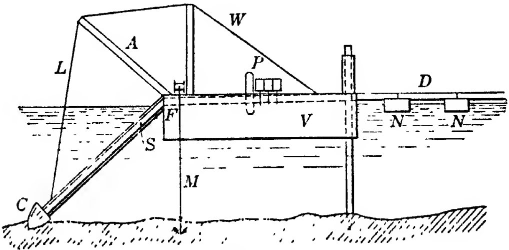

Fig. 11 (a)

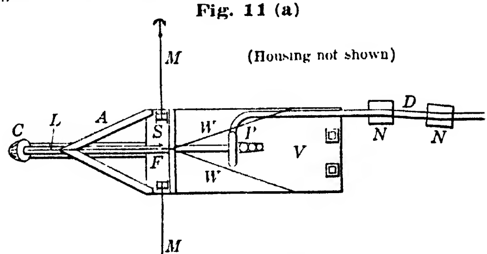

Fig. 11 (b)

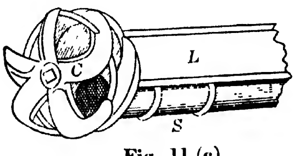

Fig. 11 (c)

of the suction pipe, cuts out the ground. The ground, along with suff'y large quantities of water, is then suckt in thru the cutter, and on into the suction pipe *S*. Where long objects, as pieces of trees or piles, may be encounterd, there is danger of damage, for as the object works its way thru the cutter and into the pipe, it comes in position to be sheard off Insted, however, some part of the machinery may be broken, unless the parts are amply strong to stall the engin. Screens are frequently inserted, either on the

rear of the cutter itself, or at the outer end of the suction-pipe The **engin** or motor for driving the cutter is of course mounted far enough back on the boom to be out of water, or it may be on the dredge itself and operate thru gears.

87. The Pump *P* is invariably of the centrifugal type, as nothing else will handle the dirt and rocks and other objects that may have to be dredged. The rotating impellor is usually of the "closed" type, *i e*, with face plates on the inner sides, which can redily be renewd, and can take the wear that would otherwise come on the pump casing itself. Some claim that the impellor blades may slope either forward or backward, (Trans Am Soc C E, 1904, LIV, Part C, p 505), or that the inner half of each alternate blade may be cut out entirely (Eng N-R, '20 Jul 22, pp 166–170) in order to permit of the free passage of large bodies, as rocks and stumps, without greatly reducing the effic'y of the pump While there has been much discussion on this point, ideally high efficiency of the pump while running may perhaps well be considerably sacrificed, if, by so doing, we can greatly reduce the time lost standing idle due to clogging and break-downs

88. The Engin is usually triple-expansion condensing. Electric motors are coming largely into use.

89. The Discharge Pipe-line *D*, Figs 11(*a*) and (*b*), is usually laid along the dredge to the rear, making as few turns as possible. It is well to provide a check-valv or other device, to prevent flooding the vessel in case of breakage of the pipe In any case, this pipe should be exceptionally strong From the rear, it is carried over the water on floating **pontoons** *NN* of some available type, placed at intervals, while flexible joints of one form or another are provided, suff to permit a considerable amount of wave action vert'y, and movement of the vessel laterally and fore-and-aft This section of pipe need not have so high a factor of safety. The pipe terminates where the dredged mat'l may be disposed of Provision must be made for **impounding** the mat'l, so that the solids will settle and remain in place while the water flows off. Where the discharge line must be so long as to result in inefficient operation of the pump on the vessel, one or more "booster" or "relay" pumps may be inserted in the pipe-line.

90. Maneuvering. (1) The vessel may be held rigidly in position by spuds, ¶47, while the lower end of the suction pipe is swung hor'y over the bottom about a pivot at its upper end The vessel is then advanst, preferably by means of a "walking spud", ¶64, at the rear, while the two forward spuds are raisd; after which the swing of the suction is repeated (2) Usually, the ladder is fixt, and two spuds *SS* are provided at the rear, as shown in Fig. 11 (*b*), so spaced that by lifting and letting down first one and then the other, the dredge may be swung as a whole from side to side by means of mooring lines *MM*, at the same time advancing suff'y for the next cut.

91. Operation of hydr dredges is comparativly simple, practically all movements, etc, being under the control of only one man The eff'y, however, depends chiefly upon his skilful handling. If the suction pipe is not fed into the cut rapidly enough, power will be wasted in pumping water with too little solid mat'l. If, on the other hand, too great a cut is attempted, or if the cutter-head is not appropriate to the work, or if water-jets (¶94) are used too sparingly, clogging may result, either at the entrance, or, more seriusly, in the delivery pipe. The action cannot, of course, be seen by the operator, but he learns of conditions thru the indications of pres and vac gages connected with the suction and delivery pipes, and by the behavior of the cutter. The delivery pipe, if long, and if terminating on land, necessitating continual attention for shifting, may require a larger crew than the dredge itself.

92. Utility. The hydr dredge is largely and effectivly used in handling sand, silt, mud and clays, in open water, on ocean bars, and in channel maint, as in cleaning out old canals, and as used on the Mississippi R To some extent, gravel and small stones may be dredged with the aid of a cutter, and, with a large machine, even stumps, large rocks and other debris may be removed directly, tho large stumps may have to be undermined and then lifted out in some other way. The pipe-line and mooring lines may offer considerable obstruction to navigation.

(B). Sea-going Hopper Dredge.

93. This dredge is similar in many ways to the pipe-line dredge. The following diffs may be noted.

94. The Vessel of this dredge is bilt with molded hull, and contains its own marine propelling engins The dredging machinery is similar, but the **orifis or "drag-head"** at the lower end of the suction pipe is usually flared laterally to form a long slot, open part down, sometimes with one edge extended to act as a cutting edge or scoop, and may be from 5 to 20 ft long and approx 1 ft wide This orifis, placed at rt-angles to the length of the vessel, drags over the bottom as the vessel moves along, acting much like a household vacuum cleaner, except that water insted of air is the medium for carrying the dirt, and the dirt is subsequently separated by sedimentation insted of being filtered out **Water jets** are often used at the orifis, arranged so as to break up the mat'l to be excavated, and to put it into better condition for being suckt up. **No ladder** is used, the pipe being its own arm. If one pipe is used, a **well** is cut thru the hold, but it is usual to employ **two pipes, one on each side** of the vessel, outside the hull. The **pump and engin** are much the same as in the pipe-line dredge, except that where two pipes are used, the centrif pump may be a double one, with the advantage of balanst hydr thrusts. **No discharge pipe line** is used, the water and dredged mat'l being deliverd into **hoppers** bilt into the hull, as in the continuus bucket sea-going hopper dredge, ¶¶ 71 and 75, provision being made for draining off the water **Maneuvering** consists merely in steaming along slowly while the lower ends or drag-heads of the suction pipes drag over the bottom, and the movements are repeated, in approx paralel courses, as may be nec'y. **Operation,** so far as control of suction and cutting are concernd, is much the same as with the pipe-line dredge (¶82). **Utility.** The advantages are, freedom in navigation, no exceptional obstruction to other vessels, sea-going ability, and lack of pipe-line. The chief disadvantage lies in the necessity for stopping when the hoppers are full, and traveling to a suitable dumping ground and returning.

Sizes, Capacities, etc.

95. Where two figures are given, they are for usual min and max; while three figs denote usual min, av and max respectivly.

	"Stationary"	General	"Sea-going hopper"
Hull, length, ft	70 – 200		140 – 460
width, ft	20 – 44		30 – 50
Digging depth, ft		15 – 50	
Ladders, length, ft		45 – 85	
Pump, pipe diam, ins		12, 20, 48	
runners, diam, ins		46 – 96	
Vel in pipe line, ft/sec		10 – 18	
Pumping cap, cu yds/day		5,000 – 50,000*	
% of solids pumpt		5*, 25*, 40	
Horse-power		up to 5000	
Hopper cap, cu yds			300 – 3000†
Speed of vessel, mi/hr			6 – 10
Cost of dredge		$100,000 – $400,000	

Sub-Aqueus Rock Excavation

Methods

96. Where rock is loose or quite soft, it may be dredged directly, as by dipper or continuus bucket dredges. But where it is hard, it must first be broken, either by rock-breaking machines or by blasting, and then dredged.

* Dredges "Delta" and "J. Israel Tarte".

† From Simon's "Dredging Engineering".

Rock Breaking, Hammering, Cutting or Chiseling Machines.

97. The Lobnitz type of breaker is similar to a pile-driver, and consists of a barge carrying machinery for operating a massiv steel ram, which is repeatedly raisd and dropt on to the rock beneath, which is thus shatterd more or less. The ram is from 30 to 60 ft long, and up to 2 ft or more in diam. Together with the special shoe on its lower end, it may weigh from 3 to 18 or 20 tons or more, tho the lighter rams are now seldom used, and rams of over 15 tons are exceptional. The ram is dropt thru a lft of from 3 to 15 ft, depending largely upon the results obtaind, and at frequencies of from 2 to 18 drops per min, 4 to 12 being usual. The rock may shatter to a depth of from 1 to 6 ft, depending upon its nature. Authorities do not recommend that rock-breakers attempt to break up a layer more than 2 ft thick at one time, tho operations are frequently forst down thru 3 ft. (For greater depths of excavation, rather than stop, dredge and break again over the same area, drilling and blasting is recommended and usually resorted to). After shattering the rock at one point, the barge is shifted from 3 to 5 ft either ahead or laterally, and the operation is repeated, attacking the rock on the corners of a checker-bord pattern. Usually, 10 to 30 blows suffice at each point, tho some rocks have yielded under 2, while others have required over 50 blows. The resulting rock-breakage may amount to from 150 to 300 or more cu yds per day (of 24 hrs). The life of the point of the ram may be from 100,000 to 500,000 blows, and is lengthened by having the point hardened at its core, so that as the ram wears, it will always remain fairly sharp.

98. The Submarine Co has patented a rock-cutter in which the ram is surrounded by a metal casing, extending down to the rock surface, from which the **water is kept expeld** by comprest air, and the point or chisel is separate from the body of the hammer, and remains upon the rock. It is claimd that the resistance which water offers to an exposed ram is so serius as to make this machine exceptionally effectiv.

99. Care must be taken to prevent the cable to which the ram is attacht, from slacking down after the ram strikes bottom, as the cable is likely to kink, and to be injured or to break when again puld taut.

100. If the bottom slopes much and is very hard, rock breakers are difficult to operate, for the ram receivs a severe hor reaction, and the mounting and barge are seriusly rackt if the lateral movement is restraind.

Blasting.

101. Unconfined Explosivs, merely laid on the surf of the rocks and there exploded, have been used with some success on soft rocks. A second charge over the same spot may do fairly efficient work, but the resulting debris acts as a cushion for any further charges, and should be cleard away before proceeding.

102. Drilling and Blasting is the usual method of breaking rock under water. **A barge or scow** is used, 80 to 130 ft long, 30 to 45 ft wide, and 7 or 8 ft deep, with a draft of about 4 ft. It must be solidly bilt to withstand the shocks of the blasts. Usually it has a strong **spud** at each corner, operated by power, so that the barge may be lifted from its normal flotation, thus practically standing on the bottom.

103. Some half dozen **drills** may be mounted on the barge, with a **spacing** of 6 or 8 ft, being either of the usual "drop" or "percussion" type, but exceptionally massiv and powerful, and delivering abt 250 strokes per min. While the mountings of the drills do not seem to have been standardized, yet in gen'l the drills are mounted on **towers or colums,** and either over-reach the side of the vessel or operate down thru wells. Provision is made for raising and lowering the drills either by sliding on the towers, or by mounting each drill on a colum or **"drill spud",** which can be raisd or lowerd as a whole, while alongside the colum are carried long gide pipes or tubes, inside which the drill operates. The pipes and drill spud extend down to the rock, and serv to stedy the drill rod, to relocate the hole when changing drills, and to reduce the inflow of soft mat'l. The towers, colums, or drill spuds, are usually so arranged that they can be moved laterally, to adjust their spacing as desired. The **bits** are from 2½ to 3½ ins in diam, 3″ or more being more usual. The **depth** of hole is abt equal to the spacing betw holes, but should eventually extend sev'l ft below the level to be attaind, to allow for irregular breakage.

104. Blasting is accomplisht by 60 to 75% dynamite, in sticks 2 to 2½ ins diam, tampt in place from the drill boat. *The drill boat is then moved* abt 50 ft away, and the charge exploded electrically. The boat is then returnd and starts a new row of holes a dist ahead abt equal to the lat'l spacing employd. One of the most notable pieces of work was that done in the **Galops Rapids** of the St Lawrence R, with a turbulent current of 8 to 12 mi/hr. The drill boat carried four 5″ drills. It was held in place by four 20″ x 20″ power-controld spuds, with the aid of five 1¼″ chains which were used also for maneuvering, and which, on account of their wt and sag, prevented the development of sudden severe stresses. 75,000 cu yds of rock were excavated; av cut 6.5 ft; 1 1/3 lbs of 75% dynamite per cu yd. The blasted rock was removed by a dipper dredge, similarly equipt with spuds and chains

105. Undermining, Mining, or Honeycombing, has been employd here and there in large operations, such as the removal of Flood Rock, East R, New York The area to be excavated is first enclosed, as by cofferdams and pumpt out (unless part of the rock is above water level, when the work may be started from dry rock), after which shafts and drifts and tunnels are drild and blasted out, and then loaded with dynamite and prepared for electric firing The area may then be flooded and blasted, often in a single blast

105½. Diving Bells have been used occasionally on small operations.

Dredging.

106. After breaking up the rock by any of the methods given above, it must be removed Dipper or continuous bucket dredges are usually employd for this work, as in ¶¶ 57 etc and 70 etc.

Miscellaneous Dredges

107. "Universal" or Combination dredges have sometimes been used, consisting merely of several kinds of dredging machines assembled together on the same vessel, and are prob seldom advisable unless there is in prospect an unusually large amount of work for which such a combination would be of servis

108. "Hydraulic Giants" may sometimes be employd to advantage to break down a bank in advance of the regular dredge on which they may be mounted A hydr giant consists of a large nozzle which may be aimd as desird, and supplied with water at very high pres thru piping, by means of a powerful centrifugal pump.

109. Pneumatic dredges have been tried, but apparently without success.

110. A "Stone Lifter" was used by the Canadian Gov't on the St. Lawrence R, consisting of a grab with tongs, operating thru a well in the center of the vessel, being capable of lifting boulders of 50 tons each. (Prelini, "Dredges and Dredging")

TRANSPORTATION OF MATERIALS

Scows or Barges

111. The vessels used for carrying the material from the dredge should be very strong, to withstand the shocks of loads of dirt dumpt into them, and probable rough treatment in maneuvering They are usually merely rectangular barges Within, however, are false sloping sides, which lead the mat'l down to the hoppers in the center of the bottom, and which, together with the vert outside walls of the vessel, form an air space which supplies most of the buoyancy. The hopper doors, when opend, should not extend below the bottom of the scow, as they may strike ground. They are usually operated by chains working over winches on frame-work above. The scows, if they are to carry economical loads, may be of considerably greater draft than that of the dredge, so that it is sometimes necessary to maintain a channel of extra depth, in advance, to accommodate them.

112. Different Materials may make trouble for scows in varius ways. Sand and silt, deposited from a hydr suction dredge, may not settle rapidly enough, and much may be lost overboard with the water. Clay often sticks when the hoppers are opend, and may have to be washt out. Gravel

gives perhaps the least trouble of any. Large rocks, of course, even when deposited carefully, hammer the scows seriusly, and shorten their lives.

113. Chutes, Shoots or Slides of varius kinds are often employd as means toward final disposal. Wet clay will slide on a chute inclined at 1-on-5 to 1-on-3; but wet sand will not move on 1-on-2 without water or pushing; and the usual slope is 1-on-1.5 to 1-on-2.2, or 1-on-1.8 to 1-on-2. However, Prelini, in his "Dredges and Dredging", quotes from 1-on-10 for soft mud, to 1-on-25 for fine sand and water, perhaps with ample lubrication by water.

POWER

114. Steam is the most usual power for dredges, tho apparently unable to give anything like continuus servis. Steam dredges are customarily shut down for a day each week on the Mississippi or other rivers where the water is bad. A condensing plant and extra boiler may be necessary.

115. Oil engins are coming into considerably greater use. They have from 2 to 6 cyls, with H-P around 100. The use of **friction clutches** is important to prevent stalling or injury by unexpected overloads. Drag-line excavators on the drainage works of the Rio Grande irrigation project (Eng N-R, '19 Sep 18) operated with fuel costs of from $0.011 to $0.022 per cu yd of mat'l excavated.

116. Electricity, where readily available, is claimd to have many advantages;—lower first cost, no boilers on dredge (making smaller hull possible) no coal to transport, control simple and good, and more nearly continuus operation than steam. Tho practis is not definitly crystalized, it is customary to carry the current from the land to the dredge at 2200 volts, by means of a **cable** designd for much higher voltage to protect it against abrasion, etc. Sometimes the cable is laid in the water on the bottom, but often it is carried over the water on pontoons, utilizing the pontoons of the discharge pipe if there is one; or the cable may be floated by buoyant sheathing. Where there may be lightning, thoro protection against it is advisable. The **motors** are either 2200-volt, operating directly from the cable, or if of less voltage, a transformer is instald on the dredge. Motors are used of from 50 to 800 HP or more, and run from abt 1100 to 360 revs/min.

SURVEYS AND COMPUTATIONS *

117. Ordinarily, a cross-section of a body of water to be dredged, will correspond broadly to Fig 12, in which *NFABEM* is the bottom. Usually all that will be needed for passage of vessels will be an enlarged cross-section such as *ABCD*. (See also ¶13 in regard to "squatting".) But if the approx triangles *FAD* and *BEC* are not excavated also, they may sooner or later fall into the area *ABCD*, and reduce its depth. These **side slopes,** *FD* and *EC*, will range from 1-on-3 to 1-on-5; under docks, 1-on-2 to 1-on-4; while river silt may flow out almost flat. Furthermore, it is impossible for any kind of dredge to excavate to precisely the plane *FDCE* desired, but must excavate somewhat below it to insure that all mat'l above has been removed. This will result in a somewhat irregular bottom, as *NfghijklmM*, which should be nowhere above *FDCE*. Inasmuch as an operator might prefer to dig out large chunks to much greater depths than called for, so as to be paid for this excessiv removal of mat'l, it is customary to specify a still lower level, *NLGM*, a given vert dist below the reqd depth *FDCE*. This is known as **"overdepth"**, and only that mat'l is to be paid for which lay above the irregular line *Nfghi*, the *straight* line *ik*, the irregular curv *klm*, and the straight line *mM*, is to be paid for; *ik* and *mM* illustrating instances in which the overdepth was exceeded in the actual dredging. The overdepth is usually taken as 2 ft for grapple and dipper dredges, and as 1 ft for continuus bucket and hydr dredges.

118. Soundings are made usually both before and after the dredging, and in front of and to the rear of the dredge, as it proceeds, by any of the usual methods. Thus, there is establisht the topografy of (1) the original bottom, represented by *NFABEM*, in the section shown in Fig 12, and (2) the excavated bottom, represented by the irregular line *NfghijklmM*. The soundings taken in advance show whether the depths are suff for flotation

* Nearly all of the information in this portion of this article, ¶¶117 to 131, has been deduced from Mr. F. Lester Simon's "Dredging Engineering".

of the scows. Owing to the great cost of **rock** excavation, surveys for it should be made with exceptional care, and **wire sweeps** should be employd after excavating, to make sure that no points remain above grade.

Place Measurement

119. Computations of volume excavated betw the irregular surfs represented by *NFABEM* and *NfghijklmM*, may be made by any one or more of the following methods **Averaging end areas** is largely used for long narrow areas, as channels The **Prismoidal formula,** as employd in RR earth work computations, is also used. In the **contour-planimeter** method, contours are plotted from the soundings obtaind, and a planimeter traced over them in order to obtain the areas, and from these the vols, assuming that the vol is composed of flat slabs with thicknesses equal to the vert intervals betw contours, and with vert edges co-inciding with the contours. Either this or the following is well adapted to large irregular areas **Square section or "unit"** method, in which the vol excavated is conceivd as composed of vert square prisms, each prism containing one sounding, or a group of soundings.

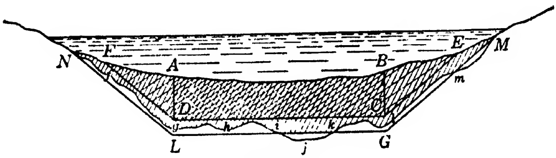

Fig. 12.

119a. Uncertainty in place measurements may arise becaus mat'l from districts outside the dredged area may drift back in while the work is going on, thus giving the contractor more mat'l to excavate than would be indicated by the two place measurements. Prelini ("Dredges and Dredging") believs this never exceeds 10% of the entire amount excavated. On the other hand, the new excavated channel, if appropriately located, may result in a scour which would help the contractor. For such reasons scow measurements are usually made where practicable, in addition to the place measurements.

Scow Measurement

119b. By Volume. For this purpose, the internal vol of the scow (or hoppers, on sea-going dredges) is ascertaind, and if the mat'l is at any time piled up above the upper level of the scow, the vol of such pile must be computed or closely estimated (say by cross-sectioning, as in RR work).

119c. By Weight. Or, by computing the external vol of the scow, its displacements may be ascertaind for given supposed added loads, so that by measuring the amount by which the scow is deprest by the added load of excavated mat'l, the wt of that mat'l can be found, (provided the water density can be made sure of, which may not be practicable in tidal estuaries). It would still remain, however, to translate the wt into vol, which can of course be done by weighing and measuring the contents of an evenly filled dipper, for example.

120. Uncertainty due to Increas in Bulk will still exist, however, becaus practically all mat'ls, after being excavated, will not settle back into as small a space as they occupied "*in situ*", and the increas in bulk is by no means the same for all mat'ls. The increas to be allowd should be establisht in the contract. It may usually be taken as betw 5 and 30%, tho usually betw 15 and 25. For a heavy mixture of mud and sand, it may be taken as 16 or 17% (an increas of 1/6), fine running sand or silt, less than this, and mud or clay more when stiff enough to hold its voids. Prelini

("Dredges and Dredging") states that in the Ambrose channel, New York harbor, the increas was found to be 22%, and recommends that 30% be allowd

120a. Probings or Borings may be desirable or necessary, to ascertain in advance the character of the mat'l to be excavated. **Probings** consist in the driving of pointed rods into the bottom, and so "feeling" the character of the mat'l. **Borings** may be either core or wash borings, and aim to bring reliable samples of the mat'l to the surf *Core* borings may be obtaind either by driving an open pipe into the bottom and then removing it with the mat'l adhering inside, or by means of a wood auger operating within the pipe Or a larger pipe may be first driven into the bottom, pumpt out, and then a smaller pipe forst down inside and then removed, thus obtaining samples nearly dry and undisturbd *Wash* borings are usually made by operating a hollow drill within a pipe, water being forst down thru the hole in the drill, with the result that at least the lighter portions of the borings of the drill are washt up to the surf, but the results are likely to be misleading

Miscellaneus

121. Range targets will be needed (unless the channel is quite narrow) to guide the dredge on its proper course Range targets are usually establisht and maintaind by the engineers of the party employing the dredge The **locations** of the targets should be laid out on the map Where possible, the targets should be on the shore, or maintaind on piles specially driven, since anchord targets can hardly be held near enough to correct positions. Each range is indicated by two targets, one a considerable dist behind and at a slightly higher elevation than the other; and for each range a diff character of target should be employd, as square, round, oblong, triangular, etc, while further diffs can be had by varying the colors If the dredge is to work at night, lights must be used at the targets The **directions** of the ranges should usually be paralel to the current, as this gives the greatest ease in placing and holding the dredge where desired The **width of cut** for diff dredges is given as follows

Grapple,	50 to 60 ft,	Hydraulic,		40 to 50 ft,
Dipper,	20 to 60 ft,	"	stern spud pivot,	150 to 200 ft

122. Tide Gages will be needed, unless the depth of water remains pract'y constant to enable the operator to adjust the depth of cut, as the stage changes, and should be set up and painted boldly, so that he can see them readily.

123. Current Drift may have to be determind to assist in proper location of a channel, or for other reasons, and may be ascertaind by any of the usual float methods.

Records.

124. Maps should of course be prepared, showing the soundings, and kept up to date, to show progress of the work

125. Material Removed by Scows or Hoppers (except for hydr pipeline dredges), and other information that may promis to be of value, should be recorded.

126. Time records should be kept of all operations, and subsequently classified or "distributed" in order to learn where wastes occur, and to otherwise supervise and adjust the methods of working (See ¶154 etc, Percentages of time spent in dredging operations)

127. Cost Estimates. See also ¶135 The elements that may be needed for estimating cost of work are summarized below from Simon's "Dredging Engineering". (Certain items would obviusly not arise on certain kinds of work.)

128. *Preliminary Engineering* Engineers and assistants and their equipment, on all steps from making surveys, to contracts, borings; overhed.

129. *Permits, Property.*

130. *Preliminary Structures* Dikes, ditches, shore pipe-lines, construction plant, transportation, field office, overhed.

131. *Dredging proper.* The engineer need concern himself with the following only when he is operating the dredge himself. Plant, labor, supplies, dump maint, and engineering and overhed connected with this work.

OPERATION AND MAINTENANCE

Operating Efficiency

132. Dredges are usually so seriusly subject to breakdowns, and the entire plant is so very expensiv, that operating efficiency while running, is usually of much less importance than **continuity of operation.** Refined machinery of high theoretical efficiency is likely to prove much less efficient in the long run, than hardy machinery that may consume more power continuusly or need more men, but which does not involv frequent shut-downs covering long periods. For this reason also

133. Duplicate Parts are strongly urged by almost all authorities. The depreciation of such parts is slight, but that, together with the interest on the investment, is likely to be far less than the cost resulting from trying to work without such parts on hand Not only is there the computable loss of time, but the delays are demoralizing to the crew, especially if men are laid off.

134. Good Management is of prime importance. Fuel must be supplied in ample time to avoid shut-downs The boilers should not be fed with water of doubtful character A clean and ship-shape dredge has a good effect upon the crew The large suction dredges of Toronto harbor (Eng Rec '16 Apr 15) have main floors of cement and composition rubber, and a min of wood trim. The tendency to run a dredge somehow, as long as possible, should be replaced by a strong policy of maintaining it continuusly at the best possible

COSTS

135. Satisfactory dredging costs are seldom available Figures often appear to refute each other. Often one or more of the following pieces of information is omitted,— type of dredge, mat'l handled, whether interest and depreciation are included, time idle, number of shifts, etc.

136. Modifying Factors. After referring to the data given below, estimates should be made of the probable or possible effects of the following,— dist to point of deposit, weather conditions, exposure of site, depth of cut, time idle, collisions and other accidents, the general skill of those operating, and efficiency of the machinery See ¶¶ 3 to 21

137. All figures are to be taken as applying **before the War of 1914.**

Classified by Dredges

138. Drag-line Scrapers. Silt, loam and hard-pan from abt 12 to 20 ¢ cu yd, while disintegrated rock may run the cost up to 45

139. Grapple dredges, (including clam-shell and orange-peel). Mud, sand and clay from abt 8 to 60 ¢ cu yd, more usually 15 to 40.

140. Dipper dredges Light mat'ls, as mud, sand, silt, gravel and clay, at from 8 to 40 or 60 ¢ cu yd, tho hard-pan may run the cost up considerably over $1 00.

141. Continuus Bucket dredges Mud, silt, clay and disintegrated rock, from 10 to 30 ¢ cu yd.

142. Hydraulic Pipe-line dredges ("stationary"). Nearly everything but rock, from 5 to 30 ¢, cu yd.

143. Hydraulic Sea-going dredges Mud, silt, sand and gravel, often as low as 2 or 3 ¢/cu yd, with maxima around 10, tho one record gives 36.

144. Rock Breakers or Cutters. Figures appear very uncertain. From 25 and 30 to 70 or 80 ¢/cu yd are usual, but these figs seem to be for breaking only. One figure* is $9/cu yd. Dredging would be extra.

145. Drilling and Blasting has been recorded as low as 18¢/cu yd for breaking only, while figs around 50 ¢ to $1 are usual. However, in Black Rock Harbor, Buffalo (report by W. L Saunders, in XIIth Internat'l Congress of Navigation), the tot cost including depreciation (but not dredging) was 48 ¢/cu yd with drill holes av'g 9'9" deep, while with holes only 3'6" deep, the cost ran up to $5.57/cu yd. Again* $6.13 is quoted. One figure is available including dredging, $1.55¢/cu yd.

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* British Columbia dredging fleet, Eng Rec '13 Aug 23, p 209. Not stated whether dredging is included.

146. Undermining and Blasting of Flood Rock cost something under $3/cu yd for breaking only; and $5.88 including dredging.

Classified by Materials

147. Mud is generally removed at from 2 to 10 ¢/cu yd, tho the cost is occasionally considerably higher—around 30 or 40 if the job is small.

148. Silt usually around 7 ¢/cu yd.

149. Sand; min around 5 ¢/cu yd, with 10 to 15 or 20 as more usual figures, occasionally running up to 30 and 40 or more for small jobs

150. Clay is seldom excavated as cheaply as 5 ¢/cu yd; more usually 10, 20, 30 and 40.

151. Hard-pan and Disintegrated Rock from 20 to 40 ¢ or 50 ¢/cu yd —possibly a dollar or more.

152. Gravel, 2 to 12 ¢/cu yd, or possibly as high as 30

153. Rock, see ¶¶ 144 etc, Rock Breakers, etc

Percentages of Time
Spent in Dredging Operations

154. The following figures have been compiled from Report of Chief of Engrs, U.S.A., 1919, being av's of a number of diff dredges of each type, and are especially valuable as indicating the ways in which serius losses of time may occur.

155. Time at Work	Bucket	Dipper	Hydraulic Pipe-Line	Sea-Going Hopper
Dredging	10 15	5 88	15 12	14 35
Waiting on tug	0 30			
Handling scows		0 37		
Handling pipe-line			1 94	
Handling swinging wires			0 98	
Turning in cut				0 79
To and from dump				7 16
Dumping				1.39
Spudding up	0 30	0.38		
To & from wharf & anchor'g			0 27	2 49
Placing dredge			0 57	
Taking fuel and supplies				2 99
Waiting for vessels to pass			0.18	
Miscellaneus	1.67	1.63		
Totals, time at work	12.42	8 26	19 06	29.17

156. Time Lost from Work	Bucket	Dipper	Hydraulic Pipe-Line	Sea-Going Hopper
Repairing	4.60	2.58	7.70	11.01
Bad Weather	1.90	5.44	2.41	3.29
Changing location of plant	2.97	1.21	2.58	0.00
Delays	4.17	4.40	0.00	0.00
Sundays and holidays	10.31	10.41	10.77	14.88
Out of commission	36.68	58.94	28.56	4.61
Lying at berths				30.75
Transferring, etc.			0.21	1.21
Miscellaneus	26.95	8.49	27.30	4.92
Totals, time lost from work	87.58	91.47	79.53	70.67

FOUNDATIONS.

A VOLUME might be occupied by this important subject alone. We have space for only a few general hints; leaving it to the student to determine how far they may be applicable in any given case. In ordinary cases, as in culverts, retaining walls, &c, if excavations, or wells, &c, in the vicinity, have not already proved that the soil is reliable to a considerable depth, it will usually be a sufficient precaution, after having dug and levelled off the foundation pits or trenches to a depth of 3 to 5 ft. to test it by an iron rod, or a pump-auger; or to sink holes, in a few spots, to the depth of 4 to 8 ft farther; (depending upon the weight of the intended structure;) to ascertain if the soil continues firm to that distance. If it does, there will rarely be any risk in proceeding at once with the masonry; because a stratum of firm soil, from 4 to 6 ft thick, will be safe for almost any ordinary structure; even though it should be underlaid by a much softer stratum. If, however, the firm upper stratum is exposed to running water, as in the case of a bridge-pier in a river, care must be taken to preserve it from gradually washing away; or from becoming loosened and broken up by violent freshets; especially if they bring down heavy masses of ice, trees, and other floating matter. These are sometimes arrested by piers, and accumulate so as to form dams extending to the bottom of the stream; thus creating an increase of velocity, and of scouring action, that is very dangerous to the stability both of the bottom and of the structure. When the testing has to be made to a considerable depth, it may be necessary to drive down a tube of either wrought or cast iron, to prevent the soil from falling into the unfinished hole. If necessary, this tube may be in short lengths, connected by screw joints, for convenience of driving, and the earth inside of it may be removed by a small scoop with a long handle.*

Borings in common soils or clay may be made 100 feet deep in a day or two by a common wood auger $1\frac{1}{2}$ inches diameter, turned by two to four men with 3 feet levers. This will bring up sample-

In starting the masonry, the largest stones should of course be placed at the bottom of the pit, so as to equalize the pressure as much as possible; and care should be taken to bed them solidly in the soil, so as to have no rocking tendency. The next few courses at least should be of large stones, so laid as to break joint thoroughly with those below. The trenches should be refilled with earth as soon as the masonry will permit; so as to exclude rain, which would injure the mortar, and soften the foundation. It is well to ram or tread the earth to some extent as it is being deposited.

If the tests show that the soil (not exposed to running water) is too soft to support the masonry, then the pits should be made considerably wider and deeper; and afterward be filled to their entire width, and to a depth of from 3 to 6 or more ft, (depending on the weight to be sustained,) with rammed or rolled layers of sand, gravel, or stone broken to turnpike size; or with concrete in which there is a good proportion of cement. On this deposit the masonry may be started. The common practice in such cases, of laying planks or wooden platforms in the foundations, for building upon, is a very bad one. For if the planks are not constantly kept thoroughly wet, they will decay in a few years; causing cracks and settlements in the masonry.

Some portions of the brick aqueduct † for supplying Boston with water gave a great deal of trouble where its trenches passed through running quicksands and other treacherous soils. Concrete was tried, but the wet quicksand mixed itself with it, and *killed* it. Wooden cradles, &c, also failed, and the difficulty was finally overcome by simply depositing in the trenches about two feet in depth of strong gravel.‡ Sand or gravel, *when prevented from spreading sideways*, forms one of the best of foundations. To prevent this spreading, the area to be built on may be surrounded by a wall; or by squared piles driven so close as to touch each other, or in less important cases, by short sheet piles only. But generally it is sufficient simply

* Subterranean caverns in limestone regions are a frequent source of trouble, against which it is difficult to adopt precautions.

† The Cochituate aqueduct, built 1846-48; egg-shape, 6 feet 4 inches × 5 feet, with semicircular invert.

‡ Smeaton mentions a stone bridge built upon a natural bed of gravel only about two feet thick, overlying deep mud so soft that an iron bar 40 feet long sank to the head by its own weight. One of the piers, however, sank while the arches were being turned, and was restored by Smeaton. Although a wretched precedent for bridge-building, this example illustrates the bearing power of a thick layer of well-compacted gravel.

to give the trenches a good width; and to ram the sand or gravel (which are all the better if wet) in layers, taking care to compact it well against the *sides* of the trench also Under heavy loads, some settlement will, of course, take place, as is the case in all foundations except rock. If very heavy, adopt piling, &c. See GRILLAGE.

When an unreliable soil overlies a firm one. but at such a depth that the excavation of the trenches (which then must evidently be made wider, as well as deeper) becomes too troublesome and expensive; especially when (as generally happens in that case) water percolates rapidly into the trenches from the adjacent strata, we may resort to piles When making deep foundation-pits in **damp clay.** we must remember that this material, being soft, has, to a certain degree, a tendency to press in every direction, like water. This causes it to bulge inward at the sides, and upward at the bottom The excavations for tunnels, or for vertical shafts, often close in all around, and become much contracted thereby before they can be lined, therefore they should be dug larger than would otherwise be necessary. The bottoms of canal and railroad excavations in moist clay are frequently pressed upward by the weight of the sides. **Dry clay** rapidly absorbs moisture from the air, and swells, producing effects similar to the foregoing Its expansion is attended by great pressure, so that retaining-walls backed with dry rammed clay will be in danger of bulging if the clay should become wet It is a treacherous material to work in **For concrete foundations.** see Concrete.

As to the greatest load that may safely be trusted on an earth foundation, Rankine advises not to exceed 1 to 1.5 tons per square foot But experience proves that on good compact gravel, sand, or loam, at a depth beyond atmospheric influences, 2 to 3 tons are safe, or even 4 to 6 tons if a few inches of settlement may be allowed, as is often the case in isolated structures without tremors Years may elapse before this settlement ceases entirely. Pure clay, especially if damp, is more compressible, and should not be trusted with more than 1 to 2.5 tons, according to the case All earth foundations must yield *somewhat.* **Equality of pressure** is a main point to aim at. **Tremor** increases settlements, and causes them to continue for a longer period, especially in weak soils. Great care must be taken not to overload in such cases, even if piled **Foundations in silty soils** will probably settle, in years, at the rate of from 3 to 12 inches per ton (up to 2 tons) per square foot of *quiet* load, if not on piles.

Figure 2 shows an easy mode of obtaining a foundation in certain cases. It is the "**pierre perdue**" (lost stone) of the French; in English, "**random stone,**" or **rip-rap.**

It is merely a deposit of rough angular quarry stone thrown into the water; the largest ones being at the outside, to resist disturbance from freshets, ice, floating trees, &c A part of the interior may be of small quarry chips, with some gravel, sand, clay, &c. When the bottom is irregular rock, this process saves the expense of levelling it off to receive the masonry. For 2 or 3 feet below the surface of the water, the stones may generally be disposed by hand, so as to lie close and firmly. Small spawls packed between the larger ones will make the work smoother, and less liable to be displaced by violence. Cramps or chains may at times be useful for connecting several of the large stones together for greater stability. **Rip-rap, however, is apt to settle.**

If the bottom is so yielding as to be liable to wash away in freshets, it may, in addition, be protected, as in Fig 2, by a covering of the same kind of stones, as at c; extending all around the structure. Or the main pile of stones may be extended as per dotted line at *d*; so that if the bottom should wash away, as per dotted line at *o*, the stones *d* will fall into the cavity, and thus prevent further damage. Sheet-piles, *s s*, may be driven as an additional precaution. For greater security, the bed of the river may be dredged or scooped under the entire space to be covered by the main deposit, as per dotted lines in Fig 3, to as great a depth as any scouring would be apt to reach;

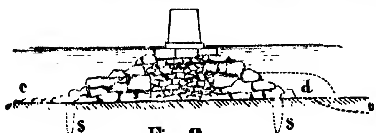


Fig 2

this excavation also to be filled with stone. Such foundations are evidently best adapted to quiet water. The masonry should rest on a strong platform.

Large deposits of stone, as in these two figs, greatly increase the velocity, and the scouring action of the stream around them, especially in freshets; unless the bottom on each side from the deposit be dredged out to such an extent that the original area of water shall not be reduced. If the bottom is treacherous, this should be done before depositing the covering stones c, Fig 2. Judgment and experience are necessary in such matters, as in all others connected with engineering. Mere study will not guard against constant failures. Theory and practice must guide each other.

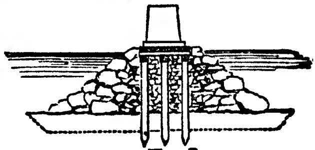
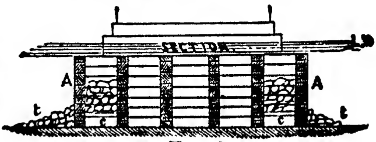


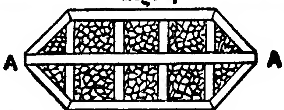
Fig 3

Fig 3 is another simple method; and when it does not create too great an obstruction to the navigation of the stream, or to the escape of its waters in time of high freshets, is a very effective one. Here the piles are first driven into the river bottom, for the support of the pier; then the deposit of stone is thrown in, for the support and protection of the piles; preventing them from bending under their loads; and shielding them from blows from floating bodies. The tops of the piles being cut off to a level, a strong platform of timber is laid on top of them, as a base for the masonry. The top of the platform should not be less than about 12 or 18 ins below ordinary low water, to prevent decay. Mitchell's iron screw pile; or hollow piles of cast iron, may be used instead of wooden ones.

Figs 4 represent a convenient method of establishing a foundation in water, by means of a **timber crib, A A, without a bottom.** It should be built of squared timbers, notched together at their crossings, as shown at Fig 5; each notch being ¼ of the depth of the stick. By this means each timber is supported throughout its entire length by the one below it; and resists pulling in both directions. Bolts also are driven at the intersections; at least in the sides of the crib, to prevent one portion from being floated off from the other. The crib is thus divided into square or rectangular cells, from 2 to 4 or 5 ft on a side, according to the requirements of the case. The partitions between the cells are put together in the same manner as those at the sides of the cribs; and consequently, like the latter, form solid wooden walls



Figs 4



PLAN.

Fig.5 t.

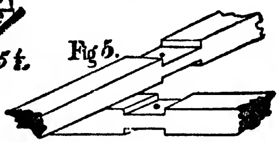


Fig 5.

The crib may be framed afloat, at any convenient spot; and when finished, may be towed to its final place, where it is carefully moored in position, and then sunk by throwing stone into a few cells provided with platforms, as at c c, for that purpose. These platforms should be placed a little above the lower edge of the cells, so as not to prevent the crib from settling slightly into the soil, and thus coming to a full bearing upon the bottom. After it has been sunk, all the cells are filled with rough stone. A stout top platform may be added or not, as the case may be; also, a protection, t t, of random stone, to prevent undermining by the current. If the sides are exposed to abrasion from ice, &c, they may be covered in whole or in part with plank, or plate iron; and the angles strengthened by iron straps, &c. In deep water, a foundation may be made partly of random stone, as in Figs 2 and 3; and on top of this may be sunk a crib, with its top about 2 ft under low water, as a base for the masonry. This is much safer than random stone alone.

On uneven rock bottom it may be necessary to scribe the bottom of the crib to fit the rock; or the crib may first be sunk by means of a loaded platform on its top, or by filling some of its cells, until its lowest timbers are within a short distance above the bottom. Being there kept in a horizontal position, small stones may be thrown into the cells, and allowed to find their way under the timbers of the crib, thus forming a level support for it. The cells may then be

A crib with only an outside row of cells for sinking it may be built; and the interior chamber may be filled with concrete under water. The masonry may then rest on the concrete alone. If the crib rests upon a foundation of broken stone, the upper interstices of this stone should first be levelled off by small stone or coarse gravel to receive the concrete of the inner chamber.

Or a crib like Fig. 4 may be sunk, and piles be driven in the cells, which may afterward be filled with broken stone or concrete. The masonry may then rest on the piles only. which in turn will be defended by the crib. If the bottom is liable to scour, place sheet-piles or rip-rap around the base of the crib.

By all means avoid a crib like ***e*****, Fig 5½,** much higher at one part than at another, *if the superstructure is to rest on the timber of the crib* instead of on piles, or on concrete independent of the timber; for the high part of the crib will compress more under its load than the low part, and will thus cause the superstructure to lean or to crack.

A crib either straight sided or circular, with only an outer row of cells for **puddling may be used as a cofferdam** (see cofferdams, p. 586). The joints between the outer timbers should be well caulked; and care be taken, by means of outside pile-planks, gravel, &c, to prevent water from entering beneath it.

The cast-iron Bridge across the Schuylkill at Chestnut St, Phila, Mr. Strickland Kneass, Engineer, affords a striking example of crib foundation. The center pier stands on a crib, an oblong octagon in plan, 31 by 87 feet at base; 24 by 80 ft at top, and (with its platform) 29 ft high. Its timbers are of yellow pine, hewn 12 ins square, and framed as at Fig 5. The lower timbers were carefully cut or scribed to conform to the irregularities of the tolerably level rock upon which it rests. These were ascertained (after the 8 ft depth of gravel had been dredged off) in the usual manner of mooring above the site a large floating wooden platform, composed of timbers corresponding in position with all those of the lower course of the intended crib, both longitudinal and transverse. Soundings were then taken close together along all these lines of timber. Most of the cells are about 3 by 4 ft on a side, in the clear. A few of them had platforms at the level of the second course from the bottom, for receiving stone for sinking the crib, the others are open to the bottom.

The crib was built in the water; and was kept floating, during its construction, with its unfinished top continually just above water, by gradually loading it with more stone as new timbers were added. The stone required for this purpose alone was 300 tons. When the crib was towed into position, and moored, 150 tons more were added for sinking it. All the cells were afterward filled with rough dry stone, and coarse gravel screenings, making a total of 1666 tons. A platform of 12 by 12 inch squared timber covered the whole, its top being 2½ ft below low water The pier alone, which stands on this crib, weighs 3255 tons, and during its construction it compressed the crib 6½ ins. The weight of superstructure resting on the pier, may be roughly taken at 1000 tons more.

An ordinary caisson is merely a strong scow, or a box without a lid; and with sides which may at pleasure be readily detached from its bottom. It is built on land, and then launched. The masonry may first be built in it, either in whole or in part, while afloat, and the whole being then towed into place, and moored may be sunk to the bottom of the river, to rest upon a foundation previously prepared for it, either by piling, if necessary; or by merely levelling off the natural surface, &c. The bottom of the caisson constitutes a strong timber platform, upon which the masonry rests; and is so arranged, that after it is sunk, the sides may be detached from it, and removed to be rebottomed for use at another pier, if needed. This detaching may be effected by some such contrivance as that shown in Fig 6, where P P *w* is the bottom of the caisson, to which are firmly attached at intervals strong iron eyes *t*; which are taken hold of by hooks *d*, at the lower end of long bolts E *n*, reaching to the top timbers S of the crib, where they are confined by screw nuts *n*. By loosening the nuts *n*, the hooks *d* can be detached from the eyes *t*; and the sides can then be removed from the bottom, there being no other connection between the two. These hooks and eyes are usually placed outside of the caisson; the screw nuts *n* being sustained by the projecting ends of cross pieces, as *t t*, Fig 9. The improper position given them in our Fig was merely for convenience of illustrating the principle. It will sometimes be necessary to have one side detachable from the others, in order to float the caisson away clear from the finished pier; unless it be floated away before the masonry has been built so high as to render the precaution useless.

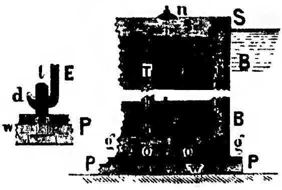


Fig 6.

Fig 6 shows one of many ways of constructing a caisson; with sides consisting of upright corner-posts, I; cap pieces S, on top; and sills *g* at bottom, resting on the bottom platform P P *w*; intermediate uprights T, framed into the caps and sills; the whole being covered outside by one or two thicknesses of planking B, which, as well as the platform, should be well calked, to prevent leaking. Tarpaulin also may be nailed outside to assist in this. The greatest trouble from leaking is where the sides join the platform. On top of the platform is firmly spiked a timber *o o*, extending all around it just inside of the inner lower edge of the sides of the caisson. Its use is to prevent the sides from being forced inward by the pressure of the water outside. The details of construction will of course vary with the requirements of the case. In deep caissons, inside cross-braces or struts from side to side, as at *c c*, Fig 7, will be required to prevent the sides from being forced inward by the pressure of the water, as the vessel gradually sinks while the masonry is being built within it. As the masonry is carried up, the struts are removed; and short ones, extending from the sides of the caisson to the masonry, are inserted in their place. When the caisson is shallow, only the upper course of braces will be required, they also support a platform for the workmen and their materials. In deep caissons, in order not to be in the way of the masons, the outer planking of the sides may, in part, be gradually built up as the masonry progresses. It may sometimes be expedient to build the masonry hollow at first, with thin transverse walls inside to stiffen it if necessary; and to com-

plete the interior after sinking the caisson. Indeed, masonry or brickwork, in cement, may thus be built hollow at first, resting on the platform; the masonry itself forming the sides of the caisson. Or the sides may consist of a water-tight casing of iron, or wood, of the shape of the intended pier, &c. This casing being confined to the platform, becomes, in fact, a mould, in which the pier may be formed, and sunk at the same time by filling it with hydraulic concrete. **For concrete foundations,** see Concrete.

On rock bottom the under timbers of the platform may be cut to suit the irregularities as already stated under "Cribs." Or the bottom may be levelled up by first depositing large stones around the area upon which the caisson is to rest; and then filling between these with smaller stones and gravel; testing the depth by sounding Or a level bed of cement concrete may, with care, be deposited in the water. If there are deep narrow crevices in the rock, through which the concrete may escape, they may be first covered with tarpaulin Diving bells may often be used to advantage, in all such operations. But in the case of very irregular rock, it will often be better to resort to coffer-dams.

Valves for the admission of water for sinking the caisson are usually introduced. If, after sinking, it should be necessary to again raise the whole, it is only necessary to close the valves, and pump out the water. Guide piles may be driven and braced alongside of the caisson, to insure its sinking vertically, and at the proper spot. Or it may be lowered by screws supported by strong temporary framework

Assuming the uprights I, T, &c, Fig 6, to be sufficiently braced, as at c c, Fig 7, the following table will show the thickness of planking necessary for different distances apart of the uprights, (in the clear,) to insure a safety of six against the pressure of the water at different depths; and at the same time not to bend inward under said pressure, more than $\frac{1}{480}$ part of the distance to which they stretch from upright to upright; or at the rate of ¼ inch in 10 ft stretch; ⅛ inch in 5 ft, &c. Such a table may be of use in other matters.

Table of thickness of white pine plank required not to bend more than $\frac{1}{480}$ part of its clear horizontal stretch, under different heads of water. (Original.)

Stretch in Ft.	HEADS IN FEET. 40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22¼	20	17½	14	11

Coffer-dams are enclosures from which the water may be pumped out, so as to allow the work to be done in the open air Their construction of course varies greatly. In still shallow water, a mere well-built bank of clay and gravel; or of bags partly filled with those materials when there is much current, will answer every purpose; or (depending on the depth) a single or double row of sheet-piles; or of squared piles of larger dimensions, driven touching each other; their lower ends a few feet in the soil; and their upper ones a little above high water, and protected outside by heaps of gravelly soil or puddle, (as at P in Fig 7,) to prevent leaking. The sheet-piles may be of wood; or of cast iron, of a strong form.

The sufficiency of a mere bank of well-packed earth in still water, is shown by the embankments or levees to prevent rivers from overflowing adjacent low lands The levees along 700 miles of the Mississippi average about 6 ft high; 3 ft wide on top; side-slopes 1½ to 1 In floods the river frequently bursts through them, doing immense damage. They are entirely too slight.

The method of a single row of 12 by 12 inch squared piles, driven in contact with each other, (close piles,) and simply backed by an outer deposit of impervious soil, is very effective; and with the addition of interior cross-braces or struts, like *c c*, Fig 7, to prevent crushing inward by the outside pressure of the water and puddle when pumped out, has been successfully employed in from 20 to 25 ft depth of water, in which there was not sufficient current to wash away the puddle. The cross-braces are inserted successively, as the water is being pumped out; beginning, of course, with the upper ones. The ends of these braces may abut on longitudinal timbers, bolted to the piles for the purpose. Another method is **a strong crib,** composed of uprights framed into caps and sills; and covered outside with squared timbers or plank, laid touching each other, and well calked; as in the caisson, Fig 6; but without a bottom. Between the opposite pairs of uprights are strong interior struts, as *c c*, Fig 7, reaching from side to side, to prevent crushing inward. The

upper series of these usually supports a platform for the workmen, windlasses, &c. The crib having been built on land, is launched, taken to its final place, and sunk by piling stones on a temporary platform resting on the cross-struts; the bottom of the stream having been previously levelled off, if necessary, for its reception.

To prevent leaking under the bottom of the crib, sheet-piles may be driven around it, their heads extending a few feet above its bottom, or a small deposit of outside puddle may be placed around it, as shown at the stone deposits *t t*, Fig 4. Or a broad flap of tarpaulin may be closely nailed around and a little above the lower edge of the crib, so arranged that it may be spread out loosely on the river bottom, to a width of a few feet all around the outside of the crib, and the puddle may be placed upon it. Such a tarpaulin is also very useful in case the river bottom is somewhat irregular, and cannot be levelled off without too great expense; in which case the crib cannot come to a full bearing upon it, and consequently the water would leak or flow beneath freely. It is especially adapted to uneven rock, where sheet-piles cannot be driven. An artificial stratum of impervious soil may however, be deposited on bare rock, in which case the sinking of the crib, and the subsequent operations will be the same as on a natural stratum. These expedients are evidently more or less applicable in other cases, where, to avoid repetition, they are not specially mentioned.

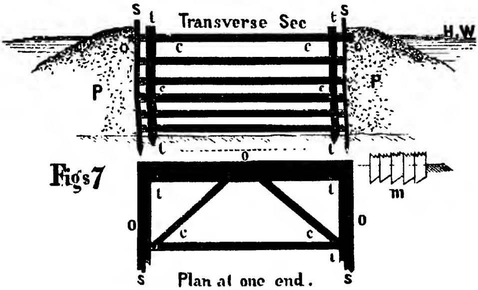
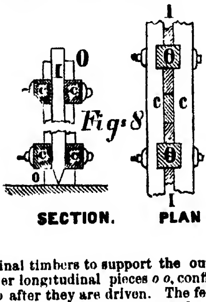
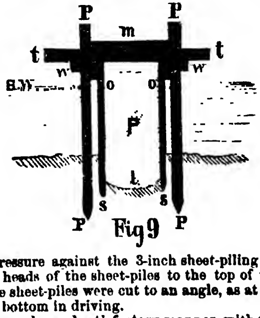


Fig 7 is another crib coffer-dam; in which the sides, instead of being planked longitudinally, as in the last instance, are sheathed with vertical sheet-piles *s*, driven after the crib is sunk. It is much inferior to the last, owing to its greater liability to leak. In one of this description, Fig 7, successfully used in 16 ft water, the dimensions of the crib were 34 ft by 80 ft. Along each long side were 7 uprights *t, t*, 19 ft long, 12 ins square, 12¾ ft apart. Into each opposite pair of these were notched, and held by dog-irons, 6 cross-braces *c c*, of 12 ins square. The distance between the two upper ones was 3 ft in the clear, gradually diminishing to 18 ins between the two lower ones, on account of the increased pressure of the water in descending. On the outside of the uprights, and opposite the ends of the braces, were bolted longi-





tudinal timbers to support the outside pressure against the 3-inch sheet-piling *s s*. Other longitudinal pieces *o o*, confine the heads of the sheet-piles to the top of the crib after they are driven. The feet of the sheet-piles were cut to an angle, as at *m*; to make them draw close to each other at bottom in driving.

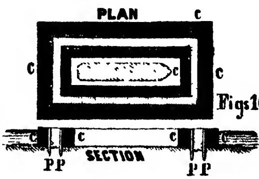
The sheet-piles will drive in a far more regular and satisfactory manner, with the arrangement shown in Figs 8. Here *o o* are the uprights; *c c* are pairs of longitudinal

pieces, notched and bolted to the uprights, near both their tops and their feet; and at as many intermediate points as may be desired. The sheet-piles I, are inserted between these; and of course are guided during their descent much more perfectly than in Fig 7.

When the current is too strong to permit the use of outside puddle, P, Fig 7, the principle of coffer-dam shown in Fig 9, is generally used; in which both sides of the puddle are protected from washing away. The space to be enclosed by the dam is surrounded by two rows of firmly-driven main piles *p p*, on which the strength chiefly depends. They may be round. In deciding upon their number, it must be remembered that they may have to resist floating ice, or accidental blows from vessels, &c. With reference to this, extra *fender*-piles may be driven. A little below the tops of the main piles are bolted two outside longitudinal pieces *w w*, called *wales;* and opposite to them two inner ones, as in the fig. The outer ones serve to support cross-timbers *t t*, which unite each pair of opposite piles, and steady them; and prevent their spreading apart by the pressure of the puddle P. The inner ones act as guides for the sheet-piles *s s*, while being driven; after which the heads of the sheet-piles are spiked to them. In deep water these sheet-piles must be very stout, say 12 ins square; to resist the pressure of the compacted puddle.

A gangway m, is often laid on top of the cross-pieces *t t*, for the use of the workmen in wheeling materials, &c. The puddle P is deposited in the water in the space, or boxing, between the sheet-piles. It should be put in in layers, and compacted as well as can be done without causing the sheet-piles to bulge, and thus open their joints. The bottom of the puddle-ditch should be deepened, as in the fig, in case it consists, as it often does, of loose porous material which would allow water to leak in beneath it and the sheet-piles. This leaking *under* the dam is frequently a source of much trouble and expense. Water will find its way readily through almost any depth and distance of clean coarse gravelly and pebbly bottom, unmixed with earth. Sand is also troublesome: and if a stratum of either should present itself extending to a great depth, it will generally be expedient to resort to either simple cribs, Fig 4; or to caissons; with or without piles in either case, according to circumstances. But if such open gravel, or any other permeable or shifting material, as soft mud, quicksand, &c, is present in a stratum but a few feet in thickness, and underlaid by stiff clay, or other safe material, leaking may be prevented, or at least much reduced, by driving the sheeting-piles 2 or 3 ft into this last, and by deepening the puddle-trench to the same extent. It may sometimes be better, and more convenient, to dredge away the bad material entirely from all the space to be enclosed by the dam, and for a short distance beyond, before commencing the construction of the latter. If the dam, Fig 9, is (as it should be) well provided with cross-braces, like *c c*, Fig 7, extending across the enclosed area, the thickness or width *o o* of the puddle, need not be more than 4 or 5 feet for shallow depths; or than 5 to 10 ft for great ones; because its use is then merely to prevent *leaking*. But if there are no braces, it must be made wider, so as to resist *upsetting* bodily; and then, with good puddle, *o o* may, as a rule of thumb, be ¾ of the vertical depth *o l* below high water; except when this gives less than 4 ft: in which case make it 4 ft; unless more should be required for the use of the workmen, for depositing materials, &c. Or if the excavation for the masonry is sunk deeper than the puddle, the dam must be wider, else it may be upset into the excavated pit.

The excavated soil may be raised in buckets by windlasses, or by hand, in successive stages. The pumps may be worked by hand, or by steam, as the case may require; as also the windlasses generally needed for lowering mortar, stone, &c. More or less leaking may always be anticipated, notwithstanding every precaution.



Figs 10

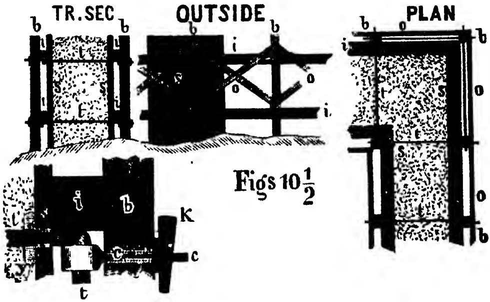
Where a coffer-dam is exposed to a violent current, and great danger from ice, &c, the expensive mode shown in Figs 10 may become necessary. The two black rectangles c c, represent two lines of rough cribs filled with stone, and sunk in position; one row being enclosed by the other; with a space several feet wide between them. Sheet-piles *p p* are then driven around the opposite faces of the two rows of cribs; and the puddle is deposited within the boxing thus provided for it, as shown in the fig.

Where the current is not strong enough to wash away gravel backing, we may, on rock especially, enclose the space to be built on, by a single quadrangle of cribs sunk by stone; and after adopting precautions to prevent the gravel from being pressed in beneath the cribs, apply the backing.*

Figs 10½ show the plan, outside view, and transverse section, to a scale of 20 ft to an inch, of a coffer-dam on rock, in 8 to 9 ft water, used successfully on the Schuylkill Navigation.

* A pure clean coarse gravel is entirely unfit for such purposes. A considerable proportion of earth is essential for preventing leaks.

Coffer-dam on rock. Uprights *b*, about 1 ft square, and 10 ft apart from center to center along the sides of the dam; and 10 ft in the clear, transversely of the dam, support two lines of horizontal stringers, *i i*; inside of which are the two lines of sheeting-piles, *s s*, enclosing between them a width of 7 ft of gravel puddle. Two flat iron bars (*t t*, of the transverse section) tie together each pair of uprights *b b*. These bars are ½ inch thick, by 2½ ins deep, and 9 ft long. Their hooked ends fit into eye-bolts *c*, which pass through the uprights *b*; outside of which they are fastened by keys, *k*, (see detail sketch.) Between the keys and *b*, were washers. At the corners of the dam (see plan) were additional tie-bars, as shown. A small band of straw, as seen at *y*, wrapped around the tie-bars just inside of the sheet-piles, and kept in place by the puddle; effectually prevented the leaking which generally proves so troublesome in such cases The stout oblique braces, *o o*, were merely spiked to the outside faces of the uprights *b*. They are not shown in the transverse section. This dam was built on shore, in sections 30 to 40 ft long. These were floated into place, and weighted down, sheet-piled, and puddled with gravel. The dam had sluices by which water was admitted when necessary for preventing the outside head from exceeding 9 ft. The lengths of the uprights *b b* were first found by careful soundings.



Figs 10½

The mooring of large caissons or cribs, preparatory to sinking them, is sometimes troublesome, especially in strong currents. It may be necessary to drive clumps of piles; or to temporarily sink rough cribs filled with stone, to which to attach the long guide-ropes by which the manœuvring into position, &c, is done. Frequently dams are left standing after the work is done; if not in the way of navigation, or otherwise objectionable; inasmuch as the materials are rarely worth the expense of removal. But if removed, the piles should not be *drawn* out of the ground; but be *cut off* close to river bottom; for if drawn, the water entering their holes may soften the soil under the masonry. It is often expedient to drive two rows of piles from the dam to the shore, for supporting a gangway for the workmen; or even for horses and carts, or for a railway for the easy delivery of large stones, &c.

Coffer-dams may be sunk through a soft to a firm soil, in shape of a box of cribwork, either rectangular or circular, and without a bottom. This being strongly put together, and provided with proper temporary internal bracing, (to be gradually removed as the masonry is built up,) is floated into place; and after being loaded so as to rest on the soft bottom, is sunk by dredging out the soft material from inside. Additional loading will sometimes be required for overcoming the friction of the soil against the outside; or it may even become necessary to dredge away some of the outer material also. **On rock** it may at times be expedient to drill holes in deep water, for receiving the ends of piles, or of iron rods, &c. This may be done by means of long drill-rods, working in an iron tube or pipe sunk as a guide to the rod; with its lower end over the spot to be bored Or a diving-bell may be used. Or **a cylinder** of staves 4 to 12 inches thick, long enough to reach above the surface, and having a broad tarpaulin flap or apron around its lower edge, to be covered with gravel to prevent leaking; may be sunk, and the water pumped out, to allow a workman to descend, and work in the open air.

Piles. When driven in close contact, as in Fig 11, for preventing leakage; for confining puddle in a coffer-dam: or for enclosing a piece of soft or sandy ground, to prevent its spreading when loaded; or if the outside soil should wash away from

around them, &c, they are called **sheet-piles.** Generally these are thinner than they are wide; but frequently they are square; and as large as bearing piles; and are then called **close piles.** To make them drive tight together at foot, they are cut obliquely as at *f*. Occasionally, when driven down to rock through soft soil, their feet are in addition cut to an edge, as at *i*, so as to become somewhat bruised when they reach the rock, and thus fit closer to its surface. Their heads are kept in line while driving, by means of either one or two longitudinal pieces *a* and *o*, called **wales** or *stringers*. These wales are supported by *gauge-piles*, or *guide-piles*, previously driven in the required line of the work, and several ft apart, for this purpose. See Figs 8.

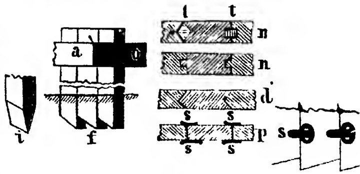


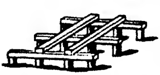
Fig 11

A dog-iron d, of round iron, may also be used for keeping the edges of the piles close at top to those previously driven, both during and after the driving. Its sharp ends, *c c*, being driven into the tops of the wales *w w*, (shown in plan,) it holds the descending pile *o* firmly in place. At *n*, *d*, *p*, Fig 11, are other modes occasionally used for keeping the piles in proper line. At *p*, the letters *s s* denote small pieces of iron well screwed to the piles, a little above their feet, to act as guides; very rarely used. At *m* are shown wooden tongues *t t*, sometimes driven down between the piles after they themselves have been driven; to assist in preventing leaks. In some cases sheet-piles are employed without being driven. A trench is first dug to their full depth for receiving them; and the piles are simply placed in these, which are then refilled. Closer joints can be secured in this manner than by driving.

Fig 12

When piles are intended to sustain loads on their tops, whether driven all their length into the ground, or only partly so, as in Fig 3, they are called **bearing piles.** They are generally round; from 9 to 18 ins diam at top; and should be straight, but the bark need not be removed. White pine, spruce, or even hemlock, answer very well in soft soils; good yellow pine for firmer ones; and hard oaks, elm, beech, &c, for the more compact ones. They are usually driven from about 2½ to 4 ft apart each way, from center to center, depending on the character of the soil, and the weight to be sustained. **A tread-wheel** is more economical than the winch for raising the hammer, when this is done by men. Morin found that the work performed by men working 8 hours per day, was 3900 foot-pounds per man, per minute by the tread-wheel; and only 2600 by a winch.

After piles have been driven, and their heads carefully sawed off to a level, if not under water, the spaces between them are in important cases filled up level with their tops with well rammed gravel, stone spawls, or concrete, in order to impart some sustaining power to the soil between the piles. Two courses of stout timbers (from 8 to 12 ins square, according to the weight to be carried) are then bolted or treenailed to the tops of the piles and to each other, as shown in the Fig, forming what is called a **grillage.** On top of these is bolted a floor or platform of thick plank for the support of the masonry; or the timbers of the upper course of the grillage may be laid close together to form the floor. The space below the floor should also, in important cases, be well packed with gravel, spawls, or concrete. **If under water,** the piles are sawed off by a diver, or by a circular saw driven by the engine of the pile-driver, and the grillage is omitted. Instead of it the masonry or concrete may be built in the open air in a caisson, which gradually sinks as it becomes filled; or on a strong platform which is lowered upon the piles by screws as the work progresses. Or a strong caisson may first be sunk entirely under water, and then be filled with concrete, up to near low water; the caisson being allowed to remain. Or the caisson may form a cofferdam, to be first sunk, and then pumped out. If the ground is liable



to wash away from around the piles, as in the case of bridge piers, &c, defend it by sheet-piles, or rip-rap, or both.

The cost of a floating steam pile driver, scow 24 ft by 50 ft. draft 18 ins, with one engine for driving, and one (to save time) for getting another pile ready; with one ton hammer, is about $6000; and $500 more will add a circular saw, &c, for sawing off piles at any reqd depth Requires engineman, cook, and 4 or 5 others. Will burn about half a ton of coal per day. Driving 20 feet into gravel, and sawing off, will average from 15 to 20 piles per day of 10 hours. In mud about twice as many. On land about half as many as in water.

In the gunpowder pile driver invented by the late Mr. Thomas Shaw, of Philadelphia, the hammer is worked by small cartridges of powder, placed one by one in a receptacle on top of the pile; and exploded by the hammer itself. It can readily make 30 to 40 blows of 5 to 10 ft per minute; and, since the hammer does not come into actual contact with the piles, it does not injure their heads at all; thus dispensing with iron hoops, &c, for preserving them. When only a slight blow is required, a smaller cartridge is used. To drive a pile 20 ft into mud averages about one-third of a pound of powder; into gravel, 4 times as much. This machine does not assist in raising the pile, and placing it in position, as is done by ordinary steam pile drivers; the latter, however, average but from 6 to 14 blows per minute.

Piles have been driven by exploding small charges of **dynamite** laid upon their heads, which are protected by iron-plates.

Steam-hammer pile drivers, operating on the principle of that devised by **Nasmyth** about 1850, are economical in driving to great depths in difficult soils where there are say 200 or more piles in clusters or rows, so that the machine can readily be moved from pile to pile.

The steam **cylinder** is upright, and is confined between the upper ends of two vertical and parallel I or channel **beams** about 6 to 12 ft long and 18 ins apart, the lower ends of which confine between them a hollow conical "**bonnet casting,**" which fits over the head of the pile. This casting is open at top, and through it the hammer, which is fastened to the foot of the piston-rod, strikes the head of the pile. Each of the vertical beams encloses one of the two upright guide-timbers, or "leaders," of the pile driver, between which the driving apparatus, above described, is free to slide up or down as a whole.

When a pile has been placed in position, ready for driving, the bonnet casting is placed upon its head, thus bringing the weight of the beams, cylinder, hammer, and casting upon the pile. This weight rests upon the pile throughout the driving, the apparatus sliding down between the leaders as the pile descends.

The steam is conveyed from the boiler to the cyl by a flexible pipe. When it is admitted to the cyl, the hammer is lifted about 30 or 40 ins, and upon its escape the hammer falls, striking the head of the pile. About 60 blows are delivered per minute. The hammer is provided with a trip-piece which automatically admits steam to the cylinder after each blow, and opens a valve for its escape at the end of the up-stroke. By altering the adjustment of this trip-piece, the length of stroke (and thus the force of the blows) can be increased or diminished. The admission and escape of steam, to and from the cyl, can also be controlled directly by the attendant. The number of blows per minute is increased or diminished by regulating the supply of steam.

In making the up-stroke, the steam, pressing against the lower cyl head, of course presses downward on the pile and aids its descent.

The chief advantage of these machines lies in the great rapidity with which the blows follow one another, allowing no time for the disturbed earth, sand, &c, to recompact itself around the sides, and under the foot, of the pile. This enables the machines to do work which cannot be done with ordinary pile drivers. They have driven Norway pine piles 42 ft into sand. They are less liable than others to split and broom the pile, so that these may be of softer and cheaper wood The bonnet casting keeps the head of the pile constantly in place, so that the piles do not "dodge" or get out of line. Their heads have, in some cases, been set on fire by the rapidly succeeding blows.

These machines consume from 1 to 2 tons of **coal** in 10 hours, and **require a crew** of 5 men. They work with a **boiler pressure** of from 50 to 75 lbs per sq inch.

Rules for the Sustaining Power of Piles.

They differ very much. No rule can apply correctly to all conditions. The ground itself between the piles, in most cases, supports a part of the load; although the whole of it is usually assigned to the piles. Again, in very clayey soils, there is greater liability to sink somewhat with the lapse of time, in consequence of the admission of water between the pile and the clay; thus diminishing the friction between them. The less firm the soil, the more will the piles be affected by tremors; which also tend in time to cause sinking. In some cases this sinking will not be that of the piles settling deeper into the earth *around* them; but that of the entire compacted mass of piles and earth into which they were driven, settling down into the less dense mass *below* them. Piles are sometimes blamed for settlements which are really due to the crushing (flatways) of the timbers which rest immediately upon their heads.

In the fine **London bridge** across the Thames, each pile under some of the piers sustains the very heavy load of 80 tons. They are driven but 20 feet into the stiff, blue London clay; and are placed nearly 4 ft apart from center to center; which is too much for such piers and arches. At 3 ft apart scant, they would have had but 45 tons to sustain. They are 1 ft in diam at the middle of their length. Ugly settlements, some of them to the extent of about a ft, have occurred under these piers. **Blackfriars bridge**, in the same vicinity, exhibits the same defect. By some this is ascribed in both cases to the gradual admission of water between the clay and the piles, perhaps by capillary action of the piles themselves; or perhaps by direct leaking. It may, however, be owing in part to the crushing of the platforms on top of the piles; or to a bodily settlement of the entire mass of piled clay, into the unpiled clay beneath, under the immense load that rests upon it. This here amounts to $5\frac{1}{2}$ tons per sq foot of area covered by a pier; and is probably too much to trust upon damp clay, when even the slightest sinking is prejudicial.

Maj J. Sanders, U. S. Engs, experimented largely at Fort Delaware in river mud; and gave the following in the Jour. Franklin Inst, Nov 1851. For the **safe** load for a common wooden pile, driven until it sinks through only small and nearly equal distances, under successive blows, divide the height of the fall in ins, by the small sinking at each blow in ins. Mult the quot by the weight of the hammer, ram, or monkey, in tons or pounds, as the case may be. Divide the prod by 8. He does not state any specific coefficient of safety.

Example. At the **Chestnut St Bridge,** Philada, the greatest weight on any pile is 18 tons. Mr Kneass had the piles driven until they sank $\frac{3}{4}$, or .75 of an inch under each blow from a 1200 ℔ hammer, falling 20 ft. Was he safe in doing so? Here we have the fall in ins = 20 × 12 = 240. And $\frac{240}{.75} = 320$; and 320 × 1200 = 384000 ℔s; and $\frac{384000}{8} = 48000$ ℔s, = 21.4 tons safe load by Maj Sanders' rule. The soil was river mud.

Our own rule is as follows. Mult together the cube rt of the fall in ft; the wt of hammer in ℔s; and the decimal .023. Divide the prod by the last sinking in ins. + 1. The quotient will be the **extreme load** that will be just at the point of causing more sinking. For the safe load take from one twelfth to one half of this, according to circumstances. Or, as a formula,

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{\begin{array}{c}\text{Cube rt of}\\ \text{fall in feet}\end{array} \times \begin{array}{c}\text{Wt of hammer}\\ \text{in pounds}\end{array} \times .023.}{\text{Last sinking in inches} + 1}.$$

Example. The same as the foregoing at Chestnut St Bridge. Here the cube rt of 20 ft fall is 2.714 ft. Hence we have

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{2.714 \times 1200 \times .023}{.75 + 1} = \frac{74.9}{1.75} = 42.8 \text{ tons.}$$

Or say half of this, or 21.4 tons, the load for a safety of 2. Major Sanders' rule makes the **safe** load 21.4. The actual one is 18 tons.

A safety of 2 is not enough for river mud.

But although Major Sanders' rule and our own agree very well in this instance *if a safety of 2 be taken for each*, they differ widely in some others. Thus at **Neuilly Bridge,** France, Perronet's heaviest hammer weighed 2000 ℔s, fall 5 ft, sinkage .25 of an inch in the last 16 blows; or say .016 inch per blow. The piles sustain 47 tons each. Our rule gives 38.8 tons for a safety of 2; while Sanders' rule gives 515 tons safe load! If, as we think probable, there was no actual sinking at the last blow, then our rule gives 39.3 tons for a safety of 2; while Sanders' gives infinity.

At the **Hull Docks,** England, piles 10 ins square, driven 16 ft into alluvial mud by a 1500 ℔ hammer, falling 24 ft, sank 2 ins per blow at the end of the driving. They sustain at least 20 tons each, or according to some statements 25 tons. Our rule gives 33.2 tons for the extreme load; or 16.6 for a safety of only 2. Sanders gives for *safety* 12.06 tons. As before remarked, 2 is not safety enough for mud. In mud, it is not primarily the piles, but the piled **soil** that settles, bodily, for years.

At the **Royal Border Bridge,** England, piles were very firmly driven from 30 to 40 ft in sand and gravel, in some cases wet. Pine was first tried, but it split and broomed so badly under the hard driving, that American elm was substituted, with success. They were driven until they sank but .08 inch per blow, under a 1700 ℔ monkey, falling 16 ft. They support 70 tons each. Our rule gives 47 tons for a safety of 2; while Sanders gives 364 tons safe load!

It is the writer's opinion, however, that the piles did not actually sink, as was (and always is, in such cases) taken for granted by the observers; but that they were merely compressed or partially crushed by overdriving. Most of the piles were driven until they sank (?) only an inch under 150 blows; but we doubt whether they were any safer, or farther in the ground, than when they had received only one of them; and consider such extreme precaution worse than useless.

In some experiments (1878) at Philada, a trial pile was driven 15 ft into soft river mud, by a 1600 ℔ hammer; its last sinking being 18 ins under a fall of 36 ft. Only 5 hours after it was driven it was loaded with 6.5 tons; which caused a sinking of but a very small fraction of an inch. Our rule

gives 6 4 tons as the extreme load. Under 9 tons it sank .75 of an inch; and under 15 tons, 5 ft. By Maj Sanders' rule its safe load would be 2.14 tons.

A U. S. Govt trial pile, about 12 ins sq, driven 29 ft through layers of silt, sand, and clay, hammer 910 lbs, fall 5 ft, last sinking .375 of an inch, bore 26.6 tons; but sank slowly under 27.9 tons. Our rule gives 26 tons extreme load.

French engineers consider a pile safe for a load of 25 tons, when it is driven to the refusal of 1344 lbs, falling 4 ft; our rule gives 24.2 tons for safety 2. They estimate the refusal by its not sinking more than 4 of an inch under 30 blows. In many important bridges &c they drive until there is no sinking under an 800 lb hammer, falling 5 ft. Our rule here gives 31.5 tons extreme load; or 15.7 for safety 2.

As to the proper load for safety, we think that not *more* than one-half the extreme load given by our rule should be taken for piles *thoroughly* driven in *firm* soils; nor more than one-sixth when in river mud or marsh; assuming, as we have hitherto done, that their feet do not rest upon rock.

If liable to tremors, take only **half** these loads.

Piles may be made of any required size as regards either length or cross section, by bolting and fishing together sidewise and lengthwise, a number of squared timbers.

Piles with blunt ends. At South Street Bridge, Phila, 1200 stout piles of Nova Scotia spruce with blunt ends were driven 15 to 35 ft. partly in strong gravel, by a common steam pile driver, at a total cost (piles and driving) of $7 to $8 each. At **Wilmington Harbor,** Cal, Mr. C. B. Sears, U S Army. (Jour. Am. Soc. C. E., Dec 1876) found that in firm compact wet sand, after the first few blows the piles would not penetrate more than .5 to 1.5 ins at a blow, no matter how far the 2400 lb hammer fell. The unpointed ones of which there were many thousands, drove quite as readily to average depths of 15 ft in this sand as the pointed ones, and with much less tendency to cant. As a high fall had no farther effect than to batter the heads he reduced it to 10 ft. which drove an average of about .72 inch to a blow. To insure straight driving, the ends must be at right angles to the length. **Instead of driving** piles to moderate depths it may at times be better to merely **plant** them butt down in holes bored by an auger like **Pierce's Well Borer.**

The ultimate friction of piles even with the bark on, and driven about 3 ft apart from cen to cen probably never much exceeds about 1 ton per sq ft even when well driven into dense moist sand or loamy gravel; nor more than .5 to .75 of a ton in common soils and clays; or than .1 to .2 of a ton in silt or wet river mud depending on the depth and density.

The friction of cast iron cylinders seems to be about .3 that of piles.

There is a great difference in the penetrability of different sands. Thus, in the Lary bridge, no special difficulty was found in driving piles 35 ft into deep wet sand, while, in other wet localities, piles of very tough wood, well shod with iron, cannot be driven 6 ft into sand, without being battered to pieces. The same difference has been found in the case of screw piles. At the Brandywine light-house these could not be forced more than 10 ft into the clean wet sand Stiff wet clay (and clean gravels) also differ very much in this respect Generally they are penetrable to any required depth with comparative ease, but we have seen stout hemlock piles battered to pieces in driving 6 ft through wet gravel, and Mr Rendel found that at Plymouth he "could not by any force drive screw-piles more than about 5 ft into the clay, which is not as stiff as the London clay," on which the forementioned new London and Blackfriars bridges were founded; and into which even ordinary wooden piles were driven 20 ft without special difficulty.

A mixture of mud with the sand or gravel facilitates driving very much; but before beginning an extensive system of piling, a few experimental ones should be driven, to remove doubt as to the trouble and expense that may be anticipated. Mere boring will often be but a poor substitute for this.

As a general rule, a heavy hammer with a low fall, drives more pleasantly than a light one with a high fall. Where a hammer of ⅔ ton (1500 lbs) falling 25 ft, in a very strong ground, shattered the piles; one of 2 tons, (4500 lbs,) with 7 ft fall, drove them satisfactorily. More blows can be made in the same time with a low fall; and this gives less time for the soil to compact itself around the piles between the blows. At times a pile may resist the hammer after sinking some distance; but start again after a short rest; or it may refuse a heavy hammer, and start under a lighter one. It may drive slowly at first, and more rapidly afterward, from causes that may be difficult to discover. The driving of one sometimes causes adjacent ones previously driven, to spring upward several feet. A pile is in the most favorable position when its foot rests upon rock, after its entire length has been driven through a firm soil, which affords perfect protection against its bending like an overloaded column; and at the same time creates great friction against its sides; thus assisting much in sustaining the load, and thereby relieving the pressure upon the foot. A pile may rest upon rock, and yet be very weak; for if driven through very soft soil, all the pressure is borne by the sharp point; and the pile becomes merely a column in a worse condition than a pillar with one rounded end. In such soils the piles need very little sharpening; indeed, had better be driven without any; or even butt end down.

The driving of a pile in soft ground or mud will generally cause an adjacent one previously driven, to lean outwards unless means be taken to prevent it.

In piling an area of firm soil it is best to begin at its center and work outwards; otherwise the soil may become so consolidated that the central ones can scarcely be driven at all.

Elastic reaction of the soil has been known to cause entire piled areas to rise, together with the piles, before they were built upon.

In very firm soil, especially if stony; or even in soft soil, if the piles are pointed, and are to be driven to rock; their feet should be protected by **shoes** of either wrought iron, as at *a*, *s*, and *b*, Figs 13; spiked to the pile by means of the iron straps *n*, forged to them; or of cast iron, as at *c*, where the shoe is a solid inverted cone, the wide flat base of which affords a good bearing for the flat bottom of the pile-point. The dotted line is a stout wrought-iron spike, well secured in the cone, which is cast around it; this holds the shoe to the pile. Regular

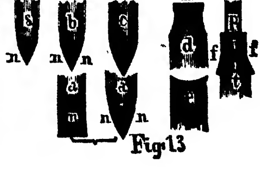


Fig 13

wrought-iron shoes will generally weigh 18 to 30 lbs; but sheet iron may be used when the soil is but moderately compact, plate iron when more so, and solid iron or steel points, from 2 to 4 ins square at the butt, and 4 to 8 ins long, when very compact and stony. **Holes may be drilled in rock** for receiving the points of piles, and thus preventing them from slipping, by first driving down a tube, as a guide to the drill, after the earth is cleaned out of the tube. **To preserve the heads** to some extent from splitting under the blows of the hammer, they are usually surrounded by a hoop *h*, Fig *d*, from ½ to 1 inch thick, and 1½ to 3 ins wide. These are, however, sometimes but imperfect aids; for in hard driving the head will crush, split, and bulge out on all sides, frequently for many feet below the hoop moreover, the hoops often split open The heads, therefore, often have to be sawed, or pared off several times before the pile is completely driven; and allowance must be made for this loss in ordering piles for any given work, especially in hard soil Capt Turnbull, U S Top Eng. states that at the Potomac aqueduct, his pileheads were preserved from injury by the simple expedient of dishing them out to a depth of about an inch, and covering them by a loose plate of sheet iron, as shown in section at *e*, Figs 13. A very slight degree of brooming or crushing of the head, materially diminishes the force of the ram. Piles may be driven through small loose rubble without much labor. Shaw's driver does not injure the heads. Piles which foot on sloping rock may slide when loaded.

To drive a pile head below water a wooden punch, or follower, as at *p*, Figs 13, may be used. The foot of this punch fits into the upper part of a casting *f f*, round or square, according to the shape of the pile, and having a transverse partition *o o* The lower part of the casting is fitted to the head of the pile *t*; and the hammer falls on top of the punch When driving piles vertically in very soft soil, to support retaining-walls, or other structures exposed to horizontal or inclined forces, care must be taken that these forces do not push over the piles themselves; for in such soils piles are adapted to resist vertical forces only, unless they be driven at an inclination corresponding to the oblique force.

A broken pile may be drawn out, or at least be started, if not very firmly driven, by attaching scows to it at low water, depending on the rising tide to loosen it Or a long timber may be used as a lever, with the head of an adjacent pile for its fulcrum Or a crab worked by the engine of the pile driver In very difficult cases the method devised by Mr J Monroe, C E, may be used. A 4 inch gas pipe 15 ft long, shod with a solid steel point, and having an outer shoulder for sustaining a circular punch, was thereby driven close to and 2 or 3 ft deeper than two piles driven 12 ft, in 37 ft water, and broken off by ice Four pounds of powder were then deposited in the lower end of the pipe, and exploded, lifting the piles completely out of place. It will often be best to let a broken pile remain, and to drive another close to it. May be drawn by hydraulic press.

Ice adheres to piles with a force of about 30 to 40 lbs per sq inch, and in rising water may lift them out of place if not sufficiently driven

Iron piles and cylinders. Cast iron in various shapes has been much used in Europe for sheet piles, especially when intended to remain as a facing for the protection of concrete work, filled in behind and against them * Cast iron cylinders, open at both ends, may be used as bearing piles; and may be cleaned out, and filled with concrete, if required The friction in driving is greater than in solid piles, inasmuch as it takes place along both the inner and the outer surfaces This may be diminished by gradually extracting the inside soil as they go down They require much care, and a lighter hammer, or less fall than wooden ones to prevent breaking, to which end a piece of wood should be interposed between the hammer and the pile, or the ram may be of wood. But it is better to use them in the shape of **screw cylinders**, which, moreover, gives them the advantage of a broad base as in the following

Brunel's process. He experimented with an open cast-iron cylinder, 3 ft outer diam; 1⅛ ins thick, in lengths of 10 ft, connected together by internal socket and joggle joints, secured by pins, and run with lead. It had a sharp edged hoop or cutter at bottom, and a little above this, one turn of a screw, with a pitch of 7 ins, and projecting one foot all around the outside of the cylinder. By means of capstan bars and winches, he screwed this down through stiff clay and sand, 58 feet to rock, on the bank of a river In descending this distance the cylinder made 142 revolutions; sinking on an average about 5 ins at each. The time occupied in actually screwing was 48¼ hours; or about $1\frac{2}{10}$ ft per hour. There were, however, many long intervals of rest for cleaning away the soil in the inside After resting, there was no great difficulty in restarting The next fig will give an idea of the arrangement of the screw.

The screw-pile of Alex. Mitchell, Belfast, consists usually of a rolled iron shaft A, Figs 14, from 3 to 8 ins diam, and having at its foot a cast-iron screw S S S, with a blade of from 18 ins to 5 ft diam. The screws used for light-houses, exposed to moderate seas, or heavy ice-fields, are ordinarily about 3 ft diam, have 1½ turns or threads, and weigh about 600 lbs The round rolled shafts are from 5 to 8 ins diam. They are screwed down from 10 to 20 ft into clay, sand, or coral, by about 30 to 40 men, pushing with 6 to 8 capstan bars, the ends of which describe a circle of about 30 to 40 ft diam. For this purpose a platform on piles has frequently to be prepared. In quiet water, this may be supported on scows; or a raft well moored may be used when the driving is easy; or the deck of a large scow with a well-hole in the center for the pile to pass through. Roughly made temporary cribs, filled with stone and sunk, might support a platform in some positions. The platform must evidently be able to resist revolving horizontally under the great pushing force of the men at the capstan bars; and on this account it is difficult to drive screws to a sufficient depth, in clean compact sand, by means of a floating platform. The feet of the piles must be firmly secured to the screws, to prevent

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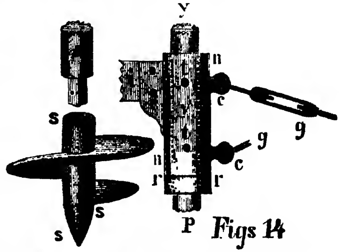
* **Cast iron, intended to resist sea-water,** should be close-grained, hard, white metal. In such, the small quantity of contained carbon is chemically combined with the metal; but in the darker or mottled irons it is mechanically combined and such iron soon becomes soft, (somewhat like plumbago,) when exposed to sea-water. Hard white iron has been proved to resist for at least 40 years without any deterioration, whether constantly under water, or alternately wet and dry. Copper and bronze are but slightly and superficially affected by sea-water; but destructive galvanic action takes place if diff metals are in contact.

their being lifted out of them by the upward force of waves against the superstructure. At *y p*, Figs 14, is shown a mode of splicing or uniting the different lengths or sections of a pile The point of junction is at *v*; *r r* is a stout iron ring forged on to the lower pile *p*, about a foot or 18 ins below its top *v*. A strong cylindrical casting *n n*, enclosing the ends of the sections, rests on this ring, and is pinned through the piles, as at *t t*. On this casting are also cast projections *c c c*, for attaching rods *g g*, and beams *i*, &c, necessary for bracing the structure from pile to pile. The time actually required for driving a screw is from 2 to 10 hours, in favorable circumstances.



Figs 14

At the Brandywine lighthouse, on a sand-bank of very pure sand covered 6 or 8 ft at low water, and from 11 to 13 ft at high, they could not be forced down, from a fixed platform, for more than 10 ft At other places 20 ft in sand is reached without much trouble, where the sand contains a good deal of mud, but its **bearing power** is then less This (ultimate) ranges between about 1 and 6 tons per sq ft according to purity, depth, compactness, &c, of the sand. In important cases the bearing power should be tested.

Mitchell's piles have been screwed about 40 feet into a mixture of clay and sand, with screws 4 ft diam. They pass through small broken stone and coral rock without much difficulty; and will push aside bowlders of moderate size. Ordinarily, clay or sand will present no great obstruction; but occasionally either of them will do so Perfectly pure clean sand, as a general rule, gives most difficulty. At the Brandywine shoal the driving was aided by a spur and pinion placed as low as the water permitted; and the levers were worked by 30 men. The danger of twisting off the shaft is the limit for screwing them They are much used for the anchoring of chains for mooring buoys, &c. On land, small screws, with short hollow shafts, make good durable supports for depot pillars, cranes, wooden telegraph poles, station signals in marine surveying, &c, &c. They can readily be unscrewed for removal Horses or oxen may be used in driving large screws. The Brandywine light-house stands on 9 screw-piles, which are surrounded by 30 others of 5 ins diam, as fenders. They have to resist not only moderate seas, but immense fields of floating ice, miles in extent. An unfinished structure was destroyed by ice, which at times injures the bracing of the standing one

Test borings should be made to ensure that the screws do not stop just above a very weak stratum which may endanger their bearing power So with *any* piles.

By means of a jet of water forcibly impelled through a tube by a force pump, the most obstinate sands will be loosened, and the sinking of screw piles, or wooden ones, or even the largest cylinders, be greatly facilitated In a government **pier at Cape Henlopen** in very compact sand, in which 6 out of 7 screws previously broke before reaching 10 ft, the use of the jet was found to remove more than three fourths of the resistance * The pile *p* to be sunk having first been placed in position as in Fig 15, the lower open ends *t t* of a bent iron tube *t s t* of one and a quarter ins bore were stood upon the upper face of the screw disk, and there held firmly by 3 or 4 men while the pile was being screwed down by the capstan *c*, which was worked by a leading rope *r*. From the bend *s* of the pipe, a hose *h*, 2 ins diam, led to the force pump, the cylinder of which was 5 ins bore, and 9 ins stroke, and worked about 80 full strokes per minute, by a mule walking on a tread wheel on a floating platform *f*. There was now no trouble in screwing the piles to any required depth Previous trials by playing the jet *beneath* the disk gave unsatisfactory results

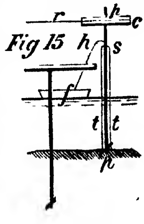


Fig 15

In Mobile Bay several thousands of wooden piles, from 18 to 48 ins diam, were sunk from 10 to 20 ft into obstinate sand, at the average sinking rate of about 1 ft per second, entirely by means of jets. The jet was propelled by a city steam fire engine, on a steamboat, through its own hose, with a one and a quarter inch nozzle. During the descent the nozzle *n n* was held loosely in its place near the foot of the pile, by two staples *s s* and by a string *t* reaching to the surface. The piles were suspended by their heads from shears, by the tackle of which their descent was regulated The sand settled firmly around the piles in a few minutes after they were sunk.†

At Tensas River, Alabama, for iron cylinders 6 ft diam (enclosing piles, in deep light shifting sand, the jet was forced by a small rotary pump of 200 to 300 revolutions per minute, through a canvas hose 3 ins diam, into a central conical cast iron vessel 10 ins diam. from which radiated 12 gas pipes 1 inch diam, and about 30 ins long. At the outer end of each of these radii was an elbow to which was attached a long vertical pipe reaching down into the cylinder, and made in 10 ft lengths with screw ends for prolonging them as the cylinder went down. This apparatus was raised and lowered by a light block and line; and by it alone each cylinder was sunk about 16 ft into the light sand in a few hours.‡

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* Report Sec of War 1872. † John W. Glenn, C E, Van Nostrand, June 1874.
‡ Gabriel Jordan, C E; Trans Am Soc C E, Feb 1874.

At the Levan Viaduct, Mr James Brunlee, England, in a light sandy marl of great depth, sunk hollow cast iron cylinders of 10 ins outer diam, to a depth of 20 ft, by means of a jet pipe 2 ins diam passing down inside of the cylinder, and through a hole in its base, which was a cast iron disk 30 ins diam, and 1 inch thick, strengthened by outside flanges. The connecting flanges of the cylinder sections are *outside*, thus impeding the descent, as did also the broad bottom disk; still 3 or 4 hours usually sufficed for the sinking of each, to 20 ft depth. Actual trial showed that their safe sustaining power was about 5 tons per sq ft of bottom disk

At Lock Ken viaduct each pier consists of two cylinders, open at both ends; of cast iron, 8 ft in diam; $1\frac{1}{8}$ ins thick; in lengths of 6 ft, weighing 4 tons each; and bolted together by inside flanges, with iron cement between them. The cylinders stand 8 ft apart in the clear; and are in 36 ft water. "A strong staging was erected; and 4 guide-piles driven for each cylinder. The several lengths being previously bolted together, these were lowered into their places Each cylinder sunk by its own weight one or two ft through the top mud, and then settled upon the sand and gravel which form the substratum for a great depth Into this last they were sunk about 8 or 9 ft farther, by excavating the inside earth under water, by means of an inverted conical **screw-pan,** or dredger, of $\frac{1}{4}$ inch plate iron. This was 2 ft greatest diam, and 1 ft deep; and to its bottom was attached a screw about 1 ft long, for assisting in screwing it down into the soil. Its sides had openings for the entrance of the soil; and leather flaps, opening inward, to prevent its escape. From opposite sides of the pan, 3 rods of $\frac{3}{4}$ inch diam projected upward 4 feet, and were there forged together, and connected by an eye-and-bolt joint to a long rod or shaft, at the upper end of which was a four-armed cross-handle, by which the pan was screwed down by 4 men on the staging."

"When a pan was full, a slide which passed over the joint at the bottom was lifted; and the pan was raised by a tackle. This pan raised about 1 cub ft at a time. A smaller one of only 1 ft diam, and 1 ft deep, raising about $\frac{1}{4}$ cub ft, was used when the material was very hard. By this means the cylinders were sunk at the rate of from 2 to 18 ins per day. The slow rate of 2 ins was caused by stones some of them of 50 lbs. These were first loosened by a screw-pick, which was a bar of iron 3 ft long, with circular arms 12 ins long projecting from the sides. After being loosened by this, the stones were raised by the pan. The expense of all this apparatus was very trifling; and the excavation was done easily and cheaply. After the excavation was finished, and the cylinder sunk, before pumping out the water, concrete (gravel 2, hydraulic cement 1 measure) was filled in to the depth of 12 feet, by means of a large pan with a movable bottom; and about 12 days were left it to harden The water was then pumped out, and the masonry built in open air In some of the cylinders, however, the water rose so fast, notwithstanding the 12 ft of concrete, that the pumps could not keep them clear, and 6 ft more of concrete had to be added in those Finally random-stone, or rough dry rubble, was thrown in around the outsides of the cylinders, to preserve them from blows and undermining."* The masonry extends 20 ft above the cylinders, and above water.

The vacuum and plenum processes. We can barely allude to the general principles of these two modes of sinking large hollow iron cylinders In the **vacuum process** of Dr. Lawrence Holker Potts, of London, the cylinder *c*, Fig 16, while being sunk, is closed air-tight at top, by a trap-door, opening upward A flexible pipe *p*, of India-rubber, long enough to adapt itself to the sinking of the cylinder, and provided with a stopcock *s*, leads from the cylinder to a vessel *v*; which may be placed on a raft, or a scow, or on land, as may suit circumstances. The cylinder being first stood up in position, as in the fig, the water is pumped out, and the interior soil removed if the cylinder has sunk some distance by its own weight The cock *s* is then closed, and the air is drawn out from the vessel *v* by an air-pump. The cock is then opened, and most of the air in the cylinder rushes into the void vessel *v*; thus leaving the cylinder comparatively empty, and therefore less capable of resisting the downward pressure of the external air upon its top This pressure, as is well known, amounts to nearly 15 lbs on every sq inch; or nearly 1 ton per sq ft of area of the top. Consequently the cylinder is forced downward in the bed of the river, by this amount of pressure, in addition to its own weight. At the same time, the pressure of the air upon the surface of the water is transmitted through the water to the soil around the open foot of the cylinder; so that if this soil be soft or semi-fluid, it will be pressed up into the nearly void cylinder, in which is no downward pressure to resist it. The descent varies from a few inches, to 4 or 5 ft each time. The process is then repeated, by admitting air again into the cylinder, opening the trap-door, removing the water and soil, as before, &c. Additional lengths of cylinder may be bolted on, by means of interior flanges.

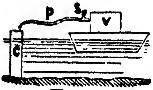

Fig 16

It is **adapted only to soft soils,** and to wet sandy ones; but is not sufficiently powerful in very compact ones; nor does it answer where obstructions from bowlders, logs, &c. occur.

***Hollow Iron Piles** either cast or wrought with solid pointed feet, to be driven by the hammer falling *inside* of them and striking against the top of the solid foot, are a recent device of great use in many cases. They are made in sections of which enough can be gradually united to reach any required depth. They avoid the danger of bending which attends striking the top. The iron feet are swelled outwardly a little to diminish earth-friction against the pile above them.

the removal of which requires men to enter the cylinder to its foot; which they cannot do in the rarefied air. The pipe *p* should be of sufficient diam to allow the air to leave the cylinder rapidly, so that the outer pressure may act upon the top as suddenly as possible.

At the Goodwin Sands light-house, England, hollow cylinders 2½ ft in diam, were sunk 34 ft into sand by this process, in about 6 hours; where a steel bar could be driven only 8 ft by a sledge-hammer. Others, 12 ins in diam, have been sunk 16 ft into sand within less than an hour. In this last instance the air-pump had two barrels, 4½ ins diam, 16 inch stroke, worked by 4 men. The pipe *p* was of lead, and only ½ inch diam.

The plenum process, invented by Mr Triger, of France, consists in forcing air into the cylinder C C, Fig 17, to such an extent as to force out the water, compelling it to escape beneath the open foot, into the surrounding water. The interior of the cylinder being thus left dry to the bottom, men pass down it to loosen and remove the soil at and below its base. When this is done, they leave; the compressed air is allowed to escape; and the cylinder, being no longer sustained by the upward pressure of the compressed air beneath its top, sinks into the cavity, or the loosened material at its foot. Fig 17 shows the simple arrangement by which workmen are enabled to enter or leave the cylinder, without allowing the compressed air to escape; as well as the general principle of the entire process.

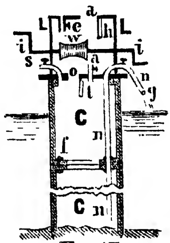

Fig 17

L L is a separate small chamber, **the air-lock**, which is removed when a new length of pipe is to be added; and afterward replaced and firmly bolted on. This chamber has a small air-tight door *d*, by which it can be entered from without, and another, *o*, opening into the cylinder. The flaps, *t*, *k*, of both doors, open inward, or toward the cylinder. This chamber also has two stopcocks; one, *a*, in its floor, communicating with the cylinder, and one *e*, above, communicating with the open air. At *s* is a bent tube, also with a cock, which passes air-tight through the side and the bottom of the air-lock. Through it the compressed air is forced into the cylinder by an air force pump or condenser, and through it the same air is allowed to escape at a later period. A siphon is shown at *n n n*. A drum *w* is used for hoisting the excavated material from the bottom, to the air-lock; its axle *i i* passes air-tight through stuffing boxes in the sides of the lock; the hoisting being done by men outside. This is the general arrangement employed by Mr W. J. McAlpine, C E, of New York, at Harlem bridge; and from his description of it, ours has been condensed. The cylinders were there 6 ft diam, 1¾ ins thick, and in lengths of 9 ft, bolted together through inside flanges *f*, as the sinking went on. The air-lock is 6 ft diam, by nearly 6 ft high; with sides of boiler iron; and top and bottom of cast iron.

Now suppose the cylinder C C to be let down, and steadied in position, as in the fig; and the air-lock L L to be adjusted on top of it. The next process is to force in air through the curved tube *s* the flap *t* of the lower door *o*, and the cock *a*, being previously closed. As the compressed air accumulates in the cylinder, it forces out the water; which escapes partly beneath the bottom of the cylinder, and partly by rising through the siphon *n n*, and flowing out at *g*. The door *o* being already closed, and that at *d* open, the air in the air-lock is in the same condition as that outside; so that workmen can enter it readily. Having done so, they close the door *d*, and the cock *e*, and open the cock *a*, through which condensed air from the cylinder rushes upward, soon filling the air-lock. When this is done, the flap *t* is opened, and the men descend through the door *o* by a ladder, or by a bucket lowered by the drum *w*, to the bottom. Here they loosen and excavate the material as deep as they can; and, filling it into a bucket or bag, they signal to those outside, who raise it to the air-lock. When done, they ascend to the air-lock, close the door *o*, and the cock *a*; and open the cock *e*, through which the condensed air in the lock soon escapes, leaving the internal air the same as that outside. The door *d* is then opened, the buckets of earth are removed, and the men go out. Finally the cock at *s* is opened, the condensed air in the cylinder escapes through it to the outside air, and the cylinder sinks by its own weight into the cavity and loosened soil prepared for it at its base, and which is now forced up into the cylinder by the rush of the returning water. The process is then repeated. The sinking will often vary from 0 to 10 or more feet at one operation. Until depths of 40 or 50 ft, most men can endure the pressure of the condensed air; but as the depth increases this becomes more difficult, and positively dangerous to life. Cast-iron cylinders 15 ft diam; and great caissons, Fig 18, have been thus sunk; but at times at great expense and trouble.

The cylinder should be guided in its descent by a strong frame, which may be supported by piles. Otherwise it will be apt to tilt, and thus give great trouble to settle it upon its exact place. Have been sunk in deep water by divers undermining inside.

The plenum process as applied at the South St bridge, Philada by Mr John W. Murphy, contracting engineer, differs materially from that described above; and moreover deserves notice on account of the great simplicity and efficacy of his plant. This consisted partly of two canal boats, decked, each 100 ft long, by 17½ ft wide, and 8 ft depth of hold. They were anchored parallel to each other, 15 ft apart. Supported by the boats, and over the space between them, was a strong four-legged shears about 50 ft high; at the top of which was attached tackle for handling the cast iron cylinders. In the hold of one of the boats was a **Burleigh Compressor** having two pistons of 10 ins diam, and 9 ins stroke; together with its boiler. On the deck of the same boat stood a vertical **air-tank or regulator** 22 ft long, by 2 ft diam, made of quarter inch boiler iron. This served to maintain a supply of compressed air in the submerged cylinder in case of an accidental stopping of the compressor; which otherwise would probably be fatal to the laborers in the cylinder. The condensed air flowed from this air tank to the air-lock of the cylinder through a hose 4 ins diam, made of gum elastic and canvas, and so long, and so placed, as to extend itself as the cylinder went down, thus maintaining the communication at all times. Entirely across both boats, and across the interval between them, extended two heavy wooden **clamps**, each 3 ft wide by 18 ins high; each compose

of three pieces of 12 × 18 inch timber strongly bolted together. At the centers of these clamps the two inner vertical sides which faced each other were hollowed out to the depth of a foot by concavities corresponding to the curve of the cylinders. The distance apart of the clamps was regulated by two strong iron rods, having screws and nuts at their ends for that purpose. Thus when a section of a cylinder was hoisted by means of the shears into its position over the space between the two boats, the two concavities of the clamps were brought into contact with it, and the nuts being then screwed up, the cylinder was firmly held in place by the clamps. The shears could then be used to raise another section of the cylinder to its place upon the first one, that the two might be bolted together. By repeating this process the height of the cylinder would soon become too great to allow the shears to place another section upon it, in which case the nuts of the screws were slightly loosened, and the cylinder was allowed to slip down slowly into the water until its top was but a little above the surface. The screws were then again tightened, and the cylinder again held fast until other sections were added and bolted to it. When there was danger that the upward pressure of the condensed air might lift a cylinder, the clamps were raised by the shears clear of the boats, then tightened to the cylinder, and a platform of planks laid upon them, and loaded with stone

The **air-lock** was so arranged as not to require to be removed when a new section was to be bolted on This was effected as follows. Sections of the cylinder were bolted together in the manner just described, until its foot rested on the bottom, with its top a few feet above high water. A heavy cast iron **diaphragm** 1¼ inches thick, to form the floor of the air-lock, was then placed on top. Then was added another 10 ft high section of the cylinder, to form the chamber of the air-lock. These were bolted together, and then another diaphragm was added at top to form the roof of the air-lock. These diaphragms were furnished with openings, and with doors and valves corresponding with those shown in Fig 17, and remained permanently in the cylinders when the work was finished If the depth of soil to be passed through before reaching rock is so great as to require other sections of cylinder to be bolted on above the top of the air-lock this may be done to any extent, inasmuch as it is immaterial whether the air-lock is under water or not. **To keep the cylinder both air- and water-tight** the faces of the flanges before being bolted together were smeared with a mixture of red and white lead and cotton fiber

South St. bridge, Philadelphia. D. M. Stauffer, Franklin Inst. Jour Nov 1872. Thirteen cast iron cylinders, in 10 ft lengths, 1¼ ins thick; 4, 6, and 8 ft diam; weighing, resp, 6800, 10800, 14600 lbs, diaframs 783, 1600, 2800 lbs resp. Inside flanges 2¾ ins wide, 1¼ ins thick, with bolt-holes 1¼ ins diam, 5 ins apart c. c. The bottom edge has no flange. The work went on, day and night, summer and winter: with no interruption from the tides, floods, or floating ice; and the thirteen cyls were sunk, filled with concrete, and **completed in 11 months**; much of which was consumed in leveling off the rock, and bolting the cyls to it by means of cast iron brackets. The want of guides caused much tilting, trouble and delay. Range of tide abt 7 ft. Water abt 25 ft deep. Depth of soil, gravel, etc, 6 to 30 ft. **Cost** of cylinders, in place, filled with concrete, $40, $64, $92 resp per ft of total length. Three gangs of men; each gang workt 4 hours at a time.

Chestnut St. bridge, Philadelphia. 1884–5. Four wrot-iron cyls, 8 ft diam, 66 ft long, **at 45° with the hor,** intended as struts to prevent the movement of one of the abut piers.

Cast iron cylinders have cracked thru, around their entire circumference, in many parts of the U. S. in very cold weather; owing to **diff of contraction** betw the iron and the concrete filling.

The shaded part of Fig 18 shows a transverse section of the **caisson of yellow-pine** timber and cement, for the **Brooklyn tower of East River** (N Y) suspension bridge, of 1600 ft clear span. It is 168 ft long at bottom, and 102 ft wide. A longitudinal section resembles the transverse one, except in being longer, and in showing more shafts J. Of these there are 6, arranged in pairs, for expedition and as a precaution against accident. Namely, two water-shafts J, each 7 ft by 6½ ft across, for removing by buckets and hoisting apparatus, the material excavated beneath the caisson; together with such water as may accumulate at *o o*; two air-shafts of 21 ins diam, through which air is forced from above, to expel the water from the chamber C S S D below the caisson, so as to allow the laborers to work there at undermining; the expelled water escaping under the foot C D of the caisson, into the river; and two supply shafts of 42 ins diam, for admitting laborers, tools, &c. The several shafts of course have air-chambers on top, on the same principle as Fig 17, to prevent the escape of the compressed air in *s s*.

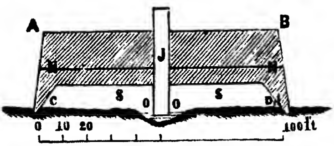

Fig 18

The shafts are of ¼ inch boiler iron. The foot C D, nine timbers high, is continuous, extending entirely around the caisson; its bottom is shod with cast iron; its four corners are strengthened by wooden knees 20 ft long.

From the bottom, up to the line N, N, 14 ft, the caisson is built of horizontal layers of timbers one foot square; the layers crossing each other at right angles; and the timbers of each layer touching each other well forced and bolted together; and all the joints filled with pitch. To aid in preventing leakage, the nuts and heads of the screws have India-rubber washers, also all outside seams, as well as all the seams of the layer of timbers N, N, are thoroughly calked; and a layer of tin, enclosed between two layers of felt, is placed outside of each outer joint; and over the entire top of the layer next below N, N.

When the caisson was built up to N, N, on land, it was launched, floated into position, and anchored; after which were added for sinking it, fifteen courses of timbers one ft square; and laid one ft apart in the clear; with the intervals filled with concrete. The top course A B is of solid timber, to serve as a floor for supporting machinery, &c. It was sunk some feet below the very bottom of the river, in order to avoid the teredo.

Cribs are sunk outside of the caisson, to form temporary wharves for boats carrying away excavated material; and for vessels bringing stone, &c.

When the caisson was sunk, and the water forced out from the chamber or space C S S D, workmen began to excavate uniformly the enclosed area of river bottom, so as to allow the caisson to descend slowly until it reached a firm substratum. The space C S S D, as well as the shafts, was then filled up solid with concrete masonry. A coffer-dam was built on top of the caisson, and in it the regular masonry of the tower was started. The total height of this tower including the caisson, is about 300 ft For full details see report, 1873, of W. A. Roebling the chief engineer

Hollow cylinders, or other forms of brickwork or masonry, with a strong curb or open ring of timber or iron beneath them, may be gradually sunk by undermining and excavating from the inside, and form very stable foundations. Under water this may be done by properly shaped scoops, with or without the aid of the diving-bell, according to the depth, &c. On land it will often be the most economical and satisfactory mode, especially in firm soils. The descent may be assisted by loading them, if, as sometimes happens, the friction of their sides against the earth outside prevents their sinking by their own weight. A brick cylinder, 46 ft outer diam, walls 3 ft thick, has been sunk 40 ft in dry sand and gravel, without any difficulty. It was built 18 ft high, (on a wooden curb 21 ins thick,) and weighed 300 tons before the sinking was begun. The interior earth was excavated slowly, so that the sinking was about 1 ft per day; the walls being built up as it sank. Tunnel shafts are at times so sunk.

On the Rhine for a coal shaft, a brick cylinder 25½ feet diam was first thus sunk by its own weight 76 ft through sand and gravel; then an interior one, 15 ft diam, was sunk in the same way to the depth of 256 ft below the surface, of which depth all the 180 ft below the first cylinder was a running quicksand. At 256 ft friction rendered the cylinder immovable. The quick sand was removed by boring; no pumping was done; but the water was permitted to keep the cyl full.

The entire foundation for a large pier of masonry has been sunk in this manner, in a single mass; a sufficient number of vertical openings being left in it for the workmen to descend, or for tools to be inserted for undermining. This is generally a very slow and tedious operation, especially under water. It may often be expedited by diving-bells or by diving-dresses. It will generally be better to make the mass wider at bottom than above it, so as to diminish friction against the outside earth. On land, water may at times be used for softening the bottom earth. By keeping the interior of such hollow masonry dry, it may even be *built downward* from the surface; by undermining only a portion of its circumference at a time, filling said portion with masonry, and then removing and filling the other portion; and so on in successive stages of 2 or 3 ft downward at a time. This mode may be adopted also when friction has stopped the sinking of a mass by its own weight when undermined.

The sand pump as used at the St Louis bridge will often be of service in raising sand from cylinders while being sunk in water. With a pump pipe of 3.5 ins bore, and a water jet under a pressure of 150 lbs per sq inch, 20 cub yds of sand per hour were raised 125 feet. A jet of air has also been successfully used in the same way, as at the East River, N Y, suspension bridge, &c.

Fascines. On marshy or wet quicksand bottoms, foundations may be laid by first depositing large areas of layers of fascines, or stout twigs and small branches, strongly tied together in bundles from 6 to 12 ft long, and from 6 ins to 2 ft in diam.

The layers or strata of bundles should cross each other. A kind of floating raft or large mattress is first made of these, and then sunk to the bottom by being loaded with earth, gravel, stones, &c In this manner the abutments and piers of the great suspension bridge at Kieff, in Russia, with spans of 440 ft, were founded in 1852, on a shifting quicksand. There the fascine mattresses extend 100 ft beyond the bases of the masonry which rests upon them

Fascines may be used in the same way for sustaining railway embankments, &c, over marshy ground, but they will settle considerably.

Sand-piles. We have already alluded to the use of sand well rammed in layers into trenches or foundation pits; but it may also be used in soft soils, in the shape of piles. A short stout wooden pile is first driven 5 to 10 feet or more, according to the case. It is then drawn out, and the hole is filled with wet sand well rammed. The pile is then again driven in another place, and the process repeated. The intervals may be from 1 to 3 ft in the clear. Platforms may be used on these piles as on wooden ones. If the sand is not put in wet, it will be in danger of afterward sinking from rain or spring water. In this case, as with fascines, it is well to test the foundation by means of trial loads. Some settlement must inevitably take place until all the parts come to a full bearing; but it will be *comparatively* trifling. The same occurs in every large work to some extent; as in a roof or arch of great span, whether of wood, iron, or masonry; so also with all tall piers, walls, &c, &c. Sandy foundations under water should be surrounded by stout well-driven sheet-piling, to prevent the enclosed sand from running out in case the outer sand is washed away; and should also be defended by a deposit of random-stone.

On bad bottoms under water, **small artificial islands** of good soil have been deposited; and the masonry founded upon them. Canal locks and other structures may at times be advantageously founded in this way in marshy soils. If necessary, a depth of several feet of the bad soil may be dredged out before the firmer soil is deposited; and the latter may be weighted by a trial load to test its stability.

The mode of laying a foundation under water, by building the masonry upon a timber platform above water, upheld by **strong screws**, and lowered into the water as the work is finished in the open air, a course or two at a time, has of late been much employed with entire success, in large bridge-piers in deep water. It however is not new. It was suggested more than 100 years ago by Belidor.

Piles are driven 6 to 10 ft apart around the space to be occupied by the pier; having their tops connected by heavy timber cap-pieces. These last uphold the screws, which work through them. The whole is braced against lateral motion.

A CLUMP OF PILES WELL DRIVEN; and then enclosed by an iron cylinder sunk to a firm bearing, and filled with concrete, is an excellent foundation. The piles may extend to the top of the cylinder, and thus be enclosed in the concrete. Such an arrangement has been patented by S. B. Cushing, C. E., Providence, R. I. The cylinder and concrete serve to protect the piles from sea-worms, and from decay above low water; and are not intended to support the load above them.

STONEWORK.

Where work is done on a large scale, blasting can sometimes be done at from 10 to 20 per cent less cost per cubic yard by means of **machine drills and dynamite,** than by **hand drills and gunpowder.** Ordinarily, however, **the cost is about the same,** and the advantage of the newer methods consists rather in economy of time, convenience, and having the work more entirely under control. In ordinary railroad work in average hard rock, and when common labor costs $1 per day of ten hours, the cost per cubic yard, for loosening, will ordinarily range between 30 and 60 cts, including tools, drilling, powder, &c.

Holes for blasting, drilled by hand, are generally from $2\frac{1}{2}$ to 4 ft deep; and from $1\frac{1}{4}$ to 2 ins diam. **Churn-drilling** is much more expeditious and economical than that by *jumping*, mentioned below. The churn-drill is merely a round iron bar, usually about $1\frac{1}{4}$ ins diam, and 6 to 8 ft long; with a steel cutting edge, or bit, (weighing about a lb, and a little wider than the diam of the bar,) welded to its lower end. A man lifts it a few inches; or rather catches it as it rebounds, turns it partially around; and lets it fall again. By this means he drills from 5 to 15 feet of hole, nearly 2 ins diam, in a day of 10 working hours, depending on the character of the rock. From 7 to 8 ft of holes $1\frac{1}{4}$ ins diam, is about a fair day's work in hard gneiss, granite, or compact silicous limestone; 5 to 7 ft in tough compact hornblende; 3 to 5 in solid quartz; 8 to 9 in ordinary marble or limestone; 9 to 10 in sandstone; which, however, may vary within all these limits. When the hole is more than about 4 ft deep, two men are put to the drill. Artesian, and oil wells, in rock, are bored on the principle of the churn-drill.

The **jumper,** as now used, is much shorter than the churn-drill. One man (the *holder*) sitting down, lifts it slightly, and turns it partly around, during the intervals between the blows from about 8 to 12 lb hammers, wielded by two other laborers, the *strikers*. It can be used for holes of smaller diameters than can be made by the churn-drill; because the holder can more readily keep the cutting end at the exact spot required to be drilled. It is also better in conglomerate rock; the hard siliceous pebbles of which deflect the churn-drill from its vertical direction, so that the hole becomes crooked, and the tool becomes bound in it. The coal conglomerates are by no means hard to drill with a jumper. The jumper was formerly used for large deep holes also, before the churn-drill became established.

Either tool requires resharpening at about each 6 to 18 inches depth of hole, and the wear of the steel edge requires a new one to be put on every 2 to 4 days. With iron jumpers, the top also becomes battered away rapidly. As the hole becomes deeper, longer drills are frequently used than at the beginning. The smaller the diameter of the hole, the greater depth can be drilled in a given time; and the depth will be greater in proportion than the decrease of diam. Under similar circumstances three laborers with a jumper will about average as much depth as one with a churn-drill.

The **hand-drill,** in which the same man uses both the hammer and the short drill, is chiefly used for shallow holes of small diam. With it a fair workman will drill about as many feet of hole from 6 to 12 ins deep, and about $\frac{3}{4}$ inch diam, as one with a churn-drill can do in holes about 3 ft deep and 2 ins diam, in the same time. Only the jumper or the hand-drill can be used for boring holes which are horizontal, or much inclined.

Cost of quarrying stone. After the preliminary expenses of purchasing the site of a good quarry; cleaning off the surface earth and disintegrated top rock; and providing the necessary tools, trucks, cranes, &c; the total neat expenses for *getting out* the rough stone for masonry, per cub yard, ready for delivery, may be roughly approximated thus: Stones of such sizes as two men can readily lift, measured *in piles*, will cost about as much as from ¼ to ½ the daily wages of a quarry laborer. Large stones, ranging from ½ to 1 cub yd each, got out by blasting, from 1 to 2 daily wages per cub yd. Large stones, ranging from 1 to 1½ cub yds each, in which most of the work must be done by wedges, in order that the individual stones shall come out in tolerably regular shape, and conform to stipulated dimensions; from 2 to 4 daily wages per cub yard. The smaller prices are low for sandstone, while the higher ones are high for granite. Under ordinary circumstances, about 1⅓ cub yds of good sandstone can be quarried at the same cost as 1 of granite; or, in other words, calling the cost of granite 1, that of sandstone will be ¾; so that the means of the foregoing limits may be regarded as rather full prices for sandstone; rather scant ones for granite; and about fair for limestone or marble.

Cost of dressing stone. In the first place, a liberal allowance should be made for **waste.** Even when the stone wedges out handsomely on all sides from the quarry, in large blocks of nearly the required shape and size, from ⅛ to ¼ of the rough block will generally not more than cover waste when well dressed. In moderate-sized blocks, (say averaging about ½ a cub yard each,) and got out by blasting, from ¼ to ⅓ will not be too much for stone of medium character as to straight splitting. About the last allowance should also be made for well-scabbled rubble. The smaller the stones, the greater must be the allowance for waste in dressing. In large operations, it becomes expedient to have the stones dressed, as far as possible, at the quarry; in order to diminish the cost of transportation, which, when the distance is great, constitutes an important item—especially when by land, and on common roads.

A stonecutter will first take out of wind; and then fairly patent-hammer dress, about 8 to 10 sq ft of plain face in hard granite, in a day of 8 working hours; or twice as much of such inferior dressing as is usually bestowed on the beds and joints, and generally on the faces also of bridge masonry, &c, when a very fine finish is not required. In good sandstone, or marble, he can do about ¼ more than in granite. Of *finest* hammer finish, *granite*, 4 to 5 sq ft.

Cost of masonry. Every item composing the total cost is liable to much variation; therefore, we can merely give an example to show *the general principle* upon which an approximate estimate may be made; assuming the **wages of a laborer** to be $2.00 per day of 8 working hours; and $3.50 for a mason. **The monopoly of quarries** affects prices very much.*

Cost of ashlar facing masonry. Average size of the stones, say 5 ft long, 2 ft wide, and 1.4 thick, or two such stones to a cub yd. Then, supposing the stone to be granite or gneiss, the cost per cub yd of masonry **at such wages** will be,

Getting out the stone from the quarry by blasting, allowing ¼ for waste in dressing; 1⅓ cub yds, at $3.00 per yard	$4.00
Dressing 14 sq ft of face at 35 cts	4.90
" 52 " beds and joints, at 18 cts	9.36
Neat cost of the dressed stone at the quarry	18.26
Hauling, say 1 mile; loading and unloading	1.20
Mortar, say	.40
Laying, including scaffold, hoisting machinery, superintendence, &c	2.00
Neat cost	21.86
Profit to contractor, say 15 per ct	3.28
Total cost	25.14

Dressing will cost more if the faces are to be rounded, or moulded. If the stones are smaller than we have assumed, there will be more sq ft per cub yd to be dressed, &c.

If in the foregoing case, the stones be *perfectly* well dressed on all sides, including the back, the cost per cub yd would be increased about $10; and if some of the sides be curved, as in arch stones, say $12 or $14; and if the blocks be carefully wedged out to given dimensions, $16 or $18; thus making the neat cost of the dressed stone *at the quarry* say $28, $31, or $35 per cub yd.

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* The blocks of granite for **Bunker Hill monument** averaging 2 cub yds each, were quarried by wedging, and delivered at the site of the monument, at a neat actual cost of $5.40 per cub yd; by the Monument Association; from a quarry opened by themselves for the purpose. The Association received no profit; their services being voluntary. The average contract offers for the same, were $24.30! The actual cost of getting out the rough blocks at the quarry was $2.70. Loading upon trucks at quarry, about 15 cts. Transportation 8 miles by railway and common road, $2.55. Total, $5.40. In 1825 to 1845; common unskilled labor averaging $1 per day.

The item of laying will be much increased if the stone has to be raised to great heights; or if it has to be much handled; as when carried in scows, to be deposited in water-piers, &c. Almost every large work presents certain modifying peculiarities, which must be left to the judgment of the engineer and contractor. The percentage of contractors' profit will usually be less on large works than on small ones.

Cost of ashlar facing masonry. If the stone be **sandstone** with good natural beds, the getting out may be put at $3.00 per cubic yard. Face dressing at 26 cts per sq ft · say $3.64 per cubic yd. Beds and joints 13 cts per sq ft, say $6.76 per cub yd. The neat cost, laid, $17.00.

And **the total cost of large well scabbled ranged sandstone masonry in mortar,** may be taken at about $10 per cub yd.

Cost of large scabbled granite rubble, such as is generally used as backing for the foregoing ashlar; stones averaging about ½ cub yd each ·

Labor at $1 per day.	Cost per cub yd of masonry.
Getting out the stone from the quarry by blasting, allowing ⅐ for waste in scabbling; $1\frac{1}{7}$ cub yds at $3.00	$3 43
Hauling 1 mile, loading and unloading	1 20
Mortar; (2 cub ft, or 1.6 struck bushels quicklime, either in lump or ground; and 10 cub ft, or 8 struck bushels of sand, or gravel; and mixing) ..	1 50
Scabbling; laying, including scaffold, hoisting machinery, &c.	2 50
Neat cost..........	8 63
Profit to contractor, say 15 per ct..........	1 30
Total cost	9 93

Common rubble of small stones, the average size being such as two men can handle, costs, to get it out of the quarry, about 80 cts per yard of pile, or to allow for waste, say $1.00. Hauling 1 mile, $1.00 It can be roughly scabbled, and laid, for $1.20 more; mortar as foregoing, $1.50. Total neat cost, $4.70, or, with 15 per ct profit, $5.40, *at the above wages for labor.*

With smaller stones, such as one man can handle, we may say, stone 70 cts, hauling $1; laying and scaffold tools &c, $1, mortar $1.50 Making the neat cost $4 20, or with 15 per ct profit, $4.83 Neat scabbled irregular range-work costs from $2 to $3 more per yd than rubble, according to the character of the stone &c. The laying of thin walls costs more than that of thick ones, such as abutments &c *

The cost of plain 8 inch thick ashlar facings for dwellings &c in Philada, in 1888, is about as follows per square foot showing, put up, including everything. Sand stone, $1.50 to $2 25. Pennsylvania marble, $2.50. New England marble, $2 75 to $3.25 Granite, $2.25 to $2.75. If 6 ins thick, deduct one-eighth part. **First class artificial stone** could be made and put up at one third the price. **North River blue stone flags,** 3 ins thick, for footwalks, put down, including gravel &c, 70 cts per sq foot. **Belgian street pavement,** with gravel, complete, $3.50 per sq yard in Eastern cities.

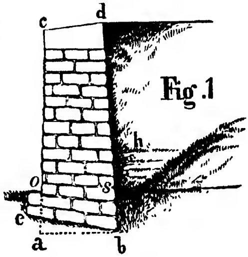
When dressed ashlar facing is backed by rubble, the expense per cub yard of the entire mass will of course vary according to the proportions of the two. Thus, if ashlar at $12 per yd, is backed by an equal thickness of rubble at $5, the mean cost will be ($12 + $5) ÷ 2 = $8.50; or if the rubble is twice as thick as the ashlar then ($12 + $5 + $5) ÷ 3 = $7.33, &c. **Such compound walls are weak** and apt to separate in time, as also walls of cut stone backed by concrete, or by brick; from unequal settlement of the two parts.

At times the contractor must be **allowed extra** in opening new quarries, in forming short roads to his work; in digging foundations; or for pumping or otherwise draining them, when springs are unexpectedly met with; for the centers for arches, &c, unless these items are expressly included in the contract per cub yd.

RETAINING-WALLS.

Art. 1. A retaining-wall is one for sustaining the pressure of earth, sand, or other *filling* or *backing*, deposited behind it after it is built; in distinction to a **face-wall,** which is a similar structure for preventing the fall of earth which is in its undisturbed natural position, but in which a vert or inclined face has been excavated. The earth is then in so consolidated a condition as to exert little or no lateral pres, and therefore the wall may generally be thinner than a retaining one.

This, however, will depend upon the nature and position of the strata in which the face is cut. If the strata are of rock, with interposed beds of clay, earth, or sand; and if they dip or incline toward the wall, it may require to be of far greater thickness than any ordinary retaining-wall; because when the thin seams of earth become softened by infiltrating rain, they act as lubrics, like soap, or tallow, to facilitate the sliding of the rock strata, and thus bring an enormous pres against the wall. Or the rock may be set in motion by the action of frost upon the clay seams; or, as sometimes occurs, by the tremor produced by passing trains. Even if there be no rock, still if the strata of *soil* dip toward the wall, there will always be danger of a similar result; and additional precautions must be adopted, especially when the strata reach to a much greater height than the wall. **A vertical wall** has both *c o* and *d s* vert.



Experience, rather than theory, must be our guide in the building of both kinds of wall. We recommend that the hor thickness *a b*, Fig 1, at the base of a vert or nearly vert retaining-wall *c d b a*, which sustains a backing of either sand, gravel, or earth, level with its top *c d*, as in the fig, should not be less than the following, in railroad practice, when the foundations are not more than about three feet deep.

When the backing is deposited loosely, as usual, *as when dumped from carts, cars, &c.*

Wall of cut-stone or of first-class large ranged rubble, in mortar.....a.b .... .35 *of its entire vert height* d b.
" *good common scabbled mortar-rubble, or brick.* .4 " " " "
" *well-scabbled dry rubble* .... .5 " " " "

With good masonry, however, we may take the height *d s* instead of *d b*, and then the above proportions of *d s* will give a sufficient thickness at the ground-line *o s*.

When the backing is somewhat consolidated in hor layers, each of these thicknesses *may* be reduced, but no rule can be given for this.

The **offset** *o e*, in front of the wall, is not included in these thicknesses.

When, however, the backing is a pure clean sand, or gravel, we should use only the full dimensions; inasmuch as the tremor, caused by passing trains, would neutralize any supposed advantage from ramming materials so devoid of cohesion. Such sand may be rammed with much advantage for the purpose of compacting it in foundations; but a diff principle is involved in that case. When it is done even with cohesive earths, with a view of saving masonry in retaining-walls, it is probable that the expense will generally be found quite equal to that of the masonry saved.

The base *a b* in Fig 1, is $\frac{4}{10}$ of the height *b d*. In the foregoing thicknesses at base, the back *d b* of the wall is supposed to be vert, and the face *c a* either vert, or battered (sloped or inclined backward) to an extent not exceeding about 1½ inches to a foot; which limit it is rarely advisable to exceed in practice, owing to the bad effect of rain, &c, upon the mortar when the batter is great. The base of a vert wall need not in fact be as thick as one with a battered face; but when the batter does not exceed 1.5 inches to a foot, the diff is very small. See Table, Art 7

Rem. 1. **A mixture of sand, or earth, with a large proportion of round bowlders** paving pebbles, &c, will weigh considerably more than the materials ordinarily used for backing, and will exert a greater pres against the wall; the thickness of which should be increased, say about one-eighth to one-sixth part, when such backing has to be used.

Rem. 2. The wall will be stronger if all the **courses of masonry be laid with an inclination inward,** as at *o e b*; especially if of dry masonry, or if time cannot be allowed (as it always should be, when practicable) for the mortar to set properly, before the backing is deposited behind it. The object of inclin-

ing the courses, is to place the joints more nearly at right angles to the direction fP, Figs 6, 7, and 8, of the pres against the back of the wall; and thus diminish the tendency of the stones to slide on one another, and cause the wall to bulge.

When the courses are hor, there is nothing to prevent this sliding, except the friction of the stones, one upon the other, when of dry masonry; or friction and the mortar, when the last is used. But if, as is frequently the case, (especially in thick and hastily built walls,) this has not had time to harden properly, it will oppose but little resistance to sliding. But when the courses are inclined, they cannot *slide*, without at the same time being *lifted up* the inclined planes formed by themselves. In retaining-walls, as in the abuts of important arches, the engineer should place as little dependence as possible upon mortar; but should rely more upon the position of the joints, for stability.

An objection to this inclining of the joints in *dry* (without mortar) walls, is that rain-water, falling on the battered face, is thereby carried inward to the earth backing, which thus becomes soft, and settles. This may be in a great measure obviated by laying the outer or face-courses hor, or by using mortar for a depth of only about a foot from the face. The top of the wall should be protected by a coping $c\,d$, Fig 1, which had better project a few ins in front. After the masonry has been built up to the surface of the ground, the foundation pit should be filled up, and it is well to consolidate the filling by ramming, especially in front of the wall.

The back $d\,b$ of the wall should be left rough. In brickwork it would be well to let every third or fourth course project an inch or two. This increases the friction of the earth against the back, and thus causes the resultant of the forces acting behind the wall to become more nearly vert; and to fall farther within the base, giving increased stability. It also conduces to strength not to make each course of uniform height throughout the thickness of the wall, but to have some of the stones (especially near the back) sufficiently high to reach up through two or three courses. By this means the whole masonry becomes more effectually interlocked or bonded together as one mass, and therefore less liable to bulge. Very thick walls may consist of a facing of masonry, and a backing of concrete.

Rem. 3. It is the pres itself of the earth against the back, that creates the friction, which in turn modifies the action of the pres, as the wt or pres of a body upon an inclined plane produces friction between the body and the plane, sufficient, perhaps, to prevent the body from sliding down it. A retaining-wall is *overthrown* by being made to revolve around its outer toe or edge e, Fig 1, as a fulcrum, or turning-point; but in order thus to revolve, its back must first plainly rise, and in doing so must rub against the backing, and thus encounter and overcome this friction. The friction exists the same, whether the wall stands firm or not, as in the case of the body on an inclined plane; the only diff is that in one case it *prevents* motion; and in the other only *retards* it.

C

O

Where deep freezing occurs the back of the wall should be sloped forwards for 3 or 4 ft below its top as at $c\,o$, which should be quite smooth so as to lessen the hold of the frost and prevent displacement.

a

Fig 2

Rem. 4. When the wall is too thin, it will generally fail by **bulging** outward, at about $\frac{1}{3}$ of its height above the ground, as at a, in Fig 2. A slight bulging in a new wall does not *necessarily* prove it to be actually unsafe. It is generally due to the newness of the mortar, and to the greater pres exerted by the fresh backing; and will often cease to increase after a few months. It need not excite apprehension if it does not exceed $\frac{1}{4}$ inch for each foot in thickness at a. See Remark 3, Art 7.

Art. 2. The young engineer need not in practice concern himself particularly about the PRECISE SP GRAV OF HIS BACKING, or about the ANGLE OF SLOPE at which it will stand; for the material which he deposits behind his wall one day, may be dry and incoherent, so as to slope at $1\frac{1}{4}$ to 1, the next lay rain may convert it into liquid mud, seeking its own level, like water; the next it may be ice, capable of sustaining a considerable load, as a vert pillar.

Moreover, he cannot foretell what may be the nature of his backing; for, as a general rule, this must consist of whatever the adjacent excavation may produce from time to time; sand to-day, rock to-morrow, &c. Retaining-walls are therefore usually built before the engineer knows the character of their backing; so that in practice, these theoretical considerations have comparatively but little weight. Theory, uncontrolled by observation and common sense, will lead to great errors in every department of engineering; but, on the other hand, no amount of experience alone will compensate for an ignorance of theory. The two must go hand-in-hand.

Again, **the settlement of the backing under its own wt,** aided by the tremors produced by heavy trains at high speed; its expansion by frost, or by the infiltration of rain; the hydrostatic pressure arising from the admission of the latter through cracks produced in the backing during long droughts; as well as its lubricating action upon it, (diminishing its friction, and giving it a tendency to slide,) &c, exert at times quite as powerful an overturning tendency as the legitimate theoretical pres does. The action of these agencies is gradual. Careful observation of retaining-walls year after year, will often show that their battered faces are becoming vertical. Then they will begin to incline outward; and eventually the wall will fail. Theory omits loads that may come on backing increasing its pres.

Assuming the theoretical views advanced by Professor Moseley to be correct as theories, the thicknesses which we have recommended in Art 1, for mortar walls, correspond to from 7 to 14 times; and for dry walls about 10 to 20 times, the pres assigned by him; and we do not consider ours greater than experience has shown to be necessary. See Table 3. Retaining-walls designed by good engineers, but in too close accordance with theory, (which assumes that a resistance equal to twice the theoretical pres is sufficient,) have failed; and the inference is fair that many of those which stand have too small a coefficient of safety.

The fact is, (or at least so it appears to us,) there must be defects in the theoretical assumptions of some of the most prominent writers who give practical rules on this subject. Thus Poncelet, who certainly is at their head, states that his tables, for practical use, give thicknesses of base for sustaining $1\frac{8}{10}$ times the theoretical pres; and this he considers amply safe. Yet, for a vert wall of cut granite, his base for sustaining dry sand level with the top, as in Fig 1, is .35 of the vert height; and for brick, .45. But the writer found that *when not subject to tremor*, a wooden model of a vert wall, weighing but 28 lbs per cub ft, and with a base of .35 of its height, balanced perfectly dry sand sloping at 1½ to 1, and weighing 89 lbs per cub ft.

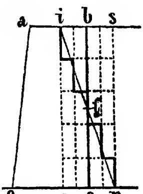


Fig 3

NOW, THE RESISTANCE OF SIMILAR WALLS, OF THE SAME DIMENSIONS, VARIES AS THEIR SPECIFIC GRAVITIES; and, since granite weighs about 165 lbs per cub foot, or 6 times as much as our model, it follows, we conceive, that a wall of that material, with a base of .35 of its height, must have a resistance of 6 times any *true theoretical* pres, instead of only $1\frac{8}{10}$ times; and that his brick wall must have about 5 times the mere balancing resistance. Our experiments were made in an upper room of a strongly built dwelling; and we found that the tremor produced by passing vehicles in the street, by the shutting of doors, and walking about the room, sufficed to gradually produce leaning in walls of considerably more than twice the mere balancing stability while quiet; and it appears to us that the injurious effects of a heavy train would be comparatively quite as great upon an actual retaining wall, supporting so incohesive a material as dry sand.

Since, therefore, Poncelet's wall is in this instance sufficiently stable for *practice*, it seems to us that his *theory*, which neglects the effect of tremors, &c, must be defective. He also gives $\frac{1}{5}$ of the height as a sufficiently safe thickness for a vert granite wall supporting *stiff earth;* but we suspect that very few engineers would be willing to trust to that proportion, when, as usual, the earth is dumped in from carts, or cars; especially during a rainy period. If deposited, and consolidated in layers, theory could scarcely assign any thickness for the wall; for the backing thus becomes, as it were, a mass of unburnt brick, exerting no hor thrust; and requiring nothing but protection from atmospheric influence, to insure its stability without any *retaining*-wall. It is with great diffidence, and distrust in our opinions, that we venture to express doubts respecting the assumptions of so profound an investigator and writer as Poncelet; and we do so only with the hope that the views of more competent persons than ourselves, may be thereby elicited. Our own have no better foundation than experiments with wooden and brick models, by ourselves; combined with observation of actual walls.

Art. 3. After a wall *a b c o*, Fig 3, with a vert back, has been proportioned by our rule in Art 1, it may be **converted into one with an offsetted back,** as *a i n o*. This will present greater resistance to overturning; and yet contain no more material. Thus, through the center *t* of the back, draw any line *i n*; from *n* draw *n s*, vert; divide *i s* into any *even* number of equal parts; (in the fig there are 4;) and divide *s n*, into *one more* equal parts; (in the fig there are 5.) From the points of division draw hor, and vert lines, for forming the offsets, as in the fig.

In the offsetted wall, the cen of grav is thrown farther back from the toe *o*, than in the other, thus giving it increased leverage and resistance; but within ordinary practical limits, the diff is very small; and since the triangle of supported earth is greater than when the back is vert, its pres is also greater, so that probably no appreciable advantage attends *that* consideration. **The increase of thickness near the base, diminishes, however, the leverage *v a*, Fig 8, of the pres *f* P, of the earth** against the back. The *center of pressure* of this pres is in both cases at ⅓ the vert height, measured from the bottom; and it is therefore plain that the farther back from the front it is applied, the shorter must *v a* become. Moreover, in the offsetted back, the *direction* of the pres becomes more nearly vert than when the back is upright. It is to these causes, rather than to the throwing back of the cent of grav, that the offsetted wall owes its increase of stability over one with a vert back.

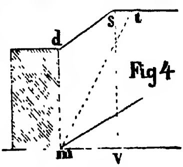


Fig 4

Art. 4. When, as in Fig 4, the backing is higher than the wall, and slopes away from its *inner* edge *d*, at the natural slope *d s*, of 1½ to 1, we are confident that the following thicknesses at base will at least be found *sufficien*

for vert walls with sand. They are deduced from the experiments just alluded to, and are but rude approximations, with no scientific basis We should not have inserted them, but for the fact that we know of no others for this case

The first column contains the vert height *s e*, of the earth as compared with the vert height of the wall: which latter is assumed to be 1, so that the table begins with backing of the same height as the wall, as in Fig 1 These vert walls may be changed to others, with battered faces, by Art 8; or without any such proceeding, their faces may be battered to any extent not exceeding 1½ inches to a foot, or 1 in 8, without sensibly affecting their stability, without increasing the base

TABLE 1. (Original)

Total height of the earth, compared with the height of the wall above ground.	Wall of Cut Stone, in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.	Total height of the earth, compared with the height of the wall above ground	Wall of Cut Stone in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.
	Thickness at Base, in parts of the height.				Thickness at Base, in parts of the height.		
1.	.35	.40	.50	2	.58	63	.73
1.1	.42	.47	57	2 5	.60	65	75
1.2	.46	.51	61	3.	.62	.67	.77
1.3	.49	.54	.64	4.	.63	.68	78
1.4	.51	.56	.66	6	64	69	79
1.5	.52	.57	67	9	.65	70	.80
1.6	.54	.59	.69	14	66	.71	81
1.7	.55	.60	70	25.			
1.8	.56	.61	.71	or more	.68	.73	83

Art. 5. But when the slope *n r*, Fig 5, of 1½ to 1, starts from the *outer* edge *n* of the wall, greater thickness is required. Poncelet gives the following for this case, for *dry sand.*

TABLE 2.

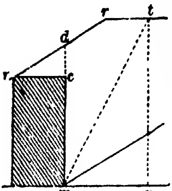


Fig. 5.

Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.	Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.
1	.35	.452	2.4	.762	1 02
1.1	.393	.498	3 0	.811	1.11
1.2	.439	.548	4 0	.852	1.18
1 3	.485	.604	6.0	.883	1 25
1 4	.532	.665	11.0	.909	1 28
1 5	.579	.726	21.0	.922	1.31
1.6	.617	.778	31 0	.926	1 32
1 7	.645	.824	Infinite.	.934	1.34
1.8	.668	847			
1.9	690	.903			
2.0	.707	.930			

When the earth reaches above the top of the wall, as in Figs 4 and 5 the wall is **surcharged:** and the earth that is *above* the top, is called the SURCHARGE When the surcharge is carefully deposited above the wall, so as to slope back at a steeper angle than 1½ to 1, as say at 1 to 1, theory does not require the wall to be as thick. Notwithstanding Poncelet's high position, the writer cannot imagine that the base of a brick wall need be so great as 1½ times its height for any height of sand whatever.

Art. 6. On the theory of retaining-walls. Let *b c a m*, Fig 6, be such a wall, upholding backing or filling *c s m g*; the upper surf *c s* of which is hor, and level with the top *b c* of the wall; and let *m s* represent the nat slope of the earth which composes the backing; *m g* being hor.

Abundant experience on public works shows that this slope, whether for sand, gravel, or earth when dry, may be practically taken at 1½ to 1; that is, 1½ hor, to 1 of vert measurement; which corresponds to an angle *s m g* of 33° 41′ with the hor; which is also about the angle at which bricks and roughly dressed masonry begin to slide on each other. This angle, however, varies considera

bly, being greatly influenced by the degree of dryness, or dampness, of the material; so that moderately damp sand or earth will stand at a slope of 1 to 1, or at an angle of 45°. Whatever it may be, it is called THE ANGLE OF NAT SLOPE of the material under consideration. In theoretical calculations for walls, it is safest to assume (as we have done throughout) that the backing is perfectly dry, since

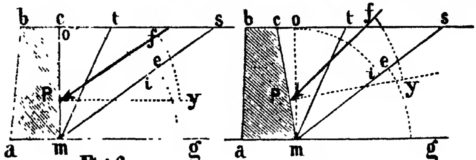


Fig. 6 Fig. 7

its pres is then greatest; unless it be supposed to be so wet as to possess some degree of fluidity. The triangle *c m s* of earth above the nat slope *m s*, tends to slide down said slope, but is prevented from so doing by the wall.

It is assumed in all cases, that the wall is secured from *sliding* along its base, Art 9, that it is thick enough to prevent failure by *bulging*, and that it will fail only by *overturning*, by rotating around its toe, *a*, as a fulcrum. The thickness necessary to insure safety against the last will also be sufficient to prevent bulging. Now referring only to Fig 6 with a vert back, if the angle *o m s*, contained between the natural slope *m s*, and a vert line *m o*, drawn from the inner bottom edge *m* of the wall, be divided by a line *m t*, into two equal angles, *o m t*, *t m s*, then the angle *o m t* is called THE ANGLE, and *m t* THE SLOPE, OF MAXIMUM PRESSURE. The triangular prism of earth, of which *o m t* is a section, or an end view, is called THE PRISM OF MAX PRES; because, if considered as a wedge acting against the back of the wall, it would produce a greater pres upon it than would the entire triangle *c m s* of earth, considered as a single wedge. For although the last is the heaviest, yet it is more supported by the earth below it. Calculation shows that if we consider the earth *o m s* to be thus div into wedges by *any* line *m t*, the wedge that will press most against the wall is that formed when *m t* divides the angle *o m s*, or the arc *o i*, into two equal parts. But see Art 11.

Since *m g* is hor, and *m o* vert, the two form an angle of 90°; consequently the angle of max pres is plainly found by taking the angle *s m g* of nat slope from 90°, and div the rem by 2. Thus a nat slope of 1½ to 1, or 33° 41′, taken from 90°, leaves 56° 18′, and $\frac{56° 18'}{2}$ = **28° 9′, the corresponding angle o m t of max pres.**

For ease of calculation, only one foot of the *length* of the wall, and of its backing, is usually considered. The number of *cub ft* of wall, or of backing, is then equal to that of the *square* feet in their respective profiles, or cross-sections.

Now, according to Moseley, if we assume the particles of earth composing the backing to be perfectly dry, and devoid of cohesion, (or tendency to stick to each other,) which is very nearly the case in pure sand; and if we suppose the wall to be suddenly removed, then the triangle of earth *c m t*, comprised between the slope *m t* of max pres, and the vert back *c m* of the wall, Fig 6, would slide down, under the influence of a force which may be represented by *y* P, acting in a *direction y* P, at right angles to the face *c m* of the triangle of earth; (or in other words, at right angles to the back of the vert wall,) its *center of force* being at P, distant ⅓ way between *m* and *c*, measured from the bottom; and its *amount* equal to either of the following:

No 1. $\frac{\textit{Perp pres}}{\text{y P}} = \frac{\textit{Wt of the triangle of earth}\ \text{c m t} \times \text{o t}}{\textit{vert depth}\ \text{o m}}$; or

No 2. $\frac{\textit{Perp pres}}{\text{y P}} = \frac{\textit{Wt of a single cub ft of the backing} \times \textit{sq of}\ \text{o t}}{2.}$

See Art. 11.

In view of the great uncertainty involved in the matter of the actual pressure of earth against retaining-walls in practice (see Art 2,), and in order to furnish a simple rule which, although entirely unsupported by theory, is still (in the writer's opinion) sufficiently approximate for ordinary practical purposes, we shall assume that No 1 of the two foregoing formulas applies near enough to walls **with inclined backs *c m*,** also, as Figs 7 and 8, (precisely as they are lettered,) at least **until the back of the wall inclines forward as much as 6 ins hor, to 1 foot vert,** or at an angle *c m o* of 26° 34′. **What follows on retaining-walls will involve this incorrect assumption, and must be regarded merely as giving safe approximation.**

Some appear to assume this perp pres to be the only one acting against the back of the wall; and hence arrive at erroneous practical conclusions. For when, in order to prevent this force from causing the triangle of earth to slide, we place a retaining-wall in front of it, then, instead of *motion*, the force will produce *pres* of the earth against the wall, causing *friction* between the pressed surfaces of the

earth and wall. That is, if a wall were to begin to overturn around its toe *a* as a fulcrum, its back *c m* must of course rise, and in so doing must rub against the earth filling in contact with it; and this rubbing would evidently act to *impede* the overturning. So long as the wall does not move, the same friction assists in *preventing* overturning. To ascertain the amount and effect of this friction, let *y* P, Fig 8, represent *by scale* the force perp to the back *c m*; and supposed to have been previously calculated by the foregoing formula No 1 Make the angle *y* P *f* equal to the angle of wall friction,* draw *y f* at right angles to *y* P, or parallel to *m c*; make P *x* equal to *y f*, and complete the parallelogram P *y f x*. Then will *x* P represent by the same scale, **the amount of the friction against the back of the wall.**

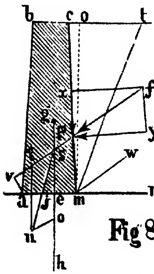


Fig 8.

Hence we have acting at P, two forces; namely, the perp force *y* P, and the friction *x* P; consequently, by comp and res of force, the diag *f* P of the parallelogram P *y f x*, if measured by the same scale, will give us the amount of their resultant; **which is the approx single theoretical force, both in amount and in direction, which the wall has to resist,** including the wall friction.

But this force, *f* P, is also always equal to the perp force *y* P, mult by the nat sec of the angle *y* P *f* of the wall friction; (or divided by its nat cosine) and of course may be ascertained thus:

$$\text{Approx theoretical pres f P} = \frac{\text{wt of triangle c m t} \times \text{o t} \times \text{nat sec of angle y P f of wall friction}}{\text{vert depth o m}} = \frac{\text{wt of c m t} \times \text{o t}}{\text{cos y P f} \times \text{o m}}$$

Or finally, if it is assumed, as we do throughout, that the earth is perfectly dry (inasmuch as its pressure is then the greatest) and that the angles of nat slope, and of wall friction are then each 33° 41′ or 1.5 to 1, then in Figs 6, 7 and 8, if the angle *c m o* between the back *c m* and the vert *o m* does not exceed about 26° 34′ we may assume

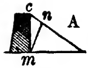
$$\textbf{Approx theoretical pres f P} = \textbf{wt of triangle c m t} \times .643$$

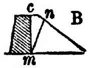
which includes the action of the friction of the earth against the back of the wall.

Rem. 1. **When the back of the wall is offsetted or stepped,** as in Fig 3, instead of being simply battered, as in Figs 7 and 8, the direction of the pres of the earth will be the same as if the back had the batter *i n*.

Rem. 2. **Now to find both the overturning tendency of the earth,** and the resistance of the wall against being overturned around its toe *a* as a fulcrum, first find the cen of grav *g* of the wall, and thru it draw a vert line *gh*. Prolong *f*P toward *v* and draw *av* perp to it By any scale make *so* = wt of wall, and *st* = calculated pres *f*P. Complete the parallelogram *s t n o*, and draw its diagonal *sn*, which will be the resultant of the pres *f*P and of the wt of the wall, and should for safety be such that *aj* be not *less* than about one-fifth of *am*, *even with best masonry and unyielding soil*. Otherwise the great pressure so near the toe *a* may either fracture the wall or compress the soil near that point so that the wall will lean forward. In walls built by our rule, Art 1, or by table, p 610, *aj* will be more than one-fifth of *am*. The pres *f*P if multiplied by its leverage *av* will give the moment of the pres about *a*; and the wt of the wall multiplied by its leverage *ea* will give that of the wall. The wall is safe from overturning in proportion as its moment exceeds that of the pres. It is assumed to be safe against *sliding*, *breaking*, or *settling* into the soil.

* **This angle of wall friction** is that at which a plane of masonry must be inclined to the horizontal so that dry sand or earth would slide down it. It is about the same as the nat slope, or 33° 41′, or 1.5 to 1; and its nat secant is 1.202, and its nat cos .832.





Rem. 4. If the earth slopes downward from C, as at A or B, instead of being hor as in Figs 6, 7, 8, use the wt of the earth *c m n* instead of *c m t*, *m n* being the slope of max pressure. In A the point of application will still be at P (at one-third of *m c*) as in 6, 7, 8; but in B it will be a little higher as explained below for Fig 9.

Surcharged walls are those in which the earth backing extends above the tops of the walls.

According to theory, when as in Fig 9, there is a surcharge *v c k* of backing, sloping away from *c* at its natural slope *c v*, the max pres against the wall is attained when the earth reaches to the level of *d*, where the slope *m t d* of max pres intersects the face of the nat slope *c v*; so that if afterward the earth is raised to *v*, or to any greater height, no additional pres is thereby thrown against the back of the wall. So also if the earth slopes from *b*, or from between *c* and *b*, except that then the slope *m d* of max pres must extend up to meet this other slope.

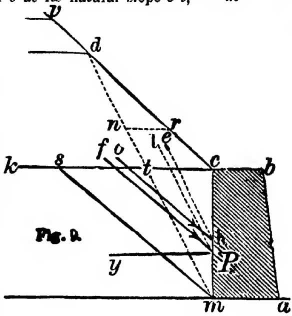


Fig. 9.

The approximate amount of the oblique pres, when the wall is surcharged, (as in any of the Figs 4, 5, 9,) may be found on the same principle as when the earth is level with its top; namely, instead of the *triangle c m t* of earth, Figs 6, 7, 8, 9, find the wt of *all* the earth *d s m t*, Fig 4, *d m t r*, Fig 5, or *c d m*, Fig 9 (if the surcharge reaches to *d* or *v*, or higher), between the slope *m d*, Fig 9, *m t*, Figs 4 and 5, of max pres, the back of the wall, and the front slope; omitting any which, like *d c n*, Fig 5, rests on the top of the wall (and thus adds to its stability) when the slope starts in front of *c*. Having found this weight, then for dry backing the

$$\left.\begin{array}{r}\textbf{Pressure}\\ \textbf{approximately}\end{array}\right\} = \textbf{Wt of the earth} \times .643,$$

including the action of the friction of the earth against the back of the wall; near enough (in the writer's opinion) for practical purposes in so uncertain a matter; but essentially empirical.

The direction of the pressure thus found will be the same as when the earth is level with the top *b c*; namely, as in Figs 6 and 7, first draw a line, as P *y*, perp to the back *c m*, whether vert or inclined. Then draw another line, as P *f*, making the angle *y* P *f* = the angle of wall friction, which we all along assume to be 33° 41′, or 1.5 to 1. Then P *f* will give the direction of the pressure. But its point of application will not always be at P (one-third of the height of the wall above *m*) as heretofore; for in all cases it will be at that point P, or at some **higher** one as *h*, where the back is cut by a line *l* P or *e h*, Fig 9, drawn from the cen of grav of the sustained earth (omitting any that rests immediately on the top *b c*), and parallel to the slope *m d* of max pres; and such a line will strike at one-third the height of the wall only when the sustained earth *t c m* or *d c m* **forms a complete triangle**, one of whose angles is at the inner top edge *c* of the wall. In all other cases said line for a surcharge will strike above P.

Art. 7. On page 603, Fig 1, we recommend that the base *o s* at the ground-line of well built **vertical** walls should not be less than .35, or .4, or .5 of the height *d s* above said line, depending on the kind of masonry. But a wall with a **battered** (inclined) front or face as found by Art 8, (by which the following table was prepared), will be as strong, and at the same time contain less masonry than a vert wall, although the battered one will have the thickest base *o s*.

Table 3, of thicknesses at base *o s*, Fig 1, and at top *c d*, of walls with battered faces, so as to be as strong as vertical ones which contain more masonry.

For the cub yds of masonry above *o s* per foot run of wall, mult the **square** of the vert height *d s* by the number in the column of cub yds. Then add the foundation masonry below *o s*.

(Original.)

All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .35 of its ht *d s*.				All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .4 of its ht *d s*.			All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .5 of its ht *d s*.		
	Cut stone.			Mortar rubble.			Dry rubble.		
Batter, in ins to a ft.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run	Base, in pts of ht	Top, in pts of ht	C yds per ft run.	Base, in pts of ht.	Top, in pts of ht	C yds per ft run.
0	.350	.350	.01296	.400	.400	.01482	.500	.500	.01852
½	.352	.310	.01226	.401	.359	01407	.501	459	.01778
1	.355	.270	.01158	.403	.320	.01339	.503	420	.01709
1½	.359	.234	.01098	.408	.283	01280	.506	.381	.01643
2	.364	.197	.01039	.413	.246	01220	.510	.343	.01580
2½	.371	.163	.00989	.419	.210	01165	.516	.308	.01526
3	.379	.129	.00941	.425	.175	.01111	522	.272	01470
3½	389	.096	.00898	.435	.143	.01070	.528	.236	.01415
4	.400	.066	.00863	.446	.110	01028	.537	.204	.01372
5	.425	.007	.00800	.468	.051	.00961	.555	.138	01283
Triangle	.429	.000	00794	.490	000	00907	.612	.000	.01133

Moseley and others quote Gadroy, for a DRY SAND SLOPING AT 21°. It would be better to cease from circulating such evident mistakes. Dry sand will stand at no less angle for a *savant* than for anybody else. For practical purposes, we may say that dry sand, gravel and earths, slope at 33° 41′ or 1½ to 1; as abundant experience on railroad embkts proves. Poncelet gives tables for walls to support dry earth sloping at 1 to 1, or 45°, but as we do not believe in the existence of such earth, we omit such tables. Sand gravel and earths may be moistened to diff degrees, so as to stand at any angle between hor and vert, and by moistening and ramming the earths may be converted into compact masses, exerting little or no pres, and may even so continue after they become dry; being then, in fact, a kind of air-dried brick. It is sometimes difficult to know whether earth or sand is perfectly dry or not; and an *exceedingly* small degree of moisture will cause them to stand at 1 to 1, in *small heaps*, such as have probably been observed by the authorities on the subject. The writer found that fine sand from the sea-shore, and under cover, would stand at 1½ to 1 during warm dry weather, and at 1 to 1 when the air was damp. Yet no diff whatever in its degree of moisture was perceptible to the feeling. Its susceptibility to dampness was of course owing to salt. A few handfuls of dry earth may perhaps be coquetted into standing at 1 to 1 on a table, but so far as our observation extends, when it is dumped in large quantities from carts and wheelbarrows, its slope is about 1½ to 1; and this we consider the proper one to be used in practical calculations, where safety is the consideration of paramount importance.

The less the nat slope, the greater is the pres: and since the slope is least when the backing is perfectly dry, (omitting of course its condition when so absolutely *wet* as to become partially fluid,) we have, on the score of safety, confined our tables to dry backing. As stated in Art 1, we cannot recommend dimensions less than those there given, when we consider the rough treatment to which masonry is exposed on public works.

In carrying a road along dangerous precipices, we should rather be tempted at times to make thicker walls. We imagine, for instance, that the centrifugal force of a heavy train, whirling around a sharp curve, convex on the dangerous side, should not be overlooked in designing walls for such localities. This force is hor; and is applied near the top of the wall: and, consequently, its leverage may be considered as equal to the height; whereas the theoretical pres of the earth is oblique; and is applied at ⅓ of the height from the bottom; so that its leverage about the toe of the wall is very short. Moreover, the simple *weight* of the train, produces pres against the wall; as well as that of the backing. All such considerations are omitted by theorists. The dangerous pres caused by tremors, &c, cannot be

assumed to be applied at $\frac{1}{3}$ of the height from the bottom; nor indeed, can it be calculated at all.

REM. 2. **Wharf walls** are an instance where the thickness should be increased, notwithstanding that the pres of the water in front helps to sustain them. The earth behind such walls, is not only liable to be very heavily loaded when vessels are discharging; but is apt to become saturated with water, especially below low-water level; and thus to exert a very great pres against the walls. Moreover, the water gets under the wall; and by its upward pressure virtually reduces its weight, and consequently its stability. The same cause of course diminishes the friction of the wall upon its base. Such walls are, therefore, very liable to slide, if the foundation is smooth, and horizontal; and have done so even when the foundation had a considerable inclination backward, as in Fig 1. See Art 9.

REM. 3. A retaining-wall is usually in greater danger for a few months after its completion, than after time has been allowed for the mortar to harden perfectly; and for the backing to settle. When there are suspicions of the safety of a new wall, it would be well to place strong temporary shores against it, at about $\frac{1}{4}$ to $\frac{1}{2}$ of its height above ground. In some cases, permanent buttresses of masonry may be built for the purpose. They should be well bonded into the wall.

REM. 4. The pres of the earth backing will be much reduced, if the first few feet of its height be made up in thin hor layers, to be consolidated by being used by the masons instead of scaffolding; as shown at *h*, Fig 1. Frequently this can be done without inconvenience; and at very trifling cost.

Art. 8. To change a vert retaining-wall, into one with a battered face, which shall present an equal resistance against overturning; although requiring less masonry. This is sometimes termed a **transformation of profile.** (Original.)

Let *a b o i*, Fig 10, be the vert wall. Mult its base *o i*, by 1.225; (1.22475 is nearer;) the prod will be the base *o e*, of a triangular wall *b o e*, possessing the same stability; and yet not requiring much more than half the masonry of the vert one. See Rem 1. This being done, suppose a wall to be desired with a face batter, of say 3 ins to a ft; or 1 in 4. From the point *n*, where the face of the triangular wall intersects that of the vert one, step off vert any 4 short equal spaces; and from the upper one *m*, step off one space hor, to *v*. Through *v* and *n* draw the dotted line *s t*, which evidently will batter 1 in 4. Then is *b s t o* approximately the reqd wall, but a little thicker than necessary. To reduce it, from *t* draw the dotted line *t b*. Mark the point *c*, where this line intersects the face *a i*, of the vert wall; and through *c* draw *d l*, parallel to *s t*. Then is *b d l o* the reqd wall. Our fig is drawn in an exaggerated manner, so as to avoid confusion in the lines. The base *o e* of the triangular wall, would not in reality be near so great as it is represented.

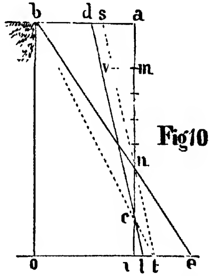
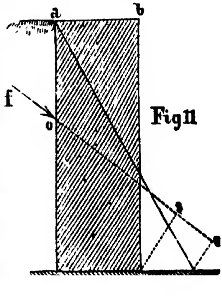


Fig 10

It will be observed that as the base increases, the quantity of masonry diminishes.

REM. 1. **The battered wall will in fact be safer than the vert one.** The battered wall has the same moment of stability as the vert one; and the pres of the earth against it also remains unchanged, but the *moment* or *tendency* of the pres to upset the wall has become less. For let *a b m n*, Fig 11, represent a vertical wall; and *f o* the *amount* and direction of pres behind it. (For ease of illustration, we have placed *o above* the true cen of pres of the earth filling, which would be at one-third of *a n* above *n*.) Now, the leverage with which this pres tends to overturn the wall around its toe *m*, is the dist *m s*, measured from the toe or fulcrum *m*, and at right angles to the direction *f o s c* of the pres, and this leverage mult by the force *f o*, gives the overturning tendency or moment of said force. See "Moments and leverage." Again, let *a n y*, represent a triangular wall of the same stability as the other, as found by our rule. Here we still have the same amount *f o*, and direction *f o s c*, of pres force against the wall; but it now acts to overturn the wall *a n y* around the toe *y*; and therefore, with the reduced leverage *y c*. Consequently, its overturning tendency is less than before. Therefore, in ordinary language, we may say that the wall is stronger than before, although its moment of stability, or standing tendency, has in itself undergone no change. If the pres *f o* against the vert back were hor, as in the case of water, then its leverage would evidently be the same in both walls; and the proportion between the overturning moment of the pres, and the moments of stability of the two walls, would be constant.



REM 2. In attempting to reduce the masonry by adopting a wall, *o b e*, Fig 10, of a triangular section; or of one nearly approaching a triangle, special attention should be given to the quality of the masonry near the thin toe *e*; which will otherwise be apt to crack, or fail under the pres.

REM 3. MOREOVER, WHEN COMMON MORTAR IS USED WITHOUT AN ADMIXTURE OF CEMENT, which it neve should be, in retaining-walls, where durability is an object, a great batter is obje tionable; inasmuch as the rain, combined with frost, &c, soon destroys the mor tar. In such cases, therefore, the batter should not exceed 1 or 1½ ins to a ft; an even then, at least the pointing of the joints, and a few feet in height of bot the upper and the lower courses of masonry, should be done with cement, o cement-mortar. We have observed a most marked diff in the corrosion of the mor tar, where, in the same walls, with the same exposure, one portion has been buil with a vert face; and another with a batter of but 1½ inch to a foot. Commo mortar will never set properly, and continue firm, when it is exposed to mois ture from the earth. This is very observable near the tops and bottoms o abuts, retaining-walls, &c; the lime-mortar at those parts will generally b found to be rendered entirely worthless. A profile somewhat like Fig 12, ma at times prove serviceable, instead of the triangular. This is the form of th Gothic buttress; which probably had its origin in the cause just spoken of.

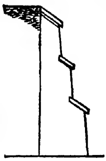


Fig 12

Art. 9. A retaining-wall may slide, withou losing its verticality; and, indeed, without any dange of being overturned. This is very apt to occur if it is built upo a hor wooden platform; or upon a level surf of rock, or clay without other means than mere friction to prevent sliding. This may be obviate by inclining the base, as in Fig 1; by founding the wall at such a depth as to pro vide a proper resistance from the soil in front; or in case of a platform, by securing one or more lines of strong beams to its upper surf, across the direction in whic sliding would take place. On **wet clay,** friction may be as low as from .2 t ⅓ the weight of the wall; on dry earth, it is about ½ to ⅝; and on sand or gravel about ⅝ to ¾. The friction of masonry on a woode platform, is about $\frac{6}{10}$ of the wt, if dry; and ¾ if wet

Fig 13

Counterforts, shown in plan at *c c c*, Fig 13, consist i an increase of the thickness of the wall, *at its back*, at regular inte vals of its length. We conceive them to be but little better than waste of masonry. When a wall of this kind fails, it almost in variably separates from its counterforts; to which it is connecte merely by the adhesion of the mortar; and to a slight extent, by th bonding of the masonry. The table in Art 7 shows that a very small addition to the base of a wall, i attended by a great increase of its strength; we therefore think that the masonry of counterfort would be much better, and more cheaply employed in giving the wall an additional thickness, alon its entire length; and for the lower third of its height. Counterforts are very generally used i retaining-walls by European engineers; but rarely, if ever, by Americans.

Buttresses are like counterforts, except that they are placed *in front* of a wall instead of be hind it; and that their profile is generally triangular, or nearly so. They greatly increase its strength but, being unsightly, are seldom used, except as a remedy when a wall is seen to be failing.

Land-ties, or long rods of iron, have been employed as a makeshift for upholding weak re taining-walls. Extending through the wall from its face, the land ends are connected with anchor of masonry, cast-iron or wooden posts; the whole being at some dist below the surface.

Retaining walls with curved profiles are mentioned here merely to cau tion the young engineer against building them. Although sanctioned by the practice of some hig authorities, they really possess no merit sufficient to compensate for the additional expense and trou ble of their construction.

Art. 10. Among military men, a retaining-wall is called a **revetment.** When th earth is level with the top, a **scarp revetment;** when above it, a **counterscar** revetment, or a *demi-revetment*. When the face of the wall is battered, a *sloping;* and when the bac is battered, a *countersloping* revetment. The batter is called the **talus.**

Art. 11. The pres against a wall Fig 6, from sand etc level with its top, is not diminished by reducing the quantity of sand, until its top width *c s* becomes less than that (*c t*) pertaining to the angle *c m t* of maximum pres. The pres then begins to di minish, but in practice *the diminution is not appreciable until the width is reduced to* about one sixth of that (*c s*) pertaining to the angle *c m s* of natural slope, or about half of *c t*. The pres then begins to decrease rapidly as the width is further reduced.

Table 4, of contents in cub yards for each foot in length of retaining-walls, with a thickness at base equal to .4 of the vert height if the back is vert. If the back is stepped according to the rule in Art 3, the proportionate thickness at base will of course be increased. Face batter, 1½ inches to a foot; or ⅛th of the height. Back either vert, or stepped according to the rule in Art 3, Fig 3. The strength is very nearly equal to that of a vert wall with a base of .4 its height. Experience has proved that such walls, when composed of well-scabbled mortar rubble, are safe under all ordinary circum stances for earth level with the top. Steps or offsets, *o c*, at foot, Fig 1, are not here included.

TABLE 4. (Original.)

Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.
1	.013	10½	1 38	20	5.00	29½	10.9	48	28 8	74	38.5
½	.028	11	1.51	½	5 25	30	11.3	49	30 0	76	72.2
2	.050	½	1.65	21	5 51	31	12.0	50	31 3	78	76.1
½	.078	12	1.80	½	5 78	32	12 8	51	32 5	80	80.0
3	.113	½	1.95	22	6 05	33	13.6	52	33 8	82	84.1
½	.153	13	2.11	½	6 33	34	14.5	53	35.1	84	88.4
4	.200	½	2.28	23	6 61	35	15.3	54	36.5	86	92 5
½	.253	14	2 45	½	6 90	36	16 2	55	37.8	88	96.8
5	.313	½	2.63	24	7.20	37	17.1	56	39.2	90	101.3
½	.378	15	2 81	½	7.50	38	18 1	57	40 6	92	105 8
6	.450	½	3 00	25	7.81	39	19 0	58	42 1	94	110.5
½	.528	16	3 20	½	8.13	40	20 0	59	43 5	96	115.2
7	.613	½	3.40	26	8.45	41	21 0	60	45.0	98	120.1
½	.703	17	3.61	½	8 78	42	22 1	62	48 1	100	125.0
8	.800	½	3.83	27	9 12	43	23 1	64	51.2	102	130.1
½	.903	18	4 05	½	9 45	44	24.2	66	54 5	104	135 2
9	1.01	½	4 28	28	9 80	45	25 3	68	57 8	106	140.5
½	1.13	19	4 51	½	10 2	46	26.5	70	61 3		
10	1.25	½	4.75	29	10 5	47	27.6	72	64 8		

STONE BRIDGES.

Art. 1. In an arch *s t s*, Fig 1, the dist *e o* is called its **span**; *i a* its **rise**; *t* its **crown**; its lower boundary line, *e a o*, its **soffit**, or **intrados**; the upper one, *r t r*, its **back**, or **extrados.** The terms soffit and back are also applied to the entire lower and upper curved *surfaces* of the whole arch. The ends of an arch, or the showing areas comprised between its intrados and extrados, are its **faces**; thus the area *s t s a* is a face. The inclined surfaces or joints, *r e*, *r o*, upon which the *feet* of the arch rest, or from which the arch *springs*, are the **skewbacks.** Lines level with *e* and *o*, at right angles to the faces of the arch, and forming the lower edges of its feet, (see *n n*, Fig 2½,) are the **springing lines**, or **springs.** The blocks of which the arch itself is composed, are the **arch-stones**, or **voussoirs.** The center one, *t a*, is the **keystone**; and the lowest ones, *s s*, the **springers.** The term *archblock* might be substituted for voussoir, and like it would apply to brick or other material, as well as to stone. The parts *t r*, *t r*, are the **haunches**; and the spaces *t r l*, *t r b*, above these, are the **spandrels.** The material deposited in these spaces is the **spandrel filling**; it is sometimes earth, sometimes masonry; or partly of each, as in Fig 1.

In large arches, it often consists of several parallel SPANDREL-WALLS, *l l*, Fig 2½, running lengthwise of the roadway, or astraddle of the arch. They are covered at top either by small arches from wall to wall, or by flat stones, for supporting the material of the roadway. They are also at times connected together by vert cross-walls at intervals, for steadying them laterally, as at *t t*, Fig 2½. The parts *g p e n*, *g p o n*, Fig 1, are the ABUTMENTS of the arch; *e n*, *o n*, the *faces*; *g p*, *g p*, the backs; and *p n*, *p n*, the *bases* of the abuts. The bases are usually widened by *feet*, *steps*, or *offsets*, *d d*, for distributing the wt of the bridge over a greater area of foundation; thus diminishing the danger of settlement. The distance *t a* in any arch-stone, is called its *depth*.

The only arches in common use for bridges, are the circular, (often called segmental); and the elliptic.

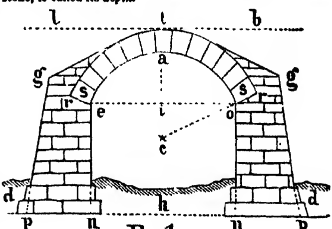


Fig 1

Art. 2. To find the depth of keystone for first-class cut-stone arches, whether circular or elliptic.*

Find the rad *c o*, Fig 1, which will touch the arch at *o*, *a*, and *e*. Add together this rad, and *half* the span *o e*. Take the sq rt of the sum. Div this sq rt by 4. To the quot add $\frac{2}{10}$ of a ft. Or by formula,

* Inasmuch as the rules which we give for arches and abuts are entirely original and novel, it may not be amiss to state that they are not altogether empirical, but are based upon accurate drawings

$$\text{Depth of key in feet} = \frac{\sqrt{\text{Rad} + \text{half span}}}{4} + 0.2 \text{ foot.}$$

For second-class work, this depth may be increased about ⅛th part; or **for brick or fair rubble,** about ⅓rd. See table of Keystones.

In large arches it is advisable to increase the depth of the archstones toward the springs; but when the span is as small as about 60 to 80 or 100 feet, this is not at all *necessary* if the stone is good; although the arch will be stronger if it is done. In practice this increase, even in the largest spans, does not exceed from ¼ to ½ the depth of the key; although theory would require much more in arches of great rise.

REM. **To find the rad** $c\,o$, whether the arch be circular or elliptic. Square *half* the span $e\,o$. Square the whole rise $i\,a$. Add these squares together; div the sum by *twice* the rise $i\,a$. Or it may be found near enough for this purpose by the dividers, from a small arch drawn to a scale.

Amount of pressure sustained by archstones. In bridges of the same width of roadway; if all the other parts bore to each other the same proportion as the spans, the *total* pres would increase as the squares of the spans, while the pressure *per square foot* would increase as the spans. But in practice the depth of the archstones increases much less rapidly than the span; while the thickness of the roadway material, and the extraneous load per sq ft, remain the same for all spans. Hence the *total* pressures, at key and at spring, increase *less* rapidly than the *squares* of the spans; but *more* rapidly than the simple *spans;* as do also the pressures *per square foot.* Thus in two bridges of the same width, but with spans of 100 and 200 ft, with depths of archstones taken from our table and uniform from key to spring; supposed to be filled up solid with masonry of 160 lbs per cub ft, to a level of about 15 inches above the crown, (including the stone paving of the roadway); with an extraneous load of 100 lbs per sq ft; the pressures will be approximately as follows:

	Span 100 ft.				Span 200 ft.			
	AT KEY.		AT SPRING.		AT KEY.		AT SPRING.	
	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.
Rise.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons	Tons.
⅙	42¾	13¾	58	18¾	126	29¼	179	42
⅕	36¾	12¼	57	19	112	27¾	181	44
¼	31¼	11	57½	20	97	24½	188	47½
⅓	25	9	61½	22½	80¼	21	207	54¾
½	18	6¾	67½	25	67¼	15½	230	61½

It will be seen that with the same span, the pres at the key becomes less, while that at the spring becomes greater, as the rise increases. Also that when the archstones are of uniform depth, the pres at either spring of a semicircular arch is about 4 times as great as at the key; whereas when the rise is but one-sixth of the span, the pres at spring averages but about one-third greater than at the key. These proportions vary somewhat in different spans.

The greater pres per sq ft at the springs may be reduced by increasing the depth of the archstones towards the springs. This however is not *necessary* in moderate spans, inasmuch as good stone will be safe even under this greater pres.

By using parallel spandrel walls, see Fig 2½, or by partly filling with earth instead of masonry, the pres on the archstones may be diminished, say, as a rough average, about ⅓ part.

and calculations made by the writer, of lines of pres, &c, of arches from 1 to 300 ft span, and of every rise, from a semicircle to $\frac{1}{16}$ of the span. From these drawings he endeavored to find proportions which, although they might not endure the test of strict criticism, would still apply to all the cases with an accuracy sufficient for ordinary practical purposes.

Table 1. Of some existing arches, with both their actual and their calculated depths (by our rule) of keystone. Where two depths are given in the column of keys the smallest is for first-class cut-stone, and the largest for good rubble, or brick. Those also which are not specified are of first-class cut-stone. C stands for circular, E for elliptic. For 2d class work, add about 1/8th part, and for brick, or fair rubble, about 1/4th

		Span	Rise.	Radius.	Actual Key.	Key by our Rule	Engineer.
		Ft.	Ft.	Ft.	Ft	Ft.	
Cabin John Washington Aqueduct	C	220	57 25	134.25	4 16	4 11	Meigs.
Grosvenor Bridge, Chester, England	C	200	42	140	4	4 07	Hartley.
London Bridge, England	E	152	29 5	112 6	4 75	3 62	Rennie
Gloucester, across the Severn, England	E	150	35	98	4 50	3 49	Telford.
Dora Riparia, Turin, Italy	C	148	18	160.1	4.92	4 03	Mosca.
Pont-y Prydd, Wales. Rough rubble	C	140	35	87 5	1.5	3 3 4 2	Edwards.
Pont de l Alma, Paris, France Small rough rubble in cement	E	141 4	28 2	102 8	4 92	3 5 4 44	Darcel.
Souppes, France An experimental arch of cut-stone in Portland cement mortar *	C	124 3	6 97	280.5	2.62	4 82	Vaudrey.
Waterloo, Across the Thames at London	E	120	32	72 2	4 5	3 07	Rennie.
Tongueland, England Turnpike	C	118	38	64 8	3 5	3 0	Telford.
Pont National, formerly Pont Napoleon R R viaduct, Paris. Small rough rubble in cement	C	115 5	14 75	120	4	3 54 4 48	Couche.
Holy Trinity, Florence. Very slightly pointed		95 5	15 1	83 9	2 75	3 07	Ammanati.
Dean Bridge, Scotland. Turnpike	C	90	30	48 9	3	2 62	Telford
Dunkeld "	C	90	30	48 9	3 17	2 62	Telford.
Licking Aqueduct Chesapeake & Ohio Canal	C	90	15	75	[2 83]	2 94	Fisk.
Posen Viaduct, Germany. Brick in cement	C	80	16	58	4 66	2.68 3.40	
Allanton, Scotland. Turnpike	C	75	11.5	67	2 5	2.75	Stevenson.
Falls Bridge, Philadelphia & Reading R R	C	76	25	43	3	2 46	
Staines, England. Turnpike	C	74	9 3	78	3†	2 98	Rennie.
Brent R R Viaduct, England. Brick in cement	E	70	17 6	43 6	3	2 42 3 02	Brunel.
Bow Bridge. Turnpike. England	E	66	13.75	46 5	2.50	2 42	Walker.
Royal Border Viaduct. England. Brick in cement	C	61.5	30 75	30 75	2.66	2.16 2.88	
Bewdly. Turnpike. England	C	60	20	33	2.2	2.18	Telford.
Chestnut Street Bridge, Philadelphia. Brick in cement	C	60	18	34	2 5	2 2 2 8	Kneass.
Monocacy Aqueduct, Chesapeake & Ohio Canal	E	54	9	45	2.5	2 32	Fisk
Llanrwast, Wales. Turnpike	C	58	17	33	1 5	2 17	Inigo Jones.
Avon Viaduct, England. Brick in cement	E	50	15	28 3	2	2 03 2 67	Vignoles.
James River Aqueduct, Virginia	C	50	7	47	2 66	2 32	Ellet.
Tonoloway Culvert, under Ches & Ohio Canal. Rubble in cement	C	40	15	21	2	1.8 2 3	Fisk.
Philadelphia & Reading R R	C	44	8	34 3	2 5	2 08	Steele.
" " " "	C	31.2	5	26.8	1.66	1.83	Steele.
Edinburg & Dalkeith R R. Scotland	E	24	4	20	.75	1 62	Jardine.

* See Experimental Arch at Souppes, France.
† Staines bridge. Some authorities give 2 ft 4 ins as the depth at key.

Experimental Arch at Souppes, France. See Table.
Span = about 18 × rise

	Span.	Rise.	Radius of intrados.	Depth of arch-stones at spring.	Depth of arch-stones at key on faces	Depth of arch-stones at key betw faces.	Width.
Meters.....	37.886	2 125	85.5	1.10	1.10	0 80	3.5
Feet........	124 30	6 97	280.52	3.61	3 61	2 624	11.5

Arch of granite. The centers rested (for four months) on sand in 16 cylinders, 1 ft diameter, 1 ft high, of $\frac{1}{32}$-inch sheet iron. The unloaded arch settled 15 millimeters (0.59 inch) on striking the centers. The additional settlements under extraneous loads were as follows:

Extraneous load.	Kilograms.	Pounds.	Increase of settlement. Millimeters.	Increase of settlement. Inches.
Distributed	367000	809000	21	0 8
Center..	4975	11000	0.3	0.012
Distributed	132600	292000	1.2	0.047

With the distributed load of 367000 kilog, a load of 4975 kilog, falling 0.3 m (11.8 ins) on key, caused vibrations of 2.8 mm (0.11 inch). *Annales des Ponts et Chaussées,* 1866 *Part* 2, 1868 *Part* 2.

The arch on the BOURBONNAIS RAILWAY, is probably the boldest:* and THE CABIN JOHN ARCH, by Capt, now Gen'l M. C. Meigs, U S Army, the grandest stone one in existence. PONT-Y PRYDD, in Wales, is a common road bridge, of very rude construction, with a dangerously steep roadway. It was built entirely of rubble, in mortar, by a common country mason, in 1750, and is still in perfect condition. Only the outer, or *showing* arch-stones, are 2.5 ft deep, and that depth is made up of two stones. The inner arch-stones are but 1 5 ft deep; and but from 6 to 9 inches thick. The stone quarried with tolerably fair natural beds, and received little or no dressing in addition The bridge is a fine example of that ignorance which often passes for boldness. PONT NAPOLEON carries a railroad across the Seine at Paris. The arches are of the uniform depth of 4 ft, from crown to spring. They are composed chiefly of *small rough quarry chips*, or *spawls*, well washed, to free them from dirt and dust; and then thoroughly bedded in good cement, and grouted with the same It is in fact an arch of cement-concrete. The PONT DE ALMA, near it, *and built in the same way*, has elliptic arches of from 126 to 141 ft span; with rises of $\frac{1}{5}$ the span. Key 4.9 ft. These two bridges, considering the want of precedent in this kind of construction, on so large a scale, must be regarded as very bold; and as reflecting the highest credit for practical science, upon their engineers, Darcel and Couche Some trouble arose from the unequal contraction of the different thicknesses of cement. They show what may be readily accomplished in arches of moderate spans, by means of small stone, and *good hydraulic cement* when large stone fit for arches is not procurable. In Pont Napoleon the depth of arch is less than our rule gives for second class cut-stone.

Art. 3. The keystones for large elliptic arches by the best engineers, are generally made about ⅓ part deeper than our rule requires, or than is considered necessary for circular ones of the same span and rise; in order to keep the line of pres well within the joints; although the elliptic arch, with its spandrel filling, has slightly less wt; and that wt has a trifle less leverage than in a circular one; and consequently it exerts less pres both at the key, and at the skew-back. See London, Gloucester, and Waterloo bridges, in the preceding table.

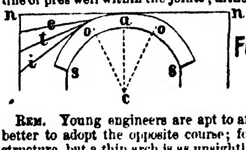


Fig. 1½

REM. Young engineers are apt to affect shallow arch-stones; but it would be far better to adopt the opposite course; for not only do deep ones make a more stable structure, but a thin arch is as unsightly an object as too slender a column. According to our own taste, arch-stones fully ⅓ deeper than our rule gives for first-class cut stone, are greatly to be preferred when appearance is consulted. Especially when an arch is of rough rubble, which costs about the same whether it is built up as arch, or as spandrel filling, it is mere folly to make the arches shallow Stability and durability should be the objects aimed at; and when they can be attained even to excess, without increased cost, it is best to do so.

* Built like that at Souppes.

Table 2. Depths of keystones for arches of first-class cut stone, by Art 2. For second class add full one-eighth part; and for superior brick one-fourth to one-third part, if the span exceeds about 15 or 20 ft. Original.

SPAN. Feet.	Rise, in parts of the span. 1/2	1/3	1/4	1/5	1/6	1/8	1/10
	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.
2	.55	.56	.58	.60	.61	.64	.68
4	.70	.72	.74	.76	.79	.83	.88
6	.81	.83	.86	.89	.92	.97	1.03
8	.91	.93	.96	1.00	1.03	1.09	1.16
10	.99	1.01	1.04	1.07	1.11	1.18	1.26
15	1.17	1.19	1.22	1.26	1.30	1.40	1.50
20	1.32	1.35	1.38	1.43	1.48	1.59	1.70
25	1.45	1.48	1.53	1.58	1.64	1.76	1.88
30	1.57	1.60	1.65	1.71	1.78	1.91	2.04
35	1.68	1.70	1.76	1.83	1.90	2.04	2.19
40	1.78	1.81	1.88	1.95	2.03	2.18	2.33
50	1.97	2.00	2.08	2.16	2.25	2.41	2.58
60	2.14	2.18	2.26	2.35	2.44	2.62	2.80
80	2.44	2.49	2.58	2.68	2.78	2.98	3.18
100	2.70	2.75	2.86	2.97	3.09	3.32	3.55
120	2.94	2.99	3.10	3.22	3.35	3.61	3.88
140	3.16	3.21	3.33	3.46	3.60	3.87	4.15
160	3.36	3.44	3.58	3.72	3.87	4.17	
180	3.56	3.63	3.75	3.90	4.06	4.38	
200	3.74	3.81	3.95	4.12	4.29		
220	3.91	4.00	4.13	4.30	4.48		
240	4.07	4.15	4.30	4.48			
260	4.23	4.31	4.47	4.66			
280	4.38	4.46	4.63				
300	4.53	4.62	4.80				

Art. 4. To proportion the abuts for an arch of stone or brick, whether circular or elliptic. (Original.)

The writer ventures to offer the following rule, in the belief that it will be found to combine the requirements of theory with those of economy and ease of application, to perhaps as great an extent as is attainable in an endeavor to reduce so complicated a subject, to **a simple and reliable working rule for practical bridge-builders.** This is all that he claims for it. Notwithstanding its simplicity, it is the result of much labor on his part. It applies equally to the smallest culvert, and to the largest bridge; whatever may be the proportions of span and rise; and to any height of abut whatever. It applies also to all the usual methods of filling above the arch; whether with solid masonry to the level *v f*, Fig 2, of the top of the arch; or entirely with earth; or partly with each, as represented in the fig; or with parallel spandrel-walls extending to the back of the abut, as in Fig 2½. Although the stability of an abut cannot remain precisely the same under all these conditions, yet the diff of thickness which would follow from a strict investigation of each particular case, is not sufficient to warrant us in embarrassing a rule intended for popular use, by a multitude of exceptions and modifications which would defeat the very object for which it was designed.

It gives a thickness of abut, which, without any backing of earth behind it, is safe in itself, and in all cases, against the pres, when the bridge is unloaded. Moreover, in very large arches, in which the greatest load likely to come upon them in practice is small in comparison with the wt of the arch itself, and the filling above it, our abuts would also be safe from the *loaded* bridge, without any dependence upon the earth behind them; but as the arches become less, and consequently the wt of the load becomes greater in proportion to that of the arch, and of the filling above it, we must depend more and more upon the resistance of the earth behind the abuts, in order to avoid the necessity of giving the latter an extravagant thickness. *It will therefore be understood throughout that, except when parallel spandrel walls are used, our rules suppose that after the bridge is finished, earth will be deposited behind the abuts, and to the height of the roadway, as usual.*

In small bridges and large culverts of first class railroads, subject to the jarring of heavy trains at high speeds, the comparative cheapness with which an excess of strength can thus be given to important structures, has led, in many cases, to the use of abutments from one-fourth to one-half thicker than by the following rule **If of rough rubble** add 6 ins to insure full thickness in every part

$$\left.\begin{array}{c}\textit{Thicks o n of abut at spring}\\ \textit{in ft, when the height o s}\\ \textit{does not exceed } 1\tfrac{1}{2} \textit{ times the}\\ \textit{base s p}\end{array}\right\} = \frac{\textit{Rad in ft}}{5} + \frac{\textit{rise in ft}}{10} + 2\,\textit{ft}.$$

Mark the points n and y thus ascertained. Next, from the center i, of the span or chord eo, lay off ih, equal to $\frac{1}{24}$ part of the span. Join ah; and through n, and parallel to ah, draw the indefinite line gnp of the abut. Do the same with the other abut. Make ym and ng each equal to half the entire height it of the arch; and from g draw a straight line gx, touching the back of the arch as high up as possible; or still better, as shown at tm, with a rad dt or dm, (to be found by trial,) describe an arc tm. Then gx or tm will be the top of the masonry filling above the arch;* and this should be completed before striking the centers; before which, also, the embkt should be finished, at least up to yn.

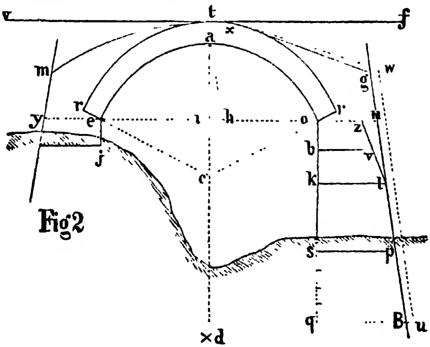


Fig 2

Now find by trial the point s, Fig 2, at which the thickness sp is equal to two-

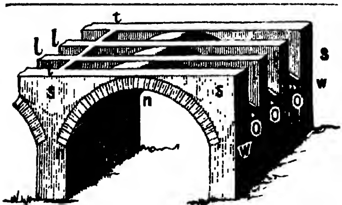


Fig $2\frac{1}{2}$

* EXCEPT WHEN THE RISE IS BUT ABOUT $\frac{1}{4}$ OF THE SPAN, OR LESS, in which case carry the masonry up solid to the level vtf, of the top of the arch. Or if the arch is a large one, exceeding say about 60 ft span, and especially if its rise is greater than about $\frac{1}{5}$ of its span, it is better to economize masonry by the use of parallel interior spandrel-walls, ll, Fig $2\frac{1}{2}$, carried up to vtf, Fig 2. Indeed, such interior walls may often be advantageously introduced in much smaller arches. When high, they are steadied by occasional cross-walls, as tt, Fig $2\frac{1}{2}$. Their feet should be spread by offsets, as shown at o o o, so as to bear upon the whole surf of the back of the arch; thus equalizing the pres upon it. On top of the walls flagstones may be laid, or small arches may be turned from wall to wall, for supporting the ballast, &c, of the roadway. The spaces below are left hollow. In Fig $2\frac{1}{2}$ the dark part ww is supposed to be a section across an abutment; but omitting the second cross-wall, similar to ss. In R R bridges, put a spandrel-wall, ll, under each rail.

thirds of the corresponding vert height *o s*, and draw *s p*. Then will the *thickness* *o n* or *e y* be that at the springing line of the given circular or elliptic arch of any rise and span; and the line *g p* will be the *back* of the abut; provided its height *o s* does not exceed 1½ times *s p*; or in other words, provided *s p* is not less than ⅔ of *o s*. In practice, *o s* will rarely exceed this limit; and only in arches of considerable rise. But if it should, as for instance at *o q*, then make the base *q u* equal to *s p*, added to one-fourth of the *additional* height *s q*; and draw the back *u w*, parallel to *g p*; and extending to the same height, &c, as in Fig 2. If, however, this addition of ¼ of *s q* should in any case give a base *q u*, less than *one-half* the total height *o q*, (which will very rarely happen in practice,) then make *q u* equal to half said total height; drawing the back parallel to *g p*, and extending it to the same height as before. The additional thicknesses thus found below *s p*, have reference rather to the pres of the earth behind the abut, than to the thrust of the arch. In a very high abut, the inner line *g p* would give a thickness too slight to sustain this earth safely.

When the height *o b*, Fig 2, of the abut is less than the thickness *o n* at spring, a small saving of masonry (not worth attending to, except in large flat arches) may be effected by reducing the thickness of the abut throughout, thus: Make *o k* equal to *o n*, and draw *k l*. Make *o z* equal to ¾ of *o n*, and draw *l z*. Then, for any height *o b* of abut less than *o n*, draw *b v*, terminating in *l z*. This *b v* will be sufficient base, if the foundations are firm. The back of the abut will be drawn upward from *v*, parallel to *g p*, and terminating at the same height as *g* or *w*.

Rem. 1. All the abuts thus found will (with the provisions in Art 6) be safe, without any dependence upon the wing-walls; no matter how high the embkt may extend above the top of the arch. If the bridge is narrow, and the inner faces of the wing-walls are consequently brought so near together as to afford material assistance to the abuts, the latter may be made thinner; but to what extent, must depend upon the judgment of the engineer.

We, however, caution the young practitioner to be careful how he adopts dimensions less than those given by our rule. There are certain practical considerations; such as carelessness of workmanship; newness of the mortar; danger of undue strains when removing the centers; liability of derangement during the process of depositing the earth behind the abuts, and over the arch; &c, which must not be overlooked; although it is impossible to reduce them to calculation.

Whenever it can be done, the centers should remain in place until the embkt is finished; and for some time afterward, to allow the mortar to set well. But for more on this see Rem 4. p. 633.

Rem. 2. A good deal of liberty is sometimes taken in reducing the quantity of masonry above the springing line of arches of considerable rise, and of moderate spans. When care is taken to leave the centers standing until the earth filling is completed above the arch, and behind its abuts, so that it may not be deranged by accident during that operation; and when good cement is used instead of common mortar, such experiments may be tried with comparative safety; especially with culvert arches, in which the depth of arch stones is great in proportion to the span. They must, however, be left to the judgment of the engineer in charge; as no specific rules can be laid down for them. They can hardly be regarded as legitimate practice, and we cannot recommend them. We have known nearly semicircular arches, of 30 to 40 ft span, to be thus built successfully, with scarcely a particle of masonry above the springs to back them. Such arches, however, are apt to fall, if at any future period the earth filling is removed, without taking the precaution to first build a center or some other support for them. Even when the embkt can be finished before the centers are removed, we cannot recommend (and that only in small spans) to do *less* than to make *n g*, Fig 2, equal to ⅙ of the total height *i t* of the arch; and from *g* so found, to draw a straight line touching the back of the arch as high up as possible.

Rem. 3. We have said nothing about **battering the faces of the abuts,** because in the crossing of streams, the batter either diminishes the water-way; or requires a greater span of arch. Such a batter, however, to the extent of from ½ to 1½ ins to a ft, is useful, like the offsets, for distributing the wt of the structure, and its embkt, over a greater area of foundation; especially when the last is not naturally very firm; or when the embkt extends to a considerable height above the arch. In our tables, Nos 3 and 5, of approximate quantities of masonry in semicircular bridges of from 2 to 50 ft span, the faces are supposed to be vert.

Art. 5. Abutment-piers. When a bridge consists of several arches, sustained by piers of only the usual thickness, if one arch should by accident of flood, or otherwise, be destroyed, the adjacent ones would overturn the piers; and arch after arch would then fall. To prevent this, it is usual in important bridges to make some of the piers sufficiently thick to resist the pres of the adjacent arches, in case of such an accident; and thus preserve at least a portion of the bridge from ruin. Such are called abutment-piers.

Our formula of $\frac{\text{Rad}}{5} + \frac{\text{Rise}}{10} + 2$ ft, for the thickness at spring; with the back battering as before, at the rate of $\frac{1}{24}$ of the span to the rise; face vert; will of itself (*without any modification for great heights*) give a perfectly safe abut pier, for any unloaded bridge; and to any height whatever; due regard being had, however, to the consideration alluded to in the next Art. Thus, for an abut-pier as high as *o q*, Fig 2; or of any greater height; it is only necessary first to find the thickness *o n* at spring as before; and then draw the battered back *g n p*; extending it down to the base at B; without adding ¼ of the additional height *s q*. This addition is made in the case of abuts, that they

may be secure from the pres of the *earth* behind them; as well as from the pres of the arch; a consideration which does not apply to abut-piers; in which only the pres of the arch is to be resisted.

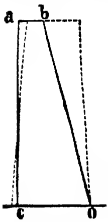


Fig 3

But although the abut-pier thus found by our formula, would be abundantly safe, yet its shape *a b c o*, Fig 3, is inadmissible. In practice it would be changed to one somewhat like that shown by the dotted lines; having an equal degree of batter on both faces. This of course requires more masonry, with but little increase of stability, but that cannot be avoided.

WHEN AN ABUT-PIER IS BUILT IN DEEP WATER, or in a shallow stream subject to high freshets, care must be taken that water cannot find its way *under* the pier, and thus produce an upward pres, which will either diminish, or entirely counteract its efficiency as an abut. See Remark 2, Art 4, of Hydrostatics.

Art. 6. Inclination of the courses of masonry below the springs of an arch. Although our foregoing rule gives a thickness of abut which cannot be *overturned*, or *upset*, by the pres of the arch, yet if the arch be of large span, and small rise, its great hor thrust may produce a *sliding* outward of the masonry near the level of the springs, if the stones are laid in *hor* courses; especially if the mortar has not set well.

This danger, it is true, could be avoided by confining the courses together by iron bolts and cramps; or by increasing considerably the thickness of the abuts; but the expense of doing either of these, leads to the cheaper expedient of inclining the masonry, as shown between *o* and *n*, Fig 4; the courses near *o* being steeper; and gradually becoming less steep near *n*. By this process the arch is virtually prolonged into the body of the abut, so far that when the inclination of the lower masonry ceases, as at *n*, the direction of the theoretical *line of thrust*, or *of pres* of the arch, (rudely represented by the dotted curved line *o n*) is nearly at right angles to the joints of the hor masonry below *n*; and consequently, said thrust is unable to produce sliding at that point. Between *o* and *n*, the line of pres is everywhere so nearly at right angles to the variously inclined joints, as to preclude the possibility of sliding in that interval also

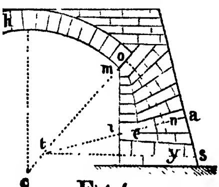


Fig. 4

The abut being thus safe throughout from both overturning and sliding, can fail only from defective foundations; or from the inferiority of the stone of which it is built; and which, if soft, may be crushed.

This inclination of the masonry is as necessary in an elliptic arch, Fig 4½, as in a circular one.

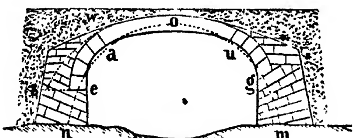


Fig. 4½

The elliptic form is plainly unfavorable for uniting the arch-stones with the inclined masonry near the springs, so as to receive the thrust properly; or about at right angles to its resultant. In ordinary cases this difficulty may be overcome by making the joints of only the outside or showing arch-stones to conform to the elliptic curve; as between *e* and *a*; while the joints of the inner or hidden ones, may have the directions shown between *g* and *u*, nearly at right angles to the line of thrust. It will rarely happen, however, that the young engineer will have to construct elliptic arches of sufficient magnitude to require either this, or any equivalent expedient. For spans less than 50 ft, with rises not less than about $\frac{1}{5}$ of the span, nothing of the kind is actually necessary, if the mortar is good, and has time to harden.†

In order to incline the masonry of any abut with sufficient accuracy, it would be necessary first to trace the curved line of pres of the given arch, so as to arrange the bed joints about at right angles to it at every point of its course; but we offer the following process as sufficing for all ordinary practical purposes, while its simplicity places it within the reach of the common mason. In actual bridges the direction of the actual thrust changes as the load is passing; therefore, in practice no given degree of inclination of the abut masonry can conform to it precisely during the entire passage. Consequently, any excess of refinement in this particular, becomes simply ridiculous, especially in small spans.

Rule for inclining the beds of the masonry in the abuts. Add together the rad *c m*, Fig 4; and the span of the arch. Div the sum by 5. To the quot add 3 ft. Make *o t*, on the rad, equal to the last sum. Then is *t* a central point, toward which to draw the directions of the beds, as in the fig. Draw *t s* hor, and from *t* as a center, describe the arc *o y*; *o* being the center of the depth of the springers. From *y* lay off on the arc the dist *y n*, equal to one-sixth part of *t y*; draw *t n a*. It will never be *necessary* to incline the masonry below this *t n a*. Neither *need* the inclination extend entirely to the face *m i* of the abut; but may stop at *e*, about half-way between *i* and *n*. From *e* upward, the inclination may extend forward to the line *e m*.

Cement should be freely used, not only in the arches themselves, and in the masonry above them, as a protection from rain-soakage; but in abuts, wing-walls, retaining-walls, and all other important masonry exposed to dampness. The entire backs of important brick arches should be covered with a layer of good cement, about an inch thick. The want of it can be seen throughout most of our public works. The common mortar will be found to be decayed, and falling down from the soffits of arches; and from the joints of masonry generally, within from 3 to 6 ft of the surface of the ground. The moisture rises by capillary attraction, to that dist above the surf of the nat soil; or descends to it from the artificial surf of embankments, &c; therefore, cement-mortar should be employed in those portions at least. The mortar in the faces of battered walls even when the batter is but 1 to 1½ inches per foot, is far more injured by rain and exposure, than in vert ones; and should therefore be of the best quality. See MORTAR, &c.

We have, however, seen a quite free percolation of surface water through brick arches of nearly 3 ft in depth, even when cement was freely used. In aqueduct bridges, we believe that cement has not been found to prevent leaks, whether the arches were of brick, or even of cut-stone. May not this be the effect of cracks produced by settlement of the arch; or by contraction and expansion under atmospheric influence? Cement at any rate prevents the joints from crumbling.

† The feet of both elliptic and semicircular arches are always made hor; but it is plain from Fig 4¾, that this practice is at variance with correct principles of stability in the case of the ellipse. It is the same in the semicircle. In ordinary bridges of the latter form, the vert pres, or weight resting on each skewback, is (roughly speaking) usually about from 3¼ to 4 times the hor pres on the same: and the total pres is about 4 times as great as the pres on the keystone. Therefore, theoretically, the skewback should usually be about 4 times as deep as the keystone; and its bed, instead of being hor, should be inclined at the rate of about 1 vert to 4 hor.

When the arch is flat, this inclination may become so steep, especially in the upper parts, that struts, or shores of some kind, must be used for preventing the masonry from sliding down, until the completion of the arch secures it from doing so. The hor courses between the face *m i*, and the line *o e*, will aid somewhat in this respect.

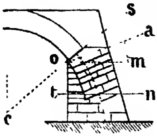


Fig 5

This method should be applied to *all very large arches* whose rise is one-third, or less, of the span. As before remarked, it is not actually necessary in arches not exceeding about 50 ft span, and not flatter than $\frac{1}{5}$ of the span. Indeed, if the earth filling can be deposited before the centers are removed, these limits may be considerably extended without danger. Still, since a certain degree of inclination is attended with very little trouble or expense, we would recommend for even such arches, a process somewhat like the following From *half* the span take the rise. Div the rem by 3. Make *o t*, Fig 5, equal to the quot Draw *t n*, and *o m*, hor. Div the angle *s o m* into two equal parts, by the line *o a*. Incline the masonry so as to be parallel to *o a*, as far down as *t n*. The inclined courses may extend out to the face *o t*, or not, at pleasure.

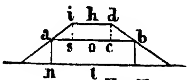


Fig 7

REM. 1. **To find the length** (*a b*, Fig 7) **from face to face of a culvert.** From the height *h t* of the embkt, take the *above ground* height *n a* of the culvert, the rem will be the height *h o* of the embkt *above* the culvert Then the reqd length *a b* is plainly equal to the top width *i d* of the embkt, added to the two dists *a s*, *c b*, which correspond to its steepness of side slopes. Thus, if the side slope is, as usual, 1½ to 1, then *a s* and *c b* will each be equal to 1½ times *o h*, or the two together will be 3 times *o h*. So that if the width *i d* is 14 ft, and *h o* 5 ft, the length *a b* will be

$$14 + (5 \times 3) = 14 + 15 = 29 \text{ ft}$$

Art. 7. The following tables, 3, 4, and 5, of quantities, will be found useful for expediting preliminary estimates, for which purpose chiefly they are intended; hence no pains have been taken to make them scrupulously correct, but rather a little in excess of the truth. The first column of Table 3 contains the total vert height *o e*, Fig 6, from the crown *o* of a semicircular arch, to the foundation or base *g m* of its abut The other columns give approximately the number of cub yds contained in each running foot, or foot in length of the culvert or bridge, measured from end to end (face to face) of the arch proper; and including only the arch and its abuts, as shown in Fig 1; or in the half section *o p m g y* in Fig 6; including footings to the abuts, but omitting the wing-walls (*w n*), and the spandrel-walls (*s*), Figs 6 and 2½. At the foot of each column is the approximate content in cub yds of the two spandrel-walls by themselves; one over each face of the arch.

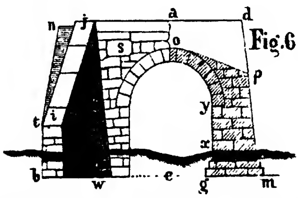


Fig. 6

These spandrel-walls are calculated on the supposition that their thickness at base, at their junction with the wing-walls, where their height is greatest, is equal to $\frac{4}{10}$ of their height at that point: except where that proportion gives a less thickness at top than 2½ ft; and that they extend 2 ft (*o a*) above the top *o* of the arch. At the top of the arch, they are all supposed to be 2½ ft thick at top; that being assumed to be about the least thickness admissible in a rubble wall in such a position. Both the back and the face are supposed to be vert. The contents of these spandrel-walls will vary somewhat, however, even in the same span, with the height of the abut and the arrangement of the wings. They, however, constitute so small a proportion of the entire contents given in Table 5, that this consideration may be neglected in preliminary estimates. They are so firmly bonded into the masonry of the wings at their highest points, and so strongly connected by mortar with the backing of the arch at their bases, that they require no greater thickness however high the emb may be.

The contents of the four wing-walls, of which *n j w b*, Fig 6, is one, will be found in a table (No. 4) immediately following that for the body of the culvert. We have also added a table (No. 5) for complete semicircular culverts of various lengths, including their spandrel and wing walls.

Rem. 1. Although the thickness of wing-walls increases in all parts with their height, they are not made to *show* thicker at *nj* than at *ti*, Fig 6; but (as seen in the fig) are offsetted at their back *n*, a little below their slanting upper surf *ij*, so as to give a uniform width for the steps or flagstones, as the case may be, with which they are covered. In the fig the covering is supposed to be of flagstones; but steps are preferable, being less liable to derangement. To prevent the flagstones from sliding down the inclined plane *j i*, the lower stone *i* should be deep and large, and laid with a hor bed. The flags are sometimes cramped together with iron, and bolted down to the wall. Steps require nothing of that kind, as seen at *s*, Fig 11.

Rem 2 **The tables show the inexpediency of too much contracting the width of water-way,** with a view to economy, by adopting a small span of arch, when a culvert of greater span can be made, *of the same total height.*

For the wings must be the same, whether the span be great or small, provided the total height is the same in both cases; and since the wings constitute a large proportion of the entire quantity of masonry, in culverts of ordinary length, the span itself, within moderate limits, has comparatively little effect upon it. Thus, the total masonry in a semicircular culvert of 3 ft span, 8 ft total height, and 60 ft long between the faces of the arch, is, by Table 5, 151½ cub yds, while that of a 5 ft span of the same height and length, is 152.4. A semicircular bridge of 25 ft span, 24 ft total height, and 40 ft between the faces of the arch, contains 1031 cub yds; while one of 35 ft span, of the same height and length, contains 1134 yds, so that in this case we may add nearly 50 per cent to the water-way, by increasing the masonry of the bridge but $\frac{1}{10}$th part.

Rem. 3. Partly for the same reason, and partly because **the culverts for a double-track road are not twice as long as those for a single-track one,** the quantity of culvert masonry for the former will not average more than about from ¼ to ⅛ part more than that for the latter; so that it frequently becomes expedient to *finish* the culverts at once to the full length required for a double track, although the embkts may at first be made wide enough for only a single one, with the intention of increasing them at a future time for a double one.

Thus, the average size of culverts for a single track may be roughly taken at 6 ft span, 30 ft long from face to face, and 10 ft total height; and such a one contains, by Table 5, 140 cub yds. For a double track, it would require to be about 12 feet longer; and we see by Table 3 that this will add 2.67 × 12 = 32 cub yds, making a total of 172 yds instead of 140; thus adding rather less than ¼ part. When the culverts are under very high embkts, and consequently much longer, the addition for a double track becomes comparatively quite trifling.

Table 3, of approximate numbers of cub yds of masonry per foot run, contained in the arches and abutments only, as shown in Fig 1 (omitting wings, and the spandrel-walls over the faces of the arches) of semicircular culverts and bridges, of from 2 to 50 ft span, and of different total heights, *ht*, Fig 1, or *oc*, Fig 6. It will be seen that in many cases, a bridge of larger span contains less masonry than one of smaller span, *when their total heights are the same.* There is a liberal allowance for footings or offsets at the bases of the abuts.

TABLE 3. (Original.)

Total Height.	Span 2 ft	Span 3 ft.	Span 4 ft.	Span 5 ft.	Span 6 ft	Span 8 ft.	Span 10 ft	Span 12 ft.	Span 15 ft.
Feet.	Cub. y.	Cub. y	Cub. y	Cub. y.	Cub. y.	Cub. y.	Cub. y	Cub. y.	Cub. y.
2	.42	......	......	......	......	......	......	......	......
3	.60	.63	.67	......	......	......	......	......	......
4	.79	.83	.87	.92	.97	......	......	......	......
5	.99	1.04	1.08	1.15	1.21	......	......	......	......
6	1.28	1.28	1.28	1.3[illegible]	1.46	1.58	1.69	......	......
7	1.62	1.59	1.55	1.64	1.72	1.85	1.97	2.12	......
8	2.01	1.96	1.91	1.95	1.99	2.13	2.26	2.38	......
9	2.45	2.38	2.31	2.29	2.27	2.42	2.56	2.65	3.02
10	2.94	2.85	2.76	2.72	2.67	2.77	2.87	2.95	3.34
11	......	3.38	3.26	3.19	3.12	3.16	3.19	3.25	3.67
12	......	3.98	3.82	3.72	3.62	3.57	3.52	3.55	4.01
13	......	......	4.42	4.29	4.17	4.10	4.02	3.86	4.36
14	......	......	5.08	4.90	4.77	4.67	4.57	4.41	4.72
15	......	......	......	5.57	5.42	5.30	5.17	5.01	5.09
16	......	......	......	6.30	6.12	5.97	5.82	5.56	5.69
17	......	......	......	......	6.87	6.70	6.52	6.26	6.34
18	......	......	......	......	7.69	7.48	7.27	7.01	7.04
19	......	......	......	......	......	8.32	8.07	7.71	7.69
20	......	......	......	......	......	9.20	8.92	8.56	8.49
21	......	......	......	......	......	......	9.82	9.46	9.34
22	......	......	......	......	......	......	10.8	10.3	10.2
23	......	......	......	......	......	......	......	11.3	11.1
24	......	......	......	......	......	......	......	12.3	12.1
25	......	......	......	......	......	......	......	......	13.2
26	......	......	......	......	......	......	......	......	14.2

Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.

	2.9	3.7	4.4	5.2	5.8	7.9	9.8	12.	16.

TABLE 3. (Continued.)

Total Height.	Span 20 ft.	Span 25 ft.	Total Height.	Span 35 ft.	Total Height.	Span 50 ft.
Feet.	Cub. y.	Cub. y.	Feet.	Cub. y.	Feet.	Cub. y.
12	4.60	....	20	10.5	27	18.0
13	4.98	...	21	11.0	28	18.7
14	5.37	6.10	22	11.6	29	19.4
15	5.77	6.41	23	12.2	30	20.1
16	6.18	6.76	24	12.7	31	20.9
17	6.60	7.16	25	13.3	32	21.6
18	7.03	7.61	26	13.8	33	22.4
19	7.47	8.10	27	14.5	34	23.1
20	8.12	8.60	28	15.1	35	23.9
21	8.82	9.02	29	15.7	36	24.7
22	9.57	9.72	30	16.3	37	25.5
23	10.4	10.4	31	17.0	38	26.3
24	11.3	11.2	32	18.1	39	27.1
25	12.2	12.1	33	19.2	40	28.0
26	13.1	13.0	34	20.4	41	28.8
27	14.1	14.0	35	21.7	42	30.0
28	15.2	15.0	36	23.0	43	31.5
29	16.3	16.1	37	24.3	44	33.0
30	17.4	17.2	38	25.7	45	34.6
31	18.6	18.4	39	27.2	46	36.3
32	19.9	19.6	40	28.7	47	38.1
33	21.2	20.9	41	30.2	48	39.8
34	22.6	22.2	42	31.8	49	41.6
35	24.0	23.6	43	33.5	50	43.6
36	25.4	25.0	44	35.2	51	45.5
37	26.9	26.5	45	36.9	52	47.4
38	28.5	28.0	46	38.7	53	49.4
39	30.1	29.5	47	40.6	54	51.6
40	31.7	31.2	48	42.5	55	53.7
41	....	32.8	49	44.4	56	55.9
42	....	34.5	50	46.4	57	58.1
43	....	36.3			58	60.4
44	....	38.1			59	62.7
45	....	40.0			60	65.1
Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.						
	28.	42		85.		135.

Art. 8. The following table of contents of wing-walls, or wings, will, like the preceding one, be useful in making preliminary estimates. The wings *so*, *no*, shown in plan at Fig 8, are supposed to form an angle *aoc*, of 120°, with the face, or end *oo* of the culvert. Their outer or small ends *nn*, are all assumed to be of the dimensions shown on a larger scale at E. Thickness at base at every part equal to $\frac{4}{10}$ of the height of the wall at said part; except when that proportion becomes too small to allow the width or thickness at top to be 2.5 ft; in which case it is enlarged at such parts sufficiently for that purpose. See Remark 2. This happens only

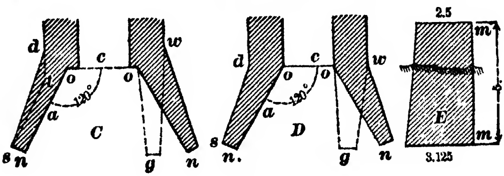


Fig. 8

when the height *m m*, Fig. E, of the wing, becomes less than 9 ft. Batter of face, $1\frac{1}{2}$ ins. to a ft.; or 1 in 8. Back vert.; but offsetted, if necessary, for a short dist. below the top, so as to give a uniform *showing* top thickness of $2\frac{1}{2}$ ft. The masonry is supposed to be good well-scabbled mortar rubble. The height given in the first column is the greatest one; or that at *oo* (or *wj*, Fig. 6), where the wing joins the face of the culvert. In the table no allowance is made for footings (offsets or steps) at the base of the wings; as these are frequently omitted in wings on good founda-

tions. In taking out quantities from the table, bear in mind that the height of the wings is usually a little greater than that of the culvert itself.

The plan shown at C is the common one, but that at D is greatly preferable for culverts; for the shoulders at *o o* in Fig. C, apart from their greater liability to catch branches of trees, etc., floating down stream, offer of themselves a much greater resistance to the flow of the water into the culvert than do the mere corners at *o o*, Fig. D.

Table 4, of approximate contents, in cub yds, of the four wing-walls of a culvert, or bridge. (Original.)

The heights are taken where greatest, as at *j w*, Fig 6

Height of wing	Length of one wing.	Cub. yds. in 4 wings	Height of wing	Length of one wing.	Cub. yds. in 4 wings.
Feet	Feet.		Feet.	Feet	
6	1.73	4.04	30	43.4	818
7	3.46	8.85	32	46.8	997
8	5.20	14.6	34	50.3	1192
9	6.93	21.5	36	53.7	1414
10	8.66	30.2	38	57.2	1661
11	10.4	40.9	40	60.7	1928
12	12.1	53.7	42	64.2	2220
14	15.6	85.2	44	67.6	2552
16	19.1	128	46	71.1	2912
18	22.5	183	48	74.6	3306
20	26.0	247	50	78.0	3741
22	29.5	329	55	86.7	4942
24	32.9	428	60	95.3	6404
26	36.4	541	65	104	8131
28	39.8	672	70	113	10155

To reduce cub yds to perches of 25 cub ft, mult by 1.080.
To reduce perches to cub yds, mult by .926, or div by 1.08.

The contents for heights intermediate of those in the table may be found approximately by simple proportion.

REM. 1. It is not recommended to actually prolong all wings until their dimensions become as small as shown at E, in Fig 8. In large ones it will generally be more economical to increase their end height *m m*, a few feet. The contents, however, may be readily found by the table in that case also. Thus suppose the height of the wings at one end to be 30 ft, and at the other end 8 ft; we have only to subtract the tabular content for 8 ft high, from that for 30 ft high. Thus, 818 — 14.6 = 803.4 cub yds required content.

REM. 2. It might be supposed that inasmuch as the wings of arches often have to sustain the pressure from embankments reaching far above their tops, they should, like ordinary retaining-walls, be made much thicker in that case. But the fact that they derive great additional stability from being united at their high ends to the body of the bridge or culvert, renders such increase unnecessary when proportioned by our rule; no matter how far the earth may extend above them; as shown by abundant experience.

Relying upon this aid, we may indeed, when the earth does not extend above the top, reduce the base at *o* to one third of the ht, as shown at *o t*, and by dotted line *t s*. Experience shows that we may also do the same even when the earth reaches to a great height above the top, provided that the wings, instead of being splayed or flared out, as at *o n*, *o n*, merely form straight prolongations of the abutments of the arch, as shown by the dotted lines at *o g w*. In this case the pressure of the earth against the wings is less than when they are splayed. We have known the thickness at *o* to be reduced in such cases to less than one-third the height, when the wings were 15 ft high, and the height of the embankment *above their tops* 16 feet in one case, and 36 ft in another. In another instance, similar wings 25½ ft high, and with 29 ft of embankment above their top, had their bases at *o* rather less than $\frac{1}{5}$ of the height. In all these cases, the uniform thickness at top was 2.5 feet; backs vertical. We mention them because this particular subject does not seem to be reducible to any practical rule. The last wall appears to us to be too thin; especially if the earth is not deposited in layers, and after allowing the mortar full time to set. The labor, however, required in compacting the earth carefully in layers, may cost more than is thereby saved in the masonry. The young practitioner must bear this in mind when he wishes to economize masonry by such means; and also that the thin wall may bulge, or fail entirely, if the earth backing is deposited while the mortar is imperfectly set.

Table 5. Approximate contents in cubic yards, of com- plete semicircular culverts and bridges of from 2 to 50 feet span; including the 2 spandrel walls, and the 4 wings; all proportioned by the oregoing directions; and taken from the two preceding tables. The height in the econd column, is from the top of the *keystone* to the bottom of the foundation. The ings are calculated as being 2 ft higher than this, including the thickness of the oping. The wings are frequently carried only to the height of the top of the arch, hus saving a good deal of masonry. Table 4, of wings alone, will serve to make the roper deduction in this case.

The several lengths are from end to end, or from face to face, of the *arch proper* The contents for intermediate lengths may be found exactly; and those for inter- nediate heights, quite approximately, by simple proportion. In this table, as in No. 3, it will be observed that *when the heights are the same in both cases*, a larger pan frequently contains less masonry than a smaller one. A semicircular culvert r bridge contains less masonry than a flatter one, when the total height is the same n both cases; therefore, the first is the most economical as regards cost; but it does ot afford as much area of water-way; or width of headway.

(Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.	Cub Y.
2	5	27	32	42	52	72	92	112	132	152	172	192	212
	6	37	43	56	69	94	120	146	171	197	222	248	274
	7	49	57	73	89	122	154	187	219	251	284	316	349
	8	63	73	93	113	153	193	233	273	313	353	393	433
	10	101	116	145	175	234	291	351	410	469	527	586	645
3	5	28	34	44	54	75	96	117	138	158	179	200	221
	6	38	44	57	70	95	121	146	172	198	223	249	275
	7	49	57	73	89	121	153	184	216	247	280	312	343
	8	63	73	93	112	152	191	230	269	308	348	387	426
	10	101	115	143	172	229	286	343	400	457	514	571	628
	12	149	169	208	248	328	407	487	567	646	726	806	885
4	5	30	35	46	57	78	100	122	143	165	186	208	229
	6	38	45	58	70	96	122	147	173	198	224	250	275
	7	49	57	73	88	119	150	181	212	243	274	305	336
	8	63	73	92	111	149	188	226	264	302	340	379	417
	10	100	114	141	169	224	279	335	390	445	500	555	611
	12	147	166	204	243	319	395	472	548	625	701	777	854
	14	209	234	285	336	437	539	641	742	844	945	1047	1149
5	6	41	47	61	75	102	130	157	184	212	239	267	294
	7	52	60	76	93	125	158	191	224	257	289	322	356
	8	65	75	94	114	153	192	231	270	309	348	387	426
	10	100	114	141	168	223	277	331	386	440	495	549	603
	12	146	165	202	239	314	388	463	537	611	686	760	835
	14	207	231	280	329	427	525	623	721	819	917	1015	1113
6	7	53	62	79	96	131	165	200	234	268	303	337	372
	8	66	76	96	116	156	196	236	276	316	356	396	436
	10	100	113	140	167	220	274	327	380	434	487	541	594
	12	146	164	200	236	308	381	453	526	598	670	743	815
	14	206	219	277	325	420	516	611	706	802	897	993	1088
	16	281	311	373	434	556	679	801	923	1046	1168	1291	1413
8	7	57	67	85	104	141	178	215	252	289	326	363	400
	8	70	81	102	124	166	209	251	294	337	379	422	464
	10	104	118	145	173	228	284	339	395	450	505	561	616
	12	147	165	200	236	308	379	450	522	593	664	736	807
	14	206	230	276	323	416	510	601	696	790	883	977	1070
	16	281	310	370	430	549	669	788	908	1027	1146	1266	1385
	18	367	405	480	554	704	854	1003	1153	1302	1452	1602	1751
10	8	74	85	108	131	176	221	266	311	357	402	447	492
	10	107	121	150	179	236	294	351	408	466	523	581	638
	12	148	166	201	236	306	377	447	518	588	658	729	799
	14	207	229	275	321	412	504	595	686	778	869	961	1052
	16	290	300	368	426	542	659	775	891	1008	1124	1241	1357
	18	366	402	475	548	693	839	984	1129	1275	1420	1565	1711
12	10	110	125	154	183	242	301	359	418	476	535	594	652
	12	151	168	204	239	310	381	452	523	594	665	736	807
	14	206	228	272	317	405	493	581	669	758	846	934	1022
	16	279	306	362	418	529	640	751	862	974	1085	1196	1307
	18	364	399	469	540	680	820	960	1100	1241	1381	1521	1661

Table 5—(Continued.) (Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft	Length. 200 Ft.
Ft.	Ft	Cub Y	Cub Y	Cub Y	Cub Y	Cub Y	Cub Y.	Cub Y	Cub Y	Cub Y.	Cub Y	Cub Y	Cub Y.
15	12	162	182	222	262	342	422	502	583	663	743	823	903
	14	215	239	286	333	427	522	616	711	805	899	994	1088
	16	285	313	370	427	541	654	768	882	996	1110	1223	1337
	18	369	404	474	545	686	826	967	1108	1249	1390	1530	1671
	20	473	515	600	685	855	1024	1194	1364	1534	1704	1873	2043
	22	595	646	748	850	1054	1258	1462	1666	1870	2074	2278	2482
20	14	237	264	317	371	478	586	693	801	908	1015	1123	1230
	16	304	335	397	458	582	706	829	953	1076	1200	1324	1447
	18	381	416	486	556	697	838	978	1119	1259	1400	1541	1681
	20	479	520	601	682	844	1007	1169	1332	1494	1656	1819	1981
	22	598	646	741	837	1028	1220	1411	1603	1794	1985	2177	2368
	24	739	795	908	1021	1247	1473	1699	1925	2151	2377	2603	2829
25	16	327	360	428	496	631	766	901	1036	1172	1307	1442	1577
	18	403	441	517	594	746	898	1050	1202	1355	1507	1659	1811
	20	500	543	629	715	887	1059	1231	1403	1575	1747	1919	2091
	22	614	663	760	857	1051	1246	1440	1635	1829	2023	2218	2412
	24	751	807	919	1031	1255	1479	1703	1927	2151	2375	2599	2823
	26	909	974	1104	1234	1494	1754	2014	2274	2534	2794	3054	3314
	28	1085	1160	1310	1460	1760	2060	2360	2660	2960	3260	3560	3860
35	22	685	743	859	975	1207	1439	1671	1903	2135	2367	2599	2831
	24	817	880	1007	1134	1388	1642	1896	2150	2404	2658	2912	3166
	26	969	1033	1181	1309	1585	1861	2137	2413	2689	2965	3241	3517
	28	1130	1205	1356	1507	1809	2111	2413	2715	3017	3319	3621	3923
	30	1327	1408	1571	1734	2060	2386	2712	3038	3364	3690	4016	4342
	32	1519	1639	1820	2001	2363	2725	3087	3449	3811	4173	4535	4897
	35	1946	2054	2271	2488	2922	3356	3790	4224	4658	5092	5526	5960
50	30	1494	1594	1795	1996	2398	2800	3202	3604	4006	4408	4810	5212
	32	1711	1819	2035	2251	2683	3115	3547	3979	4411	4843	5275	5707
	34	1956	2071	2302	2533	2995	3457	3919	4381	4843	5305	5767	6229
	36	2228	2350	2597	2844	3338	3832	4326	4820	5314	5808	6302	6796
	38	2519	2650	2913	3176	3702	4228	4754	5280	5806	6332	6858	7384
	40	2835	2975	3255	3535	4095	4655	5215	5775	6335	6895	7455	8015
	42	3197	3347	3647	3947	4547	5147	5747	6347	6947	7547	8147	8747
	45	3818	3991	4337	4683	5375	6067	6759	7451	8143	8835	9527	10219
	50	5063	5281	5717	6153	7025	7897	8769	9641	10513	11385	12257	13129

Art. 9. Especial pains should be taken to **secure an unyielding foundation for culverts and drains under high embkts;** otherwise the superincumbent weight, especially under the middle of the embkt, may squeeze them into the soil below, if soft or marshy; and thus diminish the area of waterway, or at least cause an ugly settlement at the midlength of the culvert. Also, in soft ground, the embkt may press the side walls closer together, narrowing the channel This may be prevented by an inverted arch, or a bed of masonry, between the walls. A stratum from 3 to 6 ft thick, of gravel, sand, or stone broken to turnpike size, will generally give a sufficient foundation for culverts in treacherous marshy ground; or quicksand, with but a moderate height of embkt. It should extend a few feet beyond the masonry in every direction, and should be rammed; the sand or gravel being thoroughly wet, if possible, to assist the consolidation. Piling will sometimes be necessary. If the masonry is built upon timber platforms, or a smooth surface of rock, care must be taken to prevent it from sliding, from the pres of the earth behind it. This same pres may even overthrow the piles, if they are not properly secured against it.

Art 10. Drains. Drains of the dimensions in Fig 11, contain 1 perch, of 25 cub ft; or .926 of a cub yd, per ft run.

They are frequently built of dry scabbled rubble, and paved with spawls. When there is much wash through them, with a considerable slope, it is better to continue the foundation

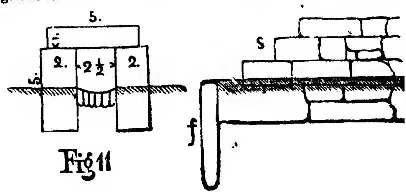


Fig 11

solid clear across. This is often done without those causes, inasmuch as the additional masonry is a mere trifle; and the excavation of a single broad foundation pit is less troublesome than that of two narrow ones. A deep flag-stone *f* at the entrance, and others at short dists of the length, may be introduced in both drains and culverts, to protect from undermining.

These drains extend under the entire width of the embkt, from toe to toe, and may terminate in steps, as in the side view at S. They are of course better when built with mortar, with an admixture of cement to prevent the water when full from leaking into and softening the embankment.

Sometimes two or three such drains may be placed parallel to each other instead of a culvert. When two are so placed, they contain only $1\frac{1}{2}$ times the masonry of one, still their use will generally involve no saving of masonry over a culvert. A man can crawl through Fig 11 to clean it.

Art. 11. The drainage of the roadways *of stone bridges of several arches, is generally effected by means of open gutters, which descend slightly from the crowns of the arches, each way, until they reach to near the ends of the respective spans.*

There they discharge into vertical iron pipes built into the masonry. The upper ends of the pipes should be covered by gratings. When inconvenience would result from the water falling upon persons passing under the arches, these pipes may be carried down the entire height of the piers; but when such is not the case, they may extend only to the soffit, or under face of the arch; allowing the water to fall freely through the air from that height.

Table 6, of approximate contents, in cub yds, of a solid pier of masonry, 6 ft by 22 ft on top; and battering 1 inch to a ft on each of its 4 faces. The contents of masonry of such forms must be calculated by the prismoidal formula, and not by taking the length and breadth of the pier at half its height as an average length and breadth, as is sometimes done. This incorrect method would give only 6492 cub yds as the content of the pier 200 ft high; instead 7178 yds, its true content. High piers may for economy be built hollow, with or without interior cross walls for strengthening them, as the case may require; and the batter is generally reduced to ½ inch or less to a foot. Hollow piers require good well bedded masonry.

(Original.)

Ht. Ft.	Lgth at base	Bdth at base	Cubic yards	Ht. Ft.	Lgth at base.	Bdth at base	Cubic yards	Ht. Ft.	Lgth at base	Bdth at base.	Cubic yards
6	23.	7.	32.5	52	30.67	14.67	537	128	43.33	27.33	2759
7	.17	.17	38.6	54	31.	15	570	130	.67	.67	2848
8	.33	.33	44.9	56	.33	33	605	132	44.	28.	2940
9	23.5	7.5	51.3	58	.67	.67	641	134	.33	.33	3032
10	.67	.67	58.	60	32.	16.	679	136	67	.67	3126
11	.83	.83	64.8	62	.33	.33	717	138	45.	29.	3222
12	24.	8	71.7	64	.67	.67	757	140	.33	.33	3320
13	.17	.17	79.	66	33.	17.	798	142	.67	.67	3420
14	.33	33	86.4	68	.33	.33	840	144	46	30	3521
15	24.5	8.5	94.	70	.67	67	884	146	.33	.33	3623
16	.67	.67	102	72	34.	18.	928	148	.67	.67	3728
17	.83	.83	110	74	.33	.33	973	150	47.	31.	3835
18	25.	9.	118	76	67	.67	1021	152	33	.33	3944
19	.17	.17	127	78	35.	19.	1070	154	67	.67	4056
20	.33	.33	135	80	.33	.33	1120	156	48.	32.	4168
21	25.5	9.5	144	82	67	.67	1171	158	.33	.33	4284
22	.67	.67	153	84	36.	20.	1224	160	67	67	4402
23	.83	.83	163	86	.33	.33	1278	162	49.	33.	4520
24	26.	10.	172	88	.67	.67	1334	164	.33	33	4640
25	.17	.17	182	90	37.	21.	1392	166	.67	.67	4763
26	.33	33	192	92	.33	.33	1451	168	50	34.	4887
27	26.5	10.5	202	94	.67	.67	1510	170	33	.33	5014
28	.67	.67	212	96	38.	22.	1569	172	.67	.67	5143
29	.83	.83	223	98	.33	.33	1631	174	51.	35.	5275
30	27.	11.	234	100	67	.67	1695	176	33	.33	5409
31	.17	.17	245	102	39.	23.	1761	178	.67	.67	5545
32	.33	.33	256	104	.33	.33	1829	180	52.	36.	5680
33	27.5	11.5	268	106	.67	.67	1899	182	.33	.33	5820
34	.67	.67	280	108	40.	24.	1968	184	.67	.67	5962
35	.83	.83	292	110	.33	.33	2041	186	53.	37.	6106
36	28.	12	304	112	.67	.67	2115	188	.33	.33	6252
38	.33	33	329	114	41.	25.	2191	190	.67	67	6400
40	.67	67	356	116	.33	.33	2269	192	54.	38.	6552
42	29.	13.	383	118	.67	.67	2346	194	.33	.33	6704
44	.33	.33	411	120	42.	26.	2424	196	.67	.67	6859
46	.67	.67	441	122	.33	.33	2504	198	55.	39	7016
48	30.	14.	472	124	.67	.67	2587	200	.33	.33	7178
50	.33	.33	504	126	43.	27.	2672	202	.67	.67	7339

Art. 12. Brick Arches. Since even good brick fit for large arches has far less crushing strength than good granite or limestone, and is inferior even to good sandstone, while its weight does not differ very materially from stone, it is plain that it cannot be used in arches of as great span as stone can. Some of those already built, and which have stood for many years, have a theoretical co-efficient of safety of but about 3; whereas the authorities direct us not to trust even stone with more than one-twentieth of its crushing load. This last, however, appears to the writer to be one of those hasty assumptions which, when once admitted into professional books, are difficult to be got rid of. It is his opinion that with good cement, and proper care in striking the centers, one-tenth of the ultimate strength is sufficiently secure against even the abnormal strains caused by the settling at crown, and rising at the haunches when the centers are struck. It is useless to attempt to fix limits of safety for bad materials poorly put together.

Rem. 1. The common practice of building brick arches in a series of **concentric rings**, as at *a c e e*, Fig 12, with no other bond between them than that afforded by the mortar, is censured by authorities, on the ground that the line of pressure in passing from the extrados to the intrados tends to separate the rings, and thus weaken the arch by, as it were, splitting it longitudinally. The reason for using these rings, instead of making the radial joints continuous throughout the depth *m n* of the arch, as at *b*, is to avoid the thick mortar-joints at the back of the arch, and shown in the Fig. If the center of an arch built as at *b* be struck too soon, the soft mortar in these thick joints will be so much compressed as to cause great settlement at the crown, throwing the arch out of shape, and creating such inequality of pressure as might even lead to its fall, especially if flat. As a compromise between rings and continuous joints, they are sometimes employed together, so as to get rid of **some** of the long radial joints; and at the same time to break **at intervals** the continuity of the rings. Thus in Fig 12, which is supposed to be brick-and-a-half deep, beginning at the abutment *a*, we may lay half-brick rings as far as say to *e o e;* then **cutting away** the brick *o* to the line *e e*, we may lay from *e e* to *m n* a block of bricks with continuous radial joints, the same as at *b;* and then start again with three rings; and so on alternately. A still better, but more expensive, mode would be to fill *e e*, *m n* with a regular cut-stone voussoir.

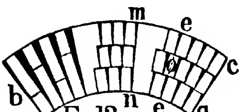


Fig.12.

The proper intervals for changing from rings to blocks will depend upon the number of the rings and the depth *c a* of the arch; reference being also had to reducing the amount of brick cutting as much as possible.

These points can be best decided on from a drawing of a portion of the arch on a scale of 3 or 4 ins to a foot. Generally the rings are made only half-brick, or about 4 to 4.5 ins thick, as at *a c;* and in Brunel's Maidenhead viaduct of two elliptic brick arches of 128 ft span, and 24 25 ft rise; the boldest brick arches yet attempted; but which have been estimated to have a co-efficient of safety of but three against crushing at the crown

So many others of from 70 to 100 ft span have been successfully built entirely in rings of either half or whole brick thick, as to justify us in attaching but little weight to the above theoretical objection, provided first class cement be used, and time allowed it to become nearly or quite as hard as the bricks themselves, before striking the centers. Under such circumstances we should not object to a series of rings even 1.5 bricks thick, laid alternately header and stretcher, as at *b*.

If the bricks were voussoir-shaped, that is, a little thicker at one end than the other, then rings a whole-brick thick could be used without any increase in thickness of mortar-joint at the back of each ring. Still with more than one ring, the radial joints would not be continuous, as at *b*, but broken as at *ac*. Such bricks however would be more expensive to make; and moreover, in order fully to answer the intended purpose, they would have to be made of many patterns, so as to conform to the many radii used in arches, and even to the radii of the different rings, when the depth of the arch required several of them.

Rem. 2. Wet the bricks before laying.

Rem. 3. When the ends or faces of a brick arch are to be finished with **cut-stone voussoirs,** these had better not be inserted until some time after the completion of the brickwork, the hardening of the mortar, and a partial easing of the centers; lest they be cracked or spawled by the unequal settlements of themselves and the bricks.

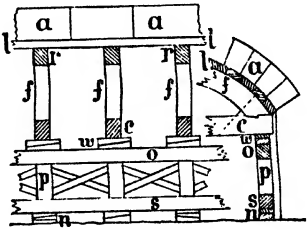
Rem. Brick arches, from their great number of joints are apt to settle much more than cut stone ones when the centers are removed, and thereby to derange the shape of the arch, and at times, without due care, even to endanger its safety, especially if it be large and flat. When the span exceeds about 30 to 35 ft, and particularly if flat, use only brick of superior quality in good **cement** mortar. With even best materials and work we advise the young engineer not to attempt brick arches for railroad bridges of greater spans than about the following. Considerably larger ones than some of them have been built, and have stood; but their coefs of safety are not in all cases satisfactory. In this table the rise is in parts of the span.

R.	S.	R.	S.	R.	S.	R.	S.	R.	S.
.5	100	1/3	88	.225	68	1/6	50	.134	35
.4	97	.29	82	1/5	60	.155	45	1/8	30
.36	93	1/4	75	.183	55	1/7	40		

On the **Filbert Street Extension of the Penna R R, in Phila,** are four brick arches of 50 ft 1 inch span, and with the very low rise of 7 ft. They are 2 ft 6 ins thick, except on their showing faces, where they are but 2 ft. The joints are in common mortar, and about 1/4 inch thick. These four arches, about 200 yards apart, with a large number of others of 26 ft span, form a viaduct. The piers between the short spans are 4 ft 3 ins thick. Those at the ends of the 50-ft spans, 18 ft 6 ins. The springing lines of all the arches are about 6 to 8 ft above the ground. One of the 50-ft arches settled 3 ins upon prer at rely striking the centers; but no further settlement has been observed, although the viaduct has, since built (1880) had a very heavy freight and passenger traffic, at from 10 to 20 miles per hour. Roadbed, about 100 ft wide, giving room for 9 or 10 tracks.

CENTERS FOR ARCHES.

Art. 1. A center is a temporary wooden structure (built lying flat, on a full size drawing, on a fixed platform, under cover or not) for supporting an arch while it is being built. It consists of a number of trusses or **frames**, *f*, *f*, Fig. 1, placed from 1 to 6 ft apart from cen to cen, and covered with a flooring *l*, *l*, of rough boards or planks, usually laid close, and called the **sheeting** or **lagging**, immediately upon which the archstones are laid. In Fig 3, the lagging is not laid close. There is no great economy in placing the frames very far apart, on account of the greater required amount of lagging, the thickness of which increases rapidly. For the thickness of lagging see Rem 9

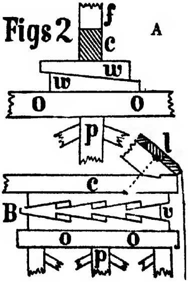


Figs 1.

The centers rest by the ends of their chords, *c*, upon wooden **striking wedges** *w*, Fig 1, supported by **standards** composed of posts *p*, whose tops are connected by **cap-pieces** *o*; and whose feet rest on **stringers** *s*; the whole being braced diagonally as shown.

If the ground is very firm, and the arch light, the standards may rest on it, with the interposition of **adjusting-blocks**, *n*, below the stringer, to accommodate irregularities of the surface of the ground, as in the Fig. These blocks shoul be somewhat double-wedge-shaped, so that by driving them the standard ma be raised at any point in case it should settle a little into the ground. But fo heavy arches the standards must rest on a much firmer foundation, such as shor blocks of brickwork sunk a few feet into the ground, or some other devic adapted to the case. Frequently projecting offsets or footings, or at times re cesses, are provided in the masonry of the abutments and piers for this expres purpose; and with a view to this it is well to design the center at the same tim as the arch.

Figs 2



Up to spans of 50 or 60 ft a single row of posts (one under each end of each frame) will suffice; but for much larger ones two or three rows, 2 or more feet apart may become expedient, as in the lower Fig 2.

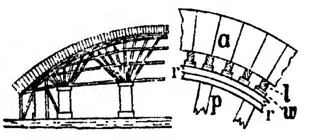
The striking or lowering-wedges before alluded to are for striking or lowering the center after the completion of the arch. They consist of pairs of wedge-shaped blocks, *w w*, at A, Figs 2, of hard wood, from 1 to 2 ft long, about half as wide, and a quarter or more as thick, (sufficient to lower the center from say 2 to 6 or more inches, according to span and other circumstances,) resting on the cap *o*, of the standard, while the chord *c* of the frame rests on *them*. When the end of a frame is supported by two or more posts *p*, as at B, Fig 2, instead of upon one, the striking-wedges are sometimes made as there shown; and where B *v* is one long wedge at right angles to the abutment, and acting as four wedges which may all be lowered together by blows against the end B.

each so long as to reach **transversely** across the entire arch. Then all the frames can be lowered at one operation, as described near end of Art 9.

If we had to consider only the friction of dry wood against dry wood, the taper of these wedges might be as steep as 1 vert to 3 hor, without any danger of their sliding upon each other of their own accord; and they would then require very moderate blows to start them, or even to entirely separate them, when the center had finally to be lowered. But it is of the utmost importance, especially in large arches, that the **centers should be lowered very slowly,** otherwise the momentum acquired by so heavy a body as the arch in descending suddenly even but 2 or 3 ins, might possibly affect its shape, or even its safety.

Therefore the wedges should not have a taper steeper than about 1 in 6 or 8 for arches of less than about 50 ft span; or than 1 in 8 or 10 for larger spans. Vertical lines at equal dists apart should be drawn on the long sides of the wedges as a guide for lowering them all to the same extent at a time; and this should not exceed in all about half an inch a day in intervals of about an eighth of an inch, for 50 ft spans; or about .1 to .25 of an inch per day in all, for spans over 100 ft. Slowness is especially to be recommended in **brick arches,** not only because their greater number of joints exposes them to greater derangement of shape, but because even good brick has much less than the average crushing strength of good granite, limestone, or sandstone, and therefore is far more liable than they to crack, or even to crush (as the writer has seen) when the strains are thrown almost entirely upon their edges, as described in Art 3.

At Gloucester Bridge, England, of first class cut stone, span 150 ft, rise 35 ft, the centers were entirely struck within the very short space of 3 hours, and the crown of the arch descended 10 ins! **At Grosvenor Bridge,** England, of first class cut stone, span 200 ft, rise 42 ft, such care was taken in easing the centers that the crown of the arch settled but 2.5 ins. This case however was marked by two or three peculiarities, all of which contributed to this favorable result. Namely, the center instead of being a series of frames supported as usual by their ends, and of course involving an appreciable, although small, degree of sagging or settlement, consisted essentially of vertical and inclined posts or struts, see Fig 3, footing on four temporary piers of masonry, 7 or 8 feet thick, built in the river, parallel to the abutments, and as long as they. These piers supported six frames (or rather six series) about 7 ft apart cen to cen, of such struts, footing on cast iron shoes. Fig 3 shows half of one series. Each frame or series consisted of four fan-like sets of posts, all in the same vertical plane. The long horizontal pieces seen extending from side to side of the arch were bolted to the struts to increase their stiffness, and other pieces for the same purpose united the six series transversely. Here each strut sustains its own share of the weight of the archstones, and transfers it directly to the unyielding foundation of the pier; whereas in the usual trussed centers, the entire load rests upon the frames, and is finally transferred to the comparatively unstable support of the posts at their ends.



Figs 3.

The tops p of the posts of a series varied about from 5 to 8 ft apart cen to cen; and were connected by a continuous curved rib, r r, of two thicknesses of 4 inch plank, bent to conform approximately to the curve of the arch. On this rib were placed pairs of striking-wedges w like Fig 2, about 16 ins long, 10 to 12 ins wide, and tapering 1.5 ins, so near together (varying about from 2.5 to 3.5 ft cen to cen) that there was a pair under each joint of the archstones, a a. On these wedges, and extending over all six of the frames, were the lagging pieces l, 4.5 ins thick.

This peculiar arrangement of the striking-wedges and lagging has, in large spans, great advantages over the usual one of placing them only at the ends of the frames. In the last the entire center and the entire arch are lowered together, without giving an opportunity to rectify any slight derangements of shape or inequality of bearing that may have occurred in the arch during its construction. This center, designed by Mr. Trubshaw, admits of lowering either the whole equally, or any one part a little more or less than the others. He had much experience in large arches, and stated that during the striking he found that he had an arch under better control, or could humor it better, by keeping the haunches a little down and the crown a little up, until near the end of

Rem. 1. Instead of piers of masonry for supporting the feet of the posts, wooden cribs or piles may often be used if the arch is over water.

The principle of supporting even trussed frames by struts at points of the chord as far from the abutments as circumstances will admit of (in addition to those at the very ends) should always be applied when possible, in order to reduce their sagging to a minimum. **Steps or offsets in the masonry** of the abutments and piers may be provided for receiving the feet of such struts, when they are inclined.

Rem. 2. Screws may be used instead of wedges for lowering centers. At the Pont d'Alma, Paris, ellipse of 141.4 ft span, and 28.2 ft rise, the frames were supported by wooden pistons or plungers, the feet of which rested on **sand confined in plate-iron cylinders** 1 ft in diam and height, and having near the bottom of each a plug which could be withdrawn and replaced at pleasure, thus regulating the outflow of the sand and the descent of the center. This device succeeded perfectly, and is well worthy of adoption under arches exceeding about 60 ft span. When much larger than this the driving of the wedges on striking requires heavy blows, and becomes a somewhat awkward operation, requiring at times a battering-ram, even when the wedges are lubricated. In railroad cuttings crossed by bridges, **the earth under the arch** has been made to serve as a center, by dressing its surface to the proper curve, and then embedding in it curved timbers a few feet apart, and extending from abut to abut, for supporting the close plank lagging.

Rem. 3. All centers must yield or settle more or less under the wt of the arch, especially when supported only near their ends; and since the arch itself also settles somewhat not only when the centers are struck, but for some time after, it is advisable to make them at first a little higher than the finished arch is intended to be. This extra height, when the supports are at the ends, may be from 2 to 4 ins per 100 ft of span for cut stone arches (according to time of striking, character of masonry, workmanship, etc.), and about twice as much in brick ones.

Rem. 4. The proper time for striking centers is a disputed point among engineers, some contending that it should be done as soon as the arch is finished and sufficiently backed up, and others that the mortar should first be given time to harden. It is the writer's opinion that inasmuch as in cut-stone arches the mortar joints should be very thin; and since, in such, the mortar is at best of very little service, it is of no importance when they are struck; provided the masonry backing, and the embkt up to $y\ n$ Fig 2, p 618, have been completed; but that in brick or rubble, the numerous joints of both of which require much mortar, (which for hardness should consist largely of cement,) 3 or 4 months, or longer, if possible, should be allowed it to harden sufficiently to prevent undue compression and consequent settlement when the centers are struck. The continuance of the centers need not interfere with traffic over the bridge.

Art. 2. The pressure of archstones against a center is very trifling until after the arch is built up so far on each side that the joints form angles of 25° or 30° with the horizontal. Theoretical discussions on this pressure make no allowance for accidental jarrings in laying the archstones, or by the accumulation of material ready for use, laborers working on it, &c. Without going into any detail, we merely advise on the score of safety not to assume it at less than about the following proportions or ratios to the weight of the entire arch, namely, in a semicircular arch .47, rise .35 span, .61; rise .25 span, .79; rise .2 span, .86; rise .167 span, or less, 1, or equal to the wt of the arch. This gives the pressure of a semicircular arch upon its centers rather less than half its wt. **The wt of the centers themselves** when supported only near the ends must be considered as part of the load borne by them.

Art. 3. We have seen that as an arch $a\ a\ a$ is being gradually built upward on both sides, after passing the points e, e, Fig 4, where its joints form angles $a\ s\ e$, of about 30° with the horizontal $a\ a$, the arch begins to press more and more upon the centers; thereby tending to flatten them at the haunches, as shown at h in the dotted line; and consequently to raise them at the crown, as shown at c. But as the building goes on still higher, the added stones press much more heavily upon the centers than those below had done, and thereby tend to a final derangement of the centers just the reverse of that caused by the lower ones; namely to depress them at the crown a, as at o; and consequently to raise the haunches as at n; and this the more because the upper stones actually tend to lift or ease the lower ones from the lagging. In some cases where this tendency has been increased by **forcing** the keystones into place by too hard driving, the lagging under the haunches could be drawn out without any trouble before the centers were eased at all. On striking the centers this tendency to sink at crown and

rise at haunches is very apt to exhibit itself more or less dangerously in the arch-stones themselves, as in Fig 5, causing those near the crown to press very hard together at the extrados, and to separate from each other at the intrados; while near the haunches the reverse takes place. Hence the angles of the stones are frequently split and spawled off near *c* and *h* by this unequal pressure. These

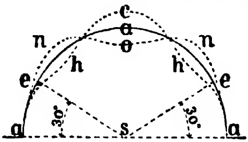


Fig. 4.

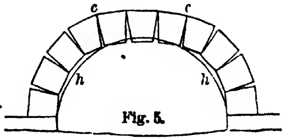


Fig. 5.

derangements are of course much more likely to be serious in high arches than in flat ones, especially if their spandrels are not sufficiently built up before lowering the centers

In the Grosvenor bridge, before alluded to, of 200 ft span, this dangerous excess of pressure near *c* and *h* was prevented by covering the skewback joint of the springing course at each abutment with a wedge of lead 1.5 ins thick at the intrados of the arch, and running out to nothing at the extrados. Beside this a strip 9 ins wide of sheet lead was laid along the intrados edge of every joint until reaching that point at which it was judged that the line of pressure would pass from the intrados to the extrados; after which similar strips were laid along the extrados edges of the joints, up to the crown. Hence when the centers were struck, this excess of pressure merely compressed the lead, and was thus enabled to distribute itself more evenly over the entire depth of the joints. See Trans Ins Civ Eng London, vol i. See also top of p 1213

At the bridge at Neuilly, France (of 5 elliptic arches of 120 ft span, and 30 ft rise), the centers were so radically defective in design that the arches sank 13.25 ins at crown during the time of building; and 10.5 ins more during and immediately after the striking; or say 2 ft in all Their construction made the striking very tedious and hazardous; greatly endangering the lives of the workmen and the existence of the arches. Some of the joints at the extrados at the haunches opened an inch each; and those at the intrados of the crown .25 of an inch By the exercise of great care and humoring in lowering the centers, these openings were much reduced.

Rem. 1. Chamfering the edges of the archstones diminishes the danger of their spawling off from unequal pressure, as does also the **scraping out of the mortar of the joints** for an inch or two in depth before striking the centers.

Rem. 2. It is evident that in order to prevent, or at least to diminish the alternate derangements of the center, those of its web members which at first acted as **struts** near the haunches, Fig. 4, to prevent them from sinking as at *h* must afterwards act as **ties** to prevent them from rising as at *n*; while those which at first acted as ties near the crown *a*, to prevent it from rising as at *c*, must afterwards act as struts to prevent it from sinking as at *o*. In other words, the principle of **counterbracing** must be attended to as well in a frame or truss for a center, as in one for a bridge.

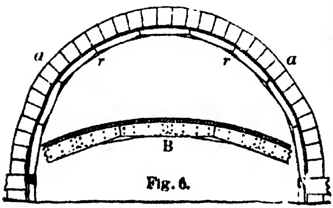


Fig. 6.

Art. 4. From the foregoing it is plain that **a simple unbraced wooden**

arch, or curved rib is, on account of its great flexibility, about as unfit a form as could be chosen for a center, except for very small spans, where a great proportional depth of rib can be readily secured. Still the writer has seen it used for a cut-stone semicircular arch of 35 ft span, with archstones 2 ft deep. Fig 6 shows one rib *r r*, and the arch, *a a*, drawn to a scale. Each rib consisted of two thicknesses of 2 inch plank in lengths of about 6.5 ft, treenailed together so as to break joint, as at B. Each piece of plank was 12 ins deep at middle, and 8 ins at each end; the top edge being cut to suit the curve of the arch. The treenails were 1.25 ins in diam; and 12 of them showed to each length. These ribs were placed 17 ins apart from cen to cen, and steadied together by a bridging piece of inch board, 13 ins long, at each joint of the planks, or about 3.25 ft apart. Headway for traffic being necessary under the arch, there were no chords to unite the opposite feet of the ribs. The ribs were covered with close board lagging, which also assisted in steadying them together transversely. As the arch approached about two-thirds of its height on each side, the ribs began to sink at the haunches, as at *h*, Fig 4; and to rise at the crown, as at *c*. This was rectified by loading the crown with stone to be used in completing the arch; which was then finished without further trouble.

A still more striking example of the use of a simple unbraced wooden rib, was in the old National Turnpike bridge over Wills Creek, at Cumberland, Md. This bridge, of which one arch with its center is shown in Fig 7 drawn to a scale, consisted of two elliptic cut stone arches 26.5 ft wide across roadway, and of 60 ft span, and 15 ft rise. The archstones were 3 ft deep at crown, and 4 ft deep at skewbacks. **Each frame of the center** was a simple rib 6 ins thick, composed of three thicknesses of 2 inch oak plank in different lengths (about 7 to 15 ft) to suit the curve and at the same time to preserve a width of about 16 ins at the middle of each length, and 12 ins at each of its ends. The thicknesses were well treenailed together, breaking joint and showing from 10 to 16 treenails to a length.

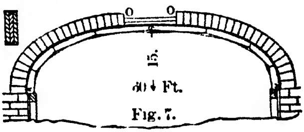


Fig. 7.

Here, as in Fig 6, there were no chords, owing to the violence of the floods in the creek. These ribs were placed 18 ins from cen to cen, and steadied against one another by a board bridging-piece 1 ft long, at every 5 ft. These were of course assisted by the lagging.

When the archstones had approached to within about 12 ft of each other near the middle of the span, the sinking at the crown, and the rising at the haunches had become so alarming that pieces of 12 × 12 oak, 00, were hastily inserted at intervals, and well wedged against the archstones at their ends. The arch was then finished in sections between these timbers, which were removed one by one as this was done.

Rem. 1. Such instances of partial failure are very instructive. It is indeed by such, rather than by theoretical deductions, that the proper dimensions are arrived at in a vast number of cases pertaining to engineering, machinery, &c.* Thus we might with entire confidence of no serious mishap, apply ribs of the foregoing dimensions to spans only half as great.

Rem. 2. Assuming the rib-planks to be 12 ins wide, it would, as a matter of detail, be better to make them about 10 ins wide at the ends instead of the 8 ins in Fig 6 making top curve 2 ins. To secure this, their lengths, depending on the **radius** of the rib, must not exceed those in **the following table:**

Rad of Arch.	Greatest Length.	Rad of Arch.	Greatest Length.
Feet.	Feet and Ins.	Feet.	Feet and Ins.
5	2 " 5	30	6 " 4
10	3 " 4	35	7 " 0
15	4 " 2	40	7 " 6
20	5 " 0	45	7 " 10
25	5 " 9	50	8 " 2

* The young engineer should make and preserve full notes in detail of all such as may fall within his notice.

If cut $1\frac{1}{2}$ times as long as this table, they will be very approximately 8 ins wide at ends; or each will on top curve 4 ins.

Art. 5. In cases where all possible headway is essential during the building of the arch, as in the two foregoing ones, the writer would suggest the expedient rudely illustrated by Fig 8; namely to **place the centers above the arch,** instead of **below** it; and after the arch is completed in sections, *a a*, instead of **lowering** the centers, **to take them apart.**

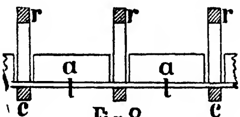


Fig 8.

Fig 8 is a transverse section through part of the center, and of the arch *a a*. Here *r c, r c, r c*, are frames of the center say 5 or 6 ft apart; and of any depth and construction whatever that may be necessary to insure absolute safety; and *l l* is the lagging Having built the arch from abutment to abutment in a series of sections *a, a, a*, necessarily separated say a foot or more by the deep frames, we may take the centers apart, and then fill in the narrow intermediate sections upon a lagging suspended by iron rods from the already completed sections Good concrete might be used for these narrow sections. In some cases it might be well to use **deep plate-iron ribs** of I section, resting the lagging on the lower flange Part of the web might be left remaining embedded in the masonry, and the upper part and both flanges removed after the arch is finished.

Art. 6. Centers with hor chords *c c* Fig 9 are objectionable (notwithstanding their strength) in large spans of great rise, as on right side of the Fig, on account of the excessive length required for the web members; and hence it will in such cases usually be found expedient to adopt something analogous to what is shown on the left hand of the Fig Here a truss *f*, shorter and shallower than that on the right hand, is substituted for the latter. At its ends provision must be made for supporting not only itself, but the archstones below it. As the pressure of these lower archstones is comparatively small, this may usually be effected by resting the end of the frame

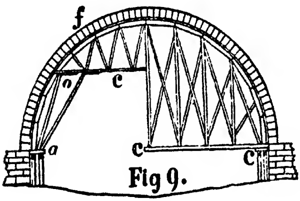


Fig 9.

f upon another and shallower frame *o a*. This may in large spans be aided by either inclined or vertical struts, either single or braced together; or as the trestles on p 1037 Sometimes one shallow truss like *f* is sustained upon another truss throughout its entire length. The striking-wedges for these various supports may be placed at either their tops or their feet, as may be most convenient.

Art. 7. For flat arches of 10 feet clear span, a mere board *o o* Fig 10, 12 ins deep, by 1.5 ins thick, with another piece *c* of the same thickness on top of it, trimmed to the curve, and confined to *o o* by nailing on two cleats of narrow board, will answer every purpose, with intervals of 18 ins from cen to cen. If the upper piece also is as much as 12 ins deep at its center, the clear span may be extended to 15 ft.

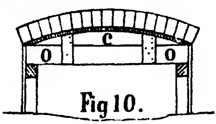


Fig 10.

For spans of 10 to 15 ft, and of any rise, two thicknesses of plank from 1 to 2 ins thick according to span; 8 to 12 ins wide at middle of each piece, in lengths as per table, Rem 2, Art 4, well **nailed** or **spiked** together, according to span, breaking joint as in Fig 6, will answer for distances of 2 to 3 ft apart cen to cen. For greater dists apart increase the thickness of the planks proportionally.

If the centers have to be moved from place to place, to serve for other arches, then, to preserve them from injury in handling, their feet should be united by nailing on one or both sides of each frame a chord piece of about 1

inch board, and also a vertical piece or pieces of the same size from the center of the chord to the top of the frame.

Even when they are not to be moved, the chord pieces are useful even in so small spans, inasmuch as they render the striking easier, by not allowing the feet of the ribs to give trouble by spreading outward and pressing against the abutments

For spans of 15 to 30 ft, and for any rise not less than one sixth of the span, the following dimensions, varying with the span, may be used for distances apart of 3 ft from cen to cen See Fig 11. **For the bow** *b*, two thicknesses of 1 to 2 inch plank from 9 to 12 ins wide at the middle, and from 7 to 10 ins at each end, well spiked together breaking joint as at B, Fig 6. **For the chord** *c*, two thicknesses of plank of same size as the bow at its middle; placed on outsides of bow, and well spiked to its ends **A vertical** *v*, in one piece as wide as a bow plank, and twice as thick Its top is placed under the bow, and is confined to it by two pieces, *o, o*, of bow plank twice as long as the bow plank is deep, and spiked to both *v* and the bow. The foot of *v* passes between the two thicknesses of the chord *c*, and is spiked to them. **Two oblique tie-struts,** *s*, each of two pieces of bow plank, outside of the bow and vertical *v*; footing against each other; and spiked to bow and *v* These with *v* divide the bow into 4 parts.

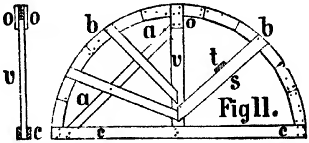


Fig 11.

Rem. 1. The above dimensions are suitable to a rise of one sixth If the rise is one fourth, the **thickness only** of the planks may be reduced one third part; and for a rise of one third or more, we may reduce to one half.

Rem. 2. If in the larger of these spans the struts *s* should show any inclination to bend sideways, nail on some pieces *t* from frame to frame. Also in the larger ones with rises exceeding one third, insert four double struts *s*, instead of two; thus dividing the bow into 6 parts, as at left side of Fig. 11. For spans of 25 to 35 ft, add also two struts like *a a*, of same size as *v*.

Art. 8. For spans greater than about 30 ft, the writer believes that as a general rule (liable to modifications according to the judgment of the engineer in charge) the following ideas will lead to safe practice. Namely, to adopt a bowstring truss with a simple Warren or triangular web, as at *f* on the left side of Fig 9. The bow to rest on the chord, and each to be of a single thickness. The web members (especially in large spans) to be also of single thickness, and placed below the bow, resting on the chords, and well strapped to both, so as to act as either ties or struts. In smaller spans the web members may each be in two thicknesses, one bolted or treenailed to each side of the bow and chord. Other modes will suggest themselves; but we have not space for such details.

Or a web of the Howe, or of the Pratt system, as on the right side of Fig 9 may be used. But in reference to both of these it may be remarked that **the use of long iron rods in centers** of large spans is highly objectionable, owing to the different rates of expansion between iron and wood. Therefore if these systems are used, all the members should be of wood. **The lattice** may be used

Even when the rise of the **arch** exceeds .25 of the span, it is better not to let that of the **centers exceed** that limit; but adopt the expedient shown at the left side of Fig 9, with a rise of about one sixth of the span.

Rem. 1. To fix on the number of web triangles in a Warren truss or frame for a center, find the square root of the span, and to it add one tenth of the span. Divide their sum by 2, and call the quotient *n*. Divide the span by *n*. If this quotient is a whole number use it; or if the quotient is partly decimal, use the whole number nearest to it, as a distance in feet to be stepped off along the chord; thus dividing the chord into a number of equal parts. All the points thus found on the chord, are the places for the **feet** of the triangles Next, from half way between each two of these points, draw vertical lines to the bow. The points thus found along the bow, are the places of the **tops** of the triangles. This rule will be used in connection with the following Table of Areas of Bows, as the two are dependent on each other.

In large arches **the timber of the bow should not be wasted** by trimming its upper edges to the curve of the arch, but should be left straight; and separate pieces so trimmed, like *c* in Fig. 10. should be spiked on top of them.

The transverse area of the bow, in square inches, may be taken from the following table; and may in practice be assumed to be uniform throughout its entire length; which in fact it is quite approximately. See Rem 2.

TABLE FOR BOWSTRING CENTERS.

Table of areas in square inches at the crown of each Bow, of **properly trussed** Bowstring frames for centers of stone or brick arches. The frames to be placed 5 feet apart from cen to cen. With these areas, the combined weights of arch, center (of oak), and lagging, will in no case in the table strain the Bow at crown of the greatest spans quite 1000 lbs per square inch; diminishing gradually to 600 or 700 lbs in the smallest spans, which are more liable to casualties.

Although centers of moderate span are usually made of white or yellow pine, spruce, or hemlock, all of which are considerably lighter than oak, we have for safety assumed them to be of oak, in preparing our table.

For spans of from 10 to 20 feet use the same sizes as for 20 feet.

								Original.
	Rise in parts of the Span.							
	.5	.4	.35	.3	.25	.2	.15	.1
Span in feet.	Areas of transverse section of Bow, in square inches.							
20	14	17	19	21	24	29	38	59
25	18	22	25	28	33	40	53	80
30	23	28	32	37	43	51	71	103
35	28	34	40	45	54	64	87	125
40	34	41	48	55	65	77	106	150
45	40	49	57	65	76	92	126	175
50	47	57	66	76	89	107	146	203
55	53	64	75	87	102	121	166	233
60	60	73	85	99	115	135	187	263
65	68	81	95	110	129	151	209	294
70	75	90	105	122	143	168	233	325
75	83	99	115	133	157	184	256	357
80	91	108	125	145	171	201	279	390
85	99	117	136	157	185	218	302	423
90	108	127	147	169	199	235	325	457
95	115	136	158	181	214	252	348	490
100	123	146	169	194	229	270	372	524
110	133	166	191	219	260	307	420	592
120	155	187	213	246	291	345	470	660
130	172	208	237	274	323	384	520	
140	190	230	263	303	357	424	572	
150	209	252	289	333	393	466		
160	229	276	315	365	430	509		
170	250	299	343	399	469			
180	272	323	373	435	511			
190	294	347	403	472				
200	318	372	435	509				

Rem. 2. The square root of any of these areas gives in inches the **side of a square bow** of that area. The distances apart of the triangles which form the web of the frame, having first been found by Rem 1 (for said Rem and this table are dependent on each other), the above areas for bows 5 ft apart from cen to cen, suffice not only to resist the pressure along the bow, but also, as **square beams**, to sustain with a safety in no case less than about 5, the load of archstones resting upon them between the adjacent tops of two triangles; and with very trifling deflections. It is therefore unnecessary to deepen the ribs for that purpose; although it may be done (preserving the same area) in case considerations of detail should render it desirable.

As before suggested, it will generally be best, in spans exceeding 30 or 40 ft, to give the bow a rise not exceeding about one fifth or one sixth of the span; and to support the frames as at *f*, Fig 9.

The size of the chord may be the same as that of the bow; and like it uniform from end to end; care however being taken that it be not materially weakened by footing the bow upon its ends; or (when too long for single timbers) by the splicing necessary to prevent its being stretched or pulled apart by

the thrust of the bow. When, however, the chord can be placed at, or a little below the springs of the arch, all danger of this kind may be avoided by simply wedging its ends well against the faces of the abutments.

As to the size of the web members, when a bowstring truss is **fully loaded** on top of the bow, (as is approximately the case with a center and its archstones,) the strains on the web members are quite insignificant, and arise chiefly from the weight of the center itself; **but while it is being so loaded,** they are not only greater, but are constantly changing, not only in amount, but also in character—being at one period compressive, and at another tensile.

Hence it would be very tedious to calculate the dimensions of the web members. Fortunately the necessity for doing so is in a great measure obviated by the fact that a center being but a temporary structure, the timber composing it is not ultimately wasted if a greater quantity of it is used than is absolutely required. Moreover facility of workmanship is secured by not having to employ timbers of many different sizes.

Hence the writer will venture to suggest, entirely as a rule of thumb, to **give each web member half the transverse area of the bow,** taking care to make each of them a **tie-strut.**

Rem. 3. As to details of joints, we refer to the Figs on pages 735, 736; merely suggesting here the use of long and wide iron shoes where timbers are subjected to great pressure sideways.

Rem. 4. To prevent the thrust of the bow when its rise is small, from splitting off the ends of the chords, the two may be united by many more bolts than are employed in roof trusses, &c, where only one is generally placed near each end of the chord. But they may when required be inserted at intervals extending to many feet from the ends. They should have strong large washers; and may have about the same inclination as the shortest web member.

Another way of securing the same end in smaller spans, is by completely encasing the two sides of the bow and chord, to a distance of a few feet from their ends, in short pieces of board or plank spiked to both of them, and having about the same inclination as just suggested for bolts.

Rem. 5. Build up both sides of the arch at once, in order to strain the centers as little as possible.

Rem. 6. When a bridge consists of more than one arch, and they are to be built one at a time, there must be at least two centers; for a center must not be struck until the contiguous arches on both sides are finished, for fear of overturning the outer unsupported pier. Therefore if there are but two arches, they must be built at once, requiring two centers.

Rem. 7. Always use supports either vertical or inclined (and provided with striking-wedges) under the frames, and intermediate of the end supports, when possible; even if they can extend out but a few feet from the abutments, as at the left side of Fig 9.

Rem. 8. The weight of large centers and their lagging is greater for flat arches than for high ones of the same span; and also approaches nearer to that of the supported arch.

Rem. 9. Thickness of lagging. The following table gives thicknesses which will not bend more than an eighth of an inch under the weight of any probable archstones **adapted to the respective spans;** and generally not so much.

TABLE OF LAGGING.—Original.

Distance apart of frames, **in the clear.**	Span of center in feet.					
	10.	**20.**	**50.**	**100.**	**150.**	**200.**
	Thickness of close lagging not to bend more than ⅛ inch.					
Feet.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
6	3¼	3⅝	4¼	4¾	5	5¼
5	2½	2⅞	3⅜	3⅝	3⅞	4
4	1⅞	2⅛	2½	2¾	2⅞	3
3	1¼	1⅜	1¾	1⅞	2	2
2	¾	⅞	1	1⅛	1⅛	1¼

With thicknesses three quarters as great as these, the bending may reach a full quarter inch; which may be allowed in dists apart of 3 or more ft.

Rem. 10. Centers are framed, or put together, (like iron bridges) on a firm, level temporary floor or platform, on which a full-size drawing of a frame

first made. As each frame is finished, it is removed to its place on the piers or abuts.

Art. 9. The Wissahickon Bridge of the Reading R R, at Philadelphia, has five arches of 65 ft span, 23 ft rise, 28 ft wide (archstones 3 ft deep, with dressed beds and joints, in cement mortar); with four cutstone piers 9.5 ft thick at top, and from 35 to 50 ft high. It contains about 15400 cub yds of masonry.* **Each center** consisted of 7 frames or trusses of hemlock timber, of the Bowstring pattern, with lattice web-members; and as nearly as may be, of the same span and rise as the arches. They were placed 4.5 ft apart from center to center; and were supported near each end *f*, Fig 13 (a transverse section to scale) by a hemlock post *p*, 12 ins square. **The bow** was of two thicknesses *b b* of hemlock plank, 6 ins apart clear, in lengths of 6 ft, with their upper edges cut to suit the curve of the arch. Each piece was 4 ins thick, by 13.5 ins deep at its middle, and 12 ins at its ends. These pieces did not break joint; but at each joint were four ¾ inch bolts, with nuts and washers, uniting them with chocks or filling-in pieces.

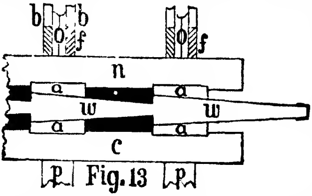


Fig. 13

The bow, *b b*, footed on top of the ends of the chords *f*; and the angle formed by their meeting (seen only in a side view) was (for about 2.5 ft horizontal and 5.5 ft vertical) filled up solid with vertical pieces, to afford a firmer base for resting the frame on *n*; beyond which it extends (in a side view) about 18 ins.

The chords *f* were of two thicknesses of 4 × 12 hemlock plank, 6 ins apart clear, and most of them in two or three lengths, breaking joint, and with two ¾ inch bolts, with nuts and washers, at each joint, for bolting them together, and to filling-in pieces. **The web members** of each frame were 26 lattices, *o*, of 3 × 12 inch hemlock, crossing each other about at right angles, at intervals of about 3.5 ft from center to center, and passing between the two thicknesses *b b* of the bow, and *f f* of the chords. A few of the lattices were in two lengths, and the joints were not at the crossings. The lattices were connected at each crossing by two hard wood treenails 9 ins long, and 2 ins diam; and one such, 18 ins long, passed through the intersection of each end of a lattice with a bow or chord. The first lattice foots about 4 ft from the end of a chord. They do not extend above the top of the bow. All the spaces between the two thicknesses of bow or chord, where not occupied by the ends of lattices, were completely filled by chocks, well spiked.

Each frame contained about 360 cub ft of timber; and weighed about 5 tons. They were very flexible laterally until in place, and braced together by 4 transverse horizontal planks spiked to their chords; and by 5 others above them, spiked to the lattices.

Until the keystones were placed, all the joints of the frames continued tight, under the pressure from the arch, and from the unfinished backing to the height of about 14 ft above the springing line; but after the keystones were set, all the joints of the chords alone opened from .25 to .75 of an inch; and at the same time the lagging under the *haunches* of the arches became slightly separated from the soffit of the masonry.

Each center sank but a full inch at the middle, under the pressure from the arch and 14 ft of backing.

The portion of the bridge above the piers was about two thirds completed before the centers were struck.

There was one wedge *w*, *w*, (32.5 ft long, of 12 × 12 inch *oak*) under each end of a center. It was trimmed to form 7 smaller ones *w*, *w*, each 4.5 ft long, and tapering 7 ins; one under each end of each frame *f*. They played between tapered blocks *a*, *a*, of *oak*, 2 ft long, 1 ft wide, let 1 inch into the cap *c*, or into the piece *n*, on which last the frames *f*, *f*, rested. The sliding surfaces were well lubricated with tallow when put in place.

The wedges were struck with ease, at one end of a center at a time, by an oak log battering-ram 18 ft long, and nearly a ft in diam, suspended by ropes, and swung and guided by 4 men. They generally yielded and moved several inches at the second blow with a 3 or 4 ft swing. Although each wedge was loosened *entirely* within 2 or 3 minutes, thus lowering the centers *very suddenly*, yet on account of the

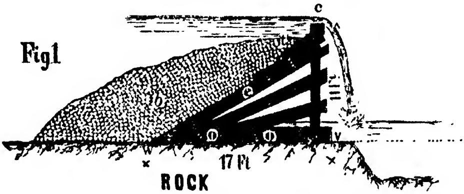
* This bridge, finished without accident, in 1882, reflects much credit on the late William Lorenz, Esq, Ch. Eng; on Mr. Charles W. Buchholz, Assistant in Charge; and on the skilful and energetic contractors, William & James Nolan, of Reading, Penna. These last most cordially assisted the writer in making observations during the entire progress of the work.

good character of the masonry, not the slightest crack of a mortar joint could afterwards be detected in any part of the work. After three days the average sinking of the keystones was only .35 of an inch; the least was ¼: and the greatest ⅝ of an inch. The heads and feet of the posts *p* compressed the hemlock caps *c*, and the sills, about ⅜ of an inch each, showing that for arches of this size the caps and sills had better be of some harder wood, as yellow pine or oak; although probably the compression was facilitated by the large mortices, 3 by 12 ins, and 6 ins deep.

TIMBER DAMS.

Primary requisites, in the erection of dams, are, a foundation sufficiently firm to prevent them from settling, and thus leaking; the prevention of leaks through their backs, or under their bases; and the prevention of wear of the bottom of the stream in front of the dam, by the action of the falling water. For the first purpose, hard level rock bottom is of course the best; and should be chosen, if possible. In that case, thick planks, *tt*, Fig 6, (single or double, as the case may be,) closely jointed, and reaching from the crest, *c*, to the back lower edge *w*, (where they should be scribed down to the rock;) with a good backing, *b*, of gravel, will suffice to prevent leaks. Gravel, or rather very gravelly soil, is far better than earth for this purpose; for if the water should chance to form a void in it, the gravel falls and stops it. To prevent this backing from being disturbed near the crest of the dam, by floating bodies swept along by freshets, a rough pavement of stones, about 15 to 18 inches deep, as shown in Fig 7, should be added for a width of about 10 to 20 feet; or until its top becomes 3 to 5 feet below the crest *c* of the dam, according to circumstances.

Fig 1

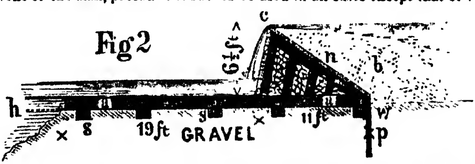


In Fig 1, (a dam on the Schuylkill navigation,) the upper timbers, *e*, are all close jointed, and laid touching, so as not to require planking in addition.

But if the bottom of the stream is gravel or earth, there must in addition to these be used two thicknesses of sheet piles, *p*, Fig 2, &c, close driven, breaking joint, to a depth of several feet, to prevent leaking through the soil beneath the base of the dam. Frequently but one thickness is used. If the bottom is soft or open for a depth of only a few feet, it is at times better to remove it, and base the dam on the firmer stratum below; still, however, using the sheet piles. Old decayed timber and other rubbish should be removed from the base. In very bad soils of greater depth, it may be necessary to support the dam entirely upon a platform resting on bearing piles. Here great precautions are necessary against leaks; but the case occurs so rarely, that we shall not stop to consider it.

As to the wearing away of the bottom of the stream by the water falling over the front of the dam, precautions should be used in all cases except that of very

Fig2



hard rock, or of medium rock protected by a considerable depth of water. The dam, Fig 1, was built upon a tolerably firm micaceous gneiss in nearly vertical strata, covered by about 2 feet of water in ordinary stages. In 30 years the rock was

worn away in front of the dam, as shown in the fig, to the average depth of 3 feet; or very nearly 1 inch per year. The depth of water on the crest c, was usually from 6 to 18 ins, rarely 5 or 6 ft during freshets, and but a few times during the whole period, 8 or 9 ft.

At Jones's dam, on Cape Fear River; height of dam, 16 ft; front vert; fall, usually 10 ft, into 6 ft depth of water, the soft shale rock, in vert strata, was, in the course of a few years, worn away 16 ft, and the dam was undermined to such an extent as to fall into the cavity

In another case, dam 36 ft high, front vert; the water falling upon nearly vert strata of hard shale rock, usually covered by but about 2 ft of water, in about 20 years wore it to an irregular depth of from 10 to 20 ft, and extending from the very face of the dam, to 70 or 80 ft in front of it.

In Fig 2, upon a stream subject to very violent freshets, the gravel was washed away for a considerable width and depth beyond the apron, as at *h*. To prevent a repetition, the cavity was filled with cribwork full of stone, clear across the river.

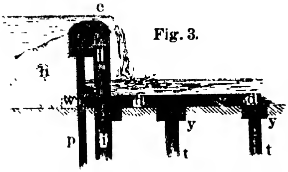


Fig. 3.

A deposit of blocks of loose stone, of even a ton weight or more, will not serve as a protection in front of a dam exposed to high freshets, but will soon be swept away. A common precaution against this wear, in low dams, is an *apron*, *a a*, Fig 2, or *d d*, Fig 3; of either rough round tree trunks, or of hewn timber, laid close together; extending under the entire base of the dam, and from 15 to 30 ft in front of its face. These are sometimes bolted to pieces, *s s*, Fig 2, or *y y*, Fig 3; laid under them across the stream. In Fig 3, with very soft bottom, these pieces *y y* are supposed to be bolted to short piles *t t*, driven for that purpose.

At times a distinct wide low timber crib, filled with stone, and covered on top with stout plank has been placed in front of the dam, to receive the fall of the water, and is effective in protecting the bottom. Also in some cases, a dam of less height, and of cheap character, has been built at a short distance down stream from the main one, in order to secure at all times a deep pool in front of the latter for breaking the force

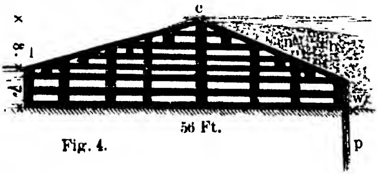


Fig. 4.

Another precaution is to substitute a sloping front like *c l*, Fig 4, or such as Figs 1 and 2 would form if reversed, for the nearly vert one of the other figs, thus to some extent reducing the force of the water. This, however, is but a partial remedy, especially for soft bottoms in shallow water, for the sliding sheet still descends with great force. The best form of dam, perhaps, in such cases, is that shown in Fig 5, in which the front consists of a series of steps of about 1 vert, to 3 or 4 hor. These effectually break the force of the water, and, with the addition of an apron *o o*, secure a satisfactory result. It is objected against this form, as also against Figs 4 and 6, that their fronts are liable to be torn by descending trees, ice and other bodies swept along during freshets, but experience shows that this objection has but little weight, for when such bodies pass, the sheet of water is thicker than usual; and protects the front timbers. On the Sch Nav, the timbers *c l*, Fig 6, scarcely wear thin at the rate of an inch in 10 to 15 years.

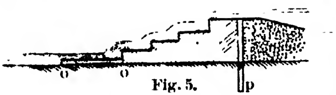


Fig. 5.

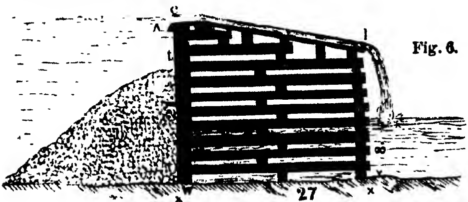


Fig. 6.

The forms of wooden dams are many; (see the figs, which show those most used,) varying with the circumstances of the case, and with the fancy of the designer. In the United States they are usually of cribwork, of either rough round logs with the bark on, or of hewn timber; in either case about a foot through. These timbers are merely laid on top of each

ther, forming in plan a series of rectangles with sides of about 7 to 12 ft. They are not notched together, but simply bolted by 1 inch square bolts (often ragged or jagged) about 2 to 2½ feet long, through two timbers at every intersection. These are not found to rust or wear seriously, even when exposed to a current. Square bolts hold best. Round logs are flattened where they lie upon each other. Experience shows that former but more expensive connections are entirely unnecessary. The cribs are usually, but not always, filled with rough stone. In triangular dams, disposed as in Figs 1, 2, and 7, this stone filling is not so essential as in other forms, because the weight of the water, and of the gravel backing, tends to hold the dam down on its base. Still, even in these, when the lower timbers are not bolted to a rock bottom, or otherwise secured in place, some stone may be necessary to prevent the timbers from floating away while the work is unfinished, and the gravel not yet deposited behind it. On rock, the lowest timbers are often bolted to it, to prevent them from floating away *during construction*, and when the water is some feet deep, this requires coffer-dams. Or, the cribs may be built at first only a few feet high; then floated into place, and sunk by loading them with stone, for the reception of which a rough platform or flooring will be reqd in the cribs, a little above their lowest timbers. The bolting to the rock may then be dispensed with. The water may flow through the open cribwork as the building higher goes on; attention being paid to adding stone enough to prevent it floating away if a freshet should happen. Or, cribs shown in plan at *c c*, Fig 8, loaded with stone, may be sunk, leaving one or more intervals like that at *o o o o*, between them, for the free escape of the water. These openings to be finally closed by floating into them closing cribs shaped like *n*.

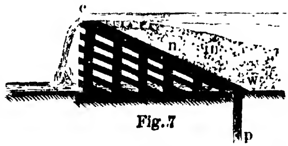


Fig.7

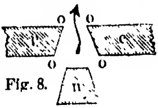


Fig. 8.

The workmanship of a dam in deep water can of course be much better executed in coffer dams, than by merely sinking cribs. The joints can be made tighter, the stone filling better packed; the sheet piling more closely fitted, &c.

When a very uneven rock bottom in deep water, or the introduction of sluices in the dam, or any other considerations, make it expedient to build dams within coffer dams, both should be carried on *in sections*; so as to leave part of the channel-way open for the escape of the water. Commencing at one or both shores, the first section of the coffer dam may reach say quarter way or more across the stream. In the section of the dam itself built within this enclosing coffer-dam, ample sluices should be left for the water to flow through when we come to build the *closing* section of the coffer-dam. When the dam has been finished, these sluices may be closed by **drop-timbers***. Before removing one section of coffer dam, the outer end of the enclosed section of dam itself must be firmly finished in such a manner as to constitute a part of the inner end of the next section of coffer dam. It is impossible to give details for every contingency; the engineer must rely upon his own ingenuity to meet the peculiarities of the case before him. In some cases of shallow water, mere mounds of earth may answer for coffer dams, or rough stone mounds backed with earth or gravel.

After the water has passed beyond the crest, *c* in the figs, there is no necessity for preventing its leaking down among the crib timbers; on the contrary, the thick sheeting planks, (or squared timbers, as occasion may require) *c l*, Figs 4 and 6, which form the slopes along which the water then flows in some dams, are usually not laid close together, but with open joints of about ½ inch wide between them, for the express purpose of allowing part of the water to fall through them, so as to keep the timbers beneath them partially wet, which, to some extent, renders them more durable. In Figs 1, 4, 6, and 7, the water of the lower pool flows freely back among the crib timbers, and rough quarry stones with which the cribs are filled either partly or entirely. In Figs 4 and 6, these stones are not shown. In the dam, Fig 1, none were used. In Fig 2 they were as shown.

A substantial, and not very expensive dam of the form of Fig 7, may be built of rough stone in cement. Some hewn timbers should be firmly built horizontally into the masonry of the sloping back *c n w*, at a few feet apart, with their tops level with the surf of the masonry. To these must be well spiked close-jointed sheeting-plank *c n w*, for protecting the masonry from the action of the water, and of floating bodies. The gravel backing *b*, may be omitted, but the sheet piles *p*, and an apron in front of the dam, will be as indispensable in yielding soils, as if the dam were of timber.

Figs 1, 2, 4, 6, and 7, are sections drawn to a scale, of existing dams in Pennsylvania, that have stood successfully the force of heavy freshets for a long series of years †. These freshets at times carry along large bodies of ice, trees, houses, bridges, &c., and have risen to 11 ft above the crests. Fig 1, on the Sch Nav was built in 1819, and served perfectly for 39 years, until in 1858 the decay of much of its timber, especially of the close-laid top ones, *e*, rendered it necessary to build a new one just in front of it. It was of extremely simple construction, and was never filled with stone. The bottom timbers, *o o*, 10 ft apart, were bolted to the rock; and immediately over each of them, was such a series of inclined timbers as is shown in the fig. The top ones, *e*, however, were close jointed and laid touching so as to form the top sheeting, instead of thinner planks. The short pieces at *t* were laid in the same way. No coffer dam was used, but the bottom pieces were first bolted to the rock, 10 ft apart, then the stringers and the sloping pieces were added. The close covering (*e*) was carried forward from each end of the dam, until at last a space of only about 60 ft was left in the center, for the water to pass. The close covering for this space being then all got ready, a strong force of men was set to work, and the space was covered so rapidly that the river had not time to rise sufficiently high to impede the operation.

* Timbers ready prepared for closing an opening through which water is flowing; and suddenly *dropped* into place by means of grooves or guides of some kind for retaining them in position. Several such timbers may at times be firmly framed together, and then be all dropped at once; closing the opening or sluice at one operation, especially when it is of small size. In some cases, a crib may be sunk on the up-stream side of such an opening, for closing it.

† Those on the Schuylkill Navigation were obligingly furnished by James F. Smith, Esq, chief engineer and superintendent of that work. Other valuable information from the same source will be found in different parts of this volume.

Fig 2 is a canal feeder dam on the Juniata. Here *s s* are timbers stretching clear across the stream, (about 800 ft.) and sustaining the apron *a a*, of stout hewn timbers laid touching. This dam was filled with stone, for the retention of which the front sheeting planks were added.

Fig 6 is on the Sch Nav, was built in 1855 It is a form much approved of on that work, for such situations, namely, firm rock foundation, with a considerable depth of water in front. The highest dam (32 ft) on the Sch Nav, is very similar to it, built in 1851. All the dams on this work are of hewn timber, chiefly white and yellow pine. The water occasionally runs from 8 to 12 feet deep over their crests, and then overflows and surrounds many of the abuts. The vertical back allows the overflowing water to leak down among *all* the lower timbers of the dam, and thus tend to their preservation

Fig 4 shows the dams on the Monongahela slackwater navigation, W. Milnor Roberts, eng. They are of round logs, with the bark on, flattened at crossings. The longest ones in the fig are 10 feet apart along the length of the dam Experience shows that such dams possess all the strength necessary for violent streams. On rock, the lowest timbers are bolted to it

Fig 7 has been successfully used to heights of 40 ft *

Fig 3 is intended merely as a hint for a very low dam on yielding bottom Its main supports are piles *i i*, from 4 to 8 ft apart, according to the height of the dam, and other circumstances, and *t t* are short piles for sustaining the apron *d d* It may be extended to greater heights by adding braces in front, which may be covered by stout planks to form an inclined slide for the overfalling water. Many effective arrangements of piles and sloping timbers for dams on soft ground, will suggest themselves to the engineer Thus, at intervals of several feet, rows of 3 or more piles may be driven transversely of the dam, the top of the outer pile of each row being left at the intended height of the crest, while those behind are successively driven lower and lower, so that when all are afterward connected by transverse and longitudinal timbers and covered by stout planking, and gravel, they will form a dam somewhat of the triangular form of Fig 7. It would be well to drive the piles with an inclination of their tops up stream

There is much scope for ingenuity both in designing, and in constructing dams under various circumstances, and in turning the course of the water from one channel to another, by means of ditches, pipes, or troughs, &c, at diff heights, aided at times by low temporary dams or mounds of earth, or of sheet piles, &c, or by coffer dams, so as to keep it away from the part being built Each locality will have its peculiar features, and the engineer must depend on his judgment to make the most of them

Abutments of dams as a general rule should not contract the natural width of the stream, or, if they must do so as little as possible, for contractions increase the height, and violence of the overflowing water in time of freshets, during which a great length of overfall is specially desirable They should be very firmly connected with the ends of the dams; and should, if the section of the valley admits of it be so high, and carried so far inland that the high water of freshets will not sweep either over them, or around their extremities, and thus endanger undermining, and destruction. In wide, flat valleys they cannot be so extended without too much expense and the only alternative is to found them so deeply and securely as to withstand such action, making their height such that they will, at least, be overflowed but seldom Their ends adjacent to the dam should be rounded off, so as to facilitate the flow of the water over the crest

They are best built of large stone in cement, for although sufficient strength may be secured by timber, that material decays rapidly in such exposures. If of earth only, they are very apt to be carried away if a freshet should overtop them.

Sluices should be placed in every important dam, in order that all the water may be drawn off if necessary, for the purpose of repairs; or of removing mud deposits; or finding lost articles of importance, &c They may be merely strong boxings, with floor, sides, and top of squared timbers, and passing through the breadth of the dam, just above the bottom. To prevent trees, &c, from entering and sticking fast in them, some kind of strong screen is expedient. In common cases a sluice should not *exceed* about 3½ ft by 5 ft in cross section; otherwise it becomes hard to work Two or more such openings may be used when much water is to be voided. They should be near the abutments. The gates or valves for opening and shutting them, should be at the up stream end; for if at the lower one, accumulations of mud, &c, will fill the sluices, and prevent them from working They are usually of timber, and slide vertically in rebates, being raised and lowered by rack and pinion, but in very important dams they may be of cast iron Two sets of sluices are desirable that one may be always ready for use if the other is stopped for repairs

The part of the apron in front of the sluice should be particularly firm, so as not to be deranged by the water rushing out under a high head.

In dams of masonry and of concrete, if the shores are of rock, **the plan** is frequently given the form of a **flat arch**, convex up-stream. By thus utilizing the banks as abutments for the (horizontal) arch, material reduction of the volume of material in the dam may properly be effected; but, unless the banks are of rock, they afford but imperfect abutments for the arch; and they are exposed to wear by the current thrown against them at its ends.

A dam, built obliquely across a stream, will have less depths of water upon its crest than one built normally to the current, and will therefore flood adjacent country to a less extent. Other things equal, the less the slope of the stream, the further will the flood heights extend up stream.

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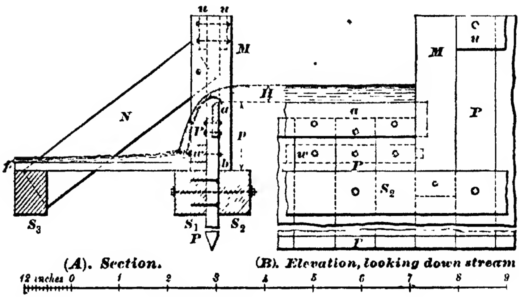
* **Cost of crib dams.** With common labor at $1.50 per day; lumber, $20 per 1000 ft. board measure, delivered; stone for filling, $1 per cub. yard; gravel 50 cents per cub. yd.; iron for bolts etc., 2 cts. per lb.,—such dams in shallow water usually cost, complete, $2.48 to $3.24 per cubic yard of crib.

Figs. 9 and 10 are **designs for small measuring weirs**, suitable for shallow streams up to say 100 feet wide; Figs. 9 for earth or gravel bottom, and Figs. 10 for rock.

In the former, the 8 × 10 inch hemlock sills S_1 and S_3 are first laid across the bottom of the stream, which is trenched where necessary, care being taken to lay S_1 in a true line. The sills should extend say from 5 to 10 feet into each bank of the stream. Tongued and grooved sheet piling P, of 3 × 10 inch hem-



Figs. 9.—Measuring Weir on Earth or Gravel Bottom.

lock, is then driven close behind the upper sill S_1 to a depth of from two to four feet, and spiked to S_1. A third sill, S_2, of the same length as S_1 and S_3, is then laid behind the sheet piling, and the two sills S_1 and S_2 and the sheet piling P are then secured together, as shown, by 1 inch bolts, spaced about 2 feet apart. The tops of the sheet piling project about a foot above the sills, and are stiffened by 4 × 4 inch timbers *w*, bolted in front of them and resting upon the flooring *f* of 2 × 10 inch spruce. This flooring, like the sills, extends several feet beyond each end of the weir into the bank, and is there loaded to its full capacity with heavy stones. Any spaces left underneath it by unevenness of the bottom should also be leveled up with stones or gravel.

A 10 × 10 inch yellow pine post M, 3 feet high, is tenoned between sills S_2 and S_1 at each end of the overflow, and braced by an 8 × 10 inch yellow pine strut N, tenoned to it and to the sill S_3. Beyond these posts the sheet piling P extends as high as the top of the posts, and is carried, at that height into the bank; the tops of the piles being held in line by two 2 × 8 inch waling pieces *u u* bolted to them, one on each side.

In Figs. 10, the hemlock sills, S_1 of 10 × 10 inch, and S_2 of 6 × 8 inch, rest upon a Portland cement masonry wall, of varying height to accommodate the inequalities of the rock bottom; and are secured to it by 1 inch bolts spaced about 4 feet apart. These bolts pass down through the masonry, as shown, and a foot or more into the rock below.

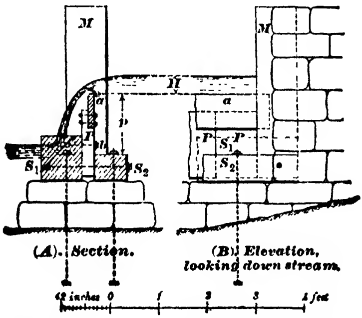
Between the two sills are bolted upright 3 × 10 inch tongued and grooved hemlock planks P, 15 inches long. At each end of the weir, a 10 × 10 inch yellow pine post M is tenoned between the sills, as in Figs. 9, and built into the masonry ends of the dam, which last extend well into the banks of the stream.

In both Figs. the crest-piece *a*, is of 2 × 8 inch oak, beveled so as to leave a horizontal top face ¼ inch wide. The crest-piece is let in flush with the back of the piles or boards P, to which it is bolted, and is let into the end posts M about 2 or 3 inches. At low stages of water, the flow may be confined to a *portion* of the length of the overfall by flash-boards placed along the rest of the dam.

A crest-piece made of 8 × ¼ inch bar iron is preferable to one of wood. It requires of course much less cutting away of the sheet-piling, and its upper edge is less subject to abrasion by drift passing over the weir. The top edge, and the abutting ends of the several lengths, should be planed smooth and square; the former

to insure a sharp inner corner at *a* for the water to pass over, and the latter in order to avoid leakage. As a further precaution against leakage, a strip or butt-strap of 8 × ⅛ inch iron, about a foot long, may be let in, *between the crest-piece and the sheet piling*, opposite each joint of the former, and overlapping both the adjoining ends, the piling being cut away ⅛ inch deeper at those points, in order to accommodate them. Such butt-straps, if placed on the *up*-stream side of the crest-piece, would break the continuity of the sheet of water passing over the weir, and thus interfere somewhat with the correctness of the gauging. Such iron is obtainable in any commercial center, in lengths of about 16 feet. 8 × ¼ weighs 6⅔ pounds per running foot; 8 × ⅛, 3⅓ pounds.

All the joints should be caulked with oakum. To apply the usual weir formulæ (see Art. 14 *f*, p. 548) the back of the weir should be vertical for a depth *p* below



Figs. 10.—Measuring Weir on Rock Bottom.

the crest *a* equal at least to twice the head H on the weir. It is therefore better to protect the back of the weir by tarpaulin rather than resort to puddling, except close to the bottom.

In a long weir with a low fall, it is difficult to secure a sufficiently free access of air to the space behind the falling sheet of water, especially when the stream is low and the sheet tends to hug the face of the dam. In such cases a partial vacuum * forms between the falling sheet and the face of the dam, and increases the discharge, thus vitiating the results. It is therefore important, in designing measuring weirs, to arrange (as far as possible) so that the sheet of water may fall clear through the entire distance between the up-stream and down-stream levels without striking any portion of the weir itself, for such striking would diminish the clear space behind the sheet and increase the difficulty of preventing a vacuum there.

* Such a vacuum causes the down-stream water near *w*, Figs. 9, to rise behind the sheet. When the rarefaction of the air behind the sheet has proceeded to a certain extent, the external air breaks in and relieves the vacuum. Then another vacuum forms, and is in turn relieved, and so on, alternately. At such times it has been noticed that light bodies, such as chips, etc., floating in the down-stream water near the ends of the weir, are drawn into the space behind the sheet and carried toward the middle of its length, and then in turn ejected at the point where they entered, thus traveling back and forth along the space behind the sheet.

Tremblings in Dams. Dams over which the water falls in a long, smooth, unbroken sheet of considerable height, are more or less subject to tremblings, caused apparently by alternate compression and rarefaction of the air by the falling sheet, especially in the space (W, Fig 20, p. 547) behind the sheet, where a partial vacuum is often formed, because the air there is entangled in the falling water and given off again by it down stream in the shape of foam. Such tremblings sometimes cause a rattling of windows half a mile or more away. We have known this to be stopped (in one case unintentionally) by building a well-covered wide crib apron, a few feet high against the front of the dam, for preventing the abrasion of the bottom. In other cases a series of oblique timbers placed against the front of the dam, and part way up it, at a slope of about 1½ to 1, and covered with plank, has been perfectly effective in stopping it. In short, any device which admits air more freely behind the falling sheet, or destroys the continuity of the latter (such as flash boards of different heights or placed at intervals along the crest), or which reduces its height and its continuous length, ought to diminish or obviate the trouble.

The proper time for building dams is of course at the longest period of low stage of water.

Table of thickness of white pine plank required not to bend more than $\frac{1}{240}$ part of its clear horizontal stretch, under different heads of water. (Original.)

Stretch in Ft.	Heads in feet. 40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8⅓
20	22⅓	20	17⅓	14	11

WATER SUPPLY.

Consumption of water. Owing largely to the proper extension of the use of water in dwellings, the quantity required in cities increases faster than the population. In other words, the per capita consumption increases.

Use. Abundant experience shows that a supply of 50 gallons (or say 7 cubic feet) per capita per day is abundant for all the needs and luxuries of well-to-do families in American cities. The manufacturing consumption, of course, bears no fixed relation to the population. In cities it is generally much less than the domestic consumption.

Waste. In American cities, the waste often amounts to two or three times the quantity really used. Of the 116 gallons per capita per day, delivered in New York in 1899, Mr. Freeman* estimates that from 31 to 56 gallons were used, 10 unavoidably wasted, and from 50 to 75 avoidably wasted.

In Philadelphia, investigations by means of the Deacon waste-water detector, on 142 modern seven-room, two-story dwellings, with bath, etc., on two intermediate streets, showed that, of 222 gallons per capita per day, furnished through 782 fixtures, 192 gallons, or 86.5 per cent. were wasted, and only 30 gallons, or 13.5 per cent. were used. The City has built extensive works for the purpose of pumping, filtering, conveying, repumping, storing, and distributing the water wasted, as well as the smaller quantity used. Of the total cost,† less than half would have sufficed for the water used and unavoidably wasted.

Sources of waste. The waste is caused by heedlessness; by allowing water to run to waste in order to prevent it from freezing in winter and in order to get cooler water in summer; by leaky and otherwise defective fixtures; by unsuspected leaks in mains and service pipes, etc.

As a "guess, tempered by judgment," Mr. Freeman* classifies the 50 to 75 gallons per capita per day, wasted in New York, as follows:

Leaks in mains	10 to 15	gals per capita per day.		
" service pipes	10 to 15	"	"	"
" defective plumbing	15 to 25	"	"	"
Careless and wilful waste	14 to 17	"	"	"

The avoidable waste is usually perpetrated by a small fraction (say from one-fifth to one-third) of the population, the remainder using water reasonably. In the Philadelphia case, above cited, of the 782 fixtures, 22 were found to be "leaking slightly," and 32 "turned on continually."

Waste restriction. Water meters. Waste is best restricted by making its avoidance a pecuniary object to the consumer; and this is best accomplished by the use of the water meter, at least on all services (domestic, industrial, and public) where waste is found to be going on. The meters should be owned and maintained by the corporation supplying the water.

Minimum charge. In order to encourage the liberal *use* of water, while discouraging its *waste*, and thus avoid undue economy (tending to uncleanliness) each consumer should be charged a minimum periodical rate, sufficient to cover amply all the water he can possibly use and enjoy.

Mr. Freeman* estimates the average cost of domestic meters, for New York and Brooklyn, mostly 5-8 inch and 3-4 inch, with a few of larger sizes, at $12.50 each, and the cost of installation by the city, working systematically and on a large scale, at $2.50 each, or a total of $15.00 each. He assumes "the average life of the ordinary domestic meter, of a good type, well cared for, and with occasional repairs and renewal of worn parts," at "not far from 20 years"; and annual expenses as follows:

	Providence, R. I. Actual, approx.	New York. Assumed.
Interest on cost of meter and setting	$0.50	$0.45
Depreciation and renewal of meter (life assumed 20 years)	0.75	0.75
Maintenance and repairs, testing and resetting	0.46	0.70
Reading meters and computing bills	0.42	0.60
Total annual cost, per meter	$2.13	$2.50

* Report upon New York's Water Supply, made to Bird S. Coler, Comptroller, by John R. Freeman, Civil Engineer, 1900.

† The total cost approximates $30,000,000.

Free water for fire protection. Cities sometimes give to manufacturers a free supply of water through special connections, to be used for fire protection only; the manufacturer giving bond not to use such connection for any other purpose, and the city placing a meter on the connection for the detection of any illicit use of the water for other purposes.

Water for city use should not be drawn from the very bottom of the reservoir, because it will then be apt to carry along the sediment; which not only injures the water, but creates deposits within the pipes; thus obstructing the flow. In fixing upon the necessary capacity of a reservoir, this must be taken into consideration; inasmuch as all the water below the level for drawing off, must be regarded as lost. When circumstances justify the expense, it is well to curve up the reservoir end of the service main, so as to provide it with valves at different heights; for drawing off only the purest stratum that may be in the reservoir. With this view, the valve-tower generally has such valves communicating with the water in the reservoir; and by this means only the purest is admitted into the tower: and from it, into the city pipes. This refinement, however, is rarely practicable. Such valves must of course be worked by watchmen.

Art. 1. Reservoirs. In important reservoirs of earth, for storing water to moderate depths for cities, experience appears not to sanction **dimensions** *bolder* than 10 feet thick at top; inner slope 2 to 1; outer slope 1½ to 1.* A top width of 15 feet to 20 feet, and inside slopes of 3 to 1, are adopted in some important cases; with outer slopes of 2 to 1. Both slopes, however, are at times made only 1½ to 1. The level water surface should be kept at least 3 or 4 feet below the top of the embankment; or more, if liable to **waves.** In a large reservoir, a quite moderate breeze will raise waves that will run 3 feet (measured vertically) up the inner slope. A low wall, or close fence, *w*, Fig 37, is sometimes used as a defence against them. The top and the outer slopes should be protected at least by sod or by grass. To assist in keeping the top dry, it should be either a little rounding, or else sloped toward the outside.† The soft soil and vegetable matter should be carefully removed from under the entire base of the embankments; which should be carried down to soil itself impervious to water, in order that leakage may not take place *under* them. To aid in this, a double row of sheet piles, or a sunk wall of cement masonry, carried to a suitable depth below the bottom may be placed along the inner toe in bad cases. If there are springs beneath the base, they must either be stopped, or led away by pipes. The embankment should be carried up in layers, slightly hollowing toward the center, and not exceeding a foot in thickness; and all stones, stumps, and other foreign material, such as *clean* gravel, sand, and decomposed mica schists, &c, that may produce leakage, carefully excluded. These layers should be well consolidated by the carts; and the easier the slopes are, the more effectively can this be done. The layers, however, should not be distinct, and separated by actual plane surfaces; but each succeeding one should be well incorporated with the one below. This has sometimes been done by driving a drove of oxen, or even sheep, repeatedly over each layer; in addition to the carting. Rollers are not to be recommended, as they tend to produce seams between the layers. This might possibly be obviated by projections on the circumference of the roller.

Gravelly earth is an excellent **material,** perhaps the best. The choicest material should be placed in the slope next to the water; and should be deposited and compacted with special care in that portion, so as to prevent the water from leaking into the main body of the dam, and thus weakening it. It is not amiss to introduce a bench, *b*, Fig 37, in the outer slope, to diminish danger from rainwash by breaking the rapidity of its descent.

If the bottom of the reservoir itself is on a leaky soil, or on fissured rock, through the seams of which water may escape, it must be carefully covered with from 1½ to 3 feet of good puddle; which, in turn, should be protected from abrasion and disturbance, by a layer of gravel; or of concrete, either paved or not, according to circumstances.

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* The writer suggests that a top width equal to 2 feet + twice the square root of the height in feet, will be safe for any height whatever of reservoir properly constructed in other respects.

† Some engineers slope the top toward the *inside.*

Reservoirs constructed with the foregoing dimensions, and with care, *may* remain safe for an indefinite period; but where serious damage would result from failure, the following **additional precautions** should be taken. The inner slopes should be carefully faced up to the very top, with at least a close dry rubble-stone pitching, not less than 15 to 18 inches thick; as a protection against wash, and against muskrats. These animals, we believe, always commence to burrow under water. If the slopes are much steeper than 2 to 1, this dry pitching will be apt to be overthrown by the sliding down of the softened earth behind it, if the water in the reservoir should for any cause be drawn down rather suddenly. It will be much more effective, but of course more costly, if laid in hydraulic cement; and still more so if laid upon a layer a few inches thick of cement and-gravel concrete; especially if this last be underlaid by a layer about 1½ to 3 feet thick of good puddle, spread over the face of the slope; the great object being to protect the inner slope from actual contact with the water. If this can be effectually accomplished, slopes as steep as 1½ to 1 will be perfectly secure; for the danger does not arise from any want of weight of the earth for resisting overthrow. **Special care should be bestowed upon the inner toe of the slope.** to prevent water from finding its way beneath it, and softening the earth so as to undermine the stone pitching. Near the top reference should be had to danger of derangement by ice, frost, rain, and waves. Flat inner slopes tend not only to prevent the displacement of the pitching; but increase the stability of the embankment, by causing the pressure of the water (which is always at right angles to the slope) to become more nearly vertical, and thus to hold the embankment more firmly to its base than if there were no water behind it. Sometimes the toes of both the inner and outer slopes abut against low retaining-walls in cement. This gives a neat finish, and tends to preservation from injury.

Many engineers, in order to prevent leaking, either through or beneath the embankment, construct a **puddle-wall**, *p*, Fig 37, of well-rammed impervious soil, (gravelly clay is the best.) reaching from the top to several feet below the base. This wall should not be less than 6 or 8 feet thick on top, for a deep reservoir; and should increase downward by *offsets* (and not by slopes or batters) at the rate of about 1 in total thickness, to 3 or 4 in depth. Other engineers object to these puddle-walls, and contend that leakage should be prevented by making both the inner slopes and the bottom of the reservoir water-tight, by means of puddle, concrete, and stone facing in cement, as just alluded to. They argue that if the embankment is well constructed, it is itself a puddle-wall throughout.

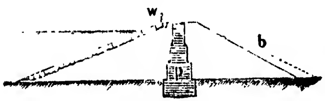

Fig. 37.

Near San Francisco, Cal. are two earthen reservoir dams built about 1864 one 95 feet high, 26 on top inner slope 2.75 to 1, outer 2.5 to 1. The other 93 high, 25 on top, inner slope 3.5 to 1, outer 3 to 1. In each the puddle-wall is carried 47 feet deeper than the base. No stone facing.

It is difficult to prevent water under high pressure from finding its way through considerable distances along seams where earth is in contact with smooth rock, wood, or metal; as, for instance, along the surfaces of iron pipes laid under reservoir embankments; or along the tie-rods sometimes used through the puddle of coffer-dams; and the same is apt to occur under the bases of embankments which rest on smooth rock. Special care should be taken that the earth used in such positions is not of a porous nature; and that it is thoroughly compacted all along the seam; and the straight continuity of the seam should be interrupted or broken as frequently as possible by projections. Faucets or flanges do this to a limited extent in the case of iron pipes; and something similar, but on a larger scale, should at short intervals be constructed in the shape of collars or yokes of cement stonework, in the case of rock or masonry.

It is usually advisable to **divide reservoirs into two parts**, so that while the water in one part is being drawn off for use, that in the other may purify itself by settling its sediment. Also, one part may remain in use, while the other is being cleaned or repaired. Many days, or even two or three weeks, sometimes, are required for the complete settlement of the very fine clayey particles in muddy water; depending on the depth of the reservoir. One or more flights of steps to the bottom of the reservoir should be provided.

Mud in Reservoirs. The reservoirs of the New River Water Co, London, England, were uncleaned for 100 years, during which mud 8 feet deep was

deposited, or about an inch annually. At Philadelphia it is about 25 inch per annum from the Schuylkill, and 1 inch from the Delaware River. At St Louis, Missouri, about 3 to 4 feet per year! Vegetation is apt to take place in shallow reservoirs and near the edges of deep ones, especially in very warm weather; and the plants, on decaying, injure the water.

Water flowing through marsh lands is sometimes unfit for drinking purposes. That, for instance, in some sections of the Concord River, Massachusetts, was reported by the eminent hydraulic engineer, Loammi Baldwin, of Boston, to be absolutely *poisonous* from this cause.

The construction of a large deep reservoir is not only a very costly, but a very hazardous undertaking. With every watchfulness and care, it is almost impossible entirely to prevent leaking; although this may not manifest itself for months, or even years. Should a break occur, especially near a city, it would probably be attended by great loss of life and property. If the water once finds its way in a stream, either across the unpaved top, or through the body of the embankment, the rapid destruction of the whole becomes almost certain.

Art. 1a. Storing Reservoirs. The entire annual yield of a stream may be much more than sufficient for supplying a certain population with water; and yet in its natural condition the stream may not be available for this purpose, because it becomes nearly dry in summer, when water is most needed; while, at other seasons, the rains and melted snows produce floods which supply vastly more than is required; and which must be allowed to run to waste. A storing reservoir is intended to collect and store up this excess of water, so that it may be drawn off as required during the droughts of summer, and thus equalize the supply throughout the entire year. This, when the locality permits, is effected by building a dam across the stream, to form one side of the reservoir, while the hill-slopes of the valley of the stream form the other sides. The stream itself flows into this reservoir at its up-stream end. When the stream is liable to become nearly dry during long summer droughts experience shows that the **capacity** of the reservoir should be equal to from 4 to 6 months' supply, according to circumstances. During the construction of the dam, a free channel must be provided, to pass the stream without allowing it to do injury to the work. If the dam were built precisely like Fig 37, entirely of earth, it would plainly be liable to destruction by being washed away in case the reservoir should become so full that the water would begin to flow over its top. To provide against this we may, by means of masonry, or of cribs filled with broken stone, or otherwise, construct either the whole, or part of the dam, to serve as an **overfall**, or a **waste-weir.** Or a side channel (an open cut, pipes, or a culvert, &c) may be provided at one or both ends of the dam, and in the natural soil, at such a level as to carry away the surplus flood water before it can rise high enough to overtop the earthen dam. Besides these, and the pipes for carrying the water to the town, there should be an outlet, with a valve or gate, at the level of the bottom of the reservoir; in order that, if necessary for repairs, or for cleaning by scouring, all the water may be drawn off. The entrances to the city pipes should be protected by gratings, to exclude fish, &c.

To facilitate repairs or renewals of all valves, &c, which are under water, the reservoir ends of the pipes or culverts to which they are attached, may be surrounded by a water-tight box or chamber, which will usually be left open to the reservoir; but may be closed when repairs are required. Access may then be had to them by entering at the outer end, after the water has flowed away from inside. In case the outlet is through a long line of pipes which cannot thus be entered, a special entry for this purpose may be cast in the pipe itself, near the outer toe of the embankment; to be kept closed except in case of repairs. Sometimes a better, but more expensive means of access to such valves, is secured by enclosing them in a **valve-tower** of masonry. This is a hollow vertical water-tight chamber, like a well; but near the toe of the inner slope; having its foundation at the bottom of the reservoir; whence the tower rises through the water to above its surface. This chamber is provided with valves or gates usually left open to the reservoir; but which may be closed when repairs are needed; and the water in the tower allowed to escape from it through the open valves of the outlets. This done, workmen can descend through the tower by ladders from the aperture at its top.

At times the outlets for the discharge of surplus flood water are, like those for scouring, placed at, or just above, the level of the bottom of the reservoir. In order that these may work in case of a sudden flood at night, &c, they must be furnished with self-acting valves, which will open of their own accord when the flood is about to rise too high. This may be effected by attaching them to floats, the rising of which, when the water is high, will pull them open. All such outlets should be large enough to let men enter them for repairs. They should by

no means be laid through the artificial earthen body of the dam itself, without being supported upon masonry reaching down to a firm natural foundation; otherwise they are very apt to be broken by the subsidence of the embankment. It is usually safer to carry them through the firm natural soil near one end of the dam. Their valves, if only single, should be at their inner or reservoir end, so as to leave the outlets themselves usually empty, for inspection; but it is better to have two valves, so that one may be used when the other needs repair; and in this case one may be placed at each end. Reservoirs which are supplied by pumps, need no precautions against overflow; because the pumping is stopped when they are filled to the proper height. Large storing reservoirs necessarily submerge more or less land, which has therefore to be purchased. By intercepting the descending water, they frequently prevent spring floods from injuring low lands farther down stream. If there are mills down stream from the reservoir, they would evidently be deprived of water for driving them, unless a portion of that stored in the reservoir be devoted to that purpose. Water thus applied to *compensate* for the loss of the natural stream, is called **compensation water**; and the reservoir, a *compensating* one.

Art. 1b. Distributing reservoirs. Frequently a valley fit for a storing reservoir can be found only at a long dist (sometimes many miles) from the town; and it then becomes expedient to construct also an additional one of smaller size than the storing one, near the town; and at as great an elevation above it as circumstances will permit; but lower than the storing one. This is called, by way of distinction, a *distributing* reservoir, because from it the water, after having flowed into it from the storing reservoir, through the long *supply pipe* which connects them, is distributed in various directions through the town, by means of the street mains, or pipes. This small reservoir should hold a supply sufficient at least for a few days; a few weeks would be better; and the end of the supply pipe which terminates in it, should be provided with a valve for shutting off the supply from the storing reservoir. These precautions permit repairs to be made along the line of supply pipe without depriving the town of water in the mean time. With a view to such repairs; as well as to scouring out sediment from the supply pipe, this last should be provided with **outlet valves** at various low points along the entire interval between the two reservoirs; especially at those at which the valves may disch into natural watercourses. On opening these valves, the out rush of the water carries away sediment; and leaves the pipe empty for inspection

In fixing upon the diams of pipes for supplying cities, it is necessary to bear in mind, that by far the greater portion of the 24 hours' yield is actually drawn from them during only 8 to 12 hours of daylight, and therefore the capacity of the pipes must be sufficient to furnish the daily supply in much less than 24 hours. Again, during the hot summer months, much more water is used than during the winter ones, and this consideration necessitates a still larger diam.

Art. 2. Systems of street pipes for supplying cities. The writer knows of no practical rules for proportioning the diams for such systems. The various complications involved, render a purely scientific investigation of little or no service. With much hesitation, he ventures the following purely empirical rules of his own, based on such limited observations as have casually fallen under his notice

RULE 1. *When at no point in a system of city pipes, is the head, or vert dist below the surface of the reservoir, compared with the hor dist from the reservoir, less than at the rate of 50 ft per mile, then the population in the last column of the following Table A, may be abundantly supplied, for all city purposes, by either one pipe of the inner diam or bore in the 1st col, or by 2, 3, &c, pipes of the diams in the other cols* These diams are given to the nearest safe ⅛ inch. The supply is assumed to be about 60 gallons per day to each inhabitant.

TABLE A. (Original.)

NUMBER OF PIPES. 1	2	3	4	6	8	12	24	Population
Diam. Ins.	Diam. Ins.	Diam Ins.	Diam Ins.	Diam. Ins.	Diam Ins.	Diam. Ins.	Diam. Ins.	
6	4⅜	3⅝	3⅓	3	2⅞	2⅝	1⅞	1647
8	6⅛	5¼	4⅝	4	3½	3⅛	2⅝	3465
10	7⅝	6½	5⅜	5	4⅞	3⅜	3	5308
12	9⅛	7¾	7	5⅝	5¼	4⅓	3½	9324
14	10⅝	9⅛	8⅝	7	6⅛	5⅝	4⅛	13706
16	12⅛	10⅜	9¼	7⅜	6⅞	6	4⅝	19141
18	13¾	11⅝	10⅓	8⅜	7⅜	6¾	5⅛	25677
20	15¼	13	11⅝	9⅜	8⅝	7½	5⅝	33426
22	16¾	14¼	12⅔	10⅜	9⅝	8¼	6⅛	42433
24	18⅓	15⅓	13⅜	11¾	10⅝	9	6⅝	52671
26	19⅝	16¾	15	12¾	11⅝	9¾	7⅛	64447
28	21¼	18⅛	16¼	13¾	12⅜	10⅝	8	77565
30	22⅞	19⅝	17⅝	14¾	13⅜	11½	8⅝	91580
32	24⅜	20⅝	18¼	15⅝	14	11⅞	9	108160
34	25⅞	23	19⅝	16¾	15	12⅝	9⅝	125840
36	27⅜	23¼	20⅞	17¾	15¾	13⅛	10⅜	144480
40	30⅛	25⅞	23⅓	19⅝	17⅝	15	11⅓	188320
44	33⅛	28⅔	25⅝	21⅝	19⅝	16⅓	12⅝	239600
48	36⅓	31	27⅝	23⅜	21⅛	18	13⅝	297600
54	41	34⅞	31⅓	26⅛	23⅜	20⅛	15⅝	391200
60	45⅝	38⅝	34⅝	29⅛	26⅝	22⅝	17⅛	511200
66	50⅛	42⅔	38⅛	32⅓	29⅛	24⅜	18⅝	650400
72	54⅝	46⅛	41⅝	35⅝	31⅝	26⅛	20⅛	800000
			[illegible]	[illegible]	[illegible]	29¼	22¾	1064000

It is well to allow in addition from ½ inch to 1 inch, or more, (depending on the character of the water,) to each diameter; for deposits and concretions.

The water, after reaching the city through one or more large main pipes from the reservoir, must be distributed through the streets by means of smaller mains branching from the larger ones. The diameters of these smaller ones also may be found by Table A. Thus, if a street, with its alleys, &c, contains about 6000 persons, (the rate of head being, as before, not less than 50 feet to a mile at any point of the system,) then we see by the table that a 10-inch pipe will answer. It would be well to lay no city street pipes of less than 6 inches diameter.

Mains which cross each other should be connected at some of their intersections, to allow the water a more free circulation throughout the entire system; so that if the supply at any point is temporarily cut off from one direction by closing the valves for repairs, or is diminished by excessive demand, it may be maintained by the flow from other directions.

Avoid dead ends when possible, as the water in them becomes foul and unwholesome.

RULE 2. *With the same diameters, different rates of head will supply the proportionate populations in column 3 of Table B. Or, to find the diameters which at different rates of head will supply the same populations given in the last column of Table A,* multiply the diameter given in Table A, by the corresponding number in column 4 of Table B; or (approximately) do as directed in column 5.

TABLE B. (Original.)

COL. 1.	COL. 2.	COL. 3.	COL. 4.	COL. 5.
Rate of Head, in Feet per Mile.	Rate of Head, compared with that in Table A.	Proportionate Populations.	Proportionate Diam. to supply the Populations in Table A.	Remarks.
5	.1	.32	1.58	
10	.2	.45	1.37	
12½	.25	.50	1.32	Add one third
15	.3	.55	1.27	Add full one-fourth
20	.4	.64	1.20	Add one fifth
25	.5	.71	1.14	Add one seventh.
30	.6	.78	1.11	Add one ninth.
35	.7	.84	1.07	Add one fourteenth.
37½	.75	.87	1.06	Add one sixteenth.
40	.8	.90	1.05	Add one twentieth.
45	.9	.95	1.02	Add one fiftieth.
50	1.0	1.00	1.00	
75	1.5	1.23	.92	Deduct one thirteenth.
100	2.0	1.41	.88	Deduct one eighth.
125	2.5	1.59	.83	Deduct full one-sixth.
150	3.0	1.73	.80	Deduct one fifth
200	4.0	2.00	.76	Deduct nearly one fourth.
250	5.0	2.25	.73	Deduct nearly two sevenths.
300	6.0	2.46	.69	Deduct three tenths.
400	8.0	2.83	.66	Deduct full one third
500	10.0	3.18	.63	

Example. By Table A we see that with the rate of head of 50 feet per mile, a 30-inch pipe will supply a population of 91580; but with three times that rate of head, or 150 feet per mile, we see by column 3, Table B, that the same pipe will supply 1.73 times as many persons, or $91580 \times 1.73 = 158433$ persons. But if, at this greater rate of head, we still wish to supply only 91580 persons, then we find in column 4, Table B, that we may diminish the diameter of the pipe from 30, down to $30 \times .80 = 24$ inches; or, by column 5, we have $30 - 6 = 24$ inches.

Again, after the water has reached the city by the 30-inch pipe of Table A, if we wish to distribute it through the city by say eight branches or smaller mains, we see by column 8, Table A, that each of them must have at least 13⅛ inches diameter. From these eight, other smaller ones may branch off into the cross streets, alleys, &c; and in estimating the supply required for any particular street main, we must evidently add what is required also for such cross streets, &c, &c.

If certain limited parts of a city pipe system have considerably less rates of head than most of the remainder, it may become expedient to supply the former by a special separate main of larger diameter; which may start either directly

from the reservoir; or as a branch from the grand leading main which feeds the lower parts, according to circumstances.

It must be remembered, that although by increasing the diameters, an abundant supply may be obtained under a small rate of head, as well as under a great one, yet the water will not rise to as great a height in the service pipes for supplying the different stories of dwellings, &c. Even with the diameters in Table A, the water, under ordinary use, will not rise in these pipes to the full height of the surface of the reservoir; and if an unusual drawing-off is going on at the same time at many parts of the system, as in case of an extensive fire, or frequently during the hot summer months, it may not rise to even one-half of that height.

Art. 3. The following has been found very effective for preventing concretions in water pipes. Formerly in Boston, cast-iron city pipes, 4 inches diameter, became closed up in 7 years; and those of larger diameter became seriously reduced in the same time. But later, during 8 years, in which this varnish was used, no concretions formed.*

Coal-pitch varnish to be applied to pipes and castings, made for the Water Department of Philadelphia, under the following conditions:

First. Every pipe must be thoroughly dressed and made clean, free from the earth or sand which clings to the iron in the moulds; hard brushes to be used in finishing the process to remove the loose dust.

Second. Every pipe must be entirely free from rust when the varnish is applied. If the pipe cannot be dipped immediately after being cleansed, the surface must be oiled with linseed oil to preserve it until it is ready to be dipped: no pipe to be dipped after rust has set in.

Third. The coal-tar pitch is made from coal tar, distilled until the naphtha is entirely removed, and the material deodorized. It should be distilled until it has about the consistency of wax. The mixture of five or six per cent of linseed oil is recommended. Pitch which becomes hard and brittle when cold, will not answer for this use.

Fourth. Pitch of the proper quality having been obtained, it must be carefully heated in a suitable vessel to a temperature of 300 degrees Fahrenheit, and must be maintained at not less than this temperature during the time of dipping. The material will thicken and deteriorate after a number of pipes have been dipped; fresh pitch must therefore be frequently added; and occasionally the vessel must be entirely emptied of its old contents, and refilled with fresh pitch; the refuse will be hard and brittle like common pitch.

Fifth. Every pipe must attain a temperature of 300 degrees Fahrenheit, before it is removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip.

All pipes of 20 inches diameter and upward, will require to remain at least thirty minutes in the hot fluid, to attain this temperature; probably more in cold weather.

Sixth. The application must be made to the satisfaction of the Chief Engineer of the Water Department; and the material be subject at all times to his examination, inspection, and rejection.

Seventh. Payment for coating the pipes will only be made on such pipes as are sound and sufficient according to the specifications, and are acceptable independent of the coating.

Eighth. No pipe to be dipped until the authorized inspector has examined it as to cleaning and rust; and subjected it thoroughly to the hammer proof. It may then be dipped, after which, it will be passed to the hydraulic press to meet the required water proof.

Ninth. The proper coating will be tough and tenacious when cold on the pipes, and not brittle or with any tendency to scale off. When the coating of any pipe has not been properly applied, and does not give satisfaction, whether from defect in material, tools, or manipulations, it shall not be paid for; if it scales off or shows a tendency that way, the pipe shall be cleansed inside before it can be recoated or be receivable as an ordinary pipe.

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* Mr. Dexter Brackett, of Boston, informs us, 1892, that while tubercles form there in uncoated pipes to a thickness of about three-quarters of an inch, rendering 4-inch pipes of little or no value for fire supply, yet no actual stoppage has been known to occur from this cause during the twenty-three years of his connection with the City Engineering Department. He states also that even their coated pipes, taken up after being in the ground for ten or fifteen years, are generally found to be pitted on their inner surfaces.

Art. 4. The pipes are laid to conform to the vertical undulations of the street surfaces. The tops of the pipes are laid not less than 3½ feet below the surface of the street; but in 3-inch pipes the water has at times been frozen at that depth.

In Philada., in 1885, there were about 784 miles of street pipes; or about 1 mile to every 1100 inhabitants. The population was about 860,000; residing in about 150,000 dwellings. **Berlin,** 1887-8; 1,400,000 inhabitants, in 20,000 houses (average 70 persons per house). Mean consumption per head, 17 U. S gallons per day; maximum, 24; minimum, 12½, all approximate. 25,000 wheel meters in use.

No galvanic action has been observed where lead pipes or brass unite with cast-iron ones. **No pipe** less than 6 inches diam should be laid in cities; and even they only for lengths of a few hundred feet. Their insufficiency is chiefly felt in case of fire. 8 ins would be a better minimum. No more leakage occurs in winter than in summer; except from the bursting of *private service-pipes* by freezing.

To compact the earth thoroughly against the pipes excludes air, and greatly impedes rust. Pipes may be corroded by the leakage of gas through the body as well as through the joints of **adjacent gas-pipes.**

WEIGHT OF CAST-IRON WATER-PIPES,

As used in Phila, and tested by hydraulic press before delivery to an internal pres of 300 lbs per sq inch. This table includes spigots, and faucets or bells. The pipes are required to be made of remelted strong tough gray pig iron. such as may be readily drilled and chipped; and all of more than 3 ins diam to be cast vertically, with the bell end down. Deviations of 5 per cent above or below the theoretical weights, are allowed for irregularities in casting, which it seems impossible to avoid.

The pipes are in lengths from 3 to 3½ ins longer than 12 ft; so that when laid they measure 12 ft from the mouth, *f*, Fig 38, of one bell to that of the next.

Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
3	$\frac{7}{16}$	158	16	⅝	1322	36	$\frac{15}{16}$	4334
4	⅜	211	20	"	1654	36	$1\frac{1}{16}$	4862
6	½	385	20	$\frac{11}{16}$	1798	36	$1\frac{3}{16}$	5366
8	"	460	30	[illegible]	3313	48	1¼	7282
10	½	667	30	.9	3610	48	1⅜	8607
12	$\frac{9}{16}$	899	30	1	3964	48	1½	9378

The following sizes **of lap-welded wrought-iron water-pipe** are made by the National Tube Works Co, McKeesport, Pa., and fitted with their **"Converse patent lock-joint."** One end of each length of pipe has the lock-joint permanently attached (leaded) to it at the works before shipping. The "weights per foot" include these joints. The weight of "lead per joint" given is that required to be poured in *laying* the pipe, or that for one side only of the joint.

Outer **diam**, ins	2	3	4	5	6	8	10	12	16
Weight per ft, lbs	1.86	3.48	5.26	7.33	8.76	13.20	17.08	25.12	47.70
Lead per joint, lbs	⅝	$1\frac{9}{16}$	2⅛	3¼	$3\frac{9}{16}$	5¾	6	8¾	16
Average **car load:**									
Number of lengths	800	380	275	145	126	128	80	56	49
" " feet	11500	5000	4500	2600	2000	2000	1200	800	630

The pipes are tested for a bursting pressure of 500 lbs per square inch, or higher if desired. They are furnished either coated with asphaltum, or "**kalameined;**" or, if desired, first kalameined and then coated with asphaltum. Kalameining consists in "incorporating upon and into the body of the iron a non-corrosive metal alloy, largely composed of tin." The surface thus formed is not cracked by blows, or by bending the pipe, either hot or cold.

The joint, or coupling, is of cast-iron, and has internal recesses which receive and hold lugs on the outside of each length of pipe, near each of its ends. The joint is then poured with lead in the usual way (see page 660), either with clay collars, or with a special pouring clamp furnished by the Co. This clamp resembles the "jointer," Figs 39 &c, except that it is in two rigid semi-circular pieces, connected together by a hinge-joint, and furnished with handles like those of a lemon-squeezer, and has a hole in one side for pouring The coupling forms a flush inner surface with the pipe at the joint, thus avoiding much of the resistance of cast-iron pipes to flow. For cases where it may be necessary to make frequent changes, the couplings are made in two pieces, which are bolted together by flanges.

Wrought-iron, for pipes, has the great **advantages** over cast-iron of lightness, toughness, and pliability. The lightness of wrought-iron pipes renders them easier to handle, and cheaper *per foot* notwithstanding that their cost *per ton* is about 25 per cent greater. They are not liable to breakage in transportation or from rough handling, and they may be bent through angles up to about 25°. They therefore require no special bend castings for such angles. The National Co supply bending machines, to be worked by two men. One machine can, by changing the dies, be used in bending all sizes of pipe. The pipes are in lengths of from 15 to 18 feet, instead of 12 feet, as in the case of cast-iron, so that **fewer joints are required per mile.**

The Co furnish special "service clamps" and tapping machines for **attaching service pipes to mains.** This may be done (as in the case of the Payn machine, while the main is under pressure. The service clamp is a cast iron saddle, which, before the main is tapped, is attached to it by means of a U bolt, and which remains permanently so attached after the tapping. A sheet-lead gasket is placed between clamp and main. The clamp has a tapped cylindrical opening through it, into which the corporation stop is screwed before the pipe is tapped. The drill of the tapping machine passes through the stop, and through the cylindrical opening in the clamp, and drills through the lead gasket and through the side of the main.

The Co furnish also pipe-cutting machines, and special castings (reducers, crosses, &c, &c) fitted with the Converse joint.

Art. 5. Wrought-iron pipes corrode much more rapidly than cast.

A gutta-percha pipe, ¼ inch thick, and ¾ inch bore, has sustained safely an internal pres of more than 250 lbs per sq inch; equal to nearly 600 feet head. It merely swelled slightly at 337 lbs In 1851 a tube of that material, 2½ ins bore, about ½ inch thick, and 1350 ft long, was sunk in the East River, New York, to carry the Croton water to Blackwell's Island. It was held down by weights It proved unsatisfactory owing to abrasion caused by tidal currents, and injury from the anchors of dragging vessels. A wrapping of canvas, confined by spun yarn was useful in preventing the former, but not the latter. This pipe was replaced in 1870 by wrought-iron pipes.

Ball's patent iron and cement pipe, is made by The Patent Water and Gas Pipe Co, of Jersey City, N J It is formed of riveted sheet-iron, and each length is dipped into, and coated with, a hot mixture of coal tar and asphalt. The lining of hydraulic cement is then applied This ranges, in thickness, from ⅛ inch for 12-inch pipes to 1 inch for 20-inch pipe. This pipe is made up to diams of 36 ins. It is laid in a bed of cement mortar, and completely covered with the same. Suitable means are provided for making all the attachments, &c, required in city pipes for water and gas. More than 1300 miles of it are in use in various towns some of it for 35 years; and it appears to give general satisfaction. Tubercles do not form in these pipes, as they are apt to do in cast-iron ones. There is every reason to suppose that they are durable. The trenches being dug, the Jersey City Co furnish pipes and lay them (including the cement).

A Wyckoff & Son, Elmira, N. Y., make **wooden water pipes.** For pressures of 15 to 20 lbs per sq inch, they furnish either plain pipes, 3½ to 7 ins square externally and from 1½ to 4 ins internal diam; or round pipes, 1 inch to 16 ins bore, coated externally with asphaltum cement. At their ends, both the square and the round pipes are banded with iron. For pressures from 40 to 160 lbs per sq inch, the round wooden pipes, before being coated with cement, are spirally wrapped, by steam power, with hoop iron, which is first passed through a preparation of coal tar. The iron is wound so tightly as to be imbedded in the pipe, leaving its outer surface flush with that of the wood. The ends of each length of pipe receive extra banding. The asphaltum cement coating is then applied. These pipes have been extensively and successfully used for both water and gas. Suitable arrangements are provided for joints and connections.

Water pipes of bored oak and pine logs, laid in Philadelphia in 1800-1820, are usually found quite sound, and still fit for use, except where outer sap wood is decayed When this is removed, many of these old pipes have been relaid in factories, &c. Clay *well packed* around wooden pipes, excludes the contact of air, and thus contributes greatly to their durability. Loose porous soils, such as gravel, &c, on the contrary, are unfavorable.

Pipes made of bituminized paper, prepared under great pressure, have been used for both water and gas. They are much less liable to break than cast-iron, and do not weigh or cost more than about half as much. Pipes of 5 ins bore and ½ inch thick, have resisted pressures of 220 lbs per sq inch; equal to a water head of 507 ft. British patent No. 2137, Sep 23, 1858, to A F Jaloureau, of Paris.

Costs of water pipe and laying. The following figures are deduced from a table kindly furnished by Mr. Allen J. Fuller, General Superintendent, Bureau of Water, Philadelphia. They represent average conditions for straight pipe, laid in earth, in that city. The cost, in any given case, may differ materially from these figures, according to circumstances.

"Laying" includes all handling of materials, after their delivery on the ground, for laying them in the trench, making joints, calking, etc. Calkers receive $2.50, lead men $2.00, and laborers $1.75 per day of 8 hours.

The costs of materials are taken as follows: Pipe castings, 1 2 cts; lead, 5 cts.; gasket, 3½ cts.; coke, 0 27 cts., per pound; blocking, 1.7 cts. per ft. B. M.

Add for stops, branches, fire hydrants, special castings, repaving, damages, foremen's wages, cost of tools, etc. For rough estimates, to cover fixtures, rock excavation, additional depth required for trench, wear and tear of tools, and ordinary repaving, but not including damages, asphalt repaving, or trestling, the costs in the table may be increased as follows:

Diameter of pipe	4	6	8	10	12	16 to 48 inches.
Add	80	70	65	60	50	40 per cent.

MATERIALS.

Pipe.		Per length of 12 feet.											Per lineal foot.
Diam.	Thickness.	Iron.		Lead.		Gasket.		Coke.		Block'g.		Total.	
Ins.	Ins.	lbs	$	lbs.	$	lbs.	$	lbs.	$	Ft. B M	$	$	$
4	3/8	214	2 57	6	0.30	0 2	0.01	4	0 01	2	0 03	2.92	0.24
6	7/16	367	4.40	10	0.50	0 2	0.01	4	0.01	2	0.03	4.96	0 41
8	7/16	490	5.88	12	0 62	0 3	0.01	4	0.01	2	0.03	6.56	0.55
12	9/16	918	11.02	18	0.90	0 6	0.02	5	0 01	2	0 03	11.98	1.00
18	5/8	1510	18.12	30	1.50	0.8	0.03	7	0.02	2	0.03	19.70	1.64
24	3/4	2458	29.50	40	2.00	1.0	0.03	8	0 02	3	0 05	31.60	2.63
30	13/16	3325	39.90	75	3.75	1.3	0.05	9	0.02	4	0.07	43.79	3 65
"	1	4009	48.11	75	3.75	1.3	0.05	9	0.02	4	0.07	52.00	4 33
36	15/16	4610	55.32	115	5.75	2.0	0.07	10	0.03	5	0.09	61.25	5.10
"	1 1/16	5680	68.16	115	5.75	2.0	0.07	10	0.03	5	0.09	74.09	6 17
48	1¼	8038	96.46	150	7.50	2 4	0.08	12	0.03	8	0.14	101.21	8.68
"	1½	9455	113.46	150	7.50	2 4	0.08	12	0.03	8	0.14	121.21	10.10

EARTHWORK.

Pipe.		Trench.			Earthwork, per lineal foot.			
Diam.	Thickness.	Width, feet.		Depth, feet.	Excavation.		Back fill and removing surplus.	Total.
Ins.	Ins.	Top.	Bottom.		Cubic yards.	$	$	$
4	3/8	2.50	2.25	4.50	0.40	0.09	0.02	0.11
6	7/16	2.50	2 25	4.50	0.40	0.09	0.02	0.11
8	7/16	2.50	2.25	4.50	0.40	0 09	0 02	0.11
12	9/16	2.75	2.50	4.50	0.47	0 11	0.06	0.16
18	5/8	2.75	2.50	4.50	0.48	0 11	0.09	0.20
24	3/4	3.70	3.00	4.75	0.64	0 14	0.15	0.30
30	13/16	4.25	3.25	5.00	0.76	0 17	0.23	0.40
"	1	4 25	3.25	5.00	0 76	0 17	0.23	0.40
36	15/16	5.00	3.75	5.50	0.97	0.22	0.30	0.52
"	1 1/16	5.00	3.75	5 50	0 96	0 22	0.30	0.52
48	1¼	7.00	4.50	6.50	1.47	0.44	0.56	0.99
"	1½	7.00	4.50	6.50	1.47	0.44	0.56	0.99

HAULING, LAYING, RECAPITULATION.

Pipe. Diam.	Pipe. Thickness	Hauling, per lineal foot, at 75 cents per ton. Pipe	Hauling, per lineal foot, at 75 cents per ton. Miscellaneous	Hauling, per lineal foot, at 75 cents per ton. Total	Laying, per lineal foot	Recapitulation of Cost, per lineal foot. Materials	Recapitulation of Cost, per lineal foot. Earthwork	Recapitulation of Cost, per lineal foot. Hauling.	Recapitulation of Cost, per lineal foot. Laying	Recapitulation of Cost, per lineal foot. Total.
Ins	Ins	$	$	$	$	$	$	$	$	$
4	3/8	0 01	0 01	0 02	0 03	0 24	0 11	0 02	0 03	0.40
6	7/16	0 01	0 01	0 02	0.04	0.41	0 11	0 02	0 04	0.58
8	7/16	0 01	0 01	0 02	0.04	0.55	0 11	0 02	0 04	0.72
12	9/16	0 03	0 01	0 04	0 05	1 00	0 16	0 04	0 05	1.25
18	5/8	0 04	0 01	0 05	0 06	1.64	0.20	0 05	0 06	1.95
24	3/4	0 07	0 01	0 08	0 08	2.63	0 30	0 08	0 08	3.09
30	13/16	0 09	0 02	0 11	0 08	3 65	0 40	0 11	0.08	4.24
"	1	0 11	0 02	0 13	0.08	4 33	0 40	0.13	0 08	4.94
36	15/16	0 13	0 02	0 15	0 08	5 10	0 52	0 15	0.08	5.85
"	1 3/16	0 16	0 02	0 18	0 09	6 17	0 52	0.18	0 09	6.96
48	1 1/4	0 22	0 03	0 25	0 12	8 68	0 99	0 25	0 12	10.04
"	1 1/2	0 26	0 03	0 29	0 12	10 10	0 99	0 29	0 12	11.50

Art. 6. Cast-Iron Pipe Joints. Philadelphia standard. The clear distance, *d*, between the spigot and the faucet, is nearly uniform for all sizes of pipe, varying only from $\frac{5}{16}$ inch for 4-inch pipe, to $\frac{7}{16}$ inch for 30-inch pipe. The depth, *m n*, of the faucet varies from 3 ins in 4-inch pipe, to 4 ins in 30-inch pipe.

The small beads at *s* and *m*, *s'* and *m'* on the spigot end of the pipe, project about ¼ inch; and prevent the calking material from entering the pipe. The calking consists of about 1 to 2 ins in depth of well-rammed, untarred gasket, or rope yarn; above which is poured melted lead, confined from spreading by means of clay plastered around the joint. The lead is afterwards compacted by a calking hammer

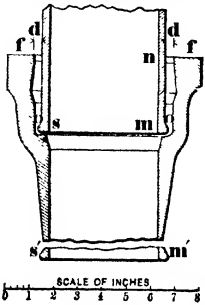
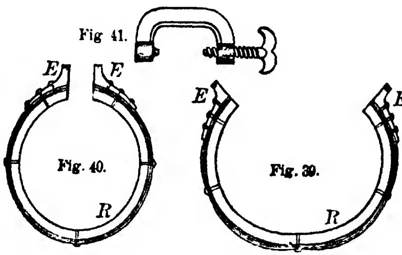


Fig 38

The lead is poured through a hole left in the clay on the upper side of the pipe. In large pipes, two additional holes are left in the clay, one at each side of the pipe, and lead is first poured into the side holes by two men at once, one man pouring into each side hole until the oint is half full. The side holes are then stopped, and, after the lead already poured has hardened, the two men finish the pouring by means of the top hole. This course is necessary, because the great weight of melted lead in the entire large joint would press away the clay at the lower side of the joint, and thus escape.

The moisture in the clay is liable to freeze in cold weather, and to render it too hard to be used. It is also liable, at all times, as is also any dampness in the pipe to be converted into steam by the heat of the melted lead. The steam sometimes breaks out, or "blows" through the clay, allowing the lead to escape.

Art. 7. The Watkins patent "**Pipe Jointer**" avoids these difficulties by dispensing with the ring of clay. It consists of a ring R, Figs 39 and 40, of square



cross-section, and made of packing composed of alternate layers of hemp cloth and India rubber. This ring is encircled by one or more thin strips of spring steel, which are riveted to it at intervals, as shown. E E are iron-elbows riveted outside of the steel bands. After the gasket has been rammed into its place, the ring is placed around the spigot near the faucet, in the position shown in Fig 40, and is held loosely by the clamp, Fig 41, one point of which enters a small pit in each of the elbows, E E. The ring is then, by means of a hammer, driven close up against the end, *f*, of the faucet, Fig 38; the screw of the clamp is tightened somewhat, so as to bring the ring close to the spigot; a small dam of clay is placed in front of the aperture between the two elbows, E E; and the joint is ready for pouring. After the lead has hardened, the "jointer" is removed, and is ready for use at another joint. Upon its removal the lead is found smooth, requiring no chipping. One can be used for several hundred joints. They dispense with the services of the men who prepare the clay collars, and supply them to the pourers. Thos. Watkins. Johnstown, Pa.

Art. 8. As a further preventive against the **escape of any of the gasket into the pipe,** a ring of lead pipe is sometimes placed in the joint before the gasket is inserted. This lead pipe is of such diameter that it can just be pushed through the space, *d*, Fig 38, between the spigot and the faucet; and of such length as just to encircle the water-pipe. It is driven as closely as possible into the narrow annular space at *o o*, Fig 38. The gasket is then rammed in, and the lead poured, as usual.

Art. 9. In John F. Ward's flexible joint, Fig. 42, for cast-iron pipes laid across the irregular beds of streams, a portion, *a o*, of the inside of the bell B is accurately turned to form the middle zone of a sphere, with center at C, and the narrow surface *m m*, on the outside of the spigot S, is turned to form a segment of a sphere accurately fitting the former. The lead is poured while the two adjacent lengths of pipe, resting on suitable vessels or floats, are in a straight line, or nearly so. The lead occupies the space *m n*, shown black, and is held in place on the spigot by the depression *d d*. As fast as the joints are thus filled, the floats are moved forward, and the pipes, if small, are passed into shallow water without further care. Suitable apparatus is used for lowering large pipes into deep water without undue strain on the joints. The joint permits a deflection of 15°, as shown; but further deflection, which would be liable to split the bell, is prevented by the stops at *o o* on the bell and *v v* on the spigot. In some cases preliminary dredging may be expedient, to diminish abrupt irregularities of the bottom. Over thirty lines of pipe furnished with this joint have been successfully laid, of diameters up to 3 feet.

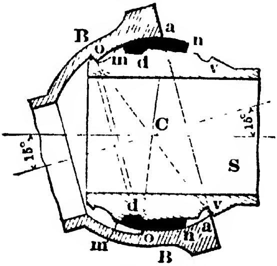
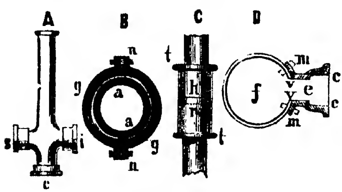


Fig. 42.

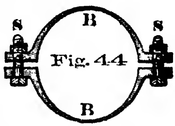
Art. 10. In Figs 43, A is **a double branch;** which is a pipe having, in addition to the faucet, *c*, at one end, two others, *s* and *i*, to which pipes leading in opposite directions (as at cross-streets) may be attached. If either *s* or *i* be omitted, the pipe is a **single branch.** The pipe is stronger when these extra faucets are near its end, than if they were at its middle. In a long line of pipes, for the sake of expedition, different gangs of men are frequently laying detached portions some distance apart; and when two ends of different portions are brought near enough together to be united, as *h* and *r*, Fig C, their junction cannot be effected by the usual spigot-and-faucet joint. In this case a cast-iron **sleeve,** *t t*, is used, which is first slid upon one of the pieces of pipe; and (after the other piece also is laid) is slid back into the position in the fig. so as to cover the joint. Sleeves are usually about a foot long; as thick as the pipe; and their diam is sufficient to allow the usual joint of gasket and lead. There is of course such a joint at each end of the sleeve.



Figs. 43

Art. 11. When a crack occurs in a pipe, *a a*, Fig B, already in use, it is repaired by means of a cast-iron sleeve, *g g*, made in two parts, bolted together by means of flanges as at *n n*. In other respects it is like the preceding sleeve. The intermediate white ring is the lead joint. If the crack is too long, or otherwise too bad to be remedied by a sleeve, the pipe is broken to pieces; and the lead joints at its ends melted out, so as to allow of its removal. Then, since an entire new pipe cannot now be inserted, owing to the overlapping of the spigot-and-faucet ends, two short pieces must be substituted for it. One end of each of these is lead-jointed to the pipes already laid; while the other two ends, which will probably be a few inches apart, are covered by a sleeve, *t t*, Fig. C.

Cracks may at times be temporarily repaired in an emergency, by a wrapping of folds of canvas thoroughly saturated with white-lead paint; and tightly confined to the pipe by a spiral banding of thin hoop-iron or wire. Or, by an iron band, made in two parts, B B, Fig 44, and clamped together by screw-bolts, S S. Such bands are useful, also, for strengthening pipes that are considered to be in danger of bursting.



Art. 12. To attach a pipe, *e*, Fig 43, to one, *f*, already in use, but in which no provision has been made for such attachment, a piece may be cut out of *f*, as at *v v*, and a casting, *e*, furnished with flanges, *m m*, bolted over the opening, by screw-bolts passing through female screws tapped in the thickness of the pipe. If the new pipe is so large that the opening, *v v*, if circular, would be inconveniently wide, it may be made *oval*, with the longest diameter in the direction of the *length* of the pipe, *f*. In that case the casting *e* will be oval at its flanges; and circular at *c c*.

Art. 13. Air valves. Air is apt to collect gradually at the high points of vertical curves along the supply pipes; and, unless removed, obstructs the flow. This may be prevented by an air valve, Fig 44A. This consists of a cast-

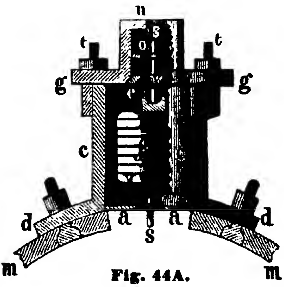


Fig. 44A.

iron box, *c c d d*, confined to the main pipe *m m*, by screw-bolts passing through its flange *d d*. It has a cover *g n g*, confined to it by screws *t t*; and at the top of which is an opening *n*, for the escape of air from within. In this box is a float *f*, which may be a close tin or copper vessel, or of layers of cork, as supposed in the fig; or &c. This float has a spindle or stem *s s*, fast to it; which passes through openings in the bridge-bars *a a*, and *o*; thereby allowing the float to rise and fall freely, but preventing it from moving sideways. When the pipe *m m* is empty, the float is down; its base *y* resting on the cross-bar *a a*. The stem *s s* has fixed to it a valve *v*, which rises and falls with it and the float. Suppose the pipe *m m* to be empty, and consequently the float and the valve *v* down. Then, if water be admitted into the pipe, it will rise and fill also the box as far up as *e*; and in doing so will lift the float *f*, and the valve *v*, to the position in the fig; thus preventing egress to the outer air by closing the opening at *v*. Now, air carried along by the water, will, on account of its lightness, ascend to the highest points it meets with.

Hence, when such air arrives under the opening *a a*, it will rise through it, and ascend to *e*; the closed valve preventing it from going farther. Thus successive portions of air ascend, and in time accumulate to such an extent as gradually to force much of the water downward out of the box. When this takes place, the float, which is held up only by the water, of course descends also; and in doing so, pulls down with it the valve *v*. The accumulated air then instantly escapes through the openings at *v* and *n*, into the atmosphere; and the water in the pipe *m m*, immediately ascends again into the box, carrying with it the float; and thus again closing the valve *v*. The valve, and the valve-seat *e*, are faced with brass, to avoid rust, and consequent bad fit. The whole is protected by an iron or wooden cover, reaching to the level of the street.

Air valves are no longer used in city pipes; their place being supplied by the fireplugs at average distances of about 150 yards apart. These, being placed as much as possible at the summits of undulations in the lines of pipes, for convenience of washing the streets, and being frequently opened for that purpose, permit also the escape of accumulated air.

The escape of compressed air through an air valve, or other opening, has been known to produce bursting of the main pipes; for the escape is instantaneous, and permits the columns of water in the pipes on both sides of the valve, to rush together with great forces, which arrest each other, and react against the pipes.

Air-Vessels. Motion is imparted to the water in a line of pipes, by the *forward* stroke of the piston of a single-acting pump; but during the *backward* stroke, this motion is stopped; and the water in the pipes comes to rest. Therefore, at the next forward stroke, all the water has to be again set in motion; and the force that must be exerted by the pump to do this is much greater than would be required if the motion previously imparted had been maintained during the time of the backstroke. The addition of an air-vessel secures this maintenance of motion, and thus effects a great saving of power; besides diminishing the danger of bursting the pipes at each forward stroke. It is merely a tall and strong air-tight iron box, usually cylindrical, strongly bolted on top of the pipes just beyond the pump, and communicating freely with them through an opening in its base. It is full of air. The forward stroke of the piston then forces water not only along the pipes, but also into the lower part of the air-vessel through the opening in its base; thus compressing its contained air. But during the backstroke, this compressed air, being relieved from the pressure of the pump, expands; and in so doing presses upon the water in the pipes, and thus keeps it in motion until the next forward stroke; and so on. An air-vessel also acts as an *air-cushion;* permitting the piston to apply its force to the water in the pipes gradually: thus preserving both the pipes and the pump from violent shocks. The air in the vessel, however, becomes by degrees absorbed and taken away by the water; and its action as a regulator then ceases. To prevent this, fresh air must be forced into the vessel from time to time by a condenser, or forcing air-pump. A *double-acting* pump does not *so much* need an air-vessel. There is no particular rule for the size or capacity of air-vessels. In practice it appears to vary from about 5 to 50 times that of the pump; with a height equal to two or more times the diameter. A stand-pipe (see below) is sometimes used instead of an air-vessel.

A stand-pipe is sometimes used for the same purpose as an air-vessel (see above). It is a tall pipe, open to the air at top; and communicating freely at its foot with the water-pipe, in the same manner as in an air-vessel. Its top must be somewhat higher than that to which the pump has to force the water through the system of pipes; otherwise the water would be wasted by flowing over its top. The area of its transverse section should be *at least* equal to that of the pipe or pipes which conduct the water *from* it; but it is at times better to have it much larger, as a stand-pipe may then answer, especially in a small town, *as a reservoir*, if the pumping should cease for a few hours. A stand-pipe should be cylindrical, not conical; for if thick ice should form on top of the water in a conical one, a sudden forcing of it upward by the pump might strain the stand-pipe seriously. The stand-pipes connected with the Philadelphia Water-Works are from 125 to 170 feet high; 5 feet diameter; and made of riveted boiler-iron about ⅜ inch thick near the base, and about ¼ inch near the top. They have no protection from the weather; nor are they braced in any manner; but retain their positions by their own inherent strength, although exposed at times to violent winds.

Art. 14. The service-pipes for supplying single dwellings, are of lead; and of ½ to ⅝ inch bore. They are connected with the street mains, n n, Fig 45, by a **brass ferrule,** *f*, here shown at ⅙ real size. The dotted lines show its ½ inch bore. The tapering ferrule is merely hard driven into a cylindrical hole reamed out of the main, as at *s*. The lead pipe, *o*, is attached to the other end of the ferrule; overlapping it about 1½ ins; and the joint soldered, *t*. The extra thickness near *f*, is for giving proper shape and strength for hammering the ferrule into the main. The pipe and solder are shown in section. Besides the stopcocks attached to each service-pipe, and to its branches through the house, there is an underground one by which the city authorities can stop off the water in case of delinquency in payment of dues; and another by which the plumber can stop it off when so required during indoor repairs. **Galvanized iron tubes** are being much used for service-pipes, especially for hot water; being less subject to contraction and expansion, which produce leaks

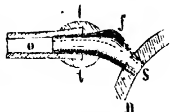


Fig 45.

Art. 15. The so-called **"corporation stops"** or "corporation cocks" are inserted into the pipe by a special machine, Fig 46. Their great advantage over the ferrule, Fig 45, is that **they can be inserted into a pipe when the latter is full of water under pressure.** Besides, inasmuch as they are *screwed* into the pipe, they are in no danger of being forced out of it by any pressure within it. As their name implies, they are furnished with a stop valve, which is kept closed while the valve is being inserted into the pipe, and is then opened, and generally remains open permanently.

Pipe-tapping machines, for drilling and tapping the pipe, and for attaching these stops, are made in a variety of forms. Fig 46 shows one made by **Walter S. Payne & Co.,** Fostoria, Ohio. Each of these machines is furnished with a number of malleable cast-iron saddles, which fit the various diams of pipe with which it is to be used. The saddle is not shown in the fig. It is fastened to the pipe by a chain slung around the latter. The chain is tightened by a bolt and nut at each end.

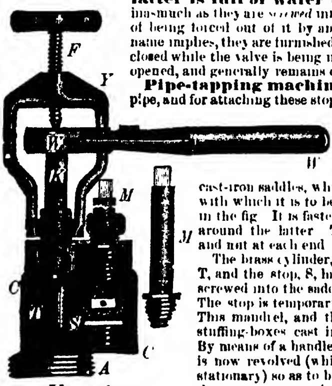


Fig. 46.

The brass cylinder, C C (into which a tap-and-drill, T, and the stop, S, have first been inserted), is then screwed into the saddle by means of the thread at A. The stop is temporarily screwed on to a mandrel, M. This mandrel, and the drill-shank, K, pass through stuffing-boxes cast in one with the head of the cyl. By means of a handle, not shown in the fig, this head is now revolved (while the body of the cyl remains stationary) so as to bring the drill, T, and stop, S into the respective positions shown in the fig. When the cyl head comes to the proper position, it is stopped by a lug inside of the cyl. The drill is then immediately over the center of a large circular opening in the base of the cyl, C C, and over a similar opening, through the saddle, to the surface of the pipe to be tapped. It is then pushed down until it touches said pipe. The ratchet-wrench, W W, is then set on the square head of the drill-shank, K; the feeder-yoke, Y, with feed screw, F, is put in position as shown; and the pipe is drilled and tapped by working the wrench; whereupon the water in the pipe, if under pressure, rushes out through the hole thus made, and fills the cylinder.

By reversing the position of the switch on the ratchet in the wrench, W, and by working the latter, the tap is now withdrawn from the hole, but remains in the cyl. The cyl head is now revolved so as to reverse the positions of S and T; the lug inside of the cyl stopping the head when the *stop* is immediately over the hole. By means of the ratchet-wrench, applied to the square head of the mandrel, M, the stop, S (*the valve of which must be closed*), is now screwed into the hole, but only far enough to hold securely, and thus prevent the further escape of the water from the pipe when the machine is now removed. The stop is now screwed firmly into place by means of a wrench applied to a square on the stop itself. When the pres in the pipe exceeds about 200 lbs per sq inch, the feeding apparatus, as used with the *drill* (see fig), may be also used in aiding the insertion of the stop.

The mandrel, M, is made in two lengths (one of which screws into the other) in order that the upper part may be out of the way of the wrench-handle while drilling. It has three or more diff threads at its foot, to suit diff sizes of stop. Stops made to suit the machine, are furnished as wanted.

The machine can work in any direction radial to the pipe, and can therefore be used for tapping a pipe in any part of its circumference.

After the stop is inserted, the service-pipe is attached to its outer end by a coupling nut passing over the thread there shown.

The machines are guaranteed to tap under a pressure of 600 lbs per square inch.

Art. 15 a. The pneumatic dome Figs 46 *a* and 46 *b*, invented by Mr. N. Monroe Hopkins, of Washington, D. C., is designed to prevent the bursting of water-pipes in freezing weather.

In unprotected pipes, the water, in freezing, is unable to expand longitudinally, and therefore frequently bursts the pipe in expanding laterally. The domes, being placed in the pipe, as shown, at intervals of about 12 feet, where freezing is to be apprehended, permit the longitudinal expansion, which pushes the ice in both directions toward each dome, where it compresses the air-cushion there provided. In the horizontal dome, Fig. 46 *a*, the double inclined planes, cast in its lower side at *c* compel the two horizontal columns of ice to rise into the dome, instead of merely abutting endwise against each other.

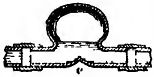


FIG. 46 *a*.

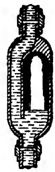


FIG 46 *b*.

In order to insure that the domes in a system (such as those for a house or mill or on a bridge) shall not be deprived of their air by the flow of the water in the pipes below them, an inspirator is placed in the pipe at the entrance to the system. The inspirator consists essentially of a constriction in the pipe, which increases the velocity of flow at that point, and thus causes an indraft of air through a valve provided for the purpose (see Venturi meter, page 532). The air, thus introduced, is carried along the pipe in bubbles between the surface of the water and the top of the pipe and is entrapped by the domes When, by closing faucets, etc, the flow in the pipe is checked, the increase of pressure closes the valve

Severe tests of both large and small pipes (4-inch and 3/4-inch), protected by these domes, have shown them to be always effective in preventing rupture.

Art. 16. Stop-valves, or **gates,** opening vertically in grooves, are placed across the street pipes at intervals of from 100 to 300 yds. Their use is to shut off the water from any section during repairs; the water of such sections being allowed to run to waste, and to soak into the ground.

The details are much varied by diff makers.

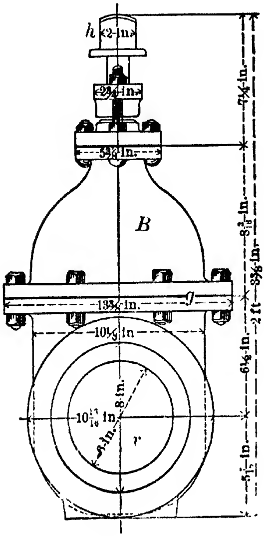


Fig. 47.

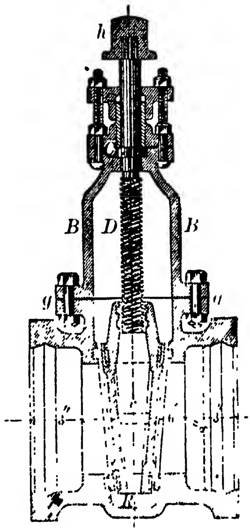


Fig. 48.

Figs 47 and 48 show such a gate. The valve, *v*, is cast in one piece. When down, as in the figs, it closes the pipe. It opens vert by means of a screw, D, the valve rising into the cast-iron case or box, B B, and leaving, when all the way up, an opening of the full diam of the pipe. The screw is turned by a wrench fitting on its square head, *h*. The screw, D, itself, is prevented from moving vert by the collar, C.

The two principal castings which compose the box or cover are bolted together by means of flanges, *g*. The joint faces of the castings are carefully smoothed; and a thin strip of lead is inserted between them, as a precaution against leaks. The recess, R, admits small particles of foreign matter which might otherwise prevent the gate from closing perfectly. The valve seats are faced with Babbitt metal. At the top of the cover, the screw stem passes through a stuffing-box, which prevents leaking at that point.

Bell-end Water-gates.

Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.
2	32	6	195	12	600	20	1706
3	55	7	245	14	843	24	2750
4	116	8	290	16	1080	30	6400
5	135	10	439	18	1475	36	8300

Art. 17. Fig 49 shows an arrangement **with outside screw** for raising and lowering the valve. Here the screw, D. does not revolve, but is attached to the valve, and rises and falls with it, being raised or lowered by turning the wheel, W, at the center of which is a nut through which the screw passes. The nut is fixed in the wheel, and is so confined that it, and the wheel, cannot move vertically.

Art. 18. A four-way stop, or **four-way valve,** Figs 50 and 51, is placed at the intersection of two mains; the four ends of which are attached, respectively, to the four openings, M M M M. At the bottom is an additional opening, connecting, by means of an elbow, H, with pipes running to a fire-hydrant at the street curb. See Arts 20 and 21. Two or more of such bottom openings may be made, if desired, for the supply of as many fire-hydrants. All of the openings are opened or closed at one time by raising or lowering the valve or plug, P, by means of a wrench or key applied to the square head, S, of the screw stem. As in Figs 47 and 48, the screw turns, but is prevented from rising and falling, and the plug moves up and down on the screw.

Inasmuch as all sediment escapes into the bottom opening which leads to the fire-hydrant, the valve is not liable to clogging through this cause. The fire-hydrant, being fed from both of the mains, obtains a fuller supply than would be possible if it were fed, as usual, through only one main.

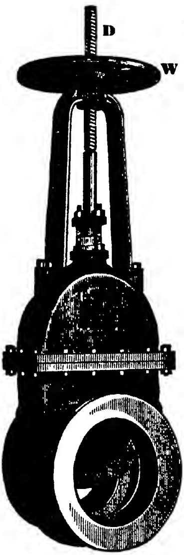


Fig. 49

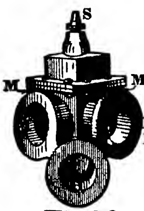


Fig. 50

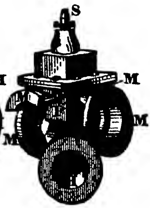


Fig. 51

Art. 19. Whatever the style of the gate may be, it is, when attached to the pipe, **protected by a surrounding box,** generally of plank or cast-iron, with four sides, which taper so that the box is of smaller hor section at top than at bottom. It is open at bottom, but has a movable iron top, level with the street. This top is taken off when the valve is to be opened or closed, or inspected. Two of the opposite sides of the box of course have openings for the passage of the pipes to or from the valve.

The gates, especially of large mains, must be closed very slowly. Otherwise, the too sudden arresting of the momentum of the flowing water would be apt to break either them or the covers; or burst the pipes. As a precaution against this, the covers for very large valves are cast with outside strengthening ribs.

No self-acting air-valves (Fig 44 A) are now placed at street summits, to allow confined air to escape. The fire-plugs answer instead. The rad for hor bends in mains is if possible not less than about 12 times their diams, they are made as large as the widths of the streets will admit: usually about 50 ft. **Fire-plugs,** Figs 52, &c are placed as much as possible at summits, so as to serve also for washing the streets: and for the escape of accumulated air. They average about 8 in number to each mile of pipe; or 1 to each block of buildings.

Fire Hydrants.

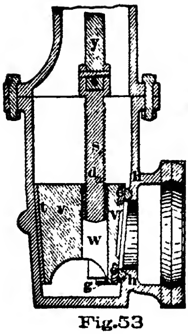


Fig.53

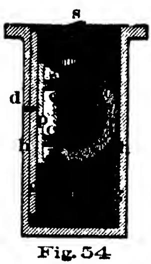


Fig. 54

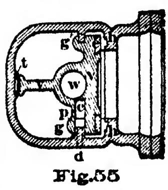


Fig.55

Art. 20. Fig 52 represents a common street **fire-plug, or fire-hydrant.**

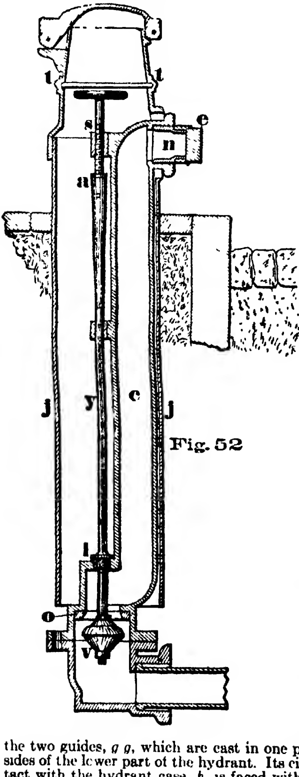


Fig. 52

The valve, *v*, is of layers of well-hammered sole-leather; and, when closed, shuts against a brass ring seat, *o*, which is confined to its place by a lead joint. The valve is opened by working down the screw, *s*, which, by means of a swivel joint at *a*, can revolve without turning the valve-rod, *y*. When the valve, *v*, is closed, after the plug has been in use, the chamber, *c*, is full of water, which, if allowed to remain, would be in danger of freezing, and of bursting the plug. But, in closing the valve, we raise the flange, *l*, on the rod, and thus allow the water to escape through the opening at *l*, whence it runs to waste into the ground, through the open lower end of the **frost-jacket,** *j j*, which is a hollow cast-iron cylinder surrounding the working parts of the hydrant. Being free to slide vert it rises and falls when the level of the ground is disturbed by frost, and the hydrant is thus protected against injury from this cause

The top, *t*, of the hydrant case, is cast in one piece with the chamber, *c*.

The stopper, *e*, screws on over the nozzle, *n*.

Art. 21. In Figs 53, 54, 55, **the valve, v v, is a *sliding* one.** The stem, *y*, Fig 53, to which the screw, *s*, is attached, is, like that in Figs 47 and 48, prevented from moving *vert* by a collar, fast to it near its top, and confined in a circular groove. When the rod and screw are made to revolve, by means of a wrench applied to the square head of the former, the valve slides up or down on the screw, admitting the water to, or shutting it off from, the hydrant. The valve slides on the two guides, *g g*, which are cast in one piece, respectively, with the two *vert* sides of the lower part of the hydrant. Its circular face, where it comes into contact with the hydrant case, *h*, is faced with a gun-metal ring, *e e*, which bears against a similar ring, made of Babbitt metal, let into the hydrant case.

The water, left in the hydrant case after closing the valve, **escapes** through a cylindrical hole, *d*, about ⅜ inch diam, bored through the guide, *g*, and case, *h*. This hole is at such a ht as to be just above the top of the loose plate, *p*, when the valve is closed, as in Fig 53. This plate lies in a vert groove in the side of the valve-casting, and is pressed against the side of the case, *h*, by two spiral springs confined in cavities, *c c*, in the valve casting. When the valve begins to rise, the hole, *d*, is closed by the plate, *p* and remains so until the valve is again entirely closed.

Test Borings. Figs 1 and 2 show a tool **for boring into soils, clay, sand,** or **gravel,** even when quite indurated, or when frozen. It will not bore through hard rock, or through large boulders. It consists of two sheet-iron cylindrical segments S S, called "pods," having their lower or cutting edges shod with steel These edges project (as shown in Fig 1) beyond the sides of the auger, and thus make the hole larger than it, so that it cannot bind or stick. The two cutting edges are equidistant from the vert cen line of the tool, and this insures a straight and vert hole. At *a* the auger is attached to the lower end of a vert boring rod composed of a number of 1½-inch square iron bars, or 2½-inch iron tubes, about 10 to 15 ft long, jointed together at their ends by means of square socket joints. At the top of this boring-rod is a swivel-hook, by means of which the entire apparatus is hung to the end of a rope, which passes over a pulley at the top of a derrick or tripod, and down to a drum worked by a windlass and gearing. By means of this drum and rope, the auger and boring-rod (which at first consists of only one bar) are lifted, and suspended over the intended hole. The auger is then lowered, and rotated hor by two men or one horse, working at the ends of levers which grip the boring-rod a few ft above the ground. The swivel at the top of the boring-rod permits this rotation to take place without twisting the rope. The shape of the auger is such that its rotation feeds or screws it into the ground; and the man at the windlass has, during the boring, merely to keep the rope tight, so as to prevent the auger from boring too fast, and becoming clogged. In about 8 revolutions the auger fills with earth. By means of the windlass it is then raised to about 2 ft above the ground; and by unkeying and removing the band *b* the auger is opened like a pair of tongs, and the earth emptied into a wooden box which has in the meantime been placed over the hole. The box is then removed and emptied, and the boring proceeds as before. When the boring has reached a depth of about 10 ft, a second bar must be added to the top of the rod. For this purpose the rod and auger are raised a few inches; a slight frame-work of boards is placed on the ground, close to the boring-rod and surrounding it; and a flange is clasped tightly to the rod just above, and close to, the framework. The framework and flange now support the rod and auger; the swivel-hook and rope are removed, and attached to the upper end of the second bar, which is then raised, and its lower end is fastened into the socket-joint upon the top of the first one. The rope is then drawn tight; the flange removed; the auger lowered to the bottom of the hole; and the boring resumed. Additional lengths of boring-rod are attached in the same way from time to time, as required by the descent of the auger.

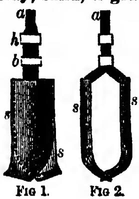


Fig 1. Fig 2.

The borers may be made from 6 to 18 inches in diameter, or larger. If desired, the boring may be made from 24 to 36 ins diam by attaching a reamer to the auger. This auger will bore to a depth of 100 ft or more at the rate of from 5 to 20 ft per hour. It removes stones as large as half the diam of the hole. In dry soils a bucketful of water is poured into the hole each time the auger is raised.

This borer may be advantageously used in boring the holes for **sand piles,** and at times, **instead of driving wooden piles,** it may be better to *plant* them (butt down if preferred) in holes bored by this auger; ramming the earth well around them afterwards. This will save adjacent buildings from the jarring and injury done by a pile driver.

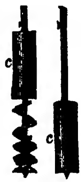
If sand, mud, or loose gravel is reached in boring with this tool, the hole is reamed out 4 ins larger, and a **tubing** of inch boards is inserted into the hole, and driven into and through the sand or gravel, which is then removed from within the tubing by means of a **sand-pump,** consisting of a hollow iron cylinder, about 5 ins diam × 30 ins long, with a valve at its foot, opening upward. The sand-pump is lowered to the bottom of the hole; covered with water to a depth of 2 to 4 ft, and churned quickly up and down 4 to 6 ins, by hand, 20 or 30 times, during which the sand fills the pump, which is then drawn up and emptied. From 10 to 20 ft in depth of sand, mud, &c, per hour can thus be taken from a 6 to 18-inch hole. This pump is also used for removing broken earth, &c, from a hole bored in compact earth by the borer first described.

The cost, with derrick, boring-rods, rope, sand-pump, &c, &c, complete, is about $175. **The auger weighs** from 150 to 200 lbs, according to size. Boring-rod 1½ ins sq, 3⅔ lbs per ft. Derrick, 150 lbs.

The sand-borer, Figs 3 and 4, like the sand-pump just described, is used inside of tubing, and for the same purpose. The hollow iron cylinder C, 10 ins diam × 30 ins long, slides vertically on the rod, but the screw is fast to the rod. While boring, the sand below and around the cyl keeps it in the position shown in

Fig 3. Six revolutions of the rod and screw fill the cyl with sand. The rod is then lifted. This first draws the screw up into the cyl, as in Fig 4; and a valve at the foot of the screw closes the bottom of the cyl, and prevents the sand from falling out when the borer is lifted from the hole. The rod is hollow, and open at top and bottom. This allows passage of the air, and thus prevents resistance from suction in withdrawing the borer. This tool is rotated and withdrawn in the same way as the earth borer first described. Cost about $30.

Steel prospecting augers, from 2 to 4 ins diam, and 2 ft long, are used for boring holes from 2½ to 6 ins diam, and to depths of 10 to 50 ft, into **clay, sand,** or **fine gravel,** of all of which they bring up samples. They are turned by wrenches, and by man or horse power. See also p 674.



Figs 3, 4

The boring tool shown in vert section by Fig 6, and in hor cross section by Fig 5, is very useful **for boring shallow holes by hand through surface soils,** clay, and gravel, and bringing up samples. The borer proper consists of a cylinder of spring steel, 3 or 4 ins diam, and 4 or 5 ins high, with sides ¼ inch thick, having a vert slit (see cross section) throughout its height, and beveled to a cutting edge all around its foot, as shown in the vert section. At its top it is riveted, as shown, or welded, to the inverted-v-shaped forging, which, by means of the socket at its top, is screwed to a length of gas-pipe which serves as a handle, and to which other pieces are joined by sockets as boring proceeds.

The boring is done by two men, who grasp the handle, and, holding the tool vert, drive it into the ground by repeatedly lifting it and forcibly bringing it down upon the same spot. As the tool strikes the ground, the beveled shape of its cutting edge causes it to open slightly, and when the downward pres is relieved in lifting it, it springs back and grasps the earth which has entered it. It soon fills, and the men, finding that it ceases to penetrate readily, lift it to the surface and empty it. The character of its contents from different depths, measured along the handle, is noted from time to time.

In six days of 8 hours each, three men (one resting at intervals) using one such auger between them, bored 20 holes, averaging 9½ ft each, in loam, gravel, clay, and decomposed mica schist, at a cost of 22 cts per foot. Wages of each man, $2 per day.

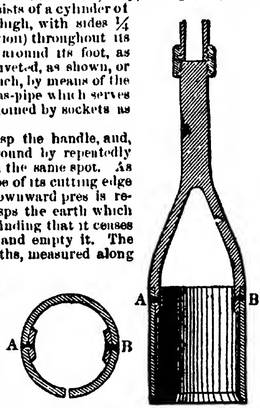


Fig 5. Fig 6.

For work in loam, clay, or non-running sand, **an effective screw-auger** can be made by any good blacksmith, by merely forming a one-inch sq bar of iron or steel into corkscrew shape about 2 ft long with 6 complete turns 6 ins in diam; its lower end sharpened to form a vertical cutting edge, which should project say .5 of an inch beyond the spiral of the screw, in order to diminish friction. It will bring up full samples. Requires a derrick, or some other simple mode of lifting, when the screw is full.

Artesian Well Drilling. Deep vert holes in earth and rock, 6 and 8 ins in diam, such as are reqd for artesian wells for water and oil, and for mining explorations, are drilled by repeatedly lifting and dropping, in the same vert line, a heavy iron bit, Fig 1, p 673, with a steel cutting-edge. The bit is partly revolved horizontally after each blow, to insure roundness of hole. The length of the cutting-edge of the bit is a little greater than the diam of the bit, and the hole is thus made sufficiently large to prevent the bit from binding in it.

The bit is the lowest one of a series of iron and steel bars, &c, screwed together at their ends, and called a **"string of tools."** The string of tools varies in length from 25 to 60 ft, according to the size and depth of the hole, and the hardness of the rock; and its diam throughout (above the cutting-edge) is an inch or two less than that of the hole. Its weight is from 800 to 4000 lbs. Its upper member is always a "rope-socket," Fig 4 (*without* a swivel), to which the lower end of the supporting rope **cable** is attached. This cable passes up out of the hole to **a hor lever,** which, by means of a horse-power or steam-engine, is kept constantly moving up and down with a see-saw motion. The string of tools, with the cutting edge of the bit at its lower end, is thus alternately lifted from 2 to 4 ft, and

let fall, from 30 to 50 times per minute, and so drills its way into the rock or earth. From 4 to 10 ft in depth of water are kept in the hole, to facilitate the drilling and the removal of debris. After water is reached, the drilling may be continued, even if the hole is full of water; but a great depth of water of course diminishes the force of the blows of the bit. A suitable arrangement must be provided for **paying out the rope** as the boring tool descends. A clamp is attached to the cable, and the man in charge, by turning the clamp, twists the rope, and thus **turns the bit horizontally** about one-fifth of a revolution after each stroke, until six or eight complete revolutions have been made in one direction. He then reverses the motion, and makes an equal number of turns, at the same rate, in the opposite direction.

After drilling a few feet, the string of tools is lifted out of the hole by means of the cable, to allow the **removal of the debris** which has accumulated in the hole. This is done by means of a **sand-pump**, which is a sheet-iron cylinder, say 4 ins diam, and 4 to 6 ft long, provided at its foot, with a valve opening upward. The pump is lowered to the bottom of the hole, and filled with the mixed water and debris by churning it up and down a number of times. Sometimes, in addition to the valve, the pump is fitted with a plunger, which is at the foot of the pump when the latter is let down to the bottom of the hole. The plunger is then drawn up into the pump, and the debris follows it. In either case, the pump, when filled, is lifted out of the hole and emptied; the string of tools is again lowered into the hole, and the drilling resumed. The debris must be removed after every 3 to 5 ft of drilling. Otherwise it would interfere too greatly with the action of the bit.

Wells are usually drilled from 6 to 8 ins diam. For diams less than 6 ins, the tools are so slender that they are liable to be broken in a deep hole.

The same apparatus is used for **drilling through the earth above the rock,** before the latter is reached. This is called "spudding." In this case the sides of the hole must be prevented from caving in. For this purpose a wrought-iron pipe of such diam as to fit the hole closely, and ¼ inch thick, is inserted into the hole, and is driven down from time to time as the drilling proceeds. The pipe is driven by means of a heavy maul of oak, or other hard wood, 14 to 18 ins square, and 10 to 16 ft long. This maul is attached, by one end, to the lower end of the same cable which, during drilling, supports the string of tools. It is thus repeatedly lifted, and dropped upon the head of the tube, which is protected by a cast-iron "driving-cap." The foot of the tube is shod with a steel cutting-edge ring, or "steel shoe." When the tube has been driven as far as it will readily go, the maul is removed from the end of the rope; the string of tools substituted; and the drilling resumed within the pipe.

The pipe is put together in lengths of from 8 to 18 ft, and the drilling and pipe-driving proceed alternately until the rock is reached, and the foot of the pipe forced into it to a depth of a few ins, or far enough to shut off quicksand or surface water.

If quicksand is encountered, the string of tools is removed, and the sand-pump is used inside of the pipe.

For reaming out, or **enlarging, holes,** or for **straightening** crooked ones, &c, special tools, such as reamers, &c, are substituted in place of the boring bit.

Special care must be taken to have **all the rubbing surfaces thoroughly lubricated.** The pulley in the mast-head, and the pinion-wheels of the horse power (if such be used) should be well oiled every two or three hours.

In very **cold or wet weather, a shed of rough boards,** or a covering of canvas, about 8 ft high, should be erected, to protect the men; and, if steam is used, 2 or 3 boards should be used as a covering for the belt, which will slip if wet.

For holes from 200 to 1000 ft deep, **portable drilling machines,*** worked by horse or steam power, are used. In these machines, the drill-rope, extending from the string of tools up out of the hole, passes over a sheave at the top of a wooden mast; down to, and around, a pulley fast to the working lever; and thence, by way of a pulley fixed at the foot of the mast, to a drum upon which it is wound. To this drum a friction and ratchet wheel is attached, for paying out the cable as the tools descend.

The mast is hinged six feet above its foot, so that its upper part may be laid hor when the machine is to be moved. When at work, it is held in position by two timber struts or braces, bolted to it near its top, and having their lower ends fastened to the "**drill-jack,**" which is a light and strong framework, 9 ft long, 8 ft wide, and 4 ft high, at the foot of the mast, containing the working lever which

* See Price-list, 3.67.

raises the rope and lets it fall, the drum on which the rope is wound, the shaft and cam which work the lever, &c. The operator stands at the foot of the mast, and by means of foot- and hand-levers within his reach, regulates all the movements of the machine. One of these governs the pawl and ratchet wheel regulating the paying out of the cable. By letting the ratchet-wheel of the drum move one notch, the bit is fed down quarter of an inch.

The operator, by moving a slide with his foot, holds the working lever down, out of reach of the cam, thus stopping the up-and-down motion of the rope and tools. By means of another lever he can now put the rope-drum in gear with the main driving-shaft, so that the rope is wound up on the drum, and the tools drawn up out of the hole. Another lever controls the separate reel on which the light rope, carrying the sand-pump, is wound. All these operations are performed by the same power (horse or steam, as the case may be), which works on without stopping; the various changes being made by merely throwing the different parts into, or out of, gear with the main driving-shaft.

Fig. 4.

One of these portable machines requires two horses or a small steam-engine, a man to attend the same, and another man to operate the machine, empty the sand-pump, change the tools, &c. It can be transported on a farm wagon over any common road. Two men can unload it, set it up, and commence drilling, in two hours; and, unless steam is preferred, the two horses used for its transportation furnish the motive power. The machine can be taken down and reloaded in the wagon in two hours.

Figs 1 to 4 show the **tools used with these machines.** For the different sizes of machine they differ chiefly in their dimensions and weights.

Fig 1 shows the **drilling bit,** which is 30 to 36 ins long, and weighs about 100 lbs. Its lower or cutting edge is 6 ins long. Its top is screwed into the foot of the "**auger-stem,**" Fig 2, which is of 3-inch round iron, 12 ft long, and weighs 350 lbs. Its use is that of a weight, giving additional force to the blows of the bit. Its top is screwed into the foot of the "**drill-jars,**" Fig 3; and to the top of these is screwed the "**rope-socket,**" Fig 4, to which the drilling cable is attached. **If the bit, or auger-stem, becomes wedged in the hole by any means,** the operator stops the churning motion of the tools, and the rope is let out about 12 ins. This permits the upper link U of the drill-jars, Fig 3, to slide down about 12 ins in the slot S in their lower link. The churning motion is then started again, and the upward jerk of the link U against the upper end of the slot loosens the tools.

Fig. 2.

These machines are made in a number of sizes, to drill holes from 200 to 1000 feet deep. The string of tools weighs from 800 to 1800 lbs; and the machine complete including tools, rope, mast, etc., but exclusive of power, from 1800 to 4500 lbs. They cost from $700 to $1500 exclusive of power. The smaller sizes may be worked by horse power. A horse power weighs about 800 lbs, and costs about $75. Steam engine, 1600 to 3600 lbs., $150 to $300.

For wells from 1000 to 3000 feet deep, a stationary machine, with a walking-beam, is used, similar to those employed in the oil regions of Pennsylvania. A square pyramidal derrick is erected, 74 feet high, 20 feet square at base, 4 feet square at top. Each of its 4 corner legs is of 2 inch × 8 inch and 2 inch × 10 inch planks, spiked together so as to form a 10 inch × 10 inch angle-piece, 2 inches thick. The legs are braced together by horizontal and diagonal timbers. The walking-beam is of timber, 26 feet long, 12 inches wide, and 26 inches deep at the middle of its length, where it is pivoted to the top of a wooden post 18 inches square and 12 feet high, called a "Samson post."

Fig. 1.

Fig. 3.

This post, at its foot, is dovetailed into the main sill of the machine, which is 18 inches wide × 24 inches deep.

The motive power is a 15-hp steam-engine, which, by means of a belt and pulley crank and pitman, working at one end of the walking-beam, gives to the latter its see-saw motion. To the other end of the beam, and immediately over the well, is suspended, by means of a hook, a "temper-screw." This last is composed of two bars of iron, about $\frac{5}{8} \times 2$ ins, 5 ft long, hung 2 ins apart, fastened together at their top ends, at which point there is an eye, which is suspended on the walking-beam hook. At the bottom of the two bars there is a sleeve-nut, and between the two bars and passing through the nut, is a screw 5 ft long, at the bottom of which there is a head, which carries a swivel, set-screw, and a pair of clamps. These grasp the cable, 2 or $2\frac{1}{4}$ ins diam, which carries at its lower end the **string of tools.** This, for a 2000-ft hole, consists of a steel bit, 3 or 4 ft long, weighing 200 to 400 lbs; an auger-stem of 4 or 5-inch round iron, from 24 to 30 ft long, and weighing from 1200 to 2100 lbs; steel-lined drill-jars 8 ft long, weighing 600 to 700 lbs; a sinker-bar of round iron of same diam as the auger-stem, 12 to 15 ft long, and weighing from 600 to 1100 lbs; and a rope-socket, $2\frac{1}{2}$ ft long, weighing 200 lbs. Total length of string of tools, 50 to 60 ft, total weight, 3000 lbs; or, for an 8-inch hole in the hardest rock, 4000 lbs. **The sinker-bar** is added to give additional wt, and thus to assist in pulling the cable down through the water, either in lowering the string of tools or in working the drill-jars. The shapes of the other tools are given by Figs 1 to 4. **Special tools** are used for recovering articles that may be accidentally dropped into the hole.

The drilling cable is wound on a drum, called a bull-wheel shaft, at the foot of, and inside of, the derrick. While drilling is going on, it passes from the bull-wheel shaft loosely over the sheave at the top of the derrick, and down to the clamps at the lower end of the temper-screw on the end of the walking-beam. As the drilling progresses, the temper-screw is turned or fed out by the man in charge, who also, by means of a clamp, twists the rope, so as to change the position of the bit after each stroke.

When the tools are to be lifted out of the hole, the cable is disengaged from the clamps on the temper-screw, and is wound upon the bull-wheel shaft, which, for this purpose, is thrown into gear with the steam-engine; the pitman being at the same time removed from the crank-pin, so that the walking-beam is at rest. As in the portable machines, the sand-pump is also raised by the same power which does the drilling.

About 10000 ft b m of rough lumber are reqd for the derrick, walking-beam, sills, &c, and about 3000 ft more for sheds over the boiler, engine, and belt

In ordinary hard limestone rock, such a **machine will drill** about $1\frac{1}{2}$ ft per hour under the most favorable circumstances. **Two men are required;** one to attend to the boiler, sharpen the bits, &c, and one to operate the machine.

In Pierce's machine for test-boring, mineral prospecting and well boring, the pipes are driven by an iron ram, like that of a pile driver, but bushed with hard wood on its lower or striking end. The ram is worked by a hand winch The pipes are in lengths of 5 to 10 feet. After each length is driven, water,* under pressure, is forced, by a hand pump, through a hollow drill rod, into the bottom of the hole, while the drill rod is churned up and down by hand. The water forces the drillings (mud, sand, gravel, etc.) to the surface. The smallest machine drives 2 to 3 inch pipes; the largest, 2 to 8 inch. The machines are in detachable parts, weighing from 10 to 65 lbs each. Four upright iron pipes, which carry the head casting and crown pulley, act as guides for the ram, their ends fitting into sockets in castings at their heads and feet. The driving rams are made in sections which are bolted together. In the smaller machines the weight of the ram may thus be made from 100 to 200 pounds, and, in the larger machines, from 100 to 2,000 pounds, as required. Borings can be made to depths of 100 to 400 feet. These machines have been extensively used in Nicaragua by the Isthmian Canal Commission. If desired, the machines can be furnished with special tools for boring in rock and for taking out solid cores (as with the diamond drill), with others for taking out dry cores in earth, and with sand-pumps and mud sockets for bringing up mud, fine sand, gravel, and detached pieces of rock and minerals.

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*In Alaska, where the frost extends to great depths, boiling or hot water is used. This is obtained by melting ice or snow in iron tanks about 4 ft square and 2 ft deep.

MACHINE ROCK-DRILLS.

Art. 1. Machine Rock-drills bore much more rapidly than hand drills, and more economically, provided the work is so great as to justify the preliminary outfit. They drill in any direction, and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam directly; or by air, compressed by steam or water power into a tank called a "receiver," and thence led to the drills through iron pipes. The air is best for tunnels and shafts, because, after leaving the drills, it aids ventilation.

Art. 2. Such drills are of two kinds: rotating drills and **percussion drills.** In the former, the drill-rod is a long tube, revolving about its axis. The end of this tube, hardened so as to form an annular cutting-edge, is kept in contact with the rock, and, by its rotation, cuts in it a cylindrical hole, generally with a solid core in the center. The core occupies the core-barrel, Art 8. The drill-rod is fed forward, or into the hole, as the drilling proceeds. The debris is removed from the hole by a constant stream of water, which is led to the bottom of the hole through the hollow drill-rod, and which carries the debris up through the narrow space between the outside of the drill-rod and the sides of the hole.

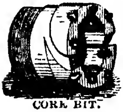
In **percussion drills,** the drill-rod is solid, and its action is that of the churn drill.

Art. 3. In the **Brandt** (European) **rotary drill,** the cutting-edge at the end of the tubular drill-rod is armed with hardened steel teeth. It is pressed against the rock under enormous hydraulic pressure, and makes but from 5 to 8 revolutions per minute.

Art. 4. The Diamond drill is the only form of rotary rock-drill extensively used in America. In it, the boring-rod consists of a number of tubes jointed rigidly together at their ends by hollow interior sleeves.

Art. 5. The boring-bit, Fig 1, is called a "core-bit." Its cutting-edge has imbedded in it a number of **diamonds** as shown. These are so arranged as to project slightly from both its inner and outer edges. Annular spaces are thus left between core and core-barrel, and between the latter and the walls of the hole These spaces permit the ingress and egress of the water used in removing the debris from the hole, and, at the same time, prevent the core from binding in the barrel, or the latter in the hole.

Fig. 1

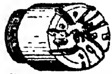


CORE BIT.

Fig. 2

CORE LIFTER.

Fig. 3



BORING HEAD.

Art. 6. Just above the "core-bit," the **"core-lifter,"** Fig 2, is screwed to the barrel. This is a tube about 8 ins long and of the same outer diam as the barrel. Inside it is slightly coned, with the base of the cone upward, and furnished with a loose split-ring, R, toothed inside, and similarly coned. While the drilling is going on, this ring encircles the core closely, and remains loose from the outer cylinder; but when the drilling is stopped, and the drill-rod begins to be raised, the ring is caught and raised by the outer cylinder: and, by reason of its beveled shape, is pressed hard against the core of rock, which is pulled apart close to its foot by the power which lifts the drill-rod.

Art. 7. This power is supplied by a **rope-drum.** fastened to the top of the frame which supports the drill and worked by the same engine which rotates the drill-rod. The rope from the drum passes up to a pulley at the top of a **derrick,** and thence down to the upper end of the drill-rod. The considerable height of the derrick enables from 40 to 60 feet of the drill-rod to be removed in one piece.

Art. 8. Above the "core-lifter" is the **"core-barrel."** This is a wrought-iron tube from 8 to 16 ft long. It is spirally grooved outside, to permit the ascent of the water and debris from the hole; and is sometimes set with diamonds on its outer surface, to prevent wear. The bit, lifter, and barrel are of uniform outer diam, a little less than the diam of the hole. The outer diam of the drill-rod varies from about $1\frac{1}{4}$ ins for 2-inch barrel to $5\frac{1}{2}$ ins for 18 inch barrel.

Art. 9. Where it is not desired to preserve the core intact, a "**boring-head,**" Fig 3, may be used instead of the "core-bit." Fig 1. This is a solid bit (except that it is perforated with holes which allow the water to pass out from the drill-rod), and is armed with diamonds, some of which project beyond its circumference.

Art. 10. The drill-rod **revolves at a speed** of from 200 to 400 revolutions per minute. **The engine,** by which it is rotated, consists usually of two cylinders, either fixed or oscillating, operated by steam or compressed air, and working at right angles to each other. By means of cranks they turn a shaft, which communicates its motion, through bevel gearing, to the drill-rod. The latter is **fed down,** as the hole progresses, either by other **bevel gearing** driven by the same engine; or by being attached to a cross-head which connects the piston rods of 2 **hydraulic cylinders,** the piston rods being parallel with the drill rod.

Art. 11. The diamond drill bores perfectly **circular holes, in straight lines and in any direction, to great depths;** from 300 to 1500 feet being not uncommon. This, with the fact that **it brings up unbroken cores,** from 8 to 16 ft long, which show the precise nature and stratification of the rock penetrated, renders it very valuable in test-boring, prospecting of mines, &c. They are also furnished of sufficient size to bore holes from 6 to 15 ins diam, for **artesian wells.** The roundness of the holes bored enables the use of casing of nearly as great diam as that of the hole; and their straightness is advantageous in case a pump has to be used.

Art. 12. In soft rock a bit may drill through 200 ft or more without resetting. On the other hand, in very hard rocks, similar drills will wear out in 10 ft or less. In 1883-4, a diamond drill by the Am'n Diamond Rock Boring Co, weighing complete about 1400 lbs. and costing about $2800, bored, in 1428 hours of actual boring, 53 holes of 2 ins diam, and aggregating 9141 lineal ft. Average length of hole 172.5 ft. **Average rate,** 6.4 lin ft per hour; greatest, 12.8. **Average total cost,** about 96 cts per lin ft. The rock was principally limestone, with some quartz and sandstone. The holes were bored at angles varying from 0° to 45° with the vertical.

As a rough average we may say that **in ordinary rocks,** as granite, limestone, and hard sandstone, these drills will bore deep holes, 2 to 3 ins diam, at **from 1 to 2 ft per hour,** and at a cost of from **$1 to $2 per ft.**

Art. 13. These drills are made of many widely **different sizes,** and with **different mountings,** depending upon the nature of the work to be done.

They are sold under restrictions as to the location and extent of the territory in which they are to be used. **The prices** depend, to a great extent, upon the nature of these restrictions. The card prices for some of the leading sizes, are as follows: Discount, see price list.

Diam of hole.	Diam of core.	Depth of hole.	Boiler H. P. required.	Card price. Drill.	Card price. Pump
Ins.	Ins.	Feet.	H. P.	$	$
$2\frac{9}{16}$	2	4000	25	4000	....
$2\frac{1}{16}$	$1\frac{1}{2}$	1500	15	2500	3400
"	"	1000	12 to 15	1900	2800
$1\frac{9}{16}$	1	600	10	1400	1900
"	"	400	hand	425	....

Art. 14. In percussion drilling machines, the drill-bar is driven forcibly against the rock by the **pressure of steam or of compressed air,** acting upon a piston, P, Fig 4, moving in a cylinder, C C, Figs 4 and 5; and makes about 300 strokes per minute. The rotation of the drill-bar is accomplished automatically, as explained in Art 27.

Art. 15. The cylinder, C C, is free to slide longitudinally in the fixed frame or shell, S S, Fig 5, to which it is attached, and which, in turn, is fixed to the tripod or other stand (see Arts 18 and 19) upon which the machine is supported.

Art. 16. The drill-rod, R, corresponding to the churn drill, is fastened, by an appropriate chuck, K, to the end of the piston-rod, O. The drilling is begun with a short drill-rod, and with the cylinder as far from the hole as the length of the shell, S, will permit. As the bit penetrates the rock, the cylinder is fed forward,* either automatically or by hand (see Art 28), as far as the length of

* By **forward,** or **downward,** we mean *toward* the hole which is being drilled. By **backward,** or **upward,** *from* the hole.

the shell permits. The drilling is then stopped, by shutting off the steam,* and the cylinder is run back, by reversing the motion of the feeding apparatus. The short drill-bar is then removed, and, if the drilling is to be continued, a longer one is substituted in its place, and the process repeated.

Art. 17. Inasmuch as the act of drilling wears the edges of the bit, thus reducing its diam somewhat, **the hole will of course be tapering,** or of slightly less diam at bottom than at top. The second bit must therefore be of slightly less diam than the first; say from $\frac{1}{16}$ to $\frac{1}{8}$ inch less; the third must be less than the second, and so on. On the other hand, in long holes, the drill-bar will seldom be in a perfectly straight line, so that the bit, instead of striking always in the same spot, will describe a circle, and thus enlarge the hole.

Art. 18. The shell, S, in which the cylinder slides, is provided with an arrangement by which it may be clamped, either to a **tripod,** as in Fig 5, or to a long **bar** or **column,** along which it may slide. The column, if hor, may rest upon two pairs of legs; or it may be braced, in any position, against the opposite sides of a narrow cut, or against the floor and ceiling of a tunnel-heading, &c, in which case one of its ends is provided with a screw which is run out so as to cause the two ends of the col to press firmly against the opposite rock walls; or rather against wooden blocks which are always placed between each end of the col and the rock. In any case, the supports of the drill are so jointed that it can bore in any direction.

Art. 19. Frequently the drill is **clamped to a short arm,** which, in turn, is clamped to the column, and projects at right angles from it. The arm may be slid lengthwise of the column, and may be revolved around it, and thus may be placed in any desired position, and there clamped. This gives the drill a greater range of motion, and enables it to bore holes over a greater space than would otherwise be possible without moving the column.

Art. 20. In tunnels, one or more drills may be mounted upon a **drill-carriage,** travelling upon a railroad track running longitudinally of the tunnel. Upon this track the carriage is moved up to the work, or run back from it when a blast is to be fired The gauge of the track may be made wide enough to admit of a second track, of narrower gauge, running underneath the drill-carriage. Upon said narrower track the cars are run which carry away the debris. Drill-carriages are less commonly used in this country than in Europe.

Art. 21. The pressure used in the cylinders of percussion drills is usually from about 60 to 70 lbs per sq inch. **In an hour, one will drill** a hole from 1 to 2 ins diam, and from 3 to 10 ft deep, depending on the character of the rock and the size of the machine at from 10 to 25 **cts per lin ft** with lab r; at $1 per day. **A bit requires sharpening** at about every 2 to 4 ft depth of hole. One blacksmith and helper can sharpen drills for 5 or 6 machines.

Art. 22. The bits are of many different shapes, varying with the nature of the work to be done. For uniform hard rock, the bit has two cutting-edges, forming a cross with equal arms at right angles to each other. For seamy rock, the arms of the cross are equal, but form two acute and two obtuse angles with each other, as in the letter X. For soft rock, the cutting-edge sometimes has the shape of the letter Z.

Art. 23. Each drill requires one man to operate it. Two or three men are required for moving the heavier sizes from place to place. One man can attend to a small air-compressor and its boiler.

Art. 24. Figs 4 and 5 represent the "**Eclipse**" percussion drill of the Ingersoll-Sergeant Drill Co. Havemeyer Building, New York Fig 5 shows the drill, mounted (as is most frequently the case) upon a tripod. Fig 4 is a longitudinal section through the cylinder, valve-chest, and piston.

Art. 25. The cylinder, C, is provided at each end with a **rubber cushion,** N, for deadening the blows of the piston, which, in all percussion drills, is liable, at times, to strike either cylinder-head. The side of each cushion nearest the piston is protected by a thin iron plate. The cushions have to be renewed from time to time.

Art. 26. The valve, V, is shaped somewhat like a spool. The bolt, B, passes loosely through its center and guides it. Steam is admitted from the boiler to the steam-chest, and occupies all of the space between the two end flanges of the valve, except *u*. It drives the valve alternately from one end of the valve-chest to the other, and back, according as one end or the other is relieved from opposing pressure by being put into communication with the exhaust, *E*, by way of the passages, D D′ and F F′. D and D′ communicate with the ends of the steam-chest through passages not shown; while F and F′ communicate, through similar passages, with the exhaust, E. The piston has an annular channel, L L′, encircling it. Whatever the position of the piston, one of the passages, D or D′, is always, by means of this channel, in communication with its corresponding passage, F or F′, leading

* To avoid repetitions, we will use the word **steam** to signify either *steam* or *compressed air* whichever happens to be used.

to the exhaust. Thus, one or the other end of the valve-chest is always in communication with the open air; and to that end the valve is driven by the pres of the steam surrounding it, admitting steam to the cyl, C, from the other end.

Art. 27. The rotation of the piston, and, with it, that of the drill-bar, is effected thus: The spirally-grooved, cylindrical steel bar, A, called a **rifle-bar,** passes through and works in, the **rifle-nut,** H, which is firmly fixed in the end of the piston, and has spiral grooves corresponding with those on the rifle-bar. Said bar is fixed, at its upper end, to the ratchet-wheel, J, the pawls of which are so arranged that, on the *down* stroke of the piston, the rifle-nut, H, acting upon the grooves on the rifle-bar, causes it, and, with it, the ratchet-wheel, to revolve about their common axis. The weight and momentum of the piston, &c, are such that it thus readily turns the ratchet-wheel without itself turning. Thus the bit is prevented from rotating while delivering its blow. But, on the *up* stroke, the tendency of the rifle-nut is to turn the rifle-bar and ratchet-wheel in the *opposite* direction; and as this is prevented by the pawls, the *rifle-bar* remains *stationary*, while the *piston, piston-rod*, and *drill* are *made to revolve* about their common axis.

Art. 28. The feed-screw, M, is collared, at its upper end, to the fixed frame, Q. It is thus prevented from moving longitudinally when revolved by means of the crank fixed to its top. Its lower end works in a nut, T, fixed to the cylinder, which last is thus moved longitudinally backward or forward as the crank is turned.

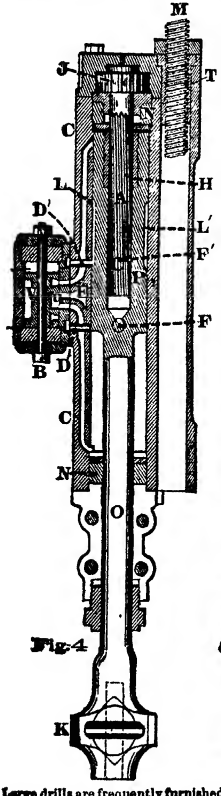


Fig. 4

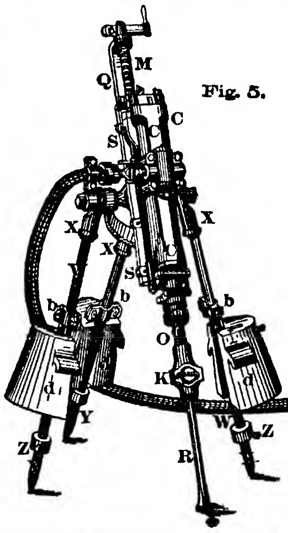


Fig. 5.

Large drills are frequently furnished with an ***automatic* feeding arrangement** in addition to the hand-crank. In this arrangement, when the cylinder requires feeding forward, and when, consequently, the piston is running nearly to

the forward limit of its stroke, the piston presses against a cam projecting into the cyl near the forward end, and presenting an inclined plane to it. The motion of this cam, by means of an exterior axle, running alongside of the cyl and furnished at its top with a dog, turns a ratchet-wheel fixed to the feed-screw. When desired, the automatic feed may be thrown out of gear, and the feed moved by hand.

Art. 29. The tripod legs consist of wrought-iron tubes, W W. These are screwed at their upper ends into sockets, X X. At their lower ends, they receive the pointed and tapering steel bars, Y Y, about 2 or 3 ft long. The legs may be lengthened or shortened by turning the set-screws, Z Z, thus regulating the distance to which the bars, Y Y, can enter the legs. The clamps, *b b*, have L-shaped hooks of ½ inch to 1 inch round iron forged to them. On these hooks **the weights,** *d d*, are hung, which hold the machine down against the upward reaction of its blows.

Art. 30. The following **table** gives the principal **dimensions** of these drills, with the **diams and lengths of holes** to which each is adapted. Size H is used for submarine work, heavy tunneling, and deep rock cutting. G and F for tunneling, street grading, quarrying, and sewer work. E, D, and C for general mining purposes. B is adapted only for very light work. In asking for estimates on drills and compressors, give the fullest possible description (accompanied by a sketch) of the work to be done, stating its present and proposed extent. State whether the work is on the surface or underground. State how far the steam or compressed air will be carried. Give depth of holes to be drilled, nature of rock, &c. Percussion drills are sold without restriction as to the purpose or extent to which they are to be used.

	Letter designating the size of the machine.							
	A	B	C	D	E	F	G	H
Inner diameter of cylinder........ins.	1¾	2½	2¾	3	3⅛	3½	4¼	5
Length of full stroke .... "	3	4	5	6	6	6½	7	7
Length of feed "	12	20	24	24	24	26	34	34
Length of machine*..... "	36	34	36	40	42	53	60	60
Wt of machine, unmounted, lbs.	80	155	195	230	250	345	605	670
Wt of tripod, without wts. "	‡	125	125	125	125	150	275	275
Wt of 3 wts for tripod legs, "	‡	250	250	250	250	350	400	400
Wt of column, arm & clamp "	‡	200	280	280	280	420	420	420
Diam of hole drilled...........ins.	½ to ⅞	¾ to 1¼	1 to 2	1 to 2	1 to 2	1½ to 2½	2 to 4	3 to 6
Max depth of vert † hole.......ft.	‡	4	8	10	12	16	30	40

* From top of handle of feed-crank to lower end of piston at the end of the down stroke.

† For greatest advisable lgth of *hor* holes, deduct one-fourth from these depths.

‡ Machine **A** is mounted on a small frame.

Art. 31. The drills of different makers differ chiefly in the methods by which the valve is operated. In some this is done, as in the Ingersoll "Eclipse" drill, Art 26, by the **pres of steam.** In others, the valve is moved by a **lever or tappet,** which projects into the cylinder so as to come into contact with, and be moved by, the piston at each stroke. As these strokes are made with great force some 300 or more times per minute, such valve-gear is necessarily subject to great wear.

Art. 32. In the "**Little Giant Drill,**" made by the **Rand Drill Co.,** the valve, V, Fig 6, is slid backward and forward, in the same direction in which the piston is moving, by the tappet, T, which is pivoted at *p*. The inclined lower corners of this tappet ride up as they come, alternately, in contact with the shoulders, *s s*, of the piston.

Art. 33. In the "**Economizer**" and the "**Slugger**" (Rand Drill Co.) the valve, as in the Ingersoll "Eclipse" drill, is moved by steam, but upon

a quite different principle. In these two drills, there is no steam cushion for the piston to strike against on the down stroke, the force of which is thus more completely expended upon the rock. The cushion behind or above the piston, on the return stroke, is formed by exhaust steam. Both of these drills cut off steam before the completion of either stroke, thus using the steam expansively. On the down stroke, the "Economizer" cuts off earlier than the "Slugger." Hence its name. In both machines the point of cut-off is fixed when the machine is made.

Art. 34. In the improved **Burleigh** drill, the valve, V, Fig 7, is moved by **two** tappets, T T, which are alternately struck by the ends of the piston, P.

Art. 35. In the "Dynamic" rock-drill, invented by **Prof De Volson Wood**, the valve is attached to a valve-piston, V, Fig 8, which is moved backward and forward by steam, which is admitted so as to act alternately upon its two ends. The admission of this steam is controlled by a small auxiliary valve, *a*. A hub on the back of the auxiliary valve fits in the spiral groove shown on the plug, *n*. This plug is constantly pressed downward (as the Fig stands) by steam pressing upon its upper shoulder, but it is lifted at each forward stroke by the conical surface of the piston, P, pressing against its foot. It thus moves constantly up and down, carrying the valve, *a*, with it. By turning the plug, *n*, by means of the adjusting-stem, *s*, the hub of the valve is made to occupy a higher or lower point in the spiral groove, and thus the stroke of the piston may be varied, or may be confined to any part of the cylinder.

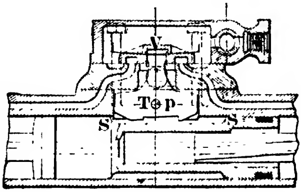


Fig. 6.

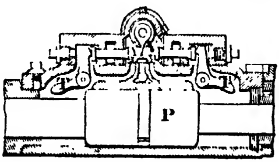


Fig. 7.

In this drill, unlike the Ingersoll, Art. 27, the **piston rotates** while making

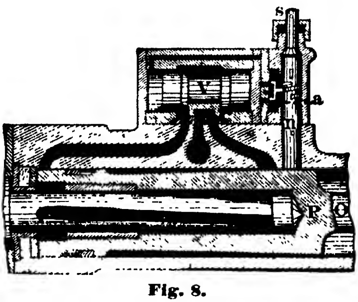


Fig. 8.

the *downward* stroke. **The piston-rod, *o*, is made lighter than in other drills.** This gives a greater surface under the piston for the pressure of the steam on the up stroke, and, consequently, greater lifting power. This is useful when the drill sticks in the hole.

The **tripod legs** are of bar iron. Their length is adjustable.

Art. 36. The **Pierce hand rock-drill** is a percussion drill worked by a crank which turns a disc about 2 ft in diam. The disc has a semi-circular slot, in which works the arm which raises the drill-rod. This arm, in rising, compresses a coil-spring, which, on the down stroke, drives the drill against the rock. An iron ball, weighing 30 lbs or more, is furnished with each machine. This ball may be screwed to the top of the drill-rod, for giving greater force to the blows of the drill. The ball may be used without the spring, by disengaging the latter.

The drill makes about 40 strokes of 10 or 12 ins per minute; and bores holes from ¾ to 2½ ins diam. It can be arranged to drill to depths of 30 ft and over. For sharpening the bits, it has an emery wheel attached, which is turned by the crank. The latter, at such times, is thrown out of gear with the disc.

The drill is mounted on a rectangular two-legged frame, about 5 ft high by 2 ft wide, made of iron tubes. To the top of this frame a third leg is attached, by adjusting which the angle of the drill-rod with the vert may be changed. Like other percussion drills worked by hand-power, this one ceases to work to advantage when said angle exceeds about 45°.

Art. 37. Channeling consists in making long, deep, and narrow cuts in the rock. In this way large blocks can be gotten out without blasting and the consequent danger of fracture. This is ordinarily done by boring a row of holes about an inch apart in the clear, and then breaking down the intermediate spaces by means of a blunt tool, called a **broach.** This is called **broach channeling.** For this purpose a steam drilling machine is mounted upon a hor bar resting upon two pairs of legs. The hor bar is placed over the intended row of holes, and the drill is slid along upon it from one hole to the next. In using the *broach*, the rotating apparatus is thrown out of gear, so that the edge of the broach maintains its position in line with the row of holes

Art. 38. The Saunders patent channeling machine, of the Ingersoll Co, consists of a rock-drilling machine, having, in place of the usual drilling-bit, a gang of tools consisting of a number of chisels, clamped together side by side, and thus forming a cutting tool about 7 ins long by ¾ inch wide. This tool has as many cutting-edges (each as long as the tool is wide) as there are chisels. The machine is supported upon a carriage, moving on a track parallel with the channel to be cut. The tool is of course not rotated; but the rifle-bar, A, Fig 4, is employed to move the carriage along the track about an inch after each blow. The carriage remains stationary while a blow is being struck. Under favorable circumstances **this machine has cut** from 80 to 100 sq ft of channel per day of ten hours. Its **weight,** including carriage, is about 5000 lbs.

A valve is provided, by which, if desired, the **steam may be shut off** from the piston on the down stroke, so that said stroke may be made with only the *weight* of the piston, rod, and drill.

Art. 39. The Ingersoll Co have a special appliance, designed by Mr. W. L. Saunders, C E, for **drilling and blasting rocks under water,** even when they are covered by a considerable depth of mud.

Art. 40. Air compressors for rock-drills, as made and used in this country, are mostly hor, direct-acting engines. That is, the axes of the steam- and air-cylinders are hor; and the piston-rod passes directly from the steam-cylinder into the air-cylinder. A fly-wheel is attached, by a crank and connecting-rod, to the piston-rod. Sometimes the steam-engine is separate from the compressor, and the power is conveyed to the latter by belts or gearing; or water-power may be used in the same way. The air is forced into a receiver, which is generally a plate-iron cylinder, 3 or 4 ft in diam, and 5 to 12 ft long.

If the air- or pumping-cylinder of the compressor is so arranged as to take in air on one stroke only, and force it out into the receiver upon the return stroke, it is **"single-acting."** If, at each stroke, it both takes in and forces out air, it is **"double-acting."** If the compressor has only one air-cylinder, it is **"single."** If it has two, and thus practically consists of two single compressors, it is **"duplex."**

The **valves** may be either **"poppet"** valves, held in place by springs, and operated by the pressure of the air itself; or **slide** valves, operated by eccentrics and rods, as in steam-engines.

The compression of the air develops **heat.** This is removed either by causing cold water to circulate through the air-piston, and through jackets surrounding the air-cylinder; or by injecting it into the air-cylinder in the form of spray. Or both methods may be used together.

Art. 41. The following partial list of Clayton compressors, compiled from data given by the makers, shows the **dimensions** and **performance** of each. We give also a list of their **receivers**.

CLAYTON DOUBLE-ACTING AIR-COMPRESSORS. Partial List.

	Number, designating the size of the machine.			
	1	2½	4	7
Duplex Direct-acting* Compressors.				
Diam of steam-cylinders....ins.	8	10	14	18
" air " ....ins.	8	10	14	18
Length of stroke....ins.	12	13	15	24
Number of revolutions per minute....	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute ....Actual.	136	210	438	900
Approximate wt of compressor ....lbs.	3000	7000	15000	25000
Approx number of rock-drills with 3-inch cyls supplied with air at 60 to 80 lbs per sq inch....	2	4	8	18
Single Direct-acting* Compressors.				
Diam of steam-cylinder....ins.	8	10	14	18
" air " .... ins.	8	10	14	18
Length of stroke....ins.	12	13	15	24
Number of revolutions per minute....	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute.... Actual.	68	105	219	450
Approx wt of compressor....	1650	3850	8250	13750
Approx number of rock-drills with 3-inch cyls supplied with air at 60 to 80 lbs per sq inch....	1	2	4	9

* The price of a compressor alone, to be worked by a separate steam-engine or water-power is of course less than that of the above compressor and engine combined.

Air-Receivers; vertical and horizontal.

Diameter Inches.	Length, Feet.	Approximate weight, lbs.	Diameter, Inches.	Length, Feet.	Approximate weight, lbs.
33	5	700	40	8	1675
30	7	890	40	10	1900
36	8	1560	40	11	2000
40	6	1600	40	12	2100

The Air-Receivers have brass-face pressure-gauge, glass water-gauge, safety-valve, blow-off valve, try-cocks, flanges and connections to automatic feed on compressor.

TRACTION.

Traction on common roads, and canals; or the power reqd to draw vehicles and boats along them.

The following table shows tolerable approximations to the force in lbs per ton, reqd to draw a stage coach and passengers, up ascents on the Holyhead turnpike road in England, (a fine road,) by horses, as ascertained by means of a dynamometer. The entire weight was 1¼ tons, but in the table, the results are given per single ton. From the nature of such cases, no great accuracy is attainable.

Proportional Ascent.	Ascent in Ft. per Mile	At 4 Miles per Hour.	At 6 Miles per Hour.	At 8 Miles per Hour.	At 10 Miles per Hour.
		Lbs.	Lbs	Lbs.	Lbs.
1 in 15½	340 7	210	216	225	240
1 " 20	264	196	202	212	229
1 " 26	203 1	155	160	166	175
1 " 30	176.	137	142	147	154
1 " 40	132.	114	120	124	130
1 " 64	82 5	109	115	120	126
1 " 118	44 7	102	107	113	120
1 " 138	38 3	99	103	109	117
1 " 156	33 9	98	101	106	112
1 " 245	21 6	93	96	101	107
1 " 600	8 8	81	85	91	96
Level	0	76	80	85	91

The following results most of them with the same instrument, are also in lbs per ton, with a four-wheeled wagon, at a slow pace, on a level, and the roads in fair condition.

On a cubical block pavement .. .. 32 lbs per ton..........to 50.
" McAdam road, of small broken stone.............. 62 " " " probably to 75.
" gravel road ... . 140 " " "
" Telford road, of small stone on a paving of spawls 46 " " " " " 75.
" broken stone on a bed of cement concrete .... 46 " " " " " 75
" common earth roads.. .... 200 to 300. On a plank road 30, to 50 lbs

The tractive power of a horse diminishes as his speed increases: and perhaps, within certain limits, say from ¾ to four miles per hour, nearly in inverse proportion to it. Thus, the average traction of a horse, *on a level*, and actually pulling for 10 hours in the day, may be assumed approximately as follows

Miles per hour.	Lbs. Traction.	Miles per hour.	Lbs. Traction.
¾	333 33	2¼	111.11
1	250.	2½	100.
1¼	200.	2¾	90.91
1½	166 66	3	83.33
1¾	142.86	3½	71.43
2	125.	4	62.50

If he works for a smaller number of hours, his traction may increase as the hours diminish; down to about 5 hours per day and for speeds of about from 1¼ to 3 miles per hour. Thus, for 5 hours per day his traction at 2½ miles per hour will be 200 lbs, &c. When ascending a hill, his power diminishes so rapidly, from having partially to raise his own weight, (which averages about 1000 to 1100 lbs,) that up a slope of 5 to 1, he can barely struggle along without any load. On such an ascent, he must exert a force equal to 439 lbs per ton, or of 196 lbs for the 1000 lbs of his own weight. Assuming that on a level piece of good turnpike, he would when hauling a cart and load, together weighing 1 ton, have to exert a traction of 60 lbs; then on ascending a hill of 4° inclination, (or 1 in 14 3; or 369¼ ft per mile,) he would have to exert 156 lbs, against the gravity of the 1 ton; and 67 lbs, against that of his own weight, or 223 lbs in all. He may, for a few mins, exert without injury, about twice his regular traction. This calculation shows that up a hill of 4°, an average horse is fully tasked in drawing a total load of one ton; and should, therefore, be allowed, in such a case, to choose his own gait; and to rest at short intervals. A fair load for a single horse with a cart, at a variable walking pace, working 10 hours per day, on a common undulating road in good order, is about half a ton, in addition to the cart, which will be about half a ton more. With two horses to this same cart, the *load* alone may be about 1¼ tons.

Rem. Since the action of gravity is the same on good roads and bad ones, it follows that **ascents become more objectionable the better the road is.** Thus, on an ascent of 2°, or 184.4 ft. per mile, gravity alone requires a traction of 78 lbs per ton'

which is about 10 times that on a level railroad at 6 miles an hour, but only about equal to that on a level common turnpike road, at the same speed. Therefore, (to speak somewhat at random,) it would require 10 locomotives instead of 1, but only 2 horses instead of 1. A grade of 1 in 35, or 150 ft to a mile; or 1° 38', is about the steepest that permits horses to be driven down a hard smooth road, in a fast trot, without danger. It should, therefore, not be exceeded except when absolutely necessary, especially on turnpikes.

On canals and other waters, the liquid is the resisting medium that takes the place of friction on level roads. But unlike friction its resistance varies as the squares of the vels; at least from the vel of 2 ft per sec, or 1.364 miles per hour; to that of 11½ ft per sec, or 7.84 m per h. As the speed falls below 1⅜ m per h, the resistance varies less and less rapidly; and this is the case whether the moved body floats partly above the surface; or is entirely immersed. In towing along stagnant canals, &c, the vel is usually from 1 to 2½ m per h, for freight most frequently from 1½ to 2. Less force is required to tow a boat at say 2 m per h, where there is no current, than at say 1¼ m per h, against a current of ¾ m per h, because in the last case the boat has to be *lifted up* the very gradual inclined plane or slope which produces the current.

The force required to tow a boat along a canal depends *greatly* upon the comparative transverse sectional areas of the channel, and of the immersed portion of the boat. When the width of a canal at water-line is at least 4 times that of the boat; and the area of its transverse section as great as at least 6½ times that of the *immersed* transverse section of the boat, the towing at usual canal vels will be about as easy as in wider and deeper water. With less dimensions, it becomes more difficult. (D'Aubuisson.) Much also depends on the shape of the bow and other parts of the boat; and on the proportion of its length to its breadth and depth. Hence it is seen that the mere weight of the load is by no means so controlling an element as it is on land. The whole subject, however, is too intricate to be treated of here. Morin states that naval constructors estimate the resistance to sailing and steam vessels at sea, at but from about .5 to .7 of a lb for every sq ft of immersed transverse section, when the vel is 3 ft per sec, or 2.046 miles per hour. It is far greater on canals.

On the Schuylkill Navigation of Pennsylvania, of mixed canal and slackwater, for 108 miles, the regular load for 3 horses or mules, is a boat of very full build; and no keel; 100 ft long, 17½ ft beam; and 8 ft depth of hold, drawing 5½ ft when loaded. Weight of boat about 65 tons, load 175 tons of coal, (2240 lbs,) total weight 240 tons, or 80 tons per horse or mule. On the down trip with the loaded boats, for 4 days, the animals are at work, *actually towing*, (except at the locks,) for 18 hours out of the 24; thus exceeding by far the limits of time usually allowed for continuous effort.

On the canal sections, (which have 60 ft water-line; and 6 ft depth,) the speed is 1¾ miles per hour; and on the deep wide pools, 2 miles.

On the up trip with the empty 65 ton boats, the average speed is about 2½ miles per hour. The empty boats draw 16 to 18 ins water, and frequently keep on without stopping to rest day or night through the entire distance of 108 miles. The animals generally have 2 or 3 days' rest at each end of the trip, but are materially deteriorated at the end of the boating season.

If our preceding assumption of 143 lbs traction of a horse at 1¾ miles per hour, is correct, the traction of the loaded boats on the canal sections is $\frac{143 \text{ lbs}}{80 \text{ tons}} = 1.83$ lbs per ton.

The intelligent engineer and superintendent of the Sch Nav, James F Smith, gives as the results of his own extensive observation, that one of these large boats loaded (240 tons in all) may, without distressing the animals, be drawn along the canal sections, for 10 hours per day, as follows: By one average horse or mule, at the rate of 1 mile; by two animals, at 1½ miles; and by three, at 1¾ miles per hour. When four animals are used the gain of time is very trifling. At a time of rivalry among the boatmen, one of them used 8 horses, but with these could not exceed 2½ miles per hour in the canal portions. Two or more horses together cannot for hours pull as much as when working separately.

If our preceding short table of the traction of a horse at diff vels for 10 hours is correct, then the traction of the above loaded coal boats (240 tons) on the canal sections of the navigation is as follows: The last column shows the traction in lbs per sq ft of area of immersed transverse section where largest; viz, about 95 sq ft.

Horses.	Miles per Hour.		Lbs. per Ton.	Lbs. per Sq Ft.
1	1	$\frac{250}{240}$	1.04	2.63
2	1½	$\frac{333}{240}$	1.39	3.50
3	1¾	$\frac{428}{240}$	1.78	4.50
3 on pools	2	$\frac{375}{240}$	1.56	3.95
8	2½	$\frac{800}{240}$	3.33	8.42
3 up trip	2½	$\frac{300}{65}$	4.61	12.50

Lachine Canal, Canada, 120 ft wide at water-line; 80 ft at bottom; depth on mitre sills 9 ft, 6 horses tow loaded schooners with ease.

Before the enlargement of the **Erie canal,**† its dimensions were 40 ft water line, 28 ft bottom; 4 ft depth of water. The average weight of the boats was about 30 tons. With 75 tons of load, or 105 tons total, they were towed by 2 horses, at the rate of about 2 miles per hour, which by our table gives a traction of nearly 2.4 lbs per ton. The boats were about 80 ft long, 14 ft beam, full 3½ ft draught loaded; hence the traction by our table would be about 5.7 lbs per sq ft of immersed transverse section.

† Length 363 miles; cost $19680 per mile. The enlarged canal has 70 ft; 42 ft; and 7 ft of water; and cost $80600 per mile for the enlargement only. The cost of the several canals in Pennsylvania has ranged between $25000 and $50000 per mile.

While, for 82-ton loaded boats on a smaller canal, (the boats nearly touching bottom,) the traction at 1¾ miles, would be 3½ ℔s per ton, or about twice as great as the above 1.78 ℔s. It also would be 5.7 ℔s per sq ft of immersed section.

ANIMAL POWER.

Art. 1. So far as regards horses, this subject has been partially considered under the preceding head, Traction. All estimates on this subject must to a certain extent be vague, owing to the diff strengths and speeds of animals of the same kind; as well as to the extent of their training to any particular kind of work. Authorities on the subject differ widely; and sometimes express themselves in a loose manner that throws doubt on their meaning. We believe, however that the following will be found to be as close approximation to practical averages as the nature of the case admits of with our present imperfect knowledge. We suppose a good average trained horse weighing not less than about ½ a ton, well fed and treated. Such a one, when actually walking for 10 hours a day, at the rate of 2½ miles per hour, on a good *level* road, such as the tow path of a canal, or a circular horse-path,* **can exert a continuous pull, draught, power, or traction, of 100 lbs.**

Now, 2½ miles per hour, is 220 ft per min, or 3⅔ ft per sec; and since 10 hours contain 600 min, his day's work of actual hauling on a level, at that speed, amounts to

$$\overset{\text{min}}{600} \times \overset{\text{ft}}{220} \times \overset{\text{℔s}}{100} = 13\,200\,000 \text{ ft ℔s per day.}$$

Or, 22000 ft-℔s per min, or 366⅔ ft-℔s per sec.† Which means that he exerts force enough during the day to *lift* 13 200 000 ℔s 1 foot high; or 1 320 000 ℔s 10 feet high; or 132 000 ℔s 100 ft high, &c. He may exert this force either in *traction* (hauling) or in *lifting* loads. If he has to raise a small load to a great height, the machinery through which he does it must be so geared as to gain speed, at the loss (commonly but improperly so expressed) of power. Whether he lifts the great weight through a small height, or the small weight through a great height, he exerts precisely the same amount of force or power.

Experience shows that within the limits of 5 and 10 hours per day, (the speed remaining the same, **the draft of a horse may be increased in about the same proportion as the time is diminished:** so that when working from 5 to 10 hours per day, it will be about as shown in the following table. Hence, the total amount of 13 200 000 ft-℔s per day may be accomplished, whether the horse is at work 5, 6, or 8, &c, hours per day.‡ This, of course, supposes him to be actually lifting or hauling *all the time;* and makes no allowance for stoppages for any purpose.

Table of draft of a horse, at 2½ miles per hour, on a level.

Hours per day.	Lbs.	Hours per day.	Lbs.
10	100	7	$142\frac{6}{7}$
9	$111\frac{1}{9}$	6	166⅔
8	125	5	200

Experience also shows that at speeds between ¾ and 4 miles an hour, his force or draught will be inversely in proportion to his speed. Thus, at 2 miles an hour, for 10 hours of the day, his draught will be

$$\overset{\text{miles}}{2} : \overset{\text{miles}}{2\tfrac{1}{2}} :: \overset{\text{℔s}}{100} : \overset{\text{℔s}}{125} \text{ draught}$$

At 1½ miles, it would be 166⅔ ℔s; at 3 miles, 83⅓ ℔s; and at 4 miles, 62½ ℔s; as per table in Traction.

Therefore, in this case also, the entire amount of his day's work remains the same;§ and within

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* To enable a horse to work with ease **in a circular horse-walk,** its diam should not be less than 25 ft, 30 or 35 would be still better.

† **A nominal horse-power** is 33000 ft-℔s per minute; this being the rate assumed by Boulton and Watt in selling their engines: so that purchasers wishing to substitute steam for horses, should not be disappointed. Their assumption can be carried out by a very strong horse day after day for 8 or 10 hours; but as the engine can work day and night for months without stopping, which a horse cannot, it is plain that a one-horse engine can do much more work than any one such horse. Hence many object to the term horse-power as applied to engines; but since everybody understands its plain meaning, and such a term is convenient, it is not in fact objectionable. Boulton and Watt meant that a one-horse engine would at any moment perform the work of a very strong horse. An *average* horse will do but 22000 ft-℔s per min.

‡ It is plain that although the *day's* labor will be the same, that of an hour, or of a min, will vary with the number of hours taken as a day's work. It must be remembered that a *working day* of a given number of hours, by no means implies, in every case, that number of hours of *actual work;* but includes intermissions and rests.

§ **This remark about speed will not apply to loads towed through the water.** Thus, if his draught at 2 miles an hour be 125 ℔s; and at 4 miles, 62½ ℔s; he will *on land* draw loads in these proportions; but in hauling a boat *through the water* at the greater speed, he has to encounter the increased resistance of the *water* itself; which resistance at 4 miles is much more than twice as great as at 2 miles; probably 4 times as great. Therefore, at 4 miles on a canal, his draught of 62½ ℔s would not suffice for *a load* half as great as he could tow with his draft of 125 ℔s at 2 miles.

all the foregoing limits of hours and speed, may be practically taken to be about 13 200 000 ft-lbs per day; or 22000 ft-lbs per min of a day of 10 hours. But it does not follow that the horse can always in practice actually *lift loads* at that rate, because generally a part of his power is expended in overcoming the *friction* of the machinery which he puts in motion; and moreover, the nature of the work may require him to stop frequently; so that in a *working day* of 8 or 10 hours, the horse may not actually be *at work* more than 5, 6, or 7 hours.

As a rough approximation, to allow for the waste of force in overcoming the friction of hoisting machinery, and the weight of the hoisting chains, buckets, &c, we may say that **the useful or paying daily net work of a horse, in hoisting by a common gin,** is about 10 000 000 ft-lbs. That is, he will raise equivalent to 10 000 000 lbs net of water, or ore, &c, 1 foot. The load which he can raise at once, including chains, bucket, and an allowance for friction, will be as much greater than his own direct force, as the diam of the horse walk is greater than that of the winding drum, and it will move that much slower than he does. His own direct force will vary according to the number of hours per day that he may be required to work, as in the foregoing table. With these data, the size of the buckets can be decided on, and of these there should be at least two, so that the empty one at the bottom may be filled while the full one at top is being emptied; so as to save time. The same when the work is done by men.

Art. 2. A practised laborer hauling along a level road, by a rope over his shoulders; or in a circular path, pushing before him a hor lever, at a speed of from $1\frac{1}{2}$ to 3 miles per hour, exerts about $\frac{1}{6}$ part as much force as a horse; or 2 200 000 ft-lbs per day; or $3666\frac{2}{3}$ ft-lbs per min of a day of 10 hours of actual hauling or pushing.

But laborers frequently have to work under circumstances less advantageous for the exertion of their force than when hauling or pushing in the manner just alluded to; and in such cases they cannot do as much per day. Thus in turning a winch or crank like that of a grindstone, or of a crane, the continual bending of the body, and motion of the arms, is more fatiguing. **The size of a winch should not exceed 18 ins,** or the rad of a circle of 3 ft diam; and against it a laborer can exert a force of about 16 lbs, at a vel of $2\frac{1}{2}$ ft per sec, or 150 ft per min, making very nearly 16 turns per min; for 8 hours per day. To these 8 hours an addition must be made of about $\frac{1}{4}$ part, for short rests. Or if a *working day* is taken at 8 or 10, &c, hours, $\frac{1}{5}$ part must generally be taken from it for such rests. On the foregoing data an hour's *work* of 60 min of *actual hoisting* would be

$$\overset{\text{lbs}}{16} \times \overset{\text{ft}}{150} \times \overset{\text{min}}{60} = 144000 \text{ ft-lbs};$$

or, deducting $\frac{1}{5}$ part for rests, 115200 ft lbs per hour of *time, including rests*. In practice, however a further deduction must be made for the fric of the machine, and for the wt of the hoisting chains; and in case of raising water, stone, ore, &c, from pits, for the wt of the buckets also. As a rough average we may assume that these will leave but 100 000 ft lbs of paying, or useful work per hour; that is, that **a man at a winch will actually lift equivalent to 100000 lbs of water, ore, &c, 1 foot high per hour's time, including rests.** This is equal to $1666\frac{2}{3}$ ft lbs per min of a day of 10 hours including rests. Therefore, in a day of 10 working hours he would raise 1 000 000 lbs net, 1 foot high, **or just $\frac{1}{10}$ part of what a horse would do with a gin** in the same time. We have before seen that in hauling along a level road, he can at a slow pace perform about $\frac{1}{6}$ of the daily duty of a horse. He may also work the winch with greater force, say up to 30 or even 40 lbs, but he will do it at a proportionately slower rate; thus, accomplishing only the same daily duty. **With a gin, like those for horses,** but lighter, with 2 or more buckets, a practised laborer will in a working day of 10 hours, raise from 1 200 000 to 1 400 000 ft-lbs net of water, ore &c. With a shallow well or pit, more time is lost in emptying buckets than in a deep one; but the deep one will require a greater wt of rope. To save time in all such operations on a large scale, there should be at least two buckets; the empty one to be filled while the full one is being emptied. It is also best to employ 2 or more men to hoist at the same time, by winches, at both ends of the axis; and the men will work with more ease if the winches are at right angles to each other. Each winch handle may be long enough for 2 or 3 men. An extra man should be employed to empty the buckets. He may take turns with the hoisters. The same remarks apply in some of the following cases.

On a treadwheel a practised laborer will do about 40 per cent more daily duty than at a winch; or in a working day * of 10 hours, including rests, he will do about 1 400 000 ft-lbs. And he can do this whether he works at the outer circumf of the wheel, stepping upon foot-boards, or tread-boards, on a level with its axis; or walks inside of it near its bottom. In both cases he acts by his wt, usually about 130 to 140 lbs, and not by the muscular strength of his arms. When at the level of the axis, his wt acts more directly than when he walks on the bottom of the wheel; but in the first case he has to perform a slow and fatiguing duty resembling that of walking up a continuous flight of steps; while in the second he has as it were merely to ascend a very slightly inclined plane; which he can do much more rapidly for hours, with comparatively little fatigue: and this rapidity compensates for the less direct action of his wt. Therefore, in either case, as experience has shown, he accomplishes about the same amount of daily duty. Treadwheels may be from 5 to 25 ft in diam, according to the nature of the work. They are generally worked by several men at once, and may at times be advantageously used in pile-driving, as well as in hoisting water, stone, &c.

By a good common pump, properly proportioned, a practised laborer will in a day of 10 working hours, raise about 1 000 000 ft lbs of water, net.†

Bailing with a light bucket or scoop, he can accomplish about 200 000 ft lbs net of water. **By a bucket and swape,** (a long lever working vertically, and weighted at one end so as to balance the full bucket hung from the other; often seen at country

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* The working day must be understood to include necessary rests, and such intermissions as the nature of the work demands; but does not include time lost at meals. A *working day* of 10 hours may, therefore, have but 8, 7, or 6, &c hours of *actual labor*. This will be understood when we hereafter speak of a working day, or simply a day.

† Desaguliers's estimates of daily work of men and horses exceed the above, but are evidently too great

wells,) 600 000 to 800 000. In the last he has only to pull down the empty bucket, and thereby raise the counterweight. **By 2 buckets at the ends of a rope suspended over a pulley,** 500 000 to 600 000. Here he works the buckets by pulling the rope by hand.

By a tympan, or tympanum,* worked by a treadwheel, about 1 200 000 to 1 400 000.

By a Persian wheel,† a chain-pump, a chain of buckets,‡ or an Archimedes screw, all worked by a treadwheel, from 800 000 to 1 000 000 ft-lbs. Of these four, the first three lose useful effect by either spilling, leaking, or the necessity for raising the water to a level somewhat higher than that at which it is discharged.

When any of the five foregoing machines are worked by men at winches, the result will be about ⅕ less than by treadwheels. They are all frequently worked also by either steam, water, or horse-power.

By walking backward and forward, on a lever which rocks on its center, a man may, according to Robison's Mech Philosophy, perform a much greater duty than by any of the preceding modes. He states that a young man weighing 135 lbs, and loaded with 30 lbs in addition, worked in this manner for 10 hours a day without fatigue; and raised 9¼ cubic feet of water, 11½ ft high per min. This is equal to 3 984 000 ft lbs per day of 10 hours; or 6640 ft lbs per min; or nearly $\frac{4}{10}$ of the net daily work of a horse in a gin.

A laborer standing still, can barely sustain for a few min, a load of 100 lbs, by a rope over his shoulder, and thence passing off hor over a pulley. And scarcely as much, when (facing the load and pulley) he holds the end of a hor rope with his hands before him. He cannot push hor with his hands at the height of his shoulders, with more than about 30 lbs force.

Weisbach states from his own observation, that 4 practised men raised a **dolly** (a wooden beetle or rammer, of wood; with 4 hor projecting round bars for handles) weighing 120 lbs, 4 ft high, at the rate of 34 times per min, for 4½ min; and then rested for 4½ min; and so on alternately through the 10 hours of their working day. Therefore, 5 of these hours were lost in rests; and the duty performed by each man during the other 5 hours, or 300 mins, was

$$\frac{120 \times 4 \times 34 \times 300}{4} = 1\,224\,000 \text{ ft-lbs.}$$

In the old mode of driving piles, where the ram of 400 to 1200 lbs suspended from a pulley, was raised by 10 to 40 men pulling at separate cords, from 35 to 40 lbs of the ram were allotted to each man, to be lifted from 12 to 18 times per min, to a height of 3½ to 4½ feet each time, for about 3 min at a spell, and then 3 min rest. It was very laborious; and the gangs had to be changed about hourly, after performing but ½ an hour's actual labor.

Hauling by horses. See Traction. When working all day, say 10 working hours, the average rate at which a horse walks while hauling a full load, and while returning with the empty vehicle, is about 2 to 2½ miles per hour; but to allow for stoppages to rest, &c, it is safest to take it at but about 1.8 miles per hour, or 160 ft per min. The time lost on each trip, in loading and unloading, may usually be taken at about 15 min. Therefore, to find the number of loads that can be hauled to any given dist in a day, first find the time in min reqd in hauling one load, and returning empty. Thus: div twice the dist in ft to which the load is to be hauled; or in other words, div the length in ft, of the round trip, by 160 ft. The quot is the number of min that the horse is in motion during each round trip. To this quot add 15 min lost each trip while loading and unloading; the sum is the total time in min occupied by each round trip. Div the number of min in a working day (600 min in a day of 10 working hours) by this number of min reqd for each trip; the quot will be the number of trips, or of loads hauled per day.

Ex. How many loads will a horse haul to a dist of 960 ft, in a day of 10 working hours, or 600 min?

Here, $960 \times 2 = 1920$ ft of round trip at each load. And $\frac{1920}{160} = 12$ min, occupied in walking. And $12 + 15$ (in loading, &c) $= 27$ min reqd for each load. Finally, $\frac{600}{27} = \frac{\text{min in 10 hours}}{\text{min per trip}} = 22.2$, or say 22 trips, or loads hauled per day.

Table of number of loads hauled per day of 10 working hours. The first col is the distance to which the load is actually hauled; or half the length of the round trip. The cost of hauling per load, is supposed to be for one-horse carts; the driver doing the loading and unloading; rating the expense of horse, cart, and driver at $2 per day.

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* The tympan revolves on a hor shaft; and is a kind of large wheel, the spokes, arms, or radii of which are gutters, troughs, or pipes, which at their outer ends terminate in scoops, which dip into the water. As the water is gradually raised, it flows along the arms of the wheel to its axis, where it is dischgd. The scoop wheel is a modification of it. It is an admirable machine for raising large quantities of water to moderate heights. We cannot go into any detail respecting this and other hydraulic machines.

† A kind of large wheel with buckets or pots at the ends of its radiating arms; revolves on a hor axis; discharges at top. The buckets are attached loosely, so as to hang vert, and thus avoid spilling until they arrive at the proper point, where they come into contact with a contrivance for tilting and emptying them. The noria is similar, except that the buckets are firmly held in place, and thus spill much water. It is therefore inferior to the Persian wheel.

‡ An endless revolving vert chain of buckets. D'Aubuisson and some others erroneously call this the noria. It is an effective machine.

Dist Feet	No. of Loads.	Cost per Load	Dist. Feet.	No. of Loads.	Cost per Load	Dist Miles	No of Loads.	Cost per Load
		Cts			Cts			Cts
50	38	5 26	1500	18	11 11	1	7	28 57
100	37	5 41	2000	15	13 33	1¼	6	33 33
200	34	5 88	2500	13	15 39	1½	5	40 00
300	32	6 25	3000	11	18 18	2	4	50 00
400	30	6 67	3500	10	20 00	3	3	66 67
600	27	7 41	4000	9	22 22	4	2	100 00
1000	22	9 09	5000	7	28 57	9	1	200 00

If the loading and unloading is such as cannot be done by the driver alone, but requires the help of cranes, or other machinery, an addition of from 10 to 50 cts per load may become necessary. Hauling can generally be more cheaply done by using 2 or 3 horses, and one driver to a vehicle. The neat load per horse, in addition to the vehicle, will usually be from ½ to 1 ton, depending on the condition, and grades of the road. From 13 to 15 cub ft of solid stone; or from 23 to 27 cub feet of broken stone, make 1 ton. **In estimating for hauling rough quarry stone for drains, culverts, &c,** bear in mind that each cub yard of common scabbled rubble masonry, requires the hauling of about 1.2 cub yds of the stone as usually piled up for sale in the quarry, or about ⅔ of a cub yd of the original rock in place. **A cub yd of solid stone, when broken into pieces, usually occupies about 1.9 cub yds perfectly loose;** or about 1¾ when piled up. A strong cart for stone hauling, will weigh about ¾ ton, or 1500 lbs, and will hold stone enough for a perch of rubble masonry, or say 1.2 pers of the rough stone in piles. The average weight of a good working horse is about ½ a ton.

Morin gives the following results from careful experiments made by him for the French Government. The draft of the same wheeled vehicle on a road, may in practice be considered to be,

1st. On hard turnpikes, and pavements: in proportion to the loads; inversely as the diams of the wheels; and nearly independent of the width of tire. It increases to uncertain extents with the inequalities of the road, the stiffness (want of spring) of the vehicle, and the speed; (considerably less than as the square roots of the last.)

2d. On soft roads, the draft is less with wide tires than with narrower ones; and for farming purposes he recommends a width of 4 ins. With speeds from a walk to a fast trot, the draft does not vary sensibly.

TRUSSES.

INTRODUCTION.

General Principles.

1. Truss Design a Specialty. The design, construction and erection of trusses have become a specialty, to which persons confine themselves more or less exclusively, and thus attain a degree of expertness beyond the reach of the general engineer.* The latter, however, should have a knowledge of the subject, sufficient at least to enable him to form a well-grounded opinion of the general merits of a design and to guard him against the adoption of one involving serious imperfections. In a volume like this we can discuss only general principles.

2. The Truss Principle. Theoretically, a truss consists of a number of straight bars, joined, near their ends, by perfectly flexible joints, loaded only at these joints, and so arranged that all its internal stresses are sustained by its members, and only the vertical † pressures, due to the weights of the truss and its load, are transmitted to the abutments.

3. Distinction between Beams and Trusses. When a solid beam (Fig. 2, ¶ 7, Transverse Strength) bends, under its own weight or under that of its load, all the fibers above the neutral axis are compressed, while all those below are extended; and the resulting change of length, in each fiber, is proportional to the distance of the fiber from the neutral axis; but, in a truss, the loads (including the weight of the truss itself) are theoretically regarded as divided into portions which are concentrated at the joints between the members and which act through the cens of grav of their cross-sections. So placed, the stresses caused by them could not act transversely of the members, as in a beam, causing so-called secondary stresses, but must act longitudinally or axially of the members, and must be uniformly distributed over their entire cross-sectional areas. This is the distinguishing feature of all trusses.

4. In such a truss the material would be used most economically, and the stresses in each piece and in each part of such piece could be readily and accurately determined.

5. In the truss of a well-designed bridge or roof, this ideal condition is approximated by using, for the principal members, straight and rather slender pieces, and by so distributing the extraneous load that it shall be applied only at the joints between the members, thus subjecting them chiefly to forces acting at their ends and in the directions of their lengths. In pin-connected trusses (see ¶ 175) the joints are practically flexible.

Most of the trusses in common use consist of two long members, usually horizontal (but see ¶ 49), called **chords,** extending throughout the span and connected by **web members,** which are sometimes all inclined, and sometimes alternately vertical and inclined. Inclined web members are called **diagonals.**

6. Ties and Struts. A member sustaining tension is called a rod or tie. One sustaining compression is called a strut or post. One capable of sustaining both tension and compression is called a tie-strut or a strut-tie.

7. The dimensions of a truss are usually measured along the center lines of its members; and, in pin-connected trusses, the pins are placed at the intersections of these lines Hence, the measurements are usually made from "center to center of pins."

8. In a plate girder, the flanges are usually regarded as performing the function of the chords of a truss, and the web as performing that of the web members of a truss.

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* Railroad companies and municipal corporations frequently prepare their own bridge specifications; but the general proportions, number of panels, etc., are often left to the judgment of the bidders.

† We here suppose the truss to be loaded vertically. If the load is otherwise applied, as in the case of the wind pressure upon a horizontal bracing truss, the pressure on the supports may be horizontal, or otherwise inclined to the vertical, but all the internal stresses are still sustained by the truss members.

Loading.

9. Dead and Live Load. In bridges, we distinguish between the "dead" and the "live" load; the dead load comprising the weight of the permanent structure—*i. e.*, of the bridge itself, with its trusses, bracing and floor system; while the live load comprises any temporary and extraneous loads, such as engines, cars, horses, vehicles, foot passengers, etc., which may come upon the bridge

10. The dead load is usually distributed uniformly along the span, but the loaded chord (that carrying the roadway) of course usually receives a greater share of it than the unloaded chord. The live load comes only upon the loaded chord. In determining stresses, it is usual to consider the weight of live load and of floor system as being on the loaded chord, and the rest of the dead load as divided equally between the two chords. It sometimes happens, however, that both the upper and the lower chords carry roadways. They must then, of course, both be treated as "loaded," though not necessarily *equally* loaded; for one may carry a railway while the other carries only a highway.

Unsymmetrical Loading. Counterbracing.

11. Unsymmetrical Loading. In Figs. 2 to 10, the loads are supposed to be placed symmetrically.

12. If this could be the case in practice, the compression members would never be called upon to resist tension, or the tension members to resist compression; and the trusses in Figs. 2 to 10 would suffice (supposing each member to have sufficient strength), even though the compression members were incapable of resisting tension and vice versa. Thus, the tension members might be flexible chains, and the compression members might be posts, merely abutting against supports at their ends.

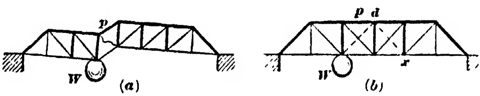

Fig. 1.

13. But in a truss, Fig. 1 (*a*), with a flexible tie in the panel, *p*, as shown, the load W, unsymmetrically placed, would cause failure, as indicated.

14. Counterbracing. To prevent this, those members which, under moving loads, may be subjected alternately to both tension and compression may be so constructed as to be able to resist both kinds of stress. That is to say, the tension members may be so stiffened as to be capable of acting as posts, and the ends of the compression members so connected to the chords that those members can also act as ties. This is the expedient usually employed in trusses without vertical web members.

15. Counters. In trusses with rectangular panels, the distortion, Fig. 1 (*a*), caused by unsymmetrical loading, is usually prevented by the introduction of additional members called counterbraces, or counters, in distinction from the "main" members, which last are designed to resist the normal stresses due to uniformly or symmetrically distributed loads. Thus, in Fig 1 (*b*) the unsymmetrical load, W, tends to convert the rectangle, *p*, into a rhomboid, by lengthening its diagonal, W *d*; and this may be prevented by the introduction of an oblique tension member (counter) in the line of that diagonal, as shown by dotted line. For a similar reason, such a counter is inserted also in the corresponding panel, *x d*.

16. Triangles. It will be noticed that the introduction of counters reduces the truss to a framework made up exclusively of *triangles*.

17. It might at first sight appear that the several parts of a bridge truss must be most strained when covered from end to end with its maximum load; but this is true only of the chord and of the *main* diagonals and verticals near the *ends* of the truss. The other web members may be more strained by a *part* of the load, placed unsymmetrically on the truss; so that, although correctly proportioned for a full load, they may be too weak for a

partial one. If all be made as strong as the end ones, they will, it is true, be safe for a passing load; but this would require an expense of material that would be justified only in the case of moderate spans, especially of wood, in which the additional trouble and expense of getting out and fitting together pieces of many different sizes may more than counterbalance the saving in material.

18. In large bridges, where the live load is small, relatively to the dead load, but little counterbracing is needed, and that at and near the center only; whereas, in a very light bridge, the counters should extend from the center, where they are most strained, to near the ends, where the strain upon them is least.

Cross-bracing.

19. Bracing between Trusses. Advantage is taken of the proximity of the two or more trusses of a bridge, standing side by side, to connect them by cross-bracing, thus giving to the entire structure far greater lateral stability than would be possible in the single trusses.

20. Thus, **lateral bracing,** Fig. 39, consists of horizontal trusses placed between the two upper chords of the main trusses, or between the two lower chords, or both; the chords of the main trusses acting also as the chords of the lateral trusses. The lateral bracing prevents lateral deflection of the chords.

21. Sway bracing, Fig. 64 (*c*) (called also diagonal, cross, vibration and wind bracing), consists of short trusses (usually vertical) crossing the bridge transversely and thus connecting the two trusses. The sway bracing has its own chords, but uses parts of the posts of the main truss as its end posts.

22. Portal bracing, Fig. 54 (*a*), consists of sway bracing (usually in an inclined plane) joining the tops of the end posts in trusses of sufficient depth to permit its use. The portal bracing, with the end posts, forms a portal through which trains, etc., enter the bridge.

Types of Trusses.

23. The simplest form of truss consists of a single triangle, Figs. 2 (*a*) and (*b*). In Fig. (*a*) the load produces compression in the rafters, tension in the chord or tie rod,* and compression (= the tension in the chord) between the heads of the rafters; in Fig. (*b*) vice versa.

24. The truss shown in Fig. 2 (*a*) is in common use for roofs of small span, as in dwellings. In practice, it is of course loaded along the rafters, and not only at the apex as in Fig. (*a*); but, in calculating the stresses in truss members, we commonly first assume that the loads are concentrated at the *intersections* of the members. The effect of their actual distribution *along* the members is then determined separately, treating the members as beams.

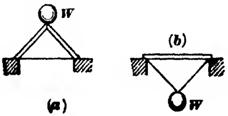

Fig. 2.

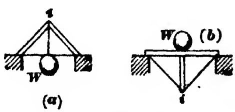

Fig. 3.

25. In Fig. 3 (*a*) (called a King truss), the vertical tie (improperly called a King post), and in Fig. 3 (*b*) the vertical post, simply carries the weight of the load to the apex, *i*, where it produces the same effect as in Figs. 2 (*a*) and (*b*).

26. Hence, neglecting the weights of the vertical tie and other members, the stresses, caused by a given load, W, in the diagonals, and in the horizontal tie, Fig. 3 (*a*), are the same (not only in character, but also in amount) as those produced by an equal load, W, in Fig. 2 (*a*). Similarly, those in Fig. 3 (*b*) correspond with those in Fig. 2 (*b*).

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* In Figs. 2 to 12, and 14 to 17, double or heavy lines indicate posts or struts, and light lines indicate ties

27. Figs. 4, 5, and 6, giving modifications of the simple forms shown in Figs. 2 and 3, illustrate in principle most of the bridge trusses in common use for spans up to 300, 400 or even 500 feet. See Figs. 7 to 10, ¶¶ 35, etc.

28. In Figs. 4, 5, and 6, there is an upper chord, in compression; and a lower chord, in tension; the shorter chord sustaining the compression between the heads of the rafters, Figs. 2 (*a*) and 3 (*a*), or the horizontal tension between the feet of the diagonals, Figs. 2 (*b*) and 3 (*b*). Figs. 4 (*a*) and 5 (*a*) are modifications of Figs. 2 (*a*) and 3 (*a*); Figs. 4 (*b*) and 5 (*b*) of Figs. 2 (*b*) and 3 (*b*): Fig. 6 (*a*) of Fig. 2 (*a*), and Fig. 6 (*b*) of Fig. 2 (*b*)

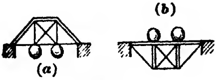


Fig. 4.

29. Figs. 4 (*a*) and 4 (*b*) may be regarded as showing Figs. 3 (*a*) and 3 (*b*) respectively, with the vertical member, as well as the load, split in two, and the two parts separated by horizontal straining pieces. If the loads are placed symmetrically, so that the horizontal pressures, Fig. 4 (*a*), or tensions, Fig. 4 (*b*), on the two ends of the shorter chord, are equal, the two diagonal counters in the center are unnecessary.

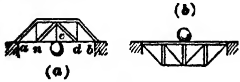


Fig. 5.

30. Howe and Pratt Systems. In Fig. 5 (*a*) the vertical web members are in tension, and the diagonals are in compression, embodying the "Howe" principle, used in bridges with wooden diagonals; while in Fig. 5 (*b*) the verticals are in compression, and the diagonals in tension, embodying the "Pratt" principle, used in bridges with metal diagonals. In such bridges long compression members are objectionable.

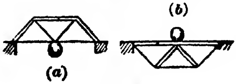


Fig. 6.

31. Warren or Triangular Trusses. In Fig. 6, illustrating the "Warren" or "triangular" truss, the web members are all diagonal, and are alternately in tension and in compression. They divide the truss profile into *isosceles* triangles.

32. Through, Deck and Pony Spans. Figs. 4 (*a*), 5 (*a*) and 6 (*a*), with the roadway on the lower chords, are called "through" spans, and Figs. 4 (*b*), 5 (*b*) and 6 (*b*), with the roadway on the upper chord, are called "deck" spans. The deck span permits the use of sway bracing (see ¶ 21) between, and throughout the depth of, the two or more trusses forming the bridge, while the through span of course does not; but the use of the through span is often required, in order to give sufficient head-room for boats, floods, trains on crossing roads, etc., below the bridge. A truss, loaded on the lower chord, but too shallow for lateral bracing (see ¶ 20) between the upper chords, is called a "pony" truss (or "pony through" truss).

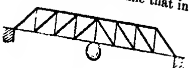
33. Panels. The points where the vertical web members meet the chords, in Figs. 4 and 5, are called **panel points;** and the rectangular spaces, *a n, n c. c d*, etc., Fig. 5 (*a*), between the verticals, are called panels.

34. The Warren truss, Fig. 6, has no verticals, as essential parts of it. See ¶¶ 45 and 46. Its subdivisions are called simply triangles; and a panel is a length of truss equal to the width of a triangle. A panel of either chord, however, is that portion of it between two panel points.

35. Further modifications of these designs, with more numerous panels, are shown in Figs. 7 to 10. Figs. 7 (*a*) and 8 (*a*), with verticals in tension, represent the Howe truss of Figs. 4 (*a*) and 5 (*a*), while Figs. 7 (*b*), 8 (*b*), 9 (*a*) and 9 (*b*), with diagonals in tension, represent the Pratt truss of Figs. 4 (*b*) and 5 (*b*). Figs. 10 represent the Warren truss of Fig. 6.

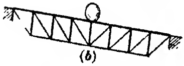
36. Fig. 8 (*a*) represents simply Fig. 7 (*a*), lowered so as to become a deck, instead of a through bridge; while Fig. 8 (*b*) represents Fig. 7 (*b*) converted from a deck to a through span by being carried on vertical end posts.

37. In Fig. 8 (*a*), the **vertical end posts, and the horizontal piece at each end of the loaded chord**, form no part of the truss proper. The latter simply act as beams, supporting the load during its passage from the abutment to the truss and vice versa. The end post in Fig (*a*) supports only one end of this beam, while that in Fig. (*b*) supports half the *truss*.



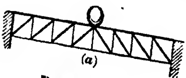
(*a*)

Through Howe.



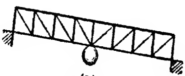
(*b*)

Fig. 7. **Deck Pratt.**



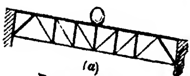
(*a*)

Deck Howe.



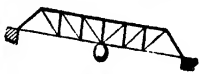
(*b*)

Fig. 8 **Through Pratt.**



(*a*)

Deck Pratt.

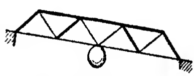


(*b*)

Fig. 9. **Through Pratt.**

(*a*)

Deck Warren.



(*b*)

Fig. 10. **Through Warren.**

38. In Figs. 8 the middle vertical carries no part of the load. Theoretically it serves merely to prevent deflection of the two unloaded middle chord panels under their own weight; but in practice such members are often inserted for the purpose of obtaining convenient connections for lateral pieces, such as floor beams.

39. In Figs. 9 (modifications of Fig. 7 (*b*)) and in Fig. 10 are shown, in principle, the most common forms of metal bridge truss, used as deck and as through spans respectively.

40. In Fig 9 (*a*), as in Fig. 8, the vertical end posts and the horizontal pieces at the ends of the loaded chord form no part of the truss; and in Fig. 9 (*b*), as in Figs. 8, the middle vertical supports only the unloaded middle chord panels.

41. Intersections. In deep trusses, two or more sets of web members are sometimes combined in one truss, with one pair of chords. Thus the two simple Pratt trusses shown in Figs. 11 (*a*) and (*b*) combine to make the "Whipple" or "double intersection Pratt" truss, Fig. 11 (*c*), recently in general use.

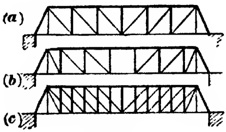


Fig. 11.

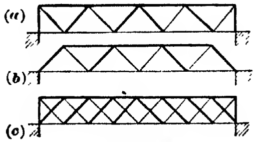


Fig. 12.

42. Similarly the two simple Warren trusses, in Figs. 12 (*a*) and (*b*), combine to form the double intersection Warren of Fig 12 (*c*).

43. A combination of *four* systems is called a "quadruple intersection" truss. See Fig 59 (*t*).

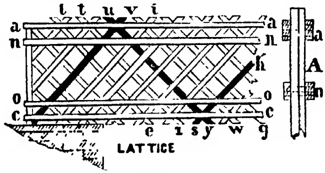


Fig. 13.

44. The old Towne "lattice" truss, Fig 13, consisting of planks crossing each other (usually at right angles) and bolted or tree-nailed together at their intersections, may be regarded as a combination of several Warren trusses.

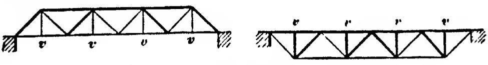


Fig. 14.

45. Sub-verticals. In deep trusses, where the horizontal spread of the panels is considerable, sub-verticals, *v*, Figs. 14 and 15, are often used, especially in Warren trusses, to support the segments of the loaded chord. See also Figs. 59 (*i*), (*r*), and (*s*).

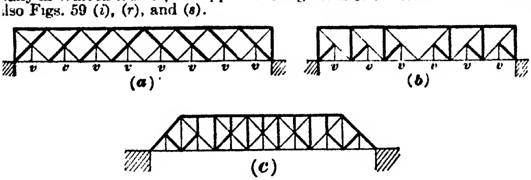


Fig. 15.

In the **"Baltimore" truss,** Fig. 15 (*b*), each diagonal is braced, at its middle point, by a short diagonal strut inclined in the opposite direction, and a sub-vertical is suspended from their junction. With very long panels, sub-verticals are sometimes used for the panels of the unloaded chord also. See Fig. 15 (*c*).

46. Collision struts, or collision posts, S, Figs. 59 (*k*), (*m*), (*o*), and (*t*), and 73 (*a*), are used for bracing long diagonal end posts against a blow from a derailed train.

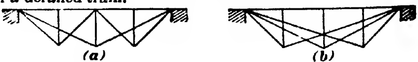


Fig. 16.

47. Fink and Bollman Trusses. Figs. 16 show two obsolete modifications of Fig. 3 (*b*), viz.. the Fink, Fig. 16 (*a*), and the Bollman, Fig. 16 (*b*). The large bridge over the Ohio River at Louisville, Ky., completed 1870, is of the Fink type. The Bollman was largely used on the Baltimore and Ohio Railroad years ago.

48. In the Fink and in the Bollman truss there was but one chord, as shown. This chord usually carried the roadway. Where the roadway was placed lower, it gave the truss the appearance of having two chords. Under uniformly distributed loads, in the Fink, and under all circumstances in the Bollman, the stress in this chord was uniform throughout. In the Bollman (see Fig.), the longitudinal stresses in the chord were all applied at its ends. Each type may be regarded as a combination of several suspension trusses like Fig. 3 (*b*). In the Bollman, the simple trusses were all of the same span and depth; and each vertical post, except the central one, divided its simple truss eccentrically. The Fink principle is still largely used in metal roo trusses. See Figs. 26.

Fig. 17.

49. Curved Chords. Trusses with curved or "broken" chords, Fig. 17, are frequently used for long spans. The members themselves, between panel points, are always straight. In the **bowstring**, Fig. 17, the panel points of the upper chord lie in a curve, convex upward. In the **crescent** truss, the lower chord also is convex upward. The bowstring truss has the advantage, over those with horizontal upper chords, of making all the chord and web stresses more nearly equal, thus simplifying the construction and reducing the weight of the trusses. It has the disadvantage of permitting no overhead bracing near the ends of the span. If the curve of the upper chord is made parabolic, the dead load stress is uniform throughout the lower chord, and in each vertical (now in tension) the stress is equal to the dead load on the lower chord. The diagonals receive no dead load stress, but are called into action only by eccentric loads.

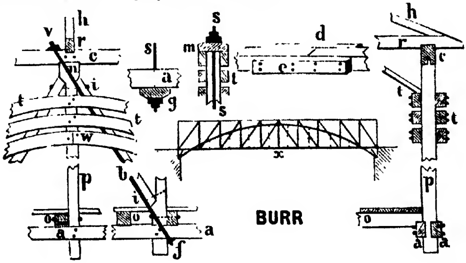


Fig. 18.

50. The Burr truss, Fig. 18, at one time much used for wooden bridges was a combination of a Howe truss and an arch.

Camber.

51. Camber. In practice, the members of the upper and lower chords of bridges are not placed perfectly in line, but so that the chords curve slightly, with the convex side upward. This curve is called the camber. Its object is to prevent the truss from bending down below a horizontal line when heavily loaded. When the chords are cambered (see *y s* and *c d*, Fig. 19),

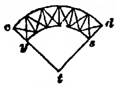


Fig. 19.

they become approximately concentric arcs of two large circles, of which the center is at *t*; and the upper one plainly becomes longer than the lower. The verticals, instead of remaining truly vertical, become portions of radii of the arcs mentioned; and, although their lengths remain unchanged, yet their tops are farther apart than their feet; and this renders it necessary to lengthen the diagonals. See ¶¶ 211–214.

Cantilevers.

52. The cantilever principle is shown in Fig 20, where A and B

Fig. 20.

represent counterweights or anchorages. Fig 21 shows the Niagara cantilever bridge. It consists of two cantilever trusses, *ab*, *a′ b′*, connected by an ordinary truss, *ba′*, which is suspended, by a vertical link at each end, from the ends of the cantilever trusses. The weight of the truss is counterbalanced by anchorages, A and B, or by weights, or both. The principal advantage of the cantilever is that it may be built outward from the piers across the channel. It thus greatly facilitates erection in cases where false-work cannot well be used.

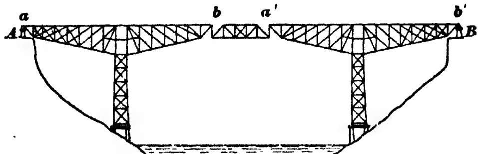


Fig. 21.

Movable Bridges.

53. Movable bridges, including draw, swing and lift bridges, are of three general classes; one in which the movable part slides horizontally, one in

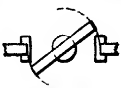


Fig. 22.

which it swings horizontally, and one in which it swings vertically. Ordinarily, the movable span is pivoted near the middle, and swings horizontally on the central "swing" or "pivot" pier, as in Fig. 22. In such cases it is

usually mounted on a central pivot, or on a nest of rollers or wheels running on a circular track. Such a bridge must be so designed that, when it is swung open, or if it is not brought to a bearing at the ends when closed, it shall sustain not only its own weight, but also any other loads that may come upon it. In addition, each half must be able to act as a bridge supported at both ends, with all possible live loads; for, as an unbalanced live load comes on either end, that end will be brought to a bearing. In elaborate bridges, provision is made for raising the ends of the draw span, when closed, thus bringing both ends to a firm bearing, and the floor flush with that on the adjacent abutment or fixed span. This raising is usually made sufficient to relieve the middle pier of only a portion of the load. The bridge then acts like a "continuous" girder (see Transverse Strength, ¶¶ 78, etc.) supported at three or at four points, depending upon the arrangement of the bearing on the pivot pier.

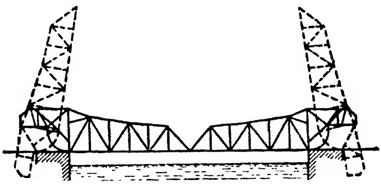


Fig. 23.

54. Drawbridges in which the movable part swings vertically (the shore end carrying a counterweight) may either revolve about a pivot, or they may roll, as in the Scherzer rolling lift bridge, Fig. 23. In the Scherzer Bridge the center of gravity of each of the two moving wings remains at a constant elevation, so that no lifting or lowering of such wing occurs, and the work done consists merely in overcoming the rolling friction of the curved end of the wing upon its support.

55. Skew bridges are used where a channel, road, etc., is crossed obliquely, and where it is inconvenient to have the abutments perpendicular to the trusses. For simplicity in making floor connections, etc., the truss is usually so designed as to bring the panel points opposite each other, as in Figs. 24 and 25. Where the skew is but slight, this necessitates a difference

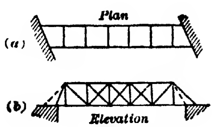


Fig. 24.

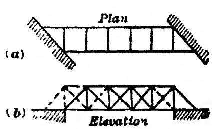


Fig. 25.

in inclination between the two end posts, as in Fig. 24, involving complication in the connections for the portal bracing. But where the skew is greater, it may be possible to make it just equal to one or more even panels, adjusting the panel length to suit, and thus leaving each truss symmetrical, as in Fig. 25. In each figure, those members which belong only to the *farther* truss (the upper one in the plan) are shown by *dotted* lines.

Roof Trusses.

56. Roof trusses are made in a great variety of forms. Those shown in Figs. 26 are common. In Fig. 26 (*a*), part of load, at *d*, compresses the rafter from *d* to *a*, while the remainder compresses the strut, *d h*, and pulls the rod *h i* and the part-chord *h a*. Similarly, part of *c* passes through *c a* to *a*, and the remainder through *c k d h i* to the apex *i*. Thus each load is eventually carried by the members, part to the apex and part along a rafter to an abutment. It will be seen that the greatest stresses in the rafters and in the chord occur near the ends.* Sometimes the members shown

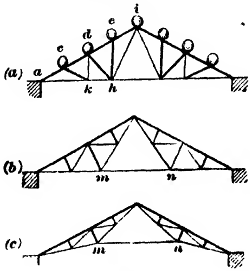


Fig. 26.

vertical in Fig. (*a*) are inclined, or the lower chord is "broken," being usually convex upward. Roof trusses are often composed, as in Figs (*b*) and (*c*), of two Fink trusses, inclined, and leaning against each other, their feet being held in position by a tie, *m n*, and the rafters forming the upper chords of the Fink trusses.

STRESSES IN TRUSS MEMBERS.

General Principles.

57. Conditions of Equilibrium. In trusses, as in beams, it is necessary and sufficient, for equilibrium, that the internal stresses, and their moments, shall balance the external forces and their moments The external forces (viz., the loads and the end reactions) and the resulting moments and shears, are discussed under Statics, ¶¶ 285, etc. We here discuss the determination of the internal stresses For the fundamental distinction between beams and trusses, see Trusses, ¶ 3.

58. In general, the stresses in the members are found by means of the principles of moments (Statics, ¶¶ 301, etc.), and of Shears (Statics, ¶¶ 325, etc.); making use of the force parallelogram (Statics, ¶¶ 35, etc) or force triangle (Statics, ¶¶ 46, etc.), the force and cord polygons (Statics, ¶¶ 72, etc., 86, etc) and the influence diagram (Statics, ¶¶ 339, etc).

59. A very convenient method, and one in common use, is that described more fully in ¶¶ 67, etc , below, where the truss is considered as being cut through by a section. We then seek to ascertain what stresses, in the members so cut, would be required to preserve equilibrium.

60. Before the stresses can be calculated, and the truss proportioned to those stresses, its weight must be known; for this constitutes a load, and therefore affects the stresses. But, on the other hand, we cannot learn its weight until we know the sizes of its different members. In this dilemma we must assume for it an approximate weight, based upon our knowledge of somewhat similar trusses already built. This becomes the more necessary as the truss increases in size, so that its own weight becomes greater in proportion to that of the load.

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* If the diagonals were parallel, their stresses, and those in the verticals, would be greatest at the center of the span and least at the abutments.

61. To distinguish between ties and struts; from the point, o, Figs. 27, where the force is applied, draw $o\,c$ to represent the applied force, in the direction in which that force tends to move the point, o; and upon $o\,c$ as a diagonal construct the force parallelogram, $a\,b$. Through o draw $i\,i$ parallel to the other diagonal $a\,b$. Then, if a piece be on the same side of $i\,i$ with $o\,c$, it is a strut; while, if it be on the opposite side, it is a tie.

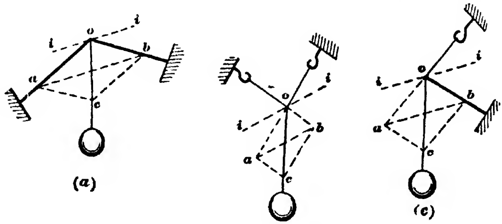
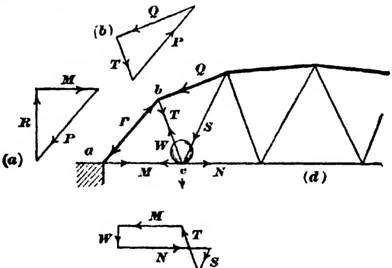


Fig. 27. (b)

62. Ties and struts may often (as in Fig. 27) be readily distinguished by inspection, by imagining the piece to be flexible, like a rope or chain. If it is seen that it would then resist the force acting upon it, the member is a tie; if not, it is a strut. Or, suppose that the piece is not secured at its ends. If, then, it is seen that it would resist the force acting upon it, the member is a strut; if not, it is a tie.

63. Or we may proceed as follows: In Fig. 28 (a), representing joint a, we begin with the known net vertical reaction, R *; and find the unknown stresses in the chord and in the end post by means of the force triangle, making their arrows follow the known direction of R. Transferring these arrows to the respective truss members, Fig. (d), we find that the chord pulls away from a, and is therefore a tie; while the end post pushes toward a, and is therefore a strut.



(c) **Fig. 28.**

64. In Fig. (b), representing joint b, we draw P upward to represent the pressure of the end post toward b; and the other two sides of the force triangle give the pressure in the chord member, Q, and the tension in the tie, T.

65. In Fig. (c), representing joint c, we know T, M, and the load, W, and we obtain the tension, N, and pressure, S, in the corresponding members.

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* Inasmuch as half of each end panel rests directly upon a support, and thus adds nothing to the stresses in the members, we must, in determining those stresses, use only the

net reaction = reaction — half panel load.

66. *Tensile* stresses, because they tend to *elongate* a member, are conventionally regarded as *positive*, and designated by +, while *compressive* stresses are regarded as *negative*, and designated by —.

Method by Sections.

67. Let Fig. 29 (*a*) represent a roof truss, with three equal loads, W, of 2 tons each, applied at *a*, *c* and *b*, respectively, and let it be required to find the stresses produced, by those loads alone, in the members *a c* and *a d*. Suppose the portion shown in Fig. 29 (*b*) to be separated from the rest of the truss, as shown, by cutting through the members *a c* and *a d*. The lower portions of those members, shown in (*b*), are, however, supposed to be held in their original positions by the stresses S_c and S_d, exerted in these members themselves. Taking moments about the right support, *b*, Fig. 29 (*a*) we have, for the upward reaction of the left abutment, *a*,

$$R = \frac{3\,W}{2}.$$

68. We have, then, at *a*, Fig. 29 (*b*), four forces, as follows: two known forces, viz.: W, vertically downward, = 2 tons, and R, vertically upward, $= \frac{3\,W}{2}$; and two unknown forces, S_c and S_d. Now S_c makes a known angle, A, and S_d a known angle, B, with the vertical. The vertical forces, W and R, have, of course, no horizontal resolutes (see Statics, ¶¶ 54, etc.); and their vertical resolutes are the forces themselves.

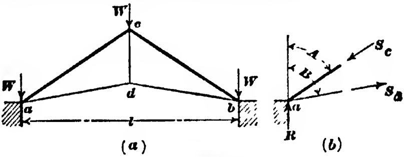

Fig. 29.

69. The horizontal resolutes of the inclined forces, S_c and S_d, are, respectively: S_c.sinA, and S_d.sinB; and their vertical resolutes are: S_c.cosin A, and S_d.cosin B.

70. We see, by inspection, that the stress, S_c, in the rafter, *a c*, is compression, and that the stress, S_d, in the lower member, is tension; but, for convenience, we may at first assume, in advance, that all of the unknown stresses are tensions or +. Then those which finally appear as + are known to be tensions, and vice versa. Their horizontal resolutes, in this case, are therefore both taken, for the present, as being right-handed, or positive; and their vertical components upward or positive also. It will be remembered (see ¶ 66) that we regard tensions as positive, and compressions as negative.

71. Now, in order that the four forces at *a*, viz.: W = 2 tons, downward, $R = \frac{3\,W}{2}$ upward, S_c and S_d, may be in equilibrium, it is necessary;

(1) that the sum of their horizontal resolutes be zero, or
S_c.sin A + S_d.sin B = 0;
(2) that the sum of the vertical resolutes be zero, or
R — W + S_c.cosin A + S_d.cosin B = 0.

Thus, let A = 45°, sin A = 0.707; cosin A = 0.707.
B = 75°, sin B = 0.966; cosin B = 0.259.

Then $0.707\,S_c + 0.966\,S_d = 0;$

$$R - W + 0.707\,S_c + 0.259\,S_d = 0;$$

$$S_c = \frac{-0.966\,S_d}{0.707} = \frac{-0.259\,S_d - R + W}{0.707}$$

$$0.966\,S_d - 0.259\,S_d = 0.707\,S_d = R - W.$$

72. Again, in Fig 30, with section $u\,v$, stress in $ed = W_1 - R = 6 - 15 = -9$. With section vy, stress in $ef = \frac{W_1 - R}{\cos\theta} = \frac{-9}{\cos\theta}$. With section ux, stress in $gd = \frac{W_1 + W_2 - R}{\cos\theta} = \frac{-3}{\cos\theta}$. It will be seen that these forces, all acting *downward* on the part truss to the *left* of the section, give *tension* in *ed*, and *compression* in *ef* and *gd*

With section *uz* we cut *two* web members, *gd*, and *gc*; but the stress in *ga* has already been found $= \frac{3}{\cos\theta}$, the vertical component of which is $= 3$. Hence, stress in $gc = W_1 + W_2 + W_3 + 3 - R = 6$.

73. It is, however, evident from inspection that the middle vertical bears simply the middle load, $W_3 = 6$, for, cutting the truss by a curved section, as at *c*, and examining the small portion thus cut out, we see that we have but two vertical forces—viz, the central load, W_3, and the stress in the vertical member; and, for equilibrium, these two must be equal.

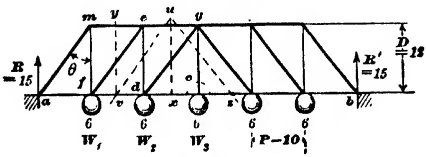

Fig. 30.

Chord Stresses, Moments.

74. For the chord stresses, Fig. 30, let P = panel length = 10 ft. Then the bending moment at the panel point, *d*, is

$$\begin{aligned} M &= 2\,R\,P - W_1\,P \\ &= 15 \times 20 - 6 \times 10 \\ &= 300 - 60 = 240. \end{aligned}$$

Cutting the truss by section $u\,v$, we find that, of the three members cut, only the upper chord member, *eg*, has a moment about *d*. Call its stress S. Its leverage is the depth, D, of the truss, = 12; and, for equilibrium, S D = M. Hence, $S = \frac{M}{D} = \frac{240}{12} = 20$.

75. Similarly, taking moments about *e*, we find the stress, in the lower chord member, *f d*, cut by the section, $u\,v$, to be $\frac{240}{12} = 20$, or the same as the stress in the upper chord panel cut by the same section. Inspection shows the correctness of this result; for the diagonal strut, *e f*, evidently delivers to the upper chord panel, *e g*, a compressive stress or "chord increment" (see ¶ 77) = the tensile stress which it delivers to the lower chord panel, *f d*.

76. If the chord members are inclined, their lever arms must of course be measured *perpendicularly to them;* and we can no longer use the vertical depth of the truss as the lever arm.

77. Chord Increments. Fig. 30. Each diag delivers a comp stress to the upper chord, and, in trusses with parallel chords, an equal tensile stress to the lower chord Find the shear, or vertical component, V_1, V_2, etc, of the stress in each diagonal, beginning with the end post. Then the "chord increments," h_1, h_2, etc, or the stresses in the chord members, *af* and *me*, *fd* and *eg*, etc., due to the several diagonals separately, are

$$\begin{aligned} h_1 &= V_1 \tan\theta \\ h_2 &= V_2 \tan\theta \\ h_3 &= V_3 \tan\theta \end{aligned}$$

and, for the *total* stress in each chord member, we have, $H_1 = h_1$ $H_2 = h_1 + h_2$; $H_3 = h_1 + h_2 + h_3$; and so on.

Shear.

78. Any portion of a shear diagram applies to all those members through which its shear travels, up to the *panel point* where the shear undergoes a change. Thus, in Fig. 31 the shear diagram on the right of the Fig. includes the vertical, *tu;* that on the left includes the diagonal, *mo;* and that between the loads includes the diagonal, *t n*, and the vertical, *m n*.

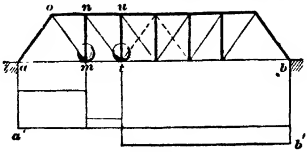

Fig. 31.

79. Shear Influence Diagram. See Statics, ¶¶ 325, etc. In a truss, Fig. 32, the ordinates, *c' g*, etc., to the line *a'' b'* (constructed as in Fig 156, Statics, ¶ 349), give the left end *reactions;* and those, *c' h*, etc., to the line *a'b''*, give the right end *reactions*, for *any position* of the load; and the resulting *shears* for a load, W (not shown), *at any panel point;* but the *shears in a panel, cd*, for a load, W, *between the panel points*, are modified by the action of the stringers in distributing the load between said adjacent panel points, as indicated by the influence lines, *gh*, etc., for the several *panels*. Thus, with W at *c* and at *d*, respectively, the shear, in the panel *cd*, is represented respectively by *c'h* (negative) and by *d'q* (positive); and, as the load passes from *c* to *d*, the shear in the panel changes from *c'h* to *d'q*.

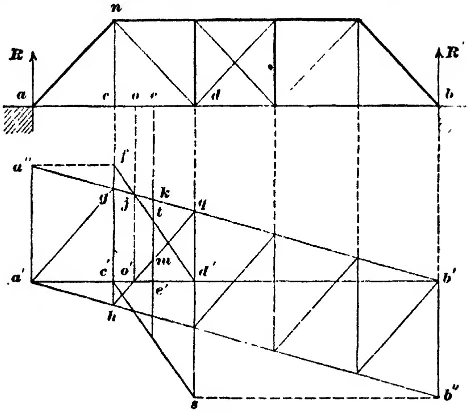

Fig. 32.

But when W is placed at any point, *e*, between *c* and *d*, its load is distributed between the panel points, *c* and *d*, by the stringer, acting as a beam.

80. Thus, drawing, for this beam, *c d* (as for the whole beam in Statics Fig. 156), the *panel* influence lines *fd'* and *c's*, we see that, as W moves from *d* into the panel *cd*, as to *e*, the truss reaction at *a* is thereby slightly increased from *d'q* to *e'k;* but at the same time a portion of W, represented by *e't*, is

carried by the stringer to c, where it diminishes the shear $e'k$ (due to the truss reaction at a), leaving tk as the value of the shear in the panel. As we place W successively at other points, farther from d, and approaching o, the load carried by the stringer to c, and represented by the ordinates from $c'd'$ to fd', continue to increase faster than does the left end truss reaction, R, represented by the ordinates from $a'b'$ to $a''b'$; and the resulting shears in the panel are represented by the ordinates from jd' to jq. At o, the part load, $o'j$, carried to c, is = the left truss reaction, and the shear in the panel is zero. With the load between o and c, the part loads carried to c, and represented by the ordinates from o' c', to jf, are *greater* than the corresponding left end truss reactions; and the result is a *negative* shear in the panel, indicated by the ordinates from j g to jf. It will be noticed that the resulting shears throughout, both positive and negative, are indicated by the ordinates from $c'd'$ to hq.*

Reversing the process, a similar argument may be applied to the panel influence line $c's$, beginning with the load at c, with negative shear in panel = $c'h$, and supposing it moved across the panel to d, where positive shear in the panel becomes = $d'q$.

81. In the case of a **uniform load,** extending on to the span from the right support, b, the point o is the position of head of load for maximum positive shear in the panel, cd; for, in the case of a uniform load, the shear, with head of load at e, is represented by the *area* (sum of all the ordinates) $e'mqb'e'$; and manifestly this area increases as the head of the load approaches o; but when it reaches o, the area above $a'b'$ can increase no further, and when it *passes* o, the *negative* shears, represented by the ordinates from $o'c'$ to $o'h$, begin to *reduce* the resultant positive shear.

82. Having found, by any method, the maximum shear, $d'q$, due to a concentrated load at d, for the diagonal, d n, Fig. 32, and the reverse maximum shear, c' h, due to the same load at c, we may draw an influence line, h q, which gives, as before, the point, o, of position of head of uniform load for maximum stress in the diagonal, d n, from which (as above) we find the corresponding position of the head of a series of concentrated loads.

In practice, the influence line for shear is of value chiefly in thus finding the position of load producing maximum stress, and the resulting stresses, in trusses with *curved* chords, such as Fig. 17. In such a truss, owing to the inclination of the members of the upper chord, those members take some of the shears in their respective panels, and the stress in the diagonal is therefore less than the shear in the panel.

Graphic Determination of Dead Load Stresses.

83. Construct first a diagram of the truss, as in Fig. 33 (a), lettering the spaces between the members, and those between the arrows representing the dead loads. Call the end post, 1–3, between A and B, "AB," the stress in it "ab," the load at 2, "cd," etc., using capital letters for panels and truss members, and small letters for loads and stresses. Adopt a suitable scale of forces, and construct the diagram, Fig. 33 (b), as follows:

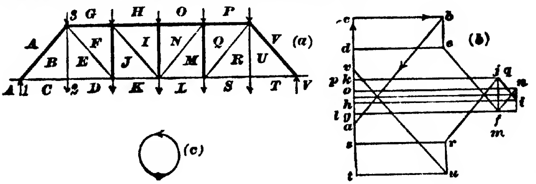

Fig. 33.

84. Consider first the point 1, Fig. 33 (a). There are here three forces in equilibrium, viz., ac, ab and bc. Find the net end reaction, R = ac, and lay it off upward (since it acts upward on 1) from any convenient point, a, to c, Fig. 33 (b). From a draw an indefinite line ab parallel to AB and from c

* Since fh, jo', qd' and ke' are parallel, me' = kt, and $c'h$ = gf.

draw cb parallel to BC, obtaining the force triangle acb of the point 1. The lengths of cb and ba then give the stresses by scale.

85. In Fig. 33 (b), the arrow on ac indicates the upward direction of that force. Following around the triangle, we affix arrows (in the same direction) to cb and ba. Supposing these arrows now to be transferred to the corresponding members in Fig. 33 (a) we see that $b\,c$ pulls from the point 1, showing that $b\,c$ is tensile, or +, while $b\,a$ pushes toward 1, showing that $b\,a$ is compressive, or —.

86. The characters of the stresses may be found more quickly as follows: Draw a circle, Fig. 33 (c), and place on it arrows pointing around in the direction (counter-clockwise in this case) followed around the truss in constructing the load line. See ¶ 92, below. Then consider any panel point, Fig. 33 (a), and follow the letters in the spaces around that point in the direction of the arrows on the circle. Note the order of the letters, and follow the corresponding equilibrium polygon, Fig. 33 (b), around in the same direction. This will give the directions in which the forces respectively act on that point.

87. Thus, consider the panel point 2. Following around 2 in the direction of the circle, we read B, C, D, E. Turning now to Fig. 33 (b), and reading b, c, d, e, we find that on bc we go from right to left (or opposite to the direction indicated by the arrow drawn for point 1); hence bc acts to the left on 2, and BC is therefore in tension, and its stress bc is +.

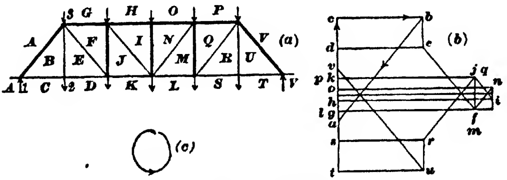

Fig. 33 (repeated).

88. Given now the stress, bc, in BC, construct, on bc, the force polygon $bcde$ for the four forces acting on the point 2. Thus, from c lay off cd downward, to represent the dead load on the lower chord at 2. Since bc acts as a pull from the left on 2, and since the forces must follow each other around the polygon, cd must evidently be drawn downward from c and not from b. From d draw an indefinite line parallel to DE, and from b another, parallel to BE. They will intersect at some point, as e, and eb and de will then represent the stresses in BE and DE.

89. Inspection would show that $be = cd$, since cd is the only force acting on 2 with a vertical component, and that $bc = de$; but the construction of the force polygon $bcde$ is necessary for the completion of the diagram.

90. Having now found the stresses in DE, BE, and AB, and knowing the panel load ($= g\,a$) at the point 3, construct the polygon $g\,a\,b\,e\,f\,g$. This gives $e\,f$ and $f\,g$, and from these the process may be continued and the diagram completed.

91. It will be noticed that, in some cases, a point on the diagram, Fig. 33 (b), is given more than one letter. Ordinarily this is simply a coincidence, arising from overlapping of the force polygons. In some cases, however, the coincidence of the letters shows that the stress in the member is zero.

92. In practice it is usual to construct first the entire load line ct, thus. draw first the net reaction, ac, upward; then, following around the truss counter-clockwise, draw all the other exterior (dead load) forces in their proper order, thus cd, dk, kl, ls, st, tv, vp, po, oh, hg, ga. The stress diagram may then be constructed, as before.

Live Loads.

93. It might at first be supposed that each member of the truss would receive its maximum stress when the train completely covered the bridge; but this is true only of the chord members. In the truss shown in Fig. 33 (*a*) each web member receives its maximum stress when the greatest possible shear occurs in a section cutting that member.

Fig. 34.

94. In Fig. 34, the *main* diagonals to the *left*, and the *counters* to the *right* of the center, C, are shown. Any one of these members receives its maximum stress from a uniformly distributed load when the load extends from it to the *right* support *b*, with head of load at a point, *o*, Fig. 32 (*a*), found as in ¶ 81; and *vice versa* for the diagonals inclined in the *opposite* direction. Each *vertical* receives its maximum stress when the load extends from the *farther* support to a point, *o* (see ¶ 81), in the panel beyond the vertical. This statement must be slightly modified when the concentrated wheel loads are considered. See ¶¶ 97, etc.

Fig. 35.

95. Assumed Uniform Live Load. As a crude approximation, the engine and train are sometimes considered as a uniform load crossing the bridge, Fig. 35; but this method, ignoring, as it does, the great concentration of weight in modern locomotives, is apt to be either unsafe or wasteful of material. This assumption is proper in connection with wind pressure on train. See ¶ 121.

Fig. 36.

96. Concentrated Excess Loads. Again, to provide for the locomotive loads, one or more concentrated excess loads, Fig. 36, are sometimes employed. The stresses due to these loads may be computed separately, and added to the stresses produced by the uniform live loads. To produce the maximum chord stresses, the excess loads should be in the middle of the span, and the train load should cover the entire bridge. This method is fairly approximate, and engineers are divided as to whether this method of concentrated excess loads should be used, or that of the actual or "typical" locomotive wheel loads as explained below.

97. Standard or Typical "Wheel" Loadings. In the method of wheel loads, the actual stresses, produced by the heaviest engines likely to cross the bridge, are considered. Even in engines of nearly the same weight, the loads may be differently spaced, and spaced at intervals of odd fractions of an inch, rendering computation very laborious. For this reason, and in order to provide for the use of heavier engines in the future, it is customary to consider an imaginary or "typical" engine, with loads and spacing given in round numbers, the stresses from which shall at least be equal to those produced by the heaviest engines likely to be used during the life of the bridge.

The live loads are ordinarily taken as consisting of two typical locomotives with their tenders, followed by a uniform train load. See Digests of Specifications.

98. The following is an example of the computation of live load stresses by the method of locomotive wheel loads:

Fig. 37 (*b*) represents the loads on one rail, corresponding to Cooper's Standard,* Class E 40, which consists of two coupled consolidation locomotives, followed by a train considered as equivalent to a uniform load of 4000 lbs. per linear foot. In the diagram, Fig. 37 (*a*), all loads are figured in thousands of pounds, moments in millions of foot-pounds, and distances in feet.

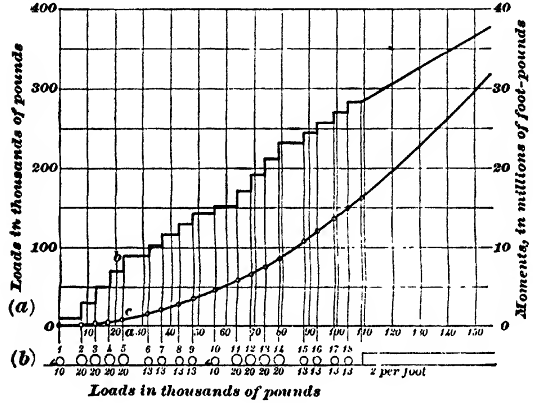

Fig. 37.

99. Live Load Web Stresses. The maximum live load stresses will occur in the web members of any panel of the truss in Fig. 34 or 38, when the live load produces the maximum shear in that panel. It can be shown that this will occur when $P = \frac{W}{n}$, where P = the live load on the panel cut by the section; W = the total live load on the truss, and n = the number of panels in the truss. This equation is called the criterion for maximum shear.

100. The following table is based upon this relation. The second column is obtained by adding successive wheel loads to P. In this case, W = 6 P, since our truss has 6 panels. Let any wheel be at a panel point Then, by moving the wheel a little to the left or right, it will be included in or excluded from P. Hence P and W have each a minimum and a maximum value for each wheel at the panel point.

No. of wheel at any given panel point.	Value, P, of load on panel to left of given point.	Corresponding value of W for maximum shear in panel.
1	0 to 10,000	0 to 60,000
2	10,000 to 30,000	60,000 to 180,000
3	30,000 to 50,000	180,000 to 300,000
4	50,000 to 70,000	300,000 to 420,000
5	70,000 to 90,000	420,000 to 540,000

101. The correct position of live load, for maximum shear in any panel, is found by successive trials. When the correct position is found, the

* "Transactions Am. Soc. Civ. Engrs.," vol. XLII, No. 858. Dec., 1899, p. 227. See Digests of Specifications.

moment about the right support is computed, and from this the shear is obtained. For example, see below.

102. These operations may be performed by computation, with or without the aid of graphic methods. As the method of computation alone is rather tedious, particularly when the form of the truss is complicated by curved chords or sub-panels, and as the graphic method is abundantly accurate for all practical purposes, and has the advantage of direct appeal to the eye, only the latter is given herewith.

103. The "wheel diagram," Fig. 37 (*a*),* gives (1) a stepped "load line" or "shear diagram," and (2) a curved "moment diagram" or "equilibrium polygon." See Statics, ¶¶ 359, etc.

104. The load line gives the total live load to the left of, and including, any point.

105. The moment line gives, at any point, the (left-handed) live load moment, about that point, of all loads to the left of and including that point. Thus, to the left of and including wheel No. 5 we have

Wheel.	Load.	Distance from wheel 5.	Moment about 5 in ft.-lbs.
1	10,000	23	230,000
2	20,000	15	300,000
3	20,000	10	200,000
4	20,000	5	100,000
5	20,000	0	0
Total,	90,000		830,000

and the ordinates, *ab* to the load curve, and *ac* to the moment curve under wheel 5, measure 90.0 and 0.830 respectively.

106. Fig. 38 represents the truss, to the same scale as Fig. 37. We may call this a "truss diagram." †

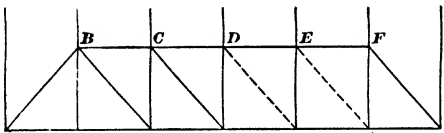

Fig. 38.

107. Example. To compute the maximum shear in the panel *bc*, Fig. 38, first find that position of the load which will produce that maximum shear. As a guess, place the truss diagram, Fig. 38,† with its point *c* under wheel 2, Fig. 37. Examining the load diagram, over the right end, *g*, of the span, we see that we now have a total load, W, of 284,000 lbs. on the span; and the load diagram, over wheel 2 (placed at point *c*) shows (see also table, ¶ 100) that the load, P, on the panel, *b c*, is now somewhere between 10,000 and 30,000 lbs.; but, for maximum shear in the panel, *b c*, the load, P, on that panel must be (see ¶¶ 99 and 100) $= W \div n =$ $284{,}000 \div 6 > 30{,}000$ lbs. Hence, P must be increased by moving the train diagram, Fig. 37, to the left (or, which is the same thing, by moving the truss diagram, Fig. 38, to the right) until wheel 3 is over *c*. We now have W = 292,000 lbs.; P = anywhere between 30,000 and 50,000; and required value of P, for maximum shear, $= W \div n = 292{,}000 \div 6 =$ 48,667 lbs. Hence, the conditions are satisfied, and panel *b c* receives its

* Method published by Ward Baldwin, "Engineering News," vol. XXII Sept. 28, 1889, p. 295. See also letter, "Eng. News," Dec. 28, 1889, p. 615

† For the following discussion it will be found convenient to make a copy of Fig. 38, or simply of the lower chord, on a separate piece of paper which may be applied, in different positions, to Fig. 37.

maximum shear, when wheel 3 is at c. The moment diagram shows, vertically over g, the live load moment about the right support = 17,516,000 ft.-lbs.; and the moment at c = 233,000 ft.-lbs.

108. Let M = the (left-handed) live load moment at the right abutment, due to all the loads on the span

L = the span.

m = the (left-handed) live load moment at the panel point on the right of the panel in question

l = the length of the panel.

V = the shear in the panel.

$$\text{Then } V = \frac{M}{L} - \frac{m}{l} = \frac{17{,}516{,}000}{150} - \frac{230{,}000}{25} = 107{,}600 \text{ lbs}^{*}$$

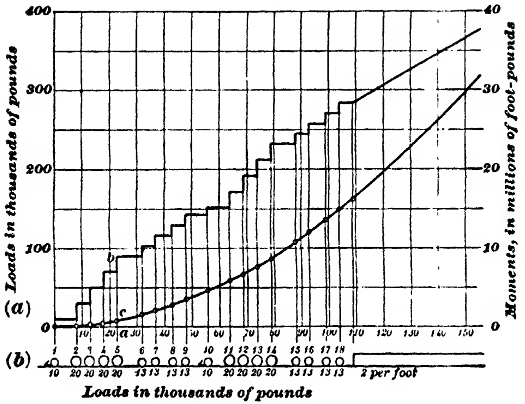

Fig. 37 (repeated).

109. The maximum live load shears in the other panels, similarly computed, are as follows, the load being, in each case, so placed as to give said maximum shear:

Panel, No and Position of Wheel.	Mom M at Rt End of Truss Ft.-lbs.	Mom m at Rt. End of Panel. Ft.-lbs.	Shear, Pounds. $V = \frac{M}{L} - \frac{m}{l}$	Member.	Stress, Pounds. $S = \frac{V}{\cos\theta}$†
ab 4 at *b*	27,176,000	480,000	162,000	*a*B	−217,200
bc 3 at *c*	17,516,000	230,000	107,600	B*c*	+144,200
cd 3 at *d*	10,816,000	230,000	63,000	C*c*	−63,000
				C*d*	+84,500
de 2 at *e*	4,936,000	80,000	29,700	D*d*	−29,700
				D*e*	+39,800
ef 2 at *f*	1,743,000	80,000	8,400	E*f*	+11,250

* Because $\frac{M}{L}$ = the reaction of the left support, a, and $\frac{m}{l}$ = so much of the panel load as goes to the left end of the panel.

† θ = angle between diagonal and vertical.

110. The live load stress in the hip suspender, Bb, is due entirely to loads upon the two lower-chord panels, *a b* and *b c*. Thus, with wheel 4 at *b*, panel length = *a b* – *b c* = 25 ft., we have:

On *a b*				On *b c*			
Wheel.	Load, W.	Dist, *d*, from *a*.	Stress on B*b* = *wd* ÷ 25.	Wheel.	Load, W.	Dist, *d*, from *c*.	Stress on B*b* = *wd* ÷ 25.
1	10,000	7	2,800	5	20,000	20	16,000
2	20,000	15	12,000	6	13,000	11	5,720
3	20,000	20	16,000	7	13,000	6	3,120
Total,	50,000		30,800	Total;	46,000		24,840

Total load on *a b*	50,000	Stress in B *b* from *ab*	= 30,800
" " " *b c* =	46,000	" " " " *b c*	= 24,840
Wheel 4 =	20,000	" " " " wheel 4	= 20,000
Total load on *ac* =	116,000	" " " Total,	75,640

111. For any given set of loads on *a c*, the *maximum* stress in B *b* occurs when the load on *a c* is *equally* divided between *a b* and *b c;* and this ordinarily occurs while some wheel (to be found by trial) is passing *b*. Thus, with wheel 4 just to the *right* of *b*, we have, on *a b*, wheels 1, 2 and 3, = 50,000; and, on *b c*, wheels 4, 5, 6 and 7, = 66,000 lbs.; but, with wheel 4 just to the *left* of *b*, we have, on *a b*, wheels 1, 2, 3 and 4, = 70,000; and, on *b c* (neglecting wheel 8, which now enters *b c*), wheels 5, 6 and 7, = 46,000 lbs. Hence, while wheel 4 is passing *b*, there is an instant when the loads on *a b* and on *b c* are equal, and at that instant the stress in B *b* reaches its maximum (75,640 lbs., see ¶ 110) for the given set of loads.

112. Live Load Chord Stresses. The criterion, for position of load for maximum bending moment in any section, and hence for maximum stress in the chord members at that section, is $\frac{W}{w} = \frac{L}{l}$, or $l = L.\frac{w}{W}$; where W = total load on the truss, w = load to the left of the section, L = span of bridge, and l = length of segment to the left of the section.

113. To find the position of load for maximum moment in any panel, by means of the moment diagram, Fig. 37; place a wheel, say wheel 2, at the panel point at the right of the given panel. From the intersection on the load line (usually coinciding with the x-axis) vertically over the left support, lay a ruler or stretch a thread to the intersection of the load line with a vertical from the right support. If the line so constructed recrosses the load line at a point vertically over the section in question, the position is a correct one; if not, it is incorrect. To facilitate this work, it is well to use a truss diagram, Fig. 38, drawn on a sheet of tracing paper, with the verticals carefully extended from the panel points as far up as the load or moment lines are likely to extend.

114. It will often be found that more than one position satisfies the criterion, and that some one of these may give greater moments than the others Hence it is well to look for all possible positions. When these are found, determine the moments, thus: On the moment curve find the two points corresponding (vertically) with the left and the right support respectively, and join these points by a straight line. When the head of train has not reached the left support, the point corresponding with the left support is in the x-axis, produced.

115. The required moment is measured by the vertical ordinate distance along the section, between the moment curve and the straight line just constructed. The stress in the chord members affected is equal to the moment divided by the depth or the truss. Using these methods, the following results are obtained:

Section.	Wheel.	Moment, ft.-lbs.	Stress, lbs.	Members.
B*b*	4	4,049,333	144,600	*ab* = *bc*
C*c*	7	6,211,667	221,800	*cd* = —BC
C*c*	8	6,207,667	Not max.	
D*d*	11	7,044,000	Not max.	
D*d*	12	7,056,500	252,000.	—CD = —DE

Wind Loads.

116. A complete bridge is subjected not only to *vertical* loads, due to dead load, to live load, and to impact caused by inequalities in track and in rolling stock, but also to *horizontal* loads. These horizontal loads are due to the transverse action of wind, or of centrifugal forces produced by the train in passing around a curve on the bridge, and to the longitudinal traction or "drag" caused by stopping or starting a train on the bridge. Hence it is necessary to supply horizontal bracing, which, with the two upper chords or the two lower chords of the two vertical trusses, form horizontal trusses, known as the upper and lower lateral systems, Figs 39 (*a*) and 39 (*b*), and sway and portal bracing, ¶¶ 21 and 22.

117. The wind is considered as blowing at right angles to the bridge.

118. The wind produces several effects, and these must be ascertained separately, and their joint effect then determined. Among these effects are:

(1) Direct stresses in both the upper and lower lateral systems, by pressing directly upon the chords; acting horizontally as a uniformly distributed load.

(2) Additional direct stresses on the lateral system of the loaded chord when a train is on the bridge, owing to pressure of wind against the train.

(3) An overturning moment upon the bridge as a whole, thus increasing the dead and live load stresses in the leeward and diminishing those in the windward vertical truss.

(4) A similar overturning effect upon the train and its wheels, which similarly modifies their pressures upon the floor beams and thus the stresses in the main trusses.

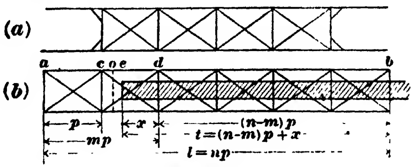

Fig. 39.

119. The wind load, acting directly upon the bridge, is assumed to be equally divided between the upper and lower chords, and between the windward and leeward trusses

120. (1) The direct wind stresses in the lateral bracing, due to the pressure of the wind on the truss, are found as are the stresses in the main trusses, due to dead load; the horizontal transverse struts of the lateral bracing corresponding to the verticals of the main trusses.

121. (2) Direct stresses in lateral system of loaded chords, Fig. 39 (*b*), due to wind on train.

Examining any panel, as $c\,d$, let

w = wind pressure, in lbs. per lineal foot of train;
p = panel length, in feet;
$w\,p$ = wind pressure, in lbs., per panel fully occupied by train;
n = number of panels in span (= 6 in this case);
l = $n\,p$ = span, in feet;
m = number of panels from left support, a, to and including the panel, $c\,d$, under consideration;
$m\,p$ = distance, $a\,d$, in feet;
x = length, in feet, of that portion of the panel, $c\,d$, which is occupied by train;
t = $(n - m)\,p + x$ = that portion of the span which is occupied by train, in feet;
= wind pressure on train for a pressure of 1 lb. per lineal foot;
R = truss wind reaction,* at a, = $w\,t^2 \div 2\,l$;
r = panel wind reaction,* at c, = $w\,x^2 \div 2\,p$;
S = wind shear * in panel, $c\,d$, = $R - r = w\,t^2 \div 2\,l - w\,x^2 \div 2\,p$.

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* See foot-note (†), ¶ 2.

The horizontal *truss* reaction,* at a, due to a concentrated horizontal pressure, $= 1$, acting at any distance, y (not shown), from d, is $= \frac{(n-m)\,p+y}{n\,p}$; and the horizontal *panel* reaction, at c, due to the same pressure, is $= \frac{y}{p}$. The maximum wind shear,* in the panel, $c\,d$, due to wind on train, occurs (see ¶¶ 79 to 81) when the head of the train reaches that point, o, at which, if the concentrated load be placed, these two reactions will be equal, or $\frac{y}{p} = \frac{(n-m)\,p+y}{n\,p}$. With head of train at o, we have $x = y = \frac{(n-m)\,p}{n-1}$.

Under any conditions, the wind shear,* S, in the panel, is $= R - r$, where $R = \frac{w[(n-m)\,p+x]^2}{2\,n\,p}$; and $r = \frac{x^2}{2\,p}$.

Substituting here the value of x, just found, for maximum shear, we obtain, as the maximum value of the wind shear* in the panel,

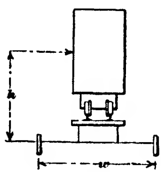
$$S_{max} = \frac{w\,p}{2\,(n-1)}\,(n-m)^2.$$

122. (3) Stresses in main truss members, due to overturning moment of wind on truss.

Overturning moment = (wind panel load at top chord) × (number of panel points in span) × height of truss.

$$\text{Vertical reaction at one support} = \frac{1}{2}\,\frac{\text{overturning moment}}{\text{width between trusses}}.$$

Since the upper lateral system carries all wind loads to the ends of the bridge, the end posts and the chords (which take the horizontal components of the end post thrusts) are the only main truss members affected.



(a)

Fig. 40.

123. (4) Stress in main truss members, due to overturning moment of wind on train, Fig. 40. Let h = height from center of gravity of lateral system of loaded chord to center of pressure of wind on train, p = wind pressure per lineal foot of train, w = width between centers of gravity of trusses, m = overturning moment, per lineal foot of train, v = added vertical load on leeward truss, per lineal foot of truss. Then $m = h\,p$, and $v = \frac{m}{w}$.

Impact, Etc.

124. The effects of impact, due to inequalities of the track; those of "drag," due to the starting and stopping of trains; and those of centrifugal force of the train on curves, are not susceptible of rigorous calculation, and engineers differ in their requirements respecting provision for them See Digests of Specifications.

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* See foot-note (†), ¶ 2.

Determination of Maximum and Minimum Stresses.

125. Where specifications make allowable unit stresses depend upon the relation between the maximum and minimum stresses in any given member, both must be computed

In computing the maximum and the minimum stress in any member, bear in mind that a condition which, of itself, would have a certain effect upon the stress, may bring with it other conditions which produce a greater effect of the opposite kind. Thus, although the action of wind on train would, of itself, reduce the stresses in certain members, this action can take place only with train on bridge, and the vertical action of the train load would ordinarily increase those stresses more than the wind action would diminish them.

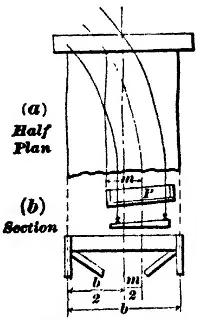
In computing minimum stresses, although the live load is usually to be neglected, we must of course not neglect the dead load, which is always present.

Curves on Bridges.

126. When the track on a bridge is curved, it is usually so laid that the center line of the bridge bisects the middle ordinate, m, of the curve. See Fig. 40 (a). The center of gravity of a panel load, P, at the center of the span (supposing it to stand over the center of the track) is thus thrown out a distance $= \frac{1}{2}\,m$ from the center line of the bridge, or a distance $= \frac{1}{2}\,b + \frac{1}{2}\,m$ from the inner truss, where b = width of bridge between centers of trusses. Taking moments about the center of the inner truss, we have, therefore, for the load, W, on the outer truss, due to P,

$$W = P\,\frac{\frac{1}{2}\,b + \frac{1}{2}\,m}{b} = P\,\frac{b + m}{2\,b}.$$

It is customary (see Digests of Specifications) to proportion the outer truss on the safe assumption that its share of the live load, at each panel point, is determined by the formula just given, and to design the inner truss like the outer one.



Figs. 40 (a) and (b).

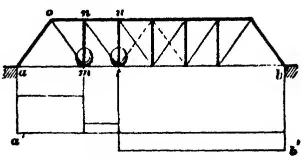


Fig. 31 (repeated).

Counterbracing.

127. In a truss of any ordinary form (like that in Fig. 31), under the action of a uniformly distributed dead or live load, or of a live load distributed symmetrically as regards the center of the span, the shears in each panel on the *left* of the center of the span are *positive*, while those in the panels on the right are *negative;* and the stresses throughout the truss are such that the ties sustain tension, and the struts, compression; the tendency, in each panel, being to *elongate* that diagonal occupied by a *tie*, and to *shorten* that diagonal occupied by a *strut*.

But the tendency of an *eccentric* load, such as those shown in Fig. 31, is to *reverse* the shears in the panels between it and the center; and, if this effort, relatively to the other forces is of sufficient magnitude to reverse the

final shear in any panel, the tendency will be to *shorten* the diagonal occupied by a *tie*, see Fig. 1 (*a*), and to *lengthen* any diagonal occupied by a *strut*.

As explained in ¶¶ 14 and 15, this condition is met, in the Warren or triangular truss, by making each web member capable of resisting both tension and compression; and, in trusses with both vertical and diagonal web members, by inserting counters.

In a drawbridge or swing bridge, not only the web stresses, but also the chord stresses, are reversed when the draw is opened or closed.

To provide against possible further increase in live loads, over those now in use, specifications sometimes require that, wherever the live and dead load stresses are of opposite character, only 70 per cent. of the dead load stress shall be considered as effective in counteracting the live load stress. For other methods of making similar provision, see Digest of Specifications for Steel Railroad Bridges.

Roof Trusses.

128. In roof trusses, **the dead load,** *i. e.*, the weight of the truss itself and that of the purlins, roof covering, etc., and the snow load, are usually taken as uniformly distributed. In many cases the sum of the dead and snow loads is divided equally between the two supports. In other words, the end reactions are equal.

129. The weights of steel trusses, in pounds per square foot of building space covered, may be taken, for preliminary estimate, at (0.05 to 0.08) × span in feet, according to design and loading. Those of **wooden trusses,** with wooden, iron or steel tension members, may be taken at from one-tenth to one-fifth less.

If it is found that the weight of a truss, as designed, considerably exceeds the weight assumed for it in advance, it should be redesigned, assuming a new weight slightly greater than that obtained from the design.

130. The weights of purlins, of steel or wood, may be taken at from 2 to 3 lbs. per square foot of building space covered.

131. The weight of roof covering may be taken approximately as follows:

Corrugated iron,	2 to 3	lbs. per sq. ft. of roof surface.
Slate,	7 to 9	" " " "
Shingles, on laths,	2 to 3	" " " "
If on boards, add	3	" " " "
If plastered below the rafters, add,	6	" " " "

132. The snow load, in States north of lat. 35°, may be taken as varying (chiefly with latitude) from 10 to 30 lbs. per sq. ft. of horizontal projection of roof surface.

133. The purlins, stringers, etc., should be so arranged as to carry the weight of roof covering and of snow directly to the panel points, and thus avoid transverse stresses in the rafters.

134. Each truss bears, besides its own weight, half the weight of roof and snow between the two trusses (or truss and wall), adjacent to it, and each panel point bears half the load between two panel points (or panel point and end support) adjacent to it.

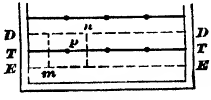


Fig. 41.

Thus, Fig. 41, truss TT carries a weight = that on the surface between the two dotted lines, DD and EE; and panel point *p* carries a weight = that on the rectangle *mn*.

135. The wind is regarded as blowing horizontally upon one side of the roof and as exerting a uniformly distributed normal pressure upon that side. In the following table of assumed normal pressures against sloping surfaces, under horizontal wind pressures of 40 lbs. per sq. ft., the values in the last column are based upon Hutton's experiments. Here *a* is the angle between the sloping roof surface and a horizontal plane.

Assumed normal wind pressure, P, in lbs. per sq. ft. Horizontal wind pressure = 40 lbs. per sq. ft. a = angle between roof surface and horizontal plane.

a.	sin a.	P 40.sin a.	P Hutton.	a.	sin a.	P 40 sin a.	P Hutton.
5°	0.087	3.5	5.1	35°	0.574	22.9	30 1
10°	0.174	7.0	9.6	40°	0.643	25 7	33.3
15°	0 259	10.4	14.2	45°	0.707	28.3	36.0
20°	0.342	13.7	18.4	50°	0.766	30.6	38 1
25°	0.423	16.9	22.6	55°	0 819	32.8	39 4
30°	0.500	20.0	26.5	60°	0 866	34.6	40 0

136. The directions and amounts of the end reactions and of the stresses in the members, due to wind, depend upon whether one or both supports are fixed. If both ends are fixed, their reactions are parallel to the normal wind pressure—*i. e.*, they are at right angles to that side of the roof upon which the wind is blowing; but, if one end is free to slide longitudinally of the truss, its reaction is taken as vertical and that of the other is more nearly horizontal than the normal wind pressure. When one end is free, the stresses must be determined for wind blowing on the fixed side (in which case it tends to flatten the roof) and also for wind blowing on the free side, in which case its horizontal component tends to shorten the tie-rod and to raise the apex. The stresses in the members of roof trusses are conveniently found by means of the method by sections, ¶¶ 67, etc., or graphically, as below.

137. Fig. 42 (a) illustrates the graphic treatment of wind stresses for Fig. 42 (b), under the three conditions named, viz :—case 1, with both ends fixed; case 2, wind blowing against the fixed side; and case 3, wind blowing against the free side *

In Fig. 42 (a), the segments ab, bc, cd and de represent the normal wind pressures at the panel points AB, BC, CD and DE respectively, and ae therefore represents the total normal wind pressure on the roof, all being exerted against the left side.*

138. In case 1 (both ends fixed) the segments fa and ef of the solid line ea represent the left and right reactions respectively.

139. In case 2 (wind blowing against fixed side) the reactions are represented by the dash line $ef'a$; and in case 3 (wind blowing against free side) by the dash-and-dot line $ef''a$.

140. The segments $f'f$ and ff'' represent the horizontal components of the right and left wind reactions respectively in case 1; and $f'f''$ that of the wind reaction of the fixed end in case 2 or case 3, or that of the total wind reaction.

141. Having found, by moments, the end reactions ef and fa for case 1; where, Fig. 42 (b),

$$ef = ae \frac{A\,s}{\text{Span}};$$

the vertical reactions, ef' and af'', for cases 2 and 3 respectively, are found by dropping perpendiculars from e and from a, Fig. 42 (a), upon gf produced. The reactions of the fixed ends are then given by the closing line, $f'a$, in case 2, and by ef'' in case 3.

* To avoid the necessity of showing two skeleton figures and two diagrams, we have supposed the wind to blow always in one direction (viz , against the left side) and first one end of the truss and then the other end to be fixed. In practice, of course, the reverse of this is the case; *i. e.*, one end or the other of the truss (if not both) is fixed, and remains so; and the wind may blow against either side. The figure and diagram will, however, answer for this latter condition also. Thus, if the wind blow against the left side, as shown, and if, as in case 2, that side is fixed, then the diagram, using the broken lines, $e\,f'\,a$, gives the stresses in the members, as they are lettered. But now (the left end remaining fixed) suppose the wind to blow against the free side; *i. e*, from the right. We may nevertheless suppose the right end fixed, and the wind blowing against the left side, as in Fig. (b), and find the stresses in the members from Fig. (a) as it stands, using the dash-and-dot diagram, $e\,f''\,a$; but we must then remember that the stresses thus found for BG, GF, etc., on the left of the truss, Fig. (b), really apply to the corresponding members, QE, QF, etc., on the right, and vice versa

142. The stresses in the web members, GH, MN, etc., Fig. (*b*), and those in the several members, BG, EM, etc., of the rafters, are given by the corresponding lines, *gh*, *mn*, *bg*, *em*, etc., in the diagram, Fig. (*a*).

143. In the leeward rafter, in this case, the stress in the three segments, ME, PE, QE, is uniform throughout, and moreover it is the same in each of the three cases, being = *me* = *pe* = *qe*.

In the four web members, LM, MN, NP, PQ, to the leeward of the center the stress, in this case, is zero, being represented by the point, *lmnpq*, Fig (*a*).

144. The stresses in the several segments, GF, JF, LF, NF, and QF, of the horizontal tie rod, Fig. (*b*), are represented, in Fig (*a*),

in case 1 (both ends fixed) by *gf*, *jf*, *lf*, *nf* and *qf*;
in case 2 (wind against fixed side) by *gf'*, *jf'*, etc.;
in case 3 (wind against free side) by *gf"*, *jf"*, etc

In each of the three cases there is uniform tension in the three leewar

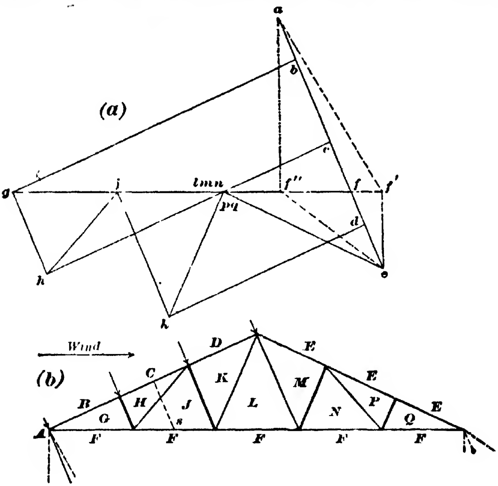


Fig. 42.

segments, LF, NF and QF, of the horizontal tie rod. This uniform tension is

in case 1 (both ends fixed) = *lf* = *mf* = *nf*;
in case 2 (wind against fixed side) = *lf'* = *mf'* = *nf'*;
in case 3 (wind against free side) = *lf"* = *mf"* = *nf"*.

145. It is thus seen that, in our Fig., with horizontal tie rod, the difference in the manner of supporting the ends affects only the horizontal stresses in the members of that rod, and, through them, the manner in which the horizontal component *f'f"* of the wind stress is distributed between the two supports.

146. If the lower chord were not straight, however, the stresses in the rafter and web members would be affected by the difference between the three cases.

147. The final or resultant stress, in any member, is the algebraic sum of the dead, snow and wind loads for that member. In some cases, the wind load may diminish or even reverse the stresses due to dead and snow loads.

148. In timber roof trusses of short span, Figs. 43 to 47, for roofs of dwellings and other small buildings, we may, with sufficient accuracy, make a liberal assumption for load, to include wind pressure. In discussing these figures, we investigate the stresses by means of the force parallelogram. For dimensions of such trusses, see ¶ 266.

149. In the wooden roof truss, Fig. 43, uniformly loaded along each rafter, let H I = the weight of one rafter and its load. Then E I = the horizontal pressure of the head and of the foot of that rafter (the latter being the tension in the chord), and H E = the inclined pressure at its foot.

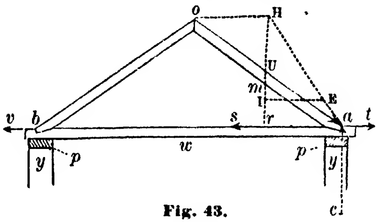


Fig. 43.

150. In Fig 44, make G R = H I. Then G L is the transverse pressure of the load against the rafter as a beam, and L R is a longitudinal pressure along the rafter, forming a *part* of the *total* longitudinal pressure

151. If G R were concentrated at G, L R would be uniform from G to *a*, and would not be exerted above G; but, as G R represents a load uniformly distributed along the rafter, from top to foot, the pressure represented by L R increases uniformly, from nothing, at the top, *o*, to L R, at the foot, *a*.

152. Of the transverse pressure, G L, one half, = *o p*, is sustained at the top, *o*, of the rafter, and the other half, = *a q*, at the foot, *a*. At the top, *o p* is resolved into the horizontal pressure, *o b* = E I, against the head of the other rafter, and a uniform thrust, *o z*, along *o a*.

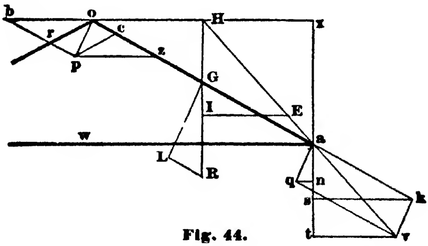


Fig. 44.

153. It is immaterial whether we thus resolve *o p directly* into *o b* and *o z* (as though the head of the rafter rested against a *vertical wall* at *o*), or whether we first resolve it between the two rafters, into *o c* and *o r*. For in the latter case we must add to *o c* a thrust (= *o r* = *c z*) produced in *o a* by the transverse pressure (similar to *o p*) of the head of the *other* rafter; and the sum of these two (*o c* and *o r*) is = *o z*.

154. The total longitudinal thrust in the rafter increases uniformly from *o z*, at the top, to *o z* + L R = *a k*, at the foot, where it combines with *a q* (= half the transverse load) to form *a v* = H E.

155. Tension in chord = I E = *t v* = *s k* — *n q*. Vertical pressure on support = H I = *a t* = *a s* + *a n* = *a s* + *s t*.

156. In Fig. 45, having found, as for Fig 44, the stresses, etc., due to the rafters and their loads, remember that the king rod, $o\ n$, supports its own weight plus the portion $y\ y$ of the chord = ½ the chord. Making $o\ t$ = this combined weight, we have $o\ m = o\ d$ = an additional pressure, uniform throughout each rafter, and $c\ m = c\ d$ = an additional tension on the chord.

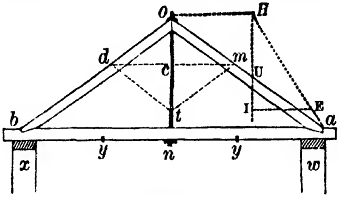


Fig. 45.

157. In Fig. 46, assuming, for safety, that the rafter, $j\ b$, is divided at its center, U, make $e\ o$ = the weight of $z\ r$ and its load ($z\ r$ = half the rafter).

158. Then $e\ i$ = an additional pressure on U b, $e\ k$ = pressure on U c; $s\ i - s\ k$ = additional tension on half chord, $c\ b$, and $e\ o = 2\ e\ s$ = load of and on $z\ r$, = additional tension on king post due to both struts. Then make $a\ g = e\ o$ + weight of king rod + weight of two struts + weight of and on $y\ y$, and proceed as in Fig 45.

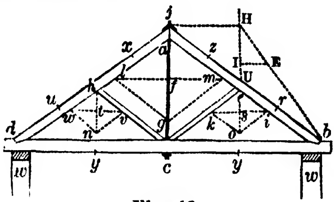


Fig. 46.

159. Each strut will thus bear *half* of the weight of and on $z\ r$, or $x\ u$, only when, as in Fig. 46, the inclination of the strut is the same as that of the rafter. If the strut is steeper than the rafter, it will bear more than half; but if it is less steep than the rafter, it will bear less than half; the remainder being in every case borne by the rafter.

160. The introduction of the struts converts each rafter, considered as a beam, into two beams of shorter span and bearing less loads.

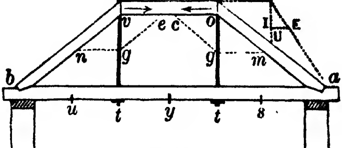


Fig. 47.

161. In the queen truss, Fig. 47, make $o\ g$ = total tension in queen rod + half weight of and on the "straining beam," $z\ w$. Longitudinal pressure on $z\ w$ = tension in chord, $b\ a$, = I E + $o\ c$.

Deflections.

162. The total deflection of a truss * comprises (1) the elastic or temporary deflection, due to the stretch† of its several members under the loading applied to the truss, and (2) the non-elastic or permanent deflection, due to looseness of its joints. In good construction the latter is relatively negligible in moderate spans.

The total elastic deflection, D, of a truss, at any point, *c*, is made up of partial elastic deflections, *d*, *d*, etc., at *c*, each due to the stretch, *k*,† in some member.

Let it be required to find the deflection at a panel point, *c* (usually the center of a span or the end of the arm of a swing bridge or other cantilever); and, for any load or system of loads, let

D = the total elastic deflection at *c*;
d = the partial elastic deflection, at *c*, due to the stretch, *k*,† in any member;
p = the unit stress in that member;
P = the total stress in that member;
l = the length of that member;
k = the stretch † in that member $= \frac{p\,l}{E}$;
W = that load which, applied at *c*, would produce the stress, P, in that member;
$u = \frac{P}{W}$;
E = the modulus of elasticity of the material, $= p \div \frac{k}{l} = \frac{p\,l}{k}$.

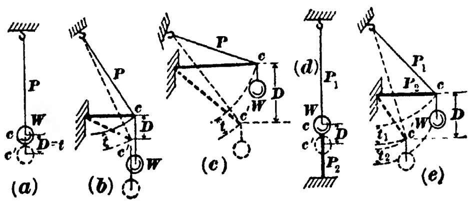


Fig. 48.

163. Equivalence of Work. In Fig. 48, let any load, W, be applied at any point, *c*, of a truss or bar. Then, for a small deflection,‡ such as may

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* See "The Application of the Principle of Virtual Velocities to the Determination of the Deflection and Stresses of Frames," by Geo. F. Swain, Jour. of the Franklin Institute, vol. LXXXV, 1883; "Trusses with Superfluous Members," by Wm. Cain, Van Nostrand's Magazine, vol. XXVII, No. 4, October, 1882; "The Graphical Solution of the Distortion of a Framed Structure," by David Molitor, Jour. Ass'n Eng'ng Societies, vol. XIII, No. 6, June, 1894; and "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan and F. E. Turneaure, New York, John Wiley & Sons.

† For brevity we here use the word "stretch" to signify any change of length, including the shortening due to compression, as well as the elongation due to tension.

‡ For the sake of clearness, the stretches and deflections, in our Figs., are exaggerated beyond the limit within which the ratio, $\frac{P}{W}$, would remain even approximately constant.

be permitted in trusses, the external work, W d,* of a partial deflection, d, due to the stretch, k, in any member, is practically = the internal work, P k, of overcoming the resisting stress, P, in that member, through the distance, k; or

$$W\ d = P\ k.$$

Hence,

$$d = \frac{P}{W}\ k = u\ k; \quad \text{or} \quad \frac{k}{d} = \frac{W}{P}.$$

In words, the stretch, k, in any one member, is, to the resulting partial deflection, d, at c, inversely as is any stress, P, in that member, to a load, W, which, if applied at c, would cause that stress.

Thus, in Fig. 48 (a), where k is in the same direction as D,† P = W, and D = k.

In Figs. (b) and (c),‡ suppose the strut incompressible. Then D is due solely to the elongation of the tie, and $D = \frac{P}{W}\ k = u\ k$.

In Fig. (c), $\frac{P}{W}$ is greater, and (for a given stretch, k, in the tie) D is therefore greater, than in Fig. (b).

164. Deflection Independent of Nature of Cause of Stretch. Now it is evident that the deflection, d, at c, depends solely upon the amount and character of the stretch, k, in the member, and is independent of the nature of the cause of that stretch. That is to say, any change, k (however caused), in the length of the member, necessarily contributes its fixed quota, $d = \frac{P}{W}\ k$, to the total deflection, D, at c. In other words, since d and k are mere distances, and since u is simply a ratio, the relation between d and k is a purely geometrical one, and is therefore not confined to deformations produced by applied loads, but is applicable also to those produced by changes of temperature, to intentional lengthening or shortening of members, or to any other cause

Hence, if a member be *in any way* lengthened or shortened, by a length, k, a corresponding change, $d, = \frac{P}{W} . k = u\ k$, takes place in the deflection at c.

For instance, if we place any system of loads upon a truss, and, by the principles of statics, determine the resulting total stress, P, and unit stress p, in any member; we have, for the partial deflection, at c, due to the stretch, k, in that member, under the given system of loads,

$$d = u\ k; \qquad \text{(For } u\text{, see ¶ 165.)}$$

and, since $k = \frac{p\ l}{E}$,

$$d = \frac{p\ u\ l}{E}.$$

165. To obtain the ratio, $u, = \frac{P}{W}$, for each member, we suppose a concentrated load applied at c; and, by the principles of statics, find the resulting total stresses, P, P, etc., in the several members. If the supposed load, at c, be taken = unity, the stresses, P, P, etc., so found, are the desired ratios, u, u, etc.

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* Strictly speaking, with a load increasing gradually from 0 to W, and with resulting stress increasing gradually from 0 to P, we should deal with the *mean* load, = ½ W, and with the *mean* stress, = ½ P, in each member; but it will be seen that this would not affect the equations derived.

† Where, as in Figs (a), (b) and (c), only one member is supposed to change its length, D = d.

‡ See note (‡) on preceding page.

166. Summation of Deflections. The total deflection, D, at c, under the given system of loads, being = the sum of the partial deflections, d, d, due (but not necessarily equal) respectively to the stretches, k, k, in the several members, we have

$$D = \Sigma\, d = \Sigma \frac{p\, u\, l}{E}.$$

Thus, in Figs. (d) and (e), we assume the tie extensible and the strut compressible. In Fig. (d), $W = P_1 + P_2$; and $W\,D = P_1\,k + P_2\,k = (P_1 + P_2)\,k$. Hence $D = \Sigma\, u\, k = \frac{P_1}{W} \cdot k + \frac{P_2}{W} \cdot k = \frac{P_1 + P_2}{W} \cdot k = \frac{W}{W} \cdot k = k$. In Fig. ($e$), $D = \Sigma\, u\, k = \frac{P_1}{W} \cdot k_1 + \frac{P_2}{W} \cdot k_2 \quad u_1\, k_1 + u_2\, k_2$.

167. Positive and Negative Stretches. In some cases it may happen that the change of length of a member diminishes, instead of increasing, the total deflection at the point, c, in question, and must therefore be taken as negative in summing up the values of $u\,k = \frac{p\,u\,l}{E}$; but when c is the middle point of a span, or the end of a cantilever, all the changes in length of the members ordinarily contribute to the deflection, and must therefore be taken as positive.

Theoretically, the formula, $D = \Sigma \frac{p\,u\,l}{E}$, applies also to the deflections of arches, dams and other structures composed of blocks; but, owing to the uncertainty of the values of E, and to the relative inaccuracy of finish in masonry work, it is of but little practical utility in such cases.

168. Redundant or Statically Indeterminate Members. Trusses frequently contain members whose stresses cannot be found by the principles of statics. Thus, in Fig. 11 (c), the two diagonal tension members meeting at the top of either end post are said to be redundant, or statically indeterminate, because the principles of statics do not enable us to determine what proportion of the total load goes to the supports through each of the two systems, Figs. 11 (a) and (b), composing Fig. 11 (c). But the deflection formula, just given, enables us to determine the stresses in such members; for, by means of it, we may find, separately, the deflection in each of the two systems, Figs. 11 (a) and (b); and the part load, transmitted to the supports through each of these two systems, is inversely proportional to their deflections.

BRIDGE DETAILS AND CONSTRUCTION.

General Principles.

169. In general, a truss **bridge** consists essentially of two or more vertical **trusses**, AB, CD, Fig. 49, placed side by side, and connected by the **floor system**, which, in turn, they support; and bracing (forming a "**lateral system**") is supplied between opposite chords, where practicable, in order to maintain the trusses parallel.

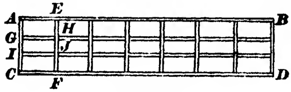

Fig. 49.

170. The **floor system** consists ordinarily of **floor beams** and **stringers.** The floor beams, AC, EF, etc., Fig. 49, are placed transversely to the bridge, and are attached to the trusses at opposite panel points. Connected with these and perpendicular to them or parallel to the trusses, are the stringers, GH, IJ, etc. In railroad bridges, there are usually two or more stringers placed side by side and running the length of the bridge, to support the ties. In city highway bridges, these stringers are usually spaced at smaller intervals, and support buckled plates or other form of flooring, on which the

paving is laid. For country highway bridges, the stringers are frequently of wood, placed quite near together, and the planks of the floor are nailed or spiked directly to them.

Solid floors (see Pencoyd floor sections) add to the rigidity and permanence of a bridge, and give increased protection to traffic below, against injury from falling bodies or in case of derailment. Their shallowness is an advantage where head-room is an object.

171. Any load, then, **is carried** first from the ties or floor, etc., to the stringers, then by the stringers to the floor beams, and finally by the floor beams to the panel points of the bridge, where it is carried through the trusses to the supports.

172. Pedestals, shoes or bed plates, Fig. 62, bolted to the piers, support the ends of the trusses. When the bridge is of long span, so that the expansion and contraction due to heat and cold are considerable, **expansion bearings,** Figs. 60, 62, must be provided at one end. See ¶¶ 205, etc. For cross-bracing, see ¶¶ 19, etc.

General Character of Design.

173. Flexible and Rigid Tension Members. Adjustable Counters. Until recently, **eye-bars** have generally been used for the tension members of trusses. These are long flat bars, liable to yield laterally under compression, and furnished, at their ends, with eyes or openings, through which pass pins connecting them with the other members of the bridge; but rigid built members, capable of sustaining some compression, as well as tension, are now much used for tension members. **Counters** were usually made in two lengths, and were **adjustable,** the two lengths being connected by turnbuckles; but these rendered it possible to bring undue and dangerous stresses in the panels, and they are now giving place to counters made each in one length.

174. Compression members are ordinarily **"built up"** of angles and plates, or of channels and plates with latticing, in hollow shapes, bringing most of the material as far as possible from the neutral axes of the cross-section and thus increasing its resisting moment.

175. Pin and Riveted Connections. The web members are connected with the chords either by pins or by rivets. In the former case the truss is said to be pin-connected; in the latter case, riveted.* Until recently, pin-connected trusses have been typical of American practice; but the Americans are now largely using riveted trusses, for spans up to from 150 to 175 ft., while the Europeans are in some cases using pins. The principal advantage claimed for the riveted joint is that it makes a stiffer bridge and one that will not rattle, and that a riveted truss, computed as if pin-connected, will have an additional margin of safety on account of added stiffness. In the pin-connected bridge, on the other hand, the stresses can be much more accurately determined, and deflection may take place without producing twisting or bending stresses in the connections themselves.

176. Tendency to Greater Rigidity. There is a growing tendency to use stiffer bracing, to design at least all short braces for compression, and to make even the longer tension members of channels or angles, forming a rigid member. Unless pin-connected eye-bars are of exactly equal length, some of them will receive more than their share of the total stress.

177. Floor-beam Connections. In the United States, floor-beam connections were formerly made by hanging the floor beams from the pins by means of hangers; but now, where possible, the ends of the floor beams are riveted directly to the inner sides of the posts.

178. In tension members, rivets are so arranged as to reduce the net effective section as little as possible.

179. Compression members are so designed as to place most of the material as far as possible from their neutral axes, and they are sometimes strengthened by auxiliary ties or posts supporting them at their middle points, in cases where the resulting saving in material for the member will be considerably greater than the expenditure of material in the auxiliary member.

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* Riveted trusses are unfortunately called, also, "lattice girders," "lattice trusses," "riveted lattice girders" and "riveted lattice trusses." The term "lattice" is often applied to shallow trusses with numerous panels.

180. So far as possible, compression members are made equally strong against bending about either of the two principal axes, AB and XY, Fig. 52, of their cross-sections.

181. Where the same member occurs many times in a bridge, and where, therefore, an excess of material in the design of such member would involve a large total waste, the computation of the member is repeated many times until the most economical section is found.

182. In metal trusses the shorter members are usually made to withstand compression, and the longer ones tension, this being more economical of material. Thus, the Pratt truss, with diagonal tension members, is used for steel bridges, while the Howe truss is now built only with wooden diagonals.

Tension Members.

183. In eye-bars, area of cross-section $= \dfrac{\text{maximum tensile stress}}{\text{allowable unit tension}}$.

184. The dimensions of the heads of eye-bars are usually determined by the manufacturers, and are so designed as to give ample excess of strength at the pin-holes, so that, if tested to destruction, fully two-thirds of the number of bars tested shall break in the body of the bar, this being usually required by specifications. It is important that the proportions of eye-bar heads should be such as to ensure thorough working of the metal in the upsetting process.

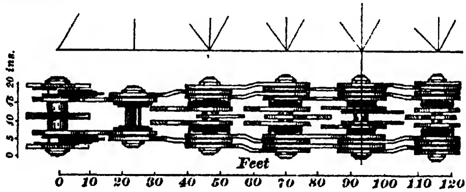


Fig. 50.

185. Fig. 50 shows, to two different scales, the "packing" (arrangement of pins and eye-bars) in the left half of the lower chord of a 150 ft. through (skew) span built by the Phœnix Bridge Co. in 1900 for the Philadelphia and Reading Railway Co. near Reynolds, Pa.

186. Built Sections. Hip vertical hangers, non-adjustable counters and their corresponding mains, are usually built up of rolled steel shapes. A section in common use shown in Fig. 51, consists of four angles, connected, at intervals, by small narrow flat bars, riveted to the angles and running across zigzag from one to the other. When single, as in Fig 51, this is called "lacing"; when double, as in Fig. 52 (*b*), "latticing." The shaded area of the angles, Fig. 51, minus that of the rivet holes, is taken as the effective section.

187. Minimum Sections. Specifications (see Digests) usually require the use of some minimum section. Thus, in a counter in which the stress is 58,000 lbs., 3.5 square inches of cross-section would suffice; but specifications frequently forbid the use, in such sections, of any angle smaller than $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$, which gives 9.20 square inches gross; or, deducting one rivet hole from each angle, 7.68 square inches net section.

Compression Members.

188. The computation of a compression member consists of a series of approximations; for the unit stress depends upon the radius of gyration, the radius of gyration on the area of section and disposal of material with regard to the axes, and this, in turn, on the unit stress. See Columns, under Strength of Materials.

189. Fig. 52 (*a*) shows a form often used for posts, and consisting of **two channels,** placed with their backs outward and riveted together by lacing. In Fig. 52 (*b*) the channels are placed with their backs inward. For economy, the channels should be so spaced as to make the radius of gyration the

same about either axis, A–B or X–Y. The radius of gyration is given in the hand-books of mfrs of structural shapes. See pp. 1174, etc.

190. The upper chord section is frequently **built up** of two channels and a plate, or in some such form as shown in Fig 53, consisting of two vertical plates or "webs," a horizontal top plate or "cover," four "angles," and flat pieces or bars on each side of the bottom Lattice bracing, or lacing, is provided along the bottom, except at panel points, where it is omitted in order that the post and the ties may enter the chord from below. In pin-connected trusses the axis of the pin lies in the line AB.

191. The interior width, *w*, depends chiefly on the space required by the post and ties which meet at the panel point, and also upon the height of the inside rivet heads. Usually, for convenience of construction, the greatest width, *w*, required is kept constant throughout the upper chord. The height, H, depends chiefly on the size of eye-bar head, and is kept constant. The thicknesses of the web plates, and sometimes also those of the bars and angles, are varied, along the chord, in order to provide, at each point, sufficient area to withstand the stress.

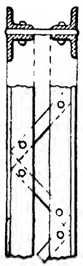


Fig. 51.

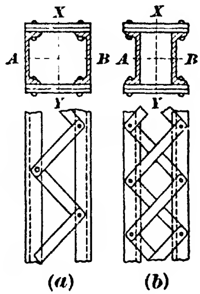


Fig. 52.

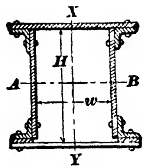


Fig. 53.

192. The **end post** is to be considered not only as a column, but also as a beam subject to shear, on account of the wind blowing against the top of the side of the truss. The design of this built-up form is much the same in principle as that given above for a post. Certain sections are tried, and then changed if necessary.

193. The end post must be safe, not only against bending about the axis, A–B, Fig 53, under compression, but also against bending about the other axis, X–Y, under the combined effect of compression and the bending moment, due to the wind blowing against the top of the truss. See Fig. 54 (*a*).

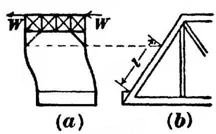


Fig. 54.

194. The portal bracing, above, and the floor beam, below, are assumed to prevent bending of the parts of the posts adjacent to them; and we may consider either half of either post ($= \frac{1}{2}\, l$) as a vertical cantilever, fixed at one end, and loaded, at the other end (which is at the middle of the post) with a load = wind pressure on half the upper portion of one truss.

195. The maximum stresses, due to compression, of course occur about the middle of the post, while those due to the wind occur near the ends. Hence it would be unreasonable to require the post to resist all of both effects simultaneously throughout its length; and specifications therefore usually allow the unit stress, due to dead, live, impact and wind loading combined, to be increased to 21,000 lbs. per sq. in., properly reduced by formula for compression.

196. Long compression members are designed with a view to their liability to failure by buckling sideways. The formulas in common use are the Rankine (often miscalled the Gordon): $p = s/(1 + m\,K^2)$, and the "straight-line": $p = s - c\,K$. Here

p = mean unit load on column = total load ÷ solid cross-section area;
s = max unit stress in cross-section;
$K = L/r$ = unsupported length ÷ least gyration radius;
m and c = coefficients.

See also pp 495, etc, 760-761, 764, 1194-1195, 1143-1148.

197. The formula for extreme fiber stress due to combined compression and bending, is

$$S = \frac{P}{A} + \frac{M_b\,T}{I - \dfrac{P\,l^2}{E\,c}}.$$

Where
P = longitudinal compressive force;
A = area of section;
M_b = bending moment due to transverse load;
T = distance from neutral axis to extreme fibers;
I = moment of inertia;
l = length of beam;
E = modulus of elasticity;
c = coefficient. See Transverse Strength, ¶ 103.

Joints.

198. Pin Plates. Where a pin passes through one or more shapes of some member, if often happens that the combined surfaces of the truss members alone, in contact with the pin, are insufficient to transmit, by bearing, all of the stresses to be delivered to the member. There is then danger of crushing the material which presses against the pin. To obviate this, other shapes, usually flats and called pin plates or reinforcing plates, are riveted to the member; giving, in all, sufficient bearing surface for the pin. See Fig. 55; where the letters denote:

AA, angles,	W, web,	F, filler,
C, cover,	P, pin,	O, outside pin plate,
B, bar,	J, jaw,	T, batten plates.

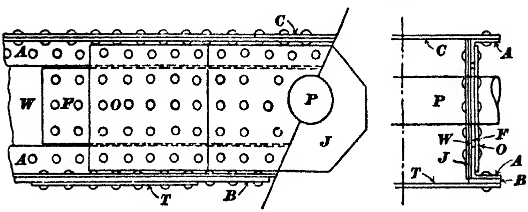


Fig. 55.

199. In Fig. 56, the two channels form the *whole* member (except the latticing, which cannot be included to resist compression) and the pin passes through both channels. In the case of a built-up chord section, or of an end post; Fig. 53, however, the webs form only a part of the section;

while the cover, the angles and the flats can receive no stress directly from the pin, but must receive it indirectly from the web and from the pin plates attached to it.

200. Where a pin plate is placed on each side of the web, the outside one must, according to most specifications, cover the angles; and there must, in addition, be a "**filler**" between it and the web.

201. Engineers differ as to the manner in which the stresses are actually transferred through the several parts of a pin connection. We may assume that the stresses in the pin plates are delivered almost directly to the shapes of the member. Thus, the outer reinforcing plate probably delivers most or all of its stress to the angles, and little or none to the web.

202. In each angle, those rivets which pass through the inner pin plate must transmit, by means of their bearing against the angle, the sum of the stresses which they take by shear from inside and from outside. In other words, these rivets are in double shear.

Pins.

203. The pin must be designed to resist bending stresses from the members through which it passes. It is also subject to shear, but this is seldom a critical point.

204. The pin requiring the greatest cross-section is usually either the one at the middle of the span and in the lower chord, where the chord stresses are greatest, or the one at the joint between the end post and the top chord; but, as the pins are relatively small members, all the other pins are, for the sake of uniformity, usually made of the same size with it.

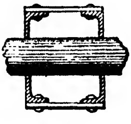


Fig. 56.

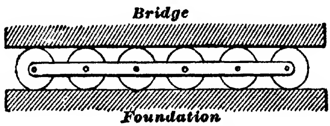


Fig. 57.

Expansion Bearings.

205. Expansion bearings usually consist of a nest of carefully turned rollers placed between two planed surfaces, shown in principle in Fig. 57.

206. The rollers are steel cylinders, from 3 to 6 ins. diam.; and 1 to 4 ft. long; planed smooth. From 4 to 8 or more of these are connected together by a frame, and one such frame is placed under at least *one end* of the truss. The rollers rest upon a strong planed cast bed-plate; bolted to the masonry below. Under the end of the truss is a similar plate by which it rests on the rollers. Since a truss of even 200 ft. span will scarcely change its length as much as 3 ins. by extremes of temperature, the play of the rollers is but small. They are kept in line by flanges cast along the side of the bed-plate. Flanges should also project downward from the upper bed-plate, so as completely to protect the rollers from dust, rain, etc.

207. The total displacement, allowed for the free end of the truss, is usually specified (see Digests); otherwise it may be taken as

$$D = \frac{(T - t)\ \text{span}}{145{,}000},$$

where T and t = the max. and min. temps. respectively, in degrees F. The min. temp. to be expected may be obtained from Weather Bureau records of temps. in the shade, but the max. should be taken 20° or 30° higher than that of the Weather Bureau; because, in bright sunshine, the bridge will become much hotter than the air.

208. Rockers. In order to restrict the length of the bearing, where the displacement is moderate, rockers, Fig. 62, are often used instead of rollers.

209. For other regulations and suggestions regarding design of roller bearings, see Digests of Specifications, and Figs. 60 and 61.

Loads, Etc.

210. Loads, Clearance, etc., for Highway Bridges. See also Digests of Specifications for Bridges.

Weights of crowds. At the Chelsea bridge, London, picked men, packed upon the platform of a weigh-bridge, gave a load of only 84 lbs. per sq. ft. At Buckingham Palace, men, wedged as closely as possible upon a space 20 ft. in diameter, the last man lowered from above, among the others, gave 120 lbs. per sq. ft. But modern experimenters have easily obtained loads of from 140 to 150 lbs. per sq. ft. With picked men, averaging 163.2 lbs. each, all facing one way, carefully packed, and confined within an enclosure 6 ft. square (0.9 sq. ft. per man), Prof. L. J. Johnson, at Harvard University, obtained a maximum of 181.3 lbs. per sq. ft.* See also pages 755, etc.

Where the enclosure of the space is such that portions of the persons, standing against the enclosure, may project beyond it, the load, per unit of space, is of course increased; and, with small areas, this increase may be relatively important.

Camber.

211. Amount of Camber. If we divide the span in feet, by 50, the quot. will ordinarily be a sufficient camber, in inches. This amounts to 1 in 600. The camber to be used is, however, usually stipulated in the specifications. See Digests. A well-built bridge, of good design, should not, under its greatest load, deflect more than about one inch for each 100 feet of its span, or 1 in 1200. Indeed, the deflection is frequently much less than this

212. The excess of length of the upper chord over that of the lower one, given the span, the depth of truss and the camber, will be =

$$\frac{8 \times \text{depth} \times \text{camber}}{3\ \text{span}}.$$

This rule applies closely with any camber not exceeding 0.02 of the span.

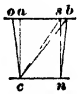


Fig. 58.

213. Length of diagonal cb, Fig. 58, $cb = \sqrt{ac^2 + ab^2}$; where ac = depth of truss, and $ab = cn + \frac{ob - cn}{2}$.

214. Sometimes the elongation or shortening, produced by the loading, is computed for each member, and the length of each member affected is correspondingly changed. See Deflections, ¶¶ 162, etc.

Examples.

215. Figs. 59 (*a*) to (*u*), to a uniform scale of 1 inch = 60 feet, serve to indicate current practice respecting the types selected for different spans, the relation between span, panel length and depth, the spacing of stiffeners in plate girders, the arrangement of chord and web members, the use of rigid and flexible members, counters and turnbuckles, in trusses, and, approximately, the dimensions of rigid members and of gusset plates, as seen in elevation.

216. In each case the left half of the span is shown, and the center line of the span is indicated by a dot-and-dash line. Through spans and deck spans are distinguished by the elevation of the roadway, as approximately indicated at the left support.

217. In Figs. *h* to *q*, representing trusses, rigid members are indicated by double lines, flexible members in verticals and main diagonals by single lines, and counters by dotted lines. In pin spans, to avoid confusion, the rigid members are shown cut off near the pins.

218. Figs. (*a*) to (*o*) represent standard designs, from 25 to 200 ft. span, by Mr. Ralph Modjeski, C.E., for the Northern Pacific Railway Co.; Fig. (*p*) a 250 ft. railroad span designed by the Pencoyd Works of the American Bridge Co.; Fig. (*q*) a 308 ft. railroad span by the Phœnix Bridge Co.; Figs. (*r*) to (*t*) designs for riveted trusses by the Elmira Works of the American Bridge Co.; and Fig. (*u*) a riveted railroad bridge, of 102 ft. span, designed by the Pencoyd Works

* Journal, Ass'n Engng Socs, Jan., 1905.

219. Fig. (*a*) represents a beam girder; Figs. (*b*) to (*g*) plate girders; and Figs. (*h*) to (*q*) riveted and pin trusses; Figs. (*h*) and (*i*) being riveted, and Figs. (*j*) to (*q*) pin.

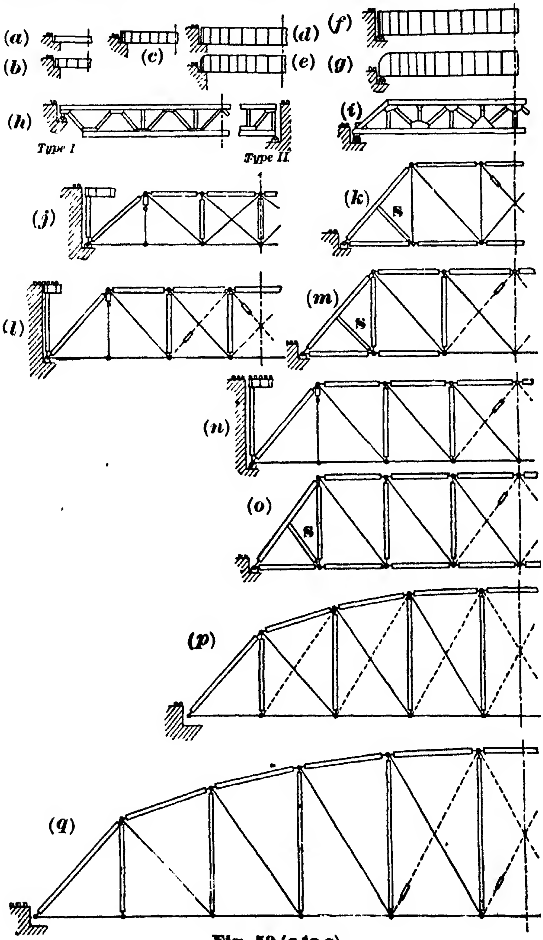


Fig. 59 (a to q).

220. Fig. (*r*) represents a 128 ft. span for the New York Central and Hudson River R. R., and Figs. (*s*) and (*t*) spans of 143 ft. and 160 ft. respectively for the Delaware, Lackawanna and Western R. R. Figs. (*r*) and (*s*) are modifications of the Baltimore truss, Fig. 15 (*b*), and Fig (*t*) is a quadruplex Warren truss. Fig. (*s*) is a skew span. Fig. (*u*) is a "pony" span.

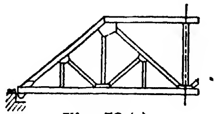


Fig. 59 (*r*).

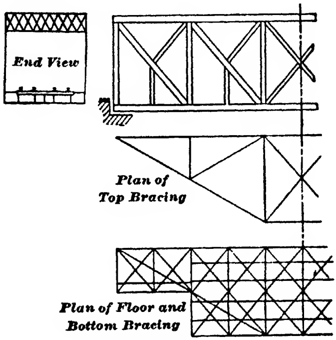


Fig. 59 (*s*).

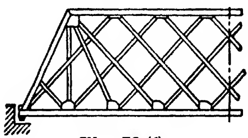


Fig. 59 (*t*).

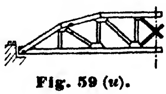
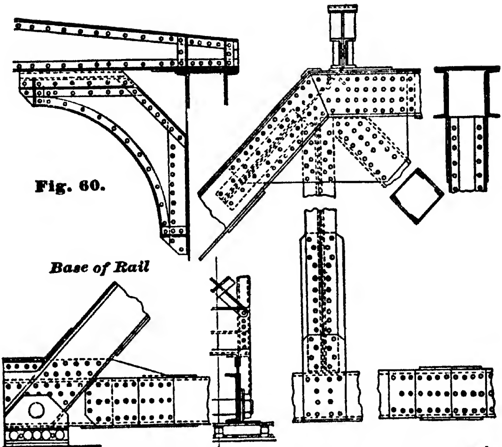


Fig. 59 (*u*).

221. Details. Figs. 60 to 65 show a few details of trusses and of plate girders.

222. Figs. 60 and 61 show left end connections of two through truss bridges (with roller bearings) designed by the Pencoyd Works; Fig. 61 representing the 250 ft. through pin span shown in Fig. 59 (*p*), and Fig. 60 a 124 riveted through span, showing the portal bracing.

Fig. 60.

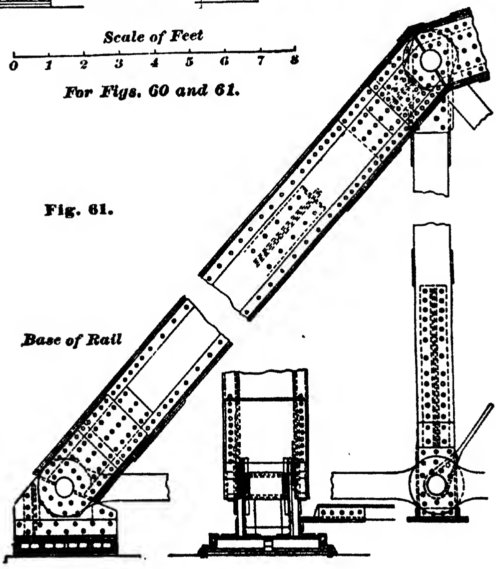


Scale of Feet

0 1 2 3 4 5 6 7 8

For Figs. 60 and 61.

Fig. 61.



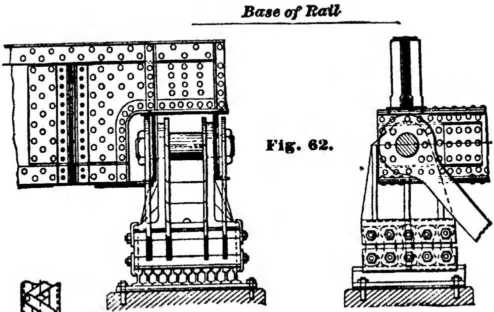


Fig. 62.

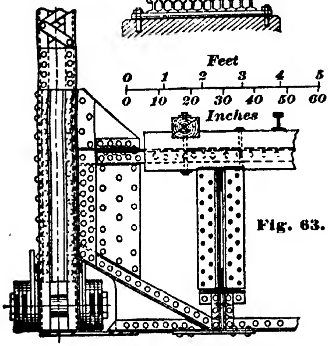
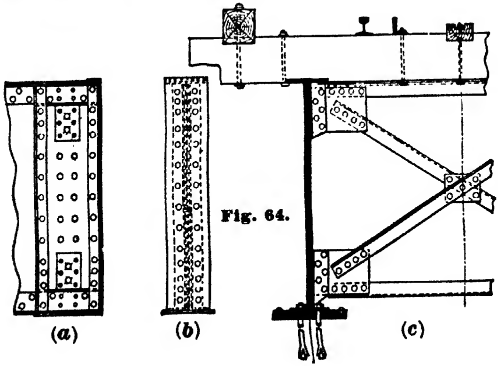


Fig. 63.

Scale for Figs. 62, 63, 64.

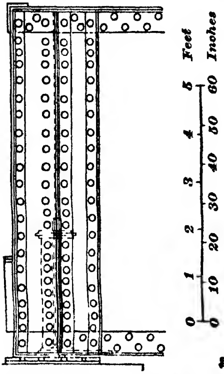
223. Fig. 62 shows the left end bearing (with rockers) and a floor beam connection for the standard 200 ft. deck pin span of the Northern Pacific Railway, Fig 59 (n); and Fig. 63 shows a floor beam connection of the 160 ft. through pin span of the same railway, Fig. 59 (m).



(*a*) (*b*) (*c*)

Fig. 64.

224. Figs. 64 and 65 represent respectively a 50 ft. deck plate girder of the N. Pac. Ry. and an 85 ft. through plate girder by the Pencoyd Works.



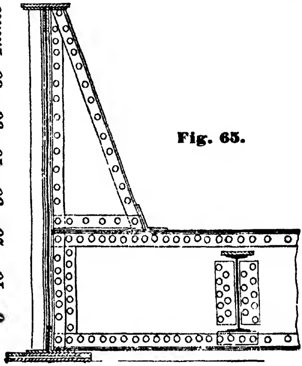


Fig. 65.

225. Weights of Steel Railroad Bridges. Fig. 66, based upon the practice of the Northern Pacific Railway, 1902,* shows the approximate

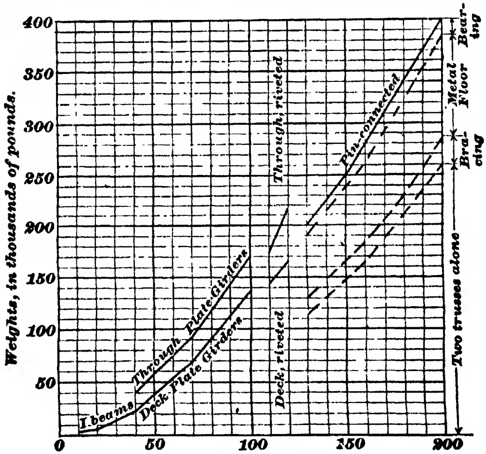


Fig. 66.

* See paper by Ralph Modjeski, C.E., in "Journal of the Western Society of Engineers," Chicago, Feb., 1901, vol. vi, No. 1.

weights of steel railroad bridges designed for two locomotives, of 146 tons each, and a uniform train load of 4000 lbs. per foot of track. The weights include the two beams, girders or trusses of one single-track span, with their bracing, metal floor system and end bearings. For wooden floor system, add 400 lbs. per lineal foot. For pin-connected spans (130 to 200 ft.), the three dash diagrams show, respectively, the weights of two trusses alone, of two trusses and bracing, and of two trusses, bracing and metal floor. The solid curve includes weights of end bearings.

For weights of combination (wood and iron) railroad bridges, see ¶ 249.

Highway bridges differ so widely, as to service and design, that it is scarcely practicable to give here useful data as to their weights.

List of Large Bridges.

Each bridge here given is believed to be (1902) the largest of its type in the world.

Type	Spanning	At	Span, ft	Built
Truss, ...	Ohio River	Louisville	553	1893
Swing, ...	Missouri River	Omaha	520	1894
Suspension, ...	East River ("Brooklyn")	New York	1595	1883
Suspension, ...	East River ("New")	New York	1600	*
Arch (metal), ...	Niagara River	Niagara Falls	840	1898
Arch (stone), ...	Petruff Valley	Luxembourg	277	*
Cantilever, ...	Firth of Forth	Queensferry	1700	1890

The highest viaduct is the Gokteik Viaduct, in Burmah, with a maximum height of 320 ft., and a total length (composed of short spans) of 2260 ft. built in 1901.

Timber Trusses.

226. Timber is now becoming so expensive, except in unsettled regions, and the labor of designing so cheap, that it is no longer found to be good practice to use unnecessarily heavy timbers, simply for the sake of being "on the safe side" and avoiding computations. Hence, in important bridges, every part of each member under stress is usually computed. On the other hand, the strength of wood is so uncertain an element that, when in doubt, it is best to adopt that assumption which will require the larger section; and ample factors of safety should be used.

227. Compression members are designed as columns (see Columns, pp 495, etc., and Wooden Columns, pp 963, etc.); and, if subjected to transverse stresses as well, these also should be carefully taken into account. All holes and other reductions of section must of course be deducted from the gross section.

228. In the **tension members** also, all reductions of section must be considered; but iron or steel rods are now generally used, in place of wood, in tension members.

229. In addition, care should be taken that the timber can withstand any **crushing or shearing** stresses that may come upon it or be set up in it. Thus, the ends of posts should be investigated, to see that they are safe against crushing. Where a post meets another member at an inclined angle and is to be notched into it, it is economy to compute the depth of notch required; as, the deeper the notch, the greater the gross section required for the notched member. Where bolts are fastened to timbers by nuts, washers should invariably be placed under the nuts, and the size of washer, necessary to prevent crushing the wood, computed.

230. Where the wood is subjected to shearing, as where a bolt, passed through a timber, transmits stress by the bearing of its side against the inside of the hole, or where there is a step or table which may be sheared off by the pressure of another piece against it, it should always be seen that there is sufficient surface along the grain of the wood to take the shear, and some allowance should be made for the possibility that the grain may run out to the surface or to some hole before all the stress can be transferred.

* Under construction. 1902.

231. Cross-section of Upper Chord. Since it would be inconvenient, in practical construction, to change the section of a timber upper chord at different points, it is designed throughout to withstand the *maximum* stress occurring between any two panel points. Assume width of chord member. Find r^2 = (least radius of gyration)$^2 = \frac{(\text{width})^2}{12}$. Find allowable unit stress according to column formula given in specifications or adopted, using the given maximum stress. Find area required for this unit stress. Find the resulting depth, which for a horizontal or inclined member is preferably somewhat greater than the width, to allow for bending moment due to its own weight. If this does not give a good commercial size, it may be well to revise, in order to obtain a better section.

232. Struts are preferably made as wide as the upper chord. Each strut must be designed separately. Obtain r^2, allowable unit stress, etc., as for the upper chord. For economy, the struts should average nearly square, even though it should be necessary to alter the section of the upper chord in order to prevent wide deviation from a square section.

233. The **vertical ties** (of iron) may now be designed. Area of cross-section $= \frac{\text{maximum stress}}{\text{allowable unit stress}}$. But see Minimum Section, ¶ 187. The size of a nut is usually fixed by the diameter of the rod, but the washers should be so designed as not to crush the wood.

234. The **bearings** or **indentations**, required in the upper and lower chords to hold the inclined members in place, may now be computed. The component (in the strut) perpendicular to the face or faces against which it presses, is computed, and the necessary depth obtained, assuming the width of the lower chord the same as that of the upper chord and the struts.

235. The section of the **lower chord** may now be decided upon, since the reduction of section, due to indentations, is known.

$$\text{Area of net cross-section} = \frac{\text{maximum stress}}{\text{allowable unit stress}}.$$

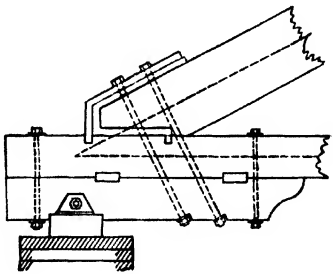


Fig. 67.

236. End Joint. Fig. 67. Many different designs for end joints have been made, proposed and discussed. The ends of the straps should enter notches in the lower chord, to such a depth that the total stress, taken by the end fibers of the sides of the notches, is equal to the stress that the ends of the straps can resist by bending. This depth can be found by successive trials, or by means of two algebraic equations, in which the maximum allowable pressure and the depth of notch are the two unknown quantities.

Determine the shearing surface required to transmit this stress to the body of the lower chord. This will also determine the space between the notches and the end of the lower chord. Compute the stress (if any) that remains to

be transferred, and design the long inclined bolts and their washers accordingly. Compute also the compressive area and depth of vertical face of lower end of upper chord, required to transmit the horizontal component of its thrust. See also that the lower bearing presents sufficient surface to resist the vertical component. The keys, between the bolster and the lower chord, must be designed to carry the horizontal component of the wind. For safety, **friction** between the two parts should be neglected.

237. Figs. 68 show **joints** adapted to most of the cases that occur in practice with wooden beams, etc. They need but little explanation. Fig. (*a*) is a good mode of splicing a post; in doing which the line *o o* should never be inclined or sloped, but be made parallel to the axis of the post; otherwise, in case of shrinkage, or of great pressure, the parts on each side of it tend to slide along each other, and thus bring a great strain upon the bolts. When greater strength is required, iron hoops may be used, as at *b*, *h* and *j*, instead of bolts. Fig. (*b*) shows a post spliced by 4 fishing pieces; which may be fastened either by bolts, as in the upper part; or by hoops, as in the lower. The hoops may be tightened by flanges and screws as at *s*; or thin iron wedges may be driven between them and the timbers, if necessary. Fig. C shows a good, strong arrangement for uniting a straining-beam *k*, or rafter *l*, and a queen-post *u*; by letting *k* and *l* abut against each other, and confining them between a double queen-post *t t*; *n n* are two blocks through which the bolts pass. A similar arrangement is equally good for uniting the tie-beam *w*, with the foot *v*, of the queens; with the addition of a strap, as in the figure. Fig. (*e*) is a method of framing one beam into another, at right angles to it. **An iron stirrup,** as at *f*, may be used for the same purpose; and is stronger. Figs. *g h*, *i j* are **built beams.** When a beam or girder of great depth is required, if we obtain it by merely laying one beam flat upon another, we secure only as much strength as the two beams would have if separate. But if we prevent them from *sliding* on one another, by inserting transverse blocks or keys, as at *g*; or by indenting them into one another, as at *i j*; and then bolt or strap them firmly together to create friction; we obtain nearly the strength of a solid beam of the total depth; which strength is as the *square* of the depth. See Horizontal Shear, ¶ 51, under Transverse Strength.

238. The strength of a built beam is increased by increasing its depth at its center, where it is most strained; as in the upper chords of a bridge. This may be done by adding the triangular strip *y y* between the two beams.

239. A piece of plate-iron may be placed at the joints of timbers when there is a great pressure; which is thus more equally distributed over the entire area of the joint; or cast iron may be used.

240. Frequently a simple strap will not suffice, when it is necessary to draw the two timbers very tightly together. In such cases, one end of each strap may, as at *x*, terminate as a screw; and after passing through a cross-bar Z, all may be tightened up by a nut at *x*. Or the principle of the *double key*, shown at K, may be applied. Sometimes, as at A, the hole for the bolt is first bored; then a hole is cut in one side of the timber, and reaching to the bolt-hole, large enough to allow the screw nut to be inserted. This being done, the hole is refilled by a wooden plug, which holds the nut in place Then the screw-bolt is inserted, passing through the nut. By turning the screw the timbers may then be tightened together.

241. When the ends of beams, joists, etc., are inserted into walls in the usual square manner, there is danger that, in case of being burnt in two, they may, in falling, overturn the wall. This may be avoided by cutting the ends into the shape shown at *m*.

242. When a strap *o*, Fig. R, has to bear a strain so great as to endanger its crushing the timber *p*, on which it rests, a casting like *v* may be used under it. The strap will pass around the back *r* of the casting. The small projections in the bottom, being notched into the timber, will prevent the casting from sliding under the oblique strain of the strap. The same may be used for oblique bolts, and below a timber as well as above it. When below, it may become necessary to bolt or spike the casting to the under side of the timber. When the pull on a strap is at right angles to the timber, if there is much strain, a piece of plate-iron, instead of a casting, may be inserted between the strap and the timber, to prevent the latter from being crushed or crippled; see I and I

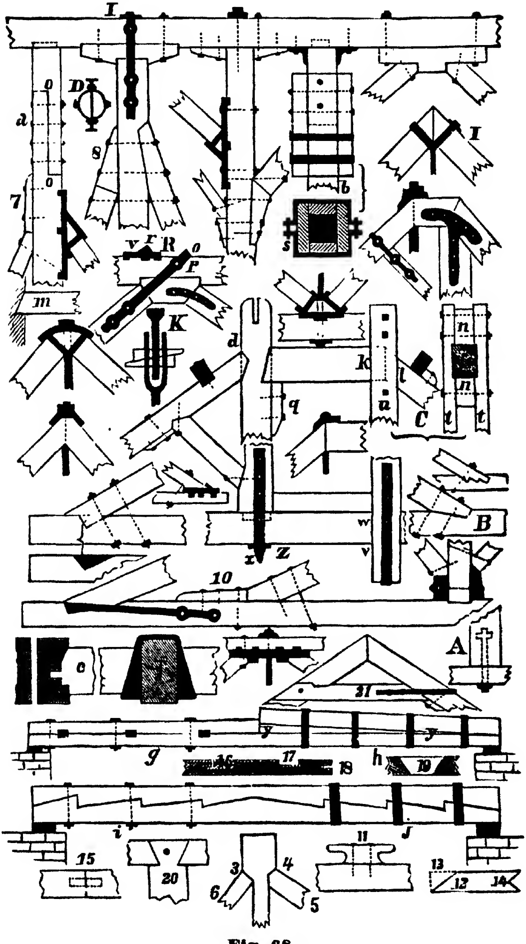

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Fig. 68.

243. Lower Chord Splice. Owing to the length of the lower chord, it may be necessary to splice it, as in Fig. 69, where the splice is a tabled fish-plate joint. The number of tables to be used is largely a matter of trial. The use of too many tables involves too much carpentry, and consequent uncertainty as to distribution of pressures, while the use of too few requires deep notches, which may too greatly reduce the section. These tables must be designed to resist bearing against their ends, and to resist being sheared off. Bolts should be passed through, especially at the ends of the fish-plates, to prevent them from bending outward; and the washers should be so designed as to transmit safely to the wood all the stress that can come on the bolt.

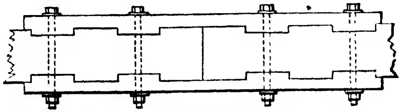


Fig. 69.

244. Fig. 70 shows a lower chord splice used in connection with the standard combination (timber and iron) Howe truss bridges of the Chicago, Milwaukee & St. Paul Railway See ¶¶ 249-251, Figs. 73 Four of the clamp-blocks shown are required for each joint, a block being placed upon each side of each stick to be spliced The two opposite blocks forming a pair are held together, and against the stick, by four through bolts; and the cylindrical lugs, cast on the surface of each block, enter corresponding holes, bored in the face of the stick. The two blocks on the same side of a stick are held together by the hooked clamp-bar. The clamp-key is driven between the left hook and the beveled key-seat on the left block.

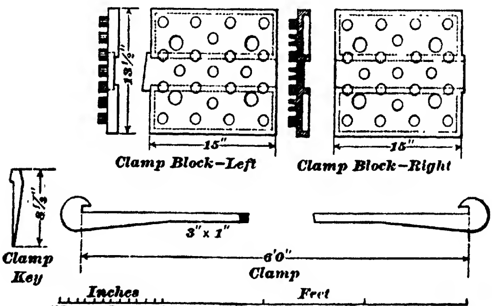


Fig. 70.

245. Figs. 71 illustrate a small wooden **Howe truss bridge.** The top and bottom chords are each made up of three or more parallel timbers *c c c*, placed a small distance apart, to let the vertical tie-rods *r r* pass between them. The main braces, *o o*, are in pairs or in threes. The pieces composing them abut, at top and at bottom, against triangular **angle blocks**, *s*; which, if of hard wood, are solid; and, if of cast iron, hollow, Fig. (*d*), and strengthened by inner ribs. These blocks extend across the three or more chord pieces. Against their centers abut also the counterbraces *c*. These are single pieces in small bridges; or in pairs, in large ones; and pass between the pieces which compose a main brace. Where the wooden braces and

counters cross, they are bolted together. In Fig. (*d*), the dotted lines show the strengthening ribs; and the lug, *x*, keeps the block in place. The vertical tie-rods *r r*, of iron, are in pairs, threes, or fours, etc., according to size of bridge; with a screw and nut at each end. The heads and feet of the braces and counters abut square against the angle blocks; and are often kept in place only by tightening the screws of the vertical ties. The end posts, *p* and *d*, the end ties, *i c* and *b y*, and the horizontal extensions, *g i* and *w b*, of the upper chord, form no part of the truss proper.

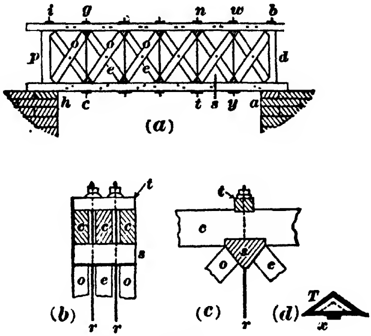


Fig. 71.

246. In **large spans**, to prevent the pressure at the ends of the diagonals from crushing the chords, the angle blocks are sometimes cast with deep projecting flanges under their bases. These flanges, passing between the pieces which compose a chord, extend to the opposite face of the chord. There the flanges bear upon broad washers at the ends of the vertical rods.

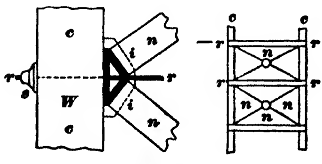


Fig. 72.

247. A common form of **lateral bracing**, Fig. 72, resembles a Howe truss laid flat on its side. In it the diagonals of the cross are struts of timber; and the pieces *r r* are round rods. One of the struts is whole, with the exception of a slight mortise on each vertical side, at its center, for receiving tenons cut on the inner ends of the two pieces which compose the other diagonal. At the sides of the chords, the ends of the diagonals rest upon a ledge (shown by the dotted line *i i*), about 1½ inches wide, cast at the bottom of the cast-iron angle block. The tie-rod *r r*, passing through the chords of both trusses, being tightened by means of the nut *s*, holds the diagonals firmly in place; and, in case of their shrinking a little in time, they can be again tightened up by the same means.

The cast angle block is as deep as a brace; its thickness need not exceed half an inch, in a large bridge. It has holes for the passage of the rod *r r*

248. Dimensions for each of two trusses of a single track **Howe bridge** for highway or suburban electric railway, with cars weighing 20 to 25 tons each. Timber not to be strained more than 800 lbs. per sq. inch; nor iron more than 5 tons per sq. inch. Iron supposed to be of rather superior quality, requiring from 25 to 27 tons (60,480 lbs.) per sq. inch to break it. The rods to be *upset* at their screw-ends. To each of the two sides of each lower chord is supposed to be added, and firmly connected, a piece at least half as thick as one of the chord pieces, and as long as three panels, at the center of the span.

Clear Span. Feet.	Rise. Feet.	No. of Panels.	An upper Chord. No. of Pieces.	An upper Chord. Size.	A lower Chord. No. of Pieces.	A lower Chord. Size.	An End Brace. No. of Pieces.	An End Brace. Size.	A Center Brace. No. of Pieces.	A Center Brace. Size.	A Counter. No. of Pieces.	A Counter. Size.	End Rod. No of Rods	End Rod. Diam.	Center Rod. No. of Rods	Center Rod. Diam.
				Ins.		Ins.		Ins.		Ins.		Ins.		Ins.		
25	6	8	2	5×6	3	5×12	2	5×8	2	5×6	1	5×6	2	1¼	2	1
50	9	9	3	6×9	3	6×14	2	6×9	2	5×8	1	5×8	2	1⅝	2	1⅛
75	12	10	3	6×12	3	6×14	2	6×11	2	6×8	1	6×8	2	2⅛	2	1⅜
100	15	11	3	6×14	3	6×16	2	8×12	2	6×10	1	6×10	2	2⅜	2	1⅜
125	18	12	4	6×14	4	6×16	2	9×14	2	6×12	1	6×12	2	8	2	1⅜
150	21	13	4	8×14	4	8×18	3	8×14	3	6×10	2	6×10	3	2⅜	3	1⅝
175	24	14	4	10×16	4	10×20	3	8×15	3	8×10	2	8×10	3	3	3	1½
200	27	15	4	12×16	4	12×20	3	9×16	3	8×14	2	8×14	3	3⅛	3	1⅝

249. Standard "combination" (wood and iron) **Howe truss railroad bridges,** Chicago, Milwaukee & St. Paul Railway, 1891-1902.

The bridges are designed for a train load of 4000 lbs. per lineal foot. Wind stresses are computed for a pressure of 500 lbs per lineal foot of train. Compressive stresses in braces, max. 505, min. 92 lbs per sq inch. Tensile stresses; in main truss rods, max. 12,450, min. 8500 lbs per sq. inch of net area at root of thread. Lateral rods, max. 15,000 lbs. per sq. inch of net section.

Timber. Cross-ties and guard rails, white oak; top chord packing blocks, white pine; all other timber, white or Norway pine or Douglas fir.

Combination Bridges, Chicago, Milwaukee & St. Paul Railway:

Total length, feet,*	82	93	103	114	125	136	147
Span, feet,*	77	88	99	110	121	132	143
Panels,	7	8	9	10	11	12	13
Upper chord,	4, 7x10	4, 7x10	4, 8x10	4, 8x10	4, 8x10	4, 8x10	4, 8x12
Lower chord,†	4, 7x14	4, 7x14	4, 8x14	4, 8x14	4, 8x15	4, 8x15	4, 8x15
Main braces, M:							
At center,‡	2, 10x10	2, 8x10	2, 10x10	2, 10x10	2, 10x10	2, 10x10	2, 10x10
At ends,	2, 10x12	2, 12x12	2, 12x12	2, 12x14	2, 12x14	2, 14x14	2, 14x14
Counters, C:							
At center,‡	2, 8x10	1, 10x12	1, 8x10	1, 8x10	1, 8x10	1, 10x12	1, 8x10
At ends,	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8
Vertical rods:							
At center,‡	3, 1½	3, 1½	3, 1⅜	3, 1½	3, 1⅝	3, 1½	3, 1⅝
At ends,	3, 2⅛	3, 2⅜	3, 2½	{3, 2⅜ 2, 2	3, 2¼ 2, 2	3, 2⅜ 2, 2	3, 2½ 2, 2
Weight, lbs.,§	130,300	155,500	182,000	209,400	233,100	259,900	286,500

* Lengths and spans are given to the nearest foot. The panel length is 10 ft. 11¼ ins. Each span is longer, by one panel, than the next preceding. Depth, in all cases, 25 feet in clear of chords. Width, 14 ft. 6 ins. in clear between chords.

† For lower chord splice, see Fig. 70, ¶ 244.

‡ Owing to the fact that the spans have alternately odd and even numbers of panels, there is some ambiguity as to the dimensions of web members at center of span.

§ Weight includes the two trusses, bracing and floor system for one single-

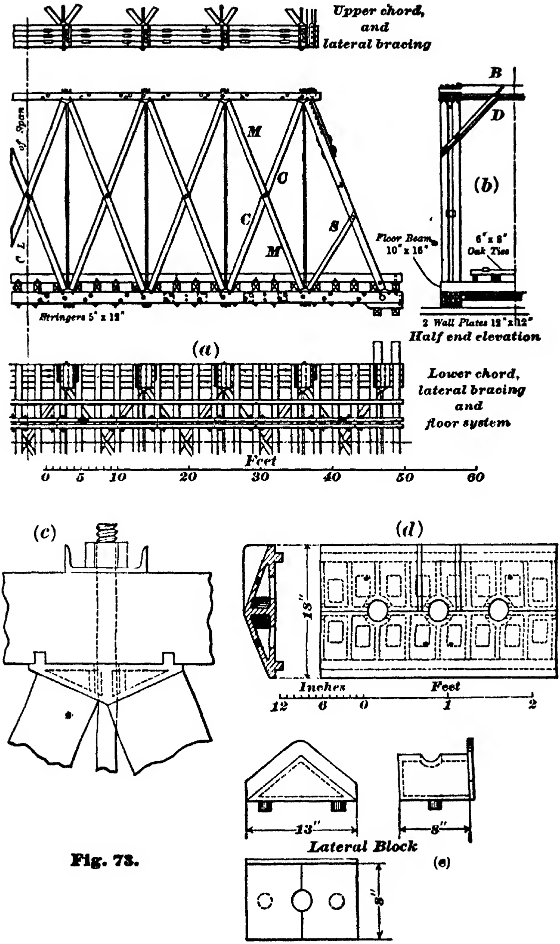


Fig. 73.

track span. Of the total weight of the bridge, the trusses have about 53 per cent., lateral systems 26, floor system 18, and wall plates 3 per cent.

250. In all spans, lateral bracing, 6 × 6 ins. at center, 8 × 8 ins. at ends, of span; lateral rods, 1¼ ins. at center, 1½ to 1¾ ins. at ends; collision struts, S (one at each end of each truss), 6 × 14 ins.; transverse portal brace, B, between ends of upper chords (one at each end of bridge), 10 × 12 ins.; diagonal portal braces, D (two at each end of bridge), 6×8 ins.

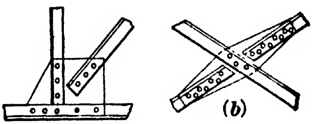
The floor beams are 10 × 16 ins., 20 ft. long, 14 ft. clear span, and 2 ft. 6 ins. apart center to center. The stringers are 5 ins. wide, 12 ins. deep, placed as shown in the end view, Fig. (*b*). Ties, 8 ins. wide, 6 ins. deep, 1 ft. apart, center to center. Guard rail, 8 ins. wide, 5 ins. deep. Under each end of the lower chord are two timber wall plates, 12×12 ins., 20 ft. long.

251. Figs. 73 show the 99 ft. span. Fig. (*a*) is a side elevation of half the span with top and bottom lateral bracing and floor system; Fig (*b*) a half end elevation, showing portal bracing; Fig. (*c*) a panel point connection (the same for upper and for lower chord); Fig. (*d*) a cast-iron angle block for same; and Fig. (*e*) a cast-iron angle block for lateral bracing.

Metal Roof Trusses.

252. Among the types commonly used for metal roof trusses are the triangular truss, Fig. 26, and the arch truss; the triangular truss being used for short spans, and the arch truss for long spans. See ¶¶ 255, etc., and Fig. 75.

253. In roof trusses of short span, carrying light loads, the minimum sections prescribed in bridge specifications will often suffice for all the members. The connections are made by means of rivets and flat plates, somewhat as shown in Fig. 74.



(*a*) (*b*)

Fig. 74.

254. In designing such trusses, no matter how light the stresses, care should be taken to avoid eccentric loadings, which are apt to occur where the members are not repeated symmetrically on both sides of the flat plates.

255. Train-shed Roof, Broad Street Station, Pennsylvania Railroad, Philadelphia. Figs. 75 (*a*) to (*f*). Built 1893 by Pencoyd Bridge and Construction Co., erected by Railroad Co. Span, 300′ 8″; rise, 108′ 6″; length, 589′ 2½″. Twenty trusses, arranged in 10 pairs, 2 pairs shown in Fig. (*c*).

256. Each truss consists of two arched **rafters**, A B and B C, and a horizontal chord, A C, with three pin joints, A, B and C, Fig (*a*).

Each rafter is composed of two chords, 14 radial braces and 26 diagonals. For the sake of appearance, the chords are extended across the top panels, occupied by the two triangular apex members, and are there connected by a sliding joint.

257. The horizontal chord AC, Fig. (*a*), lies below the floor of the train-shed, and is suspended, at intervals, from girders which support that floor and which are carried by iron columns in the lower story.

258. End bearings. One foot of each truss rests upon a fixed shoe, bolted down to the pier; the other on a nest of 11 steel rollers.

259. At each end of the roof, a horizontal wind truss, WW, Fig. (*a*), is suspended from the rafters, its ends resting upon brackets riveted to their lower chords.

260. Between these horizontal wind trusses and the rafters, and just behind the glass curtain closing each end of the roof, are placed vertical wind trusses (not shown), with horizontal and diagonal bracing, in planes normal to the main trusses. These vertical trusses are hung from the lower panel points of the rafters, Fig. (a). They resist the wind pressure on the curtain, and transmit it, through the horizontal wind trusses, to the lower chords of the rafters.

261. Two rafters, AB and BC, Fig. (a), of one truss, together weigh 138,000 lbs.; chord (two lines of eye-bars) 16,000, total 154,000 lbs. One pair of trusses, 308,000. Framework of entire train-shed, including trusses, purlins, rafters and wind bracing, about 7,000,000 lbs.

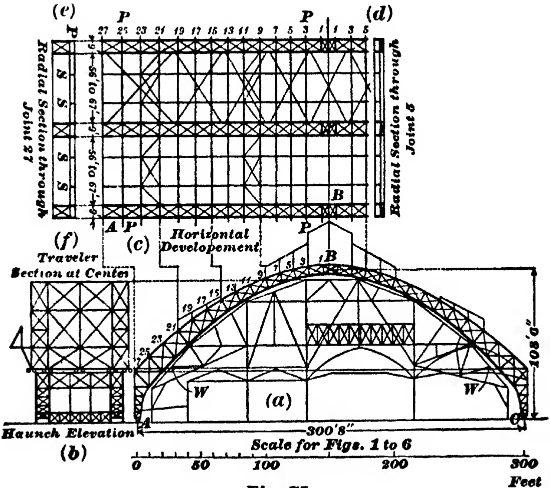


Fig. 75.

262. Assumed extraneous loads, in lbs. per sq. ft. of ground plan: rafters 9 5; laterals and purlins, 7.5; ventilator and skylight framing, 9.5; covering and skylights, 15; snow, 17; wind, 35.

263. The traveler, Figs. (a) and (f), was of timber, and was so designed as to permit the use of the old train-sheds while the new roof was being erected.

264. The top section, from apex to joint 9, Figs. (a) and (c), was riveted up at mill and placed in position as a whole, as was also the foot section from joint 27 to the top of the triangular foot. The remainder was riveted on the traveler. To erect one pair of trusses, with the purlins, etc., connecting it with the pair last erected, required about 10 days. Shifting the traveler through a distance about equal to its own length, to receive the next pair of trusses, required about 3 minutes, and was done by means of the engines used for hoisting.

265. Dimensions of Arched Roofs of Large Span.

	Arches.		Roof.	
	Span.	Rise.	Length.	Area Covered.
Train-sheds.	ft. ins.	ft. ins.	ft. ins.	sq. ft.
Pennsylvania Railroad, Philadelphia. Broad Street Station, ...	300 8	108 6	589 2	177,150
Pennsylvania Railroad, Jersey City, ...	252 8	90 0	652 6	164,900
Phila. & Reading Railroad, Philadelphia. "Reading Terminal," Market St., . .	259 0	88 3	506 8	131,250
New York Central & H. R. R. R., New York. Grand Central Station, ...	199 2	94 0	652 0	129,856
Midland Railway, London. St Pancras Station, ...	240 0	107 0	706 0	169,400
Cologne, Germany. Nave,	209 7	78 8	836 0	175,200
Exposition Buildings.				
Machinery Hall, Paris, 1889. Nave, ...	362 9	149 0	1,380 0	500,600
Manufactures and Liberal Arts Building, Chicago, 1893, ...	368 0	206 0	1,268 0	466,600

Timber Roof Trusses.

266. Dimensions for small timber roof trusses, Figs. 43 to 47, ¶¶ 148, etc., of white pine. Span = 4 × rise. Combined weight of trusses, roof and load, including snow and an allowance for wind, 40 lbs. per square foot of roof surface. Trusses 12 feet apart, center to center. Safety factor = 3; b = breadth; d = depth. For Fig. 46, struts 4.5 × 4.5 ins. would suffice; but, for practical reasons, the struts are generally made as wide as the rafters. For Fig. 47, the straining beam is 12 × 12 ins. For Figs. 45, 46 and 47, two sets of dimensions are given; the first for chord unloaded; the second for chord loaded with 100 lbs. per square foot.

Fig.	Span Ft.	Rise. Ft.	Rafters. Ins.		Chord. Timber. Ins.		Chord. Iron. Ins. Diam.	King or Queen Rod. Ins. Diam.	Chord.
			b.	d.	b.	d.			
43	30	7.5	5	10	5	10	1	..	unloaded
45	30	7.5	6.5	9	6.5	9	1	1	unloaded
			8.5	11	8.5	11	..	1¼	loaded
46	40	10	6	8	6	8	1.5	1	unloaded
			8	10	8	15	..	2	loaded
47	60	15	10	12	10	12	1.5	½	unloaded
			12	14	12	12	..	2	loaded

TRANSPORTATION AND ERECTION.

267. Girders should be loaded for **transportation** on flat cars, with web vertical, and with bearings at the points distant ¼ span from the ends. Where too long for two cars, one or more idle spacing cars are used and the points of support are pivoted.

268. Girders may be **erected** by means of gin poles, derricks, gallows, etc., or they may be skidded from the cars and lowered to place by jacks and blocking. Gin poles should have at least four guys, with tackles, for easy adjustment. Hoisting may often be done by means of a locomotive running on the tracks of that part of the structure which is already completed. Ropes are used at about ¼ their ultimate strength.

269. Viaducts are usually built from above, by means of an overhead projecting traveler. Sometimes by means of a cableway; but this method is slow In some cases the traveling tower is on the ground, and reaches to the top of the viaduct Or, the viaduct may be erected by means of false-work, or from an existing structure.

270. Long span bridges are usually built upon a platform of false-work or on a row of trestle bents, well braced.

271. Erection. Upon the false-works the lower chords are first laid, as nearly level as may be. The upper chords are then raised upon temporary supports which foot upon the one that carries the lower chord. The upper chords are first placed a few inches higher than their intended positions, in order that the web members may readily be slipped into place. When the web members are in place, the upper chords are gradually lowered until all rests upon the lower chords. The screws are then gradually tightened, to bring all the surfaces of the joints into their proper contact; and by this operation (the upper chord members having the necessary excess of length), the camber is formed, and the lower chords are lifted clear of the false-works; the truss now resting only upon its permanent supports.

272. False-work is ordinarily constructed of hemlock or pine, costing about $20 per 1000 ft. board measure. Allow about $15 per 1000 ft. B. M. for framing, etc. $5 to $15 per 1000 ft. B. M. may usually be obtained for old material ("salvage").

The main members are usually 12 × 12 ins., and the diagonals 3 × 12. Bolts, ¾ inch. Owing to the temporary nature of false-work and to the salvage which may be obtained for it if it is not too badly cut up, it is advisable to use plenty of material of good standard sizes, especially in longitudinal bracing, which may be placed between alternate pairs of bents, forming towers.

273. In soft bottoms, the false-work may rest upon piles, to which the uprights of the false-work may be notched and bolted, or banded. Not less than 4 piles per bent should be used. As many as 24 have been used. Piles should be braced below water-mark. Bents should be built in stories of from 12 to 30 feet each. Connections should be made by means of side pieces or fish-plates.

274. With rock bottom, in a strong current, it may be expedient to sink cribs filled with stone, as a foundation for the false-work.

275. The erection of **cantilevers and suspension bridges** requires much time; but their use is often necessitated by the impossibility of erecting false-work.

276. Renewal of bridges may be accomplished by displacement; either by protrusion, where the new span is skidded longitudinally along the track; by transverse displacement, where both old and new spans are placed on tracks running normally to the bridge; by vertical displacement, either rising or descending; or by pivotal displacement, the old span swinging out about a pivot, and the new span swinging into place about another pivot.

277. Cautions. In erection and renewal, consider dead weight of bridge, effect of impact of current of stream, impact of boats, ice, drift, etc.; especially when floods are to be apprehended, and strains of hoisting tackle

For rigidity, a liberal safety factor must be used. Drift may pile up and form a dam. Trestle bents, in the water, increase the velocity and scour of the current, and may thus cause undermining. False-work may be protected against drift by fender piles. Provide against eccentricities of wind stress. Numerous accidents have shown the expediency of guarding the unfinished truss itself against high winds. All lateral and other wind bracing should be in place, and secured, before the false-works are removed and the trusses allowed to rest upon their final bearings.

Avoid dropping tools, etc. Even very small pieces, falling from a great height, are dangerous to life and even to the bridge. Hooks in tackles are liable to break or to pull out. Travelers should be well guyed and clamped, and carefully watched.

TRUSS SPECIFICATIONS, PAGES 745–764. CONTENTS.

DIGESTS OF SPECIFICATIONS FOR BRIDGES AND BUILDINGS.

(1) Steel railroad, highway and electric railroad bridges.
(2) Combination (wood and steel) railroad bridges.
(3) Steel roof trusses, framework and buildings.

The following Digests of Specifications for Bridges and Buildings are intended primarily to give a general view of the essential features of current practice in such matters, and only secondarily to indicate the practice of any particular company.

(1) DIGEST OF SPECIFICATIONS FOR STEEL RAILROAD AND HIGHWAY BRIDGES.

List of Specifications Used.

A, American Bridge Company,
General Specifications for Steel Railroad Bridges, 1900.
Aa, American Bridge Company,
General Specifications for Steel Highway Bridges, 1901.
B, Baltimore & Ohio Railroad Company,
General Specifications for Railroad and Highway Bridges, Roofs and Steel Buildings, 1901.
C, Cooper, Theodore —,
General Specifications for Steel Railroad Bridges and Viaducts, 1901.
Cc, Cooper, Theodore —,
General Specifications for Steel Highway and Electric Railway Bridges and Viaducts, 1901.
D, Delaware, Lackawanna & Western R. R. Company,
Specifications for Steel Railroad Bridges, October, 1899; revised to July, 1900.
E, Erie Railroad Company,
General Specifications for Bridges, 1900.
G, General practice
Oo, Osborn Engineering Company,
General Specifications for Highway Bridge Superstructures, 1901.
P, Pennsylvania Railroad Company,
Standard Specifications for Steel Bridges, January 1, 1901.
R, Philadelphia & Reading Railway Company,
Specifications for Steel Bridges, 1898; revised February, 1901.
Y, New York Central & Hudson River R. R. Leased and Operated Lines,
General Specifications for Steel Bridges, 1900.

I. GENERAL DESIGN.

Limiting Spans for Different Types.

BEAMS AND GIRDERS.	A ft	Aa ft	B ft	C ft	Cc ft	D ft	Y ft
Rolled beams, solid floor, etc.,	up to 20	up to 40	up to 20	up to 20	up to 40	up to 20	up to 25
Plate girders,	20 to 100	25 to 80	20 to 100	20 to 120	20 to 80	20 to 100	25 to 100
TRUSSES.							
Riveted trusses,*	100 to 140	40 & over	100 to 120	75 to 150	40 & over	90 to 160	100 to 200
Pin trusses,	over 140	over 140	over 120	over 120	over 120	150 & over	200 & over

Riveted trusses,* under 100 ft; pin trusses, over 100 ft, **Oo.**
Depth of truss, min, = one-eighth of span, **Oo.**

* Unfortunately called "lattice girders," **A**; "lattice trusses" and "riveted lattice girders," **C**; "riveted lattice trusses," **D, Y.**

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie;

Classification of Highway and Electric Railroad Bridges.

Aa	Cc	
A	A1*	City bridges having buckle-plate floors, and paving on concrete base.
	A2*	City bridges having plank flooring.
B	B*	Suburban or interurban bridges for heavy electric cars.
C	C	Town or country bridges for light electric cars or **heavy loads**
D	D	Country bridges for ordinary highway traffic.
E1	E1	Bridges for heavy electric or motor cars only.
E2	E2	" " light " " "

Camber.

Top chord panels longer than lower chord panels by one-eighth of an inch in 10 ft = 1 in 960, **A, B, C, E, R, Y.**

Highway bridges, three-sixteenths of an inch to every 10 ft, **Cc.**

About three-fourths of an inch in 100 ft = 1 in 1600, **D.**

Sufficient to bring joints of compression chord to a square bearing when truss is fully loaded. Each member built longer or shorter in proportion to the stress to which it is subject under a full dead and a full live load, so that under full loading it will have its normal length, **Oo.**

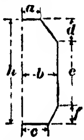
Cross Section of Bridge.

Gage, usually, 4 ft 8½ ins. **Distance, cen to cen of tracks,** 12 to 13 ft.

Width between trusses or girders in deck spans. Pin spans, min, 0.05 span. Riveted trusses (**D**), 10 ft. Plate girders (**G**), 5 to 7 ft. Spans not over 60 ft, 7 ft; 60 to 100 ft, 8 ft; over 100 ft, about one-twelfth of span, **D.** Plate girders (**Y**), over 60 ft, in proportion to height.

Clearance in through spans, on tangents, G.

a	3 to 3½ ft
b	7 ft
c	5 to 5½ ft
d	4 to 6 ft
e	10 to 14 ft
f	1 to 5 ft
h	20 to 22 ft



Minimum Clearance on Curves.
Same min clearance as on tangents, **A;** Ditto for car 74 ft long, 48 ft cen to cen of trucks, 10 ft wide, **B;** Ditto for car 75 ft long, 54 ft cen to cen of trucks. Additional clearance = 0 8 **d** ins on each side; = 1 6 **d** ins between tracks; where **d** = degree of curvature = central angle subtended by a chord of 100 ft, **C;** increase lateral clearance at top of car 2.5 ins for each inch of superelevation of outer rail, **C.** Cen line of bridge bisects middle ordinate and is parallel to chord, **Y.**

Highway Bridges. Headway, 14 ft, **Oo.** For classes A, B, C, and E, min = 15 ft, **Aa, Cc;** for a width of 6 ft over each track, **Aa.** For Class D, 12.5 ft, **Aa, Cc. Horizontal clearance.** Min 14 ins greater than width of roadway between wheel guards, **Aa.** For electric cars, 6 5 ft from cen of track, **Aa;** 7 ft, **Cc.** On curves, provide for a car of 45 ft extreme length, 8 ft wide, 20 ft between truck centers, **Aa.** Width between centers of trusses, min = 0.05 span, **G.**

Tension Members.

In general, hip verticals and one or two panels of lower chord at each end of span are required to be of rigid section, so as to resist both tension and compression.

Angles, used as tension members, must be fastened by both legs, **C, D;** or the section of one leg only will be considered effective, **C.**

Adjustment.
Avoid adjustable members, **A, Aa, D** (**P** except in counters). Avoid

* **Cc,** Classes A and B shall be designed to carry, at any future time, a double track electric railway.

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

adjustable counters, **C**, Counter rods and ties in pin spans adjustable, **Y**; screw ends upset, **C, E, Y**, screw threads U. S. Standard, **C, Cc, D, E**; diameter, at base of thread greater than in body of bar by one-sixteenth of an inch, **D**; about 10 per cent, **Y**; 17 per cent, **Oo**.

Rods with welded heads must be of wrought iron, **Oo**. Loops must develop full strength of bar, **Oo**.

Compression Members.

End posts and upper chords have 2 webs, a cover plate on top flange, batten or tie plate, and lacing on bottom flange, **G**

Not more than one plate, and that not thicker than one-half of an inch (in highway bridges three-eighths of an inch), shall in general be used as a cover plate, **C, Cc**. Cover plate must not extend more than 4 ins beyond outer row of rivets, **D**.

Joints between sections spliced on all sides with at least 2 rows of closely pitched rivets on each side of joint, **C**. Abutting surfaces faced **A** (**D, P**, except in top flanges of girders), **E, R, Y**. No reliance on abutting surfaces, **E**.

Lattice Bars.

Width, from 1.5 to 2 5 ins Thickness, in single lattice, one-fortieth, distance between rivets, in double lattice, one-sixtieth, **G**. If over seven-sixteenths of an inch, use angles, **Oo**. Angle with axis of member; in single lattice 60°, double 45°, **G**.

Pitch width of channel + 9 ins, **C, Cc**; 8 × least width of segment, **P, R**.

Double lattice bars riveted together at their intersections, **C, D**.

Batten Plates. (Tie Plates, Stay Plates) Min length generally = from 0.75 to 1 5 × its own width. Min width, 9 ins, or 0 66 × own length, or = least width of member, **Oo**. Min thickness = three-eighths of an inch or one-fiftieth to one-sixtieth of distance between centers of rivets, **G**. Rivet spacing (**D**) max 4 ins cens.

Pin Joints.

Eye Bars. Thickness, min, 0 625 inch, or 0.2 × width of bar, **Oo**. Heads upset, rolled or forged, **A, R**. No welds allowed **A, D**, except (**R**) to form loops of laterals, counters or sway rods, **R**. Upset and die-forged, **Y**. Heads not more than one-sixteenth of an inch thicker than body, **D, P**. Bars annealed, **G**; before boring, **D**. No forge work after boring, **R**. Bars to be placed side by side must be bored at the same temperature, **A, Aa, Y**. Pins must pass through without driving Eye bars working together must be clamped together, and bored at one operation, **Oo**.

Distance between pin-holes max variation, one sixty-fourth to one thirty-second of an inch; or one sixty-fourth of an inch in from 20 to 25 ft, **G**.

Built Tension Members. Net section through pin hole = 1.25 to 1.50 × net section through body of member Net section back of pin hole = 0.75 × net section through pin hole, or = 0 80 to 1 00 × net section through body of member, **G**; proportion for double shear on section from back of pin to end of plate, **Oo**; length of plate back of pin, min, 2 5 ins, **Oo**. Distance, back of eye to back of member, greater than radius of pin, **Y**.

Pin Holes. Clearance between pin and hole, from one-fiftieth to one thirty-second of an inch, **G**.

Pin Plates or Reinforcing Plates. At least one plate on each side must extend not less than 6 ins beyond edge of batten plate, **G**.

Pins. Up to 7 ins diam, rolled, **P**; over 7 ins, forged, **C**.

Diameter, min, from 0.66 × to 0.85 × largest dimension of any of its eye bars, **G**.

Plate Girders.

Min depth; about one-ninth to one-twelfth of span, **G**.

Proportions of Web and Flange. Bending moments resisted entirely by the flanges, shear resisted entirely by the web plate, **C, Cc, R**; except when the web is made in one length or is fully spliced to resist the bending stresses, in which case one-sixth of area of cross section of web plate may be considered effective as flange area, **Oo**.

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie

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One-eighth of gross area of web included in flange, **A, Aa, B;** if length = 90 ft or over, **E;** if length is less than 50 ft, only the cover plate and the horizontal legs of the flange angles are to be included in the flange area, **E,** no part of web included in flange, **C, Cc, D, P, R, Y.**

Web. Thickness, min, three-eighths of an inch, **G;** in highway bridges (**Cc, Oo**), five-sixteenths of an inch.
Total shear, acting on side next to abutment, to be taken as transferred into flange angles within distance = depth of girder, **A, Aa, B, E, Oo, P.**

Web Splices. A plate on each side of web, **G;** at least three-eighths inch thick, **A, B;** at least five-sixteenths inch, or three-fourths as thick as web, and wide enough for 2 rows of rivets on each side of splice, **Oo.**

Stiffeners. Generally required at ends and at points of concentrated load. Intermediate stiffeners required usually when unsupported distance between flange angles exceeds 50 to 60 × web thickness; when shear exceeds $10{,}000 - 75 \times \frac{\text{depth}}{\text{thickness}}$, **C;** in highway bridges when shear exceeds $12{,}500 - 90 \times \frac{\text{depth}}{\text{thickness}}$, **Cc;** when shear exceeds $\frac{20{,}000 \times 12\,t}{1 + \frac{d^2}{3{,}000\,t^2}}$, where t = web thickness, d = distance between flanges, **Oo.**
Spacing usually = depth, or = 5 or 6 ft.

Unit stress, max, $10{,}000 - 45\,\frac{l}{r}$, **C;** in highway bridges, $12{,}000 - 55\,\frac{l}{r}$, where l = length of stiffener, r = its least radius of gyration, **Cc.**
Dimensions of angles usually $3\frac{1}{2} \times 3 \times \frac{5}{16}$ to $5 \times 3\frac{1}{2} \times \frac{3}{8}$.

Flanges.
Unbraced length of flange (compression flange, **C, P**) max = 12 × width **B, P, R, Y;** 16 × width, **A, C, Cc;** 20 × width, **D;** in highway bridges = 20 × width, **Aa,** = 25 × width, **Oo.**
Comp. flange has same gross area as tension flange, **A, Aa, B, C, Cc, Oo, P.**
Cover plates must not extend more than 5 ins, or 8 × thickness of first plate, beyond outer line of rivets, **A, Aa, C, Cc.** If of unequal thickness, the heaviest plates are next the angles, and the lightest outside, **A, Aa, B, C, Cc, Y.** One must extend full length of girder, **B, E.** Others must be long enough to take 2 extra rows of rivets at each end, **C, P.**

Bracing, Riveting, Bearings. See Bracing, Riveted Joints, and Bearings, below.

Beam Girders.

Beams in groups of 2, 3 or 4 for each rail, 10-inch channel separators, about 3 ft apart, riveted to webs, **B.**

Bracing.

Composed of rigid members, riveted, **A, Aa, C, Cc, Y;** members intersect each other, and other members to which they are connected, on common center lines, passing through all centers of gravity. Attachments riveted symmetrically in all directions, **Y.**

For Beam Girders. Ten-inch channel strut at each end; with 2 or 3 beams, angle bracing between girders; with 4 beams, angle struts about 6 ft apart. Connections to have at least 3 rivets, **B.**

Lateral.

B, in through spans. Top bracing; portal struts at ends; intermediate struts as deep as the chords; single angles, $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$, intersecting in each panel, for single track; double angles, latticed, for double track.

B, in through spans, bottom bracing; end bottom strut and intermediate angles, riveted to each other and to stringers at each intersection. Not less than 4 rivets at each intersection and at each end connection.

B, in deck bridges, complete upper and lower systems at each panel.

Oo, bottom end struts in all spans, whether deck or through.

Y, top and bottom lateral bracing in all deck bridges and in all through bridges having sufficient head room. Lower lateral bracing in all through

bridges. Upper system in all stringers framed between and riveted to floor beams where length of stringers exceeds 15 times width of stringer flange.

Y, in deck bridges without metal floor system, upper bracing of cross-struts at each panel point; composed of 4 angles latticed, with same depth as upper chord; stiff diagonals intersecting in each panel and riveted to each other at each intersection

For Plate Girders.

Deck. Between upper flanges, angles with at least 4 rivets at connections. Through; lower lateral bracing of angles, intersecting in each panel, riveted to each other and to stringers at each intersection, **B**.

Lateral bracing angles generally of same size as those in stiffeners, **R**.

Cc, in highway bridges, a buckle plate floor may be considered as the required system of lateral bracing at the floor level.

Sway (diagonal, cross, vibration or wind) and **Portal.**

Proportioned to resist unequal loading of trusses in double track spans, **E**, **R**; end sway bracing to transmit all horizontal forces to abutment, **E**; to carry half the max stress increment due to wind & centrif. force, **A**, **Aa**, **B**, **P**.

In deck spans, at each panel point, **A**, **Aa**, **D**, **E**, **P**, **R**, **Y**.

Overhead bracing in through spans whose depth exceeds 25 ft, **C**, **D**, **P**, **Y**; in highway bridges, 20 ft, **Cc**; 25 ft, **Oo**.

In pony trusses and through plate girders, at ends and at each floor beam or cross strut, **A**, **Aa**, **R**; at every panel point, **D**.

In through and half through plate girders, at each floor beam and at each end, or, if there is a solid floor, not over 8 ft apart, **Y**.

In deck plate girders, rigid cross frames at ends and max 20 ft apart, **Y**; sway frames of at least 4 angles at ends and at points 12 to 14 ft apart, **B**; through, not more than 12 × flange width along top flange, **B**.

Riveted Joints.

Rivet Holes. In I beams, must be drilled, **B**.

May be punched; in steel not over $\frac{5}{8}$ to $\frac{3}{4}$ inch thick, **G**.

Sub-punched one-eighth of an inch smaller, and reamed to one-sixteenth of an inch larger, than rivet, in steel over five-eighths to three-fourths of an inch thick; in connections for floor beams and stringers to main trusses or girders, **E**.

No drifting allowed, **A**, **B**, **C**, **D**, **E**, **R**.

No interchange of pieces after reaming, **D**, **P**.

Hole larger than rivet by one-sixteenth of an inch, **G**.

Die larger than punch by max one-sixteenth of an inch, **G**,

Distance from edge of plate to center of rivet. Min, 1 25 to 1.5 ins, or 1.5 to 2 diams of rivet. Max, 4 to 5 ins, or 8 × thickness of plate, **G**.

Pitch Min = 3 × diam of rivet, general; preferably 4 × diam, **Oo**.

Max pitch in line of stress, 5 to 6 ins, or 16 × thickness of thinnest outside plate connected; normal to stress, 30 to 50 × thickness of thinnest outside plate connected. At ends of compression members (or of built members in tension, **B**); for a length of 1.5 to 2 × width or depth of member, 3.5 to 4 × diam of rivet, **G**.

In plate girders, for rivets connecting web to a top flange supporting the track, max = 3 ins, **R**.

Rivets. Diam generally three-fourths or seven-eighths inch. Heads hemispherical, **G**. Height of head, min = 0.6 diam, **R**.

Driving. Avoid hand riveting. Machines, direct-acting, worked by steam, hydraulic pressure or compressed air, capable of maintaining applied pressure after upsetting, **G**.

Floor.

Floor Beams. Depth, min, = $\frac{1}{8}$ × length, **Y**. In railroad bridges and important highway bridges, riveted to posts of trusses or to webs of plate girders, **G**. Given also a bearing on lower flange of girder or on a bracket, **G**. In default of such bearing, increase number of rivets by 25 per cent, **R**.

Hangers, when permitted, not adjustable, **C**, **Cc**. Hangers made of plates or shapes, **Oo**.

Stringers. Depth, min, = $\frac{1}{8}$ × length, **Y**. In highway bridges, Classes

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie;

A1 and A2, of steel; Classes B, C, and E, track stringers of steel; Class D, of wood or steel, **Cc.** In railroad bridges, and preferably in highway bridges, riveted to webs of floor beams and supported by their flanges or by brackets, **B, R.** Value of this bearing neglected in determining number of rivets required, **R.**

Spacing, cen to cen, 6 ft, 6 ins, **A, B, C, D, Y;** 5 ft, **E;** double track, through, generally 6 ft, **R;** single track, 8 ft, **R.**

Trough Floors. Troughs rectangular, built of plates and angles, and riveted to main girders or trusses by angles, and, when practicable, by bracket angles under the lower horizontal plates Gusset plates riveted to girders and troughs at distances of not over 8 ft, **Y.**

Bottom filled with a binder composed of 1 cu ft of clean, sharp gravel, screened to one-fourth of an inch, to 1½ gallons of No. 4 asphalt paving composition or enough to fill voids. Gravel first heated to 300° F and the whole mixed at that temperature, **Y.**

Wooden Floor. Continued over abutments, **A, B, C, R.**

Ties or Floor Beams. Long leaf yellow pine or white oak, **G.**

Width, 8 ins, **A, B;** 9 ins **R.**

Depth, 8 ins, for 7 ft span of tie, to 14 ins for 12 ft, **B;** 12 ins, **R;** 10 ins, **Y.**

Notched down one-half of an inch; max 1½ ins, **B.**

Spacing. Usually 6 ins clear; 16 ins cen to cen, **R.** Every 3d, 4th, or 5th tie fastened to stringer by ¾ inch bolt or lag screw, **G.**

Wooden Joists in Highway Bridges. Width, min 3 ins or 0 25 × depth; spacing, max, 2 to 2 5 ft Ends of joists lap past each other at bearings on floor beams, with 0 5 inch space between them for circulation of air.

Wooden Floor Beams for Electric Railroad Bridges, Classes E1 and E2 Min 6 × 6 ins, spacing max 6 ins, notched down one-half of an inch and secured to girders by three-fourths-inch bolts not more than 6 ft apart. From center of span toward end, so notched (**Cc**) as to reduce camber.

Guard Rails. 6 × 8 ins, yellow or white pine, **G.** Inner face not less than 3 ft 3 ins from center of track, **A;** 3 ft 7½ ins, **B;** 5 ft 4 ins, **Y;** 7 ft 1½ ins apart, clear, **R.** Notched ½ to 1½ ins over ties, **G.** Fastened to every 3d or 4th tie (to each tie, **R**) and at splices by three-fourth-inch bolt or lag screw, **G.** Splices over floor timbers, with half-and-half joints of 6 ins lap, **G.**

Wheel Guards and Curbs in Highway Bridges. Wheel guards 6 × 4 ins, blocked up from floor plank by blocks 2 × 6 ins, 12 ins long, not more than 5 ft apart, bolted to stringers through blocking pieces, three-fourths-inch bolts, **G.** In electric railroad bridges (**Cc**, Class E) guard timbers min 5 × 7 ins, notched 1 inch over floor timber and secured by three-fourths-inch bolt to every third floor timber and at each splice.

Buckle Plates. Min five-sixteenths of an inch thick for roadway, one-fourth of an inch for footwalk, crown 2 ins, for widths of 4 ft under roadway, 5 ft under footwalks. Preferably in continuous sheets of panel lengths. May be pressed or formed without heating.

Bearings on Abutments and Piers.

Permissible load on masonry foundations, max, pounds per sq inch. 400, **A, Aa, P;** 300, **B;** 250, **C, Cc, D, E, R;** dead load, 500; live load, 250, **Y.**

Bed Plates. Of medium steel, **C, Cc.** Min thickness, three-fourths to 1 inch; in highway bridges, one-half of an inch. Max fiber stress 12,000 lbs per sq inch, **E.**

Where ends of two spans rest on one pier, spans are tied together, or have bed plate, three-eighths to three-fourths of an inch thick, continuous under both, **G.**

Sheet lead, one-eighth to one-fourth of an inch thick, between bed plate and masonry, **G.**

Anchor bolts, 1 to 1 25 ins diam, 9 to 12 ins in masonry, **G;** fastened with sulphur, **R;** with cement, **C, Cc, Y.**

Pedestals. Of riveted plates and angles, **C;** or cast steel, **Y.** Base plate and connecting angles, min three-fourths to seven-eighths of an inch thick, **B, C, Y.** 2 rows of rivets in vertical legs, **C, Y.**

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

Expansion Bearings. Provide for temperature range of 150° F, **A, C, E, P, R**; for expansion of 1 inch in 100 ft, **D, Y.**

One end sliding, usually in spans less than 60 to 90 ft, **G.**

One end on friction rollers, usually in longer spans, **G**; in all trusses, **Y.**

Hinged bolster at each end, in spans from 80 to 100 ft, **G.**

Rollers rest on bars 3 × 1 inch, spaced 2 ins and riveted to bed plate, **B.** Free ends anchored against lifting and against moving sideways, **C. Y.**

Rollers. Of machinery steel, **C, Cc.** Min diam, 3 to 4 ins, **A, B, D, E, P, R**; 3 ins up to 100 ft span, 1 inch for each additional 100 ft, **C, Y.** Max pressure on rollers, in lbs per linear inch, 700 $\sqrt{d}$, **R**; 1200 $\sqrt{d}$, **A, B, P**; 300 d, **C, D, E**; 600 d, **Oo**; d = roller diam, ins. Length, ins, = 900 $\sqrt{d}$, **Y.**

II. MATERIAL.

Rolled and Cast Steel and Iron.

Rolled steel in superstructures in general.

Cast steel in bed plates in special cases, in machinery of movable bridges.

Rolled iron in loop-welded rods, **P**; in laterals and unimportant members, **R.**

Cast iron in bed plates in special cases and in machinery of movable bridges.

Rolled Steel, Grades.

Soft. In general, in all principal parts.

Medium. In pins, friction rollers, lateral bolts, bearing plates, eye-bars, sliding plates and bed plates; permissibly (**C**) in compression in chords, posts, and pedestals.

Rivet. In rivets.

Machinery. In expansion rollers, **C.**

Rolled Steel. Manufacture.

(See also Digest of Specifications of Internat'l Ass'n for Testing Materials.)

All to be made by open-hearth process.

Slabs for rolling plates are hammered or rolled from ingots of at least twice their cross-section, **A, B.**

Plates up to 36 ins wide rolled in universal mill, **D, R**; or have edges planed, **D.**

Rolled Steel. Manipulation.

Annealing. Eye-bars heated to uniform dark red and allowed to cool slowly, **P**; members worked at blue heat are heated to a uniform bright red (not exposed to direct flame) and allowed to cool slowly, **B.**

Steel must not be welded, **B.** No reliance upon welded steel, **C.**

No work put upon steel at or near blue heat, or between boiling-point and point of ignition of hardwood sawdust, **C.**

Rolled Steel. Shop Work.

Sheared edges of steel thicker than five-eighths inch shall be planed, **B.**

All sheared edges (in medium steel, **D**) shall be planed off to a depth of 0.25 inch, **D, Y**; except web plates of girders over 36 ins deep when covered by flange plates, and fillers where sheared edges are not seen, **D.** Grinding not accepted as equivalent of planing, except for lattice bars, **Y.**

No sharp or unfilleted re-entrant corners permitted, **D, Y.** Where a plate, angle or shape has been cut into, the fillet, as well as the cut, must be finished with sharp cutting tool, or with chisel and file, so that no sign of the punched or sheared edge remains, **D.**

Angles or bent plates, used as end connections on girders, floor beams or stringers, must be accurately fitted, so that when the member is milled to length not more than one-sixteenth of an inch will be taken off these connections at their roots, **D.**

Material bent by punching must be straightened before bolting up, **R.**

Web plates, if buckled, must be cold-rolled to remove the buckles, **D.**

Spliced chord sections must be assembled and strung out in shop in lengths of not less than three sections, and, after being drawn up into contact at

joints and lined up with splice plates in place, the field rivet holes shall be reamed to a fit before taking apart, and the assembled parts, with their splice plates, match-marked, **D.**

Riveted members must have all parts pinned up and drawn together before riveting up, **D.**

In cases of skew work, or of complicated connections, or of a large number of pieces of one and the same kind, the work shall be set up and fitted together in the shop, sufficiently to insure against any misfit, **D.**

Abutting surfaces at ends of sections of compression members, and ends of members to be framed together, are usually required to be faced.

Rolled Steel. Requirements.

See also Digest of Specifications of International Association for Testing Materials.

TENSILE TESTS.

Specimens of Medium, Soft and Rivet Steel. For tests of full size eye-bars see below.

Ultimate Strength, u, and Elastic Limit, el, in thousands of lbs per sq inch. Elongation, s (stretch), and reduction of area, a, in percentages of original dimensions. Elongation measured in a length of 8 ins

	Medium or "Pin" Steel.				Soft or "Bridge" Steel.				Rivet Steel			
	u	el	s	a	u	el	s	a	u	el	s	a
A, Aa ..	60–70	0.5 u	22	..	52–62	0.5 u	25	..	48–58	0.5 u	26	..
B	..	..	..	..	58–63	30	25	..	51–56	27	26	..
C, Cc ...	60–68	0.5 u	22	..	54–62	0.5 u	25	..	50–58	0.5 u	26	..
D	62–70	0.5 u	22	..	54–62	0.5 u	26	..	48–56	0.5 u	28	..
E ...	..	..	..	..	56–64	0.58 u	27	45	..	..	..	..
Oo ..	60–70	35	22	..	52–62	32	25	..	50–60	30	26	..
P	62–70	33	17	40	52–62	28	25	50	48–56	28	28	56
R ...	60–68	0.5 u	20	..	52–60	{ 0.5 u / 28 }	25	..	48–56	28	28	..
Y	62–70	0.6 u	25	45	56–64	0.6 u	26	50	48–56	..	28	55

Specimens from metal over five-eighths of an inch thick, el = 0.56 u, **Y.**
" " eye-bars, same requirements as for medium steel, **D.**
" " " u = 63, **B.**
" " " over 1.5 ins thick, deduct from el 1 for each one-eighth of an inch; el, min = 20, **C.**
" " " and pins u = 62–70, l = 0.6 u, s = 25, a = 45, **Y.**
" " pins, s = 15, **C, Cc.**
" " " (medium, soft or rivet) s, 5 per cent less, **A.**
" " " (soft) s = 20, **B.**
" " " and rollers, s = 10, **D.**
" " rollers and bearing plates, u = 70–78, s = 22, **Y.**

BENDING TESTS.

In medium steel, specimen to bend through an angle of 180° around a bar of diam = 1 to 1½ × thickness of specimen, without showing fracture on outside of bend; in soft and rivet steel, to bend flat upon itself.

NICKING TEST.

When nicked and bent around a bar of diam = thickness of rod, rivet steel shall show a gradual break and a fine, silky, homogeneous fracture, **D.**

DRIFTING TEST.

Center of hole as in ordinary practice, or 1.5 to 1.87 ins or 2 diams from edge of plate; enlarge to 1.25 to 1.50 diam, **G.**

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

ANGLE TEST.

Angles of all thicknesses must open flat. Angles not over one-half of an inch thick must bend shut, cold, under hammer blows without sign of fracture, **B**.

TEST PIECES.

Minimum section, usually one-half sq inch. Length, min, 8 to 12 ins Tests are usually required for each melt or blow.

TESTS OF FULL SIZE EYE-BARS.

	Ultimate lbs per sq inch min	Elastic limit lbs per sq inch min	Elongation per cent min
A, Aa	5,000 less than small specimen		10 between necks
B	55,000	0.5 ult	12 between necks § 10 between necks ‖
C, Cc	56,000		10 between necks
D	58,000	30,000	12 in 10 ft
E			15
Oo	55,000		12.5 in 15 ft
P	48,000	27,000	14¶ in 10 ft
R	58,000 * 56,000 †	0.5 ult	13 between necks 10 between necks
	48 000 ‡	27,000	15 between necks
Y	58,000	33,000	10 in 20 ft

In general not over 4 per cent of total number of bars in bridge will be tested, **R**; at least 4 per cent, and not less than 3 bars, **B**.

75 per cent of fracture must be silky, the remainder fine granular, **R**.

Break in head shall not be cause for rejection—

(a) if bar develops 10 per cent elongation (12.5 per cent in 15 ft, **Oo**) and the required ultimate strength (ultimate 56,000, **C**, 55,000, **Oo**) and if not more than one-third of all the bars tested break in the head, **A, C, Oo**.

(b) if bar stretches 14 per cent and if a second bar breaks in body and the average stretch of the two bars is not less than 16 per cent, **P**.

Company pays for bars which meet requirements, less scrap value, **G**.

TESTS OF COMPLETED STRUCTURE.

Specified loads, or their equivalent, passed over structure (in railroad bridges at a speed of not over 60 miles per hour, and brought to a stop at any point by means of air or other brakes) or maximum load rested upon structure for 12 hours. After test, structure must return to its original position and must show no permanent change in any part, **C**.

COMPOSITION

Phosphorus, max percentage.
In acid steel, 0.06 to 0.08; in basic steel, 0.04 to 0.06; in castings 0.08.
Sulphur, max percentage, 0.04 to 0.06.

MAXIMUM PERMISSIBLE VARIATION FROM SPECIFIED CROSS-SECTION OR WEIGHT.

2.5 per cent, **G**, except in extra wide plates, **D**, **Oo**, **P**.
In plates over 40 ins wide, in proportion to width, up to 5 per cent in plates 90 ins or wider, **D**.

1.5 per cent; where plates 36 ins and wider form 40 per cent of total, 2 per cent in excess, **Y**.

Long plates, ¼ inch out of line in 20 ft, ½ inch in 40 ft, **R**.

Shapes or plates, 3 per cent short in thickness; plates 80 ins wide, 5 per cent, **R**.

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* † Medium steel. * Bars not over 10 sq ins. † 20 sq ins. Proportional values for intermediate areas. ‡ Soft steel. § In bars not longer than 20 ft between necks. ‖ In bars longer than 20 ft between necks. ¶ Max. 16.

A, Aa, Am B Co **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie

Steel Castings.

Manufacture. Open hearth, **A, Aa, D, P, Y;** acid, **Y;** annealed, **P R, Y.** Carbon, per cent, 0 25 to 0.40, **G.**
Phosphorus, per cent, max, 0.08, **B, Y.**

TENSILE TESTS.

	Size of test piece, ins.	Ultimate strength, lbs per sq inch, min.	Elastic limit, lbs per sq inch, min.	Elongation per cent, in 2 ins min.	Reduction of area per cent
C, D, E, P, R..	½ square or ¾ round	65,000 to 70,000	33,000 or 0.5 ult	10 to 15	20, **P**
Y	¾ round about 6 long	(a) 55,000 to 65,000	...	20	
		(b) 72,000 to 80,000	0.5 ult	15	25

BENDING TEST.

Y (a), for general purposes, bed plates, pedestals, etc , to bend 90°, to a radius = diameter of test piece.
Y (b), for drawbridge rollers, etc.

Rolled Iron.

Requirements in Osborn's specification for highway bridges, **Oo.** Made from puddled iron or rolled from fagots or piles of No. 1 wrought iron scrap, alone or with muck bar. Tensile strength, min, 48,000 lbs per sq inch (50,-000, **R**); yield point, 25,000 lbs per sq inch (26,000, **R**); elongation, 20 per cent in 8 ins; in sections weighing less than 0.654 lb per lineal ft, 15 per cent. Specimens cut from bar as rolled must bend through an angle of 180° under a succession of light blows, when nicked and bent, fracture shall be generally fibrous and free from coarse crystalline spots; not over 10 per cent of the fractured surface shall be granular; specimens heated bright red shall bend through an angle of 180° under a succession of light blows not delivered directly on the bend; must not show red-shortness. In flat and square bars, one-thirty-second of an inch, in round iron 0.01 inch, variation either way in size will be allowed, **Oo.**

Cast Iron.

Tough gray iron, **A, D, E, R;** unless otherwise specified, **A, R.**
Transverse strength. Bar 1 inch square, 12 ins span, to bear 2,500 lbs, center load. Must deflect 0 15 inch before rupture, **G.** Bar 1 inch square, 4.5 ft span, to bear 500 lbs center load, **E, R.**

Phosphor Bronze.

1 inch cube, under compression, elastic limit, 20,000 lbs. Under 100,000 lbs, permanent set max one-sixteenth of an inch, **B.**

Timber.

Sap wood not allowed in more than 10 per cent of the pieces of one kind, and no piece will be accepted showing sap covering more than 0 25 × the width of the piece on any face at any point, or more than half the thickness of any plank at its edge, at any point, **Oo.**

III. LOADS.

1. Vertical Loads.

(Dead and Live Loads and Impact.)

Dead Loads in Steam Railroad Bridges.

Dead load = weight of metal + n lbs per lineal ft of track, **C, D, E, R, Y.** n = 400, **C, D, E**; n = 500, **R**; n = 620, **Y.**
Timber taken at 4 5 lbs per ft, B M, **G.** Ballast, 110 lbs per cu ft, **C.**
Rails, splices, and joints taken at 100 lbs per lineal ft of track, **A, B, C.**
Rails, splices, guard rails, etc, at 160 lbs per lineal foot of track, **P.**
Two-thirds of dead load assumed to be carried by loaded chord, **Y**; in spans less than 300 ft, **B**; in longer spans calculate distribution, **B.**

Dead Loads in Highway and Electric Railroad Bridges.

Iron, 3 33 lbs per lineal ft of bar 1 sq inch **area, Oo.**
Steel, 3 40 " " " " " " 1 sq inch **area, Oo.**
Timber per ft board measure, 4, **Aa**; creosoted, 5, **Oo**; oak, 4 5, **Cc, Oo**; other hard woods, 4 5, **Cc**; yellow pine, 4, **Oo**; spruce and white pine, 3.5, **Cc**; white pine and cedar, 3, **Oo.**
Concrete, etc, lbs per cu ft, 130, **Aa**; stone concrete, 125, **Oo**; cinder concrete, 100, **Oo.** Stone, 150, **Oo**, granite, 160, **Aa.**
Brick, 150, **Aa**; 125, **Oo**; sand, 100, **Oo.** Asphalt, 130, **Aa**; 90, **Oo.**
Rails, fastenings, splices and guard timbers, 100 lbs per lin ft of track, **Aa.**

Live Loads for Steam Railroad Bridges.

THEODORE COOPER'S STANDARD LOADING.

The No. (27 to 50) following the letter E (Engine) in the class designation gives the load, d, on one pair of drivers, in thousands of pounds. In each class, $d = 2b = 40t \div 26 = 10\ U$. Since these ratios are constant for all classes, the stresses, due to any class, are proportional to the class number. The weight of metal, in bridges, is, in each class, about 10 per cent greater than in the class next lighter.

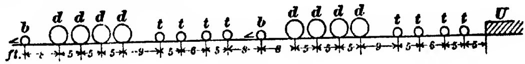


Fig. 1.

TWO CONSOLIDATION LOCOMOTIVES, WITH THEIR TENDERS AND TRAINS.

Class	Load in lbs. on one pair of wheels for each track. Truck (bogie) b	Driver d	Tender t	Train load, lbs. per lin ft. U.
E 27	13,500	27,000	17,550	2,700
E 30	15,000	30,000	19,500	3,000
E 35	17,500	35,000	22,750	3,500
E 40	20,000	40,000	26,000	4,000
E 50	25,000	50,000	32,500	5,000

A, Cooper's loading.
B, Cooper's Class E 50, unless otherwise specified
C, See above.
Y, Cooper's Class E 40.
D, E, P, R, spaces differing slightly from Cooper's. **Loads as below.†**

	b	d	t	U.
D	22,000	50,000	28,000	4,500
E	15,000	35,000	23,000	4,000
P	21,000	44,000	30,000	5,000
R	19,200	{43,200 / 48,000}†	{24,600 / 27,000}†	4,800

† Driver and tender axles unequally loaded. **R.**

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie:

ALTERNATIVE LOADINGS.

Use Fig. 1 or the alternative, Fig. 2 or 3, whichever gives the greater stresses.

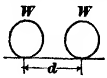


Fig. 2.

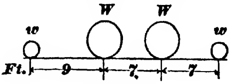


Fig. 3.

Load on one pair of wheels.

Fig. 2. { d = 6 ft; W = W = 50,000 lbs, Above E 40, 60,000 lbs, **C.**
d = 7 ft; W = W = 65,000 lbs, **D.**
d = 7 ft; W = W = 60,000 lbs; U = 4,500 lbs per lin ft, **Y.**

Fig. 3. W = W = 66,000 lbs; w = w = 30,000 lbs, **R.**

Add 30 per cent in figuring floor beams, stringers, hangers, suspenders, and other floor connections. Add 0 to 30 per cent for spans from 100 ft down to 25 ft, **D.**

ON CURVES.

Distribution of live load between the two trusses.

$W = P \frac{m + b}{2 b}$; where W = proportion of live load borne by the outer truss; P = live load at panel considered; m = middle ordinate of entire curve on span; b = dist betw cens of trusses. Make both trusses alike, **B, Y.**

SPECIAL LOADINGS.

For rivets connecting upper flange angles with web in deck girders carrying the floor directly on the top flanges, and in deck spans with wooden floor beams, when distance between trusses exceeds 6 ft, 50,000 lbs on one pair of drivers, distributed equally over three ties or floor beams, **P.**

For floors, the load on a single pair of engine wheels distributed over 4 ties, **B;** over 3 ties, **C.** For trough floors, 60,000 lbs on one pair of wheels, distributed over two troughs, **Y.**

THREE-TRUSS BRIDGES.

In double-track deck spans, all three trusses of equal strength, **C.**

In plate girder bridges of more than one track, center girder figured for 0.75 × the live load, **E.**

FUTURE INCREASE OF LIVE LOADING.

Only 70 per cent (50 per cent, **R**) of the dead load shall be considered effective in counteracting live load stress, **A, R.** Use 1.5 × live load, **E.**

"That the heavier of these engines (see 'C,' under 'Standard Loading,' above) is close to the possible maximum, considering the limitations of the permissible cross-section of existing railroads and the mechanical details of design and proportions, is not improbable. That the economical tendency toward heavier and heavier engines will in the near future reach the heavier class E 50 upon the most important roads is to be expected. The cars will also follow the same tendency for many kinds of traffic, as experience justifies the advance. There are now in use self-dumping coal cars of a nominal capacity of 100,000 pounds, which have, on four axles, a total load of 146,000 pounds (10 per cent increase over nominal capacity) on a wheel base, for two adjacent cars, of 17 ft, 2 ins. These cars on all ordinary bridges produce strains equivalent to those of E 33."—Theodore Cooper.

Members subject to reversal of stress must be so designed that a live load n per cent greater than that specified shall not increase their unit stresses more than n per cent. n = 25, C; 50, B; 100, P.

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

Live Loads for Highway and Electric Railroad Bridges.

Aa, (Am. Bridge Co.) and **Cc**, (Theo. Cooper)	For the Floor and its Supports.			For the Trusses.			
	Concentrated.		Uniform (c).	Per lin ft of single track. (Proportionally for intermediate spans.)		Per sq ft of remaining floor.	
Class.*	Wagon (a), tons	Car (b) on each track, tons	Per sq ft, lbs	Spans up to 100 ft, lbs	Spans 200 ft and over, lbs	Spans up to 100 ft, lbs	Spans 200 ft and over, lbs
A	24	..	100	1,800	1,200	100	80
B	12 or	24	100	1,800	1,200	80	60
C	12 or	18	100	1,200	1,000	80	60
						Up to 75 ft	
D	6	..	80	..	..	80	55
E1	..	24	..	1,800	1,200	..	..
E2........	..	18	..	1,200	1,000	..	..

(a) On two axles, 10 ft cens (and, **Aa**, 5 ft gage); in classes A, B, and C, assumed to occupy a width of 12 ft in single line (or, **Cc**, 22 ft in double line) on any part of the roadway.

(b) On two axles, 10 ft centers.

(c) In classes A, B, and C, on remainder of floor, including footwalks. In class D, on total floor surface.

Oo. Osborn Engineering Co. Highway. May specify any combination of the following loadings, according to character of bridge and of load.

Uniform loads, lbs per sq ft. For spans up to 150 ft, 100 on roadway and 80 on sidewalks, or 80 on both. For spans over 150 ft, 80 or 60 on both.

A steam road roller; axles 11 ft apart, forward roll 4 ft face, two rear rolls 5 ft cens and each 20 ins face. 15,000 or 9,000 lbs on forward roll and 10,000 o 6,000 lbs on each rear roll;

A horse roller, 12,000 lbs on roll, 5 ft face;

A wagon load, 10,000 lbs on two axles, 8 ft apart, 5 ft gage;

Two electric cars on each track; Fig. a

A train of electric cars on each track; Fig. b.

A train of coal cars of 60,000 lbs capacity; Fig. c.

Fig. a

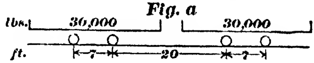
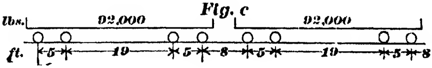


Fig. b

Fig. c



*Class A, city bridges.
" B, suburban or interurban.
" C, heavy country highway.
Class D, ordinary country highway.
" E1, heavy electric railway only
" E2, light electric railway only

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie;

FUTURE INCREASE OF LIVE LOADING. HIGHWAY.

In electric railroad bridges, Class E, only 70 per cent of dead load stress to be considered as effective in counteracting the live load stress, **Aa.** For bridges carrying electric or motor cars, counters so proportioned that a future increase of 25 per cent in the specified live load shall not increase the unit stress more than 25 per cent, **Cc.**

Impact.

$I = S \frac{300}{l + 300}$; where I = impact stress to be added to the live load stress; S = calculated max live load stress; l = length in feet of loaded distance which produces the maximum stress in the member, **A.**

$I = S \left(0.1 + \frac{56.25}{l \times 62.5}\right)$, Mr. G. Bouscaren, C. E.

In Highway Bridges; I = 25 per cent of live load stresses **Aa;** $I = L^2 \div (L + D)$, where L and D = live and dead load stresses, **Oo.**

2. Horizontal Forces.

(Drag, Centrifugal and Wind.)

(a) Longitudinal.

Drag. In bridges for steam and electric railroads, provide for a longitudinal force, at the rails, = 0.2 of the max live load. In double track (**Y**), provide for trains moving either way.

(b) Transverse.

(1) Centrifugal Force.

F = centrifugal force; W = weight of train on bridge; d = degree of curvature = central angle subtended by a chord of 100 ft; v = velocity in miles per hour; c = a coefficient.

A, F = c d W. For d up to 5°, c = 0.03. Deduct from c 0.001 for each degree over 5°. Train on each track.

B, R, F = 0.02 of the live load for each deg of curvature. **B,** up to 5°. Deduct 0.001 for each degree over 5°.

C, F, computed for v = 60 — 3 d on steam railroads, = 40 on electric railroads; force acting 5 ft above base of rail.

D, v = 60.

E, F = force due to that uniform load which would produce the max specified live load bending moment on span; v = 60.

Y, $F = W v^2 d \div 85{,}666$. Up to d = 4°, v = 60. For d over 4°, v = 60 — 2 d. Max train load on each track.

(2) Wind.

(a) ON RAILROAD BRIDGES.

Wind pressure, in lbs per sq ft, = w; in lbs per lin ft = W.

w = either 30 lbs per sq ft on exposed surface of trusses and floor and on that of a train of 10 ft average height, beginning 30 ins above rail base; or 50 lbs per sq ft on exposed surface of trusses and floor; whichever gives the greater stresses, **A, P.**

In truss spans over 200 ft & in plate girders, w = 30 lbs per sq ft of exposed surf of 1 girder and floor, + W on train for lower chord, as below, **B.**

W = L + U. L = pressure in lbs per lin ft on loaded chord, U on unloaded chord. W includes both wind on bridge and wind on train.

Wind on bridge. L = U = 150, **B, C,*** **D, R;** = 200, **E.**

L = 200, on double track 300, acting 8 ft above rail top;
U = 150, on double track 225, acting at cen of chord; } **Y.**

Wind on train. L = 300, **B, D, R,**† **Y;** = 450, **C,**‡ = 400, **E.**

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* In spans over 300 ft, add to U 10 lbs for each additional 30 ft, **C.**
† Acting 7.5 ft above rail, **R.**
‡ Acting 6 ft above rail base. Includes lateral vibrations of trains.

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

A. Wind stress, Sw, in any truss member, **C** (main truss member, **D**; chord or end post, **R**), need be considered only (1) when Sw exceeds 30 per cent, **C** (25 per cent, **D**, **R**), of max stress, S, due to dead and live loads. Then increase section to bring Sw within limit, **C**, **D**, **R**. (2) When Sw, alone or in combination with temperature stress, can balance or reverse S, **C**.

Anchorage. In determining the requisite anchorage for the loaded structure, the train is assumed to weigh 800 lbs per lineal foot, **A**, **B**, **C**, **P**; 600 lbs per lineal foot, **R**.

(b) ON HIGHWAY AND ELECTRIC RAILROAD BRIDGES.

Either 30 lbs per sq ft on the exposed surface of all trusses and floor, + 150 lbs (180, **Oo**) per lineal foot of a train covering the span; or 50 lbs per sq ft on the exposed surface of all trusses and floor; whichever gives the greater stresses, **Aa**, **Oo**.

On each chord, 150 lbs per lin ft, of span, due to bridge, and on the loaded chord 150 lbs per lin ft of span additional due to train. For spans exceeding 300 ft, add 10 lbs on each chord for each additional 30 ft, **Cc**.

Wind stresses (in truss members, **Cc**; in chords and end posts, **Oo**) to be provided for only when the wind stress exceeds 25 per cent of the max dead and live load stresses (of the sum of all other stresses, **Oo**), or when the wind stress (alone or in combination with temperature stress, **Cc**) can (neutralize or, **Cc**) reverse the stress in the member, **Cc**, **Oo**.

IV. STRESSES AND DIMENSIONS.

Effective Span and Depth.

In pin spans, span and depth are measured between centers of pins. In riveted trusses, the span is measured between centers of end bearings, and the depth between centers of gravity of chord sections. In plate girders the span is measured between centers of end bearings, and the depth between centers of gravity of flange areas or over backs of flange angles, whichever is the less. In floor beams the span is measured between centers of trusses, and in stringers between centers of floor beams, **G**.

Limiting Unit Stresses.

Tension.

Net Section. The net section of any tension member or flange is determined by a plane cutting the member square across at any point. The greatest number of rivet holes which can be cut by the plane, or come within an inch of the plane, is deducted from the gross section, **B**. The rupture of a riveted tension member is considered equally probable, either through a transverse line of rivet holes, or through a diagonal line of rivet holes where the net section does not exceed by 30 per cent the net section along the transverse line, **C**, **Cc**.

In deducting rivet holes for net section, their diameter is taken at one-eighth of an inch greater than that of the cold rivet, **G**; for countersunk rivets (**Oo**), one-fourth of an inch greater.

Maximum permissible tensile stresses, in lbs per sq inch.

		Medium Steel	Soft Steel
A, **Aa**,	Under vertical forces only or horizontal forces only	17,000	15,000
	Under vertical and horizontal forces combined	21,000	19,000

B, For "Bridge" (soft) and Rivet Steel, same as Medium Steel under **A**.

D. For soft steel:	For dead load;	live load.
Eye-bars	14,000	9,000
Built sections	12,500	8,500
Counters		8,500

	For dead and live load
Hip suspenders, floor beam hangers, members subject to sudden loading	7,500
Tension flanges of plate girders and rolled beams	9,000
Bracing	12,000

A, Aa, Am B Co; B, B & O; C, Cc, Cooper; D, D L & W; E, Erie

Main members of trusses, flanges and webs of girders and floor beams for double track, floors and girder flanges with ballast floor, add 10 per cent

For medium steel, add 10 per cent.

E. $8{,}000\left(1+\frac{\text{min stress}}{\text{max stress}}\right)$.

Oo. (Highway Bridges.) Medium steel, 22,000; soft steel, 20,000; wrought iron, 18,000

P. M = max calculated stress in member
m = min " " "

Let $r = \frac{m}{M}$; let $k = \frac{1-r}{1+r}$. Then M (1 + k) shall not exceed 15,000

Long hip verticals must have 25 per cent excess strength; short floor beam hangers 50 per cent excess, P.

Y. Soft steel. Chords and web members of trusses, and flanges of plate girders, floor beams and stringers.

Dead load and drag.................................. 16,000
Live load and centrifugal force........................... 8,000

MAXIMUM STRESSES IN TIMBER, LBS PER SQ INCH.

For Highway Bridges, Oo.	Transverse loading	End bearing	Short column *	Bearing across fibre	Shear along fibre
White oak	1,400	1,300	1,000	550	300
Long leaf pine	1,600	1,300	1,000	350	200
White pine	1,100	900	700	200	150
Hemlock	950	850	650	200	100

Extreme fibre stress, in floor beams, max, yellow pine and white oak, 1,200 lbs per sq inch; white pine and spruce, 1,000, Aa, Cc.

Compression.

p = permissible working stress in compression member, in lbs per sq inch.
f = generally the permissible stress in tension member, in lbs per sq inch.
a = a coefficient.
L = length of piece, in ins, between cens of connections.
r = least radius of gyration of cross-section of member, ins.

$$p = \frac{f}{1+\frac{L^2}{r^2 a}}$$

		f	a
Aa.	In medium steel	17,000	11,000
	In soft steel	15,000	13,500
B.	In soft steel	17,000	11,000
C.	See below.		

	Dead load f	Dead load a	Live load f	Live load a
D.	12,000 to 12,500	18,000 to 24,000	8,000 to 8,500	18,000 to 24,000*

E. $f = 8{,}000\left(1+\frac{\text{min stress}}{\text{max stress}}\right)$; a = 36,000 with both ends fixed; a = 24,000 with one end fixed; a = 18,000 with both ends hinged.

Oo. (Highway Bridges.) f = 22,000 for medium steel, 20,000 for soft steel, 18,000 for wrought iron; a as in E, above.

P. f = 15,000; a = 13,500.

R. $f = 6{,}500\left(1+\frac{\text{min stress}\dagger}{\text{max stress}}\right)$; f max = 8,000; a = 40,000 with flat ends; a = 20,000 with pin ends.

When one end is pinned, p = mean of values derived as above.

For angle iron struts, see below.

*Length not over 12 × least side. †Min stress = dead — live load stress

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

Y. Soft steel in chords and web members:

	f	a
For dead load and drag	16,000	18,000
For live load and centrifugal force	8,000	18,000

C, Cc. $p = M - c\frac{L}{r}$.

For medium steel in stationary structures:

	Dead load M	Dead load c	Live load M	Live load c
Chord segments, stiffeners.	20,000	90	10,000	45
For highway bridges	24,000	110	12,000	55
End and other posts	17,000 to 18,000	90 to 80	8,500 to 9,000	45 to 40
For highway bridges	20,000 to 22,000	90 to 80	10,000 to 11,000	45 to 40
Lateral struts, rigid bracing for railroad and highway bridges	13,000	60	8,666	40

For soft steel, deduct 15 per cent; for movable structures, deduct 25 per cent.

R. Angle iron struts.

With flat ends, $p = 9{,}000 - 30\frac{L}{r}$; with pin ends, $p = 9{,}000 - 34\frac{L}{r}$.

In lateral and cross struts, add 30 per cent.

Length of compression members, max. = 40 to 45 diameters, or 100 r to 120 r. In highway bridges, 120 r to 140 r, **Aa**; 100 r to 120 r, **Cc**; 125 r to 150 r, **Oo**, where r = least radius of gyration.

Unsupported width (distance between rivets) of plates subject to compression, max = 45 × thickness, **Oo**; 30 × thickness, **C, Cc, D**; in cover plates of top chords and end posts, 40 × thickness, **C, Cc, D**; or, if a greater width is used, effective section shall be taken as 40 × thickness, **C, Cc.** Distance between supports in line of stress, max = 16 × thickness, **Oo.**

Timber columns, whose length exceeds 12 × their least sides, in highway bridges, **Oo.**

$$\text{Max unit stress} = \frac{C}{1 + \frac{L^2}{1{,}000\,d^2}}$$

where C = 1,000 lbs per sq inch for white oak and long leaf pine, 700 for white pine, 650 for hemlock; L = length of column, between supports, ins; d = least side, ins, **Oo.**

Alternating Stresses.

Total sectional area of member to be made = sum of areas required for both stresses, **A, B.**

Area sufficient to resist either stress plus 0 8 (0 6, **R**; 1 0, **Y**) × the lesser stress, **C, Cc, D, R, Y.**

Permissible working stress, in lbs per sq inch:

$$a = 8{,}000\left(1 - \frac{\text{max stress of lesser kind}}{2 \times \text{max stress of greater kind}}\right), \textbf{ E.}$$

M = max calculated stress of greater kind,
m = max calculated stress of lesser kind,
Let $r = \frac{m}{M}$. Let $k = \frac{2 + r}{2 - r}$. Then M (1 + k) shall not exceed 15,000 lbs per sq in. } **P.**

IN BRIDGES FOR HIGHWAYS AND FOR ELECTRIC RAILROADS.

In Classes A, B, C, and D, members proportioned for that stress which requires the larger section. In Classes E 1 and E 2, make sectional area = sum of areas required for the two stresses, **Aa.** Members designed to resist either stress and given 25 per cent excess of strength in their joints and connections, **Oo.**

A, Aa, Am B Co; **B,** B & O; **C, Cc,** Cooper; **D,** D L & W; **E,** Erie

Shear and Bearing Stresses.

Shear in web plates, max, lbs per sq inch. 10,000, **B;** 4,000, **E;** 5,000, **R;** 13,000, **P;** in medium steel, 10,000. **A, Aa;** in soft steel, 9,000, **A, Aa;** across grain, 6,000, **D;** with grain, 5,000 (net section), **D;** dead load, 10,000, **Y;** live load, 5,000 (gross section), **Y.**

Shear and Bearing on Rivets, Bolts and Pins. Maximum, in lbs per sq inch

	Shear		Bearing	
	Medium	Soft	Medium	Soft
A, Aa, B	12,000	11,000	24,000	22,000
C	9,000	9,000	15,000	15,000
Cc	10,000	10,000	18,000	18,000
Oo	10,000	10,000	22,000	20,000
D, R	7,500	7,500	12,000	12,000

Y, Shear = 0.75 S ; bearing = 1.50 S. S = permissible unit stress in tension.

In field riveting, increase number of rivets 25 per cent, **A, Aa, B, Oo, P;** if machine driven, 10 per cent, **A, Aa, P;** in stringers and floor beams, one-third, **P.** Take 0 66 to 0 80 × stress as above, **C, Cc, D, R, Y.**

In floor connections, use 0.8 × stresses as above, **C, Cc;** add 20 per cent to number of rivets, **Y.**

In wind and sway bracing, use 1.25 to 1 5 × stresses as above, **C, Cc, D, R.**

Rivets with countersunk heads taken at 0.75 × value of rivets with full heads, **P.**

Bearing, on phosphor bronze disks, 5,000 lbs per sq inch, **B.**

Bending Stresses.

Stress in extreme fibres, under bending moments, max, lbs per sq inch.

In pins and bolts, 25,000, **B;** 18,000, **C;** 20,000, **Cc;** 15,000, **D, R;** 16,000, **Y;** in pins, closely packed, 25,000, **Oo;** in medium steel, 25,000; in soft steel, 22,000, **A, Aa, P.** Centers of bearings of strained members taken as points of application of the stresses, **A, Aa, R.** Applied forces considered as uniformly distributed over the middle half of the bearing of each member, **C, Cc.** Bending calculated from distances between centers of bearing, **Oo.**

In rolled beams and channels, 14,000, **P.**

In wooden floor beams, 1,000, **A, B, C, P.**

Compound Stresses.

Compound (axial and bending), maximum, lbs per sq inch.

In end posts of through spans, dead + live + wind + bending, max = 15,000, **R.**

Proportion the member to resist sum of direct stress plus 0.75 bending stress, **A, Aa, B, P, R.** Max = $\dfrac{8,000}{1 + \dfrac{L^2}{40,000\ r^2}}$, where L = length, ins; r = least radius of gyration, ins.

If pins are out of neutral axis of section, max must include the additional stress due to the eccentricity, **R.**

Bending moment at panel points assumed equal and opposite to that at the center, **A, Aa.** If fibre stress due to weight of member alone exceeds 10 per cent of the allowed unit stress on such member, the excess must be considered in proportioning the areas, **C, Cc, R.**

Minimum Dimensions.

Minimum thickness of plates, in railroad bridges, three-eighths of an inch for main members, five-sixteenths of an inch for laterals; in highway and electric railroad bridges, five-sixteenths to one-fourth of an inch. Min diam of rod, three-fourths of an inch, **Oo.** Rods and bars, min section, 1 sq inch, **D, R;** counters 1.5 sq ins, **D, P.** Posts, in pin spans, min width 10 ins, **A.** In posts of through spans, channels min 10 ins, **R.** Angle, min, 3.5 × 3 × five-sixteenths, **B.**

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way

V. PROTECTION.

At Shop. After removing loose scale and rust; 1 coat pure boiled linseed oil, **A, Aa, B, D, E, P, R**; raw linseed oil, **C, Cc**; with 10 per cent in weight of lampblack, **D**; standard red lead paint.* **Y.**

Inaccessible Parts. 2 coats iron ore paint in pure linseed oil, **A, Aa, B, C, Cc, E, R**; standard red lead paint,* **Y**; 1 coat, **D**; 1 heavy coat red lead in raw linseed oil, **P**; 2 coats, 18 lbs red lead in 1 gal boiled linseed oil, **Oo.**

Finished Surfaces. Coated with white lead and tallow. General.

Surfaces in Contact. Painted before joining, **A, Aa, B, C, Cc, R, Y**; with 2 heavy coats red lead in raw linseed oil on each surface, **Y.**

After Erection. 2 additional coats of paint in pure linseed oil, **A, Aa, B, C, Cc**; 2 coats of paint, of different colors, **R**; 2 heavy coats asphaltum varnish, **Y.**

At least 48 hours allowed for drying of each coat, **Y.**

Columns. etc., for 5 ft above surface of street, etc, 2 heavy coats asphaltum varnish; **under sides of bridges**, rest of columns, etc., 2 heavy coats standard white paint;* ballast side of trough floors, 1 part by weight refined Trinidad asphalt and 3 parts straight run coal tar pitch at 300° F, **Y.**

Wherever there is a tendency for water to collect, the spaces must be filled with a waterproof material, **C, Cc.**

First coat paint of graphite or carbon primer, **Oo.**

In highway bridges, upper surfaces of metal floor plates thoroughly coated with asphalt, **Oo.**

VI. ERECTION.

The Contractor is usually required

(1) to unload materials after delivery, to furnish falseworks and appliances, to remove the old bridge, to alter existing bridge seats;

(2) to drill and set anchor bolts, to erect and adjust the superstructure, and sometimes to furnish and place the wooden floor beams;

(3) to remove falseworks and appliances;

(4) to keep the road open for traffic and to avoid interference with any other thoroughfare by land or water and interference with other contractors; to furnish and pay watchmen; to keep material clean and in good order; and to assume all risks of damage to persons or property by reason of storms, floods or other casualties;

(5) to furnish pilot nuts for the protection of the ends of pins in driving.

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(2) DIGEST OF SPECIFICATION FOR COMBINATION RAILROAD BRIDGES.†

By Baltimore and Ohio Railroad Co., 1901.

I. GENERAL DESIGN.

Type, Howe.

Rods of steel, with upset ends; standard nut and lock nut on each end.

Cast iron joint boxes and packing spools.

Steel gib plates from 1.25 ins thick for 1.25 inch rod, to 1.75 ins thick for 2.5 inch rod.

Splices in lower chord generally of steel construction.

II. MATERIAL.

Lumber, Georgia yellow pine, white oak or white pine.

Rolled steel. Open hearth. Ultimate strength 60,000 lbs per sq inch,

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* Standard red lead paint. 5 gals contain 100 lbs pure red lead, 4 gals pure raw linseed oil, one-half pint Japan, free from benzine, **Y.**

Standard white paint. 5 gals contain 42 lbs pure white lead in oil, 21 lbs white zinc in oil, 3 gals pure raw linseed oil, **Y.**

At least 48 hours between coats, and between final shop coat and loading, **Y.**

† To be used only for temporary purposes.

permissible variation, 5 000 lbs; elastic limit, 30,000 lbs; elongation 25 per cent in 8 ins; to bend 180° flat upon itself.

III. LOADS.

Dead Load.

Timber taken at 4.5 lbs per foot board measure. Track 100 lbs per lin ft.

Live Load.

Max intended load + 25 per cent, to provide for increase and impact.

IV. STRESSES AND DIMENSIONS.

Limiting Unit Stresses.

Timber, lbs per sq inch, max	Yellow pine	White pine	White oak
Bending or direct tension	1,200	800	1,000
Columns under 17 diams in length	900	600	750
Columns over 17 diams in length	1,200–18 n	800–12 n	1,000–15 n
where n = length ÷ least thickness; n max = 40.			
Shearing, along grain	150	100	200
Bearing, in direction of grain	1,500	1,000	1,250
Bearing, perpendicular to grain	350	200	500

In columns made up of several sticks placed side by side, and bolted together at intervals, each stick treated as an independent column.

Steel rods, max unit stress = 12,000 lbs per sq inch.

Floor beams designed to carry the dead load and the heaviest engines in service without impact allowance. Reinforce for future increase of loads.

For loadings in excess of that used in designing, reduce speed from 60 to 15 miles per hour, as loads increase to limit of 25 per cent increase of load.

V. PROTECTION.

Steel rods, gibs, etc , 1 coat of paint in shop; 2 after erection.

Wood, at joints and at points of contact, to be painted.

Bolt and rod holes to be saturated with paint.

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(3) DIGEST OF SPECIFICATION FOR ROOF TRUSSES, STEEL FRAMEWORK AND BUILDINGS.

By Baltimore and Ohio Railroad Co., 1901.

I. GENERAL DESIGN.

Made principally of shapes. No adjustable members, except in lateral bracing. Lateral bracing proportioned for a full wind pressure of 30 lbs per sq ft of exposed surface, acting in any direction. Tension members in bracing must in all cases pull directly against a stiff strut. If building is enclosed and the work is exposed to the action of gases, no open spaces less than 1 inch wide left between members, or open pockets inaccessible for painting.

II. MATERIAL.

Min thickness, 0 25 inch. When subject to the action of gases, five-sixteenths inch if building is open; 0.375 inch if enclosed.

III. LOADS.

Snow, 20 lbs per sq ft of horizontal projection of roof surface. Wind, 30 lbs per sq ft, horizontal, in any direction. Min total, 40 lbs per sq ft.

Covering. For roofs, and for sides unless otherwise ordered, corrugated sheets, No. 22 gage, 26 ins wide; corrugations, 2.5 ins; 3 ins for slope of 1 on 2; 6 ins for less slope. Purlins not more than 4 ft apart between centers.

IV. STRESSES.

Columns sustaining roof are considered as hinged at base, unless so anchored as to be absolutely fixed.

Unit stresses, if subject to no moving load other than wind, see **B**, in Digest of Specifications for Steel Bridges, and Digest (2) of B & O R R Specification for Combination Bridges. Stresses given in the latter to be increased 25 per cent.

V. PROTECTION.

Three coats of paint. If exposed to gases, use bridge paint (see **B** in Steel Bridge Specifications); if not, use standard building paints.

SUSPENSION BRIDGES.

Art. 1. Table of data required for calculating the main chains or cables of suspension bridges. Original.

Deflection in parts of the Chord.	Deflection in Decimals of the Chord.	Length of Main Chains between Suspension Piers, in parts of the Chord.	Tension on all the Main Chains at either Suspension Pier, in parts of the entire Suspended Wt. of the Bridge, and its Load.	Tension at the Center of all the Main Chains; in parts of the entire Suspended Wt of the Bridge, and its Load.	Angle of Direction of the Chains at the Piers. Deg	Min.	Natural Sine of the Angle of Direction of the Chains, at the Piers.	Natural Cosine of the Angle of Direction of the Chains at the Piers.
1-40	.025	1.002	5.03	5.00	5	43	.0995	.9950
1-35	.0286	1.002	4.40	4.37	6	31	.1135	.9935
1-30	.0333	1.003	3.78	3.75	7	36	.1322	.9912
1-25	.04	1.004	3.16	3.12	9	6	.1580	.9874
1-20	.05	1.006	2.55	2.51	11	19	.1961	.9806
1-19	.0526	1.007	2.43	2.38	11	51	.2060	.9786
1-18	.0555	1.008	2.30	2.25	12	32	.2169	.9762
1-17	.0588	1.009	2.18	2.12	13	14	.2290	.9734
1-16	.0625	1.010	2.06	2.00	14	2	.2425	.9701
1-15	.0667	1.012	1.94	1.87	14	55	.2573	.9663
1-14	.0714	1.013	1.82	1.74	15	57	.2747	.9615
1-13	.0769	1.016	1.70	1.62	17	6	.2941	.9558
1-12	.0833	1.018	1.57	1.49	18	33	.3180	.9480
1-11	.0919	1.022	1.46	1.37	19	59	.3418	.9398
1-10	.1	1.026	1.35	1.25	21	48	.3714	.9285
1-9	.1111	1.033	1.23	1.12	23	58	.4062	.9138
⅛	.125	1.041	1.12	1.00	26	34	.4471	.8945
1-7	.1429	1.053	1.01	.881	29	45	.4961	.8726
3-20	.15	1.058	.972	.833	30	58	.5145	.8574
⅙	.1667	1.070	.901	.750	33	41	.5547	.8320
1-5	.2	1.098	.800	.625	38	40	.6247	.7808
	.225	1.122	.747	.555	42	0	.6690	.7433
¼	.25	1.149	.707	.500	45	00	.7071	.7071
.3	.3	1.205	.651	.417	50	12	.7682	.6401
⅓	.3333	1.247	.625	.375	53	8	.8000	.6000
.4	.4	1.332	.589	.312	58	2	.8483	.5294
9-20	.45	1.403	.572	.278	60	57	.8742	.4855
½	.5	1.480	.559	.250	63	26	.8944	.4472

These calculations are based on the assumption that the curve formed by the main chains is a parabola; which is not strictly correct. In a finished bridge, the curve is between a parabola and a catenary; and is not susceptible of a rigorous determination. **It may save some trouble in making the drawings** of a suspension bridge, to remember that when the deflection does not exceed about $\frac{1}{10}$ of the span, a segment of a circle may be used instead of the true curve, inasmuch as the two then coincide very closely; and the more so as the deflection becomes less than $\frac{1}{10}$. The dimensions taken from the drawing of a segment will answer all the purposes of estimating the quantities of materials.

The deflection usually adopted by engineers for great spans is about $\frac{1}{12}$ to $\frac{1}{15}$ the span. As much as $\frac{1}{10}$ is generally confined to small spans. The bridge will be stronger, or will require less area of cable, if the deflection is greater, but it then undulates more readily, and as undulations tend to destroy the bridge by loosening the joints, and by increasing the momentum, they must be specially guarded against as much as possible. The usual mode of doing this is by trussing the hand-railing; which with this view may be made higher, and of stouter timbers than would otherwise be necessary. In large spans, indeed, it may be supplanted by regular bridge-trusses, sufficiently high to be braced together overhead as in the Niagara Railroad bridge, where the trusses are 18 ft high; supporting a single-track railroad on top, and a common roadway of 19 ft clear width, below.*

* The writer believes himself to have been the first person to suggest the addition of very deep trusses braced together transversely, for large suspension bridges. Early in 1851 he designed such a bridge, with four spans of 1000 ft each, and two of 500; with wire cables; and trusses 20 ft high. It was intended for crossing the Delaware at Market Street, Philada. It was publicly exhibited for several months at the Franklin Institute, and at the Merchants' Exchange; and was finally stolen from the hall of the latter. Mr. Roebling's Niagara bridge, of 800 ft span, with trusses 18 ft high, was not commenced until the latter part of 1852; or about 18 months after mine had been publicly exhibited.

Another very important aid is found in deep longitudinal floor timbers, firmly united where their ends meet each other. These assist by distributing among several suspender rods, and by that means along a considerable length of main cable, the weight of heavy passing loads; and thus prevent the undue undulation that would take place if the load were concentrated upon only two opposite suspenders. With this view, the wooden stringers under the rails on the Niagara bridge are made virtually 4 ft deep. The same principle is evidently good for ordinary trussed bridges.

Another mode of relieving the main cables is by means of *cable-stays:* which are bars of iron, or wire ropes, extending like *c y*, Fig 1, from the saddles at the points of suspension *c*, *d*, obliquely down to the floor, or to some part of the truss. In the Niagara bridge are 64 such stays, of wire ropes of 1⅝ inch diam; the longest of which reach more than quarter way across the span from each tower. They transfer much of the strain of the wt of the bridge and its load directly to the saddles at the top of the towers, thereby relieving every part of the main cable, and diminishing undulation. They end at *c* and *d*, where they are attached, not to the cables, but to the saddles. They of course do not relieve the *back stays*.

The greatest danger arises from the action of strong winds striking below the floor, and either lifting the whole platform, and letting it fall suddenly; or imparting to it violent wavelike undulations. The bridge of 1010 ft span across the Ohio at Wheeling, by Charles Ellet, Jr, was destroyed in this manner. It is said to have undulated 20 ft vertically before giving way. It had no effective guards against undulation; for although its hand-railing was trussed, it was too low and slight to be of much service in so great a span Many other bridges have been either destroyed or injured in the same way. When the height of the roadway above the water admits of it, the precaution may be adopted of tie-rods, or anchor rods, under the floor at different points along the span, and carried from thence, inclining downward, to the abutments, to which they should be very strongly confined. In the Niagara Railroad bridge 56 such ties, made of wire ropes 1¼ inch diam, extend diagonally from the bottom of the bridge, to the rocks below. They, however, detract greatly from the dignity of a structure

Mr Brunel, in some cases, for checking undulations from violent winds striking *beneath* the platform, used also *inverted* or *up-curving* cables under the floor Their ends were strongly confined to the abuts several ft below the platform; and the cables were connected at intervals, with the platform, so as to hold it down.

Art. 2. The angle *a d g*, or *a c i*, Fig 1, which a tang *d g* or *c i* to the curve at either point of suspension *c* or *d*, forms with the *hor* line *c d* or chord, is called **the angle of direction of the main chains,** or cables, at those points. Frequently the ends *e h*, and *d r*, of the chains, called the **backstays,** are carried away from the suspension piers in straight lines; in which case the angles *l d r*, *e c h*, formed between the hor line *e l* and the chain itself, become the angles of direction of the backstays.

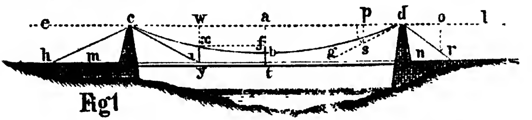


Fig 1

$$\textbf{Sine of angle of direction } a\,d\,g = \frac{\text{Twice the deflection } a\,b}{\sqrt{(\text{twice the deflection})^2 + (\text{Half the chord})^2}}$$

NOTE 1. The direction of the tang *d g* or *c i*, can be laid down on a drawing, thus: Continue the line *a b*, making it twice as long as *a b*; then lines drawn from *d* and *c* to its lower end, will be tangs to the parabolic curve at the points of suspension.

NOTE 2. **If the chord *c d* be not hor,** as sometimes is the case, the angle must be measured from a hor line drawn for the purpose at each point of suspension, as the two angles will in that case be unequal, the piers being of unequal heights.

Tension on all the main chains or cables, together, at either one of the piers, *c* or *d*, Fig 1.

$$= \frac{\text{Half the entire suspended weight of the clear span and its load}}{\text{Sine of angle of direction } a\,d\,g}$$

$$\textbf{or} = \frac{\sqrt{(\tfrac{1}{2}\text{ Span})^2 + (2\text{ Defl})^2}}{2\text{ Deflection}} \times \begin{array}{c}\text{Half the entire sus-}\\\text{pended weight of}\\\text{the clear span and}\\\text{its load.}\end{array}$$

Tension on all the main chains or cables, together, at the middle, *b*, of the span, Fig 1.

$$= \frac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Cosine of angle of direction } a\,d\,g}{\text{Sine of angle of direction } a\,d\,g}$$

$$\textbf{or} = \frac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Half the span}}{\text{Twice the deflection}}$$

The diff between the tensions at the middle, and at the points of suspension, is so trifling with the proportion of chord and deflection commonly adopted in practice, viz, from about $\frac{1}{10}$ to $\frac{1}{15}$, that it is usually neglected; inasmuch as the saving in the weight of metal would be fully compensated for by the increased labor of manufacture in gradually reducing the dimensions of the chains from the points of suspension toward the middle, and in preparing fittings for parts of many different sizes. The reduction has, however, been made in some large bridges with wrought-iron main chains; but in none with wire cables.

Art. 2A. As it is sometimes convenient to form a rough idea at the moment, of the size of cables required for a bridge, we suggest the following rule for finding approximately the area in sq ins of *solid* iron in the wire required to sustain, with a safety of 3,* the weight of the bridge itself, together with an extraneous load of 1.205 tons per foot run of span; which corresponds to 100 lbs per sq ft of platform of 27 ft *clear available* width. This suffices for a double carriage-way, and two footways. The deflection is assumed at $\frac{1}{12}$ of the span; and the wire to have an ultimate strength of 36 tons per solid square inch,

For spans of 100 ft or more,

RULE. Mult the span in feet by the square root of the span. Divide the prod by 100. To the quot add the sq rt of the span. Or, as a formula,

$$\textit{Area of SOLID metal of all the cables; in square ins; for spans over 100 feet} = \frac{\textit{span} \times \textit{sq rt of span}}{100} + \textit{sq rt of span.}$$

For spans less than 100 feet, proportion the area to that at 100 ft.

If a defl of $\frac{1}{10}$ is adopted instead of $\frac{1}{12}$, the area of the cables may be reduced very nearly $\frac{1}{7}$ part.

The following table is drawn up from this rule. The 3d col gives the united areas of all the actual wire cables, when made up, including voids. (Original.)

Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.
	Sq Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins		Sq Ins.	Sq. Ins.
1000	348	446	400	100	128	150	30.6	39.2
900	300	385	350	84	108	125	25.2	32.3
800	254	326	300	69	89	100	20	25.6
700	212	272	250	55	71	75	15	19.2
600	171	219	200	42	54	50	10	12.8
500	134	172	175	36.4	46.7	25	5	6.4

Having the areas of all the actual cables, we can readily find their diam. Thus, suppose with a span of 500 ft, we intend to use four cables. Then the area of each of them will be $\frac{172}{4} = 43$ sq ins, and from the table of circles, we see that the corresponding diam is full 7⅜ ins.

The above areas are supposed to allow for the increased wt of a depth of truss, and other additions necessary to secure the bridge from violent winds, and from undue vibrations from passing loads.

Weight, sufficiently provided for, is of great service in reducing undulation.

We do not think that diagonal horizontal bracing should, as is usual, be omitted under the floor. It may readily be effected by iron rods.

All the cables need not be at the *sides* of the bridge. One or more of them may be over its axis; especially in a wide bridge. One wide footpath in the center may be used, instead of two narrow ones at the sides.

The platform or roadway should be slightly cambered, or curved upward, to the extent say of about $\frac{1}{100}$ of the span.

* The writer must not be understood to advocate a safety of 3 against 100 lbs per sq ft, in addition to the weight of the bridge, in all cases. He believes that limit to be about a sufficient one for a properly designed wire suspension bridge for ordinary travel; but for an important railroad bridge, he would (according to position, exposure, &c) adopt a safety of at least from 4 to 6 against the greatest possible load, added to the wt of the bridge. A train of cars opposes a great surface to the action of side winds; and trains must run during violent storms, as well as during calms; but a large open bridge for common travel is not likely to be densely crowded with people during a severe storm.

Art. 3. Tension on the back-stays, $c\,h$ and $d\,r$, Fig 1. and strains on the piers, or towers, or pillars. If the angle of direction $a\,d\,g$ and the angle $l\,d\,r$, between the back-stays and the horizontal, are equal to each other, the tension on the back-stays will be equal to that on the main cables at the tops of the piers; and the pressure on the piers will be vertical, but if the two angles are unequal, then these tensions and pressures will depend, to a very important extent, upon the manner in which the chains or cables are fixed to, or laid upon, the tops of the piers.

Art. 4. In Figs 2, 3 and 4, the piers $d\,n\,m$ are supposed to be immovable, and the cables $k\,d\,u$, passing over them, rest immediately upon horizontal rollers *which have no other motion than that of revolving about their horizontal axes; the frame to which they are attached being bolted to the top of the pier.* On these rollers the cables slide, when changes of loading or of temperature produce changes in their directions.

In this case the **tension on the back-stays** is equal to that on the main cable See Funicular Machine.

To find the direction and amount of the **pressure on the pier:** from d, Fig 2, 3 or 4, lay off $d\,s$ and $d\,r$, each equal, by scale, to the tension, in tons, on the main chain at d, and from s and r lay it off to v In other words, draw the parallelogram $d\,s\,v\,r$, and its diagonal $d\,v$ Then will $d\,v$ give the *direction* and *amount* of the pressure upon the pier.

When, as in Fig 2, the angles $a\,d\,g$ and $l\,d\,u$ are *equal*, the pressure $d\,v$ will be vertical and equal to the *entire* weight of the clear span and its load.

When, as in Figs 3 and 4, the angles $a\,d\,g$ and $l\,d\,u$ are *unequal*, the pressure $d\,v$ will not be vertical but will incline from d toward the smaller angle.

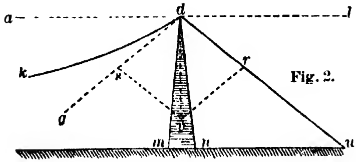


Fig. 2.

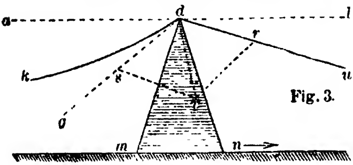


Fig. 3.

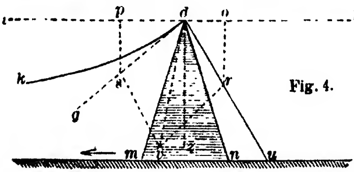


Fig. 4.

When, as in Fig 3, $a\,d\,g$ exceeds $l\,d\,u$, the pressure $d\,v$ will be *less* than the entire weight of the clear span and its load.

When, as in Fig 4, $l\,d\,u$ exceeds $a\,d\,g$, the pressure $d\,v$ will be *greater* than the entire weight of the clear span and its load.

If we suppose symmetrical piers, $d\,n\,m$, to be used in each case, the base $m\,n$ of that in Fig 2, may be much narrower than in the other two figs; because, the direction of $d\,v$ being vertical the pressure has no tendency to overturn the pier. In Fig 2, the masonry of the pier should be laid in the usual horizontal courses, in order that its bed joints may be at right angles to the pressure upon them.

But, in Figs 3 and 4, if the bases were made as narrow as in Fig 2, the lines of direction $d\,v$, of the pressure, would fall outside of them; and the piers would consequently be in danger of overturning. Also, the stones of the masonry, if laid in horizontal courses, would have a tendency to slide on each other. To prevent this, the beds should be at right angles to $d\,v$

In Fig 3 the obliquity of the pressure would tend to slide the base of the pier *outward* as shown by the arrow; but in Fig 4, *inward* This tendency is produced by the *horizontal* component of the force $d\,v$. The amount of this may be found thus, in either fig: From d downward draw a vert line as in Fig 4, and from v a hor one meeting it in z, then $v\,z$, measured by the same scale of tons as before, will give this horizontal force, and $d\,z$ will give the vertical component of the pressure $d\,v$. The effect upon the pier, of the one pressure $d\,v$ is precisely the same as would be produced upon it by one vertical force equal to $d\,z$ and a horizontal one equal to $v\,z$ acting at the same time, as explained under Composition and Resolution of Forces.

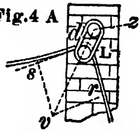
If, in either fig, we draw the vert lines $s\,p$ and $r\,o$, see Fig 4, then $d\,o$, meas'd by the foregoing scale, will give the tons of horizontal pull, and $r\,o$ the vertical pressure, produced on the pier by the back-stay; and $p\,d$ and $p\,s$ will, in like manner, give the corresponding forces produced by the main chain. If we add together $r\,o$ and $p\,s$ they will be found to be equal to $d\,z$, and if we subtract $d\,o$ from $p\,d$, their difference will equal $v\,z$. It is this difference only that tends to slide, or to upset, the pier; the other portions of $d\,o$ and $p\,d$ neutralizing each other in that respect.

The foregoing strains may all be *calculated*, thus:

Horizontal pull inward by the main chain = Tension × Cosine of $a\,d\,g$
" " **outward by the back-stay** = Tension × Cosine of $l\,d\,u$.
Vertical pressure by main chain = Tension × Sine of $a\,d\,g$.
" " " **back-stay** = Tension × Sine of $l\,d\,u$.

Art. 5. If the cables pass freely over a loose pin, d, Fig 4 A, supported by a link L, hanging from the fixed pin z, and capable of moving freely about both of its pins, the tension in the back-stay will, as before, be equal to that in the main cable; and the direction and amount of the strain on the piers will be found in the same way as for Figs 2, 3 and 4; namely: lay off $d\,s$ and $d\,r$, each equal to the tension, and draw the parallelogram $d\,s\,v\,r$. Then will $d\,v$ give the amount and direction of the strain on the piers. This last will, of course, be transmitted through the pins and the link. The amount of tension on the link will be given by the length of $d\,v$; and the link (being free to move) will be in line with this tension. The shearing strain on each pin is also given by $d\,v$.

Fig.4 A



Art. 6. But if the ends of the cable and back-stay, Figs 4 B, 4 C and 4 D, at the top of the pier, be *made fast* to a truck or wagon which is supported by rollers on a smooth platform on top of the pier, the axles of the rollers being fixed in the truck, then the strain on the back-stay will not be the same as that on the cable, unless the angles $a\,d\,g$ and $l\,d\,u$ are equal, as in Fig 4 B.

If $a\,d\,g$ exceeds $l\,d'\,u$, as in Fig 4 C, the strain on the back stay will be less than that on the cable, and vice versa (Fig 4 D).

But, in either case, if the top of the pier is horizontal, as is usually the case, the horizontal components of the strains on the cables and on the back-stays, will be equal, and will thus counteract each other, and there will consequently be no horizontal or oblique strain on the pier. That is, the strain on the pier will be vertical.

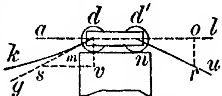


Fig.4 B

To find the amount of the tension on the back-stay, and of the pressure on the pier: on $d\,g$ in either Fig 4 B, 4 C or 4 D, lay off $d\,s$, equal, by scale, to the tension on the cable at d. Draw $d\,v$ perpendicular to the surface $m\,n$ on which the rollers rest. We assume that $m\,n$ is horizontal, as is generally, *but not necessarily*, the case and $d\,v$, therefore, vertical. Draw $s\,v$ horizontal, or parallel to $m\,n$.*

Then $s\,v$ will give the horizontal pull of the main cable on the wagon, and $d\,v$ will give the vertical pressure of the wheel d on the tower (to which that of the wheel d' has yet to be added). From d' lay off $d'\,o$ horizontal, and equal to $s\,v$ and draw $r\,o$ vertically. Then $d'\,r$ will give the amount of the pull on the back stay, and $r\,o$ will give the vertical pressure of the wheel d' on the pier; which must be added to $d\,v$ for the *total* vertical pressure.

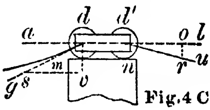


Fig.4 C

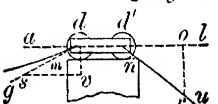


Fig.4 D

Or the various strains may be *calculated*, thus:

Horizontal pull $s\,v$ or $d'\,o$ at the top of the pier } = Horizontal pull at middle of span = Tension $d\,s$ in cable at d × Cosine of $a\,d\,g$.

Strain $d'\,r$ in back-stay } = Horizontal pull $s\,v$ or $d'o$ at top of pier, or at middle of span ÷ Cosine of $l\,d'\,u$.

Pres on pier, perp to surf on which the rollers rest } $= d\,v + r\,o = \left(\text{Tension } d\,s \text{ on main cable at } d \times \text{Sine of } a\,d\,g\right) + \left(\text{Tension } d'\,r \text{ on back stay} \times \text{Sine of } l\,d'\,u\right)$

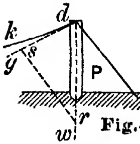


Fig.4 E

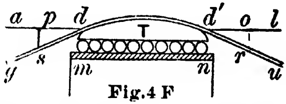


Fig.4 F

Art. 7. When, as is sometimes the case in light bridges, the piers are posts, P Fig 4 E, of wood or iron, hinged at the bottom, and having the cables and back-stays firmly fixed to their tops; from d draw $d\,s$, equal, by scale, to the tension on the main cable at d; and $d\,w$ toward the foot of the post. From s draw $s\,r$ parallel to the back stay, and meeting $d\,w$ in r. Then will $s\,r$ give the strain in the back stay, and $d\,r$ will give the amount and direction of the pressure upon the post.

Art. 8. As in the Niagara bridge, the cables often merely rest upon movable trucks, or saddles, T Fig 4 F, curved on top to avoid sudden bends in the cables and resting upon loose rollers which lie upon a thick horizontal iron plate bolted to the top of the pier, and are free to move horizontally. In such cases the angles $a\,d\,g$ and $l\,d'\,u$ are made equal; so that the pulls

* The lines $a\,l$ and $s\,v$ must be drawn *parallel to the surface* m n *on which the wagon rests*, whether said surface be horizontal or inclined.

, s and d' r are equal, as are also their horizontal components p d and d' o ; and the pressures on the pier are vertical; and if changes of temperature or of loading produce slight changes in the angles a d g and i d' u, the truck will (by reason of the inequality thus brought about between the hori zontal components) move far enough to restore the equality between the angles, and between the horizontal components, and consequently the pressure upon the pier will at all times be vertical.

Art. 9. To find, approximately, the length of a main chain c b d; Fig. 1; having the span c d, and the middle defl a b. See preceding table, Art 1.

Half length of main chain $= \sqrt{1\frac{1}{3}\ (\text{defl}^2) + (\frac{1}{2}\ \text{chord})^2}$.

In Menai bridge the chord c d is 579.874 ft. and the defl is 43 ft.

According to the above formula, the entire length is 588.3 feet. By actual measurement the chain is precisely 590 feet. The approximate rule below gives 589 764 ft.

NOTE. The lengths obtained by this rule are only approximate, because the calculation is based upon the supposition that the chains form a parabolic curve; whereas, in fact, the curve of a finished bridge is neither precisely a parabola, nor a catenary, but intermediate of the two.

The following simple rule by the writer is quite as approximate as the foregoing tedious one when, as is generally the case, the defl is not greater than $\frac{1}{12}$ of the chord, or span.

Length of main chain when defl does not exceed one-twelfth of the span = chord + .23 defl.

Art. 10. To find, approximately, the length of the vert suspending rods x y, &c, Fig 1; assuming the curve to be a parabola.

Let x, Fig 1, be any point whatever in the curve; and let x w be drawn perp to the chord c d; and x f perp to a b; then in any parabola, as $a\ c^2 : a\ n^2 :: a\ b : b\ f$. And b f thus found, added to b t, (which is supposed to be already known, being the length decided on for the middle suspending rod,) gives x y, the length of rod reqd at the point x, and so at any other point.

If b f thus found be taken from the middle deflection a b, it leaves w x; and thus any deflection w x of the main chain or cable, may be found when we know its hor dist, a w, from the center, a, of the span.

In the foregoing rule, the floor of the bridge is supposed to be straight; but generally it is raised toward the center; and in that case, the rods must first be calculated as if the floor were straight and the requisite deductions be made afterward. When it rises in two straight lines meeting in the center, the method of doing this is obvious. When an arc of a circle is used, its ordinates may be calculated and deducted from the lengths obtained by this rule. Or, having drawn the curve by the rule for drawing a parabola, the dimensions can be approximated to by a scale. The adjustments to the *precise* lengths must be made during the actual construction of the bridge, by means of nuts on their lower screw-ends. The rods require, therefore, only to be made *long enough* at first.

The towers, piers, or pillars, which uphold the chains or cables, admit of an endless variety in design. According to circumstances, they may consist each of a single vertical piece of timber, or a pillar of cast or wrought iron; or of two or more such, placed obliquely, either with or without connecting pieces; like the bents of a trestle, Or they may be made (with any degree of ornamentation) of cast iron plates, as in iron house-fronts. Or they may be of masonry, brick, or concrete; or of any of these combined.

Each of the suspending-rods, through which the floor of the bridge is upheld by the main chains, requires merely strength sufficient to support safely the greatest load that can come upon the interval between it and half-way to the nearest rod on each side of it; including the wt of the platform, &c, along the same interval.

In anchoring the backstays into the ground, it is necessary to secure for them a sufficiently safe resistance against a pull equal to the strain, upon the backstay.

As to the anchorage of the cables below the surface of the ground, natural rock of firm character is the most favorable material that can present itself. When it is not present, serious expense in masonry must be incurred in large spans, in order to secure the necessary weight to resist the pull of the cables. Our Figs 4½ give ideas of the modes most frequently adopted. For a very small bridge, such as a short foot bridge, for instance, the backstays may simply be anchored to large stones, t, Fig A, buried to a sufficient depth. Or, if the pull is too great for so simple a precaution, the block of masonry, m m, may be added, enclosing the backstay. A close covering of the mortar or cement of the masonry has a protecting effect upon the iron.

To avoid the necessity for extending the backstays to so great a dist under ground, they are usually curved near where they descend below the surface, as shown at B, D, and E; so as sooner to reach the reqd depth. This curving, however, gives rise to a new strain, in the direction shown by the arrows in Figs B and D. The nature of this strain, and the mode of finding its amount, (knowing the pull on the backstay,) are very simple; and fully explained under the head of Funicular Machine. The masonry must be disposed with reference to resisting this strain, as well as that of the direct pull of the backstay. With this view, the blocks of stone on which the bend rests should be laid in the position shown in Fig D; or by the single block in Fig B. Sometimes the bend is made over a cast-iron chair or standard, as at x, Fig F, firmly bolted to the masonry.

Fig E shows the arrangement at the Niagara railway bridge of 821⅓ ft span. The wire backstays end at c c; and from there down to their anchors, they consist of heavy chains; each link of which is composed of (alternately) 7 or 8 parallel bars of flat iron, with eye ends, through which pass bolts Each of the 7 bars of each link is 1.4 ins thick, by 7 ins wide, near the

lowest part of the chain; but they gradually increase from thence upward, until at c, c, where they unite with the wire cable, the sectional area of each *link* is 93 sq ins. These chain backstays pass in a curve through the massive approach walls. (28 ft high,) and descend vertically down shafts s, s, 25 ft deep in the solid rock. Here they pass through the cast-iron anchor-plates, to which they are confined below by a bolt 3½ ins diam. The anchor plates are 6½ feet square, and 2½ ins thick; except for a space of about 20 ins by 26 ins, at the center where the chains pass through, where they are 1

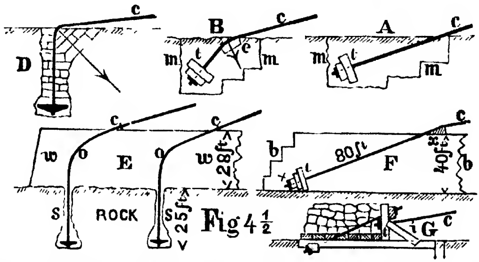


Fig 4½

foot thick. Through this thick part is a separate opening for each bar composing the lowest link. From this part also radiate to the outer edges of the lower face of the plate, eight ribs, 2½ ins thick. The shafts s, s, have rough sides, as they were blasted; and average 3 ft by 7 ft across; except at the bottom, where they are 8 ft square. They are completely filled with cement masonry, with dressed beds, well in contact with the sides of the shafts; and thoroughly grouted, thus tightly enveloping the chains at every point, as does also the masonry of the approach wall *w w*; which extends 28 ft above ground, and is 6 ft thick at top, and 10½ ft thick at its base on the natural rock.

D, Figs 4½, shows a mode that may be used in most cases, for bridges of any span. The depth and the area of transverse section of the shaft, and consequently the quantity of masonry in it, will depend chiefly upon whether it is sunk through rock, or through earth. If through firm rock, then if its sides be made irregular, and the masonry made to fit securely into the irregularities, much reliance may be placed upon it to assist the weight of the masonry in resisting the pull on the backstays. Earth also assists materially in this respect.

F is the arrangement in the Chelsea bridge of 333 feet span, across the Thames, at London; Thos. Page, eng. The space from one wall *b b*, to the opposite one, is 45 feet; and is built up solid with brickwork and concrete; except a passage-way 4 ft wide, and 5 ft high, along the backstay; and a small chamber behind the anchor-plates. It rests chiefly on piles.

The arrangement by Mr Brunel, in the Charing Cross bridge, London,* is very similar. In it also the entire abutment rests on piles; and is 40 ft high, 30 ft thick, and solid, except a narrow passage-way along the chains. The backstays extend into it 60 ft. Span 676 feet. Defl 50 feet.

G is intended merely as a general hint, which, variously modified, may find its application in the case of a small temporary, or even permanent bridge; for the number of pieces, *t*, *t*, &c, may be increased to any necessary extent; and they may be made of iron or stone, instead of wood.

In order that the backstays may be accessible, they are frequently carried through openings left in the masonry for the purpose. Thus, the masses, *m m*, of masonry, at A and B, Figs 4½, instead of being made solid, may consist of two parallel walls, between which the backstay may pass; and the anchor-stones, or anchor-plates, will extend across the space between the walls, and have their bearings against the ends of the walls. In D, E, and F, the cable may be supposed either to be tightly surrounded by the masonry and grouted to it, or else to be surrounded by a cylindrical passage-way like a culvert, so as to be at all times accessible.

Soft friable stone must be carefully excluded from such parts of the anchorage as are most directly opposed to the pull of the backstays.

If blocks of stone large enough for securing good bond are not procurable, heavy T-rails, bars of iron, or I-beams, may be advantageously introduced for that purpose.

The masses must be founded at such a depth as not to slide by the yielding of the earth in front of them.

For safety, it is well to disregard the effect of friction in diminishing the tension on the backstay, and to regard that tension as continuing uniform throughout the backstay to its end, even when the backstay is curved and imbedded in the masonry, as at E, Figs 4½.

The side parapets should be high and stout, so as to act as stiffening trusses, and should not be restricted to service as mere hand-rails or guards. As a rule of thumb their depth may be made $= \frac{1}{2}\sqrt{\text{span}}$, provided the depth be not less than that required for a hand-rail. The parapets should be stoutly constructed, with special attention to the strength of their joints, for these are exposed, by the undulations and lateral motions of the bridge, to violent deranging forces in all directions.

* Removed to Clifton, England, in 1863, and replaced by an iron truss railway and foot bridge.

RIVETS AND RIVETING.

The **weights** in the **following table** of course include the head: but the **lengths,** as usual, are taken "under the head," or are those of the shanks only. In practice, discrepancies of 5 or 6 per ct in wt may be expected.

Length of Shank. Ins.	Diameters of Rivets in inches.							
	⅜	½	⅝	¾	⅞	1	1⅛	1¼
	Weight of 100 Rivets, in pounds.							
½	3.0	8.5	....	....	....	....	....	.....
¾	3.8	9.9	17.3	....	....	....	....	.....
1	4.6	11.2	19.4	25.6	38.9	....	....	.....
¼	5.4	12.6	21.5	28.7	43.1	65.3	91.5	123
½	6.2	13.9	23.7	31.8	47.3	70.7	98.4	133
¾	6.9	15.3	25.8	34.9	51.4	76.2	105	142
2	7.7	16.6	27.9	37.9	55.6	81.6	112	150
¼	8.5	18.0	30.0	41.0	59.8	87.1	119	159
½	9.2	19.4	32.2	44.1	64.0	92.5	126	167
¾	10.0	20.7	34.3	47.1	68.1	98.0	133	176
3	10.8	22.1	36.4	50.2	72.3	103	140	184
¼	11.5	23.5	38.6	53.3	76.5	109	147	193
½	12.3	24.8	40.7	56.4	80.7	114	154	201
¾	13.1	26.2	42.8	59.4	84.8	120	161	210
4	13.8	27.5	45.0	62.5	89.0	125	167	218
¼	14.6	28.9	47.1	65.6	93.2	131	174	227
½	15.4	30.3	49.2	68.6	97.4	136	181	236
¾	16.2	31.6	51.4	71.7	102	142	188	244
5	16.9	33.0	53.5	74.8	106	147	195	253
¼	17.7	34.4	55.6	77.8	110	153	202	261
½	18.4	35.7	57.7	80.9	114	158	209	270
¾	19.2	37.1	59.9	84.0	118	163	216	278
6	20.0	38.5	62.0	87.0	122	169	223	287
½	21.5	41.2	66.3	93.2	131	180	236	304
7	23.0	43.9	70.5	99.3	139	191	250	321
½	24.6	46.6	74.8	106	147	202	264	338
8	26.1	49.4	79.0	112	156	213	278	355
9	29.2	54.8	87.6	124	173	234	306	389
10	32.2	60.3	96.1	136	189	256	333	423
11	35.3	65.7	105	148	206	278	361	457
12	38.4	71.2	113	161	223	300	388	491

The diam of rivets for bridge work is from ½ to 1 inch; usually ⅝ to ¾; and for plates more than .5 inch thick, it is about 1.5 times the thickness; and for thinner ones about twice; but these proportions are not closely adhered to. **The common form of rivets** as sold is shown at R, Figs 3, a **head** and the **shank** in one piece; and S shows the same when after being heated white hot it is inserted into its hole, and a second head (conical) formed on it by rapid hand-riveting as it cools. **When longer than about 6 ins** they are cooled near the middle before being inserted, lest their contraction in cooling should split off their heads. The hemispherical heads often seen, called **snap heads,** are formed by a machine. **The two heads alone require** about as much iron as 3 diams length of shank. **Length of a head** = about 1 diam of shank; and its **width** about 2 diams of shank.

Riveting of Steam and Water Tight Joints.

Joints for boilers and water-tight cisterns are usually proportioned about as per the following table by Fairbairn; and are made as shown either by Fig 1, or Fig 2. Fig 1 is called a **single-riveted,** and Fig 2 a **double-riveted lap-joint.** The dist *a a*, or *c c*, is the lap.

Mr Fairbairn considers the strength of the single-riveted lap-joint to be about .56; and that of the double-riveted, about .7 that of **one** of the full unholed

plates, when both joints are proportioned as in his following table. But some later experimenters consider about .5 and .6 as nearer the correct average. Experiments on the subject are quite conflicting; and it is plain that no one set of proportions can precisely suit all the different qualities of plate and rivet iron. With fair qualities of both there is every reason to rely upon .5 and .6 (or about one-seventh part less than Fairbairn's assumption) as safe for practice. These proportions include **friction** (Art 4), without which they would be *about* .4 and .5.

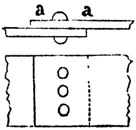


Fig 1.

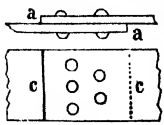
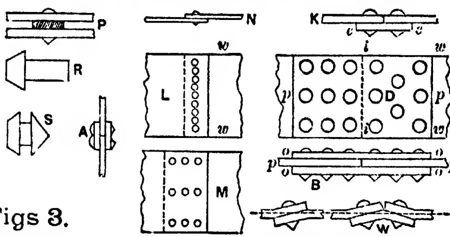


Fig 2.

Fairbairn's table for proportioning the riveting for steam and water-tight lap-joints.

Thickness of each plate	Diameter of rivets.	Length of shank before driving.	From center to center of rivets.	Lap in single riveting.	Lap in double riveting.
Ins.	Ins.	Ins.	Ins	Ins.	Ins.
3-16	⅜	⅞	1¼	1¼	2 1/16
¼	½	1⅛	1½	1½	2½
5-16	⅝	1⅜	1⅝	1⅞	3 1/16
⅜	¾	1⅝	1¾	2	3⅜
½	13-16	2¼	2	2¼	3¾
⅝	15-16	2¾	2½	2¾	4⅝
¾	1⅛	3¼	3	3¼	5½

Riveting of iron girders, bridges, &c.



Figs 3.

Art. 1. The subject of riveting is abstruse, and involved in much uncertainty; and experimental results are very discrepant. We here propose merely to confine ourselves to what is considered the best joint; and for safety we shall omit friction; see Art 4. In girder and bridge work the lap-joints above described are seldom used. Instead of them, the plates *p*, Figs 3, to be joined, are butted up square against each other, thus forming a **butt-joint,** *i i*, Fig D; and are united by either a single **covering-plate, cover, wrapper, fish-plate,** or **welt** *e e*, Fig K; or the best of all by two of them, as at A, or *o o*, *o o*, Fig B. In what follows, the term *plate* never includes the *covers.* The single cover, like the lap-joint, allows both plates and cover to bend under a strong pull, somewhat as at W, thus weakening them materially; whereas the double cover *o o*, *o o*, Fig B, keeps the pull directly along the axis of the plates, thus avoiding this bending tendency. It also brings the rivets into double shear, thus doubling their strength. When there is but one cover, it should be at least as thick as a plate; and when there are two, experience shows that each had better be about *two-thirds* as thick as a plate, although theory requires each to be but *half* as thick as a plate.

The length w w of covers across the joint is equal to that of the joint.

Butts require twice as many rivets as laps, because in the lap each rivet passes through both the joined plates; and in the butt through only one.

The rivets and plate on one side only (right or left) of the joint-line *i i* of any properly proportioned **butt-joint** D, represent the full strength of the joint, inasmuch as those on one side pull in one direction, against those on the other side, which pull in the opposite direction. Therefore in designing such joints we need keep in mind only those on one side, as is done in what follows. Thus a single, double, or triple-riveted **butt-joint** D implies one, two, or three rows of rivets **on each side** of the joint-line *i i*, and parallel to it. In a properly proportioned *lap* the strength is as **all** the rivets, because one-half of them do not pull against the other half, but one end of **every** rivet pulls in one direction, and its other end in the opposite direction.

The net iron, net plate, or net joint, is that which is left **between** the rivet holes, and **outside** of the two outer ones, all on a straight line drawn through the *centers* of the holes of *one* row. Its width and area are called the net ones of the joint. That between *other* rows does not increase the strength.

In Figs 3, N, and K, the rivets are in **single shear**, while those in A and B are in **double shear.**

Art. 2. Bridge-joints are not required to be steam or water-tight like those of boilers or cisterns; and, therefore, by increasing the breadth of the overlap, or the length of the covers, the rivets may be placed in several rows behind each other, as the 3 rows of 3 rivets each in M and D, instead of only one row of 9 rivets, as in L. By this means, without losing any of the strength of the 9 rivets, or of the **net** iron, we may narrow the width of the plate to an extent equal to the combined diams (6 in this case) of the holes thus dispensed with in the one row. Moreover, by using more than one row we lessen the weakening effect shown at W. This mode of placing the rivets directly behind each other in several rows, as at M, and at the left-hand half of Fig D, constitutes Mr Fairbairn's **chain riveting;** but the joint will be somewhat stronger if the rivets are placed in **zigzaging** order, as in the right-hand half of Fig D.

The dist apart of the rows from cen to cen should not be *less* than 2 diams. It is questionable to what extent this increase in the number of rows may be carried without an appreciable loss of strength in the rivets consequent upon the impossibility of quite equalizing the strains on the separate rows. But it is probable that if we do not exceed 2 or 3 rows in laps, or the same number *on each side* of the joint-line in butts, we may in practice assume that each row, and each rivet, is nearly equally strained.

Rivet-holes are usually of about one-sixteenth inch greater diam than the original rivet, so as to allow the hot rivet to be easily inserted The subsequent hammering swells the diam of the rivet until it fills the hole. We may either take this increased diam of rivet into consideration, as we have done, in calculating its shearing and crippling strength, as explained farther on, or with reference to increased safety we may omit it. **Drilled rivet-holes** are said to be better than punched ones, as the drilling does not injure the iron around them; but on the other hand their sharper edges are said to shear the rivets more readily. Hence, such edges are sometimes reamed off. Both these points are, however, disputed; and both modes are in common use.

The dist from the edge of a hole to the end of a plate or cover should not be *less* than about 1.2 diams, to prevent the rivets from tearing out the end of the plate; nor nearer the side edge of a plate than half the clear dist between two holes as given by the Rule in Art 5. The first is rather more than Fairbairn directs.

Rivet holes weaken the net iron left between them, not only by the loss of the part cut out, but either by disturbing the iron around them, or perhaps *by changing the shape of the net line of fracture, which may not then resist* tension as well as while it was a continuous straight line. Some deny both cause and effect entirely, each party basing its opinion on experiments. But the mass of evidence seems to the writer to show that the net iron loses on an average about one-seventh of the strength due to the net width With a view to safety, which we consider to be of paramount importance, we shall in what follows assume (until the question is definitely settled) that there **is** such a loss of strength in the net iron.

Riveted joints for resisting compression should depend, not as might be supposed upon their butting ends, but upon either the shearing or the crippling strength of the rivets; for contraction or bad work may throw the

pressure on the rivets. **Machine riveting** is somewhat stronger than that done (as is assumed in our examples) by hand. **The thickness of plates** used in girders, tubular bridges, &c, is usually .25 to .5 inch; with thicker ones up to 1 inch sparingly in large ones. **A packing piece,** as the shaded piece in P, is one inserted between two plates to prevent their being bent or drawn together by the rivets.

Art. 3. A riveted joint may yield in three ways after being properly proportioned, namely, by the shearing of its rivets; or by the pulling apart of the net plate between the rivet holes; or by the **crippling** (a kind of compression, mashing, or crumpling) of the plates by the rivets when the two are too forcibly pulled against each other. It also compresses the rivets themselves transversely, **at a less strain than the shearing one;** and this partial yielding of both plates and rivets allows the joint to **stretch,** and may thus produce injurious unlooked-for strains in other parts of a structure, considerably before there is any danger of actual fracture. Or in steam and water joints it may cause leaks, without farther inconvenience, or danger For a long time this crippling had entirely escaped notice, and it was supposed that the only important point in designing a riveted joint was that the tensile strength of the net plate, and the shearing strength of the rivets should be equal to each other.

The crippling strength of a joint is as the number of rivets, in a lap, or the number on one side of the joint-line in a butt × diam × thickness of joined plate. This product gives the crippled area of the joint. We shall here call the diam × thickness of plate, the **crippling area** of a rivet. If there are 2 or more plates (not covers) on top of each other at one joint, their united thickness is used for finding the crippling area. **The ultimate crippling unit,** by which the above product is to be multiplied for the actual ultimate crippling strength of the joint, may be safely taken at about 60000 lbs, or 26.8 tons, per sq inch.

The diam of a rivet in ins to resist safely a given single-shearing force is found thus: Mult the shearing force by the coef of safety, that is by the number, 3, 4, or 6, &c, denoting the required degree of safety. Call the product g. Mult the ultimate shearing strength per sq inch of the rivet-iron, by the decimal .7854. Call the product b. Divide g by b. Take the sq rt of the quotient. The shearing force and the shearing strength must *both* be in either lbs or tons.

Or by a formula,

$$\textbf{Diam in ins} = \sqrt{\frac{\text{Shearing force} \times \text{coef of safety}}{\text{Ult shearing strength per sq inch} \times .7854}}$$

If the rivet is to be double-sheared, first mult only half the shearing force by the coef of safety. Then proceed as before.

Or, near enough for practice, mult the diam in single shear by the decimal .7.

The ultimate shearing unit for average rivet-iron may be taken at about 45000 lbs, or 20.1 tons per sq inch of circular sheared section.

Table of ultimate single shearing strength of rivets.

(market sizes), in single shear; at 45000 lbs or 20.1 tons per sq inch.

This table is not to be used when as in our "Example," Art 5, the **crippling** strength of the rivet governs the strength of the joint.

If the rivet is in double shear it will have twice the strength in the table.

For the diam in double shear to equal the strength in the table, mult the diam in the table by the decimal .7; near enough for practice; strictly, .707.

Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam Ins.	Diam. Ins.	lbs.	Tons.
1/8	.125	552	.246		.562	11183	4.99	1	1.000	35343	15.8
	.187	1242	.554	5/8	.625	13806	6.16		1.062	39899	17.8
1/4	.250	2209	.986		.687	16705	7.46	1 1/8	1.125	44731	20.0
	.312	3452	1.54	3/4	.750	19880	8.88		1.187	49838	22.2
3/8	.375	4970	2.22		.812	23[illegible]2	10.4	1 1/4	1.250	55224	24.6
	.437	6765	3.02	7/8	.875	27060	12.1		1.312	60885	27.2
1/2	.500	8836	3 94		.937	31064	13.9	1 3/8	1.375	66820	29.8

The tensile strength of a properly proportioned joint is equally as either the sectional area of the **net** plate (not covers) across the centers of only **one** row of rivets; or as the shearing or the crippling (as the case may be) areas of **all** the rivets in a lap, or of all the rivets **on one side** of the joint-line in a butt. The tensile strength of fair quality of plate iron, before the rivet holes are made, averages about 45000 lbs, or 20.1 tons per sq inch; but we shall for safety assume, as stated in Art 2, that the making of the holes reduces the strength of the **net** iron that is left about one-seventh part, or to 38500 lbs, or 17.2 tons per sq inch.

Rem. Even this is considerably too great for laps, or for butts with one cover, owing to the weakening of the iron in such by the bending shown at W, Figs 3. But we are not speaking of such.

Art. 4. The friction between the plates in a lap, or between the plates and the covers in a butt, produced by their being pressed tightly together by the contraction of the rivets in cooling, adds much to the strength of a joint while new, perhaps as much as 1.5 to 3 tons per sq inch of circ section of all the rivets in a lap or of all on one side of a single-cover butt; or 3 to 6 tons of all on one side of a double-cover butt. In quiet structures, this friction might continue to exist, either wholly or in part, for an indefinite period; but in bridges, &c, subject to incessant and violent jarring and tremor, it is probably soon diminished, or entirely dissipated. Hence good authorities recommend not to rely on it, and it is, therefore, omitted in what follows.

Art. 5. We now give rules for finding the number of rivets required for **a double cover butt-joint** (the only kind of which we shall treat), and their clear or net distance apart. This dist + one diam is the **pitch** of the rivets, or their dist from center to center. The principle of the rule will be explained further on, at Art 7.

First, select a diam of rivet either equal to or greater than .85 times the thickness of the plate. In practice they are generally 1.5 times for plates ½ inch or more thick; and 2 for thinner than ½ in.

Second, mult the greatest total pull in pounds that can come upon the entire joint by the coef (3, 4, or 6, &c) of safety, and call the product p.

Third, multiply the crippling area of the rivet (that is, its diam $\times$ the thickness of plate) by 60000. The prod is the ult crippling strength of a rivet. Call it m.

Fourth, divide p by m. The quotient will be the number of rivets to sustain the given pull with the reqd degree of safety.

Then, **the clear distance apart** will be

$$\frac{\text{Number of rows} \times \text{Diam} \times 60000}{38500}$$

Fifth. The clear dist from either end hole of a row to the side edge of the plate, should be not less than half the clear dist between two rivets in a row.

Example. A double-cover butt-joint in .5 inch thick plate is to bear an actual pull of 33750 lbs, with a safety of 4; or not to break with less than 33750 $\times$ 4 = 135000 lbs. How many rivets must it have; and how far apart must they be?

First, Here .85 times the thickness of the plate is .5 $\times$ .85 = .425 inch; therefore, our rivets must not be less than .425 inch in diam; but we will take .75 inch diam.

Second, The greatest pull $\times$ coef of safety = 33750 $\times$ 4 = 135000 lbs = p.

Third, The crippling area of a rivet $\times$ 60000 = .75 $\times$ .5 $\times$ 60000 = 22500 = m.

Fourth, $\frac{p}{m} = \frac{135000}{22500}$ = 6 rivets required on each side of the joint-line.

And the **clear space** or net width between them will be, **if the 6 rivets are in one row:**

$$\frac{\text{Diam} \times 60000}{38500} = \frac{45000}{38500} = 1.1688 \text{ ins}$$

And the **pitch** = net space + diam = 1.1688 + .75 = 1.9188 ins, = $\frac{1.9188}{.75}$ = 2.56 diams.

In practice, to avoid troublesome decimals, we might make the net space 1.2 ins; and the pitch 1.95; but to show farther on the working of the rule, we adhere to the more exact ones.

Fifth, The clear dist from each end hole to the side edge of the plate is half of 1.1688 = .5844 ins.

The entire width of net iron is equal to one clear space $\times$ number of rivets = 1.1688 $\times$ 6 = 7.0128 ins; and the entire width of plate is equal to one pitch $\times$ number of rivets = 1.9188 $\times$ 6 = 11.5128 ins

The area of cross section of unholed plate is 11.5128 × .5 = 5.7564 sq ins; its tensile strength **before the holes are made** is 5.7564 × 45000 = 259038 lbs

The strength of our joint, omitting friction, is therefore $\frac{135000}{259038}$ = .52 of that of the original unholed plate.

If the 6 rivets are in 2 rows of 3 rivets each, the **clear dist between two rivets** in one row will be twice as great as before, or twice 1.1688 = 2.3376 ins. **Pitch** = 2.3376 + .75 = 3.0876 ins = 3.0876 ÷ .75 = 4.12 diams. **Clear dist from** end hole to side edge of plate = half of 2.3376 = 1.1688. **Entire width of net iron** = 2.3376 × 3 = 7.0128 ins. **Entire width of plate** = 3.0876 × 3 = 9.2628 ins. **Area of cross section of unholed plate**=9.2628×.5=4.6314 sq ins. **Ultimate tensile strength, unholed** = 4.6314 × 45000 = 208413 lbs. **Ult strength of riveted joint,** omitting friction = $\frac{135000}{208413}$ = .65 of that of the unholed plate.

Thus we see that the arrangement with two rows gives the same strength as one row, with a less total width and area of plate. It of course requires *longer covers.*

If the 6 rivets are in 3 rows of 2 rivets each, the **area of cross section of the unholed plate** is 4.2565 sq ins. Its **tensile strength,** 191542 lbs. Strength of riveted joint = $\frac{135000}{191542}$ = 7 of that of the unholed plate.

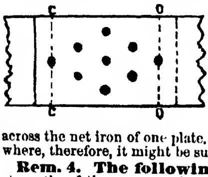
The entire width of net iron (7.0128 ins); its area (7.0128 × .5 = 3.5064 sq ins); and its ultimate tensile strength (3.5064 × 38500 = 135000 lbs), are the same in each case. The last is the required breaking strength of the joint, as in the beginning of our example; and is equal to the combined crippling strength of the six rivets.

Art. 6. The distance apart of the rows, from center to center of rivets, should not be less than two diameters of a rivet-hole.

Rem. 1. With our constants for tension, shearing, and compression, **the rivets will not yield first by shearing** in a double-cover butt (and of course in double shear), except when the diam is either equal to or **less** than .85 of the thickness of the plate, which will rarely happen. At .85 the crippling and shearing strength of a rivet are equal when using our assumed coeffs of crippling, shearing, and tension.

Rem. 2. Our example was chosen to illustrate the rule. It will rarely happen in practice that the rule will give a number of rivets without a fraction; or that may be divided by 2 and by 3 without a remainder. In case of a fraction, it is plainly best to call it a whole rivet; although the joint thereby becomes a trifle stronger than necessary. Or rivets of a slightly diff diam may be used. If the number of rivets comes out say 7 or 9, we may make 2 rows of 3 and 4, or of 4 and 5, &c. Moreover, the width of the plate is frequently fixed beforehand by some requirement of the structure, and we must arrange the rivets to suit, taking care in all cases to maintain the calculated area of net iron in one row, &c.

Rem. 3. We have (as we at first said we should do) confined ourselves to the simple butt-joint with 2 covers, and with the rivets in either 1, or in 2 or more parallel rows on each side of the joint-line; this being the strongest and the one in most common use in engineering structures. Necessity at times calls for less simple arrangements, for which we cannot afford space, and the strength of which is not so readily calculated. These sometimes yield results which appear strange to the uninitiated; thus, this lap-joint breaks across the net iron of one plate, along either c c or o o, *where there is most of it,* and where, therefore, it might be supposed to be the strongest.



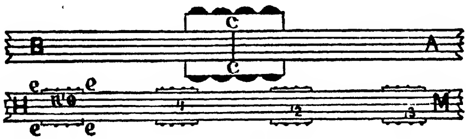
Rem. 4. The following table shows approximately the comparative strengths of the common forms of joints when properly proportioned; varying with quality of sheets, and of rivets:

	With friction.	Without friction.
The original unholed plate	1.00	1.00
Double-riveted butt with two covers	.80	.64
Double-riveted butt with one cover	.65	.52
Single-riveted butt with one cover	.50	.40
Double-riveted lap	.65	.52
Single-riveted lap	.50	.40

Rem. 5. The above tabular strengths for the **lap-joints** will be approximately attained by adopting the following proportions, according as the joint is double- or single-riveted.

	Double riv. zigzag.		**Single riv.**	
	In thicknesses.	In diams	In thicknesses	In diams.
Calling thickness of plate	1.	.6	1.	.6
Then make diam of rivet	1.67	1.0	1.67	1 0
" " breadth of lap	9 0	5.4	5 67	3.4
" " pitch from cen to cen	7.0	4.2	4 5	2.7
" " dist from *end* of plate to *edge* of holes	2.0	1.2	2.0	1.2
" " dist apart of rows from cen to cen	3.33	2.0		

Rem. 6. If two or more plates on top of each other, as the four in A B or M H, are to be jointed together so as to act as one plate of the thickness *c c*, the diams of the rivets, and the thickness of the covers *c c*, *e e* will depend upon whether the junctions of the plates are all in one line with each other as at *c c*, in A B, or whether they break joint with each other as at 0, 1, 2, 3 in M H.



It is plain that the two covers *c c* by means of their connecting rivets convey from A to B, across the joint *c c*, all the strength that partly compensates for the severance of the four plates at that joint; whereas the two covers *e e*, *e e*, and their rivets in like manner convey from *n* of one single plate, to *o* of the adjoining one, across the joint between those two letters, only the strength that partly compensates for the severance of that single plate; and so with the joints at 1, 2, and 3. Therefore the covers *c c*, and their rivets, must be four times as strong as those at any one of the four joints 0, 1, 2, 3. The first, *c c*, are to be regarded as joining two solid plates A and B, each of the fourfold thickness *c c*; and the others as joining two of the single thickness. The covers *c c* will, therefore, each be about two-thirds of the thickness *c c*; and the others each about two-thirds as thick as a single plate. Thus, suppose each of the 4 plates in A B or M H to be ¾ inch thick, making *c c* 3 ins. Then each cover, *c*, is ⅔ of 3 ins, or 2 ins thick; or the two covers, *c c*, together 4 ins, which is thus the effective thickness of the joint, *c c*. But each cover, *e e*, is only ⅔ of ¾ inch, or ½ inch thick; and the effective thickness of joint at either 0, 1, 2, or 3, is that of the 3 unbroken plates plus that of the 2 covers, or $(3 \times \frac{3}{4}) + (2 \times \frac{1}{2}) = 3\frac{1}{4}$ ins.

Art. 7. Principle of the Rule in Art 5. With our constants for shearing (45000 lbs per square inch) and for crippling (60000 lbs per square inch), and with diameter of rivet equal to, or greater than, .85 times the thickness of the plate, as by our rule, the *crippling* strength of a double cover butt joint will be equal to, or less than, its shearing strength. Therefore, to avoid waste of material, either in the plate or in the rivets, we must make

$$\text{Tensile strength of net plate across one row of rivets} = \text{Crippling strength of } all \text{ the rivets. Or.}$$

$$\text{Total net width of plate} \times \text{Thickness of plate} \times \text{Tension unit} = \text{Crippling area of one rivet} \times \text{Crippling unit} \times \text{Total number of rivets.}$$

Now, by Art 3, the crippling area of a rivet is = diam of rivet × thickness of plate. We take the crippling unit at 60000 lbs; and the tension unit at 38500 lbs. Therefore (transposing) we must make

$$\text{Total net width of plate} = \frac{\text{Diam of rivet} \times \text{Thickness of plate} \times 60000 \times \text{Total number of rivets}}{\text{Thickness of plate} \times 38500}$$

By making the clear distance between each end rivet of a row and the side edge of the plate = half the clear distance between two rivets in a row; and calling the sum of the two end distances one space, we have

$$\frac{\text{Number of } spaces}{\text{in a row}} = \frac{\text{Number of } rivets}{\text{in a row.}}$$

So that

The clear distance between two rivets in a row,

which is

$$= \frac{\text{Total net width of plate}}{\text{Number of } spaces \text{ in a row}}$$

is also

$$= \frac{\text{Total net width of plate}}{\text{Number of } rivets \text{ in a row}}$$

$$= \frac{\text{Diam. of rivet} \times \text{Thickness of plate} \times 60000 \times \text{Total number of rivets}}{\text{Thickness of plate} \times 38500 \times \text{Number of rivets in a row.}}$$

But

$$\frac{\text{Total number of rivets}}{\text{Number of rivets in a row}} = \text{Number of rows.}$$

Therefore, omitting "thickness of plate," common to both numerator and denominator, we have, as in rule in Art 5,

$$\textbf{Clear distance apart} = \frac{\text{Diam of rivet} \times 60000 \times \text{Number of rows}}{38500}$$

But if the diameter of the rivets is less than 0.85 times the thickness of the plates, the *shearing* strength of a double-cover butt join (with our assumed constants for shearing and crippling) is less than its crippling strength. In such cases, for the clear distance between two rivets in a row, say

$$\textbf{Clear distance} = \frac{\text{Circular area of a rivet} \times \text{Shearing unit}}{\text{Thickness of plate} \times \text{Tension unit}} \times 2$$

Rem. 1. Butt joints in double shear, or with 2 covers, being th only ones here considered, and inasmuch as rivets may always be used with a dian greater than .85 of the thickness of the plate, we may in practice always use th Rule in Art 5 for such joints; and, therefore, we gave it alone.

Rem. 2. When using these rules for other kinds of joint such as laps, or butts with *single* covers, remember that the rivets in such are in **single** shear; and, therefore, we can use Rule in Art 5 (for crippling) only when the diam is either **1.7 or more** times the thickness of plate. **If less,** use Rule above for **shearing;** all on the assumption that our foregoing coefs of crippling and shearing are used.

But the coef for tension must be changed for each kind of these other joints, to allow for the weakening effects of the bending shown at W, Fig 3, as deduced approximately from experiment. The writer believes that the following tension units will give safe approximate results without friction. **For double-cover butts,** double-riveted, 38500 lbs per sq inch, as adopted above **For double-riveted** laps, or one-cover butts, 28000. **For single-riveted** laps, or one-cover butts, 24000. But, as before remarked, no great certainty i attainable in riveting.

Rem. 3. A joint may fail by crippling without the facts bein known or even suspected, for it does not imply that anything **breaks,** bu merely that the joint has **stretched;** and this might not be detected even on a *slight* inspection of it. Still it might, and probably often has sufficed to endanger and even destroy both bridges and roofs by generating strains where none wer provided for.

RAILROADS.

TRACK.

GENERAL

1. Railroad track, as ordinarily constructed in the U S, consists of the following parts.

2. Rails. ¶ 83. The wheels of the rolling stock roll upon the heads of the rails, the wear of the wheel treads coming upon the upper surface of the heads, and that of the flanges coming upon the sides of the heads. The rails serv also as short girders spanning the spaces betw the ties, the heads there theoretically taking compression, the webs shear, and the bases tension; but, since the rails must act as *continuous* girders, the stresses in the heads and bases are necessarily reverst over each tie. The base of the rail servs also to distribute the load over the tie, and to afford a grip for the spikes. It offers the chief resistance against lateral flexure.

3. Joints, ¶¶ 145, etc. The rail ends are held together, and in line with each other, by means of rail-joints, splice-bars, or "fish-plates," which are clampt to the sides of the rails, near their ends, by means of bolts passing thru holes in both the plates and the rail-webs.

4. Spikes, ¶¶ 96, etc, driven into the ties beside the rails, hold the rails down to the ties by means of their overhanging heads which grip the edges of the rail bases; and their shanks resist lateral sliding of the rails on the ties.

5. Tie-plates, ¶ 135, etc, are largely used to distribute the rail pressures over a larger area of tie surface. They thus reduce the unit pressures on the ties, and the crushing effect of the loads.

6. Ties, ¶ ¶ 31, etc, serv to distribute, in and thruout the ballast, the vert loads and the hor thrusts deliverd by the rolling stock.

7. Ballast, ¶¶ 19, etc, not only takes the loads and transmits them to the roadbed or ground, but it usually has the additional function of prolonging the life of the ties by permitting water to flow away thru it from them.

8. Roadbed. The track proper, including the ballast, rests upon the ground, or roadbed; which must not only be of sufficiently firm material to hold the ballast and track up to level, but should also be so prepared as to drain away any water that may collect. For this purpose, ditches must often be provided alongside the roadbed, to carry off the water draind from the track, except where the road is on an embankment.

Specifications.

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*Am Ry Engg Assn. †P R R. ‡U P R R.

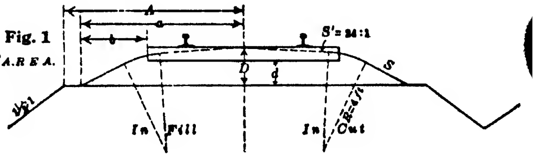
Classification of Track

Am Ry Eng Assn, Manual, 1915, p 15.

10. Railways, and portions ("districts") of railways, are classified by the A R E A, according to the volume and character of their traffic, as follows:—

Class	Class A, B or C includes all districts of a railway	Frt car mileage per year per mile	Passr car mileage per year per mile	Max speed passr trains miles/hr
A	having (1) more than one main track, or (2) single main track, with traffic $\nless$	150,000	10,000	50
B	single track, with traffic < A, and $\nless$	50,000	5,000	40
C	not meeting the traffic requirements of Classes A and B.			

Fig. 1
*A.R E A.



11. Single track.

*A R E A Fig 1

Sodding of roadbed shoulder next to ditch, and of slopes of ditch, recommended.

	Width, ins			Depth, ins		Slope
	A	a	b	D	d	S
Crusht stone and slag						
Class A§	120	...	39	19	12	2 : 1
Class B	96	...	30	16	9	2 : 1
Gravel, cinders, chats, etc						
Class A§	120	...	51	19	12	3 : 1
Class B	96	...	39	16	9	3 : 1
Cementing gravel and chert						
Class C*	84	...	18	13	6	3 : 1
‡Union Pacific, 1909						
Broken stone or slag, Fig 1 ...	96	78	30	15	8	...
Gravel, burnt clay or cinder, Fig 1	102	84	36	15	8	...
Earth,	See Fig 2					
Sand,	See Fig 3‡					

*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

§ Class A dimensions give min depths, *d*, under ties. They are recommended only on the firmest, most substantial and best draind roadbeds.

RAILROADS.

TRACK.

GENERAL

1. Railroad track, as ordinarily constructed in the U S, consists of the following parts.

2. Rails. ¶ 83. The wheels of the rolling stock roll upon the heads of the rails, the wear of the wheel treads coming upon the upper surface of the heads, and that of the flanges coming upon the sides of the heads. The rails serv also as short girders spanning the spaces betw the ties, the heads there theoretically taking compression, the webs shear, and the bases tension; but, since the rails must act as *continuous* girders, the stresses in the heads and bases are necessarily reverst over each tie. The base of the rail servs also to distribute the load over the tie, and to afford a grip for the spikes. It offers the chief resistance against lateral flexure.

3. Joints, ¶¶ 145, etc. The rail ends are held together, and in line with each other, by means of rail-joints, splice-bars, or "fish-plates," which are clampt to the sides of the rails, near their ends, by means of bolts passing thru holes in both the plates and the rail-webs.

4. Spikes, ¶¶ 96, etc, driven into the ties beside the rails, hold the rails down to the ties by means of their overhanging heads which grip the edges of the rail bases; and their shanks resist lateral sliding of the rails on the ties.

5. Tie-plates, ¶ 135, etc, are largely used to distribute the rail pressures over a larger area of tie surface. They thus reduce the unit pressures on the ties, and the crushing effect of the loads.

6. Ties, ¶ ¶ 31, etc, serv to distribute, in and thruout the ballast, the vert loads and the hor thrusts deliverd by the rolling stock.

7. Ballast, ¶¶ 19, etc, not only takes the loads and transmits them to the roadbed or ground, but it usually has the additional function of prolonging the life of the ties by permitting water to flow away thru it from them.

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BALLAST.

19. Stone. *Stone broken by artificial means into small fragments of specified sizes.* Broken stone is the universally preferd ballast for first-class conditions. The stones preferd, in approx order of preference, are *trap, granite* (or *syenite*, granite containing hornblende instead of mica), *limestone* and *sandstone*. *Screenings*, from the stone crusher, are commonly used in station platforms, and as ballast for side tracks; but they churn badly under the ties when wet. *Requirements*;—† "Trap rock, or an acceptable igneous or equally hard and suitable stone . . . . . regarded as standard material." Crushing stress $\nless$ 12,000 lbs/sq inch; limestone, $\nless$ 10,000 lbs/sq inch. Must be broken in cubical form. Must pass thru 3" ring, but not thru 1.25" ring.†

20. Gravel is cheap, when found near the work, and is easily applied. Sand and clay, in gravel, impede drainage; while sand and dust cause wear of tires and journals. *Requirements*. *Pebbles to pass a 2.5 inch ring, and be retaind on No. 10 screen. 1915.* † < 2.5 ins. 1908.†

21. Slag. Hard — . Blast furnace slag, deposited and coold in air. If it contains but little free lime, it is hard and glassy, and, when crusht, rivals broken stone as ballast material; but it sometimes affects the ties chemically. An excess of free lime leads to disintegration, and sometimes to setting, like that of mortar.

22. Slag. Granulated — . Slag into which, when molten, water has been injected. It consists of particles of very uniform size, about equal to that of very coarse sand grains. It is easily applied, and is used in yards and on side tracks. It is apt to slake, solidifying, and impeding drainage, if much free lime is present.

23. Cinder. Taken from locomotiv ash-pits. Cinder ballast drains well, but yields under heavy traffic. It sometimes affects the ties chemically, especially when wet. *Recommended for branch lines with light traffic, in sidings and yard tracks near point of production; as sub-ballast in wet, spongy places; as sub-ballast on new work where dumps are settling; and at places where the track heavs from frost. Make provisions for wetting down cinders soon as drawn. 1915.*

24. Burnt clay ballast drains well. It is easy to work, and stands well under light traffic. *Requirements*;—*Use black gumbo or other suitable clay free from sand or silt. Before establishing kiln, test material thoroly in a small test kiln. Burn hard and thoroly. Fuel should be fresh, and clean enough to burn with a clean fire. Keep enough fuel on hand to prevent interruption of burning. Cool the ballast before loading out of pit. Absorption of water $\ngtr$ 15% by wt. 1911.*

25. Chats. *Particles of quartz, about as large as wheat grains. They are the tailings from mills in which zinc and lead ores are separated from the rocks in which they occur.* Chat ballast works easily. It keeps down weeds, stands wet weather and holds its surface well; but, owing to lack of cohesion, it is easily shifted laterally. It is dusty under high speeds.

25a. Sand ballast is easily handled. It does not become plastic when wet; but it is heavd by frost, and, when dry, it is very dusty, and therefore annoying to passengers, and injurious to rolling stock, causing hot-boxes.

25b. Dirt ballast, in wet wether, becomes mud. The more sand it contains, the better.

25c. Gumbo. *A term commonly used for a particularly tenacious clay, containing no sand.*

25d. Chert. *An impure flint or hornstone occuring in natural deposits.*

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*Am Ry Engg Assn. †P R R. ‡U P R R.

Pecky tie. Made from a cypress tree affected with a fungus disease known locally as peck.

Pole tie. Made from a tree of such size that not more than one tie can be made from a section. Hewn or sawn on 2 parallel faces.

Quarterd tie. Made from a tree of such size that four ties only are made from a section.

Sap tie. Showing more than the prescribed amount of sapwood in cross-section.

Shakes. Separations of the wood fiber, due to the action of the wind.

Slab tie. A tie made from the first or outside cut of a log.

Slabbed tie. Sawd on top and bottom only.

Split tie. Made by splitting from a tree of such size that two or more ties can be made from a section.

Strict heart tie. Having no sapwood. See also Heart tie.

Substitute tie. Any tie other than a wood tie.

Switch tie. Tie of a set used to support a turnout.

Tapped tie. Made from a tree, the resin or turpentine of which has been extracted before felling.

Treated tie. A tie which has been subjected to a process designd to protect it from decay.

Wane tie. Squared and showing part of the original surface of the tree on one or more corners.*

General.

Dimensions, Bearing, etc.

32. A tie, acting as a continuous beam, serves to distribute the rail pressures to the ballast under it, and its dimensions should be such as to enable it to effect this distribution with tolerable uniformity Where a tie is more firmly supported at the middle of its length than near the rails, it is said to be *center-bound*. A tie of sufficient length transmits a fair proportion of the load to the ballast beyond the rails, correspondingly relieving the portion betw the rails. The thickness is commonly about 6 or 7 inches A 6-inch tie is hardly stiff enough for heavy traffic, especially where center binding is likely to occur; and is apt to be split by the spikes, since these, when driven, extend to very near the lower face. Since beam stiffness varies as the cube of the depth, the resistance to bending, in a 7-inch tie, is, to that in a 6-inch tie, as 343 to 216, or as 100 to 63. A thickness of 7 inches is rarely exceeded except with soft woods.

33. If more than about 40% of the rail length bears on the ties, the closeness of the ties interferes with tamping. The min clear distance, betw ties, is about 11 ins. With usual tie spacing, a tie thickness of more than 7 ins interferes with the work of tamping. With extremely wide ties, the full bearing value is not always utilized because such ties are seldom as thoroly tampt as are narrower ones.

34. When the spike enters the wood tangentially to the growth-rings, it is apt to split the tie.

35. Ties should be laid with the heart side down, in order that the growth-rings shall be in such position as to shed water.

36. Pole ties give a rail-bearing of heartwood, which resists abrasion better than sapwood and the bulging sides give additional bearing surface.

37. Cutting by rails. Moisture, collecting betw rail and tie promotes decay; and the load, aided by the undulatory motion of the rail, cuts away the decayd fibers. This *"rail-cutting"* is the principal cause of tie failures.

38. "Ties should be *protected* against failure from mechanical wear by means of *tie-plates* and *screw-spikes.*" A R E A Procs 1915 Vol 16, p 522.

"The use of treated ties wherever practicable is recommended."

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*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

Woods.

39. Cedar is light, and resists decay well; but is easily crusht by the rail. On tracks having light traffic, or where protected by tie plates, cedar ties may last from 15 to 20 years. Sound dead cedar makes as durable ties as live cedar.

White oak ties are preferd. When well seasond, they hold the spikes very firmly; and resist rail cutting until the wood decays. Life, from 5 to 10 years; average of many roads, 8¼ years. A sawd and seasond white oak tie, 7 ins × 9 ins, 8 ft long, weighs about 185 lbs.

Rock oak is less tough than white oak. Its life is about the same.

Southern yellow pine ties last from 4 to 6 years in the south; and from 8 to 12 years in the north.

Chestnut ties last from 7 to 9 years. Chestnut checks badly in the sun; but holds spikes well, and is of medium hardness

Redwood is soft, but durable. Redwood ties, with tie plates, last from 10 to 14 years.

Wild cherry, locust and walnut ties last about 8 years.

40. Specifications. *Woods usable for ties *without preservativ* treatment: white oak family, long-leaf strict heart yellow pine, cypress (other than white), redwood, white cedar, chestnut, catalpa, locust (other than honey), walnut, black cherry.*

*Woods *preferably requiring a preservativ* treatment approved by the purchaser: red oak family, beech, elm, maple, gum, loblolly, short-leaf, lodgepole, western yellow pine, Norway, North Carolina pine and other sap pines, red fir, spruce, hemlock, tamarack.*

Switch-ties. †Usable *without preservativ* treatment: white oaks, black locust, black walnut, black cherry, longleavd pines. Usable *only after preservativ* treatment; red oaks, beech, hickories, hard maples, birches. 1913.†

See also under Turnouts, Part I, ¶¶ 128, etc.

Dimensions.

41. Classification. Dimensions from **Am Ry Eng Assn** Manual, 1915, p 59.

*Class	Thick-ness, ins.	Face width, ins.	Volume of one tie, in cub ft: Length, 8 ft, cub ft	Length, 8.5 ft, cub ft	Length, 9 ft, cub ft
*A	7	10	3.889	4.132	4.375
B	7	9	3.500	3.719	3.938
C	7	8	3.111	3.305	3.500
D	6	9	3.000	3.188	3.375
E	6	8	2.667	2.833	3.000

*** Permissible excess:**—in thickness, ½ inch; in width, 2 ins; in length, 1 inch. In pole ties with rounded sides, and in half-round ties, the face width may be < above (≮ 6 ins), provided cross-section area ≮ that corresponding to tabular dimensions. Thickness, ≮ 6 ins.*

Spacing.

42. Number of ties required under one rail length :—

	On main running tracks	On side tracks & branches	In yards and on sidings
33-ft rail	18‡	16‡	16‡
‡ If ties are < standard,	20‡		
† According to wt of traffic,	16 to 20†	16†	14†
30-ft rail	16‡	14‡	14‡
If ties are < standard,	18‡		

For shorter rails, use a proportional number of ties.‡

*Am Ry Engg Assn. †P R R. ‡U P R R See also Specifications p 780.

Maximum allowable distance between bearing surfaces. †Between *joint* ties, 11 ins; betw *intermediate* ties in main running tracks, 18 ins.†

Tie Timber Conservation.

43. On account of increasing scarcity and cost of tie timber, recourse is had (1) to chemical treatment, ¶ ¶ 44, etc., (2) to forestry, ¶ ¶ 54, etc, for the growing of trees for use as ties, (3) to reduction of wear on ties, as by tie plates, ¶ ¶ 135, etc., and (4) to the use of ties of other materials, as steel or reinforced concrete, ¶ ¶ 61, etc.

Preservativs.

44. The economy of treated ties lies not only in their increast length of life, but in the reduction of the labor of tie renewals and of the resulting disturbance to the embedment.

45. Inferior timber. Chemical treatment, and the use of tie plates, permit the use of timbers not otherwise suitable, and of sound dead timber.

46. Steaming. For thoroly air-seasond ties, preliminary vacuum ≮ 24 ins mercury, maintaind ≮ 10 mins. Preservativ then admitted without breaking the vac.*

* For ties not thoroly air-seasond, a pressure of 20 lbs per sq inch must be attaind within from 30 to 50 mins, and maintaind (not exceeded) from 1 to 5 hrs, depending upon character and condition of timber. A vent must be kept open, in bottom of cyl. or in drain therefrom, for escape of air and condenst steam. After steaming, vacuum ≮ 24 ins of mercury at sea level (corresponding degs of vac at other altitudes) maintaind for half hour. Preservativ then admitted without breaking vac.*

* Unseasond ties, which are to be creosoted, may be given long steaming or seasoning in hot creosote oil.*

47. *Zinc chlorid treatment (Burnettizing). See also p 1135. Solution, heated to ≮ 140° Fahr, to be admitted without otherwise reducing vacuum. Wood must absorb 0.5 lb dry soluble chlorid per cu ft Solution must be only strong enough to give this absorption, and ≯ 5%. Steam pres maintained in steam coils of machine during treatmt.*

* Chlorid must be slightly basic, without free acid; iron ≯ 0.25%.*

* Sample borings to be taken, from time to time, from ≮ 6 ties treated in the same run. Bore-holes to be tightly and completely closed with creosoted plugs.*

* Ties must be allowed to dry (in order to harden surface) before placing in track.*

48. *Zinc chlorid, tannin and glue Treatment. See p 1135. Zinc chlorid as above. Solution then blown or run off. Ties draind 15 mins. 2% tannic acid solution (6.67 lbs of 30% extract of tannin to 100 lbs water) applied half hour under pres of 100 lbs/sq in Tannin solution run off, and 1% glue solution (2.1 lbs glue, containing 50% gelatin, in 100 lbs water) applied for half hour under same pres.*

49. *Creosote. See p 1134. Coal-tar creosote oil; at 38° Centigrade, completely liquid, and ≮ 1.03 sp grav; ≯ 3% water; heated to 160° Fahr. Pressure, 100 lbs/sq inch.*

50. *Zinc-creosote Emulsion. Emulsion to contain ≮ 10% creosote. Heated to ≮ 140° Fahr. Pres, 100 lbs/sq inch. Wood to retain an avge of 0.4 lb dry soluble chlorid and from 1.25 to 1.5 lbs creosote, per cub ft. Zinc-chlorid solution not stronger than 3.5%; to contain ≯ 0.25% iron.

***Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.**

51. * Two-injection Zinc-creosote. Zinc solution not stronger than 5%; heated to ≮ 140° Fahr. Wood to receiv 0.3 lb dry soluble chlorid per cub ft. Solution then run off, creosote oil, 140° Fahr, immediately admitted and pres applied; 3 lbs oil per cub ft. Water, exceeding 6%, to be removed.*

52. *Life.* Creosoted pine ties last about 15 yrs, creosoted oak 18 yrs, and creosoted beech 20 yrs, in main track; and giv subsequent servis, about one third as long, in side track.

Seasoning, etc.

53. Unseasond ties to be *piled for seasoning;* green ties separate from partly seasond ties; all resting on treated stringers, with ≮ 6 ins air space under lowest tier; top tier laid sloping, to form a water shed. Betw piles, leave alleys, 4 ft clear width in one direction, 1 ft in the other. Required degree of seasoning to be determind by test, finding that weight at which each wood will best receive the treatment. Ties threatend with checking must be protected by S irons, bolts or other devices Adzing or boring, for tie-plates or screw spikes, must be done before treatment.

Forestry.

54. Culture of timber should be undertaken only by expert foresters. We here outline only fundamentals of general interest to engineers.

55. *Felling season.* Whatever season is selected for felling the trees, the ties made from them should have at least six months for seasoning before going into track. However, the seasons for cutting and for tie renewals come so close together, that this is difficult to observ without holding the ties over a year. * Timber cut preferably from Oct to Mar;* †Sep to Mar.†

56. *Bark* should be peeld soon after felling, to hasten evaporation and to prevent "souring". Placing ties in track, with bark on, not only hastens decay, but renders the ties more inflammable * Bark to be removed from all ties before their delivery to the Co. 1915.*

57. *Methods of cutting* trees and of cutting ties from trees are usually wasteful. Wood is frequently lost by leaving unnecessarily large stumps, and thru failure to follow as far up as possible into the tree; large trees sometimes affording ties from their branches. Much is frequently lost by cutting trees which are only large enough to afford pole ties (one from each section); whereas, by waiting five or ten years, two ties could be obtaind from a section.

58. *Forestation.* The early practice was to plant only rapidly growing trees; but it has been found that other important considerations (such as liability to attack by parasites, low strength or short life in the track, etc) may well lead to the selection of slower-growing trees. Each tract of available ground, too, should be carefully studied as to what species it can best produce. It is frequently better, also, to develop existing imperfect timber lands, than to attempt complete reforestation from the seed.

59. *Fire prevention* may frequently warrant considerable expenditures for patroling, especially in dry wether.

Dating Nails.

60. *Iron or steel, evenly galvanized, ¼ inch diam, 2½ ins long, head ⅝ inch diam, stampt 1/16 inch deep with 2 numerals ⅜ inch long, designating the year. Nail driven in upper face of each treated tie, 10 ins inside of rail, on line side of track, on the day of laying. Each treated tie stampt also with year, on both ends, at treating plant, before treatment *

*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

Substitute Ties.

General.

61. A substitute tie is "any tie other than a wood tie." Am Ry Eng Assn Procs, 1915, Vol 16, p 522; Ties Committee Report.

62. Owing to the rapidly increasing scarcity of suitable timber, substitute ties are largely used in Europe; and their use, experimentally or in service, has been activly taken up by many American railroads, including some important systems.

The use of substitute ties involvs so radical a change in R R practice that considerable experience in their use will be required, to determin satisfactorily their relativ advantages and disadvantages.

63. Owing to their uniformity in cross-section, in strength, and in bearing surface, substitute ties (and, especially, steel ties) hold track to better line and surface, and the rail wear is more uniform. Wood ties, even those of the same wood, vary in quality when laid, compressing unequally under a given load, and they deteriorate rapidly and unequally under exposure. When fully ballasted, steel ties furnish a smoother-riding and quieter track.

Substitute ties are heavier than wood ties and more expensive in first cost. It has been found difficult to provide satisfactory rail-fastenings. See Screw-spikes. ¶¶ 107, etc.

With the growing use of electric signal circuits, the matter of insulation has become one of importance; and, in general, substitute ties offer greater difficulties in this respect.

Substitute ties, laid alternately between wood ties, hasten the deterioration of the latter.

Other things equal, the greater weight of substitute ties tends to greater stability in track.

Substitute ties are either steel, concrete (usually reinforced) or composit (composed essentially of two or more materials). Am Ry Eng Assn, Procs, 1915, Vol 6, p 522. Ties Committee Report.

Steel Ties.

64. Types. In Europe, much use has been made of *"longitudinals"* or steel *sleepers,* placed under, and parallel with, the rails, and of inverted cast iron "bowls", placed opposit each other, under the rails, at intervals, and connected by ties extending across the track; but, in America, practice and experiment with substitute ties have been practically confined to *cross ties* having functions generally similar to those of the wooden tie.

65. The earlier experiments were with *inverted trough ties;* and such ties are still used under light traffic; but the *inverted-T type,* represented by the Carnegie steel tie, ¶¶ 70, etc, is much the most largely used.

66. A flat bottom, as distinguisht from one having downward projections, facilitates tamping.

67. The fastenings consist generally either of bolts and clips, or of wedges. They should permit adjustment for differences of gage and of rail section.

68. The cost of steel ties is usually from 4 to 6 times that of good oak ties; but the steel should have a longer life than the oak tie, and considerable scrap value when no longer available for tie purposes.

69. Merits and demerits. Steel ties *hold track* absolutely *to gage.* Owing to their uniformity in cross-section, in strength and in bearing surface, they *hold track* to better *line and surface;* and the rail wear is more nearly uniform than with wooden ties.

In the absence of sufficient data from experience, their **life** is usually estimated at two or three times that of wood ties.

Owing to their small metal cross-section, steel ties require *more ballast* than do wood ties of the same external dimensions.

When steel ties have been puncht with square holes, *splitting* has been found to begin in the corners of such holes.

Steel ties are exposed to *deterioration from rust*, especially in damp situations, such as tunnels, and under brine drippings from refrigerator cars. *Protectiv coatings* have been used; but these are difficult of application to ties in service.

The all-steel tie is a satisfactory substitute under heavy medium-speed traffic. It is durable. Line and surface can be maintaind (see Maint costs, ¶ 73). It has sufficient resilience, and can be insulated. The fastenings are usually inadequate. Am Ry Eng Assn Procs 1912, Vol 13, Comm on Track.

70. Carnegie. Figs 4 represent the steel tie of the Carnegie Steel Co. *A*, plan of whole tie; *B*, side elevation; *C* and *D*, enlarged cross-secs at *m n* and *o p*, Fig *A*, respectivly; *E* and *F*, plan and elevation showing rail and standard fastenings In rock or coarse gravel ballast, and on descending grades under heavy traffic, the lower flange may be crimpt, as shown in *A*, *B* and *D*; but this interferes with ballast tamping. The tie may be puncht for two weights of rail.

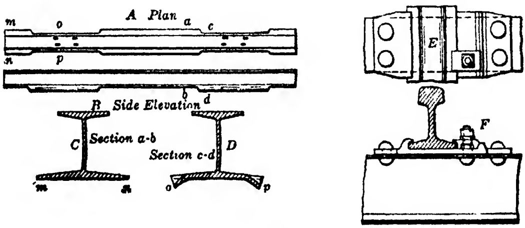

Fig. 4.

Dimensions, in ins, etc

Section	lbs/ft	Tie, 8.5' long, wt, lbs	Depth	lower flange width	upper flange width	web thickness
M 24	9.5	81	3	5	3	13/64
M 25	14.5	123	4¼	6	4	1/4
M 21	20	170	5½	8	4½	1/4
M 28	27.8	236	6½	10	5	5/16

The fastening allows about ⅛ inch play, vert and hor, in the rail.

71. *Insulation* consists of ⅛ inch fiber, of which there is (1) a plate betw the steel tie plate shown in Fig 4 *F* and the top of the tie, (2) a washer under the rivet washer, and (3) a bushing around the rivet shank.

72. *The first* Carnegie steel ties were placed on the Bessemer & Lake Erie (a Carnegie-controlled road) in 1904. They have stood well under heavy freight traffic at moderate speeds. Under high speeds, their rigidity has been found objectionable.

Owing to restriction of movement betw rail and tie, they have been found to *reduce creeping*.

In renewing, it has been found that a gang can handle more of them than of wood ties, in a given time.

As a *protection against rust,* they have been dipt in hot tar, at a cost of 5 cts each.

The insulation (see ¶ 71) has been found to work well while in good condition; but, in 3 or 4 years, the fiber plates wear, and the bolts become loose in consequence. See ¶ 73. Many have been removed upon the installation of automatic signal systems.

The cost may be taken at from $2.00 to $2.60 per tie; with 20 cts additional, per tie, for fastenings. This is rather more than double the cost of wood ties.

73. *On the Pittsburgh & Lake Erie,* Carnegie steel ties, with bolt-and-clip fastenings, were laid in new limestone ballast, Aug 1907, on 4400 ft of main-line freight tracks, speeds to 30 miles/hr. Maintenance costs as follows.

	1908	1909	1910	1911	1912	Total	$/mile per yr
White oak	$ 417	95	128	116	94	850	204
Carnegie steel	$ 280	153	428	184	348	1393	334

In 1911, the white oak tie track was renewed once; the steel-tie track three times. The first 17 fiber plates gave out, cut by rail base; 20 bolts loose in steel-tie clips. In 1912 the cost included $102 for renewing 1000 fiber plates. "Practically no signal trouble whatever"

Other roads have reported less trouble in track maintenance than with wood ties.

74. Annual *loss of weight,* due to rust, attrition, etc., has been variously reported, from diff roads, as ranging from 0.7 lb/tie, or 0.40%, in slag or gravel, to 4.5 lbs/per tie, or 2.55% in cinder. After seven years of service, they have shown little external evidence of corrosion. Laid on the Pittsburgh, Shawmut & Northern in 1907, and reported as giving "eminent satisfaction" in 1913, they appeared to be "failing quite rapidly of late" in 1915.

Carnegie steel ties, laid on the Erie in April 1909, were found rusting and *depreciating in salt air* in 1913.

75. On the Duluth, Missabe & Northern, they gave great satisfaction as substitutes for wooden ties which had given great trouble on *swampy ground.*

76. Where *wedge fastenings* are used, trouble has been experienced in keeping them tight.

77. On the Lake Shore & Mich Southern, near Sandusky, under *heavy traffic at high speed* in 1905-6, Carnegie steel ties were laid, with a wooden block fastend on top of the upper flange under each rail at each tie, at a cost, complete, of $2.25 to $2.50 per tie. The blocks were fastend by bolts, passing thru metal U-straps under the tie. This, by confining the wood fibers, increast the pulling resistance of the spikes. Up to 1908, these ties gave no more trouble than wood in regard to insulation; but trouble with the insulation was experienced later; and all of the ties had been removed, in 1915, on account of softening of the timber.

78. The L S & M S, near Toledo, in 1907, laid Carnegie steel ties from which the upper flanges had been removed, and to which two wooden blocks, one on each side, had been bolted thru the tie-web under each rail, and bearing on the tie flanges; the *rails being spiked to the wooden blocks* as to a wooden tie.

In 1910 these were reported "in fine shape", and to have "held surface and line as well as any track on the L S & M S."

79. *The inverted-trough section,* owing to its downward projections, is unfavorable to tamping. Hence, it is made shallow, and this renders the tie weak vertically. It is still used under light traffic, as in industrial, construction and mining tracks, etc.

80. Figs 5 show two **forms** of the **Carnegie steel** *trough* or *channel tie*, with their fastenings.

The following sections are furnisht, **dimensions in inches**:—

Section	lbs per ft	Depth	Extreme width	Web thickness	Fig
M 19	2.5	5/8	4	9/64	5 *a*
M 26	3.2	13/16	4 15/16	1/8	5 *a*
M 20	6	2	6	3/16	5 *b*
M 27	9	2 1/4	7	1/4	5 *b*

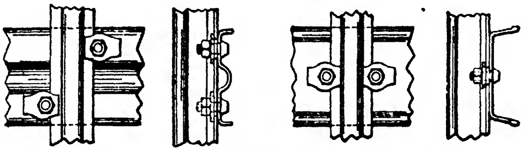

Fig. 5.

For mine work, the clips are prest out of, or riveted to, the horizontal portion of the tie itself.

In 1900-1901, *heavier* Carnegie steel trough ties, weighing 199 lbs each, were installed *on the Bessemer & Lake Erie.* After 8 years' service, they had lost 2.5 lbs/tie/yr, = 1 25%/yr, and showd no appreciable wear under the rails.

Concrete and Composite Ties.

81. Merits and demerits. Concrete and composite ties are usually heavier than steel ties for the same service; but less expensiv. Months are required for proper set of concrete. Conc surfaces, if not protected by metal, are vulnerable to blows, as in derailments. It is difficult to provide satisfactory fastenings. Their weight and the roughness of their surfs are favorable to lateral stability, and the absence of downward projections is favorable to tamping and to removal. The conc itself furnishes tolerable insulation for signal circuits; but there is danger of contact thru the reinforcing metal.

They have generally faild, by reason of brittleness, excessiv weight (rendering handling difficult) and deterioration of the filling when this is of asphalt mastic.

Am Ry Eng Assn Procs, 1912, Vol 13. Report of Tie Committee.

82. The following **records of experience** with composit ties may be of value as indicating what has been done and what should be avoided, in making such ties.

82a. 1 : 2 : 3 concrete, reinforced by 2¼ inch longitudinal wrot iron pipes and a sheet 5 × 8.5 ins of heavy wire netting, under each rail. Weight, 321 lbs; fastenings, 35 lbs additional. $1.50 to $1.75 each.

82b. Trapezoidal, with batterd ends. Reinforced by coil of wire mesh, No 16 gage, ½ inch pitch, and by ½" rods. Cost $1.50 each. In Scully yard, Penna Lines West Installed 1906, in cinder ballast, under heavy slow traffic. Rails spiked thru tie plates to wood blocks; 16 ties to 30-ft, 85-lb rail. Crumbled under rails. Bolts loosend. Rail creeping turnd clips around, unclamping rail.

82c. An inverted 65-lb scrap rail, imbedded in concrete. Batterd ends. Narrowd, at center of track, to width of rail flange. This narrowing prevents lateral movement. $0.95 + fastenings and 180 lbs scrap rail.

L S & M S. Still in track, in good condition. R R Gaz, 1908 May 1, p 594.

Pg Ft W & C. Placed 1903. All removed 1906. Conc broke away from steel.
Placed 1903. Light medium-speed traffic. Broke. Removed 1903.

"The most successful of all the conc ties I have seen, and under favorable conditions it certainly makes a fine looking track." Stands derailments well. Rock ballast may be too rigid. Gravel may be better. Am Ry Eng & M W Assn, Procs 1907, Vol 8, p 465.

A R E M W A Bulletin 108, Feb 1909, p 174 givs favorable accounts from L S & M S, and unfavorable accounts from Penna Lines, Chicago Junction, Lake Erie & Western.

RAILS, &c

General

83. Weight. Rails weighing from 75 to 85 lbs/yd are in common use on lines of heavy traffic; but 90-lb and 100-lb rails are used under extremely heavy traffic. Still heavier rails have been rolled, but difficulties in their mfr militate against their extensiv use.

84. Length, standard—, *33 ft at 60° Fahr; 1915.* †33 ft at 60° Fahr 10% of order accepted in lengths of 30 ft, 27.5 ft and 25 ft. Variation of ¼ inch from specified lengths allowd. 1912.† Rails 60 ft long are sometimes used at highway crossings, etc, to avoid the occurrence of joints under the planking and paving, where they would be inaccessible for tamping. They are used, also, in track in general, to reduce the number of joints.

Behavior

85. Wear of rails is most rapid on sharp *curvs,* where the flanges grind against the side of the rail head. It decreases as the sharpness of curvature decreases. On curvs, the wear of the top of the rail head is very small in comparison with that of the side of the head.

86. *On tangents,* rail wear occurs principally on the top of the head, and is due to the tractiv effort of the loco drivers and to the slipping of wheels of unequal diams fastened to the same axle.

87. *On grades, and at stations,* owing to necessary increase of tractiv effort, both in starting and in stopping, rail wear is greater than on level track or betw stations.

88. The *allowable limit* of wear of top of rail heads is generally taken at abt ⅝ inch; but, before this limit is reacht, the rail must, in many cases, be removed on account of roughness of running surface, causd (1) by slivering of the metal, or (2) by bending and excessiv wear at the rail ends, especially at the receiving end of the rail in double track.

89. Life of rails is variously estimated at from 100 to 250 million tons of traffic past over them, depending partly upon alinement and grades, and upon condition of track.

90. *Under heavy traffic or on sharp curvs,* the rails may have to be turnd end for end, or renewd, as often as once in two years. On rapid transit lines, renewals of rails on curvs must in many cases be made much more frequently, in some instances oftener than annually.

91. "Corrugation" of rail heads has become very annoying and expensiv, especially on curvs and in track traverst by electric rolling stock. It consists of a series of low worn spots on the rail head, the spots being usually some fraction of a foot apart. No agreement seems to have been reacht as to the cause, in spite of numerous theories put forward. A plausible theory is that, since, in rounding a curv, one wheel or the other of a pair must slip longitudinally, the slipping may take place in jerks, the truck frame and the axle being alternately twisted slightly and releast.

The low fric coef of slipping would permit an appreciable sliding (and consequent excessiv wear) along the rail top before one wheel caught up with the other; and then, when rolling of both wheels was re-establisht, the high static fric coef of rolling would prevent slipping until the curv had been traverst far enough to compel slipping once more. Wheels, traversing a curv, must slip sidewise as well, and this, like the longitudinal slipping, may take place in jerks simultaneous with the longitudinal slipping; and these slips probably resolv themselves into a diagonal slip. An other theory, based largely upon the observation that corrugations vary with tie-spacing, etc., attributes corrugation to vibrations of the track and ground.

Composition, requirements, etc.

Digest of Specifications.

R, American Railway Association. "Recommended Practice" proposed by Committee on Standard Rail & Wheel Sections, March 23, 1908.
W, American Railway Engineering & Maintenance of Way Association. "Manual of Recommended Practice," 1907.
M, American Society for Testing Materials. Standard Specifications, adopted Sep 1, 1907, Procs, 1907, Vol VII, p 44.
C, American Society of Civil Engineers. Specifications recommended by Special Committee on Rail Sections, July 9, 1907, amended Jan 1908, Procs, Aug 1907, Vol XXXIII, No 6, p 290; Feb 1908, Vol XXXIV, No 2, p 85.
A, All = **R, W, M, C.**

Composition.

Composition, R. American Railway Association.

lbs per yd	**60**	**70**	**80**	**90**	**100**
			Bessemer.		
Carbon %*	0.37–0.47	0.40–0.50	0.43–0.53	0.45–0.55	0.46–0.56
Manganese %	0.80–1.10	0.80–1.10	0.80–1.10	0.85–1.15	0.90–1.20

Phosphorus %* ≯ 0.10; sulphur % ≯ 0.075, silicon % 0.10 to 0.20.

lbs per yd	**60**	**70**	**80**	**90**	**100**
			Open Hearth.		
Carbon %†	0.50–0.60	0.55–0.65	0.60–0.70	0.65–0.75	0.70–0.80

Manganese % 0.75 to 1.00 for all weights;

Phosphorus %† ≯ 0.04; sulphur % ≯ 0.06; silicon % 0.10 to 0.20.

Composition, M. Am Soc for Testing Materials.

lbs per yd	**50 to 59**	**60 to 69**	**70 to 79**	**80 to 89**	**90 to 100**
		Bessemer and Open Hearth.			
Carbon %‡	0.35–0.45	0.38–0.48	0.40–0.50	0.43–0.53	0.45–0.55
Manganese %	0.70–1.00	0.70–1.00	0.75–1.05	0.80–1.10	0.80–1.10

Phosphorus % ≯ 0.10†; silicon % ≯ 0.20.

Composition, W, C. Am Ry Eng & M W Assn, Am Soc Civ Engrs.

lbs per yd	**70 to 79**	**80 to 89**	**90 to 100**
	Bessemer.		
Carbon %	0.50–0.60	0.53–0.63	0.55–0.65
Manganese %	0.75–1.00	0.80–1.05	0.80–1.05

Phosphorus % ≯ 0.085; sulphur % ≯ 0.075; silicon % ≯ 0.20.

Basic Open Hearth.
(Full chemical determination for each heat.)

lbs per yd	**70 to 79**	**80 to 89**	**90 to 100**
Carbon %	0.53–0.63	0.58–0.68	0.65–0.75

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* With lower phosphorus, carbon should be increased in proportion, **R.**
† With higher phosphorus, carbon should be reduced in proportion, **R.**
‡ Carbon may be reduced to suit local conditions, **W.**

Manganese % ≯ 0.90; phosphorus % ≯ 0.05; sulphur % ≯ 0.06; silicon % ≯ 0.20.

Manufacture.

Ingots kept vertical (in the pit-heating furnaces, **W, M, C**) until ready to be rolled, or until the metal in the interior has had time to solidify, **A**; use of "bled" § ingots forbidden, **A.**

"**Discard.**"¶ to be sheared from end of bloom formed from top of ingot, sufficient to insure sound rails, **R**: subject to agreement, **M**; ≮ 25%, more if necessary until steel appears solid, **W, C.**

Shrinkage. Number of passes and speed of train to be such that, on leaving the rolls at the final pass, temp of rail shall be ≯ to require, at the hot saws, a shrinkage allowance, for 33 ft 100 lb rail, of 6.5 ins, **R**; 7⁵⁄₁₆ ins, **M**: 6⁷⁄₁₆ ins, **W, C**: ⅛ inch less for each 10 lbs decrease in section, **R**: ¹⁄₁₆ inch less for each 5 lbs, **W. M, C**: allowance decreased 0.01 inch, **M** (¹⁄₁₀₀ inch, **W, C**) for each second of time between leaving finishing rolls and sawing, **W, M, C.**

Cooling. Rails must not be artificially cooled between "leading" and "finishing" passes, **R**; or after leaving the finishing rolls, **R, C**: between finishing pass and hot saws, **W, M**; or held, before sawing, to reduce temp, **R, C.**

Brand. Maker's name, weight of rail, month and year of mfr, rolled in raised letters on web; number of blow stamped on web where it will not be covered by splice bars, **A**: also "A" on rails from top of ingot, then "B," "C," etc, consecutively; "A" omitted where top discard ≮ 20%; "A" rails shipped in separate cars, **R**; open hearth rails to be marked "OH," **R.**

Straightening. Rails on hot beds to be protected against contact with water or snow, **R**: rails, varying > 5 ins, **M, C** (> 3 ins, **W, R**) from a straight line in any direction, on reaching cold-straightening machine, or having short kinks, to be classed as 2nd quality, **A**; and so marked, **R**: and so stamped, **W, M, C**: supports of rails in gagging press ≮ 42 ins apart, **R, W, M**: supports to have flat surfaces, **R**: finished rails to be straight in line and surface and smooth on head, final straightening to be done cold, sawn square on ends, variations ≯ ¹⁄₃₂ inch; saw burrs removed and ends cleaned before shipment, **A.**

Permissible Variations.

In section. In hight, ¹⁄₃₂ inch, **R**: ¹⁄₆₄ inch less, ¹⁄₃₂ inch greater **W, M, C**; in flange width, ¹⁄₁₆ inch, **A**: rail must conform to fit of splice bars, **R, W, C.**

In weight. 0.5 % on entire order; rails accepted and paid for by actual weights, **A.**

In length. 0.25 inch, **A.** Standard length, 33 ft, **A**: 10 % of order accepted in lengths of 30, 28, 26 and 24 ft, **R**; in lengths varying, by even feet, to 27 ft, **W, M, C**; all No 1 rails < 33 ft to be painted green "on the end," **W, M**; on both ends, **R, C.**

Tests.

Drop test. "Tup," 2000 lbs, **A**; striking-face radius, 5 ins, **R**; ≯ 5 ins, **W, M, C**; anvil block, 20,000 lbs, **R**; ≮ 20,000 lbs, **W, M, C**; supports to be part of, or firmly secured to, anvil, **A**: test piece, length, ≮ 4 ft, ≯ 6 ft, **A**; test piece to be taken from top of ingot, **A**: placed head upward, on supports (5 ins top radius, **R**), 3 ft apart, **A**; one drop test from each blow, **R, W**, (for Bessemer, **C**); every fifth blow, **M**; two from each heat for basic open-hearth, **C**; hight of fall:

lbs per yd	60–80	90–100	45–55	55–65	65–75	75–85	35–100	
fall, ft	16	17 18, **R**;	15	16	17	18	19,	**M**;

lbs per yd	70–79	80–89	90–100;
fall, ft	18	20	22, **W, C.**

Temperature of test pieces between 32° and 100° F, **R.** Report to state atmospheric temp, **W, M, C.**

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§Ingots from the interior of which the liquid steel has escaped, **R.**
¶Metal from top of ingot, whether cut from bloom or from rail, **R.**

Acceptance and Rejection.

If piece breaks without showing "pipe" or physical defect, all rails from that heat are rejected, **R.**

If broken piece shows "pipe" or physical defect, the top rail from each ingot of that heat is rejected, and the inspector selects a piece from a rail not from top of ingot. If this piece breaks, the rest of the rails of the heat are rejected; if not, they are accepted, **R.**

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If test piece breaks, two additional tests are made of other rails (taken from top of ingot, **W, M, C**) from same blow. If either additional test fails, all the rails of the blow are rejected. If not, all are accepted, **W, M, C.**

92. Manganese steel rails, both cast and rolled, and rails of other special steels, are used on curvs of city rapid-transit lines. Steam roads have used such rails but little, if at all, altho manganese steel is largely used in switch and crossing frogs. Some early manganese cast steel rails, used on curvs in Boston subways, altho ten times more expensiv than ordinary rails, lasted twenty and more times longer.

Where a more durable rail is used, rail renewals are less frequent. On the Boston curvs mentiond, ordinary rails required renewal twice in about 3 months.

Rail Sections.

Am Soc C E Standard Rail Sections.

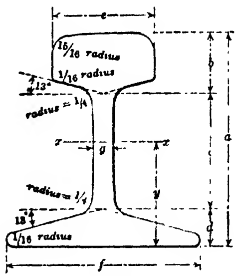

Fig. 6.

93. Fig 6. Transactions, June 1893, Vol 28, No. 6, pp 425 etc Final Report of the Committee on Rail Sections.

In all sizes,

Radius of top of head = radius of side of web = 12 inches;

Other radii (in inches) and angles, as shown in Fig;

Base-width, f = rail hight, a;

Distribution of cross-sec area; in head, 42%; in web, 21%; in base, 37%.

The following properties of A S C E rails are from Carnegie Pocket Companion:—

A = cross-section area;
y = height of grav cen above base;
I = inertia moment, p 468; }
X = section modulus, p 467; } about axis x......x
r = gyration radius = I/A }

For standard rail sections of Am Ry Assn, see pp 798, 799.

For those of Am Ry Eng Assn, see ¶ 94.

For chemical and physical requirements of rails, see pp 794-796 and 1150-1153.

Bold-faced figures give Dimensions in sixty-fourths of an inch.
A in sq ins; y and r in inches. See also ¶ and ‖.

Rail wt, lbs per yd	40	45	50	55	60	65	70	75	80	85	90	95	100
a = *f*	**224**	**236**	**248**	**260**	**272**	**284**	**296**	**308**	**320**	**332**	**344**	**356**	**368**
b	**65**	**68**	**72**	**75**	**78**	**82**	**86**	**91**	**96**	**99**	**102**	**105**	**109**
c	**119**	**126**	**132**	**139**	**145**	**152**	**158**	**163**	**168**	**176**	**183**	**191**	**197**
d	**40**	**42**	**44**	**46**	**49**	**50**	**52**	**54**	**56**	**57**	**59**	**60**	**62**
e	**120**	**128**	**136**	**144**	**152**	**154**	**156**	**158**	**160**	**164**	**168**	**172**	**176**
g	**25**	**27**	**28**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**36**	**36**	**36**
A, sq ins	3.9	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.8	8.3	8.8	9.3	9.8
y, ins	1.7	1.8	1.9	2.0	2.1	2.2	2.2	2.4	2.4	2.5	2.5	2.7	2.8
I ¶	6.6	8.0	9.8	11.9	14.5	16.9	19.6	22.9	26.2	30.0	34.0	38.6	43.8
X ‖	3.6	4.2	4.9	5.8	6.7	7.4	8.2	9.3	10.0	11.0	12.0	13.3	14.6
r, ins	1.30	1.35	1.42	1.49	1.58	1.63	1.70	1.78	1.83	1.90	1.97	2.06	2.1

94. Heavy rail sections, Am Ry Eng Assn, Manual, 1915, pp 77-83. See Fig, p 798.

Bold-faced figures give linear dimensions in 64ths of an inch.
(Areas in square inches. See also ¶ and ‖.)

	Adopted Procs, Vol 16, 1915, pp 397, 1117.				Submitted by Committee	
Lbs/yd Nomnl	90§	100	110	120	130	140
Actl	89.96	101.49	110.36	120.87	...	...
a	**360**	**384**	**400**	**416**	**432**	**448**
b	**94**	**106**	**110**	**114**	**118**	**122**
c	**202**	**210**	**218**	**226**	**236**	**246**
d	**64**	**68**	**72**	**76**	**78**	**80**
e	**164**	**172**	**178**	**184**	...	
f	**328**	**344**	**352**	**368**	**384**	**400**
g	**36**	**36**	**38**	**40**	**42**	**44**
cot *h*	4	4	4	4	4	4
j	**163**	**176**	**181**	**187**	**194**	**201**
m	**896**	**896**	**896**	**896**	**896**	**896**
n	**24**	**24**	**24**	**24**	**24**	**24**
Area, sq ins						
head	3.20	3.80	4.04	4.40	4.63	4.93
web	2.12	2.25	2.49	2.69	3.02	3.28
base	3.50	3.90	4.29	4.76	5.06	5.37
total	8.82	9.95	10.82	11.85	12.71	13.58
Inertia Mom *I* ¶	38.7	49.0	57.0	67.6	77.4	89.2
Section Mod *X* ‖						
head	12.56	15.1	16.7	18.9	20.8	23.1
base	15.23	17.8	20.1	23.1	25.6	28.4

"No new designs for sections under 100 lbs are proposed."

¶I = inertia moment, in biquadratic inches. See p 468.
‖X = section modulus in square inches. See p 478.
§Identical with Am Ry Assn's 90-lb rail, type A, p 799.

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Rail Sections.

Am Soc C E Standard Rail Sections.

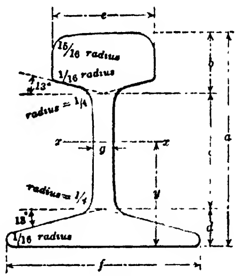


Fig. 6.

93. Fig 6. Transactions, June 1893, Vol 28 No. 6, pp 425 etc Final Report of the Committee on Rail Sections.

In all sizes,

- Radius of top of head = radius of side of web = 12 inches;
- Other radii (in inches) and angles, as shown in Fig;
- Base-width, f = rail hight, a;
- Distribution of cross-sec area; in head, 42%; in web, 21%; in base, 37%.

The following properties of A S C E rails are from Carnegie Pocket Companion:—

A = cross-section area;
y = height of grav cen above base;
I = inertia moment, p 468; } about axis x......x
X = section modulus, p 467; } about axis x......x
r = gyration radius = I/A } about axis x......x

Note. To avoid the printing of fractions, linear dimensions (except for *R;* see foot-note † †, below), are given in **bold-face,** and are in **sixty-fourths of an inch.** Areas are in square inches.

Lower-case letters *a, b, c,* etc, refer to dimensions of *rail and splice-bar. Splice-bar* dimensions are figured on the *right-hand* bar. CAPITALS refer to dimensions of *joint,* or of the combination of rail and splice-bar.

Rail-Joints. Bold-faced figures give linear dimensions in 64ths of an inch.

TYPE	A					B				
Rail lbs/yd	60	70	80	90§	100	60	70	80	90	100
Rail										
a	**288**	**304**	**328**	**360**	**384**	**268**	**291**	**316**	**337**	**361**
b	**79**	**86**	**92**	**94**	**100**	**80**	**87**	**94**	**103**	**109**
c	**157**	**160**	**174**	**202**	**216**	**132**	**145**	**158**	**168**	**183**
d	**52**	**58**	**62**	**64**	**68**	**56**	**59**	**64**	**66**	**69**
e	**144**	**152**	**160**	**164**	**176**	**136**	**152**	**156**	**164**	**170**
f	**256**	**272**	**296**	**328**	**352**	**236**	**259**	**284**	**305**	**329**
g	**30**	**32**	**33**	**36**	**36**	**31**	**33**	**35**	**36**	**36**
cot *h*	4.0	4.0	4.0	4.0	4.0	angle *h* = 13°				
i	**64**	**64**	**72**	**80**	**96**	...	...	...	...	...
j	**136.3**	**140.8**	**147.8**	**162.5**	**176.0**	**125.0**	**138.0**	**145.5**	**156.5**	**168.0**
k	**20**	**24**	**24**	**24**	**24**	**29**	**29**	**31**	**31**	**31**
l	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
m	**896**	**896**	**896**	**896**	**896**	**768**	**768**	**768**	**768**	**768**
n	**24**	**24**	**24**	**24**	**24**	**24**	**24**	**24**	**24**	**24**
p	**24**	**24**	**24**	**24**	**24**	**20**	**20**	**20**	**20**	**20**
For areas, etc, see next page.										
Bar										
q	**99.2**	**103.7**	**109.4**	**124.2**	**133.1**	**86.4**	**94.7**	**105.6**	**110.7**	**117.1**
r	**40**	**40**	**42**	**46**	**48**	**44**	**48**	**48**	**48**	**48**
s	**42**	**44**	**48**	**52**	**56**	**35**	**39**	**40**	**41**	**46**
t	**40**	**44**	**48**	**52**	**56**	**46**	**48**	**52**	**54**	**56**
u	50°	50°	50°	50°	50°	45°	45°	28°	28°	28°
cot *v*	4	4	4	4	4	4	4	4	4	4
w	23°	23°	28°	23°	23°	17°	17°	17°	17°	17°
x	**6**	**6**	**8**	**8**	**12**	**0**	**0**	**10**	**10**	**12**
y	**896**	**896**	**896**	**896**	**896**	**768**	**768**	**768**	**768**	**768**
z	**24**	**24**	**24**	**24**	**24**	**20**	**20**	**20**	**20**	**20**
For areas, etc, see next page.										
Joint										
A	**48**	**48**	**56**	**64**	**64**	**48**	**48**	**56**	**64**	**64**
B	**56**	**56**	**64**	**72**	**72**	**56**	**56**	**64**	**72**	**72**
D	**53**	**55**	**64**	**65**	**66**	**55**	**55**	**64.5**	**66**	**66**
E	**45**	**51**	**56**	**60**	**62**	**52**	**52**	**56**	**62**	**62**
F	**130.5**	**138.0**	**149.0**	**165.0**	**176.0**	**122.0**	**131.5**	**143.0**	**150.0**	**160.5**
*S***	43.41	37.67	36.59	43.64	44.34	32.33	34.51	34.62	33.68	34.86
Rail lbs/yd	60	70	80	90§	100	60	70	80	90	100
TYPE	A					B				

Continued on next page.

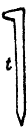
¶*I* = inertia moment, in biquadratic inches. See p 468.
‖*X* = section modulus in square inches. See p 473.
††*R* = mean radius *in inches*
= area in sq ins ÷ periphery in inches;
***S* = 100 (*I* for 2 bars) ÷ (*I* for rail)
§Adopted by Am Ry Eng Assn, Manual, 1915, p 78. See ¶ 94.

Am Ry Assn, Rails and Joints, (concluded).

(Areas in sq ins. ¶ ††)

TYPE	A					B				
Rail lbs/yd	60	70	80	90§	100	60	70	80	90	100
Rail										
Area										
head	2.21	2.68	3.05	3.20	3.64	2.28	2.76	3.07	3.56	3.95
web	1.41	1.49	1.65	2.12	2.29	1.14	1.34	1.54	1.70	1.89
base	2.24	2.65	3.16	3.50	3.91	2.45	2.79	3.30	3.61	4.01
total	5.86	6.82	7.86	8.82	9.84	5.87	6.89	7.91	8.87	9.85
%										
head	37.7	39.3	38.8	36.2	36.9	38.8	40.1	38.8	40.1	40.2
web	24.1	21.8	21.0	24.0	23.4	19.4	19.5	19.5	19.2	19.2
base	38.2	38.9	40.2	39.8	39.7	41.8	40.4	41.7	40.7	40.6
I ¶	15.41	21.05	28.8	38.7	48.94	13.3	18.6	25.1	32.3	41.3
X ‖										
head	6.50	8.21	10.24	12.56	15.04	5.90	7.79	9.38	11.45	13.70
base	7.24	9.51	12.46	15.23	17.78	6.80	8.62	11.08	13.21	15.74
R ††										
head	2.35	2.12	1.93	1.90	1.80	2.10	1.99	1.79	1.68	1.64
web	3.12	3.07	3.57	3.30	3.21	4.38	4.10	3.57	3.65	3.60
base	3.48	3.20	2.52	2.63	3.29	2.94	2.76	2.72	2.58	2.49
total	3.12	3.00	2.50	2.52	2.92	2.90	2.72	2.53	2.42	2.37
Bar										
2 bars										
lbs/ft	21.76	23.94	27.14	33.64	38.28	19.44	23.60	26.24	29.04	34.48
area	6.40	7.04	7.98	10.26	11.26	5.72	6.94	7.72	8.54	10.14
I ¶	6.69	7.93	10.54	17.47	21.70	4.30	6.42	8.69	10.88	14.40
X ‖	3.45	3.93	4.72	6.87	8.28	2.53	3.45	4.41	5.18	6.26

Railroad Spikes.



t

96. The hook-headed spikes *t*, commonly used for confining rails to the cross-ties, vary within the limits of the following table; the lightest ones for light rails on short local branches; and the heaviest ones for heavy rails on first-class roads. The spikes are sold in kegs usually of 150 lbs. For the weight of spikes of larger dimensions, we may near enough take that of a square bar of the same length. What is saved at the point suffices for the addition at the head.

Dimensions, etc.

Size in ins. Length	Side	No. per keg of 150 lbs.	No. per 100 lbs.	Size in ins. Length	Side	No. per keg of 150 lbs.	No. per 100 lbs.
4½ ×	7/16	526	350	5½ ×	½	350	233
4½ ×	½	400	266	5½ ×	9/16	289	193
5 ×	⅝	705	470	5½ ×	⅝	218	146
5 ×	7/16	488	325	6 ×	½	310	207
5 ×	½	390	260	6 ×	9/16	262	175
5 ×	9/16	295	197	6 ×	⅝	196	130
5 ×	⅝	257	171				

¶ *I* = inertia moment, in biquadratic inches. See p 468.
‖ *X* = section modulus in square inches. See p 473.
†† *R* = mean radius *in inches*
= area in sq ins ÷ periphery in inches;

97. Quantity per mile. A mile of single-track road, with 33-ft rails, and 18 ties per rail length, with 4 spikes to each tie; will have 160 rail-lengths, and 2880 ties, and 11 520 spikes, or about 39 kegs of 5½ by 9/16, which weighs a trifle more than ½ lb/spike.

98. *But an allowance must be made* for gard rails at road-crossings, which we may assume to be 33 feet wide, or the length of a rail. A gard will usually consist of 4 extra rails for protecting the track rails, and spiked to the 18 ties by which said track rails are sustaind. Consequently such a crossing requires 18 × 8 = 144 additional spikes. For turnouts, sidings, loss, etc, we may roughly average (allowing for about 1 mile of extra track in the shape of turnouts and sidings in each 15 miles of road) 900 spikes more per mile; thus making in all (assuming one road-crossing/mile) 11.520 + 144 + 900 = 12,564 spikes/mile, or say 43 kegs of 150 lbs each.

99. Adhesion of Spikes. Professor W. R. Johnson found that a plain spike .375, or ⅜ inch square, driven 3⅜ ins into seasond Jersey yellow pine or unseasond chestnut, required about 2000 lbs force to extract it; from seasond white oak, about 4000; and from well-seasond locust, about 6000 lbs. Bevan found that a 6-penny nail, driven one inch, required the following forces to extract it: Seasond beech, 667 lbs; oak, 507; elm, 327; pine, 187.

100. Very careful experiments in Hanover, Germany, by Engineer Funk give from 2465 to 3940 lbs (mean of many expts, about 3000 lbs), as the force necessary to extract a plain ½ inch square iron spike, 6 ins long, wedge-pointed for 1 inch (twice the thickness of the spike), and driven 4½ inches into *white or yellow pine*. When driven 5 ins the force required was about 1/10 part greater. Similar spikes, 9/16 inch square, 7 inches long, driven 6 inches deep, required from 3700 to 6745 lbs to extract them from pine; the mean of the results being 4873 lbs. In all cases *about twice as much force was required to extract them from oak*. The spikes were all driven *across* the grain of the wood. Experience shows that when driven *with* the grain, spikes or nails do not hold with much more than half as much force.

101. Jagged spikes, or twisted ones (like an auger), or those which were either sweld or diminisht near the middle of their length, all proved inferior to plain, square ones. When the length of the wedge point was increast to 4 times the thickness of the spike, the resistance to drawing out was a trifle less.

102. When the length of the spike is fixt, there is probably no better shape than the plain square cross-section, with a wedge-point twice as long as the width of the spike.

103. Behavior. The hook-head spike is, in time, workt upward by the undulatory motion of the rail. In being driven, it crushes the fibers of the wood; so that (particularly in soft woods) they fail to withstand the lateral pressure of the spike. The spike hole is thus so enlarged that the spike loses its hold; and water, entering the hole, hastens decay.

104. Steel. *Finisht spike must show no sign of fracture (a) when bent back on itself thru 180° and hammerd down, (b) when head is bent backward cold, (c) when body is twisted cold thru 1.5 turns.*

Max permissible variation, from dimensions shown: thickness, 1/32 inch; length under head 0 inch less, ¼ inch more; thickness of head, 1/16 inch; angle of hook, 1°.

105. Holes, 1/16 inch less in diam than the thickness of the spike, are sometimes bored for the reception of the spikes. They prevent the crippling of the wood fibers by the spike, and thus increase the resistance of the spike to vert pull, and that of the wood to the lateral thrust of the spike.

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*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

106. Tie-Plugs. Wooden tie-plugs, shaped like the body of the spike, and driven into the spike holes when the spikes are pulled, prevent decay by excluding water. If a spike is afterward driven into the same hole, the plug increases its hold in the tie. †Air-dried wood; cut longitudinally with grain; opp sides parallel; cut square at driving end; 4.5 ins long; 11/16 inch sq; chisel-pointed for 0.5 inch at one end. 1910.†

Screw-Spikes.

107. Screw-spikes are expensiv and time-consuming (see **Costs,** ¶¶ 118, etc) in installation, and in subsequent renewals or other changes; but they prolong the useful life of the tie perhaps two to three-fold, and hold the track in better shape and for a longer time, thus rendering renewal less frequent. Their use may thus reduce *total* installation and *total* maintenance costs.

On the Lackawanna, 5 years' use necessitated no increase in the number of sectionmen per mile

Resistance. In general, screw-spikes offer two or three times the resistance of cut spikes to direct pull, and greater resistance to lateral thrust, and they maintain these resistances much longer. They thus reduce liability to derailment, and damage resulting from derailment.

108. Corrosion may so reduce the size of the head as to cause loose fit in the wrench socket, and thus render removal difficult. A taperd head (Fig 8) obviates this difficulty to some extent; but machine driving (see ¶¶ 130, etc) causes alternate engaging and slipping in the socket and rounding of the corners During 5 years' use on the Delaware, Lackawanna & W'n, no screw-spike was found rusted within the tie.

When the head is broken off, removal is generally impracticable.

109. A raisd letter or device, on the top of the head, betrays any use of the hammer in driving

110. Variations in rail section increase the difficulty in the use of screw-spikes.

111. Behavior. In general, the screw-spike, by holding rail and tie in contact, compels them to rest or move (as the case may be) together. When they move together vertically, the tie may exert a prejudicial churning or a beneficial tamping action upon the ballast, according to the character of the ballast and other circumstances. Some roads prefer to hold tie-plate fast to tie, and allow some play betw rail and tie-plate.

112. As the hole must always be bored in advance, the screw-spike is *less likely to split the tie* than is the cut spike driven in an unbored tie.

113. Owing to compression of tie and to increasing closeness of contact betw tie, tie-plate and rail, in service, newly driven screw-spikes *must usually be tightened* once or twice, at intervals of some months; but thereafter they retain their hold with little or no further attention.

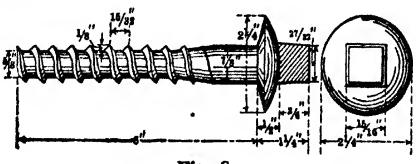

Fig. 8.

114. Dimensions. Fig 8 represents the standard screw-spike of the D L & W R R, 1915. Dimensions in inches. Other designs show head flat on the under side, and without taper. See ¶ 115.

114a. The design, once adopted, should not be changed, as any change would greatly increase the difficulty of the use of screw-spikes.

115. In order *to protect the spike against bending* under lateral pressure, its head, on each side of the spike, must be well supported, as by properly formd clips, or lugs on the tie-plate, or by fitting the spike-head to the rail-flange.

116. Dowels. Hardwood or metal dowels or plugs, driven or screwd into holes prepared for them, and tapt internally for the screw-spike thread, are advantageously used, especially in soft-wood ties, either when the spikes are first driven, or after the spike has begun to loosen. If of wood, they are treated before insertion. The large hole, required for the dowel, necessarily weakens the tie somewhat. The dowel acts as a colum, and thus diminishes indentation of the tie. If of wood, the fiber is vert. and, after driving, the top of the dowel is cut off flush with the upper surf of the tie. The dowels are indefinitely renewable during the life of the tie.

117. The Thiollier steel coild spring or helix has been used as a dowel on several Am RRs and in France. Difficulty has been found in placing and in removing the helix.

117a. The Lakhovsky cast-steel split-sleeve, screwd into the tie, and expanding as the spike is driven, has been used in France.

118. Costs. *Screw-spikes* cost from 2 to 3 cents each, or 2 to 3 times as much as cut spikes. *Hardwood dowels,* tapt outside and inside, 1.5 cts each.

119. The costs of the several **operations involvd** vary widely with conditions. Thus, *placing* 2 tie-plates, boring 4 holes and driving 4 screw-spikes, cost 13.74 cts /tie when boring and driving was done *by hand*; 4.89 cts on *scatterd work with a machine* having two drills and one driving tool; and 2.9 cts on *continuous work with machine* having two drills and two driving tools.

120. *Boring.* Mr. J. W. Kendrick, Am Ry Eng & M W Assn, Procs 1910, Vol 11, Part 1, p 625, says:—"A proper machine at the treating plant will bore and plug 600 ties with 8 plugs each, per day of 10 hrs, at a cost of 3.5 cts/tie" = 0.4375 cent/plug; but, in estimating cost/mile, Mr. Kendrick takes "boring ties for, and driving, 24,000 dowels" (8 per tie) at 1 cent each, presumably for field work.

121. Use. Screw-spikes have long been quite generally used on important lines in Europe, and they are now largely used upon a few such lines in the U S, notably upon the Santa Fé and the D L & W.

122. "Ties should be protected against failure from mechanical wear by means of tie-plates and screw-spikes." Report of Committee on Ties. Am Ry Eng Assn, Procs, 1915, Vol 16, p 522.

123. Tie-plates. The use of tie-plates without bottom flanges (see ¶ ¶ 135, etc) is generally considerd essential to satisfactory service with screw-spikes. See ¶ ¶ 115 and 111.

124. The tie-plate is sometimes screw-spiked to the tie, and the rail held by cut-spikes driven thru slots in the tie-plate.

125. Joints. During the first 2 years (1910-11) of its use of screw-spikes, the D L & W used malleable iron joint plates, and the angle bars were slotted for cut-spikes. One screw-spike was used on the outside of the outer angle bar. Later, all joint plates have been rolled, and screw-spikes used in angle bar slots, with one extra screw-spike on outside of the angle bar.

126. Driving. Any screw-spike *driving tool* should be so arranged as to release when the spike is driven home and thus prevent overdriving. This is commonly arranged by providing a *friction drive.*

127. *Hand driving,* altho inexact, and much more expensiv and time-consuming than machine driving (see ¶¶ 119, etc), must be used where only a few spikes are to be driven, and on very busy tracks (upon which a machine-car cannot be allowd) unless electric or air power can be brought to the tool.

128. For hand-driving, the *simplest* tool is a vert spindle, with a spike socket at its foot, and a hor cross-arm, or ratchet device, at its head.

129. *A crank-driven hand machine, for drilling and driving,* mounted upon an adjustable tripod, and having a hor crank-shaft bevel-geard to the vert spindle, invented by Prof. A. L. Smith, of the Worcester Polytechnic Inst, Worcester, Mass., is described in Bulletin 50, Forestry Bureau, p 53. Under test, this machine drove two screw-spikes while three cut spikes were being driven. It may be operated by power.

130. *Power-driven machines* range from small electric or pneumatic drivers, held by hand, or gasolene motor cars, which the crew can lift from the track, to formidable collections of drills and drivers, installed at the treating plant or mounted upon double-truck cars to save cost of handling ties and to facilitate their removal from the track. They are usually fitted with rail-saws, emery wheels, etc. The larger machines are provided with counters, showing the number of ties handld, and with an exhaust system for the removal of shavings.

131. The Snow gasolene motor car has a gasolene motor, driving a generator which supplies current, thru a cable, to the boring and spiking tools, which may be 1000 ft distant from the car. The car may thus be kept on a side track, out of the way of trains. It weighs, complete, 3400 lbs, and carries ten men besides the driver. It usually avs 2000 to 2500 spikes/day. It can travel 50 miles/hr. Fuses blow out when excessive resistance is encountered, thus protecting the spikes against overdriving. See ¶ 126.

132. The "au-tra-kar", a small gasolene car, can drill a spike hole in an oak tie in from 5 to 10 secs, and drive a spike in about 20 secs.

133. *Speed.* By power, one man can drill 9 holes while 1 is being drild by hand, and can drive 5 screw-spikes while 2 men drive 1 with cross-handld socket-wrench.

134. A *doweling machine,* used by the Santa Fé, had 4 tools; (1) for *boring* dowel holes, (2) for *threading* these holes, (3) for *inserting* the dowels, and (4) for *cutting off* projecting dowel-ends and facing the tie. It could plug the ties for 4000 ft of track per day, while another machine drove the spikes required for the same dist.

A single bit will drill 11,000 holes, 1500 holes betw sharpenings.

The hole should be bored *deeper than the length of the spike*
If bored thru the tie, it will *facilitate the removal of shavings.*

Tie Plates

135. Need. Where the rails bear directly upon the ties, the great unit pressure of the narrow rail base, the churning action of the rail under passing wheels, and the hastening of decay by the bruising of the wood fibers, cause rapid wear of the tie immediately under the rail.

136. Saving. Tie plates greatly lengthen the life of the tie. On curvs and bridges, the saving, in a number of cases, has been estimated at 50% in cost, and 60 to 75% in labor. The tie plate has often displaced small gangs of men whose sole duty it was to replace ties.

137. Types. The tie-plate is placed on the tie, immediately under the rail. Spikes, holding the plate and the rail in place, are driven into the tie thru holes in the plate. Some forms have two or more ribs on the lower side. These ribs stiffen the plate; and,

114a. The design, once adopted, should not be changed, as any change would greatly increase the difficulty of the use of screw-spikes.

115. In order *to protect the spike against bending* under lateral pressure, its head, on each side of the spike, must be well supported, as by properly formd clips, or lugs on the tie-plate, or by fitting the spike-head to the rail-flange.

116. Dowels. Hardwood or metal dowels or plugs, driven or screwd into holes prepared for them, and tapt internally for the screw-spike thread, are advantageously used, especially in softwood ties, either when the spikes are first driven, or after the spike has begun to loosen. If of wood, they are treated before insertion. The large hole, required for the dowel, necessarily weakens the tie somewhat. The dowel acts as a colum, and thus diminishes indentation of the tie. If of wood, the fiber is vert. and, after driving, the top of the dowel is cut off flush with the upper surf of the tie. The dowels are indefinitely renewable during the life of the tie.

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126. Driving. Any screw-spike *driving tool* should be so arranged as to release when the spike is driven home and thus prevent overdriving. This is commonly arranged by providing a *friction drive.*

***For shipment,** plates wired together in bundles of uniform number, weighing $\not>$ 100 lbs. 1915.*

Use.

142. †"Tie plates should be used on all ties in high speed tracks on curvs of 2° or over; on all ties in track subjected to heavy service; on all switch ties, and ties on turntables, ashpits, bridges and trestles; at water stations and track troughs, and thru all road crossings and station platforms. Tie plates should be used on all soft wood ties and on all ties that have been treated."†

‡Unless otherwise directed tie plates will be applied as follows whenever rail or ties are renewed. ("Soft wood" includes all treated or untreated ties except oak);

On new lines: on all soft wood ties in main track;

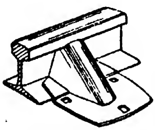
On main lines: on all soft wood ties in main track;

On branch lines; on all soft wood ties on curvs of 3° and sharper, and on all treated ties whether on curv or on tangent;

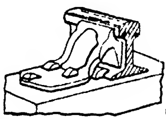
On sidings: on all switch ties and on all curvs of 3° and sharper, and on all treated ties whether on curv or on tangent.‡

Rail Braces

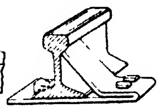
143. Rail braces are used to prevent track spreading, especially on curvs. For 5° curvs, five rail braces per 33-ft rail suffice For 10° curvs, a rail brace is used on each alternate tie.



Alkins Forged-Steel Brace



Ajax Cast-Steel Brace



Edwards Rail Brace and Tie Plate

Fig. 10.

144. Fig 10 shows three types of rail brace in common use.

Joints.

145. The tendency of a track, to yield at the joints, is detrimental both to track and to rolling-stock.

The end of a rail, upon which a loaded wheel is moving, bends more than the adjacent unloaded end of the next rail, which thus receivs a severe blow from the wheel.

146. When the ties are insecurely bedded, no rail-joint can be expected to do good service.

147. Suspended joints (those where the rail-ends meet at a point betw two ties) are generally preferd to supported joints (in which they meet directly over a tie).

148. The expansion coefficient, in steel rails, may be taken at 0.000 006 5 ft/ft/deg Fahr. Hence, a 33 ft rail, (396 ins) under an increase of 60° Fahr in temp, will lengthen by $396 \times 60 \times 0.000\,006\,5 = 0.154$ inch. The rails are also elongated slightly, at their ends, by the traffic passing over them.

*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

149. Rail creeping (See ¶¶ 163, etc.) brings additional stresses upon the joints. Creeping occurs in the direction of heaviest traffic, and where the traffic is equal in both directions, the creeping is in the direction of down-grade.

150. If, in the two lines of rails forming a track, the joints are placed opposit to each other, they are called **"even joints"**; while **"staggered" or "broken"** joints are those where each joint, in one of the lines of rails, is opp to the middle of a rail in the other line. In the latter and more usual case, the jar of passing from rail to rail is less severe than with even joints, but of course twice as frequent.

151. To lessen this jar, rails have been cut with **beveld or miterd ends**, so that the vert plane, forming the rail-end, makes an angle of 45° to 60° (instead of the usual right angle) with the longitudinal vert plane of the rail web.

152. Angle plates. Figs 7, have practically supplanted all other forms of joint. Their hor flanges give lateral stability to the joint, and carry part of the load directly to the ties, thus relieving the rail-ends to that extent.

153. The slots in the flanges of the angle bar should be so spaced that the two spikes driven into a tie for each rail, shall not be directly opp to each other, but "staggered", in order to reduce the danger of splitting the tie.

154. Usual dimensions, etc, for angle-bar rail joints.

	Angle bars		Bolt holes		Bolts	
rail lbs/yard	length ins	lbs per pair	No	diam ins	length ins	diam ins
70	40	70	6	13/16	4	¾
75	40	76	6	⅞	4⅛	"
85	40	80	6	1 1/16	4½	⅞
90	27	—	4	1	4¼	"

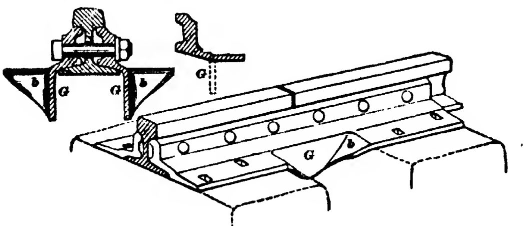
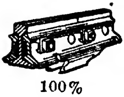


Fig. 11.

155. Bridge Joint. Fig 11. In the bridge joint, the hor flange of the angle bar is rolled wider than usual, and its middle portion is prest downward by dies, forming a girder, *G*, which extends downward betw the two joint-ties. This increases the vert strength of the joint, and the broad flanges increase the bearing surface of the load and increase the lateral strength of the joint; but the downward-projecting flanges (requiring to come between two ties) restrict the liberty of placing the joint with respect to the ties. See end of ¶ 168. Figs 11 represent the Bonzano bridge joint.

156. Other prominent types of angle-bar joints. Figs 12.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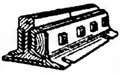
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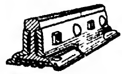
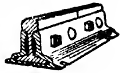
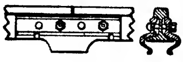
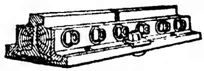

Wolhaupter


Continuous


Duquesne


Braddock

Figs. 12.

157. Abbott. In the Abbott rail-joint, invented by F. E. Abbott, Inspecting Engr, Lackawanna Steel Co, the upper edge of each angle-bar is slightly deprest, at center, in order to keep it away from the bottoms of the rail-heads, at the joint, and thus to avoid the wear and cutting of the top of the angle bar by the rail-heads at that point.

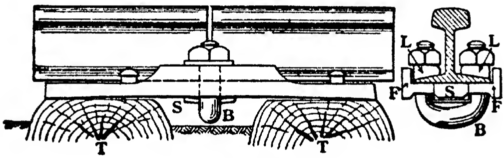


Fig. 13.

158. Fisher. Figs 13. In the Fisher joint (Fisher Rail Joint Works, Trenton, N. J.) the rail *bases*, at their ends, are supported by a flanged plate, *F*, placed under the rail base and bolted to it by a U-bolt, *B*, and clips, *L*, as shown, instead of having the support under the rail *heads* as in the angle-bar joint. A notcht and cambered piece, *S*, of spring steel, placed betw the U-bolt *B*, and the flanged plate, *F*, and held in place by the U-bolt which passes thru the notches, keeps the joint elastic, takes up looseness due to wear, cushions the stresses on the bolt, and maintains pressure betw the threads of bolt and nuts, thus acting as a nut-lock.

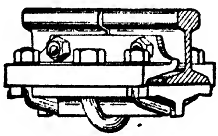


Fig. 14.

159. The "triple fish" (Fisher) joint, Fig 14, has also a supporting plate under the rail-base. It is a short joint, with three U-bolts, as shown, and specially formed angle-bars ("keepers") bolted thru the rail-web

Track Bolts

160. *Unthreaded bolt must bend cold thru 180° and flatten without fracture on outside.*

Threads; U S Standard upset, unless otherwise specified; cut or rolled; $\nless$ 2 $\ngtr$ 5 finger threads.

For shipment, bolts oild; nuts applied for $\nless$ 2 threads. 1915.

Spiral Spring Nut Locks.

(See also p 1167.)

161. *Steel; phosphorus $\ngtr$ 0.05%; sulphur $\ngtr$ 0.05%.

Finisht nut-lock (of internal diam of from $^{11}/_{16}$ to $1^{5}/_{16}$ inch), held flat for one hour, must recover $\nless$ $^{2}/_{3}$ its height or thickness if thickness < width; $\nless$ 0.5 × thickness if square; $\nless$ width if height or thickness > width.

No sign of fracture when one end is held in vise, and opp end twisted to 45° 1915.*

Metal Parts. Specifications.

Steel. Minimum requirements.

(For **rails**, see pp 795, 1152.)

162. Strength and elastic limit in lbs per sq inch. Elongation in 2 ins.

A R E A Manual 1915	Tie-plates§	Finisht Spikes Driven	Screw
Ult tensl stgth, u,	55,000	55,000	60,000
Elastic limit,	0.5 u	0.5 u	0.5 u
Elongation, in 2 ins	20%	20%	22%
Reduction of area,	40%	40%	40%

A R E A Manual 1915	Track bolts Carbon steel	Nickel or other alloy untreated	treated
Ult tensl stgth, u,	....	....	....
Elastic limit,	35,000 or 0.5 u	45,000 or 0.5 u	75,000 or 0.5 u
Elongation, in 2 ins	25%	20%	15%
Reduction of area,	50%	40%	40%

Creeping

163. Rail creeping is due to the undulatory motion of the rail under moving trains, and is most markt on roadbeds lacking in firmness, and on heavy grades. The rails creep in the direction in which the trains move; and creeping therefore gives most trouble on double-track lines. It is increast by rail expansion in hot weather, and diminisht by freezing, which restricts the undulatory motion.

164. "Cross-binding", Fig 15, has been successfully used, to prevent creeping. This consists simply in driving the outside spikes, for each rail, as shown, in advance of the inside spikes in

*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.
§For iron tie-plates, see ¶ 140.

that direction in which the rail tends to creep. Then, if, for instance, rail A creeps in the direction indicated by the arrow, carrying its end of the tie with it, the tie is thrown toward the

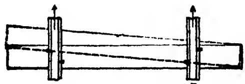


Fig. 15.

position indicated by the dotted lines; increasing (1) the lateral pressures of both its spikes against its flange; (2) the grip of the tie on the rail, and thus (3) the resistce to the sliding of the rail on the tie. And conversely for rail B.

165. Very long ties (sometimes as long as 12 ft) have also been used to reduce or prevent creeping.

166. Slot-spiking of splice-bars is effectiv against creeping, provided the joint-ties hold in the ballast. If they do not, the rails should be anchord to intermediate ties by anti-creeping straps. Or anchors may be made of old splice-bars, cut transversly into sections, leaving one bolt-hole in each section, and making a spike slot in its hor leg. A hole is then drilled in the rail web, over the tie, and the anchor is bolted to the rail, and spiked to the tie thru the slot. Resistance to creeping may then be further increast by fitting blocks betw the ties ahead of the anchord ties.

Another anti-creeping device consists in a clamp which grips the rail base without bolting, and has a lug which bears against the side of a tie.

Altho, in order to prevent rail creeping, the angle splice bars are sometimes slot-spiked to the ties, ¶ 166, it seems preferable to depend upon rail-anchors or anti-creepers; see foregoing ¶ ¶.

167. On the St. Louis bridge (steel arches) and its eastern approach (plate girders on iron columns), the rails crept, in the direction of traffic, about a ft/day, both up and down a grade of 80 ft/mile, and with such force that none of the various fastenings tried sufficed to prevent the creeping, and the track was adjusted daily to accommodate it. See paper by Prof. J. B. Johnson, Assn of Engng Socs, Journal, Vol IV, No 1, Nov 1884.

168. *In relaying rails,* when this involvs changing the positions of the joints, it has been usual to re-space the ties to conform to the new positions; but this practice has been abandoned in some cases, and with apparently beneficial results, the ties and ballast being left undisturbed.

Where angle splice-plates (such as those of the Bonzano joint, ¶ 155) with vert flanges projecting downward betw the ties, are used, tie shifting is of course unavoidable.

Continuous Rails.

169. Expansion and contraction. In street railway work, the rails are commonly **welded, cast or riveted** together at their ends, forming practically continuous rails; and ample experience has shown that, here, as in steam railroad work, a long line of continuous rail does not expand and contract seriously as a whole, under temperature changes; the tendency to expand or contract being successfully resisted by the ground, acting thru the ties and the rail-fastenings. The result is, of course, an increase of longitudinal stress in the rails, with corresponding microscopic changes in their cross-sec areas.

170. Taking the linear expansion coeff, a, of steel, (say 0.000 006 5) as the unit stretch, e, or stretch per unit length, (pp 456-7) under 1° Fahr temp change, and its elastic modulus, E, as 29,000,000 lbs/sq inch, we have unit stress $= E\,e =$ 188.5

lbs/sq inch/deg Fahr. Taking the temp range at 140° Fahr, and assuming the rails laid at mean temp, these would be subjected to a max temp change of 70° and to a max unit stress of 70 × 188.5 = say 13,200 lbs/sq inch, which is well within the allowable unit stress of rail steel.

Miscellaneous

171. Difference, in length, between inner and outer rails. Fig 16.

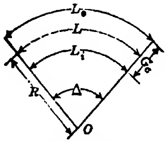


Fig. 16.

Let

R = radius of center line of track;

Δ = sweep of curv,

G_c = gage, measured between *centers* of rails,

L, L_o, L_i = length of arc, on center lines of curv, of outer rail, and of inner rail, respectively.

In a full circle (360°):

$L_o = 2\pi(R + G_c/2) = 2\pi R + 2\pi G_c/2; = 2\pi R + \pi G_c;$

$L_i = 2\pi(R - G_c/2) = 2\pi R - 2\pi G_c/2; = 2\pi R - \pi G_c;$

and, subtracting, $L_o - L_i = 2\pi G_c$.

Hence, in an arc of Δ°, we have:

$$L_o - L_i = 2\pi G_c \frac{\Delta^\circ}{360} = \frac{\pi G_c}{180}\Delta^\circ$$

With gage, G, = 4 ft 8.5 ins, we have G_c = about 4 ft 11 ins = 4.917 ft; and $L_o - L_i$ = 0.085818 Δ°; log 0.085818 = 8.93358.

In any given length, L, on center line, we have

$$\Delta^\circ = 360^\circ \frac{L}{2\pi R} = \frac{180^\circ L}{\pi R}.$$

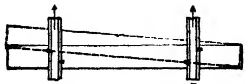
$$\text{Hence } L_o - L_i = \frac{\pi G_c \Delta^\circ}{180} = \frac{\pi G_c}{180}\cdot\frac{180\,L}{\pi R} = \frac{G_c L}{R}.$$

In other words: *for a given sweep*, Δ, the diff, $L_o - L_i$, is *independent of the radius*; whereas, *for an arc of given length*, L, the diff *varies inversely as the radius*, and is *independent of the sweep*.

Gage-widening on Curvs.

172. The necessity and the extent of gage-widening, on curvs, depend upon many variable factors, such as length of rigid wheelbase, the slack, S, or diff betw the track gage, G, (on tangents) and the *wheel* gage, W, (see Fig 2, under "Rolling Stock," p 1040) on both new and worn wheels, etc. Hence, opinions and practice differ widely.

that direction in which the rail tends to creep. Then, if, for instance, rail A creeps in the direction indicated by the arrow, carrying its end of the tie with it, the tie is thrown toward the



A B

Fig. 15.

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Then, Fig 18, **for a 4-wheel truck:**

Slack, $S_4 = G_4 - G = m\,n =$ (approx) $m'\,n'$

$= L \sin n'\,E\,m'$

$$= \text{(approx) } L \sin A\,O\,E = L \frac{B + L}{2\,(R_4 + G/2)^*}$$

$$= \frac{L\,(B + L)}{2\,R_4 + G}$$

Approximately enough, $S_4 = \dfrac{L\,B}{2\,R_4} = \dfrac{L\,B\,D}{11{,}460}$.

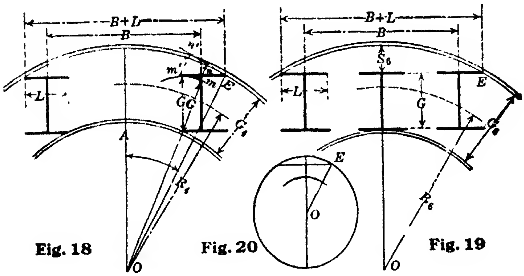

Fig. 18 Fig. 20 Fig. 19

Fig 19. **For a 6-wheel truck;** axles equally spaced, we have, approx (see small-scale Fig 20):

$$S_6 = G_6 - G = \left(\frac{B + L}{2}\right)^2 \times \frac{1}{2\,R_6 + G} = \frac{(B + L)^2}{8\,R_6 + 4\,G^*}$$

Nearly enough, $S_6 = \dfrac{B^2}{8\,R_6} = \dfrac{B^2\,D}{45{,}840}$.

If, in a six-wheel truck, the axles are **unequally spaced,** or **if** there are **more than three axles,** the equation gives S_6 greater than necessary, thus erring on the safe side.

Rail Wear on Curvs.

179. Figs 21. **Special rails** and other devices have been **used,** either to resist or to avoid the lateral forces developt on **curvs,** and the additional wear due to them.

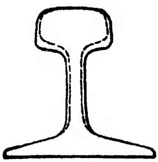

Fig. 21a.

180. Fig 21a. On sharp curvs, the Lehigh Valley RR uses, instead of its normal 100-lb rail, a 110-lb rail, of the same height and base-width, but with a web 1/32 inch thicker, and a head 5/16 inch deeper, with a special outside angle bar, which, on the outer side, projects upward, alongside of the head, to support the head. A special stept or "compromise" joint is used, to effect the transfer betw the normal and the special rail.

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*As an approximation, G is here used for G_4 or G_6 (still unknown). In practice, G may usually be neglected.

181. Fig 21b. In the **Manning** rail, used on the Balto & Ohio RR, the head thickness is unsymmetrical, the greater thickness being on the side subject to wear. When this has worn down sufficiently, the rail is shifted laterally toward the curv center, in order to take up the wear, and thus restore the gage.

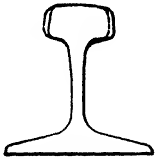

Fig. 21b.

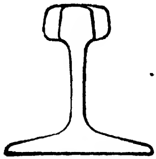

Fig. 21c.

182. Fig 21c. In order to reduce the slip of the inside wheels, in traversing curvs, the Southern Pacific Ry uses, on the inner sides of curvs, a rail with a head 25% narrower than that of the standard rail. The special rail is about 8% deeper than the normal rail A special taper rail is used, to effect the transfer betw the normal and the special rail.

Superelevation. See ¶ ¶ 172 to 193, under "Curvs", p 963.

LAYING AND MAINTENANCE

Laying

183. The complete operation of laying track is usually performed in two stages, (1) Placing the track upon the ground well enough to permit the construction train to run over it, and (2) Ballasting and general completion of work.

184. General. A vital problem of the work is that of transporting ties, rails and fittings from the source of supply to the "front" where they are to be laid. The ties are sometimes carted ahead (part of the distance, at least), on wagons, especially in prairie country, or where a highway lies near by. The rails are usually carried on flat cars in the construction train, which is pusht ahead over the new track as it is laid. The rails, and the ties, when they are carried on the train, must still be transferd from the cars to points just beyond the furthest point of track laid, where they in turn are laid, and spiked and bolted up, sufficiently to permit the construction train to progress over them.

185. Construction Train. The make-up of the construction train will depend upon the speed of laying, dist from base of supplies, grade and motive power, and possible or probable locations of sidings. If the cars used for living purposes are placed at the head of the train, the materials must all be transported around them; and if left on a siding while the work goes on, there may be serious delay in getting them back and forth to the construction crews for meals and sleeping. Frequently they are placed at the front of the train and left there. The order of the cars in the train is usually somewhat as follows, beginning with the car furthest ahead;—

"Pioneer" car; with office, and possibly shop and tools;
Store car, with miscellaneous supplies, for living, etc.;
Bunk and dining car or cars, combined or separate;
Kitchen car;
Additional bunk and dining cars, if necessary;
Tool, or feed and water car;
Rail cars; as many as needed for the day, or as can be handled over the grades and curvs by the motiv power available;

Track fittings car; splice plates, bolts, nuts, etc;
Tie cars; as necessary;
Telegraf line supplies car, (if required);
Fuel car (if the tender capacity is insufficient for a day's service ıd if other sources of supply are not available);
Locomotiv; sometimes several.

186. Forwarding Supplies. At night, or when necessary to reırn for more track material, the bunk and dining cars are left at ıe furthest end of the construction or on a siding, and the loco ›turns with the rail and tie cars (and fuel car, if any) to the base : supplies—probably a yard. Rails and other supplies are there ›aded on to the construction train. Straight and curvd rails, odd-·ngth rails, and hard and soft ties should be loaded separately. therwise, much time may be lost in rehandling and hunting them ıt as needed. By morning, this much of the train returns to the te of operations and connects up with the remainder.

187. The road bed should be well leveld, preparatory to receivıg the ties and rails; as an uneven surface may strain and bend ıe rails badly when the construction train is run over them, or inrease danger of derailment.

188. Piling ties. When ties are stored near the track, the folıwing conditions are usually required., (See also "Seasoning" nder "Ties", ¶ 53, p 788.) *Piled ≮ 10 ft from nearest rail, ft clear betw piles.* †On ground not lower than grade of RR, ıid on foundation of stone or cull ties.† *Piles of either 25 or ‹0 ties.* †Piles ≯ 12 layers high.† *Each pile markt with own-r's name and date when piled* †and number of ties of each kind f wood in pile.† *Sawd and hewd ties piled separately.* †Chest-ıut piled separately.† *Ties treated with zinc-chlorid or other vater solution to be piled in close piles on well-draind ground.*

189. Laying ties. Ties are either deliverd by teams, or are arried ahead by hand, or by the "track-laying machine" (see 199). As far as possible, hardwood ties are reservd for curvs. The ties may be thrown down, and are then laid in place, being ined and spaced by a cord or graduated rod or other means.

190. Laying rails. Rails may be skidded to the ground from he rail car, laterally over a couple of inclined rails; but it is ;enerally regarded as bad practice to let them strike each other, or o let them be thrown on to rough ground. †To be distributed base lown, with uniform bearing surf on roadbed.† They may be car-·ied ahead by men, or dragd by horses, or deliverd by the "track-aying machine" (see ¶ 199), as may appear expedient; and then aid on the ties.

†Rails laid one at a time. Ends brought squarely together ıgainst expansion shims.†

191. Rail-bending for curvs. The necessity for bending rails ›ermanently, before laying them in curvd track, increases with the veight of the rails and with the sharpness of curvature; and lecreases as the rail length increases and as the rail is held ›ecurely to its place under traffic

192. *On curvs of moderate sharpness*, rails are usually laid with-ıut previous bending, being sprung to curv, and then held by the spikes, etc. For curvs sharper than say 10°, and for turnouts, the ·ails are bent at the site, by means of a hand rail-bender, at a ·ost of from $35 to $60 or more, per mile (Camp). For special ayouts, they are sometimes bent (cold) at the mill, by power ma-·hinery.

193. *In turnouts*, the "lead rails should be curvd before they are laid; otherwise it is difficult to prevent them from twisting the headshoes around when attempting to spring a curv into them" (Camp).

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*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

181. Fig 21b. In the **Manning** rail, used on the Balto & Ohio RR, the head thickness is unsymmetrical, the greater thickness being on the side subject to wear. When this has worn down sufficiently, the rail is shifted laterally toward the curv center, in order to take up the wear, and thus restore the gage.

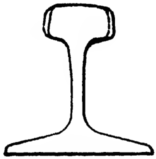


Fig. 21b.

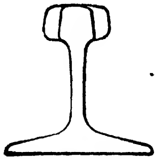


Fig. 21c.

182. Fig 21c. In order to reduce the slip of the inside wheels, in traversing curvs, the Southern Pacific Ry uses, on the inner sides of curvs, a rail with a head 25% narrower than that of the standard rail. The special rail is about 8% deeper than the normal rail A special taper rail is used, to effect the transfer betw the normal and the special rail.

Superelevation. See ¶ ¶ 172 to 193, under "Curvs", p 963.

LAYING AND MAINTENANCE

Laying

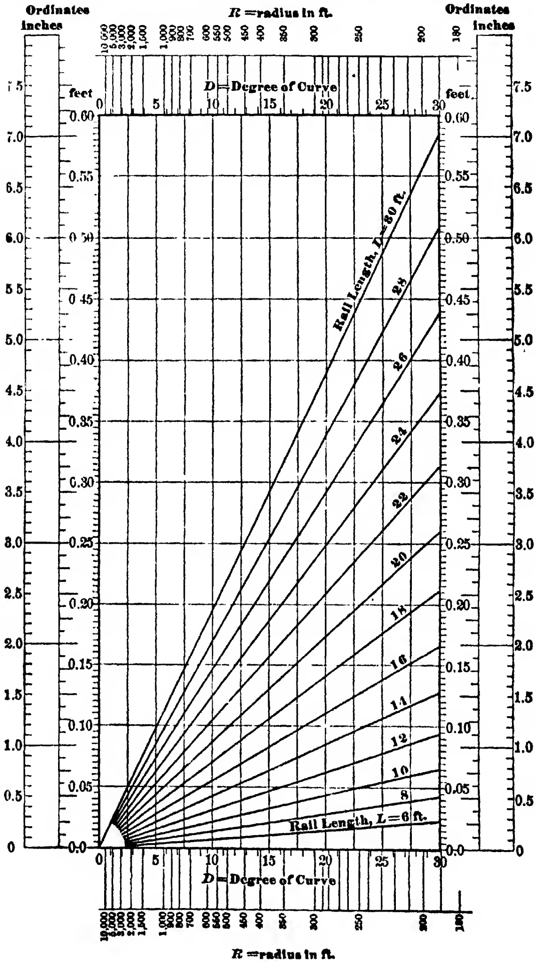
183. The complete operation of laying track is usually performed in two stages, (1) Placing the track upon the ground well enough to permit the construction train to run over it, and (2) Ballasting and general completion of work.

184. General. A vital problem of the work is that of transporting ties, rails and fittings from the source of supply to the "front" where they are to be laid. The ties are sometimes carted ahead (part of the distance, at least), on wagons, especially in prairie country, or where a highway lies near by. The rails are usually carried on flat cars in the construction train, which is pusht ahead over the new track as it is laid. The rails, and the ties, when they are carried on the train, must still be transferd from the cars to points just beyond the furthest point of track laid, where they in turn are laid, and spiked and bolted up, sufficiently to permit the construction train to progress over them.

185. Construction Train. The make-up of the construction train will depend upon the speed of laying, dist from base of supplies, grade and motive power, and possible or probable locations of sidings. If the cars used for living purposes are placed at the head of the train, the materials must all be transported around them; and if left on a siding while the work goes on, there may be serious delay in getting them back and forth to the construction crews for meals and sleeping. Frequently they are placed at the front of the train and left there. The order of the cars in the train is usually somewhat as follows, beginning with the car furthest ahead;—

"Pioneer" car; with office, and possibly shop and tools;
Store car, with miscellaneous supplies, for living, etc.;
Bunk and dining car or cars, combined or separate;
Kitchen car;
Additional bunk and dining cars, if necessary;
Tool, or feed and water car;
Rail cars; as many as needed for the day, or as can be handled over the grades and curvs by the motiv power available;

Middle ordinates for bending rails on curves. See page 816.



194. Expansion and contraction. *Expansion shims* are of many types, from special graduated wedges to wire nails; but they should be of iron or steel, not wood, and should be stampt with the temperatures at which they are to be used. They are placed temporarily betw the rail ends, and so serv to space the rail joints, and thus to provide for expansion and contraction. They are removed later.

195. Provision for expansion. (33 ft rails.) Temperatures, Fahr, taken on rail at time of laying.

Space, in inches betw rail-ends	5/16	1/4	3/16
Am Ry Eng Assn	− 20° to 0°	0° to + 25°	25° to 50°
Penna R R	− 10° to + 14°	14° to 38°	38° to 62°

Space, in inches betw rail-ends	1/8	1/16	0§
Am Ry Eng Assn	50° to 75°	75° to 100°	over 100°
Penna R R	62° to 86°	86° to 110°	over 110°
P R R, in tunnels	22°	46°	over 70°

†At insulating joints, ½ inch, irrespectiv of temp.†

196. Joints are next made up, the splice bars being put in place, some if not all of the bolts past thru the holes, and the nuts tightend up. The expansion shims may then be removed. †*On tangents*, each joint is to be opposit the middle of the opp rail of the same track. *On curvs*, variation $\ngtr$ 18 ins.†

197. Spiking the rails to the ties then follows. The tie is held up against the rail by a man with a "nipping bar", (a lever which straddles the rail and hooks under the tie), and two men drive the spikes, one on each side of the rail, striking alternately Each spike should be driven vertically and should be started with the side of the point squarely in contact with the rail flange, so that it will crowd the rail all the way down Where *screw spikes* are used, the operation is of course different. See under "screw spikes", ¶ ¶ 107 etc. Care is of course taken that the gage be properly establisht when the spikes at the other end of the tie are driven.

198. *Specifications.* †Where tie plates are not used, inside spikes to be driven near east or south edge of tie; outside spikes driven near west or north side of tie; $\nless$ 2 ins from edge of tie. 1909.†

†Spikes per tie at each rail;—

On tangents without tie plate:	1 inside, 1 outside
On tangents with flat or ribd tie plates:	2 inside, 1 outside
On curvs with flat or ribd tie plates;	2 inside, 2 outside.†

Spikes must not be straightend during driving. Outside spikes, of both rails, to be on one side of tie; both inside spikes on the opp side. Spikes ordinarily 2.5 ins from outside of tie. Old spike holes to be plugd.

199. "Track-laying machines", so called, do not actually lay track. They merely facilitate the forwarding of the rails and ties from their cars to the points ahead where wanted. They consist of some form of rollers or track mounted on the construction train, along either one side or both sides, or over the top. In some machines, the run-way is laid on a grade and the material goes ahead by gravity; while in others it is carried on traveling platforms or belts or other transfer devices driven usually by a special stationary engin. Such machines are seldom expected to expedite the work; but only to cheapen it, by reducing the number of laborers necessary.

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*Am Ry Engg Assn. †P R R. ‡U P R R See also Specifications p 780.
§Laid close, without bumping.

200. Speed of laying, according to Camp, under av conditions; about one mile in 10 hours without hurrying; employing 64 laborers, 3 foremen and 2 teams of horses.

Ballasting.

201. Delivering Ballast. Ordinary flat cars, without sideboards, loaded by steam shovel, hold about 10 cu yds each. With sideboards, 2 ft 8 ins high, a car 40 ft long will hold about 32 cu yds. The sideboards are arranged as a series of doors, hinged at top.

202. Flat cars are economically unloaded by the use of a plow, dragd along the tops of the flat cars, by means of a wire cable, attacht either to the loco, or to an engin drum on a car at end of train. With a loco, the train stands still, and each car deposits its full load within a dist equal to its own length; but, with engin and drum, the distribution of the material can be controlled by having the train in motion.

203. *Center plows* distribute the ballast on both sides of the track. They are guided either by stakes, temporarily placed in the side pockets for the purpose, or by rollers bearing on the sides of the car. The point of the center plow is usually movable sideways; so that, if desired, more ballast may be unloaded on one side of the track than on the other.

204. *Side plows* unload on one side only. They are guided by side stakes Side unloading, with either side or center plows, involves the expense of throwing the ballast back upon the track. See also Dump Cars, under Rolling Stock, ¶ 80, p 1053.

205. To prevent ballast from dropping on the track, betw cars, a hor apron of boiler plate is hinged at one end of each car; so as to overlap the space at the coupling.

206. Leveling. When ballast is unloaded (between the rails) from hopper-bottom dump-cars, it is leveled by means of a spreader plow car, attacht at the rear of the train, or by means of a crosstie fastend ahead of the front wheels of the rear truck of the car being dumpt. Several men, with bars or shovels, are required on the car, to push the material down into the hoppers.

207. Tamping. *Earth or clay ballast.* Shovel equipt with iron cuff or handle for tamping; broad-pointed tamping bars. *Tamp each tie from 18 ins (burnt clay, 15 ins), inside of the rail to end of tie with handle of shovel or tamping bar. If possible, tamp the end of the tie outside of rail first and let train pass over before tamping inside of rail; give special attention to tamping under the rail; tamp center of ties loosely with the blade of the shovel. The dirt or clay betw the ties should be placed in layers and firmly packt with feet or otherwise, so that it will quickly shed the water; the earth should not be bankt above the bottom of the ends of the ties; the filling betw the ties should not touch the rail and should be as high as, or higher than, the top of the ties in the middle of the track. With *broken stone* or *furnace slag*, do not tamp center of ties; bank ballast into shoulder about the end of the ties level with top of tie.*

208. Costs; Ballast, cts per cu yd

Machine-crusht stone, at quarry	45 to 75
Placing under track, and tamping	15 to 25
In track, completely ballasted, lined and drest	75 to 125
Gravel, in completed track	20 to 30
Including, for labor of placing in track, tamping and dressing	10 to 15
Unloading, with plow and cable, including labor of handling cable and use of equipment	0.5
Roadbed oil,	2 to 3½ cts per gallon

*Am Ry Engg Assn. †P R R. ‡U P R R See also Specifications p 780.

209. Oiling Ballast. *On dusty roadbeds,* annoyance to passengers and injury to ties and journals may be prevented by sprinkling the ballast with oil, which penetrates to a depth of several inches, holding down not only the dust already present, but also that which may settle subsequently. It evaporates so slowly that one application per season usually suffices

210. "Roadbed oil" is a product of petroleum distillation, with a spec grav of about 0.89. It is practically non-combustible under the conditions of its use, and it is said that fewer ties are burnd in oild than in unoild track.

211. Oil coating retards the growth of weeds in the ballast; and, by rendering the ballast non-absorbent, reduces heaving of the track by frost. It is believd to preserv ties by excluding moisture.

212. The sprinkling apparatus is commonly installed on a flat car, with couplings for connection to an oil-tank car. In warm weather, the oil flows freely by gravity. In cold weather, its flow may be assisted by steam or air pressure, taken from the loco. The oil is applied to the track, to the shoulder of the roadbed, and, if required, to the slopes in cuts and fills.

213. The sprinkling train can cover about 4 miles/hr, using 2,000 to 2,500 gals of oil per mile of unoild single track Subsequent sprayings require less oil The penetration increases with each spraying, until the depth reaches about 8 ins, which suffices to prevent dry and dusty ballast from being thrown up during tie renewals or tamping.

Maintenance of Way.

214. Surfacing consists of raising depressions in the track, (mostly at joints) to an even surface. It is not necessary to maintain track to the grade stakes to which it was laid when new, as long as it is maintaind fairly even.

215. Surfacing "out of face" (consisting of a resurfacing of the track as a whole) is necessary every few years.

216. *Cost* of surfacing varies from about $100 to $200/mile /annum, depending upon the density of traffic, and upon the materials of which the track is constructed.

217. Ballast. *Renewal.* Before distributing new ballast on the track, the old ballast should be removed from betw the ties as deep as their bottoms, and used to widen the shoulders of the roadbed. See also "Ballasting", ¶ ¶ 201, etc.

218. *Cleaning* ballast. *Intervals, in years, betw cleanings; in terminals, 1 to 3; heavy traffic, coal and coke lines, 3 to 5; light traffic, 5 to 8.*

Under usual conditions, clean only stone and hard slag. Use ballast forks. Clean (a) shoulder, down to sub-grade; (b) betw ties, to bottom of ties; (c) center ditch of double track to subgrade. Return clean ballast.

*Bank gravel (Percentages refer to the *original* bulk)

Road Class	To be Washt or Screend when containing >	Washt or Screend gravel should contain
A	2% dust or 40% sand	≮ 25% ≯ 35% sand
B	3% " or 60% "	≮ 25% ≯ 50% " *

219. Ties. The person inspecting ties, with a view to renewal, may indicate such ties by a spot of white paint. The usual method of putting in new ties is to dig a trench beside the old tie, slightly deeper than the tie; then to pull it sidewise into the trench, and then to haul it out. If the new tie is thicker, or the old tie was rail-cut, it is necessary to dress down the bed of the tie. In rock ballast, ties can be renewed at the rate of 8 or 10/day/man; in gravel, 14 to 18.

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*Am Ry Engg Assn. †P R R. ‡U P R R. See also Specifications p 780.

220. *Cost of Renewing Ties*

in stone ballast	about 20 cts each
in loose gravel	6 to 10 cts each
in cementing gravel	10 to 20 cts each
in hard slag	9 to 15 cts each

221. Rails. New rails, to be laid, are placed on the ties outside the old rails in place, and bolted together into such sections as can be expeditiously handled. The spikes on the inside of the rail in service are pulled, the rail is lifted out without unbolting the joints (except at ends of sections), and the new rail is set in place and spiked.

222. *On curvs,* rails are often transposed, the worn outside rails being moved to the inside of the track, and the less-worn inside rails to the outside; the inside spikes being pulled, the rails transposed, and the spikes replaced.

223. The hardest track to maintain is that on sharp curvs, elevated for fast passenger trains, and having to carry freight trains whose speed is limited, as by grade conditions.

224. Track-recording cars are of many types, there being apparently no standard. Nearly all carry a long strip of paper, moved by clock-work, or by gearing from the wheels, so that, by means of mechanical pencils or pens, plottings are made, on which the abcissae may represent either time or distance, as desired. In order to identify different portions of the record, it is customary to devote one or two pens to the recording of time by means of a clock sending impulses thru an electro-magnet which actuates a pen; or to recording distance, either automatically, by gearing from the wheels, or by impulses sent from a push-button by an observer watching for mile-posts or other land-marks. These marks are identified, either by making them with certain characteristics, as double, triple, or long marks, or by another operator recording the name by hand, opposit the mark.

225 Various elements are recorded, and in different ways. Lateral or vertical inequalities of the track are recorded either by pendulums, (free, restricted, or "dampt" by "dash-pots"), by the relativ motions of truck and car-body, or by those of an idle wheel or pair of wheels. Some cars are arranged to daub the side of the rail automatically with paint, wherever the track is uneven.

Dynamometer Cars. See under Train Resistance, p 1067. ¶¶ 56, etc.

TURNOUTS AND CROSSINGS

PART I. PRACTICE

TURNOUTS.

For the geometry of turnouts, see Part II, p 848, etc.

General

1. Elements. Figs 1. A turnout consists essentially of a switch, two "lead" rails, L and L_t*, a frog, and two gard-rails, g and g_t.

2. Facing and Trailing. Figs 1. When a train enters a turnout in the direction of the arrow (passing the switch before reaching the frog), it is said to "face" the switch When it approaches (either from the main track, M', or from the turnout, T) in the opposit direction (passing the frog before reaching the switch) it is said to "trail" the switch.

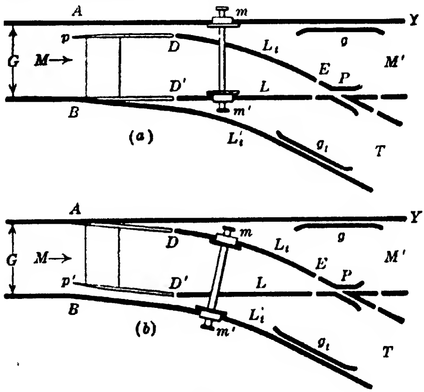


Fig. 1.

3. Double track; Fig 2; trains keeping to the right, as indicated by the arrows. Here, for the normal traffic, X and Y are facing switches; while V and W are trailing switches. In leaving a main track by a trailing switch (*i.e.*, in *facing* a *trailing* switch) a train must move in the direction contrary to the one proper to said track. Nevertheless, the superior safety of trailing switches justifies their use.

4. Fig 2. A turnout is **right-hand** or **left-hand**, accordingly as it sends a *facing* train to the right or to the left from the main track. Thus. V and W are right-hand turnouts, while X and Y are left-hand.

5. The position of the switch (Figs 1), determins which of the two tracks, M' or T, a facing train shall follow. The main lead rail or main closure rail, L, and the turnout lead rail or turnout closure rail, L_t,* lead from the switch to the frog, the flangeways of which permit the flanges of wheels, on either one of the two lead rails, L and L_t,* to pass thru (see pgf 68) the other

*Some writers call the two curvd rails, L_t and L_t', the lead rails. See Webb, Railroad Construction, pp 265 and 272.

lead rail. The two gard-rails, g and g_1, keep said wheels to their proper paths by confining the flanges of their mates while they are passing the frog.

6. The lead (sometimes called the **frog distance**), is the dist, $B\ P$, Figs 1, where P is the theoretical frog-point (¶ 42) and B is the position of the theoretical switch-point (¶ 79) on the turnout side of the main track, $M\ M'$, when the switch is set for the main track, as in Fig 1*a*.

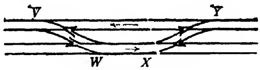


Fig. 2.

7. The lead curv, or turnout curv, $D\ E$, Figs 1, is usually (at least in theory) a simple or circular curv. Fig 13. The gage-lines, $a\ y$ and $b\ z$, of the frog are usually straight thruout the frog, or from frog heel to frog toe; as are also the switch-rails thruout their own length. But see ¶ 77.

Types of Turnouts.

Double Turnouts.

8. Figs 3. In a double turnout (often called a **three-throw or three-way** turnout), two side tracks leav the main track at the same point. As compared with two neighboring turnouts from one track, a double turnout economizes space, material, labor and time, and facilitates operation.

9. Curvature. The *main track* may be *straight or curvd,* and the *side tracks* may leav the main track *on the same or on opp sides.* When they leav from opp sides, their sharpnesses (Curvs, ¶ 11) may be equal or different. Altho, in operation, the side tracks are thus distinguisht, by their *purposes,* from the main track, it is often convenient to disregard this distinction, and to consider the three tracks with regard only to their relativ *positions,* or as *outer* and *middle* tracks. See ¶ 11. Thus, T or T' may be a curvd main track; in which case, M' is a straight turnout.

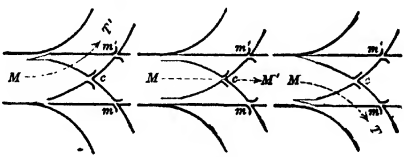


Fig. 3.

10. Elements. Figs 3. A double turnout requires *a 3-throw switch,* two main frogs, m, m', a crotch frog (middle frog), c, and six gard-rails (two for each of the three frogs), unless, as in Figs 3, the two main frog angles (see Part II, ¶ 32) are equal, and the two main frogs therefore opposit; in which case one wing-rail (see ¶ 41) of each main frog acts as gard-rail for the other main frog, and only four gard-rails proper (as shown) are required. When the two main frogs, m and m', are of but slightly diff angles, and therefore nearly opp, it is impracticable to place and maintain the two additional gard-rails of sufficient length. In Figs 3, the two rails of each three-throw switch are indicated by light lines, which show the switch set for tracks T', M' and T, respectivly, as indicated by the arrows. Where the turnout curvature is sharp, the sides of the crotch-frog are sometimes curvd to accord with it.

Main, Side and Connecting Tracks.

11. Of the two tracks, connected by a turnout, it is usually practicable to distinguish betw one, as the *main* track, and the other, as the *side* track. The track, betw the main-track frog and the side-track, is called the connecting track; but the term, "connecting track," is applied also to a track of some length connecting two tracks neither of which is a side track to the other.

12. A spur or stub track is a track (usually short), serving some special purpose, as a neighboring quarry, siding, borrow-pit or warehouse, and not returning to the main track.

12a. A transfer connects two non-adjacent tracks, and the term is used especially when these are at diff levels.

12b. A leader crosses diagonally, and connects, a series of parallel or concentric tracks.

12c. A ladder (see Part II, p 871, § 57) connects a main track with adjacent yard tracks.

Fig. 4a.

Fig. 4b.

Gauntlet and Intervolvd.

13. Fig 4. In certain special cases, as where two parallel or concentric tracks must pass thru a space too narrow for double track, as in a single-track tunnel or bridge, the arrangement shown may be used. Where the traffic, on the two tracks, is in opp directions, the arrangement is called a **gauntlet;** when it is in the same direction, the tracks are said to be **intervolvd.** Two frogs, but no switches, are required. The two tracks are laid upon the same ties.

Fig. 5.

14. Improving Switch Location. Fig 5. To avoid placing a switch on the outside of a curv it may be placed in the tangent, back of the curv-point, *A*, and the traffic carried, on a concentric intervolvd track, to the frog where the turnout, *T*, must leav the main-track curv, *A M'*. This arrangement involvs the use of one switch and one frog.

15. An arrangement of intervolvd tracks is sometimes used for the **protection of track scales** from unnecessary usage by traffic which is not to be weighd. The track, carrying such traffic, rests upon solid piers passing thru, but independent of, the scale platform. This arrangement involvs the use of two switches, without a frog. See Yards and Stations, § 60.

Diamond or equilateral turnout.

16. Where a parallel or concentric side track leavs a straight or curvd main track, the main track ordinarily continues its course unaffected by the presence of the turnout; the deflection, from this course, being effected entirely in the turnout.

17. If, however, the defl be equally divided betw the main and the side tracks, we have a "diamond" or "equilateral" turnout. This permits the use of a frog of greater angle (lower number), without sharpening the curvature. See Part II, ¶ 52.

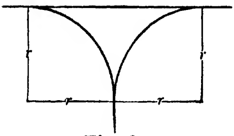


Fig. 6.

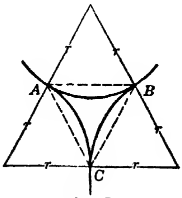


Fig. 7.

Y-Tracks.

18. A Y-track, Figs 6 & 7, is used for turning engins end for end. It thus takes the place of a turntable. Entire trains, if not too long, may be turnd on the Y, as well. The Y should cost less, for maintenance, than a turntable; and may cost less to install, depending upon the cost of ground.

19. Fig 7. Other things equal, the *Y occupies least ground when* the three tracks, or "legs," are curvs of equal radius, *r*, and of equal length. The three switches are then at the apices of an equilateral triangle, *A B C*, each side of which equals the common radius, *r*.

Derailing.

20. General. Cars, left on sidings, may be moved, by gravity, wind, carelessness, etc., to positions near enough to the main track to endanger main-track traffic by collision. Cars may start, by gravity alone, upon a down grade of about 0.4 per cent (21 ft per mile); or (under wind) on flatter grades or on level track. It is unsafe to depend upon the brakes.

21. Usually a dist of 12 ft betw track centers (say 6.5 to 7 ft clear betw rail-heads) is considered safe. The Penna R R, 1909, requires $\not<$ 16 ft from cen of siding to cen of main track, (18 ft where practicable), except on passing sidings where parallel to main tracks; the unconnected end of siding, adjacent to main track, to be curvd outward.

22. The point, where the two tracks approach closer than allowable, is often indicated by a **clearance post** of wood or of metal. If set betw the tracks, such posts are dangerous to men at night. They are therefore preferably placed 4 or 5 ft to one side of either track. Sometimes the clearance point is indicated by a whitewasht half-round tie, laid across the space betw the tracks.

23. A derail is used in main or in side tracks where derailing is preferable to allowing a car to reach a danger-point, such as a switch or a crossing, etc.

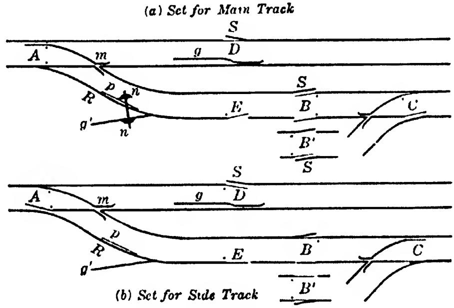
24. Figs 8*a*, 8*b*. *The heel ends* of the switch-rails are indicated by small dots on inside of track.

25. *A single derailing switch-rail*, as at *D* or *E*, suffices for mere derailment; but a *pair of switch-rails*, as at *B*, facilitates restoring derailed cars to the track. In the derailing position, the switch-rail is usually supported by two or three rail-braces. See Track, ¶ 143.

26. The derailing switches are preferably so *connected with the main-track switch*, *A*, that the derails shall always be in the positions proper to the position of the main-track switch. (Compare Figs 8*a* and 8*b*.) Otherwise, a sign, reminding the switchman to set the derail, may be placed at the main-track switch.

27. In main-track derails, as at *D*, the stock-rail, *S*, is bent outward; and a single derailing switch-rail is commonly used, heeling toward the main track frog, *m*, and opening inward, Fig 8*b*.

28. In side track, a single derailing switch-rail, *E*, placed in the outer rail, heels toward the main-track frog, *m*, and opens inward, as in Fig 8*a*.



Figs. 8.

29. Fig 8*a*. Where a car, deraild from the turnout, as at *E*, might nevertheless foul the main track (as *where the derail is near the main-line switch*, and the side-track grade is considerable), a long gard-rail, *g'*, may be used, to keep the farther wheels, *n*, away from the main track, and a plank, *p*, to carry the nearer wheels, *n'*, over the adjacent side-track rail, *R*. Or a derailing *turnout*, *C*, may be used.

30. Fig 8*b*. A main-track car, deraild at *D*, is kept near its proper course by a long gard-rail, *g*, placed about 8 ins from that rail which is opposit the derailing switch. This gard-rail may be continued backward, and bent, as shown, narrowing the flangeway opposit the derail, and thus *protecting the derail switch-point* when the derail switch is set for the main line, as in Fig 8*a*.

31. Figs 8*a*, 8*b*. **In double track,** if a single derailing switch-rail, *D* or *E*, is used, it is usually placed in an *outer* rail, in order to avoid obstruction of the other track by deraild cars.

32. Direction of Heeling. Figs 8*a*, 8*b*. Where a pair of derailing switches, *B* or *B'*, is used in the side track, they may either, as at *B*, heel *away* from the main-track frog, *m*, opening *away from* the main track; or, as at *B'*, they may heel *toward* the main-track frog and open *toward* the main track; in either case guiding a deraild car *away* from the main track.

33. Wharton Switch. A special short form of Wharton switch (see ¶¶ 119, etc.) is used for main-track derails, the derailing switch rails being thrown *against* the main track rails for *derailing*, and *away* from them for *main-track* traffic. This arrangement leavs the main track unbroken; and its wide throw avoids danger of fouling the derail when set for main line.

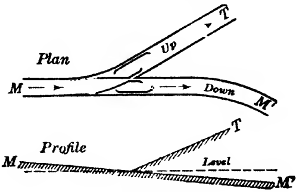


Fig. 9.

Catch Sidings.

34. Fig 9. Catch-sidings are often provided for the purpose of diverting and holding run-away cars on steep down grades, especially when approaching curvs. The catch-siding runs a short dist up the hillside; and the switch is held, by a spring, in position for the siding, as shown; so that the run-away car is diverted to the siding, and there first brought to rest by its own weight. It then runs back, by gravity, to and thru the switch, and up the main-track grade. It thus oscillates, coming finally to rest at the switch. In normal operation, the switch is thrown over to the main-track position, against the spring, by the switchman, who holds it there until the car or train has passed the switch.

For Scotch-block, see Signals, ¶ 47, p 989.

Crossovers.

35. Figs 10*a*, 10*b*. A crossover connects two parallel or concentric tracks. It consists of two turnouts (facing in opp directions) and a connecting track, which may be a tangent or a reverse curv.

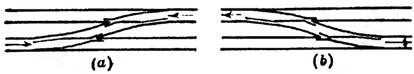
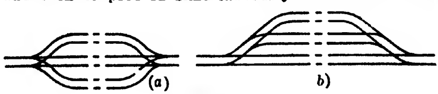


Fig. 10.

36. Right and Left. With right-hand traffic (as in **Figs 10*a*** and 10*b*), the two turnouts are both *left*-hand, if *facing*, as in Fig 10*a*; and both *right*-hand, if *trailing*, as in Fig 10*b*.

37. Double Siding. Figs 11*a*, 11*b*. On single track lines, in order to allow two meeting trains to await the passage of a third train, and then to proceed simultaneously and without backing or



Figs. 11.

interference, a double siding is provided. The side tracks may be one on each side of the main track, Fig 11*a*, or both on the same side, Fig 11*b*.

38. The lap siding, Fig 12, has the advantage that the telegraph office may be placed at the lap, and therefore convenient to the engins of both standing trains, thus facilitating operation. As will be seen, the lap siding is, in effect, a double track, of length *A B*, with a crossover midway of its length.

Fig. 12.

Gage.

39. Some roads maintain standard gage on turnouts. Others widen the gage by from ¼ to ½ inch, betw switch-point and frog-heel, beyond which points the gage is gradually narrowd to standard width within a dist of about 30 ft each way; while, on European roads, the gage is often narrowd ¼" in the turnout, in order to prevent lateral play of the wheels, and to steady vehicles while on the turnout. The Wharton switch, see ¶¶ 119, etc., requires, at the switch, a gage ½" wider than standard.

Frogs.

Rigid or Stiff Frog.

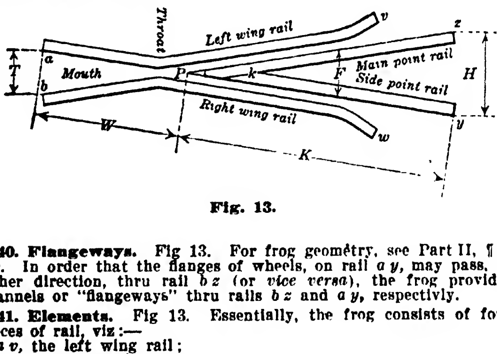


Fig. 13.

40. Flangeways. Fig 13. For frog geomêtry, see Part II, ¶ 2, etc. In order that the flanges of wheels, on rail *a y*, may pass, in either direction, thru rail *b z* (or *vice versa*), the frog provides channels or "flangeways" thru rails *b z* and *a y*, respectivly.

41. Elements. Fig 13. Essentially, the frog consists of four pieces of rail, viz:—

a v, the left wing rail;
b w, the right wing rail;
P z, the main point or long point;
k y, the side point or short point.

42. The gage lines, thru the frog, are *a y* and *b z*. Fig 13. To the *remaining parts and dimensions* of the frog are given names as follows:—

Toe spread, *T*, = *a b*, measured betw gage lines at rail heads;
Heel spread, *H*, = *z y*; measured betw gage lines at rail heads:
Throat, the point of shortest distance between wing rails;
Mouth, the trapezoidal space betw the two wing rails and betw throat and toe;
Theoretical frog point, *P*; the intersection of the gage lines, *a y* and *b z*;
Actual (or "half inch") frog point;

Tongue, the triangle betw P and k;
Flangeways or channels, the two parallel-sided channels betw the points and the wings;
Frog angle, F, $= z\,P\,y = a\,P\,b$.

43. The frog number, N, is the quotient, l/i, where l (not shown) = the length of any portion of the frog, and i (not shown) = the increase of frog width (measd betw gage-lines and perp to the frog axis) within that portion. l is measd, by some engineers, along the center-line of the frog; and, by others, along a gage-line; and the number of a given frog is of course correspondingly affected. See Part II, ¶ 6 etc.

44. Right and left. Fig 13. The *side* point, $k\,y$, is ordinarily placed in the *turnout*. Hence, a frog is called "*right*-hand" if a person, facing it, sees the *side* point on the *right*, and *vice versa*.

45. Fillers. The two point-rails are riveted together, and a steel filler is secured in place betw the point and each wing-rail. The two fillers are sometimes carried beyond the point, and joined there, forming a single straddling filler.

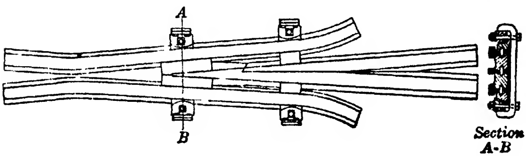


Fig. 14a.

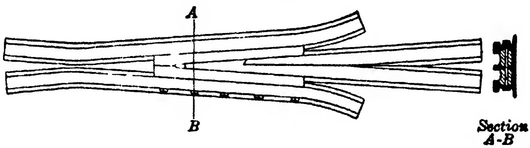


Fig. 14b.

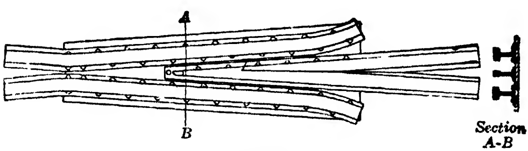


Fig. 14c.

46. Fastenings. The wing-rails and the point, are held together, in their proper relativ positions, usually either
(1) by clamps and keys, Fig 14*a*;
(2) by bolts, Fig 14*b*;
(3) by riveting to a plate, Fig 14*c*;
(4) by bolts and plate [combination of (2) and (3)]; or
(5) by riveting to tie plates.

47. Practice, respecting the choice betw these methods, is indicated by the results of two canvases, showing the number of roads using each method, as follows:—

	(1) Clampt	(2) Bolted	(3) Plate	(4) Bolt and plate	Total
Roadmasters' Assn, 1897,	7	29	9	....	45
Eng News, 1908 Jun 4,	7	32	15	10	64

Frogs are connected with the adjoining track rails by the usual rail joints. See Track, ¶¶ 145, etc.

48. Altho one side of the frog is in the turnout curv, and the other side may be in a main-line curv, yet the shortness of the sides warrants making them straight, except in very sharp turnouts and in special cases.

49. Length. The frog length must be such that the rail ends, at toe and at heel, are far enough apart to give room for the rail-joints without interference. Increast length strengthens the frog against working loose; but it also increases waste when frogs must be renewd, because the adjoining rails always outwear the frogs. The adoption of a standard frog reduces the rail cutting required when frogs are renewd. See ¶ 70.

50. Details. The wing-rail on the turnout side is sometimes made slightly longer, at the toe, than the other wing-rail, in order that the main and turnout lead rails (¶ 5) may be of equal length, and still bring the heels of the switch rails opposit.

Wear, Reinforcement, etc.

51. Special Steel. Those parts of frogs which are subjected to heavy wear are frequently made from special manganese or other steel of high wearing quality, and built into the body of the frog. Frogs so made cost about twice as much as do ordinary frogs; but their use is nevertheless economical under heavy traffic.

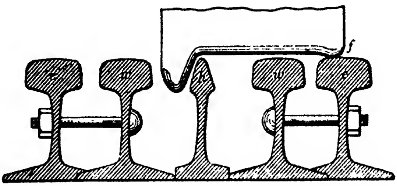


Fig. 15.

52. Easer Rails. Fig 15. False or outside wheel flanges, *f*, are formd by the wearing down of the tread, nearer to the flange. A short easer rail, *e*, bolted outside of each wing rail, *w*, and forming, with *w*, a double-headed wing, raises the wheel sufficiently to keep the worn tread out of contact with the point-rail, *h*.

53. Heel Block or Heel Raiser. Figs 16. The false flange, *f*, of a trailing wheel, dropping betw the point-rails, *M* and *S*, tends to wedge them apart. To prevent this, a heel block or heel raiser, *R*, is bolted in place betw the point rails. The raiser slopes downward toward the frog heel, as shown, in order to lift and lower the wheels gradually. It may be a solid steel casting, or an inverted piece of rail. It also servs as a foot gard. See ¶ 75. Unless securely fastend, it may be driven forward by the false flange, *f*, and may thus itself act as a destructiv wedge

*54. In the rigid or stiff frog (thus far described) all the parts are immovable, and both flangeways are always open. The wing-rails, Fig 13, by supporting the outer portion of the wheel-tread, protect the relativly slender frog-point, which otherwise would receiv hard usage from passing wheels; and the flangeways are made as narrow as possible, in order to diminish the severity of the blows deliverd by wheels passing them; but, nevertheless, these blows become serious under heavy traffic.

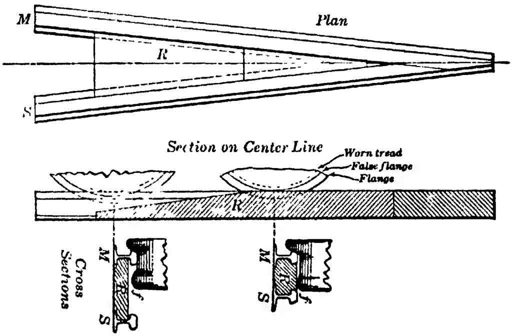


Fig. 16.

55. High Flangeways. In early days, in order to obviate this difficulty, the flangeways were made shallow, so that the wheels, traversing them, ran upon their flanges, their treads being thus lifted above contact with the rail-heads; but the flanges soon cut grooves, which lowerd the wheels until the treads again bore upon the rail-heads, striking blows as before.

Spring-rail Frogs.

56. Action. Figs 17, 18. In the spring-rail frog, the turnout wing-rail, *b s w* (called the spring-rail), altho spliced to the track rail at its toe, *b*, is spiked along only its *gage* side, and it may therefore be moved away from contact with the point, *P y*, against which it is normally held by springs, as at *h*. Any main-track wheel, *m'* (like its mate, *m*) has thus always a full bearing thru the frog. The spring-rail slides on large fixt plates, which extend under all the frog rails.

Of 59 railroads interrogated, 58 used spring-rail frogs as standard. Eng News, 1908 June 4.

57. Wheels, *n'*, *o'*, passing to or from the *turnout*, thru the frog, must cross the flangeway betw the point-rail, *P z*, and the wing-rail, *P v*. To obtain flangeway, betw the spring-rail, *s w*, and the point, *P y*, the flange of a *trailing* wheel, *o'*, merely pushes in betw the two; while a *facing* wheel, *n* (being held away from the frog by the gard-rail, *gt*), necessarily guides its mate wheel, *n'*, in that direction also, and thus forces the spring-rail, *s w*, away from the point, *P y*, and opens a flangeway for wheel *n'*. In either case, the spring, *h*, restores the spring-rail, *s w*, to its normal position, in contact with the point, *P y*, after the passage of each wheel.

58. Guide-box. Figs 17, 18. In order to prevent the free end, *w*, of the spring-rail, *b w*, from rising, when a load is concentrated at or near its other end, *b*, there is fastend, to the web of the spring-rail, near *w*, a forging, the horizontal tongue of which slides in a guide-box, *c*, riveted to a fixt slide-plate extending under all the frog rails at that point. The guide-box acts also as a stop, to prevent the free end, *w*, of the spring-rail, from moving out too far.

59. In order that the spring-rail head may fit snugly against the point-rail head, its *flange must be cut away* on the side next to the point. The spring-rail, thus weakend, is *reinforced*, usually by a strap riveted or bolted to its web.

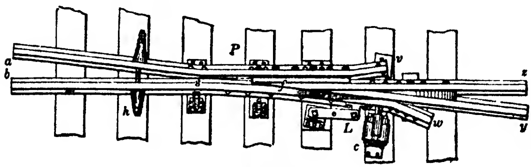


Fig. 17.

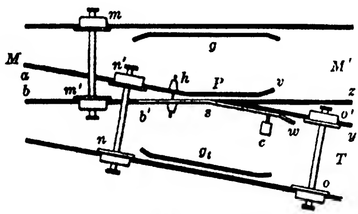


Fig. 18.

60. Provision against false flanges. Fig 18. If the bearing, betw the head of the spring-rail, *s w*, and that of the point-rail, *P y*, is too short, a false flange (¶ 52) worn on the outer edge of the tread of a trailing main-line wheel, *m'*, passing from *M'* toward *M*, may strike the spring-rail flare, *w*, and thus wedge the spring-rail away from the point, *P*. To prevent this, the spring-rail, *s w*, is given a longer bearing against the point-rail, *P y*, than would otherwise be necessary. For a similar reason, the spring-rail head, where in contact with the frog-point, is planed down to a groove parallel with the main-track gage-line, as indicated by the shading in Fig 17, providing a flangeway for false flanges.

61. For Main Track only. Since ordinary spring-rail frogs present a full bearing to the treads of only *main*-track wheels, *m'*, Fig 18, and not to those of turnout wheels, *n'* and *o'*, they are most useful where most of the traffic uses the main track, and where but few trains use the turnout. But see ¶¶ 66, 67.

62. Right and Left. Rigid frogs (except those with unequal wing-rails, ¶ 50) may be used for right-hand or for left-hand turnouts interchangeably; but spring-rail frogs must be made either right-hand or left-hand.

63. Creeping. Fig 17. To prevent the spring-rail from creeping, without restricting its lateral motion, the link, *L*, is used. The link is so set that the tendency to creep shall force the spring-rail *against* the tongue, and not *away* from it.

64. *In the hinged spring-rail frog* (designd to avoid the necessity of providing against creeping of the spring-rail) the latter is divided at *s*, Fig 17; and the portion *b s* is fixt in position, while the portion, *s w*, is hinged at *w*, instead of at *s*, where it is left free to swing outward against a spring, which restores it to its place in contact with the point, after the passage of each wheel.

65. On Curvs. Figs 17, 18. When a spring-rail frog is used on the outside of a main-track curv, the centrif force of a main-track train, pushing against the spring-rail, between *b* and *s*, tends to open it This tendency is resisted only by the gard-rail, *g*, Fig 18, opposit, but it is relievd by slightly flaring the spring-rail away from contact with the tongue near the point.

66. Double Spring-rail Frog. In the double spring-rail frog, both wing-rails are equipt as spring-rails, and each moves independently of the other. Both flangeways are closed when no wheels are passing. Such frogs are adapted to turnouts where the traffic, on main line and on turnout, is about equal. See ¶ 61.

67. Slid'ng Wing-rail Frog. In the sliding wing-rail frog, the two wing-rails are rigidly connected, at a fixt dist apart; but are free to slide together, so that either of them may rest against the fixt tongue, leaving a flangeway between the other wing-rail and the tongue. Wheels, either facing or trailing, open the flangeway (if it is not already open) by sliding the wing-rails over (facing wheels effecting this by means of the opp gard-rail). The wings then remain in the new position for following wheels. The wing-rails must slide, under high friction, while one of them carries the load of the first wheel; and an obstruction, lodged betw one wing-rail and the tongue, may hold the wing-rails fast, and cause derailment. Like the double-spring-rail frog, ¶ 66, the sliding wing-rail frog is designed for cases where the traffic, on main line and on turnout, is nearly equal. See ¶ 61.

Continuous Main Rail Frogs.

68. In order to leav the main rail always unbroken, certain frogs so elevate the turnout rail that wheel flanges on the turnout pass *over* (insted of thru) the main rail, the treads being carried by short supporting rails, which are swung temporarily into position, meeting over the main rail for this purpose, and which are afterward swung aside, in opposit directions, from the main rail, in clearing the main line. The switch is set simultaneously with the frog.

Specifications, Use, etc.

69. Requirements. In ordering frogs, specify type and details of construction; section and wt of rail; frog number or angle; length over all; length from half-inch point to heel; spacing of rail-joint holes at heel and at toe; thickness and dimensions of plate used; and, in the case of a spring-rail frog, or other frog with unequal wings at toe, whether right-hand or left-hand. In order that the parts may be interchangeable, specify also that all frogs of one kind shall be made alike as to drilling of bolt and rivet holes. Wing rails (which wear out much faster than point-rails) can then be readily replaced.

70. Dimensions, Am Ry Engng Assn, Manual, 1915, pp 168-170. **Sizes or Numbers.** (See also end of this paragraph for other than A R E A.) "Nos. 8, 11 and 16 frogs are recommended as meeting all general requirements for yards, main track switches and junctions. New work should be laid out, as far as practicable, for these three frogs, so as to effect the gradual elimination of frogs of other numbers, lessen the cost of manufacture, and decrease the amount of stock carried."

Rigid Frogs. Figs 13

Frog No.	*W*	*K*	*W + K*	*T*	*H*
8	4′ 9″	8′ 9″	13′ 6″	7 1/16″	13 1/8″
11	6′ 0″	11′ 6″	17′ 6″	6 [illegible]/16″	12 9/16″
16	8′ 0″	16′ 0″	24′ 0″	6″	12″

See also ¶ 71.

No. 11 spring frog, Fig 17, same dimensions as for No 11 rigid frog, above

Diams of bolts for all the foregoing rigid and spring frogs

Rail,	100-lb	90-lb	80-lb	70-lb	60-lb
Diam, ins,	1⅛	1⅛	1⅛	1	⅞

For spring-rail frogs, Fig 17.

Ends of wing rails chamferd at 45° with vertical

No 11 spring frog has five ⅝ inch open-end prest-steel stop-blocks.

Sizes or Nos. *For main-track slow-speed* turnouts, Nos 8 or 9 to 12 frogs are commonly used; *for high speed,* as at ends of double track, Nos 15 or 16 to 20. *In yards,* Nos 6 or 7 to 8 or 9 are used; in crowded *industrial yards,* for locomotivs of short wheel-base, Nos 4 to 6.

71. Flangeways. A canvass of 59 railroads showed that 38 used flangeways, in frogs and for gard-rails, 1¾″ wide; 18 used 1⅞″, and 3 used 2 ins.

Gard-Rails.

72. Fig 1. In passing thru the frog, each wheel is held to its proper course by the gard-rail, *g, g₁,* acting upon its mate wheel.

73. Details. On the side next the track rail, the gard-rail flange is cut away (or the gard-rail is rolled with its web inclined toward the track rail) in order to allow room for spiking, in the narrow flangeway. Instead of rails, thus trimmed or distorted, heavy steel angles have been used as gard-rails. The standard flangeway width extends for one or two feet each way from a point opp the frog point. The gard-rail ends are preferably planed down to a long bevel, to avoid their being caught by accidentally trailing objects; or the web, near the end, may be removed, and the head bent down to the desired slope. In main track, the gard-rails are from 15 to 18 ft long.

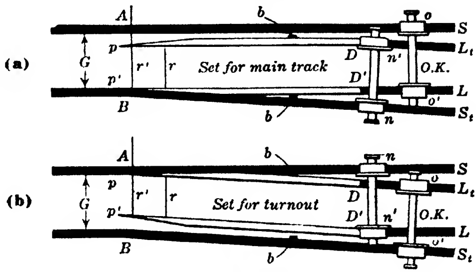
74. Fastenings. The gard-rails (sometimes weakened by the removal of one flange, ¶ 73) have to withstand severe lateral pressures; and they are therefore supported, usually by rail-braces, secured to the ties on the side opp to the track rail. Or tie-plates, extending under both the gard-rail and the main rail, may be used: or the gard-rail may be bolted to the track rail, and separated from it by filler-blocks, which, near the flaring ends, act also as foot-gards. See ¶ 75.

75. Foot Gards. Where the clear space, between adjacent rails, is betw 2½ and 5 ins, there is danger that men may have their feet caught and held in front of an advancing train. Such points are found at the ends of gard-rails, at the throat and heel of frogs, in the flare of wing rails, and in switches. Foot-gards are intended to prevent such accidents. They are required by law in some states. They are formd of pieces of plank, cut to fit the spaces and spiked to the ties; or of properly shaped wooden or cast iron blocks or steel strips, bolted to the rail webs; or of steel bars, bent into a series of angles, and placed on edge

Switches.

The Point Switch.

For the stub switch, see ¶¶ 96-99.
For the Wharton switch, see ¶¶ 119-126.



Figs. 19.

76. General. Figs 19*a* and 19*b* represent, in outline, the principle of the point or split switch, now in practically universal use on main lines. For simplicity, the rail flanges are omitted from the Figs, and only the rail heads are shown. In each Fig, the switch is shown set for the wheels marked "O. K." It is misplaced for the others. The lead-rails, L_t and L (see ¶ 1, Fig 1), and the stock-rails, S and S_t, are permanently spiked to the ties. S is the main stock-rail (or thru rail), and S_t is the turnout stock-rail (or knee-rail). At B, the knee-rail, S_t, is bent to a sharp angle = switch angle, s (Fig 21), and sometimes kinkt there, for the reception of the toe of the point-rail, $p' D'$, when set for the main track, as in Fig 19*a*.

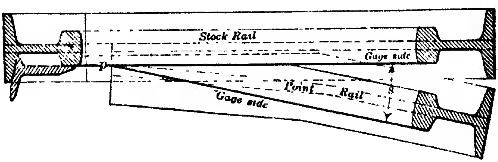


Fig. 20.

77. The switch or point rails, $p\ D$, $p'\ D'$, **Figs 19, are formd from ordinary track rails, so planed, Fig 20 (for the taper at the point), as to leav the gage side straight; and they are left straight (except for very sharp turnouts, where they are sometimes curvd). They remain straight while being set for the turnout; swinging about their heels,** D, D', **Figs 19.**

78. Length of Point. In ordinary turnouts, on lines where 30 or 33-ft rails are used in the main track, the switch-rail length (for economy in rail-cutting) is usually 15 ft or 16′ 6″, or half a rail-length; for high speeds, however, as at end of double track, switch-lengths of 20 to 30 ft are used; for yards and branches, 10 or

11 ft. In any case, the switch length must be at least such that the rail joints, at the heels, *D* and *D'*, Figs 19, shall not crowd the stock-rails.

See table, Part 2, ¶ 47

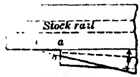


Fig. 21.

79. The Switch Angle is the angle, *s*, Figs 20, 21, between the gage sides of the switch-rail and stock-rail. On account of the thickness, *a b*, Fig 21, of the actual point or toe, the theoretical switch-point, or the vertex, *v*, of this angle is at a distance, = *v a*. from said actual point. (See table of split switches, ¶ 94.) Actual switch-point thickness, *a b*, = *v a* tan *s*.

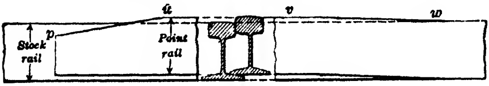


Fig. 22.

80. Details of Design. Fig 22. To protect the thin portion of each switch-rail, near the toe, the top of that portion is planed down to about ½ inch below the top of the adjacent stock-rail, at *p*; but, from *p*, the switch-rail top rises uniformly, in a varying distance, *p u* (see table ¶ 94), until, at *u*, the top of the switch-rail is about ¼ inch *higher* than that of the adjacent stock-rail, in order that the "false flange" of a gutterd wheel, Fig 23, may not foul the stock-rail, or wedge the two rails apart by dropping into the narrow space between them. From *u*, Fig 22, the top of the switch-rail maintains this superelevation for several feet, as to *v*, and then declines until the tops of the two rails are at the same level, as at *w*. Sometimes (for the same purpose) instead of thus *raising the switch-rail*, at *u*, the *stock*-rail is planed *down* to bring its top below that of the switch-rail.

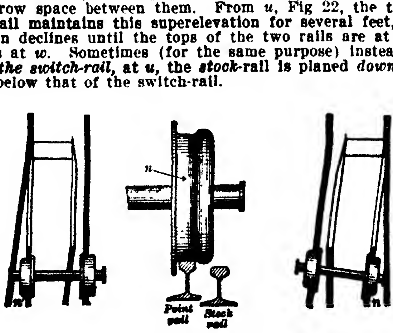


Fig. 23.

81. Protection and Reinforcement. To avoid danger of striking the open points, the *throw*, at *p*, Fig 19*a*; *p'*, Fig 19*b*, is made greater than would be necessary for merely passing the wheel flanges. The Am Ry Engng Assn, (see ¶ 94, below) specifies 5 ins.

82. Except in yards and for slow traffic, the *point-rails* are reinforced, at least thruout the planed portion, usually by a pair of strap pieces, from ¼ to ⅝ inch thick, and wide enough to fill the space betw head and flange, riveted or bolted one on each side of the rail web. An angle-bar or a channel-bar is sometimes substituted for one of the strap pieces.

83. In the *"channel" switch,* a piece of light T-rail, of proper length, is bolted alongside of the point-rail, from which it is held about 6" dist by two or three separators, placed at intervals. The resulting rigidity tends to prevent wrongly locking the switch when an accidental obstruction separates the two rails which should be brought together. See ¶ 48, under Signals, p 989.

84. Figs 19. *Stop-blocks, b,* or stop-lugs, are short metal blocks, bolted to the web of either the point-rail (Fig 19*a*) or the stock-rail (Fig 19*b*) at suitable intervals, at points where the two rails themselvs never come into contact. Their thickness is such as to afford a bearing betw either point-rail and the adjacent stock-rail, when these are at their nearest. They thus brace the point-rail against the stock-rail, enabling it better to resist the lateral pressure of passing wheels, which (owing to centrifugal force) is especially heavy on curvs. Usually, one, two or three such blocks are placed on each side of each switch.

85. **Steel Slide Plates,** 5 to 6 ins wide, and spiked to the ties, extend under both the switch-rail and the stock-rail, betw toe of switch and the point where the flanges of the switch-rail and the stock-rail separate. There are usually 6 to 8 such plates on each side of the switch. They serv the double purpose of guiding the switch-rail to its seat on the base of the stock-rail, and (by means of a raisd seat on which the switch-rail rests and slides) of elevating the switch-rail above the stock-rail, as mentioned in ¶ 80. They are generally made to receiv *rail-braces* to reinforce the stock-rail on the outside.

86. **Gage Plate.** In place of the two slide-plates, next to the toe, a single long plate, extending entirely across the track, and called a gage-plate, is sometimes used. It acts not only as a slide-plate, but also to hold the stock-rails to gage, by means of lugs attached to it.

87. **Tie Rods, Switch Rods, Tie Bars, Bridle Rods.** Figs 19. The two point-rails are connected, as indicated, by means of tie rods (switch rods, tie bars, bridle rods), *r, r',* fastend preferably to the webs of the point-rails, usually by hinged connections, to facilitate the change of angle involvd in the throwing of the switch.

88. The tie rods are sometimes made *adjustable in length,* in order to correct for wear and for accidental shifting of one or of both switch-rails.

89. *Number used* On main lines, the tie rods are usually from 2 to 5 in number. Their presence interferes with tamping; but when few rods are used, additional reinforcement of the switch-rails becomes necessary.

90. *Location.* The tie rods are placed below the levels of the tie tops, to avoid their being hit by wheels of cars, or dragd by parts of the rolling stock which may accidentally become partially detacht.

91. *Cross-section.* In point switches, the tie rods are usually flat, and are placed either vertically edgewise (in which position they resist canting of the point rails), or horizontally.

92. Figs 19. The tie rod, *r',* nearest to the toe is called the *head rod,* or *No 1* rod. At each end, it extends under the stock-rail, in order to prevent the toe end of the switch from rising out of place. At one of its ends it is jointed to the connecting rod (usually 1 to 1¼ ins round or square) leading to the switch stand.

93. Specifications Required. In ordering point switches specify gage of track; switch-rail length; switch throw, measd at head rod; rail section; drilling for rail joints; switch angle; and heel spread.

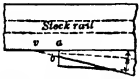


Fig. 21. (Repeated.)

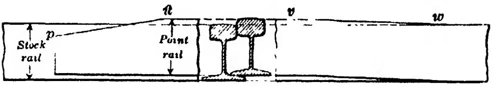


Fig. 22. (Repeated.)

94. Switch specification, Am Ry Engng Assn, Manual, 1915, pp 178-9.

Switch throw, 5 ins at center line of No 1 rod, *r'*, Figs 19.

Heel spread, at *D* and at *D'*, Figs 19, 6.25 ins betw gage lines of stock-rail and switch-rail.

Frog number, *N*			Switch length $l = pD$	*p u*	*v a*
Limitations		Recommended	Figs 19	Fig 22	Fig 21
≯ 6		. . .	11 ft 0 ins	5 ft	5.50 ins
> 6	≯ 10	8	16 ft 6 ins	7 ft	8.25 ins
> 10	≯ 14	11	22 ft 0 ins	9 ft	11.00 ins
> 14		16	33 ft 0 ins	12 ft	16.50 ins

Reinforcing bars, ⅝" thick; height to fill space betw head and flange; length as great as heel connections permit.

Two non-adjustable switch-rods, ¾" × 2½"; placed horizontally; 20" apart, cen to cen. Cen of head-rod 12" back from switch-toe.

On each tie, *two slide-plates,* ⅝" × 7", planed down to receiv stock-rail and braces.

95. A three-throw switch, Fig 3, consists virtually of two separate point switches; and, in fact, these are sometimes so arranged, the points of the two pairs of switch-rails being placed about a switch-rail length apart, or in "tandem"

Stub Switches.

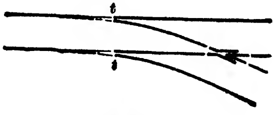


Fig. 24.

96. Fig 24. The stub switch (still used in subordinate locations, as in turnouts from branch lines) is simple in construction, and cheap in first cost, but expensiv in operation and maintenance. To avoid possibility of binding in hot weather, a considerable space must be left betw the toes, *t t*, and the ends of the adjacent track rails, and this subjects the rail-ends to serious blows from passing wheels. Such blows are injurious both to the rail-ends and to rolling-stock.

97. Danger. A lateral shifting, of either the toe or the adjacent rail-end, relativly to the other, causes a shoulder, called "lip", which may become very dangerous. But the most serious objection to the unprotected stub switch is the inevitable derailment of trailing cars when the switch is wrongly set.

98. Length. Stub-switch rails are usually full-length track rails, 30 or 33 ft long. Of this length, 4 or 5 ft, next to the heels, are spiked to the ties, leaving the remainder free to spring to a curv when thrown for the turnout.

99. The throw, or dist thru which the toes move, must be at least equal to the rail-head width (usually 2 to 2.5 ins) plus a width (1.75 to 2.5 ins) sufficient to pass the wheel flanges. Usually the throw is made from 5 to 5.5 ins

Switch Stands.

100. The lever, by means of which (thru the connecting rod) the switch is thrown, is housed in a "stand." The arrangements vary greatly. The lever may rotate in a vert or in a hor plane. If in a vert plane, this plane may be either perp to, or parallel with, the track; the movement of the lever, in the latter case, being transmitted to the connecting rod by means of gearing or a bell crank.

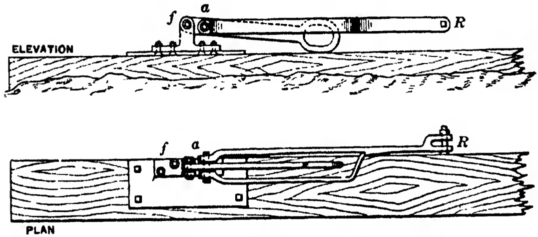


Fig. 25.

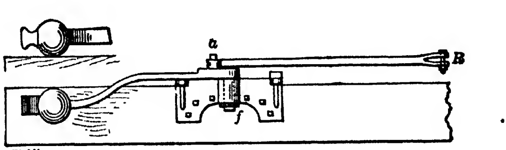


Fig. 26.

101. Ground or Tumbling Lever. Figs 25 and 26 show the "ground lever" or "tumbling lever" stand. In Fig 25, the lever moves thru an arc of a little more than 180°, in a vert plane, either perp to, or parallel with, the track; so that, when the lever is at rest, its joint, *a*, with the connecting rod, is a little below the fulcrum, *f*, about which the lever itself revolvs, or below dead center; so that the switch cannot be thrown by the lateral pressure of wheels pushing thru it. In Fig 26, the free end of the lever is weighted, and the joint, *a*, when the lever is at rest, is *above* dead center. This permits the automatic throwing of a misplaced switch by trailing wheels. See ¶ 57.

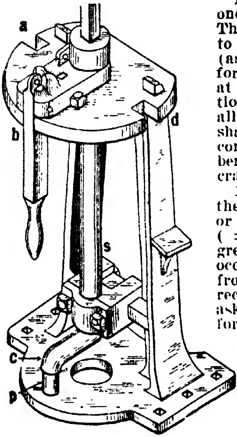


Fig. 27.

102. Revolving Stand. Fig 27 shows one of many forms of revolving stand. The lever is usually hinged, so as to hang vertically, and out of the way (as shown), when not actually in use for throwing the switch. The notches, at *a*, *b* and *d*, hold it in its three positions, respectivly. The crank, *c*, is usually formd, as shown, by bending the shaft, *s*; and the pin, *p*, to which the connecting rod is attacht, is formd by bending (up or down) the end of the crank.

103. Fig 28*a*. If, in a revolving stand, the crank-pin is moved 90° from *a* to *b*, or *vice versa*, the lateral movement (= $a' b'$ = $b' a'$) of the switch-rail, is greater than that (= $b' c'$) which occurs when the crank-pin is moved 90° from *b* to *c*, or *vice versa*. This may be rectified by setting the stand slightly askew, as indicated by the dotted lines; for this shifts b' materially toward a', while a' and c' are but very slightly shifted. Or, Fig 28*b*, the stand may be so set that the line *o d*, normal to the switch-rod, bisects the 90° throw of the crank, but this arrangement does not bring the crank to dead center in either position. For a given switch-rail throw, and given crank-throw angle, Fig 28*a* requires a longer crank than does Fig 28*b*.

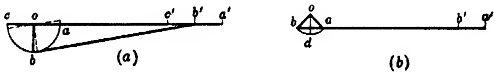


Fig. 28.

104. Locks. To prevent tampering with the switch, the lever is usually provided with a padlock and staples: but stands with self-containd and more elaborate locks have been used. See also end of ¶ 58, Signals, pp 990 and 991.

105. A target, at the top of a vertical shaft and operated simultaneously with the switch, indicates how the switch is set. The target may be a single disc, showing the switch-setting by its position (edgwise or flatwise to an approaching train); or it may have two discs, set at right angles, and differing in shape, in color or in both. See also Signals, ¶ 14, p 984.

106. Low, Pony, Intermediate or High Stands are those in which the top of the target is ≯ 2 ft, 2 to 4 ft, 6 to 8 ft, or about 18 ft, above ground, respectivly. Intermediate stands are in general use; low and pony stands are used betw neighboring tracks, and high stands where the target of a lower stand would be hidden by intervening objects.

107. Lamps. For switches which are to be used at night, the stand carries also a lamp, showing colors corresponding to those used on the target See Signals, p 984. Kerosene lamps are in general use; but, in large yards, incandescent electric lights are often used.

Automatic Switches.

108. Automatic switches are those in which the connections with the stand are such as to permit the passage of a trailing car when the switch is wrongly set for it.

109. Figs 19*a*, 19*b*. The flange of a trailing wheel, *n*, on the stock-rail, crowding into the narrowing space betw it and the adjacent switch-rail, will (if not prevented) throw the latter over, away from the stock-rail, thus providing, for wheel *n*, a flangeway betw those two rails; the other switch-rail being simultaneously thrown over against the opposit stock-rail, or into proper position for the mate wheel, *n'*.

110. A "set-over" automatic switch is one in which the point-rails, when automatically thrown, as in ¶ 109, carry over the switch lever, and remain in the new position.

111. In the "fly-back" automatic switch, the switch lever is not thrown when a trailing car passes a mis-set switch. The switch-rails are only temporarily pusht aside, and, after the passage of each pair of wheels, are returnd to their former positions by means of a spring in the head-rod connection, in the connecting rod, or in the stand. The stand throw is made a little greater than the switch throw, in order to take up lost motion.

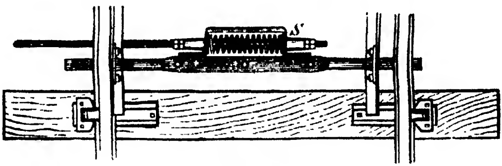


Fig. 29.

112. Objections. Both forms are open to the objection that the throwing of the switch takes place while part of the moving load is bearing upon one of the relativly slender switch-rails, which must therefore slide while loaded.

113. Lorenz spring. Fig 29 shows, in principle, the Lorenz spring, much used in "fly-back" automatic switches. The spring, *S*, is sometimes placed outside of the track.

114. The *repeated blows* of the switch-rails against the stock-rails in automatic switches are injurious, especially to the slender switch-rails; and permanent compression of the spring, resulting from long service, may leave a switch-rail (after the lever is thrown) improperly out of contact with its stock-rail, endangering facing wheels.

115. *The spring* also may permit the lever to be forced home, even when ice, a small stone, or other obstruction, lodged betw a switch-rail and a stock-rail, prevents complete setting of the switch.

116. Automatic & Non-Automatic. Frequently the rod, passing thru the coil spring, is provided with two adjustable sleevs, by means of which the switch may be arranged, at pleasure, in either one of several ways. According to this arrangement (*a*, *b*, *c*, etc), a trailing car, coming to a misplaced switch may pass it freely if coming either

(*a*) from the main line;

(*b*) from the turnout;

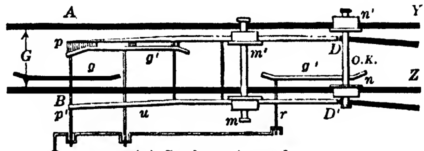
(*c*) from either the main line or the turnout;

or the sleevs may be so adjusted as to prevent the spring from acting; the switch thus being renderd non-automatic, so that a trailing car, whether coming from the main line or from the turnout, cannot freely pass the switch if the latter is misplaced.

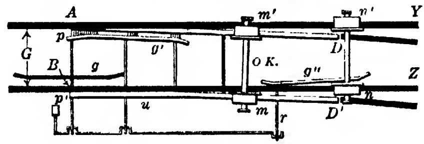
117. The *use of automatic and of non-automatic* ("rigid") switch stands, compared as follows:—

In 1900, 24 roads used automatic, and 23 roads used rigid stands; in 1908, 23 roads used automatic, and 29 roads used rigid stands Eng News, 1908 June 4.

118. Specifications Required. In ordering switch-stands, specify switch throw at point of connection, size of head rod and whether vert or hor, diam of hole in head rod, length and diam of connecting rod, style of target and its height above the ties, and dimension of lantern tip on shaft.



(*a*) *Set for main track*



(*b*) *Set for turnout*

Figs. 30.

Wharton Switch.

119. Figs 30*a* and *b*. We omit the rail flanges, showing only the rail heads. *Moving* rails are shown *white*. Wheels, for which the switch is properly set, are markt "O. K." Fig 30*a* shows the switch set for the main line; Fig 30*b* shows it set for the turnout.

120. *A Y* and *B Z* are the main-track rails. They are continuous (except, of course, that *B Z* is broken at the *frog*, beyond the Fig), and are spiked to the ties thruout. They are not moved or broken in the working of the switch. *p D* and *p' D'* are the switch-rails.

121. The Switch-rail, *p' D'* (the "elevated rail"), is blunt-ended. At its toe, *p'*, its top is level with that of *B Z*; but, at *u*, 4 or 5 ft back from the toe, its top is about 2" higher than that of *B Z*. This enables the flange of wheel *m*, in passing to or from the turnout, Fig 30*b*, to clear the main rail, *B Z*. *p' D'* retains this elevation from *u* to its heel, *D'*. Beyond *D'*, the turnout rail, continuous with *p' D'*, slopes downward until its top is at the normal level. The **"point-rail,"** *p D*, is elevated also, to prevent the lateral rocking of rolling stock which would otherwise result from the elevation of *p' D'*.

122. The reinforcing or "point" gard-rail, *g'*, is bolted to, and separated from, the point-rail, *p D*, and moves with it. Flange-way is left betw their heads. In Fig 30*b*, by confining the flange of wheel *m'*, *g'* holds the flange of wheel *m* clear of the head of rail *B Z*, in coming down the incline, *u p'*.

123. In Fig 30*b* the point, *p*, fits close against the head of the main rail, *A Y*; so that the flange of wheel *m'*, moving in either direction, clears the point, *p*; and the fixt gard-rail, *g*, confining the flange of *m*, holds *m'* away from *A Y*, thus affording additional protection to the point, *p*.

124. Early patterns were provided with special attachments, designed to receiv, from the turnout, a trailing car when the switch was set for the main line, and to guide it safely on to the main track; but this device has been abandoned, on the ground that its cost and complication are not warranted by the probability, under proper management, of such misplacement.

125. Trailing. Fig 30*b*. If a trailing car moves along the main track while the switch is set for the turnout, the flange of its first wheel, *n*, pushes aside *g''* (compare Fig 30*a*) which, pulling the rod, *r*, throws the switch into the proper position for the main track, as in Fig 30*a*.

126. Gage. For proper working of the Wharton switch, the main-track gage, at the switch, is made half-inch wider than the standard gage.

Computations.

127. Many of the approximations, permissible in computing turnouts of small angle, such as are usual on steam lines, are inadmissible in street railway turnouts and crossings, where the layouts are frequently very complex. Hence, the design of such work is preferably entrusted to mfrs who specialize in it.

Ties.

128. Turnout Ties, for standard gage, vary, in length, betw that of an ordinary tie (say 8.5 ft) and 24 ft. One or two ties, 12 to 16 ft long according to the style of switch stand, are used at the toes of split and of stub switches as a seat for the stand, and are called **Head-Blocks.** The set of ties should extend, beyond the frog, to a point where the ends of the ties of the two tracks do not interfere. Ties 7 × 9 ins are commonly used, except under the frog, where 7 × 10 ins ties are used, to give greater bearing area, and strength to resist the shocks of wheels passing over the frog. They are generally orderd in lengths varying by 6 ins. For ties in general, see under Track, p 784, etc.

Switch Ties.

129. Digest of Am Ry Engng Assn Specifications. Manual 1915, following p 176.

All 7" × 9".

Lengths for 8' 6" track-ties. For 8' 0" track-ties, the switch-tie lengths are in general 6 ins less than here given.

Length, ins	180*	108	114	120	126	132	138	144	150	156
	Number of switch-ties, beginning at switch-point									
Frog No 8,	2	8	7	5	4	3	3	3	3	3
Frog No 11,	2	12	10	8	5	5	5	3	3	4
Frog No 16,	2	20	14	10	9	6	6	5	5	5

(Table concluded)

Length, ins	162	168	174	180	186	192	198	Total	
	Number of switch-ties								
Frog No 8,	2	3	2	3	2	3	2	58 ties,	3651 ft B M
Frog No 11,	4	3	3	2	3	3	3	78 "	4814 ft "
Frog No 16,	5	5	4	5	5	4	5	115 "	7111 ft "

*Each turnout has two head-blocks, 7" × 9", 15 ft long.

130. Tie Spacing, for turnouts. Beginning at switch-point. Center to center. For use with 8′ 6″ track-ties. Ties 7″ × 9″.

										Total
Frog No 8	Number of ties,	5	4	18	3	7	9	6	5	57
	Space, ins	20	21	20	19	20	18	19	20	
	Total, ins	100	84	360	57	140	162	114	100	93′ 1″
Frog No 11	Number of ties,	10	21	9	13	5	10	4	5	77
	Space, ins	19	20	19	20	18	20	21	20	
	Total, ins	190	420	171	260	90	200	84	100	126′ 3″
Frog No 16	Number of ties,	18	19	4	16	4	20	15	18	114
	Space, ins	19	20	19	20	19	20	19	20	
	Total, ins	342	380	76	320	76	400	285	360	186′ 7″

Timber Bills.

The following are typical timber bills for double (three-way) turnouts and for crossovers.

131. For Double Turnouts. Two headblocks 7″ × 10″, 16′ long. Ties under frogs spaced 23″. Other ties spaced 24″.

Length, ins.	108	114	120	126	132	138	144	150	156	162
	Number of ties, beginning at switch point									
Frog No 6,	5	4	3	2	2	1	1	1	2*	1*
Frog No 8,	5	4	4	2	3	2	1	2	1*	1*
Frog No 10,	5	4	4	3	3	2	2	2	2*	1*

(Table continued)

Length, ins.	168	174	180	186	192	198	204	216	222	228
	Number of ties, beginning at switch point.									
Frog No 6,	1*	0	1	1	1	1	1*	1*	1*	1*
Frog No 8,	2*	1	1	1	2	1	1*	2*	0	2*
Frog No 10,	1*	2	2	2	2	0	2*	2*	0	2*

(Table concluded)

Length, ins.	240	252	264	270	276	288	Total	Ft B M
	Number of ties, beginning at switch point							
Frog No 6,	1	2	1	0	2	1	38	3069
Frog No 8,	2*	2	2	0	1	2	47	3828
Frog No 10,	3	2	2	2	2	3	57	4754

132. For Crossovers. (13 ft cens). Four headblocks 7″ × 10″, 16′ long. Ties under frogs spaced 23″. Other ties spaced 24″.

Length, ins.	102	108	114	120	126	132	138
	Number of ties, beginning at switch point.						
Frog No 6,	8	8	6	6	6	4	4
Frog No 8,	10	8	8	6	6	6	6
Frog No 10,	10	10	10	8	8	8	8

(Table concluded)

Length, ins.	144	150	156	258	258	Total	Ft B M
	Number of ties, beginning at switch point.						
Frog No 6,	4	2	4*	4*	6	62	4392
Frog No 8,	6	6	4*	4*	9	79	5544
Frog No 10,	6	6	6*	6*	11	97	6817

*Frog ties, 7″ × 10″. Other ties, 7″ × 9″.

Laying.

133. Time required. When not greatly interrupted by passing rains, "a good foreman and six fair trackmen can put in either stub- or a split-switch turnout in one day of ten hrs, including he removal of old track ties and putting in switch ties." Camp, lotes on Track, p 298.

Miscellaneous.

134. To prevent creeping of the point-rails, dependence is usully placed upon slot-spiking the four rail-joints at the ends of the rog, those at the two point-rail heels, and the intermediate joints n the lead-rails. Anchoring the switchpoint rails, at their heels, o the neighboring stock-rails, has given trouble by crowding the oints out of line.

Removal of Ice and Snow.

135. Steam Heating. Where numerous switches are in proxmity, they may be economically kept clear of ice and snow by steam pipes, fed from some central source, and laid betw several pairs of switch ties, near the point, the ballast being first removed. The pipes are provided with automatic traps, which allow the conlenst steam to escape, and thus prevent its freezing in the pipes. See Ry Age Gaz, 1910 May 13, p 1199.

136. Burning Fluid. Turnouts have been cheaply and expeditiously cleard (and kept clear) of ice and snow, by means of a fluid hydrocarbon (a by-product of the Pintsch gas plant), applied, where needed, by means of a safety distributing can, and ignited. The fluid burns, in spite of wind and snow, melting, and finally evaporating, the snow. The flame temperature is not sufficient to injure the rails. The fluid costs from 3 to 5 cts per gallon. Where several neighboring switches are to be servd, the liquid is stored in tanks, and distributed, by gravity or by comprest air, thru piping.

See Engineering-Contracting, 1908, Sep 2, p 151.

137. To protect switch rods from becoming bent or broken in cases of derailment, a sawd tie may be placed each side of each rod, leaving 2.5" clear space, betw ties, for the rod. In cold weather, bent rods should be heated before it is attempted to straighten them. Ballast should be kept clear of the rods, and well draind in their vicinity, especially in cold weather.

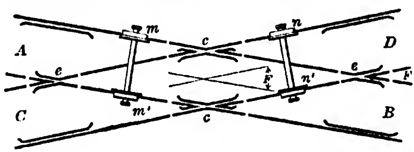


Fig. 31.

CROSSINGS

138. Fig 31. A crossing consists essentially of four frogs, *e*, *e*, *c* and *c*, and the necessary gard-rails.

139. The crossing angle, or end-frog angle, is the angle, *F*, betw center lines of the two straight crossing tracks.

140. Where F = 90°, the four frogs are all alike. In Fig 31 the two sharp "end" frogs, *e*, are alike, and similar to turnout frogs. The two blunt "middle" frogs, *c*, are alike also. The presence of the gard rails, at the middle frogs, gives the appearance of a second point, opposite the point, *c*, proper; and the middle frogs are therefore often called "double-pointed", altho the inside "points", formed by the angles in the gard-rails, carry no wheels.

141. Where one or both tracks are curvd, the frogs all have different angles; and are curvd thruout. Crossings of curvd tracks are best avoided. Problems concerning them are conveniently solvd by means of large-scale plottings.

142. Double-rail Crossing. Fig 31. If the middle gard-rails are extended, both ways, until they meet opposit the end frogs, and are properly filled, bolted and connected, the strength and durability of the crossing are greatly increast, and it is called a "double-rail" crossing. In this case, the end frogs, *e*, (as well as the middle frogs, *c*), are "double-pointed" (¶ 140).

143. An easer rail (¶ 52), to carry "false-flanges", is sometimes bolted outside the running rail.

144. When F exceeds say 35°, the webs of both of the main, gard and easer-rails of one of the two tracks, sometimes run thru the crossing uncut; the flangeways, for wheels on the other track, being cut thru the heads only of these rails.

145. When F is betw 15° and 40°, the four frogs usually meet directly; but in small-angle crossings, to avoid handling long frogs, pieces of rail are laid betw the ends of the frog rails, as in Fig 31. For convenience in shipping and handling, crossings are made with as few field joints as possible.

146. When F is less than say 10°, Fig 31, the middle frog points are nearly opp each other on either one of the two tracks, and cannot be properly garded. Thus, wheels *m* and *m'* might leav their proper track, *A B*, at the middle frogs, and take track *C D;* or wheels *n* and *n'* might leav track *D C*, and take track *B A*.

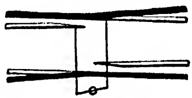


Fig. 32.

In such cases (Fig 32) the middle gard-rails are dispenst with, and a **"movable point center frog",** consisting of two pairs of short point switch-rails, facing each other, is used. These switches are, as required, seated against one of the two bent stock-rails, as shown. They may be operated by an automatic switch-stand, so that, if a train approaches a wrongly set crossing, the forward wheels force open the trailing switch-rails, and automatically throw the facing switch-rails into proper position.

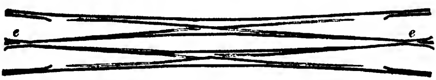


Fig. 33.

147. Slip-Switch or Combination Crossing. Fig 33. At a small-angle crossing, traffic may be intentionally diverted from one track to another by means of a slip-switch or "combination crossing", consisting of an arrangement of switch and connecting rails in addition to a regular crossing, all placed betw the two end frogs, *e*. Fig 33 shows a double slip-switch. In a single slip-switch, the switches and connecting rails are provided on one side only.

148. The slip-switch is especially useful for connecting a *leader* (see ¶ 12*b*) with the parallel tracks which it crosses. Such a combination saves much room, lengthwise, and givs a straight line, as compared with a series of separate crossovers.

149. *Movable-point center-frogs* may be used, as in Fig 33, in slip-switches; but *ordinary rigid middle frogs* are used instead, both with single and with double slip-switches, except where the angle is so small that the wings of the frogs interfere with the curvd rails connecting the switch-rails.

150. *Switch levers.* These crossings may be operated by an ordinary switch-stand for each pair of switch-rails and one for the movable-point center-frogs, if such are used; or all the switches may be connected by a system of rods and cranks, and operated from a single stand, while the movable point-frogs are operated from another stand. Or the switch-points and movable frog-points may be so arranged that they operate together.

151. Continuous Rail Crossings. In crossings of very large angle, the wheels, in crossing the flangeways, drop into them, delivering severe blows to rolling stock and to track; and devices have been constructed which afford continuous rails at such crossings, by means of a short section of rail, or its equivalent, which, thru an interlocking system, can be placed temporarily in line with the rail to be used.

152. Ties. Crossing ties, extending under both tracks, are usually laid at right angles to the long diagonal of the diamond formed by the crossing rails; sometimes at right angles with the line carrying the heavier traffic; and sometimes ordinary-length ties are used, at right angles with each track. For ties, see ¶ 128.

153. Specifications Required. In ordering crossings, specify type of construction; crossing angle, *F*, Fig 31; track gage; distance, cen to cen, betw parallel tracks, if there is more than one; rail section; lengths of arms of frogs; and bolt-hole spacing required in ends of arms to suit rail joints in use.

TURNOUTS AND CROSSINGS

PART II. THEORY

Geometry of Turnouts.

For Construction of Turnouts, see Part I, p 822.

1. Accuracy. Scrupulous accuracy is not necessary in the measurements of turnouts. A deviation of several per cent, in length of lead or of turnout radius, from the theoretical value, will seldom be noticeable. The traind eye of an expert trackman, or a large-scale drawing, may secure as good results as careful calculation. Nevertheless, the engineer should be informd as to the broader features of the geometry of turnouts. We therefore give, below, equations (both exact and approximate) covering the simpler cases.

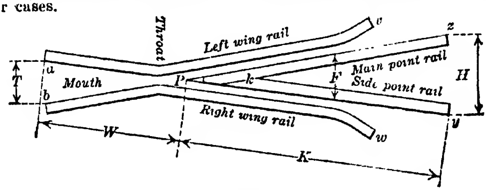


Fig. 1.

Frog angle and frog number.

For description of frog, see Part I, ¶ 40, etc.

2. Frog Angle. Figs 1 and 2. The theoretical frog point, P, is the intersection of the two gage-lines bounding the frog-tongue. The frog angle, F, is the angle, at the theoretical frog-point, betw the two gage-lines.

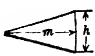
3. *How determind.* Figs 1 and 2. The frog angle, F, depends upon the sharpness-difference (¶ 29) betw the turnout curv and the main-line curv (if any) and upon the track-gage.

4. *How exprest.* Fig 2. The frog angle is usually exprest by means of the ratio, k/i, betw the length, k (not shown), of any portion of the frog, and the increase, i (not shown), of the frog width within that portion. This is also the ratio betw any given portion, (m or s, beginning at P, see below) of the frog length, and the corresponding spread, h. This ratio is called the frog number, N.

5. Frog Number. *N is determind* in two diff ways, viz:—

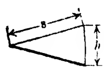
$$N = \frac{m}{h} = \frac{\cot(F/2)}{2}$$

$$2N = \cot(F/2)$$



$$N = \frac{s}{h} = \frac{1}{2\sin(F/2)}$$

$$2N = \operatorname{cosec}(F/2)$$



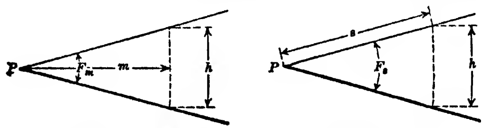
Eq (1)

6. The *Am Ry Engng Assn* makes $N = m/h = \frac{1}{2}\cot(F/2)$, as in Fig 2*a* (see values of F, in table, Manual, 1915, p 184); but railroad companies and mfrs are divided as to their practice in this

respect; many preferring the second formula, $N = s/h$, Fig 2b, because the frog length is customarily and more satisfactorily measd along a gage-line, s, than along the center line, m. Moreover, the length, so measd, is usually in even figures of ft and ins; and hence the spread is easily found by mentally dividing said length by the frog number.

7. Comparison of frog angles, F_m and F_s

N	F_m $\cot(F_m/2) = 2N$ Fig 2a	F_s $\operatorname{cosec}(F_s/2) = 2N$ Fig 2b	$F_s - F_m$ seconds	$\dfrac{F_m}{F_s - F_m}$
4	14° 15′ 00″	14° 21′ 41″	401	128
8	7° 9′ 10″	7° 10′ 00″	50	515
11	5° 12′ 18″	5° 12′ 38″	20	937
16	3° 34′ 47″	3° 34′ 54″	7	1841
24	2° 23′ 13″	2° 23′ 15″	1.87	4606

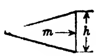
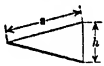


For $h=1$, $N=m$ (a)　　For $h=1$, $N=s$ (b)

Fig. 2.

8. Functions of the frog angle, F, in terms of the frog number, N.

See p 97b, ¶ 16, and p 97c, ¶ 19.

Fig 2a	Fig 2b
	
Taking $N = \dfrac{m}{h} = \dfrac{1}{2}\cot(F/2)$ we have $\tan(F/2) = \dfrac{1}{2N}$, and	Taking $N = \dfrac{s}{h} = \dfrac{1}{2\sin(F/2)}$ we have $\sin^2(F/2) = \dfrac{1}{4N^2}$, and
$\tan F = \dfrac{N}{N^2 - {}^1/_4}$ $\sin F = \dfrac{N}{N^2 + {}^1/_4}$ $\cos F = \dfrac{N^2 - {}^1/_4}{N^2 + {}^1/_4}$	$\sin F = \dfrac{\sqrt{4N^2 - 1}}{2N^2}$ $\tan F = \dfrac{\sqrt{4N^2 - 1}}{2N^2 - 1}$ $\cos F = \dfrac{2N^2 - 1}{2N^2}$ Eq (2)

9. To determin the number of an actual frog, Fig 3; find any part of the frog, where the dist, h or t, betw *gage lines* (¶ 2) is unity (say 1 ft, or the length of a pencil or pocket-knife). Then N = dist, hP or tP, from that part to the theoretical frog-point, P (see ¶ 2) measured in the same unit, along the center-line, M, or along a gage-line, S, according to the definition of N adopted (see ¶ 6). Or (since the theoretical frog-point, P, is not easily located) measure *both* widths, h and t (betw *gage-lines*), at any *two* convenient points in opp directions from the frog-point, and the dist, M or S, betw said points. Then,

$$N = \frac{M}{h + t} \quad \text{or} \quad \frac{S}{h + t}.$$ See ¶ 5

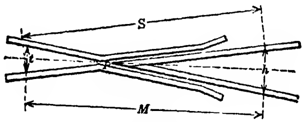


Fig. 3.

10. Actual Frog-Point. Fig 1. In practice, the tongue ends at the "actual frog-point" (or "half-inch point"), where the tongue is about ½" wide; leaving a small isosceles triangle betw the actual and theoretical points. (See ¶ 2). The length of this triangle, or the dist betw the theoretical and actual frog-points, is the product of the actual frog-point width by the frog number, N.

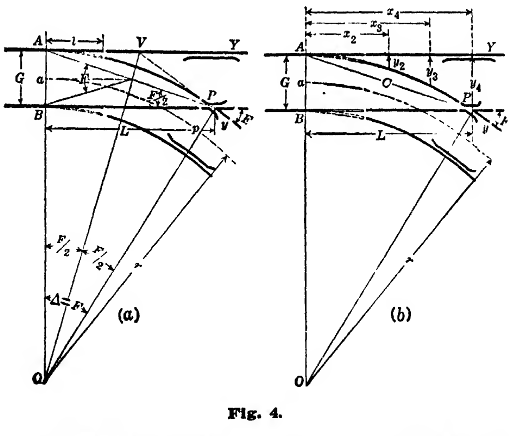


Fig. 4.

11. When angle is small. As an angle, A, approaches zero, as a limit, the values of $A \times \cot A$ ($= A/\tan A$) and of $A \times \csc A$ ($= A/\sin A$) approach unity, A being exprest in radians of 57.295 779°. (¶ 2, p 97). Hence, the same values, exprest in degrees,

approach, as a limit, 57.295 779* (log = 1.758 1226) ; and, in small angles, we have approx :—

for A in degrees, $A^\circ \;=\; 57.3^\circ/\cot A \;=\; 57.3 \sin A$;

$F \;=\; 2 \times 57.3/\cot(F/2) \;=\; 2 \times 57.3 \sin(F/2) \;=\; 57.3/N$.

For A in minutes, $A' \;=\; 3438'/\cot A \;=\; 3438 \sin A$. . (3)

STUB SWITCHES

Circular turnout curv from tangent

General.

12. Fig 4. Let the lead curv, $A\,P$, of the gage side of the outer turnout rail be a simple circular curv, from the beginning, A, of the turnout, to the theoretical frog point, P; said curv being tangential to $A\,Y$ and to $P\,y$, at A and P, respectivly. This condition is seldom perfectly realized in practice; but it is approximately realized, even in point-switches, ¶ 35, and it givs equations which, on account of their simplicity, are sometimes used, not only for stub-switch turnouts, but also (as being sufficiently approx) for point-switch turnouts. They are therefore given below, altho the stub-switch is now used only in subordinate locations.

13. Equations. Fig 4. Stub-switch turnout from tangent. Let

G	$= A\,B$	=	track gage;
r	$= O\,a = O\,A - G/2$ $= O\,B + G/2$	=	turnout center-line radius;
D	....	=	turnout curv sharpness;
Δ	....	=	turnout sweep, from switch-heel, A, to frog-point, P;
F	$= \Delta = B\,P\,V$ $= Y\,V\,P = A\,O\,P$	=	frog angle;
N	$= m/h$, Fig 2, $= \frac{1}{2}\cot(F/2)$ (See ¶ 5)	=	frog number;
L	$= B\,P$	=	lead = frog distance;
C	$= A\,P$	=	outer-rail gage-side chord;
M	....	=	middle ordinate for C;
Q	....	=	quarter-point ord for C;
y_1 (not shown), y_2, y_3, y_4	......	=	perp offsets from tangent to quarter-points of arc, $A\,P$;
x_1 (not shown), x_2, x_3, x_4	......	=	abscissas for offsets, y;
s	(Fig 10).............	=	switch angle;
l	....	=	switch length;
t	....	=	switch throw.

14. Frog angle, F, = turnout sweep, Δ.

Functions of F and of $(F/2)$.

$$\sin F = \frac{L}{r + (G/2)}; \qquad \tan F = \frac{L}{r - (G/2)};$$

$$\cos F = \frac{\sin F}{\tan F} = \frac{r - (G/2)}{r + (G/2)}$$

$$\operatorname{vers} F = \frac{G}{r + (G/2)}. \qquad F = \text{approx } D\,L/100. \quad \ldots.(4)$$

---

*This is also the number of degrees in the angle (= 1 radian) whose arc = radius, and is also the length of the radius of a curv in which an angle of 1° is covered by an arc whose length is unity.

For functions of F in terms of N, see ¶ 8.

$$\text{Cot}\,(F/2) = L/G = 2N = r/GN;$$
$$\text{Tan}\,(F/2) = G/L = 1/(2N) = GN/r. \quad \ldots\ldots\ldots\ldots\ldots (5)$$

15. Frog number, N.

$$N = \frac{\cot\,(F/2)}{2} = \frac{L}{2G} = \frac{r}{L} = \sqrt{\frac{r}{2G}} = \frac{r \tan\,(F/2)}{G}$$
$$= \text{approx } l/(2\sqrt{G\,t}) = \text{approx } 5730°/D°\,L. \quad \ldots\ldots\ldots\ldots (6)$$

16. Turnout center-line radius, r.

$$r = L/\sin F - G/2 = L/\tan F + G/2 = G/\text{vers}\,F - G/2$$
$$= G\,\frac{\cot^2\,(F/2)}{2} = G\,N \cot\,(F/2) = \frac{G\,N}{\tan\,(F/2)}$$
$$= 2\,N^2\,G = L^2/2\,G = N\,L = \text{approx } l^2/2\,t \quad \ldots\ldots (7)$$

17. Turnout center-line sharpness, D.

$$\text{Sin}\,(D/2) = 50/r = 25/N^2\,G\,.$$
$$D = \frac{100\,F}{\text{arc, } a\,p, \text{ ft}} = \text{approx}\,\frac{100\,F}{L, \text{ ft}} = \text{approx}\,\frac{57.3°}{N}\cdot\frac{100}{2NG}$$
$$= \text{approx}\,\frac{2865°}{N^2\,G} = \text{approx}\,\frac{608°}{N^2} \quad \text{for } G = 4 \text{ ft } 8.5 \text{ ins.}$$
$$= \text{approx}\,\frac{5730°}{r, \text{ ft}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

18. Track gage, G.

$$G = \frac{L}{\cot(F/2)} = \frac{L}{2\,N} = \frac{r}{2\,N^2} = \frac{L^2}{2\,r}$$
$$= 2\left(\frac{L}{\sin\,F} - r\right) = 2\left(r - \frac{L}{\tan\,F}\right)$$
$$= \text{approx}\,\frac{l^2}{4\,N^2\,t} = \text{approx}\,\frac{L^2\,t}{l^2} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

19. Lead or frog distance, $L = B\,P$. Fig. 4.

$$L = [r + (G/2)]\sin F = [r - (G/2)]\tan F = G\cot\,(F/2)$$
$$= 2\,G\,N = r/N = \sqrt{2\,r\,G} = 2\,r \tan\,(F/2)$$
$$= (\text{approx, in ft})\ 100\,F/D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

20. Long chord, C ($= A\,P$), to gage-side of outer lead-rail.

$$C = 2\,[r + (G/2)]\sin\,(F/2) = \sqrt{G^2 + L^2}$$
$$= G\sqrt{1 + 4\,N^2} = \text{approx } 2\,r \sin\,(F/2). \quad \ldots\ldots\ldots\ldots (11)$$

21. Offsets, y, from gage-side of main-track rail to gage-side of outer lead-rail.

	A. Exact values of y.	B. Approximate values of y	A/B when $F = 12°$ ($N = 4.76$)*
y_1	$= [r + (G/2)]$ vers $(F/4)$	$= G/16$	1.0034
y_2	$= [r + (G/2)]$ vers $(F/2)$	$= G/4$	1.0027
y_3	$= [r + (G/2)]$ vers $(3\,F/4)$	$= 9\,G/16$	1.0016
y_4	$= [r + (G/2)]$ vers $F = G$		(12)

*With smaller values of F (greater values of N) the approximation for y_1, y_2 and y_3 is closer.

22. Ordinates from long chord, C (= AP) to gage-side of outer rail.

22a. *Middle ordinate, M.*

M = $[r + (G/2)]$ vers $(F/2)$
= approx $G/4$ = approx $14\frac{1}{8}$ ins when G = 4 ft 8.5 ins.

When N =	4	8	16	
we find $M/(G/4)$ =	1.0039	1.0010	1.000244	......(13)

22b. *Quarter-point ordinate, Q.*

$$Q = \sqrt{[r + (G/2)]^2 - (C/4)^2} + M - (G/2) - r \quad ..(14)$$

$$\text{where } C/4 = \frac{[r + (G/2)] \sin (F/2)}{2}$$

Q = approx $3\,G/16$ = approx $0.1875\,G$
= approx 10.6 ins when G = 4 ft 8.5 ins ....(15)

When F = 12°, (N = 4.76),

$$\frac{Q}{3\,G/16} = 1.0035.$$

With smaller values of F (greater values of N), the approximation is closer.

23. In stub-switch turnouts, the middle and quarter point ordinates are *nearly independent of the frog-angle, F.* When N = 4, N = 16, and gage = 4 ft 8.5 ins the middle ords, to gage side of outer rail, compare as follows:

$$\frac{\text{mid ord, } N = 4}{\text{mid ord, } N = 16} = \frac{1.18165 \text{ ft}}{1.17737 \text{ ft}} = 1.00363$$

But see ¶ 38.

24. Switch angle, s, length, l, and throw, t. For the relation betw these we may take

$t = l \sin s$ ....(16)

25. Dimensions. Circular Turnout from Tangent. Fig 4.

Gage, 4 ft 8.5 ins = 4.70833... ft.
Switch-throw = 5.5 ins = 0.45833... ft.

N = frog number; C = AP = chord;
F = frog angle; L = BP = lead;
r = turnout center-line radius; l = switch-rail length;
D = turnout sharpness; t = throw.

N	F	r	$\log r$	D	C	L	$l = 2N\sqrt{Gt}$
4	14°15′00″	150.67	2.17802	38°46′	37.38	37.67	11.75
5	11°25′16″	235.42	2.37184	24°32′	46.83	47.08	14.69
6	9°31′38″	339.00	2.53020	16°58′	56.30	56.50	17.63
7	8°10′16″	461.42	2.66409	12°27′	65.75	65.92	20.57
8	7°09′10″	602.67	2.78008	9°31′	75.19	75.33	23.50
9	6°21′35″	762.75	2.88238	7°31′	84.62	84.75	26.44
10	5°43′29″	941.67	2.97390	6°05′	94.05	94.17	29.38
11	5°12′18″	1139.42	3.05668	5°02′	103.48	103.58	32.32
12	4°46′19″	1356.00	3.13226	4°14′	112.90	113.00	35.26
13	4°24′19″	1591.42	3.20178	3°36′	122.33	122.42	38.19
14	4°05′27″	1845.67	3.26615	3°06′	131.75	131.83	41.13
15	3°49′06″	2118.75	3.32608	2°42′	141.17	141.25	44.07
16	3°34′47″	2410.67	3.38214	2°23′	150.59	150.67	47.01

Turnout from Curv.

26a. Fig 5*b*. For stub-switch turnouts from tangents, ¶ 25 gives the sharpness, D, the radius, r, and the lead, $L, = B\,P$, for frog numbers from 4 to 16. The middle ordinate,

$$M = \left(r + \frac{G}{2}\right) \text{vers} \frac{F}{2},$$ from the chord $A\,P$ to the turnout frog-rail gage-side, is practically the same for all frog numbers. See ¶ 23.

26b. But, Figs 5*a* and 5*c*, when a given frog number is used in a turnout from a *curvd* main track, all these functions (including the ordinates) are affected; and their values depend not only upon the sharpness, D_m, of the main-track curv, but also upon whether the turnout curvs in the same direction with the main-track curv, Fig 5*a*, or in the opposit direction, Fig 5*c*.

27. Symbols. Figs 5. Let

G = gage; F = frog angle; $N = \frac{\text{Cot } (F/2)}{2}$ = frog number.

and let other symbols denote as follows:

	For curvd main track Figs 5*a*, *c*	For turnout from tang Fig 5*b*	For turnout from curv Figs 5*a*, *c*
Center	O_m	O	O_c
Center-line radius	r_m	r	r_c
Sharpness	D_m	D	D_c
Sweep, from beginning, *a*, of turnout, to frog-point, *P*,	Δ_m	Δ	Δ_c
Ordinates, from long chord, *A P*, to turnout frog-rail gage-side			
Middle-ordinate	— - -	M	M_c
Quarter-point ordinate	---	Q	Q_c
Lead, *B P*,	--- --	L	L_c

28. Relations. Then, comparing first Fig 5*a* and then Fig 5*c* with Fig 5*b*, we find, for a given frog angle, F:—

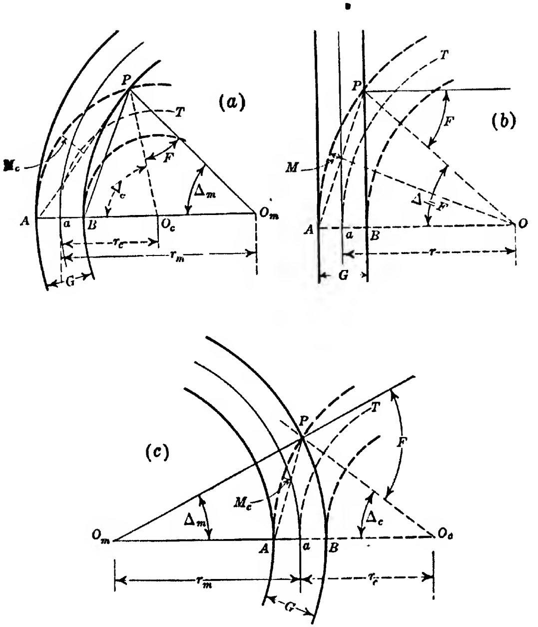
Fig 5*a*, when the two curvs are in the *same* direction,

$D_c > D;\ r_c < r;\ L_c < L;\ M_c > M.$

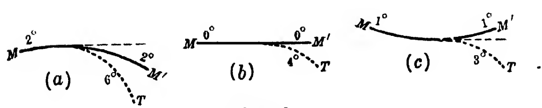
Fig 5*c*, when the two curvs are in *opposit* directions,

$D_c < D;\ r_c > r;\ L_c > L;\ M_c < M.$

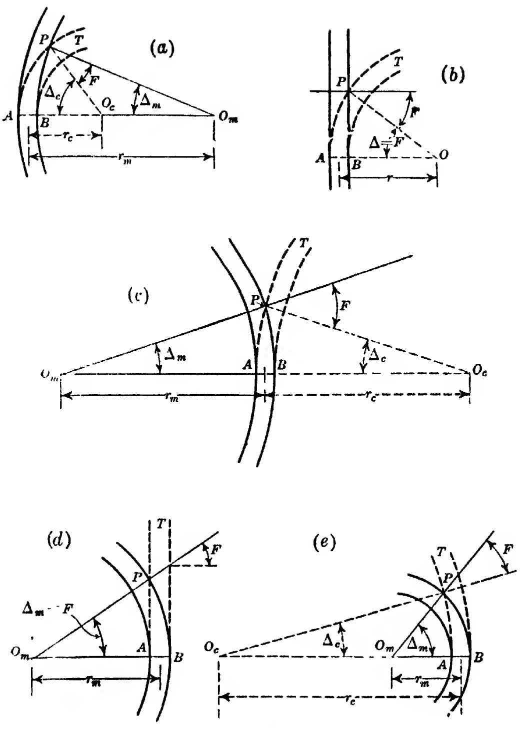
29. Sharpness Difference. Figs 6. Let the sharpness of the main-track curv, $M\,M'$, be D_m, and let that of the turnout curv, T be D_c. (In Fig 6*b*, $D_m = 0$.)



Figs. 5.



Figs. 6.



(a)
P
T
F
Δc
Δm
Oc
Om
A
B
rc
rm
(b)
P
T
F
Δ = F
A
B
O
r
(c)
T
P
F
Δm
Δc
Om
A
B
Oc
rm
rc
(d)
T
F
P
Δm = F
Om
A
B
rm
(e)
T
F
P
Δc
Oc
Om
Δm
A
B
rm
rc

Figs. 7.

Then, for their sharpness difference, D_d, we have :—

$D_d = D_c \pm D_m$; plus for Fig 6 *c;* minus for Fig 6 *a;*

or

$D_c = D_d \pm D_m$; plus for Fig 6 *a;* minus for Fig 6 *c.* (17)

If (Fig 7 *c*) the main track has the *sharper* curv, or $D_m > D_c$, we hav: D_d $(= D_c - D_m)$ negativ, and $D_c = D_m - D_d$.

Eqs (17), above, are exact, when D = sweep per *arc* unit. See Curvs p 878, ¶ 20.

·30a. Arrangements. Let Figs 7 represent a series of turnouts, T (shown dotted) each leaving the main track (shown solid) on the right, with constant frog angle, F*;* and, beginning with Fig 7*a*, let the main track deflect progressively toward the left. We then have :—

	Fig				
	7*a*	7*b* $r_m = \infty$; $\Delta = F$	7*c* $r_m = r_c$	7*d* $r_c = \infty$; $\Delta_m = F$	7*e*
Cens (and deflection from tangent in our Figs)					
Main —, O_m	right	0	left	left	left
Turnout —, O_c	right	right	right	0	left
Turnout leavs main track on	inside		outside	outside	outside
Frog rail					
Main —, BP	inner		outer	outer	outer
Turnout —, AP	outer	outer	outer		inner
Radii					
Greater	main	main		turnout	turnout
Main —, r_m	incrsg	∞	decrsg	decrsg	decrsg
Turnout —, r_c	increasing			∞	decrsg
Sweeps					
Main —, Δ_m	decrsg	0	increasing		
Turnout —, Δ_c	decreasing			0	incrsg

30b. We see, therefore, that a turnout may be a tangent, Fig 7*d*, or a curv in either direction; and that, if a curv, it may leav

Fig 7 *b*, a main-track tangent, as in our preceding articles;

Fig 7 *c*, a main-track curving in the *opp* dir'n, (cens on *opp* sides of track);

Figs 7 *a*, *e*, a main-track curving in the *same* dir'n, (cens on *same* side of track).

30c. When the turnout is a tangent (Fig 7*d*), it necessarily leavs the *outside* of the main-track curv; and the frog angle, F, is then the same (= turnout sweep, Δ_m) as for a turnout (Fig 7 *b*) of radius $r = r_m$, Fig 7 *d*, leaving a main-track tangent.

30d. When the two curvs are in opp directions (Fig 7c), (centers, O_m and O_c, on *opp* sides of the track), the turnout curv also necessarily leavs the *outside* of the main-track curv.

30e. When the two curvs are in the same direction (Figs 7 *a*, *e*) (cens, O_m and O_c, on *same* side of track) the turnout may leav either the inside or the outside of the main-track curv; but we assume (Fig 7 *a*) that, in this case, unless otherwise specified or obvious, the turnout curv has the shorter radius, so that it leavs the *inside* of the main-track curv.

31a. Equations for turnouts from curvs. For symbols, see ¶ 27.

For approximate equations, see ¶ 31 *b*.

$$\pm\,\Delta_m = F \pm \Delta_c\dagger; \qquad \pm\,\Delta_c = F \pm \Delta_m\dagger; \quad \ldots\ldots\ldots\ldots (18)$$

$$\operatorname{Tan}(\Delta_m/2) = G\,N/r_m; \qquad \operatorname{Tan}(\Delta_c/2) = G\,N/r_c; \quad \ldots\ldots\ldots\ldots (19)$$

$$\pm\,F = \Delta_c \pm \Delta_m;^* \qquad N = \tfrac{1}{2}\cot(F/2); \quad \ldots\ldots\ldots\ldots (20)$$

$$r_m = \frac{50}{\sin(D_m/2)} = \frac{G\,N}{\tan(\Delta_m/2)}; \quad \ldots\ldots\ldots\ldots (21)$$

$$r_c = \frac{50}{\sin(D_c/2)} = \frac{G\,N}{\tan(\Delta_c/2)} = \left(r_m \pm \frac{G}{2}\right)^* \frac{\sin\Delta_m}{\sin\Delta_c} - \frac{G}{2}; \quad (22)$$

$$\sin(D_m/2) = 50/r_m; \qquad \sin(D_c/2) = 50/r_c;$$

$$D_d = D_c \pm D_m;^* \quad \ldots\ldots\ldots\ldots (23)$$

$$L_c = 2\left(r_m \pm \frac{G}{2}\right)^* \sin\frac{\Delta_m}{2}; \quad M_c = \left(r_c + \frac{G}{2}\right)\operatorname{vers}\frac{\Delta_c}{2}; \quad (24)$$

$$Q_c = \sqrt{R_c^2 - x^2} + M_c - R_c \quad \ldots\ldots\ldots\ldots (25)$$

where $R_c = r_c + (G/2)$; $\quad x = A\,P/4 = \tfrac{1}{2} R_c \sin(\Delta_c/2)$

31b. Approximate equations. Fig 6. For a circular turnout from a curv, the principal functions may be found approximately, as below, by comparison with the corresponding functions in a turnout from a tangent.

With given gage, G, and given frog number, N, let the following nomenclature be observd.

---

*Plus for curvs in opposit direction; minus for curvs in same direction.

†Plus for curvs in same direction; minus for curvs in opposit direction.

	In turnout from tangt	In turnout from curv	
		Correct	Approx
Cen-line radius			
of main line,	$r_m (= \infty)$	r_m	...
of turnout,	r	r_c	r_{ca}
Sharpness			
of main line,	$D_m (= 0)$	D_m	...
of turnout,	D	D_c	D_{ca}
Sharpness-diff,	$D_d (= D)$	D_d $(= D_c \pm D_m)$*	D_{da}
Lead,	L	L_c	L_{ca}
Ordinate,			
middle—,	M	M_c	M_{ca}
quarter-point—,	Q	Q_c	Q_{ca}

Then, for a given gage, G, and given frog number, N, we have:—
Approx; see ¶ 31c

$$D_{da} = D; \quad D_{ca} = D_{da} \pm D_m\dagger; \quad r_{ca} = 50/\sin \tfrac{1}{2} D_{ca}$$

$$L_{ca} = L; \quad M_{ca} = M\,(1 \pm D_m/D)\dagger; \quad Q_{ca} = Q\,(1 \pm D_m/D)\dagger \quad (26)$$

31 c. The following table shows the *degree of approximation* in the approx equations (26), ¶ 31 *b*

						
	$D_m = 2°$	$D_m = 4°$	$D_m = 8°$	$D_m = 2°$	$D_m = 4°$	$D_m = 8°$
N	D_d/D	D_d/D	D_d/D	D_d/D	D_d/D	D_d/D
4	1.0042	1.0088	1.0196	0.99627	0.99299	0.98773
8	1.0019	1.0044	1.0115	0.99878	0.99821	0.99902
16	1.0018	1.0054	1.0176	0.99984	1.0014	1.0095
	r_c/r_{ca}	r_c/r_{ca}	r_c/r_{ca}	r_c/r_{ca}	r_c/r_{ca}	r_c/r_{ca}
4	0.99621	0.99243	0.98490	1.0038	1.0076	1.0153
8	0.99845	0.99691	0.99383	1.0015	1.0031	1.0062
16	0.99900	0.99800	0.99599	1.0010	1.0020	1.0040
	$L_c/L_{ca} = L_c/L$	$L_c/L_{ca} = L_c/L$	$L_c/L_{ca} = L_c/L$	$L_c/L_{ca} = L_c/L$	$L_c/L_{ca} = L_c/L$	$L_c/L_{ca} = L_c/L$
4	0.99916	0.99827	0.99637	1.0008	1.0016	1.0029
8	0.99909	0.99801	0.99534	1.0007	1.0013	1.0019
16	0.99883	0.99698	0.99126	1.0005	1.0003	0.99779‡
	M_c/M_{ca}		M_c/M_{ca}	M_c/M_{ca}		M_c/M_{ca}
4	1.0013	....	1.0043	0.99851	....	0.99305
16	0.99990	....	0.99285	0.99896	....	0.99368

*Plus for curvs in opposit direction; minus for curvs in same direction.

†Plus for curvs in same direction; minus for curvs in opposit direction.

‡With $D_m = 8°$, $N = 16$, the turnout, from outside of main track curv, curvs in the same direction, see Fig 7 *e*; *i. e.*, the two centers are on the same side of the track. See ¶ 30 *e*.

Double Turnout from Tangent.

From Opposit Sides.

32. Fig 8. In a double stub-switch turnout (see Part I, ¶10) from opposit sides of a tangent, let the two turnout curvs be of equal sharpness; and let

$G = AB =$ gage;

$F = F_a = F_b =$ main frog angle; $F_c =$ crotch frog angle;

$N = N_a = N_b =$ main frog number; $N_c =$ crotch frog number;

$L = A P_a = B P_b =$ main frog lead;

$L_c = a P c =$ crotch frog lead;

$r = O_a a = O_b a =$ center-line radius of either turnout curv;

$R = O_a B = O_b A = O_a P_a = O_b P_b = O_a P_c = O_b P_c = r + G/2$ = outer-rail gage-side radius of either turnout curv.

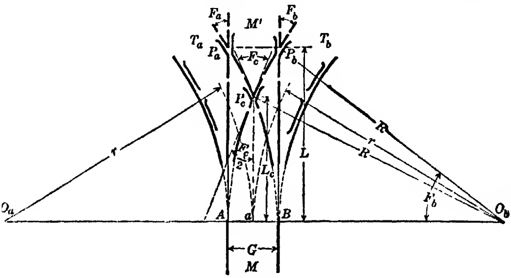


Fig. 8.

Then,

$F_c = 2 P_c O_b a$;

$L = 2 G N$ (see ¶ 19); $\quad L_c = 2 \left[G + \frac{(G/2)^2}{r} \right] N_c$

$$= \frac{G + \frac{(G/2)^2}{r}}{\tan(F_c/2)} \quad \ldots\ldots\ldots\ldots (27)$$

Or, for L_c, we have:—

$$\frac{L_c}{L} = \sqrt{\frac{R^2 - (R - G/2)^2}{R^2 - (R - G)^2}} = \sqrt{\frac{R - G/4}{2R - G}} \quad \ldots\ldots\ldots\ldots (28)$$

Hence, neglecting G, we have, approx:—

$L_c = \sqrt{1/2}\, L = 0.707\, L = \sqrt{2}\, G N$;

$L - L_c =$ approx $(1 - \sqrt{1/2})\, L =$ say $0.293\, L$;

or, practically, $L_c = 0.7\, L$; and $L - L_c = 0.3\, L$; ......(29)

$$N_c = \frac{\cot(F_c/2)}{2} = \frac{L_c}{2\,[G + (G/2)^2/r]}$$

$$= \text{app} \frac{L_c}{2G} = \frac{\sqrt{2}\, G N}{2G} = \frac{N}{\sqrt{2}} = 0.707\, N \quad \ldots\ldots\ldots\ldots (30)$$

From Same Side.

33. Fig 9. Two stub-switch turnouts, T_a and T_b, with center-line radii, r_a and r_b, respectivly, from the same side of the tan, $M\,M'$.

Let $R_a \;(= O_a A = r_a + G/2) = 2\,R_b = 2\,(O_b A) = 2\,(r_b + G/2)$.

Then, since $O_b P_b = O_b O_a$, we have:

$$F_a = F_b = F = \text{main-frog angle};$$
$$N_a = N_b = N = \text{" number}; \quad \ldots\ldots\ldots (31)$$

and the two main-frog points, P_a and P_b, are opposit, on track, T_a.

Let D_a and D_b be the center-line sharpnesses of curvs, T_a and T_b, respectivly.

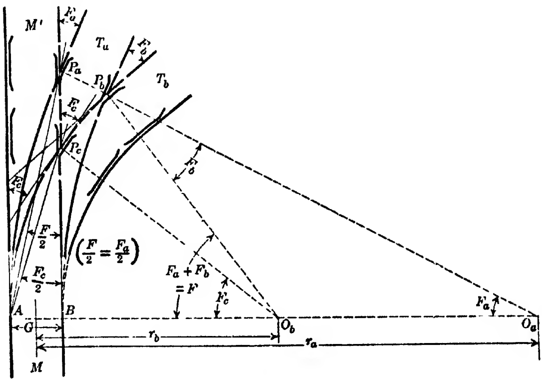


Fig. 9.

Then, approx:—

$D_b = 2\,D_a;$

$\text{Vers}\,F_c = G/R_b = 2\,G/R_a;$

$L = B\,P_a = \text{lead} = R_a \sin F_a = 2\,G\,N;$

$L_c = B\,P_c = \text{crotch-lead} = R_b \sin F_c = 2\,G\,N_c = \sqrt{0.5}\;L$
$= \sqrt{2}\;G\,N;$

$$N_c = \frac{L_c}{2\,G} = \frac{\cot\,(F_c/2)}{2} = 0.707\,N. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Double Turnouts from Curv.

34. In double turnouts from the same side or from opp sides of a curvd main track of moderate sharpness, the dimensions are practically the same as for similar turnouts from straight track, using the same frog angles. If the main-track curvature is sharp, the problem may be solvd graphically.

POINT SWITCHES

Turnout from Tangent.

35. Nomenclature. Fig 10. Point-switch turnout from tangent. Compare Am Ry Engng Assn, Manual, 1915, pp 182-3.

Let

G		=	track gage;
R	OD	=	outer-rail gage-side radius of turnout curv, DE;
r	$OD - G/2$	=	cen-line rad of turnout curv, DE;
D		=	sharpness of turnout curv, DE;
Δ	DOE	=	sweep of turnout curv, DE, betw switch-heel and frog-toe;
M		=	middle ordinate from chord, DE, to outer-rail gage-side;
F	$= BPE = tOE$	=	frog angle;
N	$= \frac{1}{2}\cot(F/2)$	=	frog number;
C	$= DE$	=	long chord of outer-rail gage-side;
L	$= BP$	=	lead, from actual switch-point to theoret frog-point;
T	$= DV = VE$	=	semitangent of turnout curv;
s	$= YAD$	=	switch angle;
l	$= AD$	=	switch length;
S		=	switch-heel distance, or spread, betw gage-lines, at D;
W	$= PE$	=	dist from frog-point to frog-toe;
e		=	$G - S - W \sin F$;
p		=	actual switch-point thickness;
t		=	actual frog-point thickness;
$x, y,$	$= ti, tk$	=	co-ordinates of any given point, as n, on outer-rail gage-side;
a	$= DOn$	=	sweep of arc, Dn.

36. Equations for point-switch turnout from tangent.

Fig 10. *Frog angle, F; turnout sweep, Δ, and switch angle, s.*

$F = \Delta + s; \quad \Delta = F - s;$

$s = F - \Delta. \quad \operatorname{Sin} s = (S - p)/l;$ ....(33)

Lead, L.

$$L = (l - W)\frac{\sin \frac{1}{2}(F - s)}{\sin \frac{1}{2}(F + s)} + (G - p)\cot\frac{F + s}{2}$$

$$= (T + l)\cos s + (T + W)\cos F \quad \text{..............(34)}$$

For *lead from vertex to theoretical switch point*, add $p \cot s$.

For *lead to actual frog point*, add $N t$, t = actual frog-point thickness.

Since $T \sin s + T \sin F = T(\sin s + \sin F) = e$, we have:—

$$T = R\tan(\Delta/2) = \frac{e}{\sin s + \sin F} \quad \text{........(35)}$$

Turnout center-line radius, r.

Since $e = DE \sin \frac{1}{2}(F + s)$

$= 2R\sin(\Delta/2) \sin \frac{1}{2}(F + s)$, we have:—

$$R = r + G/2 = T\cot(\Delta/2)$$

$$= \frac{e}{2\sin \frac{1}{2}(F + s)\sin(\Delta/2)} = \frac{e}{\cos s - \cos F}; \text{ and}$$

$= \quad R \quad - \quad G/2$ ....(36)

Chord, D E $\quad = \quad 2\,R \sin(\Delta/2)$ ....(37)

Turnout curv sharpness, D.

$\sin(D/2) \quad = \quad 50/r$ ....(38)

Arc, D E.

$DE \quad = \quad 2\,\pi\, R\,\Delta°/360 \quad = \quad 0.017\,453\,R\,\Delta°$ ....(39)

Distance, D′ E′, from switch-heel, *D′*, to frog-toe, *E′*.

$D'E' \quad = \quad L \quad - \quad (W \; + \; l)$ ....(40)

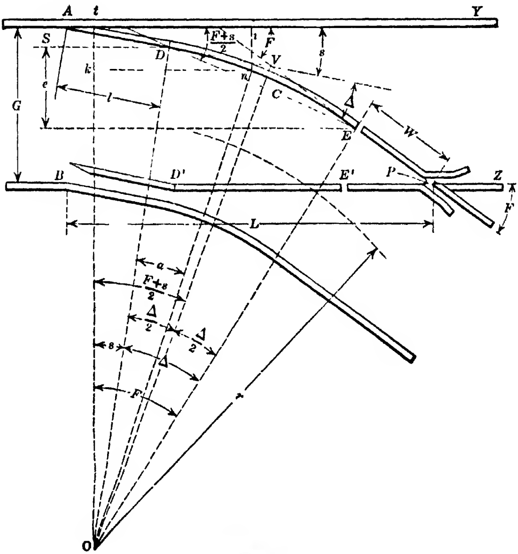


Fig. 10.

Co-ordinates, $x \quad = \quad k\,n$; and $y \quad = \quad i\,n$.

$x\;(= \;kn) \quad = \quad [l \cos s \; - \; R \sin s] \; + \; R \sin(a \, + \, s)$;

$y\;(= \;in) \quad = \quad [S \; + \; R \cos s] \; - \; R \cos(a \, + \, s)$..(41)

The quantities in brackets are constant for a given *N*, *l* and *s*.
The Am Ry Engng Assn, Manual, 1915, p 184, specifies:—

Switch-point thickness, *p*,	=	0.25 inch	=	0.020 833...ft;
Frog-point thickness, *t*,	=	0.50 inch	=	0.041 666...ft;
Switch-heel dist, or spread, *S*,	=	6.25 ins	=	0.520 833...ft.

Middle ordinate, *M*, to outer-rail gage-side.

$M = R \operatorname{vers}(\Delta/2)$ ....(42)

37. Simple approximate equations, using "switch-number", n. Fig 11. (Compare Wellington B. Lee, Eng News, 1898 Apl 21, p 252.)

Let

G	=	gage;
W	=	dist from frog-toe to frog-point;
F	=	frog angle;
l	=	switch length;
S	=	switch-heel distance;
N	=	frog number;
n	=	l/S = "switch-number";
e	=	$G - S - W \sin F$;
C	=	chord, $D\,E$, from switch-heel to frog-toe.

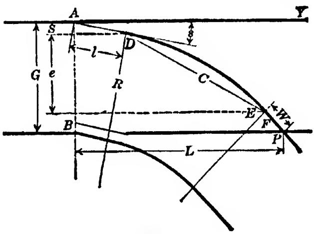


Fig. 11.

Then :—

$$C \text{ approx} = 2\,e\,\frac{n\,N}{n + N}\,; \qquad (43)$$

$$R = r + \frac{G}{2}\,; \quad R \text{ approx} = C\,\frac{n\,N}{n - N}$$

$$= \text{outer-rail gage-side radius.} \qquad (44)$$

With gage, G, = 4′ 8.5″, and S = 6.25″, calculation shows:

N =	l =	W =	F =	s =	$R - R_{app}$	R/R_{app}
4	11 ft	3 ft 2″	14° 15′ 0″	2° 36′ 19″	3.25 ins	1.0024
16	33 ft	8 ft 0″	3° 34′ 47″	0° 52′ 5″	8.53 ft	0.9958

Ordinates to chord, C.

38. Where, as assumed, for the *stub* switch, in ¶ 12, Figs 4, the outer turnout rail forms a continuous circular curv, from beginning, A, of turnout to the theoretical frog-point, P, its middle and quarter-point ordinates are approximately independent of the frog number, N (and hence independent of the radius); but, where, as in point-switches, Fig 10, the turnout curv is tangential to the switch-rail and to the frog-leg, at D and at E, respectivly, the ords diminish as the frog-angle, F, diminishes; and they would become zero if the frog-angle, F, became = the switch-angle, s; for the turnout-curv would then become a straight line.

$$\text{Mid ord} = R \text{ vers } \frac{\Delta}{2} = \left(r + \frac{G}{2}\right) \text{vers } \frac{\Delta}{2}. \quad \text{See Eq (42).}$$

"Theoretical" and "practical" dimensions.

39. To reduce rail-cutting and rail-waste, it is commonly found desirable to depart from the "theoretical" dimensions given by eqs 34-41, and represented by the *solid* lines in Figs 12, and to make the straight lead-rail, $D' \, E'$, of such length, $D_p' \, E'$, as to permit the eventual utilization of both the pieces into which a rail is cut; thus substituting "practical" for the "theoretical" dimensions.

40. The curvd lead-rail, ($D_p \, E_p$, Fig12*a*; $D_p'' \, E$, Fig12*b*) may then be made of the same number and lengths of pieces with $D_p' \, E'$, bringing the joints of these two rails opposit, if the length excess of the curvd rail may be compensated by laying it with joint-spaces wider than those of $D_p' \, E'$. But compare tables of theoretical and practical dimensions, ¶ ¶ 47-50.

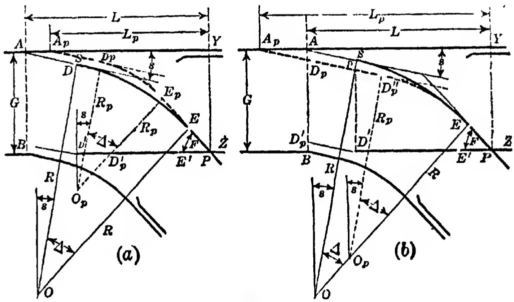


Fig. 12.

41. Diagrams. In Figs 12*a*, 12*b*, the *"theoretical"* dimensions are shown in *solid* lines; and the *"practical"* dimensions of the outer turnout rail are shown in *dotted* lines. Fig 12*a* represents the case where the "theoretical" lead, L, *exceeds* the "practical" lead, L_p; and Fig 12*b*, *vice versa.*

42. Change in lead length. Figs 12*a*, 12*b*. With a given switch-length, $l = A \, D$, given switch-angle, $s = Y \, A \, D$, and given frog-angle, F, there is but one lead length, L, and but one outer-rail gage-side radius, $R = O \, D$, for a circular arc, $D \, E$, connecting tangentially the switch-heel, at D, and the frog-toe, at E.

43. If the lead be *shortend* (as to L_p, Fig 12*a*) we must introduce an additional *"frog tangent"*, $E \, E_p$; whereas, if the lead be *lengthend* (as to L_p, Fig 12*b*), we must introduce an additional *"switch tangent"*, $D_p \, D_p''$. In *either* case, the *radius* is *shortend*, as indicated.

44. New Radius. For the new (diminisht) outer-rail gage-side radius, R_p, and for the additional tangents, we have (S. S. Roberts, Track Formulæ and Tables, pp 12-13).

Fig 12*a*

$$R - R_p = (L - L_p) \frac{\sin F \times \cot \frac{1}{2} (F - s)}{\sin (F - s)} ;$$

$$E \, E_p = (L - L_p) \frac{\sin \frac{1}{2} (F + s)}{\sin \frac{1}{2} (F - s)} \quad \ldots\ldots\ldots\ldots\ldots (45)$$

Fig 12b

$$R \; - \; R_p \; = \; (L_p \; - \; L) \; \frac{\sin s \times \cot \frac{1}{2}(F - s)}{\sin (F - s)} \; ;$$

$$D_p D_p'' \; = \; (L_p \; - \; L) \; \frac{\sin \frac{1}{2}(F + s)}{\sin \frac{1}{2}(F - s)} \; \ldots\ldots\ldots\ldots\ldots (46)$$

45. Co-ordinates. For a given point in the lead curv,

Let x = the abscissa (co-ordinate *parallel* with main track), measured from the switch-point, A_p,

y = the ordinate or offset (co-ordinate *normal* to main track) measured normally from the tangent.

In Fig 12a, where the theoretical exceeds the practical lead, the co-ords, x and y, to any point in the lead curv, $D_p E_p$, are as in eq 41, substituting R_p for R; but, in Fig 12b, where the practical exceeds the theoretical lead, these co-ords are modified, as below, by the introduction of the switch-tangent, $D_p D_p''$. (S. S. Roberts; Track Formulæ and Tables, p 13.) Here, l = switch-length, $A D$; S = switch-heel spread; s = switch-angle, $Y A D$; Δ_p = turnout sweep from switch-heel to the given point. The quantities in brackets [ ] are constant for given N, l and s.

$$x \; = \; [(l + D_p D_p'') \cos s - R_p \sin s] \; + \; R_p \sin(\Delta_p + s)$$

$$= \; (\text{approx}) \; [(l + D_p D_p'') - R_p \sin s] \; + R_p \sin(\Delta_p + s) \; ;$$

$$y \; = \; [S + D_p D_p'' \sin s + R_p \cos s] \; - \; R_p \cos(\Delta_p + s) \ldots (47)$$

46. Frogs and Switches. Am Ry Engng Assn, Manual, 1915, pp 184-5. Our Figs 1, 10 and 13. For theoretical and practical leads, see ¶¶ 48-50.

S = switch-heel spread = 6.25 ins;
N = frog number; F = frog angle;
W = dist, point to toe of frog, measured parallel with a gage-line;
K = dist, point to heel of frog; T = frog-toe spread;
H = frog-heel spread; l = switch-rail length;
s = switch angle

47. *Frog and Switch Dimensions.*

N	F ° ′ ″	W ft ins	K ft ins	$W + K$ ft ins	T ft	H ft	l ft ins	s, ° ′ ″
4	14 15 00	3 2	5 4	8 6	0.79	1.32	11 0	2 36 19
5	11 25 16	3 7	6 5	10 0	0.71	1.28	11 0	2 36 19
6	9 31 38	4 0	7 0	11 0	0.66	1.16	11 0	2 36 19
7	8 10 16	4 5	8 1	12 6	0.63	1.15	16 6	1 44 11
8	7 09 10	4 9	8 9	13 6	0.59	1.09	16 6	1 44 11
9	6 21 35	6 0	10 0	16 0	0.67	1.11	16 6	1 44 11
9½	6 01 32	6 0	10 0	16 0	0.63	1.05	16 6	1 44 11
10	5 43 29	6 0	10 6	16 6	0.60	1.05	16 6	1 44 11
11	5 12 18	6 0	11 6	17 6	0.54	1.05	22 0	1 18 08
12	4 46 19	6 5	12 1	18 6	0.53	1.01	22 0	1 18 08
15	3 49 06	7 8	14 10	22 6	0.51	0.99	33 0	0 52 05
16	3 34 47	8 0	16 0	24 0	0.50	1.00	33 0	0 52 05
18	3 10 56	8 10	17 8	26 6	0.49	0.98	33 0	0 52 05
20	2 51 51	9 8	19 4	29 0	0.48	0.97	33 0	0 52 05
24	2 23 13	11 4	23 2	34 6	0.47	0.97	33 0	0 52 05

"Nos 8, 11 and 16 frogs are recommended as meeting all general requirements for yards, main-track switches and junctions." Am Ry Eng Assn, Manual, 1915, p 168.

48. Theoretical Leads. For frog and switch dimensions, see ¶¶ 46, 47. For "practical" leads, see ¶¶ 49-50.

Am Ry Engng Assn, Manual, 1915, p 184.

N = frog number; r = center-line radius; D = sharpness;

L = lead = dist from actual switch-point to theoret frog-point;
L_s = straight rail closure, from switch-heel to frog-toe
$= L - l - W$;
L_c = curvd rail closure, from switch-heel to frog-toe.

N	r ft	D ° ′ ″	L ft	L_s ft	L_c ft
4	112.26	52 53 56	37.05	22.88	23.29
5	183.22	31 40 24	42.77	28.19	28.55
6	273.95	21 01 58	48.11	33.11	33.88
7	364.88	15 47 19	61.94	41.02	41.24
8	488.71	11 44 40	67.47	46.22	46.42
9	616.27	9 18 27	72.24	49.74	49.92
9½	699.97	8 11 33	74.90	52.40	52.58
10	790.25	7 15 18	77.51	55.01	55.17
11	940.21	6 05 48	92.06	64.06	64.20
12	1136.34	5 02 38	97.25	68.83	68.96
15	1744.45	3 17 06	130.50	89.83	89.94
16	2005.98	2 51 24	135.95	94.95	95.05
18	2587.66	2 12 52	146.38	104.54	104.61
20	3262.98	1 45 22	156.35	113.68	113.76
24	4932.77	1 09 42	175.09	130.66	130.77

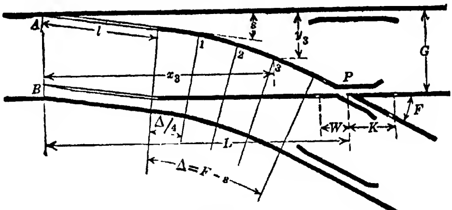


Fig. 13.

49. Practical Leads. Fig 13. (Am Ry Engng Assn, Manual, 1915, p 185. Lead-curv co-ordinates.) Theoretical leads, ¶ 48. Frog and switch dimensions, ¶¶ 46-7. Lead and closure lengths, ¶ 50.

N = frog number; r = center-line radius; D = sharpness;
x and y = co-ordinates from switch-point to the quarter points and center-point on outer-rail gage-side.

N	r ft	D ° ′ ″	x_1 ft	y_1 ft	x_2 ft	y_2 ft	x_3 ft	y_3 ft
4	110.69	53 42 24	17.74	0.97	23.44	1.67	29.75	2.79
5	174.34	33 19 57	17.78	0.95	24.54	1.61	31.27	2.62
6	265.39	21 43 04	19.07	1.01	27.13	1.74	35.15	2.72
7	362.08	15 52 29	26.72	0.97	36.93	1.71	47.11	2.74
8	487.48	11 46 27	28.37	1.02	39.91	1.78	51.45	2.91
9	605.18	9 28 42	28.75	1.02	40.98	1.76	53.19	2.75
9½	695.45	8 14 45	30.31	1.06	43.35	1.82	56.37	2.83
10	790.25	7 15 18	30.28	1.06	44.05	1.84	57.81	2.85
11	922.65	6 12 47	40.74	1.08	56.47	1.84	72.19	2.87
12	1098.73	5 12 59	43.99	1.15	60.65	1.90	77.28	2.91
15	1744.38	3 17 01	55.49	1.01	77.95	1.78	100.41	2.85
16	1993.24	2 52 59	58.16	1.04	81.76	1.82	105.35	2.87
18	2546.31	2 14 31	58.73	1.04	84.46	1.82	110.10	2.86
20	3257.26	1 45 32	61.84	1.08	90.21	1.88	118.59	2.93
24	4886.16	1 10 21	67.82	1.27	100.21	1.97	132.59	3.00

(See also ¶ 50.)

50. Figs 12. *Lengths of leads and closures, etc.*
(Am Ry Engng Assn, Manual, 1915, p 183.)

For "theoretical leads," see ¶ 48.
For frog and switch dimensions, see ¶¶ 46-7.
For practical lead-curv co-ordinates, see ¶ 49.

N = frog number; T_s = switch tangent, $D_p D_p''$, Fig 12 *b*;
T_f = frog tangent, $E E_p$, Fig 12 *a*;
(See ¶ 43.)

L = *lead, B P, from actual switch-point to actual frog-point;*
L_s = *closure, for straight rail;*
L_c = " " *curvd* " .
(*Closure* = *rail lengths required betw switch-heel and frog-toe.*)

N	L ft	L_s ft	L_c ft	T_s ft	T_f ft
4	37.94	one 23.60	one 24	1.03	0.00
5	42.47	one 27.68	one 28	0.00	0.82
6	47.98	one 32.73	one 33	0.00	0.66
7	62.10	one 13.89 one 27	one 14.11 one 27	0.00	0.19
8	67.98	one 16.40 one 30	one 16.60 one 30	0.30	0.00
9	72.28	one 16.41 one 33	one 16 59 one 33	0.00	0.57
9½	75.71	one 25.82 one 27	one 26 one 27	0.76	0.00
10	77.93	one 27 one 28	one 27.17 one 28	0.00	0.00
11	94.31	one 32.85 one 33	two 33	2.99	0.00
12	100.80	one 23.88 two 24	three 24	5.33	0.00
15	133.28	two 33 one 25.9	two 33 one 26	0.00	0.00
16	137.57	one 29.90 two 33	one 30 two 33	1.56	0.00
18	146.51	one 25.93 three 26	four 26	0.00	1.08
20	157.42	one 26.92 two 27 one 33	three 27 one 33	0.44	0.00
24	177.22	one 32.89 three 33	four 33	2.43	0.00

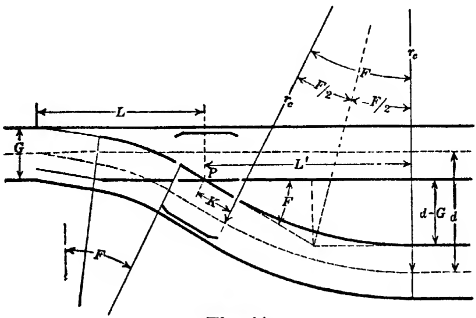


Fig. 14.

Connecting Curv.

51. Lateral Turnout. Fig 14. In the "lateral" turnout, let d = dist betw cens of tracks; $d - G$ = dist betw gage-lines of inner tracks; K = dist from theoret frog-point to frog-heel; r_c = cen-line radius of connecting curv; L' = lead from theoret frog-point to end of connecting curv. Then, remembering that

$$2N = \cot(F/2) = 1/\tan(F/2),$$

we have

$K + (r_c - G/2)\tan(F/2) = (d - G)/\sin F;$ **hence :—**

$r_c = 2N(d - G)/\sin F - 2NK + G/2;$

$L' = (d - G)\cot F + (r_c - G/2)/2N$ .... (48)

52. Diamond or Equilateral Turnout. Fig 15. In the "diamond" or "equilateral" turnout, we have :—
(compare Lateral turnout, ¶ 51.)
r = turnout curv cen-line radius, $O_t c$.

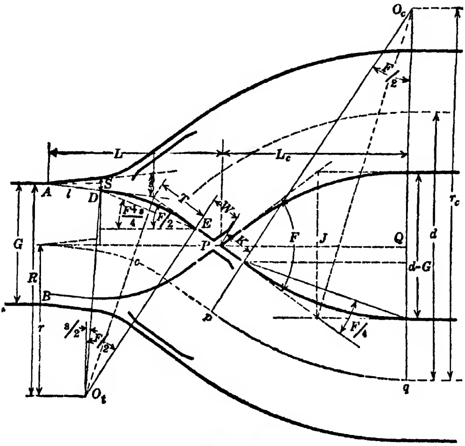


Fig. 15.

$$R\ (= r + G/2) = O_t D = T^*\cot\frac{F - s}{4} = \frac{DE^*}{2\sin\frac{1}{4}(F - s)}$$ .... (49)

$L = (T + l) \times \cos(s/2) + (T + W) \times \cos(F/2)$ .... (50)

Comparison of Dimensions between a diamond turnout and a turnout from straight track, with given frog and switch angles.
With F = 8° (N = 7.15); s = 2°; G = 4′ 8.5″ = 4.70833... ft; l = 16 ft; S = 0.5 ft; W = 5 ft;

	Diamond	Straight	Diamd/Strt
T	20.1285	20.1782	0.9975
R	768.6760	385.0226	1.9964
L	61.1903	61.0893	1.0017

$$^*T = \frac{G/2 - S/2 - W\sin(F/2)}{\sin(s/2) + \sin(F/2)};$$

$$DE = \frac{G/2 - S/2 - W\sin(F/2)}{\sin\frac{1}{4}(F + s)} = 2R\sin\tfrac{1}{4}(F - s)$$

53. Radius and Lead. Fig 15. Radius, r_c, and lead, L_c, of connecting curv, $p\,q$, for diamond turnout.

Let

$d - G$ = dist betw gage-sides of inner rails;

$r_c = O_c\,p$ = center-line radius of connecting curv, $p\,q$;

K = dist from frog-heel to theoret frog-point, P;

$L_c = P\,Q$ = lead, from theoret frog-point to end of connecting curv;

F = frog angle; N = frog number $= \frac{1}{2}\cot(F/2)$.

Then

$(r_c - G/2)\,\mathrm{vers}(F/2) = (d - G)/2 - K\sin(F/2)$. Hence,

$$r_c = \frac{d - G}{2\,\mathrm{vers}(F/2)} - \frac{K}{\tan(F/4)} + G/2; \quad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

$$L_c = P\,J + J\,Q = N\,(d - G) + (r_c - G/2) \times \tan(F/4) \quad \ldots (52)$$

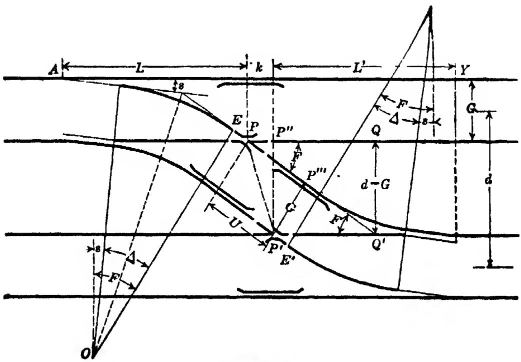


Fig. 16.

Crossovers.

54. Figs 16, 17. When the connecting track is a tangent (Fig 16), the two frog angles are necessarily equal. When it is a reverse curv (Fig 17) they are preferably equal.

With straight connecting track, $E\,E'$. Fig 16. Let

$R = O\,E$ = outer-rail gage-side radius of turnout curv;

T = semitangent of turnout curv;

d = dist betw main-track center-lines;

G = track-gage;

$d - G$ = dist betw gage-lines of inner rails;

$k = PP'' =$ dist betw theoret frog-points, P and P', $\parallel$ with main tracks;
$U = PP''' =$ dist betw theoret frog-points, P and P', $\parallel$ with connecting track;
$W =$ dist from theoret frog-point, P, to frog-toe, E;
$l =$ switch-length.

Then :—

$T = R \tan(\Delta/2)$; $L = L' = (T + l) \cos s + (T + W) \cos F$;

$$k = PQ - P'''Q = PQ - P'Q' = \frac{d - G}{\tan F} - \frac{G}{\sin F};$$

$AY = L + k + L' = k + 2L$;

$$U = PQ' - P'''Q' = \frac{d - G}{\sin F} - \frac{G}{\tan F}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

For unit change in the value of d, the corresponding changes in k and in U, respectivly, are :—in k, change $= \cot F$; in U, change $= \operatorname{cosec} F$.

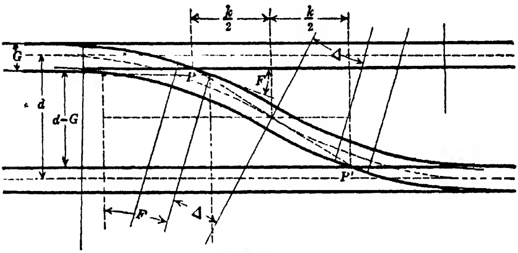


Fig. 17.

55. With reverst connecting curv. Fig 17.

The use of a reverst connecting curv economizes space, especially where the two tracks are spaced wide apart and where the frog-angle is small. The radius is usually that of the turnout curv.

Let

$F =$ frog angle;
$\Delta =$ sweep of either branch of reverst connecting curv;
$r =$ center-line radius of reverst connecting curv;
$R = r + G/2$
$=$ outer-rail gage-side radius of reverst connecting curv;
$K =$ dist, from theoret frog-point, P, to frog-heel.
$k =$ dist, parallel with track, betw frog-points, P and P'.

Then :—

$r \cos(F + \Delta) = R \cos F + K \sin F - (d - G)/2$;
$k/2 = r \sin(F + \Delta) - R \sin F + K \cos F$;
$PP' = \sqrt{k^2 + (d - G)^2}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (54)

Double Turnout.

56. Fig 18. Double point-switch turnout from opp sides of tangent. Let the two main frogs, P, be of equal angle and therefore opp, and let

r = turnout center-line radius;
R = $OE = r + G/2$ = outer-rail gage-side radius;
Δ_c = sweep, $D\,O\,P_c$;
T_c = semitangent of curv $D\,P_c$;
W = $E\,P$ = dist from main frog-toe, E, to theoret frog-point, P;
S = switch-heel spread;
s = switch angle;
l = $A\,D$ = switch length;
N, F, = main frog number and angle;
N_c, F_c = crotch frog number and angle;
$L = B\,P$ = lead to main frog, P;
$L_c = a\,P_c$ = lead to crotch frog, P_c.

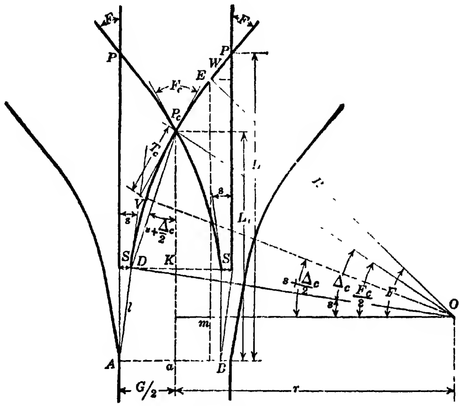


Fig. 18.

Then :—

$$\frac{F_c}{2} = \Delta_c + s; \qquad D\,P_c\,a = V\,O\,m = s + \frac{\Delta_c}{2} = \frac{F_c - \Delta_c}{2};$$

$$\operatorname{Cos}\frac{F_c}{2} = \frac{r}{R} = \cos F + \frac{G/2 - W \sin F}{R}; \qquad \cos F = \frac{O\,m}{R};$$

$$N_c = \frac{\cot(F_c/2)}{2};$$

$$T_c = \frac{G/2 - S}{\sin s + \sin(F_c/2)}$$

$$D\,P_c = \frac{G/2 - S}{\sin(s + \Delta_c/2)} = 2\,R \sin\frac{\Delta_c}{2};$$

$$R = \frac{D\,P_c}{2\sin(\Delta_c/2)} = T_c \cot(\Delta_c/2) = \frac{G/2 - S}{2\sin(\Delta_c/2)\sin(s + \Delta_c/2)}$$

$$L_c = L - W\cos F - R\,[(\sin F - \sin(F_c/2)]$$

$$= aK + KP_c$$

$$= l\cos s + (G/2 - S)\cot\frac{F_c - \Delta_c}{2}$$

$$= (T_c + l)\cos s + T_c\cos(F_c/2)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Ladders.

57. Layout. Fig 19. Ordinarily, the yard or "body" tracks, 2, 3, 4, etc., reacht by a ladder, are parallel with the main (or "drill") track, 1; the ladder is straight, from P_1 to P_4, etc; the yard tracks, 2, 3, 4, etc., are straight from the ladder frogs, P_2, P_3, etc., and the dist, *d*, betw cens of adjacent parallel tracks is constant.

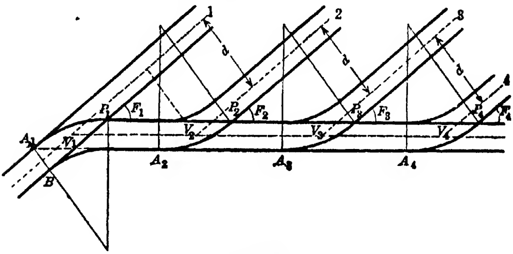


Fig. 19.

58. Distances. The frog angles, F_1, F_2, etc., betw drill track and ladder, and betw ladder and yard tracks, are then equal; and the dists, A_1A_2, A_2A_3, etc., betw consecutiv switches; those, P_1P_2, P_2P_3, etc., betw consecutiv frogs; and those, V_1V_2, V_2V_3, etc., betw consecutiv intersections, measured ‖ with the ladder, are all equal, and $V_1V_2 = d/\sin F$. (But see ¶ 60.) For unit change in the value of *d*, the corresponding change, in $d/\sin F$, is $1/\sin F = \operatorname{cosec} F$.

59. The *minimum allowable dist,* along the ladder, betw a frog point, as P_1, and a point opp the switch-toe, A_2, of the next turnout, is about 10 ft.

60. "Under ordinary conditions, *body tracks should be spaced* 13 to 14 ft centers, and, where they are parallel to main track or to other important running track, the first body track should be spaced $\nless$ 15 ft centers from such main or other important track. Ladder tracks should be spaced $\nless$ 15 ft centers from any parallel track." Am Ry Engng Assn, Manual, 1915, p 469.

61. Frog Numbers. Steam railroad yards commonly use No 7 and No 8 frogs; less commonly No 6. For general use in ladders, the Am Ry Engng Assn (Manual, 1915, p 469) recommends frogs of not lower No than No 8.

Double Turnout.

56. Fig 18. Double point-switch turnout from opp sides of tangent. Let the two main frogs, P, be of equal angle and therefore opp, and let

r = turnout center-line radius;
R = $OE = r + G/2$ = outer-rail gage-side radius;
Δ_c = sweep, $D\,O\,P_c$;
T_c = semitangent of curv $D\,P_c$;
W = $E\,P$ = dist from main frog-toe, E, to theoret frog-point, P;
S = switch-heel spread;
s = switch angle;
l = $A\,D$ = switch length;
N, F, = main frog number and angle;
N_c, F_c = crotch frog number and angle;
L = $B\,P$ = lead to main frog, P;
L_c = $a\,P_c$ = lead to crotch frog, P_c.

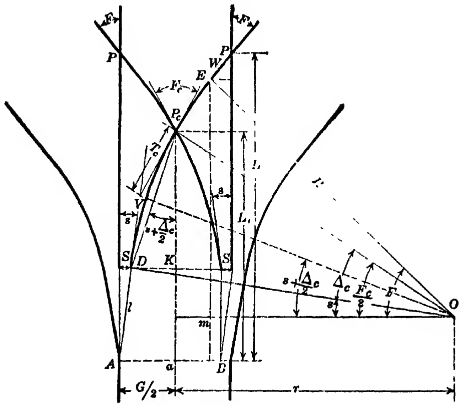


Fig. 18.

Then :—

$$\frac{F_c}{2} = \Delta_c + s; \qquad D\,P_c\,a = V\,O\,m = s + \frac{\Delta_c}{2} = \frac{F_c - \Delta_c}{2};$$

$$\text{Cos}\,\frac{F_c}{2} = \frac{r}{R} = \cos F + \frac{G/2 - W\sin F}{R}; \qquad \cos F = \frac{O\,m}{R};$$

$$N_c = \frac{\cot(F_c/2)}{2};$$

$$T_c = \frac{G/2 - S}{\sin s + \sin(F_c/2)}$$

$$D\,P_c = \frac{G/2 - S}{\sin(s + \Delta_c/2)} = 2\,R\sin\frac{\Delta_c}{2};$$

66. Double (three-throw) turnouts (see ¶¶ 32-4) are also used in connection with ladders.

In such cases, resort may be had to graphic methods.

See also ¶ 73, Yards and Stations, p 1005.

Turnout from Curv. (Point Switch).

67. Computing vs. drafting. Exact equations, for the dimensions of point-switch turnouts from curvs, would be very complicated; and problems, involving such dimensions, are best solvd by means of drawings to liberal scale. The subject is discust mathematically, in "Track Formulæ and Tables", by S. S. Roberts; New York, John Wiley & Sons.

68. Sharpness Difference. In general, and for practical purposes (even with straight switch-rails and frog-rails), the sharpness *diff*, betw the main and turnout curvs, for a given frog-number, may be taken as equal to that of a turnout from straight track, with the same frog number, as in ¶ 26.

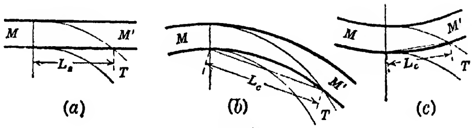


Fig. 22.

69. Lead. Figs 22. For a given frog number, the lead may be taken as approximately equal to that figured for a turnout from a tangent, by eq 10; but, in his Notes on Track, p 373, Mr. W. M. Camp gives the following:—

Let L_s = lead for turnout from tangent, Fig (*a*);
L_c = lead for turnout from curvd track with same frog, Figs (*b*), (*c*).
D = main line curv sharpness; N = frog number.

Then

$$L_c = L_s \pm D\,(N/12)^2 \;\ldots\ldots\ldots\ldots (57)$$

plus when turnout leavs *inside* of curv (Fig *b*), and *vice versa* (Fig *c*). This givs results as follows when $D = 3°$:—

	Lead		
N	L_s	L_c	
	Fig *a*	Fig *b*	Fig *c*
4	37.05	37.38	36.72
12	97.25	100.25	94.25

CURVS

SIMPLE CURVS

GENERAL

Definitions

(For equations, see ¶ ¶ 25, etc)

1. Fig 4a. A railroad line, *a b c . . . f,* is usually an alternation of curvs, *b c, d e,* etc; and of straight lines, *a b, c d,* etc, which are commonly called "tangents" because they are necessarily tangential to the curvs.

Fig 1. In a circular or simple railroad curv, the center line, *A P B* of the track (or, if not level, its projection on a horizontal plane) is a circular arc.

Fig 1. Imagin the tangents, *X A* and *B Z,* at the ends of the curv, *A B,* produced to their intersection, *V.* Then the equal dists, *A V, V B,* are called the **semitangents.**

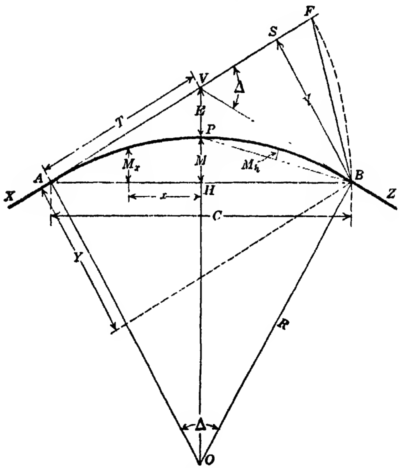


Fig. 1.

2. For the points of change from tangent to curv, etc, etc, we use the symbols adopted by the **Am Ry Eng Assn,** Manual of Recommended Practice, 1915, p 135, as follows:—

For circular curvs	Hitherto called
T. C. From tangent to curv;	Point of curv, P. C.
C. T. " curv " tangent;	Point of tangent, P. T.
C. C. " curv " curv;	Point of compound or of reverst curv, P. C. C. or P. R. C.

For spiral curvs See p 968, ¶ 201.

The point of curv, Fig 1, T. C., is the beginning, *A*, of the curv, or that end of it which is first reacht by the location in its progress.

The point of tangent, C. T., is the other end, *B*, of the curv.

The vertex, point of intersection, P. I., or apex, is the point, *V*, where the two tangents, produced, *A V* and *V B*, meet.

LOCATION.

Curv Functions and their Symbols

3. Fig 1. List of the functions of an entire curv. For that portion of a curv subtended by a unit chain, see ¶ 9.

Functions	Symbols				
Sweep (central angle, see ¶ 10)	Δ	=	$A\,O\,B$	=	$F\,V\,B$
Radius	R	=	$O\,A$	=	$O\,B$
Long Chord	C	=	$A\,B$		
Semitangent	T	=	$A\,V$	=	$V\,B$
External distance	E	=	$P\,V$		
Tangent offset, (perp to tangent $A\,V$ or $B\,V$)	Y	=	$B\,S$		
Tangent distance (making $A\,F = A\,B$)......		=	$B\,F$		
Middle ordinate	M	=	$H\,P$		
Side ordinate, distant x from M	M_x				
Middle ordinate for $\Delta/2$	M_h				

4. Locating points on Curvs. Fig 2. Where the radius *R*, is very short, it is sometimes practicable to locate the center, *O*, of the curv, and to describe the center line on the ground, using a steel tape for the radius; but, ordinarily, the center is not located, and points, *a*, *b*, etc, in the curv, are located by setting the transit at a known point in the curv (as the T. C., at *A*), laying off known peripheral angles, *v A a*, *v A b*, etc, from the tangent, *A v*, and measuring chords, *A a*, *a b*, *b c*, etc, of the proper calculated lengths.

5. Chords and Chains.* In practice, these chords (except the first and last, see ¶ 24) are of equal length, equal (or nearly equal, see ¶ 20) to that of the tape (usually 100 ft or 20 meters long) used in the location of the line, and are called "chains,"* in order to distinguish them from other chords, as *A b*, *b d*, etc, which may be drawn to the curv.

6. The equal peripheral angles, *a A b*, *b A c*, etc, subtended by these equal chains, are called **deflection angles.** Each is equal to $D/2$ = half the corresponding **central angle,** *a O b*, *b O c*, etc.

7. Tangential, Deflection and Central Angles. Samuel W. Mifflin, "Methods of Location for Railway Engineers", 1837, called the angle, *e c d* (= *c O g* = $D/2$), the "*tangential angle*", as being the angle betw a chain,* *c d*, and the *tangent*, *c e*, at its either end. In this, Mifflin followed Col. Stephen H. Long, U. S. A. ("Railroad Manual", 1829), who gave the name, "*deflection angle*", to the angle, *f c d*, betw a chain, *c d*, and the extension, *c f*, of a consecutive chain, *b c*; it being equal to the angle, *e h d* (betw the tangents, *c e* and *h d*, at the two ends of a chain, *c d*), thru which *the line deflects* within the dist subtended by a chain. It is equal to the *central* angle, *D* = *c O d*, subtended by a chain, *c d*.

8. We, in "The Field Practice of Laying out Circular Curvs for Railroads" (first edition, 1851), and Mr. William Findlay Shunk, in "The Field Engineer", 1880, followed this nomenclature. Now, however, usage calls *e c d* the **"deflection angle"** or simply the "deflection" (as indicating a deflection of the line of sight) and calls *D* (= *f c d*) the **"degree of curv".** See Sharpness, ¶ 11.

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*The use of the word "chain", to designate such a chord, survives from the days when survey *tapes* were unknown, and when the measuring was done by means of chains made up of wire links. The standard full *length* of such a chain was called "one chain."

9. Figs 1 and 2. In that portion of a curv, as *a b*, Fig 2, subtended by a unit chain*, *c*, we have sweep, $\Delta = D$, and we designate the several other functions by "lower-case" instead of by capital letters. Thus;

the letters .... *C T E Y M* M_x and M_h, Fig 1, ¶ 3,
become, respectivly .. *c t e y m* m_x and m_h, in the arc *a b*, Fig 2.

10. Fig 1. The sweep, Δ, of the curv, is the angle, *A O B*, swept out by the radius, *R*, and subtended by the curv. It is also the angle, *F V B*, swept out by the tangent in swinging from *V F* to *V B*. It is therefore the total deflection of the line, due to the curv; or it is the intersection angle betw the two tangents. Each *portion* of the curv of course has also its own sweep.

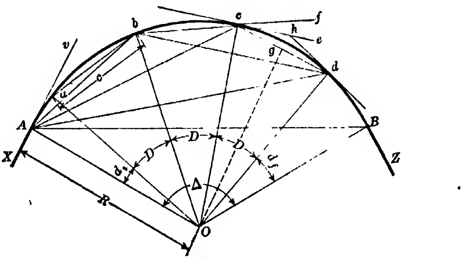

Fig. 2.

11. Fig 2. The sharpness, (commonly called the **"degree of curvature"**) is usually exprest by the sweep or *central* angle, *D*, (= *a O b* = *b O c*, etc) subtended by a *chain**, *a b* or *b c*; *i. e.*, sharpness = the deflection, *e h d*, of the line, in a length of one chain. A "one-degree", "two-degree", "three-degree-thirty-minute", etc curv is one whose sharpness is 1°, 2°, 3°30′, etc. See ¶ 14.

12. Fig 2. But sometimes (especially where the metric system is used) the curv is named for its *peripheral* chain angle, *c b d* = *c A d*, etc, or angle subtended at the *circumference*, as at *b* or *A*. A given curv, called
1° 00′, 2°, 3° 30′, etc, after its *central* chain angle, would be called 0° 30′, 1°, 1° 45′, etc, after its *peripheral* chain angle.
We follow the definition of ¶ 11.

13. Fig 3. When curvs are located by means of short chains* (see ¶ 19), the sharpness is sometimes taken as the sweep subtended by a *series* of such chains, the *sum* of whose lengths is equal to the standard unit chain. Thus, a curv, *a p b*, in which a series of two chains, *a p*, *p b*, of 50 ft each, subtends an angle of *D*°, is then called a *D*° curv, altho it is sharper (has a shorter radius, R_2 = *O b*) than the *D*° curv, *A P B* (R_1 = *O B*), in which the same angle of *D*° is subtended by a single chain, *A B*, of 100 ft.

14. Fig 1. In countries other than the U. S., the sharpnesses of curvs are often designated by the lengths of their *radii*, *R* = *O A* = *O B*, etc. The greater the sharpness, the shorter is the radius.

15. Graphic representation of curvature. In Fig 4, the center line, *a f* (Figs *a* and *b*) and the rectangles (Fig *b*) below and above it, represent the supposititious case tabulated below.

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*The use of the word "chain," to designate such a chord, survives from the days when survey *tapes* were unknown, and when the measuring was done by means of chains made up of wire links. The standard full *length* of such a chain was called "one chain."

Since the abscissas, *a b*, *a c*, etc, Fig *b*, represent dists along the line, while the ordinates represent sharpness of curvature, the *areas* of the rectangles represent the *sweeps* of the curvs, or the deflections of the tangents from each other. Similarly with *portions* of the rectangles; thus, the area of the rectangle on *b′ c* represents the sweep of the portion, *b′ c*, of the simple curv, *b c*.

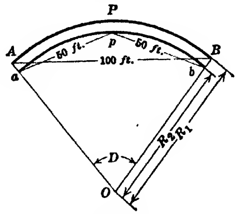


Fig. 3.

16. The inclined lines (Fig 4 *b*) represent easements of the ends of the curvs by means of spirals (see p 966). Since the positions of the tangents are not changed by the introduction of the spirals, the sweeps of the curvs, and the areas representing them, remain unchanged also. Thus, area of trapezoid on *B C* = area of rectangle on *b c*.

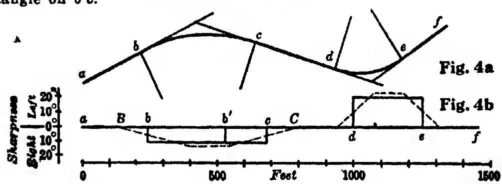


Fig. 4. Graphic Representations of Curvature

Point	*a*	*b*	*c*	*d*	*e*	*f*
Station	456 + 00	458 + 49	462 + 78	465 + 89	468 + 50	470 + 41

Stretch	*a-b*	*b-c*	*c-d*	*d-e*	*e-f*	
Length, ft ..	249	429	311	261	191	
Sharpness, *D*						
Left	0°	0°	0°	20°	0°	
Right	0°	10°	0°	0°	0°	
Sweep, Δ ..						
Stretch ...						
Left ...	0°	0°	0°	52.2°	0°	
Right ..	0°	42.9°	0°	0°	0°	
Sums						
Left ...	0°	0°	0°	52.2°	52.2°	
Right ..	0°	42.9°	42.9°	42.9°	42.9°	
Total	0°	42.9°	42.9°	95.1°	95.1°	
Diffs						
Left ...	0°	0°	0°	9.3°	9.3°	
Right ..	0°	42.9°	42.9°	0°	0°	

Chains*. Curv Length

17. The unit of railroad and of curv measurement is usually either 100 feet or 20 meters (= say 65.617 ft).

18. Full chain*. It is usual to make this (100 ft, or 20 meters) the *chain* length; and the **length of the curv** is then stated (*approximately*) as being equal to the sum of the *chain* lengths used in locating it. Thus, Fig. 6, in the curv, $A\ a\ b\ c\ B$, the length is taken as being $= A\,a + a\,b + b\,c + c\,B$, measured along these *chords*, $= 43 + 200 + 36.7 = 279.7$ ft The *true* length of the curv is of course slightly *greater* than this. See p 902.

19. Short chain.* Sharp curvs are usually located by means of a short chain whose length is half or quarter that of the standard chain, or less; these short chains subtending correspondingly smaller central angles. Thus, where the standard chain is 100 feet, chains (tape lengths) of 50 or 25 ft or less are used on such curvs. In general, the chain length used is not greater than from one-twelfth to one-eighth of the radius of the curv. The sum of a series of such short chain-lengths, subtending a given arc, is evidently greater (more nearly equal to the length of the arc itself) than is the sum of standard unit chain-lengths subtending the same arc. See ¶ 13.

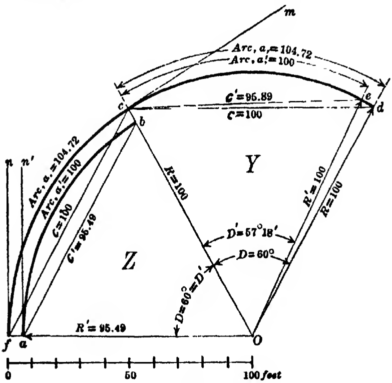


Fig. 5.

20. Diminisht chains*. If, on curvs, the rear chainman holds the tape with a properly calculated small reading (instead of the zero mark) at the stake, thus slightly diminishing the effectiv unit "chain" length, the *arc* subtended is thereby reduced to the full unit length, the *stated* length of the curv is made to correspond with its *actual* (curvd) length, and certain formulas (¶ 42) for finding curv functions (otherwise only approx) are renderd exact.

21. Sharpness, as affected by the use of diminisht chains*. Fig 5 Y. In the curv, $c\ e\ d$, let the full "chain", $c = c\,d = 100$ ft,

*The use of the word "chain," to designate such a chord, survives from the days when survey *tapes* were unknown, and when the measuring was done by means of chains made up of wire links. The standard full *length* of such a chain was called "one chain."

subtend a central angle, $D = c\,O\,d = 60°$. Then $c\,e\,d$ is a 60° curv, and its deflection angle, $D/2 = m\,c\,d = 30°$. To run **the same curv** with a *diminisht* chain, $c' = c\,e$ ($= 95.89$ ft, ¶ 22) we must use a deflection angle, $m\,c\,e$, of only 28° 39′, and D', ($= c\,O\,e$) $= 57°\,18'$. Thus, the use of diminisht chains makes a given curv *nominally flatter*.

22. Fig 5 *Z*. If, with diminisht chain, $c' = a\,b$, we use **the same deflection angle,** ($n'\,a\,b = n\,f\,c$), as with the full chain. ($c = f\,c$), we shall obtain a sharper curv, $a\,b$ (radius $= O\,a$) than with the full chain, which would give the curv $f\,c$, (radius $= O\,f$). Thus, the use of the diminisht chain makes a nominally equal curv *actually sharper*.

In Figs 5 *Y* and 5 *Z*, the functions compare as follows:

($Q = a/c$ or a'/c')

Fig	R, ft	R', ft	D	D'	a, ft	a', ft	c, ft	c', ft	Q
5 *Y*	100.00		60° 00′		104.72		100.00		1.047
		100.00		57° 18′		100.00		95.89	1.043
5 *Z*	100.00		60° 00′		104.72		100.00		1.047
		95.49		60° 00′		100.00		95.49	1.047

23. Stations. As on the tangents, the end-points of chains, on curvs, are called "stations"; but the name, "station", is often (loosely) applied to *the distance between* two such points.

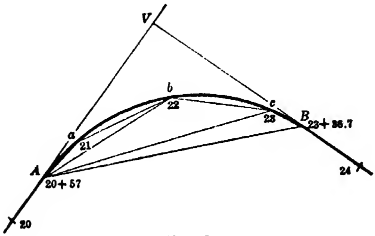


Fig. 6.

24. "Plus" stations; Sub-chains. Fig. 6. It can only accidentally happen that a curv begins or ends exactly at a station of the location. In other words, the T. C. and the C. T. nearly always fall *between* stas, *i. e.*, at a "plus" sta, as at sta A = 20 + 57, or at station B = 23 + 36.7; and this necessitates the use of a sub-chain, $A\,a$ or $c\,B$, and a sub-angle, $V\,A\,a$ or $V\,B\,c$ ($V\,B\,c = c\,A\,B$), at one or at both ends of the curv. See ¶¶ 44, etc.

GEOMETRY

Propositions of frequent application

25. Fig 7. **Similar triangles, and ratios between chord segments.** Let OV be a diam of the circle, $AVBO$; let AB ($= C$) be a chord perpendicular to OV and bisected at H; let OA, AV be chords in the semi-circle, OAV; and OB, BV, chords in the semi-circle, OBV; let AV ($= VB$) $=$ semitangent, T; and OA ($= OB$) $=$ radius, R, of the curv, APB.

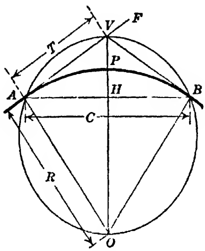


Fig. 7.

Then the 3 right triangles, OHA, AHV and OAV are similar; as are also OHB, BHV and OBV; angle, OAV ($= OBV$), betw R and T, $= 90°$; $AH = HB = C/2$; and

From similar triangles	We have		
OHB, BHV;	$OH : HB = HB : HV$;		
	$OH \times HV = (C/2)^2$;	or	$HV = (C/2)^2 \div OH$ (1)
OBV, BHV;	$OV : VB = VB : HV$;		
	$OV \times HV = T^2$;	or	$HV = T^2/OV$ ......(2)
OBV, OHB;	$OV : OB = OB : OH$;		
	$OV \times OH = R^2$;	or	$OH = R^2/OV$ ....(3)

26. Fig 8. **Angles between two chords, between two tangents, and between a chord and a tangent. Values of $\Delta/2$.** Let APB be the arc, and AB the chord, subtending the central angle, $AOB = \Delta = FVB = AOd + dOB = AOe + eOB$

Then

$\Delta/2$ = angle betw the tan, AV, at A, (or tan VB, at B) and the chord, AB

= external angle betw any two chords, as Ad and dB, or Ae and eB, drawn to the arc from A and from B respectively

= $(AOd + dOB) / 2 = (AOe + eOB) / 2$

= the peripheral angle, $AEB = VAB = VBA$, subtended by the chord, AB. ....(4)

27. Small angles, (such as the deflections usual in steam railroad work), are **approx proportional to their sines,** and less nearly proportional to their tangents. This fact may be used to facilitate many calculations.

Thus, for any small angle, A, we have,

approx: $\sin 0° 1' : \sin A = 1 \text{ minute} : A$, in mins; or

$A \text{ mins} = \sin A \div \sin 0° 1' = 3437 \sin A$;

$\sin A = A \text{ mins} \times \sin 0° 1' = 0.0002909\ A \text{ mins}$. . (5)

Similarly (approx) $\tan A = A \text{ mins} \times \tan 0° 1' = 0.0002909\ A$ mins.

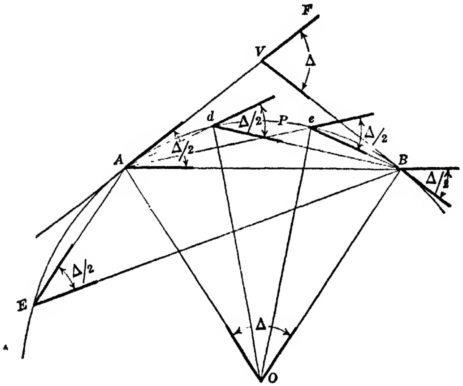


Fig. 8.

Example. With 100 ft chain; given radius, R, = 1300 ft; required the sharpness, D, of the curv. Here, $\sin (D/2) = 50/1300$; and $D = 4° 24' 30.44''$.

Approx, $D/2$ in mins

$= \sin (D/2) \div \sin 0° 1'$

$$= \frac{50}{1300} \times \frac{1}{\sin 0° 1'} = 132.221.$$

Hence, D approx $= 264.442$ mins $= D - 0.065$ min $= 0.99975\ D$.

Conversely; given $D = 4° 24'$, required the radius, R. Here

$$R = \frac{50}{\sin(D/2)} = 1302.497 \text{ ft.}$$

$$R \text{ approx} = \frac{50}{(D/2) \text{ in mins} \times \sin 0° 1'}$$

$$= 1302.177 \text{ ft} = 0.99975\ R.$$

Such operations are conveniently performd with the **slide-rule,** pp 73 etc.

Errors of Approximation.

Let A be in minutes; and let

$$S = \frac{A \sin 0^\circ 1'}{\sin A}; \qquad T = \frac{A \tan 0^\circ 1'}{\tan A}. \text{ Then:—}$$

for $A =$	$S =$	$T =$	for $A =$	$S =$	$T =$
0° 30′	1.000013	0.999995	6° 0′	1.001830	0.996342
1° 0′	1.000051	0.999898	7° 0′	1.002492	0.995020
2° 0′	1.000203	0.999594	8° 0′	1.003257	0.993493
3° 0′	1.000457	0.999086	10° 0′	1.005095	0.989825
4° 0′	1.000813	0.998375	20° 0′	1.020600	0.959050
5° 0′	1.001270	0.997460			

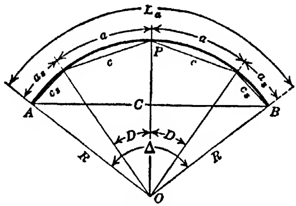


Fig. 9.

28. Relations between Δ, D, n, R, L_a and L_c. Fig 9.
In any circular curv, let

R = radius; $\pi = \dfrac{\text{circumf}}{\text{diam}}$; Δ = sweep; C = long chord;

L_a = curv length, $A\,P\,B$, measured on the arc. Then:—

πR = semi-circumf (subtending 180°);

$\dfrac{\pi R}{180}$ = arc subtending 1°; and (Δ in *degrees*)

$$L_a = \Delta \frac{\pi R}{180}; \quad \Delta = \frac{180\, L_a}{\pi R}; \quad \text{and} \quad R = \frac{180\, L_a}{\pi \Delta} \ldots. (6)$$

Let c = chain length, in ft; a = unit arc, in ft; n = number of chains, c, or of unit arcs, a, in the curv*; D = sharpness; L_c = curv length, $A\,P\,B$, measured on chains, = sum of lengths of chains. L_a = true curv length. Then:—

*Usually n is a mixt number. Thus, with two chains of 100 ft each, and sub-chains of 33.6 ft and 21.8 ft respectively, we have $n = 2 + 0.336 + 0.218 = 2.554$.

With *full* chain, ¶ 18	With *diminisht* chain, ¶ 20
$\sin\dfrac{D}{2} = \dfrac{c/2}{R}$ ....(7)	$D^\circ = \dfrac{180\,a}{\pi\,R}$ ....(11)
$\Delta = n\,D$ ....(8)	$\Delta = n\,D$ ....(12)
$R = \dfrac{c/2}{\sin(D/2)}$	$R = \dfrac{180\,L_a}{\pi\,\Delta^\circ} = \dfrac{180\,a}{\pi\,D^\circ}$ ..(13)
$= \dfrac{c}{2\sin(D/2)}$ ....(9)	
$L_c = n\,c;$ $n = L_c/c = \Delta/D$ ...(10)	$L_a = n\,a;$ $n = L_a/a = \Delta/D$ ...(14)

29. Hence, for the radius, R_f, in feet, or R_m, in meters, we have:—

	On any curv		On a 1° curv	
	Curv measured by			
¶ 18	100-ft chain	20-meter chain	100-ft chain	20-meter chain
	Radius			
	$R_f =$	$R_m =$	$R_f =$	$R_m =$
With full chain,	$\dfrac{50\text{ ft}}{\sin(D/2)}$	$\dfrac{10\text{ m}}{\sin(D/2)}$	$\dfrac{50\text{ ft}}{\sin 0^\circ\,30'}$ = 5729.650 ft log = 3.758 1281	$\dfrac{10\text{ m}}{\sin 0^\circ\,30'}$ = 1145.930 m log = 3.059 1581
	Curv measured by			
¶ 20	100-ft arc	20-meter arc	100-ft arc	20-meter arc
	Radius			
	$R_f =$	$R_m =$	$R_f =$	$R_m =$
With diminisht chain,	$\dfrac{18{,}000\text{ ft}}{\pi\,D}$	$\dfrac{3{,}600\text{ m}}{\pi\,D}$	$\dfrac{18{,}000\text{ ft}}{\pi}$ = 5729.578 ft log = 3.758 1226	$\dfrac{3600\text{ m}}{\pi}$ = 1145.916 m log = 3.059 1526
	Eqs (15)		Eqs (16)	

D	R	log R	D	R	log R	D	R	log R
0°			1°			2°		
0′	Infinite	Infinite	0′	5729.65	3.758128	0′	2864.93	3.457115
1	343775.	5.536274	1	5635.72	3.750950	1	2841.26	3.453511
2	171887.	5.235244	2	5544.83	3.743888	2	2817.97	3.449937
3	114592.	5.059153	3	5456.82	3.736939	3	2795.06	3.446392
4	85943.7	4 934214	4	5371.56	3.730100	4	2772.53	3.442876
5	68754.9	4.837304	5	5288.92	3.723367	5	2750.35	3.439388
6	57295.8	4 758123	6	5208.79	3.716737	6	2728.52	3.435928
7	49110.7	4 691176	7	5131.05	3.710206	7	2707.04	3.432495
8	42971 8	4.633184	8	5055.59	3.703772	8	2685 89	3.429089
9	38197.2	4.582031	9	4982.33	3.697432	9	2665.08	3.425710
10	34377.5	4.536274	10	4911.15	3.691183	10	2644.58	3.422356
11	31252.3	4.494881	11	4841.98	3.685023	11	2624.39	3.419029
12	28647.8	4.457093	12	4774.74	3.678949	12	2604.51	3.415727
13	26444.2	4.422331	13	4709.33	3.672959	13	2584.93	3.412449
14	24555.4	4.390146	14	4645.69	3.667051	14	2565.65	3.409197
15	22918.3	4.360183	15	4583.75	3.661221	15	2546.64	3.405968
16	21485.9	4.332154	16	4523.44	3.655469	16	2527.92	3.402763
17	20222.1	4.305825	17	4464.70	3.649792	17	2509.47	3.399582
18	19098 6	4.281002	18	4407.46	3.644189	18	2491.29	3.396424
19	18093.4	4.257521	19	4351.67	3.638656	19	2473.37	3.393289
20	17188.8	4.235244	20	4297.28	3.633194	20	2455.70	3.390176
21	16370.2	4.214055	21	4244.23	3.627799	21	2438.29	3.387085
22	15626.1	4.193852	22	4192.47	3.622470	22	2421.12	3.384016
23	14946.7	4.174547	23	4141.96	3 617206	23	2404.19	3.380969
24	14324.0	4.156064	24	4092.66	3.612005	24	2387.50	3.377943
25	13751.0	4.138335	25	4044 51	3 606866	25	2371.04	3 374938
26	13222.1	4.121302	26	3997.49	3.601787	26	2354.80	3 371954
27	12732.4	4.104911	27	3951.54	3.596766	27	2338.78	3.368990
28	12277.7	4.089117	28	3906 64	3.591803	28	2322.98	3.366046
29	11854.3	4.073877	29	3862.74	3.586896	29	2307.39	3.363122
30	11459.2	4.059154	30	3819.83	3.582044	30	2292.01	3.360217
31	11089.6	4 044914	31	3777.85	3.577245	31	2276.84	3.357332
32	10743.0	4 031125	32	3736.79	3.572499	32	2261.86	3.354466
33	10417.5	4.017762	33	3696.61	3.567804	33	2247.08	3.351618
34	10111.1	4.004797	34	3657.29	3 563160	34	2232.49	3.348789
35	9822.18	3.992208	35	3618 80	3.558564	35	2218.09	3.345979
36	9549.34	3.979973	36	3581.10	3.554017	36	2203.87	3.343187
37	9291.25	3.968074	37	3544.19	3 549517	37	2189.84	3.340412
38	9046.75	3.956493	38	3508.02	3.545063	38	2175.98	3.337655
39	8814.78	3.945212	39	3472.59	3.540654	39	2162.30	3.334916
40	8594.42	3.934216	40	3437.87	3.536289	40	2148.79	3.332193
41	8384.80	3.923493	41	3403.83	3.531968	41	2135.44	3.329488
42	8185.16	3.913027	42	3370.46	3.527690	42	2122.26	3.326799
43	7994.81	3.902808	43	3337.74	3.523453	43	2109.24	3.324127
44	7813.11	3 892824	44	3305.65	3.519257	44	2096.39	3.321471
45	7639.49	3.883065	45	3274.17	3.515101	45	2083.68	3.318832
46	7473.42	3.873519	46	3243.29	3.510985	46	2071.13	3.316208
47	7314.41	3.864179	47	3212.98	3.506908	47	2058.73	3.313600
48	7162.03	3.855036	48	3183.23	3.502868	48	2046 48	3.311008
49	7015.87	3.846082	49	3154.03	3 498866	49	2034.37	3.308431
50	6875.55	3.837308	50	3125.36	3.494900	50	2022.41	3.305869
51	6740.74	3.828708	51	3097.20	3.490970	51	2010.59	3.303323
52	6611.12	3.820275	52	3069.55	3.487075	52	1998.90	3.300791
53	6486.38	3.812002	53	3042.39	3.483215	53	1987.35	3.298274
54	6366.26	3.803885	54	3015.71	3.479389	54	1975.93	3.295771
55	6250.51	3.795916	55	2989.48	3.475596	55	1964.64	3.293283
56	6138.90	3.788091	56	2963.72	3.471836	56	1953.48	3.290809
57	6031.20	3.780404	57	2938.39	3.468109	57	1942.44	3.288349
58	5927.22	3.772851	58	2913.49	3.464413	58	1931.53	3.285902
59	5826.76	3.765427	59	2889.01	3.460749	59	1920.75	3.283470
60	5729.65	3.758128	60	2864.93	3.457115	60	1910.08	3.281051

D	R	log R	D	R	log R	D	R	log R
3°			4°			5°		
0′	1910.08	3.281051	0′	1432.69	3.156151	0′	1146.28	3.059290
1	1899.53	3.278646	1	1426.74	3.154346	1	1142.47	3.057846
2	1889.09	3.276253	2	1420.85	3.152548	2	1138.69	3.056407
3	1878.77	3.273874	3	1415.01	3.150758	3	1134.94	3.054972
4	1868.56	3.271508	4	1409.21	3.148975	4	1131.21	3.053542
5	1858.47	3.269155	5	1403.46	3.147200	5	1127.50	3.052116
6	1848.48	3.266814	6	1397.76	3.145431	6	1123.82	3.050696
7	1838.59	3.264486	7	1392.10	3.143670	7	1120.16	3.049280
8	1828.82	3.262170	8	1386.49	3.141916	8	1116.52	3.047868
9	1819.14	3.259867	9	1380.92	3.140170	9	1112.91	3.046462
10	1809.57	3.257576	10	1375.40	3.138430	10	1109.33	3.045059
11	1800.10	3.255296	11	1369.92	3.136697	11	1105.76	3.043662
12	1790.73	3.253029	12	1364.49	3.134971	12	1102.22	3.042268
13	1781.45	3.250774	13	1359.10	3.133251	13	1098.70	3.040880
14	1772.27	3.248530	14	1353.75	3.131539	14	1095.20	3.039495
15	1763.18	3.246297	15	1348.45	3.129833	15	1091.73	3.038115
16	1754.19	3.244077	16	1343.18	3.128134	16	1088.28	3.036740
17	1745.29	3.241867	17	1337.96	3.126442	17	1084.85	3.035368
18	1736.48	3.239669	18	1332.77	3.124756	18	1081.44	3.034002
19	1727.75	3.237481	19	1327.63	3.123077	19	1078.05	3.032639
20	1719.12	3.235305	20	1322.53	3.121404	20	1074.68	3.031281
21	1710.57	3.233140	21	1317.46	3.119738	21	1071.34	3.029927
22	1702.10	3.230985	22	1312.43	3.118078	22	1068.01	3.028577
23	1693.72	3.228841	23	1307.45	3.116424	23	1064.71	3.027231
24	1685.42	3.226707	24	1302.50	3.114777	24	1061.43	3.025890
25	1677.20	3.224584	25	1297.58	3.113136	25	1058.16	3.024552
26	1669.06	3.222472	26	1292.71	3.111501	26	1054.92	3.023219
27	1661.00	3.220369	27	1287.87	3.109872	27	1051.70	3.021890
28	1653.01	3.218277	28	1283.07	3.108249	28	1048.49	3.020565
29	1645.11	3.216195	29	1278.30	3.106632	29	1045.31	3.019244
30	1637.28	3.214122	30	1273.57	3.105022	30	1042.14	3.017927
31	1629.52	3.212060	31	1268.87	3.103417	31	1039.00	3.016614
32	1621.84	3.210007	32	1264.21	3.101818	32	1035.87	3.015305
33	1614.22	3.207964	33	1259.58	3.100225	33	1032.76	3.013999
34	1606.68	3.205930	34	1254.98	3.098638	34	1029.67	3.012698
35	1599.21	3.203906	35	1250.42	3.097057	35	1026.60	3.011401
36	1591.81	3.201892	36	1245.89	3.095481	36	1023.55	3.010107
37	1584.48	3.199886	37	1241.40	3.093912	37	1020.51	3.008818
38	1577.21	3.197890	38	1236.94	3.092347	38	1017.49	3.007532
39	1570.01	3.195903	39	1232.51	3.090789	39	1014.50	3.006250
40	1562.88	3.193925	40	1228.11	3.089236	40	1011.51	3.004972
41	1555.81	3.191956	41	1223.74	3.087689	41	1008.55	3.003698
42	1548.80	3.189996	42	1219.40	3.086147	42	1005.60	3.002427
43	1541.86	3.188045	43	1215.09	3.084610	43	1002.67	3.001160
44	1534.98	3.186103	44	1210.82	3.083079	44	999.762	2.999897
45	1528.16	3.184169	45	1206.57	3.081553	45	996.867	2.998637
46	1521.40	3.182244	46	1202.36	3.080033	46	993.988	2.997381
47	1514.70	3.180327	47	1198.17	3.078518	47	991.126	2.996129
48	1508.06	3.178419	48	1194.01	3.077008	48	988.280	2.994880
49	1501.48	3.176519	49	1189.88	3.075504	49	985.451	2.993635
50	1494.95	3.174627	50	1185.78	3.074005	50	982.638	2.992393
51	1488.48	3.172744	51	1181.71	3.072511	51	979.840	2.991155
52	1482.07	3.170868	52	1177.66	3.071022	52	977.060	2.989921
53	1475.71	3.169001	53	1173.65	3.069538	53	974.294	2.988690
54	1469.41	3.167142	54	1169.66	3.068059	54	971.544	2.987463
55	1463.16	3.165291	55	1165.70	3.066585	55	968.810	2.986238
56	1456.96	3.163447	56	1161.76	3.065116	56	966.091	2.985018
57	1450.81	3.161612	57	1157.85	3.063653	57	963.387	2.983801
58	1444.72	3.159784	58	1153.97	3.062194	58	960.698	2.982587
59	1438.68	3.157963	59	1150.11	3.060740	59	958.025	2.981377
60	1432.69	3.156151	60	1146.28	3.059290	60	955.366	2.980170

D	R	log R	D	R	log R	D	R	log R
6°			7°			8°		
0′	955.366	2.980170	0′	819.020	2.913295	0′	716.779	2.855385
1	952.722	2.978966	1	817.077	2.912263	1	715.291	2.854483
2	950.093	2.977766	2	815.144	2.911234	2	713.810	2.853583
3	947.478	2.976569	3	813.219	2.910208	3	712.335	2.852684
4	944.877	2.975375	4	811.303	2.909183	4	710.865	2.851787
5	942.291	2.974185	5	809.397	2.908162	5	709.402	2.850892
6	939.719	2.972998	6	807.499	2.907142	6	707.945	2.849999
7	937.161	2.971814	7	805.611	2.906125	7	706.493	2.849108
8	934.616	2.970633	8	803.731	2.905111	8	705.048	2.848219
9	932.086	2.969456	9	801.860	2.904098	9	703.609	2.847331
10	929.569	2.968282	10	799.997	2.903089	10	702.175	2.846445
11	927.066	2.967111	11	798.144	2.902081	11	700.748	2.845562
12	924.576	2.965943	12	796.299	2.901076	12	699.326	2.844679
13	922.100	2.964778	13	794.462	2.900073	13	697.910	2.843799
14	919.637	2.963616	14	792.634	2.899073	14	696.499	2.842921
15	917.187	2.962458	15	790.814	2.898074	15	695.095	2.842044
16	914.750	2.961303	16	789.003	2.897078	16	693.696	2.841169
17	912.326	2.960150	17	787.200	2.896085	17	692.302	2.840296
18	909.915	2.959001	18	785.405	2.895094	18	690.914	2.839424
19	907.517	2.957855	19	783.618	2.894105	19	689.532	2.838555
20	905.131	2.956711	20	781.840	2.893118	20	688.156	2.837687
21	902.758	2.955571	21	780.069	2.892133	21	686.785	2.836821
22	900.397	2.954434	22	778.307	2.891151	22	685.419	2.835956
23	898.048	2.953300	23	776.552	2.890171	23	684.059	2.835093
24	895.712	2.952168	24	774.806	2.889193	24	682.704	2.834232
25	893.388	2.951040	25	773.067	2.888217	25	681.354	2.833373
26	891.076	2.949915	26	771.336	2.887244	26	680.010	2.832515
27	888.776	2.948792	27	769.613	2.886272	27	678.671	2.831660
28	886.488	2.947673	28	767.897	2.885303	28	677.338	2.830805
29	884.211	2.946556	29	766.190	2.884336	29	676.008	2.829953
30	881.946	2.945442	30	764.489	2.883371	30	674.686	2.829102
31	879.693	2.944331	31	762.797	2.882409	31	673.369	2.828253
32	877.451	2.943223	32	761.112	2.881448	32	672.056	2.827405
33	875.221	2.942118	33	759.434	2.880490	33	670.748	2.826560
34	873.002	2.941015	34	757.764	2.879534	34	669.446	2.825715
35	870.795	2.939916	35	756.101	2.878580	35	668.148	2.824873
36	868.598	2.938819	36	754.445	2.877627	36	666.856	2.824032
37	866.412	2.937725	37	752.796	2.876678	37	665.568	2.823193
38	864.238	2.936633	38	751.155	2.875730	38	664.286	2.822355
39	862.075	2.935545	39	749.521	2.874784	39	663.008	2.821519
40	859.922	2.934459	40	747.894	2.873840	40	661.736	2.820685
41	857.780	2.933376	41	746.274	2.872898	41	660.468	2.819852
42	855.648	2.932295	42	744.661	2.871959	42	659.205	2.819021
43	853.527	2.931218	43	743.055	2.871021	43	657.947	2.818191
44	851.417	2.930142	44	741.456	2.870086	44	656.694	2.817363
45	849.317	2.929070	45	739.864	2.869152	45	655.446	2.816537
46	847.228	2.928000	46	738.279	2.868221	46	654.202	2.815712
47	845.148	2.926933	47	736.701	2.867291	47	652.963	2.814889
48	843.080	2.925869	48	735.129	2.866363	48	651.729	2.814067
49	841.021	2.924807	49	733.564	2.865438	49	650.499	2.813247
50	838.972	2.923747	50	732.005	2.864514	50	649.274	2.812428
51	836.933	2.922691	51	730.454	2.863593	51	648.054	2.811611
52	834.904	2.921637	52	728.909	2.862673	52	646.838	2.810796
53	832.885	2.920585	53	727.370	2.861755	53	645.627	2.809982
54	830.876	2.919536	54	725.838	2.860840	54	644.420	2.809169
55	828.876	2.918489	55	724.312	2.859926	55	643.218	2.808358
56	826.886	2.917446	56	722.793	2.859014	56	642.021	2.807549
57	824.905	2.916404	57	721.280	2.858104	57	640.828	2.806741
58	822.934	2.915365	58	719.774	2.857196	58	639.639	2.805935
59	820.973	2.914329	59	718.273	2.856290	59	638.455	2.805130
60	819.020	2.913295	60	716.779	2.855385	60	637.275	2.804327

D	R	log R
9°		
0′	637.275	2.804327
1	636.099	2.803525
2	634.928	2.802724
3	633.761	2.801926
4	632.599	2.801128
5	631.440	2.800332
6	630.286	2.799538
7	629.136	2.798745
8	627 991	2.797953
9	626.849	2.797163
10	625.712	2.796374
11	624.579	2.795587
12	623.450	2.794801
13	622.325	2.794017
14	621.203	2.793234
15	620.087	2.792453
16	618.974	2.791673
17	617.865	2.790894
18	616.760	2.790117
19	615.660	2.789341
20	614.563	2.788566
21	613.470	2.787793
22	612.380	2.787021
23	611.295	2.786251
24	610.214	2.785482
25	609.136	2.784714
26	608.062	2.783948
27	606.992	2.783183
28	605.926	2.782420
29	604.864	2.781657
30	603.805	2.780897
31	602.750	2.780137
32	601.698	2.779379
33	600.651	2.778622
34	599.607	2.777867
35	598.567	2.777112
36	597.530	2.776360
37	596.497	2.775608
38	595 467	2.774858
39	594 441	2.774109
40	593.419	2.773361
41	592.400	2.772615
42	591.384	2.771870
43	590.372	2.771126
44	589.364	2.770383
45	588.359	2.769642
46	587.357	2.768902
47	586.359	2.768164
48	585 364	2.767426
49	584.373	2.766690
50	583.385	2.765955
51	582.400	2.765221
52	581.419	2.764489
53	580.441	2.763758
54	579.466	2.763028
55	578.494	2.762299
56	577.526	2.761572
57	576.561	2.760845
58	575.599	2.760120
59	574.641	2.759397
60	573.686	2.758674

D	R	log R
10°		
0′	573.686	2.758674
2	571.784	2.757232
4	569.896	2.755796
6	568.020	2.754364
8	566.156	2.752937
10	564.305	2.751514
12	562.466	2.750096
14	560.638	2.748683
16	558.823	2.747274
18	557.019	2.745870
20	555.227	2.744471
22	553.447	2.743076
24	551.678	2.741686
26	549.920	2.740300
28	548.174	2.738918
30	546.438	2.737541
32	544.714	2.736169
34	543.001	2.734800
36	541.298	2.733436
38	539.606	2.732077
40	537.924	2.730721
42	536.253	2.729370
44	534.593	2.728023
46	532.943	2.726681
48	531.303	2.725342
50	529.673	2.724008
52	528.053	2.722677
54	526.443	2.721351
56	524.843	2.720029
58	523.252	2.718711
11°		
0′	521.671	2.717397
2	520.100	2.716087
4	518.539	2.714781
6	516.986	2.713479
8	515.443	2.712181
10	513.909	2.710887
12	512.385	2.709596
14	510.869	2.708310
16	509.363	2.707027
18	507.865	2.705748
20	506.376	2.704473
22	504.896	2.703202
24	503.425	2.701934
26	501.962	2.700671
28	500.507	2.699410
30	499.061	2.698154
32	497.624	2.696901
34	496.195	2.695652
36	494.774	2.694407
38	493.361	2.693165
40	491.956	2.691926
42	490.559	2.690692
44	489.171	2.689460
46	487.790	2.688233
48	486.417	2.687008
50	485.051	2.685788
52	483.694	2.684570
54	482.344	2.683357
56	481.001	2.682146
58	479.666	2.680939
60	478.339	2.679735

D	R	log R
12°		
0′	478.339	2.679735
2	477.018	2.678535
4	475.705	2.677338
6	474.400	2.676145
8	473.102	2.674954
10	471.810	2.673767
12	470.526	2.672584
14	469.249	2.671403
16	467.978	2.670226
18	466.715	2.669052
20	465.459	2.667881
22	464.209	2.666713
24	462.966	2.665549
26	461.729	2.664387
28	460.500	2.663229
30	459.276	2.662074
32	458.060	2.660922
34	456.850	2.659773
36	455.646	2.658628
38	454.449	2.657485
40	453.259	2.656345
42	452.073	2.655208
44	450.894	2 654075
46	449.722	2.652944
48	448.556	2.651816
50	447.395	2.650691
52	446.241	2.649570
54	445.093	2.648451
56	443.951	2.647335
58	442.814	2.646221
13°		
0′	441.684	2.645111
2	440.559	2.644004
4	439.440	2.642899
6	438.326	2.641798
8	437.219	2.640699
10	436.117	2.639603
12	435.020	2.638510
14	433.929	2.637419
16	432.844	2.636331
18	431.764	2.635246
20	430.690	2.634164
22	429.620	2.633085
24	428.557	2.632008
26	427.498	2.630934
28	426.445	2.629863
30	425.396	2.628794
32	424.354	2.627728
34	423.316	2.626665
36	422.283	2.625604
38	421.256	2.624546
40	420.233	2.623490
42	419.215	2.622437
44	418.203	2.621387
46	417.195	2.620339
48	416.192	2.619294
50	415.194	2.618251
52	414.201	2.617211
54	413.212	2.616173
56	412.229	2.615138
58	411.250	2.614106
60	410.275	2.613075

D	R	log R	D	R	log R	D	R	log R
14°			16°			18°		
0′	410.275	2.613075	0′	359.265	2.555415	0′	319.623	2.504638
2	409.306	2.612048	2	358.523	2.554517	2	319.037	2.503841
4	408.341	2.611023	4	357.784	2.553621	4	318.453	2.503045
6	407.380	2.610000	6	357.048	2.552727	6	317.871	2.502251
8	406.424	2.608980	8	356.315	2.551834	8	317.292	2.501459
10	405.473	2.607962	10	355.585	2.550944	10	316.715	2.500668
12	404.526	2.606946	12	354.859	2.550055	12	316.139	2.499879
14	403.583	2.605933	14	354.135	2.549169	14	315.566	2.499091
16	402.645	2.604923	16	353.414	2.548284	16	314.993	2.498304
18	401.712	2.603914	18	352.696	2.547401	18	314.426	2.497519
20	400.782	2.602908	20	351.981	2.546519	20	313.860	2.496736
22	399.857	2.601905	22	351.269	2.545640	22	313.295	2.495953
24	398.937	2.600904	24	350.560	2.544762	24	312.732	2.495173
26	398.020	2.599905	26	349.854	2.543887	26	312.172	2.494393
28	397.108	2.598908	28	349.150	2.543013	28	311.613	2.493616
30	396.200	2.597914	30	348.450	2.542140	30	311.056	2.492839
32	395.296	2.596922	32	347.752	2.541270	32	310.502	2.492064
34	394.396	2.595933	34	347.057	2.540401	34	309.949	2.491291
36	393.501	2.594945	36	346.365	2.539535	36	309.399	2.490518
38	392.609	2.593960	38	345.676	2.538670	38	308.850	2.489748
40	391.722	2.592978	40	344.990	2.537806	40	308.303	2.488978
42	390.838	2.591997	42	344.306	2.536945	42	307.759	2.488210
44	389.959	2.591019	44	343.625	2.536085	44	307.216	2.487444
46	389.084	2.590043	46	342.947	2.535227	46	306.675	2.486679
48	388.212	2.589069	48	342.271	2.534370	48	306.136	2.485915
50	387.345	2.588097	50	341.598	2.533516	50	305.599	2.485152
52	386.481	2.587128	52	340.928	2.532663	52	305.064	2.484391
54	385.621	2.586161	54	340.260	2.531811	54	304.531	2.483632
56	384.765	2.585196	56	339.595	2.530962	56	304.000	2.482873
58	383.913	2.584233	58	338.933	2.530114	58	303.470	2.482116
15°			17°			19°		
0′	383.065	2.583272	0′	338.273	2.529268	0′	302.943	2.481361
2	382.220	2.582314	2	337.616	2.528424	2	302.417	2.480607
4	381.380	2.581358	4	336.962	2.527581	4	301.893	2.479854
6	380.543	2.580403	6	336.310	2.526740	6	301.371	2.479102
8	379.709	2.579451	8	335.660	2.525900	8	300.851	2.478352
10	378.880	2.578501	10	335.013	2.525062	10	300.333	2.477603
12	378.054	2.577553	12	334.369	2.524226	12	299.816	2.476855
14	377.231	2.576608	14	333.727	2.523392	14	299.302	2.476109
16	376.412	2.575664	16	333.088	2.522559	16	298.789	2.475364
18	375.597	2.574722	18	332.451	2.521728	18	298.278	2.474621
20	374.786	2.573783	20	331.816	2.520898	20	297.768	2.473878
22	373.977	2.572845	22	331.184	2.520070	22	297.260	2.473137
24	373.173	2.571910	24	330.555	2.519244	24	296.755	2.472398
26	372.372	2.570977	26	329.928	2.518419	26	296.250	2.471659
28	371.574	2.570045	28	329.303	2.517596	28	295.748	2.470922
30	370.780	2.569116	30	328.681	2.516774	30	295.247	2.470186
32	369.989	2.568189	32	328.061	2.515954	32	294.748	2.469452
34	369.202	2.567264	34	327.443	2.515136	34	294.251	2.468718
36	368.418	2.566340	36	326.828	2.514319	36	293.756	2.467986
38	367.637	2.565419	38	326.215	2.513504	38	293.262	2.467256
40	366.859	2.564500	40	325.604	2.512690	40	292.770	2.466526
42	366.085	2.563582	42	324.996	2.511878	42	292.279	2.465798
44	365.315	2.562667	44	324.390	2.511067	44	291.790	2.465071
46	364.547	2.561754	46	323.786	2.510258	46	291.303	2.464345
48	363.783	2.560843	48	323.184	2.509451	48	290.818	2.463621
50	363.022	2.559933	50	322.585	2.508645	50	290.334	2.462897
52	362.264	2.559026	52	321.989	2.507840	52	289.851	2.462175
54	361.510	2.558120	54	321.394	2.507037	54	289.371	2.461455
56	360.758	2.557216	56	320.801	2.506236	56	288.892	2.460735
58	360.010	2.556315	58	320.211	2.505436	58	288.414	2.460017
60	359.265	2.555415	60	319.623	2.504638	60	287.939	2.459300

D	R	log R
20° 0′	287.939	2.459300
10	285.583	2.455733
20	283.267	2.452195
30	280.988	2.448688
40	278.746	2.445209
50	276.541	2.441759
21° 0′	274.370	2.438337
10	272.234	2.434943
20	270.132	2.431576
30	268.062	2.428235
40	266.024	2.424921
50	264.018	2.421633
22° 0′	262.042	2.418371
10	260.098	2.415134
20	258.180	2.411922
30	256.292	2.408734
40	254.431	2.405571
50	252.599	2.402431
23° 0′	250.793	2.399315
10	249.013	2.396222
20	247.258	2.393151
30	245.529	2.390103
40	243.825	2.387077
50	242.144	2.384074
24° 0′	240.487	2.381091
10	238.853	2.378130
20	237.241	2.375190
30	235.652	2.372270
40	234.084	2.369371
50	232.537	2.366492
25° 0′	231.011	2.363633
10	229.506	2.360794
20	228.020	2.357974
30	226.555	2.355173
40	225.108	2.352391
50	223.680	2.349627
26° 0′	222.271	2.346882
10	220.879	2.344155
20	219.506	2.341446
30	218.150	2.338755
40	216.811	2.336081
50	215.489	2.333424
27° 0′	214.183	2.330785
10	212.893	2.328162
20	211.620	2.325556
30	210.362	2.322967
40	209.119	2.320393
50	207.891	2.317836
28° 0′	206.678	2.315295
10	205.480	2.312769
20	204.296	2.310259
30	203.125	2.307764
40	201.969	2.305285
50	200.826	2.302820
29° 0′	199.696	2.300370
10	198.580	2.297935
20	197.476	2.295515
30	196.385	2.293108
40	195.306	2.290716
50	194.240	2.288338

D	R	log R
30° 0′	193.185	2.285974
20	191.111	2.281286
40	189.083	2.276652
31° 0′	187.099	2.272071
20	185.158	2.267541
40	183.258	2.263062
32° 0′	181.398	2.258632
20	179.577	2.254250
40	177.794	2.249916
33° 0′	176.047	2.245628
20	174.336	2.241386
40	172.659	2.237188
34° 0′	171.015	2.233035
20	169.404	2.228924
40	167.825	2.224855
35° 0′	166.275	2.220828
20	164.756	2.216842
40	163.266	2.212895
36° 0′	161.803	2.208988
20	160.368	2.205119
40	158.960	2.201288
37° 0′	157.577	2.197494
20	156.220	2.193736
40	154.887	2.190014
38° 0′	153.578	2.186328
30	151.657	2.180863
39° 0′	149.787	2.175475
30	147.965	2.170160
40° 0′	146.190	2.164918
30	144.460	2.159747
41° 0′	142.773	2.154645
30	141.127	2.149610
42° 0′	139.521	2.144641
30	137.955	2.139736
43° 0′	136.425	2.134895
30	134.932	2.130114
44° 0′	133.473	2.125395
30	132.049	2.120734
45° 0′	130.656	2.116130
30	129.296	2.111584
46° 0′	127.965	2.107092
30	126.664	2.102655
47° 0′	125.392	2.098270
30	124.148	2.093938
48° 0′	122.930	2.089657
30	121.738	2.085425
49° 0′	120.571	2.081243
30	119.429	2.077109
50° 0′	118.310	2.073022

General Curv Equations

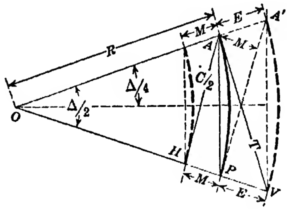
For symbols, see ¶¶ 3 and 9, pp 875, 876.

30. Fig 10. To avoid undue repetition, one equation is here frequently intended to serv, by inversion and substitution, for all the factors involvd and for others. Thus, from

$M = R \text{ vers}(\Delta/2)$, we have, also:—

$$R = M/\text{vers}(\Delta/2)\,; \quad \text{vers}\frac{\Delta}{2} = \frac{M}{R} = \frac{E\cos(\Delta/2)}{T\cot(\Delta/2)}\,;$$

etc, etc. See ¶ 69.



$$\frac{\Delta}{2} = HAV; \ \frac{\Delta}{4} = HAP = PAV = AVA'.$$

Fig. 10.

Angular Functions

31. Fig 10. **Functions of $\Delta/2$.**

$$\text{Sin}\frac{\Delta}{2} = \frac{C}{2R} = \frac{T}{R+E} = \frac{E+M}{T} \quad \text{.......... (17)}$$

$$\text{Cos}\frac{\Delta}{2} = \frac{R-M}{R} = \frac{M}{E} = \frac{R}{R+E}$$

$$= \frac{T^2-E^2}{T^2+E^2} = \frac{C^2-4M^2}{C^2+4M^2} \quad \text{.......... (18)}$$

$$\text{Tan}\frac{\Delta}{2} = \frac{T}{R} = \frac{E+M}{C/2} = \frac{C/2}{R-M} \quad \text{.......... (19)}$$

$$\text{Vers}\frac{\Delta}{2} = 1 - \cos\frac{\Delta}{2} = \frac{M}{R} = \frac{E}{R+E}$$

$$= \sin\frac{\Delta}{2} \times \tan\frac{\Delta}{4} = 2\sin^2\frac{\Delta}{4} \quad \text{.......... (20)}$$

$$\text{Exsec}\frac{\Delta}{2} = \frac{E}{R} = \frac{E-M}{M}\,; \quad \text{.......... (21)}$$

32. Fig 10. **Functions of $\Delta/4$.**

$$\sin\frac{\Delta}{4} = \frac{PH}{AP} = \frac{M}{\sqrt{M^2 + (C/2)^2}} = \frac{\sqrt{M^2 + (C/2)^2}}{2\,R};$$

$$\sin^2\frac{\Delta}{4} = \frac{AP}{2R}\,\frac{PH}{AP} = \frac{M}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

$$\tan\frac{\Delta}{4} = \frac{E}{T} = \frac{2M}{C} = \sqrt{\frac{2T - C}{2T + C}} = \sqrt{\frac{E - M}{E + M}} = \frac{\operatorname{vers}(\Delta/2)}{\sin(\Delta/2)}$$

which may be shown to be

$$= \frac{T}{2R + E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Linear functions

33. Fig 10. Relations between R, E and M.

$$R + E = \sqrt{T^2 + R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

$$R - M = \sqrt{R^2 - (C/2)^2} = \sqrt{(R + C/2)\ (R - C/2)} \ldots (25)$$

$$E + M = \sqrt{T^2 - (C/2)^2} = \sqrt{(T + C/2)\ (T - C/2)} \ldots (26)$$

34. Fig 11. Equations for R, C, T, E, Y, BF, M and M_1.
D = sharpness, *in degrees* unless otherwise stated.

$$R^* = \frac{T}{\tan(\Delta/2)} = \frac{C}{2\sin(\Delta/2)} = \frac{M}{\operatorname{vers}(\Delta/2)} = \frac{E}{\operatorname{exsec}(\Delta/2)}$$

$$= \frac{T^2 - E^2}{2E} = \frac{(C/2)^2 + M^2}{2M} = \frac{EM}{E - M};$$

$= \text{approx } R_1/D = (R \text{ for } 1° \text{ curv})/D = \text{approx } 5729.65 \text{ ft}/D$
$= \text{approx } C^2/8M$ (see table, p 893) ……………………… (27)

$$C = 2R\sin(\Delta/2) = 2T\cos(\Delta/2)$$

$$= 2M\cot(\Delta/4) = 2\sqrt{M\,(2R - M)}$$

$$= 100\,\frac{\sin(n\,D/2)^*}{\sin(D/2)}$$

$$= 2E\,\frac{\sin(\Delta/2)}{\operatorname{exsec}(\Delta/2)} = \text{approx } \sqrt{8\,R\,M} \text{ (see table, p 893)} \ldots (28)$$

$$T = R\tan(\Delta/2) = \frac{C}{2\cos(\Delta/2)}$$

$$= M\,\frac{\tan(\Delta/2)}{\operatorname{vers}(\Delta/2)} = E\cot(\Delta/4)$$

$$= (R + E)\sin(\Delta/2) = \sqrt{E\,(2R + E)}$$

$$= \sqrt{(C/2)^2 + (M + E)^2} = \frac{M + E}{\sin(\Delta/2)} \quad \ldots\ldots\ldots\ldots (29)$$

$$E = R \operatorname{exsec}(\Delta/2) = T \tan(\Delta/4) = M \sec(\Delta/2)$$
$$= R \tan(\Delta/2) \tan(\Delta/4) = C \frac{\operatorname{exsec}(\Delta/2)}{2 \sin(\Delta/2)} = \frac{R\ M}{R - M}$$
$= \text{approx } M$ (see table, p 893) .... (30)

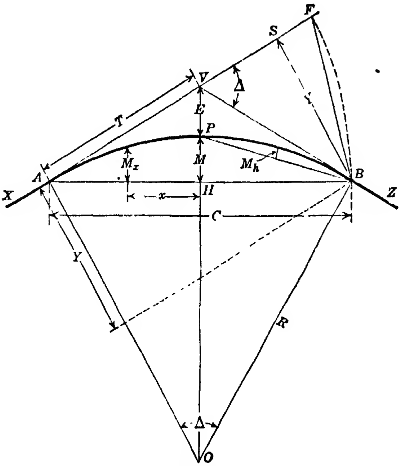


Fig. 11.

$$Y = R \operatorname{vers} \Delta = C \sin(\Delta/2)$$
$$= 2\,R \sin^2(\Delta/2) = C^2/2\,R = T \sin \Delta$$
$$= E \frac{\operatorname{vers} \Delta}{\operatorname{exsec}(\Delta/2)} = M \frac{\operatorname{vers} \Delta}{\operatorname{vers}(\Delta/2)}$$
$= \text{approx}$ (see table, p 893) $4\,M = \frac{7}{8} n^2 D^\circ$ * .... (31)

$BF = 2\,C \sin(\Delta/4)$ .... (31a)

$$M = R \operatorname{vers}(\Delta/2) = 2\,R \sin^2(\Delta/4) = \frac{PB^2}{2\,R} = T \frac{\operatorname{vers}(\Delta/2)}{\tan(\Delta/2)}$$
$$= (C/2) \tan(\Delta/4) = E \cos(\Delta/2) = R\,E/(R + E)$$
$$= R - \sqrt{R^2 - (C/2)^2} = R - \sqrt{(R + C/2)\ (R - C/2)}$$
$$= R \sin(\Delta/2) \times \tan(\Delta/4);$$
$= \text{approx}$ (see table, p 893) $4\,M_h = Y/4 = B\,F/4$
$= \frac{M + E}{2} = E = \frac{C^2}{8\,R} = \frac{7}{32} n^2 D^\circ$* .... (32)

$$M_h = R \operatorname{vers}(\Delta/4) = M \frac{1 - \cos(\Delta/4)}{1 - \cos(\Delta/2)} = M \frac{\sin^2(\Delta/8)}{\sin^2(\Delta/4)}$$

$$= \text{approx } \frac{M}{4} \text{ (see table, below)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

For side ordinate, M_x, see ¶ 35.

Multipliers for Converting Approximate Values from Equations Above **to Correct Values.** The following table of multipliers indicates the degrees of approximation of our several approximate equations in ¶ 34, and enables us to obtain the correct values from them.

Example. Given $\Delta = 30°$; required the middle ordinate, M, for the entire curv. Here eq (32) gives, approx, $M = BF/4$; but, in the following table, in the line beginning "(32) $M = BF/4$", and in the colum headed "$\Delta = 30°$", we find the correctiv multiplier, 1.0088, which gives the true value, $M = 1.0088 \times BF/4 = 0.2522\,BF$.

Eq No.	Equation.	$\Delta =$ 1°	10°	20°	30°	60°	90°
		To obtain correct values of the equations, multiply the approx value by the proper coefficient given below					
(32)	$M = 4\,M_h$	1.0000	0.9995	0 9981	0.9957	0.9830	0.9619
(33)	$M_h = M/4$	1.0000	1.0005	1.0020	1.0043	1.0173	1.0396
(31)	$Y = 4\,M$	1.0000	0.9980	0.9925	0.9831	0.9328	0.8535
(32)	$M = Y/4$	1.0000	1.0020	1.0076	1.0172	1.0720	1.1716
(32)	$M = BF/4$	1.0000	1.0008	1.0040	1.0088	1.0352	1.0824
(32)	$M = (M+E)/2$	1.0000	0.9980	0.9924	0.9826	0.9282	0.8284
(30)	$E = M$	1.0000	1.0038	1.0154	1.0353	1.1547	1.4142
(32)	$M = E$	1.0000	0.9962	0.9848	0.9659	0.8660	0.7071
(27)	$R = C^2/8\,M$	1.0000	1.0019	1.0076	1.0173	1.0718	1.1716
(28)	$C = \sqrt{8\,R\,M}$	1.0000	0.9981	0.9925	0.9830	0.9330	0.8535
(32)	$M = C^2/8\,R$	1.0000	1.0019	1.0076	1.0173	1.0718	1.1716
(32)	$M = \frac{7}{32}\,n^2D$*						
	for $D = 1°$	0.9975	0.9966	0.9947	0.9915	0.9746	0.9472
	" $D = 10°$	0.9984	0.9979	0.9961	0.9929	0.9760	0.9481
	" $D = 20°$	1.0025	1.0016	0.9998	0.9966	0.9797	0.9518
(31)	$Y = \frac{7}{8}\,n^2D$*						
	for $D = 1°$	0.9974	0.9949	0.9873	0.9747	0.9095	0.8085
	" $D = 10°$	0.9986	0.9961	0.9885	0.9760	0.9106	0.8095
	" $D = 20°$	1.0024	0.9999	0.9922	0.9798	0.9141	0.8126
(27)	$R = R_1/D°$						
When		$D=5°$	$D=10°$	$D=15°$	$D=20°$	$D=30°$	$D=40°$
Mult R_1/D by		1.0003	1.0013	1.0028	1.0051	1.0115	1.0206

*n = number of unit chains in the curv.

Long Chords, C, in ft, required to subtend from two to eight 100-ft chains, for different sharpnesses, *D*.

$$C = 2R \sin(\Delta/2)$$

For table of long-chords to a 1° curv, for diff sweeps, Δ, see ¶ 40*a*.

	2 stas	3 stas	4 stas	5 stas	6 stas	7 stas	8 stas
D ° '	C ft	C ft	C ft	C ft	C ft	C ft	C ft
0 10	200.00	300.00	400.00	500.00	599.99	699.99	799.98
20	200.00	300.00	399.99	499.98	599.97	699.95	799.93
30	200.00	299.99	399.98	499.96	599.93	699.89	799.84
40	200.00	299.99	399.97	499.93	599.88	699.81	799.72
50	200.00	299.98	399.95	499.89	599.82	699.70	799.56
1 0	199.99	299.97	399.92	499.85	599.73	699.57	799.36
10	199.99	299.96	399.90	499.79	599.64	699.42	799.13
20	199.99	299.95	399.87	499.73	599.53	699.24	798.86
30	199.98	299.93	399.83	499.66	599.40	699.04	798.56
40	199.98	299.92	399.79	499.58	599.26	698.82	798.22
50	199.97	299.90	399.74	499.49	599.11	698.57	797.85
2 0	199.97	299.88	399.70	499.39	598.93	698.30	797.44
10	199.96	299.86	399.64	499.29	598.75	698.00	797.00
20	199.96	299.83	399.59	499.17	598.55	697.68	796.52
30	199.95	299.81	399.52	499.05	598.34	697.34	796.01
40	199.95	299.78	399.46	498.92	598.11	696.97	795.46
50	199.94	299.76	399.39	498.78	597.86	696.58	794.87
3 0	199.93	299.73	399.32	498.63	597.60	696.17	794.26
10	199.92	299.70	399.24	498.47	597.33	695.73	793.60
20	199.92	299.66	399.15	498.31	597.04	695.27	792.91
30	199.91	299.63	399.07	498.14	596.74	694.79	792.19
40	199.90	299.59	398.98	497.96	596.42	694.28	791.43
50	199.89	299.55	398.88	497.77	596.09	693.75	790.63
4 0	199.88	299.51	398.78	497.57	595.74	693.20	789.80
10	199.87	299.47	398.68	497.36	595.38	692.62	788.94
20	199.86	299.43	398.57	497.15	595.01	692.02	788.04
30	199.85	299.38	398.46	496.92	594.62	691.40	787.11
40	199.83	299.34	398.34	496.69	594.21	690.75	786.14
50	199.82	299.29	398.22	496.45	593.79	690.08	785.14
5 0	199.81	299.24	398.10	496.20	593.36	689.39	784.10
10	199.80	299.19	397.97	495.94	592.91	688.67	783.03
20	199.78	299.13	397.84	495.68	592.45	687.93	781.93
30	199.77	299.08	397.70	495.41	591.97	687.17	780.79
40	199.76	299.02	397.56	495.12	591.48	686.38	779.61
50	199.74	298.96	397.41	494.83	590.97	685.58	778.41
6 0	199.73	298.90	397.26	494.53	590.45	684.75	777.17
10	199.71	298.84	397.11	494.23	589.91	683.89	775.89
20	199.70	298.78	396.95	493.91	589.36	683.02	774.58
30	199.68	298.71	396.79	493.59	588.80	682.12	773.24
40	199.66	298.65	396.62	493.26	588.22	681.20	771.86
50	199.64	298.58	396.45	492.92	587.63	680.25	770.46
7 0	199.63	298.51	396.28	492.57	587.02	679.29	769.01
10	199.61	298.44	396.10	492.21	586.40	678.30	767.54
20	199.59	298.36	395.92	491.85	585.77	677.28	766.03
30	199.57	298.29	395.73	491.47	585.12	676.25	764.49
40	199.55	298.21	395.54	491.09	584.45	675.19	762.92
50	199.53	298.13	395.34	490.70	583.77	674.12	761.31

Long Chords, C, in ft, required to subtend from two to eight 100-ft chains, for different sharpnesses, D.

$$C = 2R\sin(\Delta/2)$$

For table of long-chords to a 1° curv, for diff sweeps, Δ, see ¶ 40a.

(Continued)

D ° ′	2 stas C ft	3 stas C ft	4 stas C ft	5 stas C ft	6 stas C ft	7 stas C ft	8 stas C ft
8 0	199 51	298.05	395.14	490.31	583.08	673.02	759.67
10	199 49	297.97	394 94	489.90	582.38	671.89	758.00
20	199.47	297.89	394.73	489.49	581.65	670.75	756.30
30	199.45	297.80	394.52	489.06	580.92	669.58	754.56
40	199.43	297.72	394.30	488.63	580.17	668.39	752.79
50	199.41	297.63	394.08	488.20	579.41	667.18	750.99
9 0	199.38	297.54	393.86	487.75	578.63	665.95	749.16
10	199.36	297.45	393 63	487.29	577.84	664.70	747.30
20	199.34	297.35	393.40	486.83	577.04	663.42	745.40
30	199.31	297.26	393 16	486.36	576.22	662.12	743.48
40	199.29	297.16	392.92	485.88	575.39	660.81	741.52
50	199.26	297.06	392.67	485.40	574.55	659.47	739.54
10 0	199.24	296.96	392.42	484.90	573.69	658.11	737.52
10	199.21	296.86	392 17	484.40	572.81	656.72	735.47
20	199.19	296.76	391.91	483.89	571.93	655.32	733.39
30	199.16	296.65	391 65	483.37	571.03	653.90	731.28
40	199 13	296.54	391.39	482.84	570.11	652.45	729.14
50	199.11	296.44	391.12	482.31	569.19	650.98	726.97
11 0	199.08	296.33	390.84	481.76	568.25	649.50	724.77
10	199.05	296.21	390.57	481.21	567.29	647.99	722.54
20	199.02	296.10	390.28	480.65	566.32	646.46	720.28
30	198.99	295.99	390.00	480.09	565.34	644.91	717.99
40	198 96	295.87	389.71	479.51	564.35	643.34	715.67
50	198.94	295.75	389.41	478.93	563.34	641.75	713.33
12 0	198.90	295.63	389.12	478.34	562.32	640.14	710.95
10	198.87	295.51	388.81	477.74	561.29	638.51	708.55
20	198.84	295.38	388.51	477.14	560.24	636.86	706.11
30	198.81	295.26	388.20	476.52	559.18	635.19	703.65
40	198.78	295.13	387.88	475.90	558.11	633.50	701.16
50	198.75	295.00	387.57	475.27	557.02	631.79	698.65
13 0	198.71	294.87	387.24	474.63	555.92	630.06	696.10
10	198.68	294.74	386.92	473.99	554.81	628.31	693.53
20	198.65	294.61	386.59	473.34	553.68	626.54	690.93
30	198.61	294.47	386.25	472 68	552.55	624.76	688.31
40	198.58	294.34	385.91	472.01	551.40	622.95	685.65
50	198.54	294.20	385.57	471.33	550.23	621.12	682.97
14 0	198.51	294.06	385.23	470.65	549.06	619.28	680.27
10	198.47	293.92	384.88	469.96	547.87	617.41	677.54
20	198.44	293.77	384.52	469.26	546.67	615.53	674.78
30	198.40	293.63	384.16	468.55	545.45	613.63	671.99
40	198.36	293.48	383.80	467.84	544.23	611.71	669.18
50	198.33	293.34	383.44	467.12	542.99	609.77	666.35
15 0	198.29	293.19	383.07	466.39	541.74	607.81	663.49
10	198.25	293.03	382.69	465.65	540.47	605.84	660.60
20	198.21	292.88	382.31	464.91	539.20	603.84	657.69
30	198.17	292.73	381.93	464.16	537.91	601.83	654.76
40	198.13	292.57	381.55	463.40	536.61	599.80	651.80
50	198.09	292.41	381.16	462.64	535.30	597.75	648.82

Long Chords, C, in ft, required to subtend from two to eight 100-ft chains, for different sharpnesses, *D*.

$$C = 2\,R\,\sin(\Delta/2)$$

For table of long-chords to a 1° curv, for diff sweeps, Δ, see ¶ 40*a*.

(Concluded)

D ° ′	2 stas C ft	3 stas C ft	4 stas C ft	5 stas C ft	6 stas C ft	7 stas C ft	8 stas C ft
16 0	198.05	292.25	380.76	461.86	533.97	595.69	645.81
10	198.01	292.09	380.37	461.08	532.64	593.61	642.78
20	197.97	291.93	379.96	460.29	531.29	591.51	639.73
30	197.93	291.76	379.56	459.50	529.93	589.39	636.65
40	197.89	291.60	379.15	458.70	528.56	587.25	633.55
50	197.85	291.43	378.74	457.89	527.17	585.10	630.43
17 0	197.80	291.26	378.32	457.07	525.78	582.93	627.28
10	197.76	291.09	377.90	456.24	524.37	580.75	624.12
20	197.72	290.92	377.48	455.41	522.95	578.55	620.93
30	197.67	290.74	377.05	454.57	521.52	576.33	617.72
40	197.63	290.57	376.62	453.73	520.08	574.09	614.49
50	197.58	290.39	376.18	452.88	518.63	571.84	611.23
18 0	197.54	290.21	375.74	452.02	517.16	569.57	607.96
10	197.49	290.03	375.30	451.15	515.69	567.29	604.66
20	197.45	289.85	374.85	450.37	514.20	564.99	601.35
30	197.40	289.67	374.40	449.39	512.70	562.67	598.01
40	197.35	289.48	373.94	448.50	511.19	560.34	594.66
50	197.31	289.29	373.48	447.61	509.67	558.00	591.28
19 0	197.26	289.10	373.02	446.71	508.14	555.63	587.89
10	197.21	288.91	372.55	445.80	506.60	553.26	584.48
20	197.16	288.72	372.08	444.88	505.04	550.86	581.04
30	197.11	288.53	371.61	443.96	503.48	548.46	577.59
40	197.06	288.33	371.13	443.03	501.91	546.04	574.12
50	197.01	288.14	370.65	442.09	500.32	543.60	570.63
20 0	196.96	287.94	370.17	441.15	498.72	541.15	567.13

35. Side ordinate, M_x. Fig 12.

$$M_x = \sqrt{R^2 - x^2} + M - R = \sqrt{R^2 - x^2} - R\cos(\Delta/2) \quad \ldots (34)$$

$$= \text{approx } a\,b/2\,R \text{ (see table, below)} \quad \ldots (35)$$

where a and b are the two segments of the chord, C,

$$= \text{approx } 4\,M\,a\,b/C^2 \text{ (see table, below)} \quad \ldots (36)$$

When $C = 100$ ft ($= c$; $\Delta = D$), eq (36) becomes:—

$$m_x = \text{approx } \frac{m\,a\,b}{2500} = \text{approx } \frac{m}{0.25} \cdot \frac{a}{100} \cdot \frac{b}{100} \quad \ldots (37)$$

or, since in that case, $B\,F$ (= "tangntl dist") $= 200\sin(D/4)$ = approx $4\,m$, we have, for any curv and for any value of C:—

$$M_x = \text{approx } B\,F \cdot \frac{a}{100} \cdot \frac{b}{100} = \frac{B\,F \cdot a \cdot b}{10{,}000} \quad \ldots (38)$$

See table p 893, using coeffs for $R = C^2/8\,M$. Thus, for $\Delta = 60°$, $M_x = 1.0718\ B\ F \cdot a \cdot b/10{,}000$.

$$M_x = \text{approx } D \times M_1 \text{ (see table, below)} \quad \ldots (39)$$

where D = the sharpness of the given curv, and M_1 = the side ord for a 1° curv, at the given dist, x, from its mid ord, M, on an equal chord, C.

Coefficients for the foregoing approximate equations for side ordinates, M_x.

The following table of multipliers indicates the degree of approximation of our several approximate equations in ¶ 35, and enables us to obtain the correct values from them.

Example. Given $\Delta = 20°$, and $x/C = 1/4$; required the ordinate, M_x, dist x from the mid ord, M. Here eq (35) gives, approx, $M_x = a\,b/2\,R$; but, in the following table, in the line beginning "(35) $M_x = a\,b/2\,R$", and in the column headed "$\Delta = 20°$, $x/C = 1/4$," we find the correctiv multiplier, 1.0096, which gives the true value, $M_x = 1.0096\,a\,b/2\,R = 0.5048\,a\,b/R$.

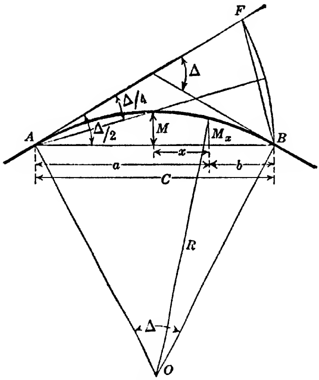


Fig. 12.

Eq. No	Equation.	Δ = 10° x/C =			Δ = 20° x/C =		
		1/4	3/8	19/40	1/4	3/8	19/40
(35)	$M_x = ab/2R$	1.0024	1.0028	1.0024	1.0096	1.0128	1.0094
(36) (37)	$M_x = 4\,Mab/C^2$ $m_x = mab/2500$	1.0005	1.0011	1.0012	1.0019	1.0051	1.0076
(39)	$M_x = D \times M_1$	0.9995	1.0003	0.9965	1.0036	1.0064	1.0053

36. For that portion of a curv which is subtended by a **chain, c, or unit arc, a,** (sweep, Δ, = sharpness, D), use the equations of ¶ ¶ 34, 35,

substituting D, t, c, m, m_x, e, y and m_h
for Δ, T, C, M, M_x, E, Y and M_h respectivly.

Middle Ordinates, m, in ft, to 100-ft chain, **for different sharpnesses,** *D.*

D Mins	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	D Mins
0	0.000	0.218	0.436	0.654	0.872	1.091	1.309	1.528	1.746	1.965	0
1	0.004	0.222	0.440	0.658	0.876	1.094	1.313	1.531	1.749	1.968	1
2	0.007	0.225	0.444	0.662	0.880	1.098	1.317	1.535	1.753	1.972	2
3	0.011	0.229	0.447	0.665	0.883	1.102	1.320	1.539	1.756	1.975	3
4	0.015	0.233	0.451	0.669	0.887	1.105	1.324	1.543	1.761	1.979	4
5	0.018	0.236	0.454	0.673	0.891	1.109	1.327	1.546	1.764	1.983	5
6	0.022	0.240	0.458	0.676	0.894	1.112	1.331	1.550	1.768	1.987	6
7	0.025	0.244	0.462	0.680	0.898	1.116	1.335	1.553	1.771	1.990	7
8	0.029	0.247	0.465	0.684	0.902	1.120	1.338	1.557	1.775	1.994	8
9	0.033	0.251	0.469	0.687	0.905	1.123	1.342	1.561	1.778	1.998	9
10	0.036	0.255	0.473	0.691	0.909	1.127	1.346	1.564	1.782	2.001	10
11	0.040	0.258	0.476	0.694	0.912	1.131	1.349	1.568	1.786	2.005	11
12	0.044	0.262	0.480	0.698	0.916	1.134	1.353	1.572	1.790	2.008	12
13	0.047	0.265	0.484	0.702	0.920	1.138	1.356	1.575	1.793	2.012	13
14	0.051	0.269	0.487	0.705	0.923	1.142	1.360	1.579	1.797	2.016	14
15	0.055	0.273	0.491	0.709	0.927	1.146	1.364	1.582	1.801	2.019	15
16	0.058	0.276	0.494	0.713	0.931	1.149	1.368	1.586	1.804	2.023	16
17	0.062	0.280	0.498	0.716	0.934	1.153	1.371	1.590	1.807	2.026	17
18	0.065	0.284	0.502	0.720	0.938	1.157	1.375	1.593	1.811	2.030	18
19	0.069	0.287	0.505	0.723	0.942	1.160	1.378	1.597	1.815	2.034	19
20	0.073	0.291	0.509	0.727	0.945	1.164	1.382	1.600	1.819	2.037	20
21	0.076	0.295	0.513	0.731	0.949	1.168	1.386	1.604	1.822	2.041	21
22	0.080	0.298	0.516	0.734	0.952	1.171	1.389	1.608	1.826	2.045	22
23	0.084	0.302	0.520	0.738	0.956	1.175	1.393	1.611	1.829	2.048	23
24	0.087	0.305	0.524	0.742	0.960	1.179	1.397	1.615	1.833	2.052	24
25	0.091	0.309	0.527	0.745	0.963	1.182	1.400	1.619	1.837	2.056	25
26	0.095	0.313	0.531	0.749	0.967	1.186	1.404	1.623	1.840	2.060	26
27	0.098	0.316	0.534	0.753	0.971	1.190	1.407	1.626	1.844	2.063	27
28	0.102	0.320	0.538	0.756	0.974	1.193	1.411	1.630	1.848	2.066	28
29	0.105	0.324	0.542	0.760	0.978	1.197	1.415	1.633	1.851	2.070	29
30	0.109	0.327	0.545	0.763	0.982	1.200	1.418	1.637	1.855	2.074	30
31	0.113	0.331	0.549	0.767	0.985	1.204	1.422	1.641	1.858	2.077	31
32	0.116	0.335	0.553	0.771	0.989	1.208	1.426	1.644	1.862	2.081	32
33	0.120	0.338	0.556	0.774	0.993	1.211	1.429	1.648	1.866	2.084	33
34	0.124	0.342	0.560	0.778	0.996	1.215	1.433	1.651	1.869	2.088	34
35	0.127	0.345	0.564	0.782	1.000	1.218	1.437	1.655	1.873	2.092	35
36	0.131	0.349	0.567	0.785	1.003	1.222	1.440	1.659	1.877	2.096	36
37	0.135	0.353	0.571	0.789	1.007	1.226	1.444	1.662	1.880	2.099	37
38	0.138	0.356	0.574	0.793	1.011	1.229	1.447	1.666	1.884	2.103	38
39	0.142	0.360	0.578	0.796	1.014	1.233	1.451	1.670	1.887	2.106	39
40	0.145	0.364	0.582	0.800	1.018	1.237	1.455	1.673	1.892	2.110	40
41	0.149	0.367	0.585	0.803	1.022	1.240	1.458	1.677	1.895	2.113	41
42	0.153	0.371	0.589	0.807	1.025	1.244	1.462	1.680	1.899	2.117	42
43	0.156	0.375	0.593	0.811	1.029	1.247	1.466	1.684	1.903	2.121	43
44	0.160	0.378	0.596	0.814	1.032	1.251	1.469	1.688	1.906	2.125	44
45	0.164	0.382	0.600	0.818	1.036	1.255	1.473	1.691	1.910	2.128	45
46	0.167	0.385	0.604	0.822	1.040	1.258	1.476	1.695	1.914	2.132	46
47	0.171	0.389	0.607	0.825	1.043	1.262	1.480	1.699	1.918	2.135	47
48	0.174	0.393	0.611	0.829	1.047	1.266	1.484	1.702	1.921	2.139	48
49	0.178	0.396	0.614	0.832	1.051	1.269	1.487	1.706	1.924	2.142	49
50	0.182	0.400	0.618	0.836	1.054	1.273	1.491	1.710	1.928	2.147	50
51	0.185	0.404	0.622	0.840	1.058	1.277	1.495	1.713	1.932	2.150	51
52	0.189	0.407	0.625	0.843	1.062	1.280	1.498	1.717	1.935	2.154	52
53	0.193	0.411	0.629	0.847	1.065	1.284	1.502	1.720	1.939	2.158	53
54	0.196	0.414	0.633	0.851	1.069	1.288	1.505	1.724	1.943	2.161	54
55	0.200	0.418	0.636	0.854	1.073	1.291	1.510	1.728	1.946	2.165	55
56	0.204	0.422	0.640	0.858	1.076	1.295	1.513	1.731	1.950	2.168	56
57	0.207	0.425	0.644	0.862	1.080	1.298	1.517	1.735	1.953	2.172	57
58	0.211	0.429	0.647	0.865	1.083	1.302	1.520	1.739	1.957	2.175	58
59	0.215	0.433	0.651	0.869	1.088	1.306	1.524	1.742	1.961	2.179	59

Middle Ordinates, m, in ft, to 100-ft chain, for different sharpnesses, *D*.

(Concluded)

D Mins	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	D Mins
0	2.183	2.402	2.620	2.839	3.058	3.277	3.496	3.716	3.935	4.155	0
2	2.190	2.409	2.628	2.846	3.065	3.284	3.504	3.723	3.942	4.162	2
4	2.198	2.416	2.635	2.854	3.073	3.292	3.511	3.730	3.950	4.169	4
6	2.205	2.423	2.642	2.861	3.080	3.299	3.518	3.738	3.957	4.177	6
8	2.212	2.431	2.650	2.868	3.087	3.306	3.526	3.745	3.964	4.184	8
10	2.219	2.438	2.657	2.876	3.095	3.314	3.533	3.752	3.972	4.191	10
12	2.227	2.445	2.664	2.883	3.102	3.321	3.540	3.760	3.979	4.199	12
14	2.234	2.453	2.671	2.890	3.109	3.328	3.547	3.767	3.986	4.206	14
16	2.241	2.460	2.679	2.898	3.117	3.336	3.555	3.774	3.994	4.213	16
18	2.249	2.467	2.686	2.905	3.124	3.343	3.562	3.781	4.001	4.221	18
20	2.256	2.475	2.693	2.912	3.131	3.350	3.569	3.789	4.008	4.228	20
22	2.263	2.482	2.701	2.919	3.138	3.358	3.577	3.796	4.016	4.235	22
24	2.270	2.489	2.708	2.927	3.146	3.365	3.584	3.803	4.023	4.243	24
26	2.278	2.496	2.715	2.934	3.153	3.372	3.591	3.811	4.030	4.250	26
28	2.285	2.504	2.722	2.941	3.160	3.379	3.599	3.818	4.038	4.257	28
30	2.293	2.511	2.730	2.949	3.168	3.387	3.606	3.825	4.045	4.265	30
32	2.300	2.518	2.737	2.956	3.175	3.394	3.613	3.833	4.052	4.272	32
34	2.307	2.526	2.744	2.963	3.182	3.401	3.621	3.840	4.060	4.279	34
36	2.314	2.533	2.752	2.971	3.190	3.409	3.628	3.847	4.067	4.287	36
38	2.321	2.540	2.759	2.978	3.197	3.416	3.635	3.855	4.074	4.294	38
40	2.329	2.547	2.766	2.985	3.204	3.423	3.643	3.862	4.081	4.301	40
42	2.336	2.555	2.774	2.992	3.211	3.431	3.650	3.869	4.089	4.308	42
44	2.343	2.562	2.781	3.000	3.219	3.438	3.657	3.877	4.096	4.316	44
46	2.351	2.569	2.788	3.007	3.226	3.445	3.664	3.884	4.103	4.323	46
48	2.358	2.577	2.795	3.014	3.233	3.452	3.672	3.891	4.111	4.330	48
50	2.365	2.584	2.803	3.022	3.241	3.460	3.679	3.899	4.118	4.338	50
52	2.372	2.591	2.810	3.029	3.248	3.467	3.686	3.906	4.125	4.345	52
54	2.380	2.598	2.817	3.036	3.255	3.474	3.694	3.913	4.133	4.352	54
56	2.387	2.606	2.825	3.044	3.263	3.482	3.701	3.920	4.140	4.360	56
58	2.394	2.613	2.832	3.051	3.270	3.489	3.708	3.928	4.147	4.367	58
60	2.402	2.620	2.839	3.058	3.277	3.496	3.716	3.935	4.155	4.374	60

Mins	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	Mins
0	4.374	4.594	4.814	5.035	5.255	5.476	5.697	5.918	6.139	6.360	0
10	4.411	4.631	4.851	5.071	5.292	5.513	5.734	5.955	6.176	6.398	10
20	4.448	4.668	4.888	5.108	5.329	5.549	5.770	5.992	6.213	6.435	20
30	4.484	4.704	4.925	5.145	5.366	5.586	5.807	6.029	6.250	6.472	30
40	4.521	4.741	4.961	5.182	5.402	5.623	5.844	6.065	6.287	6.509	40
50	4.558	4.778	4.998	5.218	5.439	5.660	5.881	6.102	6.324	6.545	50
60	4.594	4.814	5.035	5.255	5.476	5.697	5.918	6.139	6.360	6.583	60

Mins	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	Mins
0	6.583	6.805	7.027	7.250	7.473	7.696	7.919	8.143	8.367	8.592	0
20	6.657	6.879	7.101	7.324	7.547	7.770	7.994	8.218	8.442	8.667	20
40	6.731	6.958	7.175	7.398	7.621	7.845	8.068	8.292	8.517	8.741	40
60	6.805	7.027	7.250	7.473	7.696	7.919	8.143	8.367	8.592	8.816	60

Mins	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	Mins
0	8.816	9.041	9.267	9.493	9.719	9.946	10.173	10.400	10.628	10.856	0
30	8.929	9.154	9.380	9.606	9.832	10.059	10.286	10.516	10.742	10.970	30
60	9.041	9.267	9.493	9.719	9.946	10.173	10.400	10.628	10.856	11.085	60

Side ordinates, m_x, in ft, 5 ft apart, to 100-ft chain.

$m_x = \sqrt{R^2 - x^2} - R\cos(D/2)$;

x = dist, in ft, of side ordinate from middle ordinate.

D \ x, ft.	5	10	15	20	25	30	35	40	45
0° 4′	.014	.014	.013	.012	.011	.009	.007	.005	.003
8	.029	.028	.026	.024	.022	.018	.015	.010	.005
12	.043	.041	.038	.037	.033	.028	.022	.015	.008
16	.058	.056	.052	.049	.044	.037	.030	.020	.011
20	.072	.070	.066	.061	.055	.047	.037	.026	.014
24	.086	.083	.077	.074	.066	.056	.045	.031	.017
28	.101	.098	.092	.086	.077	.065	.052	.036	.019
32	.115	.112	.106	.098	.088	.075	.058	.042	.022
36	.130	.126	.119	.110	.099	.084	.066	.047	.024
40	.144	.140	.133	.123	.110	.093	.074	.052	.027
44	.158	.153	.145	.135	.121	.103	.081	.057	.030
48	.172	.167	.158	.147	.132	.112	.088	.062	.033
52	.187	.181	.171	.159	.143	.122	.095	.068	.035
56	.202	.195	.185	171	.154	.131	.103	073	.038
1	.216	.209	.198	.183	.164	.140	.111	.078	.041
4	.231	.223	.211	.196	.175	.150	.118	.083	.043
8	.245	.237	.224	.208	.186	.159	.125	.088	.046
12	.260	.252	.237	.220	.196	.168	.133	.094	.049
16	.274	.265	.251	.232	.207	.177	.140	.099	.052
20	.288	.279	.264	.244	.218	.187	.148	.104	.055
24	.303	.293	.277	.256	.229	.197	.155	.109	.057
28	.317	.307	.291	.269	.240	.206	.163	.114	.060
32	.331	.321	.304	.281	.251	.215	.171	.120	.063
36	.345	.335	.317	.293	.262	.224	.178	.125	.066
40	.360	.349	.330	.305	.273	.233	.185	.130	.069
44	.374	.363	.343	.318	.284	.242	.192	.135	.072
48	.389	.377	.356	.330	.295	.251	.200	.141	.075
52	.403	.391	.370	.342	.305	.261	.208	.147	.077
56	418	.405	.383	.354	.316	.270	.215	.152	.080
2	.432	.419	.397	.366	.327	.280	.222	.157	.083
4	.446	.433	.409	.379	.338	.289	.230	.162	.086
8	.461	.447	.425	.391	.349	.298	.237	.167	.088
12	.475	.461	.437	.403	.360	.308	.245	.173	.090
16	.490	.475	.450	.415	.371	.317	.252	.178	.093
20	.504	.489	.463	.428	.382	.326	.260	.183	.096
24	.518	.503	.476	.440	.393	.336	.267	.188	.099
28	.533	.517	.489	.452	.404	.346	.275	.194	.102
32	.547	.531	.503	.465	.415	.355	.282	.199	.104
36	.562	.545	.516	.477	.425	.364	.289	.204	.107
40	.576	.559	.529	.489	.436	.373	.297	.209	.110
44	.590	.573	.542	.501	.447	.382	.304	.214	.113
48	.605	.587	.555	.513	.458	.391	.312	.219	.116
52	.619	.601	.569	.526	.469	.401	.319	.225	.118
56	.634	.615	.582	.538	.480	.410	.326	.230	.121
3	.648	.629	.595	.550	.491	.419	.334	.235	.124
4	.662	.643	.608	.562	.502	.428	.341	.240	.127
8	.677	.657	.621	.574	.512	.438	.349	.246	.130
12	.691	.671	.635	.587	.523	.448	.357	.251	.132
16	.705	.685	.649	.599	.534	.457	.364	.257	.135
20	.720	.699	.662	.611	.545	.466	.371	.262	.138
24	.734	.713	.675	.623	.556	.475	.378	.267	.141
28	.749	.727	.688	.635	.567	.485	.386	.272	.144
32	.763	.741	.702	.648	.578	.494	.394	.278	.146
36	.777	.755	.715	.660	.589	.503	.401	.283	.149
40	.792	.769	.728	.673	.600	.512	.408	.288	.152
44	.806	.783	.741	.685	.611	.521	.415	.293	.155
48	.821	.797	.754	.697	.621	.531	.423	.298	.158
52	.835	.811	.768	.709	.632	.541	.431	.304	.160
56	.850	.825	.781	.721	.643	.550	.438	.309	.163

Side ordinates, m_x, in ft, 5 ft apart, to 100-ft chain.

(Concluded)

$m_x = \sqrt{R^2 - x^2} - R\cos(D/2)$;

x = dist, in ft, of side ordinate from middle ordinate.

D \ x, ft.	5	10	15	20	25	30	35	40	45
4° 0'	.864	.839	.794	.734	.655	.559	.445	.314	.166
10	.900	.874	.827	.764	.682	.582	.464	.327	.173
20	.936	.909	.860	.795	.709	.606	.482	.340	.179
30	.972	.944	.893	.825	.736	.629	.501	.354	.186
40	1.008	.979	.926	.855	.764	.652	.519	.367	.193
50	1.044	1.014	.959	.886	.791	.676	.538	.380	.199
5	1.080	1.048	.993	.917	.818	.699	.557	.393	.207
10	1.116	1.083	1.026	.947	.845	.722	.576	.406	.214
20	1.152	1.118	1.058	.978	.872	.746	.594	.419	.220
30	1.188	1.153	1.092	1.009	.900	.769	.613	.432	.228
40	1.224	1.188	1.124	1 039	.927	.792	.631	.445	.235
50	1.260	1.223	1.157	1.070	.954	.816	.649	.458	.241
6	1.296	1.258	1.191	1 100	.982	.839	.668	.472	.248
10	1.332	1.293	1.224	1.130	1.009	.862	.686	.485	.255
20	1.368	1.328	1.256	1 161	1.036	.886	.705	.498	.262
30	1.404	1.362	1 290	1 192	1.064	.909	.724	.511	.269
40	1.440	1.397	1 323	1.222	1.091	.932	.742	.524	.276
50	1.476	1.432	1.355	1.253	1.118	.956	.761	.537	.283
7	1.512	1.467	1.389	1.284	1.146	.979	.779	.551	.290
10	1 548	1 502	1 422	1 314	1.173	1 002	.798	.564	.297
20	1 584	1.537	1.454	1.345	1.200	1.026	.816	.576	.304
30	1.620	1.572	1 488	1 375	1 228	1 048	.835	.590	311
40	1.656	1.607	1.521	1.405	1.255	1.071	.854	.603	318
50	1.692	1.641	1.553	1.436	1.282	1.095	.872	.616	.324
8	1 728	1.677	1.587	1.467	1.310	1.118	.891	.629	.332
30	1 836	1 782	1 687	1 559	1 392	1 188	.946	.669	.353
9	1.944	1.886	1 787	1 651	1.474	1.258	1 002	.708	.373
30	2.052	1.991	1.887	1.742	1.556	1.328	1.057	.748	.394
10	2.161	2.096	1 987	1.834	1.637	1.398	1 114	.787	.415
30	2.269	2.201	2.087	1.926	1.719	1.468	1.170	.827	.436
11	2.377	2.306	2 186	2.018	1.802	1.538	1.226	.866	.457
30	2.486	2.411	2.286	2.110	1.884	1.609	1.282	.906	.478
12	2.594	2.516	2 386	2.203	1.967	1 680	1.339	.946	.499
30	2.703	2.621	2.485	2.295	2.049	1.750	1.395	.985	.520
13	2.811	2.726	2 585	2.387	2.132	1.820	1 451	1.025	.541
30	2.920	2.832	2.685	2.479	2.214	1.891	1.507	1.065	.562
14	3.028	2 937	2.785	2.571	2.297	1.961	1.564	1.105	.583
30	3.136	3.042	2.884	2.664	2.379	2.031	1.620	1.144	.604
15	3.245	3.147	2.984	2.756	2.462	2 102	1.676	1.184	.625
30	3.354	3.252	3.084	2 848	2.544	2.172	1.732	1.224	.646
16	3.462	3.358	3.184	2.941	2.627	2.243	1.789	1.264	.667
17	3.680	3.569	3.384	3.125	2.792	2.384	1.902	1.344	.709
18	3.897	3.779	3.584	3 310	2.958	2.525	2.014	1.424	.751
19	4.115	3.990	3.784	3.495	3.123	2.666	2.127	1.504	.793
20	4.332	4.201	3 984	3.680	3.288	2 808	2.240	1.583	.836
22	4.768	4.624	4.386	4.050	3.620	3.093	2.467	1.744	.922
24	5.204	5.048	4.789	4.423	3.952	3.379	2.695	1.905	1.008
26	5.642	5.473	5.192	4.798	4.286	3.665	2.924	2.068	1.094
28	6.079	5.898	5.595	5.171	4.622	3.952	3.154	2.232	1.181
30	6.517	6.323	5.999	5.544	4.958	4.239	3.385	2 396	1.268
32	6.957	6.751	6.406	5.922	5.297	4.530	3.619	2.565	1.356
34	7.398	7.179	6.813	6.300	5.637	4.822	3.854	2.733	1.445
36	7.841	7.609	7.222	6.679	5.978	5.115	4.090	2.901	1.535
38	8.286	8.041	7.633	7.060	6.320	5.410	4.327	3.069	1.626
40	8.731	8.474	8.044	7.442	6.663	5.705	4.565	3.238	1.718

37. Ratio, Q, of arc to chord. Fig. 9. In any circular curv, of radius R, and sweep Δ, we have (see ¶ 28) :—

$$\text{Eq (6) length, } L_a \text{, of arc, } = \frac{\pi R}{180} . \Delta^\circ \text{; and}$$

$$\text{Eq (28) long chord, } C, = 2 R \sin(\Delta/2).$$

Hence, for the ratio, Q, betw arc, L_a, and chord, C, we have :—

$$Q = \frac{L_a}{C} = \frac{\pi}{360} . \frac{\Delta^\circ}{\sin(\Delta/2)} ; \quad \ldots\ldots\ldots\ldots (40)$$

where π = 3.14159 . . . ;
$\pi/360$ = 0.008 726 647 ;
$360/\pi$ = 114.5916 ;
log π = 0.497 1499
log $(\pi/360)$ = 7.940 8474 :
log $(360/\pi)$ = 2.059 1526.

Hence Q is a function of Δ.

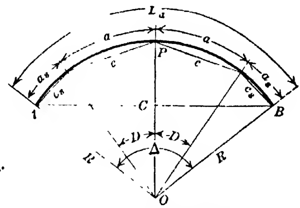


Fig. 9. (Repeated)

Values of Q = $\frac{\text{arc}}{\text{chord}}$.

Note that $Q - 1$ increases a little faster than Δ^2.

Δ	Q	Δ	Q	Δ	Q	Δ	Q
1	1.000013	16	1.003257	31	1.012302	46	1.027371
2	1.000051	17	1.003678	32	1.013116	47	1.028598
3	1.000115	18	1.004124	33	1.013957	48	1.029853
4	1.000203	19	1.004597	34	1.014825	49	1.031137
5	1.000317	20	1.005095	35	1.015719	50	1.032450
6	1.000457	21	1.005619	36	1.016641	51	1.033792
7	1.000622	22	1.006170	37	1.017590	52	1.035163
8	1.000813	23	1.006746	38	1.018566	53	1.036563
9	1.001029	24	1.007349	39	1.019569	54	1.037993
10	1.001271	25	1.007977	40	1.020600	55	1.039452
11	1.001537	26	1.008632	41	1.021659	56	1.040941
12	1.001830	27	1.009313	42	1.022745	57	1.042460
13	1.002148	28	1.010021	43	1.023860	58	1.044009
14	1.002492	29	1.010755	44	1.025002	59	1.045588
15	1.002862	30	1.011515	45	1.026172	60	1.047198

38. Fig 5 Y. Value of Q (= arc/chord) in the case of the unit chain, c, or unit arc, a' (where sweep, Δ, = sharpness, D or D'.)

Let
D° = central angle subtended by unit chain, c, and by the corresponding arc, a;
D'° = central angle subtended by unit arc, a', and by the corresponding chord, c'.

Then, from eq (40), we have, by substitution :—

$$Q = \frac{a}{c} = \frac{\pi}{360} \times \frac{D^\circ}{\sin(D/2)}$$

$$= \frac{a'}{c'} = \frac{\pi}{360} \times \frac{D'^\circ}{\sin(D'/2)} \quad \ldots\ldots\ldots\ldots (41)$$

39. Approximately :—

$$C = La\left(1 - \frac{La^2}{24\,R^2}\right); \text{ and } c = a\left(1 - \frac{a^2}{24\,R^2}\right) \quad ..(42)$$

(See Rankine, Civil Engng, p 104) When $\Delta = 50°$, this makes C only 0.031 per cent too short.

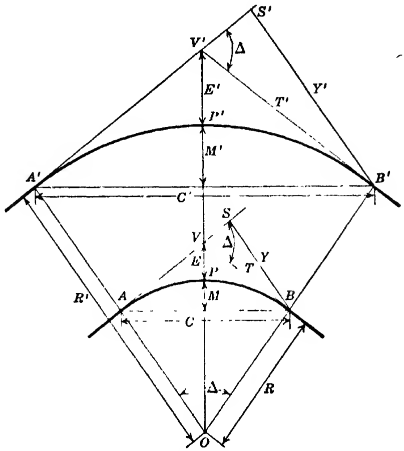


Fig. 13.

Comparisons of Curvs.

40. Fig 13 Comparison betw two curvs of **given sweep, Δ.** Ratios betw corresponding **linear functions.** Let F represent a linear function (as the radius, R, the arc, $A\,B$ (A), the semitan, T, etc) of one of two given curvs of equal sweep, Δ, but of different sharpness, D, and let F' represent the corresponding function, (as R', $A'\,B'(A')$, T', etc) of the other curv. Then, from similar triangles, we have :—

$$F/F' = R/R' = C/C' = A/A' = T/T', \text{ etc};$$
$$\text{or } F' = FR'/R = FC'/C, \text{ etc};$$
$$F = F'R/R' = F'C/C', \text{ etc} \quad \ldots\ldots\ldots\ldots(43).$$

Thus, let C be a unit chain of 100 ft, and let $C' = 200$ ft. Then, for instance, $M' = MC'/C = 2\,M$; $R' = 2\,R$; etc;

$$\frac{\sin\,(D'/2)}{\sin\,(D/2)} = \frac{50}{R'}\cdot\frac{R}{50} = \frac{R}{R'}.$$

Note that, with *given sweep*, Δ, the *sharper* curv has the *shorter* linear functions. Compare ¶ 43.

Functions of a 1° Curv.

40a. Figs 1 and 11. Table of Functions, F_1 (Semitangents, T_1; External Distances, E_1; and Long Chords, C_1); to a 1° curv, for different sweeps, Δ.

$T_1 = R_1 \tan(\Delta/2)$; $E_1 = R_1 \operatorname{exsec}(\Delta/2)$; $C_1 = 2 R_1 \sin(\Delta/2)$.

For the corresponding function, F (T, E or C) of a curv **of any other sharpness, D°,** we have approximately

$$F = F_1/D°.$$

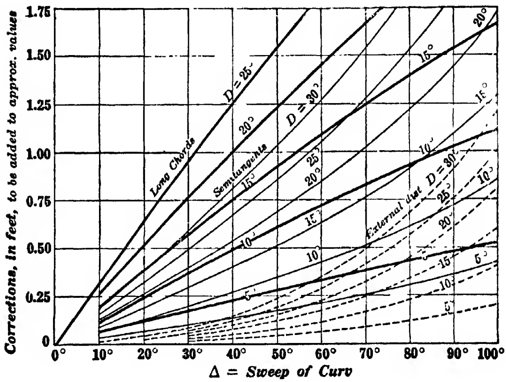


Diagram of Corrections

Corrections, in feet, to be added to approximate curv functions, F (Semitangent, T; External Distance, E; and Long Chord, C) for a $D°$ curv, as found by dividing the corresponding 1° curv function, F_1, of Table, pp 905-908, by D in degrees.

Light solid curvs give values to be added for semitangents, T;
Light dotted curvs give values to be added for external dists, E.
Heavy solid curvs give values to be added for long chords, C.

Δ	T_1, ft.	E_1, ft.	C_1, ft.	Δ	T_1, ft.	E_1, ft.	C_1, ft.
1°	50.00	0.218	100.00	**11°**	551.70	26.500	1098.3
10′	58.34	0.297	116.67	10′	560.11	27.313	1114.9
20	66.67	0.388	133.33	20	568.53	28.137	1131.5
30	75.01	0 491	150.00	30	576.95	28.974	1148.1
40	83.34	0.606	166 66	40	585.36	29.824	1164.7
50	91.68	0.733	183.33	50	593.79	30.686	1181.2
2°	100.01	0.873	199.99	**12°**	602.21	31.561	1197.8
10′	108 35	1.024	216.66	10′	610.64	32.447	1214.4
20	116.68	1.188	233 32	20	619.07	33.347	1231.0
30	125 02	1.364	249.98	30	627.50	34.259	1247.5
40	133.36	1.552	266 65	40	635.93	35.183	1264.1
50	141.70	1.752	283.31	50	644.37	36.120	1280.7
3°	150 04	1.964	299.97	**13°**	652.81	37.069	1297.2
10′	158 38	2 188	316.63	10′	661.25	38.031	1313.8
20	166.72	2 425	333 29	20	669.70	39.006	1330.3
30	175.06	2.674	349.95	30	678.15	39.993	1346.9
40	183.40	2 934	366.61	40	686 60	40.992	1363.4
50	191.74	3.207	383 27	50	695 06	42.004	1380.0
4°	200 08	3.492	399 92	**14°**	703.51	43.029	1396.5
10′	208.43	3 790	416.58	10′	711.97	44.066	1413.1
20	216.77	4.099	433.24	20	720 44	45.116	1429.6
30	225.12	4 421	449.89	30	728.90	46.178	1446.2
40	233 47	4 755	466 54	40	737.37	47.253	1462.7
50	241.81	5.100	483.20	50	745.85	48.341	1479.2
5°	250.16	5.459	499 85	**15°**	754.32	49.441	1495.7
10′	258 51	5 829	516 50	10′	762 80	50.554	1512.3
20	266.86	6.211	533.15	20	771.29	51.679	1528.8
30	275.21	6 606	549.80	30	779 77	52.818	1545.3
40	283 57	7.013	566 44	40	788 26	53.969	1561.8
50	291.92	7.432	583.09	50	796.75	55.132	1578.3
6°	300 28	7.863	599.73	**16°**	805 25	56.309	1594.8
10′	308 64	8.307	616.38	10′	813 75	57.498	1611.3
20	316.99	8.762	633.02	20	822.25	58.699	1627.8
30	325.35	9.230	649.66	30	830 76	59.914	1644.3
40	333.71	9 710	666 30	40	839 27	61 141	1660.8
50	342.08	10.202	682.94	50	847.78	62.381	1677.3
7°	350.44	10.707	699.57	**17°**	856.30	63.634	1693.8
10′	358.81	11.224	716.21	10′	864.82	64.900	1710.3
20	367 17	11.753	732.84	20	873.35	66.178	1726.8
30	375.54	12 294	749.47	30	881 88	67.470	1743.2
40	383.91	12.847	766.10	40	890 41	68.774	1759.7
50	392.28	13.413	782.73	50	898.95	70.091	1776.2
8°	400 66	13.991	799.36	**18°**	907.49	71.421	1792.6
10′	409.03	14 582	815 99	10′	916.03	72.764	1809.1
20	417.41	15.184	832 61	20	924.58	74 119	1825.5
30	425 79	15 799	849 23	30	933.13	75.488	1842.0
40	434.17	16.426	865.85	40	941.69	76 869	1858.4
50	442.55	17.066	882.47	50	950.25	78.264	1874.9
9°	450.93	17.717	899 09	**19°**	958.81	79.671	1891.3
10′	459 32	18.381	915 70	10′	967.38	81 092	1907.8
20	467 71	19.058	932 31	20	975.96	82.525	1924.2
30	476.10	19.746	948 92	30	984.53	83.972	1940.6
40	484 49	20.447	965.53	40	993.12	85.431	1957.1
50	492.88	21.161	982 14	50	1001.70	86.904	1973.5
10°	501.28	21.886	998.74	**20°**	1010.29	88.389	1989.9
10′	509 68	22.624	1015.35	10′	1018.89	89.888	2006.3
20	518.08	23.375	1031.95	20	1027.49	91.399	2022.7
30	526.48	24.138	1048 54	30	1036.09	92.924	2039.1
40	534.89	24.913	1065.14	40	1044.70	94.462	2055.5
50	543.29	25.700	1081.73	50	1053.31	96.013	2071.9
11°	551.70	26.500	1098.33	**21°**	1061.93	97.577	2088.3

Δ	T_1, ft.	E_1, ft	C_1, ft	Δ	T_1, ft.	E_1, ft.	C_1, ft.
21°	1061.9	97.58	2088.3	**31°**	1589.0	216.25	3062.4
10′	1070.6	99.15	2104.7	10′	1598.0	218.66	3078.4
20	1079.2	100.75	2121.1	20	1606.9	221.08	3094.5
30	1087.8	102.35	2137.4	30	1615.9	223.51	3110.5
40	1096.4	103.97	2153.8	40	1624.9	225.96	3126.6
50	1105.1	105.60	2170.2	50	1633.9	228.42	3142.6
22°	1113.7	107.24	2186.5	**32°**	1643.0	230.90	3158.6
10′	1122.4	108.90	2202.9	10′	1652.0	233.39	3174.6
20	1131.0	110.57	2219.2	20	1661.0	235.90	3190.6
30	1139.7	112.25	2235.6	30	1670.0	238.43	3206.6
40	1148.4	113.95	2251.9	40	1679.1	240.96	3222.6
50	1157.0	115.66	2268.3	50	1688.1	243.52	3238.6
23°	1165.7	117.38	2284.6	**33°**	1697.2	246.08	3254.6
10′	1174.4	119.12	2301.0	10′	1706.3	248.66	3270.6
20	1183.1	120.87	2317.3	20	1715.3	251.26	3286.6
30	1191.8	122.63	2333.6	30	1724.4	253.87	3302.5
40	1200.5	124.41	2349.9	40	1733.5	256.50	3318.5
50	1209.2	126.20	2366.2	50	1742.6	259.14	3334.4
24°	1217.9	128.00	2382.5	**34°**	1751.7	261.80	3350.4
10′	1226.6	129.82	2398.8	10′	1760.8	264.47	3366.3
20	1235.3	131.65	2415.1	20	1770.0	267.16	3382.2
30	1244.0	133.50	2431.4	30	1779.1	269.86	3398.2
40	1252.8	135.36	2447.7	40	1788.2	272.58	3414.1
50	1261.5	137.23	2464.0	50	1797.4	275.31	3430.0
25°	1270.2	139.11	2480.2	**35°**	1806.6	278.05	3445.9
10′	1279.0	141.01	2496.5	10′	1815.7	280.82	3461.8
20	1287.7	142.93	2512.8	20	1824.9	283.60	3477.7
30	1296.5	144.85	2529.0	30	1834.1	286.39	3493.5
40	1305.3	146.79	2545.3	40	1843.3	289.20	3509.4
50	1314.0	148.75	2561.5	50	1852.5	292.02	3525.3
26°	1322.8	150.71	2577.8	**36°**	1861.7	294.86	3541.1
10′	1331.6	152.69	2594.0	10′	1870.9	297.72	3557.0
20	1340.4	154.69	2610.3	20	1880.1	300.59	3572.8
30	1349.2	156.70	2626.5	30	1889.4	303.47	3588.6
40	1358.0	158.72	2642.7	40	1898.6	306.37	3604.5
50	1366.8	160.76	2658.9	50	1907.9	309.29	3620.3
27°	1375.6	162.81	2675.1	**37°**	1917.1	312.22	3636.1
10′	1384.4	164.87	2691.3	10′	1926.4	315.17	3651.9
20	1393.2	166.95	2707.5	20	1935.7	318.13	3667.7
30	1402.0	169.04	2723.7	30	1945.0	321.11	3683.5
40	1410.9	171.15	2739.9	40	1954.3	324.11	3699.3
50	1419.7	173.27	2756.1	50	1963.6	327.12	3715.0
28°	1428.6	175.41	2772.3	**38°**	1972.9	330.15	3730.8
10′	1437.4	177.55	2788.4	10′	1982.2	333.19	3746.5
20	1446.3	179.72	2804.6	20	1991.5	336.25	3762.3
30	1455.1	181.89	2820.7	30	2000.9	339.32	3778.0
40	1464.0	184.08	2836.9	40	2010.2	342.41	3793.8
50	1472.9	186.29	2853.0	50	2019.6	345.52	3809.5
29°	1481.8	188.51	2869.2	**39°**	2029.0	348.64	3825.2
10′	1490.7	190.74	2885.3	10′	2038.4	351.78	3840.9
20	1499.6	192.99	2901.4	20	2047.8	354.94	3856.6
30	1508.5	195.25	2917.6	30	2057.2	358.11	3872.3
40	1517.4	197.53	2933.7	40	2066.6	361.29	3888.0
50	1526.3	199.82	2949.8	50	2076.0	364.50	3903.6
30°	1535.3	202.12	2965.9	**40°**	2085.4	367.72	3919.3
10′	1544.2	204.44	2982.0	10′	2094.9	370.95	3935.0
20	1553.1	206.77	2998.1	20	2104.3	374.20	3950.6
30	1562.1	209.12	3014.2	30	2113.8	377.47	3966.3
40	1571.0	211.48	3030.2	40	2123.3	380.76	3981.9
50	1580.0	213.86	3046.3	50	2132.7	384.06	3997.5
31°	1589.0	216.25	3062.4	**41°**	2142.2	387.38	4013.1

Δ	T_1, ft.	E_1, ft.	C_1, ft.	Δ	T_1, ft.	E_1, ft.	C_1, ft.
41°	2142.2	387.38	4013.1	**51°**	2732.9	618.39	4933.4
10′	2151.7	390.71	4028.7	10′	2743.1	622.81	4948.4
20	2161.2	394.06	4044.3	20	2753.4	627.24	4963.4
30	2170.8	397.43	4059.9	30	2763.7	631.69	4978.4
40	2180.3	400.82	4075.5	40	2773.9	636.16	4993.4
50	2189.9	404.22	4091.1	50	2784.2	640.66	5008.4
42°	2199.4	407.64	4106.6	**52°**	2794.5	645.17	5023.4
10′	2209.0	411.07	4122.2	10′	2804.9	649.70	5038.4
20	2218.6	414.52	4137.7	20	2815.2	654.25	5053.4
30	2228.1	417.99	4153.3	30	2825.6	658.83	5068.3
40	2237.7	421.48	4168.8	40	2835.9	663.42	5083.3
50	2247.3	424.98	4184.3	50	2846.3	668.03	5098.2
43°	2257.0	428.50	4199.8	**53°**	2856.7	672.66	5113.1
10′	2266.6	432.04	4215.3	10′	2867.1	677.32	5128.0
20	2276 2	435 59	4230.8	20	2877.5	681.99	5142.9
30	2285.9	439 16	4246 3	30	2888.0	686.68	5157.8
40	2295.6	442.75	4261 8	40	2898 4	691.40	5172.7
50	2305.2	446.35	4277.3	50	2908.9	696.13	5187.6
44°	2314.9	449.98	4292.7	**54°**	2919.4	700.89	5202.4
10′	2324.6	453.62	4308.2	10′	2929.9	705.66	5217.3
20	2334.3	457.27	4323.6	20	2940.4	710.46	5232.1
30	2344.1	460.95	4339.0	30	2951.0	715.28	5246.9
40	2353.8	464.64	4354.5	40	2961.5	720.11	5261.7
50	2363.5	468.35	4369.9	50	2972.1	724.97	5276.5
45°	2373.3	472.08	4385.3	**55°**	2982.7	729.85	5291.3
10′	2383.1	475.82	4400.7	10′	2993.3	734.76	5306.1
20	2392.8	479.59	4416.1	20	3003.9	739.68	5320.9
30	2402.6	483.37	4431.4	30	3014 5	744.62	5335.6
40	2412.4	487.16	4446.8	40	3025.2	749.59	5350.4
50	2422.3	490.98	4462.2	50	3035.8	754.57	5365.1
46°	2432.1	494.82	4477.5	**56°**	3046.5	759.58	5379.8
10′	2441.9	498.67	4492.8	10′	3057.2	764.61	5394.5
20	2451.8	502.54	4508.2	20	3067.9	769.66	5409.2
30	2461.7	506.42	4523.5	30	3078.7	774.73	5423.9
40	2471.5	510.33	4538.8	40	3089.4	779.83	5438.6
50	2481.4	514.25	4554.1	50	3100.2	784.94	5453.3
47°	2491.3	518.20	4569.4	**57°**	3110.9	790.08	5467.9
10′	2501.2	522.16	4584.7	10′	3121.7	795.24	5482.5
20	2511.2	526.13	4599.9	20	3132.6	800.42	5497.2
30	2521.1	530.13	4615.2	30	3143.4	805.62	5511.8
40	2531.1	534.15	4630 4	40	3154.2	810.85	5526.4
50	2541.0	538.18	4645.7	50	3165.1	816.10	5541.0
48°	2551.0	542.23	4660.9	**58°**	3176.0	821.37	5555.6
10′	2561.0	546.30	4676.1	10′	3186.9	826.66	5570.2
20	2571.0	550.39	4691.3	20	3197.8	831.98	5584.7
30	2581.0	554.50	4706 5	30	3208.8	837.31	5599.3
40	2591.1	558.63	4721.7	40	3219.7	842.67	5613.8
50	2601.1	562.77	4736.9	50	3230.7	848.06	5628.3
49°	2611.2	566.94	4752.1	**59°**	3241.7	853.46	5642.8
10′	2621.2	571.12	4767.3	10′	3252.7	858.89	5657.3
20	2631.3	575.32	4782.4	20	3263.7	864.34	5671.8
30	2641.4	579.54	4797.5	30	3274.8	869.82	5686.3
40	2651.5	583.78	4812.7	40	3285.8	875.32	5700.8
50	2661.6	588.04	4827.8	50	3296.9	880.84	5715.2
50°	2671.8	592.32	4842.9	**60°**	3308.0	886.38	5729.7
10′	2681.9	596.62	4858.0	10′	3319.1	891.95	5744.1
20	2692.1	600.93	4873.1	20	3330.3	897.54	5758.5
30	2702.3	605.27	4888.2	30	3341.4	903.15	5772.9
40	2712.5	609.62	4903.2	40	3352.6	908.79	5787.3
50	2722.7	614.00	4918.3	50	3363.8	914.45	5801.7
51°	2732.9	618.39	4933.4	**61°**	3375.0	920.14	5816.0

Δ	T_1, ft.	E_1, ft.	C_1, ft.	Δ	T_1, ft.	E_1, ft.	C_1, ft.
61°	3375.0	920.14	5816.0	**71°**	4086.9	1308.2	6654.4
10′	3386.3	925.85	5830.4	10′	4099.5	1315.5	6668.0
20	3397.5	931.58	5844.7	20	4112.1	1322.9	6681.6
30	3408.8	937.34	5859.1	30	4124.8	1330.3	6695.1
40	3420.1	943.12	5873.4	40	4137.4	1337.7	6708.6
50	3431.4	948.92	5887.7	50	4150.1	1345.1	6722.1
62°	3442.7	954.75	5902.0	**72°**	4162.8	1352.6	6735.6
10′	3454.1	960.60	5916.3	10′	4175.6	1360.1	6749 1
20	3465.4	966.48	5930.5	20	4188.4	1367.6	6762.5
30	3476.8	972.39	5944.8	30	4201.2	1375.2	6776.0
40	3488.2	978.31	5959.0	40	4214.0	1382.8	6789 4
50	3499.7	984.27	5973.3	50	4226.8	1390.4	6802.8
63°	3511.1	990.24	5987.5	**73°**	4239.7	1398.0	6816.3
10′	3522.6	996.24	6001 7	10′	4252.6	1405.7	6829 6
20	3534 1	1002.3	6015.9	20	4265.6	1413.5	6843.0
30	3545.6	1008.3	6030 0	30	4278.5	1421.2	6856.4
40	3557.2	1014.4	6044.2	40	4291.5	1429.0	6869 7
50	3568.7	1020.5	6058.4	50	4304.6	1436.8	6883.1
64°	3580.3	1026.6	6072.5	**74°**	4317.6	1444.6	6896.4
10′	3591.9	1032.8	6086.6	10′	4330.7	1452.5	6909.7
20	3603.5	1039.0	6100.7	20	4343.8	1460.4	6923.0
30	3615.1	1045.2	6114.8	30	4356.9	1468.4	6936.2
40	3626.8	1051.4	6128.9	40	4370.1	1476.4	6949.5
50	3638.5	1057.7	6143.0	50	4383.3	1484.4	6962.8
65°	3650.2	1063 9	6157.1	**75°**	4396.5	1492.4	6976.0
10′	3661.9	1070.2	6171.1	10′	4409.8	1500.5	6989 2
20	3673.7	1076.6	6185.2	20	4423.1	1508.6	7002.4
30	3685 4	1082.9	6199.2	30	4436.4	1516.7	7015.6
40	3697.2	1089 3	6213.2	40	4449.7	1524.9	7028.8
50	3709.0	1095.7	6227.2	50	4463.1	1533.1	7041.9
66°	3720 9	1102 2	6241.2	**76°**	4476.5	1541.4	7055.0
10′	3732.7	1108.6	6255.2	10′	4489.9	1549 7	7068.2
20	3744.6	1115 1	6269.1	20	4503.4	1558.0	7081.3
30	3756.5	1121.7	6283 1	30	4516.9	1566.3	7094.4
40	3768.5	1128 2	6297.0	40	4530.4	1574.7	7107.5
50	3780.4	1134.8	6310.9	50	4544.0	1583.1	7120.5
67°	3792.4	1141.4	6324.8	**77°**	4557.6	1591.6	7133.6
10′	3804.4	1148.0	6338.7	10′	4571.2	1600.1	7146.6
20	3816.4	1154.7	6352.6	20	4584.8	1608.6	7159.6
30	3828.4	1161.3	6366 4	30	4598.5	1617.1	7172.6
40	3840.5	1168.1	6380.3	40	4612.2	1625.7	7185.6
50	3852.6	1174.8	6394.1	50	4626.0	1634.4	7198.6
68°	3864.7	1181.6	6408.0	**78°**	4639.8	1643.0	7211.6
10′	3876.8	1188.4	6421.8	10′	4653.6	1651.7	7224.5
20	3889.0	1195.2	6435.6	20	4667.4	1660.5	7237.4
30	3901.2	1202.0	6449.4	30	4681.3	1669.2	7250.4
40	3913.4	1208.9	6463.1	40	4695.2	1678.1	7263.3
50	3925.6	1215.8	6476.9	50	4709.2	1686.9	7276.1
69°	3937.9	1222.7	6490.6	**79°**	4723.2	1695.8	7289.0
10′	3950.2	1229.7	6504.4	10′	4737.2	1704.7	7301.9
20	3962.5	1236.7	6518.1	20	4751.2	1713.7	7314.7
30	3974.8	1243.7	6531.8	30	4765.3	1722.7	7327.5
40	3987.2	1250.8	6545.5	40	4779.4	1731.7	7340.3
50	3999.5	1257.9	6559.1	50	4793.6	1740.8	7353.1
70°	4011.9	1265.0	6572.8	**80°**	4807.7	1749.9	7365.9
10′	4024.4	1272.1	6586.4	10′	4822.0	1759.0	7378 7
20	4036.8	1279.3	6600.1	20	4836.2	1768.2	7391.4
30	4049.3	1286.5	6613.7	30	4850.5	1777.4	7404.1
40	4061.8	1293.7	6627.3	40	4864.8	1786.7	7416.8
50	4074.4	1300.9	6640.9	50	4879.2	1796.0	7429.5
71°	4086.9	1308.2	6654.4	**81°**	4893.6	1805.3	7442.2

41. English & Metric. Fig 13. Comparison betw curvs in English and metric (or other) measures. Again, let
$C = c =$ a *chain* of n meters ($\Delta = D$).
$C' = c' =$ a *chain* of 100 feet ($\Delta = D'$).

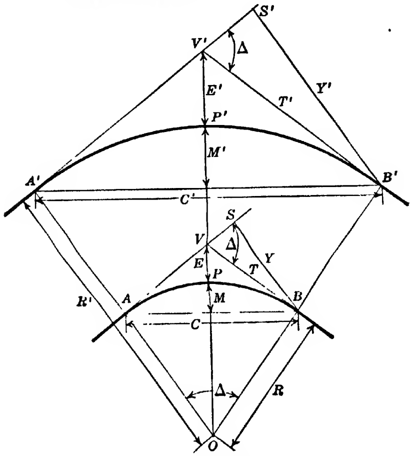


Fig. 13. (Repeated)

Then the two curvs, in Fig 13, altho of diff sharpnesses, are *called* by the same number of degrees; because, in each, the adopted chain subtends the same angle, $D = D'$.

For instance, from eq (43), we have

$$\frac{R \text{ meters}}{R' \text{ feet}} = \frac{c \text{ meters}}{c' \text{ feet}} = \frac{n}{100}\text{ ; and}$$

$$R \text{ meters} = R' \text{ ft } \frac{c \text{ meters}}{c' \text{ feet}} = R' \text{ ft } n/100. \text{ Or, in general :—}$$

$$F = F' c/c' = 0.01\, n\, F' \quad \ldots\ldots\ldots\ldots\ldots (44)$$

Example. Let $\Delta = D = D' = 6°$; $R' = 955.366$ ft. (table, p 886). Then, in a 6° metric curv ($c = 20$ meters; $n = 20$), we have :— R meters $= 0.01 \times 20\, R' = 0.2 \times 955.366 = 191.073$ meters.

Similarly, any table of linear curv functions, in any unit (as in feet, Table p 884), may be used for a curv in any other unit, by multiplying the tabular values by the ratio betw the numbers expressing the chain lengths of the two systems. The results (products) will be in said other unit.

42. Fig 13. **Effect of sharpness, D, upon linear functions** of entire curv of given sweep, Δ. Comparison betw 1° and $D°$ curvs. Ratio betw **exact and approx values.**

Let F_1 (function of 1° curv) = the radius, R_1, or the long chord, C_1, or the semitan, T_1, etc, for a 1° curv, of given sweep, Δ ; and let F_d = the corresponding function of a $D°$ curv of the same sweep, Δ.

Then, where the *"diminisht"* chain (¶ 20) is used, we have,

(D in *degrees*)

$$D = F_1/F_d; \quad F_d = F_1/D; \quad F_1 = D\,F_d \ldots\ldots (45)$$

But, where the *full* chain is used, these equations are only approx. For the ratio, q, betw the true value, F_d, and the approx value, F_1/D, we have: $R_1 = 50/\sin 0°\ 30' = 5729.65$; $\log R_1 = 3.758\,1281$.

$$q\left(= \frac{F_d}{F_1/D} = \frac{R_d}{R_1/D}\right) = \frac{50}{\sin(D/2)} \cdot \frac{D \sin 0°\ 30'}{50}$$

$$= D\,\frac{\sin 0°\ 30'}{\sin(D/2)} \ldots (46)$$

and, since (eq 41, ¶ 38) $Q = \dfrac{a}{c} = \dfrac{\pi}{360} \times \dfrac{D}{\sin(D/2)}$; we have:—

$$\frac{Q}{q} = \frac{\pi}{360} \cdot \frac{D}{\sin(D/2)} \cdot \frac{\sin(D/2)}{D \sin 0°\ 30'}$$

$$= \frac{\pi}{360} \cdot \frac{1}{\sin 0°\ 30'} = 1.000\,013 \ldots\ldots (47)$$

$$\text{Log } (Q/q) = 0.000\,0055.$$

In other words, the value of Q ($=$ arc/chord), in table, ¶ 37, for any given value of Δ, may be taken as practically identical with the value of q [$= F_d/(F_1/D)$], for the same value of D.

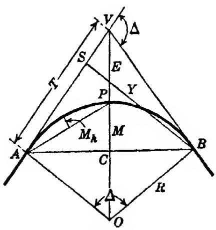


Fig. 14a.

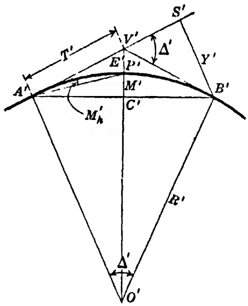


Fig. 14b.

43. Figs 14*a*, 14*b*. As between two curvs of **equal length,** L_a or L_c, the sharper (Fig 14*a*) of course has the shorter radius, R; but it has the *greater sweep*, Δ, and the *greater* values of T, M, E, M_h and Y. (Compare ¶ 40.) These values may be found by means of the equations in p 891, ¶ 34.

44. Fig 15. **Sub-chains,** c_s, c_i, c_f.* See ¶ 24. Let the curv begin at A, w ft beyond y, and end at B.

*Where it is necessary to distinguish betw the initial and the final sub-chain, we use the subscripts, i and f, respectively. Otherwise we use the subscript, s.

If a b = a diminished chain = *nearly* 100 ft, and $a\,m\,b$ = a unit arc = 100 ft (see ¶20), we have:—

initial sub-arc, $A\,a = 100 - w$;

initial sub-angle, $d_i = D \dfrac{100 - w}{100}$;

initial sub-chain, $c_i = A\,a = 2\,R \sin(d_i/2) = c \dfrac{\sin(d_i/2)}{\sin(D/2)}$ . . (48)

Having located b, we have:—

final sub-angle, $d_f = b\,O\,B = 2\,b\,A\,B$;

final sub-arc, $b\,B = 100 \dfrac{d_f}{D}$;

final sub-chain, $c_f = b\,B = 2\,R \sin(d_f/2) = c \dfrac{\sin(d_f/2)}{\sin(D/2)}$;

$x = B\,z = 100 -$ (final sub-arc, $b\,B$) . . . . (49)

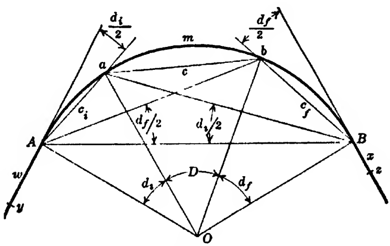


Fig. 15.

45. Fig 15 **With the full chain,** ($c = a\,b = 100$ ft; unit arc, $a = a\,m\,b$, > 100 ft) we have:—

initial sub-chain, $c_i = A\,a = 100\text{ ft} - w$;

for initial sub-angle;

$$\sin \frac{d_i}{2} = \frac{c_i}{2\,R} = c_i \frac{\sin(D/2)}{100} \quad \text{. . . . . (50)}$$

Having located b, we have, as before:—

final sub-angle, $d_f = b\,O\,B = 2\,b\,A\,B$; and

final sub-chain, $c_f = b\,B = 2\,R \sin(d_f/2) = 100 \dfrac{\sin(d_f/2)}{\sin(D/2)}$;

but $x = B\,z = 100\text{ ft} -$ final sub-*chain*, c_f . . . . (51)

46. Fig 15. **Approximate or "nominal" values of c_s* and of d. With either full or diminisht unit chain,** let

c_s = the *true* value of either sub-chain, as found above;
d = the *true* value of the corresponding sub-angle;
c = the full or diminisht chain used, as the case may be;
c_n = $c\,d/D$ = the approx or "nominal" value of c_s;
d_n = $D\,c_s/c$ = the approx or "nominal" value of d.

---

*Where it is necessary to distinguish betw the initial and the final sub-chain, we use the subscripts, i and f, respectively. Otherwise we use the subscript, s.

Then

$$c_s > c_n; \quad d_n > d; \quad \frac{c_s}{c_n} = \frac{d_n}{d} \quad \ldots\ldots\ldots (52)$$

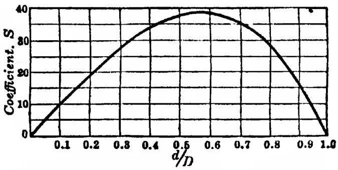
and, for 100 ft chain or 100 ft unit arc, approx,

$$c_s - c_n = S(Q - 1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

where Q = value of $\frac{\text{arc}}{\text{chord}}$ (table, ¶ 37) for the given value of D,

and S = a coefficient as per Fig 16, below.

Fig. 16.



Coefficient, S, for deducing true value, c_s, of sub-chain, from its "nominal" value, $c_n = cd/D$.

47. For any value, c, of the chain or unit arc other than 100 ft, we have:—

$$c_s - c_n = S\ (Q - 1)\ c/100 \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

48. From ¶ 27, we have, approx:—

$\sin 0°\ 1'$: $\sin(d/2)$ = 1 min : $d/2$ in mins; or

$$d/2 \text{ in mins} = \sin(d/2) \div 0.0002909 = 3437\ \frac{\text{sub-chain}}{2\ R} \ldots (55)$$

49. If, in a given circular curv, the full chain and the sub-chain be divided into the same number of equal parts, their ordinates are approx as the squares of the chain lengths.

COMPOUND CURVS

Compare Reverse Curvs, ¶ ¶ 70, etc.

Definitions

50. Figs 17, 18. Compound curvs. When two consecutiv curvs, AP and PB, of unequal radii, R_g and R_s, curv in the same direction (both to the right or both to the left), they are said to be compounded, and the entire curv, APB, is called a compound curv.

51. Branches. The two portions, AP and PB, of the compound curv, are called its branches. They lie on the same side of the common tangent, $v_g\,v_s$. Their meeting point, P, or common tangent point, is the point, "C. C." of change from *curv* to *curv*. It is also called the "P. C. C." or point of compound curvature. See ¶ 2.

52. Radii. The centers, O_g and O_s, of the two branches, are necessarily in a straight line with the P. C. C. (P): *i. e.*, the two radii, being normal to the common tangent, $v_g\,v_s$, at P, coincide in $O_s\,P$. At A and at B, the radii are normal respectively to the semitangents, $A\,V$, $V\,B$, of the entire curv.

53. The vertex, V, of the entire compound curv, is usually *not* (as it is in Fig 17) in $O_g\,P$ produced. See ¶ ¶ 57 and 146.

54. Subscripts. R_g is the *greater* radius; and its sweep, semitangent, etc., are lettered Δ_g (or simply g), T_g, etc., respectivly; and similarly for the *shorter* radius, R_s, with its sweep, Δ_s, (or s), semitangent, T_s, etc.

55. The semitangents of the entire compound curv, $A\ P\ B$, are T_g and T_s, perpendicular respectivly, at A and at B, or vice versa, to the radii, R_g and R_s. The semitangents of the two branches, respectivly, are $t_g = v_g P$, and $t_s = P v_s$.

56. If (as usual, and as in Figs 17, 18) $\Delta < 180°$, we find $T_g > T_s$; but, when $\Delta > 180° < 360°$, $T_g < T_s$. See ¶ 69. If $\Delta = 180°$, T_g and T_s are infinit. and there is no vertex, V.

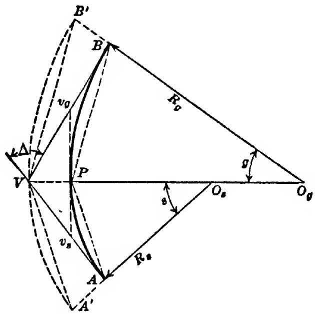


Fig. 17.

Equations

57. Fig 17. In the **special case** where the vertex, V, lies in the common radial line, $O_g\ P$, produced, we have

$$V\ P = R_g \text{ exsec } g = R_s \text{ exsec } s.$$

Hence

$$\frac{R_g}{R_s} = \frac{\text{exsec } s}{\text{exsec } g} = \frac{1 - \cos s}{\cos s} \cdot \frac{\cos g}{1 - \cos g} \ldots\ldots (56)$$

Hence, in this special case, we have $s > g$, or the *shorter* radius, R_s, has the *greater* sweep, s, and vice versa.

In this case, also, we have :—

$T_g = R_g \tan g$; $T_s = R_s \tan s$;

$$\begin{aligned} T_g \tan(g/2) &= T_g \tan V\ B\ P = T_g \tan B'V\ B \\ &= B'B = V\ P = A'A \\ &= T_s \tan(s/2)\text{ ; or} \end{aligned}$$

$$\frac{T_g}{T_s} = \frac{\tan(s/2)}{\tan(g/2)} \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Sweeps, Δ_g, Δ_s, Δ.

58. Figs 18. The sweeps, Δ_g (or g) and Δ_s (or s) of the two branches, may be equal or unequal; and, in general, either may be the greater; but see the special case of ¶ 57.

Since both branches curv in the same direction (right or left), we have, for the sweep, Δ, of the entire compound curv, $A\ P\ B$:—

$$\Delta = g + s; \quad g = \Delta - s; \quad s = \Delta - g \ldots (58)$$

See also eq (60).

In Fig 18a	In Fig 18b
Draw $Puw \perp O_s B$. Then sin s	Draw $Puw \perp O_g A$. Then sin g

$$= \frac{P\,w}{R_g} = \frac{P\,u}{R_s} = \frac{u\,w}{O_g\,O_s} = \frac{P\,w - P\,u}{R_g - R_s} \quad \ldots . (59)$$

59. Angles, A and B. Let $VAB = A$; $VBA = B$. Then: $\Delta\ (= 180° - AVB) = A + B$; $A = \Delta - B$; $B = \Delta - A$ . . . (60)

See also eq (58).

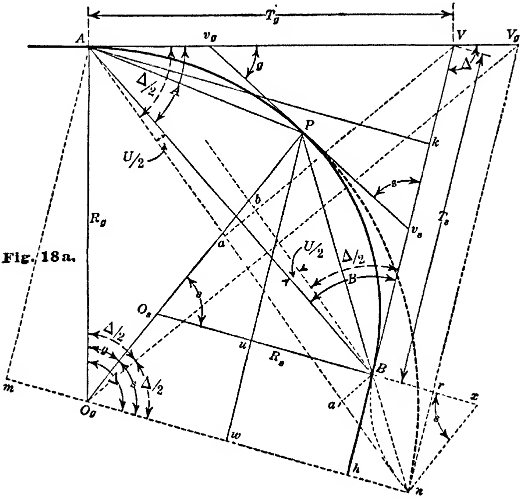


Fig. 18a.

60. Angles V A P and V B P.

$$VAP = g/2; \qquad VBP = s/2;$$
$$VBP + VAP = s/2 + g/2 = (s + g)/2 = \Delta/2$$
$$VBP - VAP = s/2 - g/2 = (s - g)/2 = \Delta/2 - g \quad . . (61)$$

61. Long chord, C_{delta} = **A B.** See also ¶ 64.

In Fig 18a	In Fig 18b
Draw $A\,k \perp B\,V$. Then $A\,k = C_{delta} \sin B = T_g \sin \Delta$	Draw $B\,k \perp A\,V$. Then $B\,k = C_{delta} \sin A = T_s \sin \Delta$

and $C_{delta} = T_g \sin \Delta / \sin B = T_s \sin \Delta / \sin A$ . . . . (62)

See also eqs (66) and (67)

Fig. 18a.	Fig 18b.
62. Produce branch $A P$ to n, making $O_g n \parallel O_s B$. Then P, B and n	Produce branch $B P$ to n, making $O_s n \parallel O_g A$. Then P, n and A

are in a straight line.

Angle, U/2.

Join $A\,n$, and draw $n\,V_g \parallel B\,V$.	Join $B\,n$, and draw $n\,V_s \parallel A\,V$.
Draw $B\,b \parallel A\,n$.	Draw $A\,a \parallel B\,n$.
Let $B\,A\,n\ (= A\,B\,b) = U/2$.	Let $A\,B\,n\ (= B\,A\,a) = U/2$.

Then $A = \Delta/2 - U/2 = (\Delta - U)/2$

$B = \Delta/2 + U/2 = (\Delta + U)/2$ ......(63)

Adding, we have $A + B = \Delta$, as in eq (60);

Subtracting, we have

$B - A = U;\quad B = A + U;\quad A = B - U;$

$U = \Delta - 2A = 2B - \Delta$ ....(64)

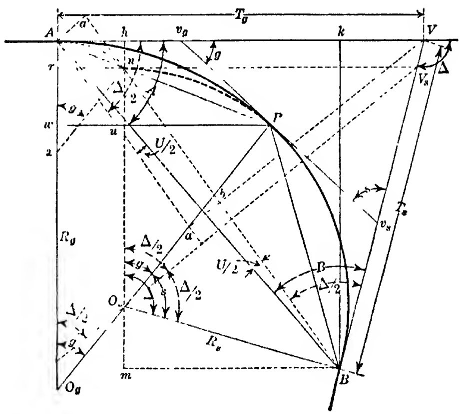


Fig. 18b.

63. Then

In Fig 18a	In Fig 18b
$O_g V_g \perp A\,n$, bisects $A\,n$; $A\,O_g\,n = \Delta$; and $A\,O_g\,V_g = V_g\,O_g\,n = \Delta/2$.	$O_s V_s \perp B\,n$, bisects $B\,n$; $B\,O_s\,n = \Delta$; and $B\,O_s\,V_s = V_s\,O_s\,n = \Delta/2$ ..(65)

64. Long chord, C_{delta} = A B. See also ¶ 61.

Fig. 18a.	*Fig. 18b.*
Draw $V\,a$ and $B\,a' \parallel O_g V_g \perp A\,n$. Then $C_{delta} \sin(U/2) = B\,a'$	Draw $V\,a$ and $A\,a' \parallel O_s V_s \perp B\,n$. Then $C_{delta} \sin(U/2) = A\,a'$

$$= V\,a - V\,b$$
$$= T_g \sin(\Delta/2) - T_s \sin(\Delta/2)$$
$$= (T_g - T_s) \sin(\Delta/2).$$

Hence

$$C_{delta} = (T_g - T_s)\,\frac{\sin(\Delta/2)}{\sin(U/2)} \;\ldots\ldots\ldots\ldots (66)$$

See also eqs (62) and (67).

We have, also

$C_{delta} = \sqrt{A\,k^2 + B\,k^2} =$ $\sqrt{(T_g \sin\Delta)^2 + (T_s - T_g \cos\Delta\dagger)^2}$	$C_{delta} = \sqrt{B\,k^2 + A\,k^2} =$ $\sqrt{(T_s \sin\Delta)^2 + (T_g - T_s \cos\Delta\dagger)^2}$ ....(67)

See also eqs (62) and (66).

65. Semitangents, T_g and T_s. Draw $n\,x \parallel = O_g O_s = R_g - R_s$,

meeting $O_s B$ produced, in x Then $B\,x\,n = s$. Draw $n\,r$ and $A\,m \parallel B\,V \perp O_s\,x$. Then $Ak = T_g \sin\Delta$ $= mn - hn$ $= mn - Br$ $= R_g \text{ vers}\Delta - (R_g - R_s) \text{ vers } s$	meeting $O_g A$ in x. Then $A x n = g$. Draw $n\,r$ and $B\,m \parallel A\,V \perp O_g\,x$. Then $B\,k = T_s \sin\Delta$ $= mn + hn$ $= mn + Ar$ $= R_s \text{ vers}\Delta + (R_g - R_s) \text{ vers } g$ ....(68)

66. Branch semitangents, t_g and t_s; and **common tangent,** $v_g\,v_s$.

$$\begin{matrix} t_g & (= v_g P) & = & R_g \tan(g/2) \\ t_s & (= v_s P) & = & R_s \tan(s/2) \end{matrix}$$

Common tangent, $v_g\,v_s$, $= t_g + t_s$ ....(69)

$B\,n = 2(R_g - R_s) \sin(s/2)$.	$A\,n = 2(R_g - R_s) \sin(g/2)$ ....(70)

67. Figs 18*a*, 18*b*. Eqs (58) and (68) enable us to find required elements of compound curvs, other elements being given. See examples, below.

Examples.

Given	**Required** elements indicated by **bold** type
$R_g\ R_s\ g\ s$	Δ is given immediately by Eq (58) and **T_g** and **T_s** by Eq (68)
$R_s\ T_s\ \Delta\ s$	**g** $= \Delta - s$, eq (58) **R_g** $\text{vers } g = T_s \sin\Delta + R_s \text{ vers } g - R_s \text{ vers }\Delta$..(71) **$T_g$** $\sin\Delta = R_g \text{ vers }\Delta - (R_g - R_s) \text{ vers } s$ ......(72)

† See ¶ 69.

R_g T_g Δ g	$s = \Delta - g$, eq (58) $R_s \text{ vers } s = R_g \text{ vers } s + T_g \sin\Delta - R_g \text{ vers } \Delta$ . . . (73) $T_s \sin \Delta = R_s \text{ vers } \Delta + (R_g - R_s) \text{ vers } g$ . . . . . . (74)
R_g R_s T_g Δ	$(R_g - R_s) \text{ vers } s = R_g \text{ vers } \Delta - T_g \sin \Delta$ . . (75) $g = \Delta - s$, eq (58) $T_s = R_g \sin \Delta - (R_g - R_s) \sin s - T_g \cos \Delta^*$ . . . (76)
R_s T_g T_s Δ	$\text{Tan } (g/2) \times (T_g + T_s \cos \Delta^* - R_s \sin \Delta)$ $= T_s \sin \Delta - R_s \text{ vers } \Delta$ . . . . (77) $s = \Delta - g$, eq (58) $R_g \sin g = R_s \sin g + T_g + T_s \cos \Delta^* - R_s \sin \Delta$. (78)
R_g T_g T_s Δ	$\text{Tan } (s/2) \times (R_g \sin \Delta - T_g \cos \Delta^* - T_s)$ $= R_g \text{ vers } \Delta - T_g \sin \Delta$ . . . . (79) $g = \Delta - s$, eq (58) $R_s \sin s = R_g \sin s - R_g \sin \Delta + T_g \cos \Delta^* + T_s$. (80)

68. Figs 18. Given angles, A and B, long chord $C_{delta} = A\ B$, and either R_g or R_s, to find **g, s and the other radius.** Find $\Delta = A + B$; T_g and T_s by eq (62).

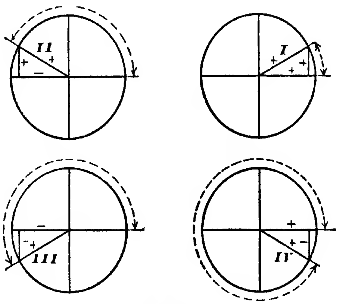


Fig. 19.

Then, having R_g (or R_s), T_g, T_s and Δ, find g, s and R_s (or R_g) by eqs (77) to (80).

69. Angles over 90°. When, owing to the considerable sweep of the curv, or to other causes, any of the angles involvd exceeds 90°, attention must be given to the fact that the algebraic signs (+ and −) vary with the several quadrants of the circle. See "Positiv and negativ signs", p 97*a*.

Thus, Fig 19

In quadrant	I	II	III	IV
Including angles from	0° to 90°	90° to 180°	180° to 270°	270° to 360°
Sine and cosecant are Tangent and cotangent are Secant and cosine are	plus plus plus	plus minus minus	minus plus minus	minus minus plus

*In our Figs 18, $\Delta > 90°$, and cos Δ is therefore ***negativ.*** See ¶ 69.

REVERSE CURVS

Definitions

70. Figs 20-23. Compare Compound Curvs, ¶ ¶ 50-69. When two consecutiv circular curvs, AP and PB, of equal or of unequal radii, R_a and R_b, curv in *opposit* directions (either one to the right, and the other one to the left) they are said to be reverst, and the combination is called a reverse curv. Reverse curvs are sometimes unavoidable, as in some crossovers and turnouts; but elsewhere they should be avoided.

71. The two branches, AP and PB, lie on *opposit* sides of the common tangent, $v_a v_b$. Their meeting point, P, or the common tangent point, is the point, "C C." of change from *curv* to *curv;* or point of reverse curvature, "P. R. C." See ¶ 2.

72. The centers, O_a and O_b, of the two branches, are necessarily in a straight line with the C. C. (P), **the two radii** being normal to the common tangent, $v_a v_b$, at P. At A and at B, the radii are normal respectivly to the tangents, AV, VB, of the entire curv. The vertex, V, is on the side of *the greater sweep.*

73. The semitangents of the entire reverse curv are $T_a = AV$ and $T_b = BV$. See ¶ 77 The semitangents of the two branches, respectivly, are $t_a = v_a P = v_a A$, and $t_b = v_b P = v_b B$. **Common tangent,** $v_a v_b = t_a + t_b$.

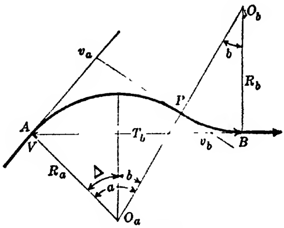


Fig. 20a.

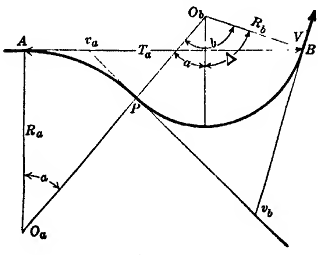


Fig. 20b.

74. Figs 20.
Fig 20*a*. If V coincides with A, we have

$$T_a \ (= AV) = 0, \text{ and } T_b \ (= VB) = AB.$$

Fig 20*b*. If V coincides with B, we have

$$T_b \ (= BV) = 0, \text{ and } T_a \ (= VA) = AB.$$

75. The sweeps, Δ_a and Δ_b (or, simply, a and b) of the two branches, may be equal or unequal, and either may be the greater.

76. Since the two branches deflect in *opposit* directions, the resultant deflection, Δ, due to the reverse curv as a whole, is

$$\Delta = \textit{difference} \text{ between } a \text{ and } b \quad \text{.......... (81)}$$

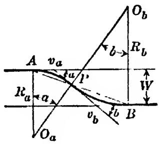


Fig. 21.

77. Fig 21. When the tangents are parallel, we have $a = b$, and Δ ($= a - b$) $= 0$. Then $T_a (= A\,V)$ and $T_b (= B\,V)$ are infinit, there being no vertex, V. See ¶ 79.
Also, P is in the line $A\,B$.

Equations

78. Figs 22 and 23. **Semitangents,** $A\,v_a$ and $v_b\,B$, **non parallel.** The P. R. C (P) is not in $A\,B$.

Radii unequal. Figs 22.	**Radii equal.** Figs 23. $R_a = R_b = R$

From the semi tan, $A\,v_a$, at A, lay off $X\,A\,D = Z\,D\,A = \Delta/2$.

Measure chord $A\,D = 2\,R_a \sin(\Delta/2)$	Measure chord $A\,D = 2\,R \sin(\Delta/2)$.

Then D is a point in the curv $A\,P$ (produced, Figs 22*b*, 23*b*). Draw $O_a\,D \parallel O_b\,B$ Then $A\,O_a\,D = \Delta$. Thru D, draw tangent, $Z\,N \parallel B\,V$, and distant $D\,B' = W$, from $B\,V$ From D lay off arc $D\,P$ ($D\,O_a\,P = b$), locating P

Radii unequal:

Measure the chords, $D\,P$ and $P\,B$. Then

$$D\,B = D\,P + P\,B$$
$$= W/\sin(b/2)$$
$$= 2\,(R_a + R_b)\sin(b/2) \quad \text{.... (82)}$$
$$R_b = R_a\,P\,B/D\,P$$
$$= \frac{D\,B}{2\sin(b/2)} - R_a \quad \text{.. (83)}$$

Common tangent,

$$v_a\,v_b = t_a + t_b$$
$$= R_a \tan\frac{a}{2} + R_b \tan\frac{b}{2} \quad \text{.... (84)}$$

Radii equal:

Measure the chords, $D\,P = P\,B$. Then

$$D\,B = D\,P + P\,B$$
$$= 2\,D\,P = 2\,P\,B$$
$$= W/\sin(b/2)$$
$$= 4\,R\sin(b/2) = 2\sqrt{W\,R} \quad \text{.... (82')}$$
$$R = R_a = R_b$$
$$= \frac{D\,P}{2\sin(b/2)} = \frac{P\,B}{2\sin(b/2)} \quad \text{.... (83')}$$

Common tangent,

$$v_a\,v_b = t_a + t_b$$
$$= R\left(\tan\frac{a}{2} + \tan\frac{b}{2}\right) \quad \text{.... (84')}$$

$$\sin D\,B\,v_b = \sin(b/2) = W/D\,B \quad \text{............ (85)}$$

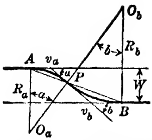
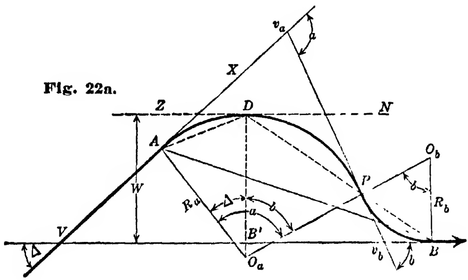


Fig. 21 (Repeated.)

79. Fig. 21. Semitangents, Av_a, v_bB, **parallel.**
$A\,B$ intersects the common tangent, $v_a\,v_b$, in the C. C. (P). $a = b$; $\Delta = a - b$ (or $b - a$) $= 0$; T_a and T_b infinit; $BAv_a = ABv_b = a/2 = b/2$.

Radii unequal, Fig 21.	Radii equal, $R_a = R_b = R$.
$W = (R_a + R_b)$ vers a	$W = 2\,R$ vers a
$= (R_a + R_b)$ vers b	$= 2\,R$ vers b
$= A\,B \sin(a/2)$	$= A\,B \sin(a/2)$
$= A\,B \sin(b/2)$ .....(86)	$= A\,B \sin(b/2)$ .....(86′)
$A\,P$	$A\,P = P\,B = A\,B/2$
$= 2\,R_a \sin(a/2)$	$= 2\,R \sin(a/2)$
$= 2\,R_a\,W/A\,B$ ......(87)	$= 2\,R\,W/A\,B$ ....(87′)
$P\,B = 2\,R_b \sin(b/2)$	$= 2\,R \sin(b/2)$
$= 2\,R_b\,W/A\,B$ ......(88)	$= 2\,R\,W/A\,B$ ....(88′)
$A\,B = A\,P + P\,B$	$A\,B = 2\,A\,P = 2\,P\,B$
$= 2\,(R_a + R_b) \sin(a/2)$	$= 4\,R \sin(a/2)$
$= \sqrt{2\,W\,(R_a + R_b)}$ ..(89)	$= \sqrt{4\,W\,R} = 2\,\sqrt{W\,R}$ ....(89′)
	$R = A\,B^2/4\,W$ ....(90)

Fig. 22a.



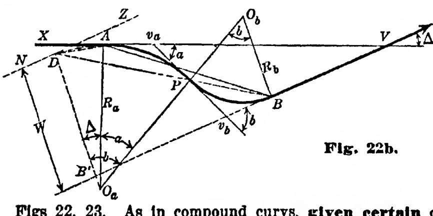


Fig. 22b.

80. Figs 22, 23. As in compound curvs, **given certain** of the seven elements, R_a, R_b, T_a, T_b, a, b, Δ; **the others** may be found. For instance:—

Given	Required quantities indicated by **bold type**
	In all cases, $\Delta = a - b$ or $b - a$.
$R_a\ R_b\ T_a\ \Delta$	$\mathbf{vers\ b} = \dfrac{R_a \text{ vers } \Delta + T_a \sin \Delta}{(R_a + R_b)}$ .... (91)
	$\mathbf{T_b} = T_a \cos \Delta + R_a \sin \Delta \pm (R_a + R_b) \sin b$ ... (92)
$R_a\ R_b\ T_b\ \Delta$	$\mathbf{vers\ a} = \dfrac{R_b \text{ vers } \Delta + T_b \sin \Delta}{(R_a + R_b)}$ .... (93)
	$\mathbf{T_a} = T_b \cos \Delta + R_b \sin \Delta \pm (R_a + R_b) \sin a$ ... (94)

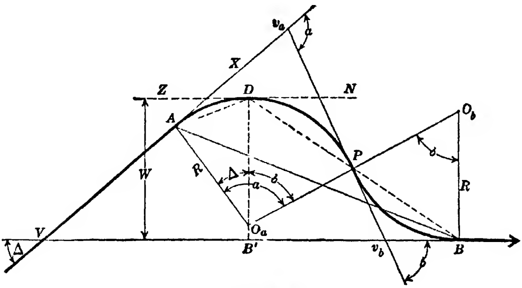


Fig. 23a.

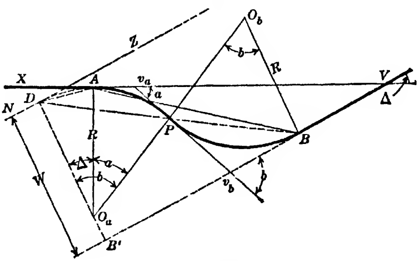


Fig. 23b.

CURV LOCATION.

81. Fig 24. By Peripheral or Deflection Angles. See ¶¶ 4, &c. Example. Instrument at the T.C. (*A*), or at the C.T. (*B*). (For other points, see ¶¶ 84 &c.)

(1) If the chaining begins at A.

To locate	Inst at A — Inst at B — Lay off angle	Chains†
A	—— $VBA^* = \Delta/2 = d_i/2 + 3\,D/2 + d_f/2$	*
a	$VAa = ABa = AOa/2 = d_i/2$	$c_i = Aa$
b	$VAb = ABb = AOb/2 = d_i/2 + 1\,D/2$	$c = ab$
c	$VAc = ABc = AOc/2 = d_i/2 + 2\,D/2$	$c = bc$
d	$VAd = ABd = AOd/2 = d_i/2 + 3\,D/2$	$c = cd$
B	VAB —— $= \Delta/2 = d_i/2 + 3\,D/2 + d_f/2$	$c_f = dB$

(2) If the chaining begins at B.

To locate	Inst at A — Inst at B — Lay off angle	Chains
B	VAB^* —— $= \Delta/2 = d_f/2 + 3D/2 + d_i/2$	*
d	$BAd = VBd = BOd/2 = d_f/2$	$c_f = Bd$
c	$BAc = VBc = BOc/2 = d_f/2 + 1D/2$	$c = dc$
b	$BAb = VBb = BOb/2 = d_f/2 + 2D/2$	$c = cb$
a	$BAa = VBa = BOa/2 = d_f/2 + 3D/2$	$c = ba$
A	—— $VBA = \Delta/2 = d_f/2 + 3D/2 + d_i/2$	$c_i = aA$

82. For the tangent, at any given station, as B, we have:—
Sighting back to the T.C., A;
$zBx = VBA = VAB =$ sum of angles laid off, at A, from tan, AV, to given sta, B;

Sighting back to another station, as b;
$zBy = VBb = cbB =$ sum of angles laid off, at b, from tan, bc, to given sta, B.
etc, etc.

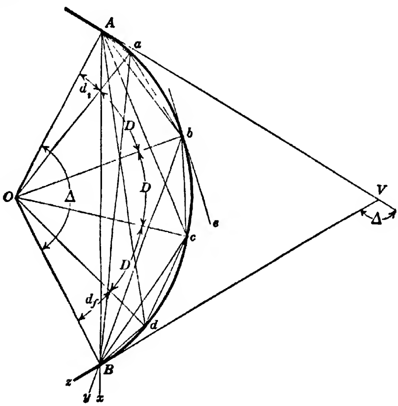


Fig. 24.

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*Sighting from either end of the curv, to the other end, where the chaining begins, the desired point is located by the intersection of the line of sight with the tan, BV or AV.

†The chains given, c_i, c, etc, refer to their several *chain* angles, VAa, aAb, bAc, etc. The *total* angle, VAc, for instance, requires chains $c_i + c + c$.

Moving the Instrument. Transit Points.*

83. Fig 24. When the sum, *V A d*, of the angles, laid off from the tangent with the inst at one point, as *A*, exceeds say 15° or 20°, then, with the rear end of the chain held at the last preceding sta, as *c*, the free end of the chain may deviate materially from its correct position, at *d*, without detection by the transitman; and this difficulty is increast by the increast length of sight.

84. In this case, or when an obstacle prevents the laying-off of the next angle, the instrument is moved to another point ("transit point"*), on the curv, and the work is continued thence.

85. Liability to error is much reduced by pursuing, thruout, a systematic method of procedure, as below.

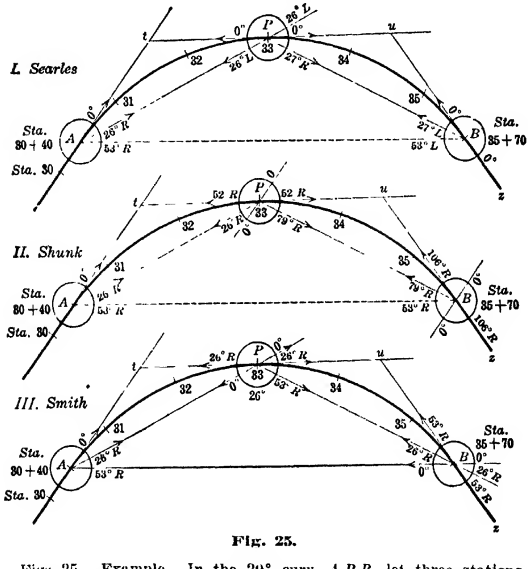


Fig. 25.

Figs 25. Example. In the 20° curv, *A P B*, let three stations, Nos. 31, 32 and 33 (*P*), be set from the T.C. (*A*, sta 30 + 40); then three stations, Nos. 34, 35 and 35 + 70 (*B*), from *P*; and then let the final tangent, *Bz*, be located from the C.T. (*B*, sta 35 + 70).

The circles, at *A*, at *P* and at *B*, in each Fig, show the orientation of the inst, the angles turnd and the resulting vernier readings.

In each of the three Figs (see Figs 25, I) let

angle $t A P = 0.6 \times 10° + 10° + 10° = 26°$;

" $u P B = 10° + 10° + 0.7 \times 10° = 27°$.

*Such points are often called "turning points," but this is liable to confusion with the "turning points" used in *leveling*.

Three methods of procedure are in common use in connection with the removal of the transit to a new inst point. These may be called :—

I. Searles. See "Field Engineering," pp 53, 57, etc.
II. Shunk. See "The Field Engineer," pp 66, etc.*
III. Smith. See Eng News, 1876 Aug 26, p 277, reprinted 1888 Sep 29, p 245 ;* also Cross's "Engineer's Field Book," pp 56 etc.

Method I (Searles) is sufficiently described in the first of the three sections of the comparativ table in ¶ 86.

Method II (Shunk) may be made clearer by the following application of it to the same case See Figs 25, II. Here all the angles are to the right (R) of zero.

Inst at	A	P	B
Sight to old point,	t	A	P
Vernier read'g on old point,	0°	$tAP = tPA = 26°$	$2\,tAP + uPB = 52° + 27° = 79°$
Add	0°	$APt = tAP = 26°$	$PBu = uPB = 27°$
Sight to tan,	At	tu	uz
Vernier reading on tan,	0°	$tAP + APt = 2\,tAP - 0° = 2 \times 26° - 0° = 52°$	$79° + 27° = 2 \times 79° - 2\,tAP = 158° - 52° = 106°$
Add	$tAP = 26°$	$uPB = 27°$	
Sight to new point,	P	B	
Vernier read'g on new point,	$tAP = 0° + 26° = 26°$	$2\,tAP + uPB = 52° + 27° = 79°$	
Move to new point,	P	B	

After removing the instrument from an old point, p (not shown), to a new point, p', the vernier reading, C, which will place the telescope in the new tangent, is determind thus :—Let

A = vernier reading on sighting new point, p', from p;
B = vernier reading on sighting along old tangent, at p;
C = vernier reading on sighting along new tangent, at p'.

Then,

$$C = 2A - B.$$

Method III (Smith) is described in ¶ ¶ 89 to 91.

86. The following table outlines the method of procedure by the three methods respectivly.
U. M. = upper motion of transit, L. M. = lower motion of transit.
R = right; L = left.

*In his 9th edition, 1890, Mr. Shunk described the "Smith" method (III), accrediting it to Mr. Robert Burgess, C.E., who described it in Eng News, 1888 Sep 22.

†For the sake of uniformity, we here assume that, in using method III (Smith), we move the inst either to P or to B, and that, after so moving, we sight to the same stations (A and P, respectivly) as in methods I and II; but, in fact, with method III, the condition, described in ¶ 90, will obtain, no matter to what station we remove, or to what station we sight from it. See ¶ 91.

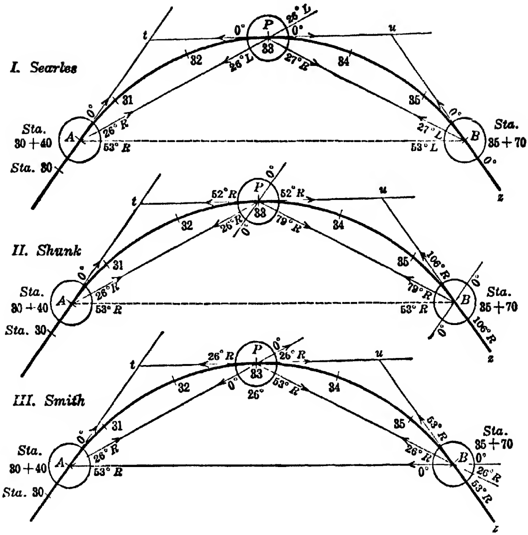


Fig. 25. (Repeated.)

1 Sight'g from	2 U. M. Vernier reads	3 Move to Sta.	4 U. M. with vern'r read'g	5 L. M. Sight to Sta.	6 U. M. Set vern'r to	7 Inst will be in tan thru
	Method I, Searles.					
A to *t*	0° + 0° = 0°					
A to *P*	0° + 26° = 26°	*P*	26° *L*	*A*	0°	*P*
P to *B*	0° + 27° = 27°	*B*	27° *L*	*P*	0°	*B*
	Method II, Shunk.					
A to *t*	0° + 0° = 0°					
A to *P*	0° + 26° = 26°	*P*	26° *R*	*A*	52° *R*	*P*
P to *B*	52° + 27° = 79°	*B*	79° *R*	*P*	106° *R*	*B*
	Method III, Smith.†					
A to *t*	0° + 0° = 0°					
A to *P*	0° + 26° = 26°	*P**	0°	*A**	26° *R*	*P*
P to *B*	26° + 27° = 53°	*B**	26° *R*	*P**	53° *R*	*B*

*See † foot-note, p. 924.
†See ¶¶ 89 to 91.

87. Fig 25. It will be noticed that, when the **vernier is at zero,**

In method	**the telescope is**
I. Searles, II. Shunk, III. Smith,	in the tangent thru the point occupied; parallel with the original tangent, *A t*; sighting back to *A*, whose recorded angle (¶ 89) is zero.
	When the inst is at *A*, these three collimation lines coincide.

88. For Figs 25, **the vernier readings,** by the three methods respectively, compare as follows:—

Sighting	Method		
	I Searles	II Shunk	III Smith*
	Vern'r readings. R = right; L = left.		
From sta 30 + 40 (*A*)			
to sta 30 + 40 (*A*) (tan)	0°	0°	0°
to sta 31	6° *R*	6° *R*	6° *R*
to sta 32	16° *R*	16° *R*	16° *R*
to sta 33 (*P*)	26° *R*	26° *R*	26° *R*
to sta 35 + 70 (*B*)	53° *R*	53° *R*	53° *R*
From sta 33 (*P*)			
to sta 30 + 40 (*A*)	26° *L*	26° *R*	0°
to sta 33 (*P*) (tan)	0°	52° *R*	26° *R*
to sta 34	10° *R*	62° *R*	36° *R*
to sta 35	20° *R*	72° *R*	46° *R*
to sta 35 + 70	27° *R*	79° *R*	53° *R*
From sta 35 + 70 (*B*)			
to sta 30 + 40 (*A*)	53° *L*	53° *R*	0°
to sta 33 (*P*)	27° *L*	79° *R*	26° *R*
to sta 35 + 70 (*B*) (tan)	0°	106° *R*	53° *R*

89. In Method III (Smith), Figs 25, III, before beginning to run the curv, calculate, and enter in the field book (as in last column of table, ¶ 88), opposite each sta (as 30 + 40, 31, etc), and opp any other points, in the curv, which, for any reason, it may be thought desirable to locate, the total angle (as *t A t* = 0°, *t A* 31, *t A P*, *t A B*, etc), betw the *original* tangent, *A t*, and the chord, *A*-31, *A*-*P*, *A*-*B*, etc) from *A* to such sta, *as tho each of the points, thruout the entire curv, were to be located from the point of curv, A.*

90. Then, after setting up at any point whatever in the *curv*, set the vernier to the angle so recorded in the field book opp *any* other convenient point, *x*, in the curv, and sight to *x*, thus orienting the inst. Then, if the vernier be set, by the upper motion, to the reading recorded, as above, opp any *other* point, *y*, in the curv, the inst will be sighted at *y*. With the vernier at the angle recorded opp the *point occupied*, the inst is in the *tan thru that point.*

*For the sake of uniformity, we here assume that, in using method III (Smith), we move the inst either to *P* or to *B*, and that, after so moving, we sight to the same stations (*A* and *P*, respectivly) as in methods I and II; but, in fact, with method III, the condition, described in ¶ 90, will obtain, no matter to what station we remove, or to what station we sight from it. See ¶ **91.**

91. If the sharpness of the curv is exprest in a whole number of degrees, if the curv begins with a sub-chain, and if many stations are to be set, it may be worth while to **make an arbitrary addition to each angle,** so as to give, to the full stations, angles exprest in a whole number of degrees (or in degs and half degs); thus simplifying the calculations.† For instance, in a 3° curv, if the actual angles be as in line A, below, we may add 0° 18′ to each angle, making them read as in line *B*.

Station	30 + 20 (*A*)	31	32	33	34	34 + 40 (*B*)
A	0°	1° 12′	2° 42′	4° 12′	5° 42′	6° 18′
B	0° 18′	1° 30′	3° 0′	4° 30′	6° 0′	6° 36′

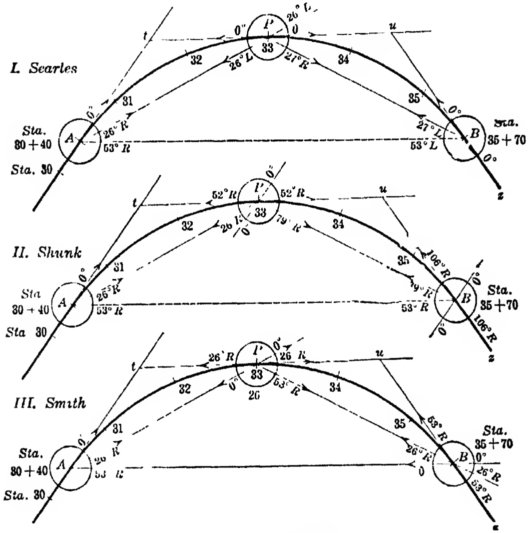


Fig. 25. (Repeated.)

Miscellaneous Methods.

92. Locating a curv without a transit. Fig 26. Let *a b*, *b c*, be chains; *a b* = *b c* = *c*; let *a t* and *b n* be tangents at *a* and at *b*, respectively. Produce chord *a b* to *f*, and make *a t* = *b f* = *a b* = *c*. Then :—

$$t\,b = 2\,c\sin(D/4)\,;\qquad f\,c = 2\,c\sin(D/2) = 2\,c\frac{c}{2\,R} = \frac{c^2}{R}\,. \quad . . (95)$$

†T. Appleton. Assn Engng Socs, Jour, March 1883.

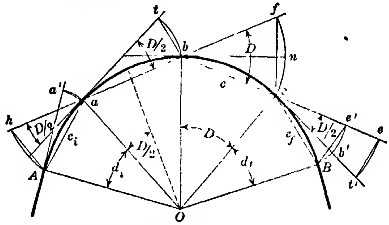
93. **The dists, $t\,b$ and $f\,c$, may be used for locating a curv without a transit.** Thus:—

(1) Having laid off $a\,t$ ($=c$) along the tangent, $a\,t$, to locate b; from a, lay off $a\,b$ ($=c$) to meet $t\,b$ laid off from t.

(2) Having laid off $b\,f$ ($=c$) along the chord $a\,b$ produced, to locate c; from b, lay off $b\,c$ ($=c$) to meet $f\,c$ laid off from f.

(3) Having laid off $c\,e$ ($=c$) along the chord $b\,c$ produced, to find the tangent, $c\,t'$, at c; from c, lay off $c\,t'$ ($=c$) to meet $e\,t'$ ($=t\,b$) laid off from e

Fig. 26.



94. For the sub-chains, $A\,a = c_i$, and $c\,B = c_f$, **sub-angles,** d_i and d_f, etc, we have the following equations.—

Distances	Angles	Approximate
$a'\,a = 2\,c_i \sin(d_i/4)$	$\sin(d_i/2) = \dfrac{c_i}{2\,R}$	$a'\,a = t\,b\,\dfrac{c_i^2}{c^2}$ . . (96)
$B\,b' = 2\,c_f \sin(d_f/4)$	$\sin(d_f/2) = \dfrac{c_f}{2\,R}$	$B\,b' = e\,t'\,\dfrac{c_f^2}{c^2}$ . . (97)
$A\,h = 2\,c_i \sin(A\,a\,h/2)$	$A\,a\,h = \dfrac{D}{2} + \dfrac{d_i}{2}$	$d_i = D\,\dfrac{c_i}{c}$ . . . . (98)
$B\,e' = 2\,c_f \sin(B\,c\,e'/2)$	$B\,c\,e' = \dfrac{D}{2} + \dfrac{d_f}{2}$	$d_f = D\,\dfrac{c_f}{c}$ . . . . (99)

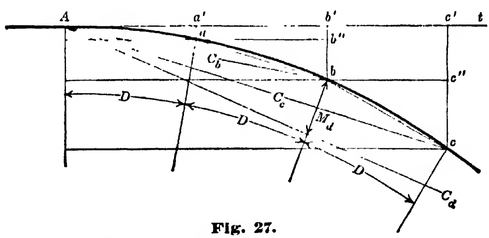


Fig. 27.

95. By Offsets from Tangent, A t. Fig 27. Let R = radius; c = chain length, $A\,a$, $a\,b$, etc; C_b, C_c, C_d, etc = long chord $A\,b$, $A\,c$, $A\,d$, etc, from A to 2d, 3d, 4th, etc station: M_b, M_c, M_d, etc = mid ord for C_b, C_c, C_d, etc; $D/2 = a'A\,a$; $3\,D/2 = b''a\,b$; $5D/2 = c''b\,c$, etc. Then

Distances on tangent

$$A\,a' = c\cos(D/2) = R\sin D \quad\ldots\ldots\ldots\ldots \quad = C_b/2 \ldots\ldots (100)$$

$$A\,b' = c\,[\cos(D/2) + \cos(3\,D/2)] = R\sin(2\,D) \quad\ldots\ldots\ldots\ldots \quad = C_d/2 \ldots\ldots (101)$$

$$A\,c' = c\,[\cos(D/2) + \cos(3\,D/2) + \cos(5\,D/2)] = R\sin(3\,D) \quad\ldots\ldots\ldots\ldots \quad = C_f/2 \ldots\ldots (102)$$

Offsets from tangent

$$a'\,a = c\sin(D/2) = c^2/(2\,R) = R \text{ vers } D = M_b \quad\ldots\ldots\ldots (100a)$$

$$b'\,b = c\,[\sin(D/2) + \sin(3\,D/2)] = C_b \sin b'\,A\,b = C_b \sin D = (C_b)^2/(2\,R) = R \text{ vers } 2\,D = M_d \quad\ldots\ldots\ldots (101a)$$

$$c'\,c = c\,[\sin(D/2) + \sin(3\,D/2) + \sin(5\,D/2)] = C_c \sin(3\,D/2) = (C_c)^2/(2\,R) = R \text{ vers } 3\,D = M_f \quad\ldots\ldots\ldots (102a)$$

Thus, let $c = 100$ ft; $D = 12°$. Then

For dist $A\,c'$, on tan	For offset, $c'\,c$ from tan
$\cos(D/2) = \cos 6° = 0.99452$	$\sin(D/2) = \sin 6° = 0.10453$
$\cos(3D/2) = \cos 18° = 0.95106$	$\sin(3D/2) = \sin 18° = 0.30902$
$\cos(5D/2) = \cos 30° = 0.86603$	$\sin(5D/2) = \sin 30° = 0.50000$
Sum of cosines = 2.81161	Sum of sines = 0.91355
$Ac' = 2.81161c = 281.16$ ft	$c'\,c = 0.91355c = 91.355$ ft

96. First, from A, line in the points, a', b', c', etc., in the tangent, $A\,t$. Then, from a', b', c', etc., lay off $a'\,a$, $b'\,b$, $c'\,c$, etc., normal to $A\,t$; or lay off $a'\,a$, $b'b$, etc, at random, to intersect $A\,a$, $a\,b$, $b\,c$, etc., each $= c$

97. If, as usual, the curv begins or ends with a sub-chain, it may be preferable to divide the curv (sweep $= \Delta$) into a convenient number, n, of equal parts, each $= \Delta/n$, and substitute Δ/n for D, and $2\,R\sin(\Delta/2n)$ for c, etc

As the work proceeds, and the angles $a\,b\,b''$, $b\,c\,c''$, etc., become more acute, the work becomes less accurate.

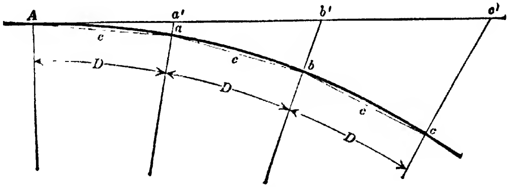


Fig. 28.

98. In Fig 28, the dists, $a\,a'$, $b\,b'$, etc., are **in line with the radii** at a, at b, etc. Here:—

$$A\,a' = R\tan D; \quad A\,b' = R\tan 2\,D; \quad A\,c' = R\tan 3\,D, \text{ etc.} \quad (103)$$

$$a\,a' = R \text{ exsec } D; \; b\,b' = R \text{ exsec } 2\,D; \; c\,c' = R \text{ exsec } 3\,D, \text{ etc.} \quad (104)$$

From A, line in a', b', c', etc. From A and from a', simultaneously, lay off c and $a'\,a$, respectivly, locating a. Then, from a and from b' respectively, lay off, simultaneously, c and $b'\,b$, locating b; and so on. See ¶ 97.

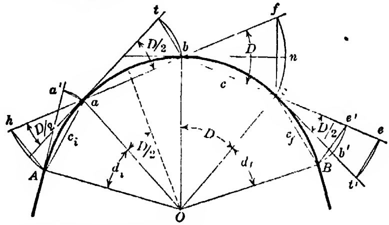
93. The dists, $t\,b$ and $f\,c$, may be used for locating a curv without a transit. Thus:—

(1) Having laid off $a\,t$ ($=c$) along the tangent, $a\,t$, to locate b; from a, lay off $a\,b$ ($=c$) to meet $t\,b$ laid off from t.

(2) Having laid off $b\,f$ ($=c$) along the chord $a\,b$ produced, to locate c; from b, lay off $b\,c$ ($=c$) to meet $f\,c$ laid off from f.

(3) Having laid off $c\,e$ ($=c$) along the chord $b\,c$ produced, to find the tangent, $c\,t'$, at c; from c, lay off $c\,t'$ ($=c$) to meet $e\,t'$ ($=t\,b$) laid off from e

Fig. 26.



94. For the sub-chains, $A\,a = c_i$, and $c\,B = c_f$, **sub-angles,** d_i and d_f, etc, we have the following equations.—

Distances	Angles	Approximate
$a'\,a = 2\,c_i \sin(d_i/4)$	$\sin(d_i/2) = \dfrac{c_i}{2\,R}$	$a'\,a = t\,b\,\dfrac{c_i^2}{c^2}$ . . (96)
$B\,b' = 2\,c_f \sin(d_f/4)$	$\sin(d_f/2) = \dfrac{c_f}{2\,R}$	$B\,b' = e\,t'\,\dfrac{c_f^2}{c^2}$ . . (97)
$A\,h = 2\,c_i \sin(A\,a\,h/2)$	$A\,a\,h = \dfrac{D}{2} + \dfrac{d_i}{2}$	$d_i = D\,\dfrac{c_i}{c}$ . . . . (98)
$B\,e' = 2\,c_f \sin(B\,c\,e'/2)$	$B\,c\,e' = \dfrac{D}{2} + \dfrac{d_f}{2}$	$d_f = D\,\dfrac{c_f}{c}$ . . . . (99)

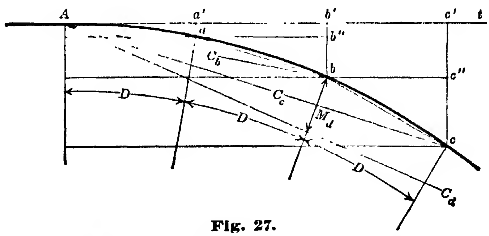


Fig. 27.

95. By Offsets from Tangent, A t. Fig 27. Let R = radius; c = chain length, $A\,a$, $a\,b$, etc; C_b, C_c, C_d, etc = long chord $A\,b$, $A\,c$, $A\,d$, etc, from A to 2d, 3d, 4th, etc station; M_b, M_c, M_d, etc = mid ord for C_b, C_c, C_d, etc; $D/2 = a'A\,a$; $3\,D/2 = b''a\,b$; $5D/2 = c''b\,c$, etc. Then

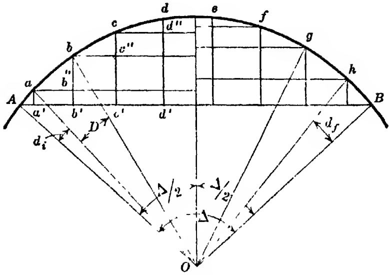


Fig. 31.

Similarly, for the ordinates, $a'\,a$, $b'\,b$, $c'\,c$, etc., we have:

$$a'\,a = c_i \sin\frac{\Delta - d_i}{2};$$

$$b''\,b = c \sin\frac{\Delta - 2\,d_i - D}{2};$$

$$c''\,c = c \sin\frac{\Delta - 2\,d_i - 3\,D}{2}; \text{ etc., and}$$

$$b'\,b = a'\,a + b''\,b; \quad c'\,c = b'\,b + c''\,c, \text{ etc.} \qquad (109)$$

102. Fig 32 *To eliminate sub-chains* (see ¶ 97), let the curv, $A\,P\,B$, be divided into a convenient number of *equal* arcs; and let

c_e = the chord, $A\,a = a\,b$, etc, subtending one of these arcs;
D_e = the central angle, $A\,O\,a$, subtended by c_e;
C, C', C'', etc = the long chords, $A\,B$, $a\,h$, $b\,g$, etc, respectivly;
M, M', M'', etc = the mid ords, $E\,P$, $E'\,P$, $E''\,P$, etc, of C, C', C'', etc, respectivly.

Then,

Abscissas

$$C/2 = A\,E = R\sin(\Delta/2);$$
$$C'/2 = a\,E' = R\sin(\Delta/2 - D_e);$$
$$C''/2 = b\,E'' = R\sin(\Delta/2 - 2\,D_e); \text{ etc.}$$

$$A\,a' = C/2 - C'/2 = c_e \cos a\,A\,a' = c_e \cos\frac{\Delta - D_e}{2};$$

$$a'\,b' = a\,b'' = C'/2 - C''/2 = c_e \cos b\,a\,b'' = c_e \cos\frac{\Delta - 3\,D_e}{2};$$

$$b'\,c' = b\,c'' = C''/2 - C'''/2 = c_e \cos c\,b\,c'' = c_e \cos\frac{\Delta - 5\,D_e}{2};$$

etc; and

$$A\,b' = A\,a' + a'\,b';$$
$$A\,c' = A\,b' + b'\,c'; \text{ etc} \qquad (110)$$

Ordinates

$$M = E\,P = R \operatorname{vers}(\Delta/2)\,;$$
$$M' = E'\,P = R \operatorname{vers}(\Delta/2 - D_c)\,;$$
$$M'' = E''\,P = R \operatorname{vers}(\Delta/2 - 2\,D_c)\,;\ \text{etc.}$$

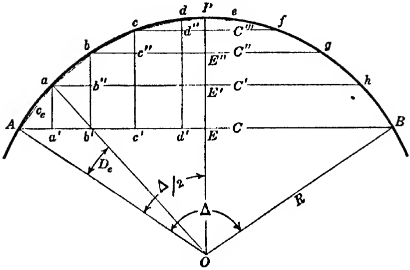


Fig. 32.

$$a'\,a = M - M' = c_c \sin\frac{\Delta - D_c}{2}\,;$$
$$b''\,b = M' - M'' = c_c \sin\frac{\Delta - 3\,D_c}{2}\,;$$
$$c''\,c = M'' - M''' = c_c \sin\frac{\Delta - 5\,D_c}{2}\,;\ \text{etc; and}$$
$$b'b = a'\,a + b''\,b = M - M''\,;$$
$$c'\,c = b'\,b + c''\,c = M - M'''\,;\ \text{etc.} \qquad (111)$$

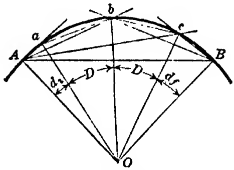


Fig. 33.

103. By Means of Two Transits, without Linear Measurement. Fig 33. Useful where chaining is difficult or impracticable, as in swamps, etc.

Given, any two stations, A and B, each visible from the other;

From each of the two stations, A and B, sight to the other, and lay off the angles found, as below, locating the points, a, b, etc., by means of the intersections of the lines of sight.

Thus, let $a\,b$, $b\,c$, be chains; $A\,a$, $c\,B$, sub-chains. Then, to locate the point b, at the intersection of $A\,b$ and $B\,b$; we have:—

$$b\,A\,B = b\,O\,B/2 = (D + d_f)/2\,;$$
$$b\,B\,A = b\,O\,A/2 = (D + d_i)/2. \qquad (112)$$

Similarly, for point c (intersection of $A\,c$ and $B\,c$,) we have:—

$$c\,A\,B = c\,O\,B/2 = d_f/2\,;$$
$$c\,B\,A = c\,O\,A/2 = (2\,D + d_i)/2 \qquad (112')$$

104. By Long Chords from the Instrument. Fig 34.

Let $a\,b = b\,c = c = 1$ chain; $A\,a = c_i =$ initial sub-chain; $c\,B = c_f =$ final sub-chain.

Then, in Fig 34;

$$t\,A\,a = d_i/2; \qquad A\,a = 2\,R \sin t\,A\,a;$$
$$t\,A\,b = (d_i + D)/2; \qquad A\,b = 2\,R \sin t\,A\,b;$$
$$t\,A\,c = (d_i + 2\,D)/2; \qquad A\,c = 2\,R \sin t\,A\,c;$$
$$t\,A\,B = (d_i + 2\,D + d_f)/2 = \Delta/2; \quad A\,B = 2\,R \sin t\,A\,B. \quad (113)$$

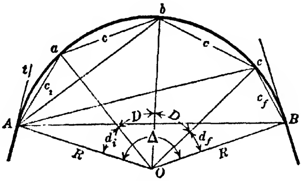


Fig. 34.

105. Platting Auxiliaries. The location of curvs upon the map is greatly facilitated by the use of *templets*, to the proper scale.

106. Wm. F. Shunk's transparent *curv protractor* shows a number of curvs to a scale of 1 inch = 400 ft

107. In Eng News 1902 Jun 19, pp 500-501, Chas H. Quimby Jr. suggests a "*curve projector and scale*" (patent applied for), which is a transparent curv protractor in which each curv is punctured at each 100 ft of its length, by scale, in order that, with a pointed instrument, corresponding points may be markt upon the map beneath.

108. Ordinates. On sharp curvs, additional points, intermediate of stations, may be located by means of ordinates to a chord joining two consecutiv stas. Usually a twine is stretched between the two consecutiv stas, and the ordinates are measured, as nearly as may be at right angles to this, by means of a graduated rod or tape. The points, along the twine, from which ordinates are to be measd (especially the middle point and the two points midway between this and the ends), are distinguisht by knots or otherwise.

109. The permissible dist apart of the ordinates of course depends upon the character of the work and upon the sharpness of the curv. For guiding the earthwork, 50 ft is short enough, or 25 ft for radii less than 1000 ft. For tracklaying, the dist may range from 25 ft on easy curvs, to 10 ft or even 5 ft on the sharpest.

See eqs (32) and (34) to (39), and table of ordinates, pp 898-901.

OBSTACLES.

Methods, other than those here suggested, may be devised to suit these and other cases.

110. Line of sight obstructed. The following suggestions refer to cases arising in the usual method of location by peripheral angles, ¶¶ 81, etc.; but the difficulty may sometimes be avoided by abandoning that method and using instead one or other of the location methods mentiond in ¶¶ 92-104. See also under Surveying, p 281, 5th "hint."

Ordinates

$$M = E\,P = R\,\text{vers}(\Delta/2)\,;$$
$$M' = E'\,P = R\,\text{vers}(\Delta/2 - D_c)\,;$$
$$M'' = E''\,P = R\,\text{vers}(\Delta/2 - 2\,D_c)\,;\ \text{etc.}$$

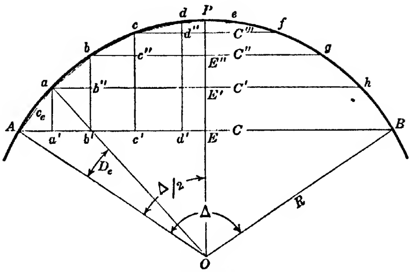


Fig. 32.

$$a'\,a = M - M' = c_c \sin\frac{\Delta - D_c}{2}\,;$$
$$b''\,b = M' - M'' = c_c \sin\frac{\Delta - 3\,D_c}{2}\,;$$
$$c''\,c = M'' - M''' = c_c \sin\frac{\Delta - 5\,D_c}{2}\,;\ \text{etc; and}$$
$$b'b = a'\,a + b''\,b = M - M''\,;$$
$$c'\,c = b'\,b + c''\,c = M - M'''\,;\ \text{etc.} \quad \ldots\ldots\ldots\ldots\ldots (111)$$

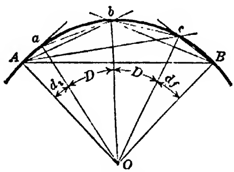


Fig. 33.

103. By Means of Two Transits, without Linear Measurement. Fig 33. Useful where chaining is difficult or impracticable, as in swamps, etc.

Given, any two stations, A and B, each visible from the other; From each of the two stations, A and B, sight to the other, and lay off the angles found, as below, locating the points, a, b, etc., by means of the intersections of the lines of sight.

Thus, let $a\,b$, $b\,c$, be chains; $A\,a$, $c\,B$, sub-chains. Then, to locate the point b, at the intersection of $A\,b$ and $B\,b$; we have:—

$$b\,A\,B = b\,O\,B/2 = (D + d_f)/2\,;$$
$$b\,B\,A = b\,O\,A/2 = (D + d_t)/2. \qquad (112)$$

Similarly, for point c (intersection of $A\,c$ and $B\,c$,) we have:—

$$c\,A\,B = c\,O\,B/2 = d_f/2\,;$$
$$c\,B\,A = c\,O\,A/2 = (2\,D + d_t)/2 \qquad (112')$$

Position of a required point inaccessible.

115. Vertex, V, inaccessible. Fig 37. Given Δ and R or D. Required the curve points, A, B. Measure the dist, $a\,b$, betw any two accessible points on the two tans respectivly. Measure the angles $z\,b\,a$ and $y\,a\,b$, and find $V\,b\,a$ ($= 180° - z\,b\,a$) and $V\,a\,b$ ($= 180° - y\,a\,b$). Then $V\,b\,a + V\,a\,b$ ($= 180° - a\,V\,b$) $= \Delta$; or $a\,V\,b = 180° - \Delta$.

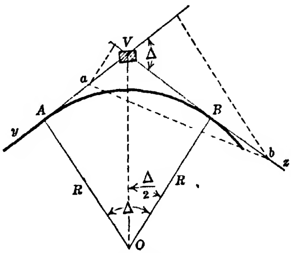


Fig. 37.

Find $V\,a = a\,b \sin V\,b\,a / \sin \Delta$;
and $V\,b = a\,b \sin V\,a\,b / \sin \Delta$. Then :—

$A\,a = A\,V - V\,a = R \tan(\Delta/2) - V\,a$;
$B\,b = B\,V - V\,b = R \tan(\Delta/2) - V\,b$.

Note that, where (as in the last case) $B\,b$ is *negativ*, b is *beyond* B; and $B\,b$ is to be laid off from b *toward* the vertex, V, to locate B.

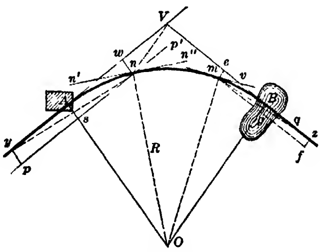


Fig. 38.

Either curv point, A or B, inaccessible.

116. T.C. (A) inaccessible. Fig 38. It is required to locate an accessible point, n, in the curv, and a tan, $n'\,n''$, thru n. The point, n, (preferably a station) must command a sight, $n\,p$, parallel to the tan, $A\,V$.

Let $A\,O\,n$ be the sweep of the arc, $A\,n$, (approx, $A\,O\,n° = D° \times A n/100$). Let $w\,n$ be normal to the tan $A\,V$, and let $n\,s = p\,s$. Then $n\,s = p\,s = w\,A = y\,A = R \sin A\,O\,n$; $w\,n = A\,s = y\,p = R \operatorname{vers} A\,O\,n$; and $V\,w = V\,A - A\,w = T - A\,w = T - n\,s$.

Find the line $n\,p$ (or $n\,p'$), and the point, n, as by one of the two following alternativ methods:—

(1) From p, run $p\,n \parallel A\,V$, making $p\,n = 2\,p\,s = 2\,R \sin A\,O\,n$. Or

(2) From V, lay off $V\,w$ to w; offset $w\,n$ to n; and, from any other point, y, on the tan, $A\,V$, offset $y\,p = w\,n$; or from n, lay off $y\,n\,p$ ($\tan y\,n\,p = wn/wy$); or, from n, lay off $V\,n\,p'$ ($\tan V\,n\,p' = wn/Vw$).

Then, for the tan, at n, lay off $n'\,n\,s = n''\,n\,p' = A\,O\,n$.

117. C.T. (B) inaccessible. Fig 38. It is required to locate a point, as z or q, in the tangent, $V\,B$, beyond B.

We give three alternativ methods, as below:—

From any point, as m, in the curv, at a known dist from B, run $m\,f \parallel$ the tangent, $V\,B$. Then:—

(1) From m, and from any other point, as f, in $m\,f$, lay off offsets $= m\,e = f\,z = R \text{ vers } B\,O\,m$, defining the tangent, $V\,B$, and locating the point, z. Or:—

(2) From m, lay off any angle, $f\,m\,q$, to a line, $m\,q$, intersecting the tan, $V\,B$, and lay off $m\,q = e\,m/\sin f\,m\,q$, to q, which will be in the tan, $V\,B$. Then $B\,q = q\,e - e\,B = me \cot fmq - R \sin BOm$ Or:

(3) From m, lay off $f\,m\,v = B\,O\,m$, locating the tan, $m\,v$, and lay off $m\,v = R \tan(BOm/2)$. Then v will be a point in the tan, $V\,B$. From v sight to m, and lay off $m\,v\,V = B\,O\,m$. Plunge the telescope, sighting along tan, $V\,B$. Then a point, z or q, beyond B, may be found, as required, by the methods given in ¶ 110, etc, for passing obstacles in straight lines.

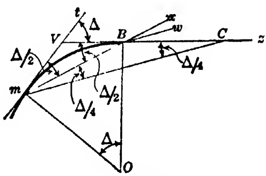


Fig. 39.

118. Fig 39. The **C.T. (B), altho accessible, is not suitable** for a transit point. It is required to find the tangent, $B\,z$.

Set over any convenient point, m, on the curv. Measure the chord, $m\,B$. Let $\Delta =$ the angle, $m\,O\,B$, $=$ sweep of curv betw m and B.

From m, lay off a tan, $m\,t$. Foresight on B. Then angle $t\,m\,B = \Delta/2$. Lay off $B\,m\,C = \Delta/4 = t\,m\,B/2$.

Lay off $B\,C = B\,m$, to meet the line $m\,C$. Then C is a point in the tangent, $B\,z$.

Demonstration.

Since $m\,B\,C$ is isosceles, we have $B\,C\,m = B\,m\,C = \Delta/4$.

Extend $m\,B$, as to x, and draw $B\,w \parallel m\,C$.

Then, from parallelism of sides,
$x\,B\,w\ (= B\,m\,C) = \Delta/4$;
$w\,B\,C\ (= B\,C\,m) = \Delta/4$; and
$x\,B\,C\ (= x\,B\,w + w\,B\,C) = \Delta/2$.

Let $B\,z$ be a tangent to the curv at B.

Then $x\,B\,z$ ($=$ the opp angle $V\,B\,m$) $= \Delta/2 = x\,B\,C$, and $B\,C$ therefore coincides with the tangent, $B\,z$.

CURV PROBLEMS

(See also Re-survey, ¶¶ 155, etc.)

Graphic Solutions

119. Most curv problems may be conveniently solved graphically, with sufficient approximation, and with the added advantage that the drawing shows the relations of the several lines more clearly than they can be shown by mere calculation, and thus reduces the probability of serious error. The scale, to be adopted, will depend upon the nature of the problem and upon the degree of approximation required. It will probably seldom be smaller than 1 inch = 20 feet. The following are given as suggestions. See also ¶ 128.

120. Example. Fig 46. Let it be required to solve graphically the problem in ¶ 131, viz:—Given two curvs, $A\,B'$ and $B\,G$, having diff given radii, R_g and R_s, and diff centers, O_g and O_s, and given the tans, $A\,V$ and $V\,B$, with their sweep, Δ; it is required to find the common radial line, $O_g\,E$, of the two curvs, their respectiv sweeps, g and s, and the shortest dist, d (which will be in the line, $O_g\,E$) betw them; as where it is desired to connect the two curvs by a spiral.

From A, draw $A\,O_g \perp A\,V$ and $= R_g$.
From B, draw $B\,O_s \perp B\,V$ and $= R_s$.
Thru O_g and O_s draw $O_g\,E$, radial to both curvs.
Measure the angles, g and s, and the dist, d.

See letter from C. M. Estabrook, Eng Record 1906 April 7, p 466.

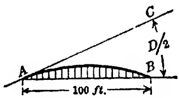


Fig. 40.

121. To draw a circular curv of large radius. Fig 40. Draw a chord, $A\,B$, to represent 100 ft by scale. Divide $A\,B$ into 20 equal spaces. At each division-point, erect an ordinate, normal to the chord, making the lengths of the 19 ordinates to correspond with those given, in table, pp 900-901, for the given sharpness, D. Join the ends of the ordinates.

122. If the curv $A\,B$ is the center line, the curvs for the rails, or for their gage sides, may be drawn at dists from $A\,B$ equal to half the gage. Ordinarily it will be sufficiently accurate if these dists are measured in the directions of the ordinates, already drawn. Otherwise, the dists must be measured radially from the curv, $A\,B$; i. e., at right angles to tangents drawn (see ¶ 124) at the ends of the 19 ordinates.

123. The angle, $C\,A\,B$, betw the 100 ft chord, $A\,B$, and a tangent, $A\,C$, at either end, A or B, of the chord, is $D/2$, where D = the sharpness of the curv = sweep (central angle) subtended by the 100 ft chord.

Fig. 41.

124. To draw a tangent to a curv, at a given point, m, Fig 41. From m, lay off two equal distances, mn, mn, to the curv. Thru m, draw $A\,B$, parallel to $n\,n$. Then will $A\,B$ be tangent to the curv at m.

Field Solutions

125. Many problems are habitually, expeditiously and with sufficient approximation, solved **by trial in the field,** aided, where necessary, by templets or curv protractor (see ¶¶ 105-107) and a field sketching board.

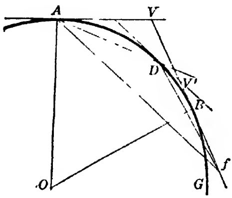


Fig. 42.

126. Example. Fig 42. In a given curv, $A\ G$, to find the point, B, from which a tangent, $B\ f$, may be run thru a given point, f.

This is solved trigonometrically in ¶ 129, Fig 44.

With instrument at any point, as A, in the curv, sight along the tangent, $A\ V$; measure the angle, $V\ A\ f$, and lay off the angle, $V\ A\ D = V\ A\ f/2$. With inst at D, where the line $A\ D$ meets the curv, repeat the process, sighting along the tan, $D\ V'$, and to f, measuring angle $V'\ D\ f$, and laying off $V'\ D\ f/2$ (not shown) to the curv at a point nearer B, and so on, approximating to B. When f is sighted in the tan, $B\ f$, the inst is at the required point, B.

Geometrical Solutions.

Below are given geometrical solutions of a few typical problems of frequent occurrence.

Caution.

127. When Δ is large, look out for necessary **changes in the signs,** + and −. See ¶ 69.

Thus, in Fig 46, ¶ 131, p 940, if $\Delta > 90° < 180°$, we have $\sin\Delta$ positiv, but $\cos\Delta$ negativ; making $B'\ I$ positiv, but $V\ i$ negativ.

In general, the use of the signs, + and −, in our text, refers to our figs, in which, for convenience, the sweep Δ is in general made $< 90°$.

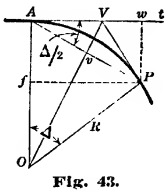


Fig. 43.

128. To pass a curv thru a given point, P. Fig 43.

Given, direction of tangent, $A\ t$; the T.C., A; and the co-ordinates, $A\ w$ and $w\ P$, of the given point, P.

Required, Δ and R (or D), for the curv, $A\ P$. Here

$$\text{Tan}\ (\Delta/2) = w\ P/A\ w;\ R = f\ P/\sin\Delta = A\ w/\sin\Delta = w\ P/\text{vers}\ \Delta.$$

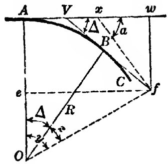
Given R instead of $A\ w$; required, Δ and $A\ w$. Here

$$\text{vers}\Delta = w\ P/R;\quad A\ w\ (= P\ f) = R\sin\Delta.$$

Graphically. Join $A\,P$, measure angle, $w\,A\,P = \Delta/2$, bisect chord $A\,P$ in v, draw a line in the direction $O\,v\,V$, perp to $A\,P$, to intersect $A\,t$ in V, and measure $A\,V$ and $V\,P$. Then V is the vertex of the curv $A\,P$ which joins A and P and is tangential to $A\,t$ at A; $A\,V$ and $V\,P$ are its semitangents; and we have

$$\Delta = 2\ w\,A\,P;\quad R = A\,V/\tan(\Delta/2) = V\,P/\tan(\Delta/2).$$

Fig. 44.



129. Fig 44. In a given curv, $A\,C$, **to find the point, B, whence a tangent, B f, can be run to a given point, f.** For approx instrumental solution, see ¶ 126, Fig 42.

Given, radius, R, curv point, A, direction of initial tan, $A\,V$, and the co-ordinates, $A\,w$ and $w\,f$, of the given point, f.

(If the co-ords, $A\,w$ and $w\,f$, are not given, see below.) Required, the angle, Δ

Thru f, draw $e\,f \perp A\,O$. Then :— $A\,e = w\,f$; $e\,f = A\,w$; and

$$\tan y = \frac{e\,f}{e\,O} = \frac{A\,w}{R - w\,f}, \quad \cos z = \frac{R}{O\,f} = \frac{R}{A\,w/\sin y}; \quad \Delta = y - z.$$

If the co-ords, A w and w f, are not given, and if they cannot easily be measured; take any convenient point, as x, on the initial tan, $A\,V$; measure $x\,f$ and $A\,x$, and the angle, $A\,x\,f$ or its supplement, $a = 180° - A\,x\,f$. Then :—

$A\,w = A\,x + x\,w = A\,x + x\,f\cos a$;
$w\,f = x\,f\sin a = x\,w\tan a$.

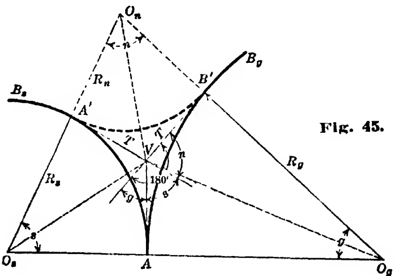


Fig. 45.

130. Fig 45. **Having two curvs,** $A\,B_s$ and $A\,B_g$, starting from a common curv point, A, it is required **to connect them by means of a third** curv, $A'\,B'$, tangent to both curvs and starting from a given point, as A', on one of the two curvs.

Given, both radii, R_s and R_g, and the sweep, Δ_s, (or s) $= A'O_s\,A$, of the curv containing the given point, A';

Required, the common semitangent, T; the sweep, Δ_g (or g) of $A\,B'$, and the sweep Δ_n (or n) and the radius, R_n, for the new curv, $A'\,B'$.

Here we have :—
$T = A\,V = A'\,V = B'\,V = R_s \tan(s/2) = R_g \tan(g/2)$
$= R_n \tan(n/2)$. Hence :—
$\mathrm{Tan}(g/2) = T/R_g$; $n = 180° - (s + g)$; $R_n = T/\tan(n/2)$.

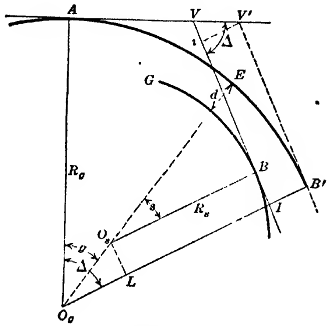


Fig. 46.

131. Fig 46. **Given two curvs,** $A\,E$ and $B\,G$, having diff given radii, R_g and R_s, and diff centers, O_g and O_s, and given the semitans, $A\,V$ and $V\,B$, with their sweep, Δ; it is required to find the common radial line, $O_g\,E$, of the two curvs, their sweeps, Δ_g (or g) and Δ_s (or s) and the shortest dist, d, betw them. (d will be in the line $O_g\,E$, which joins the centers, O_g and O_s, of the two curvs) For graphic solution, see ¶ 120.

Draw $O_g\,B' \perp V\,B$, and extend curv $A\,E$ to intersect $O_g\,B'$ in B'. Draw $V'\,B' \parallel V\,B$. From V' draw $V'\,i \perp V\,B$. Then :—

$B'\,V' = A\,V' = R_g \tan(\Delta/2)$; $V\,V' = A\,V' - A\,V$;
$B'\,I = V'\,i = V'\,V \sin\Delta$; $V\,i = V'\,V \cos\Delta$.

Draw $O_s\,L \perp O_g\,B'$. Then :—
$O_s\,L = B\,I = B'\,V' + V\,i - V\,B$;
$O_g\,L = O_g\,B' - L\,I - B'\,I = R_g - R_s - B'\,I$;

$$\mathrm{Tan}\,s = O_s\,L/O_g\,L;$$
$$\Delta_g = \Delta - \Delta_s; \text{ and}$$
$$d = R_g - R_s - (O_g\,L) \sec s.$$

See letter from Wm. R. Dunham, Jr., Eng Record, 1906, Apr 7, p 466.

Changes of Location.

132. In the following, R, T, E, etc, A, V, B, Δ, etc, indicate the given radius, semitan, ext dist, etc, T. C., vertex, C. T., sweep, etc, for the *existing* curv; and R', T', E', etc, A', V', B', Δ', etc, indicate corresponding lengths, points and angles for the *new* curv. In general, *solid* lines, in the Figs, refer to the *existing* curv; *dotted* lines to the *new* curv, or to demonstration of formulas.

Tangents Unchanged.

133. Fig 47. Change of curv joining two given tans (Δ and V constant); requiring change of given R, T and E ($E = V\,P$).

Here, T ($= A\,V$) $= R \tan(\Delta/2)$; T' ($= A'\,V$) $= R' \tan(\Delta/2)$; $T - T' = AA'$; E ($= V\,P$) $= R\,\mathrm{exsec}(\Delta/2) = T \tan(\Delta/4)$; E' ($= V\,P'$) $= R'\,\mathrm{exsec}(\Delta/2) = T' \tan(\Delta/4)$; $E - E' = P\,P'$; $R - R' = (O\,O') \cos(\Delta/2)$.

Therefore :—

1. Given T'; reqd R' and E'.

$$R - R' = (T - T') \cot(\Delta/2);$$
$$E - E' = (R - R') \operatorname{exsec}(\Delta/2) = (T - T') \tan(\Delta/4)$$

2. Given E'; reqd R' and T'.

$$R' = R\,E'/E; \quad R - R' = \frac{E - E'}{\operatorname{exsec}(\Delta/2)};$$
$$T - T' = (R - R') \tan(\Delta/2) = (E - E') \cot(\Delta/4).$$

3. Given R'; reqd T' and E'.

$$T - T' = (R - R') \tan(\Delta/2);$$
$$E - E' = (R - R') \operatorname{exsec}(\Delta/2) = (T - T') \tan(\Delta/4).$$

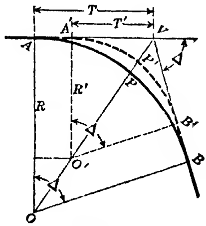


Fig. 47.

134. Changes in Tangents. Where only one of the tans is changed, we assume, for the sake of uniformity, that this is the *final* tan, the *initial* tan remaining unchanged; but, with the proper changes in the lettering, the instructions given apply equally to the reverse case; *i. e.*, where the *initial* tan is changed, the *final* tan remaining unchanged.

One tangent shifted parallel with itself (Δ constant). Tan point, B, changed.

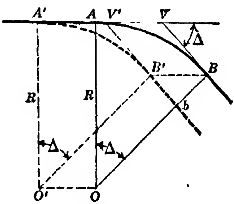


Fig. 48.

135. (1) Fig 48. Radius, R, constant; curv point, **A, shifted.** $A\,A'$, $B\,B'$, $O\,O'$, $V\,V'$ equal and parallel.

Given, dist, $B\,b$, betw tans. Reqd, the shift, $A\,A'$.

$$A\,A' = B\,B' = \frac{B\,b}{\sin \Delta}.$$

Here we have :—

$T = A\,V = A'\,V = B'\,V = R_s \tan(s/2) = R_g \tan(g/2)$
$= R_n \tan(n/2)$. Hence :—

$\text{Tan}(g/2) = T/R_g;\quad n = 180° - (s + g);\quad R_n = T/\tan(n/2)$.

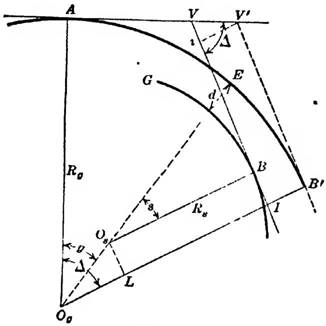


Fig. 46.

131. Fig 46. **Given two curvs,** $A\,E$ and $B\,G$, having diff given radii, R_g and R_s, and diff centers, O_g and O_s, and given the semitans, $A\,V$ and $V\,B$, with their sweep, Δ; it is required to find the common radial line, $O_g\,E$, of the two curvs, their sweeps, Δ_g (or g) and Δ_s (or s) and the shortest dist, d, betw them. (d will be in the line $O_g\,E$, which joins the centers, O_g and O_s, of the two curvs) For graphic solution, see ¶ 120.

Draw $O_g\,B' \perp V\,B$, and extend curv $A\,E$ to intersect $O_g\,B'$ in B'. Draw $V'\,B' \parallel V\,B$. From V' draw $V'\,i \perp V\,B$. Then :—

$B'\,V' = A\,V' = R_g \tan(\Delta/2)$; $V\,V' = A\,V' - A\,V$;
$B'\,I = V'\,i = V'\,V \sin\Delta$; $V\,i = V'\,V \cos\Delta$.

Draw $O_s\,L \perp O_g\,B'$. Then :—

$O_s\,L = B\,I = B'\,V' + V\,i - V\,B$;
$O_g\,L = O_g\,B' - L\,I - B'\,I = R_g - R_s - B'\,I$;

$$\text{Tan}\, s = O_s\,L/O_g\,L;$$
$$\Delta_g = \Delta - \Delta_s\text{; and}$$
$$d = R_g - R_s - (O_g\,L) \sec s.$$

See letter from Wm. R. Dunham, Jr., Eng Record, 1906, Apr 7, p 466.

Changes of Location.

132. In the following, R, T, E, etc, A, V, B, Δ, etc, indicate the given radius, semitan, ext dist, etc, T. C., vertex, C. T., sweep, etc, for the *existing* curv; and R', T', E', etc, A', V', B', Δ', etc, indicate corresponding lengths, points and angles for the *new* curv. In general, *solid* lines, in the Figs, refer to the *existing* curv; *dotted* lines to the *new* curv, or to demonstration of formulas.

Tangents Unchanged.

133. Fig 47. Change of curv joining two given tans (Δ and V constant); requiring change of given R, T and E ($E = V\,P$).

Here, $T\ (= A\,V) = R \tan(\Delta/2)$; $T'\ (= A'\,V) = R' \tan(\Delta/2)$; $T - T' = AA'$; $E\ (= V\,P) = R\,\text{exsec}(\Delta/2) = T \tan(\Delta/4)$; $E'\ (= V\,P') = R'\,\text{exsec}(\Delta/2) = T' \tan(\Delta/4)$; $E - E' = P\,P'$; $R - R' = (O\,O') \cos(\Delta/2)$.

Given, $B\,B'$. Reqd R', $A\,A'$ and $V\,V'$. Draw $O'\,G \parallel A\,K$; and $V'\,e \parallel B\,B'$. Produce radius $O\,B$ to meet tan $A\,K$ in K.

$$B\,B' = B\,K - B'K = R \text{ exsec } \Delta - R' \text{ exsec } \Delta$$
$$= (R - R') \text{ exsec } \Delta.$$

$$\therefore R - R' = \frac{B\,B'}{\text{exsec } \Delta}; \quad \text{and } R' = R - \frac{B\,B'}{\text{exsec } \Delta}.$$

$$A\,A' = G\,O' = G\,O \tan \Delta = (R - R') \tan \Delta$$
$$= \frac{B\,B'}{\text{exsec } \Delta} \tan \Delta = \frac{B\,B'}{\tan(\Delta/2)}$$

$$V\,V' = \frac{V'\,e}{\sin \Delta} = \frac{B\,B'}{\sin \Delta}.$$

138. Each tan shifted, parallel with itself. Δ constant. R constant. Curv point, A, and tan point, B, changed.

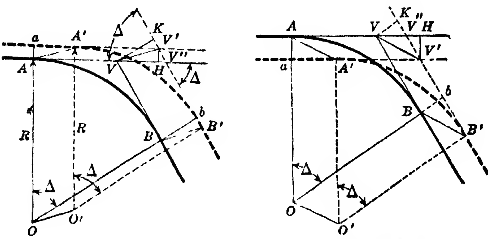


Fig. 51. Fig. 52.

(1) Fig 51. **Both tans shifted** outward, as shown, or both inward

Let tan $A\,V$ be shifted thru dist $A\,a$; tan $V\,B$ thru dist $B\,b$.

Then $A\,A'$, $B\,B'$, $O\,O'$ and $V\,V'$ are equal and parallel.

Required $a\,A'$ and $b\,B'$.

Let V'' be the intersection of $A\,V$ and $V'\,B'$.

Draw $V\,K \parallel B\,b$, and $V'\,H \parallel A\,a$. Then :—

$$a\,A' = V\,H = V\,V'' - V''\,H = \frac{B\,b}{\sin \Delta} - \frac{A\,a}{\tan \Delta}.$$

$$b\,B' = K\,V' = K\,V'' - V''\,V' = \frac{B\,b}{\tan \Delta} - \frac{A\,a}{\sin \Delta}.$$

139. (2) Fig 52. **One tan shifted inward, the other outward.**

Let one of the shifts, $A\,a$, be considered *negativ*. Then $V''\,H$ and $V''\,V'$ are negativ; and

$a\,A' = V\,V''$ *plus* $V''\,H$; $\qquad b\,B' = K\,V''$ *plus* $V''\,V'$.

Divergent tangents, $V z$ **and** $V' z'$, **forming angle,** a.

140. Figs 53, 54, 55. **Tangents intersect in tangent point, B.**

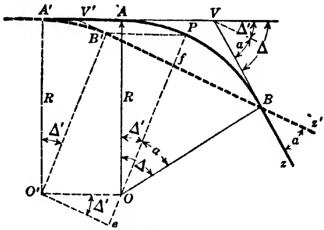


Fig. 53.

Fig 53. (1.) **R constant;** A shifted; new tan point, B'. Given curv $A\ B$, radius, R, and angles, Δ and a. $\Delta' = \Delta \pm a$.*

Required dist $A\ A'$.

Make angle $A\ O\ P = \Delta'$, and thru O, draw $e\ P$ to curv $A\ B$ at P, making $O\ e = P\ f = R$ vers a. Join $O\ O'$ and $e\ O'$. Then :—

$$\mathbf{A\ A'} = O\ O' = \frac{O\ e}{\sin \Delta'} = R\frac{\text{vers } a}{\sin \Delta'}.$$

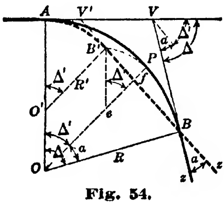


Fig. 54.

141. Fig 54. (2.) **A constant;** R changed; new tan point, B'.

Required, R'.

Note. In order that the tans shall intersect in B, we must have $R' < R$. In order that A shall remain constant, $V'\ B'$ must meet $A\ V$ at a point, V', in *advance* of A. a = angle, $z\ B\ z'$, betw the two tans. $\Delta' = \Delta - a$.

Make $A\ O\ P = \Delta'$, and draw $O\ P$, intersecting curv, $A\ B$, in P. Draw $B'\ e \parallel O\ A$. Then :—

$$\mathbf{R} - \mathbf{R'}\ (= O\ O') = B'\ e = \frac{P\ f}{\text{vers } \Delta'} = R\frac{\text{vers } a}{\text{vers } \Delta'};$$

$$R' = R\left(1 - \frac{\text{vers } a}{\text{vers } \Delta'}\right).$$

*In Fig 53, a is negativ, and $\Delta' = \Delta - a$. If, as in Fig 55, V is betw A and V', a is positiv, and $\Delta' = \Delta + a$.

142. Fig 55 (3.) **B constant; *A* shifted; *R* changed.**
Required, R' and dist, $A\,A'$.

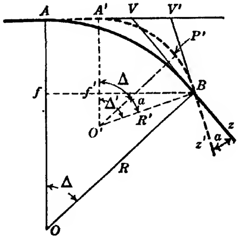


Fig. 55.

Draw $B\,f \perp O\,A$, and $O'\,P' \parallel O\,B$. Then

$$A\,f = A'\,f' = R \text{ vers } \Delta = R' \text{ vers } \Delta'.$$

$$\therefore \mathbf{R'} = R\,\frac{\text{vers } \Delta}{\text{vers } \Delta'};$$

$$\mathbf{A\,A'} = B\,f - B\,f' = R \sin \Delta - R' \sin \Delta'.$$

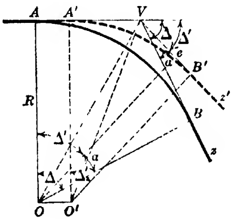


Fig. 56.

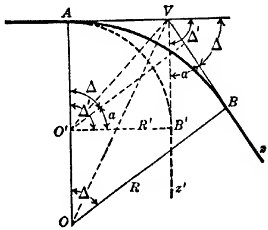


Fig. 57.

Figs 56 and 57. **Tangents intersect in vertex, V.**

143. Fig 56 (1.) **R constant;** *A* shifted; new tan point, B'.
Given, R, Δ and a.
Required, shift, $A\,A'$, for A.

$$A\,V = R \tan(\Delta/2); \quad A'\,V = R \tan(\Delta'/2);$$

$$\therefore \quad \mathbf{A\,A'} = A\,V - A'\,V = R\,[\tan(\Delta/2) - \tan(\Delta'/2)].$$

144. Fig 57 (2). **A constant;** *R* changed; new tan point, B'.
R' may be either greater or less than R.
Given, R, Δ and a.
Required, R'.

$$A\,V = R \tan(\Delta/2) = R' \tan(\Delta'/2);$$

$$\therefore \mathbf{R'} = \frac{A\,V}{\tan(\Delta'/2)} = R\,\frac{\tan(\Delta/2)}{\tan(\Delta'/2)}.$$

Divergent tangents, $V z$ **and** $V' z'$**, forming angle,** a.

140. Figs 53, 54, 55. **Tangents intersect in tangent point, B.**

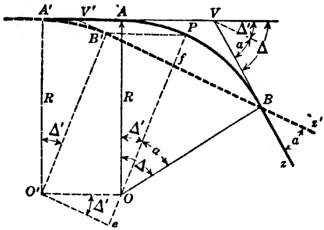


Fig. 53.

Fig 53. (1.) **R constant;** A shifted; new tan point, B'. Given curv $A\ B$, radius, R, and angles, Δ and a. $\Delta' = \Delta \pm a$.*

Required dist $A\ A'$.

Make angle $A\ O\ P = \Delta'$, and thru O, draw $e\ P$ to curv $A\ B$ at P, making $O\ e = P\ f = R$ vers a. Join $O\ O'$ and $e\ O'$. Then :—

$$\mathbf{A\ A'} = O\ O' = \frac{O\ e}{\sin \Delta'} = R \frac{\text{vers } a}{\sin \Delta'}.$$

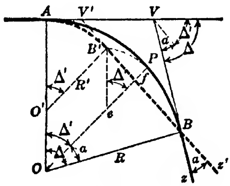


Fig. 54.

141. Fig 54. (2.) **A constant;** R changed; new tan point, B'.

Required, R'.

Note. In order that the tans shall intersect in B, we must have $R' < R$. In order that A shall remain constant, $V'\ B'$ must meet $A\ V$ at a point, V', in *advance* of A. a = angle, $z\ B\ z'$, betw the two tans. $\Delta' = \Delta - a$.

Make $A\ O\ P = \Delta'$, and draw $O\ P$, intersecting curv, $A\ B$, in P. Draw $B'\ e \parallel O\ A$. Then :—

$$\mathbf{R} - \mathbf{R}'\ (= O\ O') = B'\ e = \frac{P\ f}{\text{vers } \Delta'} = R \frac{\text{vers } a}{\text{vers } \Delta'};$$

$$R' = R \left(1 - \frac{\text{vers } a}{\text{vers } \Delta'}\right).$$

*In Fig 53, a is negativ, and $\Delta' = \Delta - a$. If, as in Fig 55, V is betw A and V', a is positiv, and $\Delta' = \Delta + a$.

$$s = \Delta/2 + (s - g)/2; \qquad g = \Delta - s;$$
$$\mathbf{R}_s = T_s/\tan s; \qquad \mathbf{R}_g = T_g/\tan g.$$

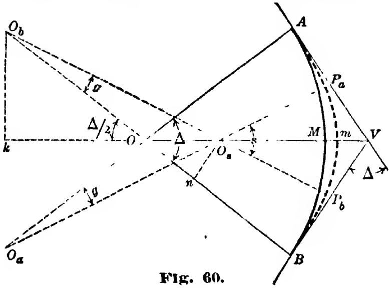


Fig. 60.

147. Figs 60-61. Given, a simple curv, $A\,M\,B$, joining two tangents, $A\,V$ and $V\,B$; Retaining the same tans, it is reqd to **substitute a three-center compound curv.**

148. Fig 60. **A. Retaining A and B.** Compound curv, $A\,P_a\,m\,P_b\,B$.

Let $R = O\,A = O\,M = O\,B$ = radius of simple curv, $A\,M\,B$;
$R_s = O_s\,P_a = O_s\,m = O_s\,P_b < R$;
$R_g = O_a\,A = O_b\,B > R$;
$s = P_a\,O_s\,P_b$; $\quad g = A\,O_a\,P_a = B\,O_b\,P_b$;
$\Delta = A\,O\,B = s + 2\,g$ = sweep of simple curv, $A\,M\,B$;
$M\,m$ = dist betw mid points of the two curvs

Produce $V\,O$ to k, and draw $O_b\,k \perp V\,K$. Then

$$O_b\,k = O_b\,O\sin(\Delta/2) = (R_g - R)\sin(\Delta/2)$$
$$= O_b\,O_s\sin(s/2) = (R_g - R_s)\sin(s/2);$$

whence :—

$$\text{Sin}\,s/2 = \frac{(R_g - R)\sin(\Delta/2)}{R_g - R_s}; \quad g = \frac{\Delta - s}{2};$$

$$\mathbf{R}_g = \frac{R\sin(\Delta/2) - R_s\sin(s/2)}{\sin(\Delta/2) - \sin(s/2)}$$

Draw $O_s\,n \perp O\,B$. Then

$$O\,O_s = O_s\,n/\sin(\Delta/2) = (R_g - R_s)\sin g/\sin(\Delta/2);$$

and

$$\mathbf{M\,m} = O\,O_s + O_s\,m - O\,M = O\,O_s - (O\,M - O_s\,m)$$
$$= (R_g - R_s)\frac{\sin g}{\sin(\Delta/2)} - (R - R_s).$$

---

*From the equations for sin $(A + B)$, cos $(A + B)$, sin $(A - B)$ and cos $(A - B)$, ¶¶ 15, 16, p 97b, it can be shown that

$$\sin\frac{s - g}{2} = \sin\frac{\Delta}{2} \times \frac{\tan(s/2) - \tan(g/2)}{\tan(s/2) + \tan(g/2)};$$

and, from $T_g = T_s\,\dfrac{\tan(s/2)}{\tan(g/2)}$, ¶ 57 and p 38, ¶ (5), we have

$$\frac{T_g - T_s}{T_g + T_s} = \frac{\tan(s/2) - \tan(g/2)}{\tan(s/2) + \tan(g/2)}$$

149. Fig 61. **B. Retaining the same middle point, M.** Compound curv, $A' P_a M P_b B'$.

Let $R = OA = OM = OB =$ radius of simple curv, AMB;
$R_s = O_s P_a = O_s M = O_s P_b < R$;
$R_g = O_a A' = O_b B' > R$;
$s = P_a O_s P_b$; $g = A' O_a P_a = B' O_b P_b$;
$\Delta = AOB = s + 2g$.

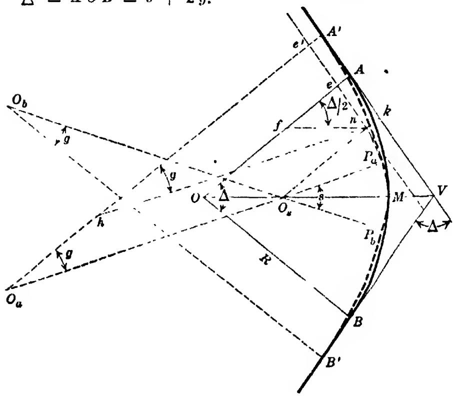


Fig. 61.

In the Fig, produce the curve, $P_b M P_a$, to meet, at n, the line, $O_s k$, $\parallel OA$; and draw $nh \parallel P_a O_a$, and $nf \parallel O_s O$. Then n is a point in the long chord, MA (not drawn), and in the long chord, $P_a A'$ (not drawn); the tan, en, thru n, is $\parallel AV$; and

$$nk = Ae = nf \operatorname{vers}(\Delta/2) = (R - R_s) \operatorname{vers}(\Delta/2)$$
$$= A'e' = nh \operatorname{vers} g = (R_g - R_s) \operatorname{vers} g;$$

whence we have:—

$$\operatorname{vers} g = \frac{nk}{R_g - R_s} = \frac{(R - R_s)\operatorname{vers}(\Delta/2)}{R_g - R_s};$$

$R_g = R_s + (nk/\operatorname{vers} g)$;

$R_s = R - [nk/\operatorname{vers}(\Delta/2)]$.

$\cot(g/2) = \cot(\Delta/4) + (AA'/nk)$; $s = \Delta - 2g$;

$AA' = BB' = kA' - kA = nk\,[\cot(g/2) - \cot(\Delta/4)]$

150. Shifting C.T. along its tangent. Fig 62. Given a compound curv, UPE, joining two tans, UV and VE. Retaining the same tangent-directions, and the given initial radius, R_u, it is desired to make the second branch join the same final tan, VE', as before, but at E', instead of at E.

Required, the new final radius, R_e', and the new distribution of the sweep, Δ, betw the two branches; *i. e.*, the new values (u' and e'), of the sweeps, u and e. The change involvs shifting the C.C. from P to a new point, as P'.

Let y ($= EE'$) be *positiv* when measured *forward* from E, as in Fig 62.

As in Fig 62, let the subscript, e, refer to that branch whose radius, R_e, is to be changed; and let the subscript u refer to the other branch.

Since the tangent-directions are unchanged, we have $\Delta = u + e = u' + e'$; and $u' = u + e - e'$.

In the Fig, produce the u branch, $U P$, to meet, at n, the line, $O_u\, n$, drawn parallel with $O_e\, E$ (n is in any straight line $P E$, $P' E'$, joining either C. C. with the corresponding tan point). Draw $n\, f$, $\parallel$ and $= O_u O_e = R_e - R_u = f E$. Draw $n\, k'$, $\perp$ $O_e\, E$ and $\perp$ $O_e'\, E'$.

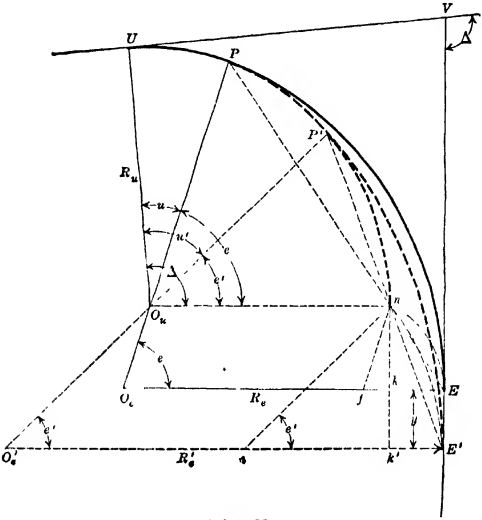


Fig. 62.

(1) Given the change ($= P\, O_u\, P' = u' - u = e - e'$) in the sweeps; reqd R_e' and y ($= E\, E'$). Here we have $e' = P'\, O_e'\, E' = P'\, O_u\, n = \Delta - (u + P\, O_u\, P')$. In the Fig, draw $n\, i$ $\parallel$ and $= O_u\, O_e' = R_e' - R_u$. Then.—

$E'\, k' = (R_e' - R_u)$ vers e' $= E\, k = (R_e - R_u)$ vers e. Hence,

$$\mathbf{R_e'}\ (= O_e'\, E') = O_e'\, i + \frac{E\, k}{\text{vers } e'} = R_u + (R_e - R_u) \text{ vers } e/\text{vers } e';$$

$$\begin{aligned} \mathbf{y}\ (= \mathbf{E\, E'} = n\, k' - n\, k) &= E'\, k' \cot(e'/2) - E\, k \cot(e/2) \\ &= E\, k\ [\cot(e'/2) - \cot(e/2)] \\ &= (R_e - R_u) \text{ vers } e\ [\cot(e'/2) - \cot(e/2)]. \end{aligned}$$

(2) Given R_e'; reqd $P\, O_u\, P'$ ($= u' - u = e - e'$) and y ($= E\, E'$). Here we have:—

$$\text{vers } e' = \frac{E'\, k'}{n\, i} = \frac{E\, k}{n\, i} = \frac{(R_e - R_u) \text{ vers } e}{R_e' - R_u}\ ; \text{ and}$$

$$\mathbf{P\, O_u P'} = e - e';\ u' = u + P\, O_u P'.$$

$\mathbf{y}$ ($= \mathbf{E\, E'}$) as above.

(3) Given y ($= E\,E'$); reqd $P\,O_u P'$ ($= e - e'$) **and** R_e'.

Since (see above) y ($= E\,E'$) $= E\,k\,[\cot(e'/2) - \cot(e/2)]$, we have:—

$$\frac{y}{E\,k} = \cot(e'/2) - \cot(e/2)\text{ ; and } \cot(e'/2) = \cot(e/2) + \frac{y}{E\,k}$$

$$= \cot(e/2) + \frac{y}{(R_e - R_u)\text{ vers }e}.\quad\text{Then :—}$$

$P\,O_u\,P'$ and R_e' may be found as above.

Plus and minus signs. See p 917, ¶ 69.

	E' beyond E (y positiv)		E' betw E and V (y negativ)	
	$R_e > R_u$ (Fig 62)	$R_e < R_u$	$R_e > R_u$	$R_e < R_u$
$P\,O_u\,P'$	positiv	negativ	negativ	positiv
e'	$< e$	$> e$	$> e$	$< e$
R_e'	$> R_e$	$> R_e$	$< R_e$	$< R_e$

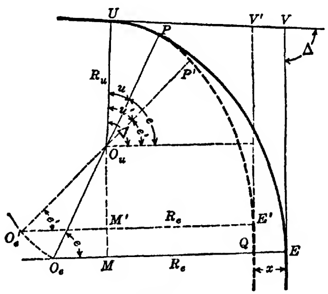


Fig. 63.

151. Fig 63. Given a compound curv, $U\,P\,E$, joining the tans, $U\,V$ and $V\,E$. It is required **to shift the final tangent,** $V\,E$, parallel with itself, to the new position, $V'\,E'$, retaining the original radii, R_u and R_e. The change involvs shifting the C. C. from P to a new point, as P'.

As in Fig 63, let the subscript, e, refer to that branch which joins the tan, $V\,E$, to be shifted; and let the subscript, u, refer to the other branch.

Since $V\,E$ and $V'\,E'$ are parallel, we have $\Delta = u + e = u' + e'$, or $u' = u + e - e'$.

Given e, R_e, R_u and the shift, x ($= E\,Q$) of the tan (call x *positiv* when measured *inward* from E, as in Fig 63);

Required the new distribution of the sweep, Δ, betw the two branches; *i. e.*, the new values, (u' and e',) of the sweeps, u and e.

Draw $O_u M \perp O_e E$ and $\perp O_e' E'$. Then

(1) $O_e' M' = R_e - E' M' = O_e' O_u \cos e' = (R_e - R_u) \cos e'$;
(2) $O_e M = R_e - E M = O_e O_u \cos e = (R_e - R_u) \cos e$.

Subtracting (2) from (1), we have

$E M - E' M' (= E Q) = x = (R_e - R_u) (\cos e' - \cos e)$

and

$$\cos e' = \cos e + \frac{x}{R_e - R_u}; \quad u' = \Delta - e'.$$

Plus and minus signs ($\Delta < 90°$). See ¶ 69.

$V' E'$ *inside* of $V E$ (x positiv)		$V' E'$ *outside* of $V E$ (x negativ)	
$R_e > R_u$ (Fig 63)	$R_e < R_u$	$R_e > R_u$	$R_e < R_u$
$e' < e$	$e' > e$	$e' > e$	$e' < e$

152. Figs 64. Given a compound curv, $U P M E$, joining the tans, $U V$ and $V E$. It is desired that the **direction of the tangent V E shall be changed** to $V' E$, $V' E$ forming, with $V E$, the angle, $a = V E V'$, and passing thru E.

Retaining the first radius, R_u, and the curv point, U, it is required to find the new radius, R_e', replacing R_e, and the changes in u and e. The change involvs shifting the C.C. from P to a new point, as P'.

Δ, Δ'	=	compound curv sweep for old and new curv respectivly.
R_u	=	initial rad (unchanged in this problem).
R_e, R_e'	=	final radius for old and new curv respectivly.
u, u'	=	sweep of R_u for old and new curv respectivly.
e	=	sweep of R_e.
e'	=	sweep of R_e'
a	=	$V E V'$

Then $\Delta = u + e = \Delta' \pm a$; $\quad u = \Delta - e = \Delta' \pm a - e$
$\quad e = \Delta - u = \Delta' \pm a - u$

$\Delta' = u' + e' = \Delta \pm a$; $\quad u' = \Delta' - e' = \Delta \pm a - e'$
$\quad e' = \Delta' - u' = \Delta \pm a - u'$.

Required, R_e', Δ', e', and u'.

153. (1) Given Δ, a, u, e, R_u and R_e. Graphic method.

From E, draw a line in the direction, $E O_e' \perp V' E$. On this line, measure $E H = R_u$. Join $H O_u$. Bisect $H O_u$ in h, and draw $h O_e' \perp H O_u$, to meet $E H$ (produced, if necessary) in O_e'. Then, $R_e' = R_u \pm H O_e' = R_u \pm O_u O_e' = E H \pm H O_e' = E O_e'$.

From O_e', thru O_u, draw $O_e' P'$, to intersect the U branch, $U P$ (produced, if necessary), in the new C.C. at P'. Then

$e' = P' O_e' E$; $\quad u' = U O_u P'$.

When	Fig	We have R_u	O_e & O_e' in	Foci	Maj axis
$R_e > R_u$	a,	$= O_e' E - O_e' O_u$ $= O_e E - O_e O_u$ $= R_e - (R_e - R_u)$	hyperbola	O_u, E	$m n = R_u$
$R_e < R_u$	b,	$= O_e' E + O_e' O_u$ $= O_e E + O_e O_u$ $= R_e + (R_u - R_e)$	ellipse	O_u, E	$m n = R_u$

154. Figs 64. (2) Given Δ, a, u, e, R_u, R_e, U V and E V.

Or, if $U V$ and $E V$ are not given, we have:—

U V $\sin \Delta = R_u \operatorname{vers} \Delta + (R_e - R_u) \operatorname{vers} e$; see eq (74).
E V $\sin \Delta = R_e \operatorname{vers} \Delta - (R_e - R_u) \operatorname{vers} u$; see eq (72).

$EV' = EV \cdot \sin\Delta / \sin\Delta'$; $VV' = EV \cdot \sin a / \sin\Delta'$;
$UV' = UV \pm VV'$;

$$\operatorname{Tan}\frac{e'}{2} = \frac{UV' \sin\Delta' - R_u \operatorname{vers}\Delta'}{EV' + UV' \cos\Delta' - R_u \sin\Delta'}, \text{ See eq (77)};$$

$u' = \Delta' - e' = \Delta \pm a - e'$;
$R_e' \sin e' = EV' + UV' \cos\Delta' - R_u \sin\Delta' + R_u \sin e'$, See Eq (78);
$P\,O_u\,P' = \pm (e - e' \pm a) = \pm (u' - u)$.

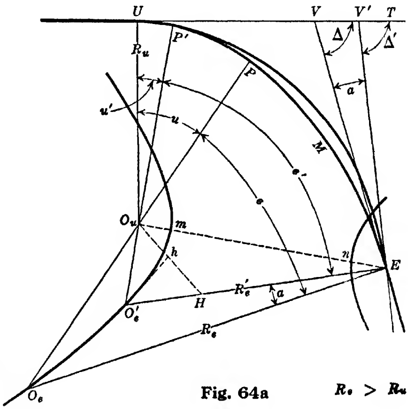


Fig. 64a $R_e > R_u$

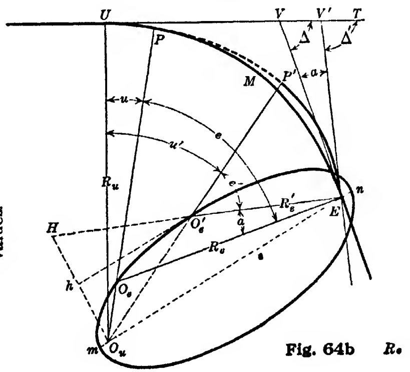


Fig. 64b $R_e < R_u$

When (as in Figs 64) $UV' > UV$, then $\Delta' > \Delta$, and $R_e' < R_e$: and vice versa. This affects the signs (+ and −) of the other values.

RESURVEY OF CURVS

To find the sharpness, D, of an existing curv.

155. Fig 65. Vertex, V, accessible, and directions of tans, VA and VB, given. Required, radius, R (or sharpness, D); A and B.

With the inst at V, measure Δ; and from either tan, lay off angle, BVP or $AVP = (180° - \Delta)/2$. Measure the external, $E = VP$, from V to cen line of existing track. Then we have :—

$R\ (= OA = OB) = E/\text{exsec}(\Delta/2)$;

$T\ (= AV = VB) = E/\tan(\Delta/4) = R\tan(\Delta/2)$;

$\sin(D/2) = 50/R$.

Approx, we have $D = 5730/R$.

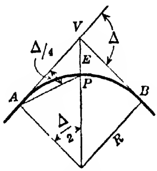


Fig. 65.

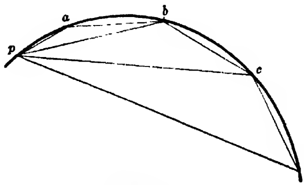


Fig. 66.

156. Fig 66. Vertex, V (not shown), **inaccessible.**

Set up over a point, as p, in the center line of the curv, preferably at or near either end; and lay off and measure peripheral angles, apb, bpc, etc., each subtended by a unit chain (as ab, bc, etc.), on the curv. The average of these angles may be taken as (approx) the peripheral angle, $D/2$, of the curv; and twice $(D/2) = D$ as the sharpness.*

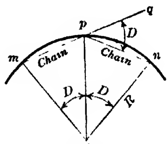


Fig. 67.

157. Fig 67. Or set over any point, p, in center line of track. Measure one chain, $pm = pn$, in each direction, to m and to n, also in center line. Measure angle $qpn = D$.*

*In all such work, the entire curv should be covered as thoroly as practicable, the method being repeated, over different portions of the curv, and an average taken; for, in curvs which hav been used for some time, the curvature is frequently found to vary from point to point, the track having been thrown out of true by traffic, and perhaps relined by eye without reference to center stakes.

158. By middle ordinates. Fig 68. In any circle,

$$(2\,R - m)m = (c/2)^2.$$

Approximately, $2\,R\,m = (c/2)^2$;

or approx, $R = \dfrac{c^2/4}{2\,m} = \dfrac{c^2}{8\,m}$.

Hence, if a cord, of any given length, c, subtending a small sweep, be stretcht betw two points, n and p, on the inner side of the head of the outer rail, and if its mid ord, m, be measured, to the rail head, we have:—

$R = (c^2/8\,m)$ — half the gage.

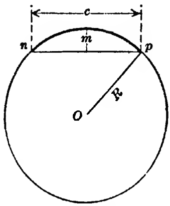


Fig. 68.

Usually the half gage may be neglected.

Then $\sin(D/2) = 50/R$; or, approx, $D = 5730/R$.

If the cord be 100 ft long (= one chain length), and the mid ord m, in ft, measured, D may be taken at once from table, pp 898-9.*

159. Conversely, **to find the mid ord, M** ins **or** m ft, corresponding to any given length of cord, c, in feet, and to any given sharpness, D, or radius, R, we have (approx)

M, ins, $= 12\,m = 12\,c^2/8\,R = 1.5\,c^2/R$.

Thus, with a 30-ft rail (c = approx 30 ft), on a 3° curv (R = 1910 ft), we have, approx:— $M = 1.5 \times 900/1910 = 0.707$ inches.

Again, (approx) $c = \sqrt{8\,m\,R} = \sqrt{8\,m\,5730/D} = 214\,\sqrt{m/D}$.

If, now, we make M = mid ord in ins $= 12\,m = D$, in degrees, or $m = D/12$, we have, approx:—

$$c = \sqrt{\frac{8\,D}{12}\,.\,R} = \sqrt{\frac{8\,D}{12}\,.\,\frac{5730}{D}} = \sqrt{3820} = 61.81 \text{ ft.}$$

Hence, with a cord, c, 61.81 (say 62) ft long, we have:—

M = mid ord in ins = sharpness, D, of curv in degrees.* Compare ¶ 178.

160. Fig 69. From any point, as a (preferably a joint), on the inner or gage side of the outer rail, sight to another such point, as b, such that the line of sight, $a\,b$, is tangent to the gage side of the inner rail, as at p.

*In all such work, the entire curv should be covered as thoroly as practicable, the method being repeated, over different portions of the curv, and an average taken; for, in curvs which hav been used for some time, the curvature is frequently found to vary from point to point, the track having been thrown out of true by traffic, and perhaps relined by eye without reference to center stakes.

Then, for the mid ord to the chord, $a\,b$, we have $p\,p' = G =$ gage.

Let l = rail-length, in ft; N = number of rail lengths in arc, $a\,p'\,b$. Then $N\,l$ = arc, $a\,p'\,b$, in ft.

Let δ = angle, $a\,O\,b$; C = chord, $a\,b$; R' = outer-rail gage-side radius; R = center-line radius; D = center-line sharpness.

Measure $C = a\,b$; or count N, find $N\,l$, and (generally near enough) assume chord, $a\,b$ = arc, $N\,l$. Then:

$\tan(\delta/4) = \tan p'\,a\,p = 2\,G/C$; $R' = C/2\sin(\delta/2)$;
$R = R' - (G/2)$; $\sin(D/2) = 50/R$; or, approx, $D = 5730/R$.

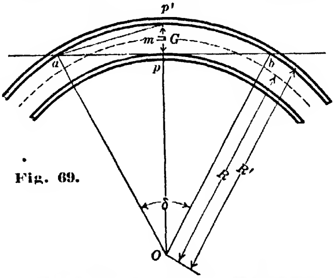


Fig. 69.

Classification of Resurvey Cases.

161. As to the choice of method of procedure, resurvey cases may be classified as follows:—

A. Vertex, *V*, accessible; short curvs;
B. Vertex, *V*, inaccessible;
  1 Curvs of moderate length;
  2. Long curvs.

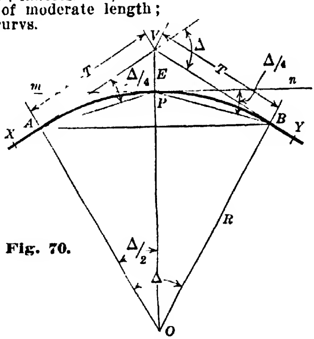


Fig. 70.

Procedure.

A. Vertex, V, accessible; short curvs.

162. Fig 70. Locate vertex, *V*, thus: with inst. at a point, *X*, on a tangent, say 50 ft back from the curv point, *A*, backsight along the tan, and plunge telescope, sighting toward *V*. Doing the same at *Y*, on the other tan, *V B*, we find the intersection, *V*, of the two tans.

Find Δ, *E*, *R*, *D* and *T* as in ¶ 155.

Then,

L (= curv length, $A\ P\ B$, in unit chains) $= \Delta/D$.

Locate the curv points, A and B, by laying off $T = V A = V B$, from V, or by laying off $L/2$, in ft, along cen line of curv, from P. $L/2$, in unit chains, $= \Delta/(2\,D)$; $L/2$, in ft. $= 50\ \Delta/D$.

With inst at P, lay off right angles, $V P m$ and $V P n$, locating the tan, $m\,n$; and verify angles $m P A = n P B = \Delta/4$; or

With inst at A and at B, verify angle $V A B = V B A = \Delta/2$, or $V A P = V B P = \Delta/4$.

If the track has shifted materially at P, giving a false value of E, the foregoing equations will of course give false values of R and D. Compare the value of D, so found, with the value recorded. If the track has shifted betw P and A or betw P and B, the measurement of $L/2$ from P, as above, may give false positions for A and B. The discrepancies must be adjusted, according to circumstances.

Run the new curv, marking it temporarily. Observe what amount of throw, and in which direction, is required at each point. Re-adjust, if necessary, and drive stakes to mark the final location

B. Vertex, V, inaccessible.

Curvs of moderate length.

a. By Trial Curv. Fig. 71.

163. Given, the sharpness, D, of an old curv, whose original position was $A\ B$, and the directions of its tans, $A\ V$ and $V\ B$. Required, Δ and the positions of the T.C. (A) and C.T. (B)

Let A'' be the supposed position of the T.C., or a point, near the T.C., on the tan or on the tan produced. Set over A'', and begin running a trial curv, $A''\,m$, of the given sharpness, D, marking the 100-ft chain points temporarily.

If A'' is materially distant from the T C. (A), as in Fig 71*, the trial curv, $A''\,m$, will soon be found deviating seriously, as at m, from the old track, $A\ B$. Measure the distance, $m\,s$, betw the trial curv, $A''\,m$, and the cen line, $A\ B$, of the old track. Then, for the shift, $A''\,A$, reqd for the T.C., we have :—

$$A''A = m\,n = \text{(approx)}\ m\,n'\ ^* = \frac{m\,s}{\sin \Delta_m}\ ^\dagger,$$

where Δ_m (or $m\,\dagger$) = sweep of trial curv betw A'' and m.*

Let $A''\,A'$ represent the approx value thus found for the shift, $A''\,A$. Move inst to A', and re-run the trial curv, with D as before, taking transit points, T_1, T_2, etc., at preferably equal dists of say 500 ft. From each transit point, locate a short line, as $T_3\,x$, parallel with the original tan, $A\ V$, marking said line with a nail, as at x.

Run the trial curv, $A'\,T_3$, thru a sweep, $s\dagger$, to a point, as T_4, where its final tan, $v'\,T_4$, is supposed to be parallel with that, $V\,B$, of the old curv, $A\ B$. Thru T_4 run a line, $T_4\,u$, parallel with $V\,B$, and measure the angle, $t\dagger$, betw $T_4\,u$ and $v'\,T_4$ produced. Then :—

$$\Delta = s \pm t.\dagger$$

*To facilitate illustration, the deviations, in Fig 71, are grossly exaggerated. In practice, the tans of the two curvs, at m and at s, would be approx parallel.

†In Fig 71, the angles, Δ_m, Δ_s and Δ_t are designated as m, s and t, respectivly, to avoid overcrowding.

Locate the terminus, B', of the trial curv, $A' B'$, accordingly. The tan, $V' B'$, is then parallel with the tan, $V B$, thru B; and angle $F V' B' = F V B = \Delta$.

Measure the dist, $B' e$, from B' to the cen line of the old track.* Then, for the shift, $A' A = B' B$, still reqd, to bring A' and B' into their proper positions, A and B, respectivly, we have:—

$$B' B = \frac{B' e}{\sin \Delta} \text{ *}$$

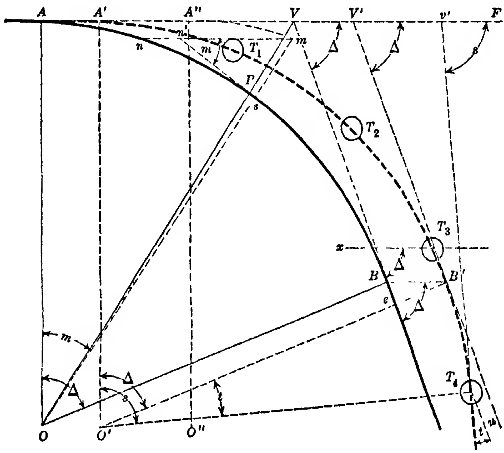


Fig. 71.

Each of the transit points, T_1, T_2, etc, is also to be shifted thru a dist $= B' B$, in the direction, $T_3 x$, parallel with the original tan, $A V$; the entire curv being thus shifted, in that direction, thru a dist $= B' B$.

If the trial curv, $A' B'$, had fallen inside (instead of outside) the existing track, this shifting of the trial curv would of course have been in the opp direction.

From each transit point, as thus shifted, and from A, the curv may now be re-run in each direction, coinciding nearly with the existing track. The max dist, from inst to stake, will be half the dist betw transit points.

---

*To facilitate illustration, the deviations, in Fig 71, are grossly exaggerated. In practice, the tans of the two curvs, at m and at s, would be approx parallel.

Then,

L (= curv length, $A\,P\,B$, in unit chains) $= \Delta/D$.

Locate the curv points, A and B, by laying off $T = V\,A = V\,B$, from V, or by laying off $L/2$, in ft, along cen line of curv, from P $L/2$, in unit chains, $= \Delta/(2\,D)$; $L/2$, in ft. $= 50\ \Delta/D$.

With inst at P, lay off right angles, $V\,P\,m$ and $V\,P\,n$, locating the tan, $m\,n$; and verify angles $m\,P\,A = n\,P\,B = \Delta/4$; or

With inst at A and at B, verify angle $V\,A\,B = V\,B\,A = \Delta/2$, or $V\,A\,P = V\,B\,P = \Delta/4$.

If the track has shifted materially at P, giving a false value of E, the foregoing equations will of course give false values of R and D. Compare the value of D, so found, with the value recorded. If the track has shifted betw P and A or betw P and B, the measurement of $L/2$ from P, as above, may give false positions for A and B. The discrepancies must be adjusted, according to circumstances.

Run the new curv, marking it temporarily. Observe what amount of throw, and in which direction, is required at each point. Readjust, if necessary, and drive stakes to mark the final location

B. Vertex, V, inaccessible.

Curvs of moderate length.

a. By Trial Curv. Fig. 71.

163. Given, the sharpness, D, of an old curv, whose original position was $A\,B$, and the directions of its tans, $A\,V$ and $V\,B$. Required, Δ and the positions of the T.C. (A) and C.T. (B)

Let A'' be the supposed position of the T.C., or a point, near the T.C., on the tan or on the tan produced. Set over A'', and begin running a trial curv, $A''\,m$, of the given sharpness, D, marking the 100-ft chain points temporarily.

If A'' is materially distant from the T.C. (A), as in Fig 71*, the trial curv, $A''\,m$, will soon be found deviating seriously, as at m, from the old track, $A\,B$. Measure the distance, $m\,s$, betw the trial curv, $A''\,m$, and the cen line, $A\,B$, of the old track. Then, for the shift, $A''\,A$, reqd for the T.C., we have :—

$$A''\,A = m\,n = \text{(approx)}\ m\,n'\,^* = \frac{m\,s\ \dagger}{\sin \Delta_m},$$

where Δ_m (or m †) = sweep of trial curv betw A'' and m.*

Let $A''\,A'$ represent the approx value thus found for the shift, $A''\,A$. Move inst to A', and re-run the trial curv, with D as before, taking transit points, T_1, T_2, etc., at preferably equal dists of say 500 ft. From each transit point, locate a short line, as $T_3\,x$, parallel with the original tan, $A\,V$, marking said line with a nail, as at x.

Run the trial curv, $A'\,T_3$, thru a sweep, s†, to a point, as T_4, where its final tan, $v'\,T_4$, is supposed to be parallel with that, $V\,B$, of the old curv, $A\,B$ Thru T_4 run a line, $T_4\,u$, parallel with $V\,B$, and measure the angle, t†, betw $T_4\,u$ and $v'\,T_4$ produced. Then :—

$$\Delta = s \pm t.\dagger$$

*To facilitate illustration, the deviations, in Fig 71, are grossly exaggerated. In practice, the tans of the two curvs, at m and at s, would be approx parallel.

†In Fig 71, the angles, Δ_m, Δ_s and Δ_t are designated as m, s and t, respectivly, to avoid overcrowding.

165. Fig 73. But points on the final curv, $A_1 B_1$, may be located **by simple measmt from the curv, A B,** thus:—

Required, to find (approx) the dist, $x x_1$, betw the two curvs, $A B$ and $A_1 B_1$, measured ∥ with the central radius, $O_1 P_1$.

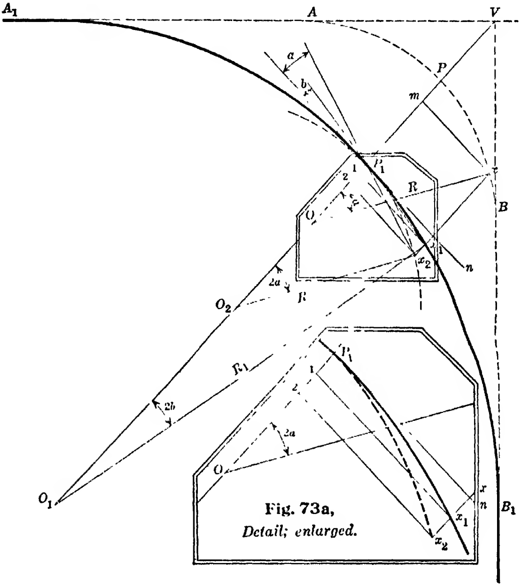


Fig. 73a, *Detail; enlarged.*

Fig. 73.

In Fig 73 (grossly exaggerated; see detail, Fig 73a), let the dotted curv, $P_1 x_2$ represent a curv having the same radius, R, as curv, $A B$, but passing thru the mid point, P_1, of curv $A_1 B_1$; and let $x_2 x_1$ be the dist (parallel with $P P_1$) from a given point, x_2, on curv $P_1 x_2$, to a required point, x_1, on the reqd curv, $A_1 B_1$. Draw $x_1 1$ and $x_2 2$, parallel to the tan, $P_1 n$, thru P_1. Then:—

$$x_2 x_1 = P_1 2 - P_1 1 = P m - P_1 1$$
$$= R \text{ vers } 2a - R_1 \text{ vers } 2b; \text{ where } a = n P_1 x_2,$$
$$\text{and } b = n P_1 x_1;$$

and the dist, $x x_1$, from the curv, $A B$, in the same direction, to the reqd point, x_1, on the final curv, $A_1 B_1$, is

$$x x_1 = x x_2 - x_2 x_1 = P P_1 - (R \text{ vers } 2a - R_1 \text{ vers } 2b.)$$

This method is sufficiently approx in practice, notwithstanding that the measurements, given by the equations, are parallel to $P P_1$, instead of being radial to either curv. As this divergence increases (with the dist of the point, x_1, from P_1) the dist, $x x_1$, betw the two curvs, diminishes.

Then,

L (= curv length, $A\,P\,B$, in unit chains) = Δ/D.

Locate the curv points, A and B, by laying off $T = V\,A = V\,B$, from V, or by laying off $L/2$, in ft, along cen line of curv, from P $L/2$, in unit chains, = $\Delta/(2\,D)$; $L/2$, in ft. = $50\ \Delta/D$.

With inst at P, lay off right angles, $V\,P\,m$ and $V\,P\,n$, locating the tan, $m\,n$; and verify angles $m\,P\,A = n\,P\,B = \Delta/4$; or

With inst at A and at B, verify angle $V\,A\,B = V\,B\,A = \Delta/2$, or $V\,A\,P = V\,B\,P = \Delta/4$.

If the track has shifted materially at P, giving a false value of E, the foregoing equations will of course give false values of R and D. Compare the value of D, so found, with the value recorded. If the track has shifted betw P and A or betw P and B, the measurement of $L/2$ from P, as above, may give false positions for A and B. The discrepancies must be adjusted, according to circumstances.

Run the new curv, marking it temporarily. Observe what amount of throw, and in which direction, is required at each point. Readjust, if necessary, and drive stakes to mark the final location

B. Vertex, V, inaccessible.

Curvs of moderate length.

a. By Trial Curv. Fig. 71.

163. Given, the sharpness, D, of an old curv, whose original position was $A\,B$, and the directions of its tans, $A\,V$ and $V\,B$. Required, Δ and the positions of the T.C. (A) and C.T. (B)

Let A'' be the supposed position of the T.C., or a point, near the T.C., on the tan or on the tan produced. Set over A'', and begin running a trial curv, $A''\,m$, of the given sharpness, D, marking the 100-ft chain points temporarily.

If A'' is materially distant from the T.C. (A), as in Fig 71*, the trial curv, $A''\,m$, will soon be found deviating seriously, as at m, from the old track, $A\,B$. Measure the distance, $m\,s$, betw the trial curv, $A''\,m$, and the cen line, $A\,B$, of the old track. Then, for the shift, $A''\,A$, reqd for the T.C., we have :—

$$A''\,A = m\,n = \text{(approx)}\ m\,n'\,^* = \frac{m\,s\ \dagger}{\sin\Delta_m},$$

where Δ_m (or m †) = sweep of trial curv betw A'' and m.*

Let $A''\,A'$ represent the approx value thus found for the shift, $A''\,A$. Move inst to A', and re-run the trial curv, with D as before, taking transit points, T_1, T_2, etc., at preferably equal dists of say 500 ft. From each transit point, locate a short line, as $T_3\,x$, parallel with the original tan, $A\,V$, marking said line with a nail, as at x.

Run the trial curv, $A'\,T_3$, thru a sweep, s†, to a point, as T_4, where its final tan, $v'\,T_4$, is supposed to be parallel with that, $V\,B$, of the old curv, $A\,B$. Thru T_4 run a line, $T_4\,u$, parallel with $V\,B$, and measure the angle, t†, betw $T_4\,u$ and $v'\,T_4$ produced. Then :—

$$\Delta = s \pm t.\dagger$$

*To facilitate illustration, the deviations, in Fig 71, are grossly exaggerated. In practice, the tans of the two curvs, at m and at s, would be approx parallel.

†In Fig 71, the angles, Δ_m, Δ_s and Δ_t are designated as m, s and t, respectivly, to avoid overcrowding.

$$\begin{aligned}
\text{lat of } d = ad'' &= ab' + bc' + cd' \\
&= ab\cos A + bc\cos B + cd\cos C; \\
aA &= ad'' - d''V - VA \\
&= ad'' - dd''/\tan\Delta - R\tan(\Delta/2).
\end{aligned}$$

(For dd'', see below. R = radius of existing curv).

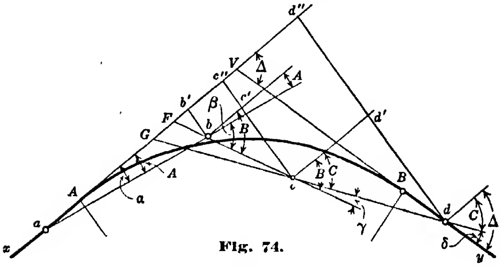
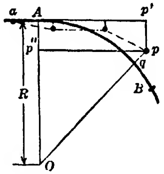


Fig. 74.

Departures (dists normal to the latitudes); for finding dist dB and location of point, B :—

$$\begin{aligned}
\text{dep of } d = dd'' &= bb' + cc' + dd' \\
&= ab\sin A + bc\sin B + cd\sin C; \\
dB &= dV - VB \\
&= dd''/\sin\Delta - R\tan(\Delta/2).
\end{aligned}$$

Fig. 75.



"Throw"

169. Fig 75. (For curvs with spirals, see end of this ¶.) Having thus, by means of the traverse, ¶¶ 166-168, Fig **74**, found the curv point, A, and tan point, B, the several traverse pts may be used for locating, by simple measurement, as many other pts in the curv; thus, Fig 75.— To find the **throw**, or radial dist, $p\,q$, from a given traverse pt, p, to the curv, $A\,B$;

Let $a\,p'$ = lat of p } referd to the origin, a, of the traverse,
$p'\,p$ = dep of p } and found as in ¶ 168.
R = radius, Oq;

Then :—

$$\text{Tan } A\,O\,p = \frac{p\,p''}{O\,p''} = \frac{a\,p' - a\,A}{R - p'\,p};$$

$$O\,p = \frac{p\,p''}{\sin A\,O\,p} = \frac{a\,p' - a\,A}{\sin A\,O\,p};$$

$$p\,q = R - O\,p.$$

If, as in Fig 75, $R < O\,p$, then p must be moved inward. If $R > O\,p$, then p must be moved outward.

Fig 76. For a point, q, in the circular portion, $B\,B_f$, of a spirald curv :— for A, in the foregoing, read A'; and, for $R - p'\,p$, read $R + H - p'\,p$. Then, as above, $p\,q = R - O\,p$.

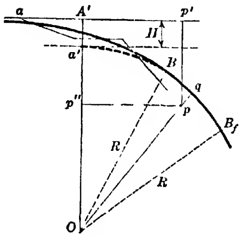


Fig. 76.

Long curvs, requiring compounding.

170. In very long curvs, especially where the track has become much out of line, it is sometimes found unpracticable to approximate satisfactorily to the existing track with a single simple curv. Resort is then had to compounding with curvs of other radii Since long curvs are seldom sharp, the required change of sharpness is seldom more than a few minutes.

The instrument is set over a point as near as may be to the middle point of the curv, and a trial curv is run in each direction, with the sharpness, D, of the existing curv, as ascertained from the records, as estimated, or as assumed

When either trial curv begins to deviate seriously from the existing track, the sharpness is to be slightly increast or diminisht, as may be necessary. This may have to be done several times during the running of the trial curv.

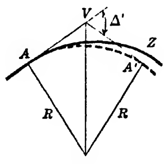


Fig. 77.

Fig 77. Suppose that one of these trial curvs, approaching from the right in the Fig, has thus reacht a point, A', say 500 ft from its destination, which is the curv pt, A, of the existing curv, $Z\,A$. Reqd, the location of A, and the sharpness, D, of the curv, $A'\,A$, which will connect tangentially with the tangent, $A\,V$, at A.

Lay off a tan, $A'\,V$, to its intersection, V, with the tan, $A\,V$, of the existing curv, $Z\,A$. Measure the intersection angle, Δ', at V, and the semitangent, $T = A'\,V = V\,A$.

Then, for the sharpness, D, of the short curv, $A'\,A$, which will complete the line, we have :—

$$R = T \cot (\Delta'/2)\;;\quad D = \text{approx } 5730/R\;;$$

and $V\,A\;(= T) = A'\,V$, measured from V, gives the curv point, A.

171. When double-tracking an existing single-track line, Fig 78, $y\,A\,B\,a\,b\,z$ (centers at O and at o), the sharpness of curvature may be reduced (if there is room on the right of way) by placing the second track, $y'\,A'\,B'$, $B''\,a'\,b'$, $b''\,z'$ (centers at O' and

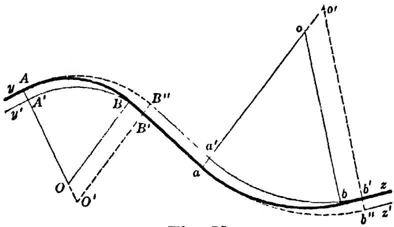


Fig. 78.

at o'), alternately on one and on the other side of the existing single track, constructing also the new curvs, $A\,B''$ and $a\,b''$ (centers at O' and at o'), and eventually removing the old curvs, $A\,B$ and $a\,b$. (Eng News 1913, Oct 23, p 802.)

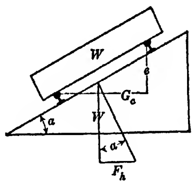
SUPERELEVATION
of Outer Rail on Curvs.

(For transition curvs, see ¶¶ 194, etc.)

Theory.

172. Let a frictionless block, W, sliding forward upon a straight track, upon which the two rails are at the same level, encounter a curv, with superelevation, e, as shown in Fig 79.

Fig. 79.



173. On the curv, its (horizontal) centrifugal force, transversely of the track, is $W\,v^2/R\,g$ (p 354); where W = wt of block; v = its vel, in ft per sec; R = rad of curv, in ft; and g = gravity accel = 32.2 ft per sec per sec.

174. The hor component of its gravitational tendency to slide, transversely of the track, down the inclined plane (*i. e.*, its centripetal force), is $F_h = W \tan a = W\,e/G_c$; where e = superelevation, in ft, and G_c = hor dist, in ft, betw rail centers.

175. In order that the block shall not be shifted laterally, we must have these two forces equal; or $W\,v^2/R\,g = W\,e/G_c$; whence

superel, e, in ft, $= G_c\,v^2/R_g = 0.03106\;G_c\,v^2/R$ .....(114)

For vel, V, in miles per hour, we have:—

superel, e, in ft, $= 0.06680\;G_c\,V^2/R$ ......(115)

Approx, e, in ft $= G_c\,D\,v^2/5730\,g = G_c\,D\,v^2/184{,}500$

$= G_c\,D\,V^2/85{,}768$ ....(116)

176. With standard gage (= 4 ft, 8.5 ins = 4.708 ft), we have, approx, G_e = 4.9 ft, and

	For v, ft/sec;	For V, mi/hour;	
superel, e, in ft	$= 0.1522\ v^2/R$	$= 0.3273\ V^2/R$	
	$= D\ v^2/37{,}652$	$= D\ V^2/17{,}504$	
	$= 0.000\ 026\ 56\ D\ v^2$	$= 0.000\ 057\ 2\ D V^2$	....(117)
superel, E, in ins	$= 1.826\ v^2/R$	$= 3.928\ V^2/R$	
	$= D\ v^2/3{,}137.7$	$= D\ V^2/1{,}458.6$	
	$= 0.000\ 319\ Dv^2$	$= 0.000\ 686\ DV^2$	......(118)

177. For other gages, since the angle, a, Fig 1, is independent of the gage, the superel, e, for a given rad and given vel, is proportional to the gage.

178. Chord, C_o, whose middle ordinate = superelevation. Compare ¶¶ 159, 186 In any circular arc, we have, approx:—

$$\text{chord} = \sqrt{8\ R \times \text{mid ord.}}$$

Hence (Eq 117), we have:

$$C_o = \sqrt{8\ R \times\ 0.000\ 057\ \text{D}\ \text{V}^2} \ldots\ldots\ldots\ldots (119)$$

and, since $R\,D$ = approx 5730. we have, approx:

$$C_o = 1.62\ V = \sqrt{1.219}\ v = 1.1\ v \ldots\ldots\ldots (120)$$

179. Thus, let the speed be 40 miles/hr. For this case, Eq (120) gives C_o = 64.8 ft. Stretch a cord, 64.8 ft long, betw any two pts on the concave side of either rail head. Then, whatever be the sharpness of curvature, the mid ord, from this cord, to the concave side of the same rail head, will be approx the superel reqd, by our equations, for a vel of 40 miles/hr.

Practice.

180. In actual trains, superel has, in view, not only the position of the trucks, transversely of the track (¶ 175), but also the **equilibrium of the car bodies.** Superel counteracts the tendency of the car bodies to swing outward under the action of the centrifugal force.

181. The foregoing equations are based upon the **ideal conditions** of Fig 79; but a railroad car, and, still more so, a train, is a complicated body, made up of many parts which are differently acted upon by diff forces; and the conditions are widely diff from those of Fig 79. Friction, betw wheel tread and top of rail head, causes the first outer wheel of a car to roll forward so that its flange presses against the head of the outer rail; and the resistance of that rail to the wheel's forward motion supplies a centripetal force, additional to that, F_h, of gravity (¶ 174), and wanting in Fig 79; and the action of the two principal forces, upon the following wheels, is complicated by the traction, exerted upon them by preceding portions of the train. See ¶ 36, p 1061. Even if an ideally perfect formula, involving the speed, could be devised, its usefulness would be limited by the fact that but one superel can be given to any piece of track; altho, on most roads, trains must travel at widely diff speeds. Nevertheless, these equations are quite generally used in practice, with modifications to suit special cases.

182. The amount and details of superel are fixt largely by individual judgment (¶¶ 183 to 187). based upon local conditions, by the nature of the traffic, etc. On the score of safety and for the comfort of passengers, the superel is usually adjusted for the *higher* speeds expected; but this may entail heavy work upon long and slow trains; and, where a train stops upon a sharp curv, a high superel causes an objectionable cant in the car bodies.

183. The superel is customarily taken at from 0.5 to 1.0 inch (in some cases, 1.25, 1.5 or even 2 ins) **per degree of sharpness;** with a max total limit of from 4 to 6 (or even 8) inches; speed being reduced where necessary Eq (114) requires a superel of 0.5 inch per deg of sharpness at about 27 miles/hr, 1 inch at about 38 m/h, and 2 ins at about 55 m/h.

184. The common practice (¶ 183) of making the superel proportional to sharpness of curvature, disregards the fact that **high speeds are usually lowered upon sharp curvs.** See ¶ 187.

185. On the New York Central (four tracks) E **max** = **6.5 ins.** For passenger trains on curvs flatter than 1°,

E = 2 ins per deg;

for passenger trains on curvs 1° and sharper,

E = 1 in per deg + 1.5 ins;

for freight trains,

E = 0.75 in per deg.

This gives:

for sharpness	0.25°	0.50°	0.75°	1.00°	1.25°	1.50°	1.75°
E, ins, passenger,	1/2	1	1½	2½	2¾	3	3¼
freight,	3/16	⅜	9/16	¾	15/16	1⅛	1 5/16
for sharpness	**2.00°**	**3.00°**	**4.00°**	**5.00°**	**6.00°**	**8.00°**	**10.00°**
E, ins, passenger,	3½	4½	5½	6½	6½	6½	6½
freight,	1½	2¼	3	3¾	4½	6	6½

186. The Phila. & Reading makes E or e = mid ord of that chord whose length, C_o, = 1.466 V, is the dist run by express trains in one second; E max = 8 inches. Compare ¶¶ 178,179.

187. Other roads use E = 1 inch per deg, plus a quantity beginning with 1 inch for a 1° curv, and diminishing by ⅛ inch for each deg.

This gives

for D =	0	1°	2°	3°	4°	5°	6°	8°	10°
E ins	0	2	2⅞	3¾	4⅝	5½	6⅜	8⅛	10

This takes account of the fact that the fastest running is apt to occur on the easiest curvs. See ¶ 184.

For ordinary practice, the **Am Ry Eng Assn,** Manual, 1915, p 158, recommends superel E = 0.00066 $D\,V^2$. Compare our eq (118) ¶ 176; E max = ordinarily 8 ins.

Cross Section.

188. In single track, superel may be effected, either

(1) by raising the outer rail;	inner rail remains at grade;	car's grav cen raisd $e/2$.
(2) by lowering the inner rail;	outer rail remains at grade;	car's grav cen lowerd $e/2$.
(3) by combining (1) and (2);	center line remains at grade;	car's grav cen maintains elev'n.

The Am Ry Engng Assn, Manual, 1915, p 159, recommends method (1).

Fig. 80.

189. Fig 80. **On double track,** three methods are used;

Plane. Favorable for drainage, and for placing cross-overs and railway and highway-grade crossings;

Saw-tooth. Generally used. Requires cross drains under one of the tracks;

Stept. Permits drainage without cross-drains.

176. With standard gage ($= 4$ ft, 8.5 ins $= 4.708$ ft), we have, approx, $G_e = 4.9$ ft, and

	For v, ft/sec;	For V, mi/hour;	
superel, e, in ft	$= 0.1522\, v^2/R$	$= 0.3273\, V^2/R$	
	$= D\, v^2/37{,}652$	$= D\, V^2/17{,}504$	
	$= 0.000\,026\,56\, D\, v^2$	$= 0.000\,057\,2\, D V^2$	....(117)
superel, E, in ins	$= 1.826\, v^2/R$	$= 3.928\, V^2/R$	
	$= D\, v^2/3{,}137.7$	$= D\, V^2/1{,}458.6$	
	$= 0.000\,319\, D v^2$	$= 0.000\,686\, D V^2$	......(118)

177. For other gages, since the angle, a, Fig 1, is independent of the gage, the superel, e, for a given rad and given vel, is proportional to the gage.

178. Chord, C_o, whose middle ordinate = superelevation. Compare ¶¶ 159, 186 In any circular arc, we have, approx:—

$$\text{chord} = \sqrt{8\, R \times \text{mid ord}}.$$

Hence (Eq 117), we have:

$$C_o = \sqrt{8\, R \times 0.000\,057\,D\,V^2} \quad \ldots\ldots\ldots\ldots (119)$$

and, since $R\,D =$ approx 5730, we have, approx:

$$C_o = 1.62\, V = \sqrt{1.219}\; v = 1.1\, v \quad \ldots\ldots\ldots (120)$$

179. Thus, let the speed be 40 miles/hr. For this case, Eq (120) gives $C_o = 64.8$ ft. Stretch a cord, 64.8 ft long, betw any two pts on the concave side of either rail head. Then, whatever be the sharpness of curvature, the mid ord, from this cord, to the concave side of the same rail head, will be approx the superel reqd, by our equations, for a vel of 40 miles/hr.

Practice.

180. In actual trains, superel has, in view, not only the position of the trucks, transversely of the track (¶ 175), but also the **equilibrium of the car bodies.** Superel counteracts the tendency of the car bodies to swing outward under the action of the centrifugal force.

181. The foregoing equations are based upon the **ideal conditions** of Fig 79; but a railroad car, and, still more so, a train, is a complicated body, made up of many parts which are differently acted upon by diff forces; and the conditions are widely diff from those of Fig 79. Friction, betw wheel tread and top of rail head, causes the first outer wheel of a car to roll forward so that its flange presses against the head of the outer rail; and the resistance of that rail to the wheel's forward motion supplies a centripetal force, additional to that, F_h, of gravity (¶ 174), and wanting in Fig 79; and the action of the two principal forces, upon the following wheels, is complicated by the traction, exerted upon them by preceding portions of the train. See ¶ 36, p 1061. Even if an ideally perfect formula, involving the speed, could be devised, its usefulness would be limited by the fact that but one superel can be given to any piece of track; altho, on most roads, trains must travel at widely diff speeds. Nevertheless, these equations are quite generally used in practice, with modifications to suit special cases.

182. The amount and details of superel are fixt largely by individual judgment (¶¶ 183 to 187), based upon local conditions, by the nature of the traffic, etc. On the score of safety and for the comfort of passengers, the superel is usually adjusted for the *higher* speeds expected; but this may entail heavy work upon long and slow trains; and, where a train stops upon a sharp curv, a high superel causes an objectionable cant in the car bodies.

183. The superel is customarily taken at from 0.5 to 1.0 inch (in some cases, 1.25, 1.5 or even 2 ins) **per degree of sharpness;** with a max total limit of from 4 to 6 (or even 8) inches; speed being reduced where necessary Eq (114) requires a superel of 0.5 inch per deg of sharpness at about 27 miles/hr, 1 inch at about 38 m/h, and 2 ins at about 55 m/h.

198. In the *ideal* transition curv, the radius diminishes proportionally as the dist from the beginning of the spiral, (*measured along the spiral itself*) increases. On such a curv, the superel, at each point, is that proper to the sharpness at that point, at the contemplated velocity. This curv was described by Mr. Ellis Holbrook, in Railroad Gazette, 1880 Dec 3, p 639. It was treated by Prof. A. N. Talbot, in Technograph (University of Illinois) No. 5, 1890-91, and afterward elaborated by him in "The Railway Transition Spiral", New York, Eng News Pub Co, 1904. In "The Transition Curve", New York, John Wiley & Sons, 1899, Prof. C. L. Crandall develops accurate methods for the application of this curv to cases of large sweep. This spiral was recommended for use by the Committee on Track of the Am Ry Eng & M W Assn, Bulletin 73, March 1906, pp 58 etc.

199. Many forms of transition curv have been proposed and elaborated. Between those in common use, the choice is governd rather by facility of computation and of location than by any mechanical differences between the behaviors of the several curvs in operation.

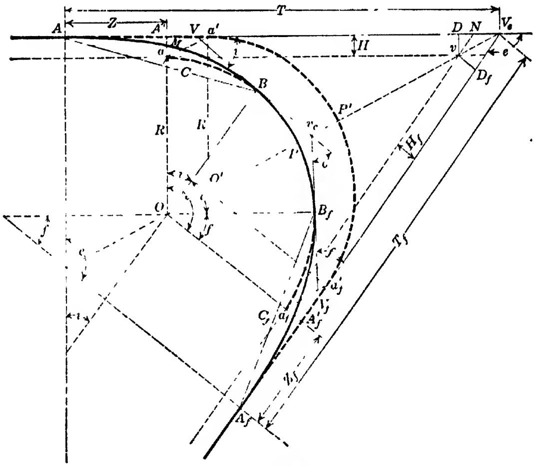


Fig. 81a.

The 10-chord Spiral

200. In the interest of simplification of the necessary formulas, the **American Railway Engineering Association** ("Manual", 1915, page 132) recommends the use of a curv, practically identical with the ideal spiral just described. Said recommended curv is subtended by ten equal spiral chains (usually much shorter than 100 ft), and is called "the ten-chord spiral". In this curv, the sharpness increases with the dist from the beginning of the spiral, measured in 100-ft chains, instead of along the curv itself, as in the true spiral. See example, ¶ 233.

Geometrical Properties. See rules for field use, ¶¶ 231 etc.

Symbols.

201. Figs 81*a*, 81*b*. Let the points, A, B, B_f, A_f, etc, of change be designated as follows (A R E A "Manual", 1915, p 135).

(A)	T. S.	From	tangent to spiral (hitherto called P. S., Point of Spiral) ;
(B)	S. C.	From	spiral to circular curv ;
(B_f)	C. S.	"	circular curv to spiral ;
(A_f)	S. T.	"	spiral to tangent ;
	S. S.	"	spiral to spiral (from one spiral to another).

202. Alfabetical list of symbols. In general, the functions of the *final* spiral are distinguisht by the subscript, $_f$.

Figs	Symbols	Meanings
81, 82	C	= the chord, $A\ B$, in ft, from T. S. to S. C. ;
	C_n	= " " , in ft, from T. S. to any given point, P, on the spiral ;
	c	= the spiral chain used, in ft ;
	c	= Δ_c. See Δ_c below.
81	D	= the sharpness of the central circular curv, $B\ B_f$
		= (practically) the sharpness of either spiral at the point, B or B_f, where it joins the circular curv ;
	d	= the sharpness of the spiral at any given point ;
81*a*	E	= the external distance $P\ V_e$ of the entire curv, $A\ B\ B_f\ A_f$, when the two spirals are equal. So defined by Am Ry Eng Assn. Compare ¶¶ 238, 239.
	e	= Δ_e. See Δ_e below.
82, 83	F	= the angle between the initial tangent, $A\ V$, and the chord, $P'\ P'''$, joining any two given spiral points ;
83	f	= the angle, $m\ P'\ P'''$, at any given spiral point, P', betw the tan, $P'\ m$, at that point, and the chord, $P'\ P'''$, to another given spiral point, P'' ;
	f	= Δ_f. See Δ_f below.
81, 82, 84, 86..	H	= the dist, $A'\ a$, betw the tan, $A\ V$ or $A'\ V_e$, and the parallel tan, $a\ v$, to the circular curv, $a\ B$ ($B\ B_f$, produced) ;
		= the ordinate of the point, a, of the circular curv produced, referd to the T S. (Abscissa = Z) ;
	i	= Δ_i. See Δ_i below.
	k	= the increase in spiral sharpness per 100-ft chord ;
	L	= the sum, in ft, of the lengths of the 10 equal spiral chains ;
	l	= the sum, in ft, of the lengths of the spiral chains betw the T. S., at A, and any given point on the spiral ;
	n	= the number (1, 2, 3....10) denoting any one of the 10 chain pts on the spiral, counting from the T. S. as zero ;
		= the number of spiral chains betw the T. S. and any given point on the spiral ;
	q	= the number of 100-ft chains betw any two given spiral points, P' and P'' ;
81, 82, 84, 86..	R	= the radius of the central circular curv ;
	S	= the number of 100-ft chains from T. S. to S. C. ;
	s	= the number of 100-ft chains betw the T. S. and any given spiral point ;

81, 82 ... T_a, T_b = the initial and final semitans, *A V* and *V B*, of the spiral;

t_a, t_b = the initial and final semitans for any portion of the spiral;

81, 84 .... T, T_f = the initial and final semitangents, $A V_e$, $V_e A_f$, of the entire curv;

82 .... X, Y = the abscissa and ordinate of the S. C. (*B*), referd to the T. S. (*A*);

x, y = the abscissa and ordinate of any given spiral point, referd to the T. S. (*A*);

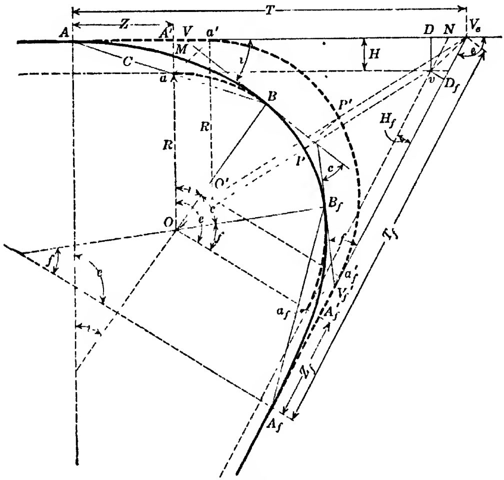


Fig. 81b.

81, 82 .... Z = $A A'$ = abscissa of the point *a*, of the circular curv produced, referred to the T. S.; (ordinate = *H*);

81, 82 .... Δ or Δ_i or *i*. See Δ_i below.

81, 86 .. Δ_c or *c* = sweep of central circular curv, $B B_f$*;

81, 84, 86. Δ_e or *e* = " " entire curv, $A B B_f A_f$;

81 ...... Δ_f or *f* = " " final spiral, $B_f A_f$*;

81, 82 ... Δ_i or *i* = " " initial spiral, $A B$*;

83 .... δ = the sweep of any given portion of the spiral, beginning at the T. S., (*A*);

= the angle betw the initial tan, *A V*, and the tan thru any given spiral pt;

See also next page.

---

*Where only the initial (or the final) spiral is under discussion, it is usually convenient to designate the sweep as Δ; but where the initial and final spirals are discust together, it may be necessary to distinguish betw their respectiv sweeps, and the subscripts are then used, with or without the letter, Δ.

81, 82 .... Θ = the peripheral or "deflection" angle, $V A B$, at the T. S. (A), betw the initial tan, $A V$, and the chord, $A B$, of the spiral;

θ = the peripheral or "deflection" angle betw the tan, $A V$, at A, and the chord from A to any given spiral point;

81, 82 .... Φ = the peripheral or "deflection" angle, $V B A$, at the S. C. (B), betw the final tangent, $V B$, thru the S. C., and the chord, $A B$;

ϕ = the peripheral or "deflection" angle, at any given spiral point, betw the tangent thru that point and the chord from the T. S.

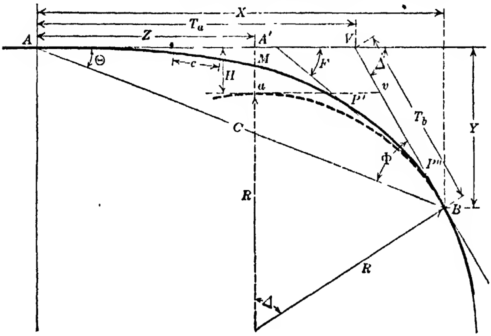


Fig. 82.

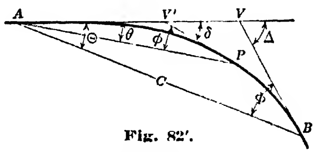


Fig. 82'.

Equations.

203. Figs 82, 82'. **Relations between L, l, S, s, c, n and q.**

$L = 100\,S = 10\,c;\quad S = 0.01\,L;\quad c = 0.1\,L = 10\,S;$

$l = 100\,s = nc;\quad n = l/c = 100\,s/c;$

$q = s'' - s' = 0.01\,c\,(n'' - n')$

$s = 0.01\,l = d/k = 0.01\,c\,n = 0.001\,n\,L$ .... (121)

204. Sharpness. D or d.

$D = k\,S = 0.01\,k\,L = 2\,\Delta/S = 200\,\Delta/L$ .... (122)

$d = k\,s = 0.01\,k\,l = 2\,\delta/s = 200\,\delta/l = n\,\Delta/5\,S$

$= 0.01\,k\,c\,n$ .... (123)

205. Rate, k, of change of sharpness.

$k = D/S = 100\,D/L = d/s = 100\,d/l = 2\,\Delta/S^2$

$=$ (approx) $6\,\Theta/S^2 = 2\,\delta/s^2 =$ (approx) $6\,\theta/s^2$ .... (124)

See also eq (133)

206. Sweep. Δ **or** δ.

$\Delta_s = \Delta + \Delta_c + \Delta_f$ .... (125)

$\Delta = D\,S/2 = D\,L/200 = k\,S^2/2 = k\,L^2/20{,}000$

$= (10/n)^2\,\delta = (S/s)^2\,\delta = (L/l)^2\,\delta$

$= \text{(approx)}\,1.854\,\sqrt{D\,H}$ .... (126)

$\delta = d\,s/2 = d\,l/200 = d^2/2\,k = k\,s^2/2 = k\,l^2/20{,}000$

$= (n/10)^2\Delta = (s/S)^2\Delta = (l/L)^2\Delta$ .... (127)

Peripheral or deflection angle, Θ **or** θ.

The angle between a tangent and a chord.

For equations for field use, see ¶ 231.

Value of deflection angle, θ, **at T. S., or beginning (A) of spiral,** betw the tan, $A\,V$, and the chord joining A with any given spiral point, P, Fig 82'

207. Let δ = the sweep of the spiral between A and the given point, P, Fig 82'. Then,

θ approx* $= \delta/3$ .... (128)

For the true value of θ, we have, very closely,

$\delta/3 - \theta$, in seconds, $= 0.00297\ (\delta \text{ in degrees})^3$.

Thus, let $\delta = 30°$; θ approx $= 30°/3 = 10°$. Then:—

$\delta/3 - \theta$, in seconds, $= 0.00297 \times 30^3$
$= 0.00297 \times 27{,}000$
$= 80.19$ seconds

or, $\theta = (\delta/3) - 0.00297\,(\delta°)^3 = 10° - 80.19$ seconds
$= 36{,}000'' - 80.19'' = 35{,}919.81'' = 9°58'39.81''$.

Taking θ approx $= \delta/3$, we have:—

θ_{approx}, in degrees $= \delta°/3 = k°s^2/6 = d°s/6$ ..... (129)

θ_{approx}, in minutes $= 10\,k°\,s^2 = 10\,d°\,s$ .... (130)

Hence, Figs 81, 82, for the angle $\Theta = V\,A\,B$, betw the tan, $A\,V$, and the chord, $A\,B$, from A to the end, B, of the spiral, we have:—

Θ approx, in minutes, $= 10\,k°\,S^2 = 10\,D°\,S$ ..... (131)

Value of deflection angle, θ_1, at the T. S. (A), betw the tan, $A\,V$, and the chord joining A with **the first spiral point,** $n = 1$.

208. Let s_1, $(= 0.01\,c)$, θ_1 and δ_1 be the values of s, of θ and of δ, respectivly, for point $n = 1$, at the end of the first of the ten spiral chains.

If Δ is less than 45° (as it always is, in practice), we have, from eq (127), for the sweep, δ_1, subtended by the first spiral chain: $\delta_1 < 0.45°$; and, from eq (128) practically:—

$\theta_1 = \delta_1/3 = d_1\,s_1/6 = s_1^2\,k/6 = (0.01\,c)^2\,k/6$ .. (132)

whence

$k = 6\,\theta_1/s_1^2 = 6\,\theta_1/(0.01\,c)^2$ .... (133)

From eq (127), we have:—

$\delta_1 = (1/10)^2\,\Delta = \Delta/100$ .... (134)

and

$\theta_1 = \delta_1/3 = \Delta/300$ .... (135)

From eq (130), we have:—

θ_1, in minutes, $= 10\,k°\,(S/10)^2 = k°\,S^2/10$
$= D°\,S/10 = D°\,s_1$ .... (136)

Values of the deflection angles, F and f, betw a tan and the chord, P' P'', joining any two spiral points. Figs 83a and 83b.

Let

F = the angle betw the initial tan, $A\,V$, and the chord, $P'\,P''$;
f = the angle, at P'', betw the tan, $P''\,m$, and the chord $P'\,P''$.

---

*Degree of approximation. Error = $0.00297\ (\delta°)^3$.

When δ =	10°	20°	30°	40°	50°
Error =	2.97	23.8	80.2	190	371 secs
1000 × error/δ =	0.08	0.33	0.74	1.32	2.06

209. Let

d', d'' = the sharpnesses at P' and at P'', respectivly;

δ' = the sweep betw A and P'
= the angle, at V', betw the initial tan, $A\ V$, thru A, and the tan, $V'\ m$, thru P';

s', s'' = the number of 100-ft chains betw A and P', and betw A and P'', respectivly;

q = $s'' - s'$ = the number of 100-ft chains betw P' and P''.

Required, F and f; ($f = F - \delta'$.)

210. Fig 83*a*.* From any spiral point, P', let a *circular* curv, $P'\ c$, be run, with the sharpness, d' of the spiral at P'. This *circular* curv diverges from the tan, $P'\ m$, at P', at the *constant* rate of d' degrees per 100-ft chord; but the *spiral*, $P'\ P''$, diverges from the same tan, $P'\ m$, at the constantly *increasing* rate of $(d' + k)°$ per 100-ft chord, where k = the increase in spiral sharpness per 100-ft chord.* Hence, the *spiral*, $P'\ P''$, diverges from the *circular* curv, $P'\ c$, at the constantly increasing* rate of $k°$ per 100-ft chord; but this k is also the rate at which the spiral, $A\ P''$, diverges from the initial tan, $A\ V$, at A. Or (since any point in the spiral may be taken as P') let P' be taken at A.* Then, at A, the initial tan, $A\ V$, the tan thru P' (now taken at A), and the circular curv (here a straight line; sharpness, d', = 0°) coincide; and we have (with P' at A): $d' = 0$; and

spiral divergence rate from the tangent thru A
= spiral divergence rate from the tangent thru P' (taken at A)
= spiral divergence rate from the circular curv beginning at P' (taken at A) = $(d' + k)°$ = $k°$.

211. In q 100-ft chords from P', the circular curv, $P'\ c$, describes a sweep of $q\ d'$ degrees, and a peripheral or "deflection" angle = half $q\ d'$; or

$$m\ P'\ c \quad = \quad q\ d'/2 \ \dots\dots\dots\dots\dots\dots (137)$$

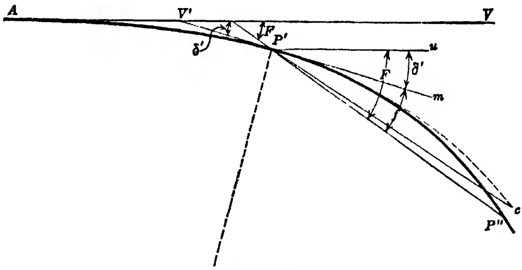


Fig. 83a.

212. In the same distance, q, the spiral increases its sharpness, over that of the circular curv, $P'\ c$, by $q\ k°$; its *mean* sharpness-increase is $q\ k°/2$; and its sweep-increase, over that of the circular

*In Fig 83*b*, the spiral sharpness increase rate, k, is *negativ*; *i. e.*, the sharpness *decreases* from P' toward P'', and the spiral diverges from the circular curv at a *decreasing* rate of $(d' - k)°$ per 100-ft chord. Also, in Fig 83*b*, if P' be taken at A, then P'' also is at A.

curv, is $q\,(q\,k°/2) = q^2\,k°/2$; but the divergence, $c\,P'\,P''$, of the spiral *chord*, $P'\,P''$, from the circular curv chord, $P'\,c$, is only one-third of $q^2\,k°/2$ (see eq 128); or

$$c\,P'\,P'' = q^2\,k/6 \quad \dots\dots\dots\dots (138)$$

213. Now

$$\begin{aligned} f = F - \delta' = m\,P'\,P'' &= m\,P'\,o + c\,P'\,P'' \\ &= \underset{\text{Eq (137)}}{q\,d'/2} + \underset{\text{Eq (138)}}{q^2\,k/6} \end{aligned}$$

$$= \frac{q}{6}\,(2\,d' + d' + q\,k) \quad \dots\dots\dots\dots (139)$$

$$= \frac{q}{6}\,(2\,d' + d'') \quad \dots\dots\dots\dots (140)$$

and

$$F = \delta' + q\,d'/2 + q^2\,k/6 = \delta' + \frac{q}{6}\,(2\,d' + d'') \quad \dots (141)$$

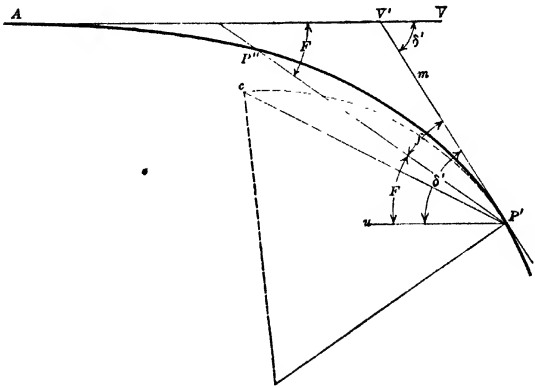


FIG. 240.

Value of f in terms of θ_1.

214. Let $m = f/\theta_1$. From eq (140) and eq (132) we have :—

$$\left.\begin{aligned} m &= \frac{f}{\theta_1} = q\,\frac{2\,d' + d''}{d_1\,s_1}; \\ \text{or } f &= m\,\theta_1 = \theta_1\,q\,\frac{2\,d' + d''}{d_1\,s_1} \end{aligned}\right\} \quad \dots\dots (142)$$

215. Thus, let $c = 20$ ft; $k = 2°$.

Then, $s_1 = 0.01\,c = 0.2$, and $d_1 = s_1\,k = 0.4°$; $d_1\,s_1 = s_1^2\,k = 0.08$; $\theta_1 = d_1\,s_1/6 = 0.0133\ldots$; and, for instance:

Sighting		$q =$	$d' =$	$d'' =$	$m = f/\theta_1 =$	$f =$
from sta	to sta	$0.01\,c \times (n'' - n')$	$0.01\,k\,c\,n'$	$0.01\,k\,c\,n''$	$q\,\dfrac{2\,d' + d''}{d_1\,s_1}$	$m\,\theta_1$
5	8	0.6	2.0	3.2	54	0 72
8	5	− 0.6	3.2	2.0	− 63	− 0.84

For θ_1, see eq (132).

216. For values of the coeff, $m = f/\theta_1$, to be used in obtaining the deflection angle, f, for locating any spiral point, P'', from any inst point, P', see Table, ¶ 233.

Values of the deflection angles F_a and f_a betw a tan and the chord $P'\,P''$ ($= 1$ spiral chain) joining two *adjacent* spiral chain points.

217. Figs 83. Let the points, P' and P'', be now two *adjacent* spiral chain points, n and $n + 1$. Then, from eqs (140) and (123), we have (since now $q = S/10$):—

$$f_a = \frac{S}{60} \times \frac{2\,n + n + 1}{5\,S}\,\Delta = \frac{3\,n + 1}{300}\,\Delta \quad \ldots\ldots (143)$$

and, from eq (127), we have:—

$$\delta' = \frac{n^2}{100}\,\Delta = \frac{3\,n^2}{300}\,\Delta \quad \ldots\ldots\ldots\ldots (144)$$

Hence:—

$$F_a \;(= f_a + \delta') = \frac{3\,n^2 + 3\,n + 1}{300}\,\Delta\,; \quad \ldots\ldots\ldots (145)$$

Let $m_a = 3\,n^2 + 3\,n + 1 = 300\,F_a/\Delta$. Then for the nine points, from $n = 0$ to $n = 9$, sighting from n to $n + 1$, or vice versa, we have, respectivly:—

$n =$	0	1	2	3	4
$n + 1 =$	1	2	3	4	5
$m_a = \dfrac{300}{\Delta}\,F_a =$	1	7	19	37	61
$m_a/300 = F_a/\Delta =$	0 0033	0.0233	0.0633	0.1233	0.2033
$n =$	5	6	7	8	9
$n + 1 =$	6	7	8	9	10
$m_a = \dfrac{300}{\Delta}\,F_a =$	91	127	169	217	271 . . . (146)
$m_a/300 = F_a/\Delta =$	0.3033	0.4233	0.5633	0.7233	0.9033

from which, and from $F_a = \dfrac{m_a}{300}\,\Delta$, the several values of F_a, for the 10 chain points, are easily found, for any given value of Δ.

218. Thus, at station $n = 7$, sighting to station $n + 1 = 8$, the angle, F_a, betw the sight, 7-8, and the initial tan, $A\,V$, is

$$F_a = \frac{m_a}{300}\,\Delta = \frac{169}{300}\,\Delta = (0.5633\ldots)\,\Delta\,.$$

219. Figs 81, 82. **Value of deflection angle, Θ**, at A, betw the tan $A\ V$ and the chord $A\ B$. Let the successiv values of F_a, thus found, be

$$F_1 = \frac{\Delta}{300}\ ; \quad F_2 = 7\ \frac{\Delta}{300}\ ; \quad F_3 = 19\ \frac{\Delta}{300};\ \text{etc};$$

and let x_1, x_2, x_3, etc, y_1, y_2, y_3, etc, be the co-ordinates for the points, $n = 1$, $n = 2$, $n = 3$, ... $n = 10$ (point B). Let $L =$ the sum of the spiral chains, in ft; $c = L/10$.

Then

$$x_1 = c \cos F_1\ ; \qquad y_1 = c \sin F_1$$
$$x_2 = c\ (\cos F_1 + \cos F_2)\ ; \qquad y_2 = c\ (\sin F_1 + \sin F_2) \quad ..(147)$$
$$\text{etc.} \qquad\qquad \text{etc.}$$

$$\text{Tan}\ \Theta = \tan V\ A\ B = \frac{Y}{X} = \frac{\sin F_1 + \sin F_2 + \ \ldots \sin F_{10}}{\cos F_1 + \cos F_2 + \ \ldots \cos F_{10}} \quad ..(148)$$

From values of functions of Θ, thus found, the value is derived:

$$\left(\frac{\Delta}{3} - \Theta\right) \text{in seconds} = 0.002\,97\ \ (\Delta \text{ in degrees})^3$$

or, $$\Theta = \Delta/3 - [0.002\,97\ (\Delta^\circ)^3]\ \text{seconds} \quad .....(149)$$

Value of the deflection angle, Φ, at the S.C. (B), betw the tan $V\ B$ and the chord $A\ B$.

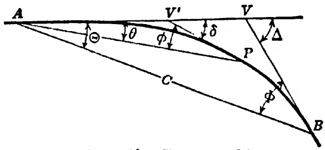


Fig. 82'. (Repeated.)

220. Figs 81, 82. Necessarily, $\Delta = \Theta + \Phi$; or

$$\Phi = \Delta - \Theta = \Delta - \left[\frac{\Delta}{3} - (0.002\,97\ (\Delta \text{ in degrees})^3 \text{ seconds*})\right].$$

Hence:—

$$\Phi = \frac{2}{3}\Delta + \left(0.002\,97\ (\Delta \text{ in degrees})^3 \text{ seconds*}\right) \quad ....(150)$$

Practically*:— $\Phi = 2\,\Delta/3 = 2\ \Theta$ ....(151)

Similarly, Fig 82', in any portion, $A\ P$, of the spiral curv, $A\ P\ B$, said portion beginning at the T.S. (A), we have, practically:—

$$\delta = \theta + \phi\ ;$$
$$\theta = \delta/3\ ;$$
$$\phi = 2\,\delta/3 = 2\,\theta \quad ....(151')$$

221. Relations between H, L and Z. Fig 82.

$$H = \text{approx}\ \frac{L^2}{24\ R} = \frac{L^2}{24} \cdot \frac{D}{5730} = 0.0727\ S^2\ D \quad ..(152)$$

$$L = \sqrt{24\ R\ H} = 4.9\sqrt{R\ H} = 4.9\sqrt{5730\ H/D}$$
$$= 371\sqrt{H/D} \quad ....(153)$$

$$\Delta = 0.005\ D\ L\ (\text{eq } 126) = 0.005\ D\ 371\sqrt{H/D}\ (\text{eq } 153)\ ;\ \text{or}$$
$$\Delta = 1.854\sqrt{D\ H} \quad ....(154)$$

---

*See *note, p 971.

215. Thus, let $c = 20$ ft; $k = 2°$.

Then, $s_1 = 0.01\,c = 0.2$, and $d_1 = s_1 k = 0.4°$; $d_1 s_1 = s_1^2 k = 0.08$; $\theta_1 = d_1 s_1/6 = 0.0133\ldots$; and, for instance:

Sighting		$q =$	$d' =$	$d'' =$	$m = f/\theta_1 =$	$f =$
from sta	to sta	$0.01\,c \times (n'' - n')$	$0.01\,k\,c\,n'$	$0.01\,k\,c\,n''$	$q\,\dfrac{2\,d' + d''}{d_1\,s_1}$	$m\,\theta_1$
5	8	0.6	2.0	3.2	54	0 72
8	5	− 0.6	3.2	2.0	− 63	− 0.84

For θ_1, see eq (132).

216. For values of the coeff, $m = f/\theta_1$, to be used in obtaining the deflection angle, f, for locating any spiral point, P'', from any inst point, P', see Table, ¶ 233.

Values of the deflection angles F_a and f_a betw a tan and the chord $P'\,P''$ ($= 1$ spiral chain) joining two *adjacent* spiral chain points.

217. Figs 83. Let the points, P' and P'', be now two *adjacent* spiral chain points, n and $n + 1$. Then, from eqs (140) and (123), we have (since now $q = S/10$) :—

$$f_a = \frac{S}{60} \times \frac{2\,n + n + 1}{5\,S}\,\Delta = \frac{3\,n + 1}{300}\,\Delta \ \ldots\ldots (143)$$

and, from eq (127), we have :—

$$\delta' = \frac{n^2}{100}\,\Delta = \frac{3\,n^2}{300}\,\Delta \ \ldots\ldots\ldots\ldots\ldots (144)$$

Hence :—

$$F_a\ (= f_a + \delta') = \frac{3\,n^2 + 3\,n + 1}{300}\,\Delta\ ;\ \ldots\ldots\ldots\ldots (145)$$

Let $m_a = 3\,n^2 + 3\,n + 1 = 300\,F_a/\Delta$. Then for the nine points, from $n = 0$ to $n = 9$, sighting from n to $n + 1$, or vice versa, we have, respectivly :—

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$n + 1 =$	1	2	3	4	5
$m_a = \dfrac{300}{\Delta}\,F_a =$	1	7	19	37	61
$m_a/300 = F_a/\Delta =$	0 0033	0.0233	0.0633	0.1233	0.2033
$n =$	5	6	7	8	9
$n + 1 =$	6	7	8	9	10
$m_a = \dfrac{300}{\Delta}\,F_a =$	91	127	169	217	271 . . . (146)
$m_a/300 = F_a/\Delta =$	0.3033	0.4233	0.5633	0.7233	0.9033

from which, and from $F_a = \dfrac{m_a}{300}\,\Delta$, the several values of F_a, for the 10 chain points, are easily found, for any given value of Δ.

218. Thus, at station $n = 7$, sighting to station $n + 1 = 8$, the angle, F_a, betw the sight, 7-8, and the initial tan, $A\,V$, is

$$F_a = \frac{m_a}{300}\,\Delta = \frac{169}{300}\,\Delta = (0.5633\ldots)\,\Delta .$$

Selection of spiral.

227. Fig 82. The selection of the length, L, of the spiral may be **restricted by the value of H,** as determind by topographical or other conditions. We then have (eq 153), for length of spiral:—

$$L = \sqrt{24\,R\,H};$$

where R = radius of circular curv.

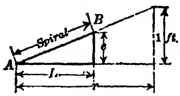

Fig. 83½.

228. Fig. 83½. Where the choice of spiral is not thus restricted, the length, L, is **determind by the superelevation, e,** in ft (or E in inches) on the circular curv, and by the rate, r, of run-off in ft of line per ft of e (¶¶ 190-193); thus (eq 117):—

$$L = e\,r = 0.3273\,V^2\,r/R = 0.1522\,v^2\,r/R = 0.1522\,v^3\,t/R \ldots\ldots (174)$$

where $t = r/v$ = time, in seconds, during which the outer wheel, on the runoff, would rise 1 foot.

The Am Ry Eng Assn rule, eqs (175), gives $t \not< 5.45$ when V = 45 miles/hr, and superel = 8 ins = 2/3 ft.

229. The Am Ry Eng Assn (Manual, 1915, pp 131-132) recommends —

On curvs which do not limit the speeds of trains:—

$$L \not< 360\,e_u; \quad \text{or} \quad L \not< 8\,V_u\,e_u \ldots\ldots\ldots\ldots (175)$$

On curvs which limit the speeds of trains:—

When $D < 6°$, $L \not< 16\,V_8 / 3$; when $D \not< 6°$, $L \not< 240 \ldots (176)$

where

L = spiral length, in ft;
V_u = ultimate speed, in miles/hr; e_u = superel, in ft, for V_u;
V_8 = speed in miles/hr calculated for an elevation of 8 ins.

$$= \sqrt{\frac{8\,R}{3.928}} \quad \text{(see eq 118).}$$

With V = 45 miles/hr, and superel, E, = 8 ins, or e = 2/3 ft, each of these rules gives $L \not< 240$ feet.

230. In practice, and in order to avoid the use of sub-chains in the spiral, a length deviating somewhat from the calculated length, L, of spiral, may be used.

For instance, with H = 9.6 ft, circular curv 5° (R = 1146.3 ft), we have (eq 153):— $L = \sqrt{24\,R\,H}$ = 513.9 ft.

With V = 50 m/h, on a 6° curv (R = 955.4 ft), rate, r, of run-off = 600, we have (eq 174):—

$$L = r\,0.3273\,V^2/R = 513.9 \text{ ft.}$$

In either case, we may use 500 ft (10 chains of 50 ft) or 510 ft (10 chains of 51 ft) or 525 ft (10 chains of 52.5 ft), etc.

Location.

Condensed results, for field use.

By deflection angles.

231. The spiral may be conveniently located by turning off "deflection" angles from a tan, as in locating a circular curv; but, in thus locating a spiral, the successiv increments, in the total deflection angle turnd off from the tan at a given inst point, increase regularly, instead of being constant as in the case of a circular curv.

215. Thus, let $c = 20$ ft; $k = 2°$.

Then, $s_1 = 0.01\,c = 0.2$, and $d_1 = s_1 k = 0.4°$; $d_1 s_1 = s_1^2 k = 0.08$; $\theta_1 = d_1 s_1/6 = 0.0133\ldots$; and, for instance:

Sighting		$q =$	$d' =$	$d'' =$	$m = f/\theta_1 =$	$f =$
from sta	to sta	$0.01\,c \times (n'' - n')$	$0.01\,k\,c\,n'$	$0.01\,k\,c\,n''$	$q\,\dfrac{2\,d' + d''}{d_1\,s_1}$	$m\,\theta_1$
5	8	0.6	2.0	3.2	54	0 72
8	5	− 0.6	3.2	2.0	− 63	− 0.84

For θ_1, see eq (132).

216. For values of the coeff, $m = f/\theta_1$, to be used in obtaining the deflection angle, f, for locating any spiral point, P'', from any inst point, P', see Table, ¶ 233.

Values of the deflection angles F_a and f_a betw a tan and the chord $P'\,P''$ ($= 1$ spiral chain) joining two *adjacent* spiral chain points.

217. Figs 83. Let the points, P' and P'', be now two *adjacent* spiral chain points, n and $n + 1$. Then, from eqs (140) and (123), we have (since now $q = S/10$):—

$$f_a = \frac{S}{60} \times \frac{2\,n + n + 1}{5\,S}\,\Delta = \frac{3\,n + 1}{300}\,\Delta \ \ldots\ldots (143)$$

and, from eq (127), we have:—

$$\delta' = \frac{n^2}{100}\,\Delta = \frac{3\,n^2}{300}\,\Delta \ \ldots\ldots\ldots\ldots\ldots (144)$$

Hence:—

$$F_a\ (= f_a + \delta') = \frac{3\,n^2 + 3\,n + 1}{300}\,\Delta\ ;\ \ldots\ldots\ldots\ldots (145)$$

Let $m_a = 3\,n^2 + 3\,n + 1 = 300\,F_a/\Delta$. Then for the nine points, from $n = 0$ to $n = 9$, sighting from n to $n + 1$, or vice versa, we have, respectivly:—

$n =$	0	1	2	3	4
$n + 1 =$	1	2	3	4	5
$m_a = \dfrac{300}{\Delta}\,F_a =$	1	7	19	37	61
$m_a/300 = F_a/\Delta =$	0 0033	0.0233	0.0633	0.1233	0.2033
$n =$	5	6	7	8	9
$n + 1 =$	6	7	8	9	10
$m_a = \dfrac{300}{\Delta}\,F_a =$	91	127	169	217	271 . . . (146)
$m_a/300 = F_a/\Delta =$	0.3033	0.4233	0.5633	0.7233	0.9033

from which, and from $F_a = \dfrac{m_a}{300}\,\Delta$, the several values of F_a, for the 10 chain points, are easily found, for any given value of Δ.

218. Thus, at station $n = 7$, sighting to station $n + 1 = 8$, the angle, F_a, betw the sight, 7-8, and the initial tan, $A\,V$, is

$$F_a = \frac{m_a}{300}\,\Delta = \frac{169}{300}\,\Delta = (0.5633\ldots)\,\Delta.$$

Example. Given

c = spiral chain length, = 20 ft;

k = increase of spiral sharpness, d, per 100-ft chord, = 3°.

Then :—

Eq (121) Length, L, of spiral, $A B$, in ft, measured along spiral chains, = 10 c = 10 × 20 = 200 ft;

Number, S, of 100-ft chords in L, = approx 0.01 L = 2;

Eq (122) Sharpness, D, of spiral at the S.C. (B), = approx $k S$ = 3° × 2 = 6°;

Eq (126) Sweep, Δ, of spiral, $A B$, = approx $DS/2$ = 6° × 1 = 6°;

Eq (129) Defl angle, Θ = $V A B$, = app $\Delta/3$ = app 2°;

Defl angle, Φ = $V B A$, = app $2\Delta/3$ = app 4°.

Required the deflection angle, f, from the tangent, $P' m$, at the point P'' to another spiral point, P'''.

Here, eq (132)

θ_1 = $(0.01\, c)^2\, k/6$ = $0.04 \times 3°/6$ = 0.02°;

and (from table above), Fig 83a, sighting from

P'', (n = 5) to P''', (n = 8), m = 54.

Hence, in this case,

f ($= m\,\theta_1$) = 54 × 0.02° = 1.08°.

Again, Fig 83b, sighting from P', (n = 8) to P'', (n = 5), we have

f ($= m\,\theta_1$) = 63 × 0.02° = 1.26°.

234. Values of m etc, in special cases. Figs 82, 83.

(a) When the inst point, P', is at the T.S. (A) or point n = 0 (Top line of table ¶ 233).

Figs 82, 83a. When sighting (forward) from A to any other spiral point, P''', of number n, eq (142) becomes

$$m = q\,\frac{2\,d' + d''}{d_1\, s_1} = 0.01\, c\, n\,\frac{k\, s''}{0.01\, k\, c\,.\,0.01\, c} = n\,\frac{s''}{0.01\, c} = n^2;$$

and θ'' (θ for point P'') $= m\,\theta_1 = n^2\theta_1$.

Hence, when P'' is at the S.C. (B, or n = 10), we have m ($= n^2$) = 100, and Θ ($= V A B$) = 100 θ_1 = approx $\Delta/3$.*

(b) Fig 83b. When the point, P'', sighted (backward), is at the T.S. (A) or point n = 0 (Column headed T.S.).

When sighting from any other point, P', of number n, to A, we have, approx, eq (151′), $\phi = 2\,\theta$.

Hence, when the inst point, P', is at the S.C. (B, or n = 10), we have Φ ($= V B A$) = 200 θ_1 = approx $2\,\Theta = \dfrac{2}{3}\Delta$.*

By offsets from the initial tangent, A V.

235. Fig 82. The offsets, x and y (eqs 147, 158, 159) to each of the points in the entire spiral $A B$, may be measured from the initial tan, $A V$.

Or, make the offset, $A' M$, for the middle point, M, of the spiral, = $A' a/2$ = $H/2$ (¶222), and the remaining offsets proportional to the cubes of their dists from A and from B respectivly, measuring the offsets for the first half of the spiral inward at right angles from the tan $A V$, and the remainder radially outward from the circular curv, $a B$.

By offsets from long chord, A B. (E. S. M. Lovelace, Canadian Soc Civ Engrs. Reprinted in Engng & Contractg, 1914 Mar 25).

236. Fig 82. Let

h = the normal offset from the long chord to any given point in the spiral;

H = dist betw parallel tans, $A V$ and $a v$;

n = number of spiral chains betw the T. S. (A) and the given point;

N = number of spiral chains in the spiral.

---

*See foot-note *, p 971.

Then, **in any spiral:**

$$\frac{h}{H} = 4\,n\,\frac{N^2 - n^2}{N^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (177)$$

This equation, derived for the lemniscate, is closely approx for the A R E A 10-chord spiral,

In the 10-chord spiral, $N = 10$; so that

$$\frac{h}{H} = 4\,n\,\frac{100 - n^2}{1000} = 0.4\,n - 0.004\,n^3 \quad \ldots\ldots\ldots (178)$$

Hence we have:

When

$n =$	0	1	2	3	4	5	6	7	8	9	10
$h/H =$	0	0.396	0.768	1 092	1.344	1.500	1.536	1 428	1.152	0.684	0

h is a max ($= 1.54\,H$) when $n = N/\sqrt{3} = 10/1.7321 = 5.77$.

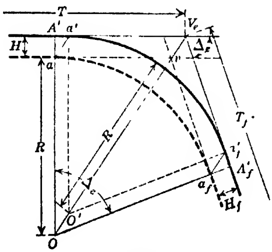

Fig. 84.

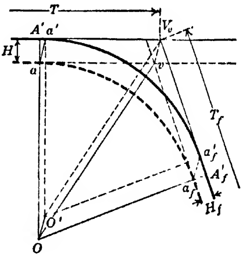

Fig. 85.

Insertion of spirals in existing track.

See also ¶ 169

237. (a) Figs 81, 84, 85. Original circular curv, $a'\,a_f'$, shifted **without change of radius,** thru a given dist, $a'\,a$, (equal and parallel to $V_e\,v$ and to $a_f'\,a_f$) to a new position, $a\,a_f$.

Required the semitangents, T and T_f, to the entire curv, ABB_fA_f, **Fig 81.**

(1) Figs 81*a*, 84. The two spirals similar. Shift, $a'\,a$, parallel to common bisector $O\,v$, of $a'\,a_f'$ and of $a\,a_f$; $T_f = T$; $H_f = H$. R = rad of circular curv.

Here $H = H_f = a'\,a\,\cos(\Delta_e/2)$ ……………………… (179)

$T = T_f = (R + H)\tan(\Delta_e/2) + Z$ …………… (180)

From eqs (153 and 157), we have

$$Z = \frac{L}{2} - \frac{L^3}{1280\,R^2} = \text{practically } \frac{\sqrt{24\,R\,H}}{2} = 2.45\,\sqrt{R\,H}.$$

(2) Figs 81*b*, 85, The two spirals dissimilar. Shift, $a'\,a$ (equal and parallel to $V_e\,v$ and to $a_f'\,a_f$), not parallel with bisector, $O\,v$ or $O'\,V_e$.

T_f and T unequal. H_f and H unequal.

For T and T_f, see eq (172).

For H and H_f, see eq (169).

238. (b) Fig 86. Retaining the original middle point, P', and shortening the radius from R' to R. The line is changed from $A\,a'\,P'$ to $A\,B\,P'$. Having determind (¶¶ 227 &c) the length, L,

and $A'\,a = H = \dfrac{L^2}{24\,R}$;

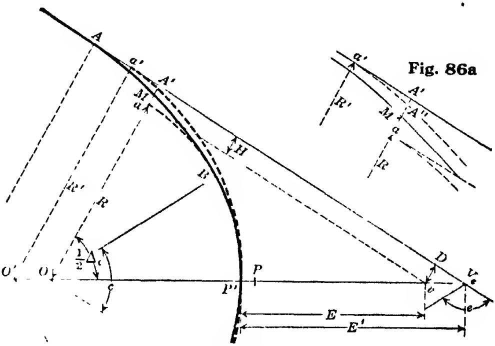

Fig. 86.

let

Δ_e = entire sweep, = 2 arc $a\,P'$ = 2 angle $a\,O\,P'$;
$A''\,M$ (Fig 86a) = max shift of center line of track;
$a'\,A'$ = shift of point a' of circular curv, toward intersection, V_e; and

Radius,	Semitan,	Ext dist,	Sharpness,	
$R' = O'\,a'$	$a'\,V_e$	$E' = P'\,V_e$	D'	of existing circular curv, $a'\,P'$;
$R = O\,a$	$a\,v$	$E = P'\,v$	D	of sharpend circular curv, $a\,P'$.

Then :

$$E = E' - V_e\,v = E' - H\ \sec(\Delta_e/2)\ ; \qquad (181)$$

$$R = \frac{E}{\text{ex sec}\ (\tfrac{1}{2}\,\Delta_e)}\ ; \qquad (182)$$

$$D = \frac{E\ \text{of}\ 1°\ \text{curv, of given sweep,}\ \Delta_e}{E} \qquad (183)$$

$$a'\,A' = (R' - R - H)\ \tan(\Delta_e/2)$$
$$= a'\,V_e - (R + H)\ \tan(\Delta_e/2) \qquad (184)$$

Practically (Fig 86a), $A''\,M = \dfrac{H}{2} - \dfrac{7}{8}\left(\dfrac{a'\,A'}{100}\right)^2 D$ ..... (185)

239. (c) Fig 86. With minimum shift of line. Shift the middle point, P' to P, making $P'\,P = H/2$. Then :

$$E = E' - H\ \sec(\Delta_e/2) - H/2 \qquad (186)$$

Use this new value of E in Eqs (182) and (183).

Then, **in any spiral:**

$$\frac{h}{H} = 4\,n\,\frac{N^2 - n^2}{N^3} \quad \ldots\ldots\ldots\ldots\ldots\, (177)$$

This equation, derived for the lemniscate, is closely approx for the A R E A 10-chord spiral,

In the 10-chord spiral, $N = 10$; so that

$$\frac{h}{H} = 4\,n\,\frac{100 - n^2}{1000} = 0.4\,n - 0.004\,n^3 \quad \ldots\ldots\ldots (178)$$

Hence we have:

When

$n =$	0	1	2	3	4	5	6	7	8	9	10
$h/H =$	0	0.396	0.768	1 092	1.344	1.500	1.536	1 428	1.152	0.684	0

h is a max ($= 1.54\ H$) when $n = N/\sqrt{3} = 10/1.7321 = 5.77$.

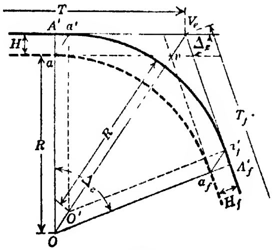

Fig. 84.

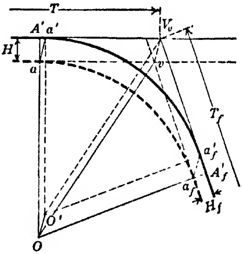

Fig. 85.

Insertion of spirals in existing track.

See also ¶ 169

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Required the semitangents, T and T_f, to the entire curv, ABB_fA_f, **Fig 81.**

(1) Figs 81*a*, 84. The two spirals similar. Shift, $a'\,a$, parallel to common bisector $O\ v$, of $a'\ a_f'$ and of $a\ a_f$; $T_f = T$; $H_f = H$. R = rad of circular curv.

Here $H = H_f = a'\ a\ \cos(\Delta_e/2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (179)$

$T = T_f = (R + H)\ \tan(\Delta_e/2) + Z \ldots\ldots\ldots\ldots (180)$

From eqs (153 and 157), we have

$$Z = \frac{L}{2} - \frac{L^3}{1280\,R^2} = \text{practically}\ \frac{\sqrt{24\,R\,H}}{2} = 2.45\,\sqrt{R\,H}.$$

(2) Figs 81*b*, 85, The two spirals dissimilar. Shift, $a'\,a$ (equal and parallel to $V_e\ v$ and to $a_f'\ a_f$), not parallel with bisector, $O\ v$ or $O'\ V_e$.

T_f and T unequal. H_f and H unequal.

For T and T_f, see eq (172).

For H and H_f, see eq (169).

much used during fogs, snow, storms, etc, when visible signals cannot well be seen. They are largely used also as an auxiliary to back-flagging, to protect the train ahed while the flagman is returning to it, or in case the following enginman fails to see him.

Fixt Signals.

7. Disc signals give their indications, by day as well as by night, solely by means of color, and not by form or position. Each disk sig is usually enclosed in a case, shaped like a banjo (whence the name, "banjo" signal) and provided with a glass face. For day use, the sig proper consists essentially of an opaque colord disc. If this disc is seen thru the glass face, it indicates "stop" when red, or "caution"* when green. To indicate "proceed", the disc is swung to one side, leaving a white background showing thru the glass face. For night use, a colord glass disc indicates "stop" or "caution" by appearing in front of a lamp; or "proceed" by being withdrawn, leaving the lamp to show thru a clear glass.

8. Semafors. Signals are usually of the semafor type, see Figs 7 to 12, incl, each consisting of a movable arm or "board", about 4′ long and 10″ wide, pivoted and mounted on a post or other suitable object.

9. Home Signal. A home sig is one at which the train must stop when the sig is set for "stop" It is placed usually at the entrance to a block, or immediately before reaching a switch, derail, crossing, drawbridge, or other object to be garded. When horizontal, such a sig indicates "stop", and by night the hor position is indicated by a red glass, which appears in front of a lantern, thus showing a red light for stop. When inclined, the sig indicates "proceed", and shows, by night, a light of some color other than red. See also "Signal aspects", ¶¶ 17, 32 and 33.

10. Distant Signal. A "Distant" or "caution"* sig, when provided, is placed at some distance (usually several thousand ft) "to the rear of" (before reaching) the "home" sig it relates to, and indicates the position of the home sig, which may be out of sight. It thus givs the enginman advance information regarding the home sig, and makes possible faster or safer running. The indications of the home and dist sigs always simultaneously correspond, unless a train is on the track between the two, in which case the dist sig shows "caution"* altho the home sig may be at "proceed".

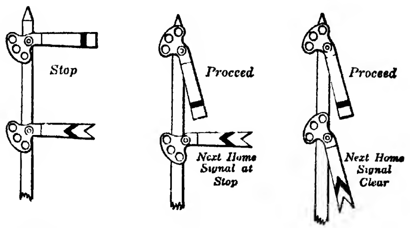

Fig. 7. Fig. 8. Fig. 9.

11. Disposition of sigs. The home sig and the dist sig for the block ahed, are usually mounted on the same post, as in Figs 7, 8 and 9; the upper one being the home, and the lower the dist sig.

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*"Caution" is a term here used somewhat loosely, tho conveniently, to denote its usual or approx meaning: "proceed prepared to stop at next signal".

Then, **in any spiral:**

$$\frac{h}{H} = 4\,n\,\frac{N^2 - n^2}{N^3} \qquad \ldots\ldots\ldots\ldots\ldots (177)$$

This equation, derived for the lemniscate, is closely approx for the A R E A 10-chord spiral,

In the 10-chord spiral, $N = 10$; so that

$$\frac{h}{H} = 4\,n\,\frac{100 - n^2}{1000} = 0.4\,n - 0.004\,n^3 \qquad \ldots\ldots\ldots (178)$$

Hence we have:

When

$n =$	0	1	2	3	4	5	6	7	8	9	10
$h/H =$	0	0.396	0.768	1.092	1.344	1.500	1.536	1.428	1.152	0.684	0

h is a max ($= 1.54\,H$) when $n = N/\sqrt{3} = 10/1.7321 = 5.77$.

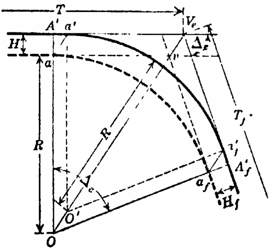

Fig. 84.

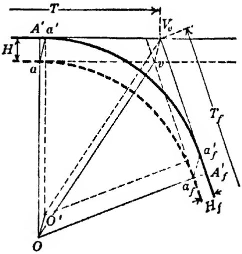

Fig. 85.

Insertion of spirals in existing track.

See also ¶ 169

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Here $H = H_f = a'\,a\,\cos(\Delta_e/2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (179)$

$T = T_f = (R + H)\,\tan(\Delta_e/2) + Z \ldots\ldots\ldots\ldots (180)$

From eqs (153 and 157), we have

$$Z = \frac{L}{2} - \frac{L^3}{1280\,R^2} = \text{practically}\ \frac{\sqrt{24\,R\,H}}{2} = 2.45\,\sqrt{R\,H}.$$

(2) Figs 81*b*, 85. The two spirals dissimilar. Shift, $a'\,a$ (equal and parallel to $V_e\,v$ and to $a_f'\,a_f$), not parallel with bisector, $O\,v$ or $O'\,V_e$.

T_f and T unequal. H_f and H unequal.

For T and T_f, see eq (172).

For H and H_f, see eq (169).

18. Raising the arm into the upper *left*-hand quadrant, is coming largely into use on electric railroads, and has the advantage of placing the sig at the same angles as in the old familiar lower right-hand quad, and is more redily placed so as to be visible among telegraf or trolley poles.

19. Flash signals, for use at night, are being tried, especially abroad. The illuminant is usually acetylene gas, and each lamp is arranged to flash periodically (much as certain light-house lamps flash) usually burning for about 0.1 second, and being extinguisht for from about 0.3 to 0.9 second. They appear to be reliable and economical, and assist in distinguishing sig lights, in general, from other lights foreign to the railroad, and for distinguishing betw different kinds of sigs.

20. Light signals, with which no semafor is used, are economical in first cost, and have no moving parts, aside from those of the relays that control them. They are used in subways and tunnels, where a semafor could not well be seen; and to some extent on electric surface roads. Where used in daylight, each light should be accompanied by a hood or other means for preventing the direct rays of the sun from falling on the lens glass, and by a screen or shield or other large dark object, to attract attention to the sig, make it more redily visible, and so reduce the chance of its being overlookt. See also "Two-light signals", ¶ 16.

21. Light and flash signals are being experimentally combined.

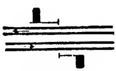


Fig. 13
Double Track

Fig. 14
Trailing Point Cross-over

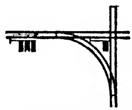


Fig. 15
Two Home and One Distant

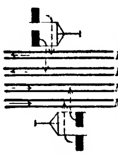


Fig. 16

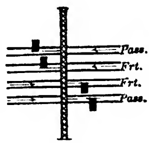


Fig. 17

Four Tracks Signaled

22. Locations of signals. Fixt visible sigs are usually placed to the right of, or directly over, the track to which they relate. Figs 13 to 17, incl, show diagrammatically certain typical sig arrangements, selected from those given in the Ry Sig Assn's "Signal Dictionary".

TRANSMISSION MECHANISM

23. Signals and switches are conveniently operated from a central point, as a sig tower; and means must be provided for connecting them with it, and for controlling them from it

Hand operated

24. Mechanical connections for hand power operation are made usually by pipes (used as rods) or by wires.

25. The pipe is usually of wrought iron or steel, one inch diam. Since it can transmit either tension or compression, only one pipe is needed (in general) for each sig or switch. The lines of pipe are usually carried upon and held in line by means of rollers held in frames. Where sharp turns must be made, the pipe ends are connected by means of bell-cranks, *B*, Fig 18 (not to scale), or by means of curvd deflecting bars, *D*, running betw rollers.

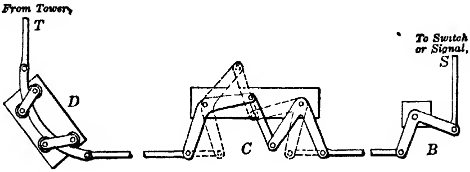


Fig. 18.

26. Compensators. Since even moderate changes in temperature materially alter the length of a pipe line, and disarrange the adjustment, the pipe lines are divided into sections, and the sects are connected by means of "compensators" or "lazy-jacks", *C*, which neutralize the effects of temp-change, without affecting the sig movements. Fig 18 represents a line of pipe, leading from the sig tower, *T*, thru a deflecting bar, *D*, a "compensator", *C*, and a bell-crank, *B*, to a switch or sig, *S*. The dotted lines show the positions of the compensator members when the pipe line is expanded by heat.

Power

27. Power operation may be effected either by comprest air, conveyd thru pipes to cylinders, in which it acts upon pistons connected with the sigs or switches; or else by electricity, conveyd by wires to electric motors (or to solenoids for small sigs). The comprest air, or electricity, which *operates* the sigs, is controld usually by means of relativly light electric circuits, or, in the all-pneumatic apparatus, by relativly low-pressure air. In either case, the power is controlled thru relays.

28. All-electric systems are coming largely into use. They have the advantage that no special air-compressing plant is needed; and various comprest air troubles are eliminated.

SYSTEMS

29. Time interval. Under this syst, a train must be given a headway of say 5, 7 or 10 mins, before a following train is permitted to proceed; the supposition being that, if the first train is unexpectedly stopt, the time interval is sufficient to permit the flagman to flag the following train; but the system, at best, is far from safe.

30. Space-interval or block. Under this syst, the length of the road is divided into sections called blocks, of suitable lengths, and where absolute blocking is practist, no two trains are permitted to be in one and the same block at the same time. The block length may vary widely from abt one mile or less to 5 or 10 miles, according to density and character of traffic.

31. Permissiv. Under permissiv blocking, a train is permitted to pass a block sig showing "stop", with the understanding that it is to run only at such diminsht speed that it can be stopt at the first point which cannot be passed with obvious and certain safety. This permits a larger train operation with a given block length; but it is less safe than absolute blocking.

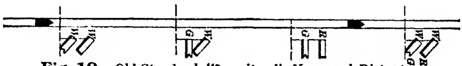


Fig. 19. *Old Standard; "2-position", Home and Distant*

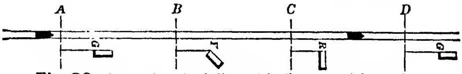


Fig. 20. *Newer Standard; "3-position", upper-right quad.*

32. Signal aspects, as frequently applied to block signaling. Figs 19 and 20 illustrate, for given conditions on a given line, the sig positions, under the **"two-position"** and **"three-position"** semafor systs, respectivly. *A B*, *B C* and *C D* represent blocks. Under either syst, the train, in block *C D*, is protected by the home sig at *C*, behind it, set at "stop"; but at the same time, a train may enter block *B C* at *B*, under the "caution" sig there; or a train may enter block *A B* at *A*, without restriction; the sig, at *A*, indicating not only "proceed", but also "next sig beyond is clear".

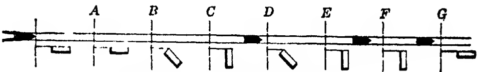


Fig. 21. *Normal "Clear"* (3-*pos.*)

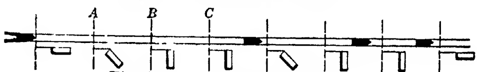


Fig. 22. *Normal "Danger"* (3-*pos.*)

33. Figs 21 and 22 illustrate, for given conditions on a given line, the semafor positions in the "three-position" syst, under the **"normal clear"** and **"normal danger"** systs, respectivly. Under the "normal clear" syst, the sig, controlling a block, always shows clear when the block is clear, whether or not a train is about to enter the block. Under the "normal danger" syst, the sig remains at danger (whether or not the block is occupied) until a train approaches the sig, to enter the block; and then, of course, the sig is cleard only if the block is actually clear. These diffs are shown in blocks *AB* and *BC*.

34. Telegraf block or manual block. Under this syst, a man is stationd at the end of each block; and, when a train passes from one block to another, the man at the passing point communicates the fact, electrically, to the men at such other points as may be necessary. The men, so advised, then display the necessary sigs.

35. The telefone is largely displacing the telegraf for such signaling, and for train dispatching generally. It is commonl, used in conjunction with selector systs, see ¶ 71.

36. Controlled Manual, or Lock and Block System. To reduce the chances of error of operation in telegraf or manual block systs, arrangements may be instald, which require the simultaneous action of both men in making or breaking electric contacts, before the sig can be set to "proceed".

37. Automatic locking. To gard against an agreement between the two men to open the block when it is not actually clear, "track instruments", or the "track circuit" (see ¶ 50) may be so arranged that this agreement cannot be made until the train has actually past out of the block.

38. Staff System. This method is especially applicable to single track roads. At each end of a block is a receptacle containing a number of metal staves or rods. The two receps are so arranged and electrically connected that only *one* staff may be taken from the receps; and this staff must be returned to one of the receps before another staff can be taken. The staff is the train's permit to travel over the block, and is carried by the train When the train leavs the block (at either end), the staff is placed in the recep at that end; rendering it again possible to remove one staff from the recep at either end of the block.

39. Special provisions may be made, to operate the machines under "permissiv blocking", and to provide for the return of "helper" engins; but this still does not permit two complete staves to be out of the receptacles at the same time.

MANIPULATION

40. Manipulation may be either *"initiativ"* or *"interlocking"*, or both. (See ¶ 3*a*.)

41. Initiativ manipulation may be either that effected by *employees*, or (either mechanical or electrical) by *the trains* themselvs

42. Interlocking manipulation (part of which may be in the "Transmission") is that of automatic devices, designed to prevent improper operations. It may be accomplisht either by purely mechanical movements, by electricity or by comprest air. Where even a few sigs and switches are operated from one point, collision of trains may be caused by errors on the part of the operators. In order to prevent this, the levers and rods are so groupt, and provided with additional sliding pieces, cald "dogs", or with electric contacts, locks, etc, that (in so far as possible) combinations or settings, which might endanger trains, are mechanically prevented.

43. Mechanical. Thus, (here considering mechanical interlocking as an illustration) at a double-track crossing, when all the sigs show "stop", any one sig may be made to show "proceed"; but, as it does so, the rod, connected with its lever, moves such dogs as will lock the levers of the sigs governing the tracks which cross its track. Thus, if the sig for a westbound train is cleard, the two sigs for northbound and for southbound movements are thereby lockt at "stop"; but the sig for the eastbound track is left clear; because both eastbound and westbound trains may of course pass over the crossing simultaneously while the northbound and southbound tracks are lockt at "stop".

44. Conversly, northbound and southbound trains can be given clear sigs only after the eastbound and westbound sigs hav been set at "stop" and lockt in that position.

45. The fundamental principle, here illustrated by a simple example, is carried out into very great elaboration.

46. Derailing switches. To prevent collisions due to an engineer inadvertently passing a signal showing "stop", a "derail" is frequently employed where practicable. This is usually a half switch (the inner "point"), which, when "open", is sufficient to gide a train off the rails and onto the ties; or, where elaborated for passenger traffic, it may be a complete switch leading to a side track buried in sand, or provided with other means for stopping

the train with a minimum of damage. Such a derail is so interlockt with switches and sigs as to prevent its being "closed" for normal running, unless all other related switches and sigs are so set as to make it safe to do so. The derail produces a very wholesome effect upon any tendency of the engineer to "take chances" or even to forget a signal set at danger.

47. Scotchblock. Other derails, commonly used on freight sidings, etc, consist of a movable metal piece, so shaped that when placed over the rail, it will lift and gide the flange over the rail to the outside of the track.

48. Switch-locking. Fig 23, (not drawn to scale, and showing essentials only). In addition to the switch lever, there is usually provided a locking lever, which can thrust a pin, *A*, thru either one of two holes in the bar, *B*, only when the switch-points, *C* and *D*, have made their complete movement in either direction. If, as by an accumulation of dirt or of ice, the switch-points have not been brought home, neither hole in the bar, *B*, is brought opposit the pin, *A*, and the pin cannot enter the hole; and, since the locking lever is in turn interlockt with the signals, these can-

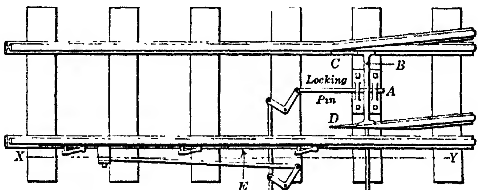


Fig. 23. *Plan*

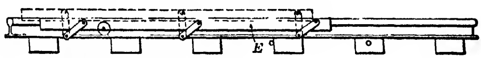


Fig. 24. *Sec. along X-Y*

not be cleard. Connected with the locking lever, is also a

49. Detector bar, *E*, Fig 24, which is so connected with the locking lever that the bar must rise above the head of the rail as the locking lever is moved. This it cannot do if there are wheels on the switch. Thus, it is practically impossible to change the setting of a switch while a train is passing over it.

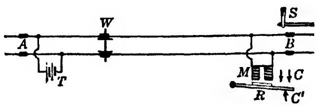


Fig. 25.

50. The Track Circuit affords a ready means of safely accomplishing many automatic sig operations, and much interlocking, or its equivalent; displacing many block sig towers, and clumsy and unwieldy all-mechanical interlocking plants. The fundamental principle of the track circuit is shown in Fig 25, which shows the two rails of a single track in a "block" or other section, *A B*. This section is electrically insulated from adjoining sections by heavy wood or fiber insulated fish-plates, indicated at *A* and *B*.

At one end of the section, is a battery, *T*, one wire of which is connected with one rail, and the other wire with the other rail. At the other end of the section is an electro-magnet, *M*, similarly connected. The armature, *R*, of this magnet, carries one or several contacts, some of which, *C*, are made, and some, *C'*, are broken when the armature is attracted by the magnet.

51. Let the track circuit, *A B*, be complete, and let there be no cars on the track. Then the current, from the battery, *T*, energizes the magnet, *M*; the armature is drawn toward the magnet, and the upper contacts, *C*, are closed, clearing the signal, *S*, at *B*. But if a train, or any pair of wheels, *W*, is on the rails between *A* and *B*, the current from the battery, *T*, will be short-circuited thru the wheels, back to the battery, thus robbing the magnet, *M*, of current, and causing its armature, *R*, to drop, and the sig, *S*, to show "stop". Also, if, thru accident or otherwise, the circuit is broken anywhere, if any short-circuit occurs, or if the battery fails, or if any metallic object touches both rails betw *A* and *B*, the sig will show "stop".

52. **The adjustment** is such that, normally, the current, energizing the magnet, *M*, will suffice to attract the armature, *R*, in spite of losses thru ballast, water, snow and ice, altho the practically total short-circuiting, caused by a pair of wheels, *W*, betw *A* and *B*, will certainly drop the armature, and set the sig at "stop".

53. **Stray direct currents** from nearby electric roads, or the direct return currents of electric roads themselvs, when the rails are used for track circuits, may "clear" the signal when it should not be cleard; but the use of alternating currents, for signaling, practically removes such danger

54. **The Relay,** *MC*, Fig 25, connected with the track circuit, is usually made to serv a number of purposes simultaneously, by means of its several contacts. When the armature drops, it may break several circuits and set several signals to "stop", and it may also share in

55. **Electric interlocking.** Where electricity is used for the control, and either electricity or air is used for the power to operate, the switch and sig levers control merely the making and breaking of electric contacts and operate a comparativly small interlocking machine; and are therefore usually less than a foot long. In different makes, the levers move in different ways. But the main features are common to all types.

56. **Train interlocking.** The mere presence of a train, on any section of track provided with a track circuit, may be arrangd, thru its relay, to break the circuits of any sig or sigs leading to that track. This does away with the necessity for the clumsy "detector bars" at switches.

57. **All-pneumatic interlocking** has been used to some extent, in which valvs take the place of electric contacts, and diaframs or pistons take the place of magnets and armatures.

Train-controld signal operation

58. **Automatic Signals** have proven very successful. They depend primarily upon "the track circuit", paragraph 50. When the head of a train enters a block, the sig it has past immediately shows "stop" against any train that may follow. When all of the train has past onto this block, the second block and sig behind it are "cleard". This sequence is produced automatically, and is continued indefinitely from block to block, operating distant sigs as well. Moreover, the mere presence of any train, or even of a pair of wheels, on any part of the track of a section of track circuit, will maintain at "stop" any sig or sigs governing any entrance to such section. In the best practice, as shown under "Electric Interlocking", ¶ 55, the track circuits are interlockt, thru relays, etc, with any switches or crossings that would be

involvd. If a train has enterd a block, no siding switch can be opend leading into that block; and conversely, once the block is cleard, and a siding switch is opend, all sigs leading to the section will show "stop".

59. Single-track Automatic. Automatic signals, as they were at first applied to single-track operation, would permit two trains at opposit ends of a stretch of single-track, to leav their sidings and run toward each other, where the single track, between sidings, was long enough to contain a number of blocks. No collision would result, however, because each train sets or holds, at "stop", not only the sigs which it passes, but also enough of the opposing signals *ahead* of it, to prevent collision; but one train or the other would eventually have to back out to the siding it had left.

60. "Absolute-permissiv". In this system, however, a train, entering either end of a length of single track, will hold, at "stop", *all* the opposing sigs ahed of it in that length. Other trains may follow it, as in ordinary double-track automatic signaling.

MISCELLANEOUS

Reliability

61. Reliability of railroad sigs generally has been developt to a very high degree. All devices, as far as possible, are so arranged that any conceivable failure will cause the sig to show "stop".

Indicators

62. Indicators of various types are used, and for several purposes. They are usually miniature models of sigs or switches or track, and indicate to the signalmen or yardmen the condition or position of the objects represented, but which may not be visible.

Other Methods

63. New methods of train control, by means of sig apparatus, are being developt, along lines radically different from those here described, and based upon other fundamental principles. These involv, in the main, various combinations or developments of the automatic stop, (¶ 64) time or speed control, and cab sigs. Some of these methods are in highly successful use; but, as this portion of the art of signaling is in a transition state, we refrain, for the present, from discussing it more fully.

64. Automatic Stop. The effect of the aut stop, upon the motorman, is similar to that of a derail (¶ 46). If he does not stop before reaching either of these devices, he is almost surely detected and disciplind; whereas, otherwise, he may repeatedly pass a sig, undetected, until the habit is developt to a dangerous degree The high cost of the aut stop, and dout as to its reliability under the severe weather conditions existing upon exposed steam roads, have retarded its adoption for such servis

65. "Smash" signals are sometimes used at the approaches of draw-bridges and at other particularly dangerous points. A smash sig consists of an arm or board, which extends out over the path of the car at such position as to clear trains when it shows "clear"; but, when set for the "stop" position, it will strike some part of the locomotiv or car. The impact smashes the sig (whence its name) and may likewise leav a mark on the loco, thus identifying the guilty driver.

66. Street and electric railway signals. The increasing size and speed of street and interurban electric rolling stock, have made desirable the replacement of primitiv methods by high-grade apparatus The use of the rails for the return of the power current, however, makes track-circuit signaling either unreliable or expensiv; so that it has been usual to utilize the passage of the trolley wheel under a special contact placed on the trolley wire, for setting the sigs.

67. Fig 26 represents a block of single-track line, with a sig and passing-place at each end. When the block is clear of cars, the apparatus is "neutral". Suppose now a car to be approaching from the left. Passing under the "contactor", at *B*, this car sets the sig, *E*, at "proceed", at the same time setting sig *F* (at the far end of the block) at "stop", and advancing a "counting wheel", at each sig, one step or notch.

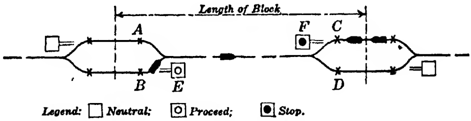


Fig. 26.

68. Sig *F* being now at "stop", no cars may enter the block from the right; but a following car, approaching from the left, may enter the block past the "proceed" sig at *E*, the motorman, however, regarding it as a caution sig, and observing that it "blinks", indicating that his car has been "counted in" And so on, with any other following cars, arriving from the left

69. But, as each car leaves the block, passing the contactor, *D*, the counting wheel at each sig is set back one notch or step; until, when the last car has left the block, the wheels have been set back to zero, and both sigs, *E* and *F*, are autom'y restored to "neutral".

70. Among recent refinements are devices for preventing the showing of "proceed" indications at both ends of a block simultaneously, even tho two cars, at opp ends of the block, make their respectiv contacts simul'y; for ensuring proper counting, even when reverse movements are made, for preventing disturbance of the counting wheels thru throwing off or restoring the power current; for applying the syst to single-track block sigs with overlap; and for the extension of the syst to include movements from a third point, as a siding.

71. Selectiv signaling enables a train dispatcher to set any one or more of a large number of sigs, visible or audible, and thus to get into communication with some one or more station agents or train crews.

72. The dispatcher is provided with apparatus with which he can send out, over a wire (which connects with all the sigs), electrical impulses at accurate predetermind intervals; or predetermind combinations of impulses, as in fire alarm systems. Each sig is differently adjusted to respond only to a given kind of impulse.

73. If the sig is a visible one, it is set to show "stop", sends a "return indication" or "answer-back" to the dispatcher so that he may be sure the sig has been set, and is autom'y lockt in that position. It can be set to "clear" only with the co-operation of the dispatcher. The conductor of a train that has been stopt by the sig, telefones the dispatcher; and the dispatcher, when satisfied that his instructions hav been understood, so operates his apparatus that the sig is unlockt, enabling the conductor to "clear" it.

74. Highway Crossings Alarms. The alarm itself may be either a bell or other sounding device, or lights or illuminated signs, or both. The alarms are usually controld electrically by means of track circuits, and various complex arrangements of relays, often interlocking with each other.

75. A crossing sig, utilizing the vertical flexibility of the rails, eliminates most of the difficulties due to irregular train movements, etc, and provides its own actuating power. The device is "direction sensitiv", requiring a train movement in the right direction to giv the alarm, and the alarm stops autom'y when the train stops.

76. Use of signals, January 1, 1911 and 1915, as given by the Interstate Commerce Commission.

(A "*line*-mile," or *road*-mile, = a mile of *roadway,* whether occupied by one track, or by two or more tracks. A "*track*-mile" = a mile of *single* track Thus, there are *two* "*track*-miles" in a "*line*-mile" of *double* track.)

Kind of signaling	In use, in 1911, on		In use, in 1915, on	
	Line-miles	Track-miles	Line-miles	Track-miles
Exposed disc	323	537	257	391
Enclosed disc ("Banjo")	1,921	3,866	1,356	2,961
Semafors,				
Electro-pneumatic	434	1,391	433	1,392
Electro-gas	919	2,018	891	1,890
Electric motor	14,168	21,339	26,575	42,409
Total	15,521	24,748	27,899	45,691
Normal clear		23,059		41,667
Normal danger		6,093		7,753
Total automatic block sigs	17,710*	29,152	29,864	49,442
Total non-automatic block sigs	53,558	63,506	66,745	74,673
Total auto & non-auto " "	71,269	92,708*	96,609	124,115
Total passr lines operated	172,390	195,922	193,180	223,081
Percentage block signald	41 4*	47 3*	50.0	55.6
Telegraf	38,613	44,542	37,938	41,174
Telefone	12,199	15,038	28,364	32,851
Electric bells and controld manual	3,212	4,357	2,883	3,622
Electric train staff	346	347	388	407
No of block sig stations		9,912		11,496
No. of block sig stations closed part of time		3,751		5,799
Number of block sections		29,881		51,690

77. Costs of sigs, prior to 1911, as given by Special Committee on Relations of Ry Operation to Legislation, 1911, Nov. 14, as result of enquiries and replies receivd from railroads operating about 80% of all track in U S equipt with block signals. From Ry Age Gaz. '11 Nov. 17.

Installing, per mile of track;
 Automatic, $1,146; Non-automatic, $248.

Maintaining, automatic, per year;
 $169 per mile of track; $69 per signal blade.

Costs for installing during 1911 were 10 to 40% greater than above, due, probably to the introduction of greater refinements.

*Substantially correct, notwithstanding insignificant discrepancies betw diff tables in the I. C. C. Reports.

YARDS AND STATIONS

1. General. Yards, stations and terminals* consist of tracks in addition to those of the main line; buildings and various facilities, for expediting the shifting and general handling of locomotivs, cars, freight, passengers, baggage, etc. They may consist merely of a small station, a single siding and a freight house; or may include large freight and passr stations, and dozens of tracks, and cover an area a mile or more in length

YARDS.

2. Definition, (Am Ry Eng Assn). "Yard.— A system of tracks, within defined limits, provided for making up trains, storing cars, and other purposes, over which movements not authorized by timetable or by train-order may be made, subject to prescribed signals and regulations."

3. General. Yards consist usually of

(a) a *Receiving Yard*, ¶ 10, upon which trains, entering from the main line, may remain until they can be sorted,

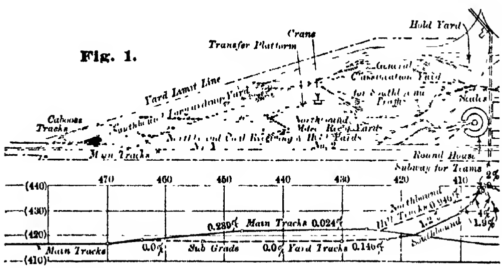
(b) the *Service Yards*, ¶ 11, in which locomotivs are coald, refrigator cars iced, etc, and rolling stock repaird,

(c) the *Separating, Sorting*, or *Classification Yard* proper, ¶ 46; and

(d) a *Forwarding Yard*, or *Departure Tracks*, ¶ 79, on which the newly made-up trains may wait before passing out onto the main line

4. The design of the yard, will depend largely upon the character, direction and amount of traffic, upon the area and shape of ground available, upon the nearness of various supplies and facilities, and upon other factors Large yards should be designd only in the light of all obtainable information on those and any other

Fig. 1.



pertinent points, and after carefully estimating future traffic and growth.

5. Example. The Missouri Pacific yd, Fig 1, at Dupo, Ill, is fairly representativ of the average ideal large yard for traffic about equally balanced.

6. Two directions. Usually, at any one place, there will be virtually two yards, one for each direction of traffic, except that there should be but one *service* yard, serving traffic in both direc-

*See foot note, p 1006.

tions. The two yards may be of approx equal, or of very unequal size, depending upon the relativ amounts of traffic in the two directions.

7. Main Line Tracks should, in general, especially in large yards, be located outside the yard, one on each side, if possible, or both on the same side if necessary; while facilities (service yard, etc) should be groupt near the center of the yard.

8. Connection with Main Line. The number of switches leading directly from the main track should be a minimum, with a cross-over if double-track in a small yard, the several *yard* tracks connecting only with *each other.* Any switches should, where possible, face away from the traffic, so that main-line trains will trail thru them, thus reducing the risk of "running into an open switch." The switch should also be interlockt with any main-line signals, and have a derail, pp 825-7, to prevent cars from wrongly getting onto the main-line.

9. Use. Freight yards are used (1) for the storage of rolling stock not in use, (2) for holding cars while being loaded or unloaded, and (3) for re-arranging or sorting cars When used for the latter purpose, they are commonly called "classification yards," are probably by far the most important, especially at large centers, and not only usually include the other two types, but serve largely to reduce their size. Various writers emphasize the importance of regarding a yard as a place in which to *handle* cars, and *not* as a place in which to *store* them.

Receiving Yards.

10. The receiving yard must be long enough to hold the longest train, and must have a number of tracks sufficient to hold all the trains coming until they can be handled. From it, the loco of each train is taken to the service yard for re-coaling, etc, and the train is pusht into the classification yard proper.

Service Yards.

11. Service yards should, in general, be centrally located. They should be provided with tracks and other means for changing cabooses and locos and for removing and holding disabled or "bad

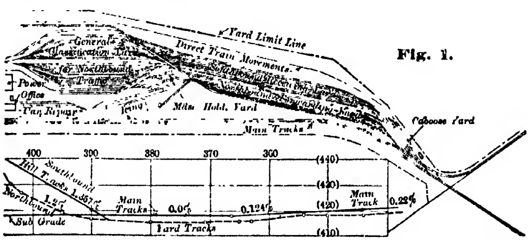


Fig. 1.

order" cars, and with running tracks to enable cars and locos to pass freely around or thru the yards. Facilities should be provided: for removing ashes from locos; for supplying them with water, coal and sand; for cleaning, oiling and housing them and making minor repairs to them; for inspecting and repairing cars and overhauling (hot) journal boxes; for cleaning passr cars; for icing refrigerator cars; for supplying water and feed to live-stock; and for such other special requirements as may exist.

12. Damaged cars of the dump or hopper-bottom type, and their contents, are sometimes conveniently handled by first running them up on a trestle and dumping their contents into good cars on tracks under the trestle.

68

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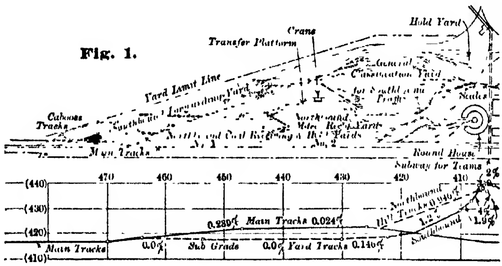
(d) a *Forwarding Yard,* or *Departure Tracks,* ¶ 79, on which the newly made-up trains may wait before passing out onto the main line

4. The design of the yard, will depend largely upon the character, direction and amount of traffic, upon the area and shape of ground available, upon the nearness of various supplies and facilities, and upon other factors Large yards should be designd only in the light of all obtainable information on those and any other

Fig. 1.



pertinent points, and after carefully estimating future traffic and growth.

5. Example. The Missouri Pacific yd, Fig 1, at Dupo, Ill, is fairly representativ of the average ideal large yard for traffic about equally balanced.

6. Two directions. Usually, at any one place, there will be virtually two yards, one for each direction of traffic, except that there should be but one *service* yard, serving traffic in both direc-

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*See foot note, p 1006.

23. *Length.* Many older tables range from 60 to 70 ft in *length;* but, of 57 important railroads, interrogated by a committee of the Am Ry Bridge & Buildg Assn* in 1912, 54 used tables ranging from 75 to 90 ft long; two, 100 ft, and one, 105 ft

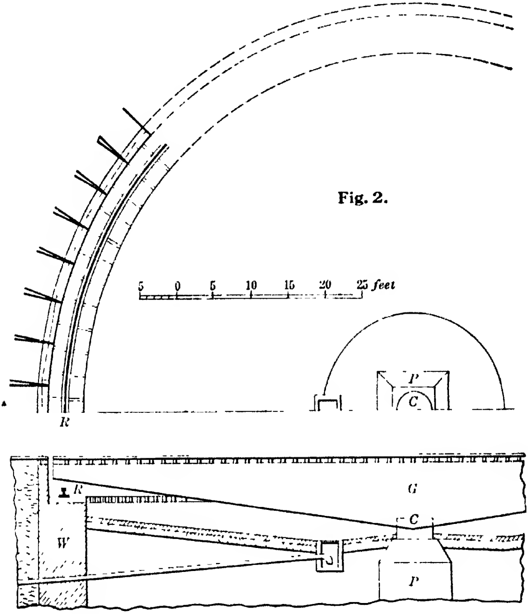

Fig. 2.

24. *Excess length* Turntables must ordinarily be considerably longer than the combined wheelbase of loco and tender, especially where unloaded tenders, with their locos, are to be "balanced" (bearing only upon the central pivot). Compare Figs 5

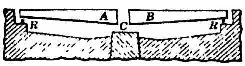

Fig. 3.

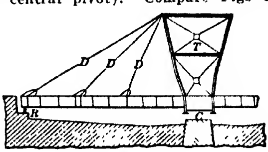

Fig. 4.

*Report, Am Ry Bridge & Bldg Assn, 1912. To this committee's elaborate report, adopted Oct 1912, we are indebted for much of the information and recommendations here given.

	Report pp	Weights		
		loco pounds	tender	
			loaded pounds	light pounds
Am Loco Co Mogul	2	187,000	143,800	55,300
Baldwin Mikado	4	305,100	166,700	50,000
Santa Fé Mallet 24-wheel	5	616,000	234,000	105,000

and 6, which indicate the turntable lengths required for a "Mogul" type loco (Rolling Stock, pgf 65), with tender; loaded, Fig 5, and light, Fig 6.

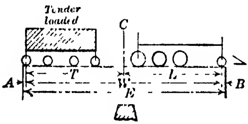

Fig. 5.

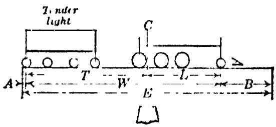

Fig. 6.

25. The table above shows weights and hor dists (to the nearest inch) for this and for two other types. The first two are the heaviest of those types made by the American and Baldwin works respectivly, up to 1912. The committee* recommends a min length of 75 ft for ordinary practice, and 90 ft for heaviest engines; preferring wyes to turntables longer than 90 ft.

26. *Mechanics. Moments and shears.* Stresses and deflections. When a turntable is "balanced," with or without load, the frame is to be treated as forming two cantilevers, each consisting of two parallel girders or trusses, *G*, the two cantilevers being connected over the central pier. For this case, but one position of the live load is possible. But, when the load is so placed that one wing of the table frame rests, at one end, upon the circle rail, *R*, and the other end upon the central pier, *C*, (as when a loco is entering or leaving the table), that wing is to be treated as a simple beam, and the other wing as a cantilever, and this is always the case with the non-tipping table, Fig 3. In either case, the moments and shears, for dead load, for live load, and for both combined, are to be found (for different positions of the live load) as on pp 440 to 453, the corresponding stresses as on pp 466, etc, and the deflections as on pp 480-481. For balanced table, the end deflections should not exceed about 1/2 inch, and must not bring the table to a bearing on the circle rail *R*.

27. According to the committee,* the practice is to use *unit stresses* of 16,000 lbs/sq inch in tension, and 10,000 in shear, when separate provision is made for impact (see Trusses, p 758); and the committee recommends 10,000 and 6,000 respectivly when no such provision is made; except that, at the ends, live load stresses should be doubled to provide for impact.

28. *Wheel loadings.* In designing a turntable, the locos, which may have to use it, should be studied, and the design should be based upon *that one which produces the greatest stresses.* By reason of their short wheelbase, *Cooper's loadings* (loco + tender = 48 ft for all classes; see Truss Specifications, p 755) are unsuited for modern heavy turntables; but they are nevertheless

*Report, Am Ry Bridge & Bldg Assn, 1912. To this committee's elaborate report, adopted Oct 1912, we are indebted for much of the information and recommendations here given.

Lengths, to nearest inch										
	Fig 5 (tender loaded)					Fig 6 (tender light)				
W	A	T	L	B	E	A	T	L	B	E
' "	' "	' "	' "	' "	'	' "	' "	' "	' "	'
56-10	0-7	28-05	28-5	0-7	58	0-8	35-4	21-06	14-6	72
67-01	0-6	36-00	31-1	5-5	73	0-9	43-9	23-04	21-2	89
108-02	2-2	57-10	50-4	9-8	120	2-3	65-3	42-11	24-7	135

sometimes used; their wheel-spacings being increased for the purpose. In the following table, the three heavier locos, by distributing their loads over a greater portion of the span, will, if placed upon a *bridge*, produce approx the same stresses as will the lighter Cooper E 50; but, on a balanced *turntable* (where each wing acts as a cantilever) their greater lengths and weights produce, at the center, greater shears and much greater negativ moments, as shown:—

Loading	Loco lbs	Tender lbs	Wheel-base, loco + tender	Moments and shears at center, in *turntable*		
				Neg mom ft-lbs.	due to Cooper	Shear, lbs
	225,000	130,000	48' 00"	2,149,260	E 50	225,000
Am L Co (tenders loaded)						
Mik	315,000	169,700	67' 10½"	4,349,000	E 100	270,000
* Pac	317,000	175,700	71' 5¾"	4,650,000	E 110	248,300
Mal	483,000	186,400	88' ½"	7,228,000	E 170	346,900

29. *Turning.* Small tables are turnd by hand; for which purpose a lever, 8 or 10 ft long, at convenient height, and fitting into a staple, is provided at each end of the table. Heavier tables are turnd by pneumatic power, or by gasoline or steam engins. Electric power is preferd, where readily available. Comprest air is used where occasional freezing, necessitating the temporary use of manual power, is not prohibitory The motor is placed sometimes next to the center, *C*, sometimes next to one end of either wing, sometimes at the end of a light wing at right angles to the main wings. With motors, tables are commonly turnd unbalanced, *i. e*, with the live load bearing partly upon the circle rail, *R*. Where ordinary tables are to be turnd without being balanced, and in non-tipping tables, it is usual to make the wing ends and their wings extra heavy, to support that part of the live load which comes upon them Balancing tables may then be turnd "balanced" when turning the shorter locos, and "unbalanced" with larger locos. Non-tipping tables, owing to the greater leverage of the resistance at their end supports, are difficult to turn, and expensiv in maintenance on account of wear in those supports.

30. Turntables are *held in position*, for passage of locos to or from them, by numerous devices; as by power brakes (especially where power is used for turning); or by bars, swinging vertically about a hinge fastend to the table ties, and catching, as they swing, in a notch fastened to the ties between the rails of the approach tracks; or by sliding latches, running in sleevs upon the table ties and entering sockets fastend between the rails of the approach tracks. The last-named are conveniently thrown into and held in the lockt positions by springs, and withdrawn by means of a hand-lever, which may withdraw simultaneously the latches at both ends of the table. The locking device is frequently connected with a signal, indicating its position.

31. *Crossing of the radial approach rails of adjacent tracks,* near the table, together with the use of frogs, is frequently necessitated by limitations as to space.

32. *Flooring.* The entire pit is now floored over only in special cases; as where floor space, or a passageway for teams, etc, is needed; or where danger of *freezing* is a serious matter; in which latter case, a stove may be mounted upon the swinging girders. A steam-pipe is sometimes laid around the pit, adjoining the circle rail, *R*, (Fig 2). The flooring, when used, is supported upon light radial trusses. Commonly the table is made wide enough to accommodate a *footway* on one or both sides, the footway being supported upon extra long ties, placed at intervals; and the footway is sometimes provided with a hand-rail. Frequently one or two light additional wings are provided, at right-angles with the main wings, to carry the motor or the stove, etc, or for other purposes; forming a cross-shape in plan For rigidity, the ends of the four wings may be connected by struts.

33. *Height of rail.* The table is usually placed at such height that, unloaded, its rails are about ¾ inch above those of the approach tracks; so as to leave ¼ inch clearance betw the circle rail and the end-wheels of the table, when the ends of the wings deflect each ½ inch under a balanced load. The tops of the table rails should come flush with those of the approach tracks at that end over which a loco may be entering or leaving the table. Steel springs have been employd, to absorb the shocks occasioned by locos entering and leaving.

34. The *foundations*, of both pivot pier and circle wall, are usually of concrete; and of course must be very carefully laid (especially that for the pivot pier), in view of the very heavy service required of them. The pivot pier is commonly provided with a stone cap. Where rock bottom is not accessible, piles are driven under the pivot pier, under the circle rail and under the parapet wall. Those under the pivot pier are usually in a square, 4 to 7 piles on a side, and covering an area of from 12 × 12 to 16 × 16 ft.

35. *The circle wall* is usually of concrete (rarely of wood), with timber coping for the support of the stub ends of the approach tracks, and radial timber ties for the support of the circle rail. When either the approach rails or the circle rails rest directly upon the concrete, the latter is apt to be disintegrated The circle wall is usually from 6 to 7 ft in greatest *width*, and the *parapet wall* from 1.5 to 2 ft. Owing to its circular form, in plan, it acts as a hor arch, and a gravity section (capable of withstanding alone the collapsing pres of the surrounding earth) is not necessary.

36. *A niche*, or recess, left in the circle wall at some point, gives convenient access to the end of the table, for inspection etc.

37. *Wooden turntables* are sometimes used, from motivs of economy, especially for temporary purposes. They are sometimes without rollers or disks at the center, and then, in turning, bear either upon their end wheels or upon a series of wheels arranged in a circle not far from the center. In the latter case, the live load bears partly upon the end wheels when entering or leaving the table. They usually require two men, with crank mechanism, to turn them. Their economy, in first cost, is apt to be offset by excessiv cost for repairs.

38. *Pit drainage,* always important, is especially so with the deep pits often required for modern heavy locos. *Water, in the pit, rusts the bearings, and thus necessitates stoppage of operations and jacking up for cleaning and oiling. Water, freezing in the pit, may entirely stop operations. Where the pit bottom is too low for convenient drainage, the pit may be made shallower by one or other of the devices mentiond in ¶¶ 20-22.

39. Figs 7 and 8. **The center* is essentially a circular steel box,

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the lid of which (carrying the frames) bears upon it usually thru the medium of conical rollers, ball bearings, or disks, of hardend steel; *conical rollers*, Fig 7, being most largely used. In the best tables, these are held in place, radially, usually by "live rings" encircling them, both at their inner and at their outer ends, and separated from them by ball bearings or frictionless washers. Conical rollers are commonly from 7 to 12 ins long, and from 4 to 8 ins diam at the larger end. They usually bear, above and below, upon relativly thin annular *track plates*, which, when worn, may be replaced, leaving box and lid intact. The design of the center is often left to the manufacturer.

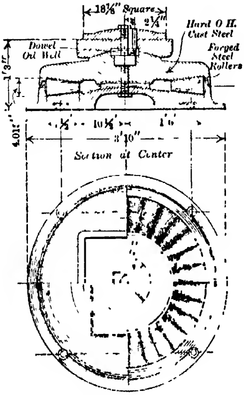

Fig. 7.

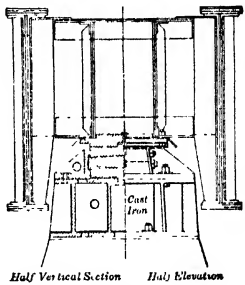

Fig. 8.

40. Notwithstanding the successful use of ***disk centers***, Fig 8, in swing-bridges, where they carry much heavier loads, they have not come into general use for turntables. The committee* recommends their serious consideration

41. *"Hydraulic" centers* (oil or glycerin being used, on account of danger of freezing, instead of water) have been suggested. They would be easily adjustable in height.

42. Centers or pivots are *very important*. It pays to use the very best obtainable. Three-quarters of RRs use conical roller or ball bearings, of manufacturers' standards. Much of the trouble with roller bearings seems to have been due to former poor design and small size, and to subsequent neglect in maintenance.

43. *The mechanism should be oild* at least annually, and as much oftener as may be required by flooding, or when the table becomes hard to turn. The frames, etc., require frequent painting. *To facilitate inspection*, repairs, etc., the table may be jacked up from the center pivot; the jacks resting upon two concrete foundations, placed diametrically on opposite sides of the center, and lifting by means of steel brackets, riveted to the table.

*Report, Am Ry Bridge & Bldg Assn, 1912. To this committee's elaborate report, adopted Oct 1912, we are indebted for much of the information and recommendations here given.

44. *The cost* of turntables varies widely with many factors, but the following may be taken as approximations. Cost of steel deck *turntable, complete,* with tractor, including pit, pier, etc., $100 per linear foot; thru tables, $150 Light *wooden* tables may cost as little as $15 to $20 per lin ft. *Pit,* lined, including pier foundation, from $0.50 to $1.30 per sq ft; *paving alone* (often omitted), from $0.15 to $0.25 per sq ft.

45. Wyes or Y-tracks. See Turnouts, ¶¶ 18, 19, p 825.

Classification Yards.

46. The main object of a classification yard is to receiv, from one or more points or lines, as *A, B, C, D,* etc., trains in each of which there may be cars for any or all of a number of other points or lines, as *M, N, O, P,* etc, and so to rearrange the cars as to make up new trains, the cars of each of which may then go directly (or with a minimum of re-arrangement or subsequent shifting) to their respectiv destinations, as *M, N, O, P.*

47. Minor objects are the similar re-classification (or "Transfer") of the *contents* of cars, and general renewals and repairs.

48. Push and pull, or flat shifting. Where gravity cannot be utilized for shifting (¶ 50), car movements must be made by locomotiv Cars may be given a *push* by the loco. and then left to go where they are wanted, sometimes assisted by a slight down grade. They should be under the control of a brakeman. Or, the cars may be *pulled,* "making flying switches," as follows:—With the train in motion, the loco is uncoupled, and runs on ahead, at increast speed, thru a switch, on to an unoccupied track; and the switch is then quickly thrown, in time to send the oncoming cars onto the required track. This requires skilful co-operation of crew and yardmen These methods, tho objectionable, and expensiv *during* operation, will, nevertheless, be usually more economical in a small or unimportant yard, than any of the following methods, which are more costly to install and maintain, but which may be well worth their cost in large or busy yards.

49. Poling. By means of a stout pole, two or three times as long as the clear space betw cars, the loco (sometimes provided with a specially constructed car, and running backward and forward on a parallel track) pushes off, from a train, one or more cars at a time, sending them, thru switches, to their destinations under their own momentum.

50. Hump or gravity classification yard

In this type of classification yard, the train is pusht up over a hump in the track, at which point the cars are uncoupled, one or more at a time, and descend, by gravity, thru the yard, to their desired destination.

51. Operation. A train enters the receiving tracks, its locomotiv is uncoupled and sent to a side track for re-coaling, etc. A special yard loco then comes up behind the train and pushes it slowly forward, over an up-grade, to a summit or "hump." As each car, or set or "cut" of cars for any one destination, passes over the hump, it is uncoupled. It then proceeds, by gravity (usually first over a track-scale, on which each car may be weighd) down quite a steep grade and onto the "ladder track," which leads by switches to the several classification tracks. As each car or "cut" starts on its way, it may be markt with a number or symbol, indicating its destination. As the car or cut comes down the ladder track, its destination, as thus markt, is noted by switchmen, who set the switches for it. It is boarded by a brakeman or "rider," who then controls its speed by means of the brakes, the grades of the ladder and classification tracks all being such as to insure the delivery of the car to the furthermost points desired. The rider then returns for another car or "cut" When it is time to make up a train from one or more classification tracks, the "road engin" backs in from the lower end of the yard, couples to the collections of cars desired, and then proceeds into the forwarding yard or out onto the main line.

52. Speed of operation. According to Mr. C. L. Bardo, Jour N Y R R Club, 1903, Dec, a train of 60 cars, with 50 cuts, requires by push and pull, 2 hours; by poling, 1 hour, 15 mins; by hump, 30 mins.

53. Grades of hump yards depend chiefly upon the av and max car resistances (usually greater in empty than in loaded cars) per unit of weight; which, in turn, depend upon length of time cars have been standing; upon temperature (the resistances being greater in cold weather); upon prevailing wind direction, or that of probable max winds; and upon condition of track, including curvature. See Train Resistance, pp 1057, etc.

54. In the following table, the first line gives the recommendations of the Am Ry Eng Assn Manual, 1911. The others (from Ry Age Gaz, 1912 Aug 9, pp 236-9) give max, av and min values reported by Mr Shelby Saufley Roberts, representing about thirty hump yards. The grades are given in ft per 100 ft.

	First grade from summit	ladders	classificatn yard
Am Ry Eng Assn	3.00	1.00	0.50
S. S. Roberts Max	4 00	1.75	1 00
" " Av	2 66	0.97	0.30
" " Min	1 00	0.50	0.00

55. *Seasonal changes.* In order to compensate for the markt differences in resistance, due to temperature, hump grades are sometimes changed, either by re-grading the hump portion of the track twice each year, or by providing two humps, side-by-side; a steeper one for winter use, and a flatter one for summer use; but the steeper hump may be advantageously used in summer for hard-running empties. For a third method, see next ¶.

56. The *mechanical hump,* designed by Mr. A. W. Epright, scale inspector, Penna R R, consists essentially of a short two-span girder bridge, the middle support of which may be raisd or lowerd by means of jacks, and then held at the desired height by blocks. In one installed at West Brownsville Junc, Monongahela Div, each span is about 20 ft, and the central support movement is about 8 ins.

57. After passing over the hump, it is usually desirable to pass the car over a

58. Track Scale. Until quite recently, very little attention appears to have been given to the design of track scales, with the result that they have been rather delicate affairs likely to get out of order, and to give erroneous readings. Later efforts, however, have produced much more satisfactory machines, altho they do not appear to have been generally standardized. They are essentially large weighing machines, on the platform of which the track is laid.

59. *Difficulties* that obtaind have been listed as follows by a committee of the Am Ry Bridge & Bldg Assn.

Carlessness of weigher; improper balance, another car partly on scale Weakness of scale; deflection of scale bridge or levers.

Misc: deck binding, rails binding, broken castings or bearings, dull bearings, bearing feet resting on angle irons, insuff clearance betw feet and scale timbers, levers out of line or loose, and foreign matter in friction with levers or scale parts. See also ¶ 63.

Many of these troubles are due primarily to insufficiently firm foundations.

60. *Dead rails.* To avoid unnecessary wear of scales, locos and cars not requiring to be weighd may, as either approaches the scale, be switcht across the scale pit, on a pair of "dead rails," laid parallel with the weighing-rails and about six ins from them, but supported rigidly on posts which pass down thru the scale without touching it and rest upon the scale-pit foundations; but modern track scales are sufficiently strong to carry all the traffic without

injury; and dead rails are going out of use, all cars and locos passing over the weighing-tracks. The dead rails endanger the trackmen, whose feet may be caught betw them and the weighing-rails, and their supports obstruct access to the scale mechanism.

61. *"Bridge-rails."* The weighing-rails are sometimes connected with the fixt rails, at each end of the scale, by short rail lengths, pivoted to each. These not only obviate the blow caused when a wheel jumps the gap betw the ends of the fixt and the weighing rails, but also transfer the load more gradually to the scale.

62. *Relieving gear* The Penna R R has employed a device, patented by Mr. A. W Epright, and consisting of a system of toggles and pistons, operated by either air or water, and under control of the weighmaster by means of a three-way valv, by means of which, in a second, betw the passage of any two cars, the load of the scale and any following car may at pleasure be taken by the toggles, or restored to the scale levers

63. *Binding of platforms* has been a frequent source of trouble and may be prevented by careful construction, by preventing inward bulging (as by frost) of the pit sides, and by so beveling the opp faces that the space, betw them, widens downward, in order that objects, falling into the space, shall not wedge and bind. Rail ends, even when properly secured against creeping, are apt to bind, under temperature changes; but this may be prevented by inserting switch-points in the rails near the scale ends.

64. The *scale length*, if each car to be weighd is to be stopt ("spotted") on the scale, should be about the length of the longest car If cars are to be weighd in motion, as they run over the scale, the scale length should be about one-third greater.

65. *Testing* Scales should be tested periodically, say every few months, by running over them, and stopping on them at various points, special loaded test cars of known weight Preferably, the weights should be adjustable, in order to see whether the indications of the scale are truly proportional to the load, for all weights likely to come upon it. Many railroads have special cars for this purpose.

66. *Drainage, heating and lighting* of the scale pit are all desirable to prevent damage by water and by freezing, and to facilitate inspection and adjustment.

67. *Speed of weighing.* The velocity of the cars over the scales, while being weighd, may vary between zero and 6 or 8 miles/hr, about 4 miles/hr being usual. It is usually practicable to weigh several cars per min; but the time required, per car, may range from several mins, down to abt 8 secs, tho this high rate can seldom be maintaind continuously.

68. Classifying. As the cars are cut off from the train, and as they pass over the hump, the switchmen must know for which track each cut is destind, in order that they may set the switches properly; and the brakeman of each cut must know this also, in order that he may properly control the speed in passing around the switch and in coupling on to any cars ahead. Sometimes the destination is chalkt upon the car bodies; sometimes (especially at night) the brakeman informs the switchman by means of arbitrary signals.

69. The *"cut list,"* recommended by the Am Ry Eng Assn, Supplement of 1913, Manual of 1911, consists of two or more duplicate lists, giving (1) the number (1st, 2nd, 3rd, etc) of the cut, (2) the number of the track to which the cut is destind, and (3) the number of cars in the cut A copy of the list is given to each switchman concernd and (if thought necessary) a copy to the brakeman of each cut.

70. *Yard switches* (Am Ry Eng Assn, Manual, 1911) for ladder tracks should have *frogs* not sharper than No. 8. See also ¶ 57 (Ladders) under Turnouts, p 871.

71. *Switch operation.* The operation of each switch by a lever placed at the switch, in the yard, is cheap in installation, but expensiv in operation, and inefficient in a large yard, as compared with control of numerous switches from a single switch tower by means of the usual switch and signal operating equipments. See Signals, pp 985, etc.

72. *Curvature compensation.* As grades, on the line, are reduced on curvs, to facilitate hauling upgrade by locos. so they are increast on curvs in gravity yards, to aid gravity in getting the cars around the curvs. See ¶ 41, under Train Resistance, p 1062. Mr. H M. North, of the L S & M S Ry, recommends an increase of about 0 05 ft per 100 ft (1/20 of one per cent) for each degree of sharpness.

73. **Ladder tracks** are those containing the switches leading to the several classification tracks (¶ 74). In small yards, one ladder track suffices, but ordinarily space is wasted when more than 8 or 10 tracks are taken from one ladder, and it is then advisable to use two or more ladders. See also "Ladders," ¶ 57, under "Turnouts," p 871.

74. **Classification tracks** are long parallel tracks, onto which cars are run from the ladders. In general, each classification track represents a diff route or destination or purpose of train Thus, track No 1 may be used for cars destind for the M R R, track No 2 for cars of the N R R, and so on; or track No 1 for cars bound for station *P*, track No 2 for station *Q*, etc, one train being subsequently made up by taking the cars successivly from tracks 1, 2, etc, in such order that they may be cut off at stations *P*, *Q*, etc, as the train proceeds, and without re-shifting; or track No 1 may be reservd for a regular train, to leav at a specified time; track No 2 for a special train; track No 3 for empties, to be returned when enough of them have accumulated to form a train; etc, etc

75. **Secondary classification** yards are used where the desired classification, especially as regards the order of the stations, cannot be attaind in the main classification yard. Such a yard is conveniently placed beyond the main classification yard, and may have a "hump" of its own These yards are called also *"reclassification yards," "station-order shifters,"* or *"grouping yards."*

76. **Capacities** of classification tracks. No such track need be longer than the longest train to be assembled. Indeed, if they be made somewhat shorter, the occasional slight inconvenience of having to make up a train from cars on two tracks is usually more than compensated by time saved in dropping cars into places from the hump, in the return of brakemen, and in general communication and operation.

77. **Spacing** of yard tracks Recommendation of Am Ry Eng Assn, Manual, 1915; c to c, min; body tracks (main tracks of classfn yard) 13 to 14 ft; ladder track and first body track, 15 ft from any adjacent track. Min spacing must enable men to see signals and to avoid colliding with electric light poles, etc.

78. **Car riders,** car droppers, or brakemen. One "cut" seldom contains more than five cars, even when more than five are bound for the same place. Each cut is (or should be) controld by a brakeman, from the hump to near its destination. The return of the men to the hump involves fatigue and loss of time if they walk, and expense if they are carried. Their transportation may be effected by means of an old light loco with a car, or by gasoline inspection or electric cars. Moving sidewalks have been suggested.

Forwarding Yards.

79. The forwarding yard is placed beyond the classifn yards. Each of its tracks should be as long as the longest train. Often two or more trains can be accommodated on one track, especially if cross-overs are provided. Comprest-air pipes should be provided, for testing brakes while loco and caboose are being coupled up.

Thorofare tracks, thru or alongside the yard, permitting free and rapid movement of locos and cabooses, around and to the trains, should be provided.

Illumination.

80. Am Ry Eng Assn, Manual, 1911, recommends, for hump and ladder tracks, arc lights of 2,000 candle power each, 28 ft or more above ground, and 140 to 150 ft apart Other authorities recommend lights of lower c p, with closer spacing The use of reflectors or lenses, and of shields, etc, to direct the light efficiently, appears to be worthy of much more careful study than it has usually receivd.

81. When the main body tracks are not illuminated, a light may be left at the rear of each collection of cars, and shifted back to the rear of each new "cut" added.

STATIONS*

Freight Stations.

82. Transfer stations. As the classification yard serves for the rearrangement of the cars in trains, so the transfer station serves primarily for rearranging the contents of cars among the diff cars, transfering it from any one car to any others, over platforms adjacent to tracks upon which the cars are run The platforms are preferably placed flush with the floors of box cars, in order that trucks may readily be wheeld over planks laid across the gap betw car and platform Storage space (preferably coverd) should be provided, on the platforms, to receiv goods for which cars are not ready.

83. Mechanical handling (electric trucks, telfers, traveling platforms, etc) is economical, both in transfer and general freight work, in large stations, if sufficiently flexible and not too cumbrous. "Where large amounts of freight are to be transferd, the use of power-driven coverd traveling platforms is recommended," Am Ry Eng Assn Manual, 1911, p 401.

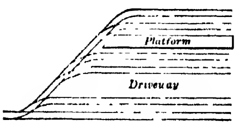

Fig. 9.

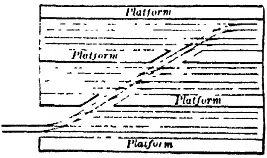

Fig. 10.

84. Freight Yards and Stations. The track layout is usually a mere series of pairs of parallel tracks, merging into one or more tracks from the main line or a classification yard, with spaces betw them, for teams or for platforms, as in Fig 9, the yards being open and the stations usually coverd. In stations, to save space, the main feeding track is often run diagonally thru the building, and the loading tracks are taken off as spurs in both directions, somewhat as indicated in Fig 10. "Inbound freight-houses should have a floor space of 50 ft width if practicable; out-bound, 25 ft." Am Ry Eng Assn.

*The Am Ry Eng Assn defines a *terminal* as "An assemblage of facilities provided by a ry at a terminus or at intermediate points on its line for the purpose of assembling, breaking up and relaying trains." To avoid confusion, however, we have refraind from using the word "*Terminal*" (except as below), and have used "*Station.*" A station that is *not a thru* station we call a "*Terminus*" (plural "*Termini*"), or a "Terminal Station."

85. Modern freight-handling devices permit economy in space by making it possible to build stations two or more stories in height; one story for inbound freight, another for outbound, and others for storage, etc; freight being transferd, upon and betw stories, by means of traveling platforms, barrel-hoists, belt-conveyors, elevators, and (for downward movements) straight or spiral shutes for non-fragile pieces, all in addition to the freight-handling devices referd to under Mechanical handling, ¶ 83.

The freight yard should have a traveling or other crane, for heavy pieces; and special apparatus (as clam-shell bucket-hoists for coal, grain elevators, etc) where much of one commodity is to be handled.

86. For team delivery yards, Am Ry Eng Assn (Manual, 1911, p 398) recommends stub tracks in pairs, 12 ft c to c of tracks, and, if practicable, not less than 30 ft c to c of pairs; tracks not more than 20 cars capacity; ingress and egress for teams at each end of each teamway; power crane, wagon scales and track scale.

Passenger Stations*

87. General. Some of the major points to be observd in station design are:—getting passengers, baggage and express to and from the trains with a minimum of delay and confusion, and, in termini,* getting the loco out of the way, turnd around, waterd and coald, and re-coupled to the train; also car cleaning, etc. Some car cleaning can be done in the station.

88. Platforms. To expedite the handling of passengers and baggage, it is recommended by the "Yards and Terminals Committee" of the Am Ry Eng Assn, 1911 March, that, if baggage platforms cannot well be provided in addition to passenger platforms on the same level, the baggage be transferd to and from another level by means of elevators so located as to keep it off the platforms as much as possible.

89. Ramps, or inclined passageways, as substitutes for stairways, greatly facilitate the movement of passengers in stations, and reduce liability to accident at rush hours. The Y & T Comm, Am Ry Eng Assn, Mar 1911, recommends a grade not exceeding 1%; but considerably steeper grades are successfully used. The surface must be sufficiently rough to avoid danger of slipping.

90. Lanes of travel. Often much can be done to reduce congestion in the layout of the station and passageways, by preventing crossing of lines of travel of passengers, keeping such lines parallel as far as possible.

91. Future requirements. Owing to the difficulty and expense involvd in reconstructing a large station when its traffic begins to exceed its capacity, the requirements of the future should be estimated for 20 years ahead, if possible, and provided for in the design of the station.

92. Large vs numerous smaller stations in large cities. Handling, at a single station, all or most of the passenger traffic of a ry at a large city, necessitates, on an average, a long journey to reach or leav the station, and much walking in the necessarily large station itself, to reach or leav the trains; and the alternativ of several smaller stations thruout the city, along the line, has been suggestd (as by Mr. Fred A. Delano, Pres Wabash Ry), even at the expense of some added time for additional stops of express trains.

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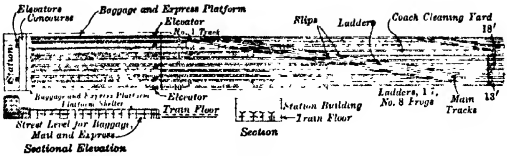


Fig. 11.

93. Plan. Fig 11 is a layout for a passenger terminal station, as recommended by a Comm of the Am Ry Engg Assn, March 1911;

Fig. 12.

and Fig 12 a station of the thru type. In the terminal station, two levels are proposed, one for passgrs and one for baggage and express, together with a simple and comprehensiv arrangement of ladder tracks, with car cleaning yard adjacent. This ideal arrangement would, of course, usually have to be alterd, on account of local restrictions. In the plan for the thru station, it will be noted that the two main-line tracks divide into 6 in all, affording two thru tracks in the middle (without deviation from their course), and two platform tracks (with an island platform betw them) on each side, thus giving ample loading and unloading facilities, without delaying trains not scheduled to stop at the station.

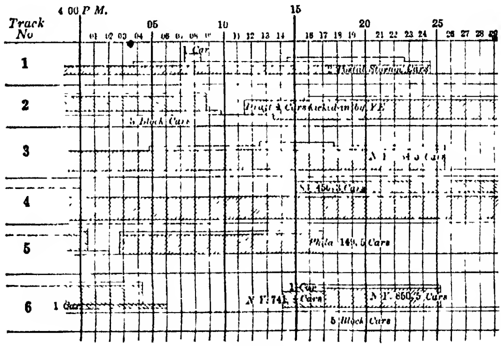


Fig. 13.

94. Study. Fig 13 shows part of a diagram recording the occupancy of the several tracks of the Penna R R at Broad St sta, Phila. (Mr. E. B. Temple, Asst Chf Engr) during a given period

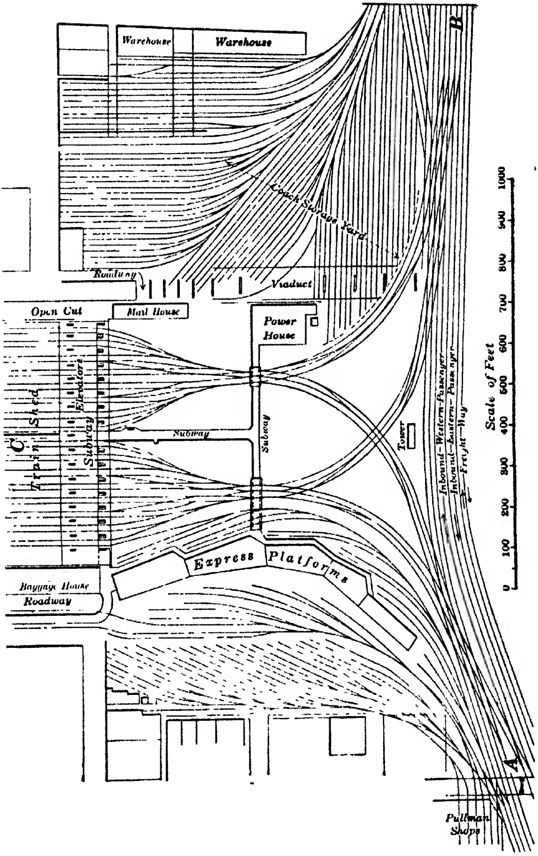


Fig. 14.

(4:00 to 4:30 p m, Jan 18, 1912; only 6 tracks shown). The entire diagram included the 16 tracks of the station, and coverd the period from 4 to 6 p m. Similar diagrams showd the conditions existing, during the same period, on portions of the yard. The several tracks are laid off to vert scale, end to end, their inner

ends down. The vert spread, allotted to each track, represents the length to which it may be occupied without fouling more than one line. Cars and locos are plotted as rectangular areas. The vert dimensions of the rectangles represent the lengths of track occupied by cars, etc; the vert positions of the rectangles represent the positions of the cars, etc, on the several tracks; and their hor dimensions and positions represent the periods during which the several tracks were so occupied. In practice, arriving and departing trains are distinguisht by diffs of shading of these rectangles; and kinds of cars and of locos by initials, as by "R.E." for "road engine." Similar diagrams have been used by the Belgian State Railways and at the Camden station of the Balto & Ohio Ry at Baltimore.

95. Fig 14, p 1009, represents a *"back-in" station.* A train, bound for the sta, coming, say, from *A*, proceeds first to *B*, and then backs in to the sta, as at *C*; thus enabling the loco to be promptly releast.

96. Speed of handling trains at termini depends largely upon the facilities provided for the rapid handling of locos, passgrs and baggage, and for cleaning cars, and upon the nature of the traffic and of the motiv power. Comparing 26 large American passgr stas the Y & T Comm, Am Ry Eng Assn (Eng News 1911, Apr 6, p 414), found that the number of trains actually handled during the busiest hour, ranges from 1 to 3 5 per av track (including movements on *all* tracks), and from 2 to 8 on the *busiest* track; while estimated possibilities ranged from 2 to 12. Some of the higher figures were those of thru stations.

97. Electric operation of trains greatly facilitates their rapid movement in termini, inasmuch as the motiv power need not leave the sta, or change its position, for reversal of loco or train, or for supplies. The tracks and yard are thus left largely free for complete train movements.

98. Rapid transit termini are frequently arranged with one or more *loop tracks,* around which the trains may be run without reversal of loco or motorman. This is often practicable with rapid transit equipment, which can readily turn sharp curvs, and a very large tract of ground is therefore not necessary for the loop.

Water Stations.

Water stations are points along a railroad, at which the engines stop to take in water. **Their distance apart** varies (like that of the fuel stations, which accompany them,) from about 6 miles, on roads doing a very large business: to 15 or 20 miles on those which run but few trains. Much depends, however, upon where water can be had. It has at times to be conducted in pipes for 2 or 3 miles or more. The object in having them near together is to prevent delay from many engines being obliged to use the same station. To prevent interruption to travel, they are frequently placed upon a side track. A supply of water is kept on hand at the station, usually in large wooden tubs or tanks, enclosed in frame tank-houses. The tank-house stands near the track, leaving only about 2 to 4 feet clearance for the cars. It is two stories high; the tank being in the upper one; and having its bottom about 10 or 12 feet above the rails. In the lower story is usually the pump for pumping up the water into the tank, and a stove for preventing the water from freezing in winter.

The tanks are usually circular; and a few inches greater in diameter at the bottom than at the top, so that the iron hoops may drive tight. **Their capacity** generally varies from 6000 to 40000 gallons, (rarely 80000 or more,) depending on the number of engines to be supplied. **A tender-tank holds** from 3000 to 7000 gallons; and **an engine evaporates** from 20 to 150 gallons per mile, depending on the class of engine; weight of train; steepness of grade, &c. Perhaps 40 gallons will be a tolerably full average for passenger, and 80 for freight engines. **The following are the contents of tanks** of different inner diameters, and depths of water. U. S. gallons of 231 cubic inches; or 7.4805 gallons to a cubic foot.

Diam.	Depth.	Contents.		Diam.	Depth.	Contents.	
Ft.	Ft.	Gallons.	Cub. Ft.	Ft.	Ft.	Gallons.	Cub. Ft.
12	8	6767	905	24	12	40607	5429
14	9	10363	1385	26	13	51628	6902
16	9	13535	1810	28	14	64481	8621
18	10	19034	2545	30	15	79310	10603
20	10	23499	3142	32	16	96253	12868
22	11	31277	4181	34	17	115451	15435

Cypress or any of the pines answer very well for tanks. The staves may be about $2\frac{1}{2}$ inches thick for the smaller ones; to 4 or 5 inches for the largest. The bottoms may be the same. The staves should be planed by machinery to suit the curve precisely. Nothing is then needed between the staves to produce tightness. A single wooden dowel is inserted between each two near the top, merely to hold them in place while being put together. The bottom is dowelled together; and simply inserted into a groove very accurately cut, about an inch deep, around the inner circumference of the tub, at a few inches above the bottoms of the staves.

One of 20 feet diameter, and 12 feet deep, may have 9 hoops of good iron; placed several inches nearer together at the bottom of the tank than at the top. Their width 3 inches; the thickness of the lower two, $\frac{1}{4}$ inch; thence gradually diminishing until the top one is but half as thick. The lower two are driven close together. These dimensions will allow for the rivet-holes for riveting together the overlapping ends; and for a moderate strain in driving the hoops firmly into place. Three rivets of $\frac{1}{2}$ inch diameter, and 3 inches apart, in line, are sufficient for a joint of a lower hoop. One of 34 feet diameter, 17 deep, may have 12 hoops; the lower ones 4 inches by $\frac{1}{2}$; with three $\frac{3}{4}$-inch rivets to a lower hoop-joint.

The bottom planks of the tank must bear firmly upon their supporting joists, or bearers.

A tank must have an **inlet-pipe** by which the water may enter it; a **waste-pipe** for preventing overflow; and **a discharge or feed-pipe** 7 or 8 inches diameter, in or near the bottom; through which the water flows out to the tender. The inner end of the discharge-pipe is covered by a valve, to be opened at will by the engine man, by means of an outside cord and lever. To

its outer end is generally attached a flexible canvas and gum-elastic hose about 7 or 8 inches diameter, and 8 or 10 feet long, through which the water enters the tender-tank. Or, instead of a hose, the feed-pipe may be prolonged by a metallic pipe, or nozzle, sufficiently long to reach the tender; and so jointed as, when not in use, to swing to one side, or to be raised to a vertical position, (in the last case it is called a drop,) so as not to be in the way of passing trains. The same tank may supply two engines on different tracks, at once. The tanks are very durable.

The patent *frost-proof* tank of John Burnham, *Batavia, Illinois,* is simply an ordinary tank, in which the water is prevented from freezing by means, 1st, of a circular roof which protects a ceiling of joists, between which is a layer of mortar; 2d, by an air-space obtained by a similar ceiling beneath the timbers on which the tank rests. *Although the sides are entirely unprotected, no house is necessary; but merely strong posts and beams on a stone foundation, for the support of the tank.** The supply pipes are in boxes made of boards and tar-paper.

Tanks are frequently made rectangular, with vertical sides of posts lined with plank, and braced across in both directions by iron rods. They are more apt to leak than circular ones. They have been made of iron; but wood seems to be preferred.

The water for supplying the tanks, may be pumped by hand, steam, horse, wind, hydraulic ram, or otherwise, from a running stream; from a pond made by damming the stream if very small or irregular; from a cistern below the tank; or from a common well. Many roads doing a business of 10 or 12 engines daily in each direction, depend entirely upon wells; and pump by hand; generally two men to a pump. Those doing a very large business, when the supply cannot be obtained by gravity, mostly use steam. **The windmill is** the most economical power; and when well made, is very little liable to get out of order. Of course it will not work during a calm; but this objection may be obviated in most cases by having the tanks large enough to hold a supply for several days. Steam, however, is most reliable.

The following table will give some idea of the **power required in a steam engine for the pumping.** In ordering an engine, specify not its number of horse-powers, but the number of gallons it must raise in a given number of hours, to a given height; with a given steam pressure, (say about 60 to 80 lbs per square inch.) The pump should be sufficiently powerful not to have to work at night; and should be capable of performing at least 25 per cent. more than its required duty.

A fair average horse should pump in 8 hours the quantities contained in the first 3 columns; to the height in the 4th column; or sufficient to supply the number of locomotives in the 5th column, with about 2000 gallons each. Two men should do about one-third as much.

Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.	Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.
1600	100000	11968	100	6	4571	285714	34194	35	17
2000	125000	14960	80	7½	5333	333333	39893	30	20
2667	166666	19946	60	10	6400	400000	47872	25	24
3200	200000	23936	50	12	8000	500000	59840	20	30
3555	222222	26596	45	13¼	10667	666667	79787	15	40
4000	250000	29920	40	15	16000	1000000	119680	10	60

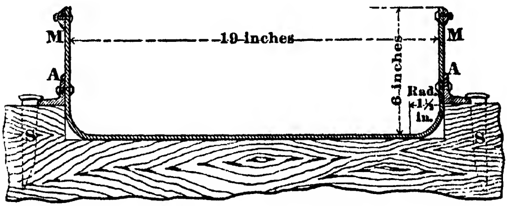
A reservoir, with a stand-pipe, or water column, is preferable to the ordinary tank, when the locality admits of it; being less liable than the pump to get out of order; and being cheaper in the end. The reservoir is supposed to be filled by water flowing into it by gravity; and to have its bottom at

*The cost of windmill alone, for railway stations, varies from about $450 for 13 feet diameter, to $1500 for 36 feet diameter, at factory.

least about 8 feet above the rails; or at any greater height whatever that the ground and the height of the water may require. It may be excavated in the ground; lined with brick or masonry in cement; with a bottom of concrete; or it may be built above ground, according to the locality. It may be roofed and covered in, or not; and it may be near the tracks, or at a considerable distance from them, according to circumstances. From its bottom, an iron pipe from 8 to 12 inches diameter, is carried (generally underground,) to within a few feet of the track. At that point it turns vertically upward to about 8 or 10 feet above the track, forming a **stand-pipe, or water-column**; from the upper end of which the water flows (through either a hose or a jointed nozzle,) as in the case of a tank. Several such pipes, or one larger one, may be laid, for the supply of two or more engines at once, through as many stand-pipes. Where the pipe makes its bend, and becomes vertical, is a valve for opening and closing it; and which may be worked by a hand-wheel placed at such a height as to be easily reached by the engine man.

On some of the more important lines, the **tenders of fast trains scoop up water, while running,** from a long **trough,** or **"track tank"** laid between the rails. The tanks are about ¼ mile long. They must of course be level, and they therefore require a level track.

As originally introduced in England, by Ramsbottom, the trough was of cast-iron, in lengths of about 6 ft. These were bolted together by means of flanges at their ends. The ends were not in contact with each other, but were separated by a strip of vulcanized rubber.



Our figure shows a track tank of $\frac{3}{16}$ inch rolled plate-iron, the sheets of which are 62 ins long. The lengths overlap each other 2 ins; leaving 5 ft as their *showing* length. The sheets are cut slightly tapering, so that at one end of each length the trough is $\frac{3}{16}$ in deeper than at the other, and the tops are thus kept flush with each other throughout. The joints are double riveted with ⅜ inch rivets, about 1½ ins from center to center, and staggered. At each end of the trough, the bottom slopes upward, and in a length of 6 ft, comes to the level of the tops of the sides. The cross-ties are notched, as shown, to receive the trough, which is loosely held to them by two spikes, S and S, in each tie. The heads of the spikes lit over the horizontal flanges of the 1½ × 1½ inch angle bars, A and A. M and M are mouldings of 1⅛ × ½ inch bar-iron. The angles and the mouldings are in lengths of 15 ft, and are riveted to the sides of the trough continuously throughout its length.

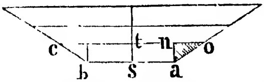
The **scoop** on the tender is **lowered** into the trough, **and raised** from it, by means of a lever on the fireman's platform, and is not permitted to touch the bottom of the trough.

The **trough is supplied** with water by means of pipes leading from an adjacent tank. The supply is regulated by a man in charge.

To prevent the water from **freezing in winter,** steam is led to the trough from the boiler of the pumping engine, through iron pipes laid under ground alongside of the track. These pipes are provided with branches which introduce the steam to the trough at every 40 ft of its length. The steam-pipes are protected by wooden boxes, and are furnished with valves for regulating the supply of steam.

EARTHWORK.

To prepare a Table, T, of Level Cuttings, for every $\frac{1}{10}$ of a foot of height, or depth.



Let the fig represent the cutting; or if inverted, the filling, in which the horizontal lines are supposed to be $\frac{1}{10}$ foot apart. First calculate the area in square feet, of the layer *a b c o*, adjoining the roadway *a b*. Then find how many cubic yards that area gives in a distance of 100 feet. These cubic yards we will call Y, they form the first amount to be put into the Table T.

Next calculate the area in square feet of the triangle *a n o*. Multiply this area by 4. Find how many cubic yards this increased area gives in a distance of 100 feet. Or they will be found ready calculated below. We will call them *y*. This is all the preparation that is needed before commencing the table.

Exam.—Let the roadbed *a b* be 18 feet, and the side slopes 1½ to 1. Then for the area of *a b c o*: since the side slopes are 1½ to 1, and *s t* is .1 foot, *c o* must be 18.3 feet, and the mean length of *a b c o* must be 18.15 feet. Consequently, the area is 18.15 × .1 = 1.815 square feet, which, in a distance of 100 feet, gives 181.5 cubic feet, which is equal to $\frac{181.5}{27} = 6.7222$ cubic yards, or Y.

Next, as to the triangle *a n o*, its height *a n* being .1 foot, and its base *n o* .15 feet; its area $= \frac{.1 \times .15}{2} = \frac{.015}{2} = .0075$ square ft. This multiplied by 4, gives .03 square feet, which, in a distance of 100 feet, gives .03 × 100 = 3 cubic feet; which is equal to $\frac{.3}{27} = .1111$ cubic yard, or *y*.

Having thus found Y and *y*, proceed to make out the table in the manner following, which is so plain as to require no explanation. The work should be tested about every 5 feet, by calculating the area of the full depth arrived at; multiply it by 100, and divide the product by 27 for the cubic yards. The cubic yards thus found should agree with the table.

Y.......	6.7222	Y. 6.722	.1
y	.1111		
	6.8333	6.8333	
y........	.1111	13.5555	.2
	6.9444	6.9444	
y........	.1111	20.5000	.3
	7.0555	7.0555	
y........	.1111	27.5555	.4
	7.1666	7.1666	
y.......	.1111	34.7222	.5
	7.2777	7.2777	
		42.0000	.6

TABLE T

Height Feet	Cub. Yds.
.1 .	6.72 Y.
.2	13.6
.3	20.5
.4	27.6
.5 .. .	34.7
.6	42.0
&c.	

The following table contains *y*, ready calculated for different side-slopes. It plainly remains the same for all widths of roadbed.

Side-slope.	*y*	Side-slope.	*y*
¼ to 1	.... .0185	1¾ to 1	.... .1296
½ to 1	.... .0370	2 to 1	1482
¾ to 1 .. .	.... .0556	2¼ to 1	1667
1 to 1	.... .0741	2½ to 1	.... .1852
1¼ to 1	.... .0926	3 to 1	. ... .2222
1½ to 1	.... .1111	4 to 1 . ..	.... .2963

Table 1. Level Cuttings.*

Roadway 14 feet wide, side-slopes $1\frac{1}{2}$ to 1.

For single-track embankment.

Height in Ft	.0	.1	.2	.3	4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu Yds	Cu. Yds	Cu Yds	Cu Yds	Cu Yds	Cu Yds	Cu Yds	Cu Yds.
0		5.24	10.6	16.1	21.6	27.3	33.1	39.0	45.0	51.2
1	57.4	63.8	70.2	76.8	83.5	90.3	97.2	104.2	111.3	118.6
2	125.9	133.4	141.0	148.6	156.4	164.4	172.4	180.5	188.7	197.1
3	205.6	214.1	222.8	231.6	240.5	249.5	258.7	267.9	277.3	286.7
4	296.3	306.0	315.8	325.7	335.7	345.8	356.1	366.4	376.9	387.5
5	398.1	408.9	419.9	430.9	442.0	453.2	464.6	476.1	487.6	499.3
6	511.1	523.0	535.0	547.2	559.4	571.8	584.2	596.8	609.5	622.3
7	635.2	648.2	661.3	674.6	687.9	701.4	714.9	728.6	742.4	756.3
8	770.3	784.5	798.7	813.1	827.5	842.1	856.8	871.6	886.5	901.5
9	916.7	931.9	947.3	962.7	978.3	994.0	1010	1026	1042	1058
10	1074	1090	1107	1123	1140	1157	1174	1191	1208	1226
11	1243	1260	1278	1295	1313	1331	1349	1367	1385	1404
12	1422	1441	1459	1478	1497	1516	1535	1554	1574	1593
13	1613	1633	1652	1672	1692	1712	1733	1753	1773	1794
14	1815	1835	1856	1877	1898	1920	1941	1962	1984	2006
15	2028	2050	2072	2094	2116	2138	2161	2183	2206	2229
16	2252	2275	2298	2321	2344	2368	2391	2415	2439	2463
17	2487	2511	2535	2559	2584	2608	2633	2658	2683	2708
18	2733	2759	2784	2809	2835	2861	2886	2912	2938	2964
19	2991	3017	3044	3070	3097	3124	3151	3178	3205	3232
20	3259	3287	3314	3342	3370	3398	3426	3454	3482	3510
21	3539	3567	3596	3625	3654	3683	3712	3741	3771	3800
22	3830	3859	3889	3919	3949	3979	4009	4040	4070	4101
23	4132	4162	4193	4224	4255	4287	4318	4349	4381	4413
24	4444	4476	4508	4541	4573	4605	4638	4670	4703	4736
25	4769	4802	4835	4868	4901	4935	4968	5002	5036	5070
26	5104	5138	5172	5206	5241	5275	5310	5345	5380	5415
27	5450	5485	5521	5556	5592	5627	5663	5699	5735	5771
28	5807	5844	5880	5917	5953	5990	6027	6064	6101	6139
29	6176	6213	6251	6289	6326	6364	6402	6440	6479	6517
30	6556	6594	6633	6672	6711	6750	6789	6828	6867	6907
31	6946	6986	7026	7066	7106	7146	7186	7226	7267	7307
32	7348	7389	7430	7471	7512	7553	7595	7636	7678	7719
33	7761	7803	7845	7887	7929	7972	8014	8057	8099	8142
34	8185	8228	8271	8315	8358	8401	8445	8489	8532	8576
35	8620	8664	8709	8753	8798	8842	8887	8932	8976	9022
36	9067	9112	9157	9203	9248	9294	9340	9386	9432	9478
37	9524	9570	9617	9663	9710	9757	9804	9851	9898	9945
38	9993	10040	10088	10135	10183	10231	10279	10327	10375	10424
39	10472	10521	10569	10618	10667	10716	10765	10815	10864	10913
40	10963	11013	11062	11112	11162	11212	11263	11313	11364	11414
41	11465	11516	11567	11618	11669	11720	11771	11823	11874	11926
42	11978	12029	12081	12134	12186	12238	12291	12343	12396	12449
43	12502	12555	12608	12661	12715	12768	12822	12875	12929	12983
44	13037	13091	13145	13200	13254	13309	13363	13418	13473	13528
45	13583	13639	13694	13749	13805	13861	13916	13972	14028	14084
46	14141	14197	14254	14310	14367	14424	14480	14537	14595	14652
47	14709	14767	14824	14882	14940	14998	15056	15114	15172	15230
48	15289	15347	15406	15465	15524	15583	15642	15701	15761	15821
49	15880	15939	15999	16059	16119	16179	16239	16300	16360	16421
50	16481	16542	16603	16664	16725	16787	16848	16909	16971	17033
51	17094	17156	17218	17280	17343	17405	17467	17530	17593	17656
52	17719	17782	17845	17908	17971	18035	18098	18162	18226	18290
53	18354	18418	18482	18546	18611	18675	18740	18805	18870	18935
54	19000	19065	19131	19196	19262	19327	19393	19459	19525	19591
55	19657	19724	19790	19857	19923	19990	20057	20124	20191	20259
56	20326	20393	20461	20529	20596	20664	20732	20800	20869	20937
57	21005	21074	21143	21212	21280	21349	21419	21488	21557	21627
58	21696	21766	21836	21906	21976	22046	22116	22186	22257	22327
59	22398	22469	22540	22611	22682	22753	22825	22896	22968	23039
60	23111	23183	23255	23327	23399	23472	23544	23617	23689	23762

* From the Author's "Measurement and Cost of Earthwork."

Table 2. Level Cuttings.

Roadway 24 feet wide, side-slopes 1½ to 1.

For double-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	6	.7	8	.9
	Cu Yds.	Cu. Yds.	Cu. Yds	Cu. Yds.	Cu. Yds	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		8.94	18.0	27.2	36.4	45.8	55.3	64.9	74 7	84.5
1	94 4	104 5	114.7	124.9	135 3	145.8	156.4	167.2	178 0	188.9
2	200 0	211.2	222.4	233.8	245.3	256.9	268.6	280.5	292.4	304 4
3	316.6	328.9	341.2	353.7	366.3	379.0	391.9	404 8	417.8	431.0
4	444 4	457 8	471.3	484.9	498 6	512.4	526.4	540 4	554 6	568.8
5	583.3	597.8	612.4	627.1	642 0	656 9	671.9	687.1	702.3	717.7
6	733.3	748.9	764.7	780.5	796.4	812 5	828.7	844.9	861.3	877.8
7	894.4	911.2	928.0	944 9	962 0	979.2	996 4	1014	1031	1049
8	1067	1085	1102	1121	1139	1157	1175	1194	1212	1231
9	1250	1269	1288	1307	1326	1346	1365	1385	1405	1425
10	1444	1465	1485	1505	1525	1546	1566	1587	1608	1629
11	1650	1671	1692	1714	1735	1757	1779	1800	1822	1845
12	1867	1889	1911	1934	1956	1979	2002	2025	2048	2071
13	2094	2118	2141	2165	2189	2213	2236	2261	2285	2309
14	2333	2358	2382	2407	2432	2457	2482	2507	2532	2558
15	2583	2609	2635	2661	2686	2713	2739	2765	2791	2818
16	2844	2871	2898	2925	2952	2979	3006	3034	3061	3089
17	3117	3145	3172	3201	3229	3257	3285	3314	3342	3371
18	3400	3429	3458	3487	3516	3546	3575	3605	3635	3665
19	3694	3725	3755	3785	3815	3846	3876	3907	3938	3969
20	4000	4031	4062	4094	4125	4157	4189	4221	4252	4285
21	4317	4349	4381	4414	4446	4479	4512	4545	4578	4611
22	4644	4678	4711	4745	4779	4813	4846	4881	4915	4949
23	4983	5018	5052	5087	5122	5157	5192	5227	5262	5298
24	5333	5369	5405	5441	5476	5513	5549	5585	5621	5658
25	5694	5731	5768	5805	5842	5879	5916	5954	5991	6029
26	6067	6105	6142	6181	6219	6257	6295	6334	6372	6411
27	6450	6489	6528	6567	6606	6646	6685	6725	6765	6805
28	6844	6885	6925	6965	7005	7046	7086	7127	7168	7209
29	7250	7291	7332	7374	7415	7457	7499	7541	7582	7625
30	7667	7709	7751	7794	7836	7879	7922	7965	8008	8051
31	8094	8138	8181	8225	8269	8313	8356	8401	8445	8489
32	8533	8578	8622	8667	8712	8757	8802	8847	8892	8938
33	8983	9029	9075	9121	9166	9212	9259	9305	9351	9398
34	9444	9491	9538	9585	9632	9679	9726	9774	9821	9869
35	9917	9965	10012	10061	10109	10157	10205	10254	10302	10351
36	10400	10449	10498	10547	10596	10646	10695	10745	10795	10845
37	10894	10945	10995	11045	11095	11146	11196	11247	11298	11349
38	11400	11451	11502	11554	11605	11657	11709	11761	11812	11865
39	11917	11969	12021	12074	12126	12179	12232	12285	12338	12391
40	12444	12498	12551	12605	12659	12713	12766	12821	12875	12929
41	12983	13038	13092	13147	13202	13257	13312	13367	13422	13478
42	13533	13589	13645	13701	13756	13813	13869	13925	13981	14038
43	14094	14151	14208	14265	14322	14379	14436	14494	14551	14609
44	14667	14725	14782	14840	14899	14957	15015	15074	15132	15191
45	15250	15309	15368	15427	15486	15546	15605	15665	15725	15785
46	15844	15905	15965	16025	16085	16146	16206	16267	16328	16389
47	16450	16511	16572	16634	16695	16757	16819	16881	16942	17005
48	17067	17129	17191	17254	17316	17379	17442	17505	17568	17631
49	17694	17758	17821	17885	17949	18013	18076	18141	18205	18269
50	18333	18398	18462	18527	18592	18657	18722	18787	18852	18918
51	18983	19049	19115	19181	19246	19313	19379	19445	19511	19578
52	19644	19711	19778	19845	19912	19979	20046	20114	20181	20249
53	20317	20385	20452	20521	20589	20657	20725	20794	20862	20931
54	21000	21069	21138	21207	21276	21346	21415	21485	21555	21625
55	21694	21765	21835	21905	21975	22046	22116	22187	22258	22329
56	22400	22471	22542	22614	22685	22757	22829	22901	22972	23045
57	23117	23189	23261	23334	23406	23479	23552	23625	23698	23771
58	23844	23918	23991	24065	24139	24213	24286	24361	24435	24509
59	24583	24658	24732	24807	24882	24957	25032	25107	25182	25258
60	25333	25409	25485	25561	25636	25713	25789	25865	25941	26018

For continuation to 100 feet, see Table 7.

Table 3. Level Cuttings.

Roadway 18 feet wide, side-slopes 1 to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds	Cu. Yds.	Cu. Yds	Cu Yds.	Cu. Yds	Cu. Yds	Cu. Yds	Cu. Yds.
0		6.70	13.5	20.3	27.3	34.3	41.3	48.5	55.7	63.0
1	70.4	77.8	85.3	92.9	100.6	108.3	116.1	124.0	132.0	140.0
2	148.1	156.3	164.6	172.9	181.3	189.8	198.4	207.0	215.7	224.5
3	233.3	242.3	251.3	260.3	269.5	278.7	288.0	297.4	306.8	316.3
4	325.9	335.6	345.3	355.1	365.0	375.0	385.0	395.1	405.3	415.6
5	425.9	436.3	446.8	457.4	468.0	478.7	489.5	500.3	511.3	522.3
6	533.3	544.5	555.7	567.0	578.4	589.8	601.3	612.9	624.6	636.3
7	648.1	660.0	672.0	684.0	696.1	708.3	720.6	732.9	745.3	757.8
8	770.4	783.0	795.7	808.5	821.3	834.3	847.3	860.3	873.5	886.7
9	900.0	913.4	926.8	940.3	953.9	967.6	981.3	995.1	1009	1023
10	1037	1051	1065	1080	1094	1108	1123	1137	1152	1167
11	1181	1196	1211	1226	1241	1256	1272	1287	1302	1318
12	1333	1349	1365	1380	1396	1412	1428	1444	1460	1476
13	1493	1509	1525	1542	1558	1575	1592	1608	1625	1642
14	1659	1676	1693	1711	1728	1745	1763	1780	1798	1816
15	1833	1851	1869	1887	1905	1923	1941	1960	1078	1996
16	2015	2033	2052	2071	2089	2108	2127	2146	2165	2184
17	2204	2223	2242	2262	2281	2301	2321	2340	2360	2380
18	2400	2420	2440	2460	2481	2501	2521	2542	2562	2583
19	2604	2624	2645	2666	2687	2708	2729	2751	2772	2793
20	2815	2836	2858	2880	2901	2923	2945	2967	2989	3011
21	3033	3056	3078	3100	3123	3145	3168	3191	3213	3236
22	3259	3282	3305	3328	3352	3375	3398	3422	3445	3469
23	3493	3516	3540	3564	3588	3612	3636	3660	3685	3709
24	3733	3758	3782	3807	3832	3856	3881	3906	3931	3956
25	3981	4007	4032	4057	4083	4108	4134	4160	4185	4211
26	4237	4263	4289	4315	4341	4368	4394	4420	4447	4473
27	4500	4527	4553	4580	4607	4634	4661	4688	4716	4743
28	4770	4798	4825	4853	4881	4908	4936	4964	4992	5020
29	5048	5076	5105	5133	5161	5190	5218	5247	5276	5304
30	5333	5362	5391	5420	5449	5479	5508	5537	5567	5596
31	5626	5056	5685	5715	5745	5775	5805	5835	5865	5896
32	5926	5956	5987	6017	6048	6079	6109	6140	6171	6202
33	6233	6264	6296	6327	6358	6390	6421	6453	6485	6516
34	6548	6580	6612	6644	6676	6708	6741	6773	6805	6838
35	6870	6903	6936	6968	7001	7034	7067	7100	7133	7167
36	7200	7233	7267	7300	7334	7368	7401	7435	7469	7503
37	7537	7571	7605	7640	7674	7708	7743	7777	7812	7847
38	7881	7916	7951	7986	8021	8056	8092	8127	8162	8198
39	8233	8269	8305	8340	8376	8412	8448	8484	8520	8556
40	8593	8629	8665	8702	8738	8775	8812	8848	8885	8922
41	8959	8996	9033	9071	9108	9145	9183	9220	9258	9296
42	9333	9371	9409	9447	9485	9523	9561	9600	9638	9676
43	9715	9753	9792	9831	9869	9908	9947	9986	10025	10064
44	10104	10143	10182	10222	10261	10301	10341	10380	10420	10460
45	10500	10540	10580	10620	10661	10701	10741	10782	10822	10863
46	10904	10944	10985	11026	11067	11108	11149	11191	11232	11273
47	11315	11356	11398	11440	11481	11523	11565	11607	11649	11691
48	11733	11776	11818	11860	11903	11945	11988	12031	12073	12116
49	12159	12202	12245	12288	12332	12375	12418	12462	12505	12549
50	12593	12636	12680	12724	12768	12812	12856	12900	12945	12989
51	13033	13078	13122	13167	13212	13256	13301	13346	13391	13436
52	13481	13527	13572	13617	13663	13708	13754	13800	13845	13891
53	13937	13983	14029	14075	14121	14168	14214	14260	14307	14353
54	14400	14447	14493	14540	14587	14634	14681	14728	14776	14823
55	14870	14918	14965	15013	15061	15108	15156	15204	15252	15300
56	15348	15396	15445	15493	15541	15590	15638	15687	15736	15784
57	15833	15882	15931	15980	16029	16079	16128	17177	16227	16276
58	16326	16376	16425	16475	16525	16575	16625	16675	16725	16776
59	16826	16876	16927	16977	17028	17079	17129	17180	17231	17282
60	17333	17384	17436	17487	17538	17590	17641	17693	17745	17796

For continuation to 100 feet deep, see Table 7.

Table 4. Level Cuttings.

Roadway 18 feet, side-slopes 1½ to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds	Cu Yds	Cu. Yds	Cu. Yds	Cu. Yds	Cu Yds	Cu Yds.	Cu. Yds.
0		6.72	13.6	20.5	27.6	34.7	42.0	49.4	56.9	64.5
1	72.2	80.1	88.0	96.1	104.2	112.5	120.9	129.4	138.0	146.7
2	155.5	164.5	173.5	182.7	191.9	201.3	210.8	220.4	230.1	240.0
3	249.9	260.0	270.1	280.4	290.8	301.3	311.9	322.6	333.4	344.5
4	355.5	366.7	378.0	389.4	400.9	412.5	424.2	436.0	448.0	460.0
5	472.2	484.5	496.9	509.4	522.0	534.7	547.6	560.5	573.6	586.7
6	600.0	613.4	626.9	640.5	654.2	668.1	682.0	696.1	710.2	724.5
7	738.9	753.4	768.0	782.7	797.6	812.5	827.6	842.7	858.0	873.4
8	888.9	904.5	920.2	936.1	952.0	968.1	984.2	1001	1017	1033
9	1050	1067	1084	1101	1118	1135	1152	1169	1187	1205
10	1222	1240	1258	1276	1294	1313	1331	1349	1368	1387
11	1406	1425	1444	1463	1482	1501	1521	1541	1560	1580
12	1600	1620	1640	1661	1681	1701	1722	1743	1764	1785
13	1806	1827	1848	1869	1891	1913	1934	1956	1978	2000
14	2022	2045	2067	2089	2112	2135	2158	2181	2204	2227
15	2250	2273	2297	2321	2344	2368	2392	2416	2440	2465
16	2489	2513	2538	2563	2588	2613	2638	2663	2688	2713
17	2739	2765	2790	2816	2842	2868	2894	2921	2947	2973
18	3000	3027	3054	3081	3108	3135	3162	3189	3217	3245
19	3272	3300	3328	3356	3384	3413	3441	3469	3498	3527
20	3556	3585	3614	3643	3672	3701	3731	3761	3790	3820
21	3850	3880	3910	3941	3971	4001	4032	4063	4094	4125
22	4156	4187	4218	4249	4281	4313	4344	4376	4408	4440
23	4472	4505	4537	4569	4602	4635	4668	4701	4734	4767
24	4800	4833	4867	4901	4934	4968	5002	5036	5070	5105
25	5139	5173	5208	5243	5278	5313	5348	5383	5418	5453
26	5489	5525	5560	5596	5632	5668	5704	5741	5777	5813
27	5850	5887	5924	5961	5998	6035	6072	6109	6147	6185
28	6222	6260	6298	6336	6374	6413	6451	6489	6528	6567
29	6606	6645	6684	6723	6762	6801	6841	6881	6920	6960
30	7000	7040	7080	7121	7161	7201	7242	7283	7324	7365
31	7406	7447	7488	7529	7571	7613	7654	7696	7738	7780
32	7822	7865	7907	7949	7992	8035	8078	8121	8164	8207
33	8250	8293	8337	8381	8424	8468	8512	8556	8600	8645
34	8689	8733	8778	8823	8868	8913	8958	9003	9048	9093
35	9139	9185	9230	9276	9322	9368	9414	9461	9507	9553
36	9600	9647	9694	9741	9788	9835	9882	9929	9977	10025
37	10072	10120	10168	10216	10264	10313	10361	10409	10458	10507
38	10556	10605	10654	10703	10752	10801	10851	10901	10950	11000
39	11050	11100	11150	11200	11251	11301	11352	11403	11454	11505
40	11556	11607	11658	11709	11761	11813	11864	11916	11968	12020
41	12072	12125	12177	12229	12282	12335	12388	12441	12494	12547
42	12600	12653	12707	12761	12814	12868	12922	12976	13030	13085
43	13139	13193	13248	13303	13358	13413	13468	13523	13578	13633
44	13689	13745	13800	13856	13912	13968	14024	14081	14137	14193
45	14250	14307	14364	14421	14478	14535	14592	14649	14707	14765
46	14822	14880	14938	14996	15054	15113	15171	15229	15288	15347
47	15406	15465	15524	15583	15642	15701	15761	15821	15880	15940
48	16000	16060	16120	16181	16241	16301	16362	16423	16484	16545
49	16606	16667	16728	16789	16851	16913	16974	17036	17098	17160
50	17222	17285	17347	17409	17472	17535	17598	17661	17724	17787
51	17850	17913	17977	18041	18104	18168	18232	18296	18360	18425
52	18489	18553	18618	18683	18748	18813	18878	18943	19008	19073
53	19139	19205	19270	19336	19402	19468	19534	19601	19667	19733
54	19800	19867	19934	20000	20068	20135	20202	20269	20337	20405
55	20472	20540	20608	20676	20744	20813	20881	20949	21018	21087
56	21156	21225	21294	21363	21432	21501	21571	21641	21710	21780
57	21850	21920	21990	22061	22131	22201	22272	22343	22414	22485
58	22556	22627	22698	22769	22841	22913	22984	23056	23128	23200
59	23272	23345	23417	23489	23562	23635	23708	23781	23854	23927
60	24000	24073	24147	24221	24294	24368	24442	24516	24590	24665

For continuation to 100 feet deep, see Table 7.

Table 5. Level Cuttings.

Roadway 28 feet wide, side-slopes 1 to 1.

For double-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		10.4	20.9	31.4	42.1	52.8	63.6	74.4	85.3	96.3
1	107.4	118.6	129.8	141.1	152.4	163.9	175.4	187.0	198.7	210.4
2	222.2	234.1	246.1	258.1	270.2	282.4	294.7	307.0	319.4	331.9
3	344.4	357.1	369.8	382.6	395.4	408.3	421.3	434.4	447.6	460.8
4	474.1	487.4	500.9	514.4	528.0	541.7	555.4	569.2	583.1	597.1
5	611.1	625.2	639.4	653.7	668.0	682.4	696.9	711.4	726.1	740.8
6	755.6	770.4	785.4	800.4	815.5	830.6	845.8	861.1	876.5	891.9
7	907.5	923.0	938.7	954.5	970.3	986.2	1002	1018	1034	1050
8	1067	1083	1099	1116	1132	1149	1166	1182	1199	1216
9	1233	1250	1267	1285	1302	1319	1337	1354	1372	1390
10	1407	1425	1443	1461	1479	1497	1515	1534	1552	1570
11	1589	1607	1626	1645	1664	1682	1701	1720	1739	1759
12	1778	1797	1816	1836	1855	1875	1895	1914	1934	1954
13	1974	1994	2014	2034	2055	2075	2095	2116	2136	2157
14	2178	2199	2219	2240	2261	2282	2304	2325	2346	2367
15	2389	2410	2432	2454	2475	2497	2519	2541	2563	2585
16	2607	2630	2652	2674	2697	2719	2742	2765	2788	2810
17	2833	2856	2879	2903	2926	2949	2972	2996	3019	3043
18	3067	3090	3114	3138	3162	3186	3210	3234	3259	3283
19	3307	3332	3356	3381	3406	3431	3455	3480	3505	3530
20	3556	3581	3606	3631	3657	3682	3708	3734	3759	3785
21	3811	3837	3863	3889	3915	3942	3968	3994	4021	4047
22	4074	4101	4128	4154	4181	4208	4235	4263	4290	4317
23	4344	4372	4399	4427	4455	4482	4510	4538	4566	4594
24	4622	4650	4679	4707	4735	4764	4792	4821	4850	4879
25	4907	4936	4965	4994	5024	5053	5082	5111	5141	5170
26	5200	5230	5259	5289	5319	5349	5379	5409	5439	5470
27	5500	5530	5561	5591	5622	5653	5684	5714	5745	5776
28	5807	5839	5870	5901	5932	5964	5995	6027	6059	6090
29	6122	6154	6186	6218	6250	6282	6315	6347	6379	6412
30	6444	6477	6510	6543	6575	6608	6641	6674	6708	6741
31	6774	6807	6841	6874	6908	6942	6975	7009	7043	7077
32	7111	7145	7179	7214	7248	7282	7317	7351	7386	7421
33	7456	7490	7525	7560	7595	7631	7666	7701	7736	7772
34	7807	7843	7879	7914	7950	7986	8022	8058	8094	8130
35	8167	8203	8239	8276	8312	8349	8386	8423	8459	8496
36	8533	8570	8608	8645	8682	8719	8757	8794	8832	8870
37	8907	8945	8983	9021	9059	9097	9135	9174	9212	9250
38	9289	9327	9366	9405	9444	9482	9521	9560	9509	9639
39	9678	9717	9756	9796	9835	9875	9915	9954	9994	10034
40	10074	10114	10154	10194	10235	10275	10315	10356	10396	10437
41	10178	10519	10559	10600	10641	10682	10724	10765	10806	10847
42	10889	10930	10972	11014	11055	11097	11139	11181	11223	11265
43	11307	11350	11392	11434	11477	11519	11562	11605	11648	11691
44	11733	11776	11819	11863	11906	11949	11992	12036	12079	12123
45	12167	12210	12254	12298	12342	12386	12430	12474	12519	12563
46	12607	12652	12696	12741	12786	12831	12875	12920	12965	13010
47	13056	13101	13146	13191	13237	13282	13328	13374	13419	13465
48	13511	13587	13603	13649	13695	13742	13788	13834	13881	13927
49	13974	14021	14068	14114	14161	14208	14255	14303	14350	14397
50	14444	14492	14539	14587	14635	14682	14730	14778	14826	14874
51	14922	14970	15019	15067	15115	15164	15212	15261	15310	15359
52	15407	15456	15505	15554	15604	15653	15702	15751	15801	15850
53	15900	15950	15999	16049	16099	16149	16199	16249	16299	16350
54	16400	16450	16501	16551	16602	16653	16704	16754	16805	16856
55	16907	16959	17010	17061	17112	17164	17215	17267	17319	17370
56	17422	17474	17526	17578	17630	17682	17735	17787	17839	17892
57	17944	17997	18050	18103	18155	18208	18261	18314	18368	18421
58	18474	18527	18581	18634	18688	18742	18795	18849	18903	18957
59	19011	19065	19119	19174	19228	19282	19337	19391	19446	19501
60	19556	19610	19665	19720	19775	19831	19886	19941	19996	20052

For continuation to 100 feet see Table 7.

Table 6. Level Cuttings.

Roadway 28 ft wide, side-slopes 1½ to 1.

For double-track excavation.

Depth in Ft	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds	Cu. Yds.	Cu. Yds.
0		10.4	21.0	31.6	42.4	53.2	64.2	75.3	86.5	97.9
1	109.3	120.8	132.5	144.3	156.1	168.1	180.2	192.4	204.8	217.2
2	229.6	242.3	255.0	267.9	280.9	294.0	307.2	320.5	334.0	347.5
3	361.2	374.9	388.8	402.8	416.9	431.1	445.4	459.9	474.4	489.1
4	503.7	518.6	533.6	548.6	563.9	579.3	594.7	610.2	625.8	641.6
5	657.5	673.4	689.5	705.7	722.1	738.5	755.0	771.7	788.4	805.3
6	822.2	839.3	856.5	873.8	891.2	908.8	926.4	944.2	962.0	980.0
7	998.1	1016	1035	1053	1072	1090	1109	1128	1147	1166
8	1185	1204	1224	1243	1263	1283	1303	1322	1343	1363
9	1383	1403	1424	1445	1465	1486	1507	1528	1549	1571
10	1592	1614	1635	1657	1679	1701	1723	1745	1767	1790
11	1812	1835	1858	1881	1904	1927	1950	1973	1997	2020
12	2044	2068	2092	2116	2140	2164	2189	2213	2238	2262
13	2287	2312	2337	2362	2387	2413	2438	2464	2489	2515
14	2541	2567	2593	2619	2645	2672	2698	2725	2752	2779
15	2806	2833	2860	2887	2915	2942	2970	2997	3025	3053
16	3081	3109	3138	3166	3195	3223	3252	3281	3310	3339
17	3368	3397	3427	3456	3486	3516	3546	3576	3606	3636
18	3667	3697	3728	3758	3789	3820	3851	3882	3913	3944
19	3976	4007	4039	4070	4102	4134	4166	4198	4231	4263
20	4296	4328	4361	4394	4427	4460	4493	4527	4560	4594
21	4627	4661	4695	4729	4763	4797	4832	4866	4900	4935
22	4970	5005	5040	5075	5111	5146	5181	5217	5253	5288
23	5324	5360	5396	5432	5469	5505	5542	5578	5615	5652
24	5689	5726	5763	5800	5838	5875	5913	5951	5989	6027
25	6065	6103	6141	6179	6218	6257	6295	6334	6373	6412
26	6451	6491	6530	6570	6609	6649	6689	6729	6769	6809
27	6850	6890	6931	6971	7012	7053	7094	7135	7176	7217
28	7259	7300	7342	7384	7426	7468	7510	7552	7594	7637
29	7680	7722	7765	7808	7851	7894	7937	7981	8024	8067
30	8111	8155	8199	8243	8287	8331	8375	8420	8464	8509
31	8554	8598	8643	8688	8734	8779	8824	8870	8915	8961
32	9007	9053	9099	9145	9191	9238	9284	9331	9378	9425
33	9472	9519	9566	9613	9661	9708	9756	9804	9851	9900
34	9948	9997	10045	10093	10142	10190	10239	10288	10337	10386
35	10435	10484	10534	10583	10633	10683	10732	10782	10832	10882
36	10933	10983	11034	11084	11135	11186	11237	11288	11339	11391
37	11443	11494	11546	11598	11649	11701	11753	11806	11858	11910
38	11963	12016	12068	12121	12174	12227	12281	12334	12387	12441
39	12494	12548	12602	12656	12710	12764	12819	12873	12928	12982
40	13037	13092	13147	13202	13257	13312	13368	13423	13479	13535
41	13591	13647	13703	13759	13815	13872	13928	13985	14042	14099
42	14156	14213	14270	14327	14385	14442	14500	14558	14615	14673
43	14731	14790	14848	14906	14965	15024	15082	15141	15200	15259
44	15318	15378	15437	15497	15556	15616	15676	15736	15796	15856
45	15917	15977	16038	16098	16159	16220	16281	16342	16403	16465
46	16526	16587	16649	16711	16773	16835	16897	16959	17021	17084
47	17146	17209	17272	17335	17398	17461	17524	17587	17651	17714
48	17778	17842	17905	17969	18033	18098	18162	18226	18291	18356
49	18420	18485	18550	18615	18680	18746	18811	18877	18942	19008
50	19074	19140	19206	19272	19339	19405	19472	19538	19605	19672
51	19739	19806	19873	19940	20008	20075	20143	20211	20279	20347
52	20415	20483	20551	20620	20688	20757	20826	20894	20963	21032
53	21102	21171	21241	21310	21380	21450	21519	21589	21659	21730
54	21800	21870	21941	22012	22082	22153	22224	22295	22366	22438
55	22509	22581	22652	22724	22796	22868	22940	23012	23085	23157
56	23230	23302	23375	23448	23521	23594	23667	23741	23814	23888
57	23961	24035	24109	24183	24257	24331	24405	24480	24554	24629
58	24704	24779	24854	24929	25004	25079	25155	25230	25306	25381
59	25457	25533	25609	25686	25762	25838	25915	25992	26068	26145
60	26222	26299	26376	26454	26531	26609	26686	26764	26842	26920

For continuation to 100 feet, see Table 7.

Table 7. Level Cuttings.

Continuation of the six foregoing Tables of Cubic Contents, to 100 feet of height or depth.

Height or Depth in Feet.	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
61	23835	26094	17848	24739	20107	26998
.5	24201	26479	18108	25113	20386	27390
62	24570	26867	18370	25489	20667	27785
.5	24942	27257	18634	25868	20949	28183
63	25317	27650	18900	26250	21233	28583
.5	25694	28046	19168	26635	21519	28986
64	26074	28444	19437	27022	21807	29393
.5	26457	28846	19708	27413	22097	29801
65	26843	29250	19981	27806	22389	30213
.5	27231	29657	20256	28201	22682	30627
66	27622	30067	20533	28600	22978	31044
.5	28016	30479	20812	29001	23275	31464
67	28413	30894	21093	29406	23574	31887
.5	28812	31313	21375	29813	23875	32312
68	29215	31733	21659	30222	24178	32741
.5	29620	32157	21945	30635	24482	33172
69	30028	32583	22233	31050	24789	33605
.5	30438	33013	22523	31468	25097	34042
70	30852	33444	22814	31889	25407	34481
.5	31268	33879	23108	33313	25719	34924
71	31687	34317	23404	32739	26033	35369
.5	32108	34757	23701	33168	26349	35816
72	32533	35200	24000	33600	26667	36267
.5	32960	35646	24301	34035	26986	36720
73	33390	36094	24604	34472	27307	37176
.5	33823	36546	24907	34913	27631	37635
74	34259	37000	25214	35356	27956	38096
.5	34697	37457	25522	35801	28282	38561
75	35139	37917	25832	36250	28611	39028
.5	35582	38379	26144	36701	28942	39498
76	36029	38844	26458	37156	29174	39970
.5	36479	39313	26774	37613	29608	40446
77	36931	39783	27092	38072	29944	40924
.5	37386	40257	27411	88535	30282	41405
78	37844	40733	27733	39000	30622	41889
.5	38305	41213	28056	39468	30964	42375
79	38768	41694	28381	39939	31307	42865
.5	39235	42179	28708	40413	31653	43357
80	39704	42667	29037	40889	32000	43852
81	40650	43650	29700	41850	32700	44850
82	41607	44644	30370	42822	33407	45859
83	42576	45650	31048	43806	34122	46880
84	43555	46667	31733	44800	34844	47911
85	44546	47694	32426	45806	35574	48954
86	45548	48733	33126	46822	36311	50008
87	46561	49783	33833	47850	37056	51072
88	47585	50844	34548	48889	37807	52148
89	48620	51917	35270	49939	38567	53235
90	49667	53000	36000	51000	39333	54333
91	50724	54094	36737	52072	40107	55443
92	51793	55200	37481	53156	40889	56563
93	52872	56317	38233	54250	41678	57694
94	53963	57444	38993	55356	42474	58837
95	55065	58583	39759	56472	43278	59990
96	56178	59733	40533	57600	44089	61155
97	57302	60894	41315	58739	44907	62331
98	58437	62067	42104	59889	45733	63518
99	59583	63250	42900	61050	46567	64716
100	60741	64444	43704	62222	47407	65926

Table 8,

Of Cubic Yards in a 100-foot station of level cutting or filling, to be added to, or subtracted from, the quantities in the preceding seven tables, in case the excavations or embankments should be increased or diminished 2 feet in width.

Cubic Yards in a length of 100 feet; breadth 2 feet; and of different depths.

Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.
.5	3.70	.5	152	.5	300	.5	448	.5	596
1	7.41	21	156	41	304	61	452	81	600
.5	11.1	.5	159	.5	307	.5	456	.5	604
2	14.8	22	163	42	311	62	459	82	607
.5	18.5	.5	167	.5	315	.5	463	.5	611
3	22.2	23	170	43	319	63	467	83	615
.5	25.9	.5	174	.5	322	.5	470	.5	619
4	29.6	24	178	44	326	64	474	84	622
.5	33.3	.5	181	.5	330	.5	478	.5	626
5	37.0	25	185	45	333	65	481	85	630
.5	40.7	.5	189	.5	337	.5	485	.5	633
6	44.4	26	193	46	341	66	489	86	637
.5	48.1	.5	196	.5	344	.5	493	.5	641
7	51.9	27	200	47	348	67	496	87	644
.5	55.6	.5	204	.5	352	.5	500	.5	648
8	59.3	28	207	48	356	68	504	88	652
.5	63.0	.5	211	.5	359	.5	507	.5	656
9	66.7	29	215	49	363	69	511	89	659
.5	70.4	.5	219	.5	367	.5	515	.5	663
10	74.1	30	222	50	370	70	519	90	667
.5	77.8	.5	226	.5	374	.5	522	.5	670
11	81.5	31	230	51	378	71	526	91	674
.5	85.2	.5	233	.5	381	.5	530	.5	678
12	88.9	32	237	52	385	72	533	92	681
.5	92.6	.5	241	.5	389	.5	537	.5	685
13	96.3	33	244	53	393	73	541	93	689
.5	100	.5	248	.5	396	.5	544	.5	693
14	104	34	252	54	400	74	548	94	696
.5	107	.5	256	.5	404	.5	552	.5	700
15	111	35	259	55	407	75	556	95	704
.5	115	.5	263	.5	411	.5	559	.5	707
16	119	36	267	56	415	76	563	96	711
.5	122	.5	270	.5	419	.5	567	.5	715
17	126	37	274	57	422	77	570	97	719
.5	130	.5	278	.5	426	.5	574	.5	722
18	133	38	281	58	430	78	578	98	726
.5	137	.5	285	.5	433	.5	581	.5	730
19	141	39	289	59	437	79	585	99	733
.5	144	.5	293	.5	441	.5	589	.5	737
20	148	40	296	60	444	80	593	100	741

REMARK. **The foregoing tables of level cuttings may also be used for widths of roadway greater than those at the heads of the tables.** Thus, suppose we wish to use Table 1, for a roadbed *m n*, 16 ft wide, instead of *c b*, which is only 14 ft, and for which the table was calculated. It is only necessary first to find the vert dist *s a*, between these two roadbeds; and to add it *mentally* to each height *t s*, of the given embkt, when taking out from the

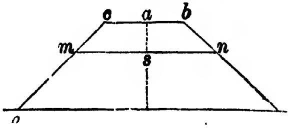


table the numbers of cub yds corresponding to the heights. By this means we obtain the contents of the embkt *c b o p*, for any required dist. Next, from these contents subtract that corresponding to the height *s a*, for the same dist. The remainder will plainly be the embkt *m n o p*.

In practice it will be sufficiently correct to take *s a* to the nearest tenth of a foot, which will save trouble in adding it mentally to the heights in the tables.

If the roadbed is narrower than the table, as, for instance, if *m n* be the width in the table, but we wish to find the contents for the width *c b*, then first find *s a*, and calculate the cubic yards in 100 feet length of *c b m n*. Then, in taking out the cubic yards from the table, first subtract *s a* mentally from each height; and to the cubic yards taken out for each 100 feet, opposite this *reduced* height, add the cubic yards in 100 feet of *c b m n*.

To avoid trouble with contractors about the measurement of rock cuts, stipulate in the contract, either that it shall conform with the theoretical cross section; or that an extra allowance of say about 2 feet of width of cut will be made, to cover the unavoidable irregularities of the sides.

Shrinkage of Embankment. Although earth, when first dug, and loosely thrown out, *swells* about ⅕ part, so that a cubic yard *in place* averages about 1⅕ or 1.2 cubic yards when dug: or 1 cubic yard dug is equal to ⅚, or to .8333 of a cubic yard in place; yet when made into embankment it gradually subsides, settles, or shrinks, into a less bulk than it occupied *before being dug*.

The following are approximate averages of the shrinkage; or, in other words, the earth measured in place in a cut, will, when made into embankment, occupy a bulk less than before by about the following proportions:

Gravel or sand	about	8 per ct;	or 1 in 12½ less.
Clay	"	10 per ct;	or 1 in 10 less.
Loam	"	12 per ct;	or 1 in 8⅓ less.
Loose vegetable surface soil	"	15 per ct;	or 1 in 6⅔ less.
Puddled clay	"	25 per ct;	or 1 in 4 less.

The writer thinks, from some trials of his own, that 1 cubic yard of any hard rock in place, will make from 1⅔ to 1¾ cubic yards of embankment; say on an average 1.7 cubic yards. Or that 1 cubic yard of rock embankment requires .5882 of a cubic yard in place. He found that a solid cubic yard when broken into fragments, made about as follows

	Cubic yards.	Of which there were Solid	Voids
In loose heap	1.9	52.6 per cent.	47.4 per cent
Carelessly piled	1.75	57 "	43 "
Carefully piled	1.6	63 "	37 "
Rubble, very carelessly scabbled	1.5	67 "	33 "
Rubble, somewhat carefully scabbled	1.25	80 "	20 "

COST OF EARTHWORK.

Art. 1. It is advisable to pay for this kind of work by the cubic yard of *excavation* only; instead of allowing separate prices for excavation and embankment. By this means we get rid of the difficulty of measurements, as well as the controversies and lawsuits which often attend the determination of the allowance to be made for the settlement or subsidence of the embankments.

It is, moreover, our opinion that justice to the contractor should lead to the **English practice of paying the laborers by the cubic yard,** instead of by the day. Experience fully proves that when laborers are scarce and wages high, men can scarcely be depended upon to do three fourths of the work which they readily accomplish when wages are low, and when fresh hands are waiting to be hired in case any are discharged. The contractor is thus placed at the mercy of his men. The writer has known the most satisfactory results to attend a system of task-work, accompanied by liberal premiums for all overwork. By this means the interests of the laborers are identified with that of the contractor; and every man takes care that the others shall do their fair share of the task.

Ellwood Morris, C E, of Philadelphia, was, we believe, the first person who properly investigated the elements of cost of earthwork, and reduced them to such a form as to enable us to calculate the total with a considerable degree of accuracy. He published his results in the Journal of the Franklin Institute in 1841. His paper forms the basis on which, with some variations, we shall consider the matter; and on which we shall extend it to wheelbarrows, as well as to carts. Throughout this paper we speak of a cubic yard considered only as solid in its place, or before it is loosened for removal. It is scarcely necessary to add that the various items can of course only be regarded as tolerably close approximations, or averages. As before stated, the men do less work when wages are high; and more when they are low. A great deal besides depends on the skill, observation, and energy of the contractor and his superintendents. It is no unusual thing to see two contractors working at the same prices, in precisely similar material, where one is making money, and the other losing it, from a want of tact in the proper distribution of his forces, keeping his roads in order, having his carts and barrows well filled, &c, &c. Uncommonly long spells of wet weather may seriously affect the cost of executing earthwork, by making it more difficult to loosen, load, or empty; besides keeping the roads in bad order for hauling.

The aggregate cost of excavating and removing earth is made up by the following items, namely:

1st. *Loosening the earth ready for the shovellers.*
2d. *Loading it by shovels into the carts or barrows.*
3d. *Hauling, or wheeling it away, including emptying and returning.*
4th. *Spreading it out into successive layers on the embankment.*
5th. *Keeping the hauling-road for carts, or the plank gangways for barrows, in good order.*
6th. *Wear, sharpening, depreciation, and interest on cost of tools.*
7th. *Superintendence, and water-carriers.*
8th. *Profit to the contractor.*

We will consider these items a little in detail, basing our calculations on the assumption that common labor costs \$1 per day, of 10 working hours. The results in our tables must therefore be increased or diminished in about the same proportion as common labor costs more or less than this.

Art. 2. Loosening the earth ready for the shovellers. This is generally done either by ploughs or by picks; more cheaply by the first. A plough with two horses, and two men to manage them, at \$1 per day for labor, 75 cents per day for each horse and 37 cents per day for plough, including harness, wear repairs &c or a total of \$3 87, will loosen, of strong heavy soils, from 200 to 300 cubic yards a day, at from 1.93 to 1.29 cents per yard; or of ordinary loam, from 400 to 600 cubic yards a day, at from .97 to .64 of a cent per yard. Therefore, as an ordinary average, we may assume the actual cost to the contractor for loosening by the plough, as follows: strong heavy soils, 1.6 cents; common loam, .8 cent; light sandy soils, .4 cent. Very stiff pure clay, or obstinate cemented gravel, may be set down at 2 5 cents; they require three or four horses.

By the pick, a fair day's work is about 14 yards of stiff pure clay, or of cemented gravel; 25 yards of strong heavy soils; 40 yards of common loam, 60 yards of light sandy soils—all measured in place, which, at \$1 per day for labor, gives, for stiff clay, 7 cents; heavy soils, 4 cents; loam, 2 5 cents; light sandy soil, 1 666 cents. Pure sand requires but very little labor for loosening; .5 of a cent will cover it.

Art. 3. Shovelling the loosened earth into carts. The amount shovelled per day depends partly upon the weight of the material, but more upon so proportioning the number of pickers and of carts to that of shovellers, as not to keep the latter waiting for either material or carts. In fairly regulated gangs, the shovellers into carts are not actually engaged in shovelling for more than six-tenths of their time, thus being unoccupied but four-tenths of it; while, under bad management, they lose considerably more than one-half of it. A shoveller can readily load into a cart one-third of a cubic yard measured in place (and which is an average working cart-load), of sandy soil, in five minutes; of loam, in six minutes; and of any of the heavy soils, in seven minutes. This would give for a day of 10 working hours, 120 loads, or 40 cubic yards of light sandy soil; 100 loads, or 33⅓ cubic yards of loam; or 86 loads, or 28.7 yards of the heavy soils. But from these amounts we must deduct four tenths for time necessarily lost; thus reducing the actual working quantities to 24 yards of light sandy soil, 20 yards of loam, 17.2 yards of the heavy soils. When the shovellers do less than this, there is some mismanagement.

Assuming these as fair quantities, then, at \$1 per day for labor, the actual cost to the contractor for shovelling per cubic yard measured in place, will be, for sandy soils, 4.167 cents; loam, 5 cents; heavy soils, clays, &c, 5.81 cents.

In practice, the carts are not usually loaded to any less extent with the heavier soils than with the lighter ones. Nor, indeed, is there any necessity for so doing, inasmuch as the difference of weight of a cart and one-third of a cubic yard of the various soils is too slight to need any attention; especially when the cart-road is kept in good order, as it will be by any contractor who understands his

own interest. Neither is it necessary to modify the load on account of any *slight inclinations* which may occur in the grading of roads. An earth cart weighs by itself about ½ a ton.

Art. 4. Hauling away the earth; dumping, or emptying; and returning to reload. The average speed of horses in hauling is about 2¼ miles per hour, or 200 feet per minute, which is equal to 100 feet of trip each way; or to 100 feet of *lead*, as the distance to which *the earth is hauled* is technically called * Besides this, there is a loss of about four minutes in every trip, whether long or short, in waiting to load, dumping, turning, &c. Hence, every trip will occupy as many minutes as there are lengths of 100 feet each in the lead; and four minutes besides. Therefore, to find the number of trips per day over any given average lead, we divide the number of minutes in a working day by the sum of 4 added to the number of 100 feet lengths contained in the distance to which the earth has to be removed, that is,

$$\frac{\textit{The number (600) of minutes in a working day}}{4+\textit{the number of 100 feet lengths in the lead}} = \textit{the number of trips, or loads removed per day, per cart.}$$

And since ⅓ of a cubic yard measured before being loosened, makes an average cart-load, the number of loads, divided by 3, will give the number of cubic yards removed per day by each cart; and the cubic yards divided into the total expense of a cart per day, will give the cost per cubic yard for hauling.

REMARK. When removing loose *rock*, which requires more time for loading, say,

$$\frac{\textit{No. of minutes (600) in a working day}}{6+\textit{No. of 100-feet lengths of lead}} = \textit{No. of loads removed, per day, per cart.}$$

In leads of ordinary length one driver can attend to 4 carts; which, at $1 per day, is 25 cents per cart. When labor is $1 per day, the expense of a horse is usually about 75 cents, and that of the cart, including harness, tar, repairs, &c, 25 cents, making the total daily cost per cart $1.25. The expense of the horse is the same on Sundays and on rainy days, as when at work; and this consideration is included in the 75 cents. Some contractors employ a greater number of drivers, who also help to load the carts, so that the expense is about the same in either case

EXAMPLE How many cubic yards of loam, measured in the cut, can be hauled by a horse and cart in a day of 10 working hours, (600 minutes,) the lead, or length of haul of earth being 1000 feet, (or 10 lengths of 100 feet,) and what will be the expense to the contractor for hauling, per cubic yard, assuming the total cost of cart, horse, and driver, at $1.25?

$$\textit{Here,}\ \frac{600\ \textit{minutes}}{4+10\ \textit{lengths of 100 feet,}} = \frac{600}{14} = 43\ \textit{loads.} \quad \textit{And}\ \frac{43\ \textit{loads}}{3} = 14.3\ \textit{cubic yards.}$$

$$\textit{And}\ \frac{125\ \textit{cents}}{14.3\ \textit{cub yds}} = 8.74\ \textit{cents per cubic yard.}$$

In this manner the 2d and 3d columns of the following tables have been calculated.

Art. 5. Spreading, or levelling off the earth into regular thin layers on the embankment. A bankman will spread from 50 to 100 cubic yards of either common loam, or any of the heavier soils, clays, &c, depending on their dryness This, at $1 per day, is 1 to 2 cents per cubic yard; and we may assume 1½ cents as a fair average for such soils; while 1 cent will suffice for light sandy soils.

This expense for spreading is saved when the earth is either dumped over the end of the embankment, or is wasted; still, about ¼ cent per yard should be allowed in either case for keeping the dumping-places clear and in order.

Art. 6. Keeping the cart-road in good order for hauling. No ruts or puddles should be allowed to remain unfilled; rain should at once be led off by shallow ditches; and the road be carefully kept in good order; otherwise the labor of the horses, and the wear of carts, will be *very greatly* increased. It is usual to allow so much per cubic yard for road repairs; but we suggest so much per cubic yard, per 100 feet of lead; say $\frac{1}{10}$ of a cent.

Art. 7. Wear, sharpening, and depreciation of picks and shovels. Experience shows that about ¼ of a cent per cubic yard will cover this item.

Superintendence and water-carriers. These expenses will vary with local circumstances; but we agree with Mr. Morris, that 1½ cents per cubic yard will, under ordinary circumstances, cover both of them. An allowance of about ½ cent may in justice be added for extra trouble in digging the side-ditches; levelling off the bottom of the cut to grade, and general trimming up. In very light cuttings this may be increased to ½ cent per cubic yard

At ½ cent, all the items in this article amount to 2 cents per cubic yard of cut.

Art. 8. Profit to the contractor. This may generally be set down at from 6 to 15 per cent, according to the magnitude of the work, the risks incurred, and various incidental circumstances. Out of this item the contractor generally has to pay clerks, storekeepers, and other agents, as well as the expenses of shanties, &c; although these are in most cases repaid by the profits of the stores; and by the rates of boarding and lodging paid to the contractors by the laborers.

Art. 9. A knowledge of the foregoing items enables us to calculate with tolerable accuracy the cost of removing earth. For example, let it be required to ascertain the cost per cubic yard of excavating common loam, measured in place; and of removing it into embankment, with an average haul or lead of 1000 feet; the wages of laborers being $1 per day of 10 working hours; a horse 75 cts a day; and a cart 25 cts. One driver to four carts

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* When an entire cut is made into an embankment, the **mean haul** is the dist between centers of gravity of the cut and embkt.

		Cents.
Here we have cost of loosening, say by pick,	*Art* 2, *per cubic yard, say,*	2 50
Loading into carts,	*Art* 3, " "	5 00
Hauling 1000 *feet, as calculated previously in example, Art.* 4, "		8 74
Spreading into layers,	*Art* 5, "	1 50
Keeping cart-road in repair,	*Art* 6, 10 *lengths of* 100 *ft,*	1 00
Various items in	*Art* 7,	2 00
	Total cost to contractor,	20 74
	Add contractor's profit, say 10 *per cent,*	2 074
	Total cost per cubic yard to the company,	22.814

It is easy to construct a table like the following, of costs per cubic yard, for different lengths of lead. Columns 2 and 3 are first obtained by the Rule in Article 4, then to each amount in column 3 is added the *variable* quantity of $\frac{1}{10}$ of a cent for every 100 feet length of lead, for keeping the road in order, and the *constant* quantity (for any given kind of soil) composed of the prices per cubic yard, for loosening, loading, spreading, or wasting, &c, either taken from the preceding articles, or modified to suit particular circumstances. In this manner the tables have been prepared.

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted	Ploughed and Spread	Ploughed and Wasted	Picked and Spread	Picked and Wasted	Ploughed and Spread	Ploughed and Wasted.
Feet.	Cu Yds.	Cts	Cts	Cts	Cts	Cts.	Cts	Cts.	Cts	Cts.
25	47 0	2 66	13.69	12 44	11.99	10.74	16 00	14.75	13 50	12 25
50	44 4	2 81	13 86	12.61	12.16	10 91	16.17	14 92	13 67	12 42
75	42.1	2 97	14 05	12 80	12.35	11 10	16.36	15 11	13 86	12.61
100	40 0	3 12	14 22	12.97	12 52	11 27	16 53	15 28	14.03	12 78
150	36 4	3 43	14 58	13 33	12.88	11 63	16 89	15 64	14 39	13 14
200	33 3	3 75	14 95	13 70	13 25	12 00	17 26	16 01	14 76	13 51
300	28 6	4.37	15 67	14 42	13 97	12 72	17 98	16 73	15 48	14.23
400	25 0	5.00	16 40	15 15	14.70	13.45	18 71	17 46	16 21	14 96
500	22.2	5.63	17.13	15 88	15.43	14 18	19 44	18 19	16 94	15 69
600	20.0	6 25	17.85	16 60	16.15	14.90	20 16	18 91	17 66	16 41
700	18 2	6 87	18 57	17 32	16 87	15 62	20 88	19 63	18 38	17 13
800	16 7	7 48	19.28	18.03	17 58	16 33	21 59	20 34	19 09	17 84
900	15 4	8 12	19.92	18 67	18 22	16 97	22 23	20 98	19 73	18 48
1000	14 3	8.74	20.74	19 49	19 04	17.79	23.05	21 80	20 55	19 80
1100	13 3	9.40	21 50	20 25	19 80	18 55	23.81	22 56	21 31	20 06
1200	12 5	10.0	22 20	20 95	20 50	19 25	24 51	23 26	22 01	20 76
1300	11.8	10.6	22 90	21.65	21 20	19.95	25.21	23 96	22.71	21.46
1400	11.1	11 2	23 60	22 35	21 90	20 65	25 91	24.66	23.41	22.16
1500	10 5	11.9	24 40	23.15	23 70	21.45	26 71	25 46	24 21	22.96
1600	10 0	12.5	25 10	23 85	23 40	22.15	27 41	26 16	24 91	23 66
1700	9 52	13 1	25 80	24 55	24 10	22 85	28.11	26 86	25 61	24 36
1800	9 09	13.7	26 50	25 25	24.80	23 55	28.81	27.56	26 31	25.06
1900	8 70	14.4	27 30	26 05	25 60	24 35	29.61	28 36	27 11	25.86
2000	8.33	15.0	28.00	26 75	26.30	25 05	30 31	29.06	27 81	26.56
2250	7 54	16.6	29.85	28 60	28.15	26 90	32.16	30 91	29 66	28.41
2500	6 90	18.1	31.00	30 35	29 90	28 65	33 91	32 66	31 41	30.16
½ mile	6 58	19.0	32.64	31 39	30.94	29.69	34.95	33 70	32 45	31.20
3000	5.88	21.2	35.20	33 95	33 50	32 25	37 51	36 26	35 01	33 76
3250	5 48	22 8	37 05	35 80	35.35	34 10	39 36	38 11	36 86	35.61
3500	5.13	24.3	38 80	37.55	37.10	35 85	41 11	39 86	38 61	37 36
3750	4 82	25 9	40.65	39.40	38 95	37 70	42 96	41.71	40.46	39 21
4000	4.54	27.5	42.50	41 25	40 80	39.55	44 81	43 56	42.31	41.06
4250	4.30	29.1	44.35	43.10	42 65	41 40	46 66	45 41	44.16	42.91
4500	4.06	30.6	46 10	44.85	44 40	43 15	48 41	47 16	45 91	44 66
4750	3.88	32.[illegible]	47 95	46 70	46 25	45 00	50 26	49 01	47.76	46.51
5000	3.70	33.8	49 80	48 55	48 10	46.85	52.11	50 86	49 61	48 36
1 mile	3.52	35.5	51.78	50 53	50.08	48.83	54.09	52 84	51.59	50.34
1¼ m.	2.86	43.8	61.40	60 15	59.70	58 45	63 71	62.46	61 21	59.96
1½ m.	2.40	52 1	71.02	69 77	69.32	68.07	73 33	72.08	70 83	69 58
1¾ m.	2.07	60 4	80.64	79 39	78 94	77.69	82 95	81 70	80 45	79.20
2 m.	1,82	68.7	90 26	89.01	88.56	87.31	92.57	91.32	90 07	88.82

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart	Cost per cubic yard in place, for hauling and emptying only.	Pure stiff Clay, or cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted	Ploughed and Spread	Ploughed and Wasted.
Feet.	Cu Yds.	Cts.	Cts	Cts	Cts.	Cts.	Cts.	Cts.	Cts	Cts.
25	47.0	2 66	19.00	17 75	14.50	13 25	11 52	10 77	10 25	9 50
50	44.4	2 81	19.17	17.92	14.67	13 42	11 69	10.94	10.42	9.67
75	42.1	2 97	19.36	18.11	14 86	13 61	11 88	11.13	10 61	9.86
100	40 0	3 12	19.53	18 28	15.03	13 78	12 05	11 30	10 78	10.03
150	36 4	3 43	19.89	18 64	15.39	14.14	12 41	11 66	11 14	10.39
200	33 3	3 75	20 26	19.01	15.76	14 51	12 78	12 03	11.51	10.76
300	28 6	4 37	20 98	19.73	15.48	15 23	13 50	12 75	12 23	11.48
400	25 0	5 00	21 71	20 46	17 21	15 96	14 23	13 48	12 46	12 21
500	22.2	5 63	22 44	21 19	17.94	16 69	14 96	14 21	13 69	12.94
600	20.0	6 25	23.16	21 91	18.66	17 41	15 68	14 93	14 41	13 66
700	18.2	6 87	23 88	22.63	19 38	18 13	16 40	15 65	15 13	14.38
800	16 7	7 48	24.59	23 34	20 09	18 84	17.11	16 36	15.84	15 09
900	15 4	8.12	25 23	23 98	20.73	19 48	17 75	17 00	16 48	15.73
1000	14 3	8 74	26 05	24 80	21.55	20 30	18 57	17.82	17.30	16.55
1100	13 3	9 40	26 81	25 56	22 31	21 06	19 33	18.58	18 06	17.31
1200	12 5	10 0	27 51	26 26	23 01	21.76	20 03	19 28	18 76	18.01
1300	11 8	10 6	28 21	26 96	23 71	22 46	20 73	19 98	19 46	18.71
1400	11.1	11 2	28.91	27 66	24.41	23 16	21 43	20 68	20 16	19.41
1500	10.5	11 9	29.71	28 46	25 21	23 96	22.23	21 48	20 96	20.21
1600	10 0	12 5	30.41	29 16	25.91	24 66	22 93	22.18	21.66	20.91
1700	9 52	13 1	31.11	29 86	26 61	25 36	23 63	22 88	22.36	21.61
1800	9 09	13 7	31.81	30 56	27 31	26 06	24 33	23.58	23.06	22.31
1900	8.70	14 4	32 61	31.36	28.11	26 86	25 13	24.38	23 86	23.11
2000	8 33	15 0	33.31	32 06	28 81	27 56	25 83	25 08	24.56	23.81
2250	7 34	16 6	35 16	33 91	30 63	29.41	27.68	26 93	26 41	25.66
2500	6 90	18 1	36.91	35 66	32 41	31.16	29 43	28 68	28.16	27.41
½ mile	6 58	19.0	37 95	36 70	33 45	32 20	30 47	29.72	29 20	28.45
3000	5 88	21 2	40 51	39 26	36.01	34 76	33 03	32.28	31.76	31.01
3250	5 48	22 8	42 36	41 11	37.86	36 61	34 88	34 13	33.61	32.86
3500	5.13	24 3	44.11	42.86	39.61	38.36	36 63	35.[illegible]8	33 36	34.61
3750	4 82	25 9	45 96	44 71	41.46	40.21	38.48	37.73	37 21	36.46
4000	4 54	27 5	47.81	46 56	43.31	42.06	40 33	39.58	39 06	38.31
4250	4.30	29.1	49.66	48 41	45.16	43.91	42 18	41.45	40 93	40 18
4500	4 08	30.6	51 41	50 16	46 91	45.66	43 93	43.18	42 66	41.91
4750	3 88	32.2	53 26	52.01	48 76	47 51	45.78	45 03	44 51	43.76
5000	3.70	33 8	55.11	53 86	50 61	49.36	47 63	46 88	46 36	45.61
1 mile	3 52	35 5	57.09	55 84	52 59	51 34	49 61	48 86	48 34	47.59
1¼ m.	2 86	43 8	66 91	65 46	62.21	60.96	59 23	58 48	57 96	57.21
1½ m.	2 40	52 1	76 33	75 08	71 83	70.58	68 85	68 10	67 58	66.83
1¾ m.	2 07	60 4	85 95	84 70	81 45	80 20	78 47	77.72	77 20	76.45
2 m.	1 82	68.7	95.57	94 32	91.07	89 82	88 09	87 34	86 82	86.07

Art. 10. By wheelbarrows. The cost by barrows may be estimated in the same manner as by carts See Articles 1, &c. Men in wheeling move at about the same average rate as horses do in hauling, that is, 2½ miles an hour, or 200 feet per minute, or . minute per every 100-feet length of lead. The time occupied in loading, emptying, &c (when, as is usual, the wheeler loads his own barrow,) is about 1 25 minutes, without regard to length of lead besides which, the time lost in occasional short rests, in adjusting the wheeling-plank, and in other incidental causes, amounts to about $\frac{1}{10}$ part of his whole time; so that we must in practice consider him as actually working but 9 hours out of his 10 working ones. Therefore

$$\frac{\textit{The number of minutes in a working day} \times .9}{1.25 + \textit{the number of 100-feet lengths of lead}} = \textit{the number of trips or of loads removed per day per barrow.}$$

See Remark, next page.
The number of loads divided by 14 will give the number of cub yards, since a cub yard, measured in place, averages about 14 loads And the cost of a wheeler and barrow per day, (say $1 per man, and 5 cents per barrow,) divided by the number of cub yards, will give the cost per yard for loading wheeling, and emptying.

Ex. How many cubic yards of common loam, measured in place, will one man load, wheel, and empty, per day of 10 working hours, (or 600 minutes;) the lead, or distance to which the earth is removed being 1000 feet, (or 10 lengths of 100 feet;) and what will be the expense per yard, supposing the laborer and barrow to cost \$1.05 per day?

$$\textit{Here, } \frac{600 \textit{ minutes} \times .9}{1.25 + 10 \textit{ lengths}} = \frac{540}{11.25} = 48 \textit{ trips, or loads per day.}$$

$$\textit{And } \frac{48}{14} = 3.43 \textit{ cub yds per day.} \quad \textit{And } \frac{105 \textit{ cents}}{3.43 \textit{ cub yds}} = 30.6 \textit{ cents}$$

per cub yard for loading, wheeling away, emptying, and returning. This would be increased almost inappreciably by the cost of the shovel, which, in the following tables, however, is included in the cost of tools.

Rem. For *rock*, which requires more time for loading, say

$$\frac{\textit{No of minutes in a working day} \times .9}{1.6 + \textit{No of 100-feet lengths of lead}} = \textit{No of loads removed per day, per barrow.}$$

Art. 11. The following tables are calculated as in the case of carts, by first finding columns 2 and 3 by means of the Rule in Art 10, and then adding to each sum in column 3, the variable quantity of .1 of a cent per cubic yard per 100 feet of lead for keeping the wheeling-planks in order, and the prices of loosening, spreading, superintendence, water-carrying, &c, per cubic yard, as given in the preceding Articles 2, 5 and 7.

By Wheelbarrows. Labor \$1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled, in feet.	Number of cubic yards in place, loaded, and wheeled per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong, Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts	Cts.
25	25.7	4 09	10.12	8.87	8.42	7.17	11.62	10.37	9.12	7.87
50	22.1	4.75	10.80	9 55	9.10	7 85	12.30	11.05	9.80	8.55
75	19.3	5.44	11.52	10 27	9 82	8 57	13.02	11.77	10.52	9.27
100	17.1	6 14	12.24	10.99	10.54	9 29	13.74	12.49	11.24	9.99
150	14.0	7.50	13.65	12 40	11.95	10 70	15.15	13.90	12.65	11.40
200	11.9	8.82	15.02	13 77	13.32	12.07	16.52	15.27	14 02	12.77
250	10.3	10 2	16.45	15.20	14 75	13.50	17.95	16 70	15 45	14.20
300	9.07	11.6	17.90	16.65	16.20	14.95	19.40	18.15	16 90	15.65
350	8.14	12 9	19.25	18.00	17 55	16 30	20 75	19 50	18 25	17.00
400	7.36	14 3	20.70	19.45	19.00	17.75	22.20	20.95	19 70	18.45
450	6.71	15.6	22.05	20.80	20 35	19.10	23.55	22.30	21.05	19.80
500	6.17	17.0	23.50	22 25	21 80	20.55	25 00	23 75	22.50	21.25
600	5.32	19 7	26.30	25 05	24.60	23 35	27.80	26.55	25 30	24.05
700	4 67	22 5	29.20	27 95	27.50	26.25	30 70	29.45	28.20	26.95
800	4.17	25.2	32 00	30.75	30.30	29.05	33.50	32 25	31.00	29.75
900	3 76	27.9	34.80	33 55	33.10	31.85	36.30	35.05	33.80	32.55
1000	3 43	30.6	37.60	36 35	35.90	34.65	39 10	37.85	36.60	35.35
1200	2.91	36.1	43.30	42.05	41.60	40.35	44.80	43.55	42 30	41.05
1400	2.53	41.5	48.90	47 65	47.20	45.95	50.40	49.15	47.90	46.65
1600	2.24	46.9	54 50	53.45	52.80	51.55	56.00	54 75	53.50	52.25
1800	2.00	52.5	60.30	59.05	58.60	57.35	61.80	60 55	59 30	58 05
2000	1.81	58.0	66 00	64.75	64 30	63 05	67.50	66.25	65.00	63.75
2200	1.66	63.3	71.50	70.25	69.80	68.55	73 00	71.75	70.50	69.25
2400	1.53	68.6	77.00	75 75	75.30	74.05	78.50	77.25	76.00	74.75
½ mile.	1.39	75.5	84.14	82.89	82.44	81.19	85.64	84.39	83.14	81.89

By Wheelbarrows. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled	Number of cubic yards in place, loaded, and wheeled, per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Pure Stiff Clay, or Cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	14.62	13.37	10.12	8.87	8.79	8.04	7.52	6.77
50	22.1	4.75	15.30	14.05	10.80	9.55	9.47	8.72	8.20	7.45
75	19.3	5.44	16.02	14.77	11.52	10.27	10.19	9.44	8.92	8.17
100	17.1	6.14	16.74	15.49	12.24	10.99	10.91	10.16	9.64	8.89
150	14.0	7.50	18.15	16.90	13.65	12.40	12.32	11.57	11.05	10.80
200	11.9	8.82	19.52	18.27	15.02	13.77	13.69	12.94	12.42	11.67
250	10.3	10.2	20.95	19.70	16.45	15.20	15.12	14.37	13.85	13.10
300	9.07	11.6	22.40	21.15	17.90	16.65	16.57	15.82	15.30	14.55
350	8.14	12.9	23.75	22.50	19.25	18.00	17.92	17.17	16.65	15.90
400	7.36	14.8	25.20	23.95	20.70	19.45	19.37	18.62	18.10	17.35
450	6.71	15.6	26.55	25.30	22.05	20.80	20.72	19.97	19.45	18.70
500	6.17	17.0	28.00	26.75	23.50	22.25	22.17	21.42	20.90	20.15
600	5.32	19.7	30.80	29.55	26.30	25.05	24.97	24.22	23.70	22.95
700	4.67	22.5	33.70	32.45	29.20	27.95	27.87	27.12	26.60	25.85
800	4.17	25.2	36.50	35.25	32.00	30.75	30.67	29.92	29.40	28.65
900	3.76	27.9	39.30	38.05	34.80	33.55	33.47	32.72	32.20	31.45
1000	3.43	30.6	42.10	40.85	37.60	36.35	36.27	35.52	35.00	34.25
1200	2.91	36.1	47.80	46.55	43.30	42.05	41.97	41.22	40.70	39.90
1400	2.53	41.5	53.40	52.15	48.90	47.65	47.57	46.82	46.30	45.55
1600	2.24	46.9	59.00	57.75	54.50	53.25	53.17	52.42	51.90	51.15
1800	2.00	52.5	64.80	63.55	60.30	59.05	58.97	58.22	57.70	56.95
2000	1.81	58.0	70.50	69.25	66.00	64.75	64.67	63.92	63.40	62.65
2200	1.66	63.3	76.00	74.75	71.50	70.25	70.17	69.42	68.90	68.15
2400	1.53	68.6	81.50	80.25	77.00	75.75	75.67	74.92	74.40	73.65
½ mile.	1.39	75.5	88.64	87.39	84.14	82.89	82.81	82.06	81.54	80.79

Art. 12. By wheeled scrapers and drag scrapers. The body of the wheeled scraper is a box of smooth sheet-steel about 3½ ft square by 15 ins deep, containing about ½ cubic yard of earth when "even full." The box is open in front (in some machines it is closed by an "end gate" when full), and can be raised and lowered, and revolved on a horizontal axis. To fill the box, it is lowered into, and held down in, the earth, while the team draws the machine forward. When full, it is raised to about a foot above ground; and, on reaching the dump, is unloaded by being overturned on its axis. All the movements of the box are made by means of levers, and without stopping the team, which thus travels constantly. The wheels have broad tires, to prevent them from cutting into the ground.

In the *drag* scraper the box, owing to the greater resistance to traction, is made much smaller. It contains about .15 to .25 cubic yard in place, and is always open in front. The operation of the drag scraper is similar to that of the wheeled scraper, except that the box, when filled, rests upon the ground and is dragged over it by the team.

Each scraper ("wheeled" or "drag") requires the constant use of a team of two horses with a driver. Besides, a number of men, depending on the shortness of the lead and the number of scrapers, are required in the pit and at the dump to load the scrapers (by holding the box down into the earth) and unload them (by tipping the box). Except in sand, or in very soft soil, it is economical to use a plow before scraping.

The severest work for the team is the filling of the box; and this occurs oftenest where the lead is shortest. Hence smaller scrapers are used on short than on long hauls. We base our calculations on the following loads:

For drag scrapers (used only on short hauls)	.2	cubic yard
For wheeled scrapers		
lead less than 100 feet	.33	"
" 100 to 300 feet	.4	"
" 400 to 500 feet	.5	"
" over 500 feet	.6	"

The daily expense per scraper, for driver's wages and the use of a 2-horse team, is about \$3.50 For loads of 400 feet and over, we add 50 cts per day for use of "snatch team" to help load the larger scrapers then used. One snatch team generally serves a number of scrapers.

Owing to the fact that the teams are constantly in motion without rest, they travel somewhat more slowly than with carts We take 150 ft per minute (or 75 ft of *lead* per minute) as an average.

In loading and unloading, the teams not only go out of their way in order to turn around, but travel more slowly than when simply hauling. To cover this we make an addition of 100 ft to each length of lead, whether long or short, for wheeled scrapers and for drag scrapers.

We add 1 cent per cubic yard for the cost of loading and dumping the scrapers; and estimate the approximate cost of the other items as follows·

Repairs of cart-road $\frac{1}{10}$ ct per cub yd in place for each 100 ft of lead

	Light Soils cts per cub yd in place	Heavy Soils cts per cub yd in place
Loosening		
by pick	*	5.
by plow	*	2.
Spreading	1.	1.5
Superintendence, wear and tear etc.	1	1.

We repeat that our figures are to be regarded merely as tolerable approximations, and subject to great variations according to skill of contractor and superintendent, strength of teams, character of material moved, state of weather etc etc.

$$\text{No. of trips per day per scraper} = \frac{\text{No. (600) of mins in a working day}}{\text{No. of 75 ft lengths in (lead + 100 ft)}}$$

$$\text{No. of cub yds in place moved per day by each scraper} = \text{No. of trips per day per scraper} \times \text{No. of cub yds in place, per scraper per trip}$$

$$\text{Cost per cub yd in place, for loading, hauling, dumping and returning} = \frac{\text{Daily expense of one scraper}}{\text{No. of cub yds in place, moved per day by each scraper}} + \text{1 ct for loading and dumping}$$

$$\text{Total cost per cubic yard in place exclusive of contractor's profit} = \text{Cost per cub yd in place, for loading, hauling, dumping, and returning} + \text{.1 ct per cub yd in place for each 100 ft of lead, for repairs of road} + \text{Cost, per cub yd in place, of loosening, spreading or wasting, and superintendence \&c.}$$

By Wheeled Scrapers. Labor \$1 per day of 10 working hours.

(a)	(b)	(c)	(d)					
Length of lead, or distance to which earth is hauled.	Quantity in place, hauled per day by each scraper.	Cost per cub yd in place for loading, hauling, dumping, and returning.	Total cost per cubic yard, in place, exclusive of contractor's profit					
			Light Soils		Heavy Soils			
			Spread	Wasted	Picked and Spread	Picked and Wasted	Plowed and Spread	Plowed and Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
50	100	4.5	6.6	5.6	12.1	10.6	9.1	7.6
100	90	4.9	7.0	6.0	12.5	11.0	9.5	8.0
150	70	6.0	8.2	7.2	13.7	12.2	10.7	9.2
200	60	6.9	9.1	8.1	14.6	13.1	11.6	10.1
300	45	8.8	11.1	10.1	16.6	15.1	13.6	12.1
400	45	9.9	12.3	11.3	17.8	16.3	14.8	13.3
600	38	11.5	14.1	13.1	19.6	18.1	16.6	15.1
800	30	14.3	17.1	16.1	22.6	21.1	19.6	18.1
1000	24	17.7	20.7	19.7	26.2	24.7	23.2	21.7

* Light soils can generally be advantageously loosened by the scrapers themselves in the act of loading.

By Drag Scrapers. Labor $1 per day of 10 working hours.

(a)	(b)	(c)	(d)					
Length of lead, or distance to which earth is hauled.	Quantity in place, hauled per day by each scraper.	Cost per cubic yard in place for loading, hauling, dumping, and returning.	Total cost per cubic yard in place, exclusive of contractor's profit.					
			Light Soils		Heavy Soils			
					Picked and		Plowed and	
			Spread	Wasted	Spread	Wasted	Spread	Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
50	60	6.9	9.0	8.0	14.5	13.0	11.5	10.0
75	50	8.0	10.1	9.1	15.6	14.1	12.6	11.1
100	45	8.8	10.9	9.9	16.4	14.9	13.4	11.9
150	36	10.8	13.0	12.0	18.5	17.0	15.5	14.0
200	30	12.7	14.9	13.9	20.4	18.9	17.4	15.9

Art. 13. By cars and locomotive, on level track. We have based our calculations upon the following assumptions: Trains of 10 cars, each car containing $1\frac{1}{2}$ cubic yards of earth measured in place. Average speed of trains, including starting and stopping, but not standing, 10 miles per hour, = 5 miles of *lead* per hour. Labor $1 per day of 10 working hours. Loosening, loading (by shovelers), spreading, wear &c of tools, superintendence, &c, the same as with carts, Arts 2, 3, 5, and 7. Loss of time in each trip for loading, unloading, &c, 9 minutes, = 0.15 hour. Therefore

$$\left.\begin{array}{l}\text{Number of trips per}\\ \text{day, per train}\end{array}\right\} = \frac{\text{The number (10) of hours in a working day}}{.15 + \text{the number of 5-mile lengths in the lead}}$$

$$\left.\begin{array}{l}\text{Number of cubic}\\ \text{yards in place, per}\\ \text{day per train}\end{array}\right\} = \begin{array}{l}\text{Number of}\\ \text{trips per day}\\ \text{per train}\end{array} \times \begin{array}{l}\text{Number (10)}\\ \text{of cars in a}\\ \text{train}\end{array} \times \begin{array}{l}\text{Number (1.5) of cubic}\\ \text{yards in place in each}\\ \text{car}\end{array}$$

$$\left.\begin{array}{l}\text{Cost per cubic yard, in place,}\\ \text{for hauling, dumping, and}\\ \text{returning}\end{array}\right\} = \frac{\text{One day's train expenses + 1 day's cost of track}}{\text{Number of cubic yards in place per day per train}}$$

One day's train expenses:

Cost of 10 cars @ $100	$1000		
" locomotive	3000		
		$4000	
One day's interest at 6 per cent, on cost of train			$0.67
Wages of engine driver (who fires his own engine)			2.00
" foreman at dump			2.00
" 3 men at dump at $1			3.00
Fuel			2.00
Water			1.00
Repairs of locomotive and cars			2.33
Total daily expense of one train			**$13.00**
Depreciation (life of rolling stock taken as 10 years) say $100 per annum per $1000, = $400 " " " train, = $ 4 " day (assuming 100 working days per year)			4.00
Daily expense and depreciation, one train			$17.00

Taking cost of track, laid, at $2500 per mile, and its life at 5 years, **the daily expense of track,** for interest, depreciation, handling and repairs, may be taken at $6.00 for each mile of lead.

Therefore

$$\left.\begin{array}{l}\text{Cost per cubic yard in place,}\\\text{for hauling, dumping, and}\\\text{returning}\end{array}\right\} = \frac{\$17 + (\$6 \text{ for each mile of lead})}{\begin{array}{l}\text{Number of}\\\text{trips per day}\\\text{per train}\end{array} \times \begin{array}{l}\text{Number (10)}\\\text{of cars in a}\\\text{train}\end{array} \times \begin{array}{l}\text{Number (1.5) of}\\\text{cubic yards}\\\text{each car}\end{array}}$$

$$\left.\begin{array}{l}\text{Total cost per cubic}\\\text{yard in place, ex-}\\\text{clusive of contrac-}\\\text{tor's profit}\end{array}\right\} = \begin{array}{l}\text{Cost per cub yd in}\\\text{place for hauling,}\\\text{dumping, and re-}\\\text{turning}\end{array} + \begin{array}{l}\text{Cost per cubic yard, in place, for}\\\text{loosening, loading, spreading or}\\\text{wasting, and superintendence, \&c}\\\text{(Arts 2, 3, 5, and 7.)}\end{array}$$

By Cars and Locomotive. Labor $1 per day of 10 working hours.

(a)	(b)	(c)	(d)							
			Total cost per cubic yard, in place, exclusive of contractor's profit							
			Light Soils.				Heavy Soils.			
Length of lead, or distance to which earth is hauled.	Quantity in place, hauled per day by each train.	Cost per cubic yard, in place, for hauling, dumping, and returning.	Picked and Spread	Picked and Wasted.	Plowed and Spread.	Plowed and Wasted.	Picked and Spread.	Picked and Wasted.	Plowed and Spread.	Plowed and Wasted.
Miles	Cu. yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
1/4	750	2.47	11.30	10.30	10.04	9.04	15.77	14.27	13 27	11.77
1/2	600	3.33	12.16	11.16	10.90	9.90	16.63	15.13	14.13	12.63
3/4	495	4.34	13.17	12.17	11.91	10.91	17.64	16.14	15.14	13.64
1	420	5.48	14.31	13.31	13.05	12.05	18 78	17.28	16 28	14.78
2	270	10.74	19 57	18.57	18.31	17 31	24.04	22.54	21 54	20.04
3	195	17.95	26 78	25.78	25 52	24 52	31.25	29.75	28 75	27.25
4	150	27 33	36.16	35.16	34.90	33 90	40.63	39.13	38.13	36.63

Additional Data of use in railroad earthwork may be found in "Hoisting, Conveying and Excavating Machinery", pp 579*y* to 581, incl.

especially the following in that article;—

"Grabs, Clam-Shells & Orange-Peels", sec 1.14,

"Dippers and Shovels", sec 1.15,

"Skimmers", sec 1.16,

"Pull-Scoops", sec 1.17,

"Scrapers", sec 1.18,

"Cableways", sec 4.0,

"Conveyors", sec 5.0,

"Vehicles", sec 6.0, and subdivisions, Wheelbarrows, Carts, Wagons, **Automobiles** (including **Trucks** and Tractors), Trailers, and Industrial Railways,

"Other Excavators", sec 7.0, and subdivisions, Plows, Rooters, Bulldozers, Graders, Elevating Graders, Trenchers and Ditchers, and Hydraulic Excavation, and

"Operation", sec 9.0 and subdivisions.

also

"Dredging", pp 581 to 581*x*, inclusiv.

Art. 14. Removing rock excavation by wheelbarrows. A cubic yard of hard rock, *in place*, or before being blasted, will weigh about 1.8 tons, if sandstone or conglomerate, (150 lbs per cubic foot;) or 2 tons if good compact granite, gneiss, limestone, or marble, (168 lbs per cubic foot.) So that, near enough for practice in the case before us, we may assume the weight of any of them to be about 1.9 tons, or 4256 lbs per cubic yard, in place; or 158 lbs per cubic foot.

Now, a solid **cubic yard, when broken up** by blasting for removal by wheelbarrows or carts, will occupy a space of about 1.8, or $1\frac{4}{5}$ cubic yards; whereas average earth, when loosened, swells to but about 1.2, or $1\frac{1}{5}$ of its original bulk in place; although, after being made into embankment, it eventually shrinks into less than its original bulk. In estimating for earth, it is assumed that $\frac{1}{14}$ cubic yard, in place, is a fair load for a wheelbarrow. Such a cubic yard will weigh on an average 2430 lbs, or 1.09 tons; therefore, $\frac{2430}{14} = 174$ lbs, is the weight of a barrow-load, of 2.31 cubic feet of loose earth. Assuming that a barrow of loose rock should weigh about the same as one of earth, we may take it at $\frac{1}{24}$ of a cubic yard; which gives $\frac{4256}{24} = 177$ lbs per load of loose rock, occupying 2 cubic feet of space.

In the following table, columns 2 and 3 are prepared on the same principle as for earth, as directed in Article 10. Column 4 is made up by adding to each amount in column 3, .2 of a cent for each 100 feet length of lead, for keeping the wheeling-planks in order; and 45 cents per cubic yard, in place, as the actual cost for loosening, including tools, drilling, powder, &c; as well as moderate drainage, and every ordinary contingency not embraced in column 3. Contractor's profits, of course, are not here included.

Ample experience shows that when labor is at $1 per day, the foregoing 45 cents per cubic yard, in place, is a sufficiently liberal allowance for loosening hard rock under all ordinary circumstances. In practice it will generally range between 30 and 60 cents, depending on the position of the strata, hardness, toughness, water, and other considerations. Soft shales, and other allied rocks, may frequently be loosened by pick and plough, as low as 15 to 20 cents; while, on the other hand, shallow cuttings of very tough rock, with an unfavorable position of strata, especially in the bottoms of excavations, may cost $1, or even considerably more. These, however, are exceptional cases, of comparatively rare occurrence. The quarrying of average hard rock requires about ¼ to ½ lb of powder per cubic yard, in place; but the nature of the rock, the position of the strata, &c. may increase it to ½ lb, or more. Soft rock frequently requires more powder than hard. A good churn-driller will drill 8 to 10 feet in depth, of holes about 2½ feet deep, and 2 inches diameter, per day, in average hard rock, at from 12 to 18 cents per foot. Drillers receive higher wages than common laborers.

Hard Rock, by Wheelbarrows.

Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place exclusive of profit to contractor	Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor
Feet.	Cubic Yds	Cents.	Cents	Feet.	Cubic Yds	Cents.	Cents.
25	12.2	8.64	51.7	600	2.96	35.5	81.7
50	10.7	9.81	54.9	700	2.62	40.1	86.5
75	9.58	11.0	56.2	800	2.34	44.8	91.4
100	8.66	12.1	57.3	900	2.12	49.5	96.3
150	7.26	14.5	59.8	1000	1.94	54.1	101.1
200	6.25	16.8	62.2	1200	1.65	63.6	115.0
250	5.49	19.1	64.6	1400	1.44	72.9	120.7
300	4.89	21.5	67.1	1600	1.28	82.2	130.4
350	4.41	23.8	69.5	1800	1.15	91.5	140.1
400	4.02	26.1	71.9	2000	1.04	100.8	149.8
450	3.69	28.5	74.4	2200	.953	110.2	159.6
500	3.41	30.8	76.8	2400	.879	119.5	169.3

Art. 15. Removing rock excavation by carts. A cart-load of rock may be taken at $\frac{1}{5}$ of a cubic yard, in place. This will weigh, on an average, 851 lbs; or but 41 lbs more than a cart-load of average soil. Since the cart itself will weigh about ½ a ton, the total loads are very nearly equal in both cases. Columns 2 and 3 of the following table are prepared on the same principle as for earth, as directed in Art. 4. Column 4 is made up by adding to each amount in column 3, the following items: For blasting, (and for everything except those in column 3; loading, and repairs of cart-road,) 45 cents per cubic yard, in place; for loading, 8 cents, per cubic yard, in place; and for repairs of road, .2, or $\frac{1}{5}$ of a cent for each 100-feet length of lead. Contractor's profit not included.

Hard Rock, by Carts.

Labor $1 per day, of 10 working hours.

Length of lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor	Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard in place, for hauling, and emptying	Total cost per cubic yard, in place, exclusive of profit to contractor
Feet.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents
25	19 2	6.51	59 6	1800	5 00	25 0	81.6
50	18 5	6 77	59 9	1900	4 80	26 0	82 8
75	17 8	7.03	60 2	2000	4.62	27 1	84.1
100	17.1	7.29	60 5	2250	4.21	29 7	87.2
150	16 0	7 81	61.1	2500	3.87	32.3	90.8
200	15 0	8 33	61 7	½ mile	3.70	33 7	92.0
300	13 3	9 37	63 0	3000	3 33	37 5	96.5
400	12 0	10 4	64 2	3250	3 12	40 1	99.6
500	10 9	11 5	65 5	3500	2 92	42 8	102.8
600	10 0	12.5	66 7	3750	2 76	45 3	105.8
700	9 23	13 6	68 0	4000	2 61	47.9	108.9
800	8 57	14 6	69 2	4250	2.47	50 6	112.1
900	8 00	15 6	70.4	4500	2 35	53 2	115.2
1000	7.50	16 7	71 7	4750	2.24	55 8	118.3
1100	7.06	17 7	72 9	5000	2.14	58 4	121.4
1200	6 67	18 7	74 1	1 mile	2 04	61.2	124.8
1300	6 32	19 8	75 4	1¼ "	1 67	75.0	141.2
1400	6 00	20 8	76 6	1½ "	1 41	88 8	157.6
1500	5.71	21 9	77 9	1¾ "	1 22	102 5	174.0
1600	5 45	22 9	79 1	2 "	1 08	116 3	190 4
1700	5 22	24 0	80 4	2¼ "	.962	130 0	206.8

"**Loose rock**" will cost about 30 cts per yd less; and even *solid* rock will *average* about 10 cts less than the tables.

Art. 16. Removing rock excavation by cars and locomotive, on level track; each car holding 1 cu yd of rock, instead of 1.5 cu yds of earth as in Art. 13. This makes hauling cost, for a given lead, 50 per cent greater than for earth. Loosening and loading costs are as in Art. 15; other assumptions as in Art. 13.

Hard Rock, by Cars and Locomotive.

Labor $1 per day of 10 working hours.

Lead, Miles.	Cu yds per day per train.	Hauling cost, cts per cu yd	Loosening, cts per cu yd	Loading, cts per cu yd	Total, cts per cu yd
¼	500	3.7	45	8	56.7
½	400	5.0	45	8	58.0
¾	333	6.4	45	8	59.4
1	286	8.0	45	8	61.0
2	182	16 0	45	8	69.0
3	133	26.2	45	8	79.2
4	105	39.0	45	8	92.0

TUNNELS.

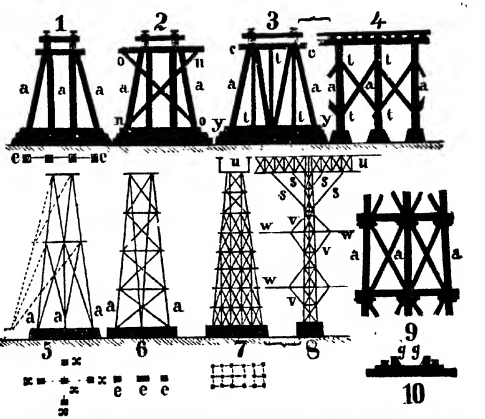
Tunnels for railroads should, if possible, be straight, especially when there is but a single track; inasmuch as collisions or other accidents in a tunnel would be peculiarly disastrous. A tunnel will rarely be expedient before the depth of cutting exceeds 60 feet. Firm rock of moderate hardness, and of a durable nature, is **the most favorable material** for a tunnel; especially if free from springs, and lying in horizontal strata. In soft rock, or in shales (even if hard and firm at first), or in earth, a lining of hard brick or masonry in cement, is necessary. A tunnel should have a **grade or inclination** in one direction, for ease of future drainage and ventilation. No special arrangement is essential for **ventilation** either during construction, or after, if the length does not exceed about 1000 feet; but beyond that, generally during construction either shafts are resorted to, or means provided for forcing air into the tunnel through pipes from its ends. But after the work is finished, except under peculiar circumstances, nothing of the kind is necessary. Shafts often draw air downwards; and frequently, even when aided by a steep, uniform grade, do not secure ventilation. The Mont Cenis tunnel under the Alps, completed in 1871, is 7½ miles long, and has no shafts, although it grades up from each end, which is the most unfavorable of all conditions for ventilation without shafts. It was made so for facilitating drainage. Its ventilation is maintained by air forced in from the ends. The Hoosac tunnel, Mass, 4¾ miles long, has shafts; one of them 1030 feet deep; but they were for expediting the work. **Shafts generally cost** from 1½ to 3 times as much per cubic yard as the main tunnel, owing to the greater difficulty of excavating and removing the material, and getting rid of the water, all of which must be done by hoisting. When through earth, they must be lined as well as the tunnel and the lining must usually be an under-pinning process. Or the lining may first be built *over* the intended shaft, and then sunk by undermining it gradually. Their sectional area commonly varies from about 40 to 100 square feet. They have the great advantage of expediting the work by increasing the number of points at which it can be carried on; but if placed too close together, their cost more than compensates for this. The air in some tunnels, while being constructed, is much more foul than in others; so that after the work has been commenced, shafts with forced air may be expedient where they were not anticipated. In excavating the tunnel itself, a **heading** or passage-way, 5 or 8 feet high, and 3 to 12 feet wide, is driven and maintained a short distance (10 to 100 feet, or more, according to the firmness of the material) in advance of the main work. In rock, the heading is just below the top of the tunnel, so that the men can conveniently drill holes in its floor for blasting; but in earth, the heading is driven along the bottom of the tunnel, that being the most convenient for enlarging the aperture to the full tunnel size, by undermining the earth, and letting it fall. In earth, the top and sides of the heading, as well as of the tunnel, must be carefully prevented from caving in before the lining is built; and this is done by means of rows of vertical rough timber props, and horizontal caps or overhead pieces, between which and the earth rough boards are placed to form temporary supporting sides and ceiling to the excavation. The props and caps are placed first; and the boards are then driven in between them and the earthen sides of the excavation. These are gradually removed as the lining is carried forward **The lining,** when of brick, is usually from 2 to 3 bricks thick (17 to 26 inches) at bottom, and from 1½ to 2½ bricks thick at top; and when of rough rubble in cement, about half again as thick. It is important that the bricks or stone should be of excellent hard quality, and laid in good cement. The bricks should be moulded to the shape of the arch. As the lining is finished in short lengths, and before the centers are removed, any **cavities or voids** between it and the earth should be carefully and compactly filled up. Even in rock, if much fissured, or if not of durable character, as common shale, lining is necessary. **The cross-section** of a single-track railroad tunnel, in the clear of everything, and for cars of 11 feet extreme width, should not be *less* than about 15 feet wide, by 18 feet high; nor a double-track one, less than 27 feet wide, by 24 feet high; unless in the last case the material is firm rock, in which a high arch is not necessary for lining. The roof may then be much flatter, so that a height of 20 feet may answer. With cars of 10 feet extreme width, the width of the tunnel may be reduced to 25 feet; or with 9 feet cars, to 23 feet. Many have been made 22 feet. The Mont Cenis is 26 feet wide, by 25 high. **The rate of daily progress** from *each* face of a tunnel varies from 18 inches to 9 feet of length per 24 hours, with three relays of workmen. On the Mont Cenis the ex-

tremes were about 4 to 9 feet daily for a whole year, from each face. Drills worked by compressed air were employed in the headings, which were 12 feet wide by 8 feet high. Ordinarily, from $1\frac{1}{2}$ to 3 feet may be taken as averages. The difference of rate of progress between a single and a double track tunnel is not so great as might be supposed; inasmuch as a larger force can be employed on the wider one. If the tunnel is in earth, the construction of the lining about makes up for the slower excavation of one in rock. In rock, with labor at $1 per day, **the cost** will usually vary with the character of the rock, from $2 to $5 per cubic yard for the main tunnel; and from $3 to $10 for the heading; while shafts will average about 50 per cent. more than heading. The cost of a single-track tunnel, when common labor is $1 per day, will generally range between $30 and $75 per foot of length. Tunnel work, however, is liable to serious contingencies which cannot be foreseen. Since the sides and roof are rough as blasted. the width and height should each be estimated to the contractor as about 18 inches or 2 feet greater than the established *clear* ones. At any rate, the mode of measurement should be clearly stated in the specifications for the work. When a tunnel is made with a uniform grade, the work generally progresses in a more satisfactory manner from the lower end, because the descent favors the drainage of the spring water that is usually met with; whereas, at the upper end, it must be removed by pumps or by bailing. The upper end has, however, the advantage of sooner getting rid of the smoke in blasting. Before commencing a tunnel, or even deciding upon one, **trial shafts** should be sunk to ascertain the nature of the material. In long ones, the greatest care and accuracy are necessary for preserving the line of direction, so that the work from both ends shall meet properly at the center.

In the heading of the Vosburg (Pa) tunnel of the Lehigh Valley R R, built 1884, cross-section $7\frac{1}{2}$ feet × 26 feet, the **average progress** per working day of 24 hours with two shifts of 12 hours each, was as follows; by hand drilling 2.8 feet and 2.4 feet respectively from each end; by machine drills (two rival drills in competition) 5.6 feet and 7.8 feet. The material was hard gray sandstone. For the whole tunnel the rate was about 2 feet per day.

For further information respecting tunnels, the reader is referred to Mr. H S. Drinker's very full treatise on the subject, published by the Messrs Wiley.

TRESTLES.



FIGS 1, 2. 3, 5, 6, 7, are **elevations of trestles**; taken across the track or roadway. We may consider Fig 1 as adapted to a height of about 10 to 20 ft; Figs 2

and 8, to heights from 20 to 30 ft; Fig 5, from 30 to 40 ft, Fig 6, from 40 to 60 ft, as rough approximations merely. A single framework, such as that shown in each of these six figures, is called a "bent." These bents of course admit of many modifications. They are usually supported by bases of masonry, as in the figures. These preserve the lower timbers from contact with the earth, which would hasten their decay. It is advisable to make these bases high enough to prevent injury from cattle, or passing vehicles, &c. Up to heights of about 40 or 50 ft, a single row of posts or uprights, *a*, *a*, *a*, Figs 1 to 9, as shown at *e e* under Figs 1 and 6, will answer. But as the height becomes greater, more posts should be introduced, as shown at *x x* under Fig 5; or two entire rows of them; or three rows, as under Fig 7; and as also in Fig 8, which is an end view of Fig 7. Figs 7 and 8 bear much resemblance to the trestles 190 ft high, with masonry bases 30 ft high (S. Seymour, C. E.), which carried the Erie Rway (now the N Y, Lake Erie & West'n R R) over the **Genesee River at Portage, N Y.** There each bent had 21 posts 14 ins square, at its base, and 15 posts of 12 × 12, at its top. The other timbers were 6 × 12; many of them were in pairs, embracing the posts. This single-track viaduct was begun July 1, 1851, and completed Aug. 14, 1852. It contained 1,602,000 ft (B M) of timber, and 108,862 lbs of iron. In the foundations were 9200 cub yds of masonry. The entire cost was about $140,000. It was burned down in 1875, and was replaced, in less than 3 mos, with a single-track **viaduct of wrought-iron trestles,** containing, in all, 1,340,000 lbs of iron, and 130,600 ft (B M) of timber; and costing, complete, above the masonry, about $95,000. Frequently the posts of trestles are in pairs; and the other timbers pass between; all bolted together.

In Fig 4, the posts *a*, *a*, *a*, are end views of three trestles or bents, such as Fig 3; and *t t* are diag braces extending from trestle to trestle: the two outer ones inclining in one direction; and the central one crossing them. These may be placed either intermediate of the posts, as in Fig 3; with the heads of the two outer ones confined to the cap *c c* of one trestle; and their feet to the sill *y y* of the next one, or they may all be spiked or bolted to the posts themselves, as in Fig 4. The last is the best, as it serves also directly to stiffen the posts: as do also the braces *o o*, *n n*, Fig 2. Such bracing is too frequently omitted. During the passage of trains, the backward pressure of the steam, exerted through the driving wheels against the track, produces a serious strain lengthwise of the road, and tending to upset the trestles; and the sudden application of brakes to a moving train, produces a similar strain in the opposite direction. These strains become more dangerous as the ht increases. Hence the need for such braces. Usually the outer **posts may lean** 1.5 to 2.5 ins to a ft.

The posts should not be less than about 12 ins square, except in quite low trestles; and even then not less than about 10 × 10 The diag bracing may generally be about as wide as the posts; and half as thick. The dist apart of the bents, when the road way is supported by simple longitudinal beams, should not exceed 10 or 12 ft, for railroads. But if these beams receive support from braces beneath, like *s s*, Fig 8; or from iron truss rods, the dist may be extended to 15 or 20 or more ft. But when the trestles become very high, and contain a great deal of timber, it becomes cheaper to place them farther apart, say 30 to 60 ft; and to carry the railway upon regular framed trusses, as at *u u*, Figs 7 and 8; as in a bridge with stone piers. In the Genesee viaduct, the trestles were 50 ft apart, center to center

When such a trestle as Fig 8 becomes very narrow in proportion to its height, we may add to its stability by introducing beams *w*, extending from trestle to trestle and still further by inserting diag braces *v v*, as in the old Genesee viaduct.

As far as practicable, arrange the pieces so that any one may be removed if it becomes decayed; and another put in its place.

On curves, additional strength should be given on the convex side; as suggested by the dotted lines in Fig 5. On very high trestles especially (as well as on bridges), wheel-guards, *g g*, Fig 10, either inside or outside of the rails, should never be omitted.

In marshy ground, piles may be driven to support the trestles; or may be left so far above ground, as themselves to constitute the posts. Such trestles may often be used advantageously, even when to be afterward filled in by embkt. They then sustain the rails at their proper level until the embkt has reached its final settlement.

They are generally used to avoid the expense of embkt; especially when earth can only be obtained from a great dist. Even when earth and timber are equally convenient, they will rarely much exceed about half the cost of embkt; even when but about 30 ft high; but owing to their liability to decay, they should be resorted to only in case of necessity; or as a temporary expedient.

Bridges. See Trusses, p 689, etc, and Stone Bridges, (Arches), p 613, etc.

ROLLING STOCK

GENERAL

1. General features, and details common to both locomotivs and cars. Under this head we aim to give only such general information as may be serviceable to *civil* engineers, who are concernd chiefly with the *fixt* plant of railways.

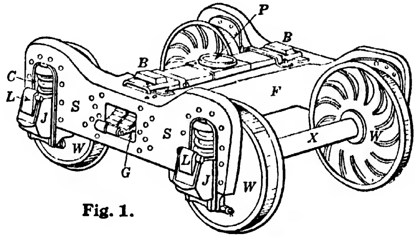


Fig. 1.

2. Trucks. A truck, Fig 1, is a frame into which the journal boxes, *C*, of the axles are fitted, and which supports the car body. In the U. S., a truck is seldom used singly, supporting the vehicle by direct and rigid attachment; but is almost invariably connected with the car or locomotiv by means of a pivot, *P*, which enables it to adapt itself to changes in direction of track. Trucks, especially the side pieces, *SS*, were formerly made up of heavy steel flat bars, forming a simple truss, and many are still in use. On newer equipment, "pressed" steel side frames are largely used (which act as girders) and also cast steel side frames, being patternd after the older flat bar trussed frames.

3. The pivot, *P*, is at the center of the truck, except in special types of locomotiv trucks, and in some electric car trucks where only one motor is used on one pair of wheels and where the pivot is placed more nearly over the driving wheels, to give better "adhesion."

4. Side-bearings. The load is usually carried by bearings at the pivot, and only those loads due to the side swing of the car are taken by side-bearings, *B, B*; but in some types, now in use, all the loads are carried by the side bearings, while the pivot takes none.

5. Springs. Elliptic, flat, or coild springs are provided between the main truck frame, *F*, and the bolster, *B P B*, which carries the pivot bearing. For passenger service, at least, a coild spring is placed also between each journal box and the truck frame. Coil springs are built in nests of two or more, one within the other, and these nests are also used in pairs, side by side, or in groups of four or more, especially where used between the truck frame and the car body. Semi-elliptical springs are usual on locomotivs.

6. Journal Boxes. The journals turn in the journal boxes. The bearing is of course upon the upper side of the journal. Each box is large enough to contain a supply of oild waste for lubrication. A lid, *L*, permits access, and keeps out much dirt. Roller bearings have been used in electric traction work.

7. Journals. Under cars, and in the axles of two-wheeld locomotiv trailer trucks, the journals are at the ends of the axles, outside the wheels; in loco driving wheel axles, between the driving wheels. Journals range from 3¾ to 6 ins in diam, and from 7 to 11 ins in length. Trailer truck journals are as large as 8 × 14 ins.

Wheels.

8. Materials. Cast iron wheels, of which the treads and flanges have been chilled to give added surface hardness, are still largely used in freight service. Steel tired iron wheels, or forged or rolled all-steel wheels, are almost exclusivly used for heavy passenger service.

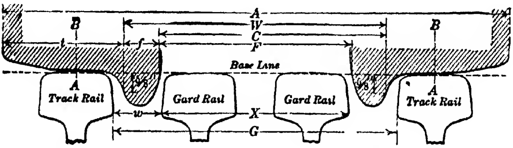


Fig. 2.

9. Tread and flange, for Master Car Builders' standard cast iron wheels, and their relations to the rail heads (including gard rails) are shown in Fig 2.

A =	overall gage	= 5′ 4⅜″;	W =	wheel gage	= 4′ 7 11/16″;
C =	check "	= 4′ 6 29/64″;	F =	flange gage	= 4′ 5 7/32″;
f =	flange thcks.	= 0′ 1 15/64″;	t =	tread	= 0′ 4 11/32″;
X =	gard rail and wing rail gage				= 4′ 5″;
G =	track gage	= 4′ 8½″;	w =	flangeway	= 0′ 1¾″.

Wheel diameter is measured on center line, *A B*, of rail.

10. Standard or usual diam, for all car wheels, 33 ins. Wt of each wheel, 620 lbs for 30-ton capacity car, to 720 lbs for 50-ton.

11. Coning of wheel treads. In order to obviate or diminish the resistances which would occur under a wheel with a cylindrical tread, when rounding curvs, (see Train Resistance, ¶ 36, p 1061), the transition, from the tread to the flange of the wheel, is made by means of a curv, or "fillet", of about 1 inch radius, which usually keeps the side of the flange away from the side of the rail head; and the wheel tread is "coned" (with its greater diam nearer the flanges), in order that the *outer* wheels (traveling always toward the outer rail) may thus be made to roll upon their *larger* diams, and the *inner* wheels upon their *smaller* diams; but, under traffic, the coning becomes reduced, and even reverst, the wheel diam, next the flange, sometimes becoming even less than that farther from the flange. Prior to 1907, the standard coning of the Master Car Builders' Association was 1/38 (1/16 inch in 2 3/8 inches). In 1907, the standard coning was increast to 1/20 (3/32 inch in 1 7/8 ins). The Pennsylvania Railroad found breakages of wheel flanges greatly reduced by the increase of coning. (See Ry Age Gaz, 1912 Jul 19, p 92.)

12. Tires. The function of a tire is to give added strength to the wheels and to resist wear. Tires are either shrunk on to the wheels by reduction of temperature, or clampt on in one or another of many different ways, or both.

Brakes.

13. *A Brake Shoe* is a metal piece so shaped on one side as to conform with the tread and flange of the wheel. Ordinarily, one brake-shoe is provided for each wheel, but "clasp" brakes, having two shoes, one on each side of each wheel, appear to have markt advantages. The two (or four) shoes for a pair of wheels are attacht by *Brake Beams* (frequently trussed to give added strength) placed parallel to the axle, either between the two pairs, or outside, according to convenience, etc, or both, as in the "clasp" brake. *Brake Hangers* are attacht to the brake-beams and trucks in such

a way as to take the forces produced by the friction of the shoes against the wheels and to hold the shoes in place. A system of *Brake Levers*, distributing the brake pressure properly to all the wheels, connects the brake-beams with the source of brake actuating force, *i. e.*, with the shaft of the hand-wheel or with the piston of the air-cylinder, or with both.

14. Air brakes. For the operation of the brakes by comprest air, each car is provided with a *Brake Cylinder*, fitted with a piston connected with the system of brake levers. Each car is equipt also with a *Brake Pipe* running its entire length. The brake pipe has a flexible hose at each end, and connections whereby the entire line may be joind together, and also with one or two steam-operated *Air Compressors*, carried on the locomotiv. A *Brake Valv*, placed in the locomotiv cab, enables the engineer to control the brakes on the locomotiv and thruout the entire train.

15. "Straight" air brake. In this brake, the comprest air passes from the compressor (or from a reservoir of comprest air) on the locomotiv, thru the engineer's brake valv, and the brake pipe, directly into the brake cylinders; thus applying the brakes. Its operation, however, is not rapid enough for long trains, and has the very great disadvantage that any failure of the air supply, as by a burst hose, will result in failure of the entire system; and this is most likely when the demand upon the system has been heaviest.

16. The "automatic" air-brake practically overcomes these objections. Each car carries an *Auxiliary Reservoir* of comprest air, and is provided with a special elaborate valv, known as a "*Triple Valve*", operated by differences in pressure between the brake pipe and the car auxiliary reservoir. The essential feature is that a reduction in pres, in the brake pipe (causd either by the operation of the engineer's brake valv, by an emergency valv on a car, or by a failure of the train line) so operates each triple valv that air passes thru it from the auxiliary reservoir of each car into the brake cylinder. To release the brakes, the engineer sets his brake-valv into such position as to restore the normal pres in the train-line; whereupon the triple valvs permit the escape of the air from each brake cylinder, and suitable springs ensure the release of the brake shoes from the wheels. The engineer's brake valv has several positions, by means of which the brakes may be either (1) gradually releast, (2) all held as they may have been set, (3) gradually applied, or (4) instantly applied.

17. "High speed brake." In this the pressures employd are nearly or quite sufficient to skid the wheels in spite of the relativly low fric coef of the brakes at high speed (see "Friction", p 412). But, as the speed of the train is reduced, the air is allowd to escape gradually, so as to avoid skidding the wheels as the fric coef increases.

18. Electric control of the valvs is largely used in "multiple unit" electric train control; and it has been advocated for very long steam trains where even the "automatic air" brake is at times troublesomely sluggish in its operation. On electric cars, an air compressor on each car is operated by an electric motor on that car.

19. Electric brakes have been largely used on electric street cars. In the Westinghouse electric brake the motors of the car are caused to generate electricity which magnetizes a shoe carried directly over each rail. The shoe then clings to the rail, and the resulting drag forces the brake shoes against the wheels. Electric brakes, depending on the rotation of the motors, cannot, however, keep a car standing on a grade.

20. New high-speed brake. The Westinghouse Air Brake Co., together with Pennsylvania Railroad (S. W. Dudley, Asst Chf Engr, Am Soc M. E., N. Y. Section, 1914 Feb 10) very thoroly tested a new air brake, with the following results:—

Train of 12 steel passr cars and modern loco; total lgth abt 1000 ft; total wt abt 1000 tons

Speed, miles per hour,	30	60	80	
Ordinary brake stopt train in		1600 to 1800		feet
New " " " "	200	1000	2000	feet

21. Vacuum brakes have been employd to a small extent. Their operation is much the same as that of the (comprest) "air brake". As it is of course impossible to obtain operating pressures of more than about 12 lbs/sq inch, very large brake cylinders are required; but the apparatus has the advantage that, in case of failure of the air service, the brakes are applied automatically, without the use of a "triple valv".

Miscellaneous.

22. Air signal and steam supply on passenger trains are operated by means of separate lines of pipe, running the length of the train, each connected, between cars and locomotivs, by individual hose connections. The fundamental principle of the air signal is similar to that of the automatic air brake, in that air pres is normally maintaind in the line of pipe, and that the work (here the blowing of the whistle, in the cab), is done by the escape of air, either (1) effected by a valv, which is operated by a signal cord on a car; or (2) from a leak, such as would be caused by the breaking in two of a train.

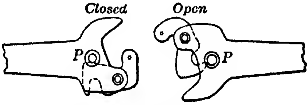


Fig. 3.

23. Couplers, one at each end of a car, are arranged symmetrically with respect to a vertical axis thru the center of the car (the open jaw facing to the left as one stands on the ground facing the car), so that cars and locomotivs may always be coupled together without any reversal of car or coupler. Figure 3 shows two couplers almost engaged, one open and one closed. The knuckles are so latcht by pins, *P, P*, that, with either one open, the cars will be coupled when they come together, without further attention from trainmen. Also, any coupler may be opend, and the cars uncoupled, by means of a lever at a corner of the car, without need for the trainman to go betw the cars. Hoses of train pipes, however, must be both coupled and uncoupled by a man who must go betw the cars; but devices are being introduced by which this also may be accomplisht automatically, especially in multiple-unit electric traction. Car couplers are pivoted to the draft gear, to permit of turning on curvs.

24. The draft gear is a combination, at each end of each car, of a longitudinally-sliding bar, with springs to take up the shock of starting. The springs make it possible, also, to approximate the starting of the train one car at a time, which is valuable because the starting resistances are higher than those of normal running. In *Friction Draft Gear,* friction devices are employd which serv to absorb the recoil of the springs, and thus prevent the accumulation of rhythmically recurring stresses.

Clearance.

25. On bridges, see p 746. **In tunnels,** the Am Ry Eng'g Assn has recommended clearances as shown in Fig 4, betw cen of track and side of tunnel.

26. On curvs an *additional clearance* is necessary and may be computed from the following formulas given by George Paaswell, Asst Engr, Pub Service Commission, New York City, in Engineering & Contracting, 1914 Mar 25, p 367. In Fig 5, let

a = dist betw centers of trucks;
L = length of car;
c = dist from middle of center line of car to center of curvd track;
e = dist from center line at end of car, to center of curvd track;
R = radius of curv.

All units preferably in feet

Then, $c = a^2/8R$; and $e = (L^2 - a^2)/8R$

See also Min clearance on curvs, p 746

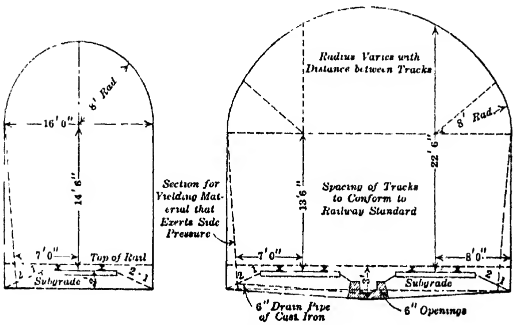


Fig. 4.

27. Where **transition curvs** or **"spirals"** are used, the radius may be taken, nearly enough, as that of the curv at the middle of the car. For the transition curv radius, R_s, at any point, we have $R_s = R_c T/t$; where R_c = circular curv radius; T = length of transition curv; t = dist of the point from beginning (T.S.) of transition curv.

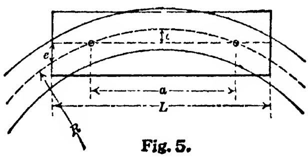


Fig. 5.

28. Tilt of car. In addition to this, allowance must be made, on the inner side of the curv, at least, for the tipping of the car due to super-elevation. This, of course, may be readily computed, given the super-elevation, gage, and height of eav of roof or other critical portion of car.

29. More precise formulas and elaborate diagrams are given for these computations by Mr. Frank H. Carter, in Engineering & Contracting, 1913 Dec 10, pp 650-2.

LOCOMOTIVS

In the following table

Wheel-base is the dist along the rail, betw points of contact of the front and back wheels with the rails.

Tractiv effort, usually betw 0.22 and 0.25 of the wt on all the drivers, is the hor force which the loco can exert, provided the wheels do not slip. It holds good for speeds up to abt 10 mi/hr.

Tractiv effort **at higher speeds** as given by Am Loco Co. Multiply the tractiv effort by the factor corresponding to the speed in question;—

Piston speed, ft/min...	300	400	500	700	900	1200	1600
Factor	.954	.863	.772	.590	.460	.337	.241

Locomotiv Data

The first four lines of the following table are made up from information receivd in 1937 from three large U S mfrs regarding 11 of their very heavy locomotivs; the last four lines from "Statistics of Rys of the U S, by the Interstate Commerce Commission," 1935.

	Loco only, lbs	Tender only, lbs	Wheel Base	Lbs/ft of Track	Tractiv Effort
Heaviest	723,400	402,000	111′11″	10,050	140,000
Av except'y heavy.	600,000	400,000	100′	10,000	100,000
Av heavy..........	486,525	339,220	95′	8,692	85,817
Av mod'y heavy...	400,000	275,000	83′	8,132	81,325
U S av freight....	280,000				54,657
U S av passenger.	250,000				36,225
U S av electric, frt	288,000				
U S av electric, pass	362,000				

Choice of a locomotiv will depend upon the service requirements, and will be limited by clearances of bridges, tunnels, etc, strength of bridges, and wt of rail, this last being taken into account on the assumption that each 10 lbs/yd of rail will carry a load of 3,000 lbs on each wheel; which, for rails weighing 90 to 100 lbs/yd, permits loads of 50,000 to 60,000 lbs per *pair* of driving wheels.

Mileage. The following figures are taken and deduced from Ann'l Report on the Statistics of Rys in the U S, for 1935, Interstate Commerce Commission:—

	Locomotiv Mileage Passenger	Freight	Other Kinds
Total Mileage........	446,148,156	344,745,112	37,711,637
Number of locos......	7,983	27,462	10,169
Mileage per loco			
per year.....	55,887	12,553	37,084
per day......	153.1	34.4	101.6

73. **To ascertain the speed** of a train or other vehicle, on which one is riding, by counting any known equal spaces, such as rail lengths, dists betw telegraf poles, or revs of a wheel, past in a given time. Rail lengths may often be counted by the sound of the wheels passing over the joints; wheel revs by any irregularity of the wheel.

74. Let L = the length of one of the known equal spaces in ft; then

Speed, in ft/sec = number of such lengths past in L secs.

Speed in mi/hr = number of such lengths past in $L \times 3600/5280$ secs, $= L \times 15/22$.

With 30-ft rails, the speed in mi/hr will be equal to the number of rail lengths traverst in 20.45 secs.

CARS

75. Practically all American RR cars, except some cabooses, ore and dump cars, are built with two trucks each, Fig 7, each truck having either two or three pairs of wheels. In a 6-wheeld truck, springs and equalizing levers distribute the load equally, or nearly so, among the three axles. Upon the pivots of both trucks, rests a long rigid "underframe" for the car body. This contains the draft gear which takes the longitudinal stresses. The sides of the cars, so far as practicable, are constructed as girders or trusses.

76. **Material.** Practically all new cars are constructed principally of steel, especially the underframes. Some wood may enter into the bodies of box and passenger cars. Aluminum is being used to a small extent.

77. **Data.** The following table gives approx max and min wts, etc, of cars ordered in the U S in 1936. (Ry Age, '37 Jan 2). Those of highest or lowest capacity were not nec'y those of greatest or least wt.

Type of Car	Length of Body Feet	Weight Empty Pounds	Capacity
Pass, Parlor, Bagg, Mail & Express	64 to 84½	57,100 to 189,000	21 to 136 pass'rs
Box	40½ to 50½	35,200 to 53,000	80,000 to 100,000 lbs
Gondola	24½ to 65½	43,100 to 71,000	80,000 to 200,000 lbs
Hopper	20 to 46	32,600 to 60,400	70,000 to 180,000 lbs
Ore	20	42,375	150,000 lbs
Flat	41 to 70	39,000 to 80,000	80,000 to 200,000 lbs
Tank	29 to 43	34,200 to 75,500	30,000 to 140,000 lbs

78. **Freight cars.** The following is from a table, publisht in Ry & Eng'g Rev, '12 Feb 10, compiled from data from 11 large RRs owning over 450,000 freight cars, for a period of 3 years; av age of all cars, abt 10 yrs. Each car is repaird on an av once a month at a cost of $6.26 each time. Av mileage per car per year, 10,400, which brings repairs to $0.007/car-mile.

79. **Passenger cars.** Av life abt 16 yrs. Av annual repairs, including painting, $300 to $700; for mail and express cars, $150 to $300. The wt of a pass'r car will av fully ten times that of its full load of passengers.

80. **Dump cars** are of very varius design. They may dump either thru the bottom, thru one or more "hoppers" (usually transvers), or thru the sides, in which case

either the sides lift up or swing out, or else the sides remain practically stationary while the entire floor of the car is tilted, thus forming an opening under the side and producing a grade for the material to slide on. In the several makes of cars, various combinations of swinging sides and bottom pieces make possible the delivery of material either between the rails, or at either or both sides of the track. Hopper bottom cars are ordinarily operated by hand, the doors or flaps being tript to open, and being closed by winding up of chains by means of cranks. Tilting-bottom cars are frequently operated by comprest air, and, in some cases, the control is quite elaborate, it being possible to tilt the entire load simultaneously either to the right or the left, or both right and left from alternate cars, under the control of one operator at any point along the train.

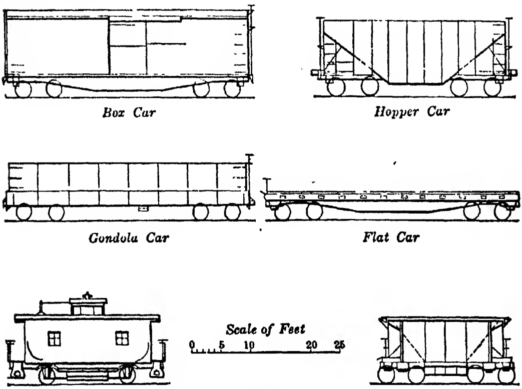


Fig. 7.

81. Ore cars (see Fig 7) are made abnormally short, and of relativly small volumetric capacity, as longer cars are necessarily very inefficient in carrying ores, which are very much heavier than coal or other materials usually carried.

82. Crane cars are used chiefly for clearing up wrecks, for excavation and earth handling generally, for shifting track, and for coaling locomotivs.

83. Locomotiv cranes appear to be coming into high favor for these purposes. They can propel themselves promptly from place to place.

84. Steam shovel cars, and pile drivers mounted on cars, are made in various styles by a number of mfrs.

85. Ballast plows or spreaders are treated under "Track", p 819.

86. Snow plows are either *plows* properly so-called, which shove the snow from the track (to both sides for single track, and to the right for double track); or they are of the *"rotary"* type, in which a large cutting wheel, at the front end, on a hor longitudinal axis, is caused to revolv by powerful engins, often driven by a full-sized locomotiv boiler, and which disposes of the snow largely by virtue of the centrifugal force generated in the cutting-wheel. On steam roads the plow is forced into the snow by one or more

pushing locomotivs behind it; whereas, on electric street roads at least, the plow or *sweeper* (which really sweeps and does not employ centrif force) is forced ahead by its own power, the brooms being rotated by independent motors. Some plows are so arranged that the deflecting surfaces may be forced outward by comprest air (when clearance is sufficient) and so throw the snow further from the track, and therefore with less chance of its return.

87. **Flangers** are pieces, usually of sheet iron or steel, so shaped and arranged as to be lowered onto the rails directly in front of the wheels, to remove any few inches of snow or ice that may be on the rail or along its inner face. They are sometimes fitted to locomotivs, and usually to plows and "rotaries".

88. **Clearance cars** are designd for readily determining the proximity of objects along the road. The clearance cars built by the Pennsylvania and Baltimore & Ohio R R's have mounted, over one of the trucks, a frame transversely of the car, the limits of which are somewhat less than the probable minimum clearances of the road. To this frame are attacht, by means of pivots, at intervals of about a half foot, movable arms, which project about two feet from the frame. Each arm is provided with a scale, so graduated as to show directly the number of inches inwardly that it may have been deflected from its extreme position. The car is run along the track, at about 4 mi/hr, where various objects, as bridges, stations, tunnels, etc, are past; the arms are deflected until they clear, and the deflections are observed and noted. Corrections must of course be made, to provide for the degree of curv, if any, and for the distance between truck pivots of any cars to be operated over the track. In addition to the clearance frame, with its fingers, one of the trucks is so connected with a pointer and dial as to show directly the degree of curvature, while a pendulum, free to swing transversely, is so connected with another pointer and dial as to show directly the super-elevation of either rail. See also "Clearance", ¶ 25.

89. **Inspection and hand cars** range, in elaborateness, from simple trucks with geard hand-levers, up to the combination locomotiv and car, used by officials of the road. Many of these, however, are now being driven by gasoline motors; which, while requiring more skill to operate them than hand cars, can make very high speeds, and can usually keep out of the way of trains. Some are so built that their wheels and axles form a metallic circuit, which operates the automatic signals and so protects the car from trains on the same track; but many of them have the wheels insulated, and must therefore run under train orders from tower-men.

90. **Track recording cars.** See under Track Laying and Maintenance, p 821

91. **Dynamometer cars.** See under Train Resistance, ¶ ¶ 56, etc, p 1067.

92. In **wrecking outfits** there is usually a powerful steam crane car and a number of other cars, carrying miscellaneous equipment, and sleeping and dining facilities. They carry rails, ties and switch-parts, car-trucks, assorted screws, nuts and bolts; tools of many kinds; hoisting equipment of pulley-blocks, chains, ropes, wire-cable, hooks, wooden blocking; also powerful portable lights, fire extinguishers, linemen's outfits with telefone, emergency medical cases, car replacers, etc.

93. **Car replacers** are made in pairs, and are castings so formed with grooves or flanges that, when placed on the ties and adjacent to the rails, they will guide each pair of wheels back onto the rails as the car is forced along the track. They are often provided with lugs, to fit around, or to be forced into, a tie, and with notches, to receiv temporary spikes, to assist in holding them in place.

94. **Loading standards,** as establisht by the Master Car Builders' Assn, are given in a pamflet obtainable from the Secretary.

Old Colony Bldg, Chicago, Ill. The aim is, of course, to load the cars as evenly as possible, to hold the load in place, and to provide for curvs and inequality of track where the load extends over more than one car. Where the load is long enough for two or more cars, it may be supported on two improvised pivots, one on the front and one on the rear car. If the material must be supported midway, metal bearing plates are provided to permit of easy sidewise sliding. Where more than two cars must be used, the extra car usually takes no load and acts merely as a spacer. Some loads of three-car lengths, as plate girders, may be loaded entirely on the central one of three cars, the other two being spacers. Piles of lath, barrels, pipe, shingles, etc, must all be suitably held in place by side verticals and lashings, etc.

TRAIN RESISTANCE

GENERAL.

1. The forces (or force components) acting upon a train in a direction parallel with the track, may be classified as

(a) "moving" forces; tending to start or accelerate the train;
(b) resistances; tending to hold or to retard the train.

2. Locomotiv tractiv force, gravity, wind and inertia may act either as moving forces (a), or as resistances (b), according to circumstances; but friction (external and internal) curvature and still air necessarily exert *resistances* (b).

3. In any given case, the train resistance, F, is the sum of the resistances acting in that case.

4. The tractiv force, exerted, by the steam, thru the loco, has to move the loco and tender, as well as the cars; but often the "resistance" is taken as that of the cars alone, or that which opposes the pull of the draw-bar behind the tender.

5. Components of train resistance. Let

F_n = normal res = the total res upon a straight level track, train at rest or at uniform speed, in still air, and at normal temperatures
= the res to which every train is necessarily subject;

F_g = grade resistance;
F_c = curv resistance;
F_w = wind resistance;
F_i = inertia resistance, due to velocity changes.

} Incidental resistances

Then, for the total resistance, we have:—

$$F = F_n + F_g + F_c + F_w + F_i \quad \ldots\ldots\ldots (1)$$

6. Unit Resistance. Let W = the weight of the train. Then the quotient, $f = F/W$, is the resistance *per unit of weight*, or unit res. Similarly, f_g, f_c, etc., denote the unit grade res, curv res, etc., respectively. Any resistance may be exprest in terms of an *equivalent grade*. See ¶ 33.

NORMAL RESISTANCE

7. The normal resistance, f_n, is the resistance, per unit of weight, on a straight, level track, at uniform speed, in still air. Of the total resistance, it is that portion to which every moving train is always necessarily subject. For incidental resistances, see ¶ 31, etc.

Components of Normal Train Resistance, f_n;

8. (a) friction betw the rail-heads and the treads and flanges of wheels;

(b) resist due to undulation of track under a moving train;

(c) internal friction of cars and locomotiv;

(d) resistance of still air.

Friction between wheels and rail-heads.

9. When the wheel-flanges run clear of the rail-heads, we have, normally, betw rails and wheel-treads, only rolling fric (see p 414, ¶ 193); and this is small in amount where track and rolling stock are in good order; but the sliding fric, betw rail-heads and wheel-flanges, may be of considerable amount, especially where there is oscillation or side wind, (see ¶ 44), and is often accompanied by lateral sliding of the treads on the heads. On curvs, the two rails are of different lengths, and we have longitudinal fric betw tread and head. The conditions vary so greatly, on different roads, and from moment to moment with the same train, that it is impracticable to establish useful rules for this item, or for the next.

10. Resistance due to undulation of track under a moving train is of course greatest on yielding rails and roadway.

11. The internal friction of rolling stock consists chiefly (that of locomotivs partly) of *journal friction*. Its value, in pounds per ton, is

$$f_j = 2000\, c\, d/D \quad \ldots\ldots\ldots\ldots (2)$$

where c = the fric coef of the journal rotating in its bearing, d = the journal diam, and D = the wheel diam. This assumes d/D constant; and, in practice, an *average* value of d/D may usually be estimated for a train.

12. *Journal and rolling resistances combined.* It is difficult to determin these, independently of the other normal resistances. Experiments, made for the purpose, appear to have been unreliable. From coasting experiments, Wellington deduced 4 to 6 lbs per ton of 2000 lbs.

13. *In locomotivs*, (Am Loco Co., Bulletin 1001, Feb. 1910), the friction of driving wheels, pistons, valvs, crossheads, etc., is taken at 22.2 lbs per 2000 lbs of wt on drivers. *Journal friction*, of loco truck and trailing wheels, and of tender wheels, is taken as equal to that for cars.

14. Normal air resistance, F_a. Air res in still air. (See also wind res, F_w, ¶ 44). The evidence is highly conflicting. Prof. W F. M. Goss, Western Ry Club Procs, 1898, April 25, p 347; reports expts made in a large closed rectangular conduit, in which air currents of known velocities impinged directly upon the front ends of fixt trains of from 1 to 25 model cars, about 3.5 ins wide, 4.5 ins high, 12 ins long, each connected, by dynamometer, with a fixt base. Prof. Goss derives the following formulas, in which it is assumed that the linear dimensions of the actual rolling stock are 32 times those of the models on each linear dimension; that freight cars are 33 ft long, and passr cars 66 ft.

Let

F_a = total atmospheric resistance on actual train, in lbs;
V = velocity, in miles per hour;
L = length, in feet, of any train, including loco & tender;
l = length, in feet, of any train, excluding loco & tender;

then, for the entire train: $F_a = 0.0003\ (L + 347)\ V^2$....(3)
for cars alone: $F_a = 0.0003\ (l + 53)\ V^2$....(4)
for loco & tender: $F_a = 0.125\ V^2$ ....(5)

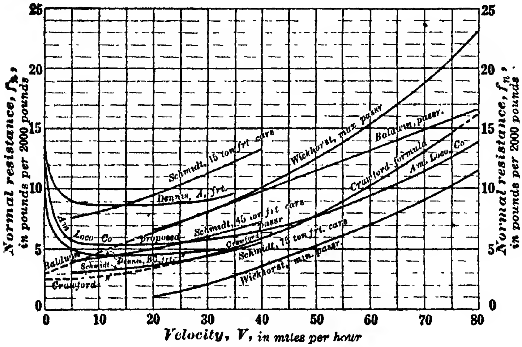


Fig. 1.

Prof. Goss regards these formulas as giving results varying not more than 15 to 20 per cent from the truth in practice.

Unit air resistance, f_a, $= F_a/W$, where W = weight of train in tons of 2000 lbs.

15. Air resistance is increast by *spaces between cars*, by *open side doors* of box cars, and by *differences in heights of cars*, as where flat and box cars alternate in position in a train. "It pays in coal to incur some trouble and delay in putting box cars together. That also enables the brakeman to get along the train more rapidly." (Angus Sinclair, "Locomotive Engine Running and Management", 1891, p 375.)

16. *Head air resistance of locomotiv* is assumed as $0.002\,V^2 A$; where V = vel, in miles per hour, A = cross-section area of locomotiv, usually taken at 120 sq ft. Am Loco Co Bulletin 1001, 1910, Feb.

Factors affecting normal resistance.

Starting.

17. When starting a train (see Fig 1), the value of f_n is much higher than when running. Starting may be facilitated by backing up sufficiently to slacken the couplings, so that the train is started one car at a time.

18. A. C. Dennis, Am Soc C E, Trans Jun 1903, Vol 50, p 1, found that, "after the train has stood for some time, the resistances are abt 2 lbs/ton above the normal until the train has been run enough to get the journals well lubricated and warmd."

19. A. K. Shurtleff, Am Ry Eng & M of W Assn, Bulletin 84, Feb 1907, pp 98, etc., found that, above the freezing pt, the av freight train can be started, on a level, with a draw-bar pull of 18

lbs per ton of 2000 lbs: but freezing weather, or careless starting, may increase this to 25 or 30 lbs per ton. On the other hand, "where a stop was but for an instant, trains were started with 6 lbs. per ton draw-bar pull."

Running.

20. Effect of distance run. With increase of dist run, after a stop, the journals become warmd, their lubrication improves, and the resistance is thus diminisht. During the first mile or two of a run, even with uniform speed and constant air temp, the normal resistance, f_n, may be as much as 50% above the min, which is not reacht until the train has run about 8 or 10 miles in warm weather, and about 35 miles in cold weather. With heavy cars and high speeds, the dist required for reaching min f_n, is less than with light cars and low speeds; the wt on the journals and increast speed aiding in the heating of the journals and in the consequent liquefaction and distribution of the lubricant. (Prof. Edw. C. Schmidt, Central Ry Club; Ry Age Gazette, 1912 Jan 12.)

21. Increase of Speed. Fig 1. The high normal res, experienced in starting, decreases very rapidly as the speed increases to say 5 miles per hour. From say 5 or 10 to 30 or 35 miles per hour (embracing usual freight train speeds) f_n increases slightly, and then more rapidly with further speed increase.

Fig 1 shows curvs of f_n according to experiments and formulas, as follows:—

Experiments.

22. A. C. Dennis, 1902. Am Soc C E, Trans, 1903 June, Vol 50, p 1. Total run, over 3000 miles. Curvs *A* and *B-C*. Runs made with

(*A*), 105 empty box cars, track frozen solid, rail good;
(*B*), 47 loaded box cars, track frozen solid, rail good; net/tare = 2;
(*C*), 52 loaded box cars, track soft; net/tare = 63/27.

Results (*B*) and (*C*) practically identical.

23. The resistances (compensated for velocity *changes*, see ¶¶ 45-52) seemed to be "greater than normal when increasing the speed, and less when decreasing". By coasting tests, locomotiv res, per unit of wt, was found to be about equal to that of *empty* cars.

24. Max H. Wickhorst, 1900, on the **Chicago, Burlington & Quincy R. R.,** between Chicago and Burlington, Ill. (206 miles). Eng News, 1901, Oct 31. Five runs, with dynamometer car, 2 baggage and 2 mail cars; 3 runs loaded, 180 tons, av 55 mi/hr; two runs light, 160 tons, av 27.5 mi/hr. Mean temp, 70° Fahr; wind very light. Very numerous observations. The two curvs show, respectivly, the approx maxima and minima

25. Prof. Edward C. Schmidt, 1910, University of Illinois Bulletin, Vol VI, No 39. Very elaborate tests of 32 ordinary freight trains, wts 747 to 2908 tons per train, 26 to 89 cars per train, averaging from abt 15 to 70 tons per car; over av track, in good condition; air temp from 34° to 82° Fahr. See three selected curvs, Fig 1; vels from 5 to 40 mi/hr.

Formulas.

26. American Locomotive Co. Bulletin 1001, 1910 Feb, p 10; R. R. Age Gaz, 1909, Sep 10, p 455:

$$f_n = 5.4 + 0.002\ (V - 15)^2 + 100/(V + 2)^2 \quad \ldots\ldots(6)$$

in which V = vel in mi/hr.

27. J. G. Crawford, Eng News, 1901, Oct 31. Based upon Wickhorst's **C, B & Q** expts, ¶ 24. For vels betw 25 and 75 mi/hr.

$$f_n = 2.5 + V^2/468 = 2.5 + 0.0021\ V^2 \quad \ldots\ldots\ldots\ldots(7)$$

28. Baldwin Locomotive Works, straight-line formula. R R Gaz, 1899, Mar 17. Based upon expts on Phila & Reading system, betw Camden and Atlantic City, 55.5 miles without stops.

$$f_n = 3 + V/6 = 3 + 0.167\ V. \quad \ldots\ldots\ldots\ldots(8)$$

Cold.

29. The American Locomotive Co. (Bulletin 1001, 1910 Feb.) ıakes the following reductions from the tonnage hauld at normal emps;—

Temperature, Fahr.	Deduct
45° to 25°	8 per cent
25° to 0°	16 " "
Below 0°	25 " "

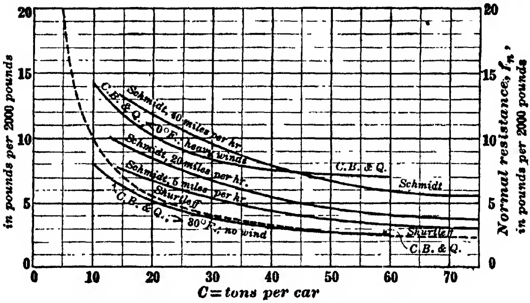


Fig. 2.

Effect of weights of cars.

30. Fig 2 shows the effect of weight of freight cars, and, inci- ɜntally, to some extent, of velocity, upon the normal resistance, ; as follows :—

C B & Q expts, R R Age Gaz, 1909, Aug 27, Sep 3;

Prof. Edward C. Schmidt, expts, Illinois Central R R. See also 25.

The dotted curv represents the formula of A. K. Shurtleff, Am y Eng & M W Assn, Bulletin 84, Feb 1907, p 99 :—

$$f_n = 1 + 90/C \quad \text{.........................} (9)$$

here C = av weight of loaded car, in tons of 2000 lbs. This ırmula is based upon expts on two railroad systems. Total runs, ver 3000 miles, 19 to 41 tons per car. Track varied from "mud ıllast, with medium line and surface, to first class track, well bal- sted, with good line and surface......from valley lines to moun- ıin lines and grades." Temp from 0° to 60° Fahr.

INCIDENTAL RESISTANCES. See ¶ 5.

31. In addition to the normal train res, we must usually take ccount of the incidental resistances:

F_s, due to grades;	F_w, due to wind;
F_c, " " curvature;	F_i, " " inertia.

Grade Resistance

32. On any grade, let a = the angle with the horizontal. hen, for the res, parallel with the slope, necessary to prevent ıovement of a wt, W, down the slope; we have;—

$$_s = W \sin a; \text{ and, since } f_s = F_s/W; \; f_s = \sin a \ldots. (10)$$

But, except on very steep railroads, the slope is so slight that, approx, $\sin a = \tan a$. Therefore, if L = the hor dist betw two points, and h = their diff of elev; we have;—

$$F_s = W \tan a, \text{ and } f_s = \tan a = h/L \quad \ldots\ldots (11)$$

33. The equivalent grade, s_e, is that grade which will produce the same tendency along the track as that due to a given resistance; or that grade where the tendency to slide downward is just balanced by the given res. Hence, any unit res, f, $= F/W$, may be exprest as a grade. For various conversions, see Fig. 3.

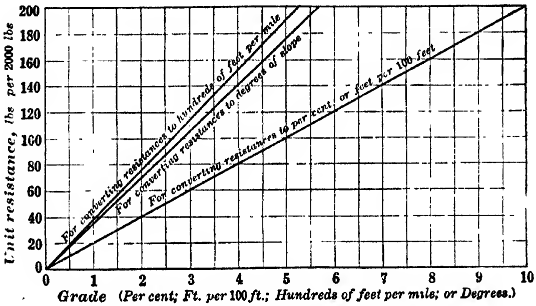


Fig. 3.

34. Units of grade resistance. Let

s = grade, in feet per 100 feet;
S = grade, in feet per mile = 52.80 s.

35. Then, for the **grade resistance,** in lbs per ton of 2000 lbs, we have;—

$$f_s = 20\,s = 0.3788\,S \quad \ldots\ldots\ldots\ldots\ldots (12)$$

and, for any unit resistance, f, due to any cause, we have;—

equivalent grade, in ft per 100 ft, $= f/20$ ..... (13)

Ruling grades; see "Operation Cost", p 1084.
Momentum grades; see p 1076, ¶ 40.
Work done and lost on grades; see p 1075, ¶ 30.
Effect of Length of Grade on Tractiv Force; see p 1076, ¶ 37.

Curv Resistance.

36. At any point on a curv, an axle, with fixt cylindrical wheels of equal diams, tends to travel forward in a direction perp to itself and tangential to the center line of the track, remaining parallel with itself. Hence, the flange of the outer front wheel tends to press outwardly against the head of the outer rail, causing sliding friction betw the sides of flange and of rail head; and this fric opposes the rotation of the wheel. Moreover, the head of the outer rail, by its hor reaction upon the wheel flange, pushes the outer wheel (with its axle and mate wheel) toward the cen of the curv, compelling both wheels to slide laterally across their respectiv rail heads, and thus causing sliding fric betw wheel tread and top of rail head. This, of course, is combined (according to the resolution of motions and forces) with any longitudinal slipping, as it takes place; (¶ 37).

All this is notably the case with the first wheels of a car or train. On the following trucks, the direction of the traction coincides, at each truck, with a chord to a portion of the curv; and thus tends to reduce the pres of said following trucks against the outer rail.

37. Again, the axles must be constantly swung, horizontally, from positions parallel with themselves, and forced into positions more nearly radial to the curv; and this requires that one or the other wheel, or both wheels, must be slid, longitudinally of the track. See also Corrugation, under "Track", p 793, ¶ 91.

38. In order to obviate or diminish the resistances thus causd, the wheel treads are "coned". See Rolling Stock, p 1040, ¶ 11.

39. The resistance, F_c, due to curvature, is influenced by many circumstances; such as diam of wheels, tightness of gage, shape of wheel treads (whether more or less conical), lengths of rigid wheel bases (see ¶ 43), kind of coupling, condition of track, elev of outer rail, length of train (affecting obliquity of traction: See end of ¶ 36).

40. Unit curv resistance is usually taken as varying, other things equal, directly with the sharpness of curvature; or, for curv resistance, in lbs per 2000 lbs, on a D° curv:—

$$f_c = D f_1 \qquad (14)$$

where D = sharpness of curvature, in degrees;
f_1 = the curv res, in lbs per 2000 lbs, on a 1° curv.

41. f_1 is usually taken as varying, (with character and condition of track and of rolling stock,) between 0.5 and 1.5 lbs per 2000 lbs. In compensating for curvature, on grades, (see p 1078, ¶ 51), it is usual to allow betw 0.6 and 0.8 lbs per ton. See Fig 4.

42. Fig 4 shows the unit curv res, f_c, by Eq (14), in various units, corresponding to sharpnesses from 0° to 20°, and for values of f_1 from 0.5 to 1.5. The *dotted* radial line gives, for diff degrees of sharpness, the sweep, in degrees, of a mile of curvd line of given sharpness; and the dotted curv gives the ft of line per degree of sweep.

$$\text{Feet per degree of sweep} = 100/D \qquad (15)$$

$$\text{Degrees of sweep per mile} = 5280\,D/100 \qquad (16)$$

These two dotted diagrams represent purely *geometrical* conditions, without regard to *resistance*.

43. Effect of length of wheel-base. Prof. Wm. G. Raymond, R R Gaz, 1906, Aug 17, makes the curv resistance,

$$f_c = 0.4 + D(0.205 + 0.035\,B) \qquad (17)$$

where D = sharpness of curvature, in degrees,
and B = length of wheel-base, in feet.

43a. Effect of velocity. It is probable that curv resistance, f_1, per degree of sharpness, (like normal res), varies with velocity, somewhat as indicated by Fig 1. In other words, we may expect to find it (1) high at very low speeds (because the value of the fric then approaches that of static fric), (2) falling to a min at moderate speeds, and (3) increasing again at higher speeds.

Wind resistance, f_w.

For *normal* resistance, f_a, due to still air, see ¶ 14.

44. Head winds of course oppose, and winds in the opposit direction assist, the forward movements of trains; and side winds, by forcing the wheel flanges against the rail heads, increase the resists; but little is known as to the extent of these influences; and, as their exertion cannot be controlled or foreseen, their effect is practically incalculable. A margin must nevertheless be made for them, according to judgment, in estimating the ability of engins to move given loads under given conditions. Where the vel of head wind, or of tail wind, can be estimated, the vel of head wind may be added to, and that of tail wind subtracted from, the train vel,

and the sum or diff used in connection with ¶ 14. For "heavy winds", the Am Loco Co deducts 8% from the tonnage hauld in still air. Bulletin 1001, 1910, Feb. See also two C B & Q diagrams in Fig 2.

Inertia.

45. To accelerate a train requires a "moving force"; and to retard it requires a resistance. Conversely, a train, undergoing accel, may be said to offer a res, due to inertia; and one being retarded may be said to exert a force, due to inertia. In order to compute the total train res, when vel is being changed, we must include the res due to inertia; and, for this, we must know or assume the weight, and either (a) the rate of accel, or (b) the vel, and the dist within which a given vel change takes place.

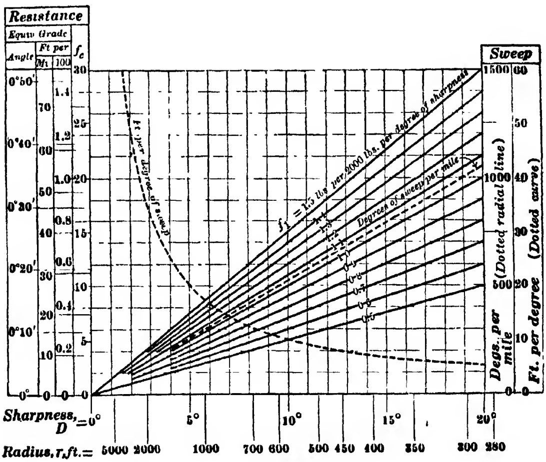


Fig. 4.

46. (a) When a weight falls freely, the accel of vel, due to the force, W, of gravity, is $g = 32.2$ ft per sec per sec; and the accel, A, of any given body, is directly proportional to the unbalanced force, F_i, acting upon it; or $g/A = W/F_i$. Hence, for the force or res, F_i, due to inertia, and producing an accel or a retardation, A, we have:—

$$F_i = \frac{W}{g} A = M A \quad \ldots\ldots\ldots\ldots (18)$$

where $M = W/g =$ the mass of the body.

(b) Let a car move on a horizontal track, and let

W = the car's weight; $M = W/g =$ the car's mass;
V = the car's velocity;
F_i = the unbalanced hor force, acting upon the car;
L = the dist, and T = the time, within which F_i can produce or destroy V;
K = the car's kinetic energy $= F_i L =$ the corresponding work.

Then (Mechanics, pp 343, etc., Art. 19)

$$K = MV^2/2 = F_t L; \quad \ldots\ldots (19)$$

and

$$F_t = K/L = \frac{\frac{1}{2} M V^2}{\frac{1}{2} T V} = M \frac{V}{T} = M A \quad \ldots (20)$$

47. Values. The following may be taken as indicating the range of accelerations, A, and of values of f_t, to be expected in practice.

A = acceleration in ft/sec/sec = $0.0161\ f_t$
f_t = force in lbs per 2000 lbs = $62.1\ A$

Starting accelerations;

	A	f_t
Accelerometer tests by Harry Egerton Wimperis, England; Mins Proc Inst C E, 1911-12, Part II, plate 8.		
Electric multiple-unit trains, when leaving stations. Average	1.5	93.2
Maximum	2.5	155.3
Steam train	0.5	31.0
A single hevy American loco, pulling a train of only two passenger cars, may be expected to attain (up to moderate vels)	1.0	62.1
Braking retardation;		
Electric and steam trains, England................	4.0	248.4
	to 4.5	279.4
Various types of modern high-speed Westinghouse air brakes. Vels 25 to 50 mi/hr. Pennsylvania R R	3.0	186.3
maximum (just before stopping)....	8.0	496.8
Assuming friction coeff, c, = 0.2 lb /lb; $A = 2000 \times 0.0161\, c = 32.2\, c = g\, c$; and assuming all brakes correctly adjusted ...	6.4	397.4

With very light trains for suburban service, where high accels are essential, the forces or resists, needed to overcome inertia, may greatly exceed all the other forces or resists.

Rotational Inertia.

48. In addition to the inertia due to motion of the car, as a whole, along the track, we have, in the revolving wheels and axles, an *additional* inertia, due to their rotation alone; and, like the linear inertia of the car, this rotational inertia of the wheels and axles exerts a resistance during acceleration, and a "moving force" during retardation. We here have to take account of the gyration radius (p 352) of the rotating parts. This is less than the wheel-tread radius.

Let

R	=	wheel-tread radius;	r	=	gyration radius of rotating parts;
V	=	rectilinear car velocity	v	=	rotational vel of point at end of r;
	=	rotational vel of point on wheel-tread about wheel axis;			
A	=	corresponding acceleration;	a	=	corresponding rotational accel;
K	=	car energy;	k	=	rotational energy of wheels, etc.;
F_t	=	hor force, applied at, and normal to, wheel axis, and producing A;	F_r	=	tangential force, at end of r, producing a;
			F_a	=	hor force, at axis, producing a;

W = car weight; w = weight of rotating parts;

M = car mass = W/g. m = mass of rotating parts = w/g.

Then (since $V/v = A/a = R/r$) we have :—

$$k = \tfrac{1}{2}\, m\, v^2; \qquad K = \tfrac{1}{2}\, M\, V^2; \qquad \frac{k}{K} = \frac{m\, v^2}{M\, V^2} = \frac{m\, r^2}{M\, R^2} = \frac{m}{M}\left(\frac{r}{R}\right)^2$$

$$F_a = F_r \frac{r}{R} = m\, a \frac{r}{R} \qquad F_i = M A; \qquad F_r = m\, a;$$

$$\frac{F_a}{F_i} = \frac{m\, a\, r}{M\, A\, R} = \frac{m\, r^2}{M\, R^2} = \frac{m}{M}\left(\frac{r}{R}\right)^2 = \frac{k}{K} \quad \text{........(21)}$$

49. Because the masses of the axles lie close to their axes of rotation, their rotational inertia is so small that it is commonly neglected. But car wheels and electric motors are of considerable wt and diam, and their rotational inertia forms a material portion of the total car inertia.

50. Values. In ordinary car wheels, r/R, ($= v/V$), is usually about 0.7, varying somewhat with wheel design; and the ratio, w/W, betw the wt, w, of the wheels, and the total wt, W, of car and load, varies (chiefly with the extent to which the car is loaded), usually betw 0.17 and 0.06.

Hence, $\dfrac{k}{K} = \dfrac{m\, r^2}{M\, R^2}$, may be expected to range from

$0.7^2 \times 0.17$ to $0.7^2 \times 0.06$, or say from 0.085 to 0.03; or, as an average, say $k/K = 0.05$; and $K + k = 1.05\, K$.

51. As determined by the Westinghouse Mfg. Co., and as computed from data furnisht by the General Electric Co, k/K, in *electric motors*, varies betw wide limits. In general, however, it lies betw 0.05 and 0.15 for av street and interurban cars, neglecting the wheels and gears, for which, in such cars, k/K appears to be about 0.10.

With electric locomotivs, the range is much wider; certain gearless electric locos having k/K less than 0.10, including wheels; while in other low-geared freight locos, k/K may be as high as 0.60.

Velocity Head.

52. Let

$H = V^2/2g$ = head due to hor vel of car;

$h = v^2/2g$ = head due to curvilinear vel of point at end of gyration radius, r.

For a given mass, $h/H = k/K$. Hence, taking $k/K = 0.05$, or $K + k = 1.05\, K$, we would have,

$$H + h = 1.05\, H.$$

From eq (20), ¶ 46, substituting $(K + k) = 1.05\, K$, for K, we have :—

$$F_i = 1.05\, K/L. \qquad \text{Hence, } F_i = 1.05\, W\, H/L; \text{ and}$$

$$f_i = \frac{2000}{W} F_i = \frac{2100\, H}{L} = \frac{2100\, V^2}{2\, g\, L};$$

where V = hor vel of car, in ft per sec = $(1.466..) \times V_m$, where V_m = vel in miles per hour. Hence, we have :—

$$f_i = 70.15\, V_m^2/L \quad \text{................(22)}$$

SUMMARY.

53. Fig 5 gives a general view of the relations betw the normal resistance, f_n, and the three calculable incidental resists, f_s, f_c and f_i, due to grades, to curvature, and to velocity increases, respectively.

The V scale refers only to the f_n and f_i diagrams; while the s and D scales refer to the f_s and f_c diagrams respectively.

Note that the right hand vertical scale relates to the f_c diagram only.

The f_n diagram represents the Am Loco Co formula of Eq (6), ¶ 26:—

$$f_n = 5.4 + 0.002(V - 15)^2 + \frac{100}{(V + 2)^3},$$

where V = speed in miles per hour.

The f_s diagram represents eq (12), ¶ 35,

$$f_s = 20\,s,$$

where s = grade in ft per 100 ft.

The f_c diagram represents eq (14), ¶ 40,

$$f_c = Df_1,$$

where D = degree of sharpness, and f_1 = 0.8.

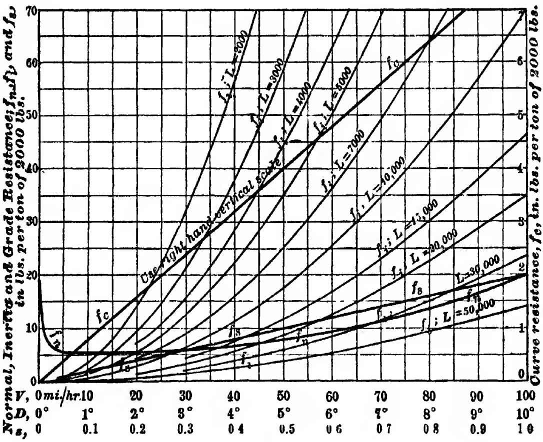


Fig. 5.

54. The several f_i curvs represent the sum of linear and rotational inertia resists, where sum = 1.05 × linear res, and L = the dist, in ft, in which the vel change is effected. In using these curvs, it is necessary to take out the two resists, one for the initial and one for the final vel (each for the dist L in question). Then, inertia res = *difference only* betw these two resists, each of which, alone, represents the res when one of the vels is *zero*.

$$f_i = 70.15\,V_m^2/L; \qquad \text{eq (22), ¶ 52.}$$

55. Example. Let the wt, W, of train be 1500 tons; the up-grade, s, = 0.05 ft per 100 ft; the curv sharpness, D = 3°; and let the vel, V, be increast in a dist, L, of 10,000 ft, from 30 to 40 mi/hr; (mean vel, 35 mi/hr).

Then, from Fig 5, we have	lbs per ton
Normal resistance, f_n, Am Loco Co (V = 35 mi/hr)	6.2
Grade " f_s (= 20 × 0.05)	1.0
Curv " f_c (= 0.8 × 3)	2.4
Inertia " f_i (= 11.2 − 6.3)	4.9
Total unit resistance, f, ...	14.5

Total resistance, $F = fW = 14.5 \times 1500 = 21{,}750$ lbs

Of which there are due
to uniform-speed resistances 9.6 × 1500 = 14,400 lbs.
to speed-increase " 4.9 × 1500 = 7,350 lbs.

DYNAMOMETER CARS

56. The chief function of a dynamometer car is to measure (and record) the **tractiv effort.** The dyn car is placed betw the loco tender and the first car of the train, and the amount of the pull upon its draw-bar then indicates the tractiv effort exerted upon the train exclusiv of the tender.

57. Recorder. The car is usually fitted with apparatus for grafically recording or plotting the various data to be obtaind. This apparatus consists essentially of a large table or frame, fitted up in the car, with a long continuous strip or band of paper, to receiv the record. This band is caused to move longitudinally over the table and under a number of pencils or pens, each of which is capable of more or less lateral movement across the paper.

The movement of the paper may be made proportional either to the dist traveld along the track, or to the time consumed. If the movement of the paper is to be proportional to *distance* traveld, the drum, onto which the paper is wound, or which drives it, is usually geard to an axle, left without brakes, since braking might cause the wheels to slide. The gearing is preferably so arranged that the paper will move in only one direction, whether the car moves forward or back, as otherwise the records would overlap when backing up and moving ahead again If the movement of the paper is to be proportional to *time,* the winding drum may be operated by powerful clock-work, or by a well-constructed fan movement such as that used for the striking action of a clock, or by means of magnets operated by electric contacts made at close intervals by a clock.

58. Pens. The lateral movements of the several pens or pencils are controld, sometimes by electro-magnets, and sometimes by mechanical connections of various kinds, as may be convenient or appropriate.

59. The draw-bar pull was formerly measured by means of a mechanism similar to a weighing scale laid upon its side, so as to measure horizontal instead of vert forces. It is the general practice now, however, to have the draw-bar connected directly with a piston, sliding in a strong cylinder containing oil. A pipe leads from this cyl to a smaller cyl placed at the recording apparatus, and fitted with a piston and spring, as in the ordinary steam-engin "indicator." This then records directly upon the paper, by moving its pen more or less to one side, depending upon the force transmitted thru the oil, from the draw-bar. Another method makes an "indicator" of the main draw-bar cylinder, by fitting it with accurate but heavy springs, and by directly transmitting its movement mechanically to the pen of the recorder. Using either method, leakage of oil past the piston in the cyl must be compensated. This is usually done by means of a special pump.

60. Acceleration. Accelerometers usually depend upon the forward or backward deflection of a pendulum or other eccentrically suspended mass, under the force due to acceleration or retardation. Difficulty has been experienced in preventing violent and misleading oscillations without unduly delaying or restricting the motion. The Wimperis accelerometer, Mins Proc Inst C E (England), 1911-12, Part II, p 420, appears to give good results, the motions of the masses being dampt magnetically. Mr. Wilfred Lewis, of Philadelphia, has used an instrument similar to the common leveling tube; and the behavior of the bubble seems fairly satisfactory; but this device can make no record. Another instrument, experimented with by the Trautwine Company, cannot possibly over-indicate, no matter how sudden the changes of accelerating force.. The apparatus (carried on the car) depends upon the constant dropping of a stream of particles, such as sand, steel balls, or drops of water, from a given point in the apparatus, thru a given height, onto a scale placed at the bottom of the apparatus and lying in the direction in which the accel is to be measured. The particles are releast in rapid succession, and each particle, falling freely, must continue falling with the hor vel it had when releast. Therefore, any change of vel of the car, which occurs during the fall of any particle, shifts the scale under it more or less, so that the particle falls upon the scale at a greater or less distance from the zero (which is immediately under the point from which the particles are releast), in direct proportion to the av accel during the time of falling.

61. Strictly speaking, an accelerometer can measure *acceleration only* when *running* on a *level*. With a train on a *grade*, the effect of the grade increases or diminishes the indications, and, unless correction can be made for this, the indication of accel is correspondingly misleading. On a car or train running without power or brakes (on *any* grade), an accelerometer's indications become a measure of the total *resistance* of the car, or train, as a whole.

62. Lateral forces, due to curvs or inequalities in the track, are frequently measured by an accelerometer placed at right angles to the track, especially on test cars.

63. Time should be recorded when the movement of the paper is proportional to the dist traverst, and a time record is useful even when the paper is moved at a time-rate. It is usually made by a pen deflected by an electro-magnet actuated once every five seconds by contacts in a clock.

64. Distance should be recorded if the paper is made to travel at a time-rate, and a dist record is useful even when the paper is moved in proportion to the *distance*. It is made by a pen, deflected by an electro-magnet, or by gearing from the wheels, every time a certain dist has been traverst. In any case, dist is usually recorded also by an observer using a push-button by means of which he can deflect another pen upon the passage of each mile-post or other object.

65. Speed may be deduced from the records of distance if the paper is moved at a time-rate or *vice versa*. It is, however, preferably plotted by a special pen, which, as the velocity changes, may be deflected from its datum line by mechanical connection with a device similar to a centrifugal governor, or by some other type of speed recorder; or the pen may be so arranged as to be drawn aside from the datum line by means of an electric contact or gearing from the wheels, proportionately with the dist traverst, releast at the end of a given time interval, and allowd to fly back to the datum line, thus giving a record of *average* velocities for successiv short intervals of time. This last device should require no calibrating. Using an additional refinement, only the stepping-up or counting mechanism returns to zero, while the pen remains at the point previously reacht, thus plotting a stept curv instead of a series of ordinates.

66. Air-brake operation is usually recorded by a pen deflected by a piston in a small cylinder connected with the brake cyl of the car.

67. Boiler pressure has been similarly recorded by means of a steam pipe connected with the loco.

68. Wind velocity has been recorded (at least on the University of Illinois Test Car) by the usual Weather Bureau Robinson cup-type anemometer, making and breaking electric contact with every 0.2 mile of wind.

69. Wind direction is recorded (on the same car) by mechanical connection with a wind-vane, whereby the deflection of the pen is kept = to the sine of the angle of the vane, right or left, while the forward or rearward position of the vane is indicated by another pen.

70. Curvs may be recorded by mechanical connection with one of the trucks.

71. Miscellaneous data are largely recorded by hand, by means of push-buttons on the car or loco; this method being used for such items as shovelfuls of coal put into fire-box, opening and closing of fire-box door, opening and closing of throttle, position of reverse lever, etc.

TRAIN DYNAMICS

GENERAL

Tractiv Force, Resistances, Speed, Energy.

1. The movements of a train are governd by the existing relation betw the tractiv force and the resistances. This relation is partly under the control of the enginman. See ¶ 4.

2. Tractiv Force. With a given loco, tractiv force (p 1056) is increast by increast steam admission to the cylinders (as by increasing the boiler output or the throttle opening, or by delaying cut-off), and by increasing the "adhesion" betw drivers and rails (as by sanding the rails). The tractiv force is diminisht by diminisht steam admission to the cylinders and by slipping betw the drivers and the rails.

3. The resistance is increast by the application of brakes or by reversing the loco engins; and is diminisht by releasing the brakes.

4. In practice, however, these operations are complicated by other phenomena. Thus, a material *speed increase* (as by increast steam admission to the cylinders) increases the difficulty of maintaining the increast cyl pressures; while, on the other hand, *as a train slows down,* it becomes easier to maintain said pressures.

5. Again, under certain conditions (Resistance, pp 1057, etc.) the frictional and atmospheric resistances increase when the speed increases. As a result of this, each grade has its limiting velocity, which cannot be exceeded by a descending train, left to itself upon such grade. See pp 1087-8, ¶¶ 37-38.

Effect of inertia.

6. Energy. When speed is being increast, kinetic energy is being stored in the train, and such storage (except when produced by a down grade) requires additional work from the loco. When speed is being diminisht, the kinetic energy, already stored in the train, assists its forward motion

7. For any change of speed, as from v_1 to v_2, the change, in kinetic energy, is

$$W \frac{v_1^2 - v_2^2}{2g} = m \frac{v_1^2 - v_2^2}{2} \text{ ; where}$$

W = the weight, and m = the mass, of the train; and g = gravity acceleration, = say 32.2 ft per sec per sec.

8. The virtual head or velocity head, = the height thru which a body must fall, in order to change its vel from v_1 to v_2, is

$$h_v = \frac{v_1^2 - v_2^2}{2g}. \quad \text{Hence,}$$

$$\text{Change in kinetic energy} = W \frac{v_1^2 - v_2^2}{2g} = W h_v.$$

Where either v_1 or v_2 = zero, we have, vel = v, and
Energy change = $W v^2/2g$ = $W h_v$.

GRADES

9. In considering the establishment of grades, their *effect upon train velocities* is of vital importance.

Virtual Profile.

10. Virtual head; virtual grade; virtual profile. In Figs 1*a*, 1*b* and 1*c*, let *A B C...G* be a given railroad profile; and, in each Fig, let a train, coming from the left, reach *A* with a vel of *v* ft per sec. Let *A a* represent, by the scale of the actual profile, the corresponding or virtual head, $h_v = v^2/2g$. For simplicity, we here neglect the rotational vel of the wheels; see p 1064. At other points, in either Fig, erect ordinates, *B b*, *C c*, *D d*, etc, representing, by the same scale, the virtual heads at those points respectivly. Then, for the condition, see ¶ 13, represented by either Fig, *a b c . . . g* is the virtual profile corresponding to the actual profile, *A B C . . . G*.

11. Let

T	=	the loco tractiv force	parallel to the track
F	=	the total or resultant resist	" "
t	=	$T - F$ = the resultant force	" "
	=	the force causing the accel	" "
F_s	=	the gravity component	" "
F_r	=	the nongravitational resist	" "
		("+" = with T; "−" = against T)	" "
W	=	train weight	acting vert downward
g	=	gravity acceleration	" " "

12. As will be shown, the behavior of the train is governd by the adjustment (partly within the control of the enginman) betw T, F, F_s and F_r.

If T is		F_s is	$T + F_s$ is	F_r is	$F =$	$t\ (= T - F) =$
↗	$< F_s$	−	−	+	$F_s - F_r$	$T - (F_s - F_r)$ $= T - F_s + F_r$
↗	$> F_s$	−	+	−	$F_s + F_r$	$T - (F_s + F_r)$ $= T - F_s - F_r$
→		0	+	−	F_r	$T - F_r$
↘		+	+	−	$F_r - F_s$	$T - (F_r - F_s)$ $= T + F_s - F_r$

13. Figs 1a, 1b and 1c represent three supposable and typical cases, as follows:—

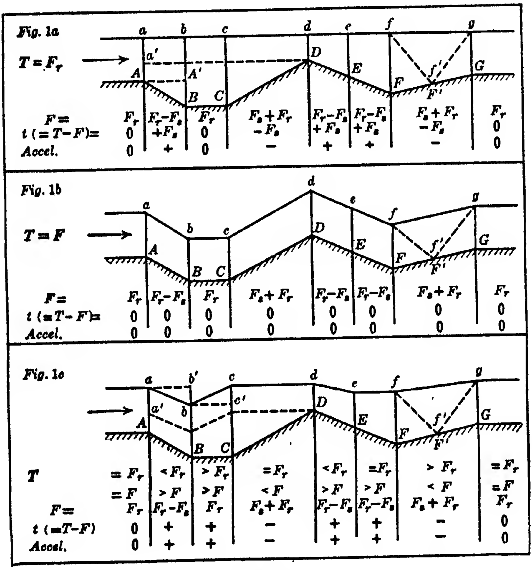
Fig 1*a*. $T = F_r$. Virtual profile horizontal;
Fig 1*b*. $T = F$. " " parallel to actual profile;
Fig 1*c*. Neither of the two foregoing conditions satisfied.

14. Fig 1a. "Frictionless train." We here assume that the tractiv force is maintaind thruout equal to the non-gravitational resistance; or $T = F_r$. The train is then mechanically in the condition of a frictionless body; *i. e.*, all vel changes are those due to gravity alone.

15. Then, in passing from *A* to *B*, the train loses *A' B* in elevation, but gains an equal amount in virtual head due to increase of vel, making its total virtual head, at *B*, $h_v = B\ b = BA' + A'\ b$. Hence, the virt profile, *a b*, betw *A* and *B*, is horizontal, as is also (in Fig 1*a*) the virtual profile, *a b c....g*, for the entire line, *A B C....G*, assuming (as above) that T and F_r are maintained

equal thruout. This is apt to be at least approx the case with light and fast passenger trains on short and moderate grades. Such a condition would continue indefinitely (assuming T maintained $= F_r$), except that if, at any point, the actual grade, $A \ldots . G$, rises to the level, $a \ldots . g$, the train comes to rest, and cannot start again without increase of T over F. See ¶ 21.

16. Fig 1b. Velocity uniform. If the tractiv force is maintained equal to the total resistance (which includes the gravity component), *i. e.*, if $T = F = \pm F_s \pm F_r$; so that $t = T - F =$ zero; then the vel (and, consequently, the vel head, h_v, represented



Figs. 1.

by $A\,a$, $B\,b$, etc) remain constant, and equal to those of the train when arriving at A; and each stretch, $a\,b$, $b\,c$, etc, of the virtual profile, $a\,b\,c \ldots . g$, is parallel with the corresponding stretch, $A\,B$, $B\,C$, etc, of the actual profile, $A\,B\,C \ldots . G$. On *level* stretches, as $B\,C$, we have $F_s =$ zero, $F = F_r$, and, in Fig 1b (as thruout Fig 1a), $T = F_r$, and $b\,c$ horizontal.

17. Fig 1c. Usual case. In general (and especially with heavy frt trains) the relation betw T, F_s, F_r and F, adjusted by the engineman (see ¶¶ 1, 2, 3) to suit existing conditions, fulfils neither the conditions of Fig 1a nor those of Fig 1b.

18. In the assumed case, represented by Fig 1*c*, we have:—

In Fig 1*c*

Section	Grade actual	Grade virtual	Tractiv force, T	Virtual and actual grades	$t\ (= T - F)$ and acceleration
. . . . . A	→	→	$= F_r = F$	parallel	0
$A - B$	↘	↘	$< F_r > F$*	divergent	+
$B - C$	→	↗	$> F_r > F$	"	+
$C - D$	↗	→	$= F_r < F$	convergent	−
$D - E$	↘	↘	$< F_r > F$*	divergent	+
$E - F$	↘	→	$= F_r > F$	"	+
$F - G$	↗	↗	$> F_r < F$	convergent	−
G	→	→	$= F_r = F$	parallel	0

In general; Figs 1*a*, 1*b*, 1*c*,

If the virtual grade is	The tractiv force, T, is	If the virtual and actual grades are	The velocity is
up	$> F_r$	divergent	accelerated
level	$= F_r$	parallel	uniform
down	$< F_r$	convergent	retarded
		coincident	zero

19. The dotted profile, $f\,f'\,g$, in each of the three figures, represents a case where, instead of proceeding continuously from F to G, the train stops, as at F'; *i. e.*, the case where the vel becomes zero, and the two grades coincide.

20. Different Profiles for different trains. Since trains differ as to speed, etc, each train may require its own separate virtual profile; thus, Fig 1*a* represents approximately the probable conditions for fast passenger trains; Fig 1*c* those for freight trains, etc.

21. The virtual profile shows the speeds, and the speed changes, which may be expected of a given train on a given profile. Thus, Fig 1*a* or 1*c*, if, at A, the vel were such that the vel head was only $A\,a'$, the train would stall at D, provided the other conditions remaind as shown; the virtual profile, $a\,d$, in that case, being simply lowered vertically and parallel with itself, thru the dist $a\,a' = d\,D$. See ¶ 15.

*On a down grade, the *total* resistance, F, becomes F_r minus F_s (see ¶ 12); and may thus be very small, or zero, or even a minus quantity. Therefore the loco tractiv force, T, even if less than F_r, may yet be greater than the "total" resistance, F.

22. Head-gain. Fig 1c. In any given dist, as $B\,C$, the non-gravitational head-gain, h_v, (or head-loss, as the case may be), represented by $c'\,c$, is directly proportional to the unbalanced non-gravitational force, t_r ($= T - F_r$ or $F_r - T$), acting upon the train thruout that dist; and the ratio, betw said unbalanced force, t_r, and the train weight, W, is equal to the ratio betw h_v and the dist, L, represented by $B\,C$. In other words,

$$\frac{t_r}{W} = \frac{h_v}{L}; \quad \text{or} \quad t_r = W\frac{h_v}{L}.$$

23. Profile steepness measures force excess. Fig 1c. Similarly, in the distance represented by $A\,B$, there is an excess ($= t_r = F_r - T$) of non-gravitational resist over tractiv force; and the increase in vel and in vel head is therefore less, in Fig 1c, than in the same stretch, $A\,B$, in Fig 1a, where $T = F_r$. The corresponding loss, h_v, of vel head, in $A\,B$, Fig 1c, as compared with Fig 1a, is represented by $b'\,b$. Here also

$$t_r = W\frac{h_v}{L}.$$

The steepness of any line in the virtual profile is thus a measure of the excess of tractiv force over non-gravitational resistance, or vice versa.

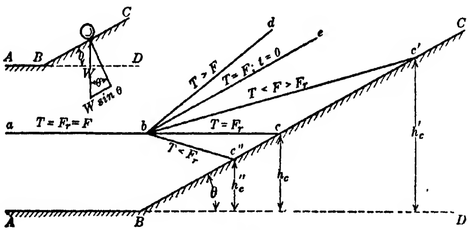


Fig. 2.

The height to which a train will ascend a grade.

24. Fig 2. Suppose a train running, at uniform speed, on a level track, $A\,B$. Then (the speed being uniform), we have, **on A B, T = F**; or t ($= T - F$) $=$ zero (¶ 18); and ($A\,B$ being level), we have: gravity component, F_s, $=$ zero; and tractiv force, T ($= F$) $= F_r + F_s = F_r + 0 =$ the nongravitational resistance, F_r

25. At B, let this train encounter an up-grade, $B\,C$; and let

θ = the angle, $C\,B\,D$, between the grade, $B\,C$, and the horizontal;
t = $T - F$ = net tractiv force
  = loco tractiv force minus total or resultant resistance;
W = weight of train and locomotiv;
F_s = $W \sin\theta$ = the gravity component;
g = gravity acceleration = 32.2 ft per sec per sec;
v = train velocity at B.

26. Now, if, **on the grade, B C** (in spite of the addition of F_s to the resistance) T is so maintaind that we have

T > F (virtual grade, $b\,d$, *diverging* from actual grade, $B\,C$),

then (see ¶ 18) the vel is accelerated, and accel $=$ $g\,t/W$.
If

T $=$ **F**, t $=$ 0 (virtual grade, $b\,c$, *parallel* with actual grade, $B\,C$), then the vel is uniform (accel $= g\,t/W =$ zero). In either of these cases, the train ascends the grade indefinitly.

27. But if, on the grade, $B\,C$, we have

T $<$ **F** (virtual grade, $b\,c$, $b\,c'$ or $b\,c''$, and actual grade, $B\,C$, *converging*),
then the vel is retarded, with negativ accel $= -\, g\,t/W$, and the unbalanced retarding force, $-\,t = T - F$, brings the train to rest
in a height, h_c, h_c' or h_c'', and
in a dist, $h_c/\sin\theta$, $h_c'/\sin\theta$, or $h_c''/\sin\theta$.
determined as follows:—

28. (1) If, on the grade, $B\,C$, we have

T ($<$ **F**) $=$ $\mathbf{F}_r$ (virtual grade, $b\,c$, *horizontal*),
then the train (see ¶ 14) moves under the action of the gravity component, $t = F_s = W\sin\theta$, alone, with neg accel $= -\,g\,t/W = -\,g\,W\sin\theta/W = -\,g\sin\theta$. Hence, the unbalanced retarding force, $-\,t = F_s = W\sin\theta$, brings the train to rest
in a height, h_c $=$ $v^2/2g$, and
in a dist, $B\,c$ $=$ $h_c/\sin\theta$.

29. (2) But if, on the grade, $B\,C$, we have either
(a) **T** ($<$ **F**) $>$ $\mathbf{F}_r$ (virtual grade, $b\,c'$, *ascending*), or
(b) **T** ($<$ **F**) $<$ $\mathbf{F}_r$ (" " $b\,c''$, *descending*) ;
then the unbalanced retarding force, $-\,t = T - F$ will bring the train to rest

in a height, (a) $h_c' = h_c\,W\sin\theta/t'$, or
(b) $h_c'' = h_c\,W\sin\theta/t''$;
in a dist, (a) $B\,c' = h_c\,W/t' = h'_c/\sin\theta$; or
(b) $B\,c'' = h_c\,W/t'' = h_c''/\sin\theta$.

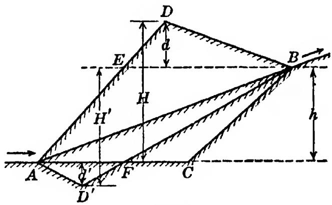


Fig. ?.

Work due to elevation differences.

30. Fig 3. Theory. In moving a train, of weight, W, from A to B (whether thru the uniform grade, $A-B$, or via $A-C-B$, or via $A-D-B$, or via $A-D'-B$), the net or resultant work done, due to elevation diff, alone, and neglecting friction, is theoretically $W\,h$, or the work of lifting the wt, W, thru the vert height, h.

31. In traversing $A-B$, or $A-C-B$, the train does not rise above B, or descend below A; and the *total* work, due to elev diff, is equal to the *net* or *resultant* work, $W\,h$.

32. In traversing $A-D-B$ or $A-D'-B$, the train wt, W, must indeed be lifted thru the greater height, H or H', respectivly, making the *total* work, due to elevation diff, $=$ $W\,H$, or $W\,H'$, respectivly; but, theoretically, the additional work, $W\,d$ or $W\,d'$, entaild by the lift, $d = H - h$, or $d' = H' - h$, respectivly, is compensated by that given out by the train on the down-grade, after passing the summit, D, or before reaching the lowest point, D'.

33. Practice. Hence, the resultant work, due to elevation diff, remains, mechanically, $= W h$; and this is frequently realized, more or less approximately, in practice; the energy, due to the down grades, being utilized to an extent depending upon the lengths and steepnesses of the grades, and the resistances upon them, and depending also upon the grade beyond *B*, and upon whether a stop is made at *B*, or at the foot (*D'*) of the down grade.

34. Any rise, *E—D* (or fall, *A—D'*), which entails work in excess of *W h*, is accompanied by a fall, *D—B* (or rise, *D'—F*) of equal vert height. But the "rise-and-fall", in *E—D—B* (*A—D'—F*) is *d* (*d'*); and not 2 *d* (2 *d'*).

Practical Considerations.

35. Grades increase the operation cost.
On an up-grade, *A B* or *C B*, Fig 3, p 1075, the lift, *h*, involvs additional fuel cost. Speed reduction, on the grade, may diminish the total normal resistance (compare Fig 1 of Train Resistance, p 1058); but reduced speed means time-loss and therefore increast labor cost.

36. Descending grades. In descending any grade, there may be a reduction of expense for fuel over that required for running on a level, and some reduction in wages cost due to traversing a given dist in less time, but steep down grades are nevertheless troublesome and expensiv, as in wear of brake-shoes and of wheel-treads.

37. Effect of grade length upon tractive force. Upon a long up-grade, unless speed is reduced, any *surplus* boiler pressure, with which the locomotiv may have enterd the grade, is drawn upon and may be exhausted, within a dist depending upon the load and upon the steepness of the grade. Thereafter, the tractiv force is limited by the steaming qualities of the boiler; and it is to be expected that the mean tractive force, on the entire grade, will diminish as the length of the grade increases. Comparing various cases, Mr. Beverley S. Randolph finds, approx:

Length of continuous up-grade, miles:	0	5	10	15	20 and over	
Mean tractiv force =	31	24	19	17	16	% of wt on drivers.

The *steepness* seems not to affect these figures appreciably. Am Soc C E, Trans Vol 70, Dec 1910, p 323.

38. Humps and sags. Notwithstanding the foregoing, minor grades (even tho steeper than ruling grade) may often be properly introduced in the profile of a railroad. Their introduction may effect great economy in construction, thru the avoidance of heavy cutting and filling. For "ruling grade," see ¶ 11, p 1084.

39. An acceleration grade is a hump introduced for the purpose of aiding in stopping and starting trains at a station placed at the grade summit, or to accelerate or retard their speed at a given point. Such grades may reduce not only the wear and tear on wheel-treads and on brake-shoes, in slowing down, and the work required in regaining speed, but the time required in both operations. They are largely used on rapid transit lines, even where their use involves increast excavation for subways, or increast climbing, by passengers, to elevated stations.

40. A momentum grade is one which, however steep, is so short that any train, with reasonable velocity, may surmount it without losing too much of its speed. It is usually so steep that a train could not surmount it without "taking a run", or re-start if stalled upon it.

41. Momentum grades must therefore be introduced only after considering every probable or possible condition which might lead to stalling; such as grade crossings, curvature, sidings, water tanks, stations, etc, on, or just before reaching, the grade; or the possible future installation of such features. If the danger of stalling is slight; or if a train, when stalled, can readily be backt off, and run on again, and if the delay to following traffic is not serious; a momentum grade is permissible.

42. Up-grade limitations. *Starting up-hill.* Where trains must be started up-hill, the grade should be such (Webb) as to permit an acceleration of at least 15 mi/hr in 1000 ft, or a ruling grade should be reduced (Alaska Central Ry) by 0.2 ft per 100 ft.

43. *In cuts,* even where other conditions would permit the use of a level grade, it is usual to establish a grade of not less than 0.2 ft per 100 ft, for the sake of drainage.

44. One *railroad specifies* that, on *sidings*, no grade shall exceed 5% (264 ft per mile) on a tangent, or 3% (158.4 ft per mile) on a curv, or 1% (52.8 ft per mile) where cars are to be loaded.

45. Down-grade limitations. The speed, acquired on a down grade, depends partly upon its vert depth, and partly upon the grade rate. See Classification of grades, p 1087, ¶ 37. The drop should never be such that freight trains (left to themselvs) will attain speeds exceeding say 30 to 35 mi/hr; nor should it reduce the speed of ascending freight trains below 10 mi/hr.

46. Vertical curvs should be employd wherever there is any markt grade change. They prevent forward and backward impacts, due to taking up and giving out slack of couplings betw cars, and avoid the resulting danger of breaking the train in two.

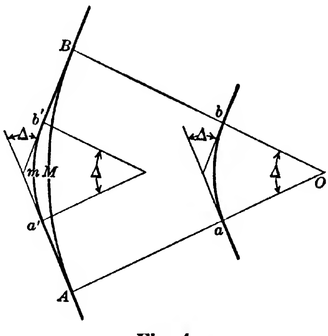


Fig. 4.

CURVS

47. In Fig 4, assuming curv resistance proportional to sharpness, D, or approx inversely as radius, R (See Train Resistance, p 1062, pgf 40); let

	For curv $A\,M\,B$	For curv $a\,b$
n = ...	R/r	R/r
Sharpness = approx.......	$d = D/n$	$D = n\,d$
Radius = ...	$R = n\,r$	$r = R/n$
Curv resistance = approx..	$f_c = F_c/n$	$F_c = n\,f_c$
Curv length, in 100-ft chains.	$L = n\,l$	$l = L/n$

Sweep = ... $\Delta = L\,d = n\,l \cdot D/n = l\,D$

48. Energy. Then, for the energy, E_c, expended (= work done) in balancing curv resistance alone, thru either of these two curvs of equal sweep, Δ, we have:—

$$E_c = F_c\,l = n\,f_c\,L/n = f_c\,L.$$

In other words, the curvature work is the same in both cases; and, plainly, if either of these curvs were shortend or lengthend, its sweep, Δ, and the curvature work required, would be proportionately affected.

49. Work proportional to sweep. Hence, assuming (as above) $F_c = n f_c$, we find that the work, due to curvature alone, on *any* curv, is proportional to its sweep, Δ, and is independent of its sharpness, D, or radius, R.

50. But, comparing the two lines, $A\ M\ B$ and $A\ m\ B$ of equal sweep, (the latter including the tangents, $A\ a'$ and $b'\ B$), we have: length $A\ m\ B$ > length $A\ M\ B$; and hence non-curvature work on $A\ m\ B$ > that on $A\ M\ B$; and, as we have seen (¶ 49), the *curvature* work is *equal* on the two lines of equal sweep, Δ. Hence, *total* work on $A\ m\ B$ (= noncurv work on $A\ m\ B$ + curv work on $a'\ m\ b'$) > *total* work on $A\ M\ B$.

Grade Compensation for Curvature.

51. On account of the added resistance, due to curvature, it is customary (where practicable) to reduce or flatten up-grades upon which curvature occurs, in order that their *total* resistance (grade + curv) shall not exceed that of a *tangent* on the original grade.

52. The compensation rate (grade-reduction rate) is the grade reduction for each degree of sharpness, D. Thus, "0.04" means a grade reduction of 0.04 per cent of grade for each degree of sharpness, D; so that, for a 5° curv, with 0.04 compensation, on a grade of 0.7 per cent, we should have.—

compensated or reduced grade

$$= 0.7\% - (0.04 \times 5)\%$$
$$= 0.7\% - 0.2\% = 0.5\%$$

53. Limiting grades (see p 1084, ¶ 11) should always be compensated, if possible; since the limiting grade, on a *tangent*, offers the maximum resistance which the standard locomotiv is expected to overcome; and any *added* resistance (as from curvature) would therefore endanger the stalling of each train.

54. Non-limiting grades (see p 1084, ¶¶ 12-14) need not be compensated beyond an extent which renders the compensated grade equivalent to the limiting grade. In many cases, this will require no compensation on non-limiting grades; but, on any non-compensated grade, the presence of the curv involvs a cost increase, due either to speed reduction or to increase of fuel expenditure. Compensation is therefore always desirable, even where not actually necessary

55. Compensation, carried to such an extent that it results in a *down*-grade, is unfavorable to traffic in the opposit direction, but this consideration is not necessarily prohibitory. Thus, the return traffic may be light. as where coal or ore cars return empty.

Practice.

56. Rate. Some authorities consider a compensation rate of 0.03 per cent, per degree of sharpness, excessiv, while others consider 0.04 insufficient. The mean of these, = 0.035, is quite generally accepted as a fair average, with modifications, as noted below, for special conditions.

57. Differences in track condition, including tightness of gage, and superelevation, apparently account for much of the diff in practice.

58. Higher rate. Factors tending to increase the rate of compensation are:—the predominance of heavy, low-speed freight trains; the probability of trains being required to slow down or even to stop and stand for some time on the curv; the existence of a stopping place immediately *before* a curv, in which case compensation as high as 0.10 per cent of grade per degree of sharpness has been recommended (Webb); probable dampness in tunnels, etc.

59. Lower rate. On the other hand, lower rates of compensation may be used where high-speed passenger trains predominate; and where there is a regular stopping place immediately *beyond* the curv, in which case the curv resistce may be utilized, along with the full grade, to assist in stopping the train.

60. Change of Rate. Some authorities suggest that the compensation rate should diminish as the sharpness increases; while others hold that the compensation rate should be proportional to the sharpness.

Maximum Sharpness

61. Theoretically, the max permissible sharpness is that which, at max permitted speed, requires the max permitted superelevation.

62. In Fig 5, the eight lower diagrams show the max permissible sharpnesses for diff speeds, and for superelevations of from 1 to 8 inches, on standard gage = 4 ft, 8.5 ins, according to equation 118, Curvs, Superelevation, p 964, ¶ 176; E, ins, $= V^2D/1458.6$, whence $D = 1458.6\ E/V^2$. The upper diagram givs the sharpnesses required by Mr. A. M. Wellington's proposed rule (Ry Location, p 270; Am Ry Eng & M W Assn, Procs, 1910, Vol II, Part I, p 681.) Centrif force $\not>$ Weight/4.

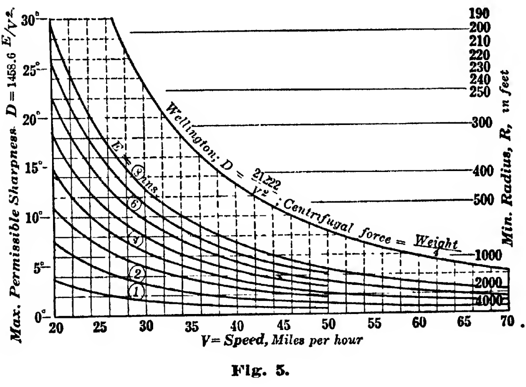


Fig. 5.

63. Practically, max sharpness is governd also by the character of the country and by the traffic. Camp ("Track," p 257) mentions the following as indicating current practice for standard-gage main-track service:—

D	R, ft	
> 4°	< 1433	considered sharp;
10°	574	" very sharp;
15°	383.1	practical limit for full speed;
20°	287.9	used, on compulsion, with speed reduction;
22°	262.0	used, on compulsion, with speed reduction.
We add		
12°	478.3	standard max sharpness (with spirals) **Alaska** Centl Ry, 1909;
14°	410.3	standard max sharpness (with spirals) **N Pac** Ry, 1903.

Curvs as sharp as 67° (90 ft radius) have been used on the elevated railways of New York City.

64. For sidings, *the following* sharpnesses are in use:—

D	*R, ft*	
17°	338	where large road locos must place cars on sidings;
29°	200	for coupled furniture cars;
39°	150	for coupled coal cars and for box cars pusht by 4-wheel or 6-wheel shifting locos;
95°	60	for cars drawn by horses or by short shifting locos on warehouse sidings, etc, with very low speeds.

65. Gard rails are commonly used with curvs of 16° sharpness (R = 359.3 ft) and sharper.

Minimum Sharpness, etc

66. It is sometimes specified that sharpness shall *not* ordinarily be *less* than 1° ($R = 5730$ ft) except where sweep, Δ, $< 3°$; that **curv length** shall be $\nless$ 300 ft; and that a **tangent,** $\nless$ 500 ft long, shall be used betw two curvs in the *same* direction, and a tangent, $\nless$ 600 ft long; betw two curvs in *opp* directions.

See also Gard rails, ¶ 65; Gage on curvs, p 811; Superelevation, p 963.

TRAIN OPERATION COST

GENERAL

1. **The design** and construction of a railroad and of its rolling stock (like its profitableness) depend largely (1) upon the amount and character of the traffic to be expected, (2) upon the resulting revenue, (3) upon the funds available for construction, and (4) upon the expenses (fixt charges plus operation cost). The first three items are usually estimated by the promoters; but the estimation of the construction and operation costs devolvs largely upon the engineer.

2. **The construction and operation costs** depend largely upon
(1) the topography,
(2) the availability of the necessary supplies of material and labor,
(3) the character and amount of the expected traffic,
(4) the character and amount of the proposed rolling stock, (motiv power and cars),
(5) the amount of money available for construction.

3. **The Problem.** Informed as to these features, the engineer is concernd chiefly with fitting, to the topography of the country, that line which best meets the requirements, the final criterion (from the investors' standpoint) being the rate of return upon the money invested.

4. **The comparison between two proposed lines,** *A* and *B*, may be based either (1) upon their respectiv construction costs, plus the capitalized values of their respectiv operation costs; or (2) upon their respectiv operation costs, plus the annual values of their respectiv construction costs. Thus, we may suppose a case where the costs per mile of the two lines, *A* and *B*, compare as follows:—(capitalization at 6%)

	(1) Capitalized		(2) Annual	
	A, $	*B*, $	*A*, $	*B*, $
Construction	25,000	20,000	1,500	1,200
Operation	100,000	125,000	6,000	7,500
Total	125,000	145,000	7,500	8,700

In this supposed case, for a given revenue, line *A* is preferable, notwithstanding its greater construction cost. For Railway Construction Cost Estimates, see p 1094.

TRAIN-MILE, AND TRAIN-MILE COST

5. **The train-mile, "t-m".** When a train (of any length, weight and character) is run (under any conditions) thru one mile, the result is a train-mile (t-m).

6. **Train-mile cost, m.** For any given series of operations (as for the operations of any one line for a day, or for all the railroad operations of a country for a year) the average **t-m cost** is

$$m = \frac{\text{total operation cost}}{\text{number of t-ms run}}$$

Similarly, the t-m cost, due to any given item or items of operation, is

$$\frac{\text{operation cost due to said item or items}}{\text{number of t-ms run}}.$$

7. Average T-m Cost in U. S. From the 24th Annual Report of the Interstate Commerce Commission, see Table 1, below, we have, as the av total t-m cost, for the three classes of lines considerd, for 1911:—

Class		Gross annual operation cost	E = total annual operation cost (Statement 41)	T = total annual train-miles run (Statement 36, p 49)	$m = E/T$ = t-m cost
I	≮	$1,000,000	$1,844,065,958	1,185,632,129	$1.555
II	< >	$1,000,000 $ 100,000	$ 57,092,361	42,315,853	$1 349
III	≯	$ 100,000	$ 13,472,094	9,552,156	$1.410
Totals and average			$1,914,630,413	1,237,500,138	$1 547

The I. C. C. reports for 1890-9 gave values of m ranging only from $0.918 to $0.984 ; avge $0.952

In our subsequent discussions, we assume m = $1.50

8. Departures. The train-mile (t-m) cost, m, varies widely, not only (from time to time), with fluctuation of unit costs, but (at a given time), as betw diff lines. See ¶ 7. Nevertheless, with given unit costs, the t-m cost, on a given line, is practically constant, because the work, required to move a train over the line, is fixt, within narrow limits, by the fact that economy forbids the use of less than the max practical train-load, and this is determined by the character of the line. See ¶ 10.

Train-mile-cost distribution.

9. Table 1 gives the percentages of the t-m cost, assignable to the diff items making up that cost, as determined by the Interstate Commerce Commission. Estimates of these percentages vary widely; but the percentages, given in Table 1, represent the av of American practice, and thus form a useful gide, subject to modification in cases where special conditions cause notable departures from normal conditions.

Operation Cost

Table 1

Deduced from the 24th Annual Report of the Interstate Commerce Commission (for the year ended June 30, 1911) on the Statistics of Railways in the United States; issued by the Government Printing Office, Washington, 1913.

Railroads of	Class I *		Class II *	Class III *
Miles of line	215,146		19,120	9,167
Miles of track	328,800		22,989	11,036
Analysis of Operation Expenses				
	a, = % of m		a, = % of m	a, = % of m
Maintenance of Way & Structures				
Superintendence	0.963	0.963	1.218	1.508
Roadway and track				
Ballast	0.423			
Ties	2.992			
Rails	0.897			
Other track material	1.021			
Roadway and track†	7.240	12.573	17.801	22.406

(Table Concluded on next page.)

*See ¶ 7, above.
†See * foot-note, p 1083.

(Table Concluded from preceding page.)

	Class I		II	III
	a_s = % of m		a_s = % of m	a_s = % of m
Track structures				
Tunnels, bridges, etc	1.682			
Crossings, signs, fences, etc.	0 369			
Signals, telegraph, etc	0.759	2.810	3.373	3.323
Buildings, docks & wharvs	1.943	1.943	1.679	0.920
Miscellaneous	0.583	0.583	0 660	0 804
Total, M W & Structures		18.872	24.731	28.961
Maintenance of Equipment				
Superintendence	0.661		0.865	0.722
Repairs				
to locomotivs	8.058		6.386	5.836
to cars	9 178		6.381	4.720
to floating equipment	0.050		0.024	0.008
to work equipment	0.223		0.210	0.136
Renewals to equipment	0.859		0.304	0.538
Miscellaneous	0.806		0.793	0.463
Depreciation of equipment	2.702	22.537	3.751	3 109
Total, Maint of Equipment		22.537	18.714	15.532
Traffic (Agencies, advertising, etc)	3.116	3.116	2.540	1.882
Transportation				
Superintendence & despatchg	2.183			
Station employees	7.062	9.245	9 373	8.248
Yard conductors & brakemen	2.869			
Yard enginmen	1 618			
Yard loco, fuel	1 601			
Yard & station operation miscel	3 085	9 173	5.461	1.962
Road loco, fuel	10.454	10.454	11.935	12.240
Road enginmen	6.256			
Road loco, other expenses	2 969	9.225	9.094	9.720
Road trainmen	6.642			
Train supplies & expenses	1.782			
Opn of interlockers & signals	0.509			
Train & signal miscel	0.415	9.348	7.749	7 818
Miscellaneous	4.307	4.307	3.998	3.441
Total Transportation		51 752	47.610	43.429
General (Administration, insurance, etc)	3 723	3.723	6.405	10.196
Total		100.000	100.000	100.000

*Applying ballast, ties, rails and other track material; track maintenance; care of roadbed; general cleaning; patrolling and watching; changing alinement and grades; bank protection; filling; train service, etc.

COMPARISON OF LINES

Limiting and Non-Limiting Factors

10. In comparing alternative lines, the factors to be considerd, as chiefly affecting operation costs, are *diffs in grade*, in *curvature* and in *length*. Taken collectivly, and considerd with regard to their effect upon operation cost, grades and curvs fall into two main classes, which must be carefully distinguisht, viz.—limiting and non-limiting, or ruling and non-ruling, or "major" and "minor", grades and curvs.

11. The ruling or limiting grade or curv, or limiting combination of grade and curvature, on a given loco division, is that which affects operation cost by *limiting the length and weight of train* which, under normal working conditions and without "momentum", one standard loco can take over the division, and by thus *limiting the minimum number of trains* (and thus the minimum *work*) required for a given traffic.

12. Total Work. With given motiv power, the *maximum train-weight,* which can be hauld over a given line (while practically independent of the *length* of the line) is thus determind by some limiting or controlling (or *"major"*) condition of grade or of curvature, or of both combined; but the total work required, to haul this max train-wt over the line, may be affected by non-limiting or "minor" conditions as to length, as well as by those of grade and curvature.

13. Thus, suppose that, with a given locomotiv, a long 12° curv (unavoidable on each of two alternativ lines, *A* and *B*) renders 500 tons the max train-wt which can be hauled over either line. Then that curv is a limiting or "major" factor. But, if line *B* has also one or more 4° curvs, or short grades, which are absent from line *A*, or if line *B* is longer than line *A* (the two lines being otherwise similar), then line *B* requires more work than does line *A* to haul the given 500-ton train over the road; and this excess of work givs line *B* the higher operation cost.

14. We thus have:—

	Affecting	Not affecting
Grades and curvs		
Limiting	max train-weight	work on max train
Non-limiting	work on max train	max train-weight
Length differences..........	" " "	" " "
Pusher grades.............	" " "	" " "

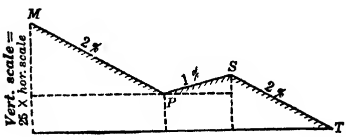


Fig. 1.

15. Steepest vs Limiting Grade. The *limiting* grade is not necessarily the *steepest* grade on the division. Thus:—

(1) A short grade, steeper than the ruling grade, may be surmounted by the standard train and loco, by means of "momentum" (See ¶ 40, p 1076).

(2) Fig 1. Where, for instance, loaded cars move from *M* to *T*, while they return empty from *T* to *M*, the *lighter and shorter* grade, *P S*, opposed to the *heavier traffic*, may be the ruling or limiting grade.

(3) The limiting condition may be a combination of moderate grade with heavy curvature.

16. Sharpest vs Limiting Curv. Similarly, the limiting curv, on a division, may be a long and comparativly easy one. Again; the limiting effect of a given curv may be eliminated by grade reduction. See p 1078, ¶ 51, etc.

For analysis of effect of limiting grades and curvs upon operation cost see ¶¶ 19 to 24.

17. Non-limiting factors are:—

(1) Length-differences;
(2) Pusher grades;
(3) Non-limiting $\begin{cases}\text{grades}\\ \text{curvs}\end{cases}$

Computation

Method of Procedure

18. In comparing the operation costs of two lines, as affected by limiting or non-limiting diffs in grades, in curvature or in length, we determin or estimate (compare Table 2):

1 The normal train-mile ("t-m") cost of each operation item;

2 The percentage, a, which each of these item costs forms of the total t-m cost, m (see Tables 1 and 2);

3. The extent (percentage, b, of a) to which the cost of each item is affected by the grade-diff, curvature-diff, or length-diff in question;

4. The resulting change ($a' = b\,a$, percentage of m) in the percentage value, a, of each item, due to the diff in question;

5 The sum, A', of the values of a', obtaind under "4"; *i. e.*, the total t-m cost of the diff in question, exprest as a percentage of the total normal t-m cost, m;

6. $m' = A'\,m$ = the t-m cost of the diff in question, exprest in dollars;

7. C_a = the annual cost of the diff in question; ¶ 43.
8. V_c = the capitalized value of the diff in question.

Example of Method

Construction of first column ("Add'l train") of Table 2.

Limiting Factors.

19. Example. Under Maint of Way & Structrs, let

t = normal number of daily round trips;	t' = number of add'l daily round trips;
m_s = supt t-m cost of normal trips, t;	m_s' = supt t-m cost of add'l trips, t';
$a_s = 100\,m_s/m$	$a_s' = 100\,m_s'/m$
= m_s as percentage of m;	= m_s' as percentage of m;
S = normal total daily supt cost;	s = increase in S, due to add'l train;

L = line length, in miles.

Let it be supposed that, with a given alinement and profile, 10 round trips per day ($t = 10$) are required to handle a given traffic, and that a proposed increase of grade or curvature, or both, due to a proposed change of line, would so reduce the max train-wt as to require 11, instead of 10, daily round trips for the same traffic. Then

$$t' = 11 - 10 = 1; \quad \text{and } t'/t = 1/10.$$

20. To find b. Let it be found or assumed that this increase of 1/10, in t, increases the total superintendence cost, S, (first item of Table 2) by only 1/100 or 1%.

Then

$$\frac{s}{S} = \frac{L\,t'\,m_s'}{L\,t\,m_s} = \frac{t'}{t} \times \frac{m_s'}{m_s} = \frac{1}{100}; \quad \text{and, (¶ 18, 4)}$$

$$b_s = \frac{a_s'}{a_s} = \frac{m_s'}{m_s} = \frac{s}{S} \times \frac{t}{t'} = \frac{1}{100} \times \frac{10}{1} = \frac{1}{10} = 10\%;$$

or, in this example, the t-m cost, m_s', of supt, for the additional train, is 10% of the normal supt t-m cost, m_s.

21. Effect on the other cost items. Then, averaging values of a for the three classes of road in Table 1, ¶ 9, using round figures for convenience, and applying assumed percentages, b (subject, of course, to wide variations under diff conditions), to the remaining items of operation cost, we obtain (merely by way of illustration of the method), the remaining values of a' ($=$ $b\,a$) given under "Add'l train" in Table 2.

22. Summing these values of a', we obtain A', $=$ 43.8, as the t-m cost of the add'l train, exprest as a percentage of the total normal t-m cost, m. Then

$$m' = A'm,$$

where m $=$ the total normal t-m cost, here assumed at \$1.50.

23. In this case, therefore, we have, as the t-m cost of the additional train required by the limiting factors,

$$m' = A'm = 0.438 \times \$1.50 = \$0.657.$$

24. To m', thus found, should be added a small amount (say from 1 to 1.5 cents) to cover interest on cost of each additional loco and tender required.

Other (non-limiting) factors.

25. In its remaining columns, Table 2 givs similar analyses for the remaining (non-limiting) operation-cost factors: viz.—length, pusher grades, rise-and-fall, and curvature. See below.

26. Length-difference. With a given max train-weight (determind by the limiting grade or curvature or both), a length-increase increases operation cost by entailing additional work on *each train, viz.—the work of hauling the max train-wt over the additional length.*

27. *Great and small length-differences.* On any length-diff (great or small), roadway and track maint cost per t-m will approx equal the normal cost of that item; but train-wages cost per t-m (which, on a great length-diff, may also equal the normal), are negligible on small length-diffs The effect of this upon the estimate may be illustrated by a supposed case, as follows; in which, assuming (for convenience) that roadway and track maint amounts normally to 15%, and train wages to 13% (compare Table 1) of the total operation cost, we assume probable values of b, find the resulting values of a', and thus compare the effects of great and of small length-diffs as affecting the sum of the costs of these two items alone.

	Normal. On entire line	On a great length-diff		On a small length-diff	
	a % of m	b % of a	a' % of m	b % of a	a' % of m
Roadway & track maint.	15	100	15	100	15
Train wages	13	100	13	0	0
% of total t-m cost, m, for these 2 items only,	28		28		15

28. Length diffs are sometimes *classified* as follows:—

	Length-difference Class		
	A	B	C
Train wages	*Not affected*	**Affected**	**Affected**
Number of stations or of sidings	*Not affected*	*Not affected*	**Affected**

The effects of length-diffs upon operation cost are indicated in Table 2.

29. *Effect of length-difference upon revenue.* Of two competing roads, betw two given points, either will lose thru business if it charges higher thru rates than the other. Hence, under competition, the rates, on both lines, betw two given points, are usually (other things equal) those proper to the shorter line, and shortness is therefore an important advantage; since the shorter line has the less operation cost.

30. But, in the absence of competition, freight and passenger rates are usually based upon the distance traveld; and the revenue of a line, betw two given points, is therefore practically proportional to length of line; whereas total operation cost usually increases less rapidly than does length of line. Hence, in the absence of competition, the longer line is in general the more remunerativ.

31. A pusher grade is a grade so steep, and so long, as to require regularly two locos or three locos (instead of one) for each train ascending it. Hence, where the pusher locos are of equal power with the road locos, the pusher grade is usually made about twice, or about three times, as steep as the limiting grade on the division, according to whether each ascending train requires two locos or three locos.

32. Theoretically, therefore, the first loco supplies, and each pusher supplies, only the power necessary to haul the max train up the limiting grade; and the work, required of each pusher outfit, is theoretically only that of hauling the max train up *an additional stretch* (= pusher-grade length) of the limiting grade.

For analysis of operation cost of pusher grades, see Table 2.

33. See Table 2. *Pusher grades affect* chiefly the items of
Roadway and track maintenance,
Repairs and renewals of locos, cars, etc,
Loco fuel and expenses.

34. The annual operation cost of a *pusher loco and tender* ranges ordinarily between $7,500 and $18,000, with $10,000 to $15,000 as more probable limits.

35. A non-limiting or minor grade is either less steep than the limiting grade (p 1084, ¶ 11), or so short that it can be surmounted, by "Momentum" (p 1076, ¶ 40), by one standard loco drawing the max train. It therefore affects operation cost by requiring additional work to lift the max train. Problems relating to non-limiting grades are usually called "rise-and-fall" problems.

A comparison of the definitions of s_e, h_d, h_e and L_s, ¶¶ 39 and 43, will show that the additional work, entaild by a minor grade, is equated with that of hauling the max train over *an additional stretch* of level track, whose length is such that the work of this level hauling, over it, is equal to that of lifting the train weight thru the vertical height of the grade.

For analysis of non-limiting grade problems, see p 1088, ¶ 39.

36. Similarly, a non-limiting or minor curv is either less sharp than the limiting curv, or so short that one standard loco can traverse it with the max train. It affects operation cost by adding to the resistance which the loco, drawing the max train, must overcome. It thus increases the work required in moving the max train over the line.

A comparison of the definitions of D_e, Δ_d, Δ_e and L_c, ¶¶ 39 and 43, will show that the additional work, entaild by a curv, is equated with that of hauling the max train over *an additional stretch* of straight track, whose length is such that the work of this hauling, over it, is equal to that due to the curvature.

For analysis of minor curv problems, see ¶ 39.

37. Classification of non-limiting grades. Fig 2, p 1091. According to the behavior of descending trains, non-limiting grades are classified as in the tabulation on p 1088, illustrated by Fig 2, p 1091. Compare J. B. Berry, Am Ry Eng & M W Assn, Bulletin 49, March 1904, p 21; and A. M. Wellington, R. R. Location, pp 330 and 374, and Table 122, pp 372-3.

	Grade Class. See Fig 2, p 1091.		
	A	B	C
Drop	$\ngtr$ 30 ft	> 30 ft; but $\ngtr$ drop indicated by solid curv	> drop indicated by solid curv
Remarks	Effect negligible	Will not overtax up-bound locos	Up-bound locos must use sand

Ordinarily an initial speed of 10 m/h will not reach 30 m/h if

	A	B	C
Steam is	on	off	off
Brakes are	releast	releast	applied

38. On a 0.6% grade, the longitudinal gravity effect about equals the train resistance at 32 mi/hr. Hence, without steam and without brakes, that speed will not ordinarily be exceeded, on such a grade or on lighter grades, however long the grade, or however great the drop.

39. Effect of non-limiting grades and curvs upon operation cost.

Let

s_e = that grade, in ft per 100 ft*	D_e = that curv sharpness, in degrees of sweep per 100-ft chain¶§

which produces an additional resistance, due to

grade alone	curvature alone

equal to the normal resistance, f_n (see p 1057) on a level tangent.

Then, for the *work* done on a given train, in balancing this additional resistance alone, we have

on a mile (5280 ft) of s_e grade, work = a train-lift of 52.8 s_e ft, on any grade;*	on a mile (5280 ft) of D_e° curv, work = that of balancing curv resist alone, thru 52.8 D_e° of sweep, § of any sharpness.

and this is equal also to the work of balancing the normal resistance, f_n, thru a mile of level tangent. But the *cost* of the work, required for

a train-lift of 52.8 s_e feet	balancing curv resistance alone thru 52.8 D_e° of sweep

is much *less* than the normal *cost*, *m* (say $1.50; see ¶ 7), of a train-mile on a level tangent; because (continued on p 1089)

*s_e is commonly taken at 0.5% = 26.4 ft per mile; or 52.8 s_e = 26.4 ft lift.

†Insted of 60% and 65%, Mr. Berry (Am Ry Eng & M W Assn, Bulletin 49, March 1904, p 22) takes 25% and 45% respectivly. The diff shows how largely such estimates depend upon individual judgment.

¶D_e is commonly taken at 10°; or 52.8 D_e° = 528° of sweep.

§Let f_1 = curv resistce on a 1° curv.
Then $D_e = f_n/f_1$.
f_n is usually assumed to range from 5 to 10 lbs, and f_1 from 0.5 to 1.5 lb, per ton of 2000 lbs.

(1) the *expenditure* of energy, in the work of train-*lift*, is often compensated (at least in part) by a *gain* of energy on the corresponding *fall;* and

(2) the normal train-mile cost, *m*, includes items which are not directly affected by train-lift or by curvature. Thus:—

Except for the item of fuel, the effect of train-lift upon operation cost is practically negligible. Of the total loco fuel cost (= say 10% of the total normal t-m cost, *m;* see Table 1), a large proportion, averaging say 45%, is due to loss in firing, in standing, stopping and starting, and in overcoming curv and grade resistce, leaving only 55% as the cost of hauling on a level tan, which work, in a mile, is equal to that reqd for 52.8 s_e ft of train-lift. To this 55% Prof. Webb adds, for Classes B and C (see ¶ 37) 5% for fuel on down grades; and, for Class C, another 5% for loss thru braking; making the average total, b = 60% for Class B, and b = 65% for Class C.* Class A grades are neglected as not materially affecting operation cost.

We have, therefore, for all items, (making crude assumptions for expenses other than loco fuel); (see Table 2)

Class B

	a	*b*	*a'*
Fuel	11	60	6.6
General	8	..	1.0
For grades ...	*A'*	=	7.6

$m' = A'm$
$= 0.076 \times \$1.50$
$= \$0.114$

Class C

	a	*b*	*a'*
Fuel	11	65	7.1
General	8	..	2.0
For grades	*A'*	=	9.1

$m' = A'm$
$= 0.091 \times \$1.50$
$= \$0.136$

Curvature affects chiefly the items of roadway and track maintenance, loco and car repairs, and loco fuel.

We assume, (see Table 2)

	a,	*b*,	*a'*,
Roadway & track maint..	16	75	12
Repairs			
loco	7	100	7
cars	8	100	8
Loco fuel	11	50	5.5
General	8	25	2
Curvature,	*A'*	=	34.5

Hence, for curvature,
$m' = A'm$
$= 0.345 \times \$1.50 = \$0.5175$

For roadway and track supt, Mr. J. B. Berry, Am Ry Eng & M W Assn Bulletin 49, March 1904, p 20, givs (instead of our a = 16%) a = 14.76%, obtaind as follows:

	a,	*b*,	*a'*,
Rails	2.28	300	6.84
Ties	3.01	50	1.50
Ballast	0.28	50	0.14
Spikes &c...	0.86	100	0.86
General	8.33	50	4.16
Supt	14.76	91	13.50

*See † foot-note, p 1088.

Table 2. Deter-

Factors		Add'l train		Length-difference For Classes, *A*, *B*, *C*, see ¶ 28.					
				A		*B*		*C*	
Items	*a*	*b*	*a'*	*b*	*a'*	*b*	*a'*	*b*	*a'*
M Way & Str									
Suptce	1	10	0.1	00	0.0	00	0.0	00	0.0
Rdwy, Trk	16	50	8.0	80	12 8	90	14.4	100	16.0
Structures	4	00	0.0	5	0.2	10	0.4	70	2 8
Miscel	1	00	0.0	100	1.0	100	1.0	100	1.0
	22								
M Equipmt									
Suptce	1	00	0.0	00	0.0	00	0.0	00	0.0
Rep, ren									
Locos	7	70	4.9	40	2.8	40	2.8	50	3.5
Cars &c	8	70	5.6	40	3.2	40	3.2	60	4.8
Miscel	1	00	0.0	30	0.3	40	0 4	50	0.5
Deprecn	3	00	0.0	00	0.0	00	0.0	00	0.0
	20								
Traffic	2	00	0.0	00	0.0	00	0.0	00	0.0
Transptn									
Suptce &c	9	20	1.8	00	0.0	00	0 0	00	0.0
Yds, Stas	7	20	1.4	00	0.0	00	0.0	80	5 6
Loco fuel	11	80	8.8	50	5.5	50	5 5	60	6 6
Loco exps	9	80	7.2	10	0.9	70	6 3	80	7.2
Operatn	8	50	4.0	20	1 6	50	4 0	60	4 8
Miscel	4	50	2.0	60	2.4	60	2.4	60	2.4
	48								
General	8	00	0.0	00	0.0	00	0.0	00	0.0
Total	100	$A' = 43.8$*		$A' = 30.7$†		$A' = 40.4$†		$A' = 55.2$†	
With m = \$1.50 we have $m' = A'm$		= \$0.657		\$0 460		\$0.606		\$0.828	

40. Table 2, at the heads of pp 1090-1. Assumed values for a (% of m), and b (% of a), and deduced values of a' ($= ba$), A' (sum of a') and m' ($= A'm$).

41. Table 2 illustrates the application of the method of ¶18, for finding the modified t-m cost, m', due to each of the several factors affecting operation cost, viz.—

1. Limiting grades and curvs (additional train);
2. Length-differences;
3. Pusher Grades;
4. { Non-limiting grades (rise-and-fall)
   Non-limiting curvs.

42. Caution. The values of b (% of a), here given, represent merely prob avs, and are used to illustrate the method of ¶ 18, rather than to represent reliable percentages. The actual values of b vary widely with diff conditions, with the classif'n of the cost items, and with the judgment of the estimator. See foot-note †, p 1088. They must, in general, be determined or estimated for each case.

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*For additional trains (first column), J. B. Berry, Am Ry Eng & M W Assn, Bull 49, March '04, p 12, finds A' = 43 29% of m. E. H. McHenry, N. Pac. Ry Rules, pp 12, 16, assumes A' = 60% of m.

†Using other classifn's of length diffs, J. B. Berry, U. P. Ry, A. R. E. M. W. A. Bull 49, Mar '04, and E. H. McHenry, N. P. Ry, '03, adopt values of A' as follows:—

		Class A	Class B	Class C
Berry, U. Pac	A' =	32%	46%	59%
McHenry, N. Pac	A' =	22%	52%	100%

mination of m'.

Factors		Pusher		Rise-and-Fall B		Rise-and-Fall C		Curv	
Items	a	b	a'	b	a'	b	a'	b	a'
M Way & Struc									
Suptce	1	00	0.0	00	0.0	00	0.0	00	0.0
Rdwy, Trk	16	50	8.0	00	0.0	00	0.0	75	12 0
Structures	4	00	0.0	00	0.0	00	0.0	00	0.0
Miscel	1	00	0.0	00	0.0	00	0.0	00	0.0
	22								
M Equipmt									
Suptce	1	00	0.0	00	0.0	00	0.0	00	0.0
Rep, ren									
Locos	7	80	5.6	00	0.0	00	0.0	100	7.0
Cars &c	8	80	6 4	00	0.0	00	0.0	100	8.0
Miscel	1	00	0.0	00	0.0	00	0.0	00	0.0
Deprecn	3	00	0.0	00	0.0	00	0.0	00	0.0
	20								
Traffic	2	00	0.0	00	0.0	00	0.0	00	0.0
Transptn									
Suptce &c	9	00	0.0	00	0.0	00	0.0	00	0.0
Yds, stas	7	00	0.0	00	0.0	00	0.0	00	0.0
Loco fuel	11	80	8.8	60	6.6	65	7.1	50	5.5
Loco exps	9	50	4.5	00	0.0	00	0.0	00	0.0
Operatn	8	20	1.6	00	0.0	00	0.0	00	0.0
Miscel	4	20	0.8	00	0.0	00	0.0	00	0.0
	48								
General	8	00	0.0	..	1.0*	..	2.0*	25	2.0
Total	100	$A' = 35.7$		$A' = 7.6$		$A' = 9.1$		$A' = 34.5$	
With $m = \$1.50$ we have $m' = A'm =$		$0.535		$0.114		$0.136		$0.517	

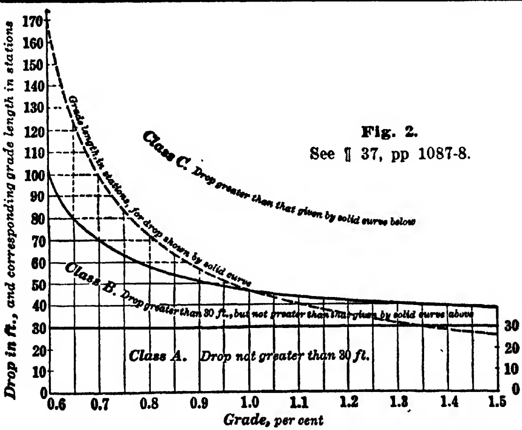

Fig. 2.
See ¶ 37, pp 1087-8.

*Covering all items other than loco fuel

43. Deduction of annual and capitalized costs. Having thus, in accordance with ¶¶ 18 to 42, estimated the modified train-mile cost, *m'*, as determind by a given factor in a given case; *we proceed, as follows, to deduce, from m', the annual operation cost, C_a, of a given difference (in limiting or in non-limiting conditions) betw two lines, and the capitalized value, V_c, of said annual cost.*

In the following schedule, an "increase" and its effects may be *negativ.*

<table>
<tr><td colspan="2">Let</td></tr>
<tr><td>s_e = that grade, in ft per 100 ft,</td><td>D_e = that sharpness, in degs of sweep per 100-ft chain,</td></tr>
<tr><td colspan="2">which produces an additional resistance equal to the normal resistance, f_n, on a level tangent;</td></tr>
<tr><td>h_d = diff, as to lift, betw two lines;
h_e = 52.8 s_e
= 1 "equivalent grade-mile"
= that lift for which the work reqd is</td><td>Δ_d = diff, as to sweep, betw two lines,
Δ_e = 52.8 D_e°
= 1 "equivalent curv-mile"
= that sweep for which the curv work reqd is</td></tr>
<tr><td colspan="2">equivalent to that of hauling the train thru a mile of level tangent
Then:</td></tr>
<tr><td>$L_s = \frac{h_d}{h_e}$ = the diff, as to rise-and-fall, betw the two lines, exprest in terms of the "equivalent grade-mile", h_e.</td><td>$L_c = \frac{\Delta_d}{\Delta_e}$ = the diff, as to sweep, betw the two lines, exprest in terms of the "equivalent curv-mile", Δ_e.</td></tr>
</table>

C_a = annual cost of a given difference between two lines;
V_c = capitalized value of C_a;
d = the number (usually 365 or 313) of train-days in a year;

<table>
<tr><td>(1)</td><td>(2)</td><td>(3)</td><td>(4)</td><td>(5)</td></tr>
<tr><td>Let t' =</td><td colspan="4">t =</td></tr>
<tr><td>No. of add'l trips. ¶ 11.</td><td colspan="4">number of daily round trips.</td></tr>
<tr><td>Then $d\,t'$ =</td><td colspan="4">$d\,t$ =</td></tr>
<tr><td colspan="5">number of train-miles made over any one mile in one year by</td></tr>
<tr><td>t' trains,</td><td colspan="4">t trains,</td></tr>
<tr><td colspan="5">each making one round trip daily.</td></tr>
<tr><td>Let L =</td><td>L_d =</td><td>L_p =</td><td>L_s =</td><td>L_c =</td></tr>
<tr><td rowspan="3">line length.</td><td colspan="4">difference between two lines as to</td></tr>
<tr><td rowspan="2">length</td><td rowspan="2">pusher-grade length.</td><td colspan="2">number of equivalent</td></tr>
<tr><td>grade miles.</td><td>curv miles.</td></tr>
</table>

(Table continued on page 1093.)

(Table continued from page 1092.)

<table>
<tr><td>(1)</td><td>(2)</td><td>(3)</td><td>(4)</td><td>(5)</td></tr>
<tr><td colspan="5">Then, for a given traffic, an increase in</td></tr>
<tr><td rowspan="2">limiting grades & curvs</td><td rowspan="2">line length</td><td rowspan="2">pusher-grade length.</td><td colspan="2">non-limiting</td></tr>
<tr><td>grades</td><td>curvature</td></tr>
<tr><td colspan="5">increases operation cost by</td></tr>
<tr><td rowspan="2">adding t' to the No. of daily rnd trips on L;</td><td rowspan="2">making each of t trains run L_a mls frthr, each single trip;</td><td rowspan="2">requiring pusher outfit for each single trip on L_p;*</td><td colspan="2">increasing resistance on</td></tr>
<tr><td>grades;</td><td>curvs;</td></tr>
<tr><td colspan="5">thus adding, to the normal work, work equivalent to that required for hauling, daily,</td></tr>
<tr><td>t' trains</td><td colspan="4">t trains</td></tr>
<tr><td colspan="5">over a distance equal to</td></tr>
<tr><td>L.</td><td>L_a.</td><td>L_p.*</td><td>L_s.</td><td>L_c.</td></tr>
<tr><td colspan="5">m' = $A'm$ = train-mile cost of this added work.</td></tr>
<tr><td colspan="5">Number of additional equivalent train-miles per annum, due to diff betw the two lines, =</td></tr>
<tr><td>$d\,t'\,L$.</td><td>$d\,t\,L_a$.</td><td>$d\,t\,L_p$.*</td><td>$d\,t\,L_s$.†</td><td>$d\,t\,L_c$.</td></tr>
<tr><td colspan="5">C_a = additional annual cost, =</td></tr>
<tr><td>$d\,t'\,L\,m'$.</td><td>$d\,t\,L_a\,m'$.</td><td>$d\,t\,L_p\,m'$.*</td><td>$d\,t\,L_s\,m'$.†</td><td>$d\,t\,L_c\,m'$.</td></tr>
<tr><td colspan="5">V_c = C_a ÷ interest rate.</td></tr>
</table>

It will be noticed that, in all five cases, the additional work is either that of train haulage, or is equated to (and stated in terms of) train haulage. In column 1 (difference in limiting conditions) said haulage is that of *additional trips* over *the same length, L*, of track in both cases; whereas, in each of the other four columns, the number of trips is unaffected by the diff betw the two lines compared, and the equivalent haulage is that of said trips (*the same* for both lines) over *an additional length, L_a, L_p, L_s* or *L_c*, of track. See ¶¶ 32, 35 and 36.

*This assumes that a pusher-outfit-mile is equivalent to a train-mile, and that pusher-outfit operation cost is the same in both directions. For any deviation from these assumptions, allowance is to be made in determining the value of b, ¶ 18, (3). See also ¶ 32.

†This assumes equality of total lift, on non-limiting grades, in both directions; and neglects advantage due to down grades.

CONSTRUCTION COST ESTIMATES

INTRODUCTORY

Variations and Limits

1. Elements affecting cost. Railroad construction cost varies between wide limits. It depends largely upon the topography and the geology of the country traverst; upon its value and the resulting right-of-way cost; upon the amount and the cost of land required for yards and terminals; upon the unit costs of the several items of construction; upon the cost of labor; upon the construction methods employd; upon the requirements as to grade and alinement; upon the amount and the character of the traffic to be carried; upon the absence or presence and the nature of competition; and upon the funds available.

2. Initial and ultimate costs. The construction cost of a given railroad, as shown by the Company's accounts, increases with its age; for the original cost (often kept at a minimum by the economic desirability of putting the road in earning condition as soon as possible) may be much less than its total subsequent construction cost ("Original Cost to Date") after desirable improvements have been made. See ¶¶ 4 and 11.

3. On the other hand; constructions, included in the original cost, are often abandoned or superseded; but their cost nevertheless enters into the total construction cost.

4. Accounting. For a given actual construction cost, the available cost *data* are affected by differences in account-keeping method. For instance; items, properly chargeable to construction, are often charged to way maintenance; as where a temporary timber trestle (part of the original construction) is afterward gradually filled in and the necessary train-service is charged to maintenance.

5. In deducting the construction cost of a proposed line from that of an existing line, allowance must of course be made for differences in condition betw the two; and, even with given unit costs, different portions of a given line may have very different costs per mile; and, therefore,

$$\text{the apparent construction cost per mile} = \frac{\text{total cost}}{\text{length}}$$

of one part of a given line, may be much higher, and that of another part much lower, than that of the entire line.

6. Example. Thus; a trunk line, running eastward from the Pacific thru several states, and burdend with heavy real estate costs at its terminals, and with heavy construction cost proper on that portion of its line crossing the mountains, may pass also thru regions of more favorable topography and requiring but little outlay for real estate. In using, for comparison, the construction-cost data of such a line, the character of the proposed road must be kept in mind. If, for instance, the proposed line traverses only favorable and inexpensiv country, and if it requires no expensiv termini; it would be misleading to apply, to it, without modification, construction-cost data covering the entire trunk line, including its more expensiv portions.

7. Tendency to increase. When using early data as a basis for the estimation of present or future costs, bear in mind that the tendency of railroad construction cost, as a whole, is to increase; for the reduction of unit cost, in some particulars, due to improved manufacturing and operating methods and apparatus, is usually more than offset by increase in wts of rolling stock, by increasingly exacting demands on the part of the traveling and shipping public, by increase of traffic, which justifies costly construction for the sake of the reduction of operating expenses, and by increasing insistence, on the part of the community, upon the avoidance of nuisance and of accident.

8. Thus, where grade crossings are still retaind, they are required to be provided with elaborate interlocking and signaling apparatus; but, more often, elimination of grade-crossings is demanded; and the abolition of a grade-crossing may cost $100,000 or more.

9. The range of variation, in total cost, increases also, and for like reasons.

10. Thus, upon a line of moderate length, joining populous centers, the cost of terminals, of modern type, may equal the remaining construction cost; whereas, upon a long line, connecting many small towns or depending chiefly upon mineral or industrial traffic, the cost of terminals may be relativly insignificant.

11. The Interstate Commerce Commission, by Act of Congress, approved 1913, March 1, is instructed to ascertain, respecting each railroad property:—

(1) Its original cost, to date;
(2) Its cost of reproduction, new;
(3) Its cost of reproduction, new, less depreciation.

12. Differences in estimates. As might be expected, there is usually a material difference betw (a) estimates prepared by a railroad company for the purpose of showing a *high* value (as in cases of proposed sale, or where it is desired to justify existing or proposed freight and passenger rates), and (b) estimates prepared by the same company for the purpose of showing a low value, as for purposes of assessment. See ¶ 139.

13. "General Expenditures." In addition to Construction Cost proper, there are the cost items of financiering, negotiation, legal service, insurance, taxes, depreciation, etc, (see III. General Expenditures. ¶ 18) which affect an appraisal of the property in its existing condition; as for purposes of taxation. Such an appraisal may properly consider the earning capacity of the plant, which, of course, may be greatly affected by considerations quite independent of construction cost. For instance, the business conditions of the country traverst may conceivably give high commercial value to a very cheaply constructed road; or, on the other hand, they may become such as to deprive an expensivly constructed road of all value.

14. All that can be usefully attempted here is to give such data (deduced from publisht records) as may facilitate the making of preliminary cost estimates for a projected line, bearing in mind the preceding cautionary suggestions.

15. Unit costs. In estimating railroad construction costs, we are concernd with

(1) unit costs (as per cu yd of grading, per acre of clearing, per lineal ft of tunnel, etc) and

(2) per-mile costs, or costs per mile of line or of single track.

16. Line-mile vs Track-mile. In figuring per-mile cost, distinction must be made between

(1) *line*-mile (or *road*-mile) cost (cost per *line*-mile)

$$= \frac{\text{total cost}}{\text{number of miles of main } \textit{line} \text{ and branch } \textit{lines}}$$

(irrespectiv of the number of tracks and of sidings), and

(2) *track*-mile cost (cost per *track*-mile, *i. e.*, per mile of *track*)

$$= \frac{\text{total cost}}{\text{number of miles of track}}.$$

17. Thus, a double-track road, L miles long, without side-tracks, has L *line*-miles, and 2 L *track*-miles.

In any given case, line-mile cost $\geqq$ *track*-mile cost.

For a given *single*-track line, without branches or sidings, we have :—

line-mile cost = *track*-mile cost.

For a given *double*-track line, without branches or sidings, we have :—

line-mile cost = 2 × *track*-mile cost;

but the *line*-mile cost of a double-track road is (other things equal) *less* than double the *line*-mile cost (= *track*-mile cost) of a single-track road.

I. C. C. Classification

18. Classification of Investment in Road and Equipment of Steam Roads, Prescribed by the Interstate Commerce Commission in accordance with Section 20 of the Act to Regulate Commerce. Issue of 1914. Effectiv on July 1, 1914. (Here condenst.)

General Accounts.

I. Road. "Cost of land, fixt improvements, and roadway machines and tools ownd by the carrier and devoted to transportation service."

II. Equipment. "Cost of the several classes of equipment ownd by the carrier, or held under equipment trust agreements for purchase."

III. General Expenditures. "Expenditures made in connection with the acquisition and construction of original road and equipment, and with extensions, additions and betterments to road and equipment property, when such expenditures cannot properly be included in any of the foregoing accounts as a part of the cost of any specific work."

Primary Accounts.

I. Road.

1. Engineering.

Including "the pay and expenses of engineers, assists. and clerks engaged in the survey and construction of new lines and extensions, or in making additions to and betterments of the carrier's road, including wharvs and docks"

2. Land for Transportation Purposes.

Including "the cost of land of necessary width acquired for roadway; the cost of land for station, office, shop and other grounds; for ingress to or egress from such grounds; for borrow pits, waste banks, snow fences, sand fences, and other railway appurtenances; and for storage of material adjoining the right of way; the cost of land for wharvs and docks and the cost of riparian or water rights necessary therefor; the cost of removing from the right of way and locating elsewhere the property of others, and the cost of the necessary land for relocation of the property, when such costs are assumed by the accounting carrier."

3. Grading.

Including "the cost of clearing and grading the roadway, and of constructing protection of a permanent character for the roadway, tracks, embankments, and cuts."

4. Underground Power Tubes.

5. Tunnels and Subways

Including "the cost of tunnels and subways for the passage of trains, including apparatus for ventilating and lighting, and safety devices therein, other than signals."

6. Bridges, Trestles and Culverts.

Including "the cost of the substructure and superstructure of bridges, trestles, and culverts which carry the tracks of the carrier over watercourses, ravines, public and private highways, and other railways."

7. Elevated Structures.

Including "the cost of elevated structures and foundations of elevated railway systems—structures other than earthwork, which are for the purpose of elevating tracks above the grade of streets, and which are not properly classable as bridges or trestles."

8. Ties.

9. Rails.

10. Other Track Material.

Including anticreepers, bumping posts, railway crossings, derails, guard rails, rail braces, rail joints, tie-plates, spikes, turnouts, etc.

11. Ballast.

12. Track Laying and Surfacing.

13. Right-of-way Fences.

14. Snow and Sand Fences and Snowsheds.

15. Crossings and Signs

Exclusiv of "railways crossing at grade", for which see Account 10.

16. Station and Other Buildings.

17. Roadway Buildings.

18. Water Stations.

19. Fuel Stations

20. Shops and Enginhouses.

21. Grain Elevators.

22. Storage Warehouses.

Including "warehouses in which merchandise is stored and which the railway companies or others operate commercially as storage warehouses."

23. Wharvs and Docks.

24. Coal and Ore Wharvs.

25. Gas Producing Plants.

26. Telegraph and Telephone Lines.

27. Signals and Interlockers.

28. Power Dams, Canals, and Pipe Lines.

29. Power Plant Buildings.

Including "the cost of the buildings of power plants erected to furnish power . . ." See Account 45, p 1098.

30. Power Substation Buildings.

Including "the cost of the buildings of power substations erected to transform power . . ." See Account 46, p 1098.

31. Power Transmission Systems.

Including "the cost of high-tension transmission systems, . . . used for transferring power from producing plants to a place where it is transformd . . ."

32. Power Distribution Systems.

Including "the cost of distribution systems, . . . for conveying low-tension electric power from producing plants or transformer stations and for conveying steam and compress air from producing plants to the place where used . . ."

33. Power Line Poles and Fixtures.

34. Underground Conduits

35. Miscellaneous Structures.

36. Paving.

37. Roadway Machines.
Including boilers, cars (hand-, lever-, motor inspection-, push-, etc), concrete mixers, ditching and dredging machines, portable engins, grading and hydraulic outfits, hydraulic jacks, pile drivers, unloading plows, rail unloaders, rock crushers, steam rollers, etc.

38. Roadway Small Tools

39. Assessments for Public Improvements.

40. Revenues and Operating Expenses During Construction.

41. Cost of Road Purchast

42. Reconstruction of Road Purchast.

43. Other Expenditures—Road.

44. Shop Machinery.

45. Power Plant Machinery.

46. Power Substation Apparatus.

47. Unapplied Construction Material and Supplies.

(Nos 48, 49 and 50 left blank.)

II. Equipment.

51. Steam Locomotivs.

52. Other Locomotivs.

53. Freight-train Cars.

54. Passenger-train Cars.

55. Motor Equipment of Cars.

56. Floating Equipment.

57. Work Equipment.

58. Miscellaneous Equipment.

(Nos 59 to 70, inclusiv, left blank.)

III. General Expenditures.

71. Organization Expenses.

72. General Officers and Clerks.

73. Law.

74. Stationery and Printing.

75. Taxes.

76. Interest during Construction.

77. Other Expenditures—General.

References, etc.

19. Acknowledgment to Gillette's "Cost Data." For most of the information, given in the following paragraphs, on the engineering features of railroad construction cost, we are indebted to Mr. Halbert P. Gillette's monumental "Handbook of Cost Data", (1910-1914, McGraw-Hill Book Co.), which we have very freely used at the author's invitation. In using the material selected, however, we have digested and arranged it to suit the purposes of "TRAUTWINE", and have frequently substituted deductions from Mr. Gillette's figures, in the place of the figures themselves. Owing to differences in methods of classification of construction cost items we have frequently been obliged to rely upon assumption for the proportions of a given reported item chargeable to this or to that "account" of the Interstate Commerce Commission, Classification of 1914 (¶18).

20. Caution. We repeat that all of our data, on railroad construction cost, are intended, not as business quotations, but merely as indicating the *range* within which the cost of a given item, or the cost per mile, may reasonably be expected to fluctuate.

21. List of References. The following are the principal statements respecting steam railroads, in Mr. Gillette's "Cost Data", to which reference is made in the text below. There, as here, the reference is indicated by the folio number, prefixt by the letter "G". Thus, "G 1418" means "See Gillette's 'Cost Data', page 1418".

	Miles of line	Miles of track
G 1289. Winchester & Beattyville R R, Kentucky. Built, 1893, in rugged country, to open up a mining district. Max grade, 1%; max curv sharpness, 6°, except for two 12° curvs. Labor, $1.25 per day. John H. Pearson, Eng. News, 1893, Sep 14, p 209.	8	8
G 1290. A logging railroad in northwestern Penna, built 1890. Ruling grade, 3.3%. Max curv sharpness, 18°. Country very rough and heavily wooded. Wages, $1.25 ? per 10-hour day. Costs given refer to the heaviest part of the line.	7	
G 1291. Texas. First-class branch line of St. Louis Southwestern Ry. Ridge line, nearly all tangent. Built, 1903.	12.13	13.7
G 1303. Great Northern in Washn. Original cost. Built by contract, at reasonable prices, in 1891-4. Crosses rugged mountain range. Cascade tunnel cost over $2,500,000.	488	527
G 1306. Fairhaven Southn branch. Easy country.	32.3	
G 1307. Spokane Falls & Northn branch, about 1890.	130.5	151.3
G 1308. Washn & Great Northn branch. 1906. Mountainous country.	83.9	91.8
G 1310. Great Northn in Washn. Original cost plus improvements to 1906.	768.0	954.8
G 1319. Great Northn in Washn. Reproduction estimate, 1906.	768.0	954.8
G 1321. Northn Pac in Washn. Original cost plus improvements to 1906. Built about 1880. Difficult and costly explorations. Built largely by company forces.	1645	2205
G 1329. Northn Pac in Washn. Reproduction estimate, 1906	1645	2205
G 1331. Oregon R R & Navigation Co., in Washn. Built 1875-1899. Original cost.	501	569
G 1332. Oregon R R & Nav Co. Reprod'n est.	501	569
G 1334. Stt of Wis. Val'n by W. D. Pence, '07	7090	10179
G 1336. Stt of Wis. Valuation by W. D. Taylor, 1903-4.	6657	9271
G 1336. Stt of Mich. Valuation by M. E. Cooley, 1900.	7813	10882
G 1347. Stt of Minn. Reprod'n est, 1908	7596	10438
G 1353. C, M & St. P in S. Dak. Prac'y no rock excav. D. J. Whittemore,* '98.	1101	1187
G 1354. Texas. Prices used in est'g cost of RRs in state.		
G 1362. N. Pac. syst. Val'n by Chf Engr,* '07-8.	5875	7695
G 1374. Gt. N. syst. Val'n by Chf Engr,* '07-8.	6635	8116

*Appraisal by Chief Engineer of road, in a rate case. Presumably not underestimated. See ¶ 12.

COST DATA

GENERAL ACCOUNT I. ROAD.

Primary Account 1.
Engineering.

Surveys.

22. Under "surveys" we here include reconnaissance, preliminary survey and location, and such other professional engineering work as may be required prior to the beginning of construction.

23. Importance. The adoption of an inferior line means increast construction cost, or increast operation cost for every train during the lifetime of the road, or both. The whole cost of surveys will usually be much < 1% of the total cost of the line; and it is poor economy (especially on important lines) to stint the expenditure upon surveys, in the matter of either salaries, subsistence or appliances and conveniences.

24. Reconnaissance consists of an exploration of *areas*, rather than of *lines*. The party includes usually a chief engineer, a transitman, a topographer, a draftsman, a rodman, one or more axmen, and a cook where required; and the monthly payroll may ordinarily range from $750 to $1,500, depending upon the importance of the line; but the time, devoted to reconnaissance, varies, betw very wide limits, with the importance of the line and the character of the country, especially as to the number of apparently favorable lines requiring investigation. The cost of reconnaissance therefore varies widely; but it will probably seldom exceed say $25 to $50 per mile of constructed line, or 0.10% to 0.25% of the construction cost.

25. Preliminary survey and location. The party will usually include

	Salaries, $/month
Locating engineer	125 to 175
Assistant engineer	100 to 125
Transitman	85 to 100
Leveler	80 to 90
Topographer	70 to 100
Draftsman	70 to 90
Rodman	50 to 65
Chainmen; 2, at $40 to $50	80 to 100
Back flagmen	30 to 40
Tapemen; 1 or 2, at $30 to $45	30 to 90
Stakemen	30 to 35
Axmen; 2 to 5, at $25 to $30	50 to 150
Total; 14 to 18 men	—— ——

Engineering salaries per party per month, when labor costs $1.50/man-day ....$800 to $1160

26. Where the party is to provision itself, add a cook and helper at $60 to $80 total per month. Provisions may range from $175 to $350 per month; or the party may be "boarded" at about the same or a little greater expense. Add other expenses as below:—

	$ per month per party		
Salaries, as above,	800	to	1160
Subsistence,	250	to	500
Team,	75	to	150
Contingencies,	25	to	125
Total,	$1150	to	$1935

27. Preliminary and location cost, per mile of survey, for a given monthly outlay for surveys, depends upon the character of the country and upon weather conditions. Rough country and

severe weather, by reducing the mileage coverd by a given outlay, increase the cost per mile surveyd.

28. In moderately rough country, prelim survey may be expected to cost from $20 to $40 per mile of prelim survey; and location from two to three times these figures per mile of location. In prairie country, half as much, or less; and, in very rugged country, twice as much, or more.

29. The survey cost, per mile of constructed line, may be as much as three or four times the survey costs, above indicated, per mile of line *surveyd*.

30. For a given cost per mile of *survey*, the *line*-mile cost of survey depends upon the character of the country and upon the limitations imposed as to max grade and max curv-sharpness.

31. A ridge, prairie, or river-bank line, offering but little if any choice betw alternate lines, will require fewer miles of prelim survey, per mile of finisht construction, than will a cross-country line, in rough country, which may require the investigation of several promising lines before the best can be selected for location. In rough country, restriction of grades and of curv-sharpness of course increases survey cost per mile of finisht construction.

Engineering.

32. Probable range of engineering cost; deduced from a comparison of eighteen tabulations in "Cost Data" —

	$/line-mile	% of total constru cost
Preliminary and location surveys,	150 to 400	½ to 1
Engineering during construction (laying out of work, and superintendence),	400 to 1300	1.5 to 3
Total "engineering",	550 to 1700	2 to 4

Primary Account 2.

Land for Transportation Purposes.

Real Estate.

33. Perhaps in no other item is greater caution required, when deducing the cost of a proposed line from the actual or estimated costs of other lines, than in that of real estate for right of way, for stations and terminals, for water supply, etc.

34. The price per acre may vary from $10 or less in waste lands, to $20,000 or more for terminal sites in cities; and the cost of such sites increases rapidly with time as a result of city development. Indeed, land is frequently *donated* to railways, by govts or business interests desirous of the services of the road in developing the country or the trade of the city; and, on the other hand, vindictiv or avaricious property-owners frequently hold out for exorbitant prices, which the companies often pay rather than resort to condemnation, which involves legal proceedings, and which restricts the company's use of the land to Ry purposes. Owing to consequential damages and inconvenience to property-holders, it is usually accepted as equitable that land, for Ry purposes, should cost the company from two to five times the market value of adjacent property. See **"the right-of-way multiple,"** ¶¶ 36, 39.

35. The right-of-way width may vary from 50 feet or less for a cheap single track line in prairie country, to 200 ft or more for a low-grade trunk line, with from two to four tracks, deep earth cuttings and high earth embankments. The area and the prices of the lands required, for yards, stations, etc., vary widely with the nature of the traffic.

1 acre per mile	=	8.25 ft right-of-way width;
66 ft width	=	8 acres per mile;
100 ft width	=	12.12 acres per mile.

36. The ratio, $\dfrac{\text{cost of land, to Ry Co.}}{\text{market value of same land, for ordinary purposes}}$ is called **"the right-of-way multiple."**

37. Promoting. The matter is further complicated by the fact that Ry companies frequently purchase a wider right-of-way than their transportn purposes require; expecting to sell the surplus at prices enhanced by the construction of the road, or being compeld to this course as a concession to recalcitrant property-holders.

Examples.

38. In 1906, the Texas R R Commission, in valuing a Ry line, assumed, (G 1354):—

Right-of-way	\$ 50 per acre
Depot grounds	\$100 " "
Reservoir grounds	\$ 25 " "

39. The Western Pacific Ry connects San Francisco, Cal., with Salt Lake City, Utah, passing thru Sacramento, Oakland and other important Cal cities. In Cal, it acquired, for its 377 miles of line in that State, 6064 acres of land = say 16 acres per line-mile, classified as follows by Walter Melvin Wells, Right-of-way Expert for the Cal R R Commissn. Eng-Contr 1913 Aug 20, p 215:—

			Av cost to Ry Co, \$/acre
Exclusiv of San Francisco terminals	Country lands,	85.2%	102
	Suburban property,	8.4%	
	City water-front property,	2.6%	4,630
	City residence property,	1.5%	
	City business property,	2.3%	
		100.0%	450
San Francisco terminals, 177 acres purchast in highly improved city warehouse district			18,862
All classes			985

The prices paid, for the portions acquired by condemnation, were

	Rural and suburban	In cities	
	per acre	per sq ft	per ft front
Market value when purchast	\$15 to \$1,000	\$0.20 to \$0.60	\$0.30 to \$0.40
Cost to Railway Co	\$37 to \$3,672	\$0.24 to \$1.50	\$0.20 to \$1.60

40. For 981 miles of steam and electric railway line in California, investigated by the State R R Commission, including the 377 line-miles of the Western Pacific (¶ 39), Mr. Wells gives

Line-Miles	Character of lands	Acres, est'd	Land Cost to R R Co. Total \$	Per acre est'd \$	Per line-mile \$
226	desert & waste lands,	3,630	37,706	10	167
310	Mt & timber lands,	4,980	89,739	18	289
445	farm lands, and lands in unincorp'd cities,	7,150	743,616	104	1,675
981	totals and averages, rural and unincorp'd,	15,760	871,061	55	890
0	within incorp'd cities including W. Pac San Fran terminals	928	6,529,360	7,036	∞
981	totals and averages,	16,688	7,400,421	444	7,540

41. The following table, referring chiefly to conditions in the middle and far west, illustrates the wide range in values of real estate, both per line-mile and in its ratio to total construction cost. Column A (Constrn) includes real estate cost (Column B). Where, for a given road, two values are stated, under B, or under 100 B/A, the first refers to right-of-way, and the second to real estate acquired for other purposes. Note the diffs in ratio betw these two.

		Line Miles	$/ line-mi A Constrn	$/ line-mi B Rl Est	100B / A
G 1289 Winchester & Beattyv, (Mine R R),	Ky*	8	13,650	428	3.14
G 1306 Fair Haven & Southern	Wash	32	22,565	2,230 84	9.90 0.37
G 1308 Washn & Grt Northn	Wash	84	36,519	1,064 3	2.92 0.01
G 1307 Spokane Falls & Northern	Wash	130	17,445	1,000	5.75
Duluth & Mankato, §	Minn	157	22,000	2,170	9.86
W Pac, † (¶ 39)	Cal	377	....	17,500	....
G 1303 Gt Northern, part. See below	Wash	488	44,412	4,056 230	9.13 0.42
G 1331 Oregon R R & Nav,	Wash	500	23,931	400 1,884	1.67 7.87
G 1332 Oregon R R & Nav. (reproducn).		500	27,122	4,487	16.55
G 1310 Gt Northern, See above, plus improvements.	Wash	768	49,848	3,759 3,432	7.54 6.90
G 1311 Gt Northern, (reproducn)	Wash	768	66,753	3,870 18,350	5.80 27.50
G 1319 20 lines (¶ 40) ‡ in	Cal	981	.....	7,540	....
G 1321 Nor Pacific, orig & imprvmnts,	Wash	1,645	38,897	1,092 827	2.80 2.16
G 1329 N P, (reprod)	Wash	1,645	54,277	3,820 16,150	7.04 29.70
G 1357 NP, entire line,¶		5,875	60,800	5,425 12,930	8.9 21.3
G 1374 Grt Northern, entire line (¶ 138) ¶,		6,635	56,500	13,160	23.30
G 1336 All lines in	Wis	6,657	24,814	3,719 0	14.9 0
G 1347 All lines in	Minn	7,596	44,726	9,637 3,552	21.6 16.45
G 1336 All lines in	Mich	7,813	21,535	113	0.52
Averages,			41,200	9,360	22.60

42. Tendency to advance. The tendency of real estate cost to advance, not only absolutely but relativly also, is illustrated by a comparison of the averages, deduced in the foregoing table, with the following estimate, publisht by Mr. J. F. Wallace in Eng Mag, 1895 Dec, Vol 10.

*Eng News, 1893 Sep 14, p 209.
†Eng-Contr, 1913 Aug 20. Includes $607,421, incidentals.
‡ " " " Excludes " "
†‡Construction costs not given.
¶Chief Engineer's estimate.
§Eng News, 1900 Dec 27, Supplement, p 210. 1905 acres @ $100. Estimated costs.

Table showing tendency of real estate cost to advance; (¶42, p 1103).

1895	Dollars per line mile A Total	B Right-of-way and Terminals		100 B / A
For trunk lines, with 90-lb rails,	36,150	Rt-of-way	2,000	5.5
		Terminals	5,000	13 8
		Total	7,000	19.3
For main lines, connecting towns and cities of minor importance; and with alinements and grades admitting of economical construction,	22,850	Rt-of-way	1,500	6.6
		Terminals	1,500	6.6
		Total	3,000	13.2
For branch lines, with two pass'r and 3 or 4 frt trains dly; alinements and grades less favorable than above,	13,600	Rt-of-way	1,000	7.4
		Terminals	500	3.7
		Total	1,500	11.1
Averages	24,200	Rt-of-way	1,500	6.2
		Terminals	2,333	9.6
		Total, 1895,	3,833	15.8
From ¶ 41	41,200		9.360	22.6

Roadbed Construction.

43. Roadbed construction (construction preliminary to track-laying) may be taken as including I.C.C. primary accounts 3 to 7 inclusiv (see ¶ 18). It is made up principally of Account 3, Grading (including clearing and grubbing); Account 5, Tunnels and Subways; and Account 6, Bridges, Trestles and Culverts.

44. With given unit costs of material and of labor, the line-mile and the track-mile cost of each of the items included under "Roadbed" varies betw very wide limits. Thus, a prairie line may require no tunnels, bridges or trestles, and little or no grading; whereas, in difficult country, nearly the entire line may well require clearing, grubbing and heavy grading, while tunnels, culverts and bridges may be of frequent occurrence; especially where the modern tendency to reduction of operation cost, by improvement in alinement and in grade, necessitates more numerous and heavier cuts and fills, culverts, bridges, etc.

Primary Account 3.
Grading.

45. Grading includes clearing and grubbing, blasting, breakwaters, bulkheading, excavation and embankment, filling in under bridges and trestles and over culverts, steam shovel operation, use of spoil banks, revetments and retaining walls, riprap, wing dams, and temporary trestles for fills.

Clearing and grubbing.

46. The cost of clearing (removing trees and brush) and that of grubbing (removing stumps and roots) are sometimes included (in bidding) with that of grading; or one price may be named for clearing and grubbing, together; but clearing and grubbing are frequently estimated separately, because often only a portion of the cleard area requires grubbing. Thus, the stumps and roots may be left in place under embankments, except where these are very shallow.

47. Units. The cost of both clearing and grubbing is usually stated in dollars per acre; sometimes in dollars per "square" of 10,000 sq ft. The cost of grubbing is sometimes given in dollars per sq rod (160 sq rods = 1 acre) and sometimes in dollars per "station", covering the right-of-way along 100 ft of line.

48. Elements affecting cost. The clearing and grubbing cost, per unit of area, depends upon the density, species and size of the timbering, upon the amount and character of the underbrush, upon the character and accessibility of the ground, and upon the disposition to be made of the material removed. This cost may be largely or quite offset, or even exceeded, by the value of the timber cut. The apparent grubbing cost is affected also by the method to be employd in the subsequent grading. Thus, a steam shovel (in earth), or blasting (in rock), does much of the work which ordinarily would be clast under grubbing. On the other hand, the use of scrapers requires careful preliminary grubbing; and that of the elevating grader is still more exacting.

49. The line-mile cost, for clearing and grubbing, depends further upon the clearing area per line-mile, and this, in turn, upon the right-of-way width, and upon the proportion of this occupied by woodland and brush.

1	5	8 25	10	16 5		feet width =
0 1212	0 606	1	1.212	2		acres per mile.
20	24.75	30	33	40	41 25	feet width =
2 424	3	3.636	4	4.848	5	acres per mile.
49 5	50	82.5	100	165	200	feet width =
6	6.06	10	12.12	20	24.24	acres per mile.

50. Usually, from 1 to 6 acres per mile (= a uniform strip from 8.25 to 49.5 ft wide) requires clearing

51. The site of the Brockton, Mass., filter beds had 480 trees (from 6 to 24 ins diam) per acre = one tree to each 91 sq ft; a dense growth. G 1045.

52. The costs of clearing and of grubbing may ordinarily be expected to range as follows:—

	Clearing	Grubbing	Clearing and grubbing
Per tree or stump	\$ 1 to 2	\$ 0 05 to 1	\$ 1 to 3
Per acre	\$25 to 150	\$25 to 150	\$ 50 to 300
Commonly assumed at	....	....	.....\$50
Per line-mile	\$50 to 600	\$50 to 300	\$100 to 1000

Earthwork.

53. "Grading" proper includes (besides clearing and grubbing), "earthwork", or the excavation, transportation, depositing and compacting of earth, in its various forms, and of gravel, hardpan (cemented gravel), loose rock and solid rock. Where the cuts furnish insufficient quantities for the fills, the deficiency is made up by taking material from "borrow-pits", and the work is then usually classified as "embankment".

54. Elements affecting cost. The cubic-yard cost of earthwork varies widely with the nature of the material, with its accessibility, with its quantities, with the facilities for handling it, with the skill of the contractor, and with the wage rate, but especially with the dist thru which it must be transported. See pp 1024-1035. On any considerable work, however, the long and the short hauls are regarded as compensating, and it is customary to contract at a fixt cu yd price for each of the several kinds of material encountered,

with a specified limit of haul-length, beyond which the contractor is paid (usually one cent per cu yd of "overhaul") for each 100 ft of excess dist.

55. Ordinarily the cubic-yard cost of earthwork (hauling included), in cents, may be taken as ranging as follows:—

		Average
Earth, ...	15 to 30	22
Hardpan and cemented gravel, ..	35 to 45	40
Loose rock, ...	40 to 55	50
Hard rock, ...	75 to 110	100
Grading, all materials, average,	30 to 45	37

56. Cost per mile. For a given cu yd cost, the mile costs, for earthwork, of course vary betw very wide limits, depending upon the quantity of grading required per mile, which, in turn, depends upon the character of the country and upon the necessity for favorable grade and alinement in the finisht line; a given country naturally requiring more earthwork for a line of closely limited grades and curvature, than for one where heavier grades, and sharper and longer curvs, may be permitted. See table of itemized costs, ¶ 145.

Earthwork Cost, Classified by Materials.

57. In the following table, under each material, the rys are arranged in the descending order of their grading cost for that material, as a percentage of the total earthwork cost. These data (mostly in round numbers) serv to give an idea as to the range of grading quantities and costs to be expected, per line-mile and per track-mile. For miles of line and of track, in the cases here used, see ¶ 21. For their total line-mile and track-mile costs, see ¶ 138.

	cu yds/mile of		$/mile of		Percent of total earthwork	
	Line	Track	Line	Track	cu yds	$
Earth						
G 1347	22230	16100	6380	4620	96.1	90.8
G 1374	25000	21000	7740	6310	74.4	56.8
G 1329	11260	8420	2490	1858	64.6	45.8
G 1308	14800	13550	2888	2639	45.5	21.9
G 1303	6290	5820	1372	1270	24.7	12.1
G 1291	5082	5082	....	....	46.2	...
Embankments from Borrow Pits						
G 1303	7670	7100	1300	1200	30.2	13.4
G 1319	4905	3945	980	780	14.6	7.8
Hardpan, etc.						
G 1362	6580	5025	2770	2110	21.9	22.3
G 1303	5740	5300	2200	2000	22.6	22.5
G 1374	5130	4190	2310	1880	15.3	17.9
G 1291	3872	3872	....	....	35.2	...
Loose Rock						
G 1362	2168	1655	1085	825	7.1	10.0
G 1329	800	600	400	300	4.6	7.4
G 1303	1510	1390	633	584	5.9	6.5
G 1347	565	410	292	212	2.5	4.1
G 1291	1210	1210	...	...	11.0	...

	cu yds/mile of		$/mile of		Percent of total earthwork	
	Line	Track	Line	Track	cu yds	$
Solid Rock						
G 1308	6900	6300	6417	5866	21.2	48.7
G 1303	4225	3900	4500	4200	16.6	45.6
G 1319	4230	3400	4643	3740	12.6	37.2
G 1329	1054	787	1160	866	6.1	21.5
G 1374	1928	1575	2120	1730	5.7	16.5
G 1362	1500	1200	1700	1300	5.1	15.0
G 1347	345	250	372	270	1.5	5.2
G 1291	836	836	...	...	7.6	...
Total Earthwork Cost					Percent of tot constr cost	
G 1291	11000	11000	5027	5027	55.4	
G 1308	32500	29600	14558	13230	40.0	
G 1362	30100	23000	11000	8500	31.2	
G 1331	.....	.....	6603	5860	25.1	
G 1374	33300	27200	13000	11000	24.9	
G 1292	.....	.....	4033	3584	23.3	
G 1347	23140	16770	7044	5100	19.0	
G 1290	.....	.....	2233	1990	18.6	
G 1334	.....	.....	5550	3870	16.1	
G 1353	11300	10500	1710	1590	11.6	
G 1336	.....	.....	2778	2000	10.6	

Primary Account 5.
Tunnels and Subways.

Tunnels

58. Elements affecting cost. Tunnel cost, per cu yd, depends upon the character of the material to be removed, and the facilities for its removal; upon the necessity for, and the required character of, lining; upon the quantities of water encounterd and the facilities for its removal; upon the accessibility of the location; upon the cost of labor at the site; upon the skill of the contractor, etc. It is usually less for contract work than for work done with the company's forces by day's labor. In long tunnels, owing to longer hauls, to the necessity for sinking shafts, to the necessity and difficulty of ventilation, and to the more expensiv character of the plant required, it is usually (other things equal) greater than in short tunnels. It is less for the bench than for the headings; and, for the same reason, it should (especially in small sections) decrease as the cross-section area increases.

59. Cost with labor at $1.00 per day. A study of numerous records, including those given in "Cost Data", indicates that, reduced to a uniform labor basis of $1.00/day, the **average contract price, per cubic unit,** for the entire section, of rock tunnels, may be expected to range, under ordinary circumstances, as follows:

Contract prices. Labor at $1 per day.	A. Cents per cu ft = cts/lin ft/sq ft of section area		$ per cu yd = 27 × A/100	
	cents		$	
Tunnel, excavated, ...	6 to 10;	av 7.8	1.60 to 2.70;	av 2.10
lined with timber, ...	8.5 to 12;	av 10.5	2.30 to 3.25;	av 2.80
conc or masonry, ...	10 to 15;	av 12.5	2.70 to 4.00;	av 3.35

60. Cost with labor at other rates. For the actual cost, in a given case, multiply the costs, in ¶ 59, by actual labor cost, in $ per day.

61. Earth tunnels, from one-third to one-half of these figures.

62. Actual cost; to skild contractors, about one-third less than contract prices; to company, by day labor, from a half to two-thirds more than contract prices.

63. Cost per linear foot. For a given cu yd cost, the lin-ft cost, for the entire cross-sec, varies (other things equal) directly with the number of cu yds per lin ft; and this depends upon the cross-sec area. Single-track standard-gage Ry tunnels usually measure from 14 to 20 ft in width, and double-track tunnels from 25 to 30 ft. The height is usually from 22 to 24 ft. A section, which would suffice for an unlined tunnel, must of course be enlarged to make room for a lining if this is required.

For given conditions, the cross-sec area, for double-track, may be from 1.6 to 1.8 times that for single track.

64. Range of variation of tunnel line-mile cost (see ¶ 145).

		Tunnel cost, in $ per line-mile of road
G 1353	Chicago, Mil & St. Paul, in Dakota, 1100 miles,	0
G 1354	Trinity & Brazos RR, Texas, 165 miles,	0
G 1347	Minnesota railways, 7596 miles,	33
G 1334	Wisconsin railways, 7090 miles,	120
G 1336	Michigan railways, 7813 miles,	147
G 1331	Oregon RR & Navigation Co., 500 miles,	260
G 1321	N Pac in Wash'n, 1645 miles,	590
G 1329	N Pac in Wash'n, 1645 miles, Reproducn, est,	1911
G 1362	N Pac, entire system, est by Chf Engr, 5875 mi,	670
G 1374	Gt Northn, entire system, est by Chf Engr, 6635 mi,	1070
G 1319	Gt Northn in Washington, 768 miles,	5559

Subways. See ¶ 144.

Primary Account 6
Bridges, Trestles and Culverts

Bridges.

65. Costs of bridges may be expected to range as follows:—

Steel bridges				
	superstructure alone, erected		including substrct	
	per lb of steelwork cents	per lin ft of strctr dollars	per lb of steelwork cents	per lin ft of strctr dollars
Single-track, moderate span, usual constr'n,	3 to 4	30 to 50	6 to 7	50 to 100
Special cases, wider roadways, long span,	to 7	to 500	to 15	to 2000

Concrete and masonry bridges		
	per sq ft of roadwy dollars	per lin ft of strctr dollars
Single-track bridges of moderate span and usual construction,	3 to 6	50 to 100
Special cases, wider roadways, long spans,	to 12	to 300

For cost of steel, see p 986.
For cost of masonry, see pp 601, 988.
For cost of concrete, see pp 988, 1375.

66. Weights of steel. The following straight-line formula, for wts of steel in highway and in single-track railway bridge superstructures, including their floor systems, is condenst from those of Messrs. J. B. Johnson and H. G. Tyrrell G 1471, 1474. For double-track railway bridges, add 90%. See our pp 731 and 738.

$$W = L(m + nL);$$

where
W = wt of structure in lbs; L = span in ft; and
m, n = coefficients, as follows:

Bridges for	Refs to notes below	m	n
Highways	A	50	2
Steam railways			
Plate Girders			
Deck		100 to 150	9 to 12
Thru		500 to 600	10 to 12
Truss bridges	B	200 to 400	5 to 7
	C	600 to 650	7
Electric Railways			
I beams (spans 5 to 20 ft)		50	5
Deck plate girders	D	30	5
Riveted trusses	E	200	0 8
	F	250	1.5

A Live load, 100 lbs/sq ft. Add or deduct 15 lbs/span-ft for each 2 ft of variation from 16 ft roadway width.
B For two 100-ton locos and 4000 lbs per lin ft of train.
C. For spans of 30 to 230 ft. Cooper loading E-50 (p 755); two 177.5-ton locos and 5000 lbs per lin ft of train.
D Live load, 2000 lbs/lin ft.
E. " ", 1000 " " 15-ton cars; spans 40 to 200 ft.
F. " ", 2000 " " 30-ton cars; " 20 to 180 ft.

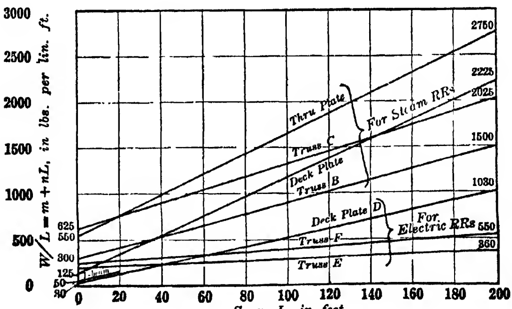

Fig. 1.

67. Fig 1 shows, approx, for each class, the average value of $W/L = m + nL$ = wt/span-ft, in railway bridge superstructures, as given by this equation.

68. For trestle bents and piers for steam railway bridges, add 9 lbs/sq ft of profile area from ground to rail-base; for electric railways, 6 lbs.

Trestles.

69. Quantities of timber in trestles, including 164 ft B M of deck timber per lin ft; and in pile trestles of 4-pile bents, 16 ft c to c, G 966.

$$F = L(mH + n) = mA + nL;$$
$$P = L(H + 20)/4$$

wherein:

F = total ft B M in structure;
L = length of trestle, in ft;
A = profile area of trestle, in sq ft, from ground to 4 ft below rail-base;
H = A/L = average height;
P = lin ft of piling in pile trestle;
m and n = coefficients as below.

	In pile trestles		In trestles			
For heights	up to 15′	15′ to 25′	25′	25′ to 50′	50′ to 75′	75′ to 125′
coefficients						
m =	0	0	6	8	9	10
n =	185	200	220	240	240	240

70. Single-track pile and timber trestles, of moderate heights, may be expected to **cost** from $5 to $10 per ft of length. For more than one track, to $15, $20 and more.

For costs of timber, see p 984.

Culverts.

For wts and costs of cast iron pipe, see pp 656, 658 and 995.
For those of vitrified pipe, and discounts, see pp 575 and 995.

71. The costs per lineal foot of culverts, in place, vary chiefly with the labor cost of placing.

Cast Iron									
Diam ins.	6	12	18	24	30	36	48	54	60
G 278	....	$2.20	3.50	...	...	...	....	....	....
to 280	....	2.50	3.75	...	...	...	....	....	....
G 1310 / G 1329 (Reproduction estimates)	$1.00	3.00	4.00	6.00	7.00	9.00	18.00	21.00	25.00
G 1713 (Pipe, $16/2000 lbs),				3.08	...	...	....	....	....
				3.80	...	...	....	....	....
G 1714 (Pipe, $14.50/ton),				2.58	3.14	4.09	6.66	....	....
G 1714 (Pipe, $15.80/ton),				2.09	3.12	4.11	7.01	....	11.87
Eng-Contr, 1916 May 31, p 494, Highway, State of Washington,				2.55	...	3.14			

Vitrified					
Diam ins.	6	12	18	24	30
Pipe, net, at factory, approx, add freight,	$0.06	0.20	0.40	0.80	1.30
Culvert, in place, G 278-280, Mass., 1907, contract prices, (Labor, $1.75 to $2.25)	0.30	0.60	1.10	2.00	3.75
G 1324, N Pac Ry in Wash, Reproduction estimate,	...	0.50	1.30	2.60	3.50
Eng-Contr, '13 Feb 5, p 163, Coal mine branch	...	0.30	0.63	1.20	2.00

72. Corrugated metal culverts, in place. G 1324, 1715, gives:— 36″, \$3.00/lin ft; 48″, 18 ft long (labor \$2.00/day) \$9.80/lin ft.

For cost of masonry, see pages 601, 988.
" concrete, " " 988, 1375.
" timber, " " 984.

73. Culverts, of these materials, vary widely, with the design and cross-section, in their costs per lin ft. Each structure should be estimated for itself.

Line-mile cost.

74. Line-mile cost of bridges, trestles and culverts on four western lines, as follows.—

G 1319. Gt Northn in Washn, 768 line-miles. Mountainous. Est'd reprodctn costs;
G 1329. N Pac in Washn, 1645 line-miles. Mountainous. Est'd reprodctn costs;
G 1374. Gt Northn system, 6635 line-miles. Chief Engineer's estimate.
G 1362. N Pac system, 5875 line-miles. Chief Engineer's estimate.

	Dollars per line-mile.			
	G 1319	G 1329	G 1374	G 1362
Trestles,				
18 ft high,	1670			
19 ft high,		1080		
Total, trestles,	1670	1080	786	610
Howe and combination trusses,				
span < 60 ft,	40	5		
60 – 100 ft,	40	16		
100 – 150 ft,	229	161		
> 150 ft,	183			
Total, Howe and comb'n,	492	182		
Draw spans,		59		
Miscellaneous,		87		
Steel bridges,				
Superstructure,	1490	1730		
Substructure	473	327		
Total, steel bridges,	1963	2057		
Howe, combn and steel bridges,	2455	2385	1315	1810
Bridges and trestles,	4125	3465	2101	2420
Culverts,				
Log,	112	26		
Timber,	74	79		
Box,	2	5		
Concrete,	67	63		
Stone box,	26	33		
Vitrified,	36	415		
Cast Iron,	46	183		
Total, culverts,	363	804	608	526
Bridges, trestles and culverts,	4488	4269	2709	2946

75. On a lumber railway, 7 miles long, in western Pennsylvania, Mr. Wm. Barclay Parsons found cost of timber trestles, of heights up to 28 ft, \$250/line-mile of road. A S C E Trans 1891, Vol 25, p 122, G. 1291.

Geographical Classification.

76. *Bridges, Trestles, Viaducts and Culverts.*

	$ per line-mile of road		$ per line-mile of road
In Washington	4662	In Michigan	1027
In Oregon	2655	In South Dakota	591
In Wisconsin	2510	Northern Pacific system	2946
In Minnesota	2576	Great Northern system	2709

Primary Accounts 8 to 12, inclusiv.
Track.

77. Trackwork may be taken as including I.C.C. Primary Accounts 8 to 12 inclusiv, as follows:—

8. Ties
9. Rails
10. Other track material (including turnouts)
11. Ballast
12. Track laying and Surfacing.

Primary Accounts 11 and 8. Ballast and Ties.

78. Ballast and ties may be expected to cost as follows:—

Ballast, cts/cu yd	Delivered	In track
Rock	50 to 70	75 to 110
Gravel and sand	20 to 30	30 to 60
Ties, cents each	30 to 60	40 to 80

79. Second-hand ties may cost from one-third to one-half as much as new ties; creosoting, from 25 to 35 cts/tie; other processes, 10 to 20 cts; switch ties, from $10 to $20 per 1000 ft B.M.; $20 to $60 per set.

80. Renewing ties may be expected to cost, approx,

in stone ballast,	about 20 cents per tie;
in loose gravel,	6 to 10 cents per tie;
in cemented gravel,	10 to 20 cents per tie;
in hard slag,	9 to 15 cents per tie.

81. Tie Spacing.

Dist, cen to cen	Ties per 30-ft rail	Ties per 33-ft rail	Ties per mile
1′ 8″ = $^5/_3$ ft	18	$19^4/_5$	3168
1′ 9″ = $^7/_4$ ft	$17^1/_7$	$18^6/_7$	3017
1′ 10″ = $^{11}/_6$ ft	$16^4/_{11}$	18	2880
1′ 11″ = $^{23}/_{12}$ ft	$15^{15}/_{23}$	$17^5/_{23}$	2755
2′ 0″ = 2 ft	15	$16^1/_2$	2640
2′ 1″ = $^{25}/_{12}$ ft	$14^2/_5$	$15^{21}/_{25}$	2534
2′ 2″ = $^{13}/_6$ ft	$13^{11}/_{13}$	$15^3/_{13}$	2437
2′ 3″ = $^9/_4$ ft	$13^1/_3$	$14^2/_3$	2347
2′ 4″ = $^7/_3$ ft	$12^6/_7$	$14^1/_7$	2263
2′ 5″ = $^{29}/_{12}$ ft	$12^{12}/_{29}$	$13^{19}/_{29}$	2185
2′ 6″ = $^5/_2$ ft	12	$13^1/_5$	2112

Primary Account 9. Rails.

82. Weights of rails, in pounds.

per yard;	50	60	70	80	90	100
per single-track mile;	176,000	211,200	246,400	281,600	316,800	352,000

Weight, in pounds per single-track mile
= 2 × 1760 × wt in lbs/yd of rail.

83. Cost. From 1888 to 1914, the price of rails, at mill, ranged only betw $24 and $32 per ton of 2240 lbs, except in 1897-8, when it fell to $18.00. Freight cost is usually about ½ ct/ton/mile. Second-hand rails usually cost a little more than half as much as new rails.

Primary Account 10. Other Track Material.

84. Joints and fastenings.

	lbs/track-mile	cts/lb	$/track-mile
Angle plates,	15,000 to 20,000	2 to 2¼	900 to 1400
Spikes,	6,000 to 7,000		
Tie plates,	25,000 to 35,000		
Bolts and nuts,	1,500 to 2,000	2½ to 3	35 to 60
Rail braces,	4,500 to 8,750	3 to 3½	135 to 300

1500 to 2500 rail braces per mile at 10 to 12 cts each, 3 to 3.5 lbs. each.

Rail joints, $2.50 to $5 00 per joint.

The two angle bars together (see p 823*b*) will weigh from 20 to 40 lbs per lin ft.

Turnouts.

85. Switches, from $20 to $30 per set.

86. The cost of frogs increases with the frog number (see Turnouts and Crossings, Part II, ¶ 4).

Rigid frogs, Nos 8 to 10, $20 to $35 each; spring-rail frogs, $45 to $55 each; crossing frogs, $225 to $325. Switch-stands, $5 to $10 each.

87. Complete turnout, $150 to $200, including ties, stand, connecting-rods, rail braces, lamp and gard-rails.

88. Sidings, from $0.80 to $1.20 per lin ft. See costs of items, ¶ 95.

Account 12. Track Laying and Surfacing.

89. Track-laying includes unloading the ties and rails, trimming earth to true grade, delivering and placing ties and rails, curving and joining rails.

90. Surfacing includes shoveling the earth in between the ties, alining the track, and tamping.

For costs of laying and surfacing, see estimates of track-mile cost, ¶ 145.

Track Cost per Mile.

91. Practically, each mile of *track* requires its 5280 lin ft of ballast, of ties, and of laying and surfacing, and its 10,560 lin ft of rails, with their joints and other fastenings. Only the item of turnouts is subject to notable variation, in cost/mile, as betw diff lines of the same general character; and this item (see tables below) is relativly insignificant; its cost ranging usually betw only 1 and 3% of the total track cost.

92. Hence, for given unit costs and given construction, the *track*-mile cost of track (given in our tables, below, ¶ 95) is practically constant; and the *line*-mile cost is practically = *track*-mile cost × number of tracks.

93. Elements affecting cost. With given material unit costs, the track-mile cost of ballast depends upon the ballast cross section; that of ties upon tie size and spacing (¶ 81); that of rails and fastenings upon wt/yd; and that of turnouts upon their construction and upon the number required per mile.

94. Increasing cost. Improvements in manufacture (and particularly large-scale mfr, the consolidation of small plants into larger systems under common control, and increast effort for efficiency) have resulted in material reduction in unit rail costs; but, as above noted (¶ 83), no great and progressiv reduction has taken place since 1887; and the unit costs of the other items (notably that of ties, following the growing scarcity of timber) have been advancing. Moreover, increase in wts of rolling stock and in carloadings have called for increasing solidity of construction. Hence, the tendency of track cost, per track-mile, is upward.

95. Table of approx costs, deduced from eighteen statements given in "Cost Data", and covering a total of over 48,000 track-miles. Practically all of the mileage, here represented, is in the western United States. See ¶ 96.

Primary Account	Dollars per track-mile			% of total track cost		
	Probable					
	Max	Min	Average	Max	Min	Average
11. Ballast,	1200	500	800	13	7	12
8. Ties,	2000	1000	1470	22	14	21
9. Rails,	4500	3000	3255	60	40	48
10. Joints, etc,	800	400	560	10	7	8
10. Turnouts,	150	80	135	3	1	2
12. Laying and surfacing,	800	400	580	11	6	9
Track	9450	5380	6800			100
=	1.79	1.02	1.29 per lin ft of single track.			

96. Geographical Tabulation, compiled from "Cost Data". Appraisd *track*-mile costs for five Western Ry systems included in the summary of ¶ 95, and having a combined mileage = 97% of the total mileage of that summary. For each item (Ballast, Ties, etc) the cost is here given (as in ¶ 95) (1) in dollars per *track*-mile, and (2) as a percentage of the total track cost of the given system.

I. C. C. Account			11		8		9	
G	State;	Track-miles	Ballast $	%	Ties $	%	Rails $	%
1333	Wis	10200	566	10.2	1095	19.7	2960	53.3
1336	Mich	10882	342	6.7	1025	20.2	2630	51.8
1347	Minn	10500	896	12.9	1665	24.0	3140	45.2
	R.R.;							
1362	N. P	7695	1355	15.4	2070	23.5	3720	42.3
1374	Gt N.	8116	1295	13.9	2300	24.7	3820	41.0
Totals & avs		47393	842	12.1	1575	22.7	3185	45.8
Tot track-miles & av costs, for 18 statements, from ¶ 95.		> 48000	800	12	1470	21	3255	48

I.C.C. Account			10				12		
G	State;	Track miles	Joints &c $	%	Turnouts $	%	Lay & Surf $	%	Track $
1333	Wis.	10200	515	9.3	116	2.1	328	5.9	5580
1336	Mich.	10882	353	7.0	135	2.7	602	11.9	5087
1347	Minn.	10500	565	8.2	132	1.9	508	7.3	6906
	R.R.;								
1362	N. P.	7695	538	6.1	260	3.0	860	9.8	8803
1374	Gt N.	8116	910	9.8	111	1.2	860	9.2	9296
Totals & avs.		47393	562	8.1	147	2.1	610	8.8	6921
Tot track-miles & av costs, for 18 statements, from ¶ 95.		>48000	560	8	135	2	580	9	6800

Primary Accounts 13-15 inclusiv.
Track Structures.

Primary Account 13. Right-of-way Fences.

97. Barbed wire fence.
Posts. Wood or concrete, 10 to 30 cts each. Usually 15 to 20. Spaced from 10 to 20 ft cens. Setting, 5 to 7 cts each.
Wire, 2.5 to 3.5 cts/lb.
Barbd wire fence, complete, 2.5 to 5.5 cts/lin ft of fence.

98. Post and rail fence.
Posts, 6 to 7 ins × 2 to 3 ins, 8 ft long, set 3 ft in ground, 6 to 20 cts each. Set at 8 to 9 ft cens.
Post and rail fence. 5 to 12 cts/lin ft of fence. Usual contract price, in Mass., with one coat white lead, 15 cts/lin ft.
Worm fences, 7 rails high, about one-fourth less than post-and-rail
Picket fence, painted, 90 cts to $1.50/lin ft. Saml Tobias Wagner, Am Soc C E, Trans, 1913 Dec, Vol 76.

99. Gas-pipe railing, with cast iron posts, Phila & Reading Ry, Phila. Contract price, attacht to walls, 58 to 75 cts/lin ft.

Primary Account 14. Snow and Sand Fences and Snowsheds.

100. Snow fences, from 10 to 20 cts/lin ft.

101. Snow sheds (not to withstand avalanches) $10 to $20/lin ft. for avalanches, $40 to $70/lin ft.
Of *concrete*, Gt Northn Ry in Washn; per lin ft of shed; 8 cu yds conc, 1500 lbs steel, $141. Ry Age Gaz, 1911 Jan 13, p 83

Primary Account 15. Crossings and Signs.

102. The line-mile cost of crossings, cattle-gards and signs, will seldom exceed $100, but may reach $200 or $300.

Primary Accounts 16 to 20, inclusiv.
Buildings.

Primary Account 16. Station and Office Buildings.

Stations.

103. Frt, passr and combination stations, of frame or brick; $1 to $2/sq ft of ground coverd, for small, plain one-story frame or brick buildings; $3.50 to $5 or over for larger and more elaborate structures. A second story may add two-fifths to the cost.
104. Columbia Ave passr sta, Phila & Reading Ry, Phila. Main building, ornamental, two stories, 170 × 150 ft; two concrete platforms, 800 ft long; umbrella sheds; four baggage elevators; total cost, $217,134 = $8.50/sq ft. Three smaller and semi-suburban, two-story passr stations, with tunnels and baggage elevators, in Phila., $43,000 to $63,000 each. Saml Tobias Wagner, Am Soc C E Transns, 1913 Dec, Vol 76, p 1866.
105. Passr stations. Average size, in towns of from 10,000 to 15,000 population, on 31 rys, about 2000 sq ft. (Comm, Am Ry Eng & M W Assn, 1904.)

106. Platforms, per sq-ft:—cinder, 5 to 7 cts; wood, 8 to 12 cts; concrete, 10 to 20 cts; brick, 20 to 30 cts.

107. Track scales, from $1200 to $3000 each, installed, according to size and capacity.

Primary Account 17. Roadway Buildings.

108. Section houses; $0.70 to $1.30/sq ft of ground occupied. Gillette, p 1121, mentions five three-room frame section-houses ("Jap houses"), of very cheap construction, about $0.50 to $0.60 per sq ft. Tool, hand-car and material store houses, $0.25 to $0.50 per sq ft.

95. Table of approx costs, deduced from eighteen statements given in "Cost Data", and covering a total of over 48,000 track-miles. Practically all of the mileage, here represented, is in the western United States. See ¶ 96.

Primary Account	Dollars per track-mile: Probable Max	Probable Min	Average	% of total track cost: Max	Min	Average
11. Ballast,	1200	500	800	13	7	12
8. Ties,	2000	1000	1470	22	14	21
9. Rails,	4500	3000	3255	60	40	48
10. Joints, etc,	800	400	560	10	7	8
10. Turnouts,	150	80	135	3	1	2
12. Laying and surfacing,	800	400	580	11	6	9
Track	9450	5380	6800			100
=	1.79	1.02	1.29 per lin ft of single track.			

96. Geographical Tabulation, compiled from "Cost Data". Appraisd *track*-mile costs for five Western Ry systems included in the summary of ¶ 95, and having a combined mileage = 97% of the total mileage of that summary. For each item (Ballast, Ties, etc) the cost is here given (as in ¶ 95) (1) in dollars per *track*-mile, and (2) as a percentage of the total track cost of the given system.

I. C. C. Account G State;	Track-miles	11 Ballast $	%	8 Ties $	%	9 Rails $	%
1333 Wis	10200	566	10.2	1095	19.7	2960	53.3
1336 Mich	10882	342	6.7	1025	20.2	2630	51.8
1347 Minn	10500	896	12.9	1665	24.0	3140	45.2
R.R.;							
1362 N. P	7695	1355	15.4	2070	23.5	3720	42.3
1374 Gt N.	8116	1295	13.9	2300	24.7	3820	41.0
Totals & avs	47393	842	12.1	1575	22.7	3185	45.8
Tot track-miles & av costs, for 18 statements, from ¶ 95.	> 48000	800	12	1470	21	3255	48

I.C.C. Account G State;	Track miles	10 Joints &c $	%	Turnouts $	%	12 Lay & Surf $	%	Track $
1333 Wis.	10200	515	9.3	116	2.1	328	5.9	5580
1336 Mich.	10882	353	7.0	135	2.7	602	11.9	5087
1347 Minn.	10500	565	8.2	132	1.9	508	7.3	6906
R.R.;								
1362 N. P.	7695	538	6.1	260	3.0	860	9.8	8803
1374 Gt N.	8116	910	9.8	111	1.2	860	9.2	9296
Totals & avs.	47393	562	8.1	147	2.1	610	8.8	6921
Tot track-miles & av costs, for 18 statements, from ¶ 95.	>48000	560	8	135	2	580	9	6800

completely furnisht with power, lathes, planing machines, scales, and all other necessary tools and appliances, say $100,000 to $150,000, exclusiv of ground. A large yard, of at least an acre, should adjoin the buildings. A moderate establishment, for the repair of a few locos only, $30,000 to $40,000.

118. A set of car shops for the Wabash Ry, at Decatur, Ill., 1906, cost $368,000. G 1147. It included

Car shop,	88 × 464 ft, at 2.7 cts/cu ft;	
Blacksmith and machine shop,	80 × 294 ft, at 3.0	"
Storehouse, with 2-story office,	40 × 464 ft, at 5.5	"
Wood mill,	80 × 238 ft, at 2.9	"
Cabinet-makers' shop	40 × 350 ft, at 4.5	"
Power house,	60 × 108 ft, at 3.4	"

Miscellaneous.

119. Transfer tables, $1000 to $2000 each. See also Turntables, ¶ 115.

120. Ash pits, $10 to $20 per lineal foot.

Primary Account 44. Shop Machinery.

121. Machinery. A comparison of five locomotiv repair shops showd costs of shop machinery ranging as follows:—$650 to $950 per machine; $450 to $850 per loco handled per year; $1.10 to $3 00/sq ft of ground coverd by buildings.

Yards.

122. Freight yards; elevated, with concrete retaining walls. Prices, exclusiv of track work.

	Master St.	Berks St.	York St.
Capacity, cars,	66	55	104
Contract price, total,	$97,868	$70,350	$67,059
" " per car,	1480	1,280	644

123. Car-cleaning yards; including service building, car repair and paint shop, power-house building and piping; but exclusiv of track work and mechanical equipment. See ¶ 124.

124. Coal-pocket yards, including stables, paving, drainage, etc. (See also Fuel Stations, ¶ 111.)

	Car cleaning yds	Coal pocket yds	
Capacity,	230 Cars	20,000 Tons	1,170 Tons
Contract price, total,	$53,327	$273,578	$42,884
" " per unit,	$232	$14	$37

Phila & Reading track elevation, Phila. Sam'l Tobias Wagner, Am Soc C E Trans, 1913 Dec, Vol 76, p 1874.

For Primary Account 21, Grain elevators,
" " " 22, Storage warehouses,
" " " 23, Wharvs and docks,
" " " 24, Coal and ore wharvs,
see table of line-mile costs, ¶ 129.

Primary Account 26.

Telegraph and Telephone Lines.

125. Poles, $1.50 to $3.00 each. Galvanized iron wire, 4 to 5 cts/lb.

126. Single-wire line, $100 to $200/mile of wire.

127. Estimated reproduction costs, in Washington. H. P. Gillette. Gt Northn, $74/line-mile; N. Pac, $151/line-mile.

Primary Account 27. Signals and Interlockers.

128. Average interlocking signal plant, $8,000, including cross-over, 4 derails, 4 high sigs and 6 dwarf sigs. G 1287.

See also Signals, p 993.

129. Tabulation of Costs of Structures in $ per line-mile.

Ref	G 1310	G 1321	G 1332	G 1334	G 1336	G 1347	G 1362	G 1374	Total Mil'ge
RR or State	Gt. N. Wash	N. P. Wash	Oreg. RR. & Nav	Wis	Mich	Minn	N. P. Sys	Gt. N. Sys	& av costs
Line-miles	768	1645	500	7090	7813	7596	5875	6635	37922
Item No. †									
16	1313	971	305	553	526	796	1138	495	720
20	334	666	165	548	276	916	675	550	580
18, 19	306	245	218	255	132	306	448	390	294
21-24	952	636	112	672	911	853	780	645	758
35	261	717	260	253	110	427*	312	369	412
13	87	166	255	227	353	200*	116	115	211
26, 27	61	153	78	123	97	259	272	390	214
44	196	179	46	242	142	241	186	270	211
14	245	79	50*	50*	50*	50*	92	112	73
Total	3755	3812	1489	2923	2597	4048	4019	3336	3473

For GENERAL ACCOUNT II. EQUIPMENT,
See ¶¶ 132, etc.

GENERAL ACCOUNT III. GENERAL EXPENDITURES.

130. Under this head may be clast such expenses as that of effecting company organization, including necessary legislation; that of obtaining a certificate of public convenience and necessity (required in some states); that of obtaining local franchises and consents; that of compensation to "promoters" (an item subject to grave abuse); and those connected with the issue and marketing of stocks and bonds, including official authorization, and discount upon those securities which must be sold below par. Their total varies widely. Mr. Walter Loring Webb ("Railroad Construction", 1905, p 394) says:—"It has been estimated that about 2% of the railway capital of Great Britain has been spent in Parliamentary expenses over the charters."

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*Assumed for our table.

†Item No.	Item.
16,	Stations.
20,	Shops and engin houses.
18, 19,	Water and fuel stations.
21-24,	Docks, wharvs, warehouses, stock yds, grain elevators.
35,	Miscellaneous buildings.
13,	Fencing, etc.
26, 27,	Telegraph, telephone and signals.
44,	Shop machinery.
14,	Snow and ice protection.

Primary Accounts 73 (Law) and 76 (Interest during construction.)

131. In statements of actual cost, the item of interest, plus legal expenses, usually ranges between 2.5 and 5% of the total construction cost exclusiv of rolling stock; but, in estimates, valuations and appraisals, the estimators, by assuming long periods of non-remunerativ conditions, sometimes arrive at much higher figures. Thus, in the case of the Gt Northn Ry System, G 1374, where the cost, exclusiv of equipment, amounted to $56,330/line-mile, the Chf Engr estimated interest at $5690 per line-mile, and legal expenses at $563/line-mile; total, $6253/line-mile.

GENERAL ACCOUNT II. EQUIPMENT.

Primary Accounts 51 (Steam Locomotivs) and 53, 54, 57 (Freight-train and Passenger-train cars and Work Equipment.)

132. Locomotiv and tender (7 to 9 cts per pound)

Passenger and switching-,	$10,000 to $12,000 each
Atlantic type, and freight-,	$15,000 to $17,000 "

133. Cars

Pullman-,	$12,000 to $15,000 "
Passenger-,	$ 6,000 to $ 8,000 "
Baggage-,	$ 4,000 to $ 5,000 "
Freight-,	$ 600 to $ 700 "

134. Costs per line-mile, etc.

	Line-Miles	$ per line-mile				
		Locos	Cars Pssr	Cars Frt	Cars Work &c.	Total Locos & cars
G 1317 Spokane Falls,	131	546	256	528	125	1455
G 1314 Gt N, Wash, repr,	768	1740	930	3030	260	5960
G 1330 N. P., Washn, repr	1645	2243	972	3444	319	6978
G 1317 Gt N System,	6635	1512	614	3070	224	5415
G 1329 N. P. System,	5875	2210	864	3650	324	7048
G 1335 Wisconsin, Pence,	7090	1625	750	4363	127	6865
G 1336 Michigan,	7813	1155	2408	2527	90	6180
G 1347 Minnesota,	7596	2250	872	6175	175	9472
Total mileage,	37,553					
Average costs,		1750	1126	3920	189	6985

	Line-miles	% of total rolling stock cost			
		Locos	Cars Pssr	Cars Frt	Cars Work
G 1317 Spokane Falls,	131	37.4	17.6	36.4	8.6
G 1314 Gt N, Wash, repr,	768	29.2	15.6	50.8	4.4
G 1330 N. P., Washn, repr	1645	32.1	13.9	49.4	4.6
G 1317 Gt N System,	6635	27.9	11.3	56.7	4.2
G 1329 N. P. System	5875	31.4	12.3	52.0	4.3
G 1335 Wisconsin, Pence,	7090	23.7	10.9	63.6	1.8
G 1336 Michigan,	7813	18.7	39.0	40.9	1.4
G 1347 Minnesota,	7596	23.7	9.2	65.2	1.9
Total mileage,	37,553				
Average costs,		25.0	16.1	56.2	2.7

For GENERAL ACCOUNT III. GENERAL EXPENDITURES

See ¶¶ 130, 131.

TOTAL COSTS PER MILE

Property Investment.

135. From the Interstate Commerce Commission's 27th Annual Report on the Statistics of Railways in the United States, for the year ended June 30, 1914, Washington, 1915, Statement No. 44 A, pp [illegible]-4, we deduce the following data respecting "property investment" in "road and equipment" for the roads represented in the statement.

Class I. Roads having annl operating revenues > $1,000,000;
Class II. " " " " " from $100,000 to $1,000,000.

For definitions of eastern, southern and western districts, see Fig. 2.

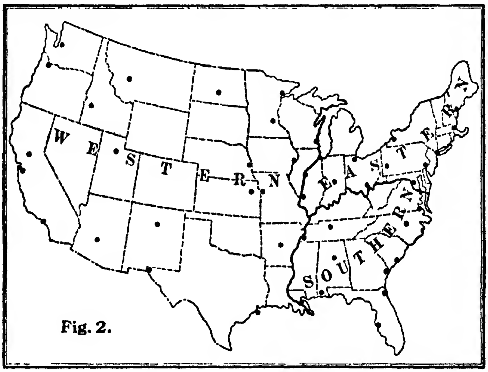

Fig. 2.

	Class I			
	Eastern	Southern	Western	Tot U. S.
Track-miles	32,121	33,418	105,964	171,503
Property investment, $ per track-mile;	138,000	67,000	61,700	77,000
	Class II	Tot Op'g	Non-op'g	U. S.
Track-miles	18,367	189,870	37,534	227,404
Property investment, $ per track-mile;	43,300	73,800	76,000	74,200

136. For the *line*-mile property investment of the 189,870 operating track-miles represented in the foregoing statement, we have (see Statistics of Rys in U. S., pp 12 and 13):

$$\frac{\text{Total track-miles operated in U. S.}}{\text{Total line-miles operated in U. S.}} = \frac{387{,}208}{256{,}547} = \text{say } 1.5$$

Hence
line-mile property investment = 1.5 × $73,800 = $110,700.

137. Funded Debt. The funded debt (1914) of the Class I and Class II roads and their nonoperating subsidiaries (a total of 243,790 line-miles), as given on p 32 of "Statistics" for 1914, amounts to $11,566,541,553, or $47,400 per line-mile. Assuming (see ¶ 136) 1.5 track-miles for each line-mile, we have: Funded debt, per track-mile = $47,400/1.5 = $31,600.

Total constrn costs, per line-mile & per track-mile.

138. The following table indicates (mostly in round numbers) the range of total construction costs (including right-of-way, unless otherwise stated) per line-mile and per track-mile. The track-mile costs are lower than would be those of single-track lines of the same character.

Dates given ("E-C, 1907 Jun 26," etc) refer to Engineering-Contracting.

For characteristics of lines, see list of Gillette data, ¶ 21.

	Tot constrn cost in $/mile of line	track
G 1336; E-C, 1907 Jun 26. Steam rys of Michigan. Estimate, 1900.	18,161	13,050
G 1347; E-C, 1909 Mar 3. Rys of Minnesota. Reproduction cost.	44,700	32,500
G 1336; E-C, 1907 Jun 26. Rys of Wisconsin. Appraisd, 1903-4.	22,000	15,800
G 1334; E-C, 1910 Jan 19. Rys of Wisconsin. Appraised, 1907,	27,600	19,200
G 1374; E-C, 1908 May 6. Gt. Northn system. Estimate of reproduction cost,	56,500	46,100
G 1303-1319; E-C, 1909 Dec 8. Gt Northn Ry lines in Washn,		
G 1303; Original cost,	44,412	41,000
G 1310; " " plus improvements,	49,848	40,000
G 1319; Reproduction cost,	66,753	53,600
G 1307; Spokane Falls & Northn branch. Estimated constrn cost,	17,000	14,500
G 1308; Washn & Gt Northn branch,	36,500	33,300
G 1306; Fairhaven Southn branch. Original constrn,	22,565	......
G 1362; E-C, 1908 Apl 15. Northn Pac system. Appraisal. (Right-of-way = $18,344/line-mile),	59,514	45,800
G 1321; E-C, 1910 Jan 12. Northn Pac Ry lines in Washn. Original cost plus improvements,	38,895	29,000
G 1329; Reproduction cost. Mr. Gillette's estimate,	54,277	40,600
G 1353; E-C, 1907 Jul 24. Chicago, Milwaukee & St. Paul Ry in S. Dakota,	14,726	13,650
G 1331; Oregon R R & Nav Co's lines in Washn. Original cost (including betterments undistributed),	26,319	23,200
G 1332; Estimate of reproductn cost,	27,122	23,900
G 1291; Branch of St. Louis Southwestern Ry in Texas. Exclusiv of land,	17,320	15,400
G 1289; Winchester & Beattyvil RR in Ky.	12,200	10,850
G 1290; Lumber ry in northwestern Penn.	9,094	9,094

139. Commercial values, and valuations for assessment, per mile of single track, as determind by the I.C.C. See Census Bulletin 21; Tables 1 and 2; 1902-1905.

See also top of next page.

	Miles of single track	Commercial Value $	Valuation for assessment for taxation purposes
Alabama,	4,669	32,200	11,550
Alaska,	28	3,600	
Arizona,	1,751	39,000	3,810
Arkansas,	4,126	30,200	8,410
California,	6,263	56,000	14,900
Colorado,	4,976	39,950	9,960
Columbia (Dist),	32	174,300	77,800
Connecticut,	1,018	103,500	118,300
Delaware,	336	51,500	
Florida,	3,556	22,500	6,150
Georgia,	6,305	24,800	10,000
Idaho,	1,462	62,900	6,920
Illinois,	11,623	69,300	36,600
Indn Terr,	2,532	31,400	
Indiana,	6,918	54,250	24,000
Iowa,	9,859	35,000	5,850
Kansas,	8,811	40,500	6,800
Kentucky,	3,253	47,850	23,800
Louisiana,	3,899	31,600	7,400
Maine,	2,022	39,600	
Maryland,	1,421	93,100	
Mass.,	2,119	118,000	
Michigan,	8,660	32,100	22,700
Minnesota,	7,811	59,800	
Mississippi,	3,480	31,000	8,570
Missouri,	7,711	40,200	12,670
Montana,	3,267	60,100	11,230
Nebraska,	5,821	45,200	7,920
Nevada,	987	44,300	14,000
New Hampshire,	1,276	62,400	17,800
New Jersey,	2,278	146,400	101,700
New Mexico,	2,505	34,500	3,400
New York,	8,297	108,300	27,700
North Carolina,	4,075	27,800	17,100
North Dakota,	3,191	38,700	6,930
Ohio,	9,197	74,900	14,600
Oklahoma,	2,611	30,000	4,560
Oregon,	1,737	43,600	
Pennsylvania,	11,023	128,900	
Rhode Island,	212	121,400	74,700
South Carolina,	3,175	23,800	9,280
South Dakota,	3,047	16,300	4,700
Tennessee,	3,481	37,700	16,900
Texas,	11,848	20,100	8,020
Utah,	1,780	50,800	11,600
Vermont,	1,063	35,200	25,700
Virginia,	3,932	53,800	16,100
Washington,	3,356	54,500	7,770
West Virginia,	2,837	71,000	10,100
Wisconsin,	7,049	40,400	30,900
Wyoming,	1,248	80,400	6,000
	213,932	52,600	

"By 'Commercial value' is meant the estimate placed upon the worth of a property regarded as a business proposition." p 8. "An effort was made, also, to test the results submitted in this report ("Commercial value") by the formal or implied valuations of railway properties made by state officials." ("Valuation for assessment"), p. 13. From Census Bulletin 21.

See "The Valuation of Public Service Corporation Property," by Henry Earle Riggs, Am Soc C E, Trans, 1911 June, Vol 72, p 15, Table 1.

140. Costs per mile of first class RRs. The following figures of cost per mile of first-class railroads are condenst and classified from R R Gaz of 1906. They include preliminary surveys, clearing right of way, grading, roadbed, ties, rails, ballast and side tracks, in shape for operation, but not real estate, stations, rolling stock and signals. *s* = single track. *d* = double track.

Six first class railroads in eastern U.S., with steel bridges, concrete abuts and piers, 80-lb rail, and stone or gravel ballast.

	Year built	Miles of line	Miles of sid'gs	$ per line-mile	Conditions
A	1902	9.06 *s*	5.45	26,300	River line. Considerable hill-side cut & fill in earth & loose rock. Culverts, C I pipe & conc. No tunnels. Few bridges.
B	1903	15.77 *s*	3.61	37,014	River line. Heavy side-hill work, earth and loose rock. 3 river crossings.
C	1901	30.08 *s*	6.64	60,628	River line. Sinuous alinement. 3 crossings of river 300-500′ wide. 4 tunnels = 1.33 miles total.
D	1903	3.60 *d*	0.00	76,336	Detour ar'd city. Sand and gravel. Heavy work at 4-track R R crossing Highway crossings. 2 deep ravines.
E	1899	1.57 *d*	0.00	105,186	Connection betw 2 main lines. 4-track R R crossing. Highway crossing. Heavy embankments. Extensiv bridging & grade-changes.
F	1898	11.00 *d*	0.00	50,000	Detour ar'd city. 4 overhead and 5 underhead trunk-line R R crossings. Heavy cuts & fills; long hauls. 3 stream crossings.

First class branch line in Texas:

	Year built	Miles of line	Miles of sid'gs	$ per line-mile	Conditions
	1903 to 1904	12.13 *s*	1.52	19,649	75-lb rail. Mostly tangent on level ridge. R R crossing. Few openings. No side-hill cuts. Considerable fill. Earth; some loose rock.

Through trunk-line work, east of Chicago.

	Year built	Miles of line	Miles of sid'gs	$ per line-mile	Conditions
A	1903	10.72 *s*	....	78,000	Low-grade line; 85-lb rail; 2/3 excavated in rock; 1/3 clay & loam.
B	1903	4.10 *s*	2.35	40,700	Ohio. Mine branch. Earth & solid rock. Ravine crossing; Single-track, timber-lined summit tunnel, 475′ long; pipe culverts; timber trestles.
C	1903-4	12.48 *s*	0.50	40,000	W Va. Bituminous coal country. River to river; crossing summit.
D	1902-3	8.10 *d*	....	154,000	W Va. Cut-off. Tunnel 4120′ long. 0.3% grade eastbound; 0.8% westbound.
E	1903-5	51.84 *d*	25.00	100,000	Ohio. Low-grade line. Heavy grading & masonry. Grade-crossings generally avoided.

141. Eng News, 1909 Dec 2, p 623. reprints, from Archiv für Eisenbahnwesen, a table from which we select the following:

	Date	Miles*	"Construction capital" in millions of dollars	$ per mile*
Europe	1904-1908†	181,853	23,588	130,000
Germany	1907	35,058	3,840	108,500
France	1905	28,973	3,520	122,000
Gt Britain & Ireland	1904	22,630	6,085	271,000
Canada	1907 Jun 30	22,447	1,295	58,000
Cuba	1905	1,534	66	43,100

Track Elevation.

142. In 1907 to 1911, the Philadelphia & Reading Railway Co. eliminated grade-crossings on its line betw Green St and Wayne Junc, Philadelphia, 4.3 line-miles. The work included

3,365 ft of 4-track steel viaduct, 27,180 tons,
11 signal bridges,
67,500 cu yds third-class masonry in retaining walls,
151,800 " concrete " in abuts and walls
430,400 " embankment,
4,322,000 ft lumber in temporary trestles,
16 stations and other buildings (including Columbia Ave station, ¶ 104, costing $217,134),
30.1 miles of permanent single track,
46 industrial connections reconstructed,
7 miles telegraph, telephone and electric light conduit,
139,620 cu yds excavation for street-grade changes,
18,306 feet of sewers, $276,679,
104,312 sq yds of street driveway and sidewalk paving.

The total estimated cost was $7,659,740, or say = $1,785,000 per line-mile = $255,300 per track-mile.

Sam'l Tobias Wagner, Am Soc C E, Trans, 1913, Dec, Vol 76.

Elevated and Subway RRs.

143. Elevated railroads. Per mile; total.

G 1376-1379. New York and Brooklyn. Double-track, exclusive of equipment, $300,000 to $350,000 per double-track mile. (No real estate cost.)

144. Subway.

G 1392. New York. A half-mile, double-track, cost the contractors about $800,000, made up approx as follows:—

Earth excavation	410,000
Rock "	5,000
Concrete	90,000
Steel	120,000
Underpinning, etc	125,000
Other items	50,000
	$800,000

For itemized and total cost per mile of surface electric lines see ¶ 146.

*Whether line-miles or track-miles is not stated.
†Includes Netherlands, 1897, with 1653 miles, $138 million.

ITEMIZED COSTS PER MILE

145. The following table gives the itemized cost of steam railroads, per line-mile and per track-mile, and the percentages of total construction cost (including equipment cost) for three typical lines, representing, respectivly, the approx max, mean and min of the usual range of cost, viz:

	G 1319			G 1347			G 1290		
	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%
I. Road—									
Surveys							130		
Engineering during construction							375		
1. Engineering	1406	1133	1.92	1250	910	2.31	505	449	3.70
Right of way,	3917								
Terminal sites,	18400								
2. Land for transportation purposes,	22317	17930	30.74	9637	7000	17.81	428	380	3.14
Clearing and grubbing,	914								
Excavation,	9730								
Embankment,	980								
Trestles and filling same,	2212								
Overhaul,	108								
Cribbing and bulkheading,	489								
	14433			7373					
Masonry,	1030								
Slope walls and retaining walls,				318					
Improvements to roadbed,	1732			1546					
3. Grading,	17195	13790	23.67	9237	6720	17.08	2233	1985	16.37
Tunnels,	5559			33					
5. Tunnels and subways,	5559	4460	7.65	33	24	0.06			

	G 1319			G 1347			G 1290		
	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%
Trestles,							1393		
Bridges									
Timber and combination—,	487								
Steel—(including foundations),	1960								
Highway—,	218								
	2665								
Culverts,	365						159		
6. Bridges, trstls & culvs,	3030	2430	4.16	2576	1875	4.76	1552	1381	11.36
8. Ties,	1870	1504	2.56	2302	1678	4.26	696	619	5.10
9. Rails,	5126	4120	7.05	4345	3163	8.05	2370	2105	17.35
Joints and other fastenings,	1001			782			506		
Turnouts,	107			183			140		
10. Other track material,	1108	890	1.52	965	704	1.78	646	574	4.73
11. Ballast,	1148	924	1.58	1240	904	2.29	330	293	2.42
12. Tracklaying and surfacing,	872	700	1.20	703	512	1.30	988	878	7.24
13. Right-of-way fences,	105	84	0.14	300	220	0.55	41	36	0.30
Snow protection,	282								
14. Snow & sand fences & snow sheds,	282	226	0.39						
15. Crossings and signs,	24	19	0.03	64	47	0.11			

	G 1319			G 1347			G 1290		
	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%
Stations; Pass'r, 714; Frt, 480	1194			772			165		
Warehouses, stockyards, etc., ...	90			98					
16. Station and office buildings, ...	1284	1030	1.76	870	635	1.61	165	147	1.21
Section horses and tool houses.									
17. Roadway buildings, ...	125	100	0.17						
18. Water stations, ...	179	144	0.25	211	154	0.39	172	153	1.26
19. Fuel stations, ...	131	105	0.18	95	69	0.18			
Enginhouses, ...	130								
Cinder pits, ...	19								
Turntables, ...	35								
	184			373					
Machine and repair shops and sheds, ..	207			543					
20. Shops and enginhouses, ...	391	312	0.54	916	666	1.69			
21. Grain elevators.	163	131	0.22						
23. Wharvs and docks, ...	845	680	1.17	779	567	1.44			
26. Telegraph and 'phone lines, ...	74	59	0.10	186	135	0.34			
27. Signals and interlockers, ...				73	53	0.13			
29. Power plant buildings, ...				105	76	0.19			
35. Miscellaneous structures, ...	314	252	0.48	572	416	1.06			

	G 1319			G 1347			G 1290		
	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%	$ per line-mile	$ per tr'k-mile	%
38. Roadway small tools,				27	20	0.05			
Freight on track materials,				478			321		
43. Other expenditures—Road,				478	348	0.87	321	286	2.35
44. Shop machinery,	236	190	0.32	241	175	0.45	83	74	0.61
47. Unapplied constrcn matl & sup,	470	378	0.65	686	500	1.27			
Legal and general expenses,	401			348			264		
73. Law,	401	323	0.55	348	253	0.64	264	235	1.94
76. Interest during construction,	2098	1686	2.88	4115	2990	7.60			
Contingencies,				2352			83		
Sidings,							2021		
77. Other expndtres—Gen'l,				2352	1705	4.34	2104	1870	15.41
Permanent construction,	66753	53700	91.80	44706	32519	82.50	12898	11465	94.49
II. Equipment—									
51. Steam locomotivs,	1740	1400	2.39	2250	1640	4.16			
53. Freight-train cars,	3020	2425	4.14	6175	4480	11.40			
54. Passenger-train cars,	930	746	1.28	872	634	1.61			
57. Work equipment,	260	209	0.36	175	127	0.32			
Cars,	4210	3380	5.78	7222	5241	13.33			
Rolling stock,	5950	4780	8 17	9472	6881	17.49			
58. Floating equipment,				6					
Equipment,	5950	4780	8.17	9478	6881	17.49	752	668	5.51
Permanent constrn and equipm't,	72703	58480	100.00	54184	39400	100.00	13650	12133	100.00

146. Surface electric railroads. The discrepancies, betw the totals of the following statements and estimates, are explaind partly by the wide diffs betw the given per-mile costs of certain items, and partly by the omission, from some of the statements and estimates, of items included in the others. The more important of these omitted items are noted below. Owing to diffs of classification betw the several authors, we have been obliged, in some cases, to resort to assumption in proportioning the costs.

G 1417. First-class third-rail suburban line, 62.5 miles long, of which 6.5 miles were laid in city streets. Exclusiv of real estate.

G 1418. Estimate. Interurban trolley R R, based upon experience in New England in 1902. Exclusive of buildings, power and equipment.

G 1419. Third-rail line. (The total, for an otherwise similar trolley line, was $832 less per line-mile). Exclusiv of land, grading, bridges, trestles and culverts. "Electric Railways," by Sydney W. Ashe, 1907

G 1438. Estimate. 3 miles of double-track trolley line. Exclusiv of grading, bridges, trestles and culverts. "Street Railways," by C. B. Fairchild, 1892.

G 1440. Estimate. Max cost of 4-track third rail suburban R R. Exclusiv of engineering. "Electric Railway Economics," by W. C. Gottschall, 1903.

Primary accounts	Dollars per single-track mile				
	G 1417	G 1418	G 1419	G 1438	G 1440
1 Engineering,	1000	600	1000	1000	
2 Land for transpt'n purposes,		1000		2083	20000
3 Grading,	1536	7230			6000
6 Bridges, trestles and culverts,	2481	2350			12000
8 Ties,	1540	1426	1980	1267	1848
9 Rails,	3600	3465	3819	7000	5100
10 Other track material,	965	686	1000	800	
11 Ballast,	1856	1500	1760	2376	4125
12 Tracklaying and surfacing,	725	1056	600	1584	1650
13 Right-of-way fences,	280	640	467		
16 Station & offc bldgs,	212		225		
20 Shops & enginhouses,	384			4834	
44 Shop machinery,				1417	
26 Teleg & teleph lines,	144	400	150		
27 Sigs & interlockers,	560	1600	500		
29 Power plant bldgs, / 45 " " mach'y,	3144		3275	20791	18000
30 " substa bldgs, / 46 " " apparat,	1432		3000		
31 " transm'n systs,	1232	1100	1404	560	2500
32 " distrib " ,	3209	2200	2600	1130	7000
33 " line poles & fixtrs,		650	450	1607	
36 Paving,				14079	
Other items,	4680	270	2304	1500	4400
Permanent constrctn,	28980	26173	24534	62028	82623
Equipment;					
54 Pass'r-train cars,	2440		5500	12437	8000
57 Work equipment,				1442	
Perman't constrn & eq'p't,	31420	26173	30034	75907	90623

For total per-mile costs of elevated RRs and subways, see ¶¶ 143, 144.

RAILROAD STATISTICS

Table 1. In the United States.

(From Reports of the Interstate Commerce Commission.)

Class I. Roads having an'l op'g revenues over $1,000,000;
Class II. " " " " " from $100,000 to $1,000,000;
Class III. " " " " " less than $100,000.

For years ending June 30	1900	1907	1914
			Classes I II & III
Plant			
Line-miles built in one year.......	4,051	5,588	*†2,454
Line-miles in op'n at end of year.	192,556	227,455	*254,555
Track-miles in op'n at end of year	258,784	327,975	*384,115
Rolling stock in operation.........			
Number of locomotives.........	37,663	55,388	*65,658
			Classes I & II
Number of passenger cars......	34,713	43,973	*39,859
" " baggage, express and postal cars	....	....	*13,607
" " freight and other cars	1,416,125	2,082,621	*2,450,356
			Class III
" " unclassified cars ...	....	....	*11,807
			Classes I II & III
" " all cars	1,450,838	2,126,594	*2,515,629
Construction Cost of Road and Equipment.			Classes I & II
Total, in millions of dollars.......	10,263	13,030	*16,937
No. of line-miles represented.....	181,437	210,793	*235,986
Cost per line-mile, $.............	56,567	61,816	*71,770
Operation For One Year.			Classes I & II
Pass'rs carried 1 mile, per line-mile	83,295	123,259	144,278
Tons of frt " " " " " "	735,366	1,052,119	1,176,923
Gross earnings from operation,‡...			
per line-mile			
from passengers, $...........	2,067	3,032	3,476
" freight, $..............	5,466	8,123	8,596
" other sources, $........	189	228	315
" all sources, $..........	7,722	11,383	12,387
per passenger-mile, from pas'rs, $	0.0200	0.0201	0.0198
per ton-mile, from freight, $....	0.0073	0.0076	0.0073
Pass'r earnings ÷ tot earnings..	0.2677	0.2664	0.2806
Fr't earnings ÷ total earnings..	0.7078	0.7136	0.6940
Other earnings ÷ total earnings	0.0245	0.0200	0.0254
Tot earn's ÷ const & equipt cost	0.1365	0.1841	0.1726
Expenses, per line-mile, $.........	4,993	7,687	8,944
Expenses ÷ constr & equipt cost...	0.0883	0.1244	0.1246
Expenses ÷ gross earnings.......	0.6465	0.6753	0.7220
Net earnings per line-mile, $......	2,729	3,696	3,443
Net earnings ÷ const & equip cost	0.0483	0.0598	0.0480

*Excluding switching and terminal companies.
†Excluding Alaska and Hawaii.
‡Clast as "Gross Earnings" in I. C. C. reports for 1900 and 1907, and as "Operating Revenue" in I. C. C. report for 1914.

Table 2. Class I Roads in the United States, by Districts, 1914.

Aggregates of such of the reports of Class I roads as were considered sufficiently complete for inclusion.

Class I roads are those having an'l op'g revenues over $1,000,000.

	Eastern District	Southern District	Western District	Total
Plant.				
Line-miles in operation.....	32,121	33,418	105,964	171,503
Construction cost of road and equipment, per line-mile, $	133,941	64,135	60,205	74,781
Operation For One Year.				
Gross earn's per line-mile, $	22,195	11,074	9,632	13,157
Expenses per line-mile, $...	16,801	8,038	6,539	9,479
Expenses ÷ gross earnings..	0.7570	0.7258	0.6789	0.7205

Table 3. Items of Operating Expenses for Maintenance and Operation of Class I Railroads, for year ending June 30, 1914.

Condenst from Report of Interstate Commerce Commission for 1914, Statement No 42 A, pp 59-60.

Compare Train Operation Cost, p 1082.

Average line-mileage operated, 225,445.43.

	Total $	$ per line-mile	Per cent of total
Maintenance of Way and Structures:			
Superintendence	20,914,050	93	0.977
Roadway and Track:			
Ballast	7,109,605	32	0.332
Ties	65,769,736	292	3.074
Rails	18,720,319	83	0.875
Other track material............	21,109,628	94	0.987
Roadway and track*............	146,481,436	650	6.846
Track Structures:			
Tunnels, bridges, etc...........	37,147,465	165	1.736
Crossings, signs, fences, etc.....	8,567,305	38	0.400
Signals, telegraph, etc..........	16,320,242	72	0.763
Buildings, fixtures and grounds..	37,389,272	166	1.747
Docks and wharves.............	3,246,420	14	0.152
Miscellaneous	20,907,115	93	0.977
Total, Maint Way & Structures..	**403,682,593**	**1792**	**18.866**
Maintenance of Equipment:			
Superintendence	15,063,195	67	0.704
Repairs			
of locomotivs	175,869,571	780	8.219
of cars	214,981,597	954	10.047
of floating equipment...........	943,622	4	0.044
of work equipment..............	4,886,329	22	0.228
Renewals of equipment...........	17,547,413	78	0.820
Depreciation of equipment........	73,737,457	327	3.446
Miscellaneous	17,171,090	76	0.803
Total, maint of Equipment......	**520,200,274**	**2308**	**24.311**
Traffic (Agencies, advert'g, etc).....	**62,366,351**	**276**	**2.914**

Table concluded on next page.

*Applying ballast, ties, rails and other track material; track maintenance; care of roadbed; general cleaning; patrolling and watching; changing alinement and grades; bank protection; filling; train service, etc.

Table 3, concluded from p 1131.

	Total $	$ per line-mile	Per cent of total
Transportation:			
Superintendence and despatching..	44,962,961	199	2.101
Station employees	147,350,812	654	6.886
Yard conductors and brakemen....	60,559,700	269	2.830
Yard enginmen	35,028,591	155	1.637
Yard loco fuel..................	33,594,011	149	1.570
Yard and station operatn, misc....	69,452,990	308	3.246
Road loco, fuel..................	201,637,470	895	9.423
Road enginmen	127,247,552	564	5.947
Road loco, other expenses........	61,328,588	272	2.866
Road trainmen	137,270,401	608	6.415
Train supplies and expenses.......	39,430,240	175	1.843
Operatn of interlockers and signals	11,374,397	50	0.532
Train operation, misc.............	8,740,448	39	0.409
Miscellaneous	96,003,219	425	4.487
Total, Transportation	1,073,981,380	4762	50.192
General (Admin'n, insnce, &c)....	79,525,390	353	3.717
Total, Maint and operation....	2,139,755,988	9491	100.000

Each of these expense items is, however, subject to wide variation, not only as betw different roads, but, on the same road, from year to year. A road with many bridges, deep cuts, high embankments, etc, to be kept in repair, will have heavier maintenance of way cost than one which has but few of these; and this item may be relativly small one year, and twice as great the next. Fuel may be cheap on one road, and dear on another; and this of course materially affects the cost of motiv power. And so with the other items. Some times the annual cost of maint of way exceeds that of motiv power and cars together. At others, the cost of conducting transportation is fully half the total expense.

Train-mile cost (see p 1081. ¶ 6) in the United States for 1911, according to the Report of the Interstate Commerce Commission for 1911, averaged as follows:

		Gross annual operation cost	Train-mile cost
On railroads of Class	I, not less than.	$1,000,000	$1.555
" " " "	II, between	$1,000,000 and $100,000	$1.349
" " " "	III, not over	$100,000	$1.410

When a road does a very large business, of such a character that the trains may be heavy and the cars full (as on coal-carrying roads) the train-mile cost is large, and vice versa, altho, on coal roads, half the train-miles are run with empty cars.

Table 4. Revenue per mile, per passenger-mile, and per ton-mile, total gross earnings and expenses per line-mile, and operating ratio of some of the Class I railroad Companies in the United States in 1914 From Report of the Interstate Commerce Commission for Year ending June 30, 1914. Class I railroads are those having annual operating revenues greater than $1,000,000.

Name of Road	Total line-miles oprtd	Passenger: Service Train Rev, per line-mi operatd $	Passenger: Recpts per passr-mile Cents	Freight: Revenue per line-mile operatd $	Freight: Recpts per ton-mile Cents	Total gross earn'gs per line-mile $	Total expenses per line-mile $	Operat'g ratio of expenses to gross earn'gs
Eastern District	58,667§	6,164	1.80	15,446	0.63	22,195	16,801	0.76
Baltimore & Ohio*	4,478	4,366	1.92	16,923	0.56	21,776	16,090	0.74
Pennsylvania Railroad Co*	4,084	11,542	1.92	31,408	0.58	43,826	32,827	0.75
New York Cen & Hudson River*	3,757†	12,738	1.76	17,449	0.63	30,017	22,829	0.76
Wabash	2,515‡	3,612	1.89	8,033	0.61	11,939	9,705	0.81
Cleveland, Cincinnati, Chi & St L	2,361	4,851	1.90	10,337	0.55	15,674	13,955	0.89
Southern District	42,055	2,867	2.17	7,876	0.67	11,074	8,038	0 73
Southern Railway	7,010	3,236	2.14	6,409	0.98	9,887	7,191	0.73
Louisville & Nashville..........	4,940	3,195	2.27	8,683	0.78	12,089	9,071	0.75
Illinois Central	4,769	3,566	1.91	9,200	0.56	13,814	10,648	0.77
Atlantic Coast Line............	4,661	2,428	2.21	5,343	1.22	7,927	5,641	0.71
Seaboard Air Line.............	3,098	2,226	2.19	5,611	1.10	8,200	5,613	0.68
Western District	126,277**	2,833	2.13	6,590	0 89	9,632	6,539	0.68
Chicago, Milwaukee & St Paul..	9,987	2,480	2.08	6.740	0 81	9.478	6,333	0.67
Chicago, Burlington & Quincy...	9,264	3,003	1.89	6,871	0.73	10,148	6,800	0.67
Atchison, Topeka & Santa Fé...	8,340	3,672	2.15	7,320	1 04	11.208	7,210	0.64
Chicago & Northwestern........	8,095	3.362	1.84	6,690	0.87	10,368	7,361	0.71
Chicago, Rock Island & Pacific..	7,852	2,832	1.91	5.479	0 86	8,459	6,325	0.75
Class I Roads, complete.....	226,999††	3,703	1.98	9,125	0.72	13,157	9,479	0.72

In the above table only the five largest railroads in each district (size being determined by mileage *operated*) are shown.

*Operations partly electric. †Includes 55 miles in Canada. ‡Includes 245 miles in Canada. §Includes 1,209 miles in Canada.
**Includes 732 miles in Canada, and 52 in Mexico. ††Includes 1,941 miles in Canada, and 52 in Mexico.

PRESERVATION OF TIMBER.

Art. 1. (a) **The decay of timber** is caused by the growth and activities of fungi. The minute spores of one of these fungi, germinating on a piece of wood, send out fine threads, which enter the wood cells and soon give off a complex compound called a ferment or enzyme, which dissolves certain parts of the wood fibre. The dissolved fibre serves as food for the fungus. The threads throw out branches and sub-branches, and soon the timber is permeated by a mass of such threads, the growing parts of which give off ferment. The action of the ferment changes the chemical and physical properties of the wood, rendering it, in some cases, like brown charcoal, in others white, soft and stringy, and the wood is said to be rotten or decayed. Eventually some of the threads grow out from the surface of the timber, and form toadstools and other excrescences. Under these are found cavities containing thousands of spores, which, when ripe, are blown off into the air and settle upon other timbers, where the process is repeated. Moisture and heat are favorable to the growth of the fungi, as are also the starches, sugars and oils found in the cells of the sapwood but wanting in the heartwood. If protected from the action of these fungi, wood will last indefinitely. Hence the accumulation of deadwood should be avoided.* If air is excluded, as when timber is kept constantly and entirely immersed in salt or fresh water, the fungi cannot thrive. Sap, confined in timber with air, ferments, producing **dry rot**; as where beams are enclosed air-tight in brickwork, etc.; and where green timber is painted or varnished, or treated with creosote, etc. The sap then not only prevents the thorough penetration of the oil, etc, but may cause the greater part of the wood to rot although its firm outer shell gives it a deceptive appearance of strength. (b) Sap should therefore be first removed by **seasoning**; *i e*, either by drying the wood in air at natural or higher temperatures, or by first steaming the wood under pres so as to vaporize the sap, and then removing the latter by means of a vacuum. Thorough seasoning of large timbers in dry air at ordinary temperatures may require years; and too rapid kiln-drying cracks and weakens the wood. But it is questionable whether steaming and vacuum remove sap as thoroughly as do the slower dry processes. (c) Alternate exposure to water and air is very destructive. It causes **wet rot**.

Art. 2. Sea-worms. The *limnoria terebrans* works from near high-water mark to a little below the surface of mud bottom; the *teredo navalis* within somewhat less limits. The teredo is said to be rendered less active by the presence of sewage in water.

Art. 3. (a) The best timber-preserving processes are practically **useless unless thoroughly well done.** If the gain in durability will not warrant the expenditure of time and money reqd for this, it is more economical to use the wood in its natural state. (b) **The woods best adapted to treatment** are those of an open or porous texture. They absorb the oil etc better than the denser woods; and their cheapness renders the use of the treatment more economical. (c) Most of the processes in common use seem to render wood less combustible. (d) After treatment by any process, the wood should be well dried before using.

Art. 4. (a) **Creosote oil, or dead oil,** is the best known preservative. Against sea-worms it is effective for 15 to 25 years, and is the only known protection. (b) As temporary expedients, piles are sometimes covered with sheet metal, or with broad-headed nails driven close together. These rust or wear away in a few years. Oak piles, cut in January, and driven with the bark on, have resisted the teredo for 4 or 5 years; and cypress piles, well charred, for 9 years. (c) For ordinary exposures on land, 8 to 10 lbs of creosote oil **per cub ft** are reqd = say 670 to 830 lbs per 1000 ft board measure = 30 to 40 lbs per cross tie of 4 cub ft. For protection against sea-worms 10 to 12 lbs per cub ft suffice in climates like those of Great Britain and the Northern U S; but in warmer waters where the teredo is very active, from 14 to 20 lbs per cub ft are used. Large timbers may not require saturation throughout, and thus may take less per cub ft. But see (i) and end of Art. 1 (a). (d) Creosote oil **weighs** about 8.8 lbs per U. S. gallon. (e) The sticks should be reduced to their intended final dimensions and framed (if framing is reqd) before treatment; especially if for exposure to teredo, which

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* See paper by Dr. Hermann von Schrenk, read before the American Railway Engineering and Maintenance of Way Association, March, 1901.

is sure to attack any spots which (as by subsequent cutting) are left unprotected. (f) Creosoted ties have remained sound after 22 years' exposure. The creosote protects the spikes from rusting. (g) Spruce and tamarack, owing to their irregular density, are unsuitable for creosoting. (h) Creosote renders wood stiffer and slightly more brittle. In hot weather it exudes to some extent and discolors the wood. Its smell excludes it from dwellings. (i) It does not wash out from the wood, but often fails to penetrate the heart-wood. Then, if any sap remains, decay begins at the center. See end of Art 1 (a). Burnettizing the cen of the stick (see Art 7) and using a coating of creosote outside, has long been suggested as the best possible method. It is the principal feature of the **Allardyce process.** This is cheaper than thorough creosoting. In the **Rütgers process,** which has been successfully employed for ties in Germany since 1874, the creosote and a solution of zinc chloride are injected simultaneously. (j) In the **creo-resinate process** * the preservative fluid consists of creosote 38 per cent, formaldehyde 2 per cent, and melted resin 60 per cent.

Art. 5. (a) **Mineral solutions** are inferior to creosote, even on land; and useless in running water or against sea-worms; but they approximately double the life of inferior timber under ordinary land exposures; and their cheapness permits their use where that of creosote is too expensive. (b) They render wood harder; and brittle if the solution is too strong. They are liable to be washed out by rain, etc. Hence the outer wood decays first. See Art 4 (i) Art 8 (b) (c) (d). (c) A committee of the American Soc of Civ Engrs,† after collating a large number of experiments, recommended **Burnettizing** (Art 7) **for damp exposure,** as that of cross ties, damp floors, etc; and **Kyanizing** (Art 6) **for comparatively dry situations** with exposure to air and sun-light, as in bridge timbers, for which it is better suited than Burnettizing because it seems to weaken wood less. In such exposures it preserves wood sometimes for 20 to 30 years.

Art. 6. (a) **Kyanizing** consists in steeping the wood in a solution of 1 lb of bi-chloride of mercury (corrosive sublimate) in 100 lbs of water. (b) It is usual to allow the wood to soak a day for each inch of the thickness or least dimension of the piece, and one day in addition, whatever the size. (c) Gen'l Cram found the process very **unhealthy,** "salivating all the men"; but Mr. J. B. Francis, at Lowell, and Mr. H. Bissell, of the Eastern R R of Mass, had little or no trouble in this respect. The sublimate, however, which is very poisonous, is apt to effloresce, and the use of the timber is thus rendered dangerous. (d) The process is valuable for timber placed in moderately damp situations, but the salt is liable to be washed out by running water. Kyanized spruce fence posts, planted 4 ft in the ground, at Lowell, Mass, in 1850, were examined in 1891, and most of them were found very sound both above and below the surface of the ground.

Art. 7. (a) **Burnettizing** consists in immersing the wood for several hours in a solution of 2 lbs **chloride of zinc** in 100 lbs of water, under a pres of from 100 to 300 lbs per sq inch.

Art. 8. Other preventives. (a) Steeping in a solution of **sulphate of copper (blue vitriol)** has been extensively used, but does not seem to have been permanently successful. The blue vitriol washes out readily. (b) In the **Barschall or Hasselmann process,**‡ introduced in Germany in 1887, in the U S in 1899, the wood is boiled, at a temperature from 212° to 284° Fahr. and under a pressure of from 15 to 45 lbs per sq inch, in a solution of iron, copper and aluminum sulphates and "Kainit," a sulphate of magnesia and potash, mined at Stassfurt, Germany. The solution is said to carry off the sap (timber being more readily treated by this process when green than when seasoned), while the copper destroys the fungi, and the iron forms an insoluble compound with the cellulose or woody fibre. It is claimed that the process greatly hardens the wood, especially the softer varieties, rendering them suitable for ties, without impairing their strength, elasticity or pliability. (c) The **Wellhouse process** injects first a solution of chloride of zinc with glue, and then one of tannin (both under pressure), in order to diminish the subsequent washing out of the chloride. In a later modification, the zinc, glue and tannin solutions are injected separately. Several millions of ties have been treated in this way. The

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* See "A Proposed Method for the Preservation of Timber," by F. A Kummer, Transactions, Am Soc C E, Vol XLIV, December, 1900.
† See Transactions, Am Soc C E, July, Aug and Sept, 1885.
‡ Railroad Gazette, February 9, 1900.

process is not recommended for sub-aqueous use. (d) Processes in which the wood is treated by **painting or soaking*** are: Carbolineum Americs (C. A. Wood Preserver) and Carbolineum Avenarius (Tar-oil, chlorine, etc), Ligni Salvor (Tar-oil, etc), Woodiline and Spirittine (chemical solutions) and a distillate of pine used by the Pennsylvania Railroad Co. for car work. (e) Fence-posts etc seem to be preserved, to some extent, by having only their lower ends dipped in **tar** well boiled to remove the ammonia, which last is destructive to wood. The upper end must be left untarred to let the sap evaporate. (f) Attempts at wood preservation by means of **vapor of creosote** etc have proved failures. (g) While wood remains thoroughly saturated with **petroleum** it does not decay. But unless the supply is kept up the oil evaporates and leaves the wood unprotected. (h) Cottonwood ties laid upon a **soil** containing about 2 per cent carbonate of lime, 1 per cent salt and 0.5 per cent each of potash and oxide of iron, on the Union Pacific R. R. in 1868, were found in 1882 "as sound and a good deal harder than when first laid," although such ties in other soils lasted but from 2 to 5 years. (i) The use of solutions of **lime** and of **salt**; and **charring** the surface; are sometimes found useful in damp situations.

* See Report by O. Chanute to the American Railway Engineering and Maintenance of Way Association, March, 1901.

Art. 4. Ultimate average tensile or cohesive strength of Timber,

Being the least weights in pounds which, if attached to the lower end of a vert rod one inch square, firmly upheld at its upper end, would break it by tearing it apart. For large timbers we recommend to reduce these constants $\frac{1}{4}$ to $\frac{1}{3}$ part.

The strengths in all these tables may readily be one third part more or less than our averages.	Lbs per sq. inch		Lbs per sq. inch.
Alder	14000	Mahogany, Honduras	8000
Ash, English	16000	" Spanish	16000
" American (author) abt.	16500	Mangrove, white, Bermuda	10000
Birch	15000	Mulberry	12000
" Amer'n black	7000	Oak, Amer'n white	
Bay-tree	12000	" " basket	
Beech, English	11500	" " red	
Bamboo	6000	" Dantzic, seasoned	10000
Box	20000	" Riga	
Cedar, Bermuda	7600	" English	
" Guadaloupe	9500	" live, Amer'n	
Chestnut	13000	Pear	10000
" horse	10000	Pine, Amer'n, white, red, and Pitch, Memel, Riga*.	10000
Cyprus	6000		
Elder	10000	Plane	11000
Elm	6000	Plum	11000
" Canada	13000	Poplar	7000
Fir, or Spruce	10000	Quince	7000
Hawthorn	10000	Spruce, or Fir	10000
Hazel	18000	Sycamore	12000
Holly	16000	Teak	15000
Hornbeam	20000	Walnut	8000
Hickory, Amer'n	11000	Yew	8000
Lignum Vitæ, Amer'n	11000		
Lancewood	23000	**Across the grain**, Oak	2300
Larch, Scotch	7000	" " " Poplar	1800
Locust	18000	" " " Larch, 900 to	1700
Maple	10000	" " " Fir, & Pines	550

THESE ARE AVERAGES. The strengths vary much with the age of the tree; the locality of its growth; whether the piece is from the center, or from the outer portions of the tree; the degree of seasoning; straightness of grain; knots, &c, &c. Also, inasmuch as the constants are deduced from experiments with good specimens of small size, whereas large beams are almost invariably more or less defective from knots, crookedness of fibre, &c, it is advisable in practice to reduce these constants as recommended above.

* **Effect of Tapping Trees for Turpentine.** Preliminary experiments by the Forestry Division of the U. S Department of Agriculture upon long-leaf pine from Alabama indicate that (contrary to the generally received impression) "turpentine timber," *i. e* the timber of trees that have been "boxed" (robbed of their turpentine), while it has slightly less tensile and shearing strength, is from 20 to 30 per cent. stronger in compression (whether with or across the grain) and under transverse strain. In the "turpentine timber," however, the resin collects in spots, gumming the tools, and thus rendering the timber harder to work than that of trees which have not been deprived of their turpentine. The specimens tested were taken mostly at heights of from 7 to 33 feet above ground. (Circular No. 8. Issued 1892.) Boxed and unboxed timber are frequently called "bled" and "unbled" respectively.

Art. 1. Compressive strengths of American woods, *when slowly and carefully seasoned.* Approximate averages deduced from many experiments made with the U S Govt testing machine at Watertown, Mass, by Mr. S. P. Sharples, for the census of 1880. **Seasoned woods** resist crushing much better than green ones; in many cases, twice as well. This must be allowed for when building bridges, &c, of timber recently cut. Different specimens of the same wood vary greatly; frequently as 5 to 8, 9, or more.

The strengths in all these tables may readily vary as much as one-third part more or less than our average.	Endwise.* lbs per sq in.	Sidewise.¶ lbs per sq in.	
		.01	.1
Ash, red and white	6800	1800	3000
Aspen	4400	800	1400
Beech	7000	1100	1900
Birch	8000	1300	2600
Buckeye	4400	600	1400
Butternut	5400	700	1600
Buttonwood (sycamore)	6000	1300	2600
Cedar, red	6000	700	1000
" white (arbor vitæ)	4400	500	900
Catalpa (Indian bean)	5000	700	1300
Cherry, wild	8000	1700	2600
Chestnut	5300	900	1600
Coffee tree, Kentucky	5200	1300	2600
Cypress, bald	6000	500	1200
Elm, Am'n or white	6800	1300	2600
" red	7700	1300	2600
Hemlock	5300	600	1100
Hickory	8000	2000	4000
Lignum vitæ	10000	1600	13000
Linden, American	5000	500	900
Locust, black and yellow	9800	1900	4400
" honey	7000	1600	2600
Mahogany	9000	1700	5300
Maple, broad-leafed, Oregon	5300	1400	2600
" sugar and black	8000	1900	4300
" white and red	6800	1300	2900
Oak, white, post (or iron) swamp white, red and black	7000	1600	4000
" scrub and basket	6000	1700	4200
" chestnut and live	7500	1600	4500
" pin	6500	1300	3000
Pine, white	5400	600	1200
" Red or Norway	6300	600	1400
" pitch and Jersey scrub	5000	1000	2000
" Georgia	8500	1800	2600
Poplar	5000	600	1100
Sassafras	5000	1300	2100
Spruce, black	5700	700	1300
" white	4500	600	1200
Sycamore (buttonwood)	6000	1300	2600
Walnut, black	8000	1300	2600
" white (butternut)	5400	700	1600
Willow	4400	700	1400

Hence it appears that *seasoned* white and yellow pines, spruce, and ordinary oaks which are the woods most employed in the United States for bridges, roofs, etc., crush endwise with from 5000 to 7000 lbs per sq inch, ***in short blocks;*** average, 6000

But it is well to bear in mind that in practice perfectly equable pressure is rarely secured. In a few trials on sidewise compression, with fairly seasoned white pine blocks, 6 ins high, 5 ins long, and 2 ins wide, we found that under an equally distributed pressure of 5000 lbs total or 500 lbs per sq inch, they compressed about from $\frac{1}{8}$ to $\frac{1}{4}$ inch; which is equal to from $\frac{1}{4}$ to $\frac{1}{2}$ inch per foot of height; or from $\frac{1}{24}$ to $\frac{1}{48}$ of the height; the mean being about $\frac{3}{8}$ inch to a foot, or $\frac{1}{32}$ of the height. Under 10000 lbs total, or 1000 lbs per sq inch, they split badly; and in some cases large pieces flew off.

The tensile or cohesive strengths of pine and oak average about 10000 lbs per sq inch, or $\frac{1}{2}$ as much as average cast-iron, or nearly double their resistance to crushing. The tensile strength does not change with the length of the piece; so that in practice we may take its safe strain at from 1000 to 2000 lbs per sq inch, depending upon the character of the structure, &c., without regard to the length, except when this is so great that two or more pieces have to be spliced together to make it; thus weakening the piece very much.

* Specimens 4 centimetres (1.57 inch) square, 32 centimetres (12.6 ins) long. When the length exceeds 10 times the least side, see Wooden Pillars.

¶ Specimens 4 centimetres (1.57 inch) square, 16 centimetres (6.3 ins) long; laid upon platform of testing machine. Pressure applied at their mid-length, by means of an iron punch 4 centimetres square, or just covering the entire width of the specimen, and one-fourth of its length. The first column (headed ".01") gives the loads producing an indentation of .01 inch. The second column (headed ".1") gives those producing an indentation of .1 inch.

Table of safe quiescent loads for horizontal rectangular beams of white pine or spruce, one inch broad. supported at both ends, and loaded at the center; together with their deflections under said loads.

The safe load is here one-sixth of the breaking load.

For the neat loads, deduct ½ the wt of the beam itself. The *deflections*, however, are the *actual* ones; the wts of the beams having been introduced in calculating them.

Loads applied suddenly will double the deflections in the table; as when, for instance, if a load is held by hand, just *touching* a beam, the hold should be *suddenly* loosed.

Caution. Inasmuch as this table was based upon well seasoned, straight grained pieces, free from knots, and other defects, we must not in practice take more than about two-thirds of the loads in the table for a safety of 6 in ordinary building timber of fair quality; and with these reduced loads should not reduce the deflections.

Observe also that our table is for safe **center** loads, but it is plain that in practice we cannot always apply the term in its utmost strictness; otherwise the load would have to be sustained by a mere knife-edge, at the *very* center of the beam. Now, in the case of Rem. p. 1140, if we attempted to sustain the center load of 6075 lbs upon such a knife-edge, it would at once cut the beam in two. If we even applied it along 3 or 4 ins of the length, it would cut into it, and we should not have a safety of 6 against crushing the top of the beam until as in the case of the ends we distributed the load along full 46 ins of length, or about 32 ins for a safety of 4.

The safe load is here ⅙ of the breakg one; and the last at 450 lbs at the center of a beam 1 inch square, and 1 foot clear length between its supports. For mere temporary purposes, ½ part may be added to the loads in the table, thus making them equal to the ¼ of the breakg load. But in important structures, subject to vibration, ¼ part should be deducted from the tabular loads, thus reducing them to ⅛ of the breaking load. This is especially necessary if the timber is not well seasoned.

With the safe loads in this table a beam may bend too much for many practical purposes. When this is the case, we may, by reducing the loads, reduce the deflections in nearly the same proportion.

All the loads in the Table are superabundantly **safe against shearing. Against crushing at the ends,** &c, see "**Cautions**" below the Table.

Original

Depth of beam.	Span 4 ft.		Span 6 ft.		Span 8 ft.		Span 10 ft.		Span 12 ft.		Span 14 ft.		Span 16 ft.		Wt. of 10 ft of beam.
Ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	lbs.
1	19	.39	13	.92	10	1.8	8	3.0	6	4.4	......	......	......	......	2
2	75	.22	50	.45	38	.82	30	1.3	25	1.9	21	2.7	19	3.7	4
3	170	.13	114	.30	85	.53	67	.84	57	1.3	48	1.7	42	2.3	6
4	300	.10	200	.22	150	.39	120	.68	100	.92	86	1.3	75	1.7	8
5	469	.08	312	.18	234	.31	187	.50	156	.72	134	1.0	117	1.3	10
6	675	.06	450	.15	337	.26	270	.41	225	.60	193	.83	168	1.1	12
7	919	.06	612	.12	460	.22	367	.35	306	.51	262	.70	230	.93	14
8	1200	.05	800	.11	600	.19	480	.31	400	.45	343	.61	300	.81	16
9	1520	.04	1014	.10	760	.17	607	.27	507	.40	434	.54	380	.72	18
10	1875	.04	1250	.09	937	.16	750	.24	625	.35	536	.49	468	.64	20
11	2270	.04	1514	.08	1135	.14	907	.22	757	.32	648	.44	567	.58	22
12	2700	.03	1800	.07	1350	.13	1080	.20	900	.29	772	.40	675	.53	24
14	3675	.03	2450	.06	1837	.11	1470	.17	1225	.25	1050	.34	918	.45	28
16	4800	.02	3200	.05	2400	.10	1920	.15	1600	.22	1372	.30	1200	.40	32
18	6075	.02	4050	.05	3037	.09	2430	.14	2025	.20	1736	.27	1518	.35	36
20	7500	.02	5000	.04	3750	.08	3000	.12	2500	.18	2145	.24	1875	.31	40
22	9075	.02	6050	.04	4537	.07	3630	.11	3025	.16	2593	.22	2268	.29	44
24	10800	.02	7200	.04	5400	.06	4320	.10	3600	.15	3088	.20	2700	.26	48

(Continued on next page.)

Table, continued. (Original.)

Depth of beam. Ins.	Span 18 ft. load lbs	Span 18 ft. def. ins.	Span 20 ft. load lbs.	Span 20 ft. def. ins.	Span 25 ft. load lbs.	Span 25 ft. def. ins.	Span 30 ft. load lbs.	Span 30 ft. def. ins.	Span 35 ft. load lbs	Span 35 ft. def. ins.	Span 40 ft. load lbs	Span 40 ft. def. ins.	Wt. of 10 ft of beam. lbs.
6	150	1 4	135	1.8	108	2.9	90	4 5	77	6.5	67	9.2	12
7	204	1.2	184	1 5	147	2.5	122	3.9	105	5 8	92	7.6	14
8	267	1.0	240	1.3	192	2.1	160	3.2	137	4.6	120	6.4	16
9	338	.92	304	1.2	243	1 9	202	2.8	174	4.0	152	5.5	18
10	417	.82	375	1.0	300	1.7	250	2.5	214	3.5	188	4.9	20
11	505	.74	454	.93	363	1.5	302	2.2	259	3.2	227	4.3	22
12	600	.68	540	.85	432	1.4	360	2.0	308	2.9	270	3.9	24
14	817	.58	735	.72	588	1.2	490	1.7	420	2.4	367	3.2	28
16	1067	.50	960	.63	768	1.0	640	1.5	548	2.1	480	2.8	32
18	1350	.45	1215	.56	972	.90	810	1.3	694	1.8	607	2.5	36
20	1666	.40	1500	.50	1200	.79	1000	1.2	857	1 6	750	2.2	40
22	2017	.37	1815	.45	1452	.72	1210	1.1	1037	1.5	907	2.0	44
24	2400	.33	2160	.41	1728	.65	1440	.96	1234	1 3	1080	1.8	48
26	2817	.31	2526	.38	2018	60	1684	.88	1449	1 2	1263	1.6	52
28	3267	.28	2940	.35	2352	.55	1960	.81	1680	1 1	1170	1.5	56
30	3750	.26	3375	33	2700	.50	2250	.76	1928	1 1	1687	1.4	60
32	4267	.25	3840	30	3072	.45	2560	.71	2194	1 0	1920	1.3	64
34	4817	.23	4335	29	3468	.44	2890	67	2477	.92	2167	1 2	68
36	5400	.22	4860	.27	3888	.43	3240	.63	2777	.86	2430	1.1	72

White oak, and best Southern pitch pine will bear loads ¼ greater.

For cast iron, mult the **loads** in the table by 4.5; and for **wrought** by 5.3 For these new loads, mult the **defs** by .4 for cast; and by 3 for wrought.

If the load is equally distributed over the span, it may be twice as great as the center one, and the defs will be 1¼ times those in the table. **If the loads in the table** be equally distributed along the whole beam, the defs will be but five-eighths as great as those in the table. When more accuracy is reqd, half the wt of the beam itself must be deduc'ed from the center load; and the whole of it from an equally distributed load. The wt of the beam, in the last column, supposes the wood to be but moderately seasoned, and therefore to weigh 28.8 lbs per cub ft.

Uses of the foregoing table. Ex. 1. What must be the breadth of a hor rect beam of wh pine, 18 ins deep, supported at both ends, and of 20 ft clear length between its supports, to bear safely a load of 5 tons, or 11200 lbs at its center? Here, opposite the depth of 18 ins in the table, and in the column of 20 feet lengths, we find that a beam 1 inch thick will bear 1215 lbs; consequently, $\frac{11200}{1215} = 9.22$ ins, the reqd breadth; for the strength is in the same proportion as the breadth

Ex. 2. What will be the safe load at the center of a joist of white pine, 18 ft long, 3 ins broad, and 12 ins deep? Here, in the col for 18 ft, and opposite 12 ins in depth, we find the safe load for a breadth of 1 inch to be 600 lbs; consequently, $600 \times 3 =$ 1800 lbs, the load reqd.

Rem. Cautions in the use of the above table. For instance, in placing very heavy loads upon short, but deep and strong beams, we must take care that the beams rest for a sufficient dist on their supports to prevent all danger from *crushing* at the ends. Thus, if we place a load of 6075 lbs at the center of a beam of 4 feet span, 18 ins deep, and only 1 inch thick, each end of the beam sustains a vert crushing force of $\frac{6075}{2} = 3037$ lbs, and that **sidewise of the grain,** in which position average white pine, spruce, and hemlock crush under about 800 lbs per sq inch, and do not have a safety of 6 until the pressure is reduced to about 133 lbs per sq inch. Therefore our beam, in order to have a safety of 6 against crushing at its ends, must rest on each support $3037 \div 133 = 23$ sq ins; or for a safety of 4 nearly 16 sq ins. When a pressure is equally distributed sidewise (that is, at right angles to the general direction of the fibres) over the entire pressed surface of a block or beam (to ensure which, the opposite surface must be supported throughout its entire length) the resulting compression might readily escape detection unless actually measured. But when a considerable pressure is applied to only a portion of the surface, as of caps and sills where in contact with the heads and feet of posts, or at the ends of loaded joists or girders, the compression becomes evident to the eye, because the pressed parts sink below the unpressed ones, in consequence of the bending or breaking of the adjacent fibres. What in the first case (especially if slight) would be called **compression**, would

in the second be called **crushing**; even when neither might be so great as to be unsafe.

Owing to the resistance which said adjacent fibres oppose to being bent or broken, it is plain that a given pressure **per sq inch**, or **per sq foot**, &c., will cause somewhat less compression or crushing when applied to only a part of a surface, than when to the whole of it.

The writer has seen 40 half seasoned hemlock posts, each 12 ins square, footing at intervals of 5 ft from center to center, upon similar 12 × 12 inch hemlock sills, to which they were tenoned, and which rested throughout their entire length on stone steps. Each post was gradually loaded with 32 tons or equal to say 500 lbs per sq inch, and their feet all crushed into the sills from ¼ to ½ inch. Their heads crushed into the caps to the same extent. **In practice** the pressure at the heads and feet of posts is rarely if ever perfectly equable, and the same remark applies to the ends of loaded joists, girders, &c., in which a slight bending will throw an excess of pressure upon the inner edges of their supports.

Art. 31. Table of greatest center loads for horizontal rectangular beams of white or yellow pine, or of spruce, 1 inch broad, supported at both ends, and required not to bend more than $\frac{1}{40}$ inch per foot of clear span, or $\frac{1}{480}$ part of the entire clear span. In practice, to allow for knots, &c, take only ⅔ rds. The loads in this table include the weight of the clear beam itself. 625 of which (or ⅝ of which) must be deducted from the tabular loads to get the neat load, when the beam is loaded at its center. When uniformly loaded, the loads will be 1 6 times as great as those in this table; but in that case the weight of the entire clear beam must be deducted. In practice this deduction need rarely be made.

Depths in Inches.	CLEAR SPANS IN FEET. (ORIGINAL.) 3	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40	Depth in Ins.	Wt. of 10 ft. Lgth of Beam.
	lbs.	lbs.	lbs.	lbs	lbs.	lbs	lbs.	lbs.	lbs.	lbs.	lbs.	lbs	lbs	lbs.	lbs.	lbs.	lbs.		lbs
1	8.4	4 8	3 0	2.1	...	...	...	...	...	...	...	...	...	...	...	...	...	1	2
½	28 7	16 2	10 4	7 2	5 3	...	...	...	...	...	...	...	...	...	...	...	...	½	3
2	68.4	38.4	24.4	17.1	12.4	9.6	...	...	...	...	...	...	...	...	...	...	...	2	4
½		75	48	33	24	19	14	...	...	...	...	...	...	...	...	...	...	½	5
3	...	130	83	58	42	32	26	21	...	...	...	...	...	...	...	...	...	3	6
½	...		131	92	67	51	40	33	23	...	...	...	...	...	...	...	...	½	7
4	...	...	196	137	100	77	60	49	34	25	...	...	...	...	...	...	...	4	8
½	...	...		192	144	108	86	70	48	36	27	...	...	...	...	...	...	½	9
5	...	...	...	267	196	150	116	96	66	49	37	29	...	...	...	...	...	5	10
½	...	...	...		260	200	156	128	89	65	50	39	32	...	...	...	...	½	11
6	...	...	...	...	336	259	204	166	115	84	64	51	41	26	...	...	...	6	12
½	...	...	...	...		332	260	207	148	109	83	65	52	33	23	...	...	½	13
7	...	...	...	...	...	412	324	263	183	134	102	81	66	42	29	21	...	7	14
½	...	...	...	...	...	511	408	324	225	166	126	100	82	52	36	26	...	½	15
8	...	...	...	...	...		484	394	273	201	154	121	96	63	44	32	24	8	16
½	...	...	...	...	...	...	585	475	330	241	184	145	117	74	52	38	29	½	17
9	...	...	...	...	...	...	692	560	389	286	219	173	140	90	62	45	35	9	18
½	...	...	...	...	...	...		662	460	336	257	202	164	104	72	52	40	½	19
10	...	...	...	...	...	...	...	769	534	392	300	237	192	123	85	63	48	10	20
½	...	...	...	...	...	...	...		619	454	348	274	221	141	96	71	54	½	21
11	...	...	...	...	...	...	...	...	711	522	399	316	256	163	114	83	64	11	22
½	...	...	...	...	...	...	...	...	813	597	456	361	291	185	129	93	71	½	23
12	...	...	...	...	...	...	...	...	923	678	519	410	332	212	148	108	83	12	24
13	...	...	...	...	...	...	...	...		874	669	528	420	273	188	138	105	13	26
14	...	...	...	...	...	...	...	...	...	1075	823	651	527	337	234	172	132	14	28
15	...	...	...	...	...	...	...	...	...		1000	801	652	411	290	212	162	15	30
16	...	...	...	...	...	...	...	...	...	...	1230	971	787	504	350	237	197	16	32
17	...	...	...	...	...	...	...	...	...	...		1175	944	604	420	309	236	17	34
18	...	...	...	...	...	...	...	...	...	...	...	1385	1121	717	498	365	280	18	36
20	...	...	...	...	...	...	...	...	...	...	...		1538	984	684	502	384	20	40
22														1309	910	668	512	22	44
24														1701	1181	867	664	24	48
26															1503	1104	845	26	52
28															1876	1378	1055	28	56
30																1695	1297	30	60

On this side of the dark lines, the safe loads of table, p 1139, would not bend the wooden beams as much as $\frac{1}{480}$ of their clear span.

Iron and Steel.

Average cast iron, with the same safe def will bear about 11½ as much as common yellow or white pine, or spruce; and wrought iron 19 times as much. The same proportion of the weight of the beam itself must, however, be deducted as stated above for wood. Average steel 29 times as much as pine.

WOODEN COLUMNS.

See columns in general, pp 495 &c.
For steel and iron columns, see pp 1189 &c.

List of References.

(1) The Elasticity and Resistance of the Materials of Engineering; by Wm. H. Burr. New York, John Wiley & Sons.
(2) Economic Designing of Timber Trestle Bridges, by A. L. Johnson, C E, Washington, 1896–1902.
(3) Cambria Steel Co, Handbook, 1907.
(4) Carnegie Steel Co, Pocket Companion, 1903.
(5) Passaic Steel Co, Manual, 1900.
(6) Trans Am Soc Civ Engrs; Vol 15, July, 1886.
(7) Trans Am Soc Civ Engrs; Vol 54, June, 1905
(8) Acts and Resolves of the Massachusetts Legislature; 1907.
(9) New York Building Code; Approved Oct 24, 1899; with amendments to April 12, 1906.
(10) The Materials of Construction; by J. B. Johnson; New York, John Wiley & Sons, 1906.
(11) "Tests of Metals, etc," Watertown Arsenal, year ending June 30, 1881.
(12) "Tests of Metals, etc," Watertown Arsenal, year ending June 30, 1882.
(13) American Architect and Building News, April 9, 1892, March 10, 1894.
(14) Am Ry Eng & M of W Assn, Comm of , Procs, Vol VII, 1906, p 694.

1. The strengths of wooden columns increase with the degree of **seasoning.** As in other columns, **eccentricity of loading** greatly diminishes the strength, and it is seldom possible to determine closely the degree of eccentricity.

2. Let all stresses be in pounds and all dimensions in inches, and let

P = total load (supposed axial) on col ;	r = least rad of gyr of cross sec ;
a = area of cross section of col ;	D = diam, or least side, of cross sec;
p = P/a = avge unit load on col ;	L = length of column ;
s = max unit stress in any cross sec ;	K = L/r = length ratio, or slenderness of col, in terms of r ;
s_e = unit stress at elastic limit ;	k = L/D = length ratio, or slenderness of col, in terms of D.
s_s = max unit stress in a short col ;	

3. Wooden columns are nearly always **solid and rectangular,** and most frequently **square,** in cross section. Hence, instead of the least radius of gyration, r, it is convenient to use the least side, D. For values of D/r and of $(D/r)^2$, see pp 353 *a*, 353 *b*.

4. In solid square or rectangular columns,

$$D = \sqrt{12}\, r ; \quad D^2 = 12\, r^2 .$$

For the slenderness, we have $k = L/D = L/(\sqrt{12}\, r) = K/\sqrt{12}$; and $k^2 = K^2/12$; $K^2 = 12\, k^2$.

Hence, the **Rankine and straight-line formulas** become, respectively :

$$\textbf{Rankine formula ; } p = \frac{P}{a} = \frac{s}{1 + m K^2} = \frac{s}{1 + 12 m k^2} .$$

$$\textbf{Straight-line formula ; } p = P/a = s_s - c\, K = s_s - k\, c\, \sqrt{12},$$

where, as in ¶ 2:

p = mean unit load on column ; **m, c** = coefficients ;
s = max unit stress in cross section ;
s_s = max unit stress in cross section for short blocks ;
K = L/r = unsupported length ÷ least *radius of gyration* ;
k = L/D = unsupported length ÷ least *side* ;
F = safety factor = ultimate load ÷ permissible load.

5. The table, pp 1144-5, gives **values commonly used,** with these two formulas, for the coefficients in connection with wooden columns. For list of authorities quoted, see above
See diagram, Fig 1, of values of p/s.

Wooden Columns.*		k max	Rankine. $p = P/a = s/(1 + 12\ mk^2)$* Ultimate s	$1/(12\ m)$	Allowed s	$1/(12\ m)$	Straight-line and stepped, $p = P/a = s_s - k\ c \div 12$* Ultimate s_s	$c \div 12$	Allowed s_s	$c \div 12$
Pine										
Yellow										
Common										
Smith (1) p 485 (a)*	1	60	5400	250	...	...	...		...	...
	2	60	8200	300	...	...	...	...	...	...
	3	60	5000	250	...				...	...
Burr (1) p 486d (b), $k = 20$ to 60		60	...	...	...	...	5800	70	...	...
$k < 20$		20	...	...	...	...	4400	0	...	...
Long-leaf										
Forestry (2) (d)			7000	(d)*	...		...		...	...
Cambria (3) (e)			5000	(d)					...	...
New York (9) § 138		30	...	...	...		...	...	1000	18
Balto. & Ohio R. R., $k > 17$		40	...	...		...	...		1200	18
(1904) $k < 17$		17	...	...					900	0
Schneider (7) p 496, $k > 10$		45	...		...	...	...	...	1000	10
$k \ngtr 10$		10	...	...		...	...		1000	0
Worcester (7) (f) Boston (8) (g)		...	...		...	...	...		...	...
M of Way (14)		...				...	...		1200	18
Short-leaf. Cambria (3) (e)		...	4500	(d)*		...			...	...
Red (Norway) Cambria (3) (e)		...	4000	(d)	...	...	...	...	...	...
New York (9) § 138		...	...	...		...	...	...	800	15
M of Way (14)		...	...	...	...	...	...	...	800	12
White										
Smith (1) p 485 (a)	1	60	5400	250		...	...	...	...	...
	2	60	8200	300		...	...	...	...	...
	3	60	5000	250	...	...	...	...	...	...
Cambria (3) (e)			3500	(d)*			...	...	...	...
Carnegie (4) p 235 (h)		...	...	...	800	1100	...	...	...	...
New York (9) § 138		30	...	...			...	...	800	15
Balt & Ohio R. R. $k > 17$		40	...	...	...	...	...	...	800	12
(1904) $k < 17$		17	...	...	...	...	...	...	600	0
Schneider (7) p 496 $k > 10$		45	...	...	...	...	...	...	600	6
$k \ngtr 10$		10	...	...	...	...	...	...	600	0
Worcester (7) (f) Boston (8) (g)		...	...	...	...	...		...	...	...
Burr (1) p 486d (b) $k = 30$ to 60		60	...	...	...	...	3800	47	...	...
$k < 30$		30	...	...	...	...	2400	0	...	...

	k max	*s*	1/(12 *m*)	*s*	1/(12 *m*)	s_s	$c\sqrt{12}$	s_s	$c\sqrt{12}$
Oak, W. H. Johnson (6) p 530	...	...	...	...	...	5400	28	...	...
New York (9) § 138	30	...	...	...	...	...	...	900	[illegible]
White									
Cambria (3) (e)	...	5000	(*d*)*	...	...	...	...	...	...
Carnegie (4) p 235 (h)*	...	...	...	925	1100	...	...	...	...
Balt. & Ohio R. R. $k > 17$	40	...	...	...	...	...	...	1000	15
(1904) $k < 17$	17	...	...	...	...	...	...	750	0
Schneider (7) p 496, $k > 10$	45	...	...	...	...	...	...	1000	10
$k \not> 10$	10	...	...	...	...	...	...	1000	0
Worcester (7) (f) Boston (8) (g)	...	...	...	...	...	...	...	...	...
M of Way (14), white, post and burr	...	...	...	...	...	...	...	1000	15
Spruce, Cambria (3) (e)	...	4000	(*d*)	...	...	...	...	...	...
Carnegie (4) p 235 (h)	...	...	...	800	1100	...	...	...	...
New York (9) § 138	30	...	...	...		...	...	800	15
Schneider (7) p 496, $k > 10$	45	...	...	...	...	...	...	600	6
$k \not> 10$	10	...	...	...	...	...	...	600	0
Worcester (7) (f) Boston (8) (g)	...	...	...	...	...	...	...	...	...
Fir, Douglas, Cambria (3) (e)	...	4500	(*d*)	...	...	...	...	...	...
Hemlock, Cambria (3) (e)	...	4000	(*d*)	...	...	...	...	...	...
New York (9) § 138	...	...	...	...	...	...	...	500	9.4
Schneider (7) p 496, $k > 10$	45	...	...	...	...	...	...	500	5
$k \not> 10$	10	...	...	...	...	...	...	500	0
Worcester (7) p 418 (f)	...	...	...	...	...	...	...	...	...
Cypress, Cambria (3) (e)	...	4000	(*d*)	...	...		...	...	...
M of Way (14)	...	...	...	...	...	...	...	800	12
Chestnut, Cambria (3) (e)	...	4000	(*d*)	...	...	...	...	...	...
New York (9) § 138	...	...	...	...	...	...	...	500	9 4
Redwood, Cambria (3) (e)	...	4000	(*d*)	...	...	...	...	...	...
Locust, New York (9) § 138	...	...	...	...	...	...	...	1200	22.5
Cedar, Cambria (3) (e)	...	3500	(*d*)	...	...	...	...	...	...
M of Way (14)	...	...	...	...	...	...	...	800	12

* **P** = total load, lbs; **a** = cross section area, sq ins; **k** = L/D = length / least side. *Nos.* in ( ) refer to list, p 1143. *Letters* in ( ) refer to **notes,** p 1146.

Notes on table, pp 1144-5.

where, as in ¶ 2:

p = mean unit load on column; **m. c** = coefficients;
s = max unit stress in cross section,
s_u = max. unit stress in cross section for short blocks;
K = L/r = unsupported length ÷ least *radius of gyration*;
k = L/D = unsupported length ÷ least *side*.
F = safety factor = ultimate load ÷ permissible load.

(**a**) **Smith.** Flat ends, firmly fixed For $k \not> 25$, $F = 5$, for $k > 25$, $F = \sqrt{k}$. (F = ult ÷ permissible load)

1. Green, half-seasoned "Good merchantable lumber."
2. Selected, reasonably straight, air-seasoned under cover 2 yrs and over.
3. Avge sticks, cut from lumber in open air service 4 yrs or over.

Coeffs based upon 1200 tests of full-sized columns, 1861-2 To guard against deterioration and defects, Mr Smith recommended the *third* set of values. $s = 5000$; $1/(12\ m) = 250$.

(**b**) **Burr.** For good avge lumber. F = ult ÷ permiss load For railway structures, $F = 8$, for temporary structures, under static loads, $F = 4$.

(**d**) **Forestry.** Square columns. $F = 7$. $1/(12\ m) = 700 + 15\ k$; hence $p = P/a = s \cdot \frac{700 + 15\ k}{700 + 15\ k + k^2}$. To allow for weathering, add 0.5 inch to each cross-sectional dimension required by formula.

(**e**) **Cambria.** See note (d) Forestry, above.

	Yellow pine F	Others F
Class A. Moisture 18%. Freely exposed (trestles, uncovered bridges, etc),	5.0	5.0
" B. " 15%. Roofed, not side-sheltered (covered bridges, etc),	4.3	4.5
" C. " 12%. Enclosed but not heated,	3.6	4.2
" D. " 10%. Enclosed and heated,	3.2	4.0

(**f**) **Worcester.**

	$k \not> 50$ p	$k \not> 10$ p
Long-leaf pine	600	1000
Spruce, white pine	300	700
White oak	600	1000
Hemlock	250	650

For each increase of 10, in k, deduct 100 from p. See Columns in General p 498 *b*, ¶ 34.

(**g**) **Boston.**

	$k \not> 30$ p	y	$k \not> 10$ p
Long-leaf pine	700	50	900
Spruce, white pine	190	35	630
White oak	330	45	510

For each increase of 5, in k, deduct y from p.

(**h**) **Carnegie and Passaic.** For long posts, $F = 5$; for short posts, $F = 4$. F = ult ÷ permiss load.

6. J. B. Johnson, (10) for his **parabolic formula** (Columns in General, ¶ 28, p 498).

$$p = \frac{P}{a} = s_e - \frac{s_e^2}{q\ \pi^2 E} K^2 = s_e - 12\ c_p\ k^2,$$

gives, as **ultimate** values:

k = length ÷ least *side*;
s_e = elastic limit.

	(10) p 367, flat ends $k \not> 60$ s_e	$12\ c_p$	(10) pp 683–4. Dry s_e	$12\ c_p$	Partially seasoned s_e	$12\ c_p$
Long leaf yellow pine	4000	0.8	6000	1.5	4500	1.0
Short leaf yellow pine	3300	0.7	......	....		
White pine	2500	0.6	3600	0.72	2500	0.5
White oak	3500	0.8	....	.....	....	....

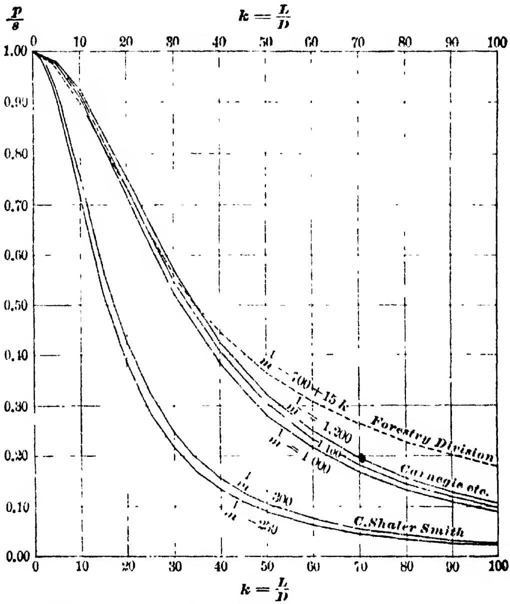


Fig. 1. Values of p/s. See Iron and Steel Cols, p 1197 ¶ 21

FORMULAS AND EXPERIMENTS.

7. The accompanying diagram, Fig 2, shows a comparison of formulas for, and experiments upon, **yellow pine columns**, as follows:

p = mean unit load on column; **k** = length ÷ least *side*.

Formulas.

Ultimate.

1. Rankine; C Shaler Smith .... $p = 5400 \div (1 + 0.004\,k^2)$
2. Rankine; C Shaler Smith .... $p = 8200 \div (1 + 0.00333\,k^2)$
3. Rankine; C. Shaler Smith .... $p = 5000 \div (1 + 0.004k^2)$
4. Rankine; U. S. Division of Forestry $p = 7000 \times \dfrac{700 + 15\,k}{700 + 15\,k + k^2}$
5. Parabolic; J. B. Johnson, p 367 ...... $p = 4000 - 0.8\,k^2$
6. Parabolic; J. B. Johnson, p 683 .... $p = 4500 - 1\,0\,k^2$, Partially seasoned.
7. Parabolic; J. B. Johnson, p 684 .... $p = 6000 - 1\,5\,k^2$, Dry.
8. Straight-line; Wm. H. Burr, . $k < 20$, $p = 4400 - 0$
   $k = 20$ to 60, $p = 5800 - 70\,k$
9. Straight-line; Jas. H Stanwood, $k < 20$, $p = 4250 - 0$
   $k > 20$, $p = 4250 - 43\tfrac{1}{8}\,k$
10. Stepped; Edward F. Ely,

k =	0 to 15,	to 30,	to 40,	to 45,	to 50,	to 60
p =	4,000	3,500	3,000	2,500	2,000,	1,500

Permissible.

8 Baltimore & Ohio R R Co,
  ($k_{max} = 40$) For $k < 17$, $p = 900 - 0$
  For $k > 17$, $p = 1200 - 18\,k$

9. Boston building code, ...

k =	0–10	10–15	15–20	20–25	25–30
($k_{max} = 30$) .... p =	900	850	800	750	700

10. New York buildigg code ($k_{max} = 30$) $p = 1000 - 18\,k$.

Experiments.

Ultimate.

✝ Average of 12 endwise compression tests (11) pp 54 etc, on **short yellow pine blocks**, from 1.5 × 1.5 ins, 3 ins long, to 10 × 10 ins, 20 ins long; k from 2 to 3.75, average = 2.36; p, average, 8,105 lbs per sq inch; in 10 of the tests, p ranged from 7,394 to 10,250 lbs per sq inch. The other 2 were

1.5 × 1.5 ins, 3 ins long, $p = 5533$; failed at knots.
1.5 × 1.5 ins, 3 ins long, $p = 6355$; season crack, nearly dividg spec'n.

● Tests of yellow pine posts, 5.5 ins square, 80 to 320 ins long, at Watertown Arsenal, (12) pp 377–8.

Each item is the average of from 2 to 4 tests.

Columns, each composed of two or more sticks, bolted together, showed, in general, no greater strength, in pounds per square inch, than did the single sticks of which they were built up.

▲ Riga and Dantzic firs, 13 ins sq, 20 ft long. Kirkaldy.

Mr. Smith's formulas represent his 1200 tests of full-size columns.

The formula of the U. S. Division of Forestry closely represents the results of 50 tests by that Division.

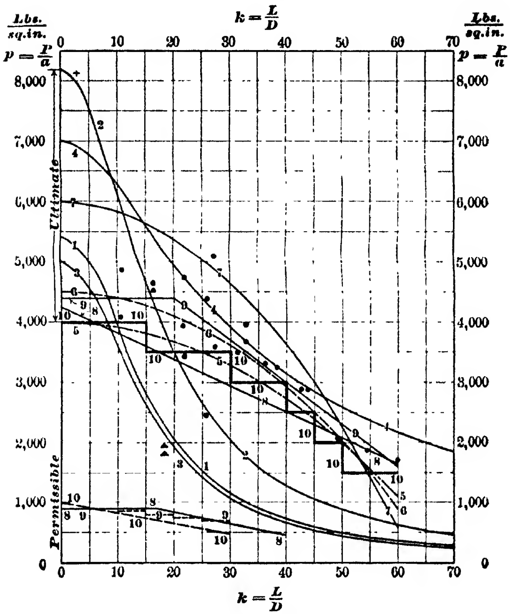

$k = \frac{L}{D}$
$\frac{Lbs.}{sq.in.}$
$p = \frac{P}{a}$
Ultimate
Permissible
0 10 20 30 40 50 60 70
8,000 7,000 6,000 5,000 4,000 3,000 2,000 1,000 0

Fig. 2.

REQUIREMENTS FOR IRON AND STEEL.

(See also Bridge Specifications.)

Digest of Specifications adopted, subject to letter ballot, at 4th Annual Meeting of the **American Section** of the **International Association for Testing Materials,** June 29, 1901 Adopted by letter ballot, August, 1901, except wrought iron, on which action was deferred

Process of Manufacture.

Wrought iron; puddled, charcoal hearth, or rolled from fagots or piles made from wrought iron scrap, alone or with muck bar added.

Steel castings. Open-hearth, crucible or Bessemer process.

Steel forgings. Open-hearth, crucible or Bessemer process.

Steel Rails. Bessemer or open-hearth. Ingots shall be kept vertical in pit-heating furnaces. No bled ingots shall be used. Sufficient material shall be discarded from the tops of the ingots to insure sound rails.

Steel Splice Bars Bessemer or open-hearth.

Boiler Plate and Rivet Steel. Open-hearth.

Structural steel for bridges and ships. Open-hearth.

Structural steel for buildings. Open-hearth or Bessemer.

Test Pieces.

For flat plates, the specimen shown in Fig. J shall be used.

For large rounds, test specimen as shown in Fig. K. The center of the specimen shall be half way between the center, and the outside of the round.

Whenever possible, iron shall be tested in full size, as rolled.

Test specimens shall be cut from bar as rolled.

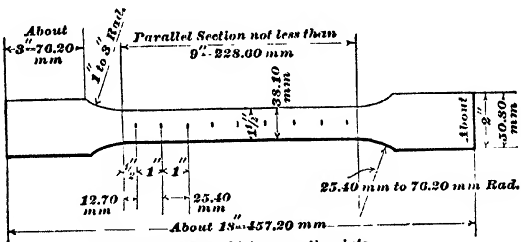


Fig. J.

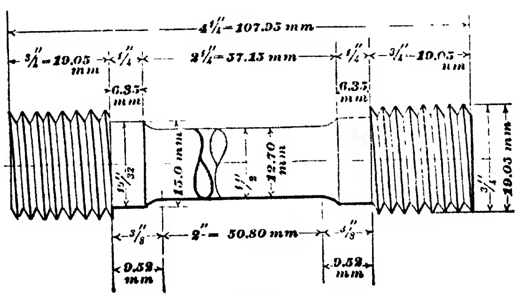


Fig. K.

Tests.

Nicking tests. The specimen shall be slightly and evenly nicked on one side, and bent back at this point through an angle of 180° by a succession of light blows.

Hot bending tests. Specimens shall be heated to a bright red, and bent by pressure or by a succession of light blows and without hammering on bend.

Cold bending tests. Specimen to be bent by pressure or by a succession of light blows.

Yield point. The yield point shall be determined by careful observation of the drop of the beam or halt in the gage of the testing machine

Drop tests. The drop testing machine for rails shall have a tup of 2000 pounds, the striking face of which shall have a radius of not more than 5 ins; and the test rail, not more than 6 feet long, shall be placed head upwards on solid supports 3 ft apart. The anvil block shall weigh at least 20,000 lbs, and the supports shall be a part of, or firmly secured to, the anvil. Height of drop from 15 ft for 45 lb rail to 19 ft for 85 lb and over. One test piece shall be selected from every fifth blow.

Homogeneity tests for fire box steel. A portion of the broken tensile test specimen is either nicked or grooved $\frac{1}{16}$ inch deep, in three places about 2 ins apart and on opposite sides. It is then clamped in a vise and broken off, by light hammer blows, bending away from each groove in succession. The specimen must not show any single seam or cavity more than $\frac{1}{4}$ inch long.

Notes to table, pp 1152 and 1153.

(a) To be bent flat.

(b) Specimen to be bent about a bar of diameter equal to its own diameter or thickness.

(c) Specimen to be bent about a bar twice its diameter.

(d) Elongation, min, per cent, in sections less than 0.654 lbs. per linear ft, grade A, 19, B, 15; C, 12

(e) Nicking test. Max per cent granular surface, grade A, 10; B, 10; C, 15.

(f) Hot bending test. Bar to be bent without cracking on outside of bend. To be bent flat in each grade; 180° in grades A and B, and sharply to 90° in C. Grade A, heated yellow and suddenly quenched in water between 80° and 90° F, to bend flat 180°. Also, heated bright red, split at end, and each part bent back 180°. Punched and drifted to hole at least 0.9 diam of rod or width of bar.

(g) Phosphorus, pieces for physical test, 0.05 for each grade.

(h) Sulphur, pieces for physical test, 0 05 for each grade.

(i) Bending. Specimen 1 inch × $\frac{1}{2}$ inch to bend cold around a diam of 1 inch without fracture on outside of bent portion.

(j) Bending. Specimen 1 inch × $\frac{1}{2}$ inch to bend cold, without fracture on outside of bent portion, around a diameter of $\frac{1}{2}$ inch.

(k) Same, around diam of $1\frac{1}{2}$ ins

(l) Same, around diam of $1\frac{1}{2}$ ins if not less than 20 ins diam; around a diam of 1 inch if less than 20 ins diam.

(m) Same, about a diameter of 1 inch.

(n) " " " " $\frac{1}{2}$ "

(o) " " " " 1 "

(p) Deduct 1 per cent for each $\frac{1}{8}$ inch in thickness above $\frac{3}{4}$ inch, and $2\frac{1}{2}$ per cent for each $\frac{1}{16}$ inch below $\frac{5}{16}$ inch.

(q) Bending. Rivet rounds to be tested of full size, as rolled. Plate specimens shall be $1\frac{1}{2}$ ins wide. For plates not over $\frac{3}{4}$ inch thick, the thickness shall be the same as that of the plate, and the specimen shall, where possible, have the natural rolled surface on two opposite sides. For plates thicker than $\frac{3}{4}$ inch, the specimen may be $\frac{1}{2}$ inch thick. Shall be subjected to both cold and quenched bending tests. For the quenched test, the material is to be heated to a light cherry red (as seen in the dark) and quenched in water of temperature between 80° and 90° Fahr. Samples shall bend flat without fracture on the outside of bent portion. Bending may be done by pressure or by blows.

(r) For pins, the elongation shall be 5 per cent less. Center of test specimen 1 inch from surface.

(s) Eye-bars shall be of medium steel. Full-sized tests shall show $12\frac{1}{2}$ per cent elongation in 15 ft of body. Min tensile strength, 55,000 lbs. per sq in. At least $\frac{2}{3}$ of eye-bars tested shall break in the body.

(t) Same as (q) but omitting quenching test.

(u) See Homogeneity Test, in text, above.

See notes, p. 1151. **Requirements for**

	Metal.	Allowable percentage of Carbon.	Phosphorus.	Sulphur.	Manganese.	Nickel.
1.	WROUGHT IRON.	Max.	Max.	Max.	Max Min.	Max. Min.
	Grade A					
	Grade B					
	Grade C					
2.	STEEL CASTINGS.					
	Hard	0.40	0.08 g	0.05 h		
	Medium	0.40	0.08 g	0.05 h		
	Soft	0.40	0.08 g	0.05 h		
3.	STEEL FORGINGS.					
	Soft or low carbon		0.10	0.10		
	Carbon, not annealed		0.06	0.06		
	Carbon, annealed		0.04	0.04		
	Carbon, oil tempered		0.04	0.04		
	Nickel, annealed					4.00 3.00
	Nickel, oil tempered					4.00 3.00
4.	STEEL RAILS.	Max. Min				
	50 to 59 lbs. per yard	0.45 0.35	0.10	0.20	1.00 0.70	
	60 to 69 " " "	0.48 0.38	0.10	0.20	1.00 0.70	
	70 to 79 " " "	0.50 0.40	0.10	0.20	1.05 0.75	
	80 to 89 " " "	0.53 0.43	0.10	0.20	1.10 0.80	
	90 to 100 " " "	0.55 0.45	0.10	0.20	1.10 0.80	
5.	STEEL SPLICE BARS	Max. 0.15	0.10		0.60 0.30	
6.	OPEN HEARTH BOILER PLATE AND RIVET STEEL.		acid basic			
	Flange or Boiler Steel		0.06 0.04	0.05	0.60 0.30	
	Fire Box Steel		0.04 0.03	0.04	0.50 0.30	
	Extra Soft Steel for Boiler Rivets		0.04 0.04	0.04	0.50 0.30	
7.	STRUCTURAL STEEL FOR BRIDGES AND SHIPS.		acid basic			
	Rivet Steel		0.08 0.06	0.06		
	Soft Steel		0.08 0.06	0.06		
	Medium Steel		0.08 0.06	0.06		
8.	STRUCTURAL STEEL FOR BUILDINGS.		Max.			
	Rivet Steel		0.10			
	Medium Steel		0.10			

Iron and Steel.

See notes, p. 1151

Tensile Tests.					Cold Bending Tests.		Other Tests. Remarks.
Strength. lbs. per sq. in.		Elastic Limit and Yield Point. lbs. per sq. in.	Elongation. Percentage in 8 ins.	Contraction of Area. Percentage	Angle of Bend.	How Bent.	
Max.	Min.	Yield Point. Min.	Min.	Min.	°		
....	50,000	25,000	25 d	....	180	a	e, f.
....	48,000	25,000	20 d	....	180	b	e, f.
....	48,000	25,000	20 d	....	180	c	e, f.
....	85,000	38,250	15	20	....	i	
....	70,000	31,500	18	25	90	i	
....	60,000	27,000	22	30	120	i	
	Average.	Average.	Avge.	Avge.			
....	58,000	29,000	28	35	180	j	
....	75,000	37,500	18	30	180	k	
....	75,000	37,500	23	32.5	180	l	
		Elastic Limit.					
....	85,000	....	21.5	42.5	180	m	
....	80,000	47,500	24.5	42.5	180	n	
....	90,000	60,000	22.5	47.5	180	o	
See Drop Test, in text, above.							
	Min.	Min.	Min.				
64,000	54,000	32,000	25	....	180	a	
		Max. and Min.					
65,000	55,000	½ Tensile Strength.	25 p	....	180	a, q	u
62,000	52,000		26 p	....	180	a, q	
55,000	45,000		28 p	....	180	a, q	
			r s.				
60,000	50,000	½ Tensile Strength.	26 p	....	180	a, t	
62,000	52,000		25 p	....	180	a, t	
70,000	60,000		22 p	....	180	b, t	
			r.				
60,000	50,000	½ Tensile Strength.	26 p	....	180	a, t	
70,000	60,000		22 p	....	180	b, t	

Digest of
"Manufacturers' Standard Specifications" for Steel,
as adopted by the **Association of American Steel Manufacturers,** Aug 9, 1895, and revised to Feb 6, 1903. In effect July 31, 1908

Words in parentheses, in Roman type, as (May be **open-hearth or Bessemer**), apply only to structural steel. Words in parentheses, in *italics*, as (*four*), (*two*), to be substituted for the preceding words (**Two**), (one), for special open-hearth plate and rivet steel. Otherwise, and except where especially stated, the provisions apply to both steels.

(May be **open-hearth or Bessemer.**)

"**Tests and inspections** shall be made at the place of manufacture prior to shipment."

Test pieces.

Standard test piece for sheared plates. See Fig J, p 1150.

Test pieces from other material, either as above or planed or turned parallel thruout length of piece.

In all cases two opposite sides of test piece to be **rolled surfaces** if possible.

"**Rivet rounds and small bars** shall be tested of full size as rolled."

"**Two** (*four*) **test pieces** shall be taken from each melt (or blow) of finished material, one (*two*) for tension and one (*two*) for bending; but in case either test develops flaws, or the tensile test piece breaks outside of the middle third of its gaged length, it may be discarded and another test piece substituted therefor."

"When material is to be **annealed or otherwise treated** before use, the specimen representing such material shall be similarly treated before testing." Otherwise, the material "shall be tested in the condition in which it comes from the rolls."

"Every finished piece of steel shall be **stamped** with the (blow or) melt number, (and steel for pins shall have the blow or melt number stamped on the ends). Rivet (and lacing) steel, (and small pieces for pin plates and stiffeners), may be shipped in bundles securely wired together, with the (blow or) melt number on a **metal tag** attached."

Maximum phosphorus and sulphur, per cent

	Phosphorus	Sulphur
Structural		
for buildings, train sheds, highway bridges, etc	0.10	..
for railway bridges	0.08	.
Special		
flange or boiler steel	0.06	0 04
extra soft and fire box steel....................	0 04	0.04

Mechanical Properties.

	Structural Steel			Special Steel		
	Rivet	Ry bridge	Medium	Extra soft*	Fire box	Flange or boiler
Bending test ‡ 180° without fracture on outside of bent portion to a diam =........	0	t	t	0†	0†	0†
Ult tensile strength in thousands of lbs per sq in	48 to 58	55 to 65	60 to 70	45 to 55	52 to 62	55 to 65
Elongation (for deductions etc see below)	1,400,000 ÷ ult strength			28%	26%	25%
Elastic Limit......................	Not less than half the ultimate strength					

*Boiler rivets to be of "extra soft" steel. †Cold or quench bends.

‡t=thickness of test piece.

Elongation, in rounds ⅝ in or less, measured in a length = 8 × diam of section tested; in other cases, measured in length of 8 ins. From the specified elongations; **deductions** to be made as follows on account of thickness: **for each increase** of ⅛ inch **above ¾ inch,** 1%, (min 20% for eye-bar material, 18% for other structural material): **for each decrease** of 1/16 inch below 5/16 inch, 2.5%: (for pins, 5%; elongation measured "on a test piece, the center of which shall be one inch from the surface of the bar.")

Percentage of allowed variation in weight or cross section of sheared plates.

When ordered to weight.

Weight per sq ft	< 12.5 lbs			≮ 12.5 lbs	
Width, ins	< 75	75 to 100	≮ 100	< 100	≮ 100
Variation, %	± 2.5	+ 5, − 3	+10, − 3	± 2.5	± 5

When ordered to gage. percentage of excess in rectangular plates. (Plates ≯ 0.01 inch light, considered up to gage.)
1 cu in rolled steel assumed = 0.2833 lb

Thickness ins.	Width, ins. < 50	50 to 70	> 70	< 75	75 to 100	100 to 115	> 115
⅛ to 5/32	10	15	20				
5/32 to 3/16	8.5	12.5	17				
3/16 to ¼	7	10	15				
¼				10	14	18	
5/16				8	12	16	
⅜				7	10	13	17
7/16				6	8	10	13
½				5	7	9	12
9/16				4.5	6.5	8.5	11
⅝				4	6	8	10
> ⅝				3.5	5	6.5	9

In other cases, a variation > 2.5% in weight or cross section will be sufficient cause for rejection.

Iron is weakened by extreme cold.

The belief (originating with Styff of Sweden) is gaining ground that iron and steel are not rendered *more brittle by intense cold,* but that the great number of breakages of rails, wheels, axles, &c, in winter, is owing to the more severe blows incident to the frozen and unyielding nature of the earth at that period of the year. But Sandberg's experiments show conclusively that although these metals may perhaps bear as much *steady* force, *gradually* applied, in winter as in summer, yet their resistance to *impulse*, or *sudden force*, is not more than ½ or ⅓ as great in severe cold; which renders them less flexible and less stretchy. It is probable that this fact does not receive as much attention as it should, in proportioning iron bridges, &c.

Some experiments with good wrought iron showed that even at 23° Fah, or only 9° colder than freezing point, there was a loss of strength of from 2½ to 4 per cent.

Malleable Cast Iron. Experiments by Mr. D L. Barnes, of Chicago, on a large number of samples of a single make of "malleable" cast iron, gave in most cases tensile strengths ranging from 21000 to 32000 lbs. per square inch, with an average of about 28000 lbs. The higher figures were obtained generally with the smallest bars (about 3 × ¼ inch) and the lower with the largest bars (about 3 × 1 inch) Pieces planed on all four sides averaged only about 24000 lbs. per square inch. This may explain the difference in favor of the smaller sections, in which the original "shell" forms a larger portion of the whole cross section.

CAST IRON.

Tensile strength	14,000 to 20,000 lbs *	per sq inch
Compressive strength (average about 100,000)	90,000 to 130,000 "	" "
Transverse strength, bar 1 in sq, 1 ft span, center load 2500 lbs. Deflection, minimum, 0.15 inch.		
Elastic limit about	6,000 lbs	per sq inch
Modulus of Elasticity "	15,000,000 "	" "

Specifications.

Tensile strength.

Bureau of Water, Philadelphia	16,000 to 20,000 lbs	per sq inch
Water Department, St. Louis, Mo	18,000 "	" "

Transverse strength.

Bureau of Water, Philadelphia.

1 in sq, 56 ins span, center load 500 lbs.

1 in sq, 36 ins span, " " 750 lbs. Deflection, minimum, 0.4 to 0.6 in.

Water Department, St. Louis, Mo.

3 in × ½ in (laid flat) 18 ins span, center load 1000 to 1250 lbs. Minimum deflection 0.3 to ⅝ inch.

Weight of Cast Iron.

Assuming 450 lbs per cub ft, specific gravity 7.2, a cub inch weighs 0.2604+ lbs; and a pound contains 3.83995+ cub ins.

Table, page 875: D = thickness or diameter, in inches.

Wt. of plate, 1 ft square, in lbs	$= 37.5\ D$ (Exact)	$\text{Log } W = 1.574\ 0313 + \text{Log } D$
" " sq bar, 1 ft long, in lbs	$= 3.125\ D^2$ (Exact)	$\text{Log } W = 0.494\ 8500 + 2 \text{Log } D$
" " rd bar, 1 ft long, in lbs	$= 2.45437\ D^2$	$\text{Log } W = 0.389\ 9400 + 2 \text{Log } D$
" " ball, in lbs	$= 0.136354\ D^3$	$\text{Log } W = \bar{1}.134\ 6551 + 3 \text{Log } D$

Weight of a spherical shell = weight of ball having outer diam of shell *minus* weight of ball having its inner diam.

Weight of pattern. A casting weighs 20 × weight of pattern of perfectly dry white pine. If not perfectly dry, although well seasoned, for 20, substitute 19 or 18.

For lead, at 700 lbs per cub ft, multiply weight of cast iron by 1.555---;
For copper, at 550 lbs, multiply by 1.222---;
For brass, at 500 lbs, multiply by 1.111---;
For wrought iron, at 485 lbs, multiply by 1.0777---;
For tin, at 460 lbs, multiply by 1.022---;
Zinc, at 450 lbs = cast iron.

* High grade irons may reach 30,000 to 40,000 lbs per sq inch, tensile.

TABLE OF WEIGHT OF CAST IRON.

At 450 lbs per cubic foot; specific gravity, 7.2.

D = Thickness or diameter, in inches. For equivalents in feet, see p 221.

D	Weights, in pounds.				D	Weights, in pounds.			
	Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.		Plate 1 ft sq	Square bar 1 ft long	Round bar 1 ft long	Ball.
1/32	1.172	0.0031	0.0024		3⅛	117.2	30.52	23.97	4.161
1/16	2.344	0.0122	0.0096		¼	121.9	33.01	25.92	4.681
3/32	3.516	0.0275	0.0216	0.0001	⅜	126.6	35.60	27.96	5.242
1/8	4.688	0.0488	0.0383	0.0003	½	131.2	38.28	30.07	5.846
5/32	5.859	0.0763	0.0599	0.0005	⅝	135.9	41.06	32.25	6.495
3/16	7.031	0.1099	0.0863	0.0009	¾	140.6	43.94	34.51	7.191
7/32	8.203	0.1495	0.1174	0.0014	⅞	145.3	46.92	36.85	7.934
1/4	9.375	0.1953	0.1534	0.0021	4.	150.0	50.00	39.27	8.727
9/32	10.55	0.2472	0.1941	0.0030	⅛	154.7	53.17	41.76	9.571
5/16	11.72	0.3052	0.2397	0.0042	¼	159.4	56.45	44.33	10.47
11/32	12.89	0.3693	0.2900	0.0055	⅜	164.1	59.81	46.98	11.42
3/8	14.06	0.4394	0.3451	0.0072	½	168.8	63.28	49.70	12.43
13/32	15.23	0.5157	0.4051	0.0091	⅝	173.4	66.84	52.50	13.49
7/16	16.41	0.5982	0.4698	0.0114	¾	178.1	70.51	55.38	14.61
15/32	17.58	0.6866	0.5393	0.0140	⅞	182.8	74.27	58.33	15.80
1/2	18.75	0.7812	0.6136	0.0170	5.	187.5	78.12	61.36	17.04
9/16	21.09	0.9888	0.7766	0.0243	⅛	192.2	82.08	64.47	18.35
5/8	23.44	1.221	0.9587	0.0333	¼	196.9	86.13	67.65	19.73
11/16	25.78	1.477	1.160	0.0443	⅜	201.6	90.28	70.91	21.17
3/4	28.12	1.758	1.381	0.0575	½	206.2	94.53	74.24	22.69
13/16	30.47	2.063	1.620	0.0731	⅝	210.9	98.88	77.66	24.27
7/8	32.81	2.393	1.879	0.0913	¾	215.6	103.3	81.15	25.92
15/16	35.16	2.747	2.157	0.1124	⅞	220.3	107.9	84.71	27.65
1.	37.50	3.125	2.454	0.1363	6.	225.0	112.5	88.36	29.45
1/16	39.84	3.528	2.771	0.1636	¼	234.4	122.1	95.87	33.29
1/8	42.19	3.955	3.106	0.1941	½	243.8	132.0	103.7	37.45
3/16	44.53	4.407	3.461	0.2283	¾	253.1	142.4	111.8	41.94
1/4	46.88	4.883	3.835	0.2663	7.	262.5	153.1	120.3	46.77
5/16	49.22	5.383	4.228	0.3083	¼	271.9	164.3	129.0	51.96
3/8	51.56	5.908	4.640	0.3545	½	281.2	175.8	138.1	57.52
7/16	53.91	6.458	5.072	0.4050	¾	290.6	187.7	147.4	63.47
1/2	56.25	7.031	5.522	0.4602	8.	300.0	200.0	157.1	69.81
9/16	58.59	7.629	5.992	0.5202	¼	309.4	212.7	167.0	76.57
5/8	60.94	8.252	6.481	0.5851	½	318.8	225.8	177.3	83.74
11/16	63.28	8.899	6.989	0.6552	¾	328.1	239.3	187.9	91.35
3/4	65.62	9.570	7.517	0.7308	9.	337.5	253.1	198.8	99.40
13/16	67.97	10.27	8.063	0.8119	¼	346.9	267.4	210.0	107.9
7/8	70.31	10.99	8.629	0.8988	½	356.2	282.0	221.5	116.9
15/16	72.66	11.73	9.213	0.9917	¾	365.6	297.1	233.3	126.4
2.	75.00	12.50	9.818	1.091	10.	375.0	312.5	245.4	136.3
1/8	79.69	14.11	11.08	1.308	¼	384.4	328.3	257.9	146.8
1/4	84.38	15.82	12.43	1.553	½	393.8	344.5	270.6	157.9
3/8	89.06	17.63	13.84	1.827	¾	403.1	361.1	283.6	169.4
1/2	93.75	19.53	15.34	2.131	11.	412.5	378.1	297.0	181.5
5/8	98.44	21.53	16.91	2.466	¼	421.9	395.5	310.6	194.1
3/4	103.1	23.63	18.56	2.836	½	431.2	413.3	324.6	207.4
7/8	107.8	25.83	20.29	3.240	¾	440.6	431.5	338.9	221.2
3.	112.5	28.12	22.09	3.682	12.	450.0	450.0	353.4	235.6

WEIGHT OF CAST-IRON PIPES per running foot.

Assuming the weight of cast-iron at 450 lbs per cub ft, or .2604 lb per cub inch. No allowance is here made for the spigot and faucet-joints used in water-pipes. As these are now commonly made, they add to the weight of each length or section of pipe of any size, about as much as that of 8 inches in length of the plain pipe as given in the table.

For lead-pipe mult by 1.6, **copper,** mult by 1.2; **brass,** add 1-7th; **welded iron,** mult by 1.0667, or add one fifteenth part.

Inner diam. or bore, in inches.	THICKNESS OF PIPE IN INCHES.												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
	Wt in Lbs	Wt in Lbs	Wt in Lbs	Wt in Lbs	Wt in Lbs	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs	Wt in Lbs	Wt in Lbs.
1.	3.07	5.07	7.38	9.99	12.9	16.2	19.7	23.5	27.7	32.1	36.9	47.4	59.1
¼	3.69	6.00	8.31	11.5	14.8	18.3	22.2	26.3	30.8	35.5	40.6	51.7	64.0
½	4.30	6.92	9.84	13.1	16.6	20.5	24.6	29.1	33.8	38.9	44.3	56.0	68.9
¾	4.92	7.84	11.1	14.6	18.5	22.6	27.1	31.8	36.9	42.3	48.0	60.3	73.8
2.	5.53	8.76	12.3	16.2	20.3	24.8	29.5	34.6	40.0	45.7	51.7	64.6	78.7
¼	6.15	9.69	13.5	17.7	22.2	26.9	32.0	37.4	43.1	49.0	55.4	68.9	86.7
½	6.76	10.6	14.8	19.2	24.0	29.1	34.5	40.1	46.1	52.4	59.1	73.2	88.6
¾	7.37	11.5	16.0	20.8	25.9	31.2	36.9	42.9	49.2	55.8	62.7	77.5	93.5
3.	7.98	12.5	17.2	22.3	27.7	33.4	39.4	45.7	52.3	59.2	66.4	81.8	98.4
¼	8.60	13.4	18.5	23.8	29.5	35.5	41.8	48.4	55.4	62.3	70.1	86.1	103.
½	9.21	14.3	19.7	25.4	31.4	37.7	44.3	51.2	58.4	65.9	73.8	90.4	108.
¾	9.83	15.2	20.9	26.9	33.2	39.8	46.8	54.0	61.5	69.3	77.5	94.7	113.
4.	10.3	16.1	22.2	28.5	35.1	42.0	49.2	56.7	64.6	72.7	81.2	99.0	118.
¼	11.1	17.1	23.4	30.0	36.9	44.1	51.7	59.5	67.7	76.1	84.9	103.	123.
½	11.7	18.0	24.6	31.5	38.8	46.3	54.1	62.3	70.7	79.5	88.6	108.	128.
¾	12.3	18.9	25.8	33.1	40.6	48.5	56.6	65.0	73.8	83.9	92.3	112	133
5.	12.9	19.8	27.1	34.6	42.5	50.6	59.1	67.8	76.9	87.2	96.0	116	138.
¼	13.5	20.8	28.3	36.1	44.3	52.8	61.5	70.6	80.0	90.6	99.6	121.	143.
½	14.2	21.7	29.5	37.7	46.1	54.9	64.0	73.3	83.0	94.0	103	125	148.
¾	14.8	22.6	30.8	39.2	48.0	57.1	66.4	76.1	86.1	97.4	107	129	153.
6.	15.4	23.5	32.0	40.8	49.8	59.2	68.9	78.9	89.2	99.8	111.	134	158.
½	16.6	25.4	34.5	43.8	51.5	63.5	73.8	84.4	95.3	107.	118	142	167.
7.	17.8	27.2	36.9	46.9	57.2	67.8	78.7	89.4	102.	113.	126	151.	177.
½	19.1	29.1	39.4	50.0	60.9	72.1	83.7	95.5	108	120.	133.	159	187.
8.	20.3	30.9	41.8	53.1	64.6	76.4	88.6	101.	114.	127.	140.	168	197.
½	21.5	32.8	44.3	56.1	68.3	80.7	93.5	107.	120.	134.	148	177.	207.
9.	22.8	34.6	46.8	59.2	72.0	85.1	98.4	112.	126	140	155	185.	217.
½	24.0	36.4	49.2	62.3	75.7	89.3	103.	118.	132.	147	163.	194	226.
10.	25.1	38.3	51.7	65.3	79.4	93.6	108	123.	138.	154.	170	202.	235.
½	26.4	40.1	54.1	68.4	83.0	97.9	113.2	129.	145.	161.	177.	211.	245.
11.	27.6	42.0	56.6	71.5	86.7	102.	118.	134.	151.	168	185.	220	255.
½	28.8	43.8	59.1	74.6	90.4	107.	123.	140.	157.	174.	192.	228	265.
12.	30.0	45.7	61.5	77.7	94.1	111.	128.	145.	163.	181.	199.	237.	275.
13.	32.5	49.4	66.4	83.8	102.	120.	138.	156.	175.	195.	214.	254	294.
14.	35.0	53.1	71.4	89.4	109.	128.	148.	168.	188.	208.	229.	271.	314.
15.	37.4	56.7	76.3	96.1	116	137.	158	179.	200.	222.	244.	289.	334.
16.	39.1	60.4	81.2	102.	124	145.	167.	190.	212	235.	258.	306.	353.
17.	42.3	64.1	86.1	108.	131.	154.	177.	201.	225.	249.	273	323.	373.
18.	44.8	67.8	91.0	115.	139	163.	187.	212.	237.	262	288.	340.	393.
19.	47.3	71.5	96.0	121.	146.	171	197.	223.	249.	276.	303.	357	412.
20.	49.7	75.2	101.	127.	153	180.	207.	234.	261.	289.	317.	375.	432.
21.	52.2	78.9	106.	133.	161.	188.	217.	245.	273.	303.	332.	392.	452.
22.	54.6	82.6	111.	139.	168.	196	227.	256	286.	316.	347.	409	471.
23.	57.1	86.3	116.	145.	175	206	236.	267.	298.	330.	362.	426	491.
24.	59.6	89.9	121.	152.	183	214.	246.	278.	311.	343.	375.	444	511.
25.	62.0	93.6	126.	158.	190.	223.	256.	289.	323.	357.	391.	461	531.
26.	64.5	97.3	131.	164	198.	231.	266.	300.	335.	370	406.	478.	550.
27.	66.9	101.	135.	170.	205.	240.	276.	311.	348.	384.	421.	495	570.
28.	69.4	105.	140.	176.	212.	249	286.	323.	360.	397	436	512	590.
29.	71.8	109.	145.	182.	220.	257.	295	334.	372.	411.	450.	530.	609.
30.	74.2	112.	150	188	227.	266	305.	345	384.	424.	465	547	629.
31.	76.7	116	155	195.	234	275	315.	356.	397.	438.	480.	564	649.
32.	79.1	120.	160.	201.	242	283	325	367.	409.	451.	495.	581	668.
33.	81.6	123.	165.	207	249.	292.	335	378	421.	465.	509.	598	688.
34.	84.1	127.	170.	213.	257.	300.	345.	389	434.	479.	524.	616.	708.
35.	86.5	131.	175.	219	264.	309.	354.	400	446.	492.	539.	633.	728.
36.	89.0	134.	180.	225.	271	318.	364.	411	458.	506.	554.	650.	746.
42.	104.	156.	210.	262.	315.	370.	423.	478	532.	588.	644.	753.	864
48.	119.	178.	239.	298.	359.	422	482.	544.	605.	669.	733.	856.	982.

For warming buildings by steam it usually suffices to allow 1 sq ft of cast or wrought pipe surface for each 120 cub ft of space to be warmed; and 1 cub ft of boiler for each 2000 cub ft of such space.

Table of Weight of WROUGHT IRON and STEEL.

At 485 lbs per cubic foot; specific gravity, 7.76. See page 1161.

D = Thickness or diameter, in inches. For equivalents in feet, see p 221.

D	Weights, in pounds.				D	Weights, in pounds.			
	Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.		Plate 1 ft sq	Square bar 1 ft long	Round bar 1 ft long	Ball.
1/32	1.263	0.0033	0.0026		3 1/8	126.3	32.89	25.83	4.485
1/16	2.526	0.0132	0.0103		1/4	131.4	35.57	27.94	5.045
3/32	3.789	0.0296	0.0232	0.0001	3/8	136.4	38.36	30.13	5.650
1/8	5.052	0.0526	0.0413	0.0003	1/2	141.5	41.26	32.40	6.301
5/32	6.315	0.0822	0.0646	0.0006	5/8	146.5	44.26	34.76	7.000
3/16	7.578	0.1184	0.0930	0.0010	3/4	151.6	47.36	37.20	7.750
7/32	8.841	0.1612	0.1266	0.0015	7/8	156.6	50.57	39.72	8.551
1/4	10.10	0.2105	0.1653	0.0023	4	161.7	53.89	42.32	9.406
9/32	11.37	0.2664	0.2092	0.0033	1/8	166.7	57.31	45.01	10.32
5/16	12.63	0.3289	0.2583	0.0045	1/4	171.8	60.84	47.78	11.28
11/32	13.89	0.3980	0.3126	0.0060	3/8	176.8	64.47	50.63	12.31
3/8	15.16	0.4736	0.3720	0.0077	1/2	181.9	68.20	53.57	13.39
13/32	16.42	0.5558	0.4366	0.0099	5/8	186.9	72.04	56.58	14.54
7/16	17.68	0.6447	0.5063	0.0123	3/4	192.0	75.99	59.68	15.75
15/32	18.95	0.7400	0.5812	0.0151	7/8	197.0	80.04	62.87	17.03
1/2	20.21	0.8420	0.6613	0.0184	5	202.1	84.20	66.13	18.37
9/16	22.73	1.066	0.8370	0.0261	1/8	207.1	88.46	69.48	19.78
5/8	25.26	1.316	1.033	0.0359	1/4	212.2	92.83	72.91	21.27
11/16	27.79	1.592	1.250	0.0478	3/8	217.2	97.31	76.42	22.82
3/4	30.31	1.895	1.488	0.0620	1/2	222.3	101.9	80.02	24.45
13/16	32.84	2.223	1.746	0.0788	5/8	227.3	106.6	83.70	26.16
7/8	35.36	2.579	2.025	0.0985	3/4	232.4	111.4	87.46	27.94
15/16	37.89	2.960	2.325	0.1211	7/8	237.5	116.3	91.30	29.80
1.	40.42	3.368	2.645	0.1470	6	242.5	121.3	95.23	31.74
1/16	42.94	3.802	2.986	0.1763	1/4	252.6	131.6	103.3	35.88
1/8	45.47	4.263	3.348	0.2092	1/2	262.7	142.3	111.8	40.36
3/16	47.99	4.750	3.730	0.2461	3/4	272.8	153.5	120.5	45.20
1/4	50.52	5.263	4.133	0.2870	7	282.9	165.0	129.6	50.41
5/16	53.05	5.802	4.557	0.3323	1/4	293.0	177.0	139.0	56.00
3/8	55.57	6.368	5.001	0.3820	1/2	303.1	189.5	148.8	62.00
7/16	58.10	6.960	5.466	0.4365	3/4	313.2	202.3	158.9	68.41
1/2	60.63	7.578	5.952	0.4960	8	323.3	215.6	169.3	75.24
9/16	63.15	8.223	6.458	0.5606	1/4	333.4	229.2	180.0	82.52
5/8	65.68	8.894	6.985	0.6306	1/2	343.5	243.3	191.1	90.25
11/16	68.20	9.591	7.533	0.7062	3/4	353.6	257.9	202.5	98.45
3/4	70.73	10.31	8.101	0.7876	9.	363.7	272.8	214.3	107.1
13/16	73.26	11.06	8.690	0.8750	1/4	373.9	288.2	226.3	116.3
7/8	75.78	11.84	9.300	0.9687	1/2	384.0	304.0	238.7	126.0
15/16	78.31	12.64	9.930	1.069	3/4	394.1	320.2	251.5	136.2
2.	80.83	13.47	10.58	1.176	10	404.2	336.8	264.5	147.0
1/8	85.89	15.21	11.95	1.410	1/4	414.3	353.9	277.9	158.8
1/4	90.94	17.05	13.39	1.674	1/2	424.4	371.3	291.6	170.1
3/8	95.99	19.00	14.92	1.969	3/4	434.5	389.2	305.7	182.6
1/2	101.0	21.05	16.53	2.296	11.	444.6	407.5	320.1	195.6
5/8	106.1	23.21	18.23	2.658	1/4	454.7	426.3	334.8	209.2
3/4	111.1	25.47	20.00	3.056	1/2	464.8	445.4	349.8	223.5
7/8	116.2	27.84	21.86	3.492	3/4	474.9	465.0	365.2	238.4
3.	121.3	30.31	23.81	3.968	12.	485.0	485.0	380.9	253.9

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot. For cast iron, deduct $\frac{1}{16}$ part; for steel, add $\frac{1}{48}$; for copper, add $\frac{1}{7}$; for cast brass, add $\frac{1}{13}$; for lead, add ½; for zinc, deduct $\frac{1}{12}$.

Width in Ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
1.	.2083	.4166	6250	8333	1 042	1 250	1.458	1.666	2.083	2.500	2.916	3.333
⅛	.2344	4688	.7033	9375	1.172	1.406	1.640	1 875	2 344	2.812	3.280	3.75
¼	2605	.5210	.7810	1.042	1 303	1 563	1 823	2 083	2 605	3 125	3.646	4.166
⅜	2865	5730	8595	1.146	1 432	1.719	2 006	2.292	2 864	3.438	4.012	4.583
½	.3125	.6250	.9375	1 250	1.562	1.875	2.188	2.500	3 125	3.750	4.375	5.006
⅝	.3385	6771	1 015	1.354	1 692	2.031	2.370	2.708	3.384	4 062	4.740	5.416
¾	.3646	7292	1.094	1 458	1 823	2.168	2.550	2 916	3.646	4 375	5.105	5.833
⅞	3906	.7812	1.172	1 562	1.953	2.344	2 735	3.125	3 906	4.688	5.470	6 25
2.	.4166	.8333	1.25	1.666	2 083	2.500	2.916	3.333	4.166	5 000	5.833	6.666
⅛	.4427	8855	1.328	1 771	2.214	2 656	3 098	3 542	4.428	5.312	6.196	7 083
¼	.4688	.9375	1.406	1.875	2 344	2.812	3 281	3.750	4.688	5 624	6.562	7.500
⅜	.4948	9895	1.484	1.979	2 474	2 968	3 463	3.958	4.948	5 936	6.926	7 916
½	5210	1.042	1.562	2 083	2 605	3 125	3.616	4.166	5.210	6.250	7 291	8.333
⅝	5470	1.094	1 641	2 187	2 735	3 282	3 829	4.375	5.470	6 564	7.658	8 750
¾	.5730	1 146	1.719	2 292	2 865	3 438	4 011	4.583	5.730	6.876	8.022	9.166
⅞	.5990	1 198	1 797	2 396	2.995	3 594	4.193	4.792	5.990	7.188	8 386	9.583
3.	.625	1 250	1 875	2 500	3.125	3 750	4 375	5.000	6 250	7.500	8.750	10.00
⅛	.6515	1 303	1 954	2 605	3.257	3.908	4.560	5 210	6.514	7.816	9.120	10.42
¼	.6770	1.354	2.031	2 708	3 385	4 062	4 739	5 416	6.770	8.124	9 478	10.83
⅜	.7031	1.406	2 109	2.812	3 516	4.218	4 921	5.625	7.032	8 436	9.842	11.25
½	7291	1.458	2.188	2.916	3 646	4.375	5 105	5 833	7.291	8.750	10.21	11.66
⅝	7555	1.511	2.266	3.021	3 777	4 533	5 288	6.042	7.554	9 066	10.58	12.08
¾	.7812	1.562	2.343	3.125	3 906	4 686	5.468	6.25	7.812	9.372	10 94	12 50
⅞	.8070	1.614	2.421	3.229	4 035	4 842	5 65	6 458	8 070	9 684	11.30	12.92
4.	.8333	1 666	2 500	3 333	4 166	5 000	5 833	6 666	8 333	10 00	11 66	13 33
⅛	.8595	1.719	2 578	3 438	4 297	5.156	6.016	6 875	8.594	10 31	12 03	13.75
¼	.8835	1.771	2.656	3 542	4.427	5.312	6.198	7.083	8 854	10.62	12 40	14.16
⅜	.9115	1.823	2.734	3 646	4 557	5 468	6 380	7.291	9 114	10 94	12 76	14 58
½	.9375	1.875	2.812	3.750	4 687	5 624	6.562	7 500	9 374	11 25	13 12	15 00
⅝	.9636	1.927	2 891	3 854	4 818	5 782	6 745	7.708	9 636	11 58	13 49	15.42
¾	.9895	1 979	2.968	3 958	4 947	5 936	6 926	7.917	9.894	11 87	13.85	15.83
⅞	1 016	2.031	3.048	4 062	5 080	6.096	7.112	8.125	10.16	12 19	14 22	16.25
5.	1 042	2.083	3.125	4.166	5 210	6 25	7.291	8 333	10 42	12.50	14.58	16 66
⅛	1.068	2.136	3 204	4 271	5.340	6 408	7 476	8 542	10 68	12 81	14.95	17 08
¼	1 094	2.188	3 282	4.375	5 470	6 564	7 658	8.750	10.94	13 13	15 31	17.50
⅜	1.120	2.240	3 360	4.479	5 600	6 720	7.840	8 958	11.20	13 44	15.68	17.92
½	1.146	2.292	3 438	4 584	5 730	6 876	8.022	9 167	11.46	13.75	16.04	18.33
⅝	1.172	2.344	3.516	4.687	5 860	7 032	8.204	9.375	11 72	14 06	16.40	18 75
¾	1.198	2.396	3.594	4.791	5 990	7.188	8.386	9 583	11.98	14 37	16.77	19 16
⅞	1.224	2.448	3.672	4.896	6.120	7.344	8.568	9 792	12 24	14 69	17.13	19.58
6.	1.250	2.500	3.750	5.000	6 250	7.500	8.750	10 00	12.50	15 00	17.50	20 00
⅛	1.276	2 552	3.828	5.104	6 380	7.656	8.932	10 21	12 76	15.31	17.86	20.42
¼	1.302	2.604	3.906	5 208	6 510	7.812	9.114	10 42	13 02	15 62	18.23	20.83
⅜	1.328	2.657	3.984	5.313	6 640	7.968	9 297	10.63	13 28	15.93	18 59	21.25
½	1.354	2.708	4 063	5 417	6.770	8.126	9.480	10.83	13.54	16.25	18.96	21.66
⅝	1.381	2.761	4.143	5 521	6.906	8.286	9 668	11.04	13.81	16 57	19.33	22.08
¾	1.406	2.813	4.218	5.625	7.030	8 436	9 843	11.25	14.06	16.87	19.69	22.50
⅞	1.432	2.864	4.296	5.729	7.160	8.592	10 02	11.46	14.32	17.18	20.04	22.92
7.	1.458	2.916	4.375	5.833	7.291	8 750	10 20	11.66	14.58	17.50	20.42	23.33
⅛	1.484	2.969	4.452	5.938	7 420	8.904	10.39	11.87	14.84	17.81	20.78	23.75
¼	1.511	3 021	4 533	6.042	7.555	9.066	10.58	12.08	15.11	18.13	21.16	24.16
⅜	1.536	3.073	4.608	6.146	7.680	9 216	10.75	12.29	15.36	18.43	21.50	24.58
½	1.562	3 125	4.686	6.250	7.810	9.372	10.93	12.50	15.62	18.74	21.86	25.00
⅝	1.588	3.177	4.764	6.354	7.940	9.528	11.12	12.71	15.88	19.05	22 24	25.42
¾	1.615	3.229	4.845	6 458	8.075	9.690	11.31	12.92	16.15	19.38	22.62	25.83
⅞	1.641	3.281	4.923	6.562	8.205	9.846	11.48	13.13	16.41	19.69	22.96	26.25
8.	1.666	3.333	5.000	6.666	8.333	10.00	11.66	13.33	16.66	20.00	23.33	26.66
⅛	1.693	3.386	5.079	6.771	8.455	10.15	11.85	13.54	16.91	20.30	23.70	27.08
¼	1.719	3.438	5.157	6.875	8.595	10.31	12.03	13.75	17.19	20.61	24.06	27.50
⅜	1.745	3.489	5.235	6.979	8.725	10.47	12.21	13.96	17.45	20.94	24.42	27.92
½	1.771	3.542	5.313	7.083	8.855	10.63	12.40	14.17	17.71	21.26	24.80	28.33
⅝	1.797	3.594	5.391	7.188	8.985	10.78	12.58	14.37	17.97	21.56	25.16	28.75
¾	1.823	3.646	5.469	7.292	9.115	10.94	12.76	14.58	18.23	21.88	25.52	29.17
⅞	1.849	3.698	5.547	7.396	9.245	11.09	12.94	14.79	18.49	22.18	25.88	29.58
9.	1.875	3.750	5.625	7.500	9.375	11.25	13.12	15.00	18.75	22.50	26.24	30.00
⅛	1.901	3.802	5.703	7.604	9.505	11.41	13.31	15.21	19.00	22.81	26.62	30.42
¼	1.927	3.854	5.781	7.708	9.635	11.56	13.49	15.42	19.27	23.12	26.98	30.83
⅜	1.953	3.906	5.859	7.812	9.765	11.72	13.67	15.62	19.53	23.44	27.34	31.25
½	1.979	3.958	5.937	7.916	9.895	11.87	13.85	15.84	19.79	23.74	27.70	31.67
⅝	2.005	4.010	6.015	8.021	10.02	12.03	14.04	16.04	20.04	24.06	28.06	32.08
¾	2.031	4.062	6.093	8.125	10.16	12.18	14.21	16.25	20.32	24.36	28.42	32.50
⅞	2.057	4.114	6.171	8.229	10.29	12.34	14.40	16.46	20.58	24.68	28.80	32.92
10.	2.083	4.166	6.250	8.333	10.41	12.50	14.58	16.66	20.82	25.00	29.16	33.33
⅛	2.109	4.219	6.327	8.438	10.55	12.65	14.76	16.87	21.10	25.30	29.52	33.75
¼	2.135	4.270	6.405	8.541	10.67	12.81	14.94	17.08	21.34	25.62	29.88	34.17

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot—(Continued.)

Width in ins	THICKNESS IN INCHES.											
	1-16	⅛	3 16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
10⅜	2 162	4.323	6.486	8 646	10.81	12 97	15.13	17 29	21.62	25.94	30 26	34.58
½	2 188	4.375	6 564	8.750	10.94	13 13	15.31	17 50	21.88	26.26	30 62	35.00
⅝	2 214	4 427	6 642	8.854	11.07	13.28	15 50	17 71	22.14	26.56	31 00	35.42
¾	2.239	4 479	6 717	8.958	11 20	13.43	15 67	17.92	22.40	26 86	31 34	35.83
⅞	2.266	4.531	6 798	9 062	11.33	13 59	15.86	18.12	22.66	27 18	31 72	36.25
11.	2 291	4 583	6 873	9 166	11.46	13 75	16 04	18 33	22.90	27 50	32 08	36.66
⅛	2 318	4.636	6.954	9 271	11.59	13.91	16.22	18 54	23.18	27.82	32 44	37 08
¼	2.344	4.688	7 032	9 375	11 72	14.06	16 40	18 75	23.44	28 12	32 80	37.50
⅜	2 370	4 740	7.110	9.479	11.85	14.22	16.59	18 96	23 70	28.44	33.18	37.92
½	2.395	4 791	7 185	9 582	11 97	14 37	16.76	19 16	23.94	28.74	33.52	38.33
⅝	2 422	4 844	7 266	9 688	12 11	14.53	16 95	19.37	24 22	29 06	33 90	38.75
¾	2.448	4 896	7 341	9 792	12 24	14 68	17 13	19 58	24.48	29 36	34 26	39.16
⅞	2 474	4 948	7 422	9 896	12 37	14 84	17 32	19 79	24.74	29.68	34 64	39.58
12.	2.500	5.000	7.500	10.00	12.50	15.00	17 50	20.00	25.00	30 00	35.00	40.00

Weight of Wrought Iron and Steel.

Assuming 485 lbs. per cub ft,* specific gravity, 7.76; a cubic inch weighs 0.28067 lbs; and a pound contains 3.5629 cubic inches.

Table, page 1159: D = thickness or diameter, in inches.

Wt. of plate, 1 ft square, in lbs,	$= 40.4167\ D$;	$\text{Log } W = 1.606\ 5605 + \text{Log } D$
" " sq bar, 1 ft long, in lbs,	$= 3.36806\ D^2$;	$\text{Log } W = 0.527\ 3792 + 2 \text{ Log } D$
" " rd bar, 1 ft long, in lbs,	$= 2.64527\ D^2$;	$\text{Log } W = 0.422\ 4693 + 2 \text{ Log } D$
" " ball, in lbs,	$= 0.146959\ D^3$;	$\text{Log } W = \bar{1}.167\ 1966 + 3 \text{ Log } D$

Weight of a spherical shell = {weight of ball having outer diameter of shell} − {weight of ball having inner diameter of shell.

Weights of equal masses.

For lead,	at 700 lbs per cub ft: weight = 1.44 ×	weight of wrought iron
For copper,	" 550 " " " " = 1.13 ×	
For brass,	" 500 " " " " = 1.03 ×	
For tin,	" 460 " " " " = 0.948 ×	
For zinc or cast iron,	" 450 " " " " = 0.928 ×	

*Very pure soft wrought iron weighs from 488 to 492 lbs per cubic foot; average rolled iron about 480. At 480 lbs, a bar 1 inch square weighs exactly 10 lbs per yard = 3⅓ lbs per foot.

Weights per square foot of **galvanized sheet iron.** Standard list adopted by the American Galvanized Iron Ass'n, at Pittsburgh, April, 1884.

No.	Ounces avoir per sq ft	Sq ft per 2240 lbs	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.
29	12	2987	24	17	2108	19	33	1086
28	13	2757	23	19	1886	18	38	943
27	14	2560	22	21	1706	17	43	833
26	15	2389	21	24	1493	16	48	746
25	16	2240	20	28	1280	14	60	597

The galvanizing is simply a thin film of zinc on both sides of the sheet, as in what is known as "tinned plates," or "tin," which are in reality sheet iron similarly coated with tin. Zinc like tin, resists corrosion from ordinary atmospheric influences, much better than iron, and hence the use of these metals as a protection to the iron. A well galvanized roof, of a good pitch, will suffer but little from 5 to 6 years exposure without being painted. It will then take paint readily, and should be painted It is better, however, always to paint tin ones at once.

Paint does not adhere well to new zinc, and this is the principal reason why new galvanized roofs are not painted; but this may be remedied by first brushing the zinc over with the following One part of chloride of copper, 1 part nitrate of copper, 1 part of sal-ammoniac Dissolve in 64 parts of water. Then add 1 part of commercial hydrochloric acid. When brushed with this solution, the zinc turns black, dries within 12 to 24 hours, and may then be painted.

Paint of some mineral oxide of a brown color is generally used, one coat being applied to both sides in the shop; and the other after being put on the roof. Repainting every 3 or 4 years will suffice afterward Ungalvanized iron (called BLACK IRON, for distinction) is also very enduring for roofs, if well painted every 1 or 2 years. The chief advantage of galvanized roofing is that it does not require painting so often as the black. The galvanizing adds about ⅛ of a lb per square foot of surface, or about ½ lb per sq ft of sheet as coated on both sides; without regard to the thickness of the sheet. Paint for roofs should not have much *dryer*. See Painting.

The sulphurous fumes from coal are very corrosive of EITHER GALVANIZED OR BLACK IRON, as may be seen in shops, railroad bridges, or engine houses, roofed with either, if efficient means are not provided for carrying off the smoke; and the same with other metals THE ACID OF OAK TIMBER is said to destroy the zinc of galvanized iron

Flat iron is usually nailed upon a sheeting of boards; but the strength of corrugated iron obviates the necessity for this, and enables it to stretch 5 or 6 ft from purlin to purlin, without intermediate support. The corrugated sheets are riveted together on the roof, by rivets of galvanized wire about one eighth inch thick, 300 to a pound, well driven (so as to exclude rain) 3 or 4 inches apart, all around the edges. The rivet holes are first punched by machinery, so as to insure coincidence in the several sheets; and the rivets are driven by two men, one above, and one beneath the roof. For black iron, ungalvanized nails, boiled in linseed oil as a partial preservative from rust, are commonly used; as also in shingling or slating. Galvanized ones, however, would be better in all these cases, or even copper ones for slating because good slate endures much longer than either shingles or iron, and therefore it becomes true economy to use durable metals for fastening it. In none of these cases, however, are the nails fully exposed to the weather

The sheets of flat iron are put together by overlapping and FOLDING THE EDGES, much the same as shown by the fig page 1208, head Tin, the joints which run up and down the roof being the same as at *a a*, and the horizontal ones as at *t t*; except that inasmuch as these are not soldered in the iron sheets the joint is made about ¾ to 1 inch wide, instead of ⅜ inch, the better to provide against leaking. Cleats are used as in tin, with 2 nails to a cleat. The iron plates are best laid on sheeting boards; but in sheds, &c, are sometimes laid directly on rafters, not more than about 18 ins apart in the clear, the plates being allowed to sag a little between the rafters, so as to form shallow gutters. In such cases it is well to bevel off the tops of the rafters slightly, as in this fig.

A serious objection to iron as a roof covering, is its rapid condensation of atmospheric moisture; which falls from the iron in drops like rain, and may do injury to ceilings, floors, or articles in the apartments immediately beneath the roof. Painting does not appreciably diminish this; it may, however, be obviated by plastering.

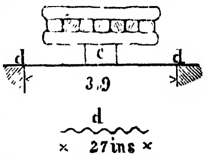
Corrugated sheet iron. The size of sheets generally used for corrugating, is 30 inches wide by 96 inches long. Corrugation reduces the width to 27½ inches. When the corrugated sheets are laid upon the roof, the overlapping of about 2½ inches along the sides, and of 4 inches along their ends diminishes the area of roof covered by a sheet, to about seven-eighths of that of the entire corrugated sheet itself; or, the weight per square foot of roof covered, will be about one-seventh greater than that per square foot of the corrugated sheet, or, the weight of corrugated iron per square foot of roof covered is about one-fifth greater than that of the flat sheets from which it is made.

About 6 inches are usually allowed for the extension over the eaves.

The weights per square foot corresponding to the different numbers of the Birmingham wire gauge, vary somewhat with the different makers. The two styles of corrugation given in the table below 5 x 1¼ and 2½ x ⅝, are those most frequently used.

No. Bmghm wire ga.	Thick-ness in ins.	Wt in lbs per sq ft of *sheets*.		Wt in lbs per sq ft of *roof*.	
Black	Black	Black	Lead cot'd or galv'd	Black	Lead cot'd or galv'd
20	.035	1 84	2.	2 12	2.3
22	.028	1.50	1.6	1.73	1 84
24	.022	1.20	1 25	1 38	1 44
26	.018	1 00	1 12	1 15	1 29

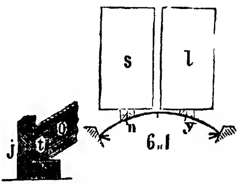
Strength of Corrugated Iron. Experiments by the author.



First. A sheet *d d*, of No. 16 iron, (about $\frac{1}{17}$ inch thick,) 27 ins wide, by 4 ft long, with five complete corrugations of 5 ins by 1 inch, was laid on supports 3 ft 9 ins apart A block of wood *c*, 9 ins wide, by 7 ins thick, and 30 ins long, was placed across the center, and gradually loaded with castings weighing 1600 lbs.

This caused a deflection at the center of precisely ½ an inch. On the removal of the load after an hour, no permanent set was appreciable. The severity of the test was purposely increased by applying the several castings very roughly, jolting the whole as much as possible.* The suspended area of the sheet was 8 44 sq ft; and since the actual *center* load of 1600 lbs is about equivalent to 3000 lbs *equally distributed*, it amounts to $\frac{3000}{8\ 44}$=355 lbs per sq ft distributed. But 3000 lbs distributed would produce a deflection of but about full ¼ of an inch. Again, 355 lbs per sq ft is about 4 times the weight of the greatest crowd that could well congregate upon a floor Consequently this iron, at 3' 9" span, is safe in practice for any ordinary crowd. Moreover, such a crowd would produce a center deflection of only the ¼th part of ¼ of an inch; or $\frac{1}{16}$ of an inch; or $\frac{1}{720}$ of the clear span; which is but two-thirds of Tredgold's limit of $\frac{1}{480}$ of the span.

In one experiment the ends of the sheets rested upon supports dressed so as to present undulations corresponding tolerably closely with the shape of the corrugations; but in the other the supports were flat, and each end of the sheet rested only upon the lower points of the corrugations. No appreciable difference was observed in the results.



Second. An arch of No. 18 ($\frac{1}{20}$ inch) iron. corrugated like the foregoing, but the depth of corrugation increased to 1¼ ins by the process of arching the sheet; clear span 6 ft 1 inch; rise 10 ins; breadth 27 ins, (of which however, only 25 ins bore against the abutments.)

Each foot *o* of the arch abutted upon a casting *j*, the inner portion *t* of which was undulated on top, to correspond with the corrugations of the arch, which rested upon it. At *y*, (one-fourth of the span,) two wooden blocks were placed, occupying a width of 9 inches, and extending across the arch; on them was piled a load, *l*, of castings, to the extent of 4480 lbs, or 2 tons. Under this load the arch descended about half an inch at *y*, becoming flatter on that side and slightly more curved upward along the unloaded side *n*. Two similar blocks were then placed at *n*, and two tons of load, *s*, were piled upon them, in addition to the 2 tons at *l*; making a total of 8960 lbs, or 4 tons. This brought the arch more nearly back to its original shape; but still slightly straightened at both *n* and *y*, and a little more curved in the center. The load was then increased to 10000 lbs, and left standing for several days. Two iron ties, each ½ by 1¾, which were used for preventing the abutment castings *j* from spreading, were found to have stretched nearly ¼ of an inch. Additional ones were inserted, and the load increased to a total of 6 tons, or 13440 lbs, parts of it on *s* and *l*, and part in the shape of long broad bars of iron at the center of the arch, below the loads *s* and *l*, and between *n* and *y*. So far as could be judged by eye, the shape of the arch was now almost perfect. *The loads s and l did not touch each other.* After standing more than a week, the load was accidentally overturned, crippling the arch The load was equal to about 1000 lbs per sq ft of the arch. Such arches have since come into common use instead of brick, **for fireproof floors.**

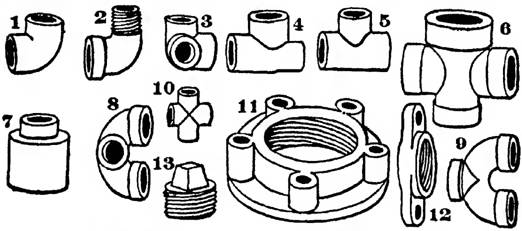
Curved roofs of 25 to 30 ft span. rising about ¼ span, may be made of ordinary corrugated iron of Nos 16 to 18, riveted as usual; and having no accessories except tie-rods a few feet apart; continuous angle-iron skewbacks; and thin vertical rods to prevent the ties from sagging.

* Without letting the deflection exceed ½ inch; which was prevented by a stop under the sheet.

Welded wrought-iron pipes, for steam, gas, and water. Usually in lengths of about 18 feet. Standard sizes.

Inner Diam. Nominal.*	Inner Diam. Actual.	Thickness.	Wt. per foot. Nominal.	Threads per inch of screw.	List price per foot run.†	Inner Diam. Nominal.*	Inner Diam. Actual.	Thickness.	Wt. per foot. Nominal.	Threads per inch of screw.	List price per foot run.†
Ins.	Ins.	Ins.	lbs.		$	Ins.	Ins.	Ins.	lbs.		$
1/8	0.270	0.068	0.24	27	0.055	3½	3.548	0.226	9.00	8	0.95
1/4	0.364	0.088	0.42	18	"	4	4.026	0.237	10.66	"	1.08
3/8	0.494	0.091	0.56	"	"	4½	4.508	0.246	12.49	"	1.30
1/2	0.623	0.109	0.84	14	0.085	5	5.045	0.259	14.50	"	1.45
3/4	0.824	0.113	1.12	"	0.115	6	6.065	0 280	18.76	"	1.88
1	1.048	0.134	1.67	11.5	0.165	7	7.023	0.301	23.27	"	2.35
1¼	1.380	0.140	2 24	"	0.225	8	7.982	0.322	28.18	"	2.82
1½	1.611	0.145	2.68	"	0 27	9	9.001	0.344	33.70	"	3.40
2	2.067	0.154	3.61	"	0.36	10	10.019	0.366	40.00	"	4.25
2½	2.468	0.204	5.74	8	0.575	11	11.000	0.375	45.00	"	4.75
3	3.067	0.217	7.54	"	0.755	12	12.000	0.375	49.00	"	5.20

Fittings for Wrought-iron Pipes. 1, Elbow. 2, Service Elbow. 3, Elbow with side outlet. 4, Reducing T. 5, T. 6, Reducing Cross. 7, Reducing Coupling or Socket. 8, Return Bend with side outlet. 9, Return Bend with back outlet. 10, Cross. 11, Flange Union. 12, Oval Flange. 13, Plug.

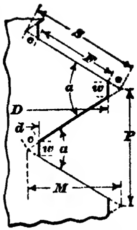


Lap-welded charcoal-iron boiler tubes, in lengths up to 20 ft.

Outer Diam.*	Thickness.		Nom Wt. per ft.	Price per ft.†	Outer Diam.*	Thickness.		Nom Wt. per ft.	Price per ft.†	Outer Diam.*	Thickness.		Nom Wt. per ft.	Price per ft.†
Ins.	Ins.	B WG	lbs.	$	Ins.	Ins.	B WG	lbs.	$	Ins.	Ins.	B WG	lbs.	$
1	0.095	13	0.90	0.30	3	0.109	12	3.33	0.35	8	0.165	8	13.65	1.50
1¼	"	"	1.15	0.28	3¼	0.120	11	3.96	0.40	9	0.180	7	16.76	1.70
1½	"	"	1.40	0.27	3½	"	"	4.28	0.44	10	0.203	6	21.00	2.10
1¾	"	"	1.66	0.22	3¾	"	"	4.60	0.50	11	0.220	5	25.00	2.50
2	"	"	1.91	0.20	4	0.134	10	5.47	0.55	12	0.229	4½	28.50	2.90
2¼	"	"	2.16	0.24	4½	"	"	6.17	0.62	13	0.238	4	32.06	3.20
2½	0.109	12	2.75	0 28	5	0.148	9	7.58	0.75	14	0.248	3½	36.00	3.65
2¾	"	"	3.04	0.34	6	0.165	8	10.16	1.00	15	0.259	3	40.60	4.10
					7	"	"	11.90	1.20	16	0.270	2½	45.20	4.60

* For *pipes* give the "nominal" *inner*, for *boiler tubes* the *outer* diam. See p 526.
† For discounts, see price list.

Screw Threads, Bolts, Nuts, and Washers.

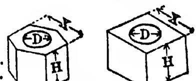


Screw threads. a = angle between two sides of a thread; P = pitch; w = width of flat top or bottom of each thread; all measured in a plane containing the axis of the screw; N = number of threads per inch, = 1/P. In the **Sellers or Franklin Institute Standard,** proposed by Mr. William Sellers and adopted by the Institute in 1864, $a = 60°$; S = P; w = P/8; F = 0.75 P; M = P cos a/2 = 0.8660 P; D (diameter) = d + 2 × 0.866 × 0.75 P = d + 1.299 P. Under the name of **United States Standard,** the U. S. Navy Department in 1868 adopted the Sellers system, except for finished heads and nuts, which it made the same as for rough heads and nuts.

D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N
¼	.185	.0062	20	1	.837	.0156	8	2	1.712	.0277	4½	4	3.567	.0413	3
5-16	.240	.0074	18	1⅛	.940	.0178	7	2¼	1.962	.0277	4½	4¼	3 798	.0435	2⅞
⅜	.294	.0078	16	1¼	1.065	.0178	7	2½	2.176	.0312	4	4½	4.028	.0454	2¾
7-16	.344	.0089	14	1⅜	1.160	.0208	6	2¾	2 426	.0312	4	4¾	4.256	.0476	2⅝
½	.400	.0096	13	1½	1.284	.0208	6	3	2.629	0357	3½	5	4.480	.0500	2½
9-16	.454	.0104	12	1⅝	1.389	.0227	5½	3¼	2.879	.0357	3½	5¼	4.730	.0500	2½
⅝	.507	.0113	11	1¾	1.491	.0250	5	3½	3.100	.0384	3¼	5½	4.953	.0526	2⅜
¾	.620	.0125	10	1⅞	1.616	.0250	5	3¾	3.317	.0413	3	5¾	5.203	.0526	2⅜
⅞	.731	.0138	9									6	5.423	.0555	2¼

Dimensions of Heads and Nuts.

	Rough.	Finished.
X	1½ D + ⅛ inch.	1½ D + 1-16 inch.
H (in *head*)	½ X.	D − 1-16 inch.
H (in *nut*)	D.	" "



Figs. 2

In the **Whitworth** (English) **standard** thread, the angle a, Fig 1, is 55°. The tops and bottoms of the threads are rounded, instead of flat as in the American standards. The number (N) of threads per inch is the same as above for diams of bolt up to three ins, except for D = ½ inch; where N = 12.

In the **International metric screw thread,** adopted at Zurich, October, 1898, the Sellers thread profile is used. The dimensions are as follows, all in millimeters:

<table>
<tr><td>Diam.</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td><td>22</td><td>24</td><td>27</td><td>30</td><td>33</td><td>36</td><td>39</td><td>42</td><td>45</td><td>48</td><td>52</td><td>56</td><td>60</td><td>64</td><td>68</td><td>72</td><td>76</td><td>80</td></tr>
<tr><td>Pitch</td><td colspan="2">1.0</td><td colspan="2">1.25</td><td colspan="2">1.5</td><td>1.75</td><td colspan="2">2 0</td><td colspan="3">2.5</td><td colspan="2">3.0</td><td colspan="2">3.5</td><td colspan="2">4.0</td><td colspan="2">4.5</td><td colspan="2">5.0</td><td colspan="2">5.5</td><td colspan="2">6.0</td><td colspan="2">6.5</td><td>7.0</td></tr>
</table>

Intermediate diameters are to be of an integral number of millimeters, and of the same pitch as the next smaller diameter in the table. Thus, for diam 65 or 69 mm; pitch = 6.0 mm.

Plate-iron washers. Standard sizes. Diameters of washers and bolt-holes in inches. Approximate thickness by Birmingham wire gauge. Approximate number in one lb.

Diams.		Ths.	No.	Diams.		Ths.	No.	Diams.		Ths.	No.
½	¼	18	450	1¼	½	14	43	2¼	15-16	9	8.6
⅝	5-16	16	210	1⅜	9-16	12	26	2½	1 1-16	9	6.2
¾	5-16	16	139	1½	⅝	12	22.5	2¾	1 ¼	9	5.2
⅞	⅜	16	112	1¾	11-16	10	13.1	3	1 ⅜	9	4.
1	7-16	14	68	2	13-16	10	10.1	3½	1 ½	9	2.8

A *square* head and nut together, weigh about as much as a length of the bolt equal to 7 or 8 times D. *Hexagon*, 6 or 7.

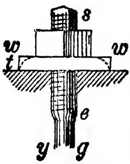


Fig 3

With the above dimensions a bolt will generally fail by breaking off between the head and the nut, where the diameter is decreased by cutting the thread, rather than by stripping off its threads. **The diam D of the thread** must of course be greater than that required to bear safely the proposed tensile strain, by an amount equal to *twice* the depth of the thread. The waste of iron, which would result from making the *entire bolt* of this greater diam, is frequently avoided by making the bolt from a bar of only sufficient dimensions to bear the strain safely, **and upsetting its ends** as in Fig 3, thus increasing their diam sufficiently to allow for the cutting of the threads.

In carpentry, as well as in ties for masonry, *washers, w w*, of either cast or wrought iron, are placed between the timber, or stone, and the head and nut in order to distribute the pressure over a greater surface, and thus prevent crushing; especially in timber.

When much strained against wood, the side of a square wrought-iron washer, or the diam *w w* of a circular one, should not be less than 4 diams of the *screw*, as in the fig; and its thickness, *t w*, ½ diam *at least*.

Two such square washers will together weigh as much as 18 diams in length of a round rod of the same diam as the screw. Two round washers will weigh together as much as 14 diams of rod of same diam as screw. In either case, a square head and nut will weigh as much as 6 diameters. Cast iron washers, being more apt to split under heavy strains, may be made about twice as thick as wrought ones. When the strain is very great, the diam of the washer may be 5 or 6 times that of the screw, and its thickness equal to diam; but 4 diams will suffice for most practical purposes, or even 2 5 when there is but little strain, and the thickness may then be but .1 or .2 diam of bolt.

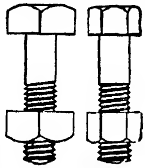
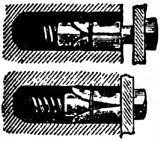


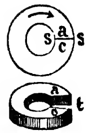
Fig. 4. Fig. 5.

Table of machine and car bolts, with square and hexagon heads and nuts, Figs 4 and 5; made by Hoopes & Townsend, 1330 Buttonwood St, Phila. All their **bolts are cut with U. S. Standard threads**, as per first table on p 1165, unless otherwise ordered. Discounts, see price list.

Diam D, Fig 1, of Bolt, ins.	Length, ins exclusive of head.		Weight, lbs of 100 bolts.		List price, $ per 100.	
	Min.	Max.	Min.	Max.	Min.	Max.
¼	1½	8	3.9	13.2	1.70	2.74
5-16	"	"	6.2	20.3	2.00	3.56
⅜	"	12	9.7	43.5	2.40	5.76
7-16	"	"	14 7	58.3	2.80	7.00
½	"	20	20.4	122.0	3.60	13.22
9-16	"	"	26	151.0	5.20	19.26
⅝	"	24	37	224.0	"	22.30
¾	"	"	58	330 0	7.20	29.70
⅞	2	"	97.7	470.0	11.20	42.00
1	"	"	145.0	625 0	16.00	55.60



Expansion bolts, for fastening plates, timbers, etc., to walls of brick or masonry. The wedge-shaped nut, traveling up the bolt, as the latter is turned, presses the wings against the sides of the hole, which, in practice, is drilled just large enough to admit the nut and wings, so as to prevent the former from turning with the bolt. If the hole is made larger, as shown, the nut must be held by a small wedge.



Lock-nut washers. When bolts are subjected to much rough jolting, as at rail-joints, &c, the *nuts are liable to wear loose, and unscrew themselves.* On railroads this is a source of great annoyance, and innumerable devices for preventing it have been tried. The **Verona** lock-nut washer* is a simple circular washer made of steel; with a slit *s s* cut through it, leaving sharp edges. On one side, *a*, of the slit, the metal is pressed upward about ⅛ inch; and that on the other side, *c*, downward, the same distance; so that a perspective view would be somewhat as at *t*. Now, when the nut is screwed down over the washer, in the direction of the arrow, the slit offers no obstruction; but if the nut afterward tends to unscrew itself, the sharp upper edge of the slit, along *a*, presents friction against the bottom of the nut, which tends to hold it in place. Besides, the washer, by its elasticity, tends to resume its original shape, and thus presses the threads of the nut against those of the bolt; and the additional friction thus produced also aids in holding the nut.

Another lock-nut washer consists of a long strip of steel, with *two* holes, each of which has its edges formed like those of a Verona washer, and through *each* of which passes one of the bolts of the rail-joint.

Another device is to cut, at the end of the screw, a few threads of a screw of less diameter than the main one, and in the opposite direction. The nut is then screwed upon the larger diameter; and after it the lock-nut is screwed in the other direction upon the smaller diam, until it comes into contact with the main nut. In the **Smith** lock-nut bolt, this second nut is only about ⅛ inch thick; and after being driven home, one of its corners is bent over the edge of the main nut.

The **Atwood** lock-nuts take advantage of elasticity in the nut itself, which is obtained either by slitting the nut, or by reducing its thickness near the bolt hole.

It is claimed that if the threads of an ordinary bolt and nut are carefully cut, so as to be in contact with each other throughout, no lock-nut contrivance is necessary, because the friction between the two threads is distributed over a larger surface, and *abrasion* does not take place so readily as if the threads touched each other at only a few points. The nuts are therefore less apt to wear loose under repeated jarring.

Owing to the difficulty of obtaining such perfect fitting bolts and nuts, due to the wear of the cutting tools used in their manufacture, bolts and nuts have been made in which the thread on the bolt differs slightly in shape from that in the nut. They also furnish nuts in which the thread, instead of being of uniform shape throughout, gradually becomes *deeper* and *thicker*, by having its side angle made more acute, and its top truncated. These nuts are used with bolts having the usual uniform thread. The bolt enters the nut upon the side where the thread is of the same shape as its own; but its thread encounters, and is forced into, the gradually narrowing and deepening path between the threads of the nut. In both devices, the enforced conformity between the two threads is relied upon to give the desired completeness of contact between them. The greater force required in screwing on the nut also increases the *friction* between the threads.

BUCKLED PLATES.

Buckled plates are usually of steel, ¼ to ½ in thick and 3 to 4 ft sq; sometimes in long plates having several buckles each. Buckle 2 to 8 ins. Flat rim or fillet, 2 to 4 ins. They are used for the floors of buildings and of highway bridges.

Total permissible load, lbs. on a single square buckled plate of any size and thickness.† Load = $4\,k\,t\,h$; where k = permissible unit stress in metal, lbs per sq in, say 6000; t = thickness of metal, ins, and h = depth of buckle, ins.

Buckled plates are stronger, and require less concrete, etc, for filling, when laid with convex side down. They weigh but little more than flat plates, or about 10 lbs per sq ft per ¼ in of thickness.

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* Invented by Mr. Thomas Shaw, M. E., of Philadelphia.

† "Steel in Construction," by Pencoyd Iron Works, Philadelphia, 1900, p 147.

WEIGHT AND STRENGTH OF IRON BOLTS. (Original.)

Diameters, weights, and approximate breaking strains, **for round bolts;** breaking strain per square inch assumed as follows: Up to 1 inch square, or 1 inch diam, 20 tons, or 44800 lbs; from 1 to 2 ins sq or diam, 19 tons; 2 to 3 ins, 18 tons; 3 to 4 ins, 17 tons; 4 to 5 ins, 16 tons; 5 to 6 ins, 15 tons.

A long upset rod is no stronger than one not upset, against *slowly applied* loads or strains. Both will then break at about midlength, under equal pulls. In such cases use columns 5 and 6.

Square bars. Strength or wt = 1.273 × strength or weight of round bar.

Copper bars. {Strength = 0.8 × strength of similar iron bar. Weight = 1.14 × weight " " " "

Ends enlarged, or upset.				Ends not enlarged.		Ends enlarged, or upset.				Ends not enlarged.	
Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.	Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain	Diam. of shank	Weight per foot run.
Ins.	Pds.	Tons.	Pds.	Ins.	Pds.	Ins.	Pds.	Tons.	Pds.	Ins.	Pds.
1/8	.0414	.245	549			1 3/4	8.10	45.7	102368	2.14	12.0
3/16	.093	.553	1239			1 13/16	8.69	49.0	109760	2.22	12.9
1/4	.165	.983	2202	.35	.321	1 7/8	9.30	52.5	117600	2.30	13.8
5/16	.258	1.53	3427	.43	.452	1 15/16	9.93	56.0	125440	2.38	14.7
3/8	.372	2.21	4950	.50	.654	2.	10.6	59.7	133728	2.45	15.7
7/16	.506	3.00	6720	.58	.897	2 1/8	12.0	63.8	142912	2.59	17.5
1/2	.661	3.93	8803	.66	1.14	2 1/4	13.4	71.6	160384	2.73	19.5
9/16	.837	4.97	11133	.73	1.41	2 3/8	14.9	79.7	178528	2.88	21.6
5/8	1.03	6.14	13754	.80	1.67	2 1/2	16.5	88.4	198016	3.02	23.9
11/16	1.25	7.42	16621	.88	2.03	2 5/8	18.2	97.4	218176	3.16	26.1
3/4	1.49	8.83	19779	.96	2.41	2 3/4	20.0	106.9	239456	3.30	28.5
13/16	1.75	10.4	23296	1.04	2.81	2 7/8	21.9	116.8	261632	3.45	31.1
7/8	2.03	12.0	26880	1.12	3.26	3.	23.8	127.2	284928	3.60	33.9
15/16	2.33	13.8	30912	1.20	3.77	3 1/4	27.9	141.0	315840	3.86	39.1
1 in.	2.65	15.7	35168	1.27	4.27	3 1/2	32.4	163.6	366464	4.12	44.4
1 1/16	2.99	16.8	37632	1.35	4.77	3 3/4	37.2	187.7	420448	4.41	51.0
1 1/8	3.35	18.9	42336	1.42	5.28	4.	42.3	213.6	478464	4.70	57.8
1 3/16	3.73	21.1	47264	1.49	5.81	4 1/4	47.8	227.0	508480	4.98	65.2
1 1/4	4.13	23.3	52192	1.55	6.39	4 1/2	53.6	254.5	570080	5.25	72.9
1 5/16	4.56	25.7	57568	1.64	7.04	4 3/4	59.7	283.5	635040	5.53	80.5
1 3/8	5.00	28.2	63168	1.72	7.74	5.	66.1	314.2	703808	5.80	88.1
1 7/16	5.47	30.8	68992	1.80	8.48	5 1/4	72.9	324.7	727328	6.08	97.0
1 1/2	5.95	33.6	75264	1.87	9.20	5 1/2	80.0	356.4	798336	6.36	106.
1 9/16	6.46	36.4	81536	1.94	9.88	5 3/4	87.5	389.5	872480	6.63	116.
1 5/8	6.99	39.4	88256	2.00	10.6	6.	95.2	424.1	949984	6.90	126.
1 11/16	7.53	42.5	95200	2.07	11.3						

The Birmingham **wire gauge** is the one in most general use for **iron**. The new British w g went into effect March 1st 1884. In the "American" w g of Darling, Brown & Sharpe, Providence R. I., each diam or thick is = the next smaller one × 1.122932. We take the wt of wrot iron per cub ft at 485 lbs in the first two; and at 486 in the last. For the **wt of steel**, mult that of iron by 1.01. For **lead**, mult iron by 1.46. For **zinc**, mult iron by .9. For **brass** (approx), mult iron by 1.06. For **copper**, mult iron by 1.134.

No.	**Birmingham** W. Ga.			**New British** W. Ga.			**American** W. Ga.		
	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets in lbs per sq ft
7-0	....	....	....	.500	.661	20.21			
6-0	....	....	....	.464	.569	18.75			
5-0	....	....	....	.432	.494	17.46			
4-0	.454	.546	18.35	.400	.423	16.17	.460000	.561	18.63
3-0	.425	.479	17.18	.372	.366	15.03	.409642	.445	16.58
2-0	.380	.383	15.36	.348	.320	14.06	.364796	.353	14.77
0	.340	.306	13.74	.324	.278	13.09	.324861	.280	13.15
1	.300	.238	12.13	.300	.238	12.13	.289297	.222	11.70
2	.284	.214	11.48	.276	.202	11.15	.257627	.176	10.43
3	.259	.178	10.47	.252	.168	10.19	.229423	.139	9.291
4	.238	.150	9.619	.232	.142	9.377	.204307	.111	8.273
5	.220	.128	8.892	.212	.119	8.568	.181940	.0877	7.366
6	.203	.109	8.205	.192	.0976	7.760	.162023	.0696	6.561
7	.180	.0859	7.275	.176	.0820	7.113	.144285	.0552	5.842
8	.165	.0721	6.669	.160	.0677	6.466	.128490	.0438	5.203
9	.148	.0580	5.981	.144	.0548	5.820	.114423	.0347	4.633
10	.134	.0476	5.416	.128	.0434	5.173	.101897	.0275	4.125
11	.120	.0382	4.850	.116	.0357	4.688	.090742	.0218	3.674
12	.109	.0315	4.405	.104	.0286	4.203	.080808	.0173	3.272
13	.095	.0239	3.840	.092	.0224	3.718	.071962	.0137	2.914
14	.083	.0183	3.355	.080	.0169	3.233	.064084	.0109	2.595
15	.072	.0137	2.910	.072	.0137	2.910	.057068	.00863	2.310
16	.065	.0112	2.627	.064	.0108	2.587	.050821	.00684	2.053
17	.058	.00891	2.344	.056	.00832	2.263	.045257	.00543	1.832
18	.049	.00636	1.980	.048	.00610	1.940	.040303	.00430	1.631
19	.042	.00467	1.697	.040	.00423	1.617	.035890	.00341	1.452
20	.035	.00325	1.415	.036	.00344	1.455	.031961	.00271	1.293
21	.032	.00271	1.293	.032	.00269	1.293	.028462	.00215	1.152
22	.028	.00208	1.132	.028	.00207	1.132	.025346	.00170	1.026
23	.025	.00166	1.010	.024	.00152	.9700	.022572	.00135	.913
24	.022	.00128	.8892	.022	.00128	.8891	.020101	.00107	.814
25	.020	.00106	.8083	.020	.00106	.8083	.017900	.000849	.724
26	.018	.000859	.7225	.018	.000857	.7275	.015941	.000673	.644
27	.016	.000678	.6467	.0164	.000712	.6628	.014195	.000534	.574
28	.014	.000519	.5658	.0148	.000579	.5982	.012641	.000423	.511
29	.013	.000448	.5254	.0136	.000489	.5497	.011257	.000336	.455
30	.012	.000382	.4850	.0124	.000408	.5012	.010025	.000266	.405
31	.010	.000265	.4042	.0116	.000357	.4688	.008928	.000211	.360
32	.009	.000215	.3638	.0108	.000309	.4365	.007950	.000167	.321
33	.008	.000170	.3233	.0100	.000265	.4042	.007080	.000133	.286
34	.007	.000130	.2829	.0092	.000224	.3718	.006305	.000105	.254
35	.005	.0000662	.2021	.0084	.000187	.3395	.005615	.0000837	.226
36	.004	.0000424	.1617	.0076	.000153	.3072	.005000	.0000662	.202
37	....	....	....	.0068	.000122	.2748	.004453	.0000525	.180
38	....	....	....	.0060	.0000952	.2425	.003965	.0000417	.160
39	....	....	....	.0052	.0000714	.2102	.003531	.0000330	142
40	....	....	....	.0048	.0000608	.1940	.003144	.0000262	.127
41	....	....	....	.0044	.0000513	.1778			
42	....	....	....	.0040	.0000428	.1617			
43	....	....	....	.0036	.0000344	.1455			
44	....	....	....	.0032	.0000271	.1293			
45	....	....	....	.0028	.0000207	.1132			
46	....	....	....	.0024	.0000152	.0970			
47	....	....	....	.0020	.0000106	.0808			
48	....	....	....	.0016	.0000068	.0647			

American gauge for sheet and plate iron and steel (**1893**). We omit *the columns of weight in kilograms per square foot and in pounds per square meter, and simplify the headings of the remaining columns.*

An Act establishing a standard gauge for sheet and plate iron and steel.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That for the purpose of securing uniformity the following is established as the only standard gauge for sheet and plate iron and steel in the United States of America, namely.

No.	Approximate thickness: Inches.	Approximate thickness: Millimeters.	Weight: Per sq. foot, in avoirdupois ounces.	Weight: Per sq. foot, in avoirdupois pounds.	Weight: Per sq meter, in kilograms
7-0	1-2 =.5	12.7	320	20.00	97 65
6-0	15-32 =.46875	11.90625	300	18.75	91.55
5-0	7-16 = 4375	11.1125	280	17.50	85.44
4-0	13-32 =.40625	10.31875	260	16.25	79 33
3-0	3-8 =.375	9.525	240	15.	73 24
2-0	11-32 =.34375	8.73125	220	13 75	67 13
0	5-16 =.3125	7 9375	200	12.50	61 03
1	9-32 = 28125	7 14375	180	11.25	54 93
2	17-64 =.265625	6.746875	170	10 625	51 88
3	1-4 =.25	6.35	160	10	48 82
4	15-64 =.234375	5 953125	150	9 375	45 77
5	7-32 =.21875	5 55625	140	8 75	42 72
6	13-64 =.203125	5.159375	130	8.125	39 67
7	3-16 =.1875	4.7625	120	7.5	36 62
8	11-64 =.171875	4.365625	110	6.875	33 57
9	5-32 =.15625	3.96875	100	6.25	30.52
10	9-64 =.140625	3.571875	90	5.625	27.46
11	1-8 =.125	3.175	80	5.	24.41
12	7-64 =.109375	2.778125	70	4.375	21 36
13	3-32 =.09375	2.38125	60	3.75	18.31
14	5-64 =.078125	1.984375	50	3.125	15 26
15	9-128 =.0703125	1.7859375	45	2.8125	13 73
16	1-16 = 0625	1.5875	40	2.5	12.21
17	9-160 =.05625	1.42875	36	2.25	10 99
18	1-20 =.05	1.27	32	2.	9.765
19	7-160 =.04375	1.11125	28	1.75	8.544
20	3-80 =.0375	.9525	24	1.50	7.324
21	11-320 =.034375	.873125	22	1 375	6.713
22	1-32 =.03125	.793750	20	1 25	6.103
23	9-320 =.028125	.714375	18	1 125	5.493
24	1-40 =.025	.635	16	1.	4.882
25	7-320 = 021875	.555625	14	875	4 272
26	3-160 =.01875	.47625	12	.75	3 662
27	11-640 =.0171875	.4365625	11	.6875	3.357
28	1-64 =.015625	.396875	10	.625	3 052
29	9-640 =.0140625	.3571875	9	.5625	2.746
30	1-80 =.0125	.3175	8	.5	2.441
31	7-640 =.0109375	.2778125	7	.4375	2.136
32	13-1280=.01015625	.25796875	6½	.40625	1.983
33	3-320 =.009375	.238125	6	.375	1.831
34	11-1280 =.00859375	.21828125	5½	.34375	1.678
35	5-640 =.0078125	.1984375	5	.3125	1.526
36	9-1280=.00703125	.17859375	4½	.28125	1 373
37	17-2560=.006640625	.168671875	4¼	.265625	1.297
38	1-160 =.00625	.15875	4	25	1.221

And on and after July first, eighteen hundred and ninety-three, the same and no other shall be used in determining duties and taxes levied by the United States of America on sheet and plate iron and steel. But this act shall not be construed to increase duties upon any articles which may be imported.

SEC. 2. That the Secretary of the Treasury is authorized and required to prepare suitable standards in accordance herewith.

SEC. 3. That in the practical use and application of the standard gauge hereby established a variation of two and one-half per cent. either way may be allowed.

Approved March 3, 1893.

CIRCULAR MEASURE.

Used in comparing cross sections of wires, etc.

A circular unit is the area of a circle whose diameter is one linear unit. Thus, a circular inch is the area (= 0.7854 square inch) of a circle whose diameter is one inch.

The following table is adapted, by permission, from Mr. Carl Hering's valuable Tables of Equivalents of Units of Measurement, New York, 1888. Inasmuch as we take 1 meter = 39.37 inches, instead of 39.37079 inches, our values differ slightly from his.

			Logarithm.
1 ○ mil*	=	0.78540 □ mil*	$\bar{1}$.895 0899
	=	0.00064516 ○ millimeter	$\bar{4}$.809 6692
	=	0.00050671 □ millimeter	$\bar{4}$.704 7591
1 □ mil*	=	1.2732 ○ mils*	0.104 9101
	=	0.00082145 ○ millimeter	$\bar{4}$.914 5793
1 ○ millimeter	=	1550.0 ○ mils*	3.190 3308
	=	1217.4 □ mils*	3.085 4207
	=	0.78540 □ millimeter	$\bar{1}$.895 0899
1 □ millimeter	=	1973.5 ○ mils*	3.295 2409
	=	1.2732 ○ millimeters	0.104 9101

EDISON STANDARD WIRE GAUGE.

Adopted by the Associated Edison Illuminating Companies.

In this table the gauge number is approximately equal to

$$\frac{1}{1000} \times \text{area of cross section in circular mils*}$$
$$= \frac{1}{1000} \times \text{square of diameter in mils*}$$

No.	Diameter, in mils.	No.	Diameter, in mils.	No.	Diameter, in mils.
3	54.78	65	254.96	160	400.00
5	70.72	70	264.58	170	412.32
8	89.45	75	273.87	180	424.27
12	109.55	80	282.85	190	435.89
15	122.48	85	291.55	200	447.22
20	141.43	90	300.00	220	469.05
25	158.12	95	308.23	240	489.90
30	173.21	100	316.23	260	509.91
35	187.09	110	331.67	280	529.16
40	200.00	120	346.42	300	547.73
45	212.14	130	360.56	320	565.69
50	223.61	140	374.17	340	583.10
55	234.53	150	387.30	360	600.00
60	244.95				

* 1 mil = $\frac{1}{1000}$ inch.

No trade Stupidity is more thoroughly senseless than the adherence to the various Birmingham, Lancashire, &c, gauges; instead of at once denoting the thickness and diameter of sheets, wire, &c, by the parts of an inch; as has long been suggested. Thus, No. $\frac{1}{8}$, or No. $\frac{1}{20}$ wire, or sheet-metal of any kind, should be understood to mean $\frac{1}{8}$ or $\frac{1}{20}$ of an inch diam, or thickness. To avoid mistakes, which are very apt to occur from the number of gauges in use; and from the absurd practice of applying the same No. to different thicknesses of different metals, in different towns, it is best to ignore them all; and in giving orders, to define the diameter of wire, and the thickness of sheet-metal, by parts of an inch. Or the weight per hundred ft for wire; or per sq ft for sheets, may be employed. We *believe* that the *foregoing* Birmingham gauge applies to zinc, copper, brass, and lead; although it is generally stated to be for iron and steel only. Another Birmingham gauge is used for sheet-brass, gold, silver, and some other metals; but we have never seen it stated what those others are. There are different gauges even for wire to be used for different *purposes;* and various firms have gauges of their own; not even according among themselves.

As Mr. Stubs makes various English gauges, the term "**Stubs gauge**" *by itself* means nothing. Generally, however, in our machine shops, it applies to the Birmingham gauge of the preceding table.

Birmingham gauge for sheet Brass, Silver, Gold, and all metals except iron and steel?

No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No	Thickn's	No.	Thickn's.	No.	Thickn's.
	Inch		Inch		Inch		Inch		Inch		Inch
1	.004	7	.015	13	.036	19	.064	25	.095	31	.135
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.012	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

The mills rolling sheet iron in the United States generally use the following, which varies slightly from the Birmingham gauge:

No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft
1	12.50	8	6.86	15	2.81	22	1.25
2	12.00	9	6.24	16	2.50	23	1.12
3	11.00	10	5.62	17	2.18	24	1.00
4	10.00	11	5.00	18	1.86	25	.90
5	8.75	12	4.38	19	1.70	26	.80
6	8.12	13	3.75	20	1.54	27	.72
7	7.50	14	3.12	21	1.40	28	.64

When wire, sheet-metal, &c., are ordered by gauge number, and it is not specified what gauge is intended; dealers in the United States fill the order as follows:

Brass, bronze or German Silver in sheets, German Silver wire, brazed brass, bronze, zinc or copper tubing, by Brown & Sharpe's (or "American") gauge.

Copper in sheets; brass and copper wire; seamless brass, bronze or copper tubing; and small brass rods; by Stubs' (or Birmingham) gauge.

Unannealed or hard **brass wire** has about $\frac{3}{4}$ths the strengths of the table p. 1173, and about $\frac{1}{6}$ more weight. If annealed, only full half the strength.

Hard copper wire may be taken at $\frac{2}{3}$ of the tabular strengths, and full $\frac{1}{4}$ more weight.

Table of Charcoal Iron Wire made by Trenton Iron Co., Trenton, N. J. The numbers in the first column are those of the **Trenton Iron Co's gauge.** The corresponding diameters in the second column will be seen to be somewhat less than those of the Birmingham gauge.

No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.
00000	.450	1.863	12598	11	.1175	27.340	1010	26	.018	1164.689
0000	.400	2.358	9955	12	105	34.219	810	27	.017	1305.670
000	.360	2.911	8124	13	.0925	44.092	631	28	.016	1476.869
00	.330	3.465	6880	14	.080	58.916	474	29	.015	1676.989
0	.305	4.057	5926	15	.070	76.984	372	30	.014	1925.821
1	.285	4.645	5226	16	.061	101.488	292	31	.018	2282.658
2	.265	5.374	4570	17	.0525	137.174	222	32	.012	2620.607
3	.245	6.286	3948	18	.045	186.335	169	33	.011	3119.092
4	.225	7.454	3374	19	.040	235.084	137	34	.010	3778.584
5	.205	8.976	2839	20	.035	308.079	107	35	.0095	4182.508
6	.190	10.453	2476	21	.031	392.772	......	36	.009	4657.728
7	.175	12.322	2136	22	.028	481.234	......	37	.0085	5222.035
8	.160	14.736	1813	23	.025	603 863	......	38	.008	5896.147
9	.145	17.950	1507	24	.0225	745.710	......	39	.0075	6724.291
10	.130	22.833	1233	25	.020	943.396	......	40	.007	7698.258

The wire in this table is supposed to be hard, bright, or unannealed. The figures in the column of tensile strength are based upon tests made with good charcoal iron wire from Trenton blooms.

The tensile strength of wire made of	is about		
Good refined iron..............	15 per cent. less		than that of bright charcoal wire, given in the above table.
Swedish charcoal iron................	10 " "		
Mild Bessemer steel....................	10 " more		
Ordinary crucible steel..................................	25 " "		
Special crucible steel....................................	30 to 120 " "		

Annealing renders wire more pliable and ductile, but less elastic; and reduces the tensile strength by from 20 to 25 per cent.

To find approximately the number of straight wires that can be got into a cable of given diameter.

Divide the diameter of the cable in inches, by the diameter of a wire in inches. Square the quotient. Multiply said square by the decimal .77. The result will be correct within about 4 or 5 per cent at most, in a *cylindrical* cable.

The solidity, or metal area of all the wires in a cable, will be to the area of the cable itself, about as 1 to 1.3. In other words, the area of the voids is nearly ¼ that of the cable; while that of the wires is fully ¾ that of the cable. All approximate.

No trade Stupidity is more thoroughly senseless than the adherence to the various Birmingham, Lancashire, &c, gauges; instead of at once denoting the thickness and diameter of sheets, wire, &c, by the parts of an inch; as has long been suggested. Thus, No. $\frac{1}{8}$, or No. $\frac{1}{20}$ wire, or sheet-metal of any kind, should be understood to mean $\frac{1}{8}$ or $\frac{1}{20}$ of an inch diam, or thickness. To avoid mistakes, which are very apt to occur from the number of gauges in use; and from the absurd practice of applying the same No. to different thicknesses of different metals, in different towns, it is best to ignore them all; and in giving orders, to define the diameter of wire, and the thickness of sheet-metal, by parts of an inch. Or the weight per hundred ft for wire; or per sq ft for sheets, may be employed. We *believe* that the *foregoing* Birmingham gauge applies to zinc, copper, brass, and lead; although it is generally stated to be for iron and steel only. Another Birmingham gauge is used for sheet-brass, gold, silver, and some other metals; but we have never seen it stated what those others are. There are different gauges even for wire to be used for different *purposes;* and various firms have gauges of their own; not even according among themselves.

As Mr. Stubs makes various English gauges, the term "**Stubs gauge**" *by itself* means nothing. Generally, however, in our machine shops, it applies to the Birmingham gauge of the preceding table.

Birmingham gauge for sheet Brass, Silver, Gold, and all metals except iron and steel?

No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No	Thickn's	No.	Thickn's.	No.	Thickn's.
	Inch		Inch		Inch		Inch		Inch		Inch
1	.004	7	.015	13	.036	19	.064	25	.095	31	.135
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.012	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

The mills rolling sheet iron in the United States generally use the following, which varies slightly from the Birmingham gauge:

No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft
1	12.50	8	6.86	15	2.81	22	1.25
2	12.00	9	6.24	16	2.50	23	1.12
3	11.00	10	5.62	17	2.18	24	1.00
4	10.00	11	5.00	18	1.86	25	.90
5	8.75	12	4.38	19	1.70	26	.80
6	8.12	13	3.75	20	1.54	27	.72
7	7.50	14	3.12	21	1.40	28	.64

When wire, sheet-metal, &c., are ordered by gauge number, and it is not specified what gauge is intended; dealers in the United States fill the order as follows:

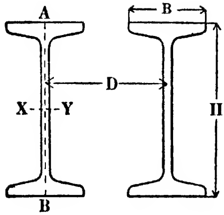
Brass, bronze or German Silver in sheets, German Silver wire, brazed brass, bronze, zinc or copper tubing, by Brown & Sharpe's (or "American") gauge.

Copper in sheets; brass and copper wire; seamless brass, bronze or copper tubing; and small brass rods; by Stubs' (or Birmingham) gauge.

Unannealed or hard **brass wire** has about $\frac{3}{4}$ths the strengths of the table p. 1173, and about $\frac{1}{9}$ more weight. If annealed, only full half the strength.

Hard copper wire may be taken at $\frac{2}{3}$ of the tabular strengths, and full $\frac{1}{7}$ more weight.

I-BEAMS.



The table gives the maximum and the minimum weight of each section. The minimum weights are standard. Others are special.

Caution.—With very short spans, the loads found by means of columns C_s and C_m, although safe against *bending*, may be so great as to endanger a *crushing* of the ends of the beam, or of the walls, etc., under them, unless the beam has, at its ends, a greater length of bearing than would otherwise be needed.

I	i	R ins.	r ins.	X	C_s lbs.	C_m lbs.	D ins.	Section index.
2380.3	48.56	9.00	1.28	198.4	2,115,800	1,653,000	17.82	B 1
2087.9	42.86	9.46	1.36	174.0	1,855,900	1,449,900	18.72	"
1655.8	52.65	7.50	1.34	165.6	1,766,100	1,379,800	14.76	B 2
1466.5	45.81	7.86	1.39	146.7	1,564,300	1,222,100	15.47	"
1268.9	30.25	7.58	1.17	126.9	1,353,500	1,057,400	14.98	B 3
1169.6	27.86	7.83	1.21	117.0	1,247,600	974,700	15.47	"
921.3	24.62	6.69	1.09	102.4	1,091,900	853,000	13.20	B 80
795.6	21.19	7.07	1.15	88.4	943,000	736,700	13.95	"
900.5	50.98	5.53	1.31	120.1	1,280,700	1,000,600	10.75	B 4
795.5	41.76	5.78	1.32	106.1	1,131,300	883,900	11.25	"
691.2	36.68	5.60	1.18	92.2	983,000	768,000	10.95	B 5
609.0	25.96	5.87	1.21	81.2	866,100	676,600	11.49	"
511.0	17.06	5.62	1.02	68.1	726,800	567,800	11.05	B 7
441.7	11.62	5.95	1.08	58.9	628,300	490,800	11.70	"
321.0	17.46	4.45	1.04	53.5	570,600	445,800	8.65	B 8
268.9	13.81	4.77	1.08	44.8	478,100	373,500	9.29	"
228.3	10.07	4.71	0.99	38.0	405,800	317,000	9.21	B 9
215.8	9.50	4.83	1.01	36.0	383,700	299,700	9.45	"
158.7	9.50	3.67	0.90	31.7	333,500	264,500	7.12	B 11
122.1	6.89	4.07	0.97	24.4	260,500	203,500	7.91	"
111.8	7.31	3.29	0.84	24.8	265,000	207,000	6.36	B 13
84.9	5.16	3.67	0.90	18.9	201,300	157,300	7.12	"
68.4	4.75	3.02	0.80	17.1	182,500	142,600	5.82	B 15
56.9	3.78	3.27	0.84	14.2	151,700	118,500	6.32	"
42.2	3.24	2.68	0.74	12.1	128,600	100,400	5.15	B 17
36.2	2.67	2.86	0.78	10.4	110,400	86,300	5.50	"
26.2	2.36	2.27	0.68	8.7	93,100	72,800	4.33	B 19
21.8	1.85	2.46	0.72	7.3	77,500	60,500	4.70	"
15.2	1.70	1.87	0.63	6.1	64,600	50,500	....	B 21
12.1	1.23	2.05	0.65	4.8	51,600	40,300	....	"
7.1	1.01	1.52	0.57	3.6	38,100	29,800	....	B 23
6.0	0.77	1.64	0.59	3.0	31,800	24,900	....	"
2.9	0.60	1.15	0.52	1.9	20,700	16,200	....	B 77
2.5	0.46	1.23	0.53	1.7	17,600	13,800	....	"

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	Inch		Inch		Inch		Inch		Inch		Inch
1	.004	7	.015	13	.036	19	.064	25	.095	31	.138
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
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4	10.00	11	5.00	18	1.86	25	.90
5	8.75	12	4.38	19	1.70	26	.80
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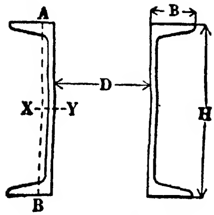
CHANNELS.



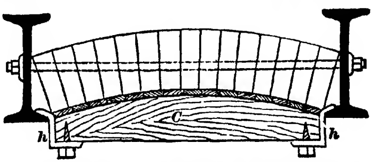
The table gives the maximum and the minimum weight of each section The minimum weights are standard Others are special.

Caution.—With very short spans, the loads found by means of columns C_s and C_m, although safe against *bending*, may be so great as to endanger a *crushing* of the ends of the beam, or of the walls, etc., under them, unless the beam has, at its ends, a greater length of bearing than would otherwise be needed.

I	i	R ins.	r ins.	X	C_s lbs.	C_m lbs.	D ins.	Section index.
430.2	12.19	5.16	0.868	57.4	611,900	478,000	8.53	C 1
312.6	8.23	5.62	0.912	41.7	444,500	347,300	9.50	"
197.0	6.63	4.09	0.751	32.8	350,200	273,600	6.60	C 2
128.1	3.91	4.61	0.805	21.4	227,800	178,000	7.67	"
115.5	4.66	3.35	0.672	23.1	246,400	192,500	5.17	C 3
66.9	2.30	3.87	0.718	13.4	142,700	111,500	6.33	"
70.7	2.98	3.10	0.637	15.7	167,600	130,900	4.84	C 4
47.3	1.77	3.49	0.674	10.5	112,200	87,600	5.63	"
47.8	2.25	2.77	0.600	11.9	127,400	99,500	4.23	C 5
32.3	1.33	3.11	0.630	8.1	86,100	67,300	4.94	"
33.2	1.85	2.39	0.565	9.5	101,100	79,000	3.48	C 6
21.1	0.98	2.72	0.586	6.0	66,800	52,200	4.22	"
19.5	1.28	2.07	0.529	6.5	69,500	54,300	2.91	C 7
13.0	0.70	2.34	0.542	4.3	46,200	36,100	3.52	"
10.4	0.82	1.75	0.493	4.2	44,400	34,700	2.34	C 8
7.4	0.48	1.95	0.498	3.0	31,600	24,700	2.79	"
4.6	0.44	1.46	0.455	2.3	24,400	19,000	1.85	C 9
3.8	0.32	1.56	0.453	1.9	20,200	15,800	2.06	"
2.1	0.31	1.08	0.421	1.4	14,700	11,500	1.07	C 72
1.6	0.20	1.17	0.409	1.1	11,600	9,100	1.31	"

The weight of a 4-inch arch, with its concrete filling and wooden flooring, but exclusive of the beams, is about 70 lbs. per square foot of floor.

A dense crowd of persons will hardly weigh more than 80 lbs. per square foot.



Each center, *C*, has fastened to it at each end a bent iron strap, *h*, forming a hook by which the center is suspended from the lower flanges of the beams.

CARNEGIE ANGLES

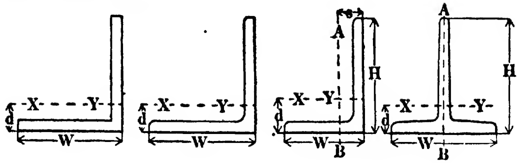
d = distance between center of gravity and back of flange W
s = " " " " " " " " H
I, i = moment of inertia; **I**, about XY; **i**, about A B
X, x = least "section modulus"; **X**, " " **x**, " " $= \frac{12\ M}{S}$†
R, R' = radius of gyration; **R**, " " **R'**, " "
r = least radius of gyration, about neutral axis forming acute angle a with each flange. In angles with equal legs, $a = 45°$

C = coefficient for uniformly distributed safe load: C_s, for static loads; fibre stress = 16,000 lbs. C_m, for moving loads; fibre stress = 12,000 lbs. } For T shapes only.

Section index.	Size H × W ins. ins.	Thickness ins.	Weight per ft lbs.	Area of section sq ins.	d ins.	s ins.	I	i
Angles with Unequal Legs.								
* A 150	7 × 3½	1	32.3	9 50	2.71	0 96	45.37	7.53
* A 159	7 × 3½	7/16	15.0	4 40	2.50	0.75	22.56	3.95
A 89	6 × 4	1	30 6	9.00	2 17	1.17	30 75	10.75
A 168	6 × 4	3/8	12.3	3.61	1.94	0.94	13.47	4.90
A 92	6 × 3½	1	28 9	8 50	2 26	1.01	29.24	7.21
A 177	6 × 3½	3/8	11 7	3.42	2.04	0.79	12 86	3.34
* A 178	5 × 4	7/8	24 2	7 11	1.71	1.21	16 42	9.23
* A 186	5 × 4	3/8	11.0	3.23	1.53	1.03	8.14	4.67
A 187	5 × 3½	7/8	22.7	6.67	1.79	1.04	15.67	6 21
A 96	5 × 3½	5/16	8.7	2.56	1.59	0.84	6 60	2.72
A 196	5 × 3	13/16	19.9	5 84	1 86	0 86	13.98	3 71
A 280	5 × 3	5/16	8.2	2.40	1.68	0.68	6.26	1.75
* A 204	4½ × 3	13/16	18 5	5 43	1.65	0 90	10.33	3.60
* A 97	4½ × 3	5/16	7.7	2 25	1.47	0.72	4.69	1.73
* A 212	4 × 3½	13/16	18 5	5.43	1.36	1.11	7.77	5 49
* A 98	4 × 3½	5/16	7.7	2.25	1.18	0.93	3.56	2.59
A 220	4 × 3	13/16	17 1	5.03	1.44	0.94	7.34	3.47
A 228	4 × 3	5/16	7.1	2.09	1.26	0.76	3.38	1.65
A 229	3½ × 3	13/16	15 7	4.62	1 23	0.98	4.98	3.33
A 237	3½ × 3	5/16	6.6	1.93	1.06	0.81	2.83	1.58
A 238	3½ × 2½	11/16	12 4	3.65	1 27	0 77	4 13	1.72
A 245	3½ × 2½	1/4	4.9	1.44	1.11	0.61	1.80	0.78
* A 246	3¼ × 2	9/16	9 0	2.64	1.21	0 59	2.64	0.75
* A 251	3¼ × 2	1/4	4.3	1.25	1.09	0.48	1.36	0.40
A 252	3 × 2½	9/16	9.5	2.78	1.02	0.77	2 28	1.42
A 257	3 × 2½	1/4	4.5	1.31	0.91	0.66	1.17	0.74
* A 258	3 × 2	1/2	7.7	2.25	1.08	0.58	1.92	0.67
* A 262	3 × 2	1/4	4.0	1.19	0.99	0.49	1.09	0.39
A 264	2½ × 2	1/2	6.8	2.00	0.88	0.63	1.14	0.64
A 269	2½ × 2	3/16	2.8	0.81	0.76	0.51	0.51	0.29
* A 270	2¼ × 1½	1/2	5.5	1.63	0 86	0.48	0.82	0.26
* A 275	2¼ × 1½	3/16	2.3	0.67	0.75	0.37	0.34	0.12
* A 276	2 × 1⅜	1/4	2.7	0.78	0 69	0.37	0.37	0.12
* A 277	2 × 1⅜	3/16	2.1	0.60	0.66	0.35	0.24	0.09
* A 278	1⅜ × 1	1/4	1.8	0.53	0.48	0.29	0.09	0.04
* A 279	1⅜ × 1	1/8	1.0	0.28	0.44	0.26	0.05	0.02

* Special sections. † For M and S see p. 1174 or p. 1176.

AND T SHAPES.



X	x	R	R′	r	C_s	C_m	Section Index.
Maximum and Minimum Weight of each Section.							
10.58	2.96	2.19	0.89	0.88	....	....	A 150*
5.01	1.47	2.26	0.95	0.89	....	....	A 159*
8.02	3.79	1.85	1.09	0.85	....	....	A 89
3.32	1.60	1.93	1.17	0.88	....	....	A 168
7.83	2.90	1.85	0 92	0.74	....	....	A 92
3.25	1.23	1.94	0.99	0 77	....	....	A 177
4.99	3.31	1 52	1 14	0.84	....	....	A 178*
2.34	1.57	1.59	1.20	0.86	....	....	A 186*
4.88	2 52	1.53	0.96	0 75	....	....	A 187
1.94	1.02	1.61	1.03	0 76	....	....	A 96
4.45	1 74	1.55	0.80	0.64	....	....	A 196
1.89	0.75	1.61	0 85	0.66	....	....	A 280
3.62	1.71	1.38	0.81	0.64	....	....	A 204*
1.54	0.76	1.44	0.88	0.66	....	....	A 97*
2.92	2.30	1 19	1.01	0.72	....	....	A 212*
1.26	1.01	1.26	1.07	0.73	....	....	A 98*
2.87	1 68	1 21	0 83	0 64	....	....	A 220
1.23	0.74	1.27	0.89	0.65	....	....	A 228
2.20	1.65	1.04	0.85	0.62	....	....	A 229
0.96	0.72	1.10	0.90	0.63	....	....	A 237
1.85	0 99	1.06	0 67	0.53	....	....	A 238
0.75	0.41	1.12	0.74	0.54	....	....	A 245
1.30	0.53	1.00	0.53	0 44	....	....	A 246*
0.63	0.26	1.04	0.57	0.45	....	....	A 251*
1.15	0.82	0.91	0.72	0.52	....	....	A 252
0.56	0.40	0.95	0.75	0.53	....	....	A 257
1.00	0.47	0.92	0.55	0.43	....	....	A 258*
0.54	0.25	0 95	0.57	0.43	....	....	A 262*
0.70	0.46	0.75	0.56	0.42	....	....	A 264
0.29	0.20	0.79	0.60	0.43	....	....	A 269
0.59	0 26	0 71	0.40	0.39	....	....	A 270*
0.23	0.11	0.72	0.43	0.40	....	....	A 275*
0.23	0.12	0.63	0.39	0.30	....	....	A 276*
0.18	0.09	0.63	0.40	0.31	....	....	A 277*
0.09	0 05	0.41	0.27	0.22	....	....	A 278*
0.06	0.03	0.44	0.29	0.22	....	....	A 279*

* Special sections.

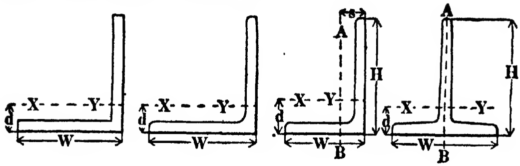
CARNEGIE ANGLES

d = distance between center of gravity and back of flange W
s = " " " " " " " H
I, i = moment of inertia; **I,** about XY; **i,** about A B
X, x = least "section modulus"; **X,** " " **x,** " " = $\frac{12\ M}{S}$ †
R, R′ = radius of gyration; **R,** " " **R′,** " "
r = least radius of gyration, about neutral axis forming acute angle *a* with each flange. In angles with equal legs, $a = 45°$
C = coefficient for uniformly distributed safe load: C_s, for static loads; fibre stress = 16,000 lbs.; C_m, for moving loads; fibre stress = 12,000 lbs. — For T shapes only.

Section index.	Size H W ins. ins.	Thick-nesss ins.	Weight per ft lbs.	Area of section sq ins.	d ins.	s ins.	I	i
Angles with Equal Legs.								
A 113	8 × 8	1⅛	56.9	16.73	2.41	....	97.97	....
A 103	8 × 8	½	26.4	7.75	2.19	....	48.63	....
A 86	6 × 6	1	37.4	11.00	1.86	....	35.46	....
A 88	6 × 6	⅜	14.8	4.36	1.64	....	15.39	....
*A 94	5 × 5	1	30.6	9 00	1.61	....	19.64	....
*A 17	5 × 5	⅜	12.3	3.61	1.39	....	8.74	....
A 18	4 × 4	13/16	19.9	5.84	1.29	....	8.14	....
A 90	4 × 4	5/16	8.2	2.40	1.12	....	3.71	....
A 26	3½ × 3½	13/16	17.1	5.03	1.17	....	5.25	....
A 99	3½ × 3½	5/16	7.1	2 09	0.99	....	2.45	....
A 34	3 × 3	⅝	11.4	3.36	0.98	....	2.62	....
A 40	3 × 3	¼	4.9	1.44	0.84	....	1.24	....
*A 41	2¾ × 2¾	½	8.5	2.50	0.87	....	1.67	....
*A 45	2¾ × 2¾	¼	4.5	1.31	0.78	....	0 93	....
A 46	2½ × 2½	½	7.7	2.25	0.81	....	1.23	....
A 100	2½ × 2½	3/16	3.1	0.90	0 69	....	0.55	....
*A 51	2¼ × 2¼	½	6.8	2.00	0 74	....	0.87	....
*A 101	2¼ × 2¼	3/16	2.8	0.81	0.63	....	0.39	....
A 56	2 × 2	7/16	5.3	1.56	0.66	....	0.54	....
A 60	2 × 2	3/16	2.5	0.72	0.57	....	0.28	....
A 61	1¾ × 1¾	7/16	4.6	1.30	0.59	....	0.35	....
A 65	1¾ × 1¾	3/16	2.1	0.62	0.51	....	0.18	....
A 66	1½ × 1½	⅜	3.4	0.99	0.51	....	0.19	....
A 102	1½ × 1½	⅛	1.2	0.36	0.42	....	0.08	....
A 70	1¼ × 1¼	5/16	2.4	0.69	0.42	....	0.09	....
A 73	1¼ × 1¼	⅛	1.0	0.30	0.35	....	0.044	....
A 78	1 × 1	¼	1.5	0.44	0.34	....	0.037	....
A 80	1 × 1	⅛	0 8	0.24	0.30	....	0.022	....
*A 81	⅞ × ⅞	3/16	1.0	0.29	0.29	....	0.019	....
A 82	⅞ × ⅞	⅛	0.7	0.21	0.26	....	0.014	....
A 83	¾ × ¾	3/16	0.8	0.25	0.26	....	0.012	....
A 84	¾ × ¾	⅛	0.6	0.17	0.23	....	0.009	....
T Shapes.								
T 50	5 × 3	....	13.6	3.99	0.75	....	2.6	5.6
T 57	4 × 5	....	15.6	4 56	1.56	....	10.7	2.8
T 61	4 × 3	....	9.3	2.73	0.78	....	2.0	2.1
T 8	3½ × 3½	....	11.7	3.45	1.06	....	3.7	1.89
T 72	3 × 4	....	11.8	3.48	1.32	....	5.2	1.21
T 77	3 × 3½	....	8.5	2.49	1.09	....	2.9	0.93
T 82	2½ × 3	....	7.2	2.10	0.97	....	1.8	0.54
T 12	2¼ × 2¼	....	4.9	1.44	0.69	....	0.66	0.38
T 16	1¾ × 1¾	....	3.1	0.90	0.54	....	0.23	0.12

*Special sections. †For M and S see p. 1174 or p. 1176.

AND T SHAPES.—Continued.



X	x	R	R'	r	C_s	C_m	Section Index.
Maximum and Minimum Weight of each Section.							
17.53	....	2.42	....	1.55	....	....	A 113
8.37	....	2.50	....	1.58	....	....	A 103
8.57	....	1.80	....	1.16	....	....	A 86
3.53	....	1.88	....	1.19	....	....	A 88
5.80	....	1.48	....	0.96	....	....	A 94*
2.42	....	1.56	....	0.99	....	....	A 17*
3.01	....	1.18	....	0.77	....	....	A 18
1.29	....	1.24	....	0.79	....	....	A 90
2.25	....	1.02	....	0.67	....	....	A 26
0.98	....	1.08	....	0.69	....	....	A 99
1.30	....	0.88	....	0.57	....	....	A 34
0.58	....	0.93	....	0.59	....	....	A 40
0.89	....	0.82	....	0.52	....	....	A 41*
0.48	....	0.85	....	0.55	....	....	A 45*
0.73	....	0.74	....	0.47	....	....	A 46
0.30	....	0.78	....	0.49	....	....	A 100
0.58	....	0.66	....	0.43	....	....	A 51*
0.24	....	0.70	....	0.44	....	....	A 101*
0.40	....	0.59	....	0.39	....	....	A 56
0.19	....	0.62	....	0.40	....	....	A 60
0.30	....	0.51	....	0.33	....	....	A 61
0.14	....	0.54	....	0.35	....	....	A 65
0.19	....	0.44	....	0.29	....	....	A 66
0.070	....	0.46	....	0.30	....	....	A 102
0.109	....	0.36	....	0.23	....	....	A 70
0.049	....	0.38	....	0.25	....	....	A 73
0.056	....	0.29	....	0.19	....	....	A 78
0.031	....	0.31	....	0.20	....	....	A 80
0.033	....	0.26	....	0.18	....	....	A 81*
0.023	....	0.26	....	0.19	....	....	A 82*
0.024	....	0.22	....	0.16	....	....	A 83
0.017	....	0.23	....	0.17	....	....	A 84
Selected Sections.							
1.18	2.22	0.82	1.19	....	12,550	9,410	T 50
3.10	1.41	1.54	0.79	....	33,070	24,800	T 57
0.88	1.05	0.86	0.88	....	9,430	7,070	T 61
1.52	1.08	1.04	0.74	....	16,210	12,160	T 8
1.94	0.81	1.23	0.59	....	20,650	15,480	T 72
1.21	0.62	1.09	0.61	....	12,910	9,680	T 77
0.87	0.43	0.92	0.51	....	9,280	6,960	T 82
0.42	0.30	0.68	0.48	....	4,480	3,360	T 12
0.19	0.14	0.51	0.37	....	2,050	1,540	T 16

* Special sections.

CARNEGIE STANDARD CAST IRON SEPARATORS FOR I BEAMS.

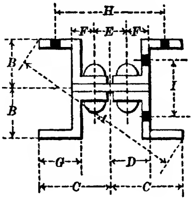
Separators for 18″, 20″ and 24″ beams are made of ⅝″ metal.
" " 6″ to 15″ beams are made of ½″ metal.
" " 5″ beams and under are made of ⅜″ metal.

Designation of Beam.			Distances.		Bolts.			Weights.			
Section Index.	Depth.	Weight.	Out to out of flanges of beams.	Center to center of beams.	Size.	Distance center to center.	Length.	Bolts and Nuts.	Increase in weight of separator bolts for 1 inch additional spread of beams.	Separator.	Increase in weight of separator for 1 inch additional spread of beams.
	Ins.	Lbs.	Ins.	Ins.	In.	Ins.	Ins.	Lbs.	Lbs.	Lbs	Lbs.
Separators with Two Bolts.											
B 1	24	80.	14¾	7¾	¾	12	9¼	3.41	.250	32.	5.50
B 2	20	80.	14¾	7¾	¾	10	9¼	3 41	"	28.	3.10
B 3	20	65.	13¼	7	¾	10	8½	3.23	"	25.	3.10
B 80	18	55.	12¾	6¾	¾	9	8¼	3.16	"	16.	2.75
B 4	15	80.	13⅝	7¼	¾	7½	9	3.35	"	15.	1.75
B 5	15	60.	12¾	6¾	¾	7½	8½	3 23	"	15.	1.75
B 7	15	42.	11¾	6¼	¾	7½	7½	2.98	"	15.	1.75
B 8	12	40.	11¼	6	¾	6	7½	2.98	"	11.	1.50
B 9	12	31.5	10¾	5¾	¾	6	7¼	2.92	"	11.	1.50
Separators with One Bolt.											
B 8	12	40.0	11¼	6	¾	....	7½	1.49	.125	10.	1.50
B 9	12	31.5	10¾	5¾	¾	....	7¼	1.46	"	10.	1.50
B 11	10	25 0	10⅛	5½	¾	....	6¾	1.40	"	8.	1.25
B 13	9	21.0	$9\frac{5}{16}$	5	¾	....	6¼	1.34	"	7.	1.20
B 15	8	18.0	8½	4½	¾	....	5¾	1.28	"	6.	1.00
B 17	7	15.0	7⅞	4¼	¾	....	5½	1.25	"	4	.75
B 19	6	12.25	$7\frac{5}{16}$	4	¾	....	5¼	1.22	"	4.	.60
B 21	5	9.75	6½	3½	¾	....	4¾	1.16	"	3.	.50
B 23	4	7.50	5⅞	3¼	¾	....	4½	1.13	"	3.	.40
B 77	3	5.50	$5\frac{5}{16}$	3	⅝	....	4¼	0.70	.09	2.	.25

CARNEGIE STEEL Z-BAR COLUMNS.

Table of dimensions, in inches.

Diameter of bolt or rivet = $\frac{3}{4}$ inch.



For area of section, weight per yard, least radius of gyration and safe load, see tables, pp. 1184 and 1185.

	Thickness of Metal.	A	B	C	D	E	F	G	H	I
6-inch columns.	$\frac{1}{4}$	$12\frac{5}{16}$	$3\frac{1}{8}$	$5\frac{5}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	$8\frac{1}{2}$	$3\frac{1}{4}$
	$\frac{5}{16}$	$12\frac{3}{8}$	$3\frac{7}{32}$	$5\frac{5}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$8\frac{3}{8}$	$3\frac{3}{8}$
	$\frac{3}{8}$	$12\frac{3}{16}$	$3\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	$8\frac{1}{4}$	$3\frac{3}{8}$
	$\frac{7}{16}$	$12\frac{1}{4}$	$3\frac{9}{32}$	$5\frac{3}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$8\frac{1}{8}$	$3\frac{1}{2}$
	$\frac{1}{2}$	12	$3\frac{1}{4}$	$5\frac{1}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	8	$3\frac{1}{2}$
	$\frac{9}{16}$	$12\frac{1}{16}$	$3\frac{11}{32}$	$5\frac{1}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$7\frac{7}{8}$	$3\frac{5}{8}$
8-inch columns.	$\frac{1}{4}$	$14\frac{11}{16}$	$4\frac{1}{8}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$9\frac{1}{2}$	$4\frac{1}{4}$
	$\frac{5}{16}$	$14\frac{3}{4}$	$4\frac{7}{32}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	$9\frac{3}{8}$	$4\frac{3}{8}$
	$\frac{3}{8}$	$14\frac{13}{16}$	$4\frac{5}{16}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$9\frac{1}{4}$	$4\frac{1}{2}$
	$\frac{7}{16}$	$14\frac{1}{2}$	$4\frac{7}{32}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$9\frac{1}{8}$	$4\frac{7}{16}$
	$\frac{1}{2}$	$14\frac{9}{16}$	$4\frac{5}{16}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	9	$4\frac{9}{16}$
	$\frac{9}{16}$	$14\frac{5}{8}$	$4\frac{13}{32}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$8\frac{7}{8}$	$4\frac{11}{16}$
	$\frac{5}{8}$	$14\frac{1}{4}$	$4\frac{5}{16}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$8\frac{3}{4}$	$4\frac{5}{8}$
	$\frac{11}{16}$	$14\frac{5}{16}$	$4\frac{13}{32}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	$8\frac{5}{8}$	$4\frac{3}{4}$
	$\frac{3}{4}$	$14\frac{3}{8}$	$4\frac{1}{2}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$8\frac{1}{2}$	$4\frac{7}{8}$
10-inch columns.	$\frac{5}{16}$	$16\frac{1}{2}$	$5\frac{5}{32}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$10\frac{1}{8}$	$5\frac{5}{16}$
	$\frac{3}{8}$	$16\frac{9}{16}$	$5\frac{1}{4}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	10	$5\frac{7}{16}$
	$\frac{7}{16}$	$16\frac{5}{8}$	$5\frac{11}{32}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{7}{8}$	$5\frac{9}{16}$
	$\frac{1}{2}$	$16\frac{3}{8}$	$5\frac{1}{4}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$9\frac{3}{4}$	$5\frac{1}{2}$
	$\frac{9}{16}$	$16\frac{7}{16}$	$5\frac{11}{32}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	$9\frac{5}{8}$	$5\frac{5}{8}$
	$\frac{5}{8}$	$16\frac{1}{2}$	$5\frac{7}{16}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{1}{2}$	$5\frac{3}{4}$
	$\frac{11}{16}$	$16\frac{3}{16}$	$5\frac{11}{32}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$9\frac{3}{8}$	$5\frac{11}{16}$
	$\frac{3}{4}$	$16\frac{1}{4}$	$5\frac{7}{16}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	$9\frac{1}{4}$	$5\frac{13}{16}$
	$\frac{13}{16}$	$16\frac{5}{16}$	$5\frac{17}{32}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{1}{8}$	$5\frac{15}{16}$
12-inch columns.	$\frac{3}{8}$	$18\frac{7}{8}$	$6\frac{3}{16}$	$7\frac{1}{8}$	4	4	2	$3\frac{1}{2}$	$11\frac{1}{4}$	$6\frac{3}{8}$
	$\frac{7}{16}$	$18\frac{15}{16}$	$6\frac{9}{32}$	$7\frac{1}{8}$	4	4	2	$3\frac{9}{16}$	$11\frac{1}{8}$	$6\frac{5}{8}$
	$\frac{1}{2}$	19	$6\frac{3}{8}$	$7\frac{1}{8}$	4	4	2	$3\frac{5}{8}$	11	$6\frac{3}{4}$
	$\frac{9}{16}$	$18\frac{11}{16}$	$6\frac{9}{32}$	$6\frac{15}{16}$	4	4	2	$3\frac{1}{2}$	$10\frac{7}{8}$	$6\frac{9}{16}$
	$\frac{5}{8}$	$18\frac{3}{4}$	$6\frac{3}{8}$	$6\frac{15}{16}$	4	4	2	$3\frac{9}{16}$	$10\frac{3}{4}$	$6\frac{11}{16}$
	$\frac{11}{16}$	$18\frac{13}{16}$	$6\frac{15}{32}$	$6\frac{15}{16}$	4	4	2	$3\frac{5}{8}$	$10\frac{5}{8}$	$6\frac{13}{16}$
	$\frac{3}{4}$	$18\frac{9}{16}$	$6\frac{3}{8}$	$6\frac{3}{4}$	4	4	2	$3\frac{1}{2}$	$10\frac{1}{2}$	$6\frac{3}{4}$
	$\frac{13}{16}$	$18\frac{5}{8}$	$6\frac{15}{32}$	$6\frac{3}{4}$	4	4	2	$3\frac{9}{16}$	$10\frac{3}{8}$	$6\frac{7}{8}$
	$\frac{7}{8}$	$18\frac{11}{16}$	$6\frac{9}{16}$	$6\frac{3}{4}$	4	4	2	$3\frac{5}{8}$	$10\frac{1}{4}$	7

CARNEGIE STEEL Z-BAR COLUMNS.

Table of Safe Loads as given by Carnegie Steel Co., for columns with square ends. Safety factor = 4. The loads given are based upon the following allowed stresses in pounds per square inch:

For lengths of 90 radii or less, 12,000.

" " over 90 radii, $17,100 - 57 \frac{l}{r}$.

Each Z-bar column is made up of four Z-bars and one web-plate (all of uniform thickness) bolted or riveted together, as shown in the figure on page 1183.

6-inch Steel Z-bar Columns.

Composed of four Z-bars about 3 inches deep and one web-plate 5¾ inches wide

Thickness of metal, inch.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
Area of section, sq. ins.	9.31	11.7	13.6	16.0	17.6	20.0
Weight per yard, pounds	95.1	119.4	138.6	162.9	179.7	203.7
Least rad of gyr, inches.	1.86	1.90	1.88	1.93	1.90	1.95
Length of column. Feet.	**Safe load of column, in pounds.**					
12 or less.	111800	140600	163200	191600	211400	239600
14	111400	140600	163200	191600	211400	239600
16	104600	133000	153200	182600	199800	229600
18	97600	124600	143400	171200	187200	215600
20	90800	116200	133400	159800	174400	201600
22	84000	107800	123600	148600	161800	187600
24	77200	99400	113800	137200	149200	173600
26	70400	91000	103800	126000	136400	159600
28	63400	82600	94000	114600	123800	145600
30	56600	74200	84000	103400	111000	131600

8-inch Z-bar Columns.

Composed of four Z-bars about 4 inches deep and one web-plate 6½ inches wide.

Thickness of metal, inch.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
Area of section, sq. ins.	11.3	14.1	17.1	19.0	21.9	24.8	26.3	29.0	31.9
Weight per yard, pounds.	114.9	144.3	174.0	194.1	221.1	252.3	267.6	296.4	325.2
Least rad of gyr, inches.	2.47	2.52	2.57	2.49	2.55	2.60	2.52	2.58	2.63
Length of column. Feet.	**Safe load of column, in pounds.**								
18 or less.	135000	169600	204800	228400	262400	297000	315000	348600	382400
20	130000	165000	201000	221000	256400	292800	306600	342600	379200
22	123800	157400	191800	210600	244800	279800	292400	327000	362600
24	117600	149600	182600	200200	233000	266800	278200	311600	346000
26	111400	142000	173600	189600	221200	253800	264000	296200	329400
28	105200	134200	164600	179200	209400	240600	249600	280800	312800
30	98800	126600	155400	168800	197600	227600	235400	265400	296400
32	92600	119000	146400	158400	186000	214600	221200	250000	279800
34	86400	111200	137400	148000	174200	201600	207000	234600	263200
36	80200	103600	128200	137400	162400	188600	192800	219200	246600
38	74000	96000	119200	127000	150600	175600	178800	203800	230000
40	67800	88200	110000	116600	139000	162600	164400	188400	213400

CARNEGIE STEEL Z-BAR COLUMNS.

Table of Safe Loads (continued).

10-inch Steel Z-bar Columns.

Composed of four Z-bars about 5 inches deep and one web-plate 7 inches wide.

Thickness of metal, inch.	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$
Area of section, sq. ins.	15.8	19.0	22.3	24.5	27.7	30.9	32.7	35.8	39.0
Weight per yard, pounds.	161.1	194.1	227.4	249.9	282.6	315.6	330.0	368.4	397.8
Least rad of gyr, inches.	3.08	3.13	3.18	3.10	3.15	3.21	3.13	3.18	3.25
Length of column. Feet.	**Safe load of column, in pounds.**								
22 or less.	189400	228400	267800	294000	332400	371200	392000	429800	468000
24	185600	225200	266200	289200	329600	370600	387200	427800	468000
26	178600	217200	256600	278400	317400	357400	373000	412400	453200
28	171600	208800	247000	267600	305400	344200	358600	397000	436800
30	164600	200400	237400	256800	293400	331000	344400	381600	420400
32	157600	192200	227600	246000	281400	317800	330000	366200	404000
34	150600	183800	218200	235200	269400	304600	315800	350800	387600
36	143600	175600	208600	224400	257400	291400	301400	335600	371200
38	136600	167200	199000	213600	245400	278200	287200	320000	354800
40	129600	158800	189400	202800	233400	265000	273000	304600	338200
42	122600	150600	179800	192000	221200	251800	258800	289200	321800
44	115400	142200	170200	181200	209200	238600	244400	273800	305400
46	108400	134000	160600	170400	197200	225400	230200	258400	289000
48	101400	125600	151000	159600	185200	212200	215800	243000	272600
50	94400	117200	141400	148800	173200	199000	201600	227600	256200

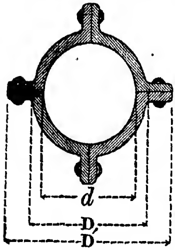
12-inch Steel Z-bar Columns.

Composed of four Z-bars about 6 inches deep and one web-plate 8 inches wide.

Thickness of metal, inch.	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
Area of section, sq. ins.	21.4	25.0	28.8	31.2	34.8	38.5	40.5	44.1	47.7
Weight per yard, pounds.	218.1	255.6	293.4	318.6	355.5	392.7	413.4	449.7	486.3
Least rad of gyr, inches.	3.67	3.72	3.77	3.70	3.75	3.73	3.68	3.66	3.64
Length of column. Feet.	**Safe load of column, in pounds.**								
26 or less.	256600	300600	345200	374600	418200	462000	486000	529000	572200
28	254000	299400	345000	372000	417800	460600	481600	522800	564200
30	246000	290200	335200	360400	405000	446600	466400	506400	546400
32	238000	281000	324800	349000	392200	432600	451400	490000	528400
34	230200	271800	314400	337400	379600	418400	436400	473400	510400
36	222200	262600	304000	325800	366800	404200	421200	456800	492600
38	214200	253400	293600	314200	354000	390200	406200	440400	474600
40	206200	244200	283000	302800	341400	376000	391200	423800	456600
42	198200	235000	272600	291000	328800	361800	376000	407400	438800
44	190200	225800	262200	279600	316000	347800	361000	391000	420800
46	182400	216600	252400	268000	303200	333600	345800	374400	402800
48	174400	207200	241400	256400	290600	319600	330800	358000	384800
50	166400	198200	231000	244800	277800	305400	315800	341400	367000

For loads greater than those given in the tables, the Z-bar columns may be re-enforced by additional plates, riveted to the flanges. The addition of such plates does not in any case diminish the least radius of gyration. Hence the same load *per square inch of cross-section* may be used.

Table of rolled-iron segment-columns of the Phœnix Iron Co, 410 Walnut St, Philada.



The **dimensions** given are subject to slight variations which are unavoidable in rolling iron shapes. The **weights** of columns given are those of the 4, 6, or 8 segments, of which they are composed The *shanks* of the rivets used in joining them together, of course, merely make up the quantity of metal punched or drilled out, in making the holes; but the rivet-*heads* add from 2 to 5 per cent to the weights given. The **rivets are spaced** 3, 4, or 6 ins apart from cen to cen.

Any desired thickness between the minimum and maximum for any given size, can be furnished. We give the dimensions, weights, &c, corresponding to the principal thicknesses. G columns have 8 segments, E, 6 segments. All others, 4 segments.

Mark.	Thickn's, ins.	Diameters, ins. d	D	D′	One column: Area of cross sec, sq ins.	Wt per ft run, lbs.	Least rad of gyr, ins	Size of Rivets.
A	3/16	3 5/8	4	6 1/16	3.8	12.6	1.45	1 1/8 × 3/8
"	1/4	"	4 1/8	6 3/16	4.8	16.	1.50	1 1/4 × "
"	5/16	"	4 1/4	6 5/16	5.8	19.3	1.55	1 3/8 × "
"	3/8	"	4 3/8	6 7/16	6.8	22.6	1 59	1 1/2 × "
B1	1/4	4 13/16	5 5/16	8 1/16	6.4	21.3	1.92	1 5/8 × 1/2
"	3/8	"	5 9/16	8 1/4	9.2	30.6	2 02	1 3/4 × "
"	1/2	"	5 13/16	8 7/16	12.	40.	2.11	1 7/8 × "
"	5/8	"	6 1/16	8 5/8	14.8	49.3	2.20	2 1/8 × "
B2	1/4	5 15/16	6 7/16	9 1/8	7.4	24.6	2.34	1 5/8 × "
"	3/8	"	6 11/16	9 5/16	10.6	35.3	2.43	1 3/4 × "
"	1/2	"	6 15/16	9 1/2	13.8	46.	2.52	1 7/8 × "
"	5/8	"	7 3/16	9 11/16	17.	56.6	2.61	2 1/8 × "
C	1/4	7 3/16	7 11/16	11 9/16	10.	33.8	2.80	1 7/8 × 5/8
"	1/2	"	8 3/16	11 15/16	18.	60.	2.98	2 1/4 × "
"	3/4	"	8 11/16	12 3/16	25.2	84.	3.16	2 5/8 × 3/4
"	1	"	9 3/16	12 9/16	33.2	110.6	3.34	3 × "
"	1 1/4	"	9 11/16	12 15/16	41.2	137.3	3.52	3 1/4 × "
E	1/4	11	11 1/2	15 7/16	16.8	56.	4.18	2 × "
"	1/2	"	12	15 7/8	26.4	88.	4.36	2 3/8 × "
"	3/4	"	12 1/2	16 5/16	37.8	126.	4.55	2 3/4 × "
"	1	"	13	16 3/4	49.8	166.	4.73	3 × "
"	1 1/4	"	13 1/2	17 3/16	61.8	206.	4.91	3 1/4 × "
G	5/16	14 3/8	15	19 1/8	24.	80.	5.45	1 7/8 × 5/8
"	1/2	"	15 3/8	19 7/16	36.	120.	5.59	2 1/4 × "
"	3/4	"	15 7/8	19 7/8	52.	173.3	5 77	2 5/8 × 3/4
"	1	"	16 3/8	20 3/8	68.	226.6	5.95	3 × "
"	1 3/8	"	17 1/8	21	92.	306.6	6.23	3 3/8 × "

THE GRAY COLUMN.

The Gray Column, designed and patented by Mr. J. H. Gray, consists in its original form, of angles, connected at intervals D (generally of 2 ft 6 ins) by transverse bent tie-plates T, usually 9 × 3/8 ins. This construction renders the parts of the column easily accessible, for painting, etc., but under transverse or buckling stresses the column must act somewhat like a rectangular frame without diagonals. To remedy this, a later form, the "twelve-angle" column, Fig 5, has been designed, having, in the square column, Figs 1 and 2, instead of the bent tie-plates T, four additional angles, running longitudinally like the others, and placed centrally, as shown. These angles supply the column with two webs, intersecting at right angles.

Figs 1 and 2 show the square column, used in the interior of buildings. It is used chiefly in the 14, 15, and 16 inch sizes. Fig 2 shows plates riveted to the outer flanges, as is done in some of the heaviest columns. Figs 3 and 4 show the wall and corner columns respectively. Fig 5 shows the "twelve-angle" column, and Fig 6 one of many forms of fireproofing, with plaster laid on terra cotta blocks. The rivets are usually 3/4 inch in diameter.

The safe load, in pounds per sq inch, of the ordinary column, is stated as $17{,}100 - 57\frac{L}{r}$, where L = length of column and r = its least rad of gyration.

The cost of the Gray column, at shop, is from 1 to 1.5 cents per lb plus the cost of the angles.

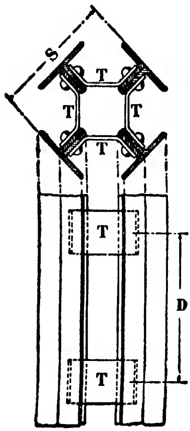


Fig. 1.

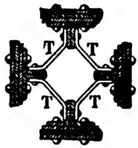


Fig. 2.

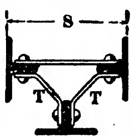


Fig. 3.

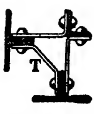


Fig. 4.

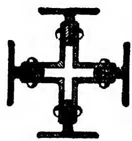


Fig. 5.

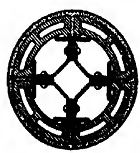


Fig. 6.

Gray Column. List of Selected Sizes.

Size S ins.	Angles ins.	Area sq in.	I Mom of in.	r Least rad. of gyr. ins.	r^2	Safe loads* for column lengths of 12 ft.	 30 ft.
	Square Columns. Figs. 1 and 2.						
9	2 × 2½ × ¼	8.48	64	2.7	7.3	115	80
"	2 × 3 × ½	18.00	119	2.6	6.8	250	165
10	2½ × 2½ × ¼	9.52	95	3.1	9.6	135	100
"	2½ × 3 × ½	20.00	179	3.0	9.0	285	205
12	3 × 3 × ⅜	16.88	241	3.8	14.4	250	195
"	3 × 5 × ½	30.00	327	3.3	10.9	435	325
13	3 × 3 × ⅜	16.88	285	4.1	16.8	255	205
"	3 × 5 × ¾	43.52	552	3.6	13.0	645	495
14	3 × 3 × ⅜	16.88	336	4.5	20.2	255	210
"	3 × 3½ × ⅝	29.36	526	4.3	18.5	445	360
"	3 × 4 × ½	26.00	444	4.2	17.6	390	315
"	3 × 5 × ½	30.00	463	4.0	16.0	450	355
"	3½ × 3½ × ⅝	31.92	597	4.4	19.4	485	395
"	3½ × 5 × ⅝	39.36	624	4.0	16.0	590	470
"	3½ × 6 × ½	36.00	526	3.8	14.4	535	420
15	3 × 3 × ½	22.00	496	4.8	23.0	335	280
"	3½ × 3½ × ⅝	31.92	693	4.7	22.1	490	405
"	3½ × 5 × ⅝	39.36	731	4.4	19.4	600	490
"	4 × 4 × ½	30.00	653	4.7	22.1	440	380
"	4 × 5 × ⅝	41.84	817	4.4	19.4	635	520
"	4 × 6 × ⅝	46 88	828	4.2	17.6	710	570
16	3 × 3 × ½	22.00	570	5.1	26.0	340	285
"	4 × 4 × ⅝	36.88	912	5.0	25.0	570	480
"	4 × 6 × ¾	55.52	1134	4.6	21.2	850	700
18	3 × 3 × ½	22.00	746	5.8	33.6	345	300
"	4 × 4 × ⅝	36.88	1182	5.7	32.5	575	495
"	5 × 5 × ⅝	46.88	1465	5.6	31.4	730	630
20	4 × 4 × ⅝	36.88	1485	6.4	41.0	580	510
"	6 × 6 × ¾	67.52	2588	6.2	38.4	1065	930
30	5 × 4 × ⅝	41.80	4147	9.9	98.0	680	630
"	6 × 6 × 1†	264.40	22688	9.2	84.6	4285	3930
	Wall Columns. Fig. 3.						
12	3 × 4 × ⅜	14.88	94	2.5	6.2	205	130
14	3 × 3½ × ½	18.00	160	3.0	9.0	255	185
"	3½ × 5 × ⅝	29.52	241	2.9	8.4	420	295
15	3½ × 4 × ½	21.00	217	3.3	10.9	305	225
"	4 × 5 × ⅝	31.38	317	3.2	10.2	455	335
16	3 × 3 × ½	16.50	200	3.5	12.2	240	185
"	4 × 6 × ⅝	35.16	375	3.3	10.9	510	380
18	4 × 4 × ⅝	27.66	434	4.0	16.0	415	330
20	4 × 5 × ⅝	31.38	562	4.2	17.6	475	380
30	5 × 4 × ⅝	31.38	1408	6.8	46.2	495	440
"	6 × 6 × 1†	198.30	8937	6.7	44.9	3150	2785
	Corner Columns. Fig. 4.						
10½	3½ × 3½ × $\frac{13}{16}$	23.48	272	3.4	11.6	345	260
12½ 13	3½ × 3½ × ½	15.75	288	4.3	18.5	240	195
13 14	4 × 5 × ⅝	24.91	425	4.2	17.6	375	305

* In thousands of lbs, by formula: Safe load = 17100 — 57 $\frac{L}{r}$ in lbs per sq in.

† Three 1-in plates riveted to each pair of angles. Fig. 2.

IRON AND STEEL COLUMNS.

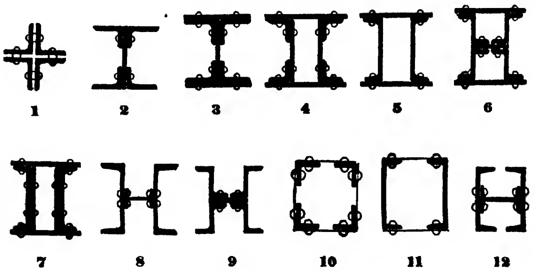
For columns in general, see pp 495, etc.

For wooden columns, see pp 1143, etc.

List of References.

(1) Steel in Construction, Pencoyd Iron Works, 12th Ed, 1900.
(2) Cambria Steel Co, Handbook, 1907.
(3) Carnegie Steel Co, Pocket Companion, 1903.
(4) Phoenix Iron Co, Hand Book, 1906.
(5) Bethlehem Steel Co, Structural Steel, 1907.
(6) Passaic Steel Co, Manual, 1903.
(7) Acts and Resolves of the Massachusetts Legislature, 1907.
(8) Trans Am Soc C E, Vol 15, p 530, July 1886.
(9) Trans Am Soc C E, Vol 20, p 258, June 1889.
(10) Trans Am Soc C E, Vol 54, June 1905.
(11) New York Building Code, Approved Oct 24, 1899, with amendments to April 12, 1906.
(12) Eng News, Jan 13, 1898, pp 27, etc.
(13) Eng News, June 30, 1898, p 424.
(14) Modern Framed Structures, by J. B. Johnson, New York, John Wiley & Sons, 1893.
(15) City of New York. Department of Bridges, Specifications for the Design of Municipal Bridges, 1907.
(16) Mitteilungen der Materialprufungsanstalt am schweizerischen Polytechnikum in Zurich, Heft VIII, 1896.
(17) "Tests of Metals etc," Watertown Arsenal. Year ending June 30, 1888.
(18) "Tests of Metals etc," Watertown Arsenal. Years ending June 30, 1886, 1890, 1894.

1. For economy of material, iron and steel columns are almost invariably **hollow.** Cast iron columns are usually round or rectangular; while structural steel columns are usually of some special or built-up shape, see Figs 1 to 12 and pp 1186, 1187.



Figs. 1 to 12.

2. Owing to displacement of the core, in casting, **cast iron columns** are apt to be much thinner on one side than on the other. They are liable to initial stresses, due to unequal contraction in cooling, and are objectionable also on account of the brittleness and relative unreliability of the material and its low tensile strength. When the metal, for very long columns, is poured at both ends of the mold, it may become so chilled that the two portions do not unite perfectly at the center. A weak place is thus left just where the max bending moment occurs. Entrained air produces blowholes and honeycomb; and impurities, collecting at the bottom of the mold, weaken the casting. See also ¶ 14.

See Columns in General, ¶¶ 9, 10, 13, and 35 to 43, pp 495, etc., 498 *c*. etc.

3. Avoid designs which, like those shown at *g* and at *a a*, Fig 13, bring part of the wall of the column out from the line of pres, inducing **bending moments** in the metal.

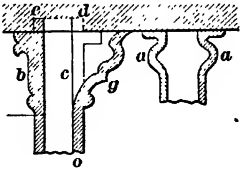


Fig. 13.

4. Use preferably a simpler form, as at *b*, or a straight col, *o c d*, to which any such ornaments, as at *g*, may be attached.

5. Figs 1 to 12 show **common forms of columns** built up of steel angles, plates, channels, and Z-bars. **"Closed" sections.** Figs 4 to 7, cannot be repainted internally. They are therefore suitable only for dry situations where temp changes are slight. In Figs 8, 10, 11, 12, the flanges should be placed far enough apart to permit machine-riveting.

6. Figs 14 to 17, from "Cambria Steel," show details of **connections** between one column and another and between columns and beams, as markt. Figs 18 to 20, from "Cambria Steel," show arrangement for **bases** of columns, as markt.

7. It is customary to provide that K, = L/r, shall not exceed about 125; but, in light structures, *K* sometimes greatly exceeds this limit. Thus, in eleven transmission towers, *K* ranged from 111 to 300. Where it reached 300, the member consisted of 1 angle, 2 × 2 × ⅛ inch, *L* = 120 ins ; *r* = 0.4 inch. (R. D. Coombs, Am Soc Civ Engrs, Trans, Vol 61, Dec 1908, p 202.)

8. In columns composed of two channels, latticed, Fig 11, the channels are usually placed so far apart that the tendency of the column will be to deflect in the plane of the webs, i. e., so that the least radius of gyration of the column is the greatest radius of gyration of the single channel. For proper distances between channels, to secure this result, see pp 1175 and 1177.

9. Angle sections being unsymmetrical, the load is usually more or less eccentric. Hence the allowable stress, for angle columns, should be less than that for symmetrical sections.

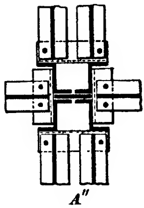
10. Rivets are usually spaced ≯ 3 inches, center to center, near the ends of columns, for a distance equal to twice the width of the column. Distance between centers of rivets, in line of stress, ≯ 16 times least thickness of metal of the parts joined. Distance between rivets, perp to line of stress, ≯ 32 times thickness of metal.

11. Lattice bars, batten plates and connections, weigh, together, 30 % or more of the weights of heavy columns, and 50 or 60 % of the weights of the lightest columns.

12. In building construction, **columns may be considered as square-ended.** Usual **factor of safety** = 4. See ¶¶ 14 and 18.

13. Columns of angles or I-beams are usually of **soft steel:** those of plates and angles, channels, plates or Z-bars are usually calculated as for **medium steel.**

14. Cast iron columns do not permit as rigid connections, to other columns or to beams, as do rolled steel and riveted columns. They should not be used, in buildings, with a safety factor less than 8. See also ¶¶ 2, 12, 18.



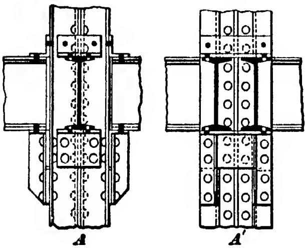


Fig. 14.

Z-bar col with beam connectns.
A, A', side elevatns;
A'', hor sec thru col.

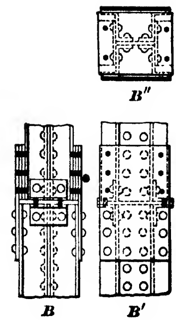
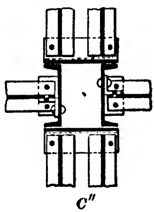


Fig. 15.

Splice betw 2 Z-bar cols of diff sizes.
B, B', side elevatns;
B'', hor sec thru col.



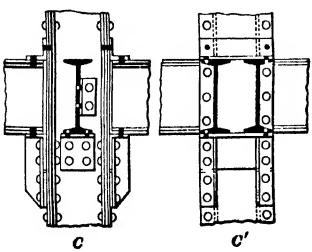


Fig. 16.

Plate & chan col with beam connectns.
C, C', side elevatns,
C'', hor sec thru col.

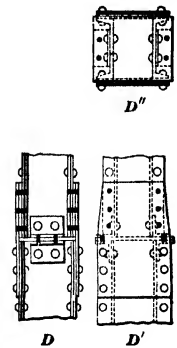
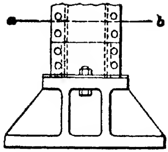
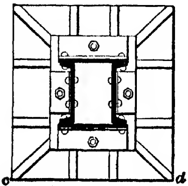


Fig. 17.

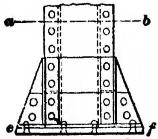
Splice betw 2 plate & chan cols of diff sizes.
D, D', side elevatns;
D'', hor sec thru splices.



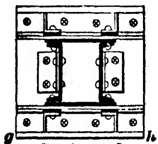
Elevation c-d
E



Section a-b
E′



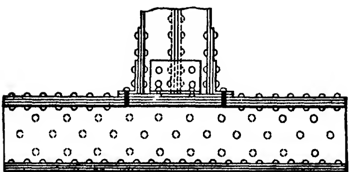
Elevation g-h
F



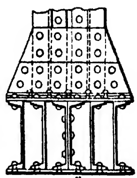
Section a-b
F′

Fig. 18.
Cast iron base for plate & chan col

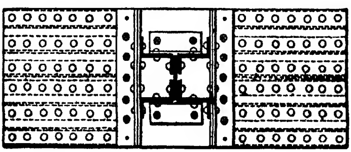
Fig. 19.
Steel base for plate & chan col.



G′



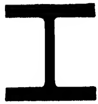
G″



G

Fig. 20.

Bolster, of beams, channels & plates, for Z-bar column.
***G*, hor sec thru col; *G′*, side elevatn; *G″*, end elevatn.**



15. In order to obviate the **weakening effect of the rivet holes** in riveted steel columns, the **solid H column,** rolled in the Grey universal mill, and shown in cross section in Fig 21, is employed.

Fig. 21.

16. Let loads and stresses be in pounds, dimensions in inches; let
P * = the total load (supposed to be axial) on the column;
a = the area of cross section of the column;
p * = P/a = the average unit load on the column;
s * = the maximum existing unit stress in any cross section;
s_e = the unit stress at the elastic limit;
s_s = the maximum unit stress in a short column;
r = the least radius of gyration of the cross section;
D = the least external diameter, or least external dimension, of the cross section;
L = the length of the column;
K = L/r = the length ratio, or slenderness, of col, in terms of r;
k = L/D = the length ratio, or slenderness, of col, in terms of D;
m = a coefficient in the Rankine formula;
c = a coefficient in the straight-line formula;
F = safety factor.

17. We then have

Rankine formula, (Columns in general, ¶¶ 21–27, pp 497–8):

$$p = P/a = \frac{s}{1 + m K^2}; \qquad s = p\,(1 + m\,K^2).$$

Straight-line formula, (Columns in general, ¶ 29, p 498).

$$p = P/a = s_s - c\,K.$$

18. Safety factors. F, proposed by Mr. James Christie (1), p 157, for structural steel columns :

For flat and fixed ends, $F = 3 + 0.010\,K$
For hinged and round ends, $F = 3 + 0.015\,K$.

See ¶¶ 12 and 14.

* Under any given conditions, P and p are the total load and the avge unit load, respectively, corresponding to the extreme fiber stress, s, existing under those same conditions. Thus, P, p and s may be those corresponding to ultimate or safe or any other loading.

19. Below are given **tables of values commonly used** in connection with various **formulas** for steel and iron columns under static axial loading. See Columns in General, pp 495 etc:

RANKINE FORMULA.

$$p = \frac{P}{a} = \frac{s}{1 + mK^2} = s \cdot \frac{1}{1 + m\left(\frac{L}{r}\right)^2};$$

$$K = L/r = \sqrt{\frac{\frac{s}{p} - 1}{m}}.$$

As in ¶ 16 :
p = mean unit load on col ; **m** = a coefficient ;
s = maximum unit stress in cross section ;
K = L/r = unsupported length ÷ least radius of gyration.

See diagram of values of $\frac{p}{s} = \frac{1}{1 + mK^2}$, Fig 22, p 1198.

Values of s and of 1/m for steel and iron columns.

For list of references, see p 1189.

TS = ultimate tensile strength; $\mathbf{S_e}$ = elastic limit; **YP** = yield point, all in thousands of pounds per square inch.

STEEL AND WROUGHT IRON.

For Buildings, Ultimate loads.

Cambria Steel Co., soft and medium steel; (2) pp 194-7.
Carnegie Steel Co. (3) pp 143-4, and Phoenix Iron Co. (4) p 88.
Safety factors; dead load, $F = 4$;
live load, $F = 5$.

	s		1/m
Steel.	Soft	Medium	
Pin ends	45,000	50,000	18,000
One pin end	45,000	50,000	24,000
Square ends	45,000	50,000	36,000

Passaic Steel Co., **wrought iron;** (6) p 153.

	s	1/m
Pin ends	40,000	20,000
Square ends	40,000	30,000
Fixed ends	40,000	40,000

For Buildings, Permissible loads.

		s	1/m
Boston building code,* (7) p 415	**Steel**	16,000	20,000
	Wrought iron	12,000	20,000

Bridge Specifications, Permissible loads.

Osborn Engineering Co., 1903-4	TS	S_e	YP	s: Steam	Electric	H'way	1/m
Wrought iron;	48	—	25	13,000	15,000	18,000	
Soft steel;	52-62	32	—	15,000	17,000	20,000	
Medium steel:	60-70	35	—	17,000	19,000	22,000	
Both pin ends							18,000
One square end							24,000
Both square ends							36,000

*The Boston code uses the Rankine formula for steel and wrought iron columns; J. R. Worcester's method for wood; and (apparently) T. H. Johnson's straight-line formula for cast iron.

Structural Steel.			s	$1/m$
Phila & Reading R R, 1906; $TS = 60$,			15,000	13,500
Pennsylvania R R, revised to Jan 1, 1907;				
Soft; TS = 52 to 62; $S_e = 28$.			16,000	13,500
Erie Railroad, 1900; revised to June 1, 1905;				
Soft; TS = 56 to 64; $S_e = 0.58\ TS$			s	
Both ends hinged;			$8000\left(1 + \frac{\text{min stress}}{\text{max stress}}\right)$	18,000
One end fixed;				24,000
Both ends fixed;				36,000
	s Dead	s Live		$1/m$
New York Central R R, 1904;				
TS = 56 to 64, $S_e = 33$	16,000	8,000		18,000
Del Lacka & W R R, Nov, 1903;				
Soft; TS = 54 to 62, $S_e = 0.5\,TS$;				
Two pin ends	12,500	8,500		18,000
One or both ends fixed	12,500	8,500		24,000
New York City, Henry B Seaman, Consulting Engineer, 1907, Highway bridges				
Wrought iron	16,000	8,000		8,000
Medium steel	20,000	10,000		8,000
Nickel steel	30,000	15,000		8,000

Prof. Mansfield Merriman believes that, for **steel** columns, best values for 1/m are about as follows:

	$1/m$
Round ends	6,000
Pin ends	8,000
Square ends	18,000
Fixed ends	24,000

CAST IRON.

For Buildings, Ultimate loads.

		s	$1/m$
Cambria Steel Co (2) p 284;	$F = 8$;	80,000	8,000*
Carnegie Steel Co (3) p 148;	Pin ends	80,000	4,000*
Passaic Steel Co (6) p 206;	One pin end	80,000	5,500*
	Square ends	80,000	8,000*

For Buildings, Permissible loads.

	s	$1/m$
Chicago building code (6) p 119;	10,000	6,000*

* The Cambria, Carnegie, Passaic and Chicago formulas, for cast iron columns, use $k = L/D$, instead of $K = L/r$. The values of $1/m$, above given, are approximate equivalents for the published coefficients. The ratio, between the two coeffs, varies with the thickness of the column See pp 353 *a*, 353 *b*.

STRAIGHT-LINE FORMULA.

$$p = P/a = s_s - cK; \quad K = L/r = \frac{s_s - p}{c}.$$

p = mean unit load on column; **c** = a coefficient;
$\mathbf{s_s}$ = maximum unit stress in a short column;
K = L/r = unsupported length ÷ least radius of gyration.

Values of s_s and of c, for steel and iron columns.

Ultimate loads.

T. H. Johnson (8)	s_s	Flat ends c	Flat ends K_t*	Hinged ends c	Hinged ends K_t*	Round ends c	Round ends K_t*
Wrought iron	42,000	128	218	157	178	203	138
Mild steel; carbon = 0.12%	52,500	179	195	220	159	284	123
Hard steel; carbon = 0.36%	80,000	337	158	414	129	534	100
Cast iron	80,000	438	122	537	99	693	77

Structural Steel
For Buildings, Ultimate loads

	s_s	c Fixed ends	c Square ends	c Pin ends
Passaic Steel Co (6) pp 149, 154				
K, 50 to 150, F = 4				
Soft (angles and beams)	54,000	185	200	225
Medium (Z-bars, channels, plates)	60,000	210	230	260

For Buildings, Permissible loads.

	s_s	c
New York building code, (11) Sec 138;	15,200	58
Chicago building code, (6) p 119, p_{max} = 13,500;	17,000	60
Carnegie Steel Co (3) pp 125-6, F = 4; medium steel (Z-bars and channels)		
$K \not> 90$;	12,000	0
$K > 90$;	17,100	57
Bethlehem Steel Co (5) p 122;		
$K < 55$;	13,000	0
$K > 55$; $\not> 125$;	16,000	55
C. C. Schneider, (10) p 494;	16,000	70
J. R. Worcester, (10) p 418; † ($K/12$) max = 16		
$K/12$ from 2 to 4	13,000	0
For each increase of 2, in $K/12$, deduct 1,000 from s_s.		
$K/12$ from 14 to 16	7,000	0

Railroad Bridge Specifications.

TS = ultimate tensile strength; s_s = elastic limit; both in thousands of pounds per square inch.

	s_s	c
American Railway Eng'ng & M of Way Assn, 1906, American Bridge Co, 1906, and Baltimore & Ohio R R, 1904; TS = 60	16,000	70
Theodore Cooper, 1906, Dead load		
Medium; TS = 60 to 70, s_s = 0.5 TS		
Chord segments, stiffeners	20,000	90
Posts	17,000 to 18,000	90 to 80
Lateral struts	13,000	60

For live loads, in lateral struts, take two-thirds the dead load stress; elsewhere, one-half.

For low steel (TS = 55 to 65) deduct 10 per cent from loads for medium steel.

For movable structures, deduct 25 per cent from loads on stationary structures.

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*Kt = value of K for the point of tangency where straight line joins Euler's curve. See Columns in General, ¶ 31, p 498 *b*.

† See Columns in General, ¶ 34, p 498 *b*.

Cast Iron

For Buildings. Ultimate Loads.

	s_s	c
Wm. H. Burr, flat ends, (13);	30,500	160
J. B. Johnson, flat ends, "Materials" p 366;	34,000	88

For Buildings. Permissible Loads.

	s_s	c
New York building code, (11) Sec 138; Min diam, 5 ins; min thickness, ¾ in, K_{max} = 70	11,300	30
Boston building code, (7) p 416; K_{max} = 70. (Cast iron columns forbidden in buildings over 75 feet high)	11,000	30

20. Prof. J. B. Johnson (14) p 150, for ult loads by his **parabolic formula** (see Columns in General, ¶ 28, p 498):

$$p \text{ ult} = s_e - \frac{s_e^2 K^2}{q \pi^2 E} = s_e - c_p K^2,$$

gives q = 6.4 for hinged ends, q = 10 for flat ends, and

	$K \not>$	s_e	c_p
Wrought iron, pin ends, ...	170	34,000	0.67
Wrought iron, flat ends, ...	210	34,000	0.43
Mild steel, pin ends, ...	150	42,000	0.97
Mild steel, flat ends, ...	190	42,000	0.62

Here, as in ¶ 16, let

s_e = elastic limit of the material; c_p = a coefficient;

K = L/r = unsupported length ÷ least radius of gyration.

21. Where the **least external dimension, D,** (instead of the least radius of gyration, r,) is given, the following method (15) for finding the unit load, p, by the Rankine formula, may be found convenient: Let

s = max permissible fiber stress;

$p = P/a = s/(1 + m K^2)$ = permissible static unit load;

$$n = \frac{D^2}{r^2} = \frac{\text{(least external dimension)}^2}{\text{(least radius of gyration)}^2}; \quad K = \frac{L}{r}; \quad k = \frac{L}{D}.$$

Then

$K^2 = n k^2$; and $p = s/(1 + m n k^2)$; or $p/s = 1/(1 + m n k^2)$.

For values of m, see ¶ 19, pp 1194–1196.

For values of p/s, see ¶ 22, p 1198.

For values of n, = $(D/r)^2$, see Columns in General, pp 353 a, 353 b

22. Diagram. Fig 22, **showing values of** $p/s = 1/(1 + mnk^2)$
(*a*) for medium and nickel steel and wrought iron;
(*b*) for cast iron.

In the scale of abscissas, find the ordinate corresponding to the given value of $k = L/D$. Find the intersection of this ordinate with the curve corresponding to the value of n, $= (D/r)^2$, for the given section, taken from the table, pp 353 *a*, *b*. Opposite this intersection, on the axis of ordinates, will be found the required value of p/s. Then $p = s\,(p/s)$.

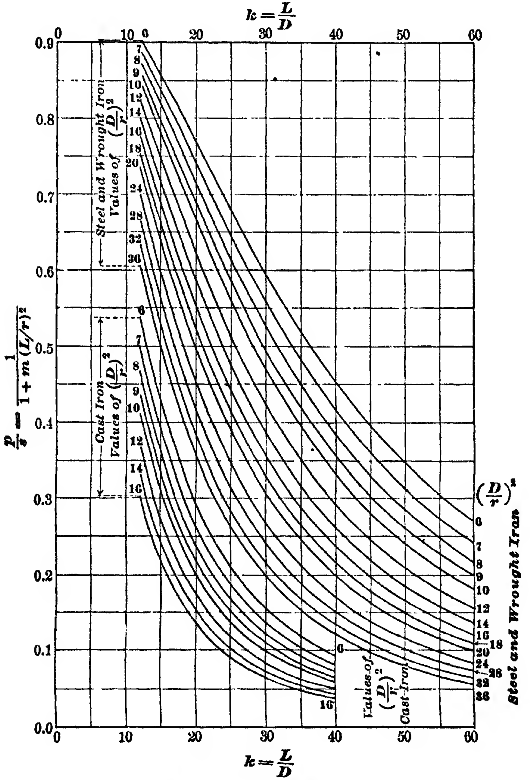


Fig. 22. Values of p/s.

23. Formulas and experiments. Figs 23 and 24 show a comparison between certain formulas for, and experiments upon, columns of **structural steel and cast iron,** as follows:

Formulas and Experiments.

Figs 23 and 24 (pp 1201 and 1203).

STRUCTURAL STEEL, Fig 23.

Formulas.

Rankine: $p = s/(1 + m K^2)$; **Straight-line:** $p = s_s - c K$.

p = mean unit load on column, **m, c** = coefficients;
s = maximum unit stress in cross section;
s_s = maximum unit stress in cross section for short blocks;
$K = L/r$ = length ÷ least radius of gyration; p and s in lbs per sq inch.

Ultimate.

1. Straight-line. T. H. Johnson, mild steel, carbon = 0.12%, square ends.	$p = 52{,}500 - 179\ K$
2. Straight-line. Passaic R. M. Co soft steel, square ends, $K \not> 30$	$p = 48{,}000 - 0$
$K > 30$	$p = 54{,}000 - 200\ K$
3. Rankine Cambria Steel Co and Carnegie Steel Co, medium steel, square ends,	$s = 50{,}000$; $1/m = 36{,}000$
4. Rankine, Cambria Steel Co, soft steel, square ends,	$s = 45{,}000$; $1/m = 36{,}000$
5. Parabolic. J. B. Johnson, mild steel, square ends,	$p = 42{,}000 - 0.62\ K^2$

Permissible.

6. Rankine. N Y. Bridge Dept, medium steel	$s = 20{,}000$; $1/m = 8{,}000$
7. Rankine. Boston building code,	$s = 16{,}000$; $1/m = 20{,}000$
8. Straight-line. C. C. Schneider and Am Ry Eng & M of W Assn, $TS = 60{,}000$,	$p = 16{,}000 - 70\ K$
9. Straight-line. Carnegie Steel Co. Medium steel, square ends. $K \not> 90$;	$p = 12{,}000 - 0$
$K > 90$;	$p = 17{,}100 - 57\ K$

Experiments.

Ultimate strength of pin-end steel columns.

TS = ult tensile strength; s_e = elastic limit.

● **Jas. G. Dagron.** Am Soc C E Trans, June, 1889, Vol 20, p 254. 8 lattice bridge columns of rectangular section, built up of plates and angles. Carbon, 0.26 to 0.27 per cent; TS = 83.7 to 84.4; s_e = 51.2 to 53.9;
6 cols of 2 plates, 8 × ½, 4 angles 2¼ × 2¼ × ¼, 16 to 24 ft;
2 cols of 2 plates, 9 × ⅝, 4 angles 2¾ × 2¾ × 5/16, 25 ft 7.5 ins.

✚ **C. P. Buchanan.** Eng News, Dec 26, 1907. 7 bridge cols, as below. Bessemer (Bess) and Open Hearth (O. H.).

No.			Carbon %	K	p
3, Post,	4 Z-bars, 3 in.	Web, 6 × ⅜, Bess	—	83	34,270
4, Post,	4 Z-bars, 3 in.	Web, 6 × ⅜, Bess	—	83	32,900
9, Post,	4 angles, 6 × 3.5	Web, 10¾ × ⅜, Bess	—	97	27,790
16, Chord,	2 angles, 3 × 3; 2 angles, 4 × 3	2 webs 16 × ⅜; cover, 22 × ⅜; Lattice, 5 × 7/16 O.H	0.21	46	30,640
17, Post,	4 angles, 3 × 3	2 webs 10 × ⅜; Lattice O. H.	0.23	45	31,680
18, Chord	4 angles, 3 × 3	1 web 18 × ⅜; 1 cover 22 × ⅜; Lattice 2¼ × 7/16; O.H.	0.15	34	31,980
19, Chord	4 angles, 3 × 3	1 web 18 × ⅜; 1 cover 22 × ⅜; Lattice 2¼ × 7/16; O.H.	0.15	35	33,950

▲ **J. A. L. Waddell,** Eng News, Jan 16, 1908. 6 nickel-steel and 6 carbon-steel bridge columns. Each of 4 angles, 3 × 3 × ⅜, 2 web plates, 12 × ⅜, and lattice 2.5 × ⅜. Lengths 10 and 30 ft r = 4.46 ins. a = 17.44 sq ins. Nickel-steel, TS = 100-115; $s_e \not< 60$. Nickel, 3.5 %; carbon 0.38 %; manganese, 0.30 %. Carbon-steel, TS = 60–70; $s_e \not< 35$.

						Average
Nickel steel,	$K = 27$,	p =	68,500,	68,500,	69,200,	68,700
" "	$K = 81$,	p =	44,400,	47,200,	42,500,	44,700
Carbon steel,	$K = 27$	p =	38,900,	38,900,	39,800,	39,200
" "	$K = 81$,	p =	29,600,	29,600,	32,400,	30,500

Quebec Bridge Commission, Eng News, April 23, 1908. Two medium steel chord members; viz: No 1, one-third size of "A 9 L" (see ¶ 26) and of similar section, and No 2, with 2, instead of 4, ribs; ribs identical with outer ribs of No 1; but the lattice-angles were 50 % heavier; rivets in lattice connections doubled; intersections of lattice-angles strengthened by gusset plates. Pins, 12 ins diam, as in "A 9 L"

No	Area sq ins	Length c. to c.	L/r *	L/r †	Failure load, in lbs/sq in indicated ‡	Failure load, in lbs/sq in net ‡
1	86.5	19′ 0″	42	35	26,850	22,150
2	42.6	11′ 4¼″	25	15	37,000	30,525

CAST IRON, Fig 24.

Formulas.

Rankine: $p = s/(1 + m K^2)$; **Straight-line:** $p = s_s - c K$.

p = mean unit load on column ; **m, c** = coefficients ;
s = maximum unit stress in cross section ;
s_s = maximum unit stress in cross section for short blocks ;
K = L/r = length ÷ least radius of gyration; s and p in lbs per sq inch.

Ultimate. Square ends.

1. Rankine. Cambria Steel Co, approx............ s = 80,000; $1/m$ = 8,000
2. Straight-line, T. H. Johnson .... p = 80,000 — 438 K
3. Straight-line, J. B. Johnson .... p = 34,000 — 88 K
4. Straight-line, W. H. Burr .... p = 30,500 — 160 K

Permissible.

5. Boston building code, K_{max} = 70 .... p = 11,000 — 30 K
6. Chicago building code (Rankine), approx...... s = 10,000; $1/m$ = 6,000

Experiments.

Hollow columns.

✚ Tests of 10 cylindrical columns at Phoenixville for New York Building Department, as follows:

Outer diam ins	Thickness ins	Length ins	K	p ultimate lbs per sq inch
15	1	190	38.3	30,800
15	1⅛	190	38.7	27,700
15	1⅛	190	38.7	24,900
15	1⅛	190	38.3	25,200
15	1 11/64	190	38.5	32,100
15	1 3/16	190	38.8	> 40,400
8	1	160	64.0	31,900
8	1 3/64	160	64.4	26,800
6	1 9/64	120	67.2	22,700
6	1 7/64	120	66.5	26,300

★ Tests of 5 cylindrical columns about 157 ins long, at Watertown Arsenal, (17) pp 730–734, as follows.

Diameter, ins outer D	Diameter, ins inner d	K	p ultimate
8.7	6.0	60	> 25,720
7.9	5.3	65.9	> 30,380
7.2	4.8	72.4	25,470
6.4	4.1	83	27,210
5.7	4.0§	90.6	25,100

* Axis parallel to pin. † Axis parallel to web.
‡ Net load taken = 82.5 % of indicated load, on account of error of testing machine. Both indicated and net loads are plotted, with L/r for axis parallel to pin, both test cols having yielded in planes perp to pins.
§ Core eccentric. Thickness, at bottom, varied from 0.78 to 1.06 inch.

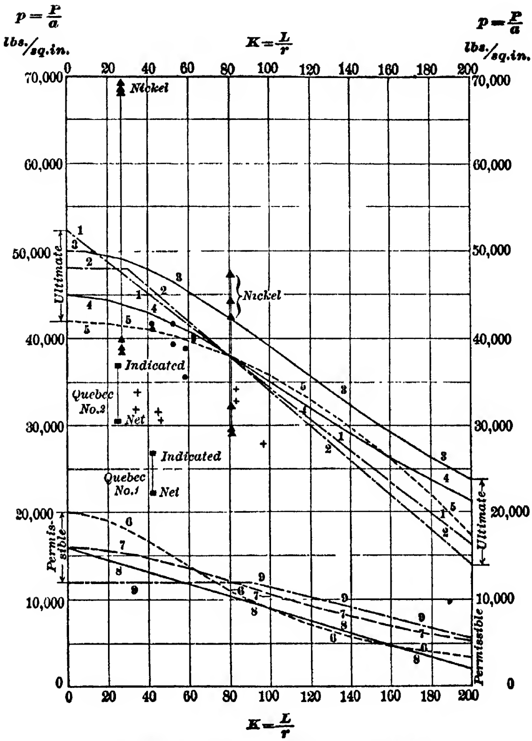


Fig. 23. Structural Steel Columns.

Solid Columns.

■ Usually accepted average crushing strength of short blocks, 100,000 lbs per square inch.
● Tests of 6 small cylinders at Watertown Arsenal, (18).

No	Diam ins	r ins	Length ins	K	p ultimate lbs per sq inch	
1	0.798	0.2	4	20	96,200	1886 p 1260. Cut from a cannon ball.
2	0.798	0.2	5	25	89,620	1890, p 738.
3	0.798	0.2	5	25	86,200	" " " "
4	0.749	0.187	5.482	29.3	76,643	1886, pp 1680-1.
5	0.750	0.1875	5.46	29	76,244	1886, pp 1680-1.
6	1.129	0.282	10.5	37.5	63,000	1894, p 105. Gun iron.

▲ Tests of 5 cylinders with short square ends, at Watertown Arsenal (17), pp 737-742. Diameter = 2 ins. r = 0.5 inch. L = length of round portion = 9 ins. K = 18. p, ult, from 61,800 to 74,780 lbs/sq in.

24. The experiments on small solid cast iron cylinders, above recorded, and many others by Tetmajer (16), indicate that the Rankine formula is correct in form; but the experiments with actual hollow cylindrical columns show that such columns fail under loads lower than those given by the formula, especially in the case of short columns. In thin hollow cols, the eccentricity of loading, which is sure to occur, even in careful experiments, and much more so in actual building operations, concentrates most of the pressure upon one side, and causes that side to act without proper support from the rest of the section. The col is thus prevented from acting as a whole.

25. Tetmajer (16), p 77, finds that **riveted columns** of structural steel **behave the same as simple rolled shapes**, provided: —
(1) the pitch of the rivets does not exceed 70 × entire flange thickness;
(2) the rivets completely fill the rivet holes;
(3) the weakening of the section by rivet holes ≯ about 12 per cent.

26. Quebec bridge failure. The collapse of the southern portion of the great steel cantilever bridge over the St. Lawrence river, near Quebec, on August 29, 1907, during construction, appears to have been due to the failure of a main compression member, "*A* 9 *L*," in the anchor arm. This member, Fig 25, 57 feet long, was composed of four webs, each from 3½ to 3¾ inches thick (made up of four plates, about ⅞ inch thick, riveted together) and 4 ft 6 ins wide, provided with flange angles 6 × 8 ins and 3½ × 8 ins, and spaced as in Fig 25, and connected with each other by latticing of 4″ × 3″ × ⅝″ and 3½″ × 3″ × ⅜″ angles on top and bottom, the angles being replaced by ½ inch batten plates for several feet at each end. The radius of gyration of the four webs alone, including the longer legs of the flange angles, acting unitedly, about the axis *a b*, is 19.5 inches. This gives, for the entire section, acting unitedly, L/r = 684/19.5 = 35.1.

27. According to the stress sheet, this member was **designed to sustain**, in service: —

Dead load, ...	11,249,000	pounds	
Live load, ...	4,019,000	"	
Dead and live load, ...	15,268,000	"	= 19,710 pounds per sq inch.
Wind	± 7,370,000	"	

28. For compression members where L/r is (as in this case) < 50, the **specification permitted** a load, in pounds per square inch, of

$$12{,}000\left(1 + \frac{\text{minimum stress}}{\text{maximum stress}}\right)^* \text{ or, in this member, of}$$

$$12{,}000 \times 1.7317^* = 20{,}780 \text{ lbs per square inch.}$$

29. The member appears to have sustained, at the time of its failure, an average pressure of about 12,500,000 ÷ 780 = say 16,000 lbs per square inch.

---

* Min stress = dead load — min live load
= 11,249,000 — 45,000 = 11,204,000.
Max stress = dead load + max live load + min live load
= 11,249,000 + 4,019,000 + 45,000 = 15,313,000.

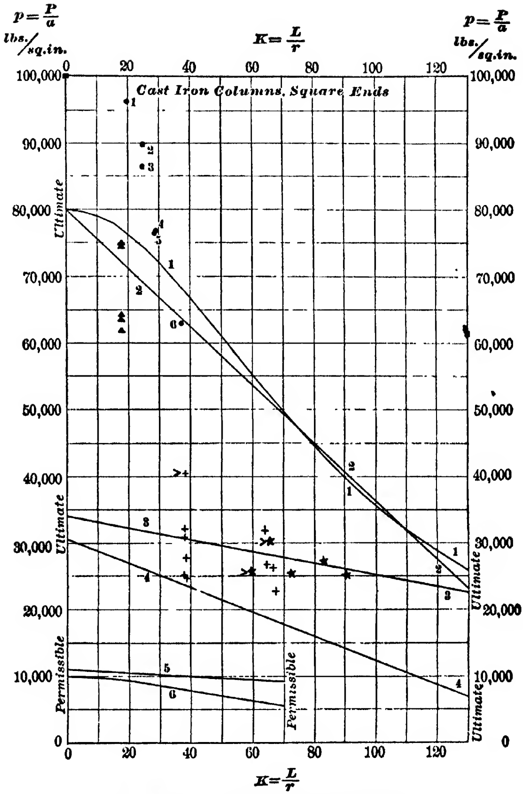


Fig. 24. Cast Iron Columns.

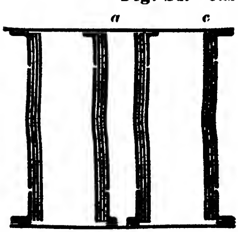


Fig. 25.
"A 9 L," Quebec bridge.

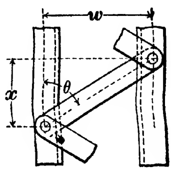


Fig. 26.
Lattice formulas.

30. Each of the four webs, including the longer leg of its flange angle, has, about its own neutral axis, cd, a radius of gyration of only about 1 inch, making its $L/r > 650$. For columns where L/r exceeds 50, the specification allows a load of only

$$(12{,}000 - 50\ L/r)\ \left(1 + \frac{\text{min stress}}{\text{max stress}}\right),^*$$

which, if applied to one of these webs, would permit only a *minus* load of some 35,000 lbs per square inch; but reliance was of course placed upon the angle latticing and upon the batten plates, to compel these four webs to act unitedly.

31. After the collapse of the structure, however, this member was found bent into the shape of a letter S, and the deflection occurred at right angles to the planes of the four webs, or in that direction in which it would have occurred if the bracing had been omitted. "The failure of lower chord $A\ 9\ L$ is an example of an insufficient lattice system." (Report of Quebec Bridge Commission.)

Stresses in Latticing.

32. "The unsatisfactory nature of the column formulas ..... is a matter of common knowledge among engineers, but **the column formulas may be considered to represent exact science in comparison with the lattice formulas.** Lattice formulas . ... fix, in a fashion, a value of the extreme fiber stress in the lattice bar, but only tests and experience can determine whether these give economical and safe results." (Report of Quebec Bridge Commission.)

33. The following **approximate method** (see communications from **Prof. Clyde T. Morris,** Ohio State University, Eng News, Nov 7, 1907, p 487, Feb 27, 1908, p 44) assumes (1) that the diagram of moments, in the columns, is a straight line, i e, that the increment of stress, per unit of length of column, is constant. This assumption makes the max stress, in the leaves, for a given max mom, less than it probably is; but, on the other hand, it is also assumed (2) that the stress, in the extreme fibers of the leaves, is uniformly distributed over the cross section of each leaf. This assumption (made because each portion of the leaf, between lattice connections, may deflect in either direction) makes the average stress greater than it is.

34. In a latticed (or "laced") column, composed of two leaves, Fig 26, let

s = max allowable unit comp stress in cross section of col
= max allowable unit comp stress in short blocks of the given material;
P = max allowable total load on column;
a = area of cross section of column;
p = P/a = max allowable avge unit comp stress, in cross section of col;
L = unsupported length of column;
r = least radius of gyration;
V = total stress transferred, by the lacing, from one leaf to the other
= half diff of stress betw the two leaves;
l = distance within which V is transferred;
v = V/l = increment of total stress, V, per unit of length of col;
w = dist betw rivet lines of lacing in the two leaves;
θ = angle betw lattice and axis of col;
x = $w\,.\cot\theta$ = longitudinal spread of one lattice bar;
n = number of lattice bars in each lattice panel;
vx/n = increment of stress, within x, in one lattice bar;
c = coeff in straight-line col formula $p = s - c\ L/r$;
= a small stress, of the same kind with s;
P_L = total longitudinal stress in one lattice bar.

* See foot-note *, p 1202.

35. Then

$V = (s - p)\ (a/2)$

$= \text{diff of unit stress} \times \text{area of one leaf}$ .... (1)

$v = V/l = (s - p)\ a\ /\ 2l$ .... (2)

$= \left(s - s + c\ \frac{L}{r}\right) a/2\,l = c\,L\,a\ /\ 2\,l\,r$ .... (3)

$v\,x/n = v\,w \cot\theta/n$ .... (4)

$$P_L = (v\,x/n) \sec\theta = \frac{v}{n} \cdot \frac{w}{\sin\theta}$$

$$= \frac{c\,L\,a}{2\,l\,r\,n} \cdot \frac{w}{\sin\theta} \quad \text{............ (5)}$$

36. In pin-end and round-end cols, the elastic curve is a simple one, and we have, for the dist, l, within which V is transferd, $l = L/2$; and $L\ /\ 2l = 1$.

In square-end and fixt-end cols, the curve reverses, with two points of contrary flexure; and we have: $l = L/4$, and $L\ /\ 2l = 2$.

Hence:

with round ends, $\mathbf{P_L} = \frac{c\,a}{r\,n} \cdot \frac{w}{\sin\theta}$ .... (6)

with square ends, $\mathbf{P_L} = \frac{2\,c\,a}{r\,n} \cdot \frac{w}{\sin\theta}$ .... (7)

37. The following **approximate method** (see **W. C. Armstrong,** Westn Soc of Engrs, Journal, June 1908, p 337) gives the total stress, P_L, in one lattice bar, **when the col is equally liable to fail in either direction.**

It is assumed that the tie plates carry 1/4, and the lacing 3/4, of the shear between the leaves. Let

d = depth of leaf;
I = mom of inertia of cross section;
$M = 2\,s\,I/d$ = resisting mom against bending in either direction; other symbols as in ¶ 34.

38. Then

$V = M\ /\ w = 2\,s\,I\ /\ d\,w$ .... (1)

$v = V\ /\ l = 2\,s\,I\ /\ d\,w\,l$ .... (2)

$v\,x\ /\ n = 2\,s\,I\,x\ /\ d\,w\,l\,n = 2\,s\,I \cot\theta\ /\ d\,l\,n$ .... (3)

$\mathbf{P_L} = \frac{3}{4} \cdot \frac{v\,x}{n} \cdot \sec\theta = \frac{1.5\,s\,I}{d\,l\,n \sin\theta}$ .... (4)

With round ends, $l = L/2$; and $\mathbf{P_L} = \frac{3\,s\,I}{d\,L\,n \sin\theta}$ .... (5)

With square ends, $l = L/4$; and $\mathbf{P_L} = \frac{6\,s\,I}{d\,L\,n \sin\theta}$ .... (6)

39. Supposing the lattice members themselves to be sufficiently strong, the integrity of the lattice system is still **limited by the possibility of rivet-slip,** the resistance to which, at any joint, is = No. of rivets × rivet section area × unit frictional resistance of rivets.

Prof. J. B. Johnson (Materials of Construction, pp 526–7) gives 12,000 lbs per sq in of rivet section as the **frictional resistance** for steel rivets; 10,000 lbs for iron rivets.

40. "**The size and strength of the pin** used have an appreciable effect on the results obtained, but the amount of this effect has not been determined." (Report of Quebec Bridge Commission.)

41. "**A compression member,** of usual design and dimensions cannot be expected to develop an ultimate strength much greater than about half that of **a tension member** made from the same material." *Ibid.*

PENCOYD FLOOR SECTIONS.

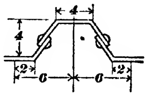
L = span, in feet.
C = coefficient.
W = distributed load, in lbs, per foot of floor width.

$$W = \frac{C}{L}.$$

Corrugated flooring, for bridges and buildings.
W = load producing fiber stress of 15,000 lbs per square inch.

SECTION 210 M.

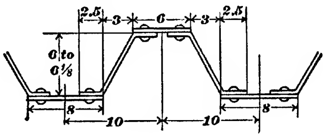
Dimensions in inches.



Thickness, inches. Web.	Flange.	Weight, lbs per sq ft.	C
5/16	1/4	14.8	44,000
11/32	5/16	18.4	55,000
3/8	3/8	21.9	66,000
13/32	7/16	25.5	77,400
7/16	1/2	29.1	88,800

SECTION 260 M.

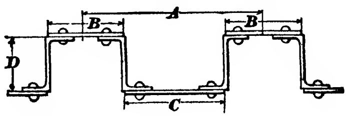
Dimensions in inches.



Thickness, inches. Web.	Flange.	Weight, lbs per sq ft.	C
1/4	1/4 to 5/8	20.0 to 30.7	105,000 to 186,000
5/16	3/8 to 3/4	26.5 to 37.2	143,000 to 224,000
3/8	3/8 to 3/4	29.4 to 40.1	153,000 to 237,000

Z Bar Flooring.

W = safe load.



Section No.	Dimensions, in inches. A	B	C	D	Thickness, ins. Z bars.	Plates.	Weight, lbs per sq ft.	C
1	15	6	9	4	1/4	1/4 to 1/2	25.9 to 36.1	93,400 to 147,400
					5/16		29.1 to 39.3	104,000 to 157,000
					3/8		32.3 to 42.5	114,400 to 167,000
2	18	8	10	5	5/16	5/16 to 9/16	32.1 to 42.3	143,000 to 209,400
					3/8		35.2 to 45.4	155,000 to 221,400
					7/16		38.4 to 48.6	166,400 to 233,000
3	21	9	12	6	3/8	3/8 to 5/8	39.3 to 49.5	203,400 to 281,000
					7/16		42.4 to 52.6	217,400 to 294,000
					1/2		45.5 to 55.7	231,000 to 307,200

WEIGHT AND STRENGTH OF IRON CHAINS.

Table of strength of chains.

Chains of superior iron will require ¼ to ⅓ more to break them. (Original.)

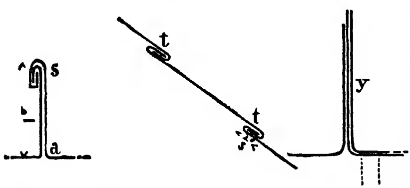
Diam of rod of which the links are made.	Weight of chain per ft run.	Breaking strain of the chain		Diam of rod of which the links are made	Weight of chain, per ft run.	Breaking strain of the chain.	
Ins	Pds.	Pds.	Tons	Ins.	Pds	Pds.	Tons.
3-16	.5	1731	.773	1	10.7	49280	22.00
¼	.8	3069	1.37	1⅛	12.5	59226	26.44
5-16	1.	4794	2.14	1¼	16.	73114	32.64
⅜	1.7	6922	3.09	1⅜	18.3	88301	39.42
7-16	2.	9408	4.20	1½	21.7	105280	47.00
½	2.5	12320	5.50	1⅝	26.	123514	55.14
9-16	3.2	15590	6.96	1¾	28.	143293	63.97
⅝	4.3	19219	8.58	1⅞	32.	164505	73.44
11-16	5.	23274	10.39	2	38.	187152	83.55
¾	5.8	27687	12.36	2¼	54.	224448	100.2
13-16	6.7	32301	14.42	2½	71.	277088	123.7
⅞	8.	37632	16.80	2¾	88.	335328	149.7
15-16	9.	43277	19.32	3	105.	398944	178.1

The links of ordinary iron chains are usually made as short as is consistent with easy play, in order that they may not become bent when wound around drums, sheaves, &c.; and that they may be more easily handled in slinging large blocks of stone, &c. U. S. Government experiments, 1878, prove that studs weaken the links.

When so made, their weight per foot run is quite approximately 3½ times that of a single bar of the round iron of which they are composed. Since each link consists of two thicknesses of bar, it might be supposed that a chain would possess about double the strength of a single bar; but the strength of the bar becomes reduced about 30 per cent, by being formed into links; so that the chain has but about 70 per cent of the strength of two bars. As a thick bar will not sustain as heavy a unit stress as a thinner one, so of course, stout chains are proportionally weaker than slighter ones. In the foregoing table, 20 tons *per square inch*, is assumed as the average breaking strain of a single straight bar of ordinary rolled iron, 1 inch in diameter or 1 inch square; 19 tons, from 1 to 2 inches; and 18 tons, from 2 to 3 inches. Deducting 30 per cent from each, we have as the breaking strain of the two bars composing each link, as follows: 14 tons *per square inch*, up to 1 inch diameter; 13.3 tons, from 1 to 2 inches; and 12.6 tons, from 2 to 3 inches diameter; and upon these assumptions the table is based. The weights are approximate; depending upon the exactness of diameter of the iron, and shape of link.

TIN AND ZINC.

The pure metal is called block tin. When perfectly pure, (which it rarely is, being purposely adulterated, frequently to a large proportion, with the cheaper metals lead or zinc,) its sp grav is 7.29; and its weight per cub ft is 455 lbs. It is sufficiently malleable to be beaten into tin foil, only $\frac{1}{1000}$ of an inch thick. Its tensile strength is but about 4600 lbs per sq inch; or about 7000 lbs when made into wire. It melts at the moderate temperature of 442° Fah. Pure block tin is **not used for common building purposes; but thin plates of sheet iron, covered with** it on both sides, constitute the *tinned plates*, or, as they are called, the *tin*, used for covering roofs, rain pipes, and many domestic utensils. For roofs it is laid on boards.



The sheets of tin are united as shown in this fig. First, several sheets are joined together in the shop, end for end, as at *tt*; by being first bent over, then hammered flat, and then soldered. These are then formed into a roll to be carried to the roof; a roll being long enough to reach from the peak to the eaves. Different rolls being spread up and down the roof, are then united along their sides by simply being bent as at *a* and *s*, by a tool for that purpose. The roofers call the bending at *s* a *double groove*, or *double lock;* and the more simple ones at *t*, a *single groove*, or lock.

To hold the tin securely to the sheeting boards, pieces of the tin 3 or 4 ins long, by 2 ins wide, called cleats, are nailed to the boards at about every 18 ins along the joints of the rolls that are to be united, and are bent over with the double groove *s*. This will be understood from *y*, where the middle piece is the cleat, before being bent over. The nails should be 4-penny slating nails, which have broader heads than common ones. As they are not exposed to the weather, they may be of plain iron.

Much use is made of what is called leaded tin, or ternes, for roofing. It is simply sheet-iron coated with lead, instead of the more costly metal tin. It is not as durable as the tinned sheets, but is somewhat cheaper.

The best plates, both for tinning and for ternes, are made of charcoal iron; which, being tough, bears bending better. Coke is used for cheaper plates, but inferior as regards bending. In giving orders, it is important to specify whether charcoal plates or coke ones are required; also whether *tinned* plates, or *ternes*.

Tinned and leaded sheets of Bessemer and other cheap **steel**, are now much used. They are sold at about the price of charcoal tin and terne plates.

There are also in use for roofing, certain compound metals which resist tarnish better than either lead, tin, or zinc; but which are so fusible as to be liable to be melted by large burning cinders falling on the roof from a neighboring conflagration.

A roof covered with tin or other metal should, if possible, slope not much *less* than five degrees, or about an inch to a foot; and at the eaves there should be a sudden fall into the rain-gutter, to prevent rain from backing up so as to overtop the double-groove joint *s*, and thus cause leaks. Where coal is used for fuel, tin roofs should receive two coats of paint when first put up, and a coat at every 2 or 3 years after. Where wood only is used, this is not necessary; and a tin roof, with a good pitch, will last 20 or 30 years.

Two good workmen can put on, and paint outside, from 250 to 300 sq ft of tin roof, per day of 8 hours.

Tinned iron plates are sold by the **box.** These boxes, unlike glass, have *not* equal areas of contents. They may be designated or ordered either by their names or sizes. Many makers, however, have their private brands in addition; and some of these have a much higher reputation than others.

Table of Tinned and Terne Plates.

Caution.—Boxes often contain considerably less weight of tin plate than the table requires; the plates being rolled thin and plated thin, in order to enable mechanics to get pay for more material than they furnish.

The *marks* indicate the *thicknesses*, approximately as follows:

Mark.	Number Birmingham wire gauge.	Ins.	Lbs per sq ft.	Mark.	Number Birmingham wire gauge	Ins.	Lbs per sq ft.
IC	30	.012	.48	DC	27	.016	.64
IX	28	.014	.56	DX	25	.020	.80
IXX	27	.016	.64	DXX	23	025	1 00
IXXX	26	.018	.72	DXXX	22	.028	1.13
IXXXX	25	.020	.80	DXXXX	21	.032	1.29

Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds	Size, inches	Mark.	No of sheets in a box	Weight per box, pounds.	Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds
9 × 18	IC	225	130	13 × 13	IXX	225	194	16 × 16	IC	225	205
"	IX	"	162	13 × 26	IC	112	135	"	IX	"	256
10 × 10	IC	"	80	"	IX	"	169	"	IXX	"	294
"	IX	"	100	"	IXX	"	194	16 × 19	IX	112	152
10 × 14	IC	"	112	14 × 14	IC	225	156	17 × 17	IX	"	141
"	IX	"	140	"	IX	"	196	"	IXX	"	166
"	IXX	"	161	"	IXX	"	225	17 × 25	DX	100	252
"	IXXX	"	182	"	IXXX	"	254	"	DXX	"	294
"	IXXXX	"	203	14 × 20	IC	112	112	18 × 18	IX	112	162
10 × 20	IC	"	160	"	IX	"	140	"	IXX	"	180
"	IX	"	200	"	IXX	"	161	"	IXXXX	"	235
11 × 11	IC	"	97	"	IXXX	"	182	20 × 20	IX	"	200
"	IX	"	121	"	IXXXX	"	203	"	IXX	"	230
11 × 22	IC	112	97	14 × 22	IX	"	154	"	IXXX	"	260
"	IX	"	121	"	IXX	"	177	"	IXXXX	"	290
"	IXX	"	139	14 × 24	IX	"	168	20 × 28	IC	"	224
12 × 12	IC	225	112	"	IXX	"	193	"	IX	"	280
"	IX	"	140	14 × 25	IC	"	140	"	IXX	"	322
"	IXX	"	161	"	IX	"	175	"	IXXX	"	364
12 × 24	IC	112	115	"	IXX	"	201	"	IXXXX	"	406
"	IX	"	144	14 × 26	IXXX	"	237				
"	IXX	"	166	14 × 28	IC	"	157				
"	IXXX	"	187	"	IX	"	196				
12½ × 17	DC	100	98	"	IXX	"	225				
"	DX	"	120	14 × 30	IXX	"	241				
"	DXX	"	147	14 × 31	IX	"	217				
"	DXXX	"	168	"	IXX	"	249				
"	DXXXX	"	189	15 × 15	IX	225	225				
13 × 13	IC	225	135	"	IXX	"	259				
"	IX	"	169	"	IXXX	"	326				

Terne Plates.

Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds
10 × 20	IC	112	80
"	IX	"	100
14 × 20	IC	"	112
"	IX	"	140
20 × 28	IC	"	224
"	IX	"	280

Sheets of larger size may be made to special order; those of tinned iron, in England; but leaded ternes are made in Philada also, and elsewhere

A box of 225 sheets of 13¾ by 10 contains 214.84 sq ft, but, allowing for overlapping, it will cover but about 150 sq ft of roof; even without any allowance for the waste which occurs in cutting away portions in order to fit at angles, &c.

To find the area of roof covered by any sheet, first deduct 2 ins from its width, and 1 inch from its length.

Zinc, in sheets, and laid in the same manner as slates, is much used in some parts of Europe for roofing. By exposure to the weather, it soon becomes covered by a thin film of white oxide, which protects it from further injury, and renders the roof very durable. Corrugated sheet zinc is also used. See Galvanized Sheet Iron.

Zinc sheets are usually about 3 ft by 7 or 8 ft. The gauge differs from that of iron; thus No 13 is .032 of an inch thick, or 1 22 lbs per sq ft; No 14 = .035 inch, and 1 35 lbs; No 15 = .042 inch, and 1 49 lbs, No 16 = 049 inch, and 1.62 lbs per sq ft. Any of these numbers may be used on roofs, for which purpose it should be very pure.

Water kept in zinc vessels is said to become injurious to health; and recently an outcry has on that account arisen against galvanized-iron service-pipes in dwellings. Yet such have been in use for many years in New England, Philada, and elsewhere, without as yet any deleterious effects. This is possibly owing to the fact that service-pipes being short, the water is usually all drawn through them several times a day; and hence does not remain in contact with the zinc or lead long enough to acquire a poisonous character. In taking possession of a house in which the water has remained stagnant in the service pipes for some considerable time, such water should all be run to waste; otherwise sickness may ensue from its use.

Roof copper is usually in sheets of 2½ feet × 5 feet; or 12½ square feet, weighing 10 to 14 lbs per sheet; and is laid on boards.* No solder is used in the horizontal joints as it is in tin roofs; but both the horizontal and the sloping joints are formed by only overlapping and bending the sheets, much as shown by the figs on page 1208; except that the horizontal joints are bent or locked together, and then flattened down close.

Sheet lead. List of **standard weights** in lbs per square foot. Thicknesses in decimals of an inch.

Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.
2.5	.042	4	.068	6	.102	8	.136	10	.170	14	.237
3	.051	5	.085	7	.119	9	.153	12	.203	16	.271

Weight of Metal Balls.

W = Weight of ball, in pounds. **D** = Diameter of ball, in inches.

Lead = (700 lbs per cub ft) **W** = $0.212106\ D^3$; **log W** = $\bar{1}.326\ 5529 + 3 \log D$

Copper = (550 lbs per cub ft) **W** = $0.166655\ D^3$; **log W** = $1.221\ 8176 + 3 \log D$

Brass = (500 lbs per cub ft) **W** = $0.151504\ D^3$; **log W** = $\bar{1}.180\ 4249 + 3 \log D$

Steel and Wrought Iron = (485 lbs per cub ft) **W** = $0.146959\ D^3$; **log W** = $\bar{1}.167\ 1966 + 3 \log D$

Cast Iron = (450 lbs per cub ft) **W** = $0.136354\ D^3$; **log W** = $\bar{1}.134\ 6674 + 3 \log D$

For steel, wrought iron and cast iron balls, see also tables, pp 875 and 877.

Lead Pipe.
Standard Sizes.

Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wt in lbs per ft.	Inner diam.	Thickness.	Wt in lbs per ft.
Ins.	Ins.		Ins.	Ins.		Ins.	Ins.		Ins.	Ins.	
⅜	.06	7 lbs R	¾	.08	16 lbs R	1½	.14	3.5	3	3-16	9
"	.08	10 oz F	"	.10	1¼ lbs F	"	.17	4.25	"	¼	12
"	.12	1 lb F	"	.12	1¾ lbs F	"	.19	5.	"	5-16	16
"	.16	1¼ lbs F	"	.16	2¼ lbs F	"	.23	6.5	"	⅜	20
"	.19	1½ lbs F	"	.20	3 lbs F	"	.27	8	3½	3-16	9 5
½	.07	9 lbs R	"	.23	3½ lbs F	1¾	.13	4	"	¼	15
"	.09	¾ lb F	"	.30	4¾ lbs F	"	.17	5	"	5-16	18.5
"	.11	1 lb F	1	.10	24¾ lbs R	"	.21	6.5	"	⅜	22
"	.13	1¼ lbs F	"	.11	2 lbs F	"	.27	8 5	4	3-16	12 5
"	.16	1¾ lbs F	"	.14	2½ lbs F	2	.15	4.75	"	¼	16
"	.19	2 lbs F	"	.17	3¼ lbs F	"	.18	6	"	5-16	21
"	.25	3 lbs F	"	.21	4 lbs F	"	.22	7	"	⅜	25
⅝	.08	12 lbs R	"	.24	4¾ lbs F	"	.27	9	4½	8-16	14
"	.09	1 lb F	1¼	.10	2 lbs F	2½	8-16	8	"	¼	18
"	.13	1½ lbs F	"	.12	2½ lbs F	"	¼	11	5	¼	20
"	.16	2 lbs F	"	.14	3 lbs F	"	5 16	14	"	⅜	31
"	.20	2½ lbs F	"	.16	3¾ lbs F	"	⅜	17			
"	.22	2¾ lbs F	"	.19	4¾ lbs F						
"	.25	3½ lbs F	"	.25	6 lbs F						

Lead service pipes for single dwellings in Philadelphia are usually of from ½ inch bore, wt 1 to 2½ lbs; to ⅝ inch bore, wt 1½ to 3 lbs per ft run, according to head. They **rarely burst** from sudden closing of stopcocks; but sometimes do so from the freezing of the contained water.

***To which it is held by copper cleats;** as at Fig y, page 1208.

ROLLED LEAD, COPPER, and BRASS: Sheets and Bars.

Thickness or Diameter, or side, in Inches.	LEAD. Sheets, per Square Foot.	LEAD. Square Bars; 1 Foot long.	LEAD. Round Bars; 1 Foot long.	COPPER. Sheets, per Square Foot.	COPPER. Square Bars; 1 Foot long.	COPPER. Round Bars; 1 Foot long	BRASS. Sheets, per Square Foot.	BRASS. Square Bars; 1 Foot long.	BRASS. Round Bars; 1 Foot long.	Thickness or Diameter, or side, in Inches.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	
1-32	1.86	.005	.004	1.44	.004	.003	1.36	.004	.003	1-32
1-16	3.72	.019	.015	2.89	.015	.012	2.71	.014	.011	1-16
3-32	5.58	.044	.034	4.33	.034	.027	4.06	.032	.025	3-32
1/8	7.44	.078	.061	5.77	.060	.047	5.42	.056	.044	1/8
5-32	9.30	.121	.095	7.20	.094	.074	6.75	.088	.069	5-32
3-16	11.2	.174	.137	8.66	.135	.106	8.13	.127	.100	3-16
7-32	13.0	.237	.187	10.1	.184	.144	9.50	.173	.136	7-32
1/4	14.9	.310	.244	11.5	.240	.189	10.8	.226	.177	1/4
5-16	18.6	.485	.381	14.4	.376	.295	13.5	.353	.277	5-16
3/8	22.3	.698	.548	17.3	.541	.425	16.3	.508	.399	3/8
7-16	26.0	.950	.746	20.2	.736	.578	19.0	.691	.541	7-16
1/2	29.8	1.24	.974	23.1	.962	.755	21.7	.903	.709	1/2
9-16	33.5	1.57	1.23	26.0	1.22	.955	24.3	1.14	.900	9-16
5/8	37.2	1.94	1.52	28.9	1.50	1.18	27.1	1.41	1.11	5/8
11-16	40.9	2.34	1.84	31.7	1.82	1.44	29.8	1.70	1.34	11-16
3/4	44.6	2.79	2.19	34.6	2.16	1.70	32.5	2.03	1.60	3/4
13-16	48.3	3.27	2.57	37.5	2.55	1.99	35.2	2.38	1.87	13-16
7/8	52.1	3.80	2.98	40.4	2.94	2.31	37.9	2.76	2.17	7/8
15-16	56.0	4.37	3.42	43.3	3.38	2.65	40.6	3.18	2.49	15-16
1.	59.5	4.96	3.90	46.2	3.85	3.02	43.3	3.61	2.84	1.
1 1/8	66.9	6.27	4.92	52.0	4.87	3.82	48.7	4.57	3.60	1 1/8
1 1/4	74.4	7.75	6.09	57.7	6.01	4.72	54.2	5.64	4.43	1 1/4
1 3/8	81.8	9.37	7.37	63.5	7.28	5.72	59.6	6.82	5.37	1 3/8
1 1/2	89.3	11.2	8.77	69.3	8.65	6.80	65.0	8.12	6.38	1 1/2
1 5/8	96.7	13.1	10.3	75.1	10.2	7.98	70.4	9.53	7.49	1 5/8
1 3/4	104.	15.2	11.9	80.8	11.8	9.25	75.9	11.1	8.68	1 3/4
1 7/8	112	17.5	13.7	86.6	13.5	10.6	81.3	12.7	9.97	1 7/8
2.	119.	19.8	15.6	92.3	15.4	12.1	86.7	14.4	11.3	2.

Seamless brass tubes. Principal sizes Extras, in cents per pound, over base price. For base price, see price list.
Copper tubes, 3 cents per pound extra.

Thickness.		Outer Diameter, inches.										
Stubs gage.	Ins.	1/4	1/2	1	1 1/2	2	3	4	5	6	7	7 3/4
4	0.238	....	12	6	3	1	1	2	5	9	13	18
11	0.120	....	"	"	"	"	"	"	"	"	"	"
16	0.065	....	13	8	4	4	4	7	11	15	19	24
18	0.049	40	15	9	6	6	7	11	15	19	23	28
20	0.035	43	18	13	10	9	11	15	19	23	27	32
22	0.0295	50	21	16	15	13	16	20	....	....	....	....
25	0.0230	65	28	22	24	....	....	....	....	....	....	....

Average ultimate tensile strength of Metals.

The ultimate tensile or pulling load per square inch of any material is frequently called its *constant*, *coefficient*, or *modulus* of tension, or of tensile strength.	Pounds per sq inch.	Tons per sq in.
Antimony, cast	1000	.45
Bismuth, cast	3200	1 4
Brass, cast 8 to 13 tons, say 18000 to 29000 lbs	23500	10 5
" wire, unannealed or hard, 80000. Annealed	49000	22
Bronze, phosphor wire, hard, 150000. Annealed	63000	28.1
Copper, cast 18000 to 30000	24000	10 7
" sheet	30000	13 4
" bolts, 28000 to 38000	33000	14.7
" wire (annealed 16 tons); unannealed	60000	26.8
Gold, cast	20000	8.9
" wire, 25000 to 30000	27500	12 3
Gun metal of copper and tin, 23000 to 55000	39000	17.4
" " cast iron, U. S. ordnance, 36000 to 40000.	38000	17
Iron, cast, English.........13400 to 22400	17900	8
" " ordinary pig..13000 to 16000	14500	6.47
American cast iron averages one-fourth more than the above.		
Average cast iron, when sound, stretches about .00018; or 1 part in 5555 of its length; or ⅛ inch in 57.9 ft. for every ton of tensile strain per sq inch, up to its elastic limit, which is at about ½ its break-strain. The extent of stretching, however, varies much with the quality of the iron, as in wrought-iron.		
Cast, malleable, annealed, 18 to 25 tons	48160	21.5
Iron and Steel, rolled.—See pp 751 to 754, 1150 to 1156.		
Lead, cast, 1700 to 2400 by author	2050	0 92
" wire, 1200 to 1600. Pipe 1600 to 1700 " "	1650	0 74
Platinum wire, annealed, 32000. Unannealed	56000	25
Steel and Iron, rolled.—See pp 751 to 754, 1150 to 1156.		
Silver, cast	41000	18 3
Tin, English block	4600	2 0
" wire	7000	3.1
Zinc, cast...3000 to 3700; (the last by author)	3350	1.5

Large bars of metal bear less per sq inch than small ones.

Iron bars re-rolled cold have tensile strength increased 25 to 50 per ct, with no increase of density. They are said to lose this strength if reheated.

The use of lead in masonry joints. See pp 634 and 1213. Under pressures usually less than the crushing stress of the stone, the lead flows laterally, and, by means of its friction with the stone, exerts, upon the latter, a tearing action normal to the pressure and tending to split the stone into prisms whose axes form slight angles with the line of pressure. This of course greatly weakens the stone.

Sheet lead is sometimes placed in the joints of stone columns, with a view to equalize the pressure, and thus increase the strength of the column. But experiments have proved that the effect is directly the reverse, and that the column is *materially weakened* thereby. See pp 634 and 1212.

Average crushing load for Metals.

It must be remembered that these are the loads for pieces but two or three times their least side in height. As the height increases, the crushing load diminishes.

	Pounds per sq inch	Tons per sq. inch.
Cast Iron, usually. It is usually assumed at 100000 lbs, or say 45 tons per sq inch. Its crushing strength is usually from 6 to 7 times as great as its tensile. Within its average elastic limit of about 15 tons per sq inch average cast iron shortens about 1 part in 5555, or ⅛ inch in 58 ft under each ton per sq inch of load, or about twice as much as average wrought iron. Hence at 15 tons per sq inch it will shorten about 1 part in 370; or full ⅛ inch in 4 feet. Different cast irons may however vary 10 to 15 per ct either way from this.	85000 to 125000	38 to 56
U. S. Ordnance, or gun metal; Some	175000	78.1
Wrought iron, within elastic limit	22400 to 35840	10 to 16
Its elastic limit under pressure averages about 13 tons per sq inch. It begins to shorten perceptibly under 8 to 10 tons, but recovers when the load is removed. With from 18 to 20 tons, it shortens *permanently*, about $\frac{1}{80}$th part of its length; and with from 27 to 30 tons, about $\frac{1}{16}$th part, as averages. The crushing weights therefore in the table are not those which absolutely mash wrought iron entirely out of shape, but merely those at which it yields too much for most practical building purposes. About 4 tons per sq inch is considered its average safe load, in pieces not more than 10 diams long; and will shorten it ⅛ inch in 30 ft. average	29120	13
Brass, reduced $\frac{1}{10}$th part in length, by 51000; and ½ by	165000	73.6
Copper, (cast,) crumbles	117000	52.2
(wrought) reduced ⅛th part in length, by	103000	46.0
Tin, (cast,) reduced $\frac{1}{16}$th in length, by 8800; and ⅓ by	15500	6.92
Lead, (cast,) reduced ½ of its length, by 7000 to 7700		3.28
" By writer. A piece 1 inch sq, 2 ins high, at 1200 lbs the compression was 1-200 of the ht, at 2000, 1-29; at 3000, 1-8, at 5000, 1-3; at 7000, 1-2 of the ht.		
Spelter or **Zinc,** (cast.) By writer. A piece 1 inch square, 4 ins high, at 2000 lbs was compressed 1-400 of its ht; at 4000, 1-200, at 6000, 1-100, at 10000, 1-38, at 20000, 1-15, at 40000 yielded rapidly, and broke into pieces.		
Steel, 224000 lbs or 100 tons shorten it from .2 to .4 part.		
" American. Black Diamond steel works, Pittsburg, Penn. experiments by Lieut W. H. Shock, U. S N., on pieces ½ in square, and 3½ ins, or 7 sides long.		
" Untempered, 100100 to 104000	102050	45.5
" Heated to light cherry red, then plunged into oil of 82° Fah, 173200 to 199200	186200	83.1
" Heated to light cherry red, then plunged into water of 79° Fah; then tempered on a heated plate, 325400 to 340800	333100	148.7
" Heated to light cherry red, then plunged into water of 79° Fah, 275600 to 400000	337800	150.8
" **Elastic limit,** 15 to 27 tons	47040	21
" **Compression,** within elas limit averages abt 1 part in 13300 or .1 of an inch in 111 ft per ton per sq inch; or .1 of an inch in 5.3 ft under 21 tons per sq inch.		
Best steel knife edges, of large R R weigh scales are considered safe with 7000 lbs pres per lineal inch of edge; and solid cylindrical steel **rollers** under bridges, and *rolling on steel* safe with $\sqrt{\text{diam in ins}} \times 3\ 100\ 000$, in lbs per lineal inch of roller parallel to axis. **And per the same, for**		

Solid cast iron wheels rolling on wrought iron, $\sqrt{\text{Diam ins}} \times 352\ 000$.

" " " " " " cast iron, $\sqrt{\text{Diam ins}} \times 222\ 222$.

Solid steel " " " steel, $\sqrt{\text{Diam ins}} \times 1\ 300\ 000$.

" " " " " wrought iron, $\sqrt{\text{Diam ins}} \times 1\ 024\ 000$.

" " " " " cast iron, $\sqrt{\text{Diam ins}} \times 850\ 000$.

From "Specifications for Iron Drawbridge at Milwaukee," by Don J. Whittemore, C. E.

Average ultimate tensile strengths of Stone, etc.

The strengths in all these tables may readily be one-third part more or less than our averages.	Pounds per sq. inch.	Tons per sq. ft.		Pounds per sq. inch	Tons per sq. ft.
Brick, 40 to 400...............	220	14.1	Marble, strong, wh. Italy.*	1034	66.5
Caen stone, 100 to 200.....	150	9.7	" Champlain, variegated *	1666	107.1
			" Glenn's F'lls. N.Y blk,* 750 to 1034	892	57.4
			" Montg'y co, Pa, gray *.....	1175	75.6
			" " white *...	734	47.2
			" Lee, Mass. white.*	875	56.3
Cement and concrete, see articles, Cement and Concrete.			" Manchester, Vt,* 550 to 800.........	675	43.4
			" Tennessee, variegated *	1034	66.5
			Oolites, 100 to 200..........	150	9.7
			Plaster of Paris, well set.	70	4.5
			Rope, Manilla, best.......	12000	771
Glass, 2500 to 9000	5750	369.6	" hemp, best...........	15000	965
Glue holds wood together with from 300 to 800...	550	35	Sandstone, Ohio *..........	105	6.75
Horn, ox......................	9000	579	" Picton, N. S*	434	27.9
Ivory.........	16000	1029	" Conn, red.*....	590	37.9
Leather belts, 1500 to 5000. Good................	3000	193	Slate, Lehigh *..............	2475	159.1
			" Peach bot'm,* 3025 to 4600	3812	245.1
Mortar, common, 6 mos old, 10 to 20.....	15	.96	Stone, Ransome's artif....	300	19.3
			Whalebone	7600	489

* **By the author's trials** with one of Riehlé's testing machines Sections broken 1½ sq inches.

Ultimate average crushing loads in tons, per square foot, for stones, &c. The stones are supposed to be ON BED, and the heights of all to be from 1.5 to 2 times the least side Stones generally begin to crack or split under about one-half of their crushing loads. In practice, neither stone nor brickwork should be trusted with more than ⅙ to $\frac{1}{10}$th of the crushing load, according to circumstances. **When thoroughly wet** some absorbent sandstones lose fully half their strength.

1 ton (2240 lbs.) per sq ft. = 15.55 lbs. per sq. inch.	Tons per sq. ft.	Mean. Tons.
Granites and Syenites.	300 to 1200	750
Basalt	....	700
Limestones and Marbles *	250 to 1000	625
Oolites, good	100 to 250	175
Brownstone · Connecticut—		
"Building"	570 to 970	775
"Bridge"	400 to 630	535
Brick*	40 to 300	170
Brickwork, ordinary, cracks with*	20 to 30	25
Brickwork, good, in cement*	30 to 40	35
Brickwork, first-rate, in cement	50 to 70	60
Slate	400 to 800	600
Caen Stone.................	70 to 200	135
" " to crack.....	....	70
Chalk, hard,..............	20 to 30	25
Plaster of Paris, 1 day old..............................	....	40

1 ton (2240 lbs.) per sq. ft. = 15.55 lbs. per sq. inch.	Tons per sq. ft.	Mean. Tons.
Cement, Portland, neat, U. S. or foreign, 7 days in water..... ..	75 to 150	112.5
Common U S cements, neat, 7 days in water	15 to 30	22.5
Concrete of Port. cement, sand, and gravel or brok stone in the proper proportions, rammed 1 m old	12 to 18	15
6 months old........... .	48 to 72	60
12 months old............	74 to 120	97
With good **common hyd cements,** abt .2 to .25 as much		
Coignet beton, 3 months old.............	100 to 150	125
Rubble masonry, mortar, rough.........	15 to 35	25
Glass, green, crown and flint	1300 to 2300	1800
or 8 times that of granite		
Ice, firm†	12 to 18	15

Crushing height of Brick and Stone.

If we assume the wt of ordinary brickwork at 112 lbs per cub ft, and that it would crush under 30 tons per sq ft, then a vert uniform column of it 600 ft high, would crush at its base, under its own wt. Caen stone, weighing 130 lbs per cub ft, would require a column 1376 ft high to crush it. Average sandstones at 145 lbs per cub ft, would require one 4158 ft high; and average granites, at 165 lbs per cub ft, one of 8145 feet. But stones begin to crack and splinter at about half their ultimate crushing load; and in practice it is not considered expedient to trust them with more than ⅙th to $\frac{1}{10}$th part of it, especially in important works; inasmuch as settlements, and imperfect workmanship, often cause undue strains to be thrown on certain parts.

The Merchants' shot-tower at Baltimore is 246 ft high; and its base sustains 6½ tons per sq ft. The base of the granite pier of Saltash bridge, (by Brunel,) of solid masonry to the height of 96 ft, and supporting the ends of two iron spans of 455 ft each, sustains 9½ tons per sq ft. The base of a brick chimney at Glasgow, Scotland, 468 ft high, bears 9 tons per sq ft; and Professor Rankine considers that in a high gale of wind, its leeward side may have to bear 15 tons The highest pier of Rocquefavour stone aqueduct, Marseilles, is 305 ft, and sustains a pressure at base of 13½ tons per sq ft.

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* **Trials at St. Louis bridge,** by order of Capt James B. Eads, C. E., showed that some magnesian limestone did not yield under less than 1100 tons per sq ft. A column 8 ins high, 2 ins diam, shortened 0 00[illegible]5 inch under pressure; and recovered when relieved

Experiments made with the Govt testing machine at Watertown, Mass. 1882-3, gave 1400 tons per sq ft ultimate crushg load for white and blue marble from Lee, Mass. 700 for blue marble from Montgomery Co, Pa, 960 for limestone from Conshohocken, Pa, 500 for limestone from Indiana, 840 for red sandstone from Hummelstown, Pa, 260 to 1000 for yellow Ohio sandstone; Phila bricks, flatwise; hard, machine made, 350 to 700 tons; hand-made, 700 to 1300; pressed, machine-made, 450 to 580; Brickwork columns, 12 ins sq and 12 ins high; in lime, 100 tons; in cement, 150.

† Experiments by Col. Wm. Ludlow, U. S. A., with Govt testing machines in 1881, gave from [illegible] to 64 tons per sq ft for pure, hard ice; and 16 to 59 tons for inferior grades. The specimens ([illegible] and 12-inch cubes) compressed ⅛ to 1 inch before crushing.

STONE BEAMS.

Table of safe quiescent extraneous loads for beams of good building granite one inch broad, supported at both ends, and loaded at the center; assuming the safe load to be one-tenth of the breaking one; and the latter to be 100 lbs for a beam 1 inch square, and 1 foot clear span. The half weight of the beams themselves is here already deducted at 170 lbs per cub ft.

Depth in ins.	CLEAR SPANS IN FEET.											
	1	2	3	4	5	6	7	8	10	12	15	20
	Safe center loads in pounds.											
1	10	5										
2	40	20	13	10								
3	90	45	29	21	17							
4	160	79	52	39	31	26	21					
5	250	124	82	61	48	40	34					
6	360	179	119	89	70	58	48	42	32			
7	490	244	162	120	96	79	67	58	45	36	27	16
8	639	319	212	158	126	104	88	76	59	47	36	22
10	999	499	331	248	197	163	139	120	94	76	58	38
12	1439	718	478	357	284	236	201	174	137	111	85	58
14	1959	978	650	487	388	322	274	238	188	153	118	81
16	2559	1278	850	636	507	421	359	312	246	201	157	109
18	3239	1618	1077	806	643	534	455	396	313	257	200	141
20	3999	1998	1329	995	794	660	563	490	388	319	249	176
22	4839	2417	1609	1205	961	860	682	594	470	387	303	216
24	6758	2877	1916	1434	1145	951	813	708	562	463	362	260
27	7288	3642	2425	1815	1450	1205	1030	898	713	588	462	332
30	8998	4496	2995	2243	1791	1489	1273	1110	882	728	573	415
33	10888	5441	3624	2714	2168	1803	1542	1345	1069	883	696	505
36	12958	6476	4314	3231	2581	2147	1836	1603	1275	1054	832	606

If uniformly distributed over the clear span, the safe extraneous loads will be twice as great as those in the table.

For good slate on bed the safe loads may be taken at about 3 times; **for good sandstone on bed** at about one-half; and for **good marble or limestone on bed** at about the same as those in the table.

CLAY. Specific Gravity.*

The specific gravity of the minerals (chiefly alumina silicates) composing clay, ranges ordinarily between 2.5 and 3; that of the solid portions of clays between 2.3 and 2.9.†

But the specific gravity of clay, *considered as a porous material,* varies between much wider limits, being affected by its porosity, the quantity of water contained, etc. See p. 211, ¶¶ 11 to 13. Thus, we find given values from 1.66 to 2.64; ‡ clay with gravel, 2.48; § potter's clay 1.8 to 2.1; dry clay, in lumps, loose, 1.0.

* See "Clays," by Heinrich Ries, New York, John Wiley & Sons, 1906.

† New Jersey Geol. Surv. Final report, 1904, Vol. VI, p. 114; Iowa Geol. Surv. 1904, Vol. XIV, p. 116.

‡ New Jersey Geol. Surv. Report on Clays, 1878; Missouri Geol. Surv. 1896, Vol. XI, pp. 562 etc.; Penna. Second Geol. Surv. Vol. B, p. 3.

§ L. M. Haupt, Engineering Specifications and Contracts.

MORTAR, BRICKS, &c.

LIME MORTAR.

Art. 1. Mortar. The proportion of 1 measure of quicklime, either in irregular lumps, or ground, and 5 measures of sand, is about the average used for common mortar, by good builders in our principal Atlantic cities; and if both materials are good, and well mixed (or *tempered*) with clean water, the mortar is certainly as good as can be desired for such ordinary purposes as require no addition of hydraulic cement. The bulk of the mixed mortar will usually exceed that of the dry loose sand alone about $\frac{1}{8}$ part.

Quantity required. 20 cub ft, or 16 struck bushels of sand, and 4 cub ft, or 3.2 struck bushels of quicklime (the measures slightly shaken in both cases), will make abt $22\frac{1}{2}$ cub ft of mortar, sufficient to lay 1000 bricks of the ordinary average size of $8\frac{1}{4}$ by 4 by 2 ins, with the coarse mortar joints usual in interior house-walls, varying say from $\frac{3}{8}$ to $\frac{1}{2}$ inch. With such joints 1000 such bricks make 2 cubic yards of massive work. Nearly one third of the mass is mortar. For outside or showing joints, where a whiter and neater looking mortar is required, house-builders increase the proportion of lime to 1 in 4, or 1 in 3. For mortar of fine screened gravel, for cellar-walls of stone rubble, or coarse brickwork, 1 measure of lime to 6 or 8 of gravel, is usual, and the mortar is good. In average rough massive rubble, as in the foregoing brickwork, about one third the mass is mortar; consequently a cubic yard will require about as much as 500 such bricks, or 10 cubic feet (8 struck bushels) of sand, and 2 cub ft, or 1.6 bushels of quicklime. Superior, well scabbled rubble, carefully laid, will contain but about $\frac{1}{5}$ of its bulk of mortar; or $5\frac{1}{2}$ cub ft sand, and 1.1 cub ft lime, per cub yard.

For public engineering works, especially in massive ones, or where exposed to *dampness*, an addition should be made in either of the foregoing mortars, of a quantity of good **hyd cement,** equal to about $\frac{1}{3}$ of the lime; or still better $\frac{1}{3}$ of the lime should be omitted, and an equal measure of cement be substituted for it. **If exposed to water while quite new,** use little or no lime outside.

With bricks of $8\frac{1}{4}$ by 4 by 2 ins, the following are the **quantities of mortar and of bricks for a cubic yard of massive work.**

Thickness of Joints.	Proportion of Mortar in the whole mass	No. of Bricks per cub yard.	No. of Bricks per cub foot.
$\frac{1}{8}$ inch	about $\frac{1}{9}$	638	23.63
$\frac{1}{4}$ "	" $\frac{1}{4}$	574	21.26
$\frac{3}{8}$ "	" $\frac{3}{10}$	522	19.33
$\frac{1}{2}$ "	" $\frac{1}{3}$	475	17.60
$\frac{5}{8}$ "	" $\frac{4}{10}$	433	16.04

In estimating for bricks in massive work, **allow 2 or 3 per ct for waste;** and in common buildings, 5 per ct, or more. Much of the waste is incurred in cutting bricks to fit angles, &c. In Philadelphia a barrel of lump lime is allowed for 1000 bricks; or for 2 perches (25 cub ft each) of rough cellar-wall rubble. Somewhat less mortar per 1000 is contained in thin walls, than in massive engineering structures, because the former have proportionally more outside face, which does not require to be covered with mortar, but thin walls involve more waste while building; so that both require about the same quantity of materials to be provided. Careful experiments show that mortar becomes harder, and more adhesive to brick or stone, if the proportion of lime is increased. Hence, on our public works the proportion of one measure of quicklime to 3 of sand, is usually specified, but probably never used.

Lime is usually sold in lump, by the barrel, of about 230 lbs net, or 250 lbs gross. **A heaped bushel** of lump lime averages about 75 lbs. **Ground quicklime,** loose, averages about 70 lbs per struck bushel; and 3 bushels loose just fill a common flour barrel, but from 3.5 to 3.75 bushels, or 245 to 260 lbs can readily be compacted into a barrel.

General remarks on mortar and lime. On too great a proportion of our public works, the common lime mortar may be seen to be rotten and useless, where it has been exposed to moisture; which will be carried by the capillary action of earth to several feet above the natural surface, or as far below the artificial surface of embankments deposited behind abutments, retaining-walls, &c. The same will frequently be seen in the soffits of arches under embankments. *Common lime mortar, thus exposed to constant moisture, will never harden properly.* Even when very old and hard, it absorbs water freely. *Cement also does so, but hardens.*

Brickdust, or burnt clay, improves common mortar; and makes it hydraulic. In localities where sand cannot be obtained, burnt clay, ground, may be substituted; and will generally give a better mortar.

Protection of quicklime from moisture, even that of the air, is absolutely essential, otherwise it undergoes the process of **air-slacking,** or

spontaneous slacking, by which it becomes reduced to powder as when slacked by water as usual, but without heating, and with but little swelling. As this air slacking requires from a few months to a year or more, depending on quality and exposure, it gives the lime time to absorb sufficient carbonic acid from the air to injure or destroy its efficacy. But **quicklime will keep good for a long time** if first ground, and then well packed in air-tight barrels. The grinding also breaks down refractory particles found in all limes and which injure the mortar by not slacking until it has been made and used. For the same reason it is better that lime should not be made into mortar as soon as it is slacked, but be allowed to remain slacked for a day or two (or even several) protected from rain sun and dust

Lime slacked in great bulk may char or even set fire to wood.

Lime paste and mortar will keep for years, and improve, if well buried in the earth. Also for months if merely covered in heaps under shelter, with a thick layer of sand. The paste shrinks and cracks in drying, but the sand in mortar prevents this.

As approximate averages varying much according to the character and degree of burning of the limestone; and to the fineness or coarseness of the sand, one measure of good quicklime, either in lump, or ground. If wet with about ⅓ a measure of water, will within less than an hour, slack to about 2 measures of dry powder. And if to this powder there be added about ⅔ more measures of water, and 3 measures of dry sand, and the whole thoroughly mixed, the result will be about 3⅓ measures of mortar. Or the same slacked dry powder, with about 1 measure of water, and 5 measures of sand, will make about 5⅔ measures of mortar. In both cases the bulk of the mortar will be about ⅓ part greater than that of the dry sand alone If ⅔ of a measure of water be used for slacking, the result, instead of a dry powder, will be about 1⅓ measures of stiff paste, or with 1 whole measure of water for slacking, the result will be about 1⅓ measures of thin paste, of about the proper consistence for mixing with the sand. Very pure, *fat* limes, slack quickly, and make about from 2 to 3 measures of powder; while poor, *meagre* ones, require more time, and swell less Slow slacking, and small swelling, in case the lime has been properly burnt, are not in general bad properties; but on the contrary, usually indicate that it is to some extent hydraulic. In this case it makes a better mortar, especially for works exposed to moisture, or to the weather. Very pure limes are the worst of all for such exposures; or are bad *weather-limes;* and in important works, should never be used without cement.

Shell lime appears to be about the same as that from the purest limestones; but that from chalk is still more inferior and will not bear more than about 1½ measures of sand; its mortar never becomes very hard. Madrepores (commonly called coral) appear to furnish a lime intermediate between those of chalk and limestone They require to be but moderately burnt.

The average weight of common hardened mortar is about 105 to 115 lbs per cub ft.

Grout is merely common mortar made so thin as to flow almost like cream. It is intended to fill interstices left in the mortar-joints of rough masonry; but unless it contains a large amount of cement, it is probably entirely worthless; since the great quantity of water injures the properties of lime; and moreover, its ingredients separate from each other; the sand settling below the lime. Besides this, it will never harden thoroughly in the interior of thick masses of masonry; indeed, the same may probably be said of any common lime mortar. In such positions, it has been found to be perfectly soft, after the lapse of many years.

Both the sand and the water for lime mortar, should be **free from clay and salt.** The clay may be removed by thorough washing; but it is extremely difficult to get rid of the salt from seashore sand, even by repeated washings. Enough will generally remain to keep the work damp, and to produce efflorescences of nitre on the surface, whether with lime, or with cement mortar. Slacking by salt water gives less paste than fresh.

Mortar should not be mixed upon the surface of clayey ground, but a rough board, brick, or stone platform should be interposed. Pit sand sifted from decomposed gneiss, and other allied rocks, is excellent for mortar; its sharp angles making with the lime a more coherent mass than the rounded grains of river or sea sand. Mortar should be applied wetter in hot than in cold weather; especially in brickwork; otherwise the water is too much absorbed by the masonry, and the mortar is thereby injured.

The tenacity, or cohesive strength, that is, the resistance to a pull of good common lime mortar of the usual proportions of lime and sand, and 6 months old, is about from 15 to 30 lbs per sq inch; or .96 to 1.9 tons per sq ft. With less sand, or with greater age, it will be stronger.

The crushing strength of good common mortar 6 months old is from 150 to 300 lbs per sq inch, or 9.7 to 19.3 tons per sq foot

The sliding resistance, or that which common mortar opposes to any force tending to make one course of masonry slide upon another, is stated by Rondelet, to be but 5 lbs per sq inch; or about one third of a ton per sq ft. in mortar 6 months old.

Transverse strength of good common mortar 6 months old. A bar 1 inch square and 12 ins clear span, breaks with a center load of 4 to 8 lbs.

The lime in mortar decays wood rapidly, especially in close, damp situations. Still the soaking of timber for a week or two in a solution of quicklime in water appears to act as a preservative. **Iron,** so completely embedded in mortar as to exclude air and moisture, has been found perfect after 1400 years; but if the mortar admits moisture the iron decays So, probably, with other metals.

The adhesion to common bricks, or to rough rubble at any age will average about ⅔ of the cohesive strength at the same age; or say 12 to 24 lbs per sq inch, or .75 to 1.5 ton per sq ft at 6 months old. If care be taken to exclude dust entirely, by dipping each brick into water before laying it, or by sprinkling the stone by a hose, &c, the adhesion will be increased. On the other hand, much dust may almost prevent any adhesion at all. The precaution of wetting is especially necessary in very hot weather to prevent the warm bricks or stone from **killing the mortar** by the rapid absorption and evaporation of its water. **The adhesion to very smooth hard pressed bricks,** or to smoothly dressed or sawed stone is considerably less.

BRICKS.

Art. 2. Bricks, size, weight, &c. A brick 8.25 × 4 × 2 ins contains 66 cub ins; or 26.2 bricks to a cub ft; or 707 bricks to a cub yard.

In ordering a large number, a minimum limit of dimension should be specified, in order to prevent fraud. A brick ¼ inch less each way than the above, contains but 52.5 cub ins; thus requiring full 25 per cent more bricks to do the same work, and 25 per ct more cost for laying, which is generally paid by the 1000.

The weight of a good common brick, 8.25 × 4 × 2 ins, will average about 4.5 ℔s; or 118 ℔s per cub ft = 3186 ℔s or 1.42 tons per cub yard; or 2.01 tons per 1000. A good pressed brick of the same size will average about 5 ℔s, = 131 ℔s per cub ft = 3537 ℔s or 1.58 tons per cub yd; or 2.23 tons per 1000. Since the weight of hardened mortar averages but little less than that of good common brick, we may for ordinary calculations assume the **weight of brickwork,** with common bricks, at 1.4 tons per cub yard, or 116 ℔s per cub ft; and, with pressed brick, at 1 56 tons per cub yd, or 129 ℔s per cub ft.

In water, either brick will in a few minutes **absorb** from ½ to ¾ ℔ of water; or 0.1 to one-seventh of the weight of a pressed brick, or ¼ to one-third of its bulk.

Number of bricks 8¼ × 4 × 2, required per sq foot of wall, allowing for the usual waste in cutting bricks to fit corners, jambs, &c.:

Wall 8¼ ins, or 1 brick	.....14 bricks	Wall 21½ ins, or 2½ brick	.....35 bricks
" 12¾ " or 1½ "	.....21 "	" 25¾ " or 3 "	.....42 "
" 17 " or 2 "	.....28 "		

Laying, per day. A bricklayer, with a laborer to keep him supplied with materials, will, in common house walls, lay on an average about 1500 bricks per day of 10 working hours. In the neater outer faces of back buildings, from 1000 to 1200; in good ordinary street fronts, 800 to 1000; or of the very finest lower story faces used in street fronts, from 150 to 300, depending on the number of angles, &c. In plain massive engineering work, he should average about 2000 per day, or 4 cub yds; and in large arches, about 1500, or 3 cub yds.

Since bricks shrink about $\frac{1}{12}$ part of each dimension in drying and burning, the moulds should be about $\frac{1}{12}$ part larger each way than the burnt brick is intended to be. Good well-burnt bricks will ring when two are struck together.

At the brick-yards about Philadelphia, a brick-moulder's work is 2333 bricks per day; or 14000 per week. He is assisted by two boys, one of whom supplies the prepared clay, moulding sand, and water; while the other carries away the bricks as they are moulded. A fourth person arranges them in rows for drying. About ¾ of a cord, or 96 cub ft of wood, is allowed per 1000 for burning. Where coal is used, the kilns are fired up with anthracite, and the finishing is done with bituminous. One ton of coal, in all, makes 4500 bricks.

For paving sidewalks the bricks are laid on a 6-inch layer of gravel, which should be free from clay, and well consolidated. With bricks of 8¼ × 4 × 2 ins, with joints from ⅛ to ¼ inch wide, a square yard requires, flatwise, 38 bricks; edgewise, 73; endwise, 149. An average workman, with a laborer to supply the bricks and gravel, will in 10 hours lay about 2000 bricks; or 53 sq yds flat, 27 edgewise, 13 endwise. When done, sand is brushed into the joints.

Art. 3. The crushing strength of bricks of course varies greatly. A rather soft one will crush under from 450 to 600 ℔s per sq inch; or about 30 to 40 tons per sq ft; while a first-rate machine-pressed one will require about 200 to 400 tons per sq ft, or about the crushing limit of the best sandstone; two-thirds that of the best marbles or limestones; or ½ that of the best granites, or roofing slates. But masses of **brickwork** crush under much smaller loads than single bricks. In some English experiments, small cubical masses, only 9 inches on each edge, laid in cement, crushed under 27 to 40 tons per sq ft. Others, with piers 9 ins square, and 2 ft 3 ins high, in cement, only two days after being built, required 44 to 62 tons per sq ft to crush them. Another, of pressed brick, in best Portland cement, is said to have withstood 202 tons per sq ft; and with common lime mortar only ¼ as much.

It must, however, be remembered, that cracking and splitting usually commence under about one-half the crushing loads. To be safe, the load should not exceed ⅛ of the crushing one; and so with stone. Moreover, these experiments were made upon low masses; and the strength decreases with the proportion of the height to the thickness.

The pressure at the base of a brick shot-tower in Baltimore, 246 feet high, is estimated at 6½ tons per sq ft; and in a brick chimney at Glasgow, Scotland, 468 feet high, at 9 tons. Professor Rankine calculates that in heavy gales this is increased to 15 tons, on the leeward side.

With our present imperfect knowledge on this subject, it cannot be considered safe to expose even first-class pressed brickwork, *in cement*, to more than 12 or 15 tons per sq ft, or good hand-moulded, to more than two-thirds as much.

Tensile strength of brick, 40 to 400 lbs per sq inch; or 2.6 to 26 tons per sq ft.

The English rod of brickwork is 306 cub feet, or $11\frac{1}{3}$ cub yards; and requires about 4500 bricks of the English standard size, with about 75 cub ft of mortar. The English *hundred of lime*, is a cub yd

Frozen mortar. There is risk in using common mortar in cold weather If the cold should continue long enough to allow the frozen mortar to set well, the work may remain safe, but if a warm day should occur between the freezing and the setting of the mortar, the sun shining on one side of the wall may melt the mortar on that side, while that on the other side may remain frozen hard. In that case, the wall will be apt to fall; or if it does not, it will at least always be weak; for mortar that has partially set while frozen, if then melted, will never regain its strength. By the writer's own trials hydraulic cements seemed not to be injured by freezing.

Experiments for rendering brick masonry impervious to water. Abstract of a paper read before the American Society of Civil Engineers, May 4, 1870, by William L. Dearborn, Civil Engineer, member of the Society

The face walls of the Back Bays of the Gate-houses of the new Croton reservoir, located north of Eighty sixth Street, in Central Park, were built of the best quality of hard-burnt brick, laid in mortar composed of hydraulic cement of New York, and sand mixed in the proportion of one measure of cement to two of sand. The space between the walls is 4 ft and was filled with concrete. The face walls were laid up with great care and every precaution was taken to have the joints well filled and insure good work They are 12 ins thick, and 40 ft high; and the Bays when full generally have 36 ft of water in them.

When the reservoir was first filled, and the water was let into the Gate-houses it was found to filter through these walls to a considerable amount. As soon as this was discovered, the water was drawn out of the Bays, with the intention of attempting to remedy or prevent this infiltration After carefully considering several modes of accomplishing the object desired, I came to the conclusion to try "Sylvester's Process for Repelling Moisture from External Walls"

The process consists in using two washes or solutions for covering the surface of brick walls; one composed of Castile soap and water, and one of alum and water. The proportions are three quarters of a pound of soap to one gallon of water; and half a pound of alum to four gallons of water both substances to be perfectly dissolved in the water before being used

The walls should be perfectly clean and dry; and the temperature of the air should not be below 50 degrees Fahrenheit, when the compositions are applied

The first, or soap wash, should be laid on when at boiling heat, with a flat brush taking care not to form a froth on the brickwork This wash should remain twenty-four hours, so as to become dry and hard before the second or alum wash is applied, which should be done in the same manner as the first The temperature of this wash when applied may be 60° or 70°, and it should also remain twenty-four hours before a second coat of the soap wash is put on, and these coats are to be repeated alternately until the walls are made impervious to water.

The alum and soap thus combined form an insoluble compound, filling the pores of the masonry and entirely preventing the water from penetrating the walls.

Before applying these compositions to the walls of the Bays, some experiments were made to test the absorption of water by bricks under pressure after being covered with these washes, in order to determine how many coats the wall would require to render them impervious to water.

To do this a strong wooden box was made put together with screws, large enough to hold 2 bricks; and on the top was inserted an inch pipe forty feet long.

In this box were placed two bricks after being made perfectly dry, and then covered with a coat of each of the washes, as before directed, and weighed. They were then subjected to the pressure of a column of water 40 feet high, and, after remaining a sufficient length of time, they were taken out and weighed again, to ascertain the amount of water they had absorbed.

The bricks were then dried, and again coated with the washes and weighed, and subjected to pressure as before; and this operation was repeated until the bricks were found not to absorb any water. Four coatings rendered the bricks impenetrable under the pressure of 40 ft head

The mean weight of the bricks (dry) before being coated, was $3\frac{1}{4}$ lbs; the mean absorption was one-half pound of water. An hydrometer was used in testing the solutions

As this experiment was made in the fall and winter, (1863,) after the temporary roofs were put on to the Gate-house, artificial heat had to be resorted to, to dry the walls and keep the air at a proper temperature. The cost was 10.06 cts per sq ft. As soon as the last coat had become hard the water was let into the Bays, and the walls were found to be perfectly impervious to water, and they still remain so in 1870, after about $6\frac{1}{2}$ years.

BRICK ARCH (FOOTWAY OF HIGH BRIDGE). The brick arch of the footway of High Bridge is the arc of a circle 29 ft 6 in radius; and is 12 in thick; the width on top is 17 ft; and the length covered was 1381 ft.

The first two courses of the brick of the arch are composed of the best hard burnt brick laid edgewise in mortar composed of one part, by measure, of hydraulic cement of New York, and two parts of sand The top of these bricks, and the inside of the granite coping against which the two top courses of brick rest was, when they were perfectly dry, covered with a coat of asphalt one-half an inch thick, laid on when the asphalt was heated to a temperature of from 360° to 518° Fahrenheit.

On top of this was laid a course of brick flatwise, dipped in asphalt, and laid when the asphalt was hot; and the joints were run full of hot asphalt

On top of this a course of pressed brick was laid flatwise in hydraulic cement mortar, forming the paving and floor of the bridge. This asphalt was the Trinidad variety: and was mixed with 10 per cent, by measure, of coal tar; and 25 per cent of sand. A few experiments for testing the strength of this asphalt, when used to cement bricks together, were made, and two of them are given below.

Six bricks, pressed together flatwise with asphalt joints, were, after lying six months, broken The distance between the supports was 12 ins; breaking weight, 900 lbs, area of single joint, $28\frac{1}{2}$ sq ins. The asphalt adhered so strongly to the brick as to tear away the surface in many places.

Two bricks pressed together end to end, cemented with asphalt, were, after lying 6 months, broken. The distance between the supports was 10 ins area of joint, 8¼ sq ins; breaking weight, 150 lbs.

The area of the bridge covered with asphalted brick, was 23065 sq ft. There was used 94200 lbs of asphalt, 33 barrels of coal tar, 10 cub yds of sand, 93800 bricks.

The time occupied was 109 days of masons, and 148 days of laborers. Two masons and two laborers will melt and spread, of the first coat, 1650 sq ft per day. The total cost of this coat was 5.25 cents per sq ft, exclusive of duty on asphalt. There were three grooves, 2 ins wide by 4 ins deep, made entirely across the brick arch, and immediately under the first coat of asphalt, dividing the arch into four equal parts. These grooves were filled with elastic paint cement.

This arrangement was intended to guard against the evil effects of the contraction of the arch in winter; as it was expected to yield slightly at these points, and at no other point; and then the elastic cement would prevent any leakage there.

The entire experiment has proved a very successful one, and the arch has remained perfectly tight.

In proposing the above plan for working the asphalt with the brickwork, the object was to avoid depending on a large continued surface of asphalt, as is usual in covering arches, which very frequently cracks from the greater contraction of the asphalt than that of the masonry with which it is in contact, the extent of the asphalt on this work being only about one-quarter of an inch to each brick. This is deemed to be an essential element in the success of the impervious covering."

A cheap and effective process for preventing the percolation of water through the arches of aqueducts, and even of bridges is a great desideratum. Many expensive trials with resinous compounds have proved failures. Hydraulic cement appears to merely diminish the evil. Much of the trouble is probably due to cracks produced by changes of temperature.

The **white efflorescence** so common on walls, especially on those of brick, is due to the presence of soluble salts in the bricks and mortar. These are dissolved, and carried to the face of the wall, by rain and other moisture. Sulphate of magnesia (Epsom Salt) appears to be the most frequent cause of the disfiguration. In many places mortar lime is made from dolomite, or *magnesian* limestone, which often contains 30 per cent or more of magnesia; which also occurs frequently in brick clay. Coal generally contains sulphur, most frequently in combination with iron, forming the well-known "iron pyrites". The combustion of the coal, as in burning the limestone or clay, in manufactures, in cooking etc, converts the sulphur into sulphurous acid gas, which, when in contact with magnesia and air, as in the lime or brick kiln, or in the finished wall or chimney, becomes sulphuric acid and unites with the magnesia, forming the soluble sulphate. We are not aware of any remedy that will prevent its appearance under such circumstances; but the formation of the sulphate may be prevented by the use of limestone and brick-clay free from magnesia.

MORTAR.

Cement.

For experiments, see p 1303.

For specifications, see pp 1229, 1232, 1234, 1352.

For Concrete, see pages 1252, etc.

For abbreviations, symbols and references, see p 1251.

1. The property of setting and hardening under water is called **hydraulicity;** and cements, which harden under water, are called **hydraulic cements;** or, more briefly, **cements. For behavior** of cement **when mixed with water,** with or without sand, see **Mortar,** p 1243.

Materials.

2. The elements, chiefly concerned in the action of lime and cem mortars, are—

Calcium,	Ca	Oxygen, O.
Aluminum,	Al	
Carbon,	C	
Silicon,	Si	
Hydrogen,	H	

3. Oxygen combines with each of the others, forming **oxides.** Thus:

Calcium oxide, CaO, is lime;
Aluminum sesqui-oxide, Al_2O_3,* is alumina;
Carbon dioxide, CO_2, is carbonic acid;
Silicon dioxide, SiO_2, is silica, or silicic acid;†
Hydrogen monoxide, H_2O, is water.

4. The materials most used in the manufacture of cements are either (a) calcareous, (b) argillaceous, or (c) both calcareous and argillaceous.

(a) Calcareous (rich in lime carbonates).

Limestone, a lime carbonate, or combination of lime and carbonic acid $CaO + CO_2$ or $CaCO_3$. Marble is limestone.

Dolomite, or **magnesian limestone,** containing about 45 per cent of magnesia carbonate, $MgO.\ CO_2$. Where strata of limestone and dolomite adjoin, the rock varies in composition between the two, containing percentages of magnesia carbonate varying from 0 to 45.

Chalk, a soft limestone, composed of remains of marine shells.

Marl, a soft and impure hydrated ‡ lime carbonate, precipitated from still water and found in the beds and banks of extinct or existing lakes.

Alkali waste, lime carbonate, precipitated, as a waste product, in the manufacture of caustic soda.

Coral. See ¶ 5

(b) Argillaceous (rich in alumina silicates).

Clay (including argillaceous minerals in general), an alumina silicate, or combination of alumina and silicic acid, $Al_2O_3 + SiO_2$.

Shale and slate, clay, solidified by geological processes.

Puzzolana, or pozzuolana, a volcanic slag, found at Puzzuoli, or Pozzuoli, near Mount Vesuvius, an impure alumina silicate.

Blast furnace slag, practically an artificial puzzolana.

Brick-dust. See ¶ 6.

(c) Rich in both **lime carbonate and alumina silicate.**

Cement rock is argillaceous (clayey) limestone. The alumina silicate usually ranges from 13 to 35 %. There is generally a considerable percentage of magnesia carbonate, amounting sometimes to 25 %.

5. A soft coral rock, from the reefs near Colon, **Panama,** mixt with clay and silt brought down by the Chagres river, or with "a pumiceous rhyolite tuff," found on the Isthmus, or with both, and crushed, burned and tested at the Lehigh Valley Testing Laboratory, at Allentown, Pa., gave a

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* The subscripts indicate the combining ratios of the several elements. Thus, in alumina, Al_2O_3 means a compound of 2 atoms of aluminum with 3 of oxygen.

† Quartz is silica; and most of the sand, used in mortar, is quartz sand.

‡ Hydrated; containing chemically combined water.

uniform cement, comparing favorably with average standard brands of Lehigh cement. The coral rock is "a remarkably pure lime carbonate." The Chagres clay and silt are "rather low in silica, but contain a relatively large amount of iron as compared with alumina." The tuff "is of approximately the same composition as the argillaceous materials used in the Lehigh district of Pennsylvania." (Ernest Howe, U. S. G S, E N, '07/Nov/21, p 544.) See ¶¶ 29, etc.

6. Mr. Ernest McCullough "mixed fine **brick dust and hydrated lime** together and made a fairly satisfactory cem for a small concrete job in a locality where Portland cem could not be obtained." (E N, '07/Nov/21, p 557.)

7. Lime. When limestone (without clay) is "burned," its CO_2 is driven off, and the remaining ("**quick**") **lime** has a strong affinity for water, absorbing it with such avidity as to develop heat sufficient to produce steam, the generation of which disintegrates and swells the mass. Combining thus with the water, the lime forms calcium hydrate, $CaO.H_2O$, or CaH_2O_2. This process is called **slaking** or **slacking**; and lime which has satisfied its affinity for water is called slaked (or slack) lime. When slaked lime is used as mortar, it gradually absorbs carbonic acid from the air, forming lime carbonate, the water being liberated and evaporated. Hardened lime mortar may thus be regarded as an artificial limestone.

Manufacture.

8. Cement. When alumina silicate, such as clay, in sufficient quantities, is "burned" with calcium carbonate, such as limestone, the burned product, called cement, is deficient in, or devoid of, the slacking property; but, on the other hand, when it is made into mortar, the combinations, formed between the elements of the lime, the alumina, the silica and the water, during the burning, and afterward in the mortar, are such that they readily proceed under water. Chemists differ as to the nature of these combinations, except that these constitute a process of crystallization, resulting chiefly in the formation of hydrated lime silicate and hydrated lime aluminate, which two compounds constitute the major portion of most cems.

Natural and Portland Cement.

9. In the manufacture of "**natural**" **cement**, cement rock, broken into lumps, is first calcined, at from 1000° to 1400° C (1800° to 2500° F) in a stationary kiln, in alternate layers with coal of about pea size, as fuel. It is then ground to a fine powder, and this is sometimes specially mixed, in order to increase its uniformity.

10. The qualities of nat cems vary widely, owing to diffs in the compositions of cem rocks found in diff localities.

11. The name **Rosendale,** originally and properly restricted to nat cems made in Ulster County, N Y, was at one time applied indiscriminately to American nat cems in general.

12. In Europe, quick-setting nat cems are called "**Roman cements.**"

13. Portland cement was so called on account of the resemblance of the hardened mortar to Portland stone, the oolitic limestone of Portland, England.

14. Portland cem **is made** from different combinations of the calcareous and argillaceous materials named in ¶ 4, and these require different preliminary treatments. Thus. hard rock is crushed; soft rock and clay are ground; marl and clay are mixed wet, and the marl is sometimes pumped to the mill. In any case, the resulting materials are dried and finely ground, mixed, and then calcined at a temperature of 1450° to 1550° C, or say 2600° to 2800° F, producing incipient vitrifaction, which consists of the chemical combination of the silica, alumina and lime, into a glassy clinker, essentially a lime silicate and aluminate. The resulting clinker is again ground to an impalpable powder, which is the finished product.

15. The proportions of the several materials are carefully adjusted. There is usually from 74 to 77.5 % lime carbonate, and about 20 % of alumina silicate and iron oxide. See ¶ 32.

16. Manipulation. The raw material is sometimes molded into bricks which are burned in a stationary kiln; but it is now more generally fed, as a fine powder, into the upper end of a nearly hor cyl (rotary kiln) 6 to 8 ft

in diam and from 60 to 100 ft or more in length. Coal dust, as fuel, is injected, by an air blast, into the other end; while most of the air, required for combustion, is admitted freely from the atmosphere thru other openings.

17. As in the case of lime, the **burning** drives off the carbonic acid and water, and more completely oxidizes any iron present.

18. The higher cost of Portland cement is due to the more careful selection of the materials and to the more elaborate and expensive treatment given them, resulting in the ultimate attainment of much greater strength and uniformity than are usually found in nat cems.

19. The improvements, which have been made in the manufacture of **Portland cement,** are driving out other makes. Owing to its greater sand-carrying capacity, it is often used, by contractors, even where the specifications permit the use of nat cem.

20. Overburning is liable to occur, if the material is deficient in lime ("over-clayed"). **Underburning** yields a soft brownish clinker, and weak, quick-setting cem, heating in water. Some cems, slow at first, become quicker after storage.

21. Portland Cement is used for structures subjected to severe or repeated stresses, for cases where high strgth must be attained in a short time, for concrete buildings, where water will be in contact with new work, for thin walls subject to water pres, and for work exposed to abrasion or to weather; while **natural cement** may be used in dry sheltered foundations under compressive loads not exceeding 75 lbs per sq inch and not imposed until 3 months after placing, for backing and filling in massive conc or stone masonry where wt and mass are desiderata, and for street and sewer foundations.

Puzzolana.

22. Slag cements (sometimes called **puzzolana** cements or puzzolana) are intimate mixtures of slaked lime and basic blast-furnace slag, both finely ground, and not calcined. As the slag leaves the blast-furnace, it is chilled and disintegrated by running it into water. A little soda is sometimes added, to hasten setting. Slag cem is not to be confounded with those *Portland* cems in which slag is one of the ingredients.

23. In dry air, the sulphides, contained in **Puzzolana cement,** oxidize, and cause superficial cracking. It sets more slowly than Portland, unless treated with soda. If so treated, the soda becomes carbonated under long storage, and the cem again becomes slow-setting. Since puzzolana cem, properly made, contains no free or anhydrous lime, it does not warp or swell, and requires less water than Portland; but, for permanency after placing, the finished work should be kept constantly moist. It is recommended for use in sea water, alone or mixed with Portland. Its mortar is tougher than Portland, but never becomes so hard. It should not be subjected to attrition or blows. (Report, Board of U S Engr officers, U. S., Prof'l Papers No 28, '01.)

24. Puzzolana cement is said to work well if used with 2 or 3 parts sand and not subjected to freezing weather. Its ingredients must be finely ground and intimately mixed. It is used where extreme strength and hardness are not required

Silica Cement.

25. Silica Cement, or sand cement, was originally made by mixing Portland cem with quartz sand (silica) and grinding the mixture to extreme fineness It was claimed that the cem thus became much more finely ground, and that "silica cement," containing one part Portland cem and three parts silica, could therefore carry, in mortar, nearly as much sand as could the pure cem alone; also that mortars, made with silica cem, were less permeable to water than those made with pure cem in the ordinary way.

26. Owing to the high cost of grinding the quartz sand, less refractory materials, such as **lime-stone,** are now substituted for it. The product, so obtained, is still called "silica cement," altho containing a less proportion of silica than Portland cem.

27. Silica cement mortar is said to work more smoothly under the trowel than that made with ordinary cems.

28. In the construction of a concrete lock at St. Paul, Minn., it was intended to use 1.5 volumes silica cem as equivalent to 1 vol Saylor's Port-

land; but experiments indicated that, at 6 mos, concrete, made with silica cem, was **as strong** as that made with **Portland.**

Other Cements.

29. White Portland cement, obtained by making certain modifications in the process of manufacture, is nearly colorless. It is suitable for making imitation marbles, etc., and capable of taking artificial coloring. It is higher in price than ordinary Portlands. See ¶ 44.

30. Iron ore cement ("Erz-cement"), Krupp Steel Co. In this cem, the argillaceous material of Portland cem is mostly replaced by iron oxide. The material is burned and ground as for Portland cem, ¶¶ 13, &c. Spec grav, 3.31. Slower setting than Portland. Sound. Low early strgths; but, in time, strgth far exceeds that of Portland. No trace of expansion or crackg in sea water under 15 atmospheres. (Wm. Michaelis, Jr., Western Soc Engrs, Aug 1907; S. B. Newberry, Cement Age, Jan 1907.)

31. Hydraulic lime is a name given to cems (much used in Europe) which, while to some extent hydraulic, do not contain enough of the hydraulic elements to prevent slaking. The slaking, however, is slower, and the swelling less, than with lime proper.

Composition.

32. Analyses of cements, in percentages.

In each group of three lines,
the *upper* line shows the *max* percentage.
" *middle* " " " *mean* "
" *lower* " " " *min* "

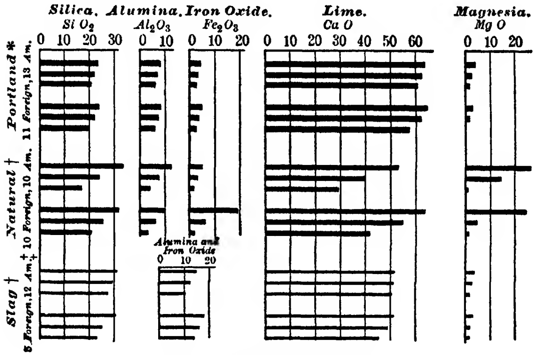

Fig 1. Analyses of Cements.

33. The ratio of the wt of alumina silicate to that of the lime, in a cem, is called its **hydraulic index.** Other things being equal, it may be used as an indication of the hydraulicity of the cem.

34. Thus, if a cem contains 30 % alumina silicate and 60 % lime, its hydraulic index is 30/60 = 0.50.

35. The hydraulic modulus is approximately the *reciprocal* of the hydraulic index; *i.e.*, the modulus is the ratio, by wt, of lime, to silica,

* Richard K. Meade, "Portland Cement," 1906, pp 16–17.
† E. C. Eckel, "Cements, Limes and Plasters," 1907, pp 253 etc., 667–8.
‡ 16 analyses of "Steel" (slag) cement, made by Illinois Steel Co., South Chicago, reported by Board of U. S. Engr Officers, 1900, gave practically the same avs, but with generally greater uniformity: silica, 29.9 to 27.8; alumina and iron, 12.1 to 11.1; lime, 52.1 to 50.3; magnesia, 3.0 to 1.6.

alumina and iron oxide. It is sometimes specified that the modulus, in Portland cement, shall be 1.7.

36. In natural cements, the modulus usually ranges from 0.667 to 1.667.

37. Mr. Spencer B. Newberry uses the ratio:

(lime — alumina) ÷ silica,

which he terms the **lime factor**, and which usually varies, in the raw material, betw 2.7 and 2.8, and, in the best commercial cems, betw 2.5 and 2.6.

38. Mr. Edwin C. Eckel (Cements, Limes and Plasters, p 170) suggests the

$$\textbf{Cementation index} = \frac{2.8s + 1.1a + 0.7i}{l + 1.4m}$$

where s, a, i, l and m are the percentages, by wt, of silica, alumina, iron oxide, lime and magnesia, respectively.

39. The most common **adulterants** of cem are ground limestone, lime, shale, slag and ashes; and Portland cem is sometimes adulterated with nat cem. Most of the adulterants commonly used are merely inert, and therefore only weaken the cem; but quick lime may do more serious mischief.

See Cement Mortar, ¶¶ 28, etc., p 1245.

Properties.

Fineness.

40. Fineness. Even in cem of standard fineness, the inner portions of the grains seem to remain inert. The finer the cem, the more sand it will carry and still produce a mortar of a given strength; but, in each case, there is a point where the cost of additional fineness offsets the additional advantage which may be gained.

41. Hence **fineness** is less important with **natural** than with **Portland** cem; for the cheapness of nat cem may render it advisable to use the cem in larger quantities, rather than pay for finer grinding, in order to secure the desired strgth.

42. Cements, ground to **extreme fineness,** in order to secure strgths beyond those of commercial products, set so quickly that they must be used immediately after adding water. (Wm. Michaelis, Jr., Western Soc of Engrs, Aug '07.)

43. The fineness of cement and sand is indicated as follows, where the large numerals represent the sieve numbers; the small numeral, to the left of each sieve number, represents the percentage retained upon that sieve; and the final small numeral, to the right of the last sieve number, represents the percentage passed by the last sieve. The sum of the small numerals = 100. Thus, ${}^{5}20\ {}^{15}30\ {}^{35}40^{45}$ means that 5 % were retained on a No. 20 sieve, 15 % on a No. 30, and 35 % on a No. 40, while the remaining 45 % passed the No 40 sieve.

Color.

44. Color. The lime silicates and aluminates, which constitute the cem proper, are colorless when pure. (See White Cement, ¶ 29.) The color of cems is therefore due to other matter which is unavoidably present, notably to the iron oxides, and may be affected by either beneficial, harmful or neutral ingredients. Hence, color, in itself, is of but little value as a guide to quality; but **variations in shade,** in a given kind of cem, may indicate diffs in the character of the rock or in the degree of burning. Thus, with nat cems, a light color generally indicates an inferior or underburned rock. A coarse-ground cem, light in color and wt, would be viewed with suspicion.

45. "With Portland cem, gray or greenish-gray is generally considered best; bluish gray indicates a probable excess of lime, and brown an excess of clay. Natural cems are usually brown, but vary from very light to very dark. Slag cem has a mauve tint—a delicate lilac." (Prof Ira O. Baker, "A Treatise on Masonry Construction," p 55.)

Weight.

46. Specific gravity and weight. See spec grav, pp. 1232, 1234. The sp gr of the solid particles of cem is not affected by fineness of grinding,

but is diminished by absorption of water and carbonic acid under exposure, and is therefore increased by drying. The sp gr of Portland cems may range from 2.9 to 3.25, ordinarily from 3 to 3.2; nat cems, 2.7 to 3.2; Puszolano cem, from 2.7 to 2.9.

47. The weight, per cu ft, of cem powder, is affected by exposure and by drying, as explained above, and is increased by compression, as in packing. It is reduced by fine grinding, the finer particles packing less closely. Faija found a loss, in wt, of about 6 % in a few days after grinding; 17 % in 6 mos, and 21 % in a year.

48. In a German Portland cem, Eliot C. Clarke found 90 lbs per cu ft when 40 % was retained on No. 120 sieve, and 75 lbs per cu ft when so finely ground that all passed the same sieve.

49. As a rude approximation, Portland cem is taken as weighing 100 lbs, nat cem 75 lbs, per cu ft.

Packages.

50. Owing to variations in the specific gravity of cems, there is corresponding variation in **sizes and weights** of packages and their contents. The trade practice is to sell a bbl of Portland cem as 400 lbs gross (including wt of bbl); nat, 300 lbs gross.

51. A Portland Cement barrel is 2 to 2.2 ft high, betw heads, 1.38 to 1.46 ft av diam. It weighs 21 to 29 lbs, and is lined with paper for ordinary transportation. Its capacity is 3.1 to 3.5 cu ft, but the cem, compressed into it, in packing, occupies 3.75 to 4.3 cu ft loose, and weighs 370 to 390 lbs. The bbl is not returnable.

52. A natural cement barrel weighs about 20 lbs. In the Western states it contains 265 lbs; in the Eastern states, 300 lbs, of cem.

53. "Domestic" barrels are used for shipment to all points in the U. S., with slight reinforcement for Gulf ports; **"standard export"** bbls for Mexico and the West Indies; **"special export barrels"** where specially severe treatment is expected.

54. The standard export barrel is of better stock than the "domestic," and is reinforced with cross pieces in the heads and with two iron hoops. It costs from 5 to 10 cts more than the "domestic" bbl, varying with cost of cooperage stock.

55. The special export barrel costs 10 to 15 cts more than the standard export bbl. It is all-hardwood, heavily hooped and reinforced, with wood cross-pieces in the heads, iron hoops, and clamps to hold the heads in place. A heavy waterproof lining is used instead of the heavy Manila paper used with the standard export bbl.

56. Most cem is now packed in "cloth" or paper bags, except for shipment by sea.

57. Cement bags are made of cloth (canvas or cotton duck) and of "rope Manila" paper. When empty, they measure about 17 × 28 ins. (See Digest of specification of the Am Soc for Testing Materials.) A "cloth" bag is usually charged to the purchaser at about 10 cts, and credited at about 7.5 cts when returned. **Paper bags** are charged at 2.5 cts each and are not returnable.

58. The use of **paper bags** obviates loss of time in emptying and returning bags, shortage on lost or damaged bags, and loss of cem in transit or by failure to empty bags completely; but paper bags are more likely to lose their entire contents by breakage, and pieces of broken bags may get into the work and weaken it.

59. For large work, cem has frequently been **shipped in cars in bulk,** with little loss or damage, provided the cars are carefully selected. This method is especially advantageous where the cem is tested at the mill, stored in "accepted bins," and shipped direct to the work, in sealed cars. The cars may be unloaded by automatic conveyors. Bags and bbls are often preferred as furnishing a convenient means for keeping account of the quantities of cem entering the work; but, in large operations, there should be no difficulty in arranging to keep such accounts with bulk shipments.

Age.

60. "Aging" consists in the slaking of the free lime remaining in the cem after burning. Good Portland cem is improved by a few weeks of

aging in dry air; and, if kept dry it deteriorates but slowly under even long storage; but nat cems usually suffer by aeration; and cems in general, being composed of compounds with a strong affinity for water, deteriorate if exposed to dampness. Hence, protection from moisture, even that of the air, is very essential for the preservation of cems, as well as of quicklime. With this precaution, the cem, altho it may require more time to set, than when fresher, does not otherwise very appreciably deteriorate in many months.

61. Storage, under pressure, tends to the caking of cems, which, therefore, does not necessarily indicate deterioration.

62. Restoration by reburning. Cems which have deteriorated by exposure, may be in great measure restored by reheating to redness.

63. If cem is **stored in warm places,** it is apt to "flash" when mixed with water, *i. e.*, to set much more rapidly than it should.

Testing.

See Digests of Specifications, A S C E, p 942; Engng Standards Comm of Gt Brit, p 940; Report of Board of U S Engr Officers, p 937.

64. *Thoro* ***chemical tests*** *of cem can of course be made only by expert chemists; but the following simple test may be made by the engineer. Treated with hydrochloric acid, "pure Port cem effervesces slightly, gives off some pungent gas, and gradually forms a bright yellow jelly, without sediment.* Powdered limestone or cem rock, mixed with the cem, causes violent effervescence, the acid giving off strong fumes until all the lime carbonate is decomposed, when the yellow jelly forms. Quartz sand remains undissolved. Reject cem containing these adulterants." Judson, "City Roads and Pavements." The presence of slag is generally indicated by the sulfur present, which causes a milky appearance, if the cem be agitated in a solution of hydrochloric acid in water.

65. Fuller and Thompson found that cems, which failed to stand this test, failed also to set properly, while cems which passed it, also passed more elaborate chemical tests. (Trans A S C E, Vol 59, '07, Dec, pp 73–4.)

Properties and Tests of Cement. Report of Board U. S. A. Engineer Officers. Properties and tests of Portland, Natural and Puzzolan* cements. Digest of a Report of Majors W. L. Marshall and Smith S. Leach and Capt. Spencer Cosby, Board of Engineer Officers, on testing Hydraulic Cements. Professional Papers, No. 28, Corps of Engineers, U. S. A., 1901.

Unfortunately, tests for acceptance or rejection must be made on a product which has not reached its final stage. A cement, when incorporated in masonry, undergoes chemical changes for months, whereas it is seldom possible to continue tests for more than a few weeks at the most.

A few tests, carefully made, are more valuable than many, made with less care.

Cement which has been in **storage** for a long time should be **carefully tested** before use, in order to detect deterioration.

A cement should be rejected, without regard to the proportion of failures among samples tested, if the samples show dangerous variation in quality or lack of care in manufacture, and resulting lack of uniformity in the product.

The practice of offering a **bonus** for cement showing an abnormal strength is **objectionable,** as it leads to the production of cements with defects not easily detected.

For Portland or Puzzolan cement, make **tests** for (**1**) fineness of grinding; (**2**) specific gravity; (**3**) soundness, or constancy of volume in setting; (**4**) time of setting, and (**5**) tensile strength. For Natural cements omit tests (2) and (3).

(**1**) **Fineness.** Cementitious quality resides principally, if not wholly, in the very finely ground particles. Use a No 100 sieve, woven from brass wire No. 40 Stubs gage; sift until cement ceases to pass through. The percentage that has passed through is determined by weighing the residue on the sieve. The screen should be frequently examined to see that no wires have been displaced.

(**2**) **Specific gravity.** The specific gravity test is of value in determining whether a Portland cement is unadulterated. The higher the burning, short of vitrification, the better the cement and the higher the specific gravity If underburned, the specific gravity of Portland cement may fall below 3; if overburned, it may reach 3.5. Natural cement has a specific gravity of about 2.5 to 2.8, and Puzzolan about 2.7 to 2 8

The temperature may vary between 60° and 80° F. Any approved form of volumenometer or specific gravity bottle may be used, graduated to cubic centimeters with decimal subdivisions. Fill the instrument to zero of scale with benzine. Take 100 grams of sifted cement that has been previously dried by exposure on a metal plate for 20 minutes to a dry heat of 212° F., and allow it to pass slowly into the benzine, taking care that the powder does not stick to the sides of the graduated tube above the fluid, and that the funnel, through which it is introduced, does not touch the fluid. The approximate specific gravity will be represented by 100 divided by the displacement in cubic centimeters. The operation requires care.

(**3**) **Soundness,** and (**4**) **setting qualities.** The temperature should not vary more than 10° from 62° F. For Portland cement use 20, for Natural 30, and for Puzzolan 18 per cent. of water by weight. Mix thoroughly for 5 minutes. On glass plates make two cakes about 3 inches in diameter, ½ inch thick at the middle and drawn to thin edges, and cover them with a damp cloth. At the end of the minimum time specified for initial set, apply needle $\frac{1}{12}$ inch diameter, weighted to ¼ pound. If an indentation is made, the cement passes the requirement for initial setting. Otherwise the setting is too rapid. At the end of the maximum time specified for final set, apply the needle $\frac{1}{24}$ inch diameter, loaded to one pound. If no indentation is made, the cement passes the requirement for final set. Otherwise the setting is too slow.

Generally speaking, both periods of set are lengthened by increase of moisture, and shortened by increase of temperature.

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* By **Portland** cement, in this report, is meant the product obtained by calcining intimate mixtures, either natural or artificial, of argillaceous and calcareous substances, up to incipient fusion. By **Natural** cement is meant one made by calcining natural rock at a heat below incipient fusion, and grinding the product to powder. By **Puzzolan** is meant the product obtained by grinding slag and slaked lime, without subsequent calcination.

Recommendations of Board of U. S. A. Engineer Officers. Continued.

In gaging Portland cement in damp weather, the samples should be thoroughly dried before adding water. This precaution is not deemed necessary with Natural cement. Sufficient uniformity of temperature will result if the testing room be comfortably warmed in winter, and if the specimens be kept out of the sun in a cool room in summer, and under a damp cloth until set. Temperatures may vary between 60° and 80° F., without affecting results more than the probable error in the observation.

Boiling test. Place the two cakes under a damp cloth for 24 hours. Place one of them, still attached to its plate, in water 28 days; immerse the other in water at about 70° F., and let it be in a rack above the bottom of the receptacle; heat the water gradually to the boiling point, maintain the heat for 6 hours and then let cool. The boiled cake should not warp or become detached from the plate, or show expansion cracks. If the cold-water cake shows evidences only of swelling, the cement may be used in ordinary work in air or fresh water for lean mixtures, but if distortion or expansion cracks appear in it, the cement should be rejected.

Accelerated tests are not generally recommended, but where a test must be made in a short time, the boiling test is considered about the best. It not only gives short-time indications, but at once directs attention to the presence of ingredients which might lead to disintegration. On the other hand, it may lead to the rejection of a cement which would behave satisfactorily in actual work and which would stand the test after air-slaking. Sulphate of lime, while enabling cements to pass the boiling tests, introduces an element of danger.

(5) **Tensile tests** are preferred to flexural or compressive tests. Sand tests are the more important and should always be made; and neat tests should be made if time permits.

A cement which tests moderately high at 7 days, and shows a substantial increase in strength in 28 days, is more likely to reach the maximum strength slowly and retain it indefinitely with a low modulus of elasticity, than a cement which tests abnormally high at 7 days with little or no increase at 28 days.

Use **briquettes** of the form recommended by the American Society of Civil Engineers,* measuring 1 inch square in cross-section at place of rupture, and held by close-fitting metal clips, without rubber or other yielding contacts. The tests should be made immediately after taking the briquettes from the water.

Neat tensile tests. Use unsifted cements. For Portland cement, use 20; for Natural, 30; and for Puzzolan, 18 per cent. water by weight. Place the cement on a smooth non-absorbent slab; in the middle make a crater sufficient to hold the water; add nearly all the water at once, the remainder as needed; mix thoroughly by turning with the trowel, and vigorously rub or work the cement for 5 minutes.

Place the briquette mold on a glass or slate slab. Fill the mold with consecutive layers of cement, each to be ¼ inch thick when rammed. Give each layer 30 taps with a soft brass or copper rammer weighing 1 pound, having a face ¾ inch diameter or 0.7 inch square, and falling about ½ inch.

After filling the mold and ramming the last layer, strike smooth with a trowel, tap mold lightly on side, to free cement from plate, remove the plate, and leave for 24 hours, covered with a damp cloth. Then remove the briquette from the mold and immerse it in fresh water, which should be renewed either continuously or twice in each week during the specified time.

Tensile tests with sand. For Portland and Puzzolan cements, use 1 part cement to 3 parts sand; for Natural or Rosendale, 1 to 1. Use crushed quartz sand, passing a No. 20 standard sieve, and being retained on a No. 30 standard sieve.

After weighing carefully, mix dry the cement and sand until the mixture is uniform, add the water as in neat mixtures, and mix for 5 minutes. The constituents should be well rubbed together.

For maximum strength in tested briquettes, Portland cements **require water** = 11 to 12½ per cent. by weight of constituent sand and cement; Natural, 15 to 17; and Puzzolan, 9 to 10.

A machine which applies the **stress** automatically and at a **uniform rate**

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* See page 1236.

Recommendations of Board of U. S. A. Engineer Officers. Continued.

of increase is preferable to one controlled entirely by hand. The stress should be increased at the rate of about 400 lbs. per minute. A rate materially greater or less than this will give different results.

The highest tensile strength from each set of briquettes made at any one time is to be considered the governing test.

Field tests are recommended, whether or not the more elaborate tests above described have been made. In connection with tests of weight and fineness, and observations of texture and hardness in the work, field tests often suffice for well-known brands, showing whether the cement is genuine and whether it is reasonably sound and active. Pats and balls of neat cement from the storehouse, and of mortar from the mixing platform or machine, should be frequently made. Estimate roughly the setting and hardening qualities by pressure of the thumb-nail; hardness of set and strength by breaking with the hand and by dropping upon a hard surface. The boiling test may also be used. Should the simple tests give unsatisfactory results, then a full series of tests should be carefully made.

A cement may be rejected if it fails to meet any of the following requirements.

Requirements.

		Portland. Slow.	Portland. Quick.	Natural.	Puzzolan.
Fineness. Percentage to pass through a No. 100 sieve as in (1)		87 to 92*		80	97
Specific gravity. Between		3.10	3 10	Not	2.7
and		3 25	3.25	given	2.8
Time of setting. Initial, not less than		45 m.	20 m.	20 m.	45 m.
nor more than		......	30 m.	......	......
Final, not less than		......	45 m.	......	......
nor more than		10 h.	2.5 h.	4 h.	10 h.
Tensile strength, neat, lbs. per sq. in.	7 days†	450	400	90	350
	28 days†	540	480	200	500
Tensile strength. With sand, as in (5). lbs. per sq. in.	7 days†	140	120	60	140
	28 days†	220	180	150	220

*92 per cent. is quite commonly attained by high-grade American Portlands, but rarely by imported brands. For the latter, use 87.

† Reject any cement not showing an increase at 28 days over 7 days.

DIGESTS OF SPECIFICATIONS.

Requirements.

American Society for Testing Materials.

Digest of Specification adopted by the Society, Nov 14, 1904. See Amendments of 1908 *

Adopted by Assn of Am Portland Cement Mfrs, June 10, 1904,* and by **Am Ry Eng & Maint of Way Assn,** Mar 21, 1905.*

1. Packages. Brand and mfr's name plainly marked thereon. Bag to contain 94 lbs net. Bbl Portland = 4 bags; nat, 3 bags.

2. Tests in accordance with recommendations of Comm of A S C E, p 1234. "Cem, failing to meet the 7-day requirements, may be held awaiting the results of the 28-day tests before rejection."

3. Qualities.	**Natural**	**Portland**
Sp gr, cem thoroly dried at 100° C.*	min 2.8 *	min 3.1
Loss of wt, on ignition	...	... *
Fineness. Percentage, by wt.		
Residue on No. 100 sieve	max 10	max 8
" on No. 200 sieve	max 30	max 25
Time of setting, mins, initial	min 10	min 30
" hard	min 30 max 180	min 60 max 600

Tensile strgth,

Min requirements,* lbs per sq inch; briquettes 1 inch square section.

Briquettes must show no retrogression in strgth during specified periods.

1 day in moist air in all cases.

	Natural	Portland
Neat		
24 hours	50 to 100	150 to 200
7 days	100 to 200	450 to 550
28 days	200 to 300	550 to 650
1 part cem, 3 parts standard sand.		
7 days	25 to 75	150 to 200
28 days	75 to 150	200 to 300
Soundness (constancy of volume) (For normal and accelerated tests, see digest of A S C E Specfns, p 945)	to stand normal test	to stand normal and accelerated tests.
Anhydrous sulfuric acid		max 1.75%
Magnesia		max 4 00%

Engineering Standards Committee of Great Britain,

Adopted Nov. 23, 1904.

1. Consignments of from 100 to 250 tons to have expert testing and chemical analysis. For consignments of less than 100 tons, makers shall, if required, give certificate, for each delivery, that cem meets this spec'n.

2. Samples. Test samples to be taken as soon as bulked at factory or on the work, at consumer's option. Samples to be taken from each "parcel," each sample consisting of cem from at least 12 diff positions in same "heap," mixed together and spread out, 3 ins deep, for 24 hours, at a temp between 58° and 64° F.

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* **Amendments** adopted by Am Soc for Testing Materials, Sep 1908·

Strength. The means of the values given shall be taken as the required minima where these are not specified.

Natural Cement. Omit specification for specific gravity.

Portland Cement. Specific gravity. For "thoroly dried at 100° C," read "ignited at a low red heat."

Loss of weight, on ignition, $\not>$ 4 %.

Requirements. Engineering Standards Committee of Great Britain. Continued.

3. Fineness.

Meshes per lin inch	Meshes per sq inch	Wire diam, ins	Residue not to exceed
76	5,776	0 0044	5.0 %
180	32,400	0.0018	22.5 %

Wire woven, not twilled.

4. Tensile strength.

Test room temperature, 58° to 64° F.

Water, fresh, renewed every 7 days. Temp 58° to 64° F.

Paste, smooth, easily worked, that will leave the trowel cleanly in a compact mass.

Briquette, filled, not rammed, into mold resting upon an iron plate, and left until cem has set. Briquette kept in damp atmosphere 24 hours; then in water until broken. Clips. See Fig. 1.

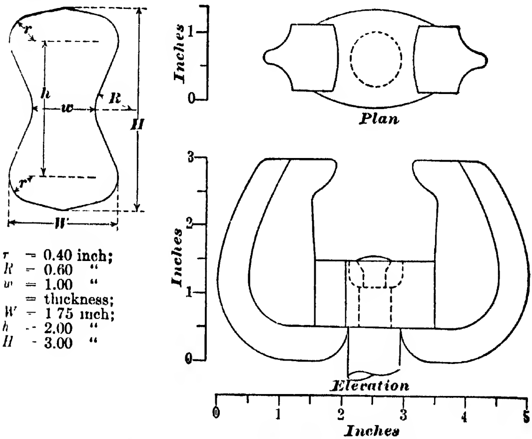


Fig 1. Briquet and Clips. British Standard.

Load, start at zero. Add 100 lbs each 12 seconds.

Neat test. 6 briquettes at 7 days, and 6 at 28 days. Av of the six accepted as the tensile strgth of the cement. 7 days, ≮ 400 lbs per sq. inch; 28 days, ≮ 500.

When 7 day test is betw	Increase, from 7 to 28 days, must be not less than
400 and 450 lbs per sq. in	25 per cent.
450 and 500 " " " "	20 "
500 and 550 " " " "	15 "
550 and over " " " "	10 "

Test with sand. By wt, 1 cem, 3 standard sand from Leighton Buzzard, thoroly washed and dried. Sand must pass No. 20 sieve of 0.0164 inch wire, and remain on No. 30 sieve of 0.0108 inch wire. Mixture thoroly wetted, but without superfluous water. 7 days, 120 lbs per sq inch; 28 days 225. Increase, from 7 to 28 days, not less than 20 %.

Requirements. Engineering Standards Committee of Great Britain. Continued.

5. Setting.	Time, mins maximum	minimum
Quick	30	10
Medium	120	30
Slow	300	120

"Set" has occurred when needle, loaded with 2½ lbs, with flat end 1/16 inch square, fails to make an impression.

6. Soundness. LeChatelier test. Expansion not to exceed 12 mm after 24 hours aeration; 6 mm after 7 days.

7. Specific gravity. Not less than 3.15, when sampled and hermetically sealed at makers'. Not less than 3.10, when sampled after delivery to consumer.

8. Analysis.
Water, ≯ 2 %, whether added or naturally absorbed from the air.
Calcium sulfate, ≯ 2 % of wt of cem, calculated as anhydrous calcium sulfate.
Lime, ≯ enough to saturate the silica and alumina.
Insoluble residue, ≯ 1.5 %. **Magnesia,** ≯ 3 %. **Sulfuric anhydride,** ≯ 2.5 %

Tests.

American Society of Civil Engineers.

Digest of report of Committee on Uniform Tests of Cement,* Jan '03, as amended Jan '04 and Jan '08.

1. Selection of samples left to discretion of engineer. Number of samples and quantity to be taken from each package depend upon importance of work, upon number of tests to be made and upon facilities for making them. Where conditions permit, sample one bbl in ten. Individual samples may be mixed, and av tested: but, where time permits, test separately.

2. Barreled cement to be sampled through a hole made in the center of a stave, midway between the heads, or in the head. Bagged cement to be sampled from surface to center.

3. Samples to be coarsely screened thru a No. 20 sieve.

4. Chemical analysis may show adulteration in the case of cems rich in inert material, but is not conclusive evidence of quality. Committee recommends method proposed by Committee on Uniformity &c , New York Section of the Society for Chemical Industry, see E N, '03, Jul 16, p 60; E R, '03, Jul 11, p 49.

5. Specific gravity test. Le Chatelier's method recommended. Fig 1. Flask, D, 120 cubic centimeters (cc); neck about 9 mm diam and 20 cm long, with bulb, C; vol, betw marks, F and E, 20 cc. Neck graduated, to 0.1 cc, above F. Neck of funnel, B, enters neck of flask, and extends to top of bulb, C. Use benzine (62° Baumé naphtha) or kerosene free from water. During the operation, in order to avoid variations in the temperature of this liquid, the flask is kept immersed in water, in a jar. Two methods, viz:

(a) Flask filled to lower mark, E. Weigh out 64 grams (2.25 oz) of the cem powder, cooled to temp of liquid. Thru the funnel, B, introduce the cem powder gradually until surf of liquid reaches the upper mark, F. Then 64 grams, minus wt of powder remaining unused, = wt, w, which has displaced 20 cc and

Specific gravity = w / 20.

(b) Fill, with liquid, to lower mark, E, as before. Add the entire 64 grams cem powder, liquid rising to some division of the graduated neck.

*Geo. S Webster, Richard L. Humphrey, Geo. F. Swain. Alfred Noble, Louis C. Sabin, Spencer B. Newberry, Clifford Richardson, F. H. Lewis, W. B. W. Howe. A S C E, Proceedings, Jan '03. Feb '04 Feb 08.

Tests. Am Soc Civ Engrs. Continued.

The reading of this division, plus 20 cc, is the vol, v, displaced by 64 grams of the powder; and

Specific gravity $= 64/v$.

6. Fineness. Sieves should be circular about 20 cm (7.87 ins) diam, 6 cm (2.36 ins) high, with pan 5 cm (1.97 ins) deep, and a cover.

Sieves should be of wire cloth,

No. 100, 96 to 100 meshes per lineal inch; wire 0.0045 inch diam.
No. 200, 188 to 200 " " " " " 0.0024 " "

Use 50 grams (1.76 oz) or 100 grams, cem; dried at 100° C (212° F). Hand sieving preferred. Use No. 200 sieve until one minute continuous sieving, at about 200 strokes per minute, passes not more than 0.1 %. Weigh residue, and treat it similarly on No. 100 sieve. A small quantity of large steel shot, placed in the sieve, expedites the work. The results should be reported to the nearest 0.1 %.

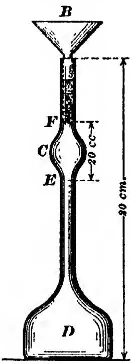


Fig 1.
Sp grav Flask.

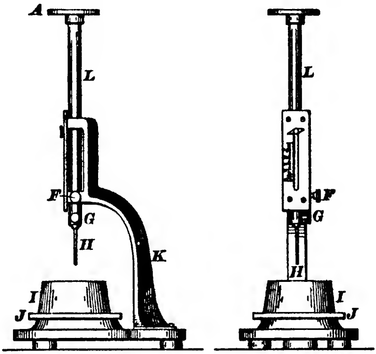


Fig 2.
Vicat Needle Apparatus.

7. Normal consistency. The percentage of water, used in making the pastes, for tests of strgth, soundness and setting, vitally affects the results. Normal consistency is determined as follows:

The quantity of cem, to be subsequently used for each batch in making the briquettes, but not less than 500 grams, is kneaded into a paste as under "Mixing," ¶ 12 quickly formed into a ball, with the hands, and tossed six times from hand to hand, held 6 ins apart. The ball is then pressed thru the larger opening of the Vicat needle apparatus into the gum ring, I, 7 cm (2.76 ins) diam, 4 cm (1.57 ins) deep, smoothed off below, and placed on the glass plate, J. Its upper surf is then smoothed off with a trowel. The point of the Vicat needle, H, is then brought into contact with the upper surf of the sample, and the cyl, L, is allowed to descend. The paste is of the normal consistency when the needle penetrates to a depth of 1 cm (0.39 in). With this rather wet paste, the committee believes that variations, in the amount of compression to which the briquette is subjected in molding, are likely to be less than with a drier paste.

8. Setting. Vicat needle, H, Fig 2, 1 mm (0.039 in) diam, loaded to 300 grams (10.58 oz). Setting has begun when needle ceases to pass a point 5 mm (0.20 in) above the upper surface of the glass plate; and has terminated when the needle does not visibly penetrate the mass. Test pieces kept damp, during test, by storage in a moist box or closet, or placed on a rack over water in a pan and covered by a damp cloth, the cloth resting upon a wire screen, so as not to touch the test pieces. Keep needle clean; as cem, adhering, seriously

Tests. Am Soc Civ Engrs. Continued.

vitiates results. Time of setting is materially affected by temp of mixing water, by temp and humidity of air, by the percentage of water used, and by the amount of molding paste receives.

9. Standard sand. Crushed quartz objectionable, "especially on account of its high percentage of voids, the difficulty of compacting in the molds, and its lack of uniformity." Comm recommends natural sand from Ottawa, Ill. Sand to pass a No. 20 sieve, with wire diam = half the diam of spaces betw wires; $\nless$ 99 % to be retained on a similar No. 30 sieve after 1 minute of continuous sifting of a 500 gram sample. The Sandusky Portland Cement Co., Sandusky, O., has agreed to furnish such a sand at actual cost of preparation.

10. Standard briquette. See Fig. 3. Am Soc Civ Engrs. Dotted lines are those recommended by earlier Comm. Trans, Vol 14, Nov. 1885.

	inch
r =	0.50
R =	0.75
R' =	1.75
h =	1.30
H =	3.00
W =	1.75
w =	1.00
	= thickness.

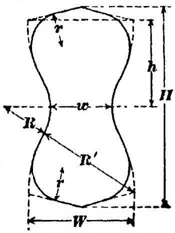


Fig 3. Briquet.

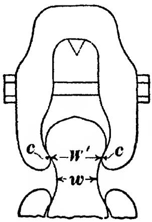
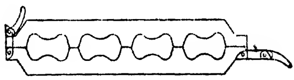


Fig 5. Clip.

W' =	1.25 ins.
c =	0.25 "
	= contact with briquet.

Fig 4. Gang Mold.



11. Molds. "of brass, bronze or some equally non-corrodible material;" sides strong enough to resist spreading. Gang mold, Fig 4, recommended, because the greater quantity of mortar, required for it, conduces to uniformity of results. Molds to be "wiped with an oily cloth before using."

12. Mixing. Proportions stated by wt; quantity of water stated as percentage of dry material.

Metric system recommended.

Temp of room and mixing water as near 21° C (70° F) as practicable.

Sand and cem thoroly mixed dry. Mixing done on some non-absorbing surf, preferably plate glass. If an absorbing surf is used, it should first be thoroly dampened.

Quantity of material, mixed at one time, depends on number of test pieces to be made; about 1000 grams (35.28 oz.) convenient to mix, especially by hand methods.

Hand mixing and hand molding recommended. Material weighed, and placed on mixing table, and a crater formed in the center, into which the proper percentage of clean water is poured; material on outer edge turned into crater by aid of a trowel. As soon as the water is absorbed, the operation is completed by vigorously kneading with the hands for an additional 1½ minutes. A sand-glass affords a convenient guide for the time of kneading. The hands should be protected by gloves, preferably of rubber.

Molds filled immediately after the mixing is completed, material pressed in firmly with the fingers and smoothed off with a trowel, without mechanical ramming; material heaped up on the upper surface of the mold. In smoothing off, the trowel should be drawn over the mold, exerting a moderate pressure on the excess material. Mold turned over and operation repeated.

Tests. Am Soc Civ Engrs. Continued.

Weigh the briquettes "just prior to immersion, or upon removal from the moist closet," and reject those varying > 3 % from the av.

13. Moist Closet. "A moist closet consists of a soapstone or slate box, or a metal-lined wooden box—the metal lining being covered with felt and this felt kept wet. The bottom of the box is so constructed as to hold water, and the sides are provided with cleats for holding glass shelves on which to place the briquettes. Care should be taken to keep the air in the closet uniformly moist."

"Where a moist closet is not available, a cloth may be used and kept uniformly wet by immersing the ends in water. The cloth should be kept from direct contact with the test pieces by means of a wire screen or some similar arrangement."

14. Immersion. "After 24 hours in moist air the test pieces for longer periods of time should be immersed in water maintained as near 21° C (70° F) as practicable. they may be stored in tanks or pans, which should be of non-corrodible material"

15. Tensile strength. Solid metal clip, Fig 5, recommended. No cushioning between clip and briquette. Briquettes broken immediately after removal from water. Center the briquette carefully in the clip, to avoid transverse stresses. Load applied at rate of 600 lbs per min. "The average of the briquettes, of each sample tested, should be taken as the test" of that sample, "excluding any results which are manifestly faulty."

16. Soundness (Constancy of Volume). "In the present state of our knowledge it cannot be said that cement should necessarily be condemned simply for failure to pass the accelerated tests (below), nor can a cem be considered entirely satisfactory, simply because it has passed these tests."

Pats of cem paste of normal consistcy (¶ 7), abt 7.5 cm (2.95 ins) diam, 1.25 cm (0.49 in) thick at center, tapering to thin edge, made on a clean glass plate about 10 cm (3.94 ins) square, 24 hours in moist air before test.

(1) Normal test. One pat immersed in water maintained as near 21° C (70° F) as possible; one in air at ordinary temp. Both observed at intervals for 28 days.

(2) Accelerated test. A pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel, for 5 hours.

Pats must remain firm and hard, and show no signs of cracking, distortion or disintegration. Warping may be conveniently detected by applying a straight edge to the surf which was in contact with the plate.

Sand.*

Composition.

1. The sand,* used in mortar, is ordinarily made up chiefly of grains of **quartz (silica)**, with some impurities, mostly grains of silicious minerals. In testing cements. in the laboratory, crushed quartz or some standard natural sand is used. (See Spec'ns A S C E, under Cement, p. 1234.)

2. The silica of the quartz, in sand, undergoes **no chemical change** in the mortar; but the use of sand, by diminishing the quantity of cem reqd, reduces also the cost of the finished work. See remarks on strength, under Mortar, p 1248.

SIZES OF GRAINS.

3. Screening. Sand and gravel are screened, usually in an inclined fixed screen, upon which the material is placed by a conveyor, or shoveled by hand; or in an inclined revolving cylindrical or hexagonal screen, into which the material is fed.

4. Method of quartering. "To obtain an average sample from a pile of sand, gravel or stone, the method of quartering is useful. Shovelfuls of the material are taken from various parts of the pile, mixed together and spread in a circle. The circle is quartered, as one would quarter a pie, one of the quarters is shoveled away from the rest, thoroughly mixed, spread, and quartered as before. The operation is repeated until the quantity is reduced to that required for the sample." (T & T, p. 281.)

Mechanical Analysis.

5. The mechanical or granulometric analysis of sands, etc., is the determination, in any given sand or broken stone, of the proportions of grains of diff sizes. It is usually performed by means of sieves or screens. See ¶ 3. Sometimes, for broken stone, &c., by hand-picking.

6. Fig. 1 shows mechanical analyses of a gravel and a sand by Mr. Allen Hazen (Mass. State Board of Health, Report 1892, pp. 546–7). In order to represent both analyses on a single diagram, we have used diff scales for diams for the two materials.

7. In Fig. 1, the diagrams show, for the two materials there represented, that

of the sand, 10 % was in grains under, and 90 % over, 0.055 mm diam
" " gravel, 10 % " " " " " 90 % " 34.5 " "

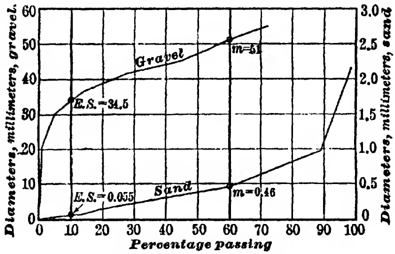


Fig 1. Sand Analyses.

***By "sand" or "gravel" we mean** a *mixture* of mineral particles with air, or water, or both; *i. e.*, an aggregation of mineral particles, with voids betw them said voids being filled with air, or with water, or with air and water, as the case may be.

Hence, the "volume" of a given quantity of sand or of gravel is the space occupied by both the solid particles and the air or water or both, filling the voids.

"Dry sand," or "dry gravel," means: not solid mineral, but a mixture of dry particles of sand (or gravel) and dry air.

The solid mineral portion of such sand or gravel, we designate as "solid."

Effective Size.

8. The effective size ("e. s.") of a sand or gravel, as defined by Mr. Hazen (Mass State Board of Health, Report 1892, p 341; Hazen, Filtration, pp 21, 240) is that size, than which 10 %, by wt, of the grains are smaller, and 90 % larger. Or, the length of the ordinate, at 10 % passing, gives the effective size. Thus, in the cases just mentioned, Fig 1, we have:

for the sand, e. s. = 0.055 mm; for the gravel, e. s. = 34.5 mm.

Uniformity Coefficient.

9. Uniformity coefficient. Similarly, let m = that diam of grain, than which 60 %, by wt, is smaller, while 40 % is larger. In Fig 1, we have

for the sand, m = 0.46 millimeters;
" " gravel, m = 51.00 "

The uniformity coefficient ("u. c."), is m/e. s.; and we have:

for the sand, u. c. = 0.46/ 0.055 = 8.4;
" " gravel, u. c. = 51.00/34.5 = 1.48.

10. With m = e. s., the unif coeff, u. c., would have its least possible value, = 1. In general the *less* nearly uniform a sand is, as to size, the *higher* is its "uniformity coeff."

11. In ordinary bank sand, the effective size, e. s., does not vary widely Hence the uniformity coefficient, u. c. = m/e. s., varies roughly with that diam, m, than which 60 % of the grains are smaller, and thus serves as an **indication of the coarseness,** as well as of the departure from uniformity, of the sand. (T & T, p. 182.)

Feret's Method.

12. Mr. R. Feret (Annales des Ponts et Chaussées, 1892, second semestre,) made elaborate experiments as to the effects of fineness of sand, and the mixture of different finenesses, upon the density, etc., of sand and upon different qualities of the mortar. He divided his sands into **three finenesses,** as follows.

Coarse,	c,	passing 5.0 mm diam	=	4 meshes / sq cm	=	5 meshes / lin in
Medium,	m,	" 2.0 " "	=	36 " / " "	=	15 " / "
Fine,	f,	" 0.5 " "	=	324 " / " "	=	46 " / "

"Coarse" grains are retained on 2.0 mm diameter; "medium" on 0.5 mm.

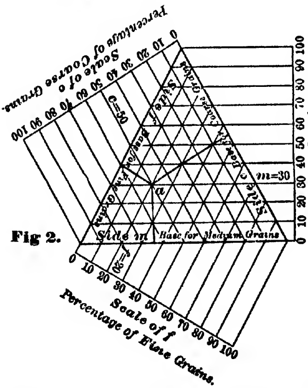


Fig 2.

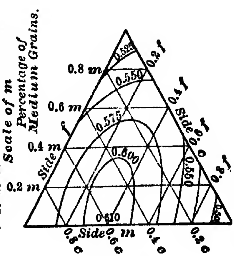


Fig 3.
Sand Analyses, Feret.
See ¶ 18.

13. The results, obtained in a certain case, with diff mixtures of these three grades of fineness, are shown in Fig 2, which is similar to diagrams used in connection with alloys of three metals.

14. After a given mixture has been analyzed, and its percentages of the three grades thus determined, it is plotted, in the triangle, by a point so placed that its perp dists, from the three sides, respectively, of the equilateral triangle, are as follows:

distance from side c = percentage of coarse grains;
" " " m = " " medium "
" " " f = " " fine "

15. The plotting of the points, and the measurements of their dists, are facilitated by the lines drawn parallel to the three sides respectively.

16. Thus, point a represents a sand having 20 % fine grains, 30 % medium and 50 % coarse, as shown by the three scales; 20, 30 and 50 being the dists of a from sides f, m and c, respectively.

17. When a series of experiments has been made, upon any given quality (as density or porosity, etc, etc) of sand or mortar, as affected by diffs in mixtures of the three finenesses, they are plotted in this way, and "contour" or **"iso"-lines** are drawn thru those points which represent equal results in the quality experimented upon. Each "iso"-line therefore represents a series of diff mixtures, each of which will give the value (as to density or porosity, etc, etc) represented by it.

18. Thus, in Fig 3 (T & T, p 144, Fig 51) the four contours and the point (0.610) represent five diff mixtures of coarse, fine and medium sands, said mixtures having densities (see ¶ 20) of 0.525, 0.550, 0.575, 0.600, 0.610, respectively.

Density.

19. Specific gravity or unit weight. Solid quartz weighs about 165 lbs per cu ft = 2.643 grams per cu cm; sp gr = 2.64 to 2.67.

20. In mechanics (see p. 338, Art. 14 a) **density is defined** as the *mass* in unit volume. In sand,* the solid portions have practically constant sp gr. Hence, for a given sand, "density" is used to designate the vol of solid in unit vol of sand, or the ratio of solid to total vol. This ratio is sometimes called the "absolute volume." Thus, in unit vol of sand, "density" = 1 — vol of voids.

21. The greater the density of sand,* the less cement will be reqd for a given quantity of mortar.

22. The weight, per cubic foot, of a sand,* of given sp gr. varies directly with its density; and this, in turn depends upon the shape of the grains, upon their *range* of size, upon the compacting accomplished, as by shaking, tamping, etc, and upon the dryness of the sand.

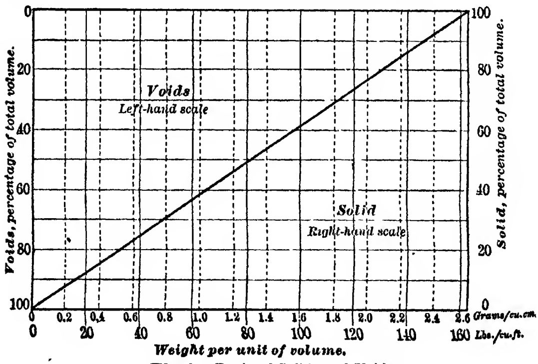


Fig 4. Ratio of Solids and Voids.

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* See foot note*, p 1238.

23. Fig 4 shows the **relation** betw (1) the **unit weight** and (2) the **percentages of solid and of voids,** solid quartz weighing as in ¶ 19.

Effect of Moisture.

24. The effect of moisture, upon the vol of a given quantity of sand,* is affected by the vol of air introduced, by the quantity of water, and by the shape of the grains.
See ¶¶ 29 to 31.

25. It is impracticable to measure the vol of **air introduced,** and its presence vitiates all observations. When sand grains are dropped, one at a time, into water, most of the air, surrounding the grains, is left behind in the atmosphere; but when sand* is thrown into water in masses, or when moist or wet sand is turned over by shoveling, considerable and unknown quantities of air are entrained with it.

26. In moist sand,* the total (or "absolute") vol of voids is usually filled partly by water and partly by air.

27. Within a certain limit, **moisture** increases the adhesion betw the grains of sand, and thus opposes their sliding, one upon the other, consequently opposing the compacting of the sand; but, beyond that limit, it acts as a lubricant and facilitates the compacting. See ¶¶ 24, 25.

28. Let

V = volume, in cu ft, of dry quartz in 1 cu ft of sand;*
v = " " " " " voids " 1 " " " " ; $V + v = 1$ cu ft;
W = wt, in lbs, of 1 cu ft of pure solid quartz = 165;
w = " " " " 1 " " " the sand (dry or moist, as the case may be);
d = " " " " dry quartz in 1 cu ft of the sand; (in dry sand, $d = w$).
P = " " " " water added to 1 lb of dry sand;*
= " " " " " in $(1 + P)$ lbs of moist sand;
p = " " " " " " 1 lb " " " ;
m = " " " " " " 1 cu ft " " " .

Then $p/P = 1/(1 + P)$; and $p = P/(1 + P)$;
$m = w\,p$; $d = w - w\,p = w\,(1 - p)$;
$V = (w - w\,p)/W = d/165$; $v = 1 - V = 1 - d/165$;
$w = W\dfrac{1 - v}{1 - p} = W\dfrac{V}{1 - p}$.

29. The proportion, p, of moisture (lbs of water in 1 lb of moist sand), is ascertained by heating a known wt of the moist sand, at not less than 100° C (212° F), until no further loss of wt takes place, and noting the loss of wt. Then:
p = loss of weight ÷ original weight of portion heated.
In dry sand (Fig 4) $p = 0$, $w\,p = 0$, $w = d$; and we have:

$$V = w/W = w/165 = d/165.$$

Effects of Shape and Size.

30. Spherical grains. If a number of spheres, of uniform diam, D, be piled as closely as possible, the ratio of vol of solid to total vol is $\dfrac{\pi\sqrt{2}}{6}$ = about 0.74; and the voids (about 0.26 × the total vol) are of two sizes, such that they can be fitted, respectively, with spheres having diams = about 0.41 D and 0.22 D. (T & T, pp 169–170.)

31. Effect of gradation of sizes. The proportion of voids may be indefinitely reduced by adding to, and mixing with, the original grains, smaller and smaller, or larger and larger, particles, in proper proportions, each size occupying a portion of the voids left between the particles of the size next coarser. With spherical particles, therefore, the voids are greatest, and the wt per unit vol least, when the grains are of uniform size. This seems to hold true also for particles of other shapes.

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* See foot-note*, p 1238.

Other Properties.

32. Turbidity test for silt in sand. *Separate the silt from a* considerable quantity of sand, and make up a special sample containing the max proportion of silt allowed by the spec'n. Place a small known portion of this mixture in a known quantity of clear water in a graduated vessel. Shake the vessel until the sample is thoroly washed. Insert a pin horizontally in the side of a stick near its end, insert that end of the stick into the vessel, lowering it until the pin is no longer visible thru the liquid, and note the depth of the pin by means of the graduation. Make several such tests and note the average depth of disappearance of pin. In testing samples, if the pin disappears at a higher elevation than the standard, the sand has more silt than the maximum allowable, and vice versa.* (W. J. Douglas, E N, '06/Dec/20, p 648.)

33. The presence of clay and loam, in sand, **may be detected** by rubbing the damp sand in the hand, and observing the condition of the hand, or by mixing the sand with clean water and noting the effect upon the water.

34. Washing. Dirty sand may be washed in a specially constructed sand washer; or, by means of a jet from a hose, in a box so arranged that the mud, clay and organic impurities are floated off, leaving the heavier sand behind.

35. Washing may carry off the finer particles of a well assorted sand, leaving it less dense than before. It is well to test a small quantity of the sand, washed and unwashed, before arranging to wash for use. (Jas. C. Hain, E R, '05/Jan/28, p 105.)

36. The degree of sharpness of a sand **may be estimated** by means of the sound emitted by it when kneaded betw the hands or more closely estimated by means of a magnifying glass.

MORTAR.*

Constituents.

1. Cement mortar consists of cem, mixt with water, with or without some inert granular material, as sand, fine gravel, stone or gravel screenings, or ground cinder. Without sand, etc., the mixture is called **neat** mortar, or **cement paste.**

Amount of Mortar Required for a Cubic Yard of Masonry.†

Description of Masonry.	Mortar. Cu yd. Min.	Max.
Ashlar, 18" courses and ¼" joints	0.03	0.04
" 12" " " "	0.06	0.08
Brickwork (bricks of standard size, 8¼ × 4 × 2¼ ins.):		
⅛" joints	0.10	0.15
⅜" to ¼" joints	0.25	0.35
⅝" to ½" joints	0.35	0.40
Rubble, of small, rough stones	0.33	0.40
" " large stones, rough hammer-dressed	0.20	0.30
Squared-stone masonry, 18" courses and ¾" joints	0.12	0.15
" " 12" " " "	0.20	0.25

2. Effect of roasting and of subsequent wetting. The materials, of which cem is made, are inert or stable compounds, remaining practically unchanged under ordinary conditions; but when, in burning, the calcareous materials are subjected to high temps, either alone or mixed with argillaceous materials, relatively unstable compounds are formed, ready to enter into new and again stable compounds when their particles are brought into intimate contact by being mixed with water, the water also entering into the new combinations. The mixture then soon "sets" (loses plasticity), and, shortly thereafter, begins to solidify and harden.
See ¶ 8, Cement, p 1223.

3. In the process of crystallization, the alumina appears to act chiefly as a **flux**, promoting the formation of the lime silicate, upon which the success of the operation depends. Iron oxide, which is generally present, seems to answer as well as alumina, as a flux, and it requires a less high temp for calcination.

4. The proportion of sand, which should be used in any given case, cannot be properly stated without stating also its range of size, or the proportion of voids to the whole mass; but, in general, good Portland cems will "carry" from 2 to 3 vols of sand; nat cems from 1.5 to 2 vols.

5. Approximate quantities of Portland cement and loose sand per cu yd of mortar.

	Neat	1:1	1:2	1:3	1:4	1:5	1:6
bbls cem	8.0	4.6	3.1	2.3	1.8	1.5	1.3
cu yds loose sand	0	0.65	0.87	0.97	1.02	1.06	1.10

Cement in Mortar.

See also CEMENT, p 1222.

6. Owing to the cheapness with which **cements** are now manufactured, and the superiority of the mortars made from them, the latter have to a great extent **superseded lime mortars,** even in ordinary building operations.

7. In selecting cem, a **reputation,** gained by years of successful use and experiment, is of greater value than the results of a few tests; but such **tests** are of value for excluding inferior parcels of such accepted brands.

8. High grade cements are usually economical, even at a higher cost, as they allow the use of a larger proportion of the cheaper ingredients, sand, gravel and broken stone.

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*As the strgth, permeability, etc, of a conc depend largely upon those of its mortar, we discuss, under "mortar," many of its properties commonly discussed under "concrete"

† Taken, by permission, from "A Treatise on Masonry Construction," by Prof. Ira O. Baker. New York, John Wiley & Sons. 9th edition, 1907.

9. Free Lime. Cem may contain "free" (uncombined) lime as a result (1) of insufficient manipulation of the raw materials, (2) of insufficient burning, (3) of an excess of lime carbonate ($CaCO_3$) in the raw materials, or (4) of adulteration after burning and grinding.

10. This lime may be present either as quick lime, CaO, or as slacked lime $Ca(OH)_2$, either of which may be washed out (the CaO first becoming $Ca(OH)_2$) by infiltrating water. This, of course, weakens the cem.

11. Slacked lime takes no part in the hardening process, but remains as an inert filling material.

12. Quick lime slacks by absorption of the water used in mixing; and, when the burning has been at a high temp, the slacking is delayed. If it takes place during the setting of the cem, the swelling of the lime weakens the cem by rendering it porous. If slacking is delayed until after hardening, and if the expansive force is sufficient, the cem is disintegrated.

13. Excess of lime retards setting, and reduces soundness.

14. Free Magnesia. Much uncertainty exists as to the effect of free magnesia, in diff proportions, in cem. Like lime, it expands when wet, but much more slowly; and its presence may therefore remain unsuspected until too late. **Dolomite,** or magnesian limestone, contains about 45 % of magnesia. Formerly, 1.5 % of free magnesia, in cem, was considered dangerous. It is now generally believed that more than from 3 to 5 % weakens the cem, and that 8 % or more causes cracking. In any proportion, it is probably objectionable, at least as displacing an equal quantity of the more valuable lime.

Sand* in Mortar.

See also SAND, pp 1238, &c.

15. The quality of the concrete depends upon the strength of the mortar, and this, in turn, depends largely upon the character of the sand

16. For a given proportion by wt, the best sand is that which produces the smallest vol of plastic mortar.

17. Weight. As betw two sands, of a given material, the heavier of course has the smaller vol of voids.

18. Fineness. A fine sand, well assorted as to sizes of grain, and therefore dense, may make better mortar than a coarser sand, with grains of more nearly uniform size and therefore less dense.

19. Extreme fineness prevents penetration of the paste betw the grains, and delays setting.

20. Mortars made with fine sand, altho less permeable than those made with coarse sand, are apt to be more easily acted upon by **sea water.**

21. Shrinkage. Mortars, with coarse sand, shrink less than those with fine sand.

22. Sharpness. It has been customary to insist upon sharpness of grain, in sand used for mortar, probably owing to the impression that sharp grains form a better bond with the cem or that sharpness indicates freedom from impurities; but the advantage is doubtful. Sands with rounded grains are commonly used, and with entirely satisfactory results; and the laboratory tests generally indicate that sharp-grained sands have no marked superiority Roundness of grain facilitates the packing, and thus increases the density of the sand.

23. The Board of Public Works of Porto Rico, with briquettes of 1 : 2 mortar, found 25 % greater strgth with **washed** than with **unwashed** sand. Sand, containing much foreign matter, should be tested before being accepted.

24. In general, the evidence, as to the **relative values of sand and of screenings,** appears to be favorable to the use of screenings (see Experiments), but opinion is divided. The **hydraulicity of the dust,** in the screenings, may add to the strength of the mortar.

25. Harry Taylor, Capt, Corps of Engrs, U S A, tested 1650 briquettes of 1 : 3, 1 : 4 and 1 : 5 mortars, at 1, 3, 6 and 12 mos, with standard crushed quartz, Plum Island sand and **crusher dust.** Crusher dust gave briquets

* See foot-note, SAND, ¶ 1, p 1238.

2.3 times stronger than sand, and 72 % stronger than quartz. 1 : 5, with stone dust, stronger than 1 : 3 quartz.

26. G. J. Griesenauer, E N, '03/Apr/16, p 342. Chicago, Mil & St P RR, 225 tests, as follows:

Limestone screenings, 1 : 3, passing No 12, held on No 40 sieve, averaged 74 % better than Hammond pit sand, 1 : 3; with all sizes used, they averaged 115 % better. Mortar of 1 : 6 screenings was 23 % stronger than 1 : 3 sand. **Gravel screenings** were not much better than sand.

27. Maryland highways. Briquettes, made with **stone screenings,** were 34 to 62 % stronger than with Potomac River sand.

Lime in Mortar.

28. The substitution of 10 % to 20 % **lime paste** for an equal vol of cem paste, reduces the cost of the mortar, renders it less "short", and slightly retards setting, without seriously diminishing its strgth. Larger quantities reduce strgth. (Baker, Masonry Construction.)

29. Feret found the effect of lime dependent upon the richness of the cem mortar. With 1 : 4 cem mortar, the addition of 4 to 5 % of dry slaked lime increased the strgth; while, with 1 : 1.25 cem mortar, the addition of lime lowered the strgth. (Chimie Appliquée, 1897, p 481.)

Clay in Mortar.

30. Laboratory tests indicate that **a small admixture of clay** increases rather than diminishes the strgths of mortar, and diminishes its permeability; but, in actual work, the clay particles tend to adhere and thus to form lumps having but slight cohesion.

31. Laboratory conditions, as to dryness, pulverization, etc., cannot be reproduced in practice.

32. When the clay occurs naturally in the sand, it may not be practicable to effect a perfect mixture and distribution.

33. Clay, etc, are more likely to give trouble with dry than with wet mixtures.

Consistency.

34. Relative strengths of dry and wet mortars, 1 : 1. Alfred Noble. over 5000 experiments. Strength of dry mortar taken as 100.

	Portland cement				Natural cement			
Age	30 days	3 mos	6 mos	1 yr	30 days	3 mos	6 mos	1 yr
Dry Mortar....	100	100	100	100	100	100	100	100
Moderately stiff.	97	94	97	97	78	89	95	90
Grout.........	90	92	91	95	63	77	86	82

35. Use dry conc when it is to be heavily loaded at once. Tests indicate that wet and dry conc will be equal in strgth within a year.

36. Wet conc bonds better to old work than does dry conc. Excess of water increases efflorescence and laitance.

37. Rule for percentage, W, of water. H. P. Gillette, Cost Data, p 266.

Let S = parts of sand to 1 part cem. Then

$$W = (8\,S + 24) \div (S + 1).$$

This gives

when S =	1	1.5	2.0	2.5	3.0	3.5	4.0
W =	16	14.4	13.3	12.6	12.0	11.5	11.2

Falk finds that mortars, thus proportioned, adhere well to steel.

38. Slag cement requires plenty of water for its proper hardening. Therefore, if used in air, slag cem mortar should be kept damp.

Setting and Hardening.

39. Setting, or the loss of plasticity, usually occurs within a few hours (sometimes within a few minutes) after mixing cem with water; whereas **hardening** and **increase of strength** (which appear to result from a different set of chemical processes) often proceed for months or even years.

40. Molded blocks of Portland conc, of even 50 tons wt, can generally be handled and removed to their places in from 1 to 2 weeks

Initial and Final Set.

41. Initial and final set are stages of the setting process, arbitrarily distinguished by means of the resistance, of the mortar, to penetration by cylindrical wires, of standard diams and loaded with standard wts, the blunt ends of the wires resting upon the surf of a pat of the mortar formed in a flat cylindrical mold on a glass plate. See ¶ 8, p 1235.

Determination of Set.

42. Genl Totten, (Genl Q. A. Gillmore, Limes, Hydraulic Cements and Mortars, p 80,) at Fort Adams, R. I., prior to 1830, used a 1/12 inch wire, loaded with 0.25 lb, and a 1/24 inch wire, loaded with 1 lb; initial and final set being taken as the conditions when these wires, respectively, failed to make an impression upon the mortar.

43. Vicat used but one wire, or "needle." The A S C E (see specifications, p 943) prescribes, for this needle, a diam of 1 mm (0.039 inch) and a load of 300 grams (10.58 oz). Initial set occurs when the end of the needle, penetrating a pat of mortar 4 cm (1.57 ins) deep, can no longer approach within 5 mm (0.2 in) of the glass plate; and final set when the needle fails to sink visibly into the mortar. The mortar, under the setting test, must be of "**normal consistency,**" or such that a cylindrical rod, 1 cm (0.39 inch) in diam, loaded with 300 grams, its end resting upon the mortar, penetrates 1 cm into it.

Speed.

44. Speed. Some of the best cems are the slowest setting. A layer of very quick-setting cem may partially set, especially in warm weather, before the masonry is properly lowered and adjusted upon it, and **any disturbance, after setting** has commenced, is prejudicial. On the other hand, **quick-setting cements** are best in certain cases, as when exposed to running water, etc. They may be rendered slower by adding a bulk of lime paste equal to 5 or 15 % of the cement paste, without weakening them seriously. Nat cems usually set quickly. Slag cem sets slowly.

45. In general, **setting is accelerated** by high alumina and by soda and potash in the cem, by freshness and fineness of the cem, by the use of warm water and warm sand in mixing, and by warm weather. **Setting is retarded** by excess of lime and silica in the cem, by the presence of sand, by wetness of mixture, by cold, by retempering, by salt or sulfuric acid in the mixing water, by the presence of 1 or 2 % of lime sulfate, either hydrated (gypsum) or anhydrous (plaster of Paris) or of slaked lime, in some cases by hard burning, and in general, by the age of the cement, but the storage of new cem in warm places accelerates setting.

45 a. Gypsum. $CaSO_4$. Time of setting (initial and final) increased rapidly with additions of gypsum up to about 2 %, and remained constant, or increased slightly, up to 4 %. E. Candlot, "Ciments et Chaux Hydrauliques."

45 b. Time of setting (initial and final) increased, up to about 1.5 % gypsum, but then decreased, as the gypsum was increased to 7 %. Kniskern and Gass, Sibley Jour of Engng, '05, Jan.

45 c. Calcium chloride, $CaCl_2$. A *weak* solution *retards*, but a *concentrated* solution *accelerates*, the setting of Port cems. Thus, with 10 to 40 grammes per liter, the time of setting reached 500 to 850 mins; while, with 200 to 300 grammes per liter, it was reduced to from 2 to 25 mins. Cems with very high or very low alumina are but little affected by $CaCl_2$. A weak solution (30 to 60 grammes per liter) may render sound a cem containing free lime, by facilitating the hydration of the lime. E. Candlot, "Ciments et Chaux Hydrauliques."

45 d. From ½ to 1½ % dry $CaCl_2$, ground with cem clinker and made into pats of normal consistency (See Tests, ¶ 7, p 1235) increased the time of initial set from 2 to 167 mins, and that of final set from 52 to 275 mins. With 6 %, the times were 68 and 145 mins respectively, Kniskern and Gass, Sibley Jour of Engng, '05, Jan.

46. Setting is attended by an **increase of temperature.** In quick setting, this increase may amount to 10° C (18° F) or more.

47. Slow setting cems are apt to *harden* more rapidly than quick setting.

48. In warm air, setting cem, in drying, loses the moisture upon which the operation of hardening depends. It therefore **sets without hardening.** In hot weather every precaution should be taken against this.

49. Cems of the same class differ much in their **rapidity of hardening.** At the end of a month one may gain nearly one-half of what it will gain in a year, and another not more than one-sixth; yet at the end of a year both may have about the same strength. Hence, tests for 1 week or 1 month are by no means conclusive as to the final comparative merits of cements.

50. Many years are required to attain the greatest hardness; but after about a year the increase is usually very small and slow, especially with neat cem. Moreover, any subsequent increase is a matter of little importance, because generally by that time, and often much sooner, the work is completed and exposed to its max loads.

51. Cems which are slow-setting when made, are apt to **become quick-setting** (or "**flashing**") **when stored,** especially in warm places, and if the cem is underlimed. This is attributed to disintegration of the particles and consequent increase in fineness. The change sometimes takes place very quickly. This difficulty can usually be overcome, without reducing the strgth, by storage in cool places and by adding 1 to 2% of slaked lime. On small jobs, a few lumps of lime may be added to each bbl of mixing water.

52. The requirement, not uncommon in specfns, that a certain percentage of **increase of strength** must take place **between 7 and 28 days,** tempts the mfr to grind the cem coarsely, or to adulterate it with inert material, in order that it may not gain too much of its strgth within the first 7 days.

Properties.

Soundness.

53. Unsoundness, in cem mortar, is the tendency to expand, contract or disintegrate in air or water, or under heat and cold. See Specifications.

54. Cem, of any established brand, will seldom be found deficient in strength; but may be deficient in soundness, upon which durability depends.

55. Unsoundness is generally **due to** excess of free lime, arising from incorrect proportioning, overburning, lack of seasoning, or coarseness of grinding; the latter preventing perfect hydration. The presence of **lime sulphate** (gypsum plaster of Paris) is favorable to soundness. Unsound cem is improved by **storage.**

56. Change of dimensions during hardening of concrete. Conc, placed **in air, shortens** or shrinks during the first two or three months; while conc, **in water, expands** during about the same time. These changes are greater with those concs having the larger proportions of cem.

57. Shrinkage of mortar set **in air.**

	per cent.	ins. per 100 ft.
Neat cement,*........................	0.132 to 0.140	1.58 to 1.68
Mortar, 1 : 1,*........................	0.080 to 0.170	0.96 to 2.04
Lean mortars,†........................	0.030 to 0.050	0.36 to 0.60

The expansion **in water** is somewhat less than the contraction in air. The total change in dimensions is the algebraic sum of that due to setting, and that due to temperature changes.

58. Conc shrinks less when it sets **under pressure. Fineness of sand** is conducive to shrinkage.

* Trans. A S C E, Vol xvii, 1887, p 214.

† Considère. Experimental Researches on Reinforced Concrete. Translation by Moissieff, p 87.

Strength.

59. Cem mortars are usually tested (by means of briquets) for **tensile strength.**

60. Factors affecting strength. The strengths of samples, under test, are much affected by the temperature of the air and water, as also by the force with which the cem is pressed into the molds; by the extent of setting before being put into the water, and of drying when taken out; and still more by the pres under which it sets, which increases the strength materially. On this account, cems, in actual masonry, may, under ordinary circumstances, give better results than in tests of samples. The causes named, together with the degree of thoroness of the mixing, the proportion of water used, and other considerations, may easily affect the results 100 % or even much more. Hence the discrepancies in the reports of different experimenters. Specimens of the same cem, tested under apparently similar conditions, may give widely diff results.

61. Personal equation. In connection with the building of the Croton Aqueduct, New York, one set of testers, testing 835 briquets, obtained an av strgth of 62.3 lbs per sq in; while another set of testers, testing 2434 exactly similar briquets by the same methods and under the same circumstances, obtained an av strgth of 85.2 lbs per sq in, or 36 % greater.

62. Owing to such uncertainties, a series of tests, to be of value, **must cover a large number of specimens**, in order that the accidental diffs may be averaged.

63. Diffs in comparative results with diff materials may be due to one or other of several diffs betw the materials. Thus, in comparing mortars made with clean and with dirty sands, the strgths may be more affected by diffs in density than by the diffs in cleanness of the sand.

64. Effect of age. The diagram,* Fig 1, illustrates approx the strengths of av Portland and of av nat cems, neat and with 2 and 3 parts

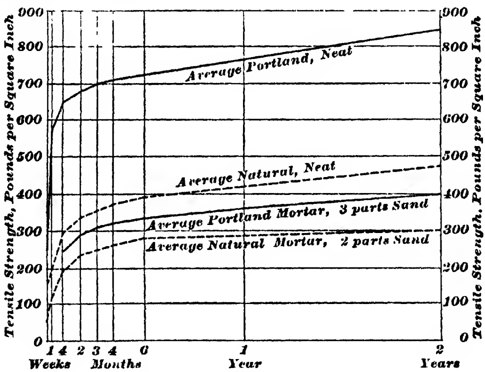

Fig 1. Age and Strength of Mortar.

of sand, up to an age of two years. Tests may readily vary 10 per cent or more eitherway from the average.

* See Richard L. Humphrey, in "Cement," Chicago, May, 1899.

65. Fig 2* shows, approximately, the effect of sand, in diff proportions, upon the strengths of Portland and natural cements, at diff

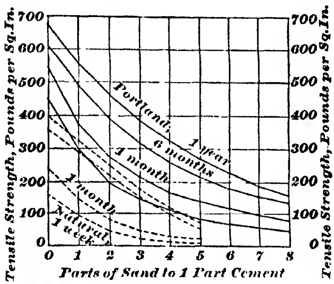

Fig 2. Effect of Sand upon Strength.

ages from 1 week to 1 year. The four solid curves represent average Portland cements, and the four dotted curves represent average natural cements For each kind of cement, the curves represent ages of 1 year, 6 months, 1 month and 1 week, respectively, beginning at the top. The curves for natural cement are carried only to 5 parts sand.

66. The compressive strengths of cem mortars, in cubes, appear to be about 8 to 10 times their tensile strengths, and their shearing strgths about ¼ their tensile strgths.

67. The adhesion of cem mortars to bricks or rough rubble, at diff ages, and whether neat or with sand, may be taken at an av of about ¾ the tensile strength of the mortar at the same age. If the bricks and stone are moist and entirely free from dust when laid, the adhesion is increased; whereas, if very dry and dusty, especially in hot weather, it may be reduced almost to nothing. The adhesion to very hard, smooth bricks, or to finely dressed or sawed masonry, is less than the adhesion to rough and porous surfs.

68. Dr. Bohme, Berlin, found tensile strgth ÷ adhesive strgth = '10, with 1 : 3 and 1 : 4 mortars, and = 6 to 8, with neat and 1 : 2 mortars.

Finish.

69. Lime mortar and cems, when used as mortar for brickwork, often disfigure it, especially near sea-coasts, and in damp climates by white **efflorescence,** which sometimes spreads over the entire exposed face of the work, and also injures the bricks. This occurs also, to some extent, with Portland cems; also in the mortar joints of stone masonry, but to a much less extent. It injures only porous stone. It is usually a hydrous soda or potash carbonate, or magnesia sulfate (Epsom salts) often with other salts. As a preventive, General Gillmore recommends to add to every 300 lbs (1 bbl) of the cem powder, 100 lbs of quicklime, and from 8 to 12 lbs of any cheap animal fat; the fat to be well incorporated with the quick-lime before slacking it, preparatory to adding it to the cem. This addition will retard the setting, and somewhat diminish the strength of the cem. It is said that linseed oil, at the rate of 2 gals to 300 lbs of dry cem, either with or without lime, will, in all exposures, prevent efflorescence; but, like the fat, it greatly retards setting, and weakens the cem. See also Bricks, p 1221.

70. For pointing, the best Portland cem should be used, and is best used neat, but it is often used with from 1 to 2 parts of sand. Mix under shelter, and in quantities of only 2 or 3 pints at a time, using very little water; so that the mortar, when ready for use, shall appear rather incoherent, and quite deficient in plasticity. The joints being previously scraped out

*Compiled, by permission, from Prof. Baker's "Masonry Construction."

to a depth of at least half an inch, the mortar is put in by trowel; a straight-edge being held just below the joint, if straight, as an auxiliary. The mortar is then to be well calked into the joint by a calking-iron and hammer, then more mortar is put in and calked, until the joint is full. It is then rubbed and polished under as great pressure as the mason can exert. If the joints are very fine, they should be enlarged by a stonecutter, to about ¼ inch, to receive the pointing. The wall should be well wet before the pointing is put in, and kept in such condition as neither to give water to, nor take it from, the mortar. In hot weather the pointing should be kept sheltered for some days from the sun, so as not to dry too quickly.

Behavior in Water.

71. Laitance. "When conc is deposited in water, especially in the sea, a pulpy gelatinous fluid exudes from the cem, and rises to the surface. This causes the water to assume a milky hue; hence the French term, *laitance*. As it sets very imperfectly, and, with some varieties of cems, scarcely at all, its interposition betw the layers of conc, even in moderate quantities, will have a tendency to lessen, more or less sensibly, the continuity and strgth of the mass. It is usually removed from the inclosed space by pumps, which must be used cautiously, to avoid disturbance of the conc by currents. The proportion of laitance is greatly diminished by reducing the area of conc exposed to the water, as by using *large* boxes, say from 1 to 1.5 cu yds capacity, for immersing the conc." (Gillmore, "Limes, Hyd. Cems & Mortars.")

72. Authorities differ as to the **effect of sea water.** H. LeChatelier (Internatl Assn for Testg Materials, Procs, 1906), finds that the active ingredients of cem (lime, aluminates, silicates) are decomposed by the magnesium salts of sea water, yielding soluble calcium chlorides and lime sulfates. The latter, with lime aluminate, forms a compound whose crystallization tends to swell and crack the material.

73. In view of the notable **puddling effect** of percolating water, it would appear that sea water especially, with its numerous salts, ought shortly to block its own passage into the conc.

74. The substitution of iron for alumina, in cem, is found to remove one of the most active reagents in the deteriorating effects of the salts in sea water.

See Cement, ¶ 30, p 1225.

75. The disintegration of conc in water (salt or fresh) appears to be due less to action of the water itself than to the repeated action of frost where the conc is alternately exposed to freezing temps between high and low water.

76. Mortar of puzzolano and lime has remained in perfect condition for 15 to 20 centuries in Italian harbor works.

77. At the dock at Kobe, Japan, to avoid possible injury, the salt water, inside the dam, was replaced with fresh water, which entered at the surface, while the heavier salt water was pumped out from the bottom.

For Concrete, see pages 1252, etc.

Abbreviations, symbols and references, in general use in the articles on Cement, Sand and Mortar, pp 1222–1250, and on Concrete pp 1252–1378.

For references to specifications, see pp 1352–3.

agg	aggregate
A S T M	American Society for Testing Materials
A S C E	American Society of Civil Engineers
Assn Eng Socs.	Association of Engineering Societies
cem	cement
conc	concrete
constr	construction
c c	cubic centimeter
d	day
elas	elastic
E N	Engineering News
E R	Engineering Record
expt	experiment
h, hr	hour
Instn C E	Institution of Civil Engineers
Jour	Journal
kg	kilogram
km	kilometer
m	meter
mm	millimeter
mo	month
mod	modulus
mom	moment
nat	natural
Port	Portland
Procs	Proceedings
reinfd	reinforced
reinfmt	reinforcement
specfn	specification
standd	standard
surf.	surface
T & M	Turneaure and Maurer, "Principles of Reinforced Concrete Construction," 1907.
T & T	Taylor and Thompson, "Concrete, Plain and Reinforced," 1905.
Trans	Transactions
transv	transverse
U. S. A	Report, Chief of Engrs, U. S. Army.
wk	week
/	per
□	square
□"	square inch
$>$	greater than, more than
$<$	less than
$\not>$	not more than, equal to or less than.
$\not<$	not less than, equal to or greater than, at least.

CONCRETE.

For Cement, Sand and Mortar, see pages 1222, etc.

For abbreviations, symbols and references, see p 1251.

AGGREGATES.*

Constituents.

1. Order of value. (1) Trap, (2) granite, (3) gravel, (4) marble, (5) limestone, (6) slag, (7) sandstone, (8) slate, (9) shale, (10) cinders.

2. The strgth of conc, with good sandstone, is about 0.75 × strength with trap. With slate, less than half strength with trap. Good cinders nearly equal to slate and shale. Hardness of agg increases in importance with the age of the conc "because, as the cem becomes hard, there is greater tendency for the stones themselves to shear thru, and the hardness of the agg thus comes into play." (Sanford E. Thompson, E R, '06/Jan/27, p 109.)

3. The choice of agg is of course a matter of **cost,** as well as of strength, &c, of product. Thus, with gravel sufficiently cheap, as compared with broken stone, it may be economical to use the gravel, or a mix of gravel & stone, obtaining the reqd total stigth by using a larger mass of conc In foundations, on weak ground, this is advisable because it distributes the load over a greater area.

4. In many cases, the choice of sand and agg depends largely upon what material can be had, and upon its distance from the work.

5. Where cem is cheap, it may be economical to use materials nearest at hand, and to depend, for quality, upon excessive use of cem.

6. Stone which breaks into nearly cubical fragments packs better than that which splinters into long pieces, and the fragments are less apt to break in the finished work.

7. Good broken stone is usually preferred to gravel. The roughness of the stone particles is believed to give better adhesion. Gravel conc cannot well be tooled.

8. Cinders are sometimes used for the agg. They are ordinarily those resulting from the burning of **bituminous** coal under boilers. The material is mostly a fine ash, containing considerable unburned coal.

9. Anthracite cinders are less extensively used, the supply being less abundant.

10. Cinder conc. weighing only from 80 to 100 lbs per cu ft, is of advantage where **lightness** is reqd. Broken stone or gravel conc weighs from 140 to 145 lbs per cu ft.

11. Clay or loam, adhering to gravel particles, destroys or weakens the adhesion of the mortar to the stones. The Boston Transit Commission, Report for 1901, page 39, found the ratio of strength, betw conc with clean and dirty gravel, about 60 : 45.

See "Clay and Loam," under "Sand" and "Accidental ingredients," p 1135.

Size.

12. In beams, arches, &c, the **size of aggregate** should not exceed 1.5 to 2 ins on any edge; but, if it is well freed from dust by screening or washing, and if the mortar completely fills the voids, all sizes, from 0.5 to 4 ins. on any edge, may be used in mass work, as foundations, dams, piers, etc.

13. With large agg, coarse sand should be used, and vice versa.

14. It is usually economical of cem to **screen** sand from gravel or fine material from crusher stone, and then remix in the required proportions.

Density.

15. When a solid body is reduced to a mass consisting of broken pieces separated by voids, the increase in bulk is due solely to the **voids,** and is

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* By "aggregate," we mean the solid materials of conc, other than the cem and sand. The term "aggregate" is sometimes used as including the sand also

equal to the space occupied by them. Hence the ratio, betw the **increase of bulk**, or "**swelling**," and the original bulk, is that of the voids to the *original*, and not to the final bulk. Thus, if a solid cu yd of stone, after being broken into pieces, occupies twice as much space as before, then the increase in bulk, or the space occupied by the voids, is = that occupied by solid pieces = half that occupied by the entire broken mass.

16. In sharp and angular broken stone, having all its pieces of nearly uniform size, about 50 **per cent** of the vol, when measured loose, will be **voids.** If the sizes of the stones vary betw somewhat wide limits, as from 2 ins down to ¼ inch, the vol, occupied by the voids, will be less, often as little as from 28 to 30 % of the whole.

17. Tests by Mr. Wm. M. Hall (Trans A S C E, Vol 42, 1899, p 132) of **voids** in crushed Green River blue limestone, 2.5 inch, screened; very clean Ohio River gravel, 1.5 inch, and mixtures of the two, resulted as follows:

Percentage of stone	100	80	70	60	50	0
" " gravel	0	20	30	40	50	100
" " voids	48	44	41	38½	36	35

These are avs of a number of tests of several bargeloads of materials, but there was little variation betw the mixtures.

18. Stone Crushers. See Price-list, p 1400.

Cyclopean Concrete.

19. "Cyclopean" conc, consisting of large, rough stones ("displacers" or "plums") laid in cem mortar, is largely, economically and advantageously used in mass work, especially in dams, where wt and hor shearing strgth are desiderata. The stones need not be flat. They are usually dropt into the wet mortar, without other bedding than that due to their fall and wt. Wet conc facilitates the bedding of the stones, and bonds better with them than does dry conc.

20. At Chaudiere water power dam, Canada, the "plums" were obtained from hard ledges in the river bed, in good shape for bedding. Their agg vol av'd betw 25 and 30 % of the vol of the dam; max, 40 %.

21. At Transmere Bay Development Works (Procs Inst C E, Vol 171, 1908, p. 145) the "plums" were of sandstone, 9 ins apart hor'y. Near the bases of the walls, they weighed a ton or more. The proportion of plums decreased, with wall thickness, from 10 to 7 % of the whole mass.

22. Unnecessary restrictions, imposed upon contractors, may eliminate the profit due to the use of "plums." See ¶ 19.

PLAIN CONCRETE.

1. Cement Concrete is composed of broken stone, gravel, cinders, slag, shells, or other hard and inert * material (the aggregate), held together by cement mortar, composed of cement and sand.

Advantages.

2. The principal advantages of conc are the convenience with which it may be placed, particularly in otherwise difficult situations or under water; its availability for subaqueous work; its cheapness, due largely to convenience of placing and to its use of stone too small for masonry; and its fire-resisting qualities, as compared with limestone (which calcines) and with granite (which splinters).

3. The availability of conc has been very greatly extended by the practice of **reinforcement,** which permits its use (heretofore often impracticable) in members subject to tension as well as to compression, as in beams, in cantilevers (including dams and retaining walls), in columns, and in arches where the rise is either very great or very small, relatively to the span. Reinforcement permits the use of much lighter sections than would have been safe when use was made only of the *compressive* strength of the material.

For reinforced concrete, see p 1278.

4. Disadvantages. Conc is rather weaker than good rubble masonry, and has only about half the strength of first class ashlar masonry of granite with thin joints in cem. Like both the stone and the mortar in masonry, it is subject to deterioration, especially in sea water; but this difficulty is being eliminated by the care which is being given to the manufacture of cem and which is fostered by its extensive use and by the conduct of its manufacture upon a large scale. As in all human work, and notably in the laying of masonry, care is necessary in order to secure faithful performance, upon which the success of the structure so intimately depends. The quality of the finished work may, however, be tested by borings.

5. Conc is used for **bringing up uneven foundations** to a level before starting the masonry. By this means the number of hor joints in the masonry is equalized, and unequal settlement is thereby prevented.

6. On railroad work, the use of conc may **obviate the use of derricks,** which are a source of interference with, and danger to, trains.

7. Conc is used to advantage in reinforcing and **protecting old stone masonry;** but, unless special precautions are taken, the two constructions are liable, in time, to separate, owing to unequal settlement, especially if the ramming has not been thoro.

Natural Cement.

8. Natural cement is now seldom used in conc, except in mass work where it is not subjected to the wearing action of water or frost, and where early strength is not reqd. It is suitable for footings and for low retaining walls not subject to serious vibration.

9. In dams, breakwaters, etc, the **core** is frequently *of natural cement* conc; with a substantial outer shell of Portland cem conc.

Proportions.

10. The proportions of cement, sand and aggregate should theoretically be **determined,** either all by wt, or all by measure in loose condition; but, in practice, the cem is measured by the number of packages used (the contents of the packages being known; see "packages," under "Cement") and the sand and agg are measured loose.

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* Without chemical affinity for other materials.

"Natural Mix."

11. It is customary to designate the quantities of cem, sand and agg, in a conc, by proportions. Thus: 1 : 2 : 4 means 1 part cement to 2 parts sand and 4 parts aggregate. Such designation is necessary in instructions to workmen; and, where the ranges of size of the particles are known, it indicates the character of the conc. The proportions are of course governed by the character of the work; but it is inadvisable to affect distinctions between nearly similar classes of work.

12. Usual proportions for Portland cement concrete:

Exceptionally massive work (leveling for foundations, dams, breakwaters) 1 : 1.5 : 8 to 1 : 5 : 10; with nat cem, 1 : 2 : 5.

Foundations, ordinarily, 1 : 3 : 6; sometimes as poor as 1 : 4 : 8.

Piers, pedestals, abutments, 1 : 2.5 : 5.5 to 1 : 3.5 : 7.

Piers and vaulting in filters, 1 : 2.5 : 5.5.

Reinforced walls and beams, 1 : 3 : 6; light sections, 1 : 2.5 : 5.

Foundation walls, 1 : 2.5 : 5.5; retaining walls, 1 : 2.5 : 5.5 to 1 : 3 : 6.

Spandrel walls, 1 : 3 : 6.

Conduits, drains, sewers, 1 : 2.5 : 5.5 to 1 : 3 : 6.

Reservoir, filter and tank walls, 1 : 1.5 : 3.5 to 1 : 2.5 : 5.5.

Subaqueous work, 1 : 2 : 3.

Floor systems (girders, beams, slabs) 1 : 2 : 4 to 1 : 2.5 : 5.5

Stairways and roofs, 1 : 2 : 4.

Arches, 1 : 2.5 : 5; light sections, 1 : 2 : 4.

Copings and bridge seats, 1 : 1 : 2 to 1 : 2 : 4.

But the essential requisite is that all the voids, between the particles of sand and agg, be filled with cem mortar. Hence, unless the grading of sizes, of sand and of agg, is known or assumed, the bare statement of proportions, of cem, sand and agg, in a mixture, gives but little useful information as to the value of the conc.

13. In reinforced work, in general, richer mixtures should be used than those that would be permissible in large mass work. In order to obtain proper and reliable adhesion, which is of the first importance, the bars must be completely surrounded by cem.

Materials Required.

14. Materials required for a cu yd of rammed Portland cement concrete. c = cement, bbls; s = sand, cu yds; a = aggregate, cu yds. Dust screened out. Stones not larger than 1 inch.

Mixture	c	s	a
1 : 2 : 4	1.46	0.44	0.89
1 : 2 : 5.5	1.19	0.46	0.91
1 : 3 : 5	1.11	0.51	0.85
1 : 3 : 6	1.01	0.46	0.92
1 : 3 : 7	0.91	0.42	0.97
1 : 4 : 7	0.83	0.51	0.89
1 : 4 : 8	0.77	0.47	0.93

With 2.5 inch stone, the quantities of all the materials, per cu yd conc were increased from 2 to 5 %. With gravel, $\not>$ ¾ inch, they were decreased about 9 %. (Chas. A. Matcham, Natl Builders' Supply Assn, 1905.)

15. Let

B = No. of barrels of cement reqd per cu yd conc
= No. of times 0.141 cu yd cement reqd per cu yd conc;
P = parts of sand (or agg) to 1 part cem.

Then

$1/B$ = No. of cu yds conc from 1 bbl cem;
$0.141\ P$ = No. of cu yds sand (or agg) to 1 bbl cem;
$0.141\ PB$ = No. of cu yds sand (or agg) to 1 cu yd conc

Voids. *See Weight, p 1271.*

16. Reduction of voids. *If stone having 50 % voids, and sand having 50 % voids, be used, with cem, in the proportions:*

Cement, 1 part = 0.25 cu yd
Sand, 2 parts = 0.50 cu yd
Stone, 4 parts = 1.00 cu yd

the resulting conc will measure something more than 1 cu yd, and yet it will contain unfilled voids.

17. These proportions, however, are not economical By selecting a sand having a **range of size,** or by mixing two or more sands having grains of diff sizes, the voids in the sand can be reduced to say 33 %. Similarly, the voids in the stone can be reduced to say 35 %. We should then have, say:

Cement, 1 part = 0.12 cu yd
Sand, 3 parts = 0.36 cu yd
Stone, 8 parts = 1.00 cu yd,

with results as good as with the 1 : 2 : 4 mixture above, although using only half as much cement.

18. Mr. Geo. W. Rafter (Trans A S C E, Dec, 1899, Vol 42, p 106) recommends that the proportions be stated by means of the ratio of the vol of the mortar to the vol of agg. Thus: a conc containing 75 vols of agg and 25 vols of mortar, would be a 33⅓ % conc.

19. Under usual conditions, the **voids** in the agg **should be filled** with as rich a mortar as the strength of the work demands. A better conc may result from the use of a lean mortar which fills the voids, than with a richer mortar but partially filling the voids.

20. The mortar cannot be perfectly distributed thru the agg, and some of the voids are too small to admit the sand grains. Moreover, the mixture is liable to disturbance in depositing. Hence, there will be voids in the conc unless there is an excess of mortar over the measured voids of the agg.

21. In practice, the **excess of volume of mortar required,** over the measured voids in the agg, in order to secure the filling of the voids, is usually from 15 to 25 % of the vol of the voids. But by 15 exp'ts with limestone, Prof. Baker found that the voids were not entirely filled unless the vol of the mortar exceeded the vol of the voids by 40 %. (Table 13 c, p 112 b, Baker's Masonry Construction, 1907.)

22. Mr. John Watt Sandeman (Procs, Instn C E, Vol 121, p 219, 1895) believes that, **to insure watertightness,** the vol of mortar should be 50 % of the vol of agg having 35 % voids; or, excess mortar = 43 % vol of voids.

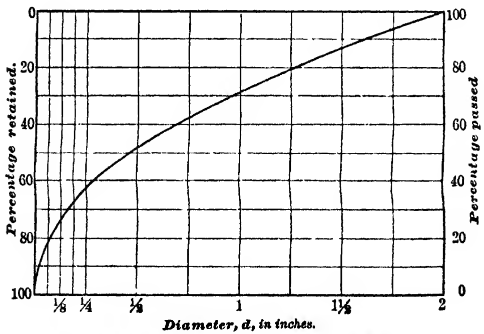


Fig 1. Parabola of Maximum Density. See ¶ 23, p 1257.

Density. See Weight, p 1271.

23. Mr. Wm. B. Fuller (T & T, p 197) finds that the **greatest density** is obtained, and consequently the smallest amount of cem reqd, when the agg and the sand are so graded that the percentages, by wt, passing the various sieves, are as represented by the ordinates of the parabola in Fig. 1, where the abscissas represent the diams, *d*, of the openings in the sieves while the ordinates *below* the parabola represent the percentages *passed*, and those *above* the parabola the percentages *retained*, by these openings respectively.

24. In this parabola $d = P^2 M$; where d = a given diam; P = proportion of particles smaller than d; M = max diam of stone (= 2 ins in the Fig).

25. Exp's (Trans A S C E, Vol 59, pp 67, &c, 1907) show that a saving of 12 % in quantity of cem may be effected, and a more impervious product obtained, by thus grading the sizes of the sand and agg; but the reduction may sometimes be offset by the additional cost of so grading, especially on small work.

26. In the lining of the tunnel for the Sudbury aqueduct, Boston Water Works, the proportions were

1 cask of Portland cem as it came from the dealer	=	3.425 cu ft
2⅙ casks of loose sand	=	7.35 cu ft
5½ casks of loose crushed stone	=	18.56 cu ft
Total		29.335 cu ft.

By slightly shaking the sand and stone, the proportions became practically 1 : 2 : 5.

These 29.335 cu ft produced from 20 to 21 cu ft conc, rammed in place: or say 38 cu ft materials = 1 cu yd conc

27. Mr. Wm. B. Fuller (Natl Assn of Cem Users, Procs, '07, p 95) tested conc beams, 30 days old, of 1 : 2 : 6, 1 : 3 : 5, 1 : 4 : 4, 1 : 5 : 3, 1 : 6 : 2 1 : 8 : 0, (all 1 : 8). The strgths compared as in Fig 2.

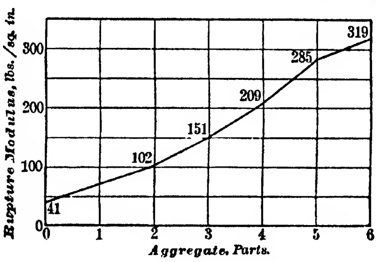


Fig 2. Proportions; strength.

28. From this it appears that, so long as the voids in the agg are filled with mortar, the comp strength of conc seems rather to increase than diminish as the proportion of stone increases, and to depend largely upon the richness of the mortar.

29. Proportioning by trial mixtures: (Wm. B. Fuller, Trans A S C E, Vol 59, pp 77, &c).

Having determined the particular sand and stone to be used on any work, provide a strong and rigid cylinder, such as a short piece of 10 inch wrought iron water pipe capped at one end.

30. On a piece of sheet steel or other non-absorbent material, weigh out and mix together all the ingredients, to the consistency required for the work. Place the mixture in the cylinder, tamping carefully and continu-

ously, and note the height to which the cyl is filled. Before the mixture has time to set, empty and clean the cyl

31. Make up another batch, using the same wts of cem and of water as before, and the same *total weight* of sand and stone, but with a slightly *diff ratio of weights* of the sand and stone.

32. Note the height, in the cyl, reached by this second and by subsequent mixtures The best mixture is that *which gives the least height in the cyl*, provided that it works well while mixing, and that its appearance in the cyl shows that all the stones are covered with mortar.

33. This method enables the engineer to select the best from the materials available in any given case.

Consistency. See also Mortar, p 1245.

34. Skill and care, in placing, and *uniformity* of consistency are more mportant than the consistency itself.

35. The extremes of **practice** are: (1) Conc with mortar about as moist as damp earth; only enough water used to show on the top surf after prolonged and hard tamping, (2) enough water used to cause the conc to quake when first placed, and to allow only of spading into place. The proper consistency depends largely upon the character and purpose of the work.

36. Dry conc is generally preferable in large open work where it can be thoroly rammed, and where early strength is reqd, as in arch skew-backs. When thoroly tamped, it develops much higher compressive strength at its early ages, and may have somewhat greater permanent strength, than wetter mixtures; but imperfect tamping of such mixtures may result in very weak conc, while thorough tamping may render the work more expensive than the increased strength will justify.

37. Medium. Present practice favors the use, in general, of mixtures wet enough to require only spading; but, even in such work, ramming may be reqd from time to time for occasional dry batches.

38. Wet conc is more easily mixt with thoroness, more readily and more cheaply laid, and more easily forced into the narrow spaces betw reinforcing bars. It comes into more perfect contact with the molds, thus giving smoother and more nearly watertight surf. It is therefore generally preferable (as in buildings) in forms of complicated shape, or in thin sections, or where smooth surfaces are reqd.

39. Wetness retards setting, gives better bond between successive courses, gives a compact mass with less tamping, and provides the surplus water reqd by absorption in wooden forms. Wet conc is less liable than dry to injury by bad workmanship; but an excess of water reduces the strgth, and increases efflorescence.

40. In "cyclopean" conc, more "plums" can be used with wet conc, which allows them to settle down into it, and which bonds better with them.

41. Mixtures, **wet enough to be poured** into the forms for columns of floors, are frequently used.

42. The quantity of water required, for a given consistency, is materially reduced by wet weather.

43. Water works upward thru placed conc. Hence a less proportion of mixing water may suffice toward the end of a day's work.

HANDLING AND MIXING.

Handling Ingredients.

1. In designing a plant for **handling and mixing** conc, the quantities to be handled, the areas over which they must be distributed, the facilities for procuring and receiving the raw materials, and the working space available, must be considered; and each case will present other factors, peculiar to itself,

2. The **arrangements** of such a plant are as various in character as are the different kinds of work. In general, these arrangements must be specially designed for each important work; and success and economy depend largely upon the excellence of the design of the handling plant.

3. Materials may reach the site by cars, boat or team. Be on guard against mud and dirt in bottom of vehicle. Sand and agg may be dredged from stream at the site.

4. After reaching the work, the materials are carried to the bins, by carts, barrows, small cars, dredge buckets, or belt or chain conveyors. From the bins they are usually carried by gravity, thru hoppers, to the mixer.

5. Storing. Cem is commonly stored in sheds or other warehouses, and is handled, separately from sand and agg, in bags or bbls, often by means of chain conveyors.

6. For bringing the materials from the **bins to the mixers,** and the conc from the mixers to the work, carts, barrows or small cars are used.

7. Where the work covers a limited hor area, as in the case of a building, or of a pier or abut, **the mixer** need not be frequently moved, and the arrangements for handling are relatively simple.

8. Where the work covers a large hor area, as in a slow filtration plant, or where it crosses a valley, as in a dam; cable conveyors, with towers, are used; or one or more mixing plants are installed in central positions.

9. Where the work extends along a line of considerable length, as in walls, sewers or aqueducts, a railway track, often of broad gage and with three or more lines of rails, is laid alongside, and the materials handled from derrick cars, often of designs specially prepared for the work in hand.

10. The work is facilitated by having the cars, barrows, buckets, etc, of known capacity, so that they may serve as measures in proportioning the sand and agg. Thus, the cars may hold enough sand or agg for one batch, and may dump into larger boxes, each holding enough sand and agg for one batch. The cem is usually measured separately, by counting the bags or bbls emptied.

11. Where **cars** are used, they may be moved by locomotive or by cable, reaching the bins by means of an inclined plane.

12. In the case of a **belt conveyor,** sand and stone, each enough for a batch or other known quantity of conc, and afterward the cem for the same quantity, are dropped upon the belt from their respective bins.

13. Commonly the **measuring platform** (or the measuring hopper for batch machines) is placed directly over the mixer.

14. For max output, there should be two sets of measuring hoppers, one to be dumping into the mixer while the other is filling.

For washing sand, see SAND, ¶ 34, p 1242.

15. Agg may be washed in a revolving cylindrical screen, by a jet of water under high pressure.

16. Work is often done **at night** by means of electric or other artificial illumination.

17. Portable (flat-car) conc mixing plant. Two 6 × 8 timbers, 58 ft long, 4 ft apart, laid upon floor of a 34 ft standard-gage flat car, their ends projecting 12 ft beyond each end of car, and guyed to an elevated framework on center of car. Each projecting end carried a 2 cu yd hopper. Sand and gravel were shoveled into this hopper and discharged from it upon a belt conveyor, running hor'y under the hopper and then upward to a hopper (3 cu yds) 15 ft above the car floor, over the center of the car. This elevated hopper discharged the sand and gravel into a ¾ cu yd Smith mixer, placed at the center of the car. Cem supplied to the mixer by hand; water from a pipe, laid along the work and provided with hose connections. A bbl, filled with water, was carried on the elevated framework, to ensure a supply for immediate use. The conveyor belt, 2 ft wide, consisted of two link-belt chains, with a heavy double-thickness canvas belt between them. Belt supported by wrought-iron pipe cross-pieces 18 ins apart. The belt forms pockets between the cross-pieces. Conveyors, driven by a 9 × 16 inch single-cylinder steam engine, mounted on one end of the car. Average capacity, 275 cu yds per day. One lower hopper was found sufficient to supply the mixer. (The Chalmette Docks of the New Orleans Terminal Co, E. R, '06/Jul/28, p 90.)

18. In constructing works which are circular in plan, the mixt conc for floors, columns, girders and roof, may be carried to the forms by means of a truss bridge, spanning the work from a central tower to a track on the

circumferential wall. The bridge then forms a **revolving crane,** carrying mixers at its outer end.

Mixing.

19. General. Each sand grain should be coated with cem, and the mortar should coat every fragment of stone in the agg and should be evenly distributed thru the whole vol. The stone, if dry, should be wetted before adding it to the mortar.

20. Thoroness of mixing is of the greatest importance; especially when the conc is poor in cem or of dry consistency.

21. The great strgth of the conc in the Munderkingen bridge is attributed to its thoro mixing. The materials were mixt 2 mins dry and 3 mins wet.

22. Variation in **color** of mixture indicates change in the proportions of the ingredients.

23. See that any cem, thrown out as defective, is replaced by good cem.

24. Lifting concrete. Where the mixing platform cannot be built near the level of the top of the structure, the conc may be raised by a power lift to the proper level, and then wheeled on level runways. For low lifts and small quantities, horsepower lifts are used; for higher lifts and larger quantities, a small steam or gasoline engine.

25. In some cases, the mixer and its enclosing frame are lifted bodily by the derrick which supplies materials, and deposits them over or near the work.

26. Hand mixing is inadvisable and uneconomical, except on small jobs.

27. In hand mixing, it is usual to mix the sand and cem dry, usually by turning with shovels two or three times, until the mixture is of uniform color, and each sand grain is coated with cem.

28. Water is then added, and the mortar is mixed before the agg is added; or the agg may be spread over the dry mixed sand and cem, or these thrown upon the agg, and the whole then wet and mixed by two or more turnings with shovels, until the water is thoroly incorporated.

29. Mixing the cem and sand first, as above, reduces the total labor by omitting unnecessary manipulation of the agg.

30. Weather. Hand mixing should be well protected against wind and rain. Wind blows away the finest (and therefore best) of the cem, and rain prevents proper (dry) mixing of cem and sand.

31. For the sub-station of the Brooklyn Rapid Transit Co., two bottomless rectangular frames were provided, one of which had a capacity of ½ cu yd, and was first filled with sand. Seven bags of cement were then emptied on top of it, and the mass was turned several times by five shovelers until the color was uniform. It was then leveled, the other frame (1 cu yd capacity) was placed on top and filled with broken stone, and water was put on with a hose. The mass was then turned four times, shoveled into wheelbarrows and deposited in the forms.

32. With equal care, **machine mixing** gives better and more reliable results than hand mixing, and is more economical on large work.

33. The **output must be carefully watched,** as the accidental and unsuspected choking of a hopper may change its character.

Mixers.

34. Mixers are of **two principal types;** "continuous," and "batch."

35. In **continuous mixers,** the raw materials are fed continuously into the machine at one end, and the mixed conc is delivered continuously from the other end.

36. The gravity (continuous) mixer is a stationary shute or trough, set nearly vert, and equipped with fixed projecting pins or baffles, against which the material impinges as it descends, and upon which the mixing depends. Water is admitted by a spray pipe, at the top of the shute. Power is required only to elevate the materials to the top of the mixer, usually a lift of about 8 feet.

37. Other continuous mixers are in the form of open troughs, nearly hor, and having a longitudinal revolving shaft, with screw-like blades

attached, which convey the material, fed in at the upper end, thru the length of the trough, to the lower or discharging end. Water is provided by means of perforated pipes along the sides of the trough.

38. **Measuring.** Continuous mixers require some means of proportioning the ingredients of the conc. Various automatic measurers have been used to a limited extent. Sometimes the sand, cem and agg are spread, in layers, on the platform of the mixer, and shoveled into the mixer. Sometimes, dependence is placed upon assigning, for instance, one shoveler for the cem, three for the sand and six for the stone; but this method is much too crude for most cases.

39. **Batch mixers** deliver the conc in batches, the size of which is determined by the capacity of the mixer. They have a wider range than gravity mixers, and give better control of the proportioning of the ingredients.

40. The oldest and simplest batch mixer consists of a revolving **cubical** iron box, plain inside, mounted on bearings at its diagonally opposite corners, and provided, on one side, with a sliding gate, for admitting the raw materials and discharging the conc. Power is applied thru gearing on the shaft. The ingredients may be mixed dry for a number of turns, and the water then added thru the hollow trunnions; or the water may be added before any mixing is done. The older cubical mixers had to be stopped, both at the time of charging and when delivering the conc.

41. At Superior Entry, Wis., the U. S. Govt used a cubical conc mixer, charging and discharging without stopping and without variation of speed. It was operated by a 7 × 10 inch vertical single steam engine, and turned out a batch of very perfectly mixt conc in 80 secs. The conc was plainly visible during the entire process. (Clarence Coleman, Rept of Chf of Engrs, U. S. A., 1904, Part IV, p 3784.)

42. In later batch mixers the cubical box is replaced by a **drum** (either cylindrical or made up of two cones), rotated by means of a chain on a ring encircling the drum, and provided with vanes or blades fixed upon the inside. These blades first carry up and then drop the material, mixing it by the agitation so caused. The discharge is effected, in the Smith (double cone) machine, by tilting the machine (like a Bessemer steel converter) about its trunions, placed at cen of grav of drum; and, in the Ransome (cylindrical drum) machine, by inserting a tilting trough, which, in the discharging position, catches the material as it falls from the blades.

43. To provide against **break-downs, extra parts** should always be furnished with each mixer.

44. **Mounting.** Mixers are either stationary, or mounted on skids or wheeled trucks, with or without steam engine, engine and boiler, gasoline engine or electric motor.

45. The mixer, with its framing, is sometimes lifted bodily from its old location, and deposited in a new one, by a derrick or cableway.

46. **Wheeled conc mixers,** with revolving drums, into which the ingredients are loaded, and in which they are mixt by means of the forwd movemt of the vehicle, have been used. The motive force may be given by hand, by horse-power or by gasoline engine; and the relation, betw forward speed and speed of rotation, may be regulated by gearing.

47. Small **hand-power** batch mixers are furnished; capacity claimed > 450 cu ft per day.

48. In the **choice of a mixer,** reliability, as established by successful use, is of prime importance, especially where continuity of work is essential.

49. Shortage of output may be due to **shortage of power** behind the mixer, as well as to the mixer itself.

50. The mixer **should be cleaned** after each day's work.

PLACING.

51. The best conc may be rendered almost worthless by carelessness or improper method in the placing.

52. When conc is **dumpt from a considerable height,** there would seem to be danger that the even distribution of materials may be disturbed. Hence, if lowered in buckets, these should be brought close to the work already done, before dumping. However, in the construction of

conc piers for a bridge at Bethlehem, Pa., by Cramp & Co. (E R, '09/Mar/6, p 280) conc was delivered, thru an inclined wooden shute, lined with sheet iron, at a point vert'y 74 ft below the mixer; and the method was found to be economical, and the conc uniformly good, and there was no difficulty from separation of ingredients.

53. In work that will show, the **layers** are usually restricted to about 6 ins in depth, owing to the difficulty of spading the face work when the layers are thicker; but in foundations, and in heavy work above ground, if to be faced with masonry, or if appearance is not important, layers of wet conc as deep as 2 feet may be used.

54. If the conc, after placing, is found to be **too wet**, it is better to correct the trouble by placing drier conc upon it. When surplus water is bailed out, some cem is carried with it and thus wasted.

55. Excessive face spading brings up water from below, and this washes cem from the face.

56. Works of considerable length, such as dams and walls, are commonly built in sections alternately, thus: secs 1, 3, 5, etc, are first built separately, and, when they have hardened, sec 2 is built betw secs 1 and 3, section 4 betw secs 3 and 5, etc. The sides of secs 1, 3, 5, etc, thus serve as part of the forms for secs 2, 4, etc. This method facilitates bonding betw the secs, by means of vertical dove-tail grooves, formed, by the molds, in the sides of the secs first built. The conc of the remaining secs, placed later, enters and fills these grooves.

57. In freezing weather, conc can be laid in large masses in water or below the ground surf. In excavations, if the ground water is permitted to rise over the work during the night, it will usually prevent frost from reaching the conc.

58. At Chaudière water power dam, conc was laid in **temps as low as −20° F.** A mixing house was erected, and the temp, within, was kept, by stoves, above freezing. Materials were lowered into the house by derricks thru hatchways in the roof. Water was kept in casks, and kept lukewarm by steam jets. Sand was heated outside the house. Stone, in piles 3 to 4 ft deep, was heated (but not dried) by steam jets from a perforated pipe, passing under the piles. After placing, the conc was loosely covered with canvas, under which the nozzle of a steam hose was introduced.

Forms.

59. In wall foundations, the trench itself may constitute the form; and, in dams and arches of conc blocks, the first blocks, placed alternately, often serve as parts of the forms for the remaining blocks; but ordinarily a considerable amount of timber framing is required. See ¶ 56.

60. The **economy** of the work depends so largely upon the **design** of the forms, that it is often advisable to modify the design of the work itself, or to use more conc than would otherwise be nec'y, in order to secure economy. The design should be such that commercial sizes of lumber may be used, and with a min of wasteful cutting; and such that the forms may be readily erected and removed with a minimum of damage to themselves and no damage to the work, and used repeatedly. Where practicable, the forms are made in sections, small enough to be conveniently moved and handled separately. Cutting is economically done by power saw benches.

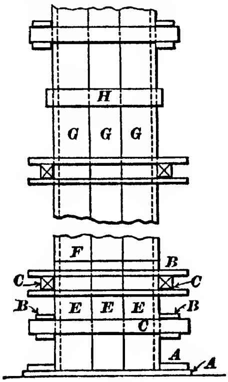
61. Even in building work, where much of the "centering" must be built in place, and where it can be removed only by taking it to pieces, the lumber may be used two or three times before it is discarded. Where the forms can be assembled in **panels**, and these panels removed as units, they may be used many times.

62. The requirements of different works, executed under diff conditions, vary so widely, that no useful details, as to the construction of the forms, etc, except for buildings (see ¶¶ 63 etc), can be given within the limits at our disposal. The designer should witness the *removal* of his forms before estimating their success.

Forms for Buildings.

63. In reinfd building construction, the forms are chiefly:
(a) Column forms,
(b) Beam, slab, floor and roof forms,
(c) Wall forms.

64. A typical **column form,** Figs 1 and 2. The boards, G, 1¼ ns thick, are held in place by cleats, H, 1¼ × 5 ins, and by "column clips," C, made of pieces 4 × 4 ins, and boards, B, 1¼ × 5 ins. These 'column clips" must be spaced to take the pres due to the conc. At the bottom of a column 18 ft high, they should be ≯ 10 ins, cen to cen. At the bottom, 4 boards, A, are used, to hold the form in shape, and the boards, G, are cut, on one side of the box, at F, 2 or 3 ft from the bottom, to form a door (cleats, on door, not shown), thru which all rubbish may be brushed. The door is then held shut by the lower two "column clips," and the form is filled Triangular fillets, T, are used to bevel the corners of the col.



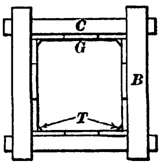


Fig 2.

Fig 1.

Figs 1 and 2. Column Form.

65. Column forms should be so designed that they may be removed without disturbing the forms for the beams and girders. The col forms may then be bared for inspection, before being loaded.

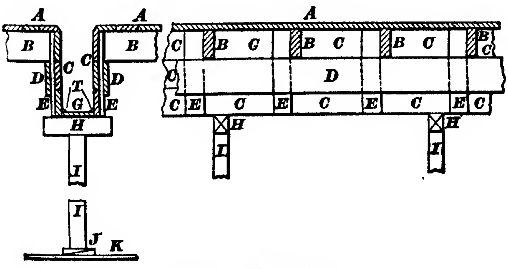


Fig 3. Beam Form.

66. Typical **beam or girder forms.** Fig 3. The forms, or beam-boxes, often miscalled "centers," are supported, betw columns, by temporary struts or shores, *I*, 4 × 4 ins, about 6 ft apart, resting on wedges, *J*, and the plank *K*. Corbels, *H*, 4 × 4 ins, are placed directly under the bottoms, *G* (1¼ ins thick) and sides, *C* (1¼ ins thick), of the beam boxes. The sides, *C*, are held together by cleats, *E*, 1¼ × 5 ins, 2 ft apart, to which are nailed the strips, *D* (1¼ × 6 ins), upon which rest the ledgers, *B*, 2 × 6 ins, about 27 ins apart. These support the panel boarding, *A*, 1¼ ins thick; and this, in turn, supports the slabs. Small triangular fillets, *T*, in the corners of the beam boxes, make the box tight and give beveled corners to the beam. Beam forms should be given a slight camber.

67. Typical **forms for floors** betw steel beams, Figs 4 to 6, vary with span and load. The forms are hung from the bottom flange of the I-beams, by "hanger bolts," *A*, Figs 4 and 6, ⅝ inch diam, with washers and handle nuts. These bolts secure the pieces, *E*, of 2 × 4 or 3 × 4, upon

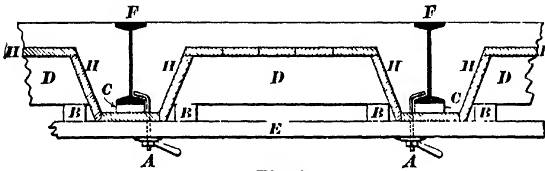


Fig 4.

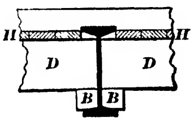


Fig 5.

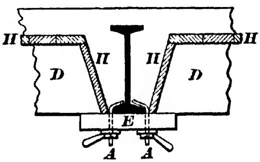


Fig 6.

Figs 4, 5 and 6. Floor Forms.

which the boards, *H H H* are supported by 2 × 6 or 2 × 8 ledgers, *D* (about 27 ins c to c, for ⅞ inch boards). Wooden blocks or sticks, *B*, Figs 4 and 5, are sometimes used under the ledgers to reduce their depth. Short conc blocks, *C*, Fig 4, are used, to keep the forms away from the lower flange of the steel beam. These remain permanently in the work. In order to promote adhesion betw the lower flanges of the I-beams and the thin mass of conc below them, the flanges are often wrapped with metal lath, before the blocks, etc, are placed.

68. Wall forms are usually made up in panels, so that they can be used several times. The panels are cleated together, and are usually about 3 × 12 ft. The panels are kept at the proper dist apart by separators, of wood or conc, and are held in place by bolts or wire ties. When wood separators are used, they must be removed just ahead of the concreting. Conc block or tube separators are sometimes used. These remain in the wall. When bolts are used that are to be later withdrawn and used again, they should be loosened by means of a wrench, about 24 hours after concreting; otherwise it will be difficult to remove them.

69. In the **Wiederholdt** system of reinfd conc wall construction. the conc is deposited within small hollow tile blocks, which form the finished exterior surface, and no wooden or other temporary forms are used. The blocks are shaped to meet the requirements of the work. Tiling and concreting are carried up simultaneously.

70. To reduce the cost of forms in reinfd building construction, columns, beams, slabs, etc, may be **cast on the ground**, and afterward erected and placed as desired; at the sacrifice, however, of the rigidity due to the monolithic character of ordinary reinfd work.

71. Metal forms. When the structure is of small and uniform cross section, permitting the repeated use of the same forms, as in sewers, conduits, tunnels, etc, the lagging, for the wooden forms, may be of sheet metal. In tunnels and similar works, of considerable extent, and in small ornamental work, forms composed entirely of metal may be used.

72. Both careless and over-careful **alignment** are to be avoided. Mr. W. J. Douglas (E N '06/Dec/20, p 646) suggests the allowance of "⅜ inch departure from established lines on 'finished' work, 2 ins on 'unfinished' work."

73. Avoid fine detail, and detail with sharp angles. Corners should be rounded or beveled, to facilitate the flow of conc and the removal of forms, and to render the corners less liable to subsequent injury.

74. Wooden forms, within which the conc is to be placed, should be fairly watertight, smooth, and of sufficient strgth and stiffness to hold to line under the pres of the green conc.

75. The forms are usually of dimensioned timber, faced with planed boards or planks. The opening of joints betw the planks may be partially prevented by the use of matched boards or of tongued-and-grooved plank.

76. Mortar, exuding thru open joints, leaves voids or stone pockets on the surface. Hence, in forms for facework, joints **should be made tight**, if necessary, by the use of mortar, putty, plaster of Paris, sheathing paper or thin metal.

77. If the lumber is very dry, when fastened in place, its swelling, due to its absorption of moisture, may bulge the boards and produce unsightly work. In such cases, the boards should not be matched, but should have their edges slightly beveled, and the sharp angle of the edges of adjacent boards placed in contact. Swelling will then crush the edges rather than bulge the board.

Lumber for Forms.

78. White pine is best for fine face-work, and quite essential for ornamental construction when cast in wooden forms.

79. Spruce, fir, Norway pine and the softer kinds of Southern pine are more liable to warp than white pine, but are generally stiffer and therefore better for struts and braces.

80. Partially dry lumber is usually best. Kiln dried lumber is unsuitable, as it swells when the wet conc touches it. In very green lumber, especially Southern pine, the joints are apt to open. Green lumber is heavy, and does not hold nails well.

81. For wall-panel forms, tongued-and-grooved or bevel-edge stuff is preferable to square-edge. Tongued-and-grooved gives smoother surface and less opening of joints, than square or bevel edge, but is more expensive, owing to waste in dressing, and there is more wear at joints if the forms are used often.

82. Even for rough forms, planing on one side may save money by reducing the cost of cleaning after using. Studs should always be planed on one side, to bring them to size.

83. Thickness. For ordinary walls, 1½ ins; for heavy construction, using derricks, 2 ins. For floor panels, 1 inch boards are most used; but, in tall buildings, they become much worn, and give bad finish to under sides of floors. For sides of girders, 1 inch or 1½ inch answers, but 2 inch is better for bottoms. Col forms usually of 2 inch plank.

84. Studding is usually from 3 × 4 to 4 × 6 inch; 4 × 4 inch is the most useful size. Spacing, usually 2 ft for 1 inch boards, 4 ft for 1½ inch, 5 ft for 2 inch.

85. Since beams and columns sustain greater stresses than floor slabs, their forms should be left in place longer, and should therefore be independent of the slab forms.

86. Sides of beam forms should be clamped or wedged together, to pre-

vent their springing away from the bottom boards, under the pressure of the conc.

87. Hardwood **wedges,** at tops and bottoms of struts facilitate the setting and removing of the struts, and testing for deflection.

88. Light joists (say 2 × 8 or 2 × 10), with frequent shores, are preferable to heavier sizes, difficult to handle.

Strength of Forms.

89. The **strength, required** for the forms, may be estimated, where wet conc is used, by assuming the pres of the conc as equal to that of a liquid weighing about 150 lbs per cu ft.* If dry and hard-rammed conc be used, the wedging of the stone, due to the tamping, will considerably increase the pressure.

90. Permissible loads, in lbs, on wooden struts for floor construction.

Unsupported length, ft	Cross section of strut, inches							
	3 × 4 = 12		4 × 4 = 16		6 × 6 = 36		8 × 8 = 64	
	per sq in	total	per sq in	total	per sq in	total	per sq in	total
14	—	—	700	11200	900	32400	1100	70400
12	600	7200	800	12800	1000	36000	1200	76800
10	700	8400	900	14400	1100	39600	1200	76800
8	850	10200	1050	16800	1200	43200	1200	76800
6	1000	12000	1200	19200	1200	43200	1200	76800

91. In timber beams, calculated for strgth, the **extreme fiber stress** is to be taken at 750 lbs per sq inch.

92. Construction live load, liable to come upon conc while setting, 75 lbs per sq ft on slabs; 50 lbs per sq ft in figuring beam and girder forms. This includes weight of men, barrows filled with conc, and structural material piled on floor, but not piles of cem sand or stone, which should not be permitted unless specially provided for.

93. Floor forms should be based upon allowable *deflection*, rather than upon *strength*. Formula:

$$d = \frac{3\,W\,L^3}{384\,E\,I}; \qquad I = \frac{b\,h^3}{12}.$$

where

d = deflection, ins;
W = total load on plank or timber;
L = distance, ins, between supports;
E = elastic modulus of lumber used = 1,300,000 lbs per sq inch;
I = moment of inertia of cross section of plank or joist;
b = breadth of plank or joist;
h = depth of plank or joist.

In the usual formula for deflection (see p 480) 1/384 is the coeff for beams with fixed ends, while 5/384 is that for merely supported ends.

Weight of conc, including reinforcemt, 154 lbs per cub ft.

(Sanford E. Thompson, Assn Am Portland Cem Mfrs, Bulletin 13, 1907.)

Details of Forms.

94. Too much **nailing** increases the difficulty of taking the forms apart without injury. Wire nails can be pulled with less damage to the wood than can cut nails.

* Mr. W. J. Douglas (E N, '06/Dec/20, p 646) assumes that the conc is a liquid of ½ its own weight, or 75 lbs per cub ft.

95. Iron or steel wall ties, extending thru the wall and fastening the forms in place, are usually removed and used again, if > ¼ inch in diam. If ≯ ¼ inch diam, they are usually allowed to remain; but, if their ends reach to the outer surface of the wall, they produce unsightly rust stains. To prevent this, the conc, surrounding their ends, is chipped out, and the rods are cut off, back from the surface. The holes, thus formed, are afterward plugged with mortar.

96. Separators (patented by Wm. T. McCarthy, 1 Madison Ave., New York city), molded of cem mortar, in the form of hollow cylinders, and in lengths of 4 and 6 ins, encircling the bolts, are sometimes used After the bolt is withdrawn, the hole in the cyl is filled with mortar.

97. Forms are liable to disturbance by blows from the conc bucket, or by the running of machinery in contact with the forms.

98. Any conc, adhering to a form, must be removed before the form is again used.

Adhesion to Forms.

99. Adhesion to forms. If the wood is new, and if the forms are thoroly wet before conc is placed, the conc, if hard, is not apt to adhere to the forms when these are removed. If the forms are to be removed before the conc is hard, they should, before concreting, be greased with material thin enough to flow and fill the grain of the wood. Crude oil, linseed oil, soft soap and other lubricating substances are used

100. New work is apt to adhere to old sticks, where conc has previously adhered, even tho this has been cleaned off.

101. Oil, applied to forms (to prevent their absorption of water or to facilitate their removal, ¶ 99), is apt to find its way to joints betw old and new work, and prevent the formation of a satisfactory bond. Soap and soft soap are of course harmless in this respect.

Removal of Forms.

102. Premature removal of forms and props has caused **many failures** of conc buildings; but undue delay, in their removal, means delay in the work and increase in the number of forms reqd.

103. The French law requires that **test blocks** and sample beams be made for every section cast. These enable the engineer to judge intelligently as to the condition of the actual work.

104. Props should be removed from *one beam or girder only* at a time, and should be at once replaced after the forms for that beam have been removed. This permits the discovery and repair of defects.

105. The **forms may be removed earlier** in warm and dry than in cold and damp weather, earlier from under light than from under heavy loads, earlier with quick-setting than with slow-setting cem, and earlier with dry than with wet mixtures. See Specifications, p 1359.

106. To release the beam boxes, the posts may be supported on wedges and capped. The posts and caps should not be removed, from more than one beam at a time. After the beam boxes have been removed, the posts and caps should be replaced before removing the forms from any other beams. Or, the posts may be supported solidly, and capped with a corbel forming the bottom and supporting the side-boards of the beam boxes. The side-boards may then be removed, leaving the posts and corbels undisturbed.

107. Prying against the conc, in removing the forms, may injure it.

Joints in Concrete.

108. Difficulty. In large work, the joints, betw work done on diff days or even before and after an hour's interval, are apt to give trouble, especially where watertightness is reqd.

109. Causes. The difficulty appears to be due partly to a surface skin or glaze, on the surf of the hardened conc, and partly to the presence of oily or dusty materials, laitance or sawdust, betw the two surfs. Oil, used upon the forms, or saturating the clothing of the workmen, is apt to find its way to the joints. Sawdust is particularly difficult to remove. The bond is especially weak if the older surf is frozen.

110. Remedies. Many remedies have been proposed, advertised and used, but none has been fully tested by time. See Specifications, p 1358. Cleanliness of surface and the use of wet mixtures are probably the best preventives. Water, used in scrubbing joints, should be rinsed off with clean water. A jet of live high-pres steam is very effective, removing even sawdust. Hydrochloric acid is used to advantage. Patented methods of securing bond, at joints, include the use of metallic binders, with their ends left projecting from the older surf, to bond with the newer. Another method employs a layer of prepared honey-comb slag, sprinkled over the still soft older surf; loose slag being removed after the hardening of the older surf and before the placing of the newer material.

111. Where conc is used in **reinforcing** and protecting **old stone masonry**, a stone should be removed here and there from the old masonry, and the joints cleaned out and washed. Key-bolts, with large washers on their heads, may also be driven into the face and left projecting into the concrete. The conc should also be carried far enough down the back of the wall to prevent water from working down into the horizontal joints on the tops of the wing walls and main walls.

Ramming.

112. Ramming of conc is necessary only with relatively dry mixtures. When properly done, it consolidates the mass about 5 or 6 %, rendering it less porous, and very materially stronger. For rammers, see spec'ns, p 1357 The men, using them, if standing on the conc, should wear gum boots.

113. Under water, ramming can be done only partially, and when the conc is enclosed in bags. A rake may be used gently for leveling loosely deposited conc under water.

114. Ramming should be discontinued before setting commences. Excessive ramming disturbs the homogeneity of the conc.

Placing under Water.

115. Concrete may readily be deposited under water in the usual way of lowering it, soon after it is mixed, in a dredge bucket, or in a V-shaped box of wood or plate iron, with a lid that may be closed while the box descends. The lid, however, is often omitted. This box is so arranged that, on reaching bottom, a pin may be drawn out by a cord reaching to the surf, thus permitting one of the sloping sides to swing open below, and allow the conc to fall out. The box is then raised to be refilled. In large works the box may contain a cu yd or more, and should be suspended from a traveling crane, by which it can readily be brought over any required spot in the work. The conc may if necessary be gently leveled by a rake soon after it leaves the box. Its consistency and strgth will of course be impaired by falling thru the water from the box; and moreover it cannot be rammed under water without still greater injury. Conc has been safely deposited in the above-mentioned manner in depths of 50 ft.

116. The Tremie, sometimes used for depositing conc under water, is a box of wood or of plate iron, round or square, open at top and bottom, and of a length suited to the depth of water. It may be about 18 ins diam. Its top, which is always kept above water, is hopper-shaped, for receiving the conc more readily. It is moved laterally and vertically by a traveling crane or other device suited to the case. In commencing operations, its lower end resting on the river bottom, it is first entirely filled with conc, which (to prevent its being washed to pieces by falling through the water in the tremie) is lowered in a cylindrical tub, with a bottom somewhat like the box described in ¶ 115, which can be opened when it arrives at its proper place. When filled, the tremie is kept so by fresh conc, thrown into the hopper to supply the place of that which gradually falls out below, as the tremie is lifted a little to allow it to do so. The weight of the filled tremie compacts the conc as it is deposited. A tremie had better widen out downward to allow the conc to fall out more readily.

117. The area upon which the conc is deposited must previously be surrounded by some kind of inclosure, to prevent the conc from spreading beyond its proper limits; and to serve as a mold to give it its intended shape. This inclosure must be so strong that its sides may not be bulged outward by the weight of the conc. It is usually a close crib of timber or plate iron without a bottom; and will remain after the work is done. If of timber it may require an outer row of cells, to be filled with stone or gravel for sink-

ng it into place. Care must be taken to prevent the escape of the conc hrough open spaces under the sides of the crib or inclosure. To this end he crib may be scribed to suit the inequalities of the bottom when the latter annot readily be leveled off. Or inside sheet piles will be better in some ases; or an outer or inner broad flap of tarpaulin may be fastened all around he lower edge of the crib, and be weighted with stone or gravel to keep it in place on the bottom. Broken stone or gravel or even earth (the last two where there is no current), heaped up outside of a weak crib, will prevent the bulging outward of its sides by the pressure of the conc. After the conc has been carried up to within some ft of low water, and leveled off, the masonry may be started upon it by means of a caisson, or by men in diving suits. Or, if the conc reaches very nearly to low water, a first deep course of stone may be laid, and the work thus brought at once above low water without any such aids.

118. The concrete should extend out from 2 to 5 ft (according to the case) beyond the base of the masonry. All soft mud should be removed before depositing conc.

119. Bags partly filled with concrete, and merely thrown into he water, are used in certain cases. If the texture of the bags is slightly open, a portion of the cem paste oozes out, and binds the whole into a tolerably compact mass. Such bags, by the aid of divers, are employed for stopping leaks, underpinning, and various other purposes, that may suggest hemselves. Such bags may be rammed to some extent.

120. Tarpaulin may be spread over deep seams in rock to prevent the loss of conc, and, in some cases, to prevent it from being washed away by springs.

121. Concrete, placed in water, should be in **large batches,** in order that the ratio of exposed surface to vol may be small. In running water, lead off the flow in pipes or shutes or by means of bulkheads (for which bag conc is suitable). If water is pumped out of the pit while concreting, it is apt to take cem with it. Observe the water flowing from the pump for indications of loss of cem.

122. Conc **dock foundation** on rock 14 to 19 ft below low water and covered with mud. Laid with assistance of diver. Mud washed off by jet. Rock not leveled. Wooden forms built on rock. Spaces, under forms, filled with bags of conc. Forms held down by means of boxes loaded with broken stone, anchored, by wire cables, near bottom, to neighboring piles, and braced, at top, by cross pieces nailed to existing dock. Conc lowered, by derrick, in ½ yd bottom-dump bucket, and dumped when close to work. The only cem lost is the little which washes from top of bucket load as bucket is submerged. The work has smooth faces along the forms, and appears to be perfectly homogeneous. (E R, '05/Oct/21, p 468.)

123. Placing conc **in 90 ft water,** in shaft, to stop inrush of water at bottom of shaft. Conc fed, by hopper, into 8 inch screw-jointed wrought iron pipe, lower end stopt with wood plug and resting on bottom of shaft. When the pipe was raised slightly, the plug refused to move and release conc. Pipe withdrawn, taken apart, and each section emptied. Plug, not tight, had allowed lowest section to fill with water, which disintegrated the conc, leaving, at top of lowest section, a plug of neat cem, which prevented the conc, above, from pushing out the wood plug as intended. Expt repeated, with tight plug. Inside the 8 inch pipe was placed a 1½ inch pipe, by means of which the wood plug was knocked out, allowing conc to descend. Rate regulated by changing dist of foot of pipe above bottom of shaft. Mass of conc, 10 or 12 ft thick, deposited. The upper 6 or 8 ins never set; but the remainder appeared to be solid and homogeneous. (Assn C E, Cornell Univ, Trans, 1898, p 74.)

124. In a case where hollow iron piles, in clean sandy bottom, were filled with conc, some of the mortar leaked out, and formed, with the surrounding sand, masses of conc, which adhered most tenaciously to the piles; suggesting the use of **hollow piles, purposely perforated,** in their lower portions, with small holes, thru which grout, poured into them, at top, can escape into the sand. (Chas List, Jour Assn Engg Socs, March, 1903, Vol 30, No 3, p 124.)

125. Superior Entry, Wis. Mixer discharges into a **sub-hopper,** with a cut-off shute, which discharges into depositing buckets on cars under the platform. Upon reaching the work, the buckets are lowered into the sub-

110. Remedies. Many remedies have been proposed, advertised and used, but none has been fully tested by time. See Specifications, p 1358. Cleanliness of surface and the use of wet mixtures are probably the best preventives. Water, used in scrubbing joints, should be rinsed off with clean water. A jet of live high-pres steam is very effective, removing even sawdust. Hydrochloric acid is used to advantage. Patented methods of securing bond, at joints, include the use of metallic binders, with their ends left projecting from the older surf, to bond with the newer. Another method employs a layer of prepared honey-comb slag, sprinkled over the still soft older surf; loose slag being removed after the hardening of the older surf and before the placing of the newer material.

111. Where conc is used in **reinforcing** and protecting **old stone masonry**, a stone should be removed here and there from the old masonry, and the joints cleaned out and washed. Key-bolts, with large washers on their heads, may also be driven into the face and left projecting into the concrete. The conc should also be carried far enough down the back of the wall to prevent water from working down into the horizontal joints on the tops of the wing walls and main walls.

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115. Concrete may readily be deposited under water in the usual way of lowering it, soon after it is mixed, in a dredge bucket, or in a V-shaped box of wood or plate iron, with a lid that may be closed while the box descends. The lid, however, is often omitted. This box is so arranged that, on reaching bottom, a pin may be drawn out by a cord reaching to the surf, thus permitting one of the sloping sides to swing open below, and allow the conc to fall out. The box is then raised to be refilled. In large works the box may contain a cu yd or more, and should be suspended from a traveling crane, by which it can readily be brought over any required spot in the work. The conc may if necessary be gently leveled by a rake soon after it leaves the box. Its consistency and strgth will of course be impaired by falling thru the water from the box; and moreover it cannot be rammed under water without still greater injury. Conc has been safely deposited in the above-mentioned manner in depths of 50 ft.

116. The Tremie, sometimes used for depositing conc under water, is a box of wood or of plate iron, round or square, open at top and bottom, and of a length suited to the depth of water. It may be about 18 ins diam. Its top, which is always kept above water, is hopper-shaped, for receiving the conc more readily. It is moved laterally and vertically by a traveling crane or other device suited to the case. In commencing operations, its lower end resting on the river bottom, it is first entirely filled with conc, which (to prevent its being washed to pieces by falling through the water in the tremie) is lowered in a cylindrical tub, with a bottom somewhat like the box described in ¶ 115, which can be opened when it arrives at its proper place. When filled, the tremie is kept so by fresh conc, thrown into the hopper to supply the place of that which gradually falls out below, as the tremie is lifted a little to allow it to do so. The weight of the filled tremie compacts the conc as it is deposited. A tremie had better widen out downward to allow the conc to fall out more readily.

117. The area upon which the conc is deposited must previously be surrounded by some kind of inclosure, to prevent the conc from spreading beyond its proper limits; and to serve as a mold to give it its intended shape. This inclosure must be so strong that its sides may not be bulged outward by the weight of the conc. It is usually a close crib of timber or plate iron without a bottom; and will remain after the work is done. If of timber it may require an outer row of cells, to be filled with stone or gravel for sink-

sion of the forms, and exposing the clean stone and sand of the conc. A few rubs of an ordinary house scrubbing brush, with a free flow of water to cut and to rinse clean, suffice; but a little additional rubbing improves the effect. The necessity for early removal of the forms, when this method is used, necessitates special care in their construction, increasing their cost. When applied to surfaces forming square corners, the projecting sand particles produce a ragged effect. Hence care should be taken not to extend the treatment to such corners.

136. An effect similar to that obtained by Mr. Quimby's method, may be produced, after hard set, by **washing with an acid solution,** which is afterward removed by the use of an alkaline wash, followed by water. This method attacks limestone in the agg.

137. Color effects are best produced by using *agg* of the desired color.

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138. The difficulty of **making oil paint adhere** to fresh conc surfs is due to moisture and free lime. A wash of dilute acid neutralizes the lime, but is unsatisfactory, muriatic (hydrochloric) acid forming highly hygroscopic salts, such as calcium chloride, and sulfuric acid having only a superficial effect. Dissolve 10 lbs ammonia carbonate (salts of hartshorne) in 45 gals water, and apply once with a brush, or give several coats of a weaker solution, or apply as spray. The ammonia is liberated, and the carbonic acid forms, with the free lime, an insoluble carbonate, which soon becomes dry and hard. After exhaustive trials this was found the only method which satisfies every requirement. The amm carb keeps, for any length of time, in fairly tight vesesls. (Fred J. Bosse, "Cement Age," '09/Jan, p 48.)

PROPERTIES OF CONCRETE.

Weight. See Voids, p 1256, and Density, p 1257.

1. Weights of concrete, in pounds per cubic foot.
Broken stone or gravel **concrete,** 130 to 160; ordinarily 140 to 150.*

One foot B M = vol of a solid 1 ft square and 1 inch thick, = 144 cu ins = 1 cu ft/12.

144 lbs per cu ft = 1 lb per 12 cu ins = 1 lb per prism 1 inch square and 12 inches long.

Hence, **at 144 lbs per cu ft,** the wt of any prism in pounds = area of cross section in square inches, multiplied by length in feet, = vol in cubic inches/12.

Wt, lbs/cu ft..........	100	110	120	125	130	140	150	160
Kilograms/ cu meter....	1600	1760	1920	2000	2080	2240	2400	2560

Cinder concrete, ...	110 to 120;
Sandstone "	143
Limestone "	148
Gravel "	150
Trap "	155

With **natural** cem, 4 to 5 lbs lighter per cu ft

2. The unit weight varies not only with character of constituents, but also with proportions, consistency, degree of compacting, etc.

Permeability.

3. Even where the primary object of the conc is not the prevention of percolation by water, impermeability is of great importance in promoting the durability of the conc, and especially in protecting metal reinfmt from corrosion and from loss of adhesion with the conc.

4. Water may pass thru conc, etc, so slowly that evaporation, from the outside, proceeds more rapidly than the water can reach it, so that the outside of the **wall may appear dry,** altho percolation is actually taking place.

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*144 lbs per cu ft = 12 lbs per ft B M (Board measure).
120 " " " " = 10 " " " "
108 " " " " = 9 " " " "

5. When made into hardened mortar, well trowelled down on all surfaces which come into contact with water, **neat cement is as nearly impermeable** as the best of natural rocks used for building purposes. (Wm. B. Fuller, Trans, A S C E, Vol 51, pp 133-4, Dec 1903.)

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6. Mortar or conc, so proportioned as to obtain the **max practicable density**, and mixt rather wet, is impervious under ordinary conditions.

7. Small blocks of conc, carefully made from materials so graded as to insure great density, or with an excess of cem, have been repeatedly found to be as nearly impervious as the best natural stones. See Expts, p 1306.

8. In **large masses**, in actual construction, it is difficult to produce an absolutely tight structure without the addition of a **lining** of material more nearly impervious than the conc. Variations in the mixture, carelessness in manipulation or placing, or in bonding betw successive days' works (an hour's interruption, in the middle of a hot day, has been known to cause leakage), or insufficiency of water, will render conc permeable, in spite of proper theoretical proportioning and the addition of lime. The mix should be at least wet enough to settle into place with but little ramming

9. Conc, impervious in itself, may develop **cracks** thru which water may permeate. Reinfmt, properly placed, opposes such cracking.

10. Water may permeate thru the mortar, thru the particles of agg, or betw mortar and agg. Probably most of the percolation takes place thru the mortar. See Mortar. We here deal with those aspects of permeability which can better be discussed in connection with the conc as a composite material.

11. When the leakage consists of mere percolation thru the **minute pores** of conc, etc (*i e*, when there are no actual fissures), leakage generally diminishes with use, the water (even when apparently clear) blocking its own passage by depositing, in the pores of the material, either its own natural sediment, or (in the form of "laitance") lime and other compounds dissolved out of the conc itself.

12. This action depends upon many factors, notably the pressure, the sizes and shapes of the pores, the hardness and solubility of the material, and the character of the sediment carried by the water. Thus, under high pres, if the material is easily scoured, or if the pores are large and relatively straight, leakage may be expected to increase, rather than diminish, with time.

13. Where the nature of the case permits, as in floors, retaining walls, etc., it is better to lead the water off by proper **drainage**, than to attempt to block its passage by rendering the structure watertight.

14. Where watertightness is required, as in dams, the constituents must be carefully proportioned for max density, there must be an excess of rich mortar over vol of voids, dry mixtures should be avoided, the mixing must be thoro, and the construction should be, as nearly as possible, monolithic.

15. The application of **waterproofing materials** may be either (a) internal, mixt with the ingredients of the conc, (b) superficial, filling the pores near the surf; (c) external, preventing contact betw water and conc.

16. Internal. For water tight work, the vol of mortar should be 40 to 45 % of the vol of agg, or 40 to 42 % if the agg is graded. (Geo. W. Rafter, Trans, A S C E, Vol 42, p 149, Dec 1899.)

17. With agg having 35 % voids, the vol of mortar should be ≮ 50 % of vol of agg; vol of dry sand and cem ≮ ⅔ vol of agg; vol of sand ≯ 2 × vol cem. For cem leaving ≯ 10 % on No. 120 sieve, ordinary sands, and agg with 35 % voids, the following proportions are given:

cem	sand	agg	(sand + agg) ÷ cem
1	1.0	3.00	4.00
1	1.5	3.75	5 25
1	2.0	4.50	6.50

See Plain Concrete, ¶ 22, p 1256.

18. Every particle of sand must be coated with cem, and every particle of stone with mortar, so that the stones or the sand grains do not touch.

19. To insure this result, mix by means of one of the newer types of ma-

chine, introducing first the measured quantity of water and then the cem, making a liquid grout which will run easily into the most minute voids of the sand, which, being next introduced, becomes coated in the shortest space of time. The resulting mortar is still quite liquid, and flows into all the voids of the stone. (Wm. B. Fuller, Trans A S C E, Vol 51, p 135, Dec 1903.)

For the use of lime, see Expt. 82 *a*, p 1345.

20. In making thin slabs with a conc of 2 parts cem to 5 of fine bituminous ash, reinfd with poultry mesh, Mr. W. K. Hatt (Trans, A S C E, Vol 51, p 129, Dec 1903) employed a 5 % solution of ground alum, in place of one half of the gaging water, and a 7 % solution of soap in place of the other half. This strengthened and hardened the ash conc by about 50 %, and diminished its absorption by about 50 %. The soap solution alone diminished absorption, but did not strengthen the conc. Sand mortar was not greatly strengthened by the **soap and alum** treatmt, but its absorption was diminished about 50 %.

21. If joints are inevitable, they may be first wet, and then covered with neat cem paste or 1 : 1 cem mortar, upon which the new work is to be placed before the binding course hardens.

22. The permeability of conc linings of aqueducts &c may be diminished by drilling holes thru them and **forcing in grout** behind them by means of grout pumps. The grout sometimes appears at many points, indicating that it is passing not only thru the cracks but also thru the body of the conc. This method was successfully used in the Torresdale filtered water conduit, Philadelphia.

23. Superficial. For plastering the inside of a covered clear water well, Mr. Edwd Cunningham used 1.25 lbs of soft soap for each 5 buckets of water, and 3 lbs of alum per bag of cem. The mortar was easy to handle with the trowel, but had a nauseating odor. 2 coats, not more than 0.5 inch in all. 18-inch dividing wall showed no leak when one side held 16 ft of water. The soap was made of clarified fats, and cost 7.5 cts per lb; much too high. With 1 part cem to 2 parts sand, 6 to 9 gals of water and 12 lbs of alum were required for each bbl of cem. (Trans, A S C E, Vol 51, pp 127-8, Dec 1903.)

24. As an **external** treatment, Mr. Richd H. Gaines, New York Board of Water Supply (Trans, A S C E, Vol 59, p 160, Dec 1907) found the **Sylvester soap and alum process** (p 928), "fairly effective, but very expensive for large work."

25. Asphalt can be successfully applied only to dry surfaces. It becomes brittle and loses its efficiency upon oxidation; but it will often prevent leakage until the structure has become tight thru infiltration. See ¶ 11, p 1272.

26. The conc surface must be clean, and must first be treated with a thin wash of liquid asphalt, thinned with benzine. This enters the pores of the conc, and acts as a binder. Without this, the asphalt coating will not adhere to the conc.

27. Asphalt coatings should be made continuous, and should be protected against decay, from creeping and from abrasion, by being placed between alternate layers of conc, or by being covered with brickwork or masonry.

28. Tunnels, subways and basements, below water level, have been thoroughly waterproofed by continuous layers of heavy **roofing papers**, well mopped with tar or asphalt, and placed between outer and inner conc walls.

29. The two basins of Queen Lane reservoir, Philadelphia, originally lined with cem conc on sandy clay puddle, and holding 383 million gals of water 30 ft deep, were re-lined with Bermudez asphalt in 1896-7. The floor received 2 inches of asphalt conc, with a thin top layer of hot liquid asphalt; the slopes, two layers of hot liquid asphalt, with burlap between them; the burlap being anchored at top by being lapped around horizontal iron or wooden bars, let into the asphalt paving. While this work was in progress, the south basin of the Roxborough reservoir (147 million gals, 25 ft deep) was similarly lined. In the north basin, Alcatraz (California) asphalt was used, and the slopes, as well as the sides, were treated with asphalt conc. All four of these basins have since been in continuous use, without sensible leakage.

Elastic Modulus, E. See ¶¶ 12 and 13, p 1279.

30. When conc is subjected to compressive test, its stress-strain diagram is in general curved throughout its length; its elastic modulus,

$$E = \frac{\text{stress, per unit of area}}{\text{shortening, per unit of length}}, \text{ diminishing as the stress increases.}$$

Strength.

31. Conc being weak in tension, and brittle, its **tensile strength** is usually and properly **neglected:** dependence is placed chiefly upon its comp strgth, and its tensile and shearing strgths are usually exprest as fractions of the comp strgth.

32. The **compressive strength** is preferably determined experimentally by means of cubic specimens. The unit comp strgth decreases when the ratio, length/side, increases, and, in similar specimens, when their dimensions increase.

33. Conc **prisms, tested in endwise compression,** usually fail by shearing on planes oblique to the axes of the prisms. Upon these oblique planes, the unit shear is about half the ult comp stress.

34. The strgth varies widely with the character of the conc

35. For **12 inch cubes** of Portland cem mixtures having from 6 to 18 volumes of (sand + agg) to 1 vol cem, Mr. **Edwin Thacher** deduces, from the data of Expt 18 a, the straight-line formula,

$$S = M - N X$$

where

S = ult comp strgth, lbs per sq inch;
X = No of parts of sand to 1 part cem;
M and N = values as below:

Age =	7 days	1 month	3 months	6 months
M =	1800	3100	3820	4900
N =	200	350	460	600

Mr. Thacher holds that, for practical mixtures, "the strgth of conc depends principally on the strgth of the mortar, and not, to any great extent, upon the amount of stone." In these tests, the vol of stone was always twice the vol of sand.

36. But few tests have been made to determine the **tensile strength** of conc. It is usually taken as approximately from one-tenth to one-eighth the comp strength, and the **shearing strength** as from 1.2 to 1.5 times the tensile.

37. Prof. L. J. Johnson (Jour, Assn Eng Socs, Vol 38, No 6, p 310, June, 1907) tested 25 reinfd beams, 3 ins × 9 ins × 8 ft, loaded 6 ins from each support; 19 of the beams were of 1 : 2 : 2⅔ scaly trap; 6 of 1 : 2.5 : 5. All the **beams failed by slip of reinfmt:** the 1 : 2 : 2⅔ beams, 137 to 143 days old, successfully resisted shears of 233 to 573 lbs per sq in, av 470; and the 1 : 2.5 : 5 beams, 488 to 750; av 628.

38. In beams, owing to the rising of the neutral axis, under loading, the **ult unit fiber stress,** or rupture modulus, is about 1.6 × the unit tensile strgth.

Setting.

39. Setting is of course a function of the *cement paste.* See Mortar. We here treat of setting, as affecting the conc as a composite body

40. Temperature. In hot weather, conc sets very much faster than in cool weather, and the load may therefore be applied sooner in hot weather; but the time required varies with the class of structure and of conc.

41. Gradual loading. Where the loading is static or gradually increased, the time may be shorter than where the load is applied suddenly or is subject to impact.

42. "As a general rule, **bridge abutments and piers** of Portland cem conc should be allowed to set at least a month before using, if built during ordinary warm weather. If built during cold weather, their use should, if possible, be deferred until warm weather sets in." (W. A. Rogers, RR Gaz, '00/Jul/27. p 514.)

43. Steel girder spans have been placed upon Portland cem conc abutments without injury 2 weeks after the completion of the abuts in hot weather; but work of the same character, finished early in Dec, was found not very solid inside, early in the following March.

Effects of Heat and Cold.

44. **Freezing** nearly always damages **nat** cem mortar or conc to such an extent that it must be replaced by new material.

45. With **Portland** cem conc, **freezing suspends the setting** and hardening of the mortar, for the length of time during which the material has been frozen. The apparent loss of strgth, in frozen specimens, may often be due merely to such delay in setting.

46. While freezing seldom results in material reduction of the ult strgth of Port cem conc, yet it **may produce serious results** by giving the conc an apparent hardness; thus causing the premature removal of forms, or the imposition of undue loads, which may produce failure when the conc thaws out, if it had not already set sufficiently before being frozen.

47. If, soon after the mortar, thru the entire thickness of a wall, is frozen, the sun shines on one face of it, so as to soften the mortar of that face, while the mortar behind it remains hard, it is plain that the wall will be liable to settle at the heated face, and at least bend outward if it does not fall.

48. If the freezing does not take place until after the cem has taken its **initial set**, there is little danger. Thin work should not be done at < 28° F on a rising, or at < 32° on a falling temp.

49. **A thin scale** is likely to crack from the surface of conc walks or walls which have been frozen before the cem has hardened. Granolithic or troweled finish sometimes spalls up in small patches, when frozen.

Protection.

50. **Protection against freezing** is expensive and uncertain. Hence the placing of conc in freezing weather should be avoided when possible.

51. **Housing and heating** the finished work. Tents or screens may be used; but wooden sheds are more effective.

52. **Covering** the conc, as soon as placed, with canvas, cem bags or tar paper, or with a thick layer of sand, straw, manure, sawdust or other poor heat-conductors. Straw should be ≮ 1 foot deep. Manure is the best, but it discolors the work. Canvas etc should be kept an inch or two away from the conc, leaving an air space. Otherwise use two layers.

53. **Heating the materials.** Stone is frequently heated by piling it over a pipe or improvised oven, and building a fire inside; or over a coil of pipe containing numerous small holes, and then forcing steam thru the pipe. The conc must be used before the steam is condensed and frozen. Sand is heated over a long sheet iron stove.

54. **Lowering the freezing point of the mixing water,** by the addition of chemicals.

55. **Salt** is the cheapest and most commonly used material. It lowers the freezing point about 1.5° F for each 1 % salt added to the water. A 10 % solution (12 lbs salt per bbl of cem) reduces the freezing point to 17° F and does not injure the strgth of the conc. For 32° F, dissolve 1 lb salt in 18 gals water; add 3 oz salt for each 3° below 32° F. (Ch of Engrs, U. S. A. Report, 1895.) Larger percentages of salt appear to weaken the conc.

56. **Calcium chloride,** 15% solution, or 1.25 lbs per gal of water, lowers the freezing point to about 20° F, and does not weaken the mortar. It rapidly absorbs moisture, and it is possible that, if ground dry with the Portland cem clinker, even to the amount of 0.5 %, it would cause the material to gather dampness. The chloride dissolves with extreme rapidity and may be added to the mixing water. (Prof. R. C. Carpenter, Cornell Univ, Sibley Jour of Eng, Jan 1905.)

57 The major portion of a pile of sand or stone may be in condition for use altho the surface is frozen.

58. In winter, we may **reduce the areas** of the exposed layers of the work, by placing the bulkheads closer together. A day's work will then run to a greater elevation, and will necessitate the use of stronger forms.

59. **Mortars**, placed in open air, are more or less **injured, by drying** instead of *setting*, when the temperature exceeds about 65° to 70°; but if mixed only in small quantities at a time, and quickly laid in masonry of dampened stone, so as to be sheltered from the air, the injury is much reduced. The sand and stone should both be *damp*, not wet, in hot weather, and a *little* more water may be used in the cem paste; also, if possible, not *only the mortar*, while being mixed, but the masonry also, should then be *shaded.*

Expansion.

60. In variable climates, **cast iron cylinders, filled with concrete,** are frequently split horizontally by unequal expansion and contraction. In such structures it is safest to consider the cylinders as mere molds for the conc; and to depend only upon the conc for sustaining the load.

For **expansion coeffs**, see Reinforced Conc, ¶ 9, p 1278.

61. **Cracks and joints.** In abutments or culverts over 60 ft long, divide the wall into sections of about 40 ft, and finish one section before beginning the other. Contraction will cause the joint to open, and irregular cracks thru the body of the wall will thus be avoided. Short sections may be completed without stopping, and horizontal joints thus avoided "Very small cracks, which, in stone masonry, would be difficult to find, show up very plainly in conc." (W. A. Rogers, R R Gaz, '00/July 6, p 461.)

62. **Effect of high temperatures.** During calcination of the materials for Portland cem, the chemically combined water is driven off. When, in mixing, this water is returned to the material, hardening takes place; but the re-application of temperatures, sufficiently high to drive off the water again, reverses the hardening process and disintegrates the material.

Chemical Effects.

63. "**Dehydration** of the water of crystallization of conc probably begins at about 500° F and is completed at about 900° F"; but this cools surrounding masses, and thus increases the heat resistance of the conc. J. C.*

64. **Rehydration.** Briquets, kept, for 6 to 8 hours, at 1000° to 1200° F (not in contact with flame) and allowed to cool, showed practically no strgth; but 28 days immersion in water restored their strgth to that of unheated briquets.

65. **Fire resistance.** In quartz sand the expansion coeff is twice that of feldspar; and the expansion, in one direction, is twice that in the direction perp to it.

66. At the Baltimore fire the conc, exposed to flames, was seldom damaged to a greater depth than ½ inch, altho projecting corners were at some places rounded off by flames to a radius of about 2 inches.

67. **Sea water** has apparently but little effect upon conc so proportioned as to secure maximum density, and thoroly mixt. Damage by sea water, reported as taking place at the water line, has probably been due, in part, to freezing. J. C.*

68. Destructiv action upon conc by **electrolysis** appears to be due to abnormal conditions seldom occurring in practice. J. C.

69. Green conc is injured by **acids**; but first class conc, thoroly hardened, is appreciably affected only by strong acids which seriously injure other materials. J. C.

70. In the reclamation of arid land, where the soil is heavily charged with **alkaline salts**, conc, stone, brick, iron and other materials are injured under certain conditions, at ground water level. Such action can be prevented by the use of an insulating coating. J. C.

71. Conc properly made, and having its surface carefully finished and hardened, resists the action of petroleum and ordinary engine **oils**. Oils containing fat acids appear to injure conc. J. C.

72. Sulphurous and sulphuric acid **gases**, combined with moisture, corrode conc, especially if heated

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* J. C. Report of Joint Comm, A S C E, A S T M, Am Ry Eng & M W Assn and Assn of Am Port Cem Mfrs. '09, Jan.

Tests of Concrete in place.

73. **Tests of concrete in place** may be made by analysis of a core of conc, obtained with a **core drill,*** using chilled steel shot for cutting. The bore holes are afterward grouted.†

74. **The ratio of cement to sand,** in the mortar, is found by means of the amounts remaining undissolved in hydrochloric acid; sand and cem, of the kinds used, and mortar, taken from the core, being tested separately in this way. (Prof. R. L. Wales, in E N, '08/Jan 9, p 46.)

75. **The ratio of mortar to stone,** in the conc, is found (1) by actual separation and by weighing the stone and the mortar separately, or (2) by ascertaining separately, and comparing, the specific gravities of the stone, the mortar, and the conc.

* Made by Cyclone Drill Co., Orrville, O., including small drills, worked by hand.

† B. G. Cope, in E N, '08/Jan/9, p 41.

REINFORCED CONCRETE.

1. The tensile and shearing strengths of conc are low as compared with its comp strgth. Hence metal rods or shapes are embedded in conc structures in those portions subject to tensile and shearing stresses, and in such positions as to take those stresses.

2. Uses. Reinfmt is used chiefly in the tension-sustaining portions of beams and girders, (including floor-slabs), cols, walls, retaining walls, dams, etc; but it is useful also in many other cases; as for preventing hair cracks in surfaces, for which purpose a light web of metal (wire mesh expanded metal, etc) is placed a few inches back from the face; for preventing fracture due to unavoidable sudden changes in cross-section; for joining walls meeting at an angle and liable to settle away from each other; and in culverts, enabling them to withstand hor tension due to the outward pressure of the embankment. For this purpose old chains may be used, or light rails, with bolts driven thru the bolt-holes, to increase adhesion.

3. Safety. Modern reinfd conc buildings are practically monolithic, and therefore more rigid than skeleton steel construction.

4. On the other hand, in the steel building, the details are more accurately worked out, and the work is usually erected by skilled men, often employed by the steel mfrs; so that there is but little chance of damage to the material in erection; whereas, in reinfd conc work, the best material may be injured in the using, and the work thus rendered unsafe.

5. Good conc protects imbedded steel from **corrosion**, both above and below fresh or sea water level; but water may penetrate porous conc and corrode the metal. Conc laid very dry is apt to be porous.

6. The **steel,** used in reinfg conc, has its ult strgth usually betw 50,000 and 70,000 lbs per sq inch, and its elastic limit between 25,000 and 35,000 lbs per sq inch, but cold working may raise the elastic limit to 40,000 or 50,000 lbs per sq inch. "Deformed" bars are often rolled of steel with much higher elastic limit (50,000 to 65,000 lbs per sq in claimed) for the sake of economy of steel; but see Bar Reinforcement, pp 1296, etc. As in rolled iron and steel in general, the elastic modulus may be taken as averaging approximately 30,000,000 lbs per sq inch. See ¶ 11.

7. Concrete. In general the necessity of working the conc around the reinfg bars requires that the agg for the conc in reinfd work shall be smaller than would be permissible in unreinfd mass work; and the vital importance of adhesion requires that all the materials for the conc shall be of the best, and the mortar not too lean or too dry.

Expansion, Contraction, Etc.

8. The **shrinkage** of conc, while setting in air, produces comp stress in the reinfmt and tensile stress in the conc itself. Setting under water, the **expansion** of the conc produces the opposite effects.

9. The linear **expansion coefficient**, a, of a material, is that fraction of its original length which a bar of it gains or loses for each degree of change in its temp. Approximately:

	Per degree, Centigrade	Per degree, Fahrenheit
In steel .... 10,000 a =	0.117	0.065
In concrete .... 10,000 a =	0.108	0.060*

10. The large number of reinfd conc structures which have been exposed, for years, to wide extremes of temp, without injury thru difference in expansion, confirms the results of experiments, quoted above, as indicating that the diff, betw the expansion coefficients of the two materials, is negligible

Elastic Modulus.

11. The **elastic modulus, E_s, of rolled iron and steel,** of all kinds (p 460,) is remarkably uniform and constant, ranging ordinarily betw 27 and 31 (av, say 30) millions of lbs per sq inch = approx 1.9 to 2.2 (av, say 2.1) millions of kgs per sq cm.

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* W D. Pence, 1 : 2 : 4 conc, Jour Westn Soc of Engrs, 1901, Vol. 6, p. 549, 10,000 a = 0.055 Fahr, results nearly uniform. Columbia Univ. 1 : 3 : 6 conc, 10,000 a = about 0.065 Fahr.

12. On the contrary, the **elastic modulus, E_c, of concrete** varies widely, not only as betw diff mixtures differently manipulated, and betw diff specimens made under like conditions from like materials, but in one and the same specimen under diff intensities of loading; so that, in stating the results of expts, it is usual to specify the range of unit stress within which the observations were made.

13. In **stone concrete, E_c** ranges from 1.5 to 4 (av, say 3) million lbs per sq inch, = 0.1 to 0.28 (av, say 0.21) million kgs per sq cm. See Expt 81 *a*, p 1172. In cinder conc, E_c is ordinarily from 20 to 50 % less than in stone conc. See ¶ 30, p 1274.

14. The ratio, n (sometimes called *r* and *R*), = E_s/E_c, betw the elastic moduli of steel and of conc respectively, is usually taken betw 10 and 15 for stone conc, with higher values for cinder conc. See Specifications, ¶ 107, p 1363. Owing to the variability of E_c (see ¶ 12), it cannot be a constant quantity, even during the range of a single experiment carried from zero load to rupture.

15. The ratio, *n*, is, however, of constant and **important** use in all calculations respecting the mutual behavior of conc and steel.

16. Considère's experiments (Expt 16 *a*, p 1314) seemed to show that conc, when reinfd (being constrained, by its adhesion to the steel, to share in its movemts), actually underwent, without fracture, far greater **elongations** than were possible in unreinfd conc, but later expts (36, 38, 81 *e*, 81 *f*), in which the conc surface was more closely observed have indicated that the supposed elongation of the conc was in fact due to the formation of cracks which had before escaped observation. If the adhesion, betw the conc and the steel, is uniform, the cracking must be evenly distributed over the area of contact, and the cracks must therefore be very numerous and very fine, probably so fine as not to endanger the materials thru the percolation of water.

Adhesion. See ¶ 58, p 1294.

17. With **rich and wet mixtures,** such as are used in reinfd construction, the cem adheres very closely to the steel.

18. After the adhesion proper has been overcome, the removal of the steel from the conc is still opposed by **friction** betw the two.

19. Upon the ability of this adhesion and friction to resist the forces tending to overcome them, depends of course the safety of the structure.

20. Both adhesion and friction, and particularly the friction, are greatly affected by the character of the conc and by its behavior under stress and under temp changes, by the method of testing, etc.

21 In direct **tests for adhesion,** whether the steel is pulled or pushed, the conc is always under comp, which causes some lateral expansion of the conc, and therefore increased pressure upon the reinfmt. Hence, the adhesion may be found higher than (other things equal) in beams, where this condition does not obtain.

22. On the other hand, where the hor reinfg bars, in a beam, are bent upward, near the ends, and pass up into the region of compression and (as is often the case) to a point over the support, the high pressures upon the bar, in those portions, may give it greater adhesion, as a whole, than could be the case with a straight bar under direct test.

23. With **great lengths of imbedment,** the stretch, in the steel, under high tensile stresses, may be such as to contract the steel laterally, sufficiently to reduce adhesion. Hence, tests where the steel is *pushed into* the conc, show higher adhesions.

24. Ultimate adhesion. In general, expts (see Expts 64 *a*, *b*) give, as the ultimate adhesion of good conc to plain round rods, from 200 to 300 lbs per sq inch of contact surface. With smooth round rods, in a beam, Kleinlogel (Beton und Eisen, 1904, pp 227 et seq) obtained 560 lbs per sq inch. The conditions of practice generally differ greatly from those obtaining in the laboratory.

25. Working bond stress. In beams subject to shock, about 50 lbs per sq inch; for quiet loading, about double this is sometimes allowed. See Specifications, ¶¶ 113–115.

REINFORCED CONCRETE COLUMNS.

1. A concrete column usually has longitudinal steel rods embedded, near the circumference, thruout its length. If there is no deflection, and no slip between the concrete and the steel, the two materials must shorten equally under load. Hence (p. 458, Eq (3)) if L = original length, l = change of length, a_s and a_c = cross section areas; s_s and s_c = unit stresses, E_s and E_c = elastic moduli, of steel and of conc. respectively; we have

$$s_s = E_s\, l/L; \quad s_c = E_c\, l/L, \qquad (1)$$

and, since l/L is necessarily the same for both materials,

$$s_s/s_c = E_s/E_c = n; \quad s_s = s_c\, n, \qquad (2)$$

and

total stress in steel $= a_s\, s_s = a_s\, s_c\, n$ ....(3)

" " " conc $= a_c\, s_c$ ....(4)

" " " column $= P = a_s\, s_s + a_c\, s_c = s_c\,(a_c + a_s\, n)$ ...(5)

$$a_c = P/s_c - a_s\, n \qquad (6)$$

$$s_c = P/(a_c + a_s\, n) \qquad (7)$$

2. Example. A square conc col 16 ins × 16 ins, 12 ft long has, embedded in each corner, a round steel rod 1 inch diam; cross section area of each rod = 0.785 sq inch. Permissible unit comp stress, s_c, on concrete, = 500 lbs per sq inch. Required the load which may be carried by the col. Here

Area, a_s, of steel = 4 × 0.785 = 3.14 sq ins;

Area, a_c, of conc = 16 × 16 — 3.14 = 253 sq ins;

E_s = 30,000,000 lbs per sq inch;

E_c = 2,500,000 lbs " " ";

$n = E_s/E_c = 12$,

Total stress taken by conc $= a_c\, s_c$ = 253 × 500 = 126,500 lbs

" " " " steel $= a_s\, s_c\, n$ = 3.14 × 500 × 12 = 18,840 lbs

" " " " column.......................145,340 lbs

3. Here the steel takes 100 × 18,840 ÷ 145,340 = about 13 % of the entire load, a safe proportion. This proportion should not exceed 20 %, or, at most 30 %.

4. A convenient rule is to count each sq inch of steel, in cols, as worth n sq ins of concrete.

5. Conservative designers load conc cols approximately as follows:

Length / diam	Mixture 1:1.5:3	1:2:4	1:2.5:5	1:3:6
	$p = P/a$ = Load, in lbs per sq inch.			
< 12	600	500	350	350
12 to 18	550	450	300	300

6. Longitudinal reinfg rods or **bars are usually placed** symmetrically near the outside of the conc, and are covered by from 1½ to 2 inches of conc. The rods should be tied together, by smaller rods or by wires, at intervals not exceeding the diam of the col.

7. Specifications usually require that the aggregrate cross-section **area of compression rods** shall not exceed from 2 to 3 % of the cross-section area of the col.

8. In buildings of say three or four stories, the **rods of each section are bent in, near their tops,** to form a cylinder, 18 or 20 ins high, of smaller diam than the main cyl below; and the section next above fits down over this portion, so that the two sections overlap the length of the reduced portion.

9. Owing to their much greater cross-section areas, and to the lower unit stresses in their materials, reinfd conc cols are much less liable to failure by deflection than are steel cols.

10. For **ultimate loads on longitudinally reinforced concrete columns liable to deflection,** we have the Rankine formula

$$p = \frac{P}{a} = \frac{s}{1 + m K^2} \quad \ldots\ldots (8)$$

where

P = ult total load on col;
a = cross section area of col;
p = P/a = ult unit load on col;
s = ult comp unit strgth of conc cubes;
K = L/r = length/least radius of gyration;
Prof. Morsch gives m = 0.0001. Eisenbetonbau, '08, p 73.

Hooped Columns.

11. Columns reinforced with hoops (or spirals) of steel, or with web reinforcement bent into cylindrical form, show high ult strgths and are largely used; but they undergo considerable deformation before the strgth of the hoops is developed, the hoops acting much like a steel cylinder, filled with sand, such cylinders being unable to act until the sand is compressed.

12. Expts at Watertown (Tests of Metals, 1905) show that, when the col is subjected to loads of from 100 to 1000 lbs per sq inch, the unit lateral deformation is less than one-fourth the unit longitudinal deformation. Thus, if the col shortened 0.0004 of its length, its diam increased less than 0.0001 of its original dimension.

13. From tests at the Univ of Illinois (Am Soc Testg Matls, Procs, 1907, p 382) Prof. A. N. Talbot derives the following **formulas** for the ult strgths of hooped cylindrical conc cols, 1 : 2 : 4, wet mixture, av age, 60 days; cols 12 ins diam, 10 ft long. Covering, over the hoops, generally < ¼ inch. Hoops, 1 inch wide, gage Nos 8, 12, 16, electrically welded, spaced generally 2 ins c. to c. Let

p = ult strgth of col, lbs per sq inch;
c = ratio of hooping to conc core;
1600 = comp strgth of conc, lbs per sq inch.

Then,

For mild steel, $p = 1600 + 65{,}000\, c$; ....(9)
" higher " $p = 1600 + 100{,}000\, c$. ....(10)

14. Assuming that the ult unit stress, in longitudinal col reinfmt, is 25 times that in the conc, the hooping gave additional ult strgth from 2 to 4 times that given by longitudinal reinfmt.

15. M. Considère's expts (Génie Civil, Nov 1902), with spirally reinforced conc cols, indicate that the bars, forming the hoops, should have a diam of approximately 1/40 of the diam of the col; that the pitch of the spirals (distance between hoops) should be from ⅛ to ⅙ the diam of the col; and that the steel, in the hoops or spirals, adds, to the ult resistance of the col, 2 4 times as much as the same weight of metal used as longitudinal reinfg. He gives the formula

Ultimate total load on col = $1.5\, a_c\, c + s_e\, (a + 2\,4\, A)$ ....(11)

where

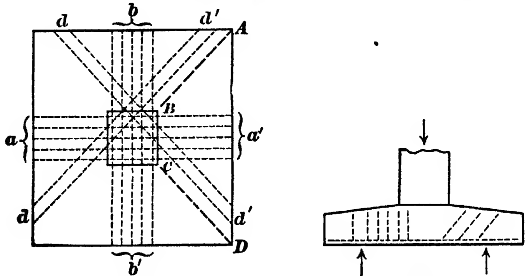
a_c = cross section area of col inside of spiral;
c = ult comp unit strgth of plain conc in short blocks;
s_e = elastic limit of steel;
a = cross section area of existing longitudinal reinfmt;
A = " " " " longitudinal reinfmt of equal wt with the spiral.

$1.5\, a_c$ is taken as representing the area of the *entire* conc cross sec.

Column Footings.

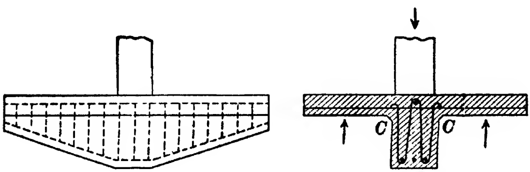
16. In a **column footing**, the stresses are analogous to those in a floor slab resting upon a col; but, owing to the relatively limited spread of the footing, the moments and shears are heavy, requiring considerable depth. The heaviest stresses are under the edges of the col. Hor rods, in the footing, are analogous to rods near the top of a beam over the support; *i. e.*, they take negative moms, and some of them should be bent upward, or provided with stirrups, just beyond the edges of the col.

17. Figs 1 and 2 (T & M, pp 261, 262). Fig 1: Two series of main reinfg rods, *a a′*, *b b′*, crossing at right angles under the col, with diag rods,



(*a*) (*b*)

Fig 1. Column Footing.



(*a*) (*b*)

Fig 2. Column Footing.

d′, *d d′*. Fig 2: Combined beam and slab. Side wings of slab tend to end upward, breaking away from the beam at *C* and *C*.

REINFORCED CONCRETE BEAMS.

1. Conc is ordinarily from eight to ten times as strong in comp as in tension. Hence, in an unreinforced conc beam of rectangular section, under bending stresses, failure occurs on the tension side.

2. The ease with which steel can be embedded in conc, the practical equality of the expansion coeffs of the two substances, the strong adhesion between conc and steel and the practicability of supplementing this adhesion by lugs or other lateral projections from the surface of the steel, facilitate combinations in which the principal service of the conc is to resist comp, while that of the steel is to resist tension.

3. The method of manufacture of conc is such that its behavior, in a given case, is less certain than that of steel.

Owing to this and to uncertainty, as to the degree of adhesion betw conc and steel, on which their united action depends, the theory of such beams is at once more complicated and less exact than that of steel beams of economical sections. In the design of reinfd conc beams, proper allowance must be made for this fact, and extreme refinement is out of place.

General Theory.

4. Simple reinfd conc beam, of rectangular section, Fig. 1.

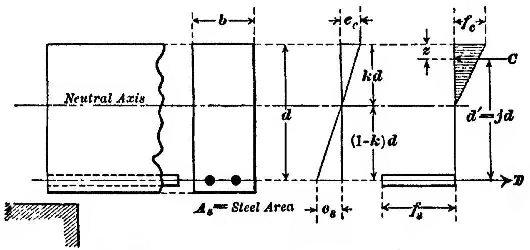


Fig 1. Reinforced Concrete Beam. Theory.

Fundamental assumptions.

1. Cross sections, plane before flexure, remain plane under flexure.
2 Initial stresses (from shrinkage, etc) are neglected.
3. No slipping occurs between conc and steel. Hence they deform equally.
4. The tensile resistance of the conc is neglected.
5. The elastic moduli, E_s and E_c, of steel and of conc respectively, and hence their ratio, $n = E_s/E_c$, remain constant.

5. Notation. Referring to Fig 1, let·

b = breadth of cross section of beam, perp to the paper;
d = dist from comp side of beam to cen of grav of steel;
kd = " " " " " " " neutral axis;
z = " " " " " " " resultant of comp forces;
$(1-k)\,d$ = " " cen of steel to neutral axis;
$d' = jd$ = " " " " " " resultant of comp forces
= leverage of resisting couple;
j = d'/d;

E_s = elastic modulus of steel; E_c = elastic modulus of concrete;
e_s = unit elongation of steel; e_c = unit shortening of concrete;*
f_s = unit tensile stress in steel †; f_c = unit comp stress in concrete;*†

* In the *outermost* fibers on the compression side of the beam.
† f_s and f_c are the *actual* unit stresses. See ¶ 13, p 1286.

a_s = cross-section area of steel; $a_c = b\,d$ = cross-section area of conc above cen of steel;
T = sum of tensile stresses in steel; C = sum of comp stresses in concrete;
$n = E_s/E_c$ = ratio of elastic moduli of steel and conc;
$p = a_s/a_c$ = ratio of steel area to that portion of conc area which is above cen of steel;*
$a_s = p\,a_c = p\,b\,d.$
M_s = resistg mom, based upon the max allowable value** of f_s;
M_c = resistg mom, based upon the max allowable value** of f_c;
M = actual resisting moment = c. b d².
For values of c (= M/bd^2) see Figs 2 and 3, pp. 1285, 1289.

Stresses, Moments, Design.

6. Figs 1 and 2§ and ¶¶ 7 to 20 illustrate the **relations existing between the important factors.** k, j, f_s, f_c, p, M_s, M_c and M; when neither f_s nor f_c exceeds the elastic limit. When they exceed that limit, see ¶¶ 21, 22, p 1290.

7. In equilibrium, the **bending moment** of the load (see p 474) is balanced by the equal **resisting moment** of the couple composed of the two equal hor forces, T and C, these forces being the resultants respectively of the tensile stresses in the steel and of the compressive stresses† in the conc.

8. The tensile stresses f_s, in the steel, are assumed to be uniformly distributed over its entire cross section, a_s; and their resultant, T, is therefore taken as acting at the grav cen of the steel area; but the **compressive stresses,** in the conc, in any cross sec, decrease uniformly‡ from a max, f_c, at the upper surf of the beam, to zero, at the neutral axis. Their resultant, C, is therefore applied at a point distant $kd/3$ below the top of the beam, kd being the distance from top of beam to neutral axis, and d the distance from top of beam to grav cen of steel.

9. Value of "j." The lever arm, d′, of the resisting couple is therefore

$$d' = jd = d - kd/3 = d\,(1 - k/3) \qquad (1)$$

and we have

$$j = d'/d = 1 - k/3 \qquad (2)$$

For approx values of j, see ¶ 12.

10. Value of "k." From assumption 1, ¶ 4 we have

$$e_c/e_s = k/(1-k) \qquad (3)$$

From assumption 5, we have

$$f_c = e_c E_c; \quad f_s = e_s E_s \qquad (4)$$

Hence

$$\frac{f_c}{f_s} = \frac{e_c\,E_c}{e_s\,E_s} = \frac{k}{1-k}\cdot\frac{E_c}{E_s} = \frac{k}{n\,(1-k)} \qquad (4a)$$

For equilibrium, $C = T$; but

$$C = f_c\,b\,k\,d/2 = e_c\,E_c\,b\,k\,d/2 \qquad (5)$$

and $$T = f_s\,a_s = f_s\,p\,b\,d = e_s\,E_s\,p\,b\,d \qquad (6)$$

Hence, $k = 2\,p\,\dfrac{e_s\,E_s}{e_c\,E_c} = 2\,p\,n\,\dfrac{1-k}{k}$;

or:

$$\mathbf{k} = \sqrt{(p\,n)^2 + 2\,p\,n} - p\,n \qquad (7)$$

---

* See ¶¶ 15, 16, p 1286. ** See ¶ 13, p 1286.
† Below the neutral axis, the conc is in *tension*, but its tensile stress is neglected. See assumption 4, ¶ 4, p 1283. ‡ See ¶¶ 21, 22.
§ Figs 2 and 3 are by Prof A. W. French, A S C E, Trans, Vol 56, '06, pp 362, etc.

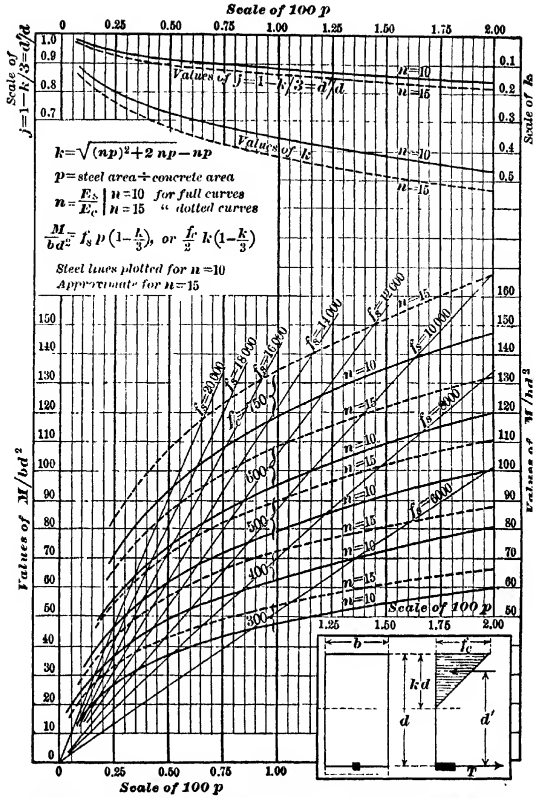


Fig 2. For Working Stresses. (For ultimate stresses, see Fig 3.)

k = $\sqrt{(p\,n)^2 + 2\,p\,n} - p\,n$, **j** = d' / d,

$\mathbf{f_s}$ = unit stress in steel, $\mathbf{f_c}$ = unit stress in conc at top of beam,

p = a_s/a_c = ratio of steel area to conc area,

$\mathbf{M_s}$, $\mathbf{M_c}$ = resistg mom, based upon allowed value of f_s, f_c, resp,

M = resistg mom, actual.

$$\frac{M}{b\,d^2} = f_s\,p\,(1 - \frac{k}{3}) \quad \text{or} \quad \frac{f_c}{2}\,k\,(1 - \frac{k}{3}).$$

$n = E_s/E_c$. **Solid curves** represent $n = 10$; **dotted curves,** $n = 1$
Steel lines plotted for $n = 10$; approx for $n = 15$.

a_s = cross-section area of steel; $a_c = b\,d$ = cross-section area of conc above cen of steel;

T = sum of tensile stresses in steel; C = sum of comp stresses in concrete;

$n = E_s/E_c$ = ratio of elastic moduli of steel and conc;

$p = a_s/a_c$ = ratio of steel area to that portion of conc area which is above cen of steel;*

$a_s = p\,a_c = p\,b\,d$.

M_s = resistg mom, based upon the max allowable value** of f_s;

M_c = resistg mom, based upon the max allowable value** of f_c;

M = actual resisting moment = c. b d².

For values of c (= M/bd^2) see Figs 2 and 3, pp. 1285, 1289.

Stresses, Moments, Design.

6. Figs 1 and 2§ and ¶¶ 7 to 20 illustrate the **relations existing between the important factors.** k, j, f_s, f_c, p, M_s, M_c and M; when neither f_s nor f_c exceeds the elastic limit. When they exceed that limit, see ¶¶ 21, 22, p 1290.

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Hence

$$\frac{f_c}{f_s} = \frac{e_c\,E_c}{e_s\,E_s} = \frac{k}{1-k} \cdot \frac{E_c}{E_s} = \frac{k}{n\,(1-k)} \qquad (4a)$$

For equilibrium, $C = T$; but

$$C = f_c\,b\,k\,d/2 = e_c\,E_c\,b\,k\,d/2 \qquad (5)$$

and $$T = f_s\,a_s = f_s\,p\,b\,d = e_s\,E_s\,p\,b\,d \qquad (6)$$

Hence, $k = 2\,p\,\dfrac{e_s\,E_s}{e_c\,E_c} = 2\,p\,n\,\dfrac{1-k}{k}$;

or:

$$\mathbf{k} = \sqrt{(p\,n)^2 + 2\,p\,n} - p\,n \qquad (7)$$

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* See ¶¶ 15, 16, p 1286. ** See ¶ 13, p 1286.

† Below the neutral axis. the conc is in *tension,* but its tensile stress is neglected. See assumption 4, ¶ 4, p 1283. ‡ See ¶¶ 21, 22.

§ Figs 2 and 3 are by Prof A. W. French, A S C E, Trans, Vol 56, '06, pp 362, etc.

The dotted and solid curved lines, for conc, represent $n = 15$ and $n = 10$, respectively. The nearly straight lines, for steel, are plotted for $n = 10$, but are sufficiently approx also for $n = 15$.

18. The upper portion of **Fig 2** gives **values of**

$$\mathbf{k} = \sqrt{2\,p\,n + (p\,n)^2} - p\,n,$$

(see ¶ 10) and of

$$\mathbf{j} = 1 - k/3 = d'/d,$$

corresponding to given values of p, for $n = 10$ and $n = 15$. Note that j varies but slightly with p.

Examples.

I. Investigation.

Required the resisting moments, M_s, M_c and M.

19 a. Given a rectangular reinfd conc beam: $b = 8''$; $d = 20''$; $a_c = bd = 8 \times 20 = 160$ sq ins; $n = E_s/E_c = 15$. Let $F_s = 16{,}000$, and $F_c = 500$ lbs per sq inch, be the *max allowable* values of the unit stresses, f_s and f_c, in steel and in conc respectively; and let P be the value of p based upon these max allowable stresses

Then $F_s/F_c = 32$; $\dfrac{F_s}{nF_c} + 1 = 3.133$; and, from Eq (11), ¶ 15, we have:

$$P = \frac{0.5}{32 \times 3.133} = 0.004987,$$

as given by the intersection, in Fig 2, of radial line, for $f_s = 16{,}000$, with dotted curve for $f_c = 500$.

19 b. (Case 1) Reinforced with two round rods, ¾″ diam;

$a_s = 2 \cdot \pi\, 0.375^2 = 0.884$ sq ins;

$p = a_s/a_c = 0.884/160 = 0.005525 > P$;

$pn = 15 \times 0.0055 = 0.0825$;

$k = \sqrt{(pn)^2 + 2\,pn} - pn$

$\quad = \sqrt{0.0825^2 + 0.1650} - 0.0825 = 0.3322$;

$d' = d\,j = d\,(1 - k/3) = 20\,(1 - 0.1107) = 20 \times 0.89 = 17.8$ **ins;**

$C = F_c\, b\, k\, d/2 = 500 \times 8 \times 0.3322 \times 10 = 13{,}288$ lbs;

$M_c = C\,d' = 13{,}288 \times 17.8 = 236{,}526$ inch-lbs;

$T = F_s\, a_s = 16{,}000 \times 0.884 = 14{,}144$ lbs;

$M_s = T\,d' = 14{,}144 \times 17.8 = 251{,}763$ inch-lbs;

$M = M_c = 236{,}526$ " " .

Notice that where, as in this case and in Case 2, $\mathbf{P} < \mathbf{p}$, the mom $\mathbf{M_c}$, based upon the max allowable stress, F_c in the *conc*, is the actual mom, M Where $\mathbf{P} > \mathbf{p}$, $\mathbf{M_s}$ is the actual mom.

19 c. By Fig 2. The intersection of the vert line, on $100\,p = 0.55$ with radial line for $f_s = 16{,}000$ lbs per sq inch, gives $M_s/bd^2 = 78.7$; and $M_s = 78.7\,b\,d^2 = 78.7 \times 8 \times 20^2 = 251{,}840$ inch-lbs; but the intersection of vert line on $100\,p = 0.55$, with dotted curve ($n = 15$) for $f_c = 500$ lb per sq inch, gives $M_c/bd^2 = 74$; and $M = M_c = 74\,bd^2 = 74 \times 8 \times 20$ = 236,800 inch-lbs

19 d. (Case 2) Reinforced with 3 round rods, 1" diam;

$a_s = 3\pi\, 0.5^2 = 2.356$ sq ins;

$p = a_s/a_c = 2.356/160 = 0.01473 > P$;

$pn = 15 \times 0.01473 = 0.2209$;

$k = \sqrt{(pn)^2 + 2\,pn} - pn$

$\quad = \sqrt{0.22^2 + 0.44} - 0.22 = 0.48$;

$d' = dj = d\,(1 - k/3) = 20\,(1 - 0.16) = 20 \times 0.84 = 16.8$;

$C = F_c\, b\, k\, d/2 = 500 \times 8 \times 0.48 \times 10 = 19{,}200$ lbs;

$M_c = C\, d' = 19{,}200 \times 16.8 = 322{,}560$ inch-lbs;

$T = F_s\, a_s = 16{,}000 \times 2.356 = 37{,}696$ lbs;

$M_s = Td' = 37{,}696 \times 16.8 = 633{,}293$ inch-lbs;

$M = M_c = 322{,}560$ " ".

19 e. By Fig 2. The intersection of the vert line on $100\,p = 1.473$, with radial line for $f_s = 16{,}000$ lbs per sq inch, would give (on a sufficiently accurate diagram) $M_s/b\,d^2 = 198$, and $M_s = 198\,b\,d^2 = 198 \times 8 \times 20^2 = 633{,}600$ inch-lbs; but the intersection of vert line on $100\,p = 1.473$, with dotted curve ($n = 15$) for $f_c = 500$ lbs per sq inch, gives $M_c/b\,d^2 = 101$; and $M = M_c = 101\,b\,d^2 = 101 \times 8 \times 20^2 = 323{,}200$ inch-lbs.

19 f. It will be noticed that, in these cases, **an increase of 166.5 %.** in the amt of steel, has **increased the resisting mom** (which still depends upon the conc) **by less than 38 %:** and the steel, in Case 2, is stressed to only about 8,000 lbs per sq inch or half the max allowable stress (intersection of vert for $100\,p = 1.473$, with dotted curve for $f_c = 500$, is nearly intersected by radial line for $f_s = 8{,}000$). See ¶ 13.

19 g. In both cases, (1) and (2), the intersection of radial line for $f_s = F_s = 16{,}000$, with dotted curve for $f_c = F_c = 500$, would give (on a sufficiently accurate diagram) $p = P = 0.004987$; $M/bd^2 = 71.5$, and $M = 71.5\,bd^2 = 228{,}800$ inch-lbs, the actual mom, for the given b and d, in the **ideal case** where f_s and f_c = respectively F_s and F_c = 16,000 and 500.

II. Design.

20 a. Conversely, given the bending moment, 236,500 inch-lbs; $F_s = 16{,}000$; $F_c = 500$ lbs per sq inch; whence $P = 0.004987$, as before. **Required b and d.**

Let K and J = the values of k and of j respectively, corresponding tc $f_s = F_s$ and $f_c = F_c$.

Here we have

$Pn = 15 \times 0.004987 = 0.075$;

$K = \sqrt{(Pn)^2 + 2\,Pn} - Pn$

$\quad = \sqrt{0.075^2 + 0.150} - 0.075 = 0.3193$;

$J = 1 - K/3 = 1 - 0.1064 = 0.8936$;

$$bd^2 = \frac{M}{F_s PJ} = \frac{2\,M}{F_c KJ} = \frac{2 \times 236{,}500}{500 \times 0.3193 \times 0.8936} = 3315.$$

20 b. An infinite number of section areas, bd, **giving the same resisting moment,** M, **may be found from** bd^2.

20 c. Thus, in the example of ¶ 20 a, with $bd^2 = 3315$, we may have

b	d^2	d	
6	552	23.5	
8	414	20.3	
10	331	18.2	etc, etc.

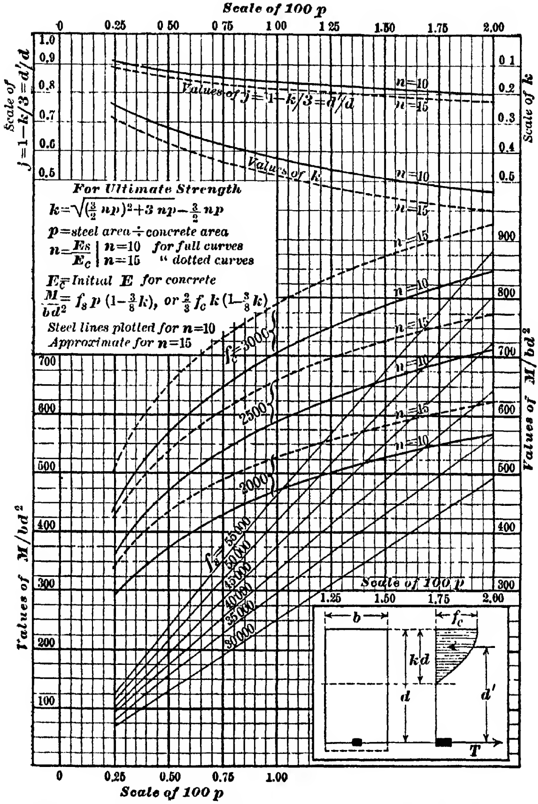

Fig 3. For Ultimate Stresses. (For allowable stresses, see Fig 2.)

$$\mathbf{k} = \sqrt{\left(\frac{3}{2}pn\right)^2 + 3pn} - \frac{3}{2}pn, \qquad \mathbf{j} = d'/d$$

$\mathbf{f_s}$ = unit stress in steel, $\mathbf{f_c}$ = unit stress in conc at top of beam

$\mathbf{p}$ = a_s/a_c = ratio of steel area to conc area,

$\mathbf{M_s}$, $\mathbf{M_c}$ = resistg mom, based upon max allowed value of f_s, f_c resp,

$\mathbf{M}$ = resistg mom, actual.

$$\frac{M}{bd^2} = f_s p\left(1 - \frac{3}{8}k\right) \quad \text{or} \quad \frac{2}{3} f_c k\left(1 - \frac{3}{8}k\right).$$

$n = E_s/E_c$. **Solid curves** represent n = 10; **dotted curves,** n = 15. **Steel lines** for n = 10; approx for n = 15. E_c = *initial* E for conc

20 d. It can be shown (T & M, pp 175–6) that, with given M, given unit stresses, and given unit prices, the **cost** of a reinfd conc beam, per unit of length, **varies** inversely as d, directly as $\sqrt{b}$, and directly as $\sqrt[3]{b/d}$. Hence, for a given bd, the deeper the beam, the less is the cost; but practical considerations (such as practical limits to reduction of b, requirements as to head room, etc) often limit the extent to which this economy can be carried in practice.

21. Within the limit of allowable working stresses. Fig 2, the stresses and deformations, in the several fibers, are taken (assumption 1, ¶ 4) as proportional to the dists of the fibers from the neutral axis, as represented by the shaded triangle in the small figure above the diagrams (said triangle representing approx the lower portion of the parabolic area shown in Fig 3); and we have, Eq (7), ¶ 10,

$$\mathbf{k} = \sqrt{(pn)^2 + 2\,pn} - pn.$$

22. For stresses exceeding the allowable workg stresses, up to the ult, Fig 3, assumption 1 is inadmissible, we must employ the entire parabolic area, its vertex corresponding with the ult comp strgth of the conc; and we have

$$\mathbf{k} = \sqrt{(3\,p\,n\,/\,2)^2 + 3\,p\,n} - 3\,p\,n\,/\,2 \qquad (14)$$

Fig 3 gives values of j, k and $M/b\,d^2$, for ult values of f_s and f_c.

23. Note that, for steel stresses, f_s, not exceeding the usual elastic limit, and with f_c ultimate $\not<$ 2000 lbs per sq inch, the ult resistg **mom increases directly with the amount of reinfmt** until this reaches 2 % or over. Thus, Fig 3, with f_s = 30,000 lbs per sq inch, f_c ult $\not<$ 2000, and p = 0 to 2 %, we have M/bd^2 = approx 25,000 p.

Tee Sections.

24. Tee sections. Fig 4. b = flange width; b' = stem width; t = flange thickness; d = depth from top of flange to cen of steel; $k\,d$ = depth of neut axis; $d' = j\,d$ = leverage of T and C

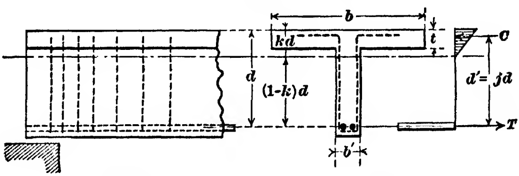

Fig 4. Reinforced Tee Section. Theory.

25. When the tops of rectangular beams are connected by slabs, the whole being placed at one time and properly bonded, all or a part of the slab may be considered as a **compression flange,** in some respects similar to those, composed of angles and plates, of steel plate girders.

26. The **width of slab,** b, Fig 4, which acts as flange, is sometimes taken as the distance between beams, but should not exceed ⅓ of the span of the beams. See Specifications, ¶¶ 168–170.

27. Exact analysis of such a section is hardly possible, but it is believed that the following **method** is reasonable and safe.

28. Determine the ratio, $p = a_s/a_c$, of steel area to conc area as tho the beam were rectangular, with depth = d, and width = the *flange* width, b. With this value of p, determine the position of the neutral axis. If this falls within the slab or just at its lower side, the resisting moment is found exactly as with any rectangular section. See Case 1, ¶ 19.

29. If the neutral axis falls below the bottom of the slab, the position of the neutral axis will not be exactly given by the equation for rectangular beams; but the difference will not be important.

30. The resisting moment is Cd' or Td', whichever is the less.

31. Examples.

(1) **Neutral axis within the slab.**

Let $b = 60$ ins; $b' = 8$ ins; $d = 20$ ins; $t = 5$ ins; max allowable unit stresses, $F_c = 500$, $F_s = 16{,}000$ lbs per sq in; $E_c = 3{,}000{,}000$; $E_s = 30{,}000{,}000$; $n = 10$. Let there be 3 round steel rods, diam = 1 inch.

Then

$$p = \frac{3 \times 0.785}{60 \times 20} = 0.002;$$

$$k = \sqrt{(pn)^2 + 2\,p\,n} - p\,n$$

$$= \sqrt{(10 \times 0.002)^2 + 2 \times 10 \times 0.002} - 10 \times 0.002 = 0.18;$$

$$k\,d = 0.18 \times 20 = 3.6 \text{ ins};$$

$$C = F_c\,b\,k\,d/2 = 500 \times 60 \times 0.18 \times 20/2 = 54{,}000 \text{ lbs};$$

$$T = 3 \times 0.785\,F_s = \text{say } 37{,}650 \text{ lbs}.$$

Using the smaller value (that for the steel) we have:

$$M = T\,d' = T\,(d - d\,k/3) = 37{,}650\,(20 - 3.6/3) = 707{,}000 \text{ inch-lbs.}$$

(2) **Neutral axis below the slab.**

Let $b = 60$ ins; $b' = 10$ ins; $d = 30$ ins; $t = 4$ ins; F_c, F_s, E_c, E_s and n as in Example (1); 6 round steel rods, diam = 1 inch. Then

$$p = \frac{6 \times 0.785}{60 \times 30} = 0.0026, \text{ and } k = 0.2;\ k\,d = 0.2 \times 30 = 6.$$

32. Since the comp unit stress, in the outer fibers of conc, is assumed to be $F_c = 500$ lbs per sq inch, the stress, at the lower side of the slab, is $500\,(k\,d - t)/k\,d = 500 \times 2/6 = 167$; and the **average stress, in the slab,** is $\frac{500 + 167}{2} = 333$ lbs per sq in.

33. The 2 inches of stem, which lie between the neutral axis and the lower side of the slab, exert some comp resistance, but this is neglected, with a small error on the safe side.

34. The **position of the center of gravity** of the compressive forces in the slab may be found as for a trapezoid; but it is usual, safe, and sufficiently approximate, to assume that it is at the cen of the slab, or, in this example, at a distance of $d - t/2 = 30 - 2 = 28$ ins above the cen of the steel. The mom of these forces is then $M_c = 333 \times 60 \times 4 \times 28 = 2{,}238{,}000$ inch-lbs; but the moment of the tensile resistance of the steel is only $M_s = 6 \times 0.785 \times 16{,}000 \times 28 = 2{,}110{,}000$ inch-lbs; and this mom, being the less of the two, is to be taken as the actual mom, M.

Shear.

35. Shear. In addition to the hor stresses, resisted by compression in the conc and by tension in the longitudinal steel reinfmt, the vertical shearing stresses require attention in relatively deep beams under heavy loads.

36. For the total shear, V, in any vert section, distant x from a support, we have:

$$V = R - W \quad\text{...}(15)$$

where R = upward reaction at the support;
W = the total of any loads in the distance, x.

37. The vert shear is sometimes provided for by using a large safety factor with the ult shearing strgth of conc, which is usually taken at from 500 to 800 lbs per sq inch, while the working shearing stress is often restricted to from 30 to 50 lbs per sq inch. But see Stirrups, ¶¶ 38, etc.

Shear Reinforcement. Stirrups.

38. Shear Reinforcement. Where the loading produces a shearing stress exceeding the limit assumed for plain conc, the beam is often reinfd by vert **stirrups,** which consist of rods, bent into the shape of a letter U, and passing under the hor bars and up to near the top of the beam; or, in the case of Tee beams (Fig 4), into the slab.

39. The distance between stirrups is sometimes made such that, within a hor length = d', there shall be an aggregate sectional area of vert steel bars sufficient to carry the vert shear by means of the permissible unit tension in the steel.

40. Example.

Consider the T beam of example (1) ¶ 31, Fig 4; $b' = 8$ ins; $b = 60$ ins; $d = 20$ ins; $k = 0.18$; $d' = 20 - k\,d/3 = 20 - 1.2 = 18.8$; safe mom of resistce, $M = 707{,}000$ inch-lbs. Let span $L = 20$ ft $= 240$ ins. Then, for a uniform load, we have $W = 8\,M/L = 8 \times 707{,}000/240 = 23{,}600$ lbs.

Shear at ends $= W/2 = 11{,}800$ lbs.

With safe unit shearing stress = 50 lbs per sq inch, we have safe shear resistance of plain conc in section $= 50\,b'\,d' = 50 \times 8 \times 18.8 = 7{,}500$ lbs

Under uniform load, this shear occurs at a dist, from the ends,

$$= \frac{(11{,}800 - 7{,}500)\,L}{2 \times 11{,}800} = 3.65 \text{ ft.}$$

From this point to the center of the span, the conc is able to care for the shear, and no stirrups are there reqd. But see ¶¶ 41, 45.

Between this point and each support, let the stirrups be of $^3/_8$ inch round steel; aggregate cross section area of the two limbs of each stirrup = 0.22 sq inch.

Allowing 16,000 lbs per sq in, one stirrup will sustain $16{,}000 \times 0.22 = 3{,}520$ lbs.

The total shear, 11,800 lbs, at the support, divided by 3520, gives 3.3 as the **number of stirrups** required, in 18.8 ins of length of beam; or the **spacing, next to the ends,** should be $\frac{18.8}{3.3} = 5.5$ ins.

Let the load, W, = 23,600 lbs, be uniformly distributed. Then, at a point 3 ft from the end, $V = \frac{10 - 3}{10} \times 11{,}800 = 8260$ lbs; $8260/3520 = 2.35$; and **stirrup spacing** $= 18.8/2.35 = 8$ ins.

41. The **spacing** may be made to **vary uniformly** betw these limits; and it would be well for the vert reinft to extend beyond the theoretical stopping point (3.6 ft from end, see ¶ 40), by one or two stirrups spaced a foot apart. See ¶ 45.

42. Let

A = aggregate vert cross sec area of hor rods, sq ins;
L = span, ft;
z = dist from end of beam to stirrup, ft;
S = aggregate cross section **area reqd in the 2 limbs of the stirrup,** sq ins.

Then, when the stirrups are 1 ft apart,

$$S = \frac{4\,A}{L}\left(1 - \frac{2\,z + 1}{L}\right) \quad \text{...(16)}$$

(J. W Schaub, E N, '03/Apr/16, p 348.)

43. In general, spacing betw stirrups $\ngtr d'$.

44. The conc, in each sec, has to act as a connecting medium between the hor and the vert reinft. It is also subjected to comp forces, in transferring the shear from one stirrup to the next. The action here is complex, and an **ample safety factor should be used.**

45. In order to provide against excessive loadings, which may come temporarily upon the beams during construction it is advisable to use stirrups, even where not actually required by the shearing stresses determined theoretically as above for the completed structure in use. The stirrups being light, the cost of using them is principally for labor; so that if *any* are reqd, it is well to be liberal with them. See ¶ 41.

Unit Shear.

46. Unit shear, v. In any hor section of a beam, Fig 5, under uniform or central loading, the hor tensile or comp stresses increase from the ends, where they are zero, toward the middle of the beam, where they are a max. Hence, of any two vert plane secs, 1 and 2, the section, 2, nearer the cen of the beam, will have the greater hor stresses, s.

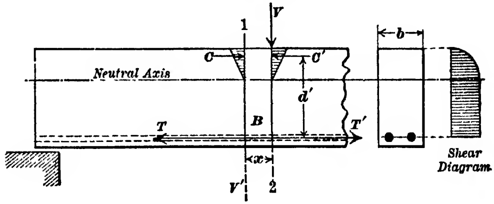

Fig 5. Unit Shear.

47. Consider the forces acting upon the rectangular body, B, between the two sections, 1 and 2.

48. At the left section, 1, the vert shear, V', coming from the left support, pushes B upward; and the tension, T, of the steel pulls B horizontally toward the left; while the total comp, C, acting at the cen of the comp forces, pushes B toward the right.

49. At the right sec, 2, the vert shear, V, pushes B downward; while T' and C' are in line with T and C respectively, but opposite to them. Note that $T' > T$, and $C' > C$. Let $T' - T = t$.

50. Let there be **no load** on B. Then $V' = V$.* Since the vert forces are distant by x, their moment $= Vx = V'x$.* The mom of $T' - T$ is $(T' - T)\, d' = td'$. Hence, **for equilibrium,**

$$Vx = t\,d'; \quad \text{or} \quad t = Vx/d' \quad \text{.......... (17)}$$

51. In a reinfd conc beam, Fig 5, we neglect the tensile strgth of the conc. Hence, the diff, $T' - T = t$, of tension, between secs 2 and 1, must be transmitted, from the steel to the neut axis, by a total shear, $= t$, uniform* in each hor sec; and, since the hor sec of the body, B, is $b\,x$, we have, for the **unit shear:**

$$v = t/b\,x = V\,x/d'\,b\,x = V/b\,d' = V'/b\,d'^{*} \quad \text{.......... (18)}$$

Diagonal Stresses.

52. As a matter of fact, the longitudinal tensile stresses and the vert and hor shearing stresses, combine to form, and are replaced by, **diagonal stresses;** and reinfmt, against shear, is more rationally designed by determining, as nearly as may be, the directions and intensities of these resultant diagonal stresses (See ¶ 53), and so placing the reinfmt as best to resist them.

53. From "Maximum Unit Stresses in Beams," p 494 e, we have, in homogeneous beams, for **the angle**, A, betw the neutral axis and the resultant normal (tensile and comp) or "principal" stresses, s_p, at any point:

$$\tan 2A = 2\,v/s; \quad \text{.......... (19)}$$

and, for the **max stress,**

$$s_p = s/2 + \sqrt{(s/2)^2 + v^2}; \quad \text{.......... (20)}$$

where v = the unit vert or hor shear, and s = the unit hor tensile or comp stress, at the given point.

---

* **If there is a load,** L, upon B (as, for instance, in the case of uniform loading) we have $V' > V$, and $V' - V = L$; and there are *two* couples of vert forces, with moms, respectively: Vx and $(V' - V)\,x'$, where x' = dist from sec 1 to gravity center of L. Here we have, for sec 1, $v' = V'/b\,d'$; and, for sec 2, $v = V/b\,d'$.

54. But, neglecting the tensile strgth of the conc, we have, in beams with tension reinfmt of straight bars, and for points betw the neutral axis and the steel, $s = 0$; whence:

$$\tan 2A = \infty; \quad 2A = 90°; \quad A = 45°;$$

$$s_p = \sqrt{v^2} = v = V/bd' \quad \ldots\ldots(21)$$

55. Hence, betw the neut axis and the steel, we should provide against **tensile unit stresses,** $s_p = V/b\,d'$, acting in parallel directions forming angles of 45° with the neut axis.

56. Other things being equal. this provision is preferably made by means of **rods, placed like the diag tension members** of a Pratt bridge truss, Figs 7*b*, 8*b*, 9*b*, p 693, and forming angles of 45° with the hor.

57. Very commonly, the tension rods, at each end, in a hor dist about equal the depth of the beam, are bent upward to form an angle of 45° or thereabouts with the axis of the beams.

Adhesion. See p 1279.

58. Unit of adhesion. Let

x = a given portion of the length of the beam;
t = $T' - T$ = the increase, in total tension, T, in the steel, in the lgth, x;
V = the total vert shear in the cross section;
d' = the dist betw T and the cen of comp of the conc;
U = t/x = the bond stress, per unit of x;
m = the number of rods;
a = the circumference of one rod
= the circumferential contact area of one rod, per unit of x;
u = $U/m\,a$ = the bond stress, per unit of a.

Then (see ¶ 50), $t\,d' = V\,x$; $t = V\,x/d'$; $U = t/x = V\,x/d'\,x = V/d'$; and

$$u = U/ma = V/d'\,m\,a \quad \ldots\ldots(22)$$

59. For given values of the bond stress, U, per unit of length, and of the bond stress, u, per unit of circumferential contact area, the product, $m\,a = U/u$ (= total circumferential area per unit of length) in a given case, is constant; but the cross sec area, weight and cost of the rods increase as the *square* of a. Hence, for a given total adhesion, numerous **small rods are more economical** than fewer larger rods; but there is, of course, for each case, a practical limit to this economy.

Continuous beams.

60. Floor systems are usually composed of slabs and beams continuous over supports; and, if the negative bending moments over the supports (producing *tension* at *top* of beam) are amply provided for, by reinfmt near the top, and if the supports are unyielding, or exactly equal in their yielding, advantage is usually taken of the reduction in the positive bending moms (at and near cen of span) due to continuity.

61. Where floor slabs are laid continuously over the supporting beams, it is usual to assume $WL/10 = wL^2/10$ as the max bending mom, where L = span; W = total load on span; $w = W/L$ = load per unit of L. Beams, continuous over the supports, may have a like value used in design, if the beams are amply reinfd at top and over the supports.

62. On the score of safety, it is frequently specified that beams, slabs, etc, shall be regarded as non-continuous over supports, this practice requiring us to provide, at cen of span, against greater (positive) bendg moms than if the beam were continuous over supports; but, on the other hand, few if any beams are wholly non-continuous; i e, even where the beam is supposed to be non-continuous, there are negative bendg moms over the supports, due to the width of the support and to the presence of loading upon the beam over the support. Such moms require reinfmt at top, over and near supports.

63. Hence, while it is advisable, in the case of non-continuous beams, to calculate the positive center bendg mom upon the assumption of absolute non-continuity, the condition of even non-continuous beams, over their supports, should be carefully investigated, and provision made for any negative moms there found.

64. Double Reinforcement. The necessity, under certain conditions, of reinfg against negative, as well as against positive moments (¶ 62) gives rise to cases (Fig 6) where reinfmt appears near both top and bottom of the section. For brevity, that on the side which, under positive mom, is under compression, will be called "compression reinft."

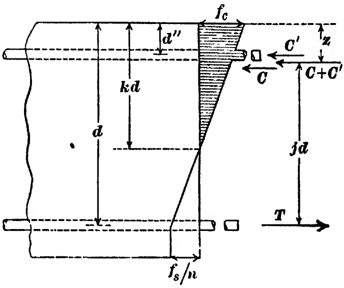

Fig 6. Double Reinforcement.

65. In addition to the symbols of ¶ 5, p 1283, let

a_s' = cross section area of comp reinft;

$p' = a_s'/a_c = a_s'/b\,d$ = steel ratio for comp reinft;

f_s' = unit stress in comp reinft;

C' = total stress " " " ;

d'' = dist from " " to nearest face of beam;

z = " " comp resultant, $C + C'$, to nearest face of beam.

66. Then, (neglecting the slight diminution of a_c by the presence of a_s') for **position of neutral axis:**

$$k = \sqrt{2\,n\,(p + p'\,d''/d) + n^2\,(p + p')^2} - n\,(p + p'); \qquad (24)$$

for position of compression resultant:

$$z = \frac{k^3\,d/3 + 2\,p'\,n\,d''\,(k - d''/d)}{k^2 + 2\,p'\,n\,(k - d''/d)}; \qquad (25)$$

for arm of resisting couple:

$$j\,d = d - z; \qquad (26)$$

for fiber stresses:

$$f_c = \frac{6\,M\,k/b\,d^2}{3\,k^2 - k^3 + 6\,p'\,n\,(k - d''/d)\,(1 - d''/d)} \qquad (27)$$

$$f_s = M/p\,j\,b\,d^2 = n\,f_c\,(1 - k)/k \qquad (28)$$

$$f_s' = n\,f_c\,(k - d''/d)/k \qquad (29)$$

METHODS OF REINFORCEMENT.

1. The **commonly accepted theory** of reinfd conc beams requires longitudinal tension reinfmt near the bottom* of the beam, and diag tension reinfmt at 45°, not only betw the hor reinfmt and the neutral axis, but extending upward into the region of compression, in order to take advantage of the superior adhesion due to the compression there. It also requires, usually, tension reinfmt near the top,* at points over or near the supports.

See ¶¶ 60, etc, p 1294.

*The terms "bottom" and "top" are here used as referring to a beam supported at the ends, and loaded on top, where the major portion of the bottom is in tension. In a cantilever, of course, this is reversed.

54. But, neglecting the tensile strgth of the conc, we have, in beams with tension reinfmt of straight bars, and for points betw the neutral axis and the steel, $s = 0$; whence:

$$\tan 2A = \infty; \quad 2A = 90°; \quad A = 45°;$$

$$s_p = \sqrt{v^2} = v = V/b\,d' \quad\text{.......... (21)}$$

55. Hence, betw the neut axis and the steel, we should provide against **tensile unit stresses,** $s_p = V/b\,d'$, acting in parallel directions forming angles of 45° with the neut axis.

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V = the total vert shear in the cross section;

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U = t/x = the bond stress, per unit of x;

m = the number of rods;

a = the circumference of one rod

= the circumferential contact area of one rod, per unit of x;

u = $U/m\,a$ = the bond stress, per unit of a.

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[illegible]

elastic limit. by permitting the use [illegible] higher unit stresses, renders elongation [illegible] panying cracking of the conc, and lateral c[illegible] of the steel, which **endangers the adhesion.** On this account, it is sometimes specified that, where the elastic limit exceeds a certain min (say 40,000 lbs/sq inch) deformed bars, ¶¶ 15 etc, shall be used. At 30,000 lbs/sq inch, steel stretches about 0.10 per cent; at 50,000 lbs/sq inch, about 0.17 per cent.

Cold working raises the ultimate strength and the elastic limit, but slightly lowers the elastic modulus; see Fig 1, representing tests at Watertown Arsenal (Tests of Metals, 1904, p 397) on plain and cold-twisted steel bars, ¾ inch square. Gaged lengths, 10 inches. The twisted bar had 1 twist in 8 inches. Similar results were shown in tests made at Watertown Arsenal, July 12, 1902, and published by Ransome Concrete Co, See ¶ 21.

Square bars, of interior steel, are twisted hot, and are more brittle.

10. Plain round steel bars are very generally used for reinforcement in America, and still more generally in Europe. **Square bars** also are used, but are less conveniently handled. **Flat bars** have been found deficient in adhesion.

11. In order **to increase the resistance of plain bars** to being pulled thru the conc, they are frequently bent up at right angles (or bent over at 180° so as to form a hook) at their ends.

12. "Anchorage, furnisht by short bends at a right angle, is less effective than hooks consisting of turns at 180°." **J. C.**

13. For the same purpose, (¶ 11), the bars may be threaded at their ends, and provided with steel **anchor plates,** secured by nuts. Such plates should be large enough and thick enough to withstand pulls due to the full tensile strength of the rods. In designing such plates, Prof. L. J. Johnson assumes a crushing strgth, in the conc, of 900 lbs/sq inch, and a fiber stress in the anchor plate, of 25,000 lbs/sq inch. Several rods, side by side, pass thru a common large plate at each end, which serves, also, to hold the rods in their relative positions while the conc is being placed. Nuts, on the inside, holding the anchor plate to a firm bearing against the outside nuts, are an important provision. Room, for such plates, is usually found in a wall or column, or over a knee-bracket, etc. Otherwise, in order to give room for the anchor plate, the beam may be deepened locally, or the rods bent up, near their ends. When bent up, the rod exerts an upwd pres upon the conc, near the bend. This increases the friction, in the bent portion, and thus reduces the pull transmitted to the anchor plate.

14. "Adequate bond strgth, thruout the length of a bar, is preferable to end anchorage." **J. C.**

15. Also for the purpose of increasing adhesion (or rather to substitute, for it, a "mechanical bond") **"deformed bars,"** of various shapes are used.

16. The **principal claim,** in favor of deformed bars, is that the "mechanical bond," which they offer, is the sole reliance of the reinfmt, after its adhesion proper has been destroyed, as by a stress exceeding the adhesion, by infiltration of water, by concussion either during or after construction, or by constant and rapid alternations or reversals of loading, in service. Vert rods especially, during construction, are liable to accidental blows upon their projecting upper ends; and such blows may affect the adhesion of the portions already imbedded in conc.

17. On the other hand, it is pointed out that innumerable structures, with plain bars, have satisfactorily withstood, for years, service involving such vibration; and it is claimed that whatever advantage arises from deformation is more than offset by the slight increase of cost. Plain bars are of course free from patent claims, and they are at all times readily obtainable in the general metal market.

18. The projections, on the surfs of some deformed bars, may injure the conc covering unless this is of considerable thickness.

19. In studying comparative tests of plain and deformed bars, attention should be given to the richness of the conc mixture. Unless this is sufficiently rich to insure the **complete covering of each bar** with cem over its entire surf, the adhesion proper will not be fairly developed, and the pulling test will exhibit chiefly the diff in "mechanical bond," in which, of course, the deformed bars are superior.

20. "Deformed bars offer a suitable means for supplying high bond resistance." **J. C.**

The following deformed rods, Figs 2, are in more or less general use:

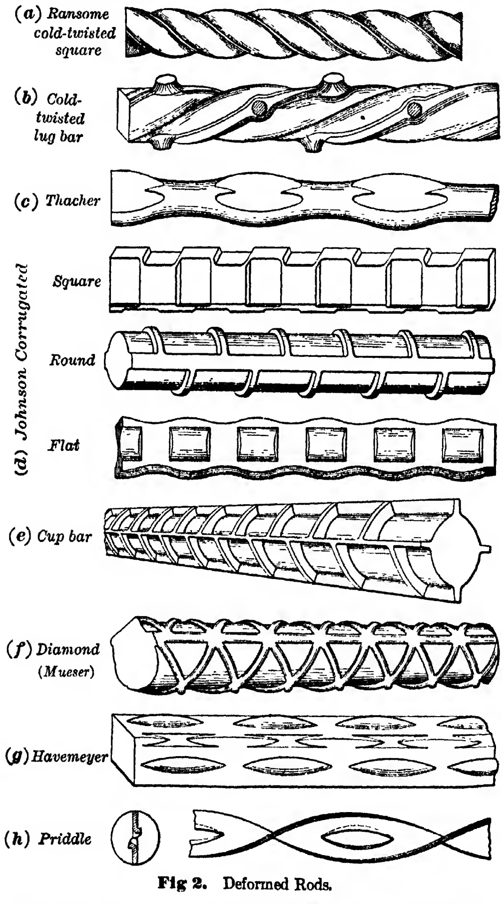


Fig 2. Deformed Rods.

21. Ransome. (*a*) Square steel rods, twisted cold. Twisted either at mill, or (conveniently and inexpensively) on the work.

22. Cold-twisted lug-bar. (*b*) Square bar, with angles rounded, to prevent the starting of cracks in the conc, twisted cold. The lugs are designed to resist any tendency of the bar to untwist under tension. For effect of cold working, see ¶ 9, p 1297.

23. Thacher. (*c*) Round rods, deformed by flattening at short intervals. Cross sec area practically constant. Changes in shape made by means of gradual curves.

24. Corrugated bars; (*d*) ordinarily of steel with yield point 50,000 lbs/sq inch or over. Square, round and flat.

25. Cup bars, (*e*).

26. Diamond bar. (*f*) Rolled round, with two spiral projecting ribs of equal pitch and in opp directions (dividing the surface into four rows of diamond-shaped recesses) and two opp longitudinal ribs, at the points where the upper and lower rolls meet in manufacture. Cross-section area and weight = those of plain square bars of like denomination. Claims: uniform cross section area, uniform elongation, uniform distribution of bond; projecting ribs aid in resisting tension; edges rounded; no tendency to untwist under tension.

27. Havemeyer bar. (*g*) Square, with rounded corners and projections.

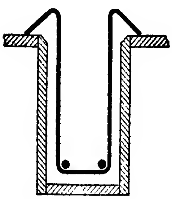
28. Priddle Internal-bond Bar. (*h*) Flat bar, perforated and twisted, and the slit flanged, as shown. Small sizes worked cold; larger sizes, hot, A web may be formed by passing smaller bars, of same or other pattern, thru the slits.

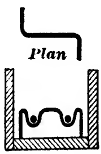
29. The monolith bar consists of a hor tension member with separate diag links. In section, the hor member resembles a heavy rail with two heads instead of head and flange. Each link is a bar of round steel, bent over at top and thus forming two parallel diag legs, which, at bottom, are bent hor, and their hor portions, one on each side of the hor member, are gripped between its heads, which are swedged in, at those points, for the purpose.

Supports.

30. It is of course of the first importance that the longitudinal **reinforcing bars** be placed and kept **in their proper positions.** If, as finally located, they are too high, their resisting leverage, d', and the resistg moment of the beam, are diminished. If they are too low, they have an insufficient protective depth of conc below them. Various devices are in use for holding the bars in position.

31. Stirrups, Fig 3, act as hangers for the main rods.





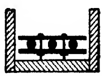


Fig 6.

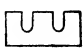


Fig 3. Fig 4. Fig 5.

Supports for Reinforcing Bars.

32. Light rods are sometimes held by **wire supports,** Fig 4, or by **conc blocks,** about 1.2 or 2 ins thick, Fig 5.

33. Heavier rods may be supported by **clamps,** Fig 6, made of pieces of ¾″ or 1″ channel iron, held together by round-headed stove bolts, ¼″ or ⅜″ diam, placed in the forms, and 6 or 8 ft apart.

"Web" Reinforcement.

34. Web reinforcement is used in broad and shallow slabs, in thin walls, in sewers and conduits, in columns, etc.

35. The simplest form consists of **rods, placed at right angles,** and wired or welded together at their intersections. The heavier or main rods are of course so placed as to take the greater stresses. The transverse rods hold the main rods in position during construction, and afterward distribute their tension across the intervening conc. They thus offer a mechanical bond. The mesh must be large enough to pass the particles of the agg used in making the conc.

36. Jean Monier, of Paris, used such webbing in the reinforcement of arches.

37. Expanded metal. Fig 7. Sheet steel, slitted and opened out into diamond-shaped panels. In sheets, 12 to 72 ins wide, 8 to 12 ft long; mesh from ½" to 6"; metal, Stubs gage, No. 18 to No. 4.

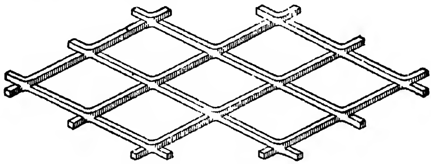


Fig 7. Expanded Metal.

38. When slab reinforcement is furnisht in short sheets, these must **overlap** sufficiently to transmit the tension from one sheet to the next. The lapping uses about 10 % of the area of the metal.

39. Clinton wire lath, in rolls of 100 or 200 ft or more, of drawn steel wires, crossing at right angles, 2½ inch mesh, electrically welded and reinfd by longitudinal reinfg warp strands, 6 ins apart, and made up each of two wires cross-looped and twisted over each crossing strand; and, when desired, by transverse V-shaped stiffeners of No. 24 gage steel, fastened to the wires at intervals of about 8 ins. Furnisht plain, japanned or galvd, in 36 inch width.

40. Clinton welded wire; No 3 to No. 10 drawn steel wire, plain or galvd; mesh, 3 × 8, 2 × 12, 3 × 12, 4 × 12 ins.

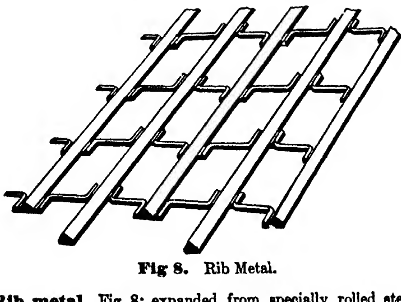


Fig 8. Rib Metal.

41. Rib metal, Fig 8; expanded from specially rolled steel plates, ribbed longitudinally. Mesh varying, by single inches, from 2 to 8 ins. Sheets up to 16 ft long

42. Rib lath, Fig 9.

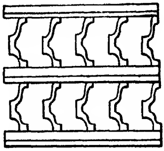
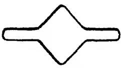


Fig 9. Rib-Lath.

Trussed Reinforcement.

43. In general, trussed reinforcement is slightly more expensive than plain bar reinfmt; and, if shipped in rigid built-up units, it incurs higher freight charges and is more liable to damage en route; but it has the great advantage of holding the bars in position while the conc is being placed, and of obviating the omission or misplacement of stirrups, etc, either by accident or by design. The trusses may be made up of either plain or deformed bars. They should be provided with means for connecting them, over the supports.

44. In the **Kahn trussed bar,** Fig 10, the projecting side fins are slit away, in places, from the central portion, and bent up, as shown. The same bar, inverted, is used over the supports.



Cross sec at cen.

Fig 10. Kahn Bar.

45. Fig 11 shows the **collapsible Economy Unit frame.**

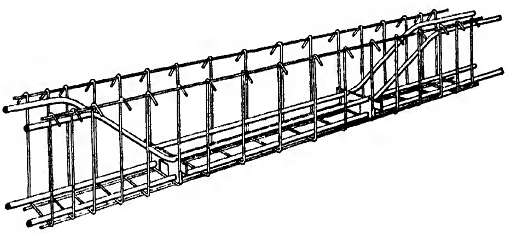


Fig 11. "Economy" Collapsible Truss.

Reinforcement with Structural Shapes.

46. The Melan system, invented by Joseph Melan, of Austria-Hungary, in 1892, and patented in the United States in 1893, comprises a concrete arch in which iron or steel beams are embedded. For small spans, the beams are usually rolled I-beams; while, for spans of considerable length, they usually consist of four angles latticed.

47. Where a structural shape, of considerable size, is imbedded in conc, to form a beam, so that **the steel predominates** and furnishes most of the strgth reqd, **the conc acts chiefly as a protecting cover** for the steel; and the case is hardly one of reinfmt properly so called.

48. It is **difficult to secure perfect filling,** with conc, of the spaces under the flanges of rolled or built-up shapes. In such cases, each day's work should be stopped either well above or well below the flange. Otherwise, shrinkage, under the flanges, will aggravate the difficulty.

Column Reinforcement.

49. Columns are reinfd by means of **vertical rods,** placed near the circumf and usually wired together at intervals, **or by circumferential** (hooped or spiral) **wrapping,** or both.

See Reinfd conc cols, pp 1280, etc.

50. In tall buildings, the column rods are often **faced at the ends** to give good bearing, and connected by loose sleeves, which keep the ends in proper contact; and an iron or steel plate is placed under the feet of the rods in the footing, to distribute the load more evenly over the conc of the foundation.

51. In Mr. C. A. P. Turner's **mushroom system** of columns and floors, the cols are splayed, at top, to increase their bearing area, and the floor reinfmt consists essentially of straight members (hor or nearly so) radiating from the cols, and joined, at intervals, by circular or polygonal members, which cross the radial members generally at right angles. Beams and ribs are dispensed with, and the floor is of uniform thickness. See E N, '09, Feb 18, p 178.

EXPERIMENT AND PRACTICE.

Directory of Selected Results, pp 1308, etc.

Words in **bold-face** type, preceding a semicolon, refer to one of two related matters; words in plain type, following the semicolon, to the other one. Numerals and letters refer to the records of experiment, etc.

Example. Under **SAND** (below), "**Sand, character;** density of mortar, 8*c*, *e*, 9*d*, 86*c*" refers to Experiments 8*c*, etc, which give information respecting the effect of (1) character of sand upon (2) density of mortar. Conversely, on p 1304, we find "**Mortar, density of —;** character of sand, 8*c*, *e*, 9*d*, 86*c*."

CEMENT.

SAND.

ACCIDENTAL INGREDIENTS.

Directory to Experiments, pp 1308–1351.

MIXING WATER.

MORTAR.

PROPORTIONS.

AGGREGATE.

Directory to Experiments, pp 1308–1351.

AGGREGATE.—Continued.

CONCRETE.

MIXING.

FORMS, PLACING, COMPACTING.

SETTING.

AGE.

LAITANCE.

REGRINDING.

FINISH.

PROPERTIES, BEHAVIOR.

Strength.

Directory to Experiments, pp 1308-1351.

CONCRETE.—Continued.

Directory to Experiments, pp 1308–1351.

REINFORCEMENT, METALS, ADHESION, CORROSION.

Experiment and Practice.

Selected Results.

See Directory, pp 1303, etc.

Order of arrangement.

The features entering into the manufacture and behavior of concrete are so numerous, and in the reports of experiments, etc, they are unavoidably so interlaced, that it has been found impracticable to group the several items in the body of the text in satisfactory order below.

Most of our "selected results" are therefore here placed approx in the order of their dates of publication, and furnisht with a directory, pp 1303, etc, by means of which any particular subject may be promptly found. The directory is arranged rationally (i e, not alphabetically), and, as far as practicable, in the order followed in the text (pp 1222–1250, 1252–1302), referring to cement, sand, mortar, aggregate and concrete, plain and reinforced. The items, covered by any one publisht statement, are given a common number, and, under this common number, the several paragraphs are indicated by letters. These letters usually distinguish also betw the several features covered by the common number.

Thus, under Expt 8, we have a number of conclusions reached by R. Feret: under 8 *a*, conclusions respecting strength of mortar as affected by proportion of cement and fineness of sand, under 8 *c*, conclusions respecting porosity and permeability as affected by fineness of sand and richness of mortar, etc, etc.

In the directory, semicolons, in general, are used to distinguish between two different but related ideas. Thus, "**Strength:** fineness of sand" and "**Sand, fineness of** —: strength," refer to items giving information respecting the effect of fineness of sand upon strength of mortar or conc.

— 1 —

1. Bouniceau, Annales des Ponts et Chaussées, 1863, p 181.

1 a. Expansion Coefficient.

Bar iron....	0.000 0123 5 per deg C;	0.000 006 86 per deg F
Port cem conc.	0.000 0137 0 " " "	0.000 007 60 " " "

— 2 —

2. John C. Trautwine, Civil Engr's Pocket Book, 1872.

2 a. Sand, density; moisture, compacting.

Specimens. Ordinary pure sand from the seashore, both dry and moist (not wet), see table. Sand B was of much finer grain than A. C consisted of the finest grains sifted from B.

Treatment. The dry sands were compacted by thoro shaking and jar ring; the moist sands by ramming in thin layers.

Results.

	Sand A (coarse)				Sand B (finer)				Sand C (finest)		
	Dry			Moist	Dry			Moist	Dry		
	lbs per cu ft	Solid %	Void %	lbs per cu ft	lbs per cu ft	Solid %	Void %	lbs per cu ft	lbs. per cu ft	Solid %	Void %
Loose.....	97	59	41	86	88	53.4	46.6	69	82	50	50
Compacted	112	68	32	107.5	101.6	61.6	38.4	103.5	98.5	60	40
Increase...	15	9	—9	21.5	13.6	8.2	—8.2	34.5	16.5	10	—10
Per cent...	15.5	15.2	22	25	15.5	15.3	17.6	50	20.1	20	20

2 b. Corrosion. 10 years' trial. Dampness absolutely excluded after setting. **Cements protect** iron, lead, zinc, copper, brass. **Plaster of Paris** protects all these except ungalvanized iron.

For abbreviations, symbols and references, see p 1251.

— 3 —

3. John Watt Sandeman. Inst C E, Vol. liv, 1878, p 260.

3 a. Aggregates; density.

Results

No.		lbs per cub ft	Percentage of voids
1.	Broken limestone, mostly 3 inch	. 95	50.9
2.	Screened gravel, from small pebbles to 2.5 inch	111½	33.6
3.	Equal parts of Nos. 1 and 2, well mixed .. .	113½	34.0
4.	Broken sandstone, 4 to 8 inch	74	50.0
5.	" " from sand to 4 inch	92	34.0
6	Equal parts of Nos. 4 and 5, mixed	. 91¼	36.0

— 4 —

4. Eliot C. Clarke, A S C E Trans, Apr, '85, Vol 14, p 163. Expts for Boston Main Drainage Works.

Results.

4 a. Clay. The addition of not exceeding one part of clay to 2 of cem, gave a "much more **dense, plastic** and **water-tight** paste, convenient for plastering surfaces or stopping leaky joints." and, in general, had no markt effect upon the **strength** of Portland and natural cem. Mortars, made with sand containing 10% of loam, were of normal strgth at 6 and 12 mos, tho of only about half normal strgth up to 1 mo. Clay, in cem, is "an almost impalpable powder, with particles fine enough to fill the spaces between the particles of cem."

4 b. A year's saturation in **fresh or salt water,** and in contact with **oak, hard pine, white pine, spruce or ash,** did not affect the mortars.

4 c. Salt, either in the water used for mixing, or in that in which the cem is laid, retards **setting** somewhat, but has no important effect upon the **strength.**

4 d. Consistency. Excess of water retards **setting. Nat** cems need more water than **Port; fine-ground** more than coarse; **quick-setting** more than slow.

4 e. The finer the sand, the less the **strength.**

4 f. With sand, **fine-ground** cems are strongest; **coarse-ground** are strongest neat, especially with Portlands.

4 g. Port resisted **abrasion** best when mixt with 2 parts sand; nat with 1 part. Resistance diminished rapidly with slight variations from these proportions.

4 h. In setting, mortars expand ≯ 1 part in 1000.

— 5 —

5. Allen Hazen, Mass. State Board of Health, Report '92, p 550. Sharp-grained sand.

5 a. Uniformity coefficient (u. c.) p 947:	<2	<3	6 to 8
Voids, per cent, approx,.	45	40	30

— 6 —

6. E. Carey, Inst C E Procs, Vol 107, '92, p 55.

6 a. Sulfuric acid; strength. Neat cem, gaged with water containing 5 % acid, had, at 7 days, only 27 % of the **strength** of neat cem gaged with water free from acid.

— 7 —

7. Dr. Wilhelm Michaelis, Inst C E Procs, Vol 107, '92, pp 372, 375.

7 a. Disintegration of porous cem **in sea water** shown to be **due to** the action of sulfuric and hydrochloric (muriatic) acids, contained in the magnesium sulfates and chlorides of sea water. These acids leave the weaker base, magnesium (which is deposited as a hydrate), and combine with the lime of the cem, expanding and disintegrating the conc.

For Directory to Experiments, see pp 1303–7.

— 8 —

8. R. Feret. Annales des Ponts et Chaussées, 7e série, Tome IV, '92.

8 a. Results. Strength of mortar increases with proportion of cem, and, in general (especially at the beginning of hardening) with size of sand.

8 b. Mortars vary widely as to **porosity.** Compare 9 *d*, 9 *e*.

8 c. Porosity increases
with fineness of sand,
with richness of mortar

8 d. Permeability increases
with coarseness of sand,
with richness of mortar.

8 e. Mortars made with a mixture of **coarse and fine sands** are less porous and less permeable than others.

8 f. The **permeability** of mortars subjected to continuous percolation of fresh or sea water, **diminishes rapidly;** but, in certain cases, the mortar disintegrates or cracks.

8 g. To avoid disintegration in **sea water,** use coarse sand and plenty of cem. Mix wet.

8 h. Density of sand; moisture and tamping. Fig. 1.

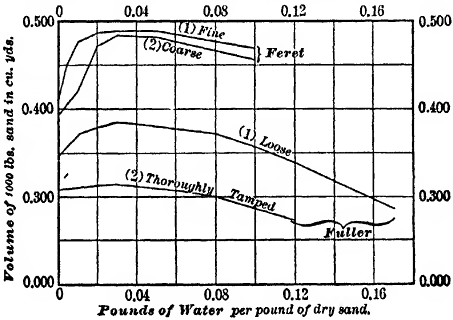


Fig 1. Moisture and Tamping.

M. Feret used (1) a very fine dune sand and (2) a coarser sea sand. Wm. B. Fuller, E N, '02, Jul 31, p 81, used a bank sand, (1) loose and (2) tamped. From these results, it appears that the addition of water affects the vol of the sand* in two opposite ways; (1) by insinuating itself betw the sand particles, thus *increasing* the vol for a given wt; (2) by decreasing the friction between the grains, allowing them more readily to take up the positions of closest contact, and thus *diminishing* the vol. When only small vols of water have been added, the first of these effects seems to prevail, the bulk increasing until the vol of water reaches from 2 to 5 % of the vol of dry sand.* With more water, the lubricating effect prevails, the vol diminishing.

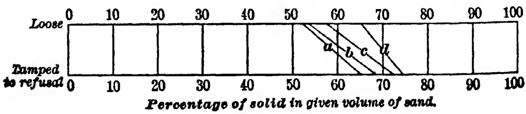


Fig 2. Compacting.

8 i. Shape of grain and tamping. Fig. 2.

* See foot-note *, p 1238.

For abbreviations, symbols and references, see p 1251.

Specimens. Four materials, as follows:
a. Granitic sand, rounded grains; *c.* Broken shells, flat grains;
b. Ground quartzite, angular grains; *d.* Residue from *b*, lamellar grains.
Each of the four materials screened to the same granulometric composition, viz: *c*, 0.5; *m*, 0.3; *f*, 0.2.† (See p 1238.)

Results. See Fig. 2.

8 j. Effect of size of grain. Fig. 3.

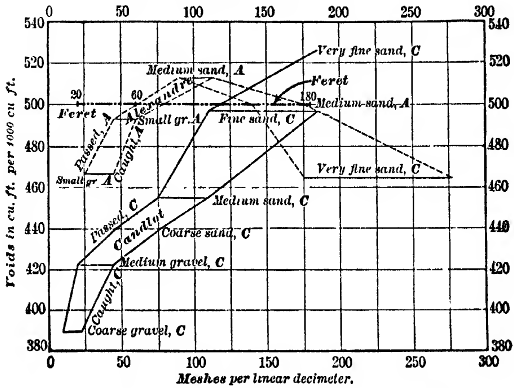


Fig 3. Size and Density. **A** = Alexandre; **C** = Candlot.

Theoretically, the density, in a sand* or gravel,* composed of grains of *uniform* size, should be independent of the *absolute* size (¶ 30, p 1241); but experimenters have obtained contradictory results, showing unimportant variations of density with size. Thus (T & T, p 170), if sand (except very fine sizes, such as pass a sieve with 74 meshes per linear inch) and broken stone, with irregular particles of approx uniform *shape*, be separated into portions containing particles of uniform *size*, these several portions will show approx equal percentages of voids. This agrees with R. Feret's experiments (T & T, pp 171 and 142), Fig 3, according to which each of the 3 sizes (coarse, medium and fine†) contained 50 % voids. M. Feret's results are represented by the hor line in Fig 3. On the other hand (Fig 3) M. Candlot (Feret, Ann des Ponts et Chaussées, 1892, 2e sem) found the voids increasing continuously, and M. Alexandre (*ibid*) found them first increasing and afterward decreasing as the size grew smaller.

8 k. Effect of sizes of grains, and shaking or tamping. Loose sand* shows densities ranging from 0.525 to 0.610, the max density occurring when 60 % of coarse sand† is mixed with 40 % of fine sand, without medium sand. In **sand shaken to refusal**, the densities range from 0.600 to 0.793, the max density occurring with a mixture of 55 % coarse with 45 % fine; no medium.

* See foot-note *, p 1238.
† Classification of sizes.

	Passed	Retained on	
c. Coarse	20	60	meshes per lineal decimeter.
m. Medium......	60	180	" " " "
f. Fine..........	180	...	" " " "

For Directory to Experiments, see pp 1303-7.

8 l. Densities of loose unscreened sands and gravels; shapes and sizes of grains; moisture.

	Wt of pebbles con-tained, %	Mechanical Analysis of sand proper			Dry sand Kg per cu M.	Moist sand	
		Coarse	Med.	Fine		Mois-ture %	Kg per cu M
Granitic rounded grains ..	1.0	0.136	0.723	0.141	1,586	0.8	1,495
Schistose	25.4	0.359	0.293	0.348	1,753	1.2	1,650
"	6.6	0.259	0.412	0.329	1,600	1.8	1,332

— 9 —

9. **Luigi Luiggi** and **Valentino Cardi,** "Esperimenti sulle Calci, etc;" Genio Civile, Rome, '93.

Porosity, permeability, etc. **Safe loads.** Twelve years' expts in connection with harbor works at Genoa, Italy.

Results.

9 a. In mortar, **voids are due** partly to air adhering to particles of sand and agg, partly to evaporation of the water used in mixing.

9 b. In mortar, **volume of voids** may vary from 12 to 46 % of vol of mortar.

9 c. Minimum voids (5 %) in conc formed with 700 lbs Port cem, 1 cu yd mixt sand, 1 ¼ cu yds small gravel.

9 d. Porosity increases
with fineness of sand;
" richness of mortar;
greatest with neat cem.
Compare 8 *c*, 8 *d*.

9 e. Permeability increases
with coarseness of sand;
" poorness of mortar;
least with neat cem.

9 f. Concrete of 1150 lbs Port cem, 1 cu yd mixt sand, 1 ¼ cu yds small gravel, carefully mixt with just enough water (about ⅛ cu yd) to work it up, was **impermeable** under 40 ft head (17.3 lbs/□").

9 g. Concrete of 700 lbs Port cem, 1 cu yd mixt sand, 1 ¼ cu yds small gravel, made into a hollow cyl with shell 2½" thick, was **impermeable** under 13 ft head (5.64 lbs/□") and barely permeable under 27 ft (11.7 lbs/□"). Similar cyls, of same mixture, **without the gravel,** leaked somewhat under 13 ft and easily under 27 ft.

9 h. Safe load in compression. In the floors of the graving docks, 1 : 2 : 3 conc of Port cem, sand and small gravel, safely carries 107 lbs/□" ; safety factor, 15.

— 10 —

10. **Dr. Keller,** Thonindustriezeitung '94, No. 24.

10 a. Expansion Coefficient. Temps from — 16° to + 72° C = + 3° to + 162° F. Gravel (20 mm) and sand, in equal parts.

	Mixture of sand and gravel, parts			
Proportions (1 part cem) to	0	2	4	8
Coefficient, per degree C...	0.000 012 6	0.000 010 1	0.000 010 4	0.000 009 5
" " " F...	0.000 007 0	0.000 005 6	0.000 005 8	0.000 005 3

— 11 —

11. **Geo. W. Rafter,** 2d Report on Genesee R Storage Project, '94. See E R, '06, Jan 27, p 109.

11 a. Concrete with **hard sandstone, gave strength** 50 % greater than where **shale** was substituted.

For abbreviations, symbols and references, see p 1251.

— 12 —

12. Leibbrand. E R, '94, Nov 3.

12 a. Comp strength; age. Bridge over Danube at **Munderkingen.** Conc 1 : 2.5 : 5, wet. Cubes 20 cm (8").

Very thoroly mixt in an iron cylinder revolving on a hor axis and containing 40 steel balls weighing together 660 lbs. Mixt 2 mins dry, 3 mins wet.

Age in days...........	7	28	150	970	3285 (= 9 years)
Comp strgth, kg/sq cm..	202	254	332	520	570
lbs/sq in.............	2870	3610	4720	7400	8100

12 b. Max existing pressures, in bridge, 500 to 560 lbs/□"

— 13 —

13. J. Watt Sandeman, Inst C E Procs, Vol 121, '95, p 220.

13 a. Watertight concrete walls (pres not stated) made with
1 part cem leaving 10 % on No. 120 sieve,
2 parts sand with 27 % voids,
4.5 " large and small gravel with ≯ 35 % voids.

13 b. Where agg has 35 % **voids, vol of mortar** should be 50 % of vol of **agg.**

— 14 —

14. A. W. Dow, U. S. Inspector of Asphalt and Cem. Report of Engr Commsr, Dist of Columbia, '97, p 165.

14 a. Compressive strength.

Specimens. 12-inch conc cubes, dry; rammed in cast iron molds; thoroly wet twice daily.

The results for one year are means of five cubes; the rest are means of two cubes. Deduct from 3 to 8 per cent. for friction of press.

The materials were as follows:

Cement.	Portland	Natural
Per cent. retained on sieve of 100 meshes per linear inch,	8.5	14
Time for initial set, minutes........................	190	20
" " hard " "	305	36

Tensile strength as follows, lbs. per square inch:

	1 Day.	7 Days.	1 Mo.	3 Mos.	6 Mos.	1 Year.
Portland, neat,........	441	839				
" 3 parts standard broken quartz,		248	429	398	428	474
Natural, neat, ...	96	180				
" 2 parts standard broken quartz,		91	188	327	414	485

Sand used in concrete.

No residue on a No. 3 sieve; 0.5 per cent. passed No. 100. Voids 44 per cent., with 4 4 per cent. water.

Broken Stone. Gneiss. Of Nos. 6 and 12 (table below) 3 per cent. retained on 2.5 inch mesh; all on 1¼ inch. Others, 0 retained on 2.5 inch; nearly all on 0.1 inch. For voids, see table, below.

Gravel. Clean quartz, passing a 1¼-inch mesh, 2 per cent. passing a No. 10 mesh. Voids, 29 per cent.

Water. With Portland cement, 0.09 cu. ft. (= 5.7 lbs) per cu. ft. of rammed concrete; with natural cement, 0.12 cu. ft. (= 7.5 lbs.).

For **Results,** see p 1314.

For Directory to Experiments, see pp 1303–7.

Crushing Strength of 12 in. Concrete Cubes, in lbs. per sq. in.
Experiments by A. W. Dow, as above:
Parts by volume; cement, 1; sand, 2; aggregate, 6.

	No.	Aggregate: Broken Stone, Parts.	Aggregate: Gravel, Parts.	Voids in Aggregate: Per Cent. of Vol.	Voids in Aggregate: Mortar, in percentage of Voids.	Crushing Strength, lbs. per sq. in., after 10 Days.	45 Days.	3 Mos.	6 Mos.	1 Year.
Portland	7	6	..	45.3	83.9	908	1790	2260	2510	3060
	8	3	3	35.5	107.0	950	1850	..	2070	2750
	9	4	2	37.8	100.6	..	..	..	..	2840
	10	6	..	39 5	96.2	..	..	..	..	2700
	11	..	6	29.3	129.1	694	1630	2680	1840	2820
	12	6	..	45.7	83 9	..	..	1630	1530	1850
Natural	1	6	..	45.3	83.9	228	539	375	795	915
	2	3	3	35.5	107.0	108	364	593	632	841
	3	4	2	37 8	100.6	..	..	..	..	915
	4	6	..	39 5	96.2	..	..	..	..	890
	5	..	6	29 3	129.1	87	421	361	344	763
	6	6	..	45.7	83 9	..	..	596	..	829

— 15 —

15. Tests of Metals, '98, p 572.

15 a. Cinder Conc with Port cem; **ult comp strength.**

Specimens; 12-inch cubes; water 10 to 12½ lbs per cu ft of conc

Results;

Proportions by volume:

Cement	Sand	Cinders	Age, days	No. of tests	Lbs/sq inch
1	1	3	30–38	18	1541
1	1	3	90	18	2053
1	2	3	39	3	1098
1	2	3	102	3	1634
1	2	4	38	3	904
1	2	4	98	3	1325
1	2	5	30–38	15	724
1	2	5	90–99	15	1094
1	3	6	29	3	529
1	3	6	91	3	788

— 16 —

16. Considère, Génie Civil, '99.

16 a. Ductility.

Specimens and results;

Conc cantilevers, 1 : 3, 6 cm sq, 60 cm long, tension side reinfd by 3 round iron bars 4¼ mm diam

Treatment. Loading such that bendg mom was the same for all cross secs. In one of the prisms, load increased until unit stretch = 0.002. Then loads, = 44 to 71 % of this original load, were applied 139,000 times; stress returning to 0 each time.

Results. Unit stretches, 0.000545 to 0.00125; strgth but little reduced. Similar tests of unreinfd specimens gave unit stretch, at rupture, only 0.0001 to 0.0002; the **reinforcement** apparently enabling the conc to endure far greater deformation than when not reinfd. But see Expts 36, 38.

For abbreviations, symbols and references, see p 1251.

— 17 —

17. C. E. Fowler, A S C E, Trans, '99, Vol 42, p 117.

17 a. Results. Proportions, assuming that
1 bbl Portland cem = 3.8 cu ft.
34 cu yds concrete = abt 27 cu yds after ramming.

Those concs, for which the vols of stone appear in bold-face type (as **1.00**), have their voids filled or more than filled; while, in those printed in plain type (as 1.04), the voids are not filled and the conc is porous and deficient in strgth.

	Quantities in 1 cu. yard of concrete:			
Proportions	Cement, Barrels	Sand, cu yds	Stone with 40 % voids, cu yds	Stone with 50 % voids, cu yds
1 : 2 : 3	1.77	0.51	**0.87**	1.05
1 : 2 : 4	1.59	0.47	**0.95**	1.15
1 : 2 : 5	1.39	0.42	1.04	1.26
1 : 3 : 4	1.30	0.57	**0.83**	**1.00**
1 : 3 : 5	1.16	0.52	**0.92**	1.11
1 : 3 : 6	1.04	0.48	**1.00**	1.20
1 : 4 : 6	1.00	0.55	**0.91**	1.09
1 : 4 : 7	0.92	0.51	**0.97**	1.17
1 : 4 : 8	0.83	0.47	1.03	1.25

The foregoing figures agreed well with the results of practice. The column for stone with 40 % voids closely represents broken limestone, which breaks into pieces of various sizes; while the column with 50 % voids represents trap rock, which breaks into pieces of more nearly uniform size.

— 18 —

18. Tests of Metals, '99.

18a. Compressive Strength of 12″ cubes of dry **Portland cement concrete,** for Geo. A. Kimball, Chief Engr Boston El Ry Co.

Specimens:

Sand. Coarse, clean, sharp. Voids, measd loose and moist, 33 %; measd after settling by saturation with water, 25 %.

Stone. Conglomerate from Roxbury, Mass. Voids, measd loose, 49.5 %.

4.8 % passed 2½″ ring, caught on 2″ ring;
76.7 % " 2″ " , " 1″ " ;
18 % " 1″ " , " ½″ " ;
0.5 % " ½″ " .

Treatment. Mixt by hand. **Water** barely showed after ramming. Cubes, except those tested at 7 days, **buried** in wet ground until within one wk of testing. In general, 5 cubes of each mix of each brand were tested at each of the ages.

Results. Ultimate compressive strengths, lbs/□″. Each max or min is the mean of five or more tests, upon cubes made from one of the four brands of cem, and thus refers to the cem giving max or min strgth under the stated conditions. The avs are those of such results for the 4 brands.

Age	1 : 2 : 4			1 : 3 : 6			1 : 6 : 12		
	max	av	min	max	av	min	max	av	min
7 ds	2219	1525	904	1550	1232	892	759	583	417
1 mo	2642	2440	2269	2174	2063	1816	1218	1042	873
3 mos	3123	2944	2608	2538	2432	2349	1257	1066	844
6 mos	4411	3904	3612	3170	2969	2750	1583	1313	815

For formulas, deduced from these results **by E. Thacher,** see ¶ 35, p **1274.**

— 19 —

19. W. A. Rogers, Chic, Mil and St P Ry, Westn Soc Engrs, Jour, 1899, Jun, Vol 4, No. 3, p 262, R R Gaz, '00, June 15, p 402, July 27, p 1274.

19 a. Effect of **cold,** and of mixing with **salt water. Specimens; comp strength** of 12-inch cubes of Port and nat cem conc. 8 cubes

For Directory to Experiments, see pp 1303-7.

Atlas Port, 1 cem, 3 gravel (2 sand, 1 pebbles), 4 hard crusher run limestone; 8 cubes Louisv nat, 1 cem, 2 gravel, 3 stone.

Same as used in track elevation masonry by Chic, Mil and St P Ry.

Treatment. All the cubes made by same person in molds of 1" lumber, and left in molds until broken.

Results.

	Portland		Natural	
	Temp, F	lbs/sq in.	Temp, F	lbs/sq in
1 cube in warm office 28 days	80° to 18°	>1290†	85° to 40°	300
1 " " " " 28 "	"	>1290†	"	defective
1 " outdoors* 28 "	57° to −24°	902‡	57° to −10°	200
1 " " 28 "	"	690‡	"	256
1 " " 28 "				
in office 28 "	85° to 32°	>1290†	85° to 40°	376
1 " outdoors* 28 "	57° to −24°		57° to −10°	
in office 28 "	85° to 32°	>1290†	85° to 40°	352
1 " outdoors * ** 28 "	57° to −24°	>1290†	57° to −10°	237
1 " " 28 "	"	>1290†	"	247

19 b. Character of aggregate; comp strength.

Specimens. 12" cubes of Port cem, gravel and stone. Gravel, 2/3 coarse, sharp sand, 1/3 pebbles from sand to 1½". Each result the average of 3 cubes. Age 28 days.

Results.		lbs/sq in
1 : 3 : 4.5	hard crusher-run limestone	1270
1 : 3 : 4.5	soft screened "	1170
1 : 3 : 4.5	washed gravel ⅜ to 2 in	1050
1 : 4 : 7	soft screened limestone	714
1 : 4 : 3.5 / 3.5	" " " / washed gravel ⅜ to 2 in	642

19 c. Dirt in sand and aggregate; comp strength.

Specimens. "Dirty" sand and gravel contained apparently abt 10% dirt "which had the appearance of containing a large amount of iron."

Results.	With sand, tensile, 90 days, lbs/□"			With gravel, comp, 12" cubes, 28 days, lbs/□"	
	1 : 1	1 : 2	1 : 3	1 : 2 . 5	1 : 2.5 · 5
Clean........	457	492	349	1097	838
Dirty........	627	541	430	988	928
Dirtier.......	515	514	396	1020	...

— 20 —

20. Edwin Thacher, E N, '99, Sep 21.

20 a. "Several brands of Port cem were improved, in tensile **strength,** by a **delay** of from 1 to 4 hrs betw **mixing and laying.**" Ransome

— 21 —

21. Geo. W. Rafter, A S C E, Trans, Dec '99, Vol 42, p 104.

21 a. Volume; consistency, richness and proportion of mortar.

Specimens; 544 12" cubes, broken on the U. S. Govt testing machine at Watertown, Mass. Port cem; sand, 86.5 to 93.5 lbs/cu ft; agg, broken stone. Cubes abt 2 years old.

"Dry," only a little more moist than damp earth;

"Plastic," ordinary consistency used by masons;

"Excess," under moderate ramming the conc quaked like liver.

* During the first part of the 28 days, temp fell to −10° and −20° F; afterward, thawing during day, freezing at night.

† Flaked slightly. Strgths exceeded capacity (185,000 lbs) of machine

‡ Cold believed to have retarded setting.

** Mixed with salt water, 1 pint salt to 10 qts water.

For abbreviations, symbols and references, see p 1251.

S = vol of sand in mortar to 1 vol cem;
M = " " mortar " conc " 1 " "
A = " " agg " " " 1 " "
C = " " conc made with 1 " "

Results.

Consistency*	Volume									
	Mortar = 33 % agg					Mortar = 40 % agg				
	Proportions				Shrkg	Proportions				Shrkg
	S	M	A	C	†	S	M	A	C	†
D.	1	1.57	4.74	4.30	9.3	1	1.64	4.10	3.82	6.8
P.	1	1.83	5.51	5.01	9.1	1	1.66	4.14	3.82	7.7
E.	1	1.70	5.11	4.64	9.2	1	1.70	4.24	3.97	6.4
D.	2	2.42	7.29	6.74	7.4	2	2.44	6.12	5.89	3.8
P	2	2.45	7.28	6.62	9.1	2	2.50	6.28	5.83	7.2
E	2	2.35	7.02	6.36	9.4	2	2.60	6.47	5.97	7.7
D.	3	3.15	9.49	8.78	7.5	3	3.21	8.03	7.36	8.4
P.	3	3.30	9.92	8.89	10.4	3	3.31	8.23	7.62	7.4
E	3	3.25	9.72	8.83	9.2	3	3.43	8.57	7.90	7.8
D	4	4.18	12.69	11.75	7.4	4	4.24	10.71	9.84	8.1
P	4	4.28	12.94	11.66	9.0	4	4.35	10.96	10.09	7.9
E.	4	4.37	13.14	11.78	10.4	4	4.33	10.84	9.64	11.1
D.	5	5.04	15.05	14.29	5.1	5	4.42	11.25	...	...
P.	5	5.00	15.00	13.66	9.1	5	5.00	12.50	11.56	7.5
E.	5	5.08	15.20	13.60	10.5	5	5.24	12.90	...	...

21 b. Density of concrete: thoro **ramming.**

Vol of 1 : 1 mortar,	Vol of rammed conc, approx,
0.33 × vol of agg,	0.91 × vol of agg,
0.40 × " " "	0.93 × " " "

21 c. Density of aggregate; compacting. Portage stone, broken to pass a 2″ ring, and having 43.3 % voids when slightly shaken in the measure, had only 37.4 % voids, as a mean of 5 trials, after being packed in the measure with a tamping iron, used about as forcibly as in ordinary ramming of conc.

— 22 —

22. Tests of Metals, '00, pp 1109, &c. For Contractors Plant Co.

22 a. Specimens; Port cem, sand, crushed stone, 1 : 3 : 5. Stone passed thru a 2½″ ring; pieces passing a ⅛″ ring screened out.

A, hand-mixt; B and C mixt in a portable gravity mixer 8 ft long, consisting of a steel trough containing numerous rows of steel pins, staggered. Water from a spray pipe strikes the mixer about midway its length. Hence conc is mixt dry in the upper half, and wet in the lower.

Stone spread evenly on a platform in front of mixer
Sand " " " top of stone
Cem " " " " " sand.

Material then shoveled into mixer.

B. Allowed to form a cone-shaped pile, stones accumulating around edges.

C. Material, as discharged, levelled off with hoe.

12″ cubes; beams from 4″ × 6″ to 6″ × 6″ 30″ span. All, 2 days in air, 2 mos in water, 1 mo in air.

* Consistency: D = dry; P = plastic; E = excess

† Shrinkage = $\frac{100\ (A - C)}{A}$.

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Treatment. All the cubes made by same person in molds of 1" lumber, and left in molds until broken.

Results.

	Portland		Natural	
	Temp, F	lbs/sq in.	Temp, F	lbs/sq in
1 cube in warm office 28 days	80° to 18°	>1290†	85° to 40°	300
1 " " " " 28 "	"	>1290†	"	defective
1 " outdoors* 28 "	57° to −24°	902‡	57° to −10°	200
1 " " 28 "	"	690‡	"	256
1 " " 28 "				
in office 28 "	85° to 32°	>1290†	85° to 40°	376
1 " outdoors* 28 "	57° to −24°		57° to −10°	
in office 28 "	85° to 32°	>1290†	85° to 40°	352
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1 " " 28 "	"	>1290†	"	247

19 b. Character of aggregate; comp strength.

Specimens. 12" cubes of Port cem, gravel and stone. Gravel, 2/3 coarse, sharp sand, 1/3 pebbles from sand to 1 ½". Each result the average of 3 cubes. Age 28 days.

Results.		lbs/sq in
1 : 3 : 4.5	hard crusher-run limestone	1270
1 : 3 : 4.5	soft screened "	1170
1 : 3 : 4.5	washed gravel ⅜ to 2 in.	1050
1 : 4 : 7	soft screened limestone	714
1 : 4 : { 3.5 / 3.5 }	" " " } washed gravel ⅜ to 2 in }	642

19 c. Dirt in sand and aggregate; comp strength.

Specimens. "Dirty" sand and gravel contained apparently abt 10% dirt "which had the appearance of containing a large amount of iron."

Results.	With sand, tensile, 90 days, lbs/□"			With gravel, comp, 12" cubes, 28 days, lbs/□"	
	1 : 1	1 : 2	1 : 3	1 : 2 . 5	1 : 2.5 · 5
Clean........	457	492	349	1097	838
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— 20 —

20. Edwin Thacher, E N, '99, Sep 21.

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21 a. Volume; consistency, richness and proportion of mortar.

Specimens; 544 12" cubes, broken on the U. S. Govt testing machine at Watertown, Mass. Port cem; sand, 86.5 to 93.5 lbs/cu ft; agg, broken stone. Cubes abt 2 years old.

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"Excess," under moderate ramming the conc quaked like liver.

* During the first part of the 28 days, temp fell to −10° and −20° F; afterward, thawing during day, freezing at night.

† Flaked slightly. Strgths exceeded capacity (185,000 lbs) of machine

‡ Cold believed to have retarded setting.

** Mixed with salt water, 1 pint salt to 10 qts water.

For abbreviations, symbols and references, see p 1251.

— 27 —

27. G. Y. Skeels, Asst City Engr, Sioux City, Iowa. E N, '02, Nov 6, p 382.

27 a. Avs of 2 and 4 briquets, 1 day in air, 14 ds in water. Port cem. Under **continuous mixing** for 8 or 10 hrs, neat cem mortar lost about ⅛ of its **tensile strength;** 1 : 2 lost about ⅙.

— 28 —

28. Thos. S. Clark, Resident Engr in Chg of Construction of Manhattan R R Power Station, New York. E N, '02, Jul 24, p 68.

28 a. Retempering; strength. Neat nat cem mortar mixed initially with 28 % water; sand nat cem mortar with 14 %. Retempered an hour after mixing, "enough water being added, as in practice, to bring the mass back to its original consistency." One day specimens 3 hours in air, the others 24 hours. Retempered specimens showed, in general, about half the normal strgth.

Similar results were obtained when the cem was moistened every 15 mins during the hour. In such cases, in practice, the strgth is sometimes increased by adding a little fresh cem.

Port cem mortars, retempered after standing an hour, failed to show marked deterioration, probably because Port cem sets more slowly than nat cem.

— 29 —

29. W. Purves Taylor, A S T M, Vol 3, p 376, '03.

29 a. Age; soundness. Ageing of finely ground cem permits hydration of the free lime, nearly always present, rendering it inert and preventing expansive action. Specimens, made with cem one wk old, were unsound; but, as the age of the cem increased, the soundness of the specimens improved until, when the cem was 5 wks old, the specimens were sound.

29 b. Fineness; soundness. The larger particles of coarsely ground cem are not readily hydrated. A cem, of which 33 % remained on a No 200 sieve and 13 % on No 100, checked and cracked in the boiling test; but became sound when reground until all passed the No 100 sieve and allowed to season for 2 weeks.

— 30 —

30. French Government Commission, Beton und Eisen, '03, Vol 5.

30 a. Ductility. Conc 1 : 2 : 4. Results similar to Considère's (see Expt 16 *a*). Ductility greater when hardened in water than when hardened in air.

— 31 —

31. Chas. List, Assn Eng Socs, Jour, Mar, '03, Vol 30, No. 3, p 128.

31 a. Effect of **sea water** at Gautemala, Central America

Hollow piles, in sea water, filled with conc in which sea water had been used for mixing. Some of the mortar leaked out, and formed, with the surrounding sand, masses of conc which adhered to the piles. When piles were removed, conc was found perfectly hard and adhering tenaciously to the piles.

31 b. Railway bridge foundation, built 1895. Lean conc mixt with and standing in brackish water. Of excellent quality in '03.

31 c. Regrinding. Cem brought from Hamburg, Germany, in bbls. Vessel sprang a leak; cem considered a loss, and value refunded. Cem stored under the floor of a warehouse with open sides and exposed to moisture of ground and to spray from sea. Cem caked hard enough to be used as foundations for wooden posts in buildings. This caked cem was broken as fine as possible, and mixt with sharp beach sand and brackish water. Conc perfectly hard in 3 days and used in bridge foundations in brackish water.

— 32 —

32. Geo. W. Lee, Jr., E N, '03, Mar 19, p 246.

Finish.

32 a. New York Central R R. **Forms** (2″ tongued and grooved pine) coated with **soft soap;** openings in joints filled with **hard soap.** Conc deposited and drawn back from mold with a square-pointed shovel, and 1 : 2

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1 : 4 : 3.5 / 3.5	" " " / washed gravel ⅜ to 2 in	642

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Specimens. "Dirty" sand and gravel contained apparently abt 10% dirt "which had the appearance of containing a large amount of iron."

Results.	With sand, tensile, 90 days, lbs/□"			With gravel, comp, 12" cubes, 28 days, lbs/□"	
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† Flaked slightly. Strgths exceeded capacity (185,000 lbs) of machine

‡ Cold believed to have retarded setting.

** Mixed with salt water, 1 pint salt to 10 qts water.

For abbreviations, symbols and references, see p 1251.

cracks, corresponding to the lowest unit stretches, were invisible on dry conc, but were detected, in moist conc, by the appearance of narrow wet streaks about 1/8″ wide. A little later, they showed as dark, hair-like cracks.

— 37 —

37. Prof Bauschinger. "Beton und Eisen," '04, Vol IV, p 193.

37 a. Corrosion: adhesion.

Fragments of reinfd conc plates, broken, in testing, '87; **exposed outdoors** until examined in '92. Adhesion; conc broken off by hammer blows, breaking only in immediate vicinity of blows. Corrosion; steel rust-free, even close to the exposed surfs of fracture.

37 b. Tank, injured by **rough treatment;** cracked; reinfmt laid bare in places. Rust only where so exposed. Adhesion as in 37 (*a*).

37 c. Fragments of Monier plates 6 to 8 cm thick. Exposed, at intervals for about 4 yrs, to **sewage-polluted water.** Conc remained hard; reinfmt rust-free 1 cm from exposed surface; adhesion excellent.

— 38 —

38. A. Kleinlogel. Beton und Eisen, '04, Vol 2.

38 a. Ductility. Reinfd conc beams 15 × 30 cm, 220 cm long. 1 : 1 . 2, cem, sand, limestone screenings. Kept under moist sand 6 mos. Bendg mom constant thruout measd portion. Unit stretches in conc; reinfd, 0.000148 to 0.000196; plain, 0.000143.

— 39 —

39. Clarence Coleman: Report, Chf of Engrs, U S A, '04. Part IV. Universal Port cem made from blast furnace slag.

	Sand*	Mix	Water‡‡	Av tensile strgth, lbs/□″ 7 da	28 da	6 mo	1 yr	3 yr
39 a.								
Cem in good condition	Q	1:3	12.5	176	298	424	..	..
Cem exposed in sacks to **dampness**	Q	1:3	12.5	173	260	411	..	...
Caked hard. Not set. Reground	Q	1:3	12.5	199	274	424	..	...
39 b.								
Cem as received on works	Q	1:3	12.5	1.00 †	1.00 †	..	.	..
Cem after 4 to 10 mos in sacks in **warehouse**	Q	1:3	12.5	1.17 †	1.09 †	.	..	...
39 c.								
Conc **hand-mixt** on platform‡	S	1:10	Random	134	211	324	343	..
Conc mixt in cubical **batch-mixer**‡§	S	1:10	Random	253	274	385	391	...
39 d.								
As in laborat'y, 24 hours in damp closet, then immersed until broken ¶	S	1:10	Random	262	366	420	462	394
As on work, 10 days under damp cloth, then in air until broken¶	S	1:10	Random	222	388	415	643	834
39 e.								
8.25 % water **	S	1:3	8.25	254	289	380	399	...
9.25 % water **	S	1:3	9.25	244	317	398	437	..
39 f.								
Pebbles 1/16 to 1/4 inch	S	1:10	Random	164	275	446	445	...
Pebbles 1/4 to 3/4 inch	S	1:10	Random	184	314	458	464	...
39 g.								
Clean sand	S††	1:3	8.25	183	259	361	340	...
Sand with small % **clay**	S††	1:3	8.25	183	272	392	359	...

* Q = Standard crystal quartz.
S = Superior Entry sand; passing sieve......No. 4 10 20 30 50
% 100 72.3 46.1 26.5 5.1

† *Relative* strgths. ‡ Briquets made of conc taken from the works.
§ A batch of very perfectly mixt conc in 80 secs.
¶ Conc taken from mixing platform Stones larger than 3/4″ removed.
** In order to approx working conditions, the mortar was allowed to stand 30 mins longer than under ordinary treatment.
†† Passing No 10 sieve. ‡‡ Water in percentage of dry agg

For Directory to Experiments, see pp 1303–7.

— **40** —

40. Prof Chas. L. Norton, E N, '02, Oct 23, '04, Jan 14.

Corrosion. Several hundred briquets of various mixes and consistencies with steel imbedded, subjected to air, steam and carbonic acid.

40 a. Steel clean when imbedded. 3 wks exposure.

Steel perfectly protected by neat cem in all cases, and where the mortar was mixt wet, so as to cover the steel with thin grout.

In conc, rust found only where voids or other defects existed.

40 b. Steel rusted when imbedded. 1 to 3 mos exposure. Changes, in size of steel, occurred only where conc had been poorly applied.

— **41** —

41. John S. Sewell, on **Baltimore fire,** E N, '04, Mar 24.

41 a. Results. "Concrete undergoes more or less **molecular change** in fire, subject to some **spalling.** Molecular change very slow. Calcined material does not spall off badly except at exposed square corners. **Efficiency,** on the whole, is high. Preferable to commercial hollow **tiles** for both floor arches or slabs, and col and girder coverings."

41 b. Reinfd conc cols, beams, girders, and floor slabs, at least as desirable as **steel work** protected with the best commercial hollow tiles.

41 c. "**Stone** conc spalls worse than any other kind, because the pieces of stone contain air and moisture cavities, and the contents of these rupture the stone, when hot. **Gravel** is stone that has had most of these cavities eliminated by splitting through them, during long ages of exposure to the weather. It is therefore better than stone for fire-resisting conc."

41 d. "**Broken bricks, broken slag, ashes and clinker** all make good fire-resisting conc."

41 e. "**Cinders** containing much partly burned coal, are unsafe, because these particles actually burn out and weaken the conc. Locomotive cinders kill the cem, besides being combustible. Cinder concrete is safe only when subjected to the most rigid and intelligent supervision; when made properly, of proper materials, however, it is doubtful whether even brickwork is much superior to it in fire-resisting qualities, and nothing is superior to it in lightness, other things being equal."

— **42** —

42. Emile Low, A S C E, Trans, June '04, Vol 52, p. 96. Buffalo Breakwater.

42 a. Shrinkage.

Cement	258	cu yds	
Sand	365	"	
Pebbles	1175	"	
Broken Stone	972	"	
Total Materials	2770	"	
Blocks made	2054	"	
Shrinkage	716	"	= 25.8 %

— **43** —

43. Alex. B. Moncrieff, Engr in Chief, South Australian Govt Letter to authors, June 7, '04

43 a. Permeability.

Specimens. Conc blocks, 2 ft cubes (8 cu ft), for expts in connection with construction of **Barossa dam.** Ingredients same as used on dam. Agg ⅛" to 2", with varying voids. Preparation of aggs very carefully watched.

Treatment. Water brought to cen of block in ½" wrought iron pipe terminating in a T piece, wrapped with hemp which formed a bulb abt 4" diam.

Results. All the blocks became practically tight. Conc used in dam "was based upon the results of the expts principally with blocks

For abbreviations, symbols and references, see p 1251.

Nos 7 and 8." There is "practically nothing that could be called a leak" thru the dam.*

Q = vol of mixing water, % of volume of conc;

X = excess mortar = $100 \frac{\text{vol of mortar} - \text{vol of voids}}{\text{vol of voids}}$;

A = age of block, in weeks, when subjected to pres;

I = interval in mins, betw application of pres and appearance of water on surf of block;

Head = 100 ft = 43.4 lbs/□." Under 200 ft (86.8 lbs/□") "the effect closely resembled the results obtained from the head of 100 ft."

No.	Cem.	Sand	Agg	Q %	X %	A Weeks	I Mins.	Observed Leakage* Pints	Observed Leakage* Mean rate U. S gals/m(
1	1	1.84	5.26	16.65	5	11	†	†	†
2	1	1.84	5.26	15.45	5	11	34	¾ in 7 wks.	0.065
3	1	1.50	4.63	16.04	5	10	18	1/80 " 4 "	0.005
4	1	2.00	4.50	16.04	15	10	14	14 " 2 "	4.000
5	1	1.75	4.13	16.65	15	9	12	27 " 7 "	2.353
6	1	1.50	4.12	16.04	10	8	35	1/50 " 2 "	0.006
7	1	1.50	3.90	14.26	12.5	6	28	⅛ " 2 "	0.037
8	1	1.50	3.70	13.68	15	5	30	1/50 " 2 "	0.006

— 44 —

44. Edwin Thacher, A S C E, Trans, '05, Vol 54, pp 425, &c.

44 a. Effect of cold. Melan arch bridge, at Mishawaka, Ind, 3 spans, 110 ft each, built in temps ranging from 0° to 55° F. **Hot water** admitted to mixer. Conc laid at blood heat; warm enough to melt snow 48 hours later. Center arch completed with temp about 25° F. The next day, temp fell to 0° F. Two wks later, an ice jam carried out the centering and left the **arch unsupported.** No bad effects observed; settlement but little greater than with the other arches, centering under which was removed later and in the usual way.

44 b. Finish.

Bridge at Oconomowoc, Wis. Mortar face, 1 cem : 1 granite screenings : 1 torpedo sand. On the second day after completion, molds removed and surf rubbed with a soft stone and water.

Inman arch, Hohenzollern. 1 cem : 5 broken limestone. After setting 12 hrs, the loose cem was removed by water and brushes.

Pacific Borax Co's factory, Bayonne, N. J. Finished to represent coursed ashlar, by inserting wooden strips in the molds and dressing the faces with a pneumatic hammer. One man could dress from 300 to 600 sq ft in 10 hours by machine, 100 to 200 by hand. Good effect.

"Mr. Cummings produced a good finish by going over the surf with a wire brush while the cem was still green."

Utica & Mohawk Valley Ry viaduct, Herkimer, N. Y., and viaduct over rys at Jacksonville, Fla. "A very superior finish." For a hard wall, wet the surface and apply a thin 1 : 2 mortar with a brush. Rub surface with a piece of grindstone or carborundum, removing board marks, filling pores and producing a lather on the surf. Go over this lather, before it dries, with a brush dipped in water.

For a green wall (molds removed in less than 7 days,) use a thin grout of neat cem, instead of the 1 : 2 mortar. Remainder of process as above.

Use smooth molds, deposit wet conc directly against them. After removing molds, float the surf with a wooden float, using only sufficient mortar to fill the pores and give a smooth finish.

44 c. Corrosion.

Chicago. Iron rods, in limestone conc slabs which had covered sidewalk vaults for 8 or 10 yrs, rust-free. E. L. Ransome.

* See ¶ 4, p 1271. † Unreliable.

For Directory to Experiments, see pp 1303–7.

— **40** —

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Treatment. Water brought to cen of block in ½" wrought iron pipe terminating in a T piece, wrapped with hemp which formed a bulb abt 4" diam.

Results. All the blocks became practically tight. Conc used in dam "was based upon the results of the expts principally with blocks

For abbreviations, symbols and references, see p 1251.

46 d. **Conc did not break or chip** under fire; but lost practically all strgth to a depth of 4″ from sides and bottom, and softened perceptibly thruout. The cem and most of the stone were thoroly calcined at surf, and, to a diminishing extent, to a depth of 4″. In all cases, a little **water appeared** in cracks running across the beams, especially with the richest mixtures and with temp at 212° F.

46 e. **Recommendations.** Materials should be well mixt, wet, by machine, and well tamped. Imbedment should be ≮ 2″; in important cases, 3″.

— 47 —

47. **John H. Quinton,** U. S. Geol Surv, "Expts on Steel-conc Pipes on a Working Scale," U. S. Water-Supply and Irrigation Paper 143, '05.

47 a. **Permeability.** To determine availability of such pipes under pres, for U. S. Reclamation Service.

Specimens. Seven reinforced hand-mixed conc pipes, 5 ft diam, 6″ thick, 20 ft long; each made in one section; one, same dimensions, in 4 secs. Skilled workmen. In 3 of the 7 pipes, and in 3 of the 4 secs of the 8th pipe, lime was used in the mixture.

The pipes varied greatly in texture. One of them "seemed to be of a crumbly nature, and it would have been easy to cut a hole through it." Another was "exceedingly hard."

Treatment. The pipes were tested with and without inside linings of cem and sand, etc, with and without lime paste. The Sylvester soap-and-alum wash (p 928), P and B waterproof paint, and other paints were tried: and clay was stirred up in the water within the pipes. Pressures up to 70 lbs/□″ = 161.5 ft. head.

Results.

47 b. In spite of all precautions, the pipes leaked, especially along tamping seams. **Leakage decreased greatly under pres,** as percolating water filled the pores with laitance; but in the mean time the leakage may be sufficient to damage foundations of pipe.

47 c. **Dry mixtures** gave the more permeable conc.

47 d. With carefully **graded gravels,** it was found difficult to secure uniform **distribution** of the diff sizes.

47 e. **Keep conc shaded** while mixing and placing.

47 f. Interruptions to work are least dangerous with **wet** mixtures. In tamping, avoid **displacement of reinforcement.**

47 g. Make **reinforcemt** strong enough to protect conc against tensile stress.

47 h. **Soap and alum** mixture of advantage in making conc; but ¾″ **plaster** found advisable on inside, in two coats, the first with lime paste, to retard setting; the second (applied when the first is dry) to be troweled smooth. When dry, apply thick neat cem wash.

47 i. Reinfd conc pipes not recommended for **heads** over 70 ft (30 lbs /□″). For short dists, special precautions may justify 100 ft (43 lbs/□″).

47 k. Conc pipes **liable to crack,** especially along tamping seams; but, even if cracked, probably drier and more durable than other kinds.

47 l. When the pipes were broken up, **rust appeared** upon only 1 rod, which was rusted all around for a length of about 1½″, where a large and long-continued leak had occurred. The pipe had been lined with a mortar containing sal ammoniac (ammonium chloride) and iron filings.

— 48 —

48. **Considère.** Beton und Eisen, '05, Vol 3

48 a. **Ductility.**

Specimens. Mixture, 400 kg Port cem, 0.4 cu m sand, 0.8 cu m limestone screenings. Beams 15 × 20 cm, 3 m long. Tension side reinfd with 2 iron bars 16 mm round, and 3, 12 mm rd. Bendg mom constant thruout measd length.

Treatment. One beam kept in water, one under damp sand, 6 mos

For Directory to Experiments, see pp 1303-7.

Results. Max unit stretches

kept under water	0.00107
" " damp sand	0.00050

No cracks discovered, altho the surf was smoothed with cem.

Strength unaffected.

— **49** —

49. R. Feret, "A Treatise on Concrete, Plain and Reinforced," by Taylor and Thompson, '05.

49 a. The injurious action of **sea water** is due chiefly to the **sulfuric acid** of the dissolved sulfates; hence, the cem should contain as little gypsum (lime sulfate) as possible. Port cem should be low in aluminum and in lime. The presence of puzzolanic material is advantageous. The conc should be dense and impervious.

— **50** —

50. Prof Ira H. Woolson, Report to Astoria Light, Heat and Power Co., '05.

50 a. Character; strength.

Strengths in lbs/□″

Port Cem, 1 : 2 : 4.	Tensile			Compressive		
	Max	Av	Min	Max	Av	Min
Sand & broken limestone	176	161	153	2000	1753	1441
Crushed* & broken limestone	282	194	138	3400	2449	2040

50 b. Sand contained < 1 % **loam;** all past ⅛″ sieve; 75 % past 20 mesh sieve. Hudson R bluestone (limestone) passing 1 ¼″ screen, retained on ⅜″ screen. Conc tampt wet in molds, 1 or 2 days in air, 5 or 6 in water. Air dried 4 to 7 wks. **Results,** see 50 a.

— **51** —

51. Prof R. C. Carpenter, Cornell Univ, Sibley Jour of Eng'g, Jan, '05.

51 a. Retardation of setting; gypsum (lime sulfate) $CaSO_4$, and **calcium chloride,** $CaCl_2$. Both ground dry with the clinker.

Initial set; paste bears a rod 1/12 inch diam, loaded with ¼ lb.
Final set; " " " " 1/24 " " " " 1 lb.

Time, in both cases, reckoned from time of mixing, and given in mins.

Results. Percentage by weight†

	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0
	Time in minutes											
Initial $CaSO_4$	2	6	.	80	24	29	30	27	28	27	19	18
" $CaCl_2$	2	115	160	167	127	103	45	97	..	73	68	..
Final $CaSO_4$	52	87	.	157	114	79	69	72	45	59	37	59
" $CaCl_2$	52	274	272	234	212	180	182	185		160	145	..

51 b. E. Candlot (Ciments et Chaux Hydrauliques) found that **concentrated** solutions of $CaCl_2$ (such as 100 to 400 grams per liter) **accelerated** setting and hardening.

51 c. Addition of slaked lime to a cem containing gypsum which, with time, has lost its retarding effect.

	Initial, mins	Final, mins
2 % gypsum, no lime	12	15
" " + 5 % "	120	300

2 to 5 % of lime is useful in this respect, but not without the gypsum. The lime does not diminish the strgth.

— **52** —

52. Jas. C. Hain, Chic, Mil and St P Ry. E N, '04/Apr 28, p 413 E R, '05, Jan 28, p 103. **Sand; size and cleanliness.**

* ⅜″ crusher screenings; 87 % past ¼″ sieve, 40 % past ⅛″ sieve.
† 1 % = about 4 lbs $CaCl_2$ to a barrel of Port cem.

For abbreviations, symbols and references, see p 1251.

Specimens.

52 a. Impure sands.

1 3 Port cem mortars, made with

(a) sand of smooth rounded quartz grains, mixt with larger fragments of limestone shells. 92 % past No 24 sieve, 28 % past No 50;

(b) "St Paul standd sand," 54 % past No 24; 11 % past No 50;

(c) "Ottawa standd sand."

Results:

Relative tensile strgths (a) 100; (b) 137; (c) 107.5.

Sand (a) made excellent conc in a draw-span center pier.

1 : 3 Port cem mortars, with sand containing 3.2 to 15 % clay; strgths ≮ with clean sand With nat cem 1 : 3, and Port 1 : 2, the results were generally favorable to the cleaner sand.

Sand with 6 % clay gave stronger mortars before than after washing

Sands, to which 2 to 20 % rich loam had been artificially added, gave mortar testing somewhat irregularly but in general higher than those with clean sand.

52 b. Fine sand, with clay. A sand, all passing No. 100 sieve, 93.2 % passing No. 200 (therefore finer than most cem. See Specfs), and containing 12 % clay, gave a 1 : 3 Port cem mortar showing, at 6 mos and 1 yr, nearly the same tensile strgth as similar mortar made with "Ottawa standard sand," but the mortar was weaker at shorter periods.

— **53** —

53. Jas. C. Hain, Engr of Masonry Constn, Chic, Mil and St P Ry, E N, '05, Mar 16.

Oil. Tests by **Geo. J. Griesenauer.**

53 a. A neat Port cem briquet, 2 yrs old, exposed to occasional drippings of signal oil, began to disintegrate in 10 mos; but no recent conc structures were found perceptibly injured by oil. **A conc floor,** upon which lubricating and lighting oils had been stored for 6 yrs, was apparently unaffected Oil penetrated about 1/16". A piece of this floor, in oil 10 mos, still sound.

53 b. Port cem; neat; 1 : 3 sand; 1 : 3 limestone screengs; 18 briquets each; 4 days in air. Then saturated daily with signal oil; later less frequently. Cracks appeared in the 1 : 3 specimens in 2½ mos; in neat specimens in 5 mos. All the briquets disintegrated eventually.

53 c. Port cem; 54 briquets, neat; 36 briquets 1 : 3 sand. 7 d in air. Then saturated daily with oil; later, less frequently. Oils used; extract lard, whale, castor, boiled linseed, crude petroleum, signal. Cems made from limestone and clay, marl and clay, limest and slag. Lard oil disintegrated most of the briquets in from 2 wks to 2½ mos, but some remained sound for 9 mos. Signal oil (animal and mineral mixt) had nearly the same effect. Whale and castor oil affected only a few briquets; while petroleum and boiled linseed disintegrated no briquets. Petroleum diminished strgth somewhat. Boiled linseed formed a protective coating and did not affect strgth. As a rule, the neat briquets yielded first In general, briquets of limestone and slag yielded most; those of limestone and clay least.

53 d. Silica cem; neat, 1 : 1, 1 : 2, 1 : 3, sand. 1 briquet each. 2 yrs in water; 20 days in warm air. Signal oil 2 yrs. First 3 briquets sound; 1 briquet (1 : 3) disintegrating.

53 e. Linseed oil, Sylvester's process (p 1220), paraffine, and water glass (soda silicate) were applied, as **coatings,** to the briquets, but **all failed** to protect them against the action of the oils.

53 f. Rich conc, well made of good materials and well set and seasoned, is best for resisting oil. In practice, **conc structures** are rarely, if ever, saturated with oil, as were these specimens.

— **54** —

54. Chas. A. Matcham, Nat Builders' Supply Assn, E R, '05, Apr 15, p 435.

54 a. Corrosion.

For Directory to Experiments, see pp 1303-7.

Specimens and treatment. 6-inch conc cubes, 3 yrs old, with 3" steel cubes embedded

Two cubes, with **unpainted** 3" steel cubes embedded, exposed to summer and winter **weather,** and sometimes covered with **snow and ice.**

Results.

Steel uninjured. Crushing strgths, 2920 lbs and over 4166 lbs/□". One 6" cube, with 3" steel cube (painted with metallic paint) embedded, placed in bottom of river. Steel uninjured **Paint disappeared.** Crushing strgth. 2907 lbs/□".

— 55 —

55. Prof Ira H. Woolson. E N, '05, Jun 1.

55 a. Absorption.

Specimens. 8" cubes, 1 : 2 : 4, 3 weeks old, kiln dried 13 days at 120° F.

Part with sand with < 1 % loam; all past 0.125" screen ; 75 % past 20-mesh sieve. Part with 3/8" limestone crusher screenings; 87 % past 1/4" screen; 40 % past 0.125" screen; sand and dust, enough to fill voids. Stone past 1 1/2" ring.

Results.

Av absorption; 4 hours, 2.87 %; 24 hrs, 2.95 %; 48 hrs, 3.33 %. No marked diff betw **sand and screenings.**

— 56 —

56. W. C. Hoad, Univ of Kansas. E N, '05, Aug 10.

Clay and Loam; strength and absorption.

56 a. Port cem with (a) standd Ottawa sand, 1 : 3; (b) 2 to 20 % of the sand replaced by clay or loam. At 90 days, relative strgths; in general: (a) 100; (b) 94 to 125.

56 b. Up to 6 or 8 % clay or loam, there was no increase of **absorption,** with loam; and about 10 % decrease, with clay. With **higher percentages,** the absorption increased somewhat.

— 57 —

57. Eng News, '05, Sep 28.

57 a. Permeability.

Reinforced concrete cistern, 75,000 gals. 1 : 2 : 4, Port cem, river sand, gravel. 1" layer of 1 : 1 **mortar on bottom. Walls** washed with 3 coats **neat cem grout,** cream consistency, put on with whitewash brush after walls were well wetted. Each coat dried for 24 hrs. If too wet, the coating crackt. If too dry, it could not be brusht on. For a few days after filling, lost 5/16" in depth per day. Perfectly tight since. Cistern built with outside air at **temp below 20° F;** but was covered with boards, and two coke salamanders were used.

— 58 —

58. Prof Ira H. Woolson, E N, '05, Nov 2.

58 a. Flow.

Specimens. Cols, 4" diam, 12" long, formed in steel tubes, 1/8" to 1/4" thick, and allowed to set and remain there for 17 days, when the conc appeared very hard. Conc remained in tubes during tests.

Results. Under loads of 150,000 lbs, the cols in the stouter tubes were merely shortened < 1/4"; but under loads of 120,000 to 150,000 lbs, the cols, in some of the lighter tubes, were bent out of shape and shortened by 3 1/2", their diam increasing from 4" to about 5". *Upon removal of the tubes, the conc was found unbroken, solid and perfect!*

— 59 —

59. J. M. Braxton, U. S. Asst Engr. Reports, '05-6 E N, '08, May 14 p 525.

Corrosion in sea water.

For abbreviations, symbols and references, see p 1251.

59 a. ½″ steel rods imbedded in 4 conc blocks made with coral sand and broken brick. 2 blocks in 4 ft of sea water; 2 in a dry closet, both for more than a yr. The rod in one of the dry blocks showed signs of rusting. The others were as bright and smooth as when placed.

59 b. 30 blocks, 12″ × 12″ × 6″; Port cem, 1 : 3 : 5, broken brick. Made under usual working conditions. ⅝″ twisted steel rod, 8″ long, in cen of each block. 20 blocks with coral sand, 10 with ordinary quartz sand. Half of each placed in ocean, half in air without roof. Broken after 1 yr, 3 wks. In all the blocks placed in the ocean, the rods were found in perfect condition. All the others were more or less rusted.

— **60** —

60. Wm. R. Baldwin-Wiseman, Instn C E Procs, '06, Vol 163, p 319.

60 a. Puddling effect of water flowing thru conc discs, 13″ diam, 6″ thick, 1 : 4 Port cem, crushed gravel passing 1″ ring. Sp gr of conc 2.23, 140 lbs/cu ft. In wooden molds 10 wks. Water, for pres, pumped from chalk formation, hardness reduced from 18° to 6°. Air temp 12° to 15° C = 54° to 59° F. Pressures, 24 to 60 lbs/□″ = 55 to 139 ft. Leakage as per Fig 4. Toward the close of the expts, small **stalactitic growths**

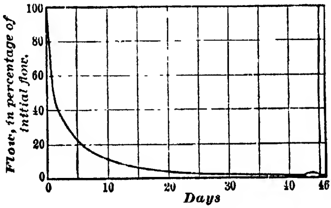

Fig 4. Puddling.

formed on bottom of test piece, and leakage was absorbed by evaporation. Near the surf, the water, under high pres, dissolved out some of the material, but deposited it in the pores farther on, where the pres had been reduced by passage thru the block.

— **61** —

61. Sanford E. Thompson. A S T M, Procs, Vol VI, 1906, p 379.

61 a. Consistency; effect upon density,* permeability and compressive strength.

Density and permeability specimens, 21 days old; comp strgth specimens 5¼ mos.

Specimens.

Atlas Portland cem; Newburyport sand, sp gr = 2.65; trap, sp gr = 2.78. 1 : 2.3 : 4.6 by vol; 1 : 2 : 4 by wt.

Consistencies used.	Water, % †
Dry. Like damp earth; water glistened on surf under hard ramming	5.4
Medium. Looked wet when mixed. Did not flow in mixing box. Slightly quaking	6.9
Wet	9.2
Very wet. Like thick soup. Settled to a level in mixing box. Required scoop shovels for handling. Slightly wetter than usual in building work	11.0
Extremely wet	13.7

* Density = vol of solid particles in unit vol of conc.
† Percentage of weight of cem, sand and stone.

For Directory to Experiments, see pp 1303-7.

Results. See Fig 5.

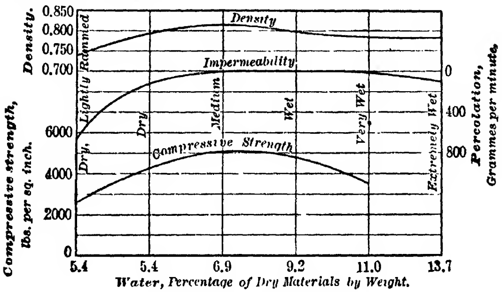

Fig 5. Consistency.

For a given consistency, the **percentage of water** depends upon the nature of the cem, and upon the size and dryness of the sand grains. A fine sand, or one with many fine grains, may require twice as much water as coarse sand requires.

61 b. Elastic modulus. Twelve 12″ cubes, deformations measd in 5″ gaged length. Averages of 4 specimens, 1 : 2 : 4; approx 1, 2, 6 and 17 mos old. Dry, 4,450,000 lbs/□″; medium, 4,200,000; very wet, 3,000,000 No appreciable increase of modulus with age.

61 c. Age; permeability. Blocks tested at 21 and 84 days, showed permeabilities abt as 2 : 1. Pressure, 80 lbs/sq in = 185 ft of water.

61 d. An excess of water washes out fine cem, forming **laitance,** reducing strgth and increasing permeability. **Thickness** of laitance formation, ⅛″ in very wet mixtures.

61 e. Mr. Thompson concludes that, in building and other reinfd work, the conc should be only wet enough "to flow sluggishly around and thoroly imbed the steel and permit smooth surfaces against the forms," and that medium or quaking conc is suitable for ordinary mass conc, such as foundations, heavy walls, large arches, piers and abuts.

— 62 —

62. A. Black, E N, '06, Aug 30, p 236.

62 a. Character of sand; strength and absorption.

Specimens.	Passing No 170 sieve.
A, crushed gneiss, screened thru ½″ mesh	90.8 %
B, Cowe Bay sand, much used in and about New York	95.8 %
C, fine clean silicious sand	95.5 %

Results. In 7 and 28 days, 1 : 2 and 1 : 3 mortars, A and B gave, in general, from 20 to 50 % greater tensile and comp **strengths** than C. In general, the **stronger** mortars showed the **higher absorptions.**

— 63 —

63. Alex. B. Moncrieff, E N, '06, Aug 30, p 227.

63 a. Briquets in water 2 yrs, in air 7 days and in oil 6 mos. In general, neat cem lost from 0 to 36 % strgth, while 3 : 1 gained from 0 to 65.5 %, by air drying and immersion in oil.

63 b. Briquets in air 7 days; then 6 mos in either oil or water. The neat cem briquets in oil were from 0 to 55 % weaker than the neat cem in water; the 3 : 1 briquets in oil were 49 to 79 % weaker than those in water.

For abbreviations, symbols and references, see p 1251.

63 c. Briquets in water 9 wks; others in water 4 wks, in air 1 wk and in oil 4 wks. With few exceptions, the neat cem briquets in oil were from abt 0 to 40 % stronger than like briquets in water, while the 3 : 1 briquets were from abt 0 to 63 % stronger than like briquets in water. Many of the oil-treated briquets "snapped like flint."

— **64** —

64. Prof Arthur N. Talbot, Univ of Ill. Bull, Vol IV No. 1, '06, Sept 1.

64 a. Adhesion and friction. '04.

Specimens and results.

Mix, 1 : 3 : 6.
Pull, in lbs/□" of net section;
Elastic limit, in lbs/□";
Adhesion, in lbs/□" of imbedded surf:

	Johnson bars		Round bars		Square bars		
	½"	¾"	⅜"	¾"	⅜"	½"	¾"
Pull............	71,412	34,500	31,500	21,500	35,656	26,510	20,860
Elas lim	60,000	58,300	42,500	40,500	45,000	33,300	35,000
Adhesion	595	420	249	315	297	286	325

With all the **Johnson bars**, the specimens split or broke. All the **plain rods** slipped. 6 of the 11 Johnson bars, and 4 of the 11 bars ⅜" square, were "struck 6 quarter-swing **blows** with a 10-lb sledge," reducing their adhesion by abt 5 to 20 %.

Specimens.

64 b. '05–6. Cylinders, 6" diam, 6" and 12" long; 60 days old Mixture of Am Port cems, tensile strgth, neat, 723 lb/□" at 7 days; 1 : 3, 354 at 7 ds, 533 at 75 ds, coarse mortar sand; broken limestone, screened thru 1" and over ¼" screen. Metal, elas lim, lbs/□"; Mild steel (M), Round 38,000; Flat, 45,000; Cold rolled shafting (C), 87,000; Tool steel (T) 53,000.

Results.

No. of tests	Steel	Size	Mix	Imbedded length, ins.	Lbs/□" imbedded surface: Adhesion a	Friction f	f/a
6	M	½" round	1 : 3 : 5.5	6	372	210	0.57
6	"	"	1 : 2 : 4	"	412	227	0.55
6	"	⅝" round	1 : 3 : 5.5	"	355	227	0.64
4	"	"	1 : 2 : 4	"	465	297	0.64
3	"	½" round	1 : 3 : 5.5	12	373	268	0.72
4	"	"	1 : 2 : 4	"	404	266	0.65
3	"	⅝" round	1 : 3 : 5.5	"	402	228	0.57
3	"	"	1 : 2 : 4	"	414	223	0.54
3	"	1½ × 5/16"	1 : 3 : 5.5	6	125	84	0.67
3	C	1" round	"	"	136	67	0.49
3	"	½" round	"	"	157	50	0.32
3	T	¾" round	1 : 3 : 6	"	147	...	...

Rich mixture generally superior. **Cold rolled** shaftg and **tool steel** generally inferior, owing to uniformity of sec and smoothness of surf.

— **65** —

65. Jos. W. Ellms, Chemist, Commissrs of Water Works, Cincinnati. E R, '06, Oct 27, p 467.

65 a. Permeability.

Specimens. Port and nat (Louisville) cem; Ohio River quartz sand, clean, rather fine, quite uniform in size; limestone screenings, with much very fine dust.

3" cubes;

Port cem; (*a*) 1 cem : 2 sand, 10 % water; (*b*) 1 cem : 1 sand : 1 screenings, 11 % water; (*c*) 1 cem : 2 screenings, 14 % water.

For Directory to Experiments, see pp 1303-7.

Results. See Fig 5.

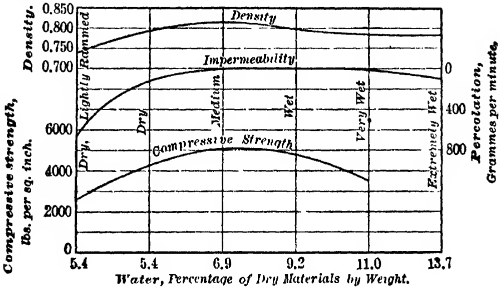

Fig 5. Consistency.

For a given consistency, the **percentage of water** depends upon the nature of the cem, and upon the size and dryness of the sand grains. A fine sand, or one with many fine grains, may require twice as much water as coarse sand requires.

61 b. Elastic modulus. Twelve 12″ cubes, deformations measd in 5″ gaged length. Averages of 4 specimens, 1 : 2 : 4; approx 1, 2, 6 and 17 mos old. Dry, 4,450,000 lbs/□″; medium, 4,200,000; very wet, 3,000,000 No appreciable increase of modulus with age.

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62. A. Black, E N, '06, Aug 30, p 236.

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— 63 —

63. Alex. B. Moncrieff, E N, '06, Aug 30, p 227.

63 a. Briquets in water 2 yrs, in air 7 days and in *oil* 6 mos. *In general,* neat cem lost from 0 to 36 % strgth, while 3 : 1 gained from 0 to 65.5 %, by air drying and immersion in oil.

63 b. Briquets in air 7 days; then 6 mos in either oil or water. The neat cem briquets in oil were from 0 to 55 % weaker than the neat cem in water; the 3 : 1 briquets in oil were 49 to 79 % weaker than those in water.

For abbreviations, symbols and references, see p 1251.

68 b. Tensile strength.

Oil in mix	Tensile strength, lbs/□″		
	1 day	7 days	28 days
None	430	696	743
2 % linseed	180	493	572
2 % engine	332	689	696

— 69 —

69. M. R. Barnett, Inst C E, Procs, '07, Vol 167, p 153.

69 a. Action of soft water upon limestone conc. Thirlmere aqueduct, water supply of Manchester, Eng. Section of aqueduct, made with limestone conc. Floor, 9″ thick, reduced about ¼″ in thickness, honeycombed, eaten thru in many places, and leaking badly.

69 b. Samples of the limestones, from which the conc was made, were kept, for 6 mos, in running soft water, in the aqueduct, and were found to lose wt at rates ranging from 6.8 to 18.1 % per year, while sample blocks of neat and 1 : 1 Port cem **mortar,** gained 5.5 and 3.6 % respectively. Deg of hardness of water, 2.18.

— 70 —

70. Prof Ira H. Woolson, A S T M, Procs, '05, p 335; '07, p 404.

High temperatures and thermal conductivity.

70 a. Mixture, 1 : 2 : 4; with cinder, 1 : 2 : 5. Cem, an equal mix of 3 Portlands. Sand, sharp, fair qual, "not especially clean"; 90 % past a 12-mesh sieve. Agg, fair quality boiler cinder, with most of the fine ashes removed; ¾″ clean quartz gravel; crusht trap. Mixt moderately wet; tampt in molds until water flusht to surf.

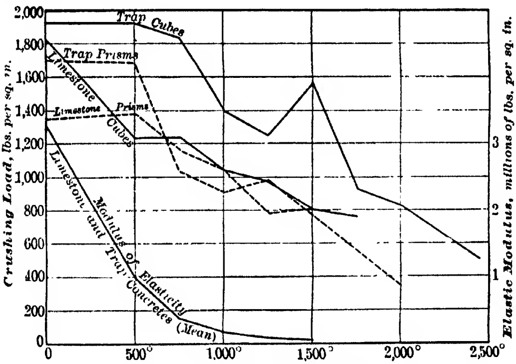


Fig 6. Heat Resistance.

70 b. High temperatures. '05, p 335. Fig 6.
Specimens. For comp strgth, 4″ cubes; for elasticity, prisms 6″ × 6″ × 14″. 3 cubes and 3 prisms tested without heating; 3 cubes and 2 prisms of each agg (trap and limestone) at each temp.

Results.

70 c. Elastic modulus, E. For *E*, the trap and limestone curves nearly coincided.

70 d. After heating to 2000° and 2250° F, the limestone cubes appeared sound while hot, but **disintegrated when cooled.**

For Directory *to* Experiments, see pp 1303-7.

70 e. After cooling from 750° F, both trap and limestone prisms were covered with minute cracks. Under higher temps, these cracks increased in number and in size, and the prisms warped and disintegrated after cooling from 1500° F.

70 f. The **trap and cinder** conc specimens remained **sound**, while the **gravel** conc specimens **cracked** and crumbled in pieces, probably owing to high **expansion** coeff of quartz, and to the fact that this coeff in one direction, is double that in the perp direction.

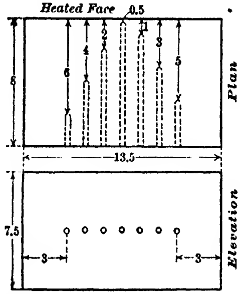
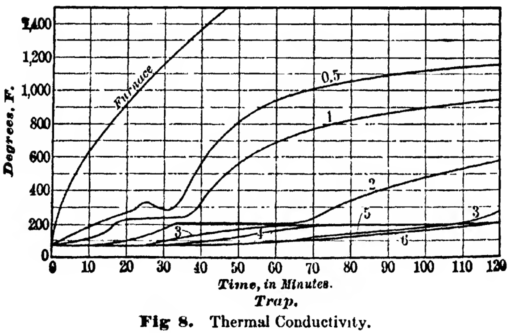


Fig 7. Thermal Conductivity. Dimensions in inches.



Trap.

Fig 8. Thermal Conductivity.

70 g. Thermal conductivity, '07, p 404. Figs 7 and 8.

Specimens. Conc blocks, with holes as in Fig 7. Dimensions in inches. Thermo couple in each hole. Mixture as in **70 a.**

Treatment. Specimens in molds 24 hrs, in water 48 hrs, kept moist 2 or 3 wks, allowed to dry well. Age, at test, about 2 mos. Blocks placed in furnace doorway.

Results. Fig 8 shows, for one of the trap conc specimens, the **times**, in mins, reqd **to transmit the furnace temps** thru diff thicknesses of conc. Each curve is marked with this thickness in ins. Drop of curves, at and near 200° F, attributed to **steam generation.**

For abbreviations, symbols and references, see p 1251.

70 h. 2 to 2½″ of **conc** (if it remains in place) **will protect** reinfg metal during any ordinary conflagration.

70 i. Exposed reinforcing metal will not conduct heat injuriously to imbedded portion.

70.5. Wm. B. Fuller and Sanford E. Thompson, "The Laws of Proportioning Concrete," A S C E, Trans, '07/Dec, Vol 59, pp 139–143.

Elastic modulus, E, under compression.

Specimens. 6″ sq conc prisms, 18″ long; age, abt 140 ds. Giant Port cem. Agg: Cowe Bay sand (CS), Jerome Park screenings (JSc). **Agg:** Cowe Bay gravel (CG), Jerome Park stone (JSt).

Results.

Effect of maximum size of stone.

Mix	1 : 9*	1 : 3 : 6	1 : 2.81 : 5.62	1 : 2.92 : 5.88
Stone	Elastic modulus, *E*, in millions of pounds per square inch.			
2.25 ins.	2 1	2 4	3.3	3.0
1.00 "	1.7	1 8	3.1	2.6
0.50 "	1 4	0.9	...	2.2

Effect of quantity of cement, in % of total dry material.*

Elastic modulus, *E*, in millions of pounds per square inch.

	With JSc and JSt.				With CS and CG				With JSc and CG		
Cem..	8	10	12.5	15	8.5	10.6	13.25	15.9	10.2	12.75	15.3
E	1.8	2.1	2.3	4.7	2 3	3.9	3.7	4.3	3.5	3.8	3.5

— 71 —

71. Richard L. Humphrey, U. S. G S Bull, No. 324, '07. Report on **San Francisco fire** of Apr 18, '06.

Results.

71 a. Conc probably the **best material** for fireproofing cols. Its stiffness supports the steel within, softened by the heat.

71 b. "**Conc** proved **superior to brick** as a fireproofing medium."

71 c. At high temps, conc loses its **water of crystallization.**

71 d. Conc, especially when reinfd, resisted both **earthquake** and **fire.** The **conc dam,** at San Mateo, altho within a few hundred yds of the fault, was uninjured. Solid **conc floors,** altho of very poor quality, proved satisfactory The **cinder conc** used, in floors and elsewhere, was high in sulfides, and injurious to reinfmt.

— 72 —

72. Wm. B. Fuller, Natl Assn of Cem Users, Procs, '07, pp 95–7.

Grading and proportions.

72 a. Tests of 6 beams, 6″ square, 6 ft long; 1 cem to 8 of sand and stone; **rupture moduli** in lbs/□″: 1 : 2 . 6, 319; 1 : 3 : 5, 285; 1 : 4 : 4, 209; 1 5 : 3, 151; 1 : 6 : 2, 102; 1 : 8 : 0, 41.

72 b. With a given percentage of cem, the **densest** mixture of sand and agg gives the strongest, the least permeable and therefore the most durable conc, and that which works most easily and therefore best fills up voids and corners.

— 73 —

73. Commission du ciment armé, Paris, '07.

73 a. Shrinkage and expansion. Conc shrinks while hardening in air, and expands under water.

* Material, larger than 0.2″ diam (abt 62 to 68 % of total) graded in accordance with the recommendations of the authors. See Plain Concrete, ¶¶ 23 to 25, p 1257.

For Directory to Experiments, see pp 1303-7.

70 e. After cooling from 750° F, both trap and limestone prisms were covered with minute cracks. Under higher temps, these cracks increased in number and in size, and the prisms warped and disintegrated after cooling from 1500° F.

70 f. The **trap and cinder** conc specimens remained **sound**, while the **gravel** conc specimens **cracked** and crumbled in pieces, probably owing to high **expansion** coeff of quartz, and to the fact that this coeff in one direction, is double that in the perp direction.

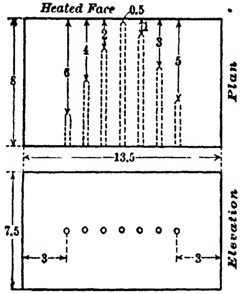


Fig 7. Thermal Conductivity. Dimensions in inches.

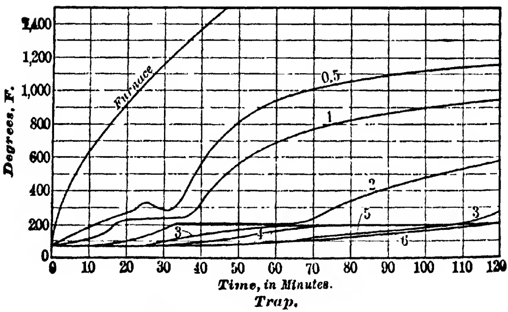


Fig 8. Thermal Conductivity.

70 g. Thermal conductivity, '07, p 404. Figs 7 and 8.

Specimens. Conc blocks, with holes as in Fig 7. Dimensions in inches. Thermo couple in each hole. Mixture as in **70 a.**

Treatment. Specimens in molds 24 hrs, in water 48 hrs, kept moist 2 or 3 wks, allowed to dry well. Age, at test, about 2 mos. Blocks placed in furnace doorway.

Results. Fig 8 shows, for one of the trap conc specimens, the **times**, in mins, reqd **to transmit the furnace temps** thru diff thicknesses of conc. Each curve is marked with this thickness in ins. Drop of curves, at and near 200° F, attributed to **steam generation.**

Louis, 80 to 95 lbs/cu ft, passing 1⅛" screen; abt half the stones larger than 1", about one-tenth of the stones less than ½"; voids 42 to 48 %. Voids, in 3 sand + 5 agg, 16 to 19 %.

Treatment. Comp specimens left in molds in air 1 day, beams 2 ds; then all in water 2 wks; then in air, protected from drafts, until tested. Comp specimens, 1 mo and 1 yr old, loaded 4 to 8 times per min; beams, 1 mo, 6 mos and 1 yr, loaded 2 to 4 times per min.

Results. Effect of rate of repetition insignificant; but believed to increase rapidly with rates above 10 per min.

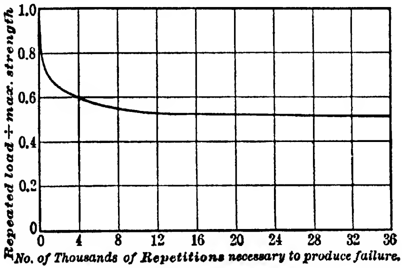


Fig 9. Fatigue.

Fatigue. The curve, Fig 9, fairly represents the results obtained under these varying conditions.

76 c. Conc, repeatedly stressed, below the fatigue limit (i. e., below about half max strgth, see Fig) "has imparted to it a **definite elastic limit,** within which stresses are proportional to strains" (i. e., within which the **elastic modulus,** E, is constant).

76 d. Fatigue and Adhesion.

Specimens. Plain ⅝" square steel bars imbedded in conc as above. Specimens made with great care and very thoroly tamped.

Treatment. In molds 2 days, in water 7 ds, in air 3 wks. 30 fatigue specimens subjected to "a combined blow, pressure and the accompanying vibration"; 150 blows per min, each blow = 740 inch-lbs. Av, 50,000 blows to each specimen.

Results. Av initial **adhesion,** 125 lbs/□" of imbedded surf; **friction** (after slip) 90 lbs/□". **Unfatigued specimens,** 150 and 100 lbs/□" respectively.

76 e. Fatigue under continued load. p 318. 2 conc prisms remained unaffected for a month under 90 % of their crushing strgth. "A few conc blocks failed in comp in a few hours under constant pres of higher %." A reinfd conc beam failed in 10 mos under 90 % of its breakg load.

— **77** —

77. Henry S. Spackman. Assn Am Port Cem Mfrs, New York, 07, Dec.

77 a. Mortar reground after hardening. Briquets of Port cem, broken in testing. Reground and made into new briquets. These showed, in general, **about half the tensile strengths** of the original briquets. Of the original cem, 91.5 % past a No. 100 sieve, 76.2 % past No. 200. The reground material had abt the same fineness.

— **78** —

78. R. Feret, A S C E, Trans, '07, Dec, Vol 59, p 152.

78 a. Permeability. "**Experiments give** in general **uncertain results.** It is not unusual to see many blocks of the same conc

For Directory to Experiments, see pp 1303-7.

which, altho treated in an identical manner, permit very diff quantities of water to filter thru them."

78 b. Age of block, days	5	29	30	365
	Flow, in grams/min per lb/□"			
Pres from 71 to 284 lbs/□"; Avge	0.554	0.044	0.159	0.294
After remaining under 284 lb/□" 2 hrs	0.349	0.034	0.133	0.278

78 c. Percolation "very nearly **proportional to pressure.**"

78 d. 3 blocks, 1 year old. Block	A	B	C
	Flow, in grams/min per lb/□"		
At 284 lb/sq in	0.067	0.111	0.108
Raised to 412 lb/□"	0.077	0.114	0.126
Reduced to 284 lb/□"	0.068	0.114	0.111

"as if the effect of the momentary increase of pres had been to open new passages for the water, or partly to clear out the passages already existing."

— 79 —

79. Wm. B. Fuller and S. E. Thompson, A S C E, Trans, '07, Dec, Vol. 59, p 67.

Strength, density and permeability, as affected by **proportions and character of sand and agg.** Expts at Jerome Park Reservoir, New York.

79 a. Specimens. Port cem, as received for use on the reservoir; agg (1) stone and screenings from crushers at reservoir, mica schist, 35 % mica, which, in mortar or conc, "does not form planes which affect the strgth seriously." (2) Cowe Bay gravel and sand, dredged from river ("water-worn rounded bank gravel and sand, thoroly clean, and consisting almost entirely of quartz particles." Sp gr abt 2.65). Max size of stone, 2¼", 1", ½".

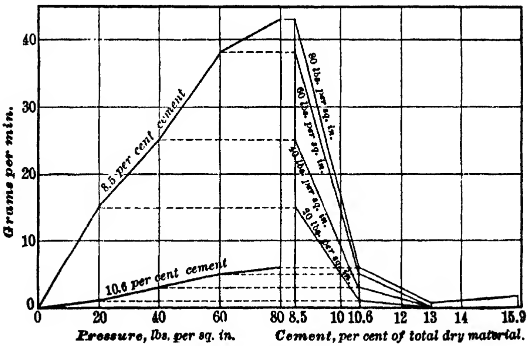


Fig 10. Permeability.

Tests were made with "**graded mix**" (proportions giving max density of agg) and "**natural mix**" (1 : 2.5 : 6.5, 1 : 3 : 6, 1 : 3.5 : 5.5).

Results.

79 b. Size of aggregate; strength and density.

Max stone size, inches	2¼	1	½
Relative strength.			
Compression	1.00	0.83	0.72
Transverse	1.00	0.91	0.75
Cem reqd for equal strgth, relative	1.00	1.17	1.32
Relative **density**	1.00	0.96	0.93

For abbreviations, symbols and references, see p 1251.

79 c. Kind of aggregate. Sand vs screenings. Relative strengths and densities.

	Comp strgth	Transv strgth	Density
Sand and stone	100	100	100
" " gravel	94	89	102
Screenings and stone	67	85	98

79 d. Graded mix gave **density** = 1.14 × density with natural mix; for equal strgth, graded mix reqd 0.88 × the **cem reqd** with nat mix.

(This means an **av saving** of about 25 cts **per cu yd** of conc. Allen Hazen, Trans, A S C E, Vol 59, p. 150, Dec, '07.)

79 e. An excess of fine or of medium **sand,** or a deficiency of fine sand in a lean conc, diminishes strgth and density.

79 f. Strength and density max when mortar just fills voids.

79 g. Permeability. See Fig 10. "Little is known of the action of conc in resisting the flow of water." As betwn "diff proportions and diff sizes of the same class of materials, the laws of watertightness are somewhat similar to those of strgth." With given percentage of cem, the **densest** specimens are usually **most watertight.** With equal densities, the **richest** specimens are **most watertight** (See Fig). The ratios, however, are very diff from those of either density or strgth, a slight diff in the composition producing a great effect upon the watertightness. "**Diff kinds of agg** produce very diff results in watertightness." Fig shows effect of **pressure** upon **permeability.**

79 h. Conc with Jerome Park stone and screenings gave very much higher rates of percolation thruout (max, 369 grams per min) than that with Cowe Bay sand and gravel. Conc with stone and sand gave about half the rates shown in Fig 10.

79 i. Permeability is sometimes greater with **large** and sometimes with **small stones.** Results especially erratic with the Jerome Park reservoir broken stone and screenings.

79 j. "**Permeability** decreases materially with **age;**" increases much more rapidly than the **thickness of the conc** decreases,

less with sand and gravel than with stone and screenings;
" " " " " " " " " sand;
" " " " stone " " " " screenings;

— 80 —

80. Richd H. Gaines, New York Board of Water Supply, A S C E, Trans, Vol 59, '07, Dec, p. 159.

80 a. Permeability and strength; Clay and alum.

Specimens. Mortar, 1 : 3, Portland, Cowe Bay sand. Tensile tests on standard briquets; comp and tensile tests on 2″ cubes. Age of specimens, 28 to 30 days. Pressures, 40 and 80 lbs/□″.

Results. (1) Replacing the mixing water with a 2.5 to 5 % (1 to 2 % sufficient) **alum** solution gave nearly complete **impermeability.**

(2) Replacing 5 to 10 % of the sand with dried and finely ground **clay,** and (3) combining (1) and (2), gave still better results.

The clay specimens (with and without alum) showed from 12 to 18 % gain in **strength** over those without clay.

The process is based upon a **theory of physico-chemical action** between ions of the electrolyte (alum) and the colloid (glue-like) molecules of the clay.

None of the **processes hitherto in use,** and examined, were found suitable for extensive use.

Slaked lime slightly decreases **permeability,** but this advantage is more than offset by loss of **strength.** There is no chemical reason why this should be otherwise.

For Directory to Experiments, see pp 1303–7.

— **81** —

81. Prof E. Mörsch, Zurich; for Wayss and Freytag A.-G., Neustadt. "Der Eisenbetonbau," Stuttgart, Konrad Wittwer, '08, to which the pages given refer.

81 a. Elastic relations, pp 27–32. **Specimens:** Square prisms; measured length, 35 cm (13¾"). 1 part Mannheim Port cem, with 3 parts of a mixture of Rhine sand and gravel consisting of 3 parts sand, 0–5 mm; 2 parts gravel, 5–20 mm. (0.197"–0.78"). Water, 14 %. Each stress maintained 3 mins. Some of the specimens tested in tension; the others in comp.

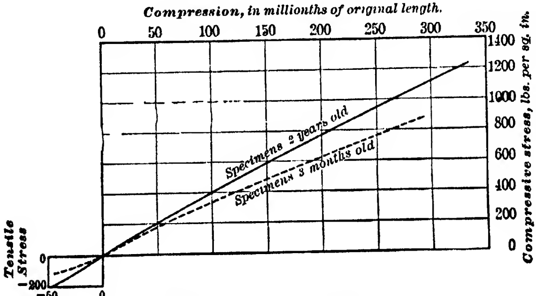


Fig 11. Stress and Stretch.

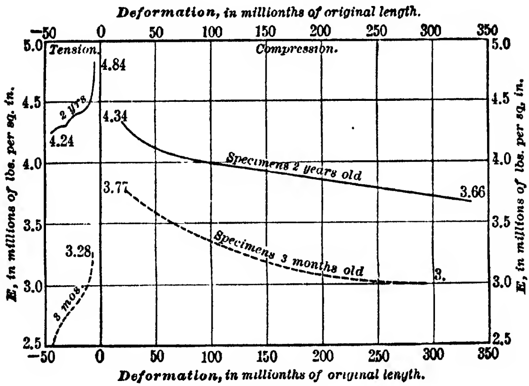


Fig 12. Elastic Modulus.

Results. **Unit stresses and stretches** as in Fig 11. Ult tensions, lbs/□" : 3 mos, 149; 2 yrs, 224.

Elastic Modulus, E, See Fig 12.

With mix 1 : 4, for a given stress in comp. **E** was in general from 15

For abbreviations, symbols and references, see p 1251.

to 20 % less than with 1 : 3. In tension, **E** was more nearly the same for both mixes.

With water 8 %, for a given stress, **E** was in general from 10 to 20 % higher than with water 14 %.

81 b. Shear. Fig 13. Dimensions in centimeters. Prisms, 18 cm square, 40 cm long, p 40. Mixture of sand and gravel as in Expt 81 *a*.

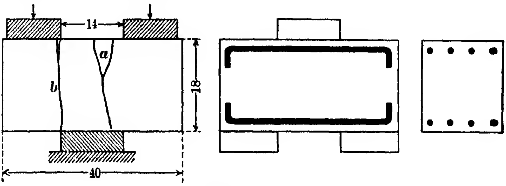


Fig 13. Shear.

Plain. Specimen first cracked, as beam, at *a*. Pres then increased until shearing crack, *b*, appeared.

Mix	Water %	Age	No. of Specimens	Ult av shear, lbs/□″* Observed	Calculated†
1 : 3	14	2 yr	3	936	980
1 : 4	14	1.5 m	3	530	550

Reinforced. The bars (1 cm diam) served merely to hold the specimens together, so that the pres could be increased as desired. The conc sheared first.

Mix	Water %	Age	No of specimens	Ult Av shear, lbs/□″ Concrete	Steel
1 : 4	14	1.5 m	2	522	46400
1 : 4	14	1.5 m	3	484	50800

81 c. Torsion, p 45. Mix, 1 : 4. **4 solid cylinders,** 79 to 98 days old; 26 cm diam; length under exp, 34 cm. Hexagonal heads. M = torsional moment; R = radius of cyl;

t = torsional stress in extreme fibers (see p 500, this book) = $2\ M/\pi R^3$
t, in lbs/□″; max, 275; mean, 243; min, 189.

3 hollow cyls, as above, 52 to 55 days old; inner diam abt 15 cm; r = inner radius.

$t = 2\ M\ R/\pi\ (R^4 - r^4)$,
t, in lbs/□″; max, 134; mean, 126; min, 112.

The much higher unit strength of the solid cylinders as given by the formulas, is attributed partly to their somewhat greater age, but chiefly to the increase in unit stress from the circumf inward, owing to which the material near the center transmits more than its share of the torsional stress, and thus relieves the outer portions.

* = ½ total force applied ÷ area of one shearing surf.

† From ult tensile strgth, t, and ult comp strgth, c, of test pieces of same mix and age, and formula, shear = $\sqrt{tc}$.

For Directory to Experiments, see pp 1303-7.

— 81 —

81. Prof E. Mörsch, Zurich; for Wayss and Freytag A.-G., Neustadt. "Der Eisenbetonbau," Stuttgart, Konrad Wittwer, '08, to which the pages given refer.

81 a. Elastic relations, pp 27-32. **Specimens:** Square prisms; measured length, 35 cm (13¾"). 1 part Mannheim Port cem, with 3 parts of a mixture of Rhine sand and gravel consisting of 3 parts sand, 0-5 mm; 2 parts gravel, 5-20 mm. (0.197"-0.78"). Water, 14 %. Each stress maintained 3 mins. Some of the specimens tested in tension; the others in comp.

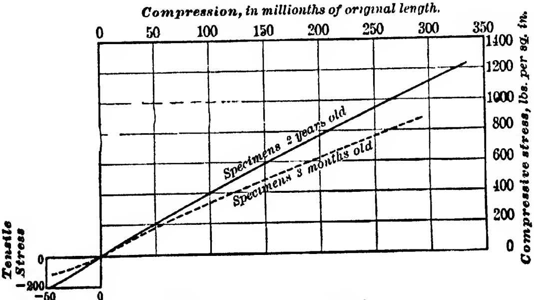


Fig 11. Stress and Stretch.

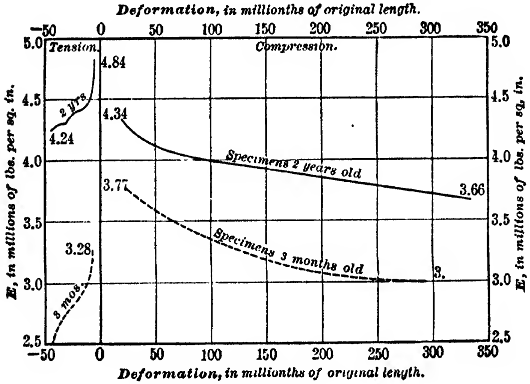


Fig 12. Elastic Modulus.

Results. **Unit stresses and stretches** as in Fig 11. Ult tensions, lbs/□" : 3 mos, 149; 2 yrs, 224.

Elastic Modulus, E, See Fig 12.

With mix 1 : 4, for a given stress in comp. E was in general from 15

For abbreviations, symbols and references, see p 1251.

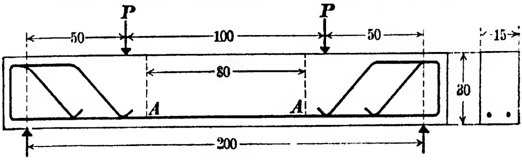


Fig 16. Ductility.

Results. Stretch per unit of length at instant of first cracking of conc:

	Steel	Conc, under tension, max
Bars 10 mm (0.39″) diam = 0.4 %	0.00042	0.00050
" 16 " (0.63″) " = 1.0 %	0.00033	0.00040
" 22 " (0.86″) " = 1.9 %	0.00030	0.00038

81 g. Steel and concrete stresses, p 97.

Specimens. Flat reinforced beams, Fig 17.

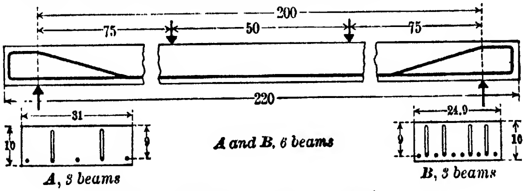


Fig 17. Stresses. Dimensions in centimeters.

Bendg mom constant betw loads. Mix 1 : 4. Length, 2.2 m; span 2 m.

Results. Failed by crushing of conc near and betw the 2 loads. Steel, 10 mm diam.

Unit stresses, *s*, in steel, and *c*, in conc, in lbs/□″, deduced under the assumption of $n = E_s/E_c = 15$.

	Age	Steel	After appearance of first cracks *s*	After appearance of first cracks *c*	At rupture *s*	At rupture *c*
3 beams A Fig 17	13 mo	1.4 %	22300	1315	54000	3180
3 " B " 17	13 "	3.3 %	20900	2250	39100	4210
3 " A " 17	2 "	1.4 %	18600	1095	·44800	2630
3 " B " 17	2 "	3.3 %	17000	1820	28000	3000

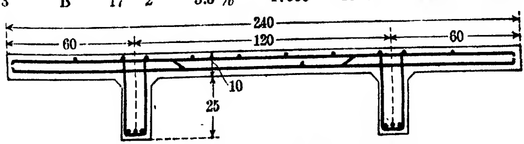


Fig 18. Shear. Dimensions in centimeters.

81 h. Shear in beams. 12 specimens, each consisting of a flat plate with two similarly reinfd ribs, Fig 18. Ribs of 2.7 m span normal to the paper. Der Eisenbetonbau, p 158.

For Directory to Experiments, see pp 1303-7.

Types of web reinforcement, neglecting slight variations. See Fig 19, and 3d col of table below.

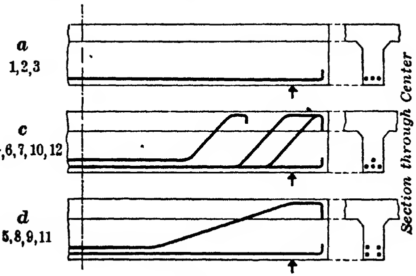


Fig 19. Shear.

Stirrups: 4th col, table below: a, thruout span; b, in one half of span; c, no stirrups.

Bars: diam in mm: a, 18; b, 16; c, 3 bars 15, and 1 bar 18; d, 2 bars 15, and 2 bars 16. Beam No. 3 had 3 straight Thacher bars, 18 mm diam.

Ends; 6th col, table below: a, hook; b, plain; c, 3 bars 45°, 1 hooked; d, 2 bars bent, 2 hooked; e, 3 bars 45°, 1 plain.

In No. 2 the **webs** were 0.28 m wide; in No. 8, 0.10 m; in the others. 0.14 m.

Age, about 3 mos. Heidelberg cem 1 : 4.5 (72 % Rhine sand 0-7 mm; 28 % gravel, 7-20 mm).

Results.

Cracks developed, following, in general, the curves convex upward, Fig 20. Stresses, in lbs / □″.

s = tensile, in steel; c = comp, in conc; a = adhesion; v = shearing, at support.

Load	Beam No.	Type	Stirrups	Bars, diam	Ends	At appearance of diagonal cracks which lead to rupture			At rupture				Beam	Load
						s	a	v	c	s	a	v		
Uniform	1	a	b	a	a	17900	123	149	540	29300	198	239	1	Uniform
	2	a	b	a	a	34300	234	142	824	44800	302	183	2	
	3	a	b	..	b	19500	103	132	398	27800	146	187	3	
	4	c	c	c	c	36600	382	309	881	46300	476	384	4	
	5	d	b	d	d	17900	205	146	686	37000	418	299	5	
	6	c	a	c	e	.....	232	186	795	42000	432	348	6	
2 concentd*	7	c	a	b	c	.....	...	...	924	48600	448	318	7	2 concentd
	8	d	b	b	d	15800	152	152	676	34800	324	324	8	
	9	d	b	b	d	22500	216	141	742	38200	352	251	9	
1 central	10	c	b	b	c	.....	...	...	1100	55000	362	257	10	1 central
	11	d	c	b	d	.....	...	...	1180	54000	357	255	11	
	12	c	c	b	e	.....	...	...	1060	53200	348	249	12	

* The positions of the 2 concentrated loads divided span into 3 equal parts

For abbreviations, symbols and references, see p 1251.

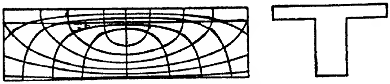


Fig 20. Diagonal Stresses.

— 82 —

82. Sanford E. Thompson. A S T M, Procs, '08, Vol 8, p 500.

82 a. Permeability. Effect of admixture of **slacked lime.**

Specimens. Cylindrical blocks, 20″ diam, 16″ thick; Lehigh cem, good av bank sand, conglomerate rock resembling trap in character; "a soft, mushy mix, such as would be adopted in construction." Pine Cone lime from Rockland, Me. Lime stated in % of wt of dry cem. Mixtures as follows.

1 : 2 : 4 conc with 0 %, 4 %, 7 % and 10 % lime; 8 % preferred;
1 : 2.5 : 4.5 " " 0 %, 6 %, 10 % " 14 % " ; 12 % " ;
1 : 3 : 5 " " 0 %, 8 %, 14 % " 20 % " ; 16 % " .

Treatment. Water, under pres, introduced into cen of block.

Results, 1 : 2 : 4 and 1 : 3 : 5, see Fig 21. 1 : 2.5 : 4 gave results intermediate betw the other two.

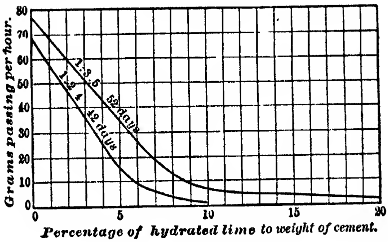


Fig 21. Permeability; Lime.

82 b. Coarser sand requires more lime, and vice versa.

82 c. If pressure is to be applied within a month, it will be better to use say 10 %, 15 % and 20 % respectively, instead of 8 %, 12 % and 16 % as recommended under Expt 82 a.

82 d. Lime paste occupies about $2\frac{1}{4}$ times the **bulk of paste** made with equal wt of Port cem, "and is therefore very efficient in void filling." The **cost of large waterproof work may be reduced** by using, with lime, a leaner conc than would otherwise be suitable.

— 83 —

83. Richard L. Humphrey, plain conc beams, cubes and cylinders, comp and transv strgths and the elas relations. "The Strgth of Conc Beams," U S G S Bull No. 344, '08. Tests to determine the effect, upon **transverse and compressive strength,** of (1) **age** of specimen, (2) **consistency** of mix, (3) character of **aggregate.**

83 a. Specimens. Unreinfd conc beams, cubes and cyls. Cem, a mix of 9 Port cems. Meramec R sand, "composed of flint grains having comparatively smooth surfs." "The granulometric analysis, p 1346, shows the sand to be rather finer than desirable."

Properties of sand and aggregates used.

	Sp gr	lbs/ cu ft	voids %	Meshes per inch of screen: 200	100	50	30	10	Size of mesh, ins: ¼	½	¾	1¼
				Percentage passing sieve or screen								
Cinders	1.53	47	51	2.84	4.17	6.5	10.5	21.1	37	60	81	100
Granite	2.59	95	41	1.59	2.29	3.2	4.4	8.5	20	58	99	100
Gravel	2.45	102	33	0	0	0	0	1.0	43	79	95	100
Limestone	2.49	98	37	2.96	3.48	4 2	5.2	10.7	29	61	96	100
Sand	2.60	101	38	0.20	1.30	13.9	64.0	97.0	100	..	..	...

Proportions. 1 : 2 : 4, by vol, except the cinder conc, which was nearer 1 : 2 : 5. All conc mixed in a mortar-driven cu-yd mixer, equipped with charging hopper. Mixed 2 mins dry, 3 mins wet, then dumped on cem floor, shoveled into barrows and wheeled to molding floor. Each batch sufficient for 2 beams, 8″ × 11″, 12 ft span, two 6″ cubes and 2 cyls, 8″ dia, 16″ long.

"**Wet:**" smooth and somewhat viscous immedy before dumping. Flows back from ascending side of mixer without tendency to break at top. When dumped, shows neither voids nor individual stones. Splashes when tamped. When finished, water stands ¼″ to ½″ deep over surf of mold.

"**Medium**": smooth, but tending to lump. Flows less smoothly than "wet," part flowing back smoothly and part breaking over in lumps. When dumped, looks somewhat lumpy, showing stones, but no voids. Stones evenly coated with mortar. No water collects on surf in mold. Surf easily finished with trowel.

"**Damp**"; granular. But little tendency to lump. Carried to top of mixer on ascending side; falls in individual stones and fragments of mortar. When dumped, shows stones and voids. Resists tamping. Compacts under hand tamping. Cannot be finished smooth with trowel.

Conc placed in oiled steel molds, in 3 nearly equal layers, and hand-tamped. "Great care was taken to tamp all the concs in the same manner."

Treatment. All molds were removed at end of 24 hrs, and pieces transferred to moist room. Sprinkled 3 times daily.

The beams were so supported, just prior to test, that the sums of moments and stresses, then existing in the measd length, were equalized, so that all fibers, in that length, then had same length as when unstressed, and the deformations, within the measd length, were thus measd from zero.

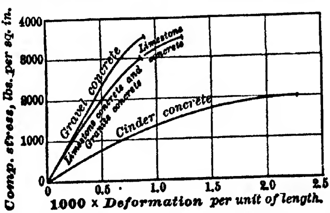


Fig 22. Stress-stretch curves for different aggregates.

Results.

Stretches and comp stresses as in Fig. 22. Medium consistency. Age, 26 weeks.

For abbreviations, symbols and references, see p 1251.

Strength of Concrete.

Results, in general, averages of 3 specimens.

	Water %	Beams, 8″ × 11″, 12 ft span: Neut axis* 100 m	Rupt mod† 4 wks	Rupt mod† 26 wks	Max comp strgth, lbs/□″: 6 in cubes 4 wks	6 in cubes 26 wks	Cylinders 8″ dia, 16″ long 4 wks	Cylinders 26 wks
Cinder								
Wet	21 9	43.3	175	246	1,256	2,320	1,081	2,021
Medm	20.6	39.9	198	277	1,191	2,765	1,201	2,203
Damp	18.9	38.2	198	250	1,378	2,488	1,118	1,945
Granite								
Wet	9.0	49.9	375	539	3,156	4,753	2,683	3,966‡
Medm	8.3	47.2	475	566	4,089	4,949	3,480	3,972‡
Damp	7.0	48.3	499	618	4,518	5,465	4,000	3,969‡
Gravel								
Wet	9.7	49.9	391	435	2,299	3,814	2 060	3,486
Medm	8.9	48.4	451	520	3,547	4,808	2,961	3,972‡
Damp	7.9	47.5	426	496	4,612	4,884	3,407	3,969‡
Limestone								
Wet	10.9	48.8	422	507	5,141	3,460	3,072	3,216
Medm	10.0	50.7	458	566	2,975	3,896	2,910	3,691
Damp	8.5	48.1	537	589	4,367	5.025	2,894	3,942‡

— 84 —

84. R. G. Clark, Inst C E, Procs, Vol 171, '08, p 115.

84a. Time of setting increased by **aeration** and by **addition of agg.** A cem, which, neat, sets in an hr, will make a conc requiring 4 or 5 hrs to set.

— 85 —

85. Hanisch and Spitzer, Morsch, Der Eisenbetonbau, '08, pp 32–33.

85a. Rupture modulus, 6 $M / b d^2$, **and direct compressive and tensile strength.**

Specimens.

Conc, 1 : 3.5. Six plates, 268 days old, 60 cm (24″) wide, 7.8 to 11 cm (3 to 4.5″) thick; span, 150 cm (60″).

Treatment. Plate broken transversely; comp and tension test pieces made from the fragments.

Results. Stresses in lbs / □″.

	Rupture modulus	compression	tension
max	775	5000	412
mean	682	4380	356
min	614	3640	284

Comparison of the values for tension with the rupture modulus shows that the **formula,** rupture mod = 6 $M / b d^2$, is not applicable to materials in which, as in conc, the elas mod varies widely, and that the rupture moduli, obtained by means of the formula, are to be used only as a means of comparison.

— 86 —

86. Richard L. Humphrey and Wm. Jordan, Jr., U S G S, Bull No. 331, '08. Results of Tests made at the Structural-Materials Testing Laboratories, St. Louis, '05-7.

86 a. Gravel screenings. In general the tensile and comp strgths of mortars seem to increase with density of screenings.

* m = (depth of neut ax below top of beam) ÷ (total depth of beam).

† "Rupture modulus" = 6 $M / b d^2$, lbs / □″; M = moment under max load.

‡ Cylinder did not break.

For Directory to Experiments, see pp 1303–7.

86 b. Stone screenings. In general, strgth of mortar was greatest with screenings most nearly uniform in grading. The **strength of the stone** itself, from which the screenings are derived, has an important bearing on the strgth of the resulting mortar.

86 c. Density of mortars is greatest with densest **sand.**

86 d. Sand mortars. Tensile, compressive and transverse **strengths** were invariably much **greater with dense sands** than with those having a larger percentage of voids.

86 e. Greatest strgth obtained when sand is uniformly graded.

86 f. A "typical mix" of 7 Port cems, like the separate brands, reached max tensile **strength** in 90 days. Like the best of these, it maintained this max to 180 ds, and its subsequent loss, at one yr and later, was no greater than for the best of the separate brands.

86 g. Age of briquet. Tests after 180 days showed greater uniformity than at 90 days and shorter periods.

86 h. After the 180 and 360 day tests, the strgths of all the sand mortars were reasonably close to one another, showing that considerable variation in **early strength** does not seriously affect the **later strength.**

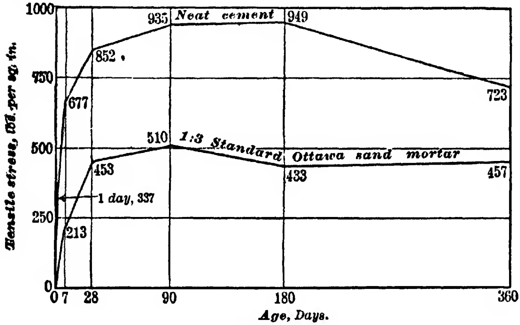


Fig 23.

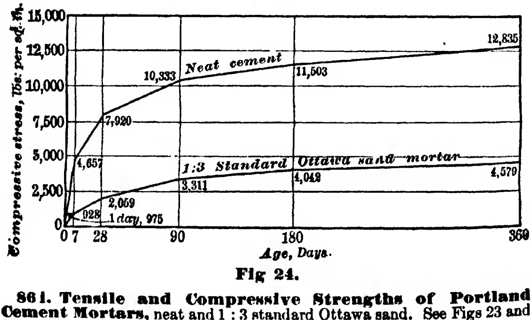


Fig 24.

86 i. Tensile and Compressive Strengths of Portland Cement Mortars, neat and 1 : 3 standard Ottawa sand. See Figs 23 and 24. Each curve represents an av of 10 tests.

For abbreviations, symbols and references, see p 1251.

Specimens. The cem was a mixture of equal parts of 7 diff brands. See Expts 86 f, 86 g and 86 h.

Test pieces, in molds, stored in moist closet 24 hrs; then kept in running water, abt 70° F, until tested. Tension briquets 1 sq inch section. Compression specimens, 2″ cubes.

Results as in Figs 23 and 24.

— 87 —

87. W. N. Willis, South & Western R. R. E R, '08, Jan 18; E N, '08, Feb 6, p 145.

87 a. Mica; water required; strength.

Specimens.

Sieve No	10	20	50	100
% of mica passing	100	29	10	4.5

Sand, Ottawa standd. Mortar 1 : 3 sand, or 1 : 3 sand and mica by wt.

Results.

Mica; % of weight of sand	0	5	10	15	20
Voids, % in Ottawa sand	37	...	...	...	67
Relative **sp gr** of Ottawa sand	100	...	...	...	80
Mixing Water required; relative	100	..	...	...	300
Tensile strength, 6 mos, relative	100	64	62	59	40

The smoothness of surf of the mica particles renders their adhesion low.

— 88 —

88. Prof J. L. Van Ornum, Washington Univ, St. Louis; for Reinforced Concrete Constr Co., St. Louis. E N, '08, Feb 6, p. 142.

88 a. Adhesion.

Specimens. Plain round steel rods, diams, ½ to 1¼″, imbedded in 12″ × 12″ prismatic blocks of 1 : 2 : 4 conc, 90 days old. Medium steel rods imbedded 25 diams; high carbon steel rods, 40 diams.

Results. See table below, in which.

for Steel:

s = Ult strgth, in thousands of lbs/□″;
s_e = Elastic limit, in thousands of lbs/□″;
e = Elongation, %;
E = Elastic mod, in millions of lbs/□″.

for Steel and concrete:

a = Area of imbedded surf, □″;
B = Adhesion, lbs/□″ of a;
F = Friction after slipping, lbs/□″.

Steel	Steel				Steel and Conc.			
	s	s_e	e	E	a	B	F	F/B
Medium								
Max	60 9	40.5	29.0	29.9	126.8	460	380	0.826
Av	58.6	39.1	26.1	29.5	62.1	408	342	0.838
Min	55.6	38.4	22.5	28.6	21.7	370	310	0.838
High Carbon								
Max	109.6	60.7	20.7	30.6	198.3	470	280	0.596
Av	92.6	56.1	17.6	29.8	92.1	392	240	0.613
Min	83.9	53.1	15.7	28.9	32.7	330	200	0.606

In all cases, the total pull which overcame the adhesion exceeded that which brought the steel to its elas lim.

— 89 —

89. W. S. Reed. Engrs' Club of Phila., Procs, Vol 25, No 3, p 290, '08, Jul.

89 a. Friction of sand. Exp by More and Harris Tabor. Top pres, lbs/□″, reqd to give 10 lbs/□″ at bottom of box.

For Directory to Experiments, see pp 1303–7.

	Depth of sand, ins			
	2.5	5	7.5	10
Box	Top pressure, lbs / □″			
4″ × 4″	12.5	17.5	34	42
6″ × 6″	11.5	.	. .	26

89 b. Fusing point of quartz sands. Exp by Prof Heinrich Ries, Cornell Univ. 3254° F.

— **90** —

90. Eng News, '08, Aug 27, p. 238.

90 a. Sea water. Charlestown, Mass, Navy Yard.

Nonreinforced arches, built '01, by Bureau of Yards and Docks **Tidal salt water**, not highly polluted, but often **freezing;** range of tide 10 ft. Specification called for "continuous construction from pier to pier of the arch rings." 3″ mortar face, 1 : 1. Mass conc 1 : 2 : 4 for 2 ft back from face, 1 : 3 : 6 interior; "a standd cem and a local gravel." Probably porous. No special effort toward density or waterproofing. Specfn provided: "The contractor must furnish satisfactory evidence of the durability in sea water of the brand of cem he proposes to furnish." The showing spandrel walls were built after completion of arch ring. Dry, well-tamped. Serious disintegration. Damage mainly betw H W and L W. Conc backing considerably affected.

— **91** —

91. U. James Nicholas, Melbourne, Victoria. E N, '08, Dec 24, p 710.

91 a. Electrolysis in cement mortars.

Specimens. 16 cylinders, 8″ diam, 8″ high. Standd Port cem; coarse sand, voids 51 %. Mortar tamped in 1½″ layers until a little water flushed to surf. Positive electrode, normally a 1″ steel pipe, 12″ long, lower end corked, immersed, in axis of cyl, to depth of 5″ in conc.

Treatment. Cyls set in fresh water ≮ 28 days. 8 cyls tested with **constant current** of about 0.1 ampere; 5 with **constant potential** of about 115 volts (higher currents, one with reversed current); 3 not subjected to current. For current, cyls placed in 3 % salt solution in separate metal pails (which normally formed the negative electrodes), and connected in series. Cyls from 29 to 57 days old at beginning of test.

Results.

All cylinders, under current, **cracked.** Cracks attributed to accumulation and pres of liberated gases. Cracks at first hair-like, exuding moisture, which dampened adjacent surf. Cracks widened under continued current. With constant current, cracks appeared when resistance reached max. Resistance in general inversely proportional to **percentage of sand.** Cyls Nos 1 and 2 easily pried open. In Nos 2 and 9, **steel pipe** was **rusted** and pitted on outside, adjacent to crack. With (const potential) reversed current (No 12), no rust or pitting.

Cyls not subjected to current were not cracked. They reqd about 20 blows, with heavy hammer and cold chisel, to break them. No rust.

	Constant Current, 0.1 ampère No of Specimen.								Constant Potential, 115 volts No of Specimen.				
	1	2	9	10	13	14	5	6	3	11	12	15	7
Mix	1 : 3	1 : 3	1 : 1	1 : 1	1 : ⅓	1 : ⅓	1 : 0	1 : 0	1 : 3	1 : 1	1 : 1	1 : ⅓	1 : 0
Sand, %	75	75	50	50	25	25	0	0	75	50	50	25	0
Days*	7	7	10	16	15	15	28	15	...	...	...	...	..
Mins*	...	...	...	...	...	...	...	...	5	19	20	9	9
Ohms†	80	90	420	270	230	270	2900	1080	120	130	240	163	190

* To first crack. † Approximate maximum resistance.

For abbreviations, symbols and references, see p 1251.

— 92 —

92. "H," of Lafayette, Ind. *Letter in E N, '08, Dec 31, p. 751.*

92 a. **Clay.** In conc for cols, gravel contained 5 % clay, which floated to top in churning, and left 1 ½" of worthless material near top of col.

— 93 —

93. **A. Q. Campbell,** Ogden, Utah. E N, '08, Dec 31, p 751.

93 a. **Grading and impermeability. Finish.** 2 million gal rectangular reinfd conc water tank, 20 ft deep. Floor, 6" thick; walls 8 to 18". 1 cem, 2 ordinary sand, 4 stone (quartzite boulders, porphyry and flinty limestone) crushed to 1", with dust; "a heavy percentage of crushed dust and sand"; machine mixt; "consistency that would almost pour." Floor laid in blocks about 15 ft sq, "allowing a half-lap of 2 ft;" walls in continuous 20" layers. Finish of 1 : 1 cem and crusher dust, applied with ordinary broom trimmed short. Clear water. No perceptible checking in surf. Apparently no seepage.

— 94 —

94. **John C. Trautwine, Jr.** '09.

94 a. **Density of sand: shape of grain.** 100 measures of rounded sand grains, or of angular crushed quartz grains, poured very slowly into 60 measures of water. Exps Nos 1 and 2 were made with sand grains; Nos 3 and 4 with crushed quartz grains. The left side of each diagram, Fig 25, represents the bottom of the vessel; and the numerals, 94, 121, etc., show the elevations of the surfs of sand and of water respectively, after the sand grains had been poured into the water.

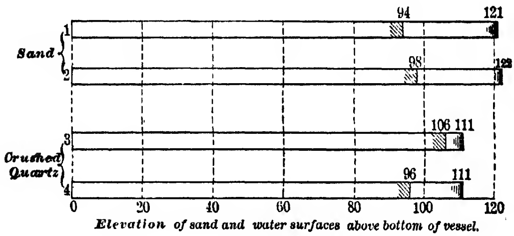


Fig 25.

In No 4, the crushed quartz, in the water, was stirred, from time to time, during the pouring, in order to liberate any air which, in spite of the slowness of pouring, might have been carried into the water with the sand grains. The fact that the water stands at practically the same ht in 4 as in 3, indicates that no more air was carried down in 3 than in 4, and that the stirring merely brought the grains into closer contact than when left to themselves.

DIGEST OF SPECIFICATIONS, ETC.

FOR GENERAL CONCRETE WORK,

Pages 1354 to 1369.

LISTS OF SPECIFICATIONS, ETC, USED.

Alphabetical List.

See Classified List, p 1353.
(For additional abbreviations, see also p 1251.)

AH, Algoma Harbor, Wis., Caisson breakwater, etc, U. S. Engrs, '08, Jan 24.

BB, Breakwater, Buffalo, N. Y., Emile Low, U. S. Engrs A S C E, Trans '04, Jun, Vol 52, p 73.

BR, Black Rock Harbor and Channel, Buffalo, N. Y. Ship lock walls U. S. Engrs, '07, Dec 19.

Bu, Burlington, Vt., Mechanical filter plant, Hering and Fuller, '07.

Ch, Chicago, '08; proposed amendments to Building Code of '05-6.

Ci, Cincinnati, O, Geo. H. Benzenberg;
a, Filters, etc, '05; **b,** Head-house, etc, '06.

Co, Columbus, O, John H. Gregory;
a, Filters, etc, '05; **b,** Pumping station and intake, '06.

CR, Columbia River impvmts, Ore. and Wash., Canal. U. S. Engrs, '08, Aug 1.

CS, Concrete-Steel Engineering Co., Edwin Thacher, genl specfns; Melan, Thacher and von Emperger patents, '03.

F, Wm. B. Fuller, Filters, specification received, '08.

FP, Pensacola, Fla., repair and protection of sea walls. U. S. Engrs, 08, Apr 18.

FW, Fort Williams, Me., Wharf, Ship Cove. U. S. Engrs, '08, April 14.

G, General practice.

Hb, Harrisonburg, La., Lock and dam No. 2. U. S. Engrs, '08, May 13.

IM, Illinois & Mississippi Canal, Locks, Eastern Section. U. S. Engrs, Jas. C. Long, Western Soc of Engrs, '01, Apr, Vol 6, No. 2, p 132.

JC, Recommendations in Report of Joint Comm of A S C E, A S T M, Am Ry Eng & M W Assn, and Assn of Am Port Cem Mfrs, '09, Jan.

L, Louisville, Ky., Building Ordinance, '07.

Lp, Liverpool Harbor Improvement, Geo Cecil Kenyon, A S C E, Trans, '04, Jun, Vol 52, p 36.

Lv, Louisville, Ky., Southern Outfall Sewer, '07.

Mc, McCall Ferry dam, Susquehanna River, Pa., '08.

Mh, Manhattan, Borough of —, Regulations of Bureau of Bldgs, '03, Sep.

Ms, Massachusetts Legislature, Acts and Resolves of the —, '07.

NO, New Orleans, La., Water Purification Stations, '06, Sep 5.

NY, New York. Building Code approved '99, Oct 24, with amendments to '06, Apr 12.

OD, Ohio R below Pittsburg, Pa., Dam No. 19, Abutment. U. S. Engrs, '08, Jul 25.

Ph, Philadelphia. Regulations of Bureau of Bldg Inspection, approved '07, Oct 8. Engrs' Club of Phila., Oct '07, Vol 24, No 4.

SE, Superior Entry, Wis., South Pier, Clarence Coleman, Asst Engr. Report, Chief of Engrs, U. S. A., '04, Part 4, pp 3779, etc.

TR, Tennessee R, below Chattanooga, Tenn., River wall. U. S. Engrs, '08, May 27.

T & T, Taylor and Thompson, "Concrete, Plain and Reinforced," publisht by John Wiley and Sons, New York, '05, pp 33-37.

Un, Underwriters, National Board of Fire —, Building Code recommended, New York, '07.

WH, Waddell and Harrington, general specifications, received '07, Dec.

Wv. Wellsville, O., Navigation pass, Dam No. 8, near —. U S. Engrs, '08, Feb 27.

Yo, Yonkers, N. Y., covered masonry filters, '07.

Classified List.

See Alphabetical List, p 1352.

U. S. Govt work, **AH, BB, BR, CR, FP, FW, Hb, IM, SE, Wv.**
Breakwaters, **AH, BB, SE.**
Sea walls, **FP, SE, TR.**
Locks and canals, **BR, CR, Hb, IM.**
Harbor improvement, **Lp, SE.**
Wharves, **FW, Lp.**
Dams, **Hb, MC, OD, Wv.**
Pumping stations, etc, **Ci b, Co b.**
Filter plants, **Bu, Ci a, Co a, F, NO, Yo.**
Sewers, **Lv.**
Bridges, **CS.**
Building codes, **Ch, L, Mh, Ms, NY, Ph, Un.**
General, **CS, JC, T & T, WH.**

Outline of Contents.

For lists of Specifications for Concrete, see pp 1352, 1353.

In order to compare intelligently the requirements of diff specfns, the character of the work involved must of course be taken into account.

DIGEST.

Cement.

1. Brand. Portland or natural, **NY**; Port just under lower miter sill, nat elsewhere in foundations, Port in lock walls except for a backing, 2 ft deep, at base, Port and nat bonded together, **IM**; for reinforced work, Portland, **G**; Am Port, **CS, BR, Hb, FW**; "Universal" Portland cement, **SE**; cem made by mfr of established reputation (in successful operation not less than 2 yrs, **F**), brand in continuous successful use (in America, **F**) for the last 5 yrs (3 yrs, **CS**) **G**; in satisfactoryuse in similar quantities by U.S. Engr Dept at Large, **TR**; of tried uniformity, in use not less than 3 yrs in similar climate, **CR, Hb**; only one brand to be used, **G**; except for good reasons, **F**; only one brand in any monolith, **FP**. Portld in reinfd work and where subject to shocks or vibrations or to stresses other than direct comp; nat in massive work where weight is more important than strgth, and where economy is the governing factor; puzzolan only for foundations underground, not exposed to air or to running water, **JC**.

2. Requirements. For Strengths, etc., see Digest of Specfn for cem, by A S T M, p 940, Report of Board of U. S. Engr Officers, Prof'l Papers No 28, Corps of Engrs, U.S.A., '01, p 937, and Digest of Specfn by Engng Standards Comm of Great Britain, p 940. **For tests,** see Digest of Specfn of A S C E, p 942. Slow setting, **FP**; must have been tested ≮ 6 mos, ≯ 12 mos, prior to issue of permit, **L**; must meet requirements of Prof'l Paper No. 28, Corps of Engrs, U.S.A., '01, p 940, **BR, AH, TR, CR, FW, Wv, FP, Hb.**

3. Shipment. Packages to "contain either 380 lbs or some even division of 380 lbs," **Lv**; in cooperage or in cloth bags, **NO**; bag, 93 lbs (94 lbs, **Co**) net, bbl = 4 bags, **NO**; in bbls, lined with paper, **CR, WH**; in cloth bags, **Ci**; may be delivered in paper bags, **Wv.**

4. Storage at site of work. In weather-tight bldg, with floor raised (≮ 6", **T & T**) above ground, **G**; and holding ≮ 2 wks' supply under av conditions of work, **Ci**; cem in bags may be used after 3 mos storage, rejected if it becomes lumpy or otherwise deteriorated within that time, **BR**; cem, kept over winter, re-tested before using, **Wv.**

Sand.

5. General. Silica, hard, clean, sharp, **G.** Reasonably clean, coarse, **F**; water worn, voids = 35 %, **SE.** "Sharpness" purposely omitted, **T & T.** River sand, **Ci, a.**

6. Size. Well graded, with fine, medium and coarse grains, **F, Lv, NO, Co.** Coarse, or coarse and fine, mixed, **CS, T & T.** Coarse predominating; coarse preferred at double or treble cost, **T & T.** Medium, **Ci, a.** Largest to pass screen of ¼" mesh, **G.** ≯ 10 % coarser than ⅛", **NO**; ≮ 50 % retained on No. 30 sieve (holes 0.022" □), **WH.** ≯ 40 % to pass No. 50 sieve (2500 meshes / □"), **Hb.** ≯ 3 % very fine, **NO, Co, Ci, a.** ≯ 5 % very fine, **Bu.**

Foreign matter (clay, loam, sticks). None, **CS, T & T**; ≯ 2 % **NO**, ≯ 3 %, **Co, Lv**; ≯ 5 %, **Wv, OD, TR, CR, Bu.** ≯ 10 % clayey, **AH.** ≯ 3 % clay, etc, ≯ 2 % mica, **FW**; ≯ 4 % free loam, **Hb**; sand may be moist, not wet, **TR**; stored on a board platform, **CR**; or in bins, **Wv.**

7. Screenings. Crusher dust, passing ¼" screen, from broken stone, may be substituted for part or all of the sand, **T & T**; "screenings & crusht stone may be substituted for sand and gravel under special conditions," **F**; screenings permitted, **BR, CR**; if passing ¼" screen, **TR**; screenings preferred to sand, **AH.**

Aggregate ("Ballast").

8. Kind. Sand grit, gravel or broken stone, **BB**; gravel or broken stone, **G**; or both, **BR**; gravel, **Lv**; (see Screenings); sea-washed gravel, **Lp**; water-worn pebbles of igneous rock, **SE**; clean stone, gravel, broken hard bricks, terra cotta, furnace slag or hard clean cinders, **Un**; broken stone preferred, gravel permitted for interior of piers, pedestals and abuts, **WH**; broken stone, **AH.**

For abbreviations, symbols and references, see p 1251

9. Requirements. Clean, hard, durable; free from dust, loam, clay and perishable matter; washed or screened if reqd, **G**: approx cubical, **CS**, **AH**; free from long thin pieces, **BR**, **NO**, **CS**; ≮ 125 lbs/cu ft, **FP**; ≮ 130 lbs/cu ft, **Hb**; voids = 31 %, **SE**; drenched before using, **G**; but not to carry water, **Wv**; kept thoroly sprinkled, **IM**, **Hb**.

10. Sizes, inches: min, ¼, **G**; ⅛, **FW**, **Mc**: max ¾, **Un**: 1½, **Bu**; 2, **G**; 2½, **Hb**; 3, **NO**, **Co**, **Ci**, **a**, **FP**, **SE**: gravel, 3, **F**; stone, run of crusher, **F**, **Mc**, **AH**; 1 to 2½, according to grade of work, **AH**; for foundations, 2; for superstructure, 1½; for beams, cols and girders, 1, **L**; gravel, ≮ 90 % over 1½, ≯ 10 % sand, **Hb**.

	1 cubic foot of stone, gravel or sand grit contained					
Agg	cu ft	lbs	cu ft	lbs	cu ft	lbs
Stone;.. . coarse,	0.63	53.8;	fine, 0.33	30.4;	dust, 0.11	11
Gravel;. . pebbles, ⅜″,	0.80	81.5;	sand, 0.29	29.2;	...	..
Sand grit; gravel, ⅛″ to ¼″,	0.47	47.2;	sand, 0.59	59.3;	.	..

BB.

11. Storage. Stored on wooden platforms, **CR**, **Wv**; or in bins, **Wv**.

12. Cinder concrete. Allowed only for floors, roofs and filling, **Ms**. Reinfd cinder conc to be used only upon special permit of Inspector of Bldgs, **L**.

13. "May be used for all bldgs in which fireproof construction is mandatory by this Chapter, or where ordinary constr, mill constr or slow burning constr may be used," not for cols, piers or walls. Clean, thoroly burnt steam-boiler cinders; mix, Port cem, not poorer than 1 : 7. Cinders must pass 1″ sq mesh, **Ch**.

14. "All other special requirements and methods of calculation for reinfd conc as reqd in this Chapter shall modify and regulate the use of cinder conc in bldgs," **Ch**.

15. Large Stones.
Hard, sound, durable, as large as can be conveniently handled; washed clean; placed wet; one dimension ≮ 12″; no dimension less than 4″; no stone less than 2″ from faces exposed in finished work, conc joggled into place with light rammers, **Co**.

16. ≯ 100 lbs, ≮ 3″ from forms or from other large stones. (From Specfn for a Soldiers' Home.)

17. Permitted in walls > than 18″ thick, diam ≯ quarter of the thickness of wall, vol of stone ≯ one-fifth vol of wall, **Yo**.

18. One-man stones and larger, roughly cubical; long flat pieces to be broken or rejected; stones somewhat uniformly scattered thruout the work; ≮ 8″ apart, ≮ 2 ft from crest or down-stream face; dropped separately into bed of wet conc, pounded down if necessary; if necessary, conc spaded under and around the stones; each stone to be covered with conc before other stones are deposited. Use as many stones as possible without violating these conditions, **Mc**.

19. "Plums." Stones, from one-man to several tons (sometimes from old masonry), aggregating abt 30 % of the finished work ≮ 1 ft from wall surf. Set in top layer of conc and so as to form bond with next layer by projecting upward into it, **Lp**.

20. Proportions, see pp 1254 to 1258.

Measurement of Ingredients.

21. Cem measd "as if compacted so that 380 lbs of dry Port shall have a vol of 3.8 cu ft," **Lv**; cem measd loose, **CS**, **WH**; 1 bag cem ≮ 93 lb = 1 cu ft, **NO**, **Ci**. Cem measd as packt by mfr, **OD**, **L**, **T & T**. Sand and agg measd as thrown loosely into measuring box, **G**. All measd loose, **CS**, **WH**; 100 lbs cem considered to occupy the vol of 1 cub ft, **F**.

Consistency.

22. In general, "very wet," **NO**; water to come to surf with moderate ramming, **CS**; without serious quaking, **OD**, **TR**; sufficiently fluid to require no ramming, **Mc**; little or no tamping, **Hb**.

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5. General. Silica, hard, clean, sharp, **G.** Reasonably clean, coarse, **F**; water worn, voids = 35 %, **SE.** "Sharpness" purposely omitted, **T & T.** River sand, **Ci, a.**

6. Size. Well graded, with fine, medium and coarse grains, **F, Lv, NO, Co.** Coarse, or coarse and fine, mixed, **CS, T & T.** Coarse predominating; coarse preferred at double or treble cost, **T & T.** Medium, **Ci, a.** Largest to pass screen of ¼" mesh, **G.** ≯ 10 % coarser than ⅛", **NO**; ≮ 50 % retained on No. 30 sieve (holes 0.022" □), **WH.** ≯ 40 % to pass No. 50 sieve (2500 meshes / □"), **Hb.** ≯ 3 % very fine, **NO, Co, Ci, a.** ≯ 5 % very fine, **Bu.**

Foreign matter (clay, loam, sticks). None, **CS, T & T**; ≯ 2 % **NO**, ≯ 3 %, **Co, Lv**; ≯ 5 %, **Wv, OD, TR, CR, Bu.** ≯ 10 % clayey, **AH.** ≯ 3 % clay, etc, ≯ 2 % mica, **FW**; ≯ 4 % free loam, **Hb**; sand may be moist, not wet, **TR**; stored on a board platform, **CR**; or in bins, **Wv.**

7. Screenings. Crusher dust, passing ¼" screen, from broken stone, may be substituted for part or all of the sand, **T & T**; "screenings & crusht stone may be substituted for sand and gravel under special conditions," **F**; screenings permitted, **BR, CR**; if passing ¼" screen, **TR**; screenings preferred to sand, **AH.**

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For abbreviations, symbols and references, see p 1251.

(2) Stone shoveled upon mixture of cem and sand. In (1) or (2), turn 3 times, adding water in first turning.

(3) Mixture of cem and sand made into mortar and spread upon stone. Mass of mortar and stone turned twice, **T & T.**

32. In any case, result must be a loose conc of uniform color and appearance, stones thoroly incorporated into mortar. Consistency uniform thruout, **T & T.**

33. "As the gravel box was being filled, the cement was added to it gradually, so that, when the gravel box was full, the cement box was empty. The box was then removed, and the heap leveled off to a uniform thickness of ≯ 1 ft, and was then mixed by casting backward and forward twice," water added at time of second casting, **Lp.**

Forms.

34. Lagging. Of well seasoned boards, 2″ thick, drest all over, tongued and grooved, **Co, b;** 2″ × 6″ pine, drest on all sides, **Hb:** boards planed on one side and two edges; one edge slightly beveled and placed against the square edge of the next plank, **Yo;** boards preferably 2″ × 6″, dressed-and-matched flooring, **WH;** forms for exposed faces, of planed lumber, tongued and grooved or beveled; wall forms to be braced, and, where possible, to have their sides wired together, **Ci;** butt joints square, and either on posts or reinfd, **Hb;** joints, showing spaces, to be filled with stiff clay immedy before placing conc, **Hb.**

Used lagging, if not scarred, may be used again; but, for exposed work, must be cleaned and oild, **Hb.**

Posts. Generally 3″ × 8″ pine, drest on both edges, of full height of wall, ≯ 4 ft apart, **Hb.**

Centers and forms to be wet, **IM;** if reqd, before laying, **NO, Ci, b;** or oiled, **NO.** According to circumstances, forms to be wetted (except in freezing weather) or greased with crude oil, before placing conc, **T & T;** oild just before use, **Hb:** painted or oild before re-using, **CR;** dampend just before placing conc and kept damp until work has hardened, **TR, Wv.**

For removal of forms, see p 1191.

35. On up-stream face of dam, molds need be only smooth enough to give good substantial work, free from voids. On crest and down-stream face, molds must have planed surfs, so as to leave the finished work smooth, **Mc.**

36. Tie rods, left in conc, must not come nearer to conc surf than 2″, **CR;** projecting ends of iron bolts and rods to be cut off smooth and flush with conc face, **BR, AH;** not chiseled, but sawn or otherwise removed without jarring the work, **AH;** aids for holding molds not to be inserted within 4 ft of top of walls, **BR;** no bolts, etc, to show in the completed work, **OD.**

Placing, Churning and Ramming.

37. Night work prohibited in general, **TR. Time of placing;** conc must be placed within 30 mins after mixing, **AH, NO, Ci, b;** ≯ 30 mins "betw wetting the cem and the undisturbed conc in final place," **F;** before initial set, **TR, OD, CR, Wv, FW, Hb, Bu;** after mixing, mass kept in motion until placed in vehicle for transportation, **TR.** No **retempering** or rehandling permitted, **TR, CR, NO, Bu, Co, Ci,b, JC.** Conc, in which the materials have separated, must be remixt (by hand mixing, **BR, AH**): before laying, **T & T.**

Manipulation. In very wet conc, air must be churned out, stones workt back from face, and conc workt **under rods,** etc., **G;** by means of thin steel or iron blades, about 4″ × 6″, with handles of adjustable length, so that workmen need not stand in conc, **NO, Ci,b.** Conc to be joggled or worked into place by light ramming, **Bu, Co;** ram until mortar comes to surface, **AH, BR;** until all voids are filled and water flushes to the surf, **CS;** one tamper to not more than 2 cu yds per hr, **BR;** rammers with striking area not less than 36□″, weighing not more than 10 lbs, **Co;** face 6″ sq, weight, with handle, about 20 lbs, **CR;** 30-lb iron-shod rammers, face area not more than 30□″, **IM;** 40-lb rammers, **SE;** conc placed without ramming, **FP.**

For lists of Specifications for Concrete, see pp 1352, 1353.

38. Dry conc moistend by sprinkling, not pouring, **CR.**

39. Conc must be continuously **worked around reinfmt,** with uitable tools, as put in place. Complete filling of forms, and subsequent uddling, prohibited. Partly set conc must not be subjected to shocks, **Ch.**

40. Placing, in layers. Care taken to remove all scum, arising rom the cem, before laying the next layer, **Lp, JC.**

41. Conc **dumped** from receiving box or car, or shoveled directly into lace, use of slides and shutes forbidden, **OD, Wv, FP, TR, CR;** not ropt further than 6 ft, **FP;** 3 ft, **Wv.**

42. No walking on finished wall until set, **OD, Co.**

43. Thickness of layers. Not over 6", **Wv, BR, OD;** about ", **CR;** about 6" after ramming, **TR;** 6 to 8", **CS;** ≯ 6", **F;** ≯ 4", **SE;** ith dry mix, on slopes, ≯ 4", **F;** ≯ 4" in foundations, about 6" in back alls, **IM;** ≯ 9", **Hb;** ≯ 12", **WH;** such that each layer can be incororated with the preceding one, **T & T.**

44. No layers permitted, **Bu, Co;** layers not run out to thin edge, **FP;** ach layer completed (rammed, **CR**) before the next is laid, **FP, CR;** ach layer of a day's work laid before the layer next below has set, **TR.**

45. On rock foundation. Rock cleaned and washed with wire rooms, roughened if reqd, covered with thick neat cem grout, **CR;** bed wet mortar, **FW;** ½" thick, **TR;** conc anchored to rock with steel rods, reqd, **CR.**

Joints.

46. Avoidance of horizontal joints. Walls, etc, built in alterate sections, so short that they can be constructed as monoliths; these ections keyed together by vertical tongue-and-groove joints, **G** for gov't pecfns; joints continuous from foundation to coping, **CR;** "joints shall be ormed betw adjoining sections of conc for 4 ft down from the deck, by a yer of tarred paper," **BR;** dovetailing to have a thin coat of mortar, 1 : 5 weaker, to set before new conc is placed against it, **Hb.**

47. Joints between old and new work. Exposed surfs shaded nd kept moist until work is resumed, **CR;** chipped or broken edges cut way, **CR;** old surf to be left stepped, to form bond, and to be cleaned and et before adding new work, **FW, G;** cleaned with stiff wire brush and ream of water, **FP, BR, Hb;** if reqd, **F, Lv;** roughed up with a pick, reqd, **BR;** wooden strips, 4 to 6" wide, with beveled sides, to be embedded 3", and removed before conc has thoroly hardened, **NO;** between old and ew work, bed of 1 : 3 cem mortar 1" thick, **NO, Co;** ½" layer of mortar, **P;** old surf covered with neat cem grout of molasses consistency, **BR;** or y cem, **OD;** dry cem, brushed in, **Hb;** with layer of mortar, **TR, W;** old surf mopped with 1 : 2 mortar, **CS;** with heavy neat cem grout orked into surf with brooms, **CR;** keyed as directed, **FW.**

48. In hor joints in thin walls, or in walls to sustain **water pres,** in other important locations, mortar joint may be reqd. Tanks, etc, ith thin walls to hold water, should be built as monoliths, without interption, the work proceeding, if necessary, night and day, **T & T.**

49. When work is suspended for more than an hour, the outer lges of the last layer are to be leveled, and the center portion of the surf to be left about 6" lower than the edges, **CR.**

50. Bond betw new conc and old wall. Dovetailed pockets, 24" wide at ce, 33" at back, 15" deep, cut vert in old masonry, 4 ft apart, **Lp.**

51. Last layer deposited to be left as rough as possible, imbedded boulds projecting. Surf to be cleaned, washed, and sprinkled with neat cem, **c.**

Placing under Water.

52. Under water. No conc to be laid under water (without explicit rmission, **F;** except to stop leaks and springs, **TR;**) water not allowed rise on new work until thoroly set, **IM, Wv, TR, OD;** not less than hrs after set, **Lv, NO, Co, Cl,b;** if placed under water before setg, mixture to be 1 : 2 : 3, **WH;** 80 % of work built in place below (fresh) ater level, **SE;** conc, placed in water, must be semi-dry, **Ph;** bags to be wered to within a few ins of surf on which conc is to be deposited, **FW.**

For abbreviations, symbols and references, see p 1251.

53. When forms extend down to below high water, leaks under forms to be stopped, in order to prevent undermining before set; bags, filled with sand, placed outside; or jute canvas, underlying the conc 12″, nailed along bottom of form on the inside, **FW.**

Rain.

54. Rain. During rain storms, no new work to be laid, **IM, Bu, CR, AH, FP;** freshly laid work to be protected by canvas, **Bu.**

Frost.

55. Freezing. No concrete or mortar to be made when temp is below 35° F. in shade; conc work stopped from Nov 20 until April 1; during freezing weather, no conc to be mixed or deposited without engineer's consent, **IM, Bu:** ice and frost to be removed, water and sand heated, gravel steamed, work covered and kept warm by steam pipes, **Lv:** conc not to be placed when frozen, if reinfd, must be kept above 32° F for $\nless$ 48 hours after placing, use of frozen sand and agg prohibited, **Ch.** No laying permitted when temp $\ngtr$ 32° F., **Un, AH, BR,** < 32° F, **OD:** < 30° F, **CR,** < 34° F., **TR, FP;** when likely to freeze before set, **Wv:** before final set, **OD;** before set sufficiently to prevent injury, **BR, CR.** Conc, frozen in place, to be removed, **Un.** No conc to be laid when temp is below 20° F; water to be heated when temp is below 35° F, **Mc.** Use of icy materials prohibited; placed conc must be protected against freezing, **Ph.**

56. Natural cement concrete must never be exposed to frost until thoroly hard and dry, **T & T.**

57. "No conc, except that laid in large masses, or heavy walls having faces whose appearance is of no consequence, shall be exposed to frost until hard and dry. Materials employed in mass conc in freezing weather shall contain no frost. Surfs shall be protected from frost Portions of surf conc, which have frozen, shall be removed before laying fresh conc upon them." **T & T.**

58. Forms, under conc placed in freezing weather, "**to remain** until all evidences of frost are absent from the conc, and the natural hardening of the conc has proceeded to the point of safety." **Ch, Ph.**

Moistening.

59. Moistening. Freshly laid conc to be protected from the sun (by boards or tarpaulins, **FP, Hb, IM;**) and kept wet, **Mc, IM;** $\nless$ two weeks, or until covered with earth, **F;** $\nless$ 10 days, **SE, AH;** 6 ds, **CR:** 3 ds, **FW;** 48 hrs, **BR;** until set, **Wv;** until hard set, **Hb;** unfinished surfs until work can be resumed, **CR;** with wet tarpaulins $\nless$ 3 days, **CR.** When a section of wall is completed, coping to be covered with a thick layer of wet sand, mass of wall kept sprinkled until conc is thoroly set, **IM;** conc to be drenched twice daily, Sundays included, for a week after placing, in hot weather, **Ch, Ph.**

60. Moisten by sprinkling with fine spray at short intervals or by covering with moistened burlap, or etc, **G.**

Removal of forms.

61. Forms must be left in place $\nless$ 4 days, **IM;** $\nless$ 7 ds; longer if reqd by engineer, **Lv;** 72 hrs, **OD;** 48 hrs, **AH, BR;** until conc has stood at least 36 hrs, **WH;** until removal is authorized by engineer, or until conc has become hard, **Cl,b;** until conc can carry its load safely, **Ms;** forms removed after 48 hrs, **SE.**

62. Props, under floors and roofs, to remain in place $\nless$ 2 weeks. Forms, for cols, $\nless$ 4 days; for slabs, beams and girders, $\nless$ 1 wk and at least until the floor can sustain its own weight. "No load or wt shall be placed on any portion of the constr where the said centers have been removed." **Ch, Ph.**

63. Time for removal of forms and centering, 24 hrs to 60 days, depending upon temp and other atmospheric conditions and upon the commissioner of bldgs, **Un.**

For lists of Specifications for Concrete, see pp 1352, 1353.

64. Not until conc is hard.

	Min time, days:	
	Apr 1 to Dec 1	Dec 1 to Apr 1
Slabs and lintels, cols and monolithic walls	10	15
Posts and bottom supports for joists, beams and girders	14	21 **L.**

65. Forms, under conc placed **in freezing weather,** "to remain until all evidences of frost are absent from the conc and the natural hardening of the conc has proceeded to the point of safety." **Ch, Ph.**

Surface finish, waterproofing, etc.

66. Finish kept smooth by manipulation during placing, not by subsequent plastering, etc. Conc, free from large agg, to be placed next the mold, and prest back from mold by means of a flat shovel, inserted betw conc and mold (mold sprinkled with water, **BR**), conc rammed with an iron rammer, lower face 2″ × 6″, **AH, BR**; finish by working gravel back from face by means of forks, **Hb**; or shovels, **FP**; faces rubbed smooth, **TR, Hb**; with a piece of wood or soft stone, **TR**; voids filled up with mortar, **Hb, TR, CR**; plastering permitted only for an occasional and accidental cavity where the plastering is not apt to be disturbed by frost, **CR.** See p 1361, ¶ 79. 1 : 3 Port cem mortar, placed simultaneously with backing, **CR.** For wall, 1 : 2 Port cem mortar, very dry, 1½″ thick, **TR.**

67. For exposed faces, forms to be removed before conc has hardened; surf (1) rubbed with mortar of 1 vol Port cem, 2 vols sand, applied with a burlap swab and brushed down with a plasterer's brush, or (2) rubbed with stiff wire brush and a thin coat of neat Port cem grout, brushed down with plasterer's brush, **NO, Co**; smooth finish of sides produced by thoro ramming against inside surfs of molds, **SE.**

68. Surfs, not built against forms, screeded and troweled to smoothness, **NO.**

69. Voids or other imperfections, appearing upon removal of forms, to be corrected at expense of contractor, who shall remove and replace unsatisfactory work if reqd, **F.**

70. For floors and roof of mixing tank. Stiff mortar, of 1 vol Port, 1 vol sharp stone screenings to pass ⅜″ ring, free from dust, loam, etc, 1″ deep, laid before conc has initial set. Screeded, floated and troweled to smooth surf. Covered and sprinkled 3 days, **Co.**

71. Promenades and tops of parapets finished with a layer of mortar $\ngtr$ ¼″ thick, consolidated with the conc "by superimposing heavy planks 4″ thick and ramming them with 40-lb cast iron rammers until their ends are in contact with the ends of the molds," **SE.**

72. For piers, pedestals, abutments. Surfs exposed to air or water, 1½″ Port cement mortar, 1 cement, 2 sand, carried up simultaneously with the conc, 10 or 11″ in depth at a time, by means of ¼″ steel plate forms, 12″ wide, 4 to 5 ft long, placed around the work, 1½″ from the forms, and blocked out every 12″ by wooden blocks, the ends of the plates lapping slightly, **WH.**

73. For inverts, 1 cem, 2 sand, not more than ½″ thick, laid at same time as conc, **Lv.**

74. Moldings, cornices, etc. Plastic mortar placed against finely constructed molds, as conc is being laid; no exterior plastering permitted, **SE, T & T**; no plastering to be done unless expressly permitted, **F.**

75. Top finish. Conc brought up to 3½″ from reqd elevation; while this is still unset and plastic, 3″ of finer conc added, tamped and kneaded to form a monolith with the underlying conc; then ½″ of 1 : 3 (1 : 2, **AH**) cem mortar added and worked down to reqd grade by rubbing with a long wooden straight-edge, **AH, BR.**

76. Coping. While conc base is still soft, unset and adhesive, mortar (to be 1″ thick when finished) spread, leveled off and beaten with wooden battens or mauls; floated with wooden float and smoothed with plasterer's trowel; covered with boards or tarpaulins until hard set; then covered with sand; to be kept damp several days, **FP**; mortar, $\nless$ 1″ thick, of 375 lbs Port cem to 10.5 cu ft sand; tamped in place on top of rammed conc before the latter has begun to set; raked with straight-edge, rubbed with wooden

For abbreviations, symbols and references, see p 1251.

floats and finished with plasterer's trowel, **CR**; 1 : 2 Port cem mortar, 1″ thick, **TR**; surf formed by working the stones back from face, **Hb.**

77. **Granitoid surface finish** for tops of piers, pedestals and abuts; 1 part Port, 2 parts clean coarse granite sand or fine granite screenings, 3 parts granite chips, passing ½″ iron ring. Finished with a floated surf. **WH.**

78. **Water-proofing.** Heavy coat of semi-liquid mortar 1 part cem, ¼ part slaked lime, 3 parts sand. This coat to be given a smooth finish. When this has set hard, add a heavy coat of pure cem grout, **CS.**

79. **Plastering with cement.** None permitted on exposed faces, **AH, CS.** Inside faces of spandrel walls, covered by fill, to be well dampened and plastered with mortar of 1 cem : 2.5 sand, **CS.** See p 1360, ¶ 66.

Artificial stone.

80. (a) **For fine moldings, etc.** Molds plastered with semi-liquid mortar, 1 cem, 2 fine sharp sand, backed with earth-damp conc 1 : 2 : 4, or 1 cem to 6 gravel passing ¾″ ring. Conc backing rammed in thin layers. (b) **For plain flat surfaces.** Conc rammed in mold. Mold removed. Exposed surfs floated to smooth finish with mortar as in (a). No body of mortar to be left on face. Use only enough to fill pores and give smooth finish, **CS.**

Strength, etc, required.

(Strengths, etc, in lbs / □″, unless otherwise stated.)

81. **Ultimate comp,** after hardening for 28 days, ≮ 2000, **Un, Mh.**

82. **Ult shear** corresponding to 2000 comp, 200, **Un.**

Maximum allowable loads.

83. **For static loads** upon a 1 : 6 Port cem conc.

	Max allowable load lbs / □″†	
Compressn, conc surface > loaded area	0.325.s*	= 650
" in columns, length ≯ 12 diams	0.225.s	= 450
" " " with longitudinal reinfmt only	0.225.s	= 450
" " " hooped	0.270.s	= 540
" " " " , with 1 to 4 % long'l bars	0.325.s	= 650
" " " with structural steel col units thoroly encasing conc core	0.325.s	= 650
Rupture modulus (elas mod, E, constant)	0.325.s	= 650
" " adjacent to supports, (E constant)	0.375.s	= 750
Pure shear (no comp normal to shearing surf; reinfmt taking the normal tension)	0.060.s	= 120
Shear, combined with equal comp	0.162.s	= 325
Adhesion, plain bars	0.040.s	= 80
" drawn wire	0.020.s	= 40

JC.

84. **Compression.** See also ¶ 146, p 1366.
A, exclusive of temp stresses,
B, including stresses due to temp changes of 40° F

In arches for bridges, lbs / □″:	A	B
for highways and electric railways	500	600
for steam railways	400	500

CS.

85. On first-class Port cem conc, with agg properly graded:
1 : 6 or less, 60,000 lbs / sq ft = ...417 lbs / □″;
1 : 5 or less, in beams or slabs ....500 "

"In case a richer conc is used, this stress may be increased with the approval of the commissioner to not more than" 600 lbs / □″, **Ms.**

* s = ult comp strgth in lbs / □″ at 28 days when tested, under laboratory conditions, in the form of cyls 8″ diam, 16″ long, of same consistency as used in the field.

† When s = 2000 lbs / □″.

For lists of Specifications for Concrete, see pp 1352, 1353.

86. Portland, 1 : 2 : 4 .... 230 lbs / □″.
1 : 2 : 5 .... 208 "
Rosendale or equal,
1 : 2 : 4 .... 125 "
1 : 2 : 5 .... 111 " **N Y.***

87. Portland, lbs / □″.

	Mix, 1 : 2 : 4	1 : 2.5 : 5	1 : 3 : 6
machine-mixed	400	350	300
hand-mixed	350	300	250
Natural	...	150	...

Cinder, 700 ;
Port, in reinfd conc; direct, 0.2 × ult; in bending, 0.35 × ult. **Ch.**

88. Port, direct, 350 lbs / □″; in reinfd work, 350 lbs / □″ simultaneously with 6000 lbs / □″ tension in steel, **Un.**

89. Port, direct, 350; in bending, 500, **Mh.**

90. Port,

	Aggregate Stone or gravel	Slag	Cinder
In bending	600	400	250 lbs / □″
Direct, in cols length ≯ 15 diam	500	300	150 "

In hooped cols, 1000 lbs / □″ on area within hooping, **Ph.**

	1 : 2 : 4	1 : 2 : 5	1 : 3 : 6
Port	700	650	600 lbs / □″
Nat	400	...	... " **L.**

91. Tension, lbs / □″.
A, exclusive of temp stresses,
B, including stresses due to temp changes of 40° F.

	A	B
In reinforced arches	50	75
In reinforced slabs, girders, beams, etc	0	0 **CS.**

On diagonal plane, 0.02 × ult comp strgth, **Ch.**

92. Shear, lbs / □″.
75, **CS**; 50, **Mh**; 60 when uncombined with comp upon the same plane "unless the bldg commissioner with the consent of the board of appeal shall fix some other value," **Ms**; stone or gravel conc, 75; slag, 50; cinder, 25, **Ph.**

Elastic modulus.

93. 1,500,000 lbs / □″, **CS.**

Adhesion.

94. See p 1279, and p 1364, ¶ 113.

Safety factors.

$$\text{Safety factor} = \frac{\text{ultimate load}}{\text{allowed load}}.$$

95. At end of 1 mo, in subways and girder bridges for highways and electric rys, also bldgs, roofs, culverts, sewers, 4; in subways and girder bridges for steam rys, 5, **CS.**
Port, in reinfd conc, comp, direct, 5; in beams, 1/0.35; **Ch.**
In reinfd beams, 1 for dead load, plus 4 for live load, = 5;
In iron or steel in latticed or open work cols, beams or girders, encased in conc which extends ≮ 2″ beyond metal (with no allowance for the conc), 3 **L.**

Reinforcement.

96. Bars, unpainted, but free from scale, rust and grease, **G.**

97. Shape. Plain round or square, or corrugated, **Lv**; plain or twisted, **NO**; deformed, **AH**; twisted or deformed, **Bu**; Square machine-

*Corresponding with loads proposed by C. C. Schneider, Trans, A S C E, Vol 54, Jun '05, p 384. On p 493 Mr. Schneider proposes, instead, for Port cem conc only:

	per sq ft		per sq inch
1 : 2 : 5	20 tons	= 40,000 lbs	278 lbs.
1 : 2 : 4	25 "	= 50,000 "	347 ".

For abbreviations, symbols and references, see p 1251.

twisted. **Co**; Ransome twisted square preferred, **F**; Ransome or equal, **Hb**; Thacher bar, **CS**; square, twisted cold, or Johnson corrugated bar; in Johnson bar, net section = that reqd, by the plans, for twisted bars; plain bars to be used in comp only, **Cl.**

98. Twisted bars.

Size, ins	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	
Twists per ft.....	12	8	5	3.5	2.5	2	1.75	1.5	1.5,	**NO, Co**;
	6	...	...	...	...	...	1.5	...	...,	**Cl.**

One turn in 5 to 7 times nominal size, **F.**

Twisted uniformly by machinery; min cross sec area to vary not more than 2.5 %, **NO, Co.**

99. Round, corrugated, etc, bars to have same agg net sec area as square or twisted bars, **NO.**

Requirements.

100. Iron and steel "to meet the 'Manufacturers' Standard Specfns,' revised Feb 3, '03," **Ph.** See pp 1154, 1155.

101. Steel. Mfr and hardness. Medium open-hearth, **NO, Bu, Co, Cl**; mild, **Lv**; soft or medium, **CS.**

102. Ultimate tensile strength, in thousands of lbs / □″. 52 to 62, **F**; 54 to 64, **Un, Mh**; medium, 50 to 65, **Cl,a**; medium, 60 to 68, **CS**; soft, 54 to 62, **CS**; 55 to 65, **Lv, T & T**; ≮ 55, **NO**; 57 to 65, **Co,a**; 60 to 70 before twisting, **Co,b**; 60 to 70, **Bu.**

103. Ult comp strength.

Mixture		1:1:2	1:1.5:3	1:2:4	1:2.5:5	1:3:6
lbs/□″	=	2900	2400	2000	1750	1500
$n = E_s / E_c$	=	10	12	15	18	20

Ch.

104. Fracture, silky, uniform in color and texture, **Co.**

105. Elastic limit ≮ half ult tensile strgth, **G.**

106. Elastic modulus, 30,000,000 lbs/□″, **CS.**

107. Ratio, n, of elastic moduli. $n = \frac{E_s}{E_c} = \frac{\text{elas mod for steel}}{\text{elas mod for conc}}$.

n = 12, **Mh.** "If not shown by direct tests," in beams and slabs, n = 15; in cols, n = 10, **Ms**; with ult comp strgth = 2000 lbs / □″, n = 18, **Un.** Stone or gravel conc, n = 12; slag, n = 15, **Ph**; cinder, n = 30, **Ph, Ch.**

108. Elongation. %, minimum, in 8″, 25, **F, Lv, NO, Co,a**; 22, **Co,b, Cl,a**; 20, **Un, Mh**; soft, 25; medium, 22, **CS**; $\frac{1,400,000}{\text{tensile strgth}}$, **T & T.**

109. Bending test. Cold, **F, Lv, Bu, CS**; hot, cold or quenched, **NO, Co,a**; 180° about a diam = the thickness of the bar, **F, NO, Bu, Co, CS**; (before deforming, **F**); about a diam = twice the thickness of the bar, **Lv**; (after deforming, **F**); soft steel, flat, **CS**; cold, 90° over a diam = twice the thickness of the bar in steel ≯ ¾″ diam; over a diam = 3 × thickness of bar in steel > ¾″ diam, **Ch.**

Maximum stresses allowed in steel.

Stresses in lbs / □″ unless otherwise stated.

110. Tension, 16,000, **Mh, Ph, JC**; (iron, 12,000, **Ph**); one-third elas lim, but not over 18,000, **Ch**; mild, 12,000; medium, 15,000; high-carbon, 18,000, **L.**

111. Shear, 10,000, **Mh**; 12,000, **Ch.**

112. Comp = comp in conc × $\frac{\text{elas mod in steel}}{\text{elas mod in conc}}$, **Ch.**

"In arches, the steel ribs under a stress not exceeding 18,000 lbs per square inch must be capable of taking the entire bending moment of the arch without aid from the conc, and have flange areas of ≮ the 150th part of the total area of the arch at crown. The actual stress when imbedded in and acting in combination with conc shall not exceed 20 times the allowed stress on the conc."

For lists of Specifications for Concrete, see pp 1352, 1353.

"In slabs, girders, beams, floors, and walls, subjected to transv stress, the steel shall be assumed to take the entire tensile stress without aid from the conc, and shall have an area sufficient to equal the comp strgth of conc composed of 1 part Port cem, 3 parts sand, and 6 parts of broken stone, of the age of 6 mos."

"In walls or posts subjected to comp only, no allowance will be made for the strgth of imbedded steel, which will be used only as a precaution against cracks due to shrinkage or changes of temp."

"In tanks, the imbedded steel under a stress not exceeding 15,000 lbs / □" shall be capable of taking the entire water pres without aid from the conc," **CS.**

Elongation in service not more than 0.2 %, **Ch.**

113. Adhesion between steel and concrete. Assumed ⋟ allowed shear on conc, **Mh, Ms;** ⋞ shear on conc, **Un;** in stone or gravel conc, 50 lbs / □"; slag, 40; cinder, 15, **Ph.**

114. In 1 : 2 : 4 conc, max, lbs / □":

on plain round or square bars, structural steel	70	
high carbon steel	50	
on plain flat bars, ratio of sides ⋟ 2 : 1	50	
on twisted bars, ⋞ 1 twist in 8 diams	80	
on specially formed bars, 0.25 × ult adhesion as determined by test; max	= 100	**Ch.**

115. When the allowed adhesion is exceeded, "provision must be made for transmitting the strgth of the steel to the conc," **Un, Mh, Ph.**

116. Length and lapping.

Longitudinal bars **not less than 30 ft,** if possible, **Lv.**

In beams, rods of **single length,** if possible, **NO, Co, Ci.**

If lapped

Size of rod, ins	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	
Lap, ins	6	10	13	18	20	22	26	30	32	**NO.**
	6	9	12	15	18	20	22	24	27	**Co.**

Lap = 25 diams of rod, **Bu.**

Lap ⋞ 20 × diam of rod, ⋞ 1 foot, **Ci.**

In parallel rods, **joints staggered, Bu, Ci.**

Ends, not less than 2" from any surf, **Lv.**

Rods extend to extreme edges of unfinished surfs.

" " " within 1" of finished surfs. **Co.**

Floor rods extend 4" beyond face of wall supporting the floor;

Beam " " ⋞ 8" beyond face of wall supporting the floor, **NO, Ci.** See Clearance, below.

117. Protection. If work is interrupted, bars, already placed, must be protected, as with canvas or tarred paper. Ends, projecting for a considerable time, to be painted with heavy coat of neat cem grout, **F, Lv.**

Permit.

118. Complete detailed plans and specfns, giving composition of conc, to be filed with the Commissioner of Bldgs, **Ch, Un, Mh, Ph.**

Issue of permit does not involve acceptance of constr, **Ch.** For tests required, see pp 1362–3.

Clearance. See also ¶¶ 116, 134, 144, 149.

Distance, t, between steel and surf of conc.

119. In cols, beams and girders, t ⋞ 1½", **Ch, Ms;** in slabs, t ⋞ ½" ⋞ diam of bar, **Ch;** t ⋞ ¾", **Ms;** t ⋞ 1.5 × diam of bar, **JC.**

Axis of rods dist from outside of conc ⋞ diam of rod, **CS.**

For fireproof buildings, see ¶¶ 120–128.

Clear dist betw bars ⋞ 1.5 × max sectional dimension of bar, **Ch, JC. Clear dist betw two layers of bars,** ⋞ ½", **JC.**

120. For fireproof buildings (¶¶ 120–128), reinfd conc constr not approved "unless satisfactory fire and water tests shall have been made under the supervision of this Bureau," **Mh.**

May be accepted if designed as prescribed in code, provided that:

(1) Agg shall be "hard-burned broken bricks, or terra-cotta, clean furnace

For abbreviations, symbols and references, see p 1251.

clinkers entirely free of combustible matter, clean broken stone, or furnace slag, or clean gravel, together with clean siliceous sand, if sand is reqd to produce a close and dense mixture;" **Un.** (The other codes quoted specify fewer permissible varieties of agg.) Agg to pass ¾ in sq mesh, **Ch**; 1" ring, and 25 % of agg ≯ half max size, **Ph.**

(2) Min thickness, t, of conc, surrounding the reinfg members, shall be as follows, where d = diam parallel to t:

121. When $d \not> \frac{1}{4}''$, $t = 1''$; when $d > \frac{1}{4}''$, $t = 4\,d$. In any case $t \not> 4''$; $t \not< $ thickness required for structural purposes plus a, $a = 1''$ in cols and girders, $a = \frac{3}{4}''$ in floor slabs "but this shall not be construed as increasing the total thickness of protecting conc as herein specified." **Un.**

122. In girders and columns, $t = 2''$; in beams, $t = 1\frac{1}{2}''$; in floor slabs, $t = 1''$, **JC.**

123. In monolithic cols, the outer 1½" to be considered as protective covering, and not included in effective section, **JC.**

124. For beams and girders; on bottom, $t = 2''$; on sides, $t = 1\frac{1}{2}''$. Under slab rods, $t = 1''$. In cols, $t = 2''$, **Ch, Ph.**

125. "If a supplementary metal fabric is placed in the conc surrounding the reinfg, simply for holding the conc, the thickness of conc under the reinfg may be reduced by ½", such fabric shall not be considered as reinforcg metal," **Ch.**

126. On floor and roof beams, $t = 1''$; on floor and roof girders, and on beams carrying masonry, on top, $t = 1''$; elsewhere, 2", on cols, carrying only floors, $t = 3''$, on cols built into or carrying walls, 4", **Ms.**

127. Cinder concrete, for fireproof constr, t same as for stone conc; for slow-burning or mill constr, on cols, $t = 2''$; "on beams, girders and other structural steel or iron members," $t = 1\frac{1}{2}''$. Covering to have "metal binders or wire fabric imbedded in and around" such members; binders, if of wire, not less than No. 8, not less than 16" apart, **Ch.**

128. Corners of cols, beams and girders, to be beveled or rounded, **JC.**

Columns.

129. Columns must be allowed ≮ 2 hrs for settlement and shrinkage before girders are constructed over them, **JC.**

130. "**Rules** for the computation of reinfd conc cols may be **formulated from time to time** by the bldg commissioner with the approval of the board of appeal," **Ms.**

131. Concrete and steel assumed to shorten "in the same proportion", **Ms.**

132. Conc and steel stressed in ratio, n, of their elastic moduli, **JC.**

133. Rods tied together at intervals sufficiently short to prevent buckling, **Ms.** See ¶ 136.

134. Outer 1½" to be considered as protective covering and not included in effective section, **JC.**

Reinforced columns.

L = length; d = diameter or least side.

135. Reinfd conc may be used for cols when $L \not> 12\,d$, **Ch, Un, Mh;** $\not> 15\,d$, **JC;** and where cross section area ≮ 64 □", **Ch.** If $L > 15\,d$, allowable stress to be decreased proportionally, **Ph.**

136. Requirements. Rods to be tied together at intervals not more than d, **Un, Mh, Ph;** not more than 12 d, not more than 18", **Ch.** See ¶ 133.

137. Longitudinal rods not considered as taking **direct compression, Ph.**

138. Combined **cross section area** of comp rods ≯ 3 % of cross sec area of col, **Ch.**

139. When comp rods are not reqd, combined cross sec area of rods to be ≮ 0.5 % of cross sec area of col; not less than 1 □", **Ch.**

140. Least dimension of smallest rod to be not less than ½", **Ch.**

For lists of Specifications for Concrete, see pp 1352, 1353.

141. Rods to extend into the col above or below, lapping the rods there sufficiently to develop the stress in the rod by the allowed unit for adhesion, **Ch.**

142. Eccentric or transverse loading. Max fiber stress, including (1) direct comp, (2) bending due to direct comp, (3) eccentricity and (4) transverse load, not more than allowable comp stress. Eccentric load "shall be considered to affect eccentrically only the length of col extending to the next point below at which the col is held securely in the direction of the eccentricity," **Ms.**

143. A column, monolithic with or rigidly **attached to a beam or girder,** must resist, in addition to direct loads, a moment = max unbalanced moment in the beam or girder at the col, **Ch.**

144. Hooped columns. Conc may be stressed to 25 % of ult strgth, provided

(1) Cross sec area of vert reinfmt ≮ area of spiral reinfmt, ≯ 5 % of area within hooping;
(2) Percentage of spiral hooping ≮ 0.5, ≯ 1.5;
(3) Pitch of spiral hooping uniform and ≯ 0.1 × diam of col, ≯ 3″;
(4) Spirals so secured to verticals, at every intersection, as to maintain form and position;
(5) Spacing of verticals ≯ 9″, ≯ 1/8 circumference of col within hooping.

Hooping "may be assumed to increase the resistance of the conc equivalent to 2.5 × the amount of the spiral hooping figured as vert reinfmt." Conc, outside of hooping, not considered as part of effective col sec, **Ch.**

145. "The working stresses will be a subject for special consideration by the Commissioner of Bldgs," **Un.**

146. Allowed unit compression = 1000 lbs/□″ of area within hooping, **Ph.**

147. Percentage of long'l rods and spacing of hoops to be such that the conc may develop this stress with a safety factor of 4, **Ph.**

148. "Hoops or bands not to be counted upon directly as adding to the strgth of the col," **JC.**

149. Clear spacing of bands and hoops ≯ 0.25 × diam of enclosed col, **JC.**

150. Structural steel reinforced columns. Conc may be subjected to 1/4 ult stress, provided (1) cross sec area of steel is not less than 1 □″; (2) spacing of lacing or battens not more than least width of col, **Ch.**

Beams and floors.

151. The common theory of beams is applicable. **Un, Ch, Mh, Ph.**

152. The steel is assumed to take all the direct tensile stresses, **L, Un, Ch, Ms, Mh, Ph.** Tensile stress in conc to be considered in calculating deflections, **JC.**

153. The stress-stretch curve of conc in comp is assumed to be a straight line, **Ch, Ph.** $n, = E_s/E_c, = 15$; for deflections, $n = 8$ to 12, **JC.**

154. At 2000 lbs/□″ extreme fiber stress, this curve may be taken as (a) a straight line; (b) a parabola, with axis vert, and vertex on neutral axis of beam; or (c) an empirical curve, enclosing an area 1/4 greater than if curve were a straight line, and with cen of grav at same height as that of area in (b), **Un.**

155. Stresses. A load, = 4 × the total working load, stresses the steel to its elas lim, and the conc to 2000 lbs/□″, **Un.** Design "based on the assumption of a load 4 times as great as the total load, **Ph.** (Total load = ordinary dead load plus ordinary live load, **Un, Ph.**)

156. The adhesion, betw conc and steel, is assumed to be sufficient to make them act unitedly, **Un, Ch, Mh, Ph.**

157. Exposed metal not considered in figuring strgth, **Un, Ch, Ph.**

158. Span = dist c to c of bed plates or other bearings, **Ms, JC.** If beam is fastened to side of a col, span is measured to cen of col, **Ms.** Span ≯ (clear span + depth of beam or slab), **JC.**

For abbreviations, symbols and references, see p 1251.

159. Shrinkage and thermal stresses to be provided for by introduction of steel, **Ch, Ph.** "Initial stress in the reinfmt, due to contraction or expansion in the conc, may be neglected," **JC.**

160. When the **shear** developed exceeds the allowed limit for conc, steel must be introduced to take the excess, **Un, Mh, Ph, JC.**

161. Allowable values for shearing stresses:

	lbs/□"
(a) With horizontal bars only	40;
(b) With part of the hor reinfmt in the form of bent-up bars, "arranged with due respect to the shearing stresses"	≯60;
(c) With thoro reinfmt for shear	≯120, **JC.**

Under (c), conc may be taken as carrying ⅓ of the shear; the remaining ⅔ being carried by bent rods or stirrups (preferably both) carrying their share within a hor dist = depth of beam, **JC.**

162. Longitudinal spacing of stirrups or bent rods ≯ 0.75 × depth of beam, **JC.**

163. Cement finish, added to the tops of slabs, beams and girders, **not to be included** in figuring strgth "unless laid integrally with the rough conc," and to be allowed no greater unit stress than that on the rough conc, **Ch.**

164. Web reinforcement. "Where the vertical shear, measured on the sec of a beam or girder, betw the centers of action of the hor stresses, > 0.02 × the ult direct comp stress/□", web reinfmt shall be supplied, sufficient to carry the excess. The web reinfmt shall extend from top to bottom of beam and loop or connect to the hor reinfmt. The hor reinfmt, carrying the direct stresses, shall not be considered as web reinfmt," **Ch.**

165. Steel in the compression sides of beams and girders. "When steel is used in the comp side of beams and girders, the rods shall be tied in accordance with requirements of vert reinfd cols with stirrups connecting with the tension rods of the beams or girders," **Ch.**

166. "When steel or iron is in the comp sides of beams the proportion of stress taken by the steel or iron shall be in the ratio of the mod of elas of the steel or iron to the mod of elas of the conc; provided, that the rods are well tied with stirrups connecting with the lower rods of the beams;" **Ph.**

167. Where slabs are used with girders and beams, the girders and beams are treated as T-beams, a portion of the slab acting as flange; **G.**

168. Portion, F, of width of slab, acting as flange.

t = thickness of slab; L = span of beam or girder;
b = breadth of beam or girder; S = dist c to c betw beams or girders.

F to be "determined by assuming that, in any hor-plane sec of the flange, the stresses are distributed as the ordinates of a parabola, with its vertex in the stress-stretch curve and with its axis in a longitudinal vert plane thru the cen of the rib of the T." Said portion to be reinforced with bars near the top, at right angles to the girder. **Un.**

169. F dependent upon hor shearing stress, $F \ngtr 20t$, **Ph**; $F \ngtr 10b$, **Mh.**

170. F governed by shearing resistce betw slab and rib; $F \ngtr S\left(1 - \frac{S^2}{L^2}\right)$ $\ngtr L/3$, $\ngtr \frac{3}{4}S$. To be assumed as thus acting, slab must be cast at same time with rib, **Ch.**

$F \ngtr L/3$, $\ngtr S$, **Ms**; $\ngtr L/4$, $\ngtr 8t + b$, **JC.**

171. T-beams to be reinfd against shear along plane of junction between rib and flange, **Un, Ph**; using stirrups thruout length of beam, **Ph.**

172. Ribs of girders and beams to be **monolithic** with floor slabs **Un, Ph.**

173. "Where reinfd conc girders carry reinfd conc beams, the portion of the floor slab acting as flange to the girder must be reinfd with bars near

For lists of Specifications for Concrete, see pp 1352, 1353

the top, at right angles to the girder, to enable it to transmit local load directly to the girder and not thru the beams, thus avoiding an integration of comp stresses due to simultaneous action as floor slab and girder flange **Un. Ph.**

Moment, M. See also ¶¶ 178, 179

174. W = load per sq ft, L = span, in ft In freely supported slabs L = free opening + depth, in continuous slabs, L = distance bet n centers of supports.

175. With concentrated or special loadings, calculate and provide for moments and shears for critical condition of loading, **Ch.**

For dead load, M obtained from the actual dead load covering all
" live load, over supports, M obtained from the actual live load spans at same time
" " " between supports M may obtain from live load covering 2 consecutive or 2 alternate spans at same time

When all spans are equal, let M_c = mom live load moment at middle of span. Then,

$$\text{for intermediate spans, } M_c = \frac{W L^2}{12}$$

$$\text{for end spans, } M_c = \frac{W L^2}{10}$$

Sum of live load moments over one support and at cen of span, $\nless \frac{W L^2}{6}$, **Ch.**

Continuity. See also ¶ 175.

176. Beams and girders considered as simply supported at ends, no allowance made for continuity, **Un. Mh.**

177. Beams, etc, calculated as simply supported, or as continuous, according to the facts, **Ch. Ms.**

178. Continuous **floor plates**, reinfd at top over supports, may be treated as continuous beams Under uniformly distributed loads, mom, M, taken at not less than $0.1\ W\ L$, $0.05\ W\ L$ with square floor plates, reinfd in both directions and supported on all sides, **Un. Mh. Ph.**

179. In floor slabs adjoining walls: if slab is reinfd in one direction, $M = \frac{W L}{8}$; if square and reinfd in both directions, $M = \frac{W L}{16}$, **Ph.**

180. Floor slabs designed and reinfd as continuous over the supports. If length of slab > 1.5 × its width, the entire load should be carried by transverse reinfmt. "Square slabs may well be reinfd in both directions." **JC.**

181. For beams and slabs continuous for > 2 spans, bending moms at cen and at support, for both live and dead loads, as follows:

In floor slabs and in interior spans of continuous beams, $M = w\ L^2/12$,
in end spans of continuous beams, ... $M = w\ L^2/10$
w = load per unit of span, L = span, **JC.**

182. In continuous spans, provide, at supports, for

negative mom = 0.8 positive mom at cen of a simply supported span.

Pos mom, at cen of continuous span, may be taken = neg mom at support, **Ms.**

Tests.

183. Bldg Commissioner may require tests of materials before or after incorporated into bldg, **Ms.** Contractor must be prepared to make load tests in any portion of bldg within a reasonable time after erection, and as often as may be reqd by engineer, **Ch, Ph, Mh, Un.** Tests must show that the constr will sustain loads as follows:

For abbreviations, symbols and references, see p 1251.

load = 2 × sum of proposed dead and live loads, **Ch**;
" = 2 × proposed live load, **Ph**;
" = 3 × proposed load, **Mh**.

184. Construction may be considered as part of the test load, **Ch.**

185. Each test load shall cover 2 or more panels, and remain in place not less than 24 hrs, **Ch.**

186. Deflection of slabs not more than $\frac{\text{span}}{800}$.

Deflection of girders $\frac{\text{span}}{800}$ × ratio of slab depth to girder depth, **Ch.**

187. Test, 45 days after completion.
Load 1.5 live load + 1.5 × dead load of finished area.
Deflection ≯ 0.001 × length of member, **Ci,b.**

CONCRETE SIDEWALKS.

Abstract of Specification

Adopted by

National Association of Cement Users

Philadelphia, January, 1908.

1. Cement. Portland, to meet specification of A S T M, adopted Jan, 1906. See p 940.

2. Sand. To pass No. 4 screen. May contain ≯ 5 % loam and clay, if these do not coat the sand grains.

∢ 60 % of the sand to pass No 10 sieve, or
35 % to pass No 10 20 30 40 sieve,
and remain on No 20 30 40 50 " , respectively.

≯ 20 % of the sand to pass No 50 sieve, or
70 % to pass No 10 20 sieve,
and remain on No 40 50 " , respectively.

3. Screenings, from crushed stone as below, and meeting sand requirements, may be substituted for sand.

4. Aggregate. Stone, crushed from clean, sound, hard, durable rock, screened dry thru ¾″ mesh, retained on ¼″ mesh.

5. Gravel, clean, hard, ranging from that retained on ¼″ mesh, to that passing ¾″ mesh.

6. Unscreened gravel, clean, hard. No particles larger than ¾″. Proportion of fine and coarse particles to conform to requirements below for conc.

7. Water, "reasonably clean, free from oil, sulfuric acid and strong alkalies."

Sub-base.

8. Sub-base to be **thoroly rammed. Soft spots** removed and replaced by hard material.

9. Fills > 1 ft thick, to be thoroly compacted by flooding and tamping in layers ≯ 6″ thick, "and shall have a slope of ∢ 1 : 1.5." "The top of all fills shall extend ∢ 12″ beyond the sidewalk."

10. "While compacting, the sub-base shall be **thoroly wetted** and shall be maintained in that condition until the conc is deposited."

Base.

11. Voids. Cem must overfill voids in sand by ∢ 5 %.

12. Mortar must overfill voids in agg, by ∢ 10 %. Proportions 1 : ≯ 8 sand and agg.

13. When the voids are not determined, 1 : 3 sand or screenings : 5 stone or gravel. "A sack of cem, 94 lbs, shall be considered to have a vol of 1 cu ft."

Mixing.

14. Hand. Sand evenly spread on a level water-tight platform, cem spread on sand. Mix dry to uniform color. Water sprayed and mass turned until homogeneous and of uniform consistency. Drenched agg added and all mixed until agg is thoroly coated with mortar.

15. Hand. With unscreened gravel. Cem and gravel "mixed dry, until no streaks of cem are visible." Water sprayed and mixed. Mortar must be equivalent to that specified above.

16. Water may be added while mixing, but conc must be turned ≮ once immediately afterward.

17. "Machine mixing will be acceptable when a conc equivalent in quality to that specified above is obtained."

18. Retempering prohibited.

Grade.

19. Grade of sidewalk ≮ sufficient for drainage, ≯ ¼"/ft, "except where such rise shall parallel the length of the walk."

Forms.

20. Lumber. clean, free from warp, ≮ 1¾" thick.

21. Upper edges to conform with finished grade of sidewalk.

22. Cross forms. "At each block division, cross forms shall be put in the full width of the walk and at right angles to the side forms," except as in ¶ 23.

23. Expansion joint. A metal parting strip ½" thick to replace a cross form ≮ once in 50 ft. "When the sidewalk has become sufficiently hard, this parting strip shall be removed and the joint filled with suitable material prior to opening the walk to traffic. Similar joints shall be provided where new sidewalks abut curbing or other artificial stone sidewalk."

24. "All forms shall be thoroly wetted before any material is deposited against them."

25. Dimensions of blocks.

Size, feet	6 × 6	5 × 5	4.5 × 4.5	4 × 4	3 × 3
Thickness, ins:					
In business districts,	6	5.5	5	4	...
In residence districts,	6	5	...	4	3

In residence sidewalks, edges may be 25 % thinner than center; min = 3".

26. Separating tool ≯ 6" wide, ¼" thick. Groove cut thru into sub-base; groove filled with dry sand before the top coat is spread; top coat cut thru to the sand after floating and troweling, "and a jointer run in the groove"; trowel then drawn thru groove again "so as to insure a complete separation of the block."

Depositing.

27. Conc carried to forms in watertight wheelbarrows. Conc must not slop over. Barrows must not be run over freshly laid conc.

28. Conc must be deposited within 1 hour after mixing, spread evenly, and tamped until water flushes to the top.

Protection.

29. Workmen must not walk on freshly laid conc.

30. Sand or dust, collecting on the base, to be "carefully removed before the wearing surface is applied."

Wearing surface.

31. Minimum thickness, ¾".

32. Mortar, 1 : 2 sand or screenings, mixed as for base, but wet enough not to require tamping, and so as to be readily floated with a straight-edge. "A thin coat of mortar shall be floated on to the base before spreading the wearing surf." Mortar spread on base within 30 mins after mixing, and floated within 50 mins after base conc is mixed.

33. Marking. "After being worked to an approximately true surf, the block markings shall be made directly over the joints in the base with a tool which shall cut clear through to the base and completely separate the wearing courses of adjacent blocks."

34. Surface edges rounded to a radius $\nless$ ¼".

35. "When partially set, the surf shall be **troweled smooth.**"

36. On grades > 5 %, surf to be **roughened** by a suitable tool "or by working coarse sand or screenings into the surf."

37. Only mineral colors shall be used, and these shall be **incorporated** with the entire wearing surf.

Single coat work.

38. Proportions, 1 : 2 sand . 4 gravel or crushed stone. **Blocks separated** as in two-coat work. Conc to be firmly **compacted** by tamping, and evenly **struck off and smoothed** to the top of the mold. "Then, with a suitably grooved tool, the coarser particles of the conc tamped to the necessary depth so as to finish the same as two-coat work."

Protection.

39. "When completed, the sidewalk shall be **kept moist and protected** from traffic and the elements for at least 3 days. The forms shall be removed with great care, and upon their removal **earth shall be banked** against the edges of the walk."

Grading adjacent to sidewalk.

40. On curb side, 1½" below sidewalk, slope $\nless$ ¼"/ft. On property side, "the ground should be graded back $\nless$ 2 ft and not lower than the walk."

CONCRETE BLOCKS.

1. Buffalo harbor. Blocks 6 ft long, abt 4 ft sq, 88.75 cu ft = 3.3 cu yds, made in wooden molds. ½ bbl Port, 2.5 cu ft sand, 7.5 cu ft pebbles, 7.5 cu ft broken stone, made a layer of conc, in mold, about 6" thick. Faces, 6" thick, of blocks on lake-face of breakwater, of finer material. Face placed first; backing placed before face had set. (Emile Low, A S C E, Trans, June '04, Vol LII, p 96.)

2. Zeebrugge breakwater, Belgium. Blocks 25 m (82 ft) long, 9 m (29.5 ft) wide, 8.75 m (28.7 ft) high, 2000 cu m (2616 cu yds), 4500 tons each. Outer conc shell, with cutting lower edge, three compartments, formed in iron framework and floated to place; placed between guides and block last sunk; sunk by admission of water, and filled up with conc, 1 cem. 2.5 sand : 6.1 broken porphyry, by means of skips of 10 cu m (13 cu yds). Top meter, rich in cem, placed above water at low tide. Seaward toe immediately protected by rubble rip-rap.

Superstructure of 55-ton blocks, laid above water; these surmounted by conc blocks, formed in place.

3. Molds for isolated monolithic sub-aqueous concrete blocks, from 150 to 222 cu yds, forming pier of trapezoidal cross-sec. The molds are bottomless boxes of trapezoidal cross-sec, composed of two sides and two end pieces, held together by 1¼" turnbuckle tie-rods acting on beams placed outside of the mold. The tie rods have, at each end, eyes in which wedge-bolts are inserted at time of erection. To remove the molds, the wedge-bolts are removed by turning up a nut on the rods which form an integral part of the wedge-bolts. This pulls the wedge-bolt from the eyes of the tie-rods and releases the walls of the molds, which are then picked up by the mold traveller, and re-assembled on the traveller ready for re-setting. Weight of mold, 40 tons. Time reqd for removing mold from a block and re-assembling for re-setting, from 45 to 60 mins. Buoyancy of timber overcome by cast iron ballast wts. Alternate blocks placed first. For intermediate blocks only the two side pieces of a mold are used. These are held in place and at their proper batter by six turnbuckle tie-rods, each passing thru a hollow square box of one-inch plank, acting as a strut. (South Pier at Superior Entry, Wisconsin. Report of Clarence Coleman, Asst. Engr Report Chf Engr, U S A, 1904, Part IV, page 3781.)

4. "Lewis holes should be cast in the blocks where practicable" and so "as not to bring excessive pres on the conc, particularly near the mortar facing or near the arrises of the block." Lewises and dogs may pull out of green blocks. Provide wooden blocks and rag cushions for use in turning over the blocks, otherwise the corners may be damaged.

5. Casting position. Blocks should be cast with the most important face down, their showing faces as nearly vert as practicable, and the back of the block on top, so that laitance, etc, rising to the surf, may appear there.

HOLLOW CONCRETE BUILDING BLOCKS.

Abstract of Specification

Adopted by

National Association of Cement Users,

Philadelphia, January, 1908.

1. Cement, Portland, to meet specification of A S T M, adopted Jan, 1906. See p 1232.

2. Sand, silicious, clean, gritty, to pass ¼" mesh sieve.

3. Aggregate, clean broken stone, free from dust, or clean screened gravel, passing ¾" mesh sieve, refused by ¼".

4. Unit of measurement for cem. Bbl = 380 lbs net; cu ft $\not>$ 100 lbs. Cem either measd in original package, or weighed; not measd loose in bulk.

5. Proportions. For exposed exterior or bearing walls.
(a) Machine-made. Semi-wet, 1 : $\not>$ 3 sand : $\not>$ 4 agg.
(b) Slush (or wet) conc (quaking or flowing), made in individual molds and allowed to harden in them, 1 : $\not>$ 3 sand : $\not>$ 5 agg.
If stone is omitted, proportion of sand may be increased if tests show no increase in voids or in absorption, and no loss of strength.

6. Water enough to perfect the crystallization of the cem.

7. Mixing. "Thoro and vigorous mixing is of the utmost importance."
(a) Hand. Cem and sand mixt dry. Water added slowly and workt in. Moistened agg spread upon mortar, or mortar upon agg. Mix.
(b) Machine preferred. Cem and sand, or cem, sand and agg, mixt dry. Water added and workt in. With wet conc, "this procedure may be varied with the consent of the bureau, etc."

8. Molding. Top surf of tampt blocks, after striking off, to be "troweled or otherwise finisht to secure density and a sharp and true arris."

9. Curing. After molding, blocks to be "carefully protected from wind currents, sunlight, dry heat or freezing for at least 5 days," and supplied with additional moisture during that time "and occasionally thereafter until ready for use."

10. Minimum age before using. 1 : 3 sand, 3 weeks; 1 : 2 sand, 2 weeks "with the special consent of the bureau, etc"; special blocks, for closures, 7 days "with the special consent of the bureau, etc."

11. Marking. All blocks to be markt with maker's name or brand, day, month and year of mfr, and proportions, as "1 : 2 : 3," etc.

12. Mortar. "All walls, where blocks are used, shall be laid up with Portland cem mortar."

13. Maximum load, including wt of wall, 8 tons per sq ft of area of blocks.

14. Thicknesses of walls. Bearing walls "may be 10 % less than is reqd by law for brick walls." In curtain or partition walls same as for hollow tile, terra cotta or plaster blocks.

15. Offsets. "Wherever walls are decreased in thickness, the top course of the thicker wall shall afford a full solid bearing for the webs or walls of the course of blocks above."

16. Under girders or joists, blocks to be made solid for $\not<$ 8" *from inside face.* If concentrated load, W, on block, > 2 tons, this applies to the blocks supporting the girder, etc; if $W > 5$ tons, it applies to blocks for $\not<$ 3 courses below, and to a dist of $\not<$ 18" each side of girder, etc.

17. In party walls, blocks must be filled solid.

18. Bond. "Where the walls are made entirely of conc blocks, but where said blocks have not the same width as the wall, every 5th course shall extend thru the wall, forming a secure bond, when not otherwise sufficiently bonded."

19. Block facing, on brick backing, "must be strongly bonded to the brick, either with headers projecting 4″ into the brick work, every 4th course being a header course, or with approved ties, no brick backing to be less than 8″."

20. Thickness of web of block (in bearing walls) $\nless$ 0.25 × ht of block.

21. Hollow space. In bearing walls, min percentage of hollow space:

Buildings of	1st	2d	3d	4th	5th	6th story
1 & 2 stories..............	33	33				
3 & 4 "	25	33	33	33		
5 & 6 "	20	25	25	33	33	33

22. Sills and lintels to be "reinforced by iron or steel rods in a manner satisfactory to the bureau, etc." When span > 54″, lintel "shall rest on block solid for $\nless$ 8″ from face next the opening and for $\nless$ 3 courses below bottom of lintel."

23. Prior to use, **application must be filed** with bureau or with chief of proper department, giving "a description of the material and a brief outline of its manufacture and proportions used," with "name of the firm or corporation, and the responsible officers thereof," "and changes in same thereafter promptly reported."

24. Certificate of approval to remain in force $\ngtr$ 4 mos, "unless there be filed with the bureau of building inspection, at least once every 4 mos following, a certificate from some reliable physical testing laboratory showing that the av" of $\nless$ 3 comp tests and $\nless$ 3 transverse tests comply with requirements; "the said samples to be selected by a building inspector or by the laboratory from blocks actually going into construction work."

25. Preliminary test. Maker to submit product to tests required, and file certificate, from a reliable testing laboratory, giving in detail the results of the tests made. Results of all tests, satisfactory or otherwise, to be filed in the bureau, open to inspection, but not necessarily for publication.

26. Additional tests. Maker or user or both "shall, at any and all times, have made such tests of the cems used in making such blocks, or such further tests of the completed blocks, or of each of these, at their own expense and under the supervision of the bureau of building inspection, as the chief of said bureau may require."

Failure to stand these tests involves immediate revocation of the certificate issued to maker.

27. Test requirements. Blocks must be subjected to transverse, compression and absorption tests, "and may be subjected to the freezing and fire tests." Freezing and fire tests not at cost of mfr.

28. Approval tests made at expense of applicant.

29. Not less than 12 samples to be selected by bureau, etc.

30. "Samples must represent the **ordinary commercial product,** of the regular size and shape used in construction. The samples may be tested as soon as desired by applicant" but $\ngtr$ 60 days after mfr.

31. Blocks, failing to stand tests, to be marked "condemned" by mfr or user, and destroyed.

32. "Tests shall be made in **series** of at least 3, except that in the fire tests a series of 2 (4 samples) are sufficient."

33. "Half samples may be used for the crushing, freezing and fire tests. The remaining samples are kept in reserve, in case duplicate or confirmatory tests be reqd "

4. "Lewis holes should be cast in the blocks where practicable" and so "as not to bring excessive pres on the conc, particularly near the mortar facing or near the arrises of the block." Lewises and dogs may pull out of green blocks. Provide wooden blocks and rag cushions for use in turning over the blocks, otherwise the corners may be damaged.

5. Casting position. Blocks should be cast with the most important face down, their showing faces as nearly vert as practicable, and the back of the block on top, so that laitance, etc, rising to the surf, may appear there.

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If stone is omitted, proportion of sand may be increased if tests show no increase in voids or in absorption, and no loss of strength.

6. Water enough to perfect the crystallization of the cem.

7. Mixing. "Thoro and vigorous mixing is of the utmost importance."
(a) Hand. Cem and sand mixt dry. Water added slowly and workt in. Moistened agg spread upon mortar, or mortar upon agg. Mix.
(b) Machine preferred. Cem and sand, or cem, sand and agg, mixt dry. Water added and workt in. With wet conc, "this procedure may be varied with the consent of the bureau, etc."

8. Molding. Top surf of tampt blocks, after striking off, to be "troweled or otherwise finisht to secure density and a sharp and true arris."

9. Curing. After molding, blocks to be "carefully protected from wind currents, sunlight, dry heat or freezing for at least 5 days," and supplied with additional moisture during that time "and occasionally thereafter until ready for use."

10. Minimum age before using. 1 : 3 sand, 3 weeks; 1 : 2 sand, 2 weeks "with the special consent of the bureau, etc"; special blocks, for closures, 7 days "with the special consent of the bureau, etc."

11. Marking. All blocks to be markt with maker's name or brand, day, month and year of mfr, and proportions, as "1 : 2 : 3," etc.

12. Mortar. "All walls, where blocks are used, shall be laid up with Portland cem mortar."

13. Maximum load, including wt of wall, 8 tons per sq ft of area of blocks.

14. Thicknesses of walls. Bearing walls "may be 10 % less than is reqd by law for brick walls." In curtain or partition walls same as for hollow tile, terra cotta or plaster blocks.

15. Offsets. "Wherever walls are decreased in thickness, the top course of the thicker wall shall afford a full solid bearing for the webs or walls of the course of blocks above."

16. Under girders or joists, blocks to be made solid for $\not<$ 8″ *from inside face.* If concentrated load, W, on block, > 2 tons, this applies to the blocks supporting the girder, etc; if $W > 5$ tons, it applies to blocks for $\not<$ 3 courses below, and to a dist of $\not<$ 18″ each side of girder, etc.

COST.

1. The following data respecting prices and costs are compiled from records of actual construction as carried out by men presumably skilled in the art, and employing labor at about the usual rates. They afford only approx estimates of what may ordinarily be expected The cost of materials, transportation, and especially of labor, varies from time to time and from place to place.

2. Not only does the rate per hour for labor vary; but the amt of work turned out in a given time varies much more widely. A well matcht gang, presided over by an efficient foreman, will produce usually from two to four times the output of an indifferent gang. Even a well-meaning worker will frequently let his efficiency drop to 75 % of what may reasonably be expected; indifferent workers will produce only 30 or 20 %. The methods of payment, the character of superintendence, and the way in which the work is arranged and handled, are all very important; and a bungler, or one unfamiliar with conc operations, would probably find difficulty in keeping the total costs within double those given.

3. The **principal items**, making up the cost of conc (plain and reinfd) may be classified as follows:

Materials; Cem, sand, gravel, stone, reinfmt
Transportation to storage; Hauling, freight
Storage.
Screening, washing.
Mixing; Loading and transporting to mixer, mixing machine and power, labor and depreciation connected with it, auxiliary apparatus as mixing board, barrows, shovels, etc., and transporting conc to forms.
Forms; Erection, shifting, depreciation, material, labor.
Depositing; Dumping, spreading and ramming.
Finishing; plastering, brushing, etc.
Inspection and superintendence.
Plant (besides mixer and forms); Interest, depreciation, repairs, insurance.

Cost of Materials.

4. For *prices* of cem, sand, etc, see "Price List," under **1**, and its subdivisions, pp 1401, etc.

5. The cost of any one material, *per cu yd of conc*, varies greatly in diff cases, due to wide variations in the percentages employed for diff grades of conc, and can therefore be approximated only betw wide limits.

6. Roughly stated, the **total** cost, for **materials** alone, may be expected to fall somewhere between \$2.50 and \$7.50/cu yd of conc. The av would probably be \$4 or a little more, exclusive of reinfmt.

7. Cement. For prices, see "Price List," **1.34**, p 1403. Per cu yd of conc, betw \$1.50 and \$4, \$2 and \$3 being the more usual limits; affected chiefly by grade of cem and richness of mixture.

8. Sand. For prices, see "Price List," under **1.32**, p 1402. Per cu yd of conc, betw 15 cts and \$1, usually below 25; affected chiefly by grade, dist from bank, natural monopoly, and proportion used in mixture.

9. Gravel. In the pit, exclusive of screening, loading and hauling, from 20 cts to 75 cts per team load; affected chiefly by quality, and natural monopoly.

10. Stone. For prices, see "Price List," under **1.32**, p 1402. Av price for stone, broken to reqd size, at quarry, exclusive of cartage, about \$1 or \$1.50 / cu yd stone. Per cu yd conc, betw 50 cts and \$1. Affected chiefly by quality, dist from quarry, natural monopoly, and proportion of mixture.

11. Reinforcement. Cost will vary with the design and type employed. For iron and steel bars, see "Price List," **1.43**, p 1404.

Plain rods, 50 ton lots, at mill, cts per lb, approx:

∢ ¾", 1½; ∢ ½", 1¾; ∢ ⅜", 2; ∢ ¼", 2¼.

Ransome twisted rods, about ⅙ ct per lb more.

Other deformed bars, ¼ to ½ ct per lb more.

12. The percentage of reinfmt usually varies from about ½ % to 1½ % of the cross-sec of a beam or slab.

Cost of Transportation to Storage.

13. Freight. Cem, by rail. Freight rates vary greatly in diff localities, often due to no other apparent reason than arbitrary discrimination, running as low as ½ ct / ton-mile, and above 2 cts; in general, 1 to 2 cts.

14. By Canal. Boat loads of 100 tons of 2000 lbs each, cem, 1 to 2 cts/ ton-mile, according to dist; stone and sand, ¾ to 1½.

15. Coastwise freight. In carload lots, 0.4 to 0.6 ct / ton-mile, approx.

Cost of Storage, etc.

16. Storage. Ordinary cem barrels may be stored about 5 layers high, which requires about 1½ □ ft floor space per bbl.

17. Screening. Cost, by hand, betw 10 and 25 cts or more / cu yd of material handled. Machine screening, betw 4 and 8 cts / cu yd. To obtain the cost per cu yd of the screened material, multiply cost per cu yd by the ratio of total quantity handled to quantity accepted.

18. Washing. Cost of washing sand, gravel and crusht stone may be 5 cts or more / cu yd of material handled, for mechanical washers, handling large quantities. For small quantities, washt under unfavorable conditions, as high as 40 cts.

Cost of Mixing and Placing.

19. Mixing and placing. Total cost, exclusive of forms, from $1 to $2.50 / cu yd of conc.

20. Labor required, for fairly large quantities, on an av, one man for each 2 or 3 cu yds mixt and placed per day. On small jobs, each man will turn out much less.

21. Dry conc costs about $1 more per cu yd to mix and place than wet conc. Herman Conrow, Jr, A S C E, Trans, Vol 42, 1899, p 124.

22. Loading. From 12 to 24 cu yds of sand loaded into carts per man per day. 12 appears to be usual, but 24 not unreasonable.

23. Transportation. Av load broken stone, gravel or sand.

Wooden wheelbarrows.......2¼ to 2½ cu ft = 0.09 cu yd.
Iron wheelbarrows ... . 1.9 cu ft = 0.07 cu yd.

Cost of transportation per cu yd conc ordinarily betw 11 and 25 cts, depending largely upon the length of haul and the industry of the laborers.

Cost of Mixing.

24. Mixing (only). Much depends upon the diligence of the laborers, and the size of the mixer. Several examples indicate costs less than 10 cts / cu yd, counting labor only, while others indicate, quite regularly, about 25 cts. Sabin says "The cost of mixing conc in large quantities is seldom less than 30 cts / cu yd if allowance is made for plant."

25. As far as practicable, the course of the material should be downward; the mixer being kept above the work if possible. If an elevator is used for the conc, its entrance should be below the mixer. In subway or sewer work, the mixer can sometimes be placed below the street level and yet above the level of the work, so that it becomes unnecessary to raise the materials again after dumping them onto the street from the wagons. Much may be lost if the supply of materials and the demand for conc are not kept nearly equal, or if the conditions are such that the men cannot keep out of each other's way.

26. Ordinarily, more than half a dozen men cannot be disposed about a mixer to operate it to advantage, measuring materials, cleaning up platforms, etc (besides those actually engaged in getting the materials to and from the mixer). Cost, for labor only, should not be much over 15 cts per cu yd of conc, even with small machines.

27. Mixers, turning out from 10 to 40 cu ft of concrete per batch (or, assuming one batch every 2 mins, 10 to 40 cu yds per hour) will cost from $500 to $1000, and will require from 5 to 10 HP. to operate. Hand power machines, with a capacity of 5 cu ft per batch, about $250.

28. Cost of **setting up** a mixer, and taking it down, including carting a few miles, and depreciation, betw $50 and $100.

Up to 100 or 200 cu yds of conc, hand mixing is usually more economical than machine mixing.

29. The first cost of a **hand mixing plant,** to be operated by 8 or 10 men, estimated as follows:

8 square-pointed shovels, size No. 3	$10
3 iron wheelbarrows	35
2 rammers	5
1 mixing platform, 15 × 15 ft	10
Total	$60

30. Performance. When material is promptly delivered, batch mixers turn out, on an av, one batch in from 2 to 3 mins. A batch in one min is extremely fast working. Sometimes 4 or 5 mins are reqd. For capacities and power reqd, see under "Mixers," ¶ 27.

31. The cost of a mixing plant for conc work is variously estimated at from 3 to 5 % or more of the cost of the work.

32. The life of a mixer, under av conditions, is from 30,000 to 40,000 batches. Thus, a mixer, turning out 120 batches per day, will require renewal in about a year. A new drum will generally be needed after turning out two-thirds the total quantity.

33. Mixer to forms. Time to fill a barrow from a mixer, about 10 secs; to discharge the entire mixer at one operation, 15 to 20 secs.

34. Av barrow load of mixt conc, 1½ to 1¾ cu ft = 0.06 cu yd. One-horse carts hold about ½ cu yd; two-horse, 1 to 2 cu yds. To compute costs of hauling, etc., see Art 4 under "Cost of Earthwork," p **1025.**

35. About 10 or 15 cu yds of conc per man per 10 hour day can be loaded by shoveling.

Cost of Forms.

36. Cost, including material and labor, varies chiefly with the character of the structure; simple forms for mass work being relatively cheap, while those for detailing walls and floors of bldgs, especially in reinfd conc, are about the most expensive.

37. Material for forms betw 10 and 80 cts / cu yd of conc in place.

38. Fabrication and erection will cost from $4 to $10 **per 1000 ft B.M.** for the simpler forms of construction, in buildings, from $10 to $20.

39. The cost of forms may be as low as 10 and as high as 50 **per cent** of the total cost of the conc in place; 25 to 35 % for forms for ordinary reinfd work, 50 % or over for detailed building work.

40. The cost, **per sq ft of surface** (as one side of a wall) can be best computed for the work in hand, given the cost of the lumber and labor available; but will usually be betw 4 cts and 20 cts.

41. The cost of forms, **per cu yd of concrete,** in building constr, is stated betw $3 and $10, from $4 to $6 being sufficient for floor construction, and $5 to $7 being more usual limits for forms for reinfd work.

42. Shifting and depreciation. The figures given for cost of forms assume that the material is not used again. For special work, involving difficult and unusual details, the forms are practically worthless after they have been used. Ordinarily the lumber can be used 2 or 3 times before it is discarded. On large buildings, the forms for which are carefully designed, and where the detailing is similar thruout, forms may be used a half dozen times.

43. The labor of shifting forms will be not much less than the labor of first erecting them.

44. Cost of **labor, for placing** forms, betw 3 or 4 % and 20 % of the cost of conc in place.

Cost of Placing.

45. Cost of **fabricating** (bending, framing, &c) **and placing reinfmt,** from about ½ to 1½ cts / lb of reinfmt. Unit systems, 33 to 50 % more.

46. Depositing. The actual labor required, for depositing only, seldom amounts to more than an extra man to help dump carts, move shutes, etc; not more than a few cts per cu yd of conc placed. Records indicate from 7 cts up, but these probably include transportation from mixer to forms.

47. Spreading and ramming. Cost varies greatly with the character of the work; being as low as 15 cts / cu yd in fairly rough mass work (5 cts if the mixture is very wet); and as high as $1 or more where much care is taken in placing, tamping, ramming and spading. Less if conc is dumpt from carts or buckets in large quantities.

48. For ramming alone, from 5 to 15 or 20 cts / cu yd; seldom over 40 cts.

Miscellaneous Costs.

49. Inspection and superintendence, as usually done, about 1 to 3 % of the cost of the work. In view of the gross inefficiencies that are likely to result if the work is not well arranged or the men not kept up to standard, it may pay to expend as much as 5 or 10 %, or more.

50. Finishing. Data very variable, due probably to diff in method.

51. Washing with brush, ⅓ ct to 7 cts / sq ft of surf; with dilute hydrochloric acid, to remove efflorescence, about 20 cts / sq ft.

52. Bush hammering; 3 to 26 cts / sq ft. Pneumatic, less than 1 ct. Pointing up and brush coating, 25 cts / sq ft or more.

Total Costs.

53. Plain. For total costs, see "Mass," etc, ¶ 56.

54. Dry conc, about $1 more per cu yd than wet, due to additional labor of ramming.

55. Gravel conc $1 to $2 / cu yd cheaper than stone conc, given the same ratio of (sand + stone) to cem, the greater diff obtaining in mixtures low in cem.

56. Mass. Breakwaters, fortifications, etc, cost betw $5 and $7 / cu yd of conc in place, the av being very close to $6. Extremes as low as $1 and as high as $8.

57. Reinforced. Where work is well organized, reinfd buildings may be built for as low as $10 / cu yd of conc in place; but the general av is nearer $18, while some builders estimate roughly on $1 / cu foot ($27 / cu yd) altho few records run so high.

58. The cost depends chiefly upon the forms (see "Forms," ¶ 36). If these are well designed, so that they are easily shifted and can be used repeatedly, the cost is low; as compared with special jobs, where refinements in designing would not pay.

59. Retaining walls, foundation walls, abutments, locks, piers, etc, vary greatly, apparently owing to the widely varying difficulties of construction likely to be encountered. The extremes run from $4 to $16 / cu yd of conc in place. Quite often, however, the price will be betw $6 and $9. Reinfd walls from $3 to $10 more.

60. Arches of moderate span, say up to 30 ft, for culvert work, etc, from $5 to $10 / cu yd.

61. Buildings. Cost may be expected to fall betw $6 and $12 / cu yd of conc in place, with the av about $8 for plain, and $10 to $15 or $20 for reinfd construction.

62. For any given type of constr, all portions of a building (except foundations), such as the floors, walls, and columns, cost practically the same per cu yd.

63. Mr. L. C. Wason (E R, '09, Feb 27, p 233) gives, as **cost of buildings:**

	$ per cu ft of space enclosed			$ per sq ft of floor		
	max	av	min	max	av	min
Offices and stores	0.197	0.131	0.084	2.42	1.77	1.12
Factories	0.129	0.102	0.060	1.70	1.34	0.90
Garages	0.118	0.102	0.085	..	...	1.23
Filters	0.333	0.233	0.134	3.82	2.43	1.04
Storehouses	0.083	0.076	0.069	0.84	0.71	0.58
Mills, etc, 2d class	0.122	0.069	0.045	1.51	0.90	0.54

PLASTERING.

THE plastering of the inside walls of buildings, whether done on laths, bricks, or stone, generally consists of three separate coats of mortar. The first of these is called by workmen the *rough* or *scratch coat;* and consists of about 1 measure of quicklime, to 4 of sand; (which latter need not be of the purest kind;) and ⅓ measure of bullock or horse hair; the last of which is for making the mortar more cohesive, and less liable to split off in spots. This coat is about ⅜ to ½ inch thick; is put on roughly; and should be pressed by the trowel with sufficient force to enter perfectly between and behind the laths; which for facilitating this should not be nailed nearer together than ½ an inch. In rude buildings, or in cellars, &c, this is often the only coat used. When this first coat has been left for one or more days, according to the dryness of the air, to dry slightly, it is roughly *scored*, or *scratched*, (hence its name,) with a pointed stick, or a lath, nearly through its thickness, by lines running diagonally across each other, and about 2 to 4 ins apart. This gives a better hold to the second coat, which might otherwise peel off. If the first coat has become too dry, it is well also to dampen it slightly as the second one is put on.

The second coat is put on about ¼ to ⅜ inch thick, of the same hair mortar, or *coarse stuff.* Before it becomes hard, it is roughed over by a hickory broom, or some substitute, to make the third coat adhere to it better.

The third coat, about ⅛ inch thick, contains no hair. and for giving it a still whiter and neater appearance, more lime is used, say 1 of lime, to 2 of sand; and the purest sand is used. This mortar is by plasterers called **stucco;** a name also applied to mortar when used for plastering the outsides of buildings. Or instead of stucco, the third coat may be, and usually is, of *hard finish*, or *gauge stuff;* which consists of 1 measure of ground plaster of Paris, to about 2 of quicklime, without sand. Hard finish works easier; but is not as good as stucco, for walls intended to be painted in oil. The plaster of Paris is for hastening the hardening.

Either of these third coats is smoothed or polished to a greater or less extent, according to whether it is to show, or to be papered, painted, &c. The polishing tools are merely, the trowel; the hand-float, (a kind of wooden trowel,) and the water-brush, (a short-handled brush for wetting the surface part at a time with water, in order to polish more freely.) For finer polishing, a float made of cork is used. The smooth piece of board about 10 to 12 ins square, with a handle beneath, on which the plasterer holds his mortar until he puts it on to the wall with his trowel, is called a *hawk.*

The more thoroughly each coat is gone over with the water-brush and trowel, (which process is called *hand floating,*) the firmer and stronger will it be. Frequently only two coats of plastering are put on in inferior rooms; or where great neatness of appearance is not needed. The first is of hair mortar, or coarse stuff; this is scratched with the broom, and then covered by the finishing coat of finer mortar, (stucco.) If this last is nearly all lime, or with but very little sand, to make it work easier, it is called a *slipped coat.* Without any sand it is called *fine stuff.* Neither is as good as stucco, if the wall is to be papered. When this is the case, the third coat also may have a little hair, to give it more strength; but this is not absolutely necessary.

A very good effect may be produced in station houses, churches, &c, by only two coats of plaster in which fine clean screened gravel is used instead of sand. When lined into regular courses, it resembles a buff-colored sandstone, very agreeable to the eye.

In purchasing plastering hair, care must be taken that it has not been taken from salted hides, inasmuch as the salt will make the walls damp. For the same cause sea-shore sand should not be used. It is almost impossible to wash it entirely free from salt.

In brick walls intended to be plastered, the mortar joints should be left very rough, to let the plaster adhere. If it is put on smooth walls, without first raking out the mortar to the depth of nearly an inch, it is very apt to fall off; especially from outside walls; as can be seen daily in any of our cities. As this raking out of brick joints is tedious and expensive, it would generally be better to use paint rather than plaster. The walls should also be washed clean from all dust; and should be slightly dampened as the plaster is put on.

To imitate granite on outer walls: after the second or smooth coat of plaster is dry, it receives a coat of lime wash, slightly tinted by a little umber, or ochre, &c. After this is dry, in case it appears too dark, or too light, another may be applied with more or less of the coloring matter in it. Finally, a wash of lime and mineral-black is *sprinkled* on from a flat brush, to imitate the black specks of granite. By this simple means, a skillful workman can produce excellent imitations. The horizontal and vertical joints of the imitation masonry, may be ruled in by a small brush, using the same black wash, and a long straight-edge.

The rough surfaces of all walls are more or less warped, or out of line; and it is not possible for the plasterer to rectify this perfectly by eye, as may be seen in almost every house. Even in what are called first-class ones, a quick eye can generally detect unsightly undulations of the plastered surfaces.

To prevent this, the process of **screeding** is resorted to. Screeds are a kind of gauge or guide formed by applying to the first rough coat, when partly dried, horizontal strips of the plastering mortar, about 8 ins wide, and from 2 to 4 ft apart all around the room. These are made to project from the first coat, out to the intended face of the second one; and while soft are carefully made perfectly straight, and out of wind with each other, by means of the plumb-line, straight-edge, &c. When they become dry, the second coat is put on, filling up the broad horizontal spaces between them; and is readily brought to a perfectly flat surface, corresponding with that of the screeds, by means of long straight-edges extending over two or more of the latter.

A day's work at plastering.

A plasterer, aided by one or two laborers to mix his mortar, and to keep his hawk supplied, can average from 100 to 200 square yards a day, of first coat; about ⅔ as much of second; and half as

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Mills, etc, 2d class	0.122	0.069	0.045	1.51	0.90	0.54

laid horizontally from rafter to rafter; or sloping, from purlin to purlin as the case may be; or to stout laths *t t t* about 2 to 3 ins wide, and from 1 to 1¼ thick, nailed to the rafters at distances apart to suit the gauges of the slates. Two nails are used to each slate; one near each upper corner. They may be either of copper, (which is the most durable, but most expensive,) of zinc, or of either galvanized or tinned iron. The last two are generally used; or in inferior work, merely plain iron ones, previously boiled in linseed oil, as a partial preservative from rust. Rust, however, sometimes weakens them so much that they break; and the slates are blown off in high winds, to the danger of passers by. Since good slate endures for a long series of years, it is true economy to use nails that are equally durable. In iron roofs, the slates, instead of being nailed to boards, are sometimes tied directly to the iron purlins, by wire **A square** of slating, shingling, &c, is 100 sq ft.

In laboratories, chemical factories, &c, subject to **acid fumes**, it is difficult to provide a metal fastening that will not be eaten away In such cases it is best to depend chiefly upon a layer of mortar between the slates. This will harden before the metal fastenings give way; and will hold the slates in place, while new fastenings are being inserted.

The **least pitch** considered advisable for a roof, to prevent rain or snow from being driven through the interstices between the slates, is about 26½°; or 1 vert to 2 hor; which corresponds to a rise of ¼ the span in a common double pitched roof But even at steeper pitches, rain, and more particularly snow, will be forced through the roof by violent winds; especially if laths alone be used, or even boarding alone. To avoid this, a layer of mortar about ¼ inch thick, may be spread over the touching surfaces of the slates if on laths. If on boards, the same process may be adopted; or the more common one of first covering the boards with a layer of what is called *slating felt*, but which in reality is merely thick brown paper, soaked in tar. This is sold in long continuous rolls, 28 ins wide, and weighing from 40 to 50 lbs. A 50 lb roll will cover about 300 sq ft of roof. With proper precautions against the admission of rain and snow, a pitch as flat as 1 in 2½, or even 1 in 3 may be adopted.

The thickness of slate on a roof is double; except at the *laps i s, i s*, &c, where it is triple. The lap is measured from the nail hole (under *i*) of the lower slate, to the lower edge or *tail*, *s*, of the upper one, and is usually about 3 ins In order that the showing lower edges of the slates shall, when laid, form regular straight lines along the roof, the nail holes are made at equal distances from said *lower* edges, so that any irregularity of length is concealed from view at the hidden *heads* of the slates. The slater estimates the length of his slate from the nail hole to the tail; discarding the narrow strip between the nail hole and the head If from this reduced length the lap be deducted, then one-half of the remainder will be the *gauge*, *weathering*, or *margin*, of the slating, or, in other words, the *showing* or *exposed* width of the courses of slates. The gauge in ins multiplied by the width of a slate in ins, gives the area in sq ins of finished roof covered by a single slate; and if 144 (the sq ins in a sq foot) be divided by this area, the quotient will be the number of slates required per sq ft of roof. The upper side of a slate is called its *back*, the lower one, its *bed*.

Slating, like shingling, must evidently be commenced at the eaves, and extended upward Since the beds of the slates are not exactly parallel to the boarding, and consequently do not rest flat upon it, those at the lower edge *w* would easily be broken. To prevent this, a *tilting strip* (a stout wide lath, with its upper side planed a little bevelling, to suit the slope of the slates) is first nailed around near the eaves, for the tails of the lowest course of slates to rest on. This is shown on a larger scale at T.

Slate of the best quality has a glistening semi-metallic appearance, somewhat like that of a surface of paper rubbed with black-lead pencil. That of a dull earthy aspect, is softer, more absorbent, and consequently more liable to yield to atmospheric influences, rain, frost, &c. Iron pyrites frequently occurs in slate, and since it always decomposes and leaves holes, should never be admitted on a roof. Of two qualities of slate, that which absorbs the least weight of water, when pieces of equal size are soaked for an hour or two, is generally the best; being least liable to split by frost, and become weather-worn. This test is easily applied.

In England the **different sizes** are distinguished by absurd names of no meaning. In the United States they are called 6 by 12's; 16 by 24's, &c, according to their measures in inches. They may be cut to order, of almost any prescribed dimensions, or shape. Those in common use vary from about 7 by 14, to 12 by 18. The first forms about 5 to 6 inch courses; and the last about 7 to 8 inch; depending upon how far from the head the nail holes are pierced. The farther this is, the firmer will the slating be.

Slate roofs, like iron ones, heat the rooms immediately below them very much. This is somewhat diminished when the slates are on boards, instead of laths; and still more by a coat of plaster beneath They are also liable to break when walked on; less so when bedded in mortar.

Weight of slate roofs. Slate weighs about 175 lbs. per cub foot; therefore, a sq ft, ⅛ inch thick, weighs about 1.8 lbs, $\frac{3}{16}$, 2.7 lbs; and ¼ thick, 3.6 lbs. But owing to the overlapping, a square foot of roof requires about 2¼ sq ft of slate of ordinary sizes; and if the slate is laid on boards an inch thick, the weight per sq ft of roof will be increased about 2¼ lbs; or with 1¼ inch boards, 2.8 lbs. Laths will weigh about ¼ lb per sq ft of roof.

Hence,

			Approx Weight of one sq ft of Slating, in lbs.
Slate	⅛ inch thick	on laths	4.75
"	"	on 1 inch boards	6.75
"	"	on 1¼ " "	7.30
"	3-16 "	on laths	7.00
"	" "	on 1 inch boards	9.00
"	" "	on 1¼ " "	9.55
"	¼ "	on laths	9.25
"	" "	on 1 inch boards	11.25
"	" "	on 1¼ " "	11.80

If slating felt is used, add ⅓ lb; or if the slates are bedded in ¼ inch of mortar, add 3 lbs

For the total weight borne by the *roof trusses*, that of the purlins also must be added. This will not vary much from the limits of 1½ to 3 lbs per sq ft in roofs of moderate span. Add for wind and snow, say 20 lbs per sq ft; and finally add the weight of the truss itself.

For stopping the joints between slates (or shingles, &c) and chimneys, dormer windows, &c, a mixture of stiff white lead paint, as sold by the keg, with sand enough to prevent it from running, is very good; especially if protected by a covering of strips of lead, or copper tin, &c, nailed to the mortar-joints of the chimneys, after being bent so as to enter said joints; which should be scraped out for an inch in depth, and afterward refilled. Mortar protected in the same way, or even unprotected, is often used for the purpose; but is not equal to the paint and sand. Mortar a few days old, (to allow refractory particles of lime to slack,) mixed with blacksmith's cinders, and molasses, is much used for this purpose, and becomes very hard, and effective.

SHINGLES.

White cedar shingles are the best in use; and when of good quality will last 40 or 50 years in our Northern States. They are usually 27 ins long; by from 6 to 7 ins wide; about ¼ inch thick at upper end; and about ⅝ at lower end or butt; and are laid in courses about 8⅛ ins wide; so that not quite ⅓ of a shingle is exposed to the weather.

They are usually laid in three thicknesses; except for an inch or two at the upper ends, where there are four. They are nailed to sawed shingling-laths of oak or yellow pine; about 16 ft long; 2½ ins wide, and 1 inch thick; placed in horizontal rows about 8⅛ ins apart. These are nailed to the rafters, or purlins; which, for laths of the foregoing size, should not be more than 2 ft apart from center to center. Two nails are used to each shingle, near its upper end. They should not be of less size than 400 to a lb. Wrought nails being the strongest, are the best; cut ones are apt to break by the warping of the shingles. Two pounds of such nails will suffice for 100 sq ft of roof, including waste. An average shingle 7½ ins wide, in 8⅛ inch courses, exposes 63¾ sq ins; making 2¼ shingles to a sq ft of roof, but to allow for waste, and narrow shingles, it is better in practice to allow about 3 shingles to a sq ft.

Shingling like slating, must plainly be begun at the eaves; and extended upward. For closing the joints between the shingles, and chimneys, dormer windows, &c, see at end of Slating.

Cypress and white pine are also much used for shingles, being much cheaper, but scarcely half as durable. All shingles wear quite thin in time by rain and exposure In warm damp climates they all decay within 6 to 12 years

PAINTING.

The principal material used in house-painting, is either white lead, or oxide of zinc, ground in raw (unboiled) linseed oil, by a mill, to the consistency of a thick paste. In this condition, it is sold by the manufacturers in kegs of 25, 50, and 100 lbs. To prepare it for actual use, merely requires the addition of more linseed oil, say 3 or 4 pints to 10 lbs of the keg paint, for thinning it sufficiently to flow readily under the brush.

Good painting requires 4 or 5 coats; but usually only 4 are used in principal rooms; and 3 in inferior ones. Each coat must be allowed to dry perfectly before the next one is put on. One lb of the keg paint will, after being thinned, cover about 2 sq yds of first coat; 3 yds of second, and 4 yds of each subsequent coat; or 1 sq yd of 3 coats will require in all, 1.08 lbs; of 4 coats, 1¼ lbs; of 5 coats, 1.58 lbs. The reason why the first coats require so much more than the subsequent ones, is that the bare surface of the wood absorbs it more.

When, as is usual, *raw* or unboiled oil is used for thinning, *dryers* must be added to it; otherwise the paint might require several weeks to harden; whereas, with dryers, from 1 to 3 days, according to the weather, suffice for each coat to become hard enough to receive the next one. The dryers most commonly used, are powdered litharge, in the proportion of one heaped teaspoonful; or Japan varnish, 1 table-spoonful, to 10 lbs of the keg paint. Either sugar of lead, or sulphate of zinc, may also be used instead of litharge; and in the same proportion. Although both litharge and Japan varnish are dark-colored, yet the quantity is so small as not to appreciably affect the whiteness of the paint. If the varnish is used in excess, as is often done in the hurry to have work finished, it produces cracks all over the surface. No dryer is necessary if painters' boiled oil be used for thinning. Mere boiling will not cause oil to harden more rapidly; but that intended for painters, has litharge added to it previously to boiling, in the proportion of 1½ lbs to each 10 gallons of raw oil In some works written for the use of house painters, it is asserted that boiling renders the oil too thick for any but coarse outdoor work. But this is entirely a mistake; for if the boiling be properly done, the oil will be quite thin enough for the best inside work; and will moreover be clearer than while raw; and

will impart to the painted surface a more shining appearance. The heat should be barely sufficient to produce boiling, or about 600° Fah. The boiling should continue about 1½ hours; the oil being thoroughly stirred at short intervals, to prevent the litharge from settling at the bottom. The fire may then be allowed to subside; when the operation will be completed. A sediment will then form at the bottom; which must be left behind when the oil is poured off. Although no dryer is necessary with this oil, still a little litharge may be added when great expedition demands it. Painters rarely use this oil, on account of its trifling increase of cost.

Another substance much used with the thinning oil, (except for the first coat,) is spirits of turpentine, called "turp" by the workmen. The quantity of oil may be diminished, to the extent of the added turp. This being more fluid than oil, causes the paint to work more pleasantly under the brush. It moreover diminishes the tendency of the paint to become yellow, especially in rooms kept closed for some time. It is also much cheaper than oil. It should not be used, or but sparingly, for exposed outdoor work; inasmuch as its tendency is to impair the firmness of the paint, and although its effects are scarcely appreciable indoors, they are quite apparent when the work has to resist the weather. As the fashions change in house-painting, the surface is at times required to present a *shining* or *glossy* finish, at other times a *dead* one is in vogue. The glossy one is that which the paint will naturally have, provided that no more turp than oil be used in the thinning. The dead finish is obtained by using no oil, but turp alone, for the last coat, which in that case is called a *flatting coat*. Although turp is not properly a dryer, still, as it evaporates quickly, it facilitates the hardening of the paint.

In outdoor work it is usually advisable to use more dryer than inside, so that the paint may sooner become hard enough not to be injured by dust or rain. Otherwise less would be better.

When, instead of a white finish, one of some other color is required the coloring ingredient is mixed with the white paint to be used in the last coat only; although two coloring coats are sometimes found to be necessary before a satisfactory effect is produced. The coloring ingredients may be indigo, lampblack, terra sienna, umber, ochre, chrome yellow, venetian red, red lead, &c, &c, which are ground in oil, ready for sale, by the manufacturers of the white-lead and zinc paints. They are simply well stirred into the white paint.

All surfaces to be painted, should first be thoroughly dry, and free from dust. If on wood, all plane marks, and other slight irregularities, should first be smoothed off by sand-paper when the neatest finish is required. Also, all heads of nails must be punched to about ⅛ inch below the surface. To prevent knots from showing through the finished work, (as those in white or yellow pine would do, on account of the contained turpentine,) they must first be *killed* as it is termed. A usual and effective way of doing this, is by covering them with two coats of shellac varnish, which, when dry, should be smoothed by sand paper. Another mode, not quite so certain, is by one or two coats of white lead mixed with thin glue water, or *size*, as it is called.

After these preparations, the first, or *priming* coat, is put on; in which there should be no turp; because it would sink at once into the bare wood, leaving the white lead behind it, in a nearly dry friable condition. After this the nail holes, cracks, &c, must be filled with common glaziers' putty, made of whiting (fine clean washed chalk) and raw linseed oil, boiled oil will not answer; the putty would be friable. The putty would be apt to fall out, if put in before priming, because the wood would absorb the oil, and the putty would then shrink. After the first coat is perfectly dry, the second one is put on; and for it about 1 measure of turp may be mixed with 3 measures of the thinning oil. In the third, and any subsequent coats, equal measures of turp and oil, may be used for thinning, if the work is required to dry *with a gloss*, but if it is to finish *dead*, the last coat must be a *flatting* one; or one in which the thinning oil is entirely omitted, and turp alone substituted for it.

Painters generally clean their brushes by merely pressing out most of the paint with a knife; and then keep them in water until further use. If to be put away for some time, they may be thoroughly cleaned by turp; or by soap and water. To prevent a hard skin from forming on the top of their paint when not used for some days, they pour on a little oil.

The best paints for preserving iron exposed to the weather, appear to be pulverized oxides of iron such as yellow and red iron ochres; or brown hematite iron ores finely ground, and simply mixed with linseed oil and a dryer. White lead applied directly to the iron, requires incessant renewal; and indeed probably exerts a corrosive effect. It may, however, be applied over the more durable colors, when appearance requires it. Red lead is said to be very durable, when pure. An instance is recorded of pump-rods, in a well 200 ft deep near London, which, having first been thus painted, were in use for 45 years; and at the expiration of that time their weight was found to be precisely the same as when new; thus showing that rust had not affected them.

When the size of the exposed iron admits of it, its freedom from rust may be very much promoted by first heating it thoroughly, and then dipping it into, or washing it well with, hot linseed oil, which will then penetrate into the interior of the iron. For tinned iron exposed to the weather, on roofs, rain pipes, &c, Spanish brown is a very durable color. The tin is frequently found perfectly bright and protected, when this color has been used, after an exposure of 40 or 50 years. White paint washes off in a few years by rain.

Plastered walls should if possible be allowed to dry for at least a year, before being painted in oil; otherwise the paint will be liable to blister. They may, if preferred, be frescoed (water color mixed with size) to the desired tint during the interval.

The painting of unseasoned wood hastens its decay. If the surface to be painted is greasy, the grease must first be removed by water in which is dissolved some lime.

Washes for outside work. Downing, in his work on country houses, recommends the following: *For wood work:* in a tight bushel, slack half a bushel of fresh lime, by pouring over it boiling water sufficient to cover it 4 or 5 ins deep, stirring it until slacked. Add 2 lbs of sulphate of zinc (white vitriol) dissolved in water. Add water enough to bring all to the consistence of thick whitewash. Apply with a whitewash brush. This wash is white; but it may be colored by adding powdered ochre, Indian red, umber, &c. If lampblack is added to water-colors, it

should first be thoroughly dissolved in alcohol. The sulphate of zinc causes the wash to become hard in a few weeks.

For brick, masonry, or rough-cast. Slack ½ a bushel of lime as before; then fill the barrel ⅔ full of water, and add a bushel of hydraulic cement. Add 3 lbs of sulphate of zinc, previously dissolved in water. The whole should be of the thickness of paint; and may be put on with a whitewash brush. The wash is improved by stirring in a peck of white sand, just before using it. It may be colored, if desired like the preceding.

He also gives the following cheap oil-paint for outside work on wood, brick, stone, &c; and says it becomes far harder and more durable than common paint. One measure of *ground* fresh quicklime; add the same quantity of fine white sand, or fine coal ashes; and twice as much fresh wood ashes; all the foregoing to be passed through a fine sieve. Mix well together dry. Mix with as much raw linseed oil as will make the mixture as thin as paint. Apply with a painter's brush. It may be colored like the foregoing, taking care to mix the colors well with oil before adding them. It is best to put on two coats; the first thin, and the second thick.

Also, another, said to stand 15 to 20 years: 50 lbs best white lead; 10 quarts raw linseed oil; ½ lb dryer: 50 lbs finely sifted sharp clean sand; 2 lbs raw umber. Add very little, say ½ pint of turpentine. Apply with a *large* brush.

Cement for stopping joints, such as around chimneys, &c, &c. White lead ground in oil, as sold by the keg; mixed with enough pure sand to make a stiff paste that will not run. It grows hard by exposure, and resists heat, cold, and water. Pieces of stone may be strongly cemented together by it, allowing a few months for proper hardening.

Whitewash for inside work, according to Mr. Downing, "is made more fixed and permanent, by adding 2 quarts of thin *size* to a pailful of the wash, just before using. The best size for this purpose is made of shreds of glove leather; but any clean size of good quality will answer," as thin glue-water. We will add, that the common practice of mixing salt with whitewash, should not be permitted. Paper pasted on a wall which has previously been covered with salt whitewash, is very apt to become wet, and loose, and to fall off during damp weather. The whitewash should be scraped off, and the wall or partition covered with a coat or two of thin size, to protect the paper from the effect of the salt that may still adhere to the plaster.

GLASS, AND GLAZING.

WINDOW glass is sold by the box. Whatever may be the size of the panes, a box contains as nearly 50 sq ft of glass as the dimensions of the panes will admit of.

Panes of any size may be made to order by the manufacturers. The sizes given in the following table, as well as many others, are generally to be had ready made. Ordinary window glass of all the sizes in the table, is about one-sixteenth of an inch thick, and this is the thickness supposed to be intended when a greater one is not specified. *Double-thick* glass is nearly ⅛ inch; and its price is 50 per ct more than the *single* thick. It is of course much stronger than the single.

The panes are confined to the sash by glaziers' putty, made of whiting (powdered chalk) and raw linseed oil; and by small triangular pieces of thin tin, about ⅛ inch on a side, which uphold the glass while the putty is being put on; and are allowed to remain afterward, as a protection while the putty continues soft.

TABLE OF NUMBERS OF PANES IN A BOX.

Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.
6 × 8	150	12 × 36	17	16 × 42	11	24 × 24	12	30 × 66	4
7 × 9	115	13 × 14	40	48	9	26	12	70	3
8 × 10	90	16	35	54	8	30	10	32 × 34	7
12	75	18	31	60	8	36	9	36	6
9 × 12	67	20	28	18 × 20	20	42	7	42	6
14	57	24	23	22	18	48	6	48	5
16	50	32	17	24	17	54	6	60	4
18	45	14 × 16	32	30	14	60	5	66	3
10 × 12	60	18	29	36	11	66	5	34 × 36	6
14	52	20	26	42	10	26 × 28	10	44	5
16	45	24	22	50	8	32	9	48	5
18	40	30	17	60	7	36	8	54	4
20	36	36	14	20 × 22	17	42	7	60	4
24	30	42	12	24	15	48	6	66	3
30	24	46	11	30	12	54	5	36 × 40	5
11 × 12	55	15 × 16	30	38	10	60	5	44	5
14	47	18	27	42	9	28 × 30	9	48	4
16	41	20	24	48	8	36	7	54	4
18	37	24	20	54	7	42	6	60	3
20	33	30	16	64	6	56	5	70	3
24	27	36	13	22 × 24	14	66	4	38 × 44	4
12 × 14	43	40	12	30	11	30 × 34	7	52	4
16	38	16 × 18	25	36	9	36	7	40 × 46	4
18	34	20	23	42	8	42	6	54	3
20	30	24	19	48	7	48	5	72	3
24	25	30	15	56	6	54	4	44 × 50	3
28	22	36	13	60	5	60	4	56	3
30	20								

The best qualities of American glass made in the vicinity of Philadelphia, Boston, Pittsburg, &c, are for most purely *useful* purposes, as good as those from foreign countries; but when the highest degree of *beauty* is required, as in the lower front windows of first-class dwellings, fancy stores, &c, polished plate-glass of England, France, or Germany, must be used, although the price for moderate sized panes is from 5 to 8 times as great as that of the best quality single-thick American. Its perfectly *smooth* surface, free from distorted reflections, also makes it the best for covering pictures; still, if carefully selected American panes be used for this purpose, few except critics in glass will detect the difference.

A thick glass is made expressly for flooring, up to 1 inch thick, and up to 50 inches by 9 feet dimensions. Also, for skylights, from ¼ to ½ inch thick. This can be furnished to order of any size up to 40 inches by 8 or 10 feet. The smaller sizes can also be had *ground*. Grinding prevents the entrance of the full glare of the sun; and, moreover, diffuses the light over a much greater width of space below.

Strength of glass. Tensile 2500 to 9000 lbs per square inch. Boston rods by author, 3500 to 5200. Crushing strength, 6000 to 10000 lbs per square inch. Transversely, (by the writer's trials) flooring glass, 1 inch square, and 1 foot between the end supports, breaks under a center load of about 170 lbs; consequently, it is considerably stronger than granite, except as regards crushing; in which the two are about equal.

REMARK. Window and other glass which contains an excess of potash or of soda is very liable to become dull in time, owing to the decomposition of those ingredients by atmospheric influences.

ROPE.

The strength of rope varies greatly. Pieces from the same coil may vary 25 per cent. The table below supposes an average quality **Manila.** Good **Italian hemp** is considerably stronger. The **tarring** of ropes is said to lessen their strength; and, when exposed to weather, their durability also. We believe that its use in standing rigging is partly to diminish contraction and expansion by alternate wet and drying weather. A few months of **exposed work** weakens ropes 20 to 50 per cent.

Table of Manilla rope.

Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load. Tons.	Breaking load. lbs.	Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load. Tons.	Breaking load. lbs.
.239	¾	.019	.25	560	1.91	6	1.19	11.4	25536
.318	1	.033	.35	784	2.07	6½	1.39	13.0	29120
.477	1½	.074	.70	1568	2.23	7	1.62	14.6	32704
.636	2	.132	1.21	27.33	2.39	7½	1.86	16.2	36288
.795	2½	.206	1.91	4278	2.55	8	2.11	17.8	39872
.955	3	.297	2.73	6115	2.86	9	2.67	21.0	47040
1.11	3½	.404	3.81	8534	3.18	10	3.30	24.2	54208
1.27	4	.528	5.16	11558	3.50	11	3.99	27.4	61376
1.43	4½	.668	6.60	14784	3.82	12	4.75	30.6	68544
1.59	5	.825	8.20	18368	4.14	13	5.58	33.8	75712
1.75	5½	.998	9.80	21952	4.45	14	6.47	37.0	82880

Working loads. For manila ropes from 1 to 1¾ ins diam, running at different speeds over sheaves of the diams stated, Mr. C W. Hunt (Trans Am Soc Mechl Engrs, Vol. XXIII, 1901) gives a table embodying approximately the following results of experience. Working load = C × ultimate strength of new rope. D = minimum diam of sheave, in ins.

Speed	ft per min	as for work on	C	1″ rope D	1¾″ rope D
Slow	50 to 100	derrick, crane, quarry	0.140	8	14
Medium	150 to 300	wharf, cargo	0.056	12	18
Rapid	400 to 800		0.028	40	70

Such ropes wear out rapidly. A rope 1½ ins diam wears out in lifting from 7,000 to 10,000 tons of coal. On the other hand, 1½ inch transmission ropes, running 5000 ft per min and carrying 1000 H. P. over sheaves 5 ft and 17 ft in diam, last for years.

Mr. Hunt's figures for ultimate strength, based upon tests of full-sized specimens of manila rope made by three independent rope-walks and purchased in open market, are practically identical with those given in our table above, as are also those of Prof. B. Kirsch, of the Imperial Royal Technological Industrial Museum, Vienna, quoted by Mr. Hunt.

WEIGHTS AND STRENGTHS OF WIRE ROPES.

Wire Rope manufactured by **John A. Roebling's Sons Co., Trenton, N. J.,** and others. See price list The prices and weights given are for ropes with *hemp* centers When made with *wire* centers, the prices per foot are 10 per cent. higher, and the weights 10 per cent. greater. **A. Leschen & Sons Rope Co., St. Louis, Mo.** "Hercules" rope; strengths and prices average about 50 per cent. higher than for cast steel rope.†

Trade No.	Diam. in ins.	Approx. circum. in ins.	Wt. per ft. in lbs	Approx. breaking strength* in tons of 2000 lbs.		Minimum diam. of drum in feet.		Price in cents per foot.†	
				Iron.	C. steel.	Iron.	C. steel.	Iron.	C. steel.
Standard Hoisting Rope, with 6 strands of 19 wires each.									
1	2¼	7⅛	8.00	78	156	13	8½	117	142
2	2	6¼	6.30	62	124	12	8	92	111
3	1¾	5½	4.85	48	96	10	7¼	80	93
4	1⅝	5	4.15	42	84	8½	6¼	63	74
5	1½	4¾	3.55	36	72	7½	5¾	57	66
5½	1⅜	4¼	3.00	31	62	7	5½	48	56
6	1¼	4	2.45	25	50	6½	5	40	46
7	1⅛	3½	2.00	21	42	6	4½	33	38
8	1	3	1.58	17	34	5¼	4	26	30
9	⅞	2¾	1.20	13	26	4½	3½	20	23
10	¾	2¼	0.89	9.7	19.4	4	3	16	18
10¼	⅝	2	0.62	6.8	13.6	3½	2¼	12	14
10½	9/16	1¾	0 50	5.5	11 0	2¾	1¾	10	12
10¾	½	1½	0.39	4.4	8.8	2¼	1½	8	11
10 *a*	7/16	1¼	0.30	3.4	6.8	2	1¼	7½	10
10 *b*	⅜	1⅛	0.22	2.5	5.0	1½	1	7	9½
Transmission or Haulage Rope, with 6 strands of 7 wires each.									
11	1½	4¾	3.55	34	68	13	8½	51	60
12	1⅜	4¼	3.00	29	58	12	8	43	51
13	1¼	4	2.45	24	48	10¾	7¼	36	43
14	1⅛	3½	2.00	20	40	9½	6¼	29	36
15	1	3	1.58	16	32	8½	5¾	23	28
16	⅞	2¾	1.20	12	24	7½	5	17½	22
17	¾	2¼	0.89	9.3	18.6	6¾	4½	14	16
18	11/16	2⅛	0 75	7.9	15 8	6	4	12	13½
19	⅝	2	0.62	6.6	13.2	5¼	3½	10	11
20	9/16	1¾	0.50	5.3	10.6	4½	3	8	9
21	½	1½	0.39	4.2	8.4	4	2½	6½	7½
22	7/16	1¼	0.30	3.3	6.6	3¼	2¼	5½	6½
23	⅜	1⅛	0.22	2.4	4.8	2¾	2	4½	5½
24	5/16	1	0.15	1.7	3.4	2½	1¾	3¾	4½
25	9/32	⅞	0.125	1.4	2.8	2¼	1½	3¼	4

Notes on the Use of Wire Rope, by the Roebling's Company.

The ropes with 19 wires per strand are the more pliable, and therefore best adapted for **hoisting** and **running** rope. The others are stiffer and better adapted for **guys,** &c. Ropes of iron or steel, up to 3 inches diameter, made to order. Hemp center rope is more pliable than wire center. Wire rope **must not be coiled or uncoiled** like hemp rope. When on a reel, the reel should be mounted on a spindle or flat turn-table in order to pay off the rope. When forwarded in a small coil without a reel, roll the coil on the ground like a wheel, and thus run off the rope. Avoid untwisting and short bends. **To preserve wire rope,** apply raw linseed oil (which may be mixed with an

* **For the safe working load,** take one-fifth to one-seventh of the breaking load, according to speed.

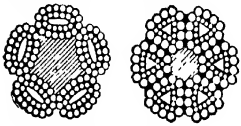
† For discounts, see price list

equal quantity of Spanish brown or lamp-black) with a piece of sheepskin, keeping the wool against the rope. **If for use in water or underground,** add 1 bushel of fresh-slacked lime and some sawdust to 1 barrel of tar. Boil the mixture well and saturate the rope with it while hot.

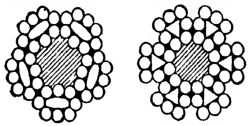
Galvanized wire rope for rigging is cheaper and more durable than hemp rope; and does not stretch permanently under great strains. Its bulk is one-sixth and its weight one-half that of hemp rope. Crucible cast-steel wire ropes are much more durable than iron ones. They should be kept well lubricated.

Patent Flattened Strand Wire Rope.

Manufactured by **A. Leschen & Sons Rope Co., St. Louis, Mo.**



Hoisting Ropes.



Haulage and Transmission Ropes.

Diameter inches.	Average weight, per ft., lbs.	Breaking* strength in tons of 2000 lbs. "Hercules."‡	Crucible cast steel.	Swedes iron.	Minimum diam. of drum in feet. "Hercules."‡	Crucible cast steel.	Swedes iron.	List price† per foot, in cents. "Hercules."‡	Crucible cast steel.	Swedes iron.	Diameter inches.
Hoisting Rope.											
2¼	9 25	260	176	75	12	8½	11½	285	182	152	2¼
2	7.50	211	140	66	11	8	10¾	225	144	120	2
1¾	5.40	168	109	54	9	7¼	9	208	121	104	1¾
1⅝	4 75	140	94	45	8½	6¼	7½	156	96	82	1⅝
1½	4.00	124	81	40	8	5¾	6¾	137	86	74	1½
1⅜	3 40	106	69	34	7½	5½	6¼	112	73	62½	1⅜
1¼	2.80	84	56	28	7	5	5¾	89	59½	52	1¼
1⅛	2 30	67	47	21	6	4½	5¼	71	50	43	1⅛
1	1.80	56	38	17	5	4	4¼	60	39½	34	1
⅞	1.35	40½	29	13	4½	3½	4	49	30	26	⅞
¾	1.00	32	21	9	4	3	3½	37½	24	21	¾
⅝	0 73	22½	15	6	3½	2¼	3	28	18½	15½	⅝
9/16	0 54	19	—	—	3	--	2½	25	—	13	9/16
½	0 44	13½	9½	4	2¾	1½	2	20¾	14½	10½	½
Haulage and Transmission Rope.											
1¼	2.80	80	54	27	9¼	7¼	9½	88	54	45	1¼
1⅛	2 30	64	45	22½	8	6¼	8½	70	45	36½	1⅛
1	1.80	53	36	18	6¾	5¾	7¾	58	35	29	1
⅞	1.35	38	27	13½	6	5	6¾	44	27½	22	⅞
¾	1.00	30	20	10	5¼	4½	6	35	20½	17½	¾
⅝	0 73	21	14	7½	4½	3½	4¾	25	14	12½	⅝
½	0.44	13	9	4½	3¾	2½	3½	16¼	10	8¼	½

* Working load = 0.2 × breaking strength.

† For discounts, see price list, pp 983 etc.

‡ "Hercules" is a registered trade mark rope, "made from a specially drawn and patent tempered steel, manufactured solely for this brand of rope."

PAPER.

24 sheets 1 quire. 20 quires 1 ream.

Sizes of drawing papers.

	Ins. Ins.		Ins Ins
Antiquarian	31 × 52	Super Royal	19 × 27
Double Elephant	26 × 40	Royal	19 × 24
Atlas	26 × 34	Medium	17 × 22
Imperial	21 × 30	Demy	15 × 20
		Cap	13 × 17

The English drawing-papers are stronger and superior to the American. Those by Whatman have a high reputation; they are, however, of different qualities. When paper is pasted on muslin, the difference in quality is not so important. Of paper in rolls, the German makes are the best. There is but little of other makes imported.

Both white and tinted papers, for the use of engineers, are made in continuous rolls, without seams. Widths 36, 42, 54, 58, and 62 ins; usual lengths 40 yds; but can be had to order to 400 yds or more. These may also be had mounted on muslin, in rolls 10 to 40 yds long.

Cartridge or **pattern** paper is furnished in long rolls, of same lengths as white paper, mounted or not; widths up to 54 ins. Color, a light buff.

Tracing paper. Most of that sold, whether domestic or foreign, tears so readily as to be of comparatively little service. Parchment paper, 37 and 38 ins wide, rolls of 20 and 33 yds, is better, but does not take ink perfectly.

Tracing cloth, usually called *tracing muslin*, and sometimes *vellum cloth*, is altogether preferable to tracing paper, on account of its great strength. Widths 18, 30, 36, and 42 ins; lengths to 24 yds.

Profile paper is made in widths of 9 ins and 20 ins, and in single sheets or in long, continuous rolls.

Cross section paper, mounted or unmounted, tracing paper and cloth, are furnished in sheets and in rolls, ruled in quarters, fifths, eighths, tenths, twelfths, and sixteenths of an inch, or in millimeters.

Colors. Since the introduction of blue printing, tinted drawings are seldom made, except for architectural effect; but colors may be used to advantage on black-line prints from tracings, p 1390. A good draughtsman needs but few colors; say India ink, Prussian blue, lake, or carmine, light red, burnt umber, burnt sienna, raw sienna, gamboge, Roman ochre, sap green. Winsor & Newton's colors are among the best in use. Purchase none but the very best India ink. Cakes of colors should always be wiped dry on paper, after being rubbed in water; and but little water should be used while rubbing; more being added afterward.

Lead pencils. *Genuine* A. W. Faber's Nos. 2, 3, and 4, are very good. The hardness increases with the number. Nos. 3 and 4 are good for field-book use: which to prefer, will depend on the character of the paper; No. 3 for smooth, and No. 4 for the coarser or more granular papers. His *lettered* pencils are of a higher grade and better suited for draughting. "H" stands for "hard," "B" for "soft." The degree of hardness or of softness is indicated by the number of H's or of B's. "F" (intermediate) corresponds with No. 3. Dixon's American pencils are good. The office draughtsman should have a flat file, or a piece of fine emery paper glued to a strip of wood, upon which to rub his lead to a fine point readily, after using the knife.

BLUE-PRINTS, ETC.*

Art. 1. (a) In order to obtain the best results, all **unnecessary exposure**, either of the sensitized paper or of the solutions, to sunlight or to other white light should be avoided.

(b) **Cleanliness** is of the first importance. The vessels in which the solutions are made and mixed must be scrupulously clean, and, if washed with soap, must be carefully rinsed with clean water. They should be left full of water when not in use. The presence of free alkali of any kind is fatal to good results, and immediately destroys the blue color of a finished print. See Art. 19 (b). The solutions must not be allowed to come in contact with iron.

Art. 2. (a) **The Solution** used in sensitizing the paper for blue-prints is usually that of *ferri*cyanide of potassium (*red* prussiate of potash) † and ammoniocitrate of iron (citrate of iron and ammonia) in water.

(b) The two salts are usually **dissolved separately** and the two solutions then mixed. The potassium salt should be broken up fine. The iron salt is usually quite pure and dissolves very rapidly. It may be kept indefinitely in a solid state if perfectly dry, but it readily absorbs moisture, and then becomes sticky and unfit for use; and the solution is apt to become mouldy after a few days, either alone or when mixed with the potassium solution. Hence, it should be prepared (in a dark room) in small quantities as required.

Art. 3. (a) The following is an average of several **recipes** that give excellent results:

Solution A. 1 ounce of red prussiate of potash to 6 ounces of water, or 2½ ounces of the salt to a pint of water.

Dissolve thoroughly and filter. The solutions may be sufficiently filtered through raw cotton, and much more rapidly than through paper.

Solution B. 1½ ounces of ammoniocitrate of iron to 6 ounces of water, or 4 ounces of the salt to 1 pint of water.

Dissolve thoroughly. Filter, unless the solution is perfectly clear.

(b) Keep the two solutions in separate glass-stoppered bottles in a dark place until they are to be used. Then mix them in equal parts, and filter the mixture. Take care that no undissolved particles of the red prussiate get into the double solution. It must be rejected when its brown color changes to bluish green.

(c) The combined solution will **cost** amateurs from 1 to 2 cents per ounce to make. About 4 ounces **will suffice for** coating 100 square feet of paper.

(d) If a few drops of strong ammonia solution be added to the citrate solution, B, until the odor is quite perceptible, the addition of a saturated solution **of oxalic acid** in water to the double solution **will hasten the printing in cloudy weather.** 10 per cent of the oxalic-acid solution will increase the rapidity of printing about 2½ times; 20 per cent., 5 times; 30 per cent., 10 times; but with more than 20 per cent., it is difficult to get clear white lines. In sunlight the difference is much less marked. (*Engineering News*, Dec. 15, 1892.)

Art. 4. (a) Where fine work is not essential, any well-sized **paper**, sufficiently tough to bear the washing, will answer. For important work use paper of fine uniform texture and smooth hard surface, free from injurious chemical substances. If the solution penetrates below the surface, a portion of the chemicals may remain in the paper in spite of the washing, and damage the result. Many papers are made especially for this purpose. The Saxe (German) and Rives (French) papers are considered among the best. Johannot and Steinbach papers give good prints, but are not very strong. Weston's and Scotch linen papers are stronger, and the latter gives excellent prints. Before sensitizing a large quantity of paper of a new kind, try a small sheet of it. **Linen** for sensitizing is also sold by dealers in photographic material and engineers' supplies.

Art. 5. (a) **The solution is applied** (in the dark room of course) to one side only of the paper. This is sometimes done by "**floating**" the paper upon the solution, taking care that none gets upon the *back* of the sheet.

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* See "Modern Heliographic Processes," by Ernst Leitze; D. Van Nostrand Co., New York, $3.00; a work to which we are indebted for many valuable suggestions.

"Modern Reproductive Graphic Processes," by Lieut. J. S. Pettit, D. Van Nostrand Co., Science Series, No. 76, 50 cents, deals chiefly with artistic photography, lithography, etc.

See also paper by Benj. H. Thwaite, Proc's, Inst'n Civ. Eng'rs, Vol. lxxxvi, p. 312, reprinted in Engineering News, Nov. 27, 1886.

† *Not* the *ferro*cyanide or *yellow* prussiate.

The paper is held by two diagonally opposite corners, and the diagonal joining the other two corners is then allowed to touch the surface of the liquid. Then the two corners held in the hand are dropped first one and then the other. The paper should then be lifted, one half at a time, to see whether any air bubbles have been formed under it. If so, they may be removed by drawing over the solution that half of the sheet under which they occur, while the other half is held up from the liquid. One or two minutes suffice for floating, and the paper is then drawn out over an edge of the bath draining off the surplus liquid. This process requires a tray larger than the sheet, and the inner surface of the tray must be not only water proof, but also proof against chemical action from the solution. Considerable care is required in the manipulation.

Art. 6. (a) The solution is usually applied by means of a soft wide **brush** (such, for instance as those used for wetting the leaves in letter-copying books) or a large soft **sponge** entirely free from sand or other grit.

Art. 7. (a) In applying the solution, the paper may be laid upon a board covered with soft smooth **oil-cloth,** which, after each sheet is sensitized, should be wiped off, to avoid smearing the back of the next sheet.

(b) **The operation** must be quickly performed, so that no portion of a sheet may become dry before its entire surface has been coated. For very large sheets it may be necessary, for this reason, to employ two persons. First cover the sheet by strokes of the wet sponge or brush, moved in the direction of the length of the paper, and then, immediately, by light strokes at right angles to these and with the sponge or brush squeezed out, so that the solution may be uniformly and thinly distributed over the entire surface. Wash out the sponge immediately in the dark room.

Art. 8. (a) The paper is then hung up **to dry** in the dark room, by means of clips, of any convenient form and free from iron. Small sheets may be hung by one corner; larger sheets by two adjacent corners, or by three or more places (according to size) along one edge, taking care to buckle this edge slightly, so that the paper may not be stretched in drying. If the sheets are hung over a rod or rail the solution will dry unevenly at the bend. In order that the whites in the print may be clear, the air should be warm, so that the paper may dry quickly and the solution be thus prevented from penetrating it deeply.

Art. 9. (a) Make sure that the paper is **perfectly dry** before it is used or put away, and see that it is kept both *dry* and *dark* until it is wanted for use. If carefully prepared and preserved it will retain sensitiveness for a long time, but the best results are obtained with fresh paper, and it is best not to keep it more than a month or two.

Art. 10. (a) **The tracing paper** or tracing cloth should be of a bluish cast (a yellow paper delays printing), thin (see Art. 15,), and as nearly transparent as possible. It should be preserved, both before and after drawing, from long exposure to light, which tends to render it opaque.

(b) Both before and after drawing, it should be kept either flat or rolled, and not folded, because folds render it difficult to bring the drawing into perfect contact with the sensitive paper in printing.

Art. 11. (a) **The drawing** or tracing should be made with the best India ink, rubbed very black. The addition of a little gamboge or chrome yellow increases the opacity. Lines drawn in chrome yellow and in gamboge print well; but Prussian blue or carmine should be rendered more opaque by the addition of a little Chinese white or flake white. Hold the tracing up to a strong light, in order to detect any weak places in the lines.

Art. 12. (a) **Printing** consists in exposing the sensitive paper to the action of light, the drawing being placed between the light and the sensitive surface. The arc **electric light** prints more slowly than direct sunlight, but has the advantage of constancy in all weathers and at all hours, and of fixedness of position. See Art. 16 (a).

(b) Place the frame with its face perpendicular to the rays of light, as nearly as may be, and see that no shadows, as of trees, buildings, etc., are allowed to fall upon a portion of the drawing.

(c) All handling of the paper, such as cutting it to size or placing it in the frame, should be done in a weak light.

Art. 13. (a) To secure close contact between the tracing and the sensitive paper (see Art. 15,) they are usually placed in a **printing-frame.** The essential parts of an ordinary frame are: the frame proper, a plate of clear glass for the passage of the light, and a padded back, which, by means of clamps and springs, presses the two sheets closely together and against the glass.

(b) The tracing is laid in the frame, with its drawn side next to the glass (but see Art. 16 b), and then the sensitive paper, with the sensitive side next to the tracing. Finally, the padded back is placed in the frame.

(c) The back is often made in two halves, hinged together and each provided with a spring, so that one half may be raised to permit examination of the progress of the exposure, while the other half, remaining clamped, holds the tracing and the sensitive paper in position.

(d) By using a frame left open at both ends long strips of sensitive paper may be used, a part at a time, the rest being rolled up at the ends of the frame and wrapped for protection from light.

(e) In any frame it is important that the glass be sufficiently thick to withstand the pressure required in order to secure close contact between the two papers (see Art. 15, below), of excellent quality, and free from defects which would obstruct or unequally refract the light. The glass should be carefully cleaned before printing.

(f) Improved forms of printing-frames have rubber air-cushions in place of flannel pads. In others the necessary pressure is secured by means of a vacuum produced between the tracing and the glass by means of a pump.

(g) Printing-frames are supplied by dealers. The prices, including glass, vary from about $2 for frames 10 x 12 inches, to $30 or $45 for frames 36 x 60 inches. Frames running on rollers, with fittings for exposing them outside of windows, are also furnished, at prices varying with the dimensions and the requirements.

(h) For large blue-prints, Prof E C. Cleaves, of Cornell University, uses, instead of a frame, a wooden cylinder covered with felt and revolving on its axis. Upon this cylinder the tracing and sensitive paper are stretched by means of a suitable clamping device, and the cylinder is then revolved in the sunlight. This method dispenses with the use of glass. It of course requires a longer exposure than the ordinary method. (*Trans Am. Soc. Mech. Eng*, vol. viii p. 722.)

(i) For still larger prints, Prof. R H Thurston stretches the two papers upon a thin board, which is then sprung into a curve and held in that shape, keeping the papers in tension upon the convex side This method also dispenses with the use of glass, and, the curvature of the board and the papers being but slight the whole of the paper is exposed to the light at one and the same time (*Trans. Am. Soc. Mech. Eng.*, vol. ix, p. 696)

Art. 14. (a) **The time required for exposure** varies with the color, directness, and intensity of the light, with the thickness and opacity of the tracing paper, with the blackness of the drawing, with the materials and the care used in sensitizing the paper, and with the freshness of the latter, from two or three minutes to hours or even days. Roughly, we may say that in full sunlight, in Philadelphia, about three minutes ordinarily suffice from noon to 2 P. M., and ten minutes at 10 A. M. or 4 P M , in the shade, thirty to forty-five minutes at noon; but no fixed rules can be given Experience must decide in each case. A preliminary experiment may be made with a small frame. If the back of the frame is in two or more pieces, the process may be inspected from time to time.

(b) If perfectly opaque ink be properly used, the blue background may be printed very dark without spoiling the lines, but over-exposure in printing renders the background first blackish and then of a dingy shade See Art 17 (c) and (d). Drawings in pale ink must be printed very lightly, in order that the lines may remain white and it is best to use with them a weak light, or to protect them by tissue paper or ground glass. See Art. 18 (a).

Art. 15. (a) To obtain perfectly sharp impressions, the side of the tracing upon which the drawing is made should be in **immediate contact** with the sensitized surface of the blue-print paper, especially if, as with sunlight, the direction of the light is variable; for, if any appreciable distance intervenes between the two, as in printing through cardboard (see Art. 16, below), the shadows cast by the lines of the tracing will move over the sensitized surface as the direction of the light changes, and thus give a blurred impression. In most cases, however, it is practically out of the question to place the two surfaces in this way, because that position gives a reversed impression as regards right and left.* Hence a thin tracing paper or linen is recommended in Art. 10 (a). For the same reason it is imperative that the two papers be firmly and evenly pressed against the glass.

Art. 16. (a) By using a light which is constant in position relatively to the surface of the tracing, such as an arc electric light, it is possible, by prolonging the exposure for hours or even days, to obtain blue-prints **from drawings made upon stout drawing paper** or even upon **bristol board.**

(b) With sunlight the same object may be accomplished. either by placing the original with its back to the glass, and the sensitive paper (which should be very

* A print, thus reversed in position, may of course be easily read by means of a mirror. This is commonly done with Patent Office drawings.

thin) with its back to the sunlight, or by placing the printing frame in the bottom of a deep and narrow box, so that the light can shine directly upon the frame only when approximately parallel with the long sides of the box To print rapidly, the sunlight must be kept full upon the frame by frequently moving the box.

Art. 17. (**a**) The print, when sufficiently exposed, is taken from the frame, and both its face and back are **washed thoroughly** in clean water until the characteristic blue color is perfectly developed.

(**b**) The washing should be done in a tray with a flat bottom larger than the largest print to be washed, and care should be taken not to injure the surface of the prints by hard rubbing or by sharp bending, or otherwise. It is better to have a circulation of water in the tray, not only to keep the water clean, but also to bring about the necessary agitation of the prints without handling them.

(**c**) The washing may be hastened, and dark or "over-exposed" prints may be lightened somewhat, by having the water warm, say at 90° or 100° Fahrenheit.

(**d**) Over-exposed prints may also be lightened by immersing them in water rendered slightly alkaline by ammonia. In this bath they at once assume a purple tint, which soon becomes weaker. At the proper moment, which must be learned by experience, the alkaline action must be stopped by drawing the print rapidly through a solution of 1 part of hydrochloric ("muriatic") acid (H. Cl) in 100 parts of water.

(**e**) Continue washing until the water has for some time come off perfectly clear. Then hang the prints up smoothly to dry.

Art. 18. (**a**) After washing, the application of a solution of from 1 to 5 per cent. of hydrochloric acid, or of oxalic acid, in water, intensifies the blue color, and is therefore useful in bringing out pale or "under-exposed" prints; but the prints must then be afterward washed again in pure water Hydrochloric acid applied *before* washing, or to imperfectly washed prints, will make the *lines* show blue.

Art. 19. (**a**) **To erase** a (white) line on a blue-print, go over the line with the sensitizing solution applied with a clean brush or *quill* pen. This should be done in a weak light. Then expose the entire print and re-wash

(**b**) **White lines are added** to blue prints, usually in Chinese white; but the blue color may be removed, showing the white paper beneath, by applying a saturated solution of concentrated lye (caustic soda or potash) or of carbonate of soda* or carbonate of potash, with a fine clean pen nearly dry. If laid on too freely, it spreads rapidly. Even if the pen is perfectly clean, the surface thus produced has a yellowish cast as compared with the white of the paper. The carbonate solutions act more slowly than the lye, but not less surely, and they are not injurious to the skin, whereas the lye burns badly. The ordinary lime-water sold by druggists makes little or no impression upon the blue color. **If red, instead of white,** lines are desired, mix with the soda or potash solution ordinary carmine writing-ink, in such quantity (to be ascertained by trial) as will give the desired color.

Art. 20. (**a**) Blue prints which are to be subjected to much handling should be **mounted** upon cloth, or the prints may be made, in the first place, upon sensitized tracing linen.

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*Either carbonate ("washing-soda") or bicarbonate ("baking-soda") will answer.

MODERN EXPLOSIVES.

Art. 1. Most of the explosives, which, of late years, have been taking the place of gunpowder, **consist of** a powdered substance, partly saturated with nitro-glycerine, a fluid produced by mixing glycerine with nitric and sulphuric acids.

Art. 2. Pure **nitro-glycerine,** at 60° Fah, has a sp grav of 1.6. It is odorless, nearly or quite colorless, and has a sweetish, burning taste. It is poisonous, even in very small quantities. Handling it is apt to cause headaches. It is insoluble in water. At about 306° Fah it takes fire, and, if unconfined, burns harmlessly, unless it is in such quantity that a part of it, before coming in contact with air, becomes heated to the exploding point, which is about 380° Fah.

N-G, and the powders containing it, are always **exploded by means of** sharp percussion. See Arts 36, &c. After N-G is made, great care is required to wash it completely from the **surplus acids remaining in it** from the process of manufacture. Their presence, either in the liquid N-G, or in the powders containing it, renders the N-G liable to spontaneous decomposition, which, by raising the temperature, increases the danger of explosion.

Art. 3. N-G freezes at about 45° Fah. It is then very **difficult of explosion,** and must be thawed *gradually*, as by leaving it for a sufficient length of time in a comfortably warm room, or by placing the vessel containing it in a second vessel containing hot water, not over 100° Fah; but never by exposing it to intense heat, as in placing it before a fire, or setting it on a stove or boiler. Extra strong caps are made for exploding N-G and its powders when frozen.

Art. 4. N-G, owing to its incompressibility, is **liable to explosion through accidental percussion.** This, and its **liability to leakage,** render it inconvenient to transport and handle. Hence it is rarely used in the liquid state in ordinary quarrying and other blasting. In the oil regions of Penna, it is largely used in **oil wells,** in order to increase the flow. For this purpose it is confined in cylindrical tin casings, from 1 to 5 inches diam, called torpedo-shells. These are suspended from, and lowered into the well by means of, a cord or wire wound on a reel; and are destroyed when the charge is exploded. They are about 1 inch less in diam than the well, and contain usually from one to twenty quarts = 3 lbs, 5⅛ oz to 66 lbs, 6¼ oz of N-G. They are pointed at their lower ends, in order to facilitate their passage through the oil or water which may be in the well. When a greater charge than about 66½ lbs is required, two or more of these shells are placed in the well, one on top of another, the conical point on the lower end of each one fitting into the top of the one next below. In this case, the N-G is fired by means of a cap or series of caps placed in the top of the charge before it is lowered. When the charge is in place, the caps are exploded by electricity led to them by conducting wires, as in Art 37, or (as in the method more commonly practised) by letting a weight fall on them.

When a well has been repeatedly torpedoed, and a cavity has thus been formed in it so large that the space surrounding a torpedo would interfere too greatly with the effect of the explosion of the N-G on the walls of the well, the latter is placed directly in the well, by lowering a tin cylinder, filled with it, and provided with an automatic arrangement which allows the N-G to escape when at the bottom of the well. The N-G is then fired by a torpedo suspended on a line, and having caps placed in its top. These caps are exploded by a leaden or iron weight sliding down the line, or by electricity. When the rock is seamy, the N-G is confined in short cylindrical tin shells, lowered into the cavity, and fired by a torpedo. It is also used for increasing the flow of springs of *water*. It of course cannot be used in **hor or upward** holes, such as often occur in tunneling, &c.

Art. 5. N-G explodes so suddenly that **very little tamping is required.** Moist sand or earth, or even water, is sufficient. This, with the fact that N-G is unaffected by immersion in water, and is heavier than water, render it particularly **suitable for sub-aqueous work,** or for holes containing water, provided the rock has no seams which would permit the N-G to escape. **If the rock is seamy,** the N-G must be confined in a water-tight casing. Such casings, however, necessarily leave some spaces between the rock and the explosive, and these diminish considerably the effect of the latter.

Art. 6. The great **explosive force of N-G is due** partly to the very large volume of gas into which a small quantity of it is converted by explosion, and

partly to the *suddenness* with which this conversion takes place, the gases being liberated almost instantaneously,* while with gunpowder their liberation requires a longer time. The suddenness of the explosion increases its effect, not only by applying all of its force practically at one instant, but also by greatly *heating* the gases produced, and thus still further increasing their *volume*.

Art. 7. The liquid condition of N-G is useful in causing it to **fill the drill-hole completely,** so that there are no vacant spaces in it to waste the force of the explosion. On the other hand, the liquid form is a disadvantage, because, when thus used without a containing vessel in seamy rock, portions of the N-G leak away and remain unexploded and unsuspected, and may cause accidental explosior at a future time.

Art. 8. N-G is stored in tin cans or earthenware jars. If properly washed from acid it does not injure tin. For transportation, these cans or jars are packed in boxes with sawdust, or in padded boxes, and loaded in *wagons*. The R R companies do not receive it.

Art. 9. When N-G and its compounds are *completely* exploded, **the gases given out** are not troublesome, but those resulting from *incomplete* explosion such as generally takes place, or from combustion, are very offensive.

Art. 10. For convenience, we apply the name "**dynamite**" to any explosive which contains nitro-glycerine mixed with a granular absorbent; "**true dynamite**" to those in which the absorbent of the N-G is "Kieselguhr,"† or some other *inert* powder which takes no part in the explosion; and "**false dynamite**" to those in which the absorbent itself contains explosive substances other than N-G.

Art. 11. The absorbent, by its granular and compressible condition, **acts as a cushion to the N-G,** and protects it from percussion, and from the consequent danger of accidental explosion.

N-G undergoes no change in composition by being absorbed; and it then freezes, burns, explodes, &c, under the same conditions as to pressure, temperature, &c, as when in the liquid form. The cushioning effect of the absorbent merely renders it more difficult to bring about sufficient percussive pressure to cause explosion. The absorption of the N-G in dyn enables the latter to be used in *hot* holes, or in holes drilled *upward*.

Art. 12. N-G and dyn **explode much more readily when rigidly confined,** as by a *metallic* vessel, or by the walls of a hole drilled in rock, than when confined by a *yielding* substance, as wood. Therefore the fact that dyn, not being liquid, can be packed in *wooden* boxes, renders it safer than N-G which has to be kept in stone or metal vessels.

Art. 13. True dynamites must contain at least about 50 per cent of N-G. Otherwise the latter will be too completely cushioned by the absorbent, and the powder will be too difficult to explode. False dynamites, on the contrary, may contain as small a percentage of N-G as may be desired; some containing as little as 15 per cent. The added explosive substances in the false dynamites generally contain large quantities of oxygen, which are liberated upon explosion, and aid in effecting the complete combustion of any noxious gases arising from the N-G.

Art. 14. Dynamites which contain large percentages of N-G explode (like the liquid N-G Art 6) with great suddenness, tending to *shatter* the rock in their vicinity into small fragments. They are most useful in *very hard* rock. In such rock, **No 1 dynamite**, or that containing 75 per cent of N-G, is roughly estimated to have **about 6 times the force of an equal wt of gunpowder.**

For soft or decomposed rocks, sand, and earth, the **lower grades** of dynamite, or those containing a smaller percentage of N-G, are more suitable. They explode with less suddenness, and their tendency is rather to upheave large masses of rock, &c, than to splinter small masses of it. They thus more nearly resemble gunpowder in their action.

Judgment must be exercised as to the **grade and quantity of explosive** to be used in any given case. Where it is not objectionable to break the rock into small pieces, or where it is desired to do so for convenience of removal, the higher, *shattering* grades are useful. Where it is desired to get the rock out in large masses, as in quarrying, the lower grades are preferable.

For very difficult work in hard rock, and for submarine blasting, the highest grades, containing 70 to 75 per cent of N-G, are used. A small charge of these does the same execution as a larger charge of lower grade, and of course does not require

* Such sudden liberation of gas is called "**detonation.**"

† Kieselguhr is an earthy, silicious limestone, composed of the fossil remains of small shells. Each shell acts as a minute receptacle for nitro-glycerine. Kieselguhr is found in Hanover, Germany and in New Jersey.

the drilling of so large a hole. In submarine work their sharp explosion **is not** deadened by the water.

For general railroad work, ordinary tunneling, mining of ores, &c, the **average grade**, containing 40 per cent of N-G, is used; **for quarrying**, 35 per cent; **for blasting stumps**, trees, piles, &c, 30 per cent; **for sand and earth**, 15 per cent.

Art. 15. Dynamite, like N-G, **can be readily exploded under water**, provided it is so immersed as not to be *scattered;* but **long exposure to water is injurious** to it. In the higher grades, the water, by its greater affinity for the absorbent, drives out the N-G. In the lower grades it is apt to wash away the salts used as additional explosives.

Art. 16. In dyns containing a large percentage of N-G, the latter is **liable to exude** in liquid form, or to "leak," especially in warm weather, and then to explode through accidental percussion. The same danger exists, even though the percentage of N-G be small, if the absorbent has but small absorbing power, and is, consequently, easily saturated.

Art. 17. True dyn resembles moist brown sugar. **Its properties are** generally those of the N-G contained in it. Thus, it takes fire at about 350° F, and burns freely. It freezes at 45° F, and is then difficult to explode. It is not exploded by friction, or by ordinary percussion, but requires, for general purposes, a strong cap, or exploder, containing fulminating powder, see Arts 36, 38, &c. It may, however, be exploded by a priming of gunpowder, tightly tamped, and fired by an ordinary safety-fuse.

Art. 18. The charge should fill the cross section of the hole as completely as possible. If water is not standing in the hole, the cartridge should be cut open before insertion, so that the powder may escape from it and fill the hole; or the powder may be simply emptied from the cartridge into the hole.

Art. 19. For **blasting ice** in place, holes are cut in it, and a number of dyn cartridges (one of which must contain an exploding cap) are tied together and lowered from 1 to 5 ft into the water. They are fired as soon as possible after immersion, to avoid the danger of freezing. Electrical exploders (Arts 37, &c,) are best for sub-aqueous work.

Art. 20. Dyn is useful for **breaking up pieces of metal,** such as old cannon, condemned machinery, "salamanders" (masses of hardened slag) in blast furnaces, &c. In cannon, the dyn is of course exploded in the bore. In other pieces, small holes are generally drilled to receive it; but plates, even of considerable thickness, may be broken by merely exploding dyn upon their surface.

Art. 21. For **blasting trees or stumps,** one or more cartridges are fired in a hole bored in the trunk or roots, or under the latter. This shatters both trunk and roots. **A tree may be felled neatly** by boring a number of small radial holes into it, at equal short dists in a hor line around its circumf, and, by means of an electric battery (Arts 37, &c), exploding simultaneously a small charge of dyn in each. Or a single long cartridge may be tied around the trunk of a small tree, and fired.

Art. 22. Piles may be blasted in the same way as trees; or a hole may be bored for the cartridge in the axis of the pile; or the cartridge may be simply tied to the side of the pile at any desired ht.

Art. 23. The higher grades of dyn, like N-G, **require but little tamping.** Use a *wooden* **tamping-bar,** *never* a *metallic* one, for any explosive. If a charge of dyn "**hangs fire,**" it is dangerous to attempt to remove it. Remove the *tamping,* all but a few ins in depth, on top of which insert another cartridge, containing an exploder, and try again. See electrical exploders, Arts 37, &c. Dyn, like N-G, if frozen, must be thawed *gradually,* by leaving it in a warm room, far from the fire; or by placing it in a metallic vessel, which is then placed in another vessel containing hot water. The water should not be hotter than can be borne by the hand. Otherwise the N-G is liable to separate from the absorbent. The N-G in dyn may freeze without cementing together the particles of the absorbent; in which case the powder of course is still soft to the touch. **An overcharge** of N-G, or of dyn, is liable to be burned, and thus wasted, giving off offensive gases.

Art. 24. Dyn is sold in cylindrical, paper-covered cartridges. from ⅞ to 2 ins in diam, and 6 to 8 ins long, or longer. They are furnished to order of any required size, and are packed in boxes containing 25 lbs or 50 lbs each. The layers of cartridges are separated by sawdust.

Art. 25. Some of the **R R companies** decline to carry dyn or N-G in any shape Others carry dyn under certain restrictions, based upon State laws; providing that it must be dry (*i e,* that no N-G shall be exuding from it); that boxes and cars containing it shall be plainly marked with some cautionary words, as "explosive," "dangerous," &c; that the cartridges shall be so packed in the boxes, and the boxes so loaded in the cars, that both shall lie upon their *sides,* and the boxes

be in no danger of falling to the floor; that caps, &c, shall not be loaded in the same car with dyn, &c, &c.

Art. 26. A great many varieties of dyn are made. They differ (generally but slightly) in the composition of the absorbent, and in the method of manufacture. Each maker usually makes a number of grades, containing different percentages of N-G, &c, and gives to his powders some fanciful name.

Art. 27. The following **table of explosives,** made by the Repauno Chemical Co, Wilmington, Del, and known as "**Atlas**" **powders,** gives the percentage of N-G in each.

Brand.	Percentage of N-G.,	Brand.	Percentage of N-G.
A	75	D+	33
B+	60		
B	50	E+	27
C+	45	E	20
C	40		

The absorbents contain: in "A" brand, 18 per cent wood pulp and 7 per cent carbonate of magnesia; in "C" brand (the average grade), 46 per cent nitrate of soda (soda saltpetre), 11 per cent wood pulp, and 3 per cent carbonate of magnesia; in "E" brand, 62 per cent nitrate of soda, 16 per cent wood pulp, &c, and 2 per cent carbonate of magnesia.

Art. 28. "Miner's Friend" powder contains nitrate of soda, wood pulp, resin, and carbonate of magnesia. It freezes at 42°, and is then, like other dyn, difficult to explode. When used under water, the cartridges should not be broken, because the powder is injured by direct contact with water. Their **"Hecla" powder** is a lower grade. It is in granulated form, like ordinary blasting powder, but is said to be much stronger. It is intended as a substitute for it.

Art. 29. "Giant" powder is dyn proper, containing 75 per cent N-G, and 25 per cent Kieselguhr obtained near their works in New Jersey. The lowest grade, branded "M," contains 20 per cent N-G. The name "giant powder" was originally applied to dynamite in general.

Art. 30. Other brands are "**Hercules**" powder and "**Judson R R P powder,**" a substitute for ordinary blasting powder. It is put up in waterproof paper bags, of 6¼, 12½, and 25 lbs each, and these are packed in wooden boxes holding 50 lbs each. "**Judson F F F dynamite**" is a higher grade, in cartridges of the usual shape, packed in 50-lb boxes.

Art. 31. "Rackarock" cartridges are said to contain no N-G, and to be entirely inexplosive until immersed, for a few seconds, in an inexplosive liquid furnished by the same Co. They are then allowed to stand for 15 mins, after which they may be used at any time. They are fired in the same way as dyn, and can be used under water. The mfrs claim that they "approximate N-G in strength, and are stronger than dyn."

Art. 32. The following explosives are made and used in Europe, but have not yet been regularly imported into the U. S.

Compressed gun-cotton, is cotton dipped in a mixture of nitric and sulphuric acids, then reduced to a fine pulp, and made into discs 1 to 2 ins thick, and ⅞ to 2 ins diam, or larger. It is generally used wet, for the sake of greater safety. It then requires extra strong caps or primers. Roughly speaking, it is about as strong as dyn No 1, but is less *shattering* in its effect. Being lighter than dyn, it requires larger holes; and, owing to its rigidity, is less easily inserted, and does not fit the hole so completely. When dry, it is very inflammable, but, if not confined, it burns harmlessly. It contains no liquid, to freeze or to exude; and is safe to handle.

Art. 33. Tonite consists of finely divided gun-cotton mixed with nitrate of baryta. It is compressed into candle-shaped cartridges having, at one end, a recess for the reception of an exploder containing fulminate of mercury. The cartridges weigh about the same as dyn. They are generally made waterproof.

Art. 34. Forcite, Lithofracteur, and Dualin are foreign makes of nitro-glycerine explosives. In Dualin the absorbent is sawdust. It has greater bulk than dyn for a given wt, and requires larger holes.

Art. 35. Explosive gelatine is a transparent, pale yellow, elastic substance, and is composed of 90 per cent N-G and 10 per cent gun-cotton. It is less sensitive than dyn to percussion, friction, or pressure, and is not affected by water. Its specific gravity is 1.6. It burns in the open air. For complete detonation a special primer is required. The addition of a small proportion of camphor renders it still less sensitive, and increases its explosive force. The camphor evaporates to some extent.

In some experiments on the power of different explosives to increase the contents of a small cavity in a leaden block, explosive gelatine caused an increase 50 per cent greater than that caused by dyn No 1. In hard rock the diff would probably have been greater The increase was 10 per cent less than that caused by N-G.

Art. 36. The cap or exploder, used with ordinary safety fuse for exploding N-G and dyn, is a hollow copper cylinder, about ¼ inch diam. and an inch or two in length. It contains from 15 to 20 per cent, or more, of fulminate of mercury, mixed with other ingredients into a cement, which fills the closed end of the cap. The cap is called "single-force," "triple-force," &c, according to the quantity of explosive it contains.

The end of the fuse, cut off square, is inserted into the open end of this cap, far enough to touch the fulminating mixture in it. In doing this, care must be taken not to roughly scratch the latter. The neck of the cap is then pinched, near its open end, so as to hold the fuse securely. The cap, with the fuse thus attached, is then inserted into the charge of N-G or dyn, care being taken not to let the fuse come into contact with the explosive, which would then be burned and wasted. If a dyn cartridge is used, the fuse, with cap, is first inserted into it. The neck of the cartridge is then tied around the fuse with a string, and the cartridge is then ready to be placed in the hole and fired.

Art. 37. The Siemens magneto-electric blasting apparatus, now in general use, consists of a wooden box about as large as a transit-box. Outside it has two metallic binding-posts with screws, for attaching the two wires leading to the exploder. From the top of the box projects a handle at the end of a vert bar This bar, which is about as long as the box is high, is made so as to slide up and down in it, and is toothed, and gears with a small pinion inside the box. When a blast is to be fired, the bar is drawn up, by means of the handle, as far as it will come. It is then pressed quickly down to the bottom of the box. In its descent it puts into operation, by means of the pinion, a magneto-electric machine inside the box. This generates a current of electricity, which increases in force with the downward motion of the bar, but which is confined to a short circuit of wire *within the box*, until the foot of the bar strikes a spring near the bottom of the box, breaking the short circuit and forcing the electricity to travel through the two longer "leading wires," which lead it from the two binding-posts on the outside of the box to the cap or exploder placed in the charge.

Art. 38. The cap used with this machine is similar to that used with safety fuse (Art 36), except that its mouth is closed with a cork of sulphur cement, through which pass the two wires leading from the electric machine. The ends of these wires project into the fulminating mixture in the cap. They are ⅛ inch apart, but are connected by a platinum wire, which is so fine as to be heated to redness by the current from the battery. Its heat ignites the fulminate and thus explodes the cap.

Art. 39. Where a number of holes are to be fired simultaneously (thus increasing their effect), each hole has a platinum cap inserted into its charge, and one of the short wires attached to each cap is joined to one of those of the next cap, so that at each end of the series of caps there is one free end of a short wire. Each of these two ends is fastened to the end of one of the leading wires, placing the whole series "in one circuit." Where the holes are too far apart for the caps to be thus joined by the short wires attached to them, the ends of the latter are connected by cotton-covered "**connecting wires.**"

Art. 40. The magneto-electrical **machine weighs** about 16 lbs. It can fire about 12 caps at once.

Caps for ordinary fuse and for electrical firing, fuses, wires, electrical machines, &c, are sold by most of the makers of, and dealers in, explosives, rock-drilling machines, &c.

Art. 41. Simultaneous firing of a number of holes can be conveniently accomplished only by electricity. Electric blasting apparatus is specially useful for blasting under water, where ordinary fuses are apt, especially at great depths, to become saturated and useless.

If an electrical machine fails to fire a charge, it is known that the charge cannot explode until the attempt is repeated. Therefore no time need be lost, and no risks run, on account of "hanging fire."

GUNPOWDER.

The explosive force of powder is about 40000 lbs, or 18 tons, per square inch. Its **weight** averages about the same as that of water, or 62½ lbs per cubic foot; hence, 1 lb = about 28 cubic inches. In ordinary quarrying, a cubic yard of solid rock in place, (or about 1.9 cubic yards piled up after being quarried,) requires from ¼ to ¾ lb. In very refractory rock, lying badly for quarrying, a solid yard may require from 1 to 2 lbs. In some of the most successful great blasts for stone for the Holyhead Breakwater, Wales (where several thousands of lbs of powder were usually exploded by electricity at a single blast,) from 2 to 4 cubic yards solid were *loosened* per lb; but in many instances not more than 1 to 1½ yards. Tunnels and shafts require 2 to 6 lbs per solid yard; usually 3 to 5 lbs. Soft, partially decomposed rock frequently requires more than harder ones. Usually sold in kegs of 25 lbs.

Weight of powder in one foot depth of hole.

Diameter of hole	1 in	1¼ ins	1½ ins	2 ins	2½ ins	3 ins
Weight of powder avoirdupois	0lb 5oz	0lb 8oz	0lb 11oz	1lb 4oz	2lb	2lb 13oz
Diameter of hole	3½ in	4 ins	4½ ins	5 ins	5½ ins	6 ins
Weight of Powder avoirdupois	3lb 14oz	5lb 0oz	6lb 6oz	7lb 14oz	9lb 8oz	11lb 5oz

PRICE LIST

For a work of this kind, any attempt to present a list of exact or even of closely approximate prices would be useless. We aim merely to give indications of the average costs or of the ranges of cost. On account of the extreme reluctance or refusal of many dealers and manufacturers to submit figures, it has been impossible to give any figures at all for many items that the engineer may need. In general, the figures given represent normal prices prevailing during 1925 and 1926. Caution:—Prices in Gulf or Southern cities are often or usually quite considerably higher than those given.

Abbreviated Outline of Classification.

(See also in Index of this book.)

For principle of classification, see Bibliography, p. 1420.

PRICE LIST.

1.0 Materials and Elementary Shapes.

1.1 Chemicals, etc.

Sulfate of aluminum, $1.45 per 100 lbs.*
Sulfate of copper, $4.45 per 100 lbs.
Soda ash, 58%, $1.25 per 100 lbs.
Hypochlorate of lime, $2.00 per 100 lbs.
Chlorine, liquid—, $4.50 per 100 lbs.

*Engineering News-Record, 1925.

1.13 Preservativs.

1.131 Paints.

0.5 cts per sq ft per coat of paint.
Paints, in oil, $2.50 per gallon.
In cents per pound:
Lead: white foreign, 18; American (white), 14 to 15; red foreign, 20; red American, 14.
Zinc oxide, American, 18; Paris, 20 to 25; Antwerp, 16 to 20.
Note; foreign material subject to duty
Lampblack, 25 to 40 cts per lb.
Colors cts per lb.
Blue Chinese, 100. Prussian, 60. Ultramarine, 40.
Brown Vandyke, 25 to 30. Greene Chrome, 30 to 35.
Sienna, Burnt—, 30 to 35 Umber burnt raw, 30 to 35.
Metal coatings, $3.00 to $5.00 per gal.
Fillers, Oils, etc., $0.60 to $1.00 per gal up to $2.00
Linseed oil, $1.00 to $1.20 per gal.*
Turpentine, $0.90 to $1.20 per gal.
Plain varnish, $2.00 to $2.50 per gal.
Linseed varnishes, $3 per gal.
Shellac, $3 per gal
Graphite pipe joint compound, 24 cts per lb.

1.132 Creosoting, Impregnating, etc.

Creo-resinate and creosote process, 15 to 25 cts per cu ft.
Creosoting varies from 25 to 40 cts per cu ft of material for various degrees of saturation. Subject to market price fluctuations

1.14 Explosives.

Gunpowder, 20 cts per lb. Smokeless powder, 60 cts per lb.
Dynamite, 15 to 25 cts per lb for grades varying between 20 and 75% nitroglycerin.*
Blasting caps, $7 00 per 1000.

1.2 Wood, Lumber, Timber.

Lumber in dollars per 1000 ft B.M. (Board Measure)
Black walnut, 250.
Douglas fir, 30 to 50 and 75, varying with location.
Hemlock, 33 to 75.
Long leaf yellow pine, 3″ x 4″ to 12″ x 12″, 48 to 56.
12″ x 12″ to 16″ x 16″, 56 to 71.
Southern pine, 57. Spruce, 35 to 88.
White ash, 130. White oak, 180.
Yellow pine, 25 to 60, average, 35 to 40.
Yellow poplar, 85 to 130.
Shingles, Cypress—, 5 to 10 cents per sq ft.
Red cedar—, 6 to 7 cents per sq ft.
Asbestos and composition—, see 1.9.
Clearing and grubbing, see 2.11.
Piles, see 1.23. Ties, see 9.14.

1.23 Piles.

Piles, 50 to 70 cts per lineal foot.
Piling, round or sheet—, 70 to 125 cents per lineal foot

1.3 Stone, Concrete, Asphalt.

Earthwork, Dredging, Foundations, see 2.1.

1.31 Road Materials.*

Cents per gallon, carload lots,
Binder, 7, Flux, 7, Liquid Asphalt, 7. See also 1.33.
Granite Blocks (28 to 30 per sq yd) $132 to $142 per 1000
Flagging, 4″ to 5″ wide, 25 cts per sq ft.
Curbing, $1 per lineal foot.
Wood block, $2 00 to $2.85 per sq yd.
See also 1.33, Asphalt.

*Engineering News-Record, 1925.

1.32 Stone.

Sand, $1.00 to $1.60, southern cities, $2.00 per cu yd.*
Gravel, $1.50 to $2.25 per cu yd, southern cities, $2.50 or more.
Broken stone, $1.10 to $2.50 per cu yd.*
Trap rock, $2.00 per ton at quarry
Ordinary building stone, $3 to $10 per cu yd
Granite, $25 to $75 per cu yd.
Roofing slate, No. 1 ribbon, 11½ cts per sq ft.
Slag roofing, $1.50 to $2.50 per ton in carload lots
Slag, crusht—, carload lots, $1.30 per cu yd *

1.33 Asphalt.

5 to 7 cts per gallon.*
Mexican—, bulk, $18 to $23 per ton, in packages, $23 to $27 *
Texas —, bulk, $21 to $25 per ton; in packages, $27.
Paving, see 2.4.

1.34 Cement, Lime, Plaster, etc.

Sand, see 1.32
Natural cement, $1.35 to $2.80 per bbl, for 500 bbls or over, exclusiv of bags.*
Portland (artificial) cements, $2 15 to $2.60 per bbl in carload lots, exclusiv of bags.*
Bags, 10 cts each, 40 cts per bbl.*
Lime, lump—, $1.50 to $3 50 per bbl.
Hydrated lime, $15 to $25 per ton.*
Plaster, $2.25 to $3.50 per bbl.
Gypsum plaster, neat, $20 per ton.
Plaster board, see 1.9.
Concrete construction, see 2.35.

1.35 Brick, Tile, Glass.

Sewer pipe, see 9 255.

1.351 Brick.

Paving, see 2.4
Building brick per 1000.
Common—, $12 to $20.*
Salmon—, $18 to $22. Hard—, $22 to $25. Dressed—, $30 to $55 Colored—, $35 to $60. Iron spots—, $60.
Fire brick, $40 to $55.
Vitrified paving brick, $25 to $45.

1.352 Tiling.

Hollow tile, 8" x 12" x 12", carloads, 11 to 20 cts per block;* smaller lots, 15 to 30 cts
Roofing tile, $15 to $50 per 100 sq ft.

1.4 Iron and Steel.

(Freight rates, cts per 100 lbs, from New York City, in carloads of 36,000 lbs; to Baltimore, Md., 31 Birmingham, Ala., 58, Boston, 37; Buffalo, 27, Chicago, 34, Cincinnati, 29; Detroit, 29; Kansas City 74; New Orleans, 67; Norfolk, 34; St. Louis, 43, St. Paul, 60.)

1.41 Cast Iron and Steel.

Cast iron pipe, see 9.251.
Pig iron per ton of 2240 lbs at foundry, $40. Bessemer—, $40 to $50 Gray Forge—, $40, Lake Superior Charcoal—, $45 to $55.

*Engineering News-Record, 1925.

1.43 Rolled and Structural Iron and Steel.

Iron and Steel:—Refined iron bars and steel bars, angles, ordinary sizes, T-shapes, beams and channels, structural shapes, tank plates, structural plates, bessemer machinery steel, 3 to 4½ cts per lb.*

Reinforcing bars, dollars per 100 lbs, warehouse,*

Size	¾″	½″	¼″
From	3.00	3.20	4.00
To	3.50	3.50	4.25

Expanded metal

Pounds per sq ft,	0.25	0.50	1.02	2.04
Cents per sq ft,	11	8	7	7

Metal lath, 4 cts per sq ft.

Steel rails, see 9.11.

1.431 Sheet and Plate Iron and Steel.

Gage,	14 to 16	22	25	28	29	30
Cts per lb,	30 to 33	35	40	42	47	52

Discount, 50%.

Blue anneald—, 3½ to 4 cts per lb.

Black iron, 4 to 4½ cts per lb.

Galvanized iron sheets, 4 to 4½ cts per lb.

1.45 Fastenings.

1.451 Nails and Spikes.

Nails, $3.50 to $4.50 per keg.

Spikes, railway—, 4 cts per lb.

1.452 Rivets.

⅜″ and 7/16″, 7 cts per lb.

½″ and larger, 6 cts per lb, to as low as 2.6 cts per lb in carload lots.

1.454 Bolts and Nuts.

Bolts and nuts (machine) per 100, square or button heads, length under head, 2″. Discount, 50%.

Diam in inches,	¼	½	¾	1
Dollars per 100,	2 40	7.80	17.00	35.00
Extra per inch over 2″,	0 80	0.60	5.00	12.00

1.455 Turnbuckles.

Discount, 60%.

Size	Take-up in inches	Price
¾″	12	$2.00
¾″	24	$5.30
⅞″	12	$3.40
⅞″	24	$6.40
1″ to 2″	48	$42.00
1″ to 2″	72	$52.60

*Engineering News-Record, 1925.

1.457 **Chains.**

American coil chain,

Inches,	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{7}{8}$
Cts per lb,	20	15	10	9	8

1.46 **Tubes.**

See 9.25, &c.

1.47 **Wire, Wire Rope, Fencing.**

Hoisting and conveying machinery, see 3.5.

1.471 **Wire.**

Hoisting Rope, extra strong cast steel, 5½ cts per ft length per ton of working load, for diams 1″ to 2¾″. 6½ cts per ft length per ton of working load for diams as small as ½″.

Extra pliable, 6½ to 8½ cts per ft per ton of working load, for diams 2¾″ to 1″.

1.472 **Wire Fencing.** 75 cts to $2.00 per 16½ ft length.

1.473 **Wire Rope.**

60 cts per sq inch of cross section per ft of length

See also 1.471.

1.5 **Other Metals and Alloys.**

Sheet brass, 30 to 60 cts per lb, depending upon width.

Soft copper, 60 cts per lb.

Pig lead, 9 cts per lb. Galvanizing, 2 cts per lb.

1.6 Paper.
Tar—, $2.00 to $2.25 per roll of 108 sq ft.
Tard felt, $2.50 to $3.25 per roll.
See also 1.9, Other Materials.

1.7 Ropes.
Wire ropes, see 1.471-3.
Manila, plain or tard, 25 cts per lb. Sisal, 20 cts per lb.
Twine, 15 cts per lb. Oakum, 12 to 15 cts per lb

1.8 Packing, Gaskets, Belting.
Rubber belting, cts per ft one inch wide,
2-ply, 6; 3-ply, 8; 4-ply, 9.
Leather belting,

width, ins,	1	6	12	24	72
$ per ft,	0.09	0.82	2.00	5 00	20.00

Pipe jointing supplies, see 3.263.

1.9 Other Materials.
Gypsum plaster board, $^3/_8$", $45 to $50 per 1000 sq ft.
Tar felt, 14 lbs per 100 sq ft, $84 per ton.
Tar pitch, $1.85 per bbl. Asphalt roofing, $38 per ton.
Asbestos shingles, 15 to 29 cts per sq ft.
Composition shingle, crusht slate surfaced, 6 to 7 cts per sq ft.

2.0 Constructions.
Waterworks supplies, see 9 2.
Railroad Supplies, see 9.1.

2.1 Earthwork, Dredging, Foundations.

2.11 Excavation and Embankment.
See also page 1024, etc.
Clearing and grubbing, $50 to $250 per acre.
Earth excavation, $0.50 to $2.50 per cu yd. In trenches, $2.00 up, according to depth.
Rock excavation in large masses, $1.25 to $4.00 per cu yd In small or difficult places up to $8 per cu yd.
Embankment, $1 to $3 per cu yd
Rock filling, $3 to $8 per cu yd.
Sodding, $0.60 to $1.00 per sq yd.

2.12 Dredging.
10 cts to $2.00 per cu yd, according to material, haul, etc.
See also p. 581w.

2.13 Foundations.
Piles, see 1.23.

2.2 Masonry.

2.21 Brick Masonry.
$10 to $15 per 1000 plus cost of bricks.
Brick chimneys, dollars equal 8.3 x height x diam (in feet) for abt 50′ ht, to 5.2 x height x diam (in feet) for abt 200′ ht.
Bricks, see 1.351.

2.22 Stone Masonry.
Rubble, dry, $4 to $15 or more per cu yd.
in cement, $7 to $20 per cu yd.
Granite coping, $50 to $70 per cu yd
Plain cellar masonry, $8.
Stone, see 1.32.

2.23 Concrete and Cement Masonry.
In place, dollars per cu yd,—
Class A, 18 to 30; Class B, 14 to 20; Class C, 11 to 18; Class D, 8 to 10.
Walls and chambers, $12 to $15 per cu yd.
Floors and Sidewalks, $9 to $11 per cu yd.
Cement, see 1.34.
Concrete Mixers, see 3.64.

2.24 Plastering.
Three-coat work, \$1.25 to \$1.75 per sq yd.
Lathing, 25 to 35 cts per sq yd.
Wall board, 4 cts per sq ft, plus labor.
Sheet plaster, 6 to 8 cts per sq ft.
Wire lath and plaster partition wall, \$4 per sq yd.

2.3 Metal Structures.
3½ to 4½ cts per lb.

2.33 Tanks, Stacks, etc.
Pipes, see 9.25.
Standpipes, 22 x 60 ft, including foundations, \$12,000.

2.34 Boilers.
Upright tubular boilers with base and fixtures.

IHP	4	12	30	50	200
\$	300	450	700	900	3000

2.35 Fireproofing, Concrete-Metal Construction.
Covering colums and girders, 50 cts per sq ft.
Walls, \$1 40 to \$3 00 per sq yd.
Wall furring, 80 cts to \$1 25 per sq yd
Metal lath, etc., see 1.43.

2.4 Paving.
Dollars per sq yd.

Asphalt, 3 to 4.50.	Belgian Block, 3.50 to 5.00.
Brick, 2 to 2 75	Macadam, 1.25 to 2.00.
Cellar floors, 2 25 to 3.75.	Sidewalks, 2.00 to 5.50.

2.5 Sewers.
From 6″ to 18″ diam, dollars per ft run.

Depth below surface	5 ft	10 ft.
Excavating only	\$0.60 to \$2.00	\$1 to \$3.

Laying pipe, exclusive of excavating, per ft.
15 inch, \$0.50 to \$1 00. 4 inch, \$0 40 to \$0.80.
Brickwork in sewers, \$25 per cu yd.
Excavating, using large excavator, 7 ft deep, 20″ wide, 60 cts per ft of length, or 5 cts per cu ft.

2.7 Roofing.
Asbestos, 40 cts per sq ft
Shingle, 25 cts per sq ft
Slag, 10 cts per sq ft and up.
Slate, 20 cts per sq ft and up.
Skylights, \$1.10 per sq ft and up.

3.0 Machinery.

3.2 Tools, Machine Tools.

3.21 Hammers.
Riveting, \$65; chipping, \$50. Concrete breaking tool, \$165.

3.23 Drills.
Portable electric drills, for metal, from \$25 for $\frac{5}{16}$″ hole to \$200 for 1½″ to 2″ hole.
Pneumatic stone drills, \$12 to \$17.
Air drills, \$80 to \$200, according to capacity.
Rock drills.

Depth of hole, ft.	1	10 to 15	25 to 35
Price,	\$200	\$300	\$400

Electric hammer drills, for holes in stone, from \$150 to \$225 for 2″ holes.
Diamond rotary drills,

Depth of hole, ft.	4000	1500	1000	800
Boiler HP	25	15	12	10
Price	\$8000	\$5000	\$3500	\$3000

Explosives, see 1.14

3.26 Pipe Cutting, Tapping and Jointing Machinery.

3.262 Tappers.
Dry pipe, \$30 to \$50 each. Under pressure, \$125 and up.

3.263 Jointers.

Diam, ins	2	12	36	72
Price, each	\$3	\$15	\$25	\$50

3.3 Engines.

Pumps, see 3.4. Boilers, see 2.34.

Portable engines, 10 HP, $600 and up. 25 HP, $850 and up. 50 HP, $1300 and up.

3.31 Stationary and Hoisting Engines.

Air Hoists, geared, from $200 for 1-ton cap,. to $800 for 10-ton.

From $25 to $35 per IHP for medium sizes.

Single cylinder stationary engines,

HP	12	30	50	100	200	300
Price,	$840	$1500	$1700	$2400	$4000	$6000

3.4 Blowing and Pumping Machinery.

3.41 Compressors.

Portable gasoline-driven compressors, from $1200 to $1400 for 100 cu ft of free air per min, to $3500 for 300 cu ft

3.45 Hydraulic Rams.

$3 to $5 times square of diam of drive pipe in inches.

3.46 Pumps.

$25 to $50 per HP.

For unwatering excavations, $100 to $135 per HP

Pulsometers, $25 to $45 per HP

Artesian well cylinders (not including the pump at the surface),

Cap, gals per stroke,	¼	½	1	2	4	7	10
Price	$20	$35	$75	$140	$250	$400	$1200

Centrifugal, $20 to $30 per HP

3.5 Hoisting and Conveying Machinery.

Belt conveyors, portable, $500 to $1800.

Bucket loaders, about $4000.

Electrical—, see 3 1.

Excavating machinery, see 3.6.

Hoisting engines, see 3.31

Wire rope, see 1.473.

3.54 Elevators, Hoists, Derricks.

Hoisting crabs, on winches, ½ to 2½ tons cap, $70 to $150.

Differential hoists, ¼ to 3 tons, $35 to $90.

Derrick, guy—,

10-ton capacity, $400 to $800.

20-ton capacity, $800 to $1700.

Derrick, stiff-leg, hand-power, 1-ton, $200

Derrick, stiff-leg, without hoisting or power devices, 10-ton, $750; 20-ton, $1400

Derrick, stiff-leg with bull-wheel, 10-ton, $1200, 20-ton, $2300.

Hand winches, pull of 2 men on single line, 900 lbs, $60, 3000 lbs, $80 to $150.

Electric hoist, without motor, double friction drum. Pull on line, 3000 lbs, $1400 to $2500, 12,000, $9,000

Steam hoisting engine, drum—, without boiler,

HP,	8	20	45
	$1000	$1500	$2400

3.6 General Construction Machinery, Excavators, etc.

Hoisting and conveying, see 3.5. Hoisting engines, see 3.31 Drills, see 3.23. Explosives, see 1.14. Excavation and Embankment, see 2.11.

Trucks or trailers average $250 each.

3.61 Trench excavators.

Trench Excavators, $5500 to $12,000.

3.62 Pile Drivers.

Pile Drivers, $500 to $2500.

3.63 Shovels, Scrapers, Plows, etc.

Steam shovels, about $10,000 each.

Wheeled Scrapers, $40 to $60 each.

Drag scrapers, $10 to $15.

Plows up to $55 each. Hardpan plow, $70.

3.64 Concrete Mixers.

Concrete mixers, $300 to $1500.

3.65 Diving and Diving Apparatus.

Complete outfit. For shallow water, $300; for moderate depths, $600; for deep sea, $1000.

3.66 Cement Guns.

Cement gun only, $7 to $14 per cu ft of free air per min.
Compressors, see 3.41.

3.67 Wells and Well Driving Machinery.

Well strainers, $2 to $4 per lin ft per inch diam of pipe up to 10″ diam.
Foot valves, $2 to $10 per inch diam inside of well casing, 2″ to 10″.
Drills, see 3.23.

3.68 Road Making Machinery.

3.681 Rollers.

Tandem, $2500 to $4500. Three-wheel, $2500 to $7500.

3.683 Rock and Ore Crushers.

Receiving Cap, ins	Capacity tons per hr	HP required	Cost, dollars
8 x 14	10 to 15	10 to 12	1000
10 x 18	16 to 24	15 to 20	1500
14 x 36	45 to 60	60 to 75	5500

Concrete mixers, see 3 64.

4.0 Engineering, Surveying and Scientific Instruments and Supplies.

Aneroid barometers, $36.
Current meters, $65 to $300.
Direction meters, $200 to $250.
Velocity register and time-piece, $15 to $60.
Hook gages, $15.
Thermometers for asphalt and sand, $3 to $5.25.
Maximum—, $4

4.2 Surveying Instruments.

4.21 Transits, Plane Tables, Compasses, etc.

Transits, plain, $250 to $300.
Engineers' complete transits, $300 to $500.
Mining transits, $300 to $400 and up to $750.
Mountain transits, $150 and up to $400.
Theodolites and portable alt-azimuth astronomical instruments, $600 to $1200.
Solar attachments, $60 to $80.
Sextants, $70 to $150. Pocket sextants, $60
Plane tables, complete, $100 to $450.
Compass, pocket—, $15 to $30

4.22 Levels.

Engineers' levels, $125 to $225.
Hand levels, $6 to $20, usually about $10.

4.23 Rods, Tapes, etc.

Leveling rods, $15 to $20 each.
Range poles, $3 to $6.
Tapes from 6 to 30 cts per ft, depending upon graduations and length.
Chains, 8 to 12 cts per ft.

4.3 Computing Instruments.

Planimeters, $15 to $375, average, $40.
Slide rules, Mannheim, $5. Other forms, $8 to $12 and up to $50.
Computing machines, $100 to $300, and special electric drive up to $700.

4.4 Drawing Instruments and Materials.

Drawing paper, "detail," 15 cts to 20 cts per sq inch of sheet per ream.
Tracing paper, 42″ wide, 4 cts to 12 cts per yd.
Cross-section paper, 8½ x 11, $1.35 per 100 sheets.
Drawing instruments, $10 to $30 per set—very elaborate sets up to $100.
Drawing pens, $1 to $3 each.
Compass, $3 to $9. Dividers, $1.50 to $4 50.
Triangular boxwood scales, 12″, $1 to $3
Metal straight edges, 36 inches long, $3 to $5
T-Squares, 36″ with celluloid edges, about $2.25. wood, $1 to $2.40; steel, $7 to $10, usually with adjustable angle.
Triangles, celluloid. 6″, 40 to 65 cts. 12″, $1 to $1 50
French curves, celluloid, $0 50 to $1 50
Protractors, German silver, 4″, $0.50 to $2.00; 6″, $1.50 to $5, with arms 6″, $9; 8″, $12.
Drafting machines, $60 to $125.
Drawing inks, 25 cts per bottle.

4.5 Heliography.

Blue prints, 2 to 3 cts per sq ft.
Blue print paper, 1¼ to 2 cts per sq ft.

9.0 Miscellaneous Supplies.

9.1 Railroad Supplies.

9.11 Rails.

Steel rails, standard, $43 per ton; Light (8 to 45 lbs), $30 to $36, Rerolled, $28,* Old, $25 per ton.

9.12 Joints.

3 to 5 cts per lb.

9.14 Ties.

Chestnut, $3 each, White Oak, $1 to $2 each; Yellow Pine, $1.50 to $2 00 each; Douglas fir, $1.
Miscellaneous treated ties, about $1 50 to $2 each.
Tie plates, 10 to 30 cts each.

9.15 Spikes.

Spikes, 2¼ to 4 cts per lb.*

9.16 Bolts.

Bolts, 3¾ to 6 cts per lb.

9.17 Angle Bars.

Standard section angle bars, 2¾ to 4 cts per lb.*

9.2 Hydraulic Supplies.

Gaskets, see 1.8.
Stand Pipes, Tanks, see 2.33.
Pipe Cutting and Tapping Machines, see 3 26.

9.23 Chlorinating Apparatus.

Chlorinator, $400 to $800.

9.24 Water Meters.

80 cts to $1.20 per gal per min.
From ⅝″ to 1″, $12 to $27; 2″, $61 to $72; 4″, $205 to $240; 6″, $360 to $407; 10″, $825, 16″, over $2000.

9.25 Pipe.

Pipe Cutting Machinery, etc., see 3.26.
Pipe laying, see 3.6.

9.251 Cast Iron Pipe.

$40 to $60 per ton.

9.252 Steel Pipe.

Steel Pipe at works, 5½ cts per lb.
Spiral rivetted—, plain end, dollars per foot.

Diameter	3″	6″	12″	18″	24″	30″
From	0.24	0.57	1 45	2.75	3.70	5.90
To	0.34	1.10	2.82	4.30	5.70	7.10

depending upon thickness or strength.
Asphalted, 10% more for large sizes, 25% more for small.
Galvanized, 35% more for large sizes, 55% more for small sizes.

*Engineering News-Record, 1925.

9.253 Wrought-Iron Pipe.

Wrought Iron Pipe, 15 cts per lb.

Ingot Iron Culverts, dollars per foot,

Diam,	8″	15″	20″	24″	36″	42″	48″	60″	72″	84″
From	1	1½	2	3	5	5½	6	10½	12½	18
To	•				6	7	8½	13½	15½	22½

9.254 Wood Pipe.

Machine Banded 50 ft head, 10 cts per inch per ft diam.
100 ft head, 20 cts per inch per ft diam

9.255 Sewer Pipe.

Drain Tile, dollars per thousand lineal feet,*

Size	3″	4″	5″	6″	8″
From	40	40	70	70	160
To	50	70	100	130	200

Sewer Pipe, dollars per foot, standard pipe.*

Size	3″	6″	12″	18″	24″	30″	36″
From	0.10	0.13	0.36	0.65	1.21	2.66	3.95
To	0.12	0.21	0.66	1.53	2.16	3.75	5.42

9.256 Hose.

Water Hose, cents per inch of internal diam per ft of length,

2-ply	4-ply	6-ply
25	40	60

Air, Hot Water and Steam Hose, cts per inch of internal diam per ft of length,

4-ply	6-ply	8-ply
75	110	140

*Engineering News-Record, 1925.

9.26 Valves.

Single or double gate valves, iron body, bronze mountings.

	Pressure, lbs per sq in.		
Tested to	200	400	600
For Water	100	250	400
For Steam	35	125	250
Diameter	Approximate prices in dollars, without discounts.		
2"	6- 7	8- 15, 23	11- 27, 35
6"	28-31	32- 50, 65	50-100, 115
12"	75-82	115-185, 215	150-335, 390
Working pressure, lbs/sq in,	30-40,		100-125.

Diameter	Approximate prices in dollars.	
18"	150	470 (double gate)
36"	700	

Check valves, Horizontal swing.

Working pressure, 100 lbs/sq in for steam.
175 lbs/sq in for water.

Diameter,	½"	1"	2"	4"	6"	12"	18"	24"	36"
	Prices each in dollars.								
Screwed ends,	1.40	2.15	8.50	20-37	31-43	97-175	230	400	1900
Flanged ends,	3.40	4.15	14.00	21-47	33-45	95-170	225	390	1900

for Working pressures of 250 lbs/sq in for steam,
400 lbs/sq in for water,
increase figures above by 20% to as high as 45% for small sizes.

Fire Hydrants. From $20 for 2" valv and one 2" nozzle, to $50 for 6" valv and three 2½" nozzles

9.9 Labor.

Bricklayers.*† $1.55 per hour in a belt about 400 miles wide, from Springfield, Ill., to New York, with maxima of $1.75 at those two cities, diminishing to $1.63 in Colorado and $1.30 on Pacific coast. Southern States and upper Lake region, about $1.25.

Carpenters.*† $1.50 per hour in a belt about 200 miles wide from New York city to central Illinois, and about $1.60 in Colorado, grading off to $1.00 on the Pacific coast, and down to $0.70 or less in the east Gulf states $1.00 in the Lake region.

Structural Iron Workers.*† $1.50 per hour from Illinois to New York city, except Ohio about $1.35, tapering rapidly to $1.10 and $1.00 on the Great Lakes, and to $0.80 in Georgia. New Orleans is exceptional with $1.25. West of Mississippi R to Pacific coast, about $1.10.

Common Labor.* $0.80 to $0 90 from Chicago to New York city, grading down to $0.30 in Georgia and Alabama, and to $0.60 on the Great Lakes. West of Mississippi R, $0.50 to $0.60.

According to Bureau of Labor Statistics, on May 15, 1925, the following averages were in effect:—Plasterers, $1.485; Plasterers' laborers, $1.00, Bricklayers, $1.475, Stone Masons, $1.401; Slate and Tile Roofers, $1.419, Tile Layers, $1.325; Lathers, $1.398, Plumbers, $1.281; Plumbers' laborers, $0.954; Inside Wiremen, $1.272; Steamfitters, $1.271; Structural Iron Workers, $1.271; Hoisting Engineers, $1.265; Cement finishers, $1.238; Painters, $1.232; Sheet Metal Workers, $1.209; Carpenters, $1.184 Building laborers, $0.954.

*Engineering News-Record, 1926, July 1.
†About 5 to 10% higher in 1928.

BIBLIOGRAPHY

The following list of books makes no pretensions to completeness It aims simply to be usefully suggestiv to the general civil engineer. The list is arranged according to the Decimal Classification of Melvil Dewey. In this classification (see outline below) all subjects are arranged under ten general heads, and to each of these heads is assignd a number in the hundreds place, as Natural Science, 500; Useful Arts, 600, etc. Then each general head is divided into ten subheads, to each of which a number in the tens place is assignd Thus Natural Science (500) is subdivided into Mathematics, 510; Astronomy, 520; Physics, 530; etc Again, each of these is subdivided, and decimally numberd, and this successiv subdivision and decimal enumeration may be continued indefinitely. **To find a subject**, refer to the table below, and decide, first, under which of the general heads it belongs, then under which sub-head, and so on Thus, Plane Geometry is seen to belong (1) under Nat Sc, 500, (2) under Math, 510, and (3) under Geom, 513. However, matter on a special subject is often containd in books on a more general subject which embraces the special one Thus, matter pertaining to Geom (513) is found in many works which would be classified under Mathematics (510) Conversely, in looking for books on Mathematics, the sub-heads, Algebra, Geometry, etc. should be consulted as well. In general, it is advisable to look under all heads that may contain the information wanted. Thus, information on Locomotivs may be found under Mechanical Engineering, 621, or under Railroads, 625. To avoid duplication in such cases, however, one such head has usually been selected, and reference to it made under the other.

Outline of Classification.

000	**GENERAL**	622	**Mining, Exc'v'n,** etc
500	**NATURAL SCIENCE**	622 26	Tunneling
510	**Mathematics**	622.32	Hydraulic Mining
512	Algebra	624	**Bridges and Roofs**
513	Geometry	624 2	Girders
514	Trigonometry	624 3	Trusses
515	Descriptive Geometry	624 6	Arches
516	Analytical Geometry	624 8	Draw Bridges
517	Calculus	624 9	Roofs
519	Probabilities. Least Squares	625	**Railroads** and Roads
		625 1	Route Track
520	**Astronomy**	625 1a	R. R Surveying
526	Geodesy	[illegible] 2	Trains. Cars
526.9	Surveying	625.8	**Roads & Pavements**
530	**Physics**	[illegible] 7	**Hydraulic** Eng
531	Mechanics	[illegible] 8	Irrigation
532	Hydromechanics	627.7	Dredging
533	Pneumatics	627 8	Dams
536	Heat	628	**Sanitary** Engineering
537–8	Elec'y and Magnetism	628.1	**Water Works**
540	Chemistry	630	Forestry
550	Geology	660	Chem. Tech'y Explosives
600	**USEFUL ARTS**	670	Manufactures Iron & St'l
610	**Medicine**	690	**Building.** See 721
620	**Engineering**	691	**Constr'n & Materials**
620 c	**Civil** Engineering	691.1	Wood
620.1	Strength of Mater'ls	691.2	Stone
621	**Mechanical** Engin'g	691 3	Concrete
621.1	Steam Engineering	691.37	Reinforced —.
621.2	Water Eng & Motors	691.7	Iron and Steel
621 3	Electrical Engin'g	693	Masonry
621.4	Miscell's Motors	721	**Arch'l Construction**
621.6	Pumps & Blowers	740	**Drawing**
621.8	Transmis'n M'ch'm	770	**Photography**

Abbreviations.

A— D. Appleton & Co., 29-35 W. 32nd St., New York; 25 Beford St., (Covent Garden), London

AB—Allyn & Bacon, 50 Beacon St., Boston, Mass; 11 E. 36th St, New York; 1006 S. Michigan Ave, Chicago, Ill.

AC—Constable & Co., Ltd., 10 Orange St., Leicester Sq., London, W. C.

B— Henry Carey Baird & Co., 810 Walnut St, Philadelphia, Pa

CH—Chapman & Hall, Ltd., 11 Henrietta St., Covent Garden, London, W C. 2.

CL—Crosby Lockwood & Son, 7 Stationers' Hall Court, Ludgate Hill, London, E C.

G— Ginn & Co., Boston, Mass.

L— J. B. Lippincott Co, E. Washington Sq., Philadelphia, Pa.

LG—Longmans, Green & Co., Fourth Ave. & 30th St., New York; Boston & Chicago.

M— The Macmillan Co., 64-66 Fifth Ave., New York.

McG— McGraw Hill Book Co, 239 W. 39th St., New York.

R—Renouf Publishing Co. 25 McGill College Ave., Montreal, Canada

SB— Simmons-Boardman, 30 Church St., New York, N. Y.

SC—Spon & Chamberlain, 123-125 Liberty St, New York.

T—Trautwine Company, Box 26, Wallingford, Pa

US—Government Printing Office Washington, D C.

VN—D. Van Nostrand Co., 8 Warren St., New York.

W— John Wiley & Sons, Inc, 432 Fourth Ave, New York.

Book Sizes

The following book sizes have been recommended by the American Library Association —

Name	32mo	24mo	16mo	12mo	8vo	4to	Folio
Ht. ins	4 to 5	5 to 6	6 to 7	7 to 8	8 to 10	10 to 12	12 to 14

When width is 3/5 height, book is called "narrow"; ¾ height "square"; = height "oblong."

000 General.

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510.8 Tables and Mathematical Instruments.

Barlow. Barlow's Tables of **Squares, Cubes,** Square **Roots,** Cube Roots, **Reciprocals** of all Numbers up to 10,000. 200 pp. 12mo. Cloth. $1.25. 1905. SC.

Bruhns, C.—. New Manual of **Logarithms** to Seven Places of Decimals. 9th ed. 633 pp. 7 x 10. Cloth, $2.00. Half Morocco, $2.50. VN.

Chambers. Chambers's Mathematical Tables, consisting of **Logarithms** of Numbers 1 to 108,000, **Trigonometrical,** Nautical and Other Tables. 8vo. Cloth. $1.75. VN.

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Hewes, L. T.—, and Seward, H. L.— Design of **Diagrams,** and the Theory of **Nomography.** 112 pp. 82 ills. 9 x 12 $5.00. 1923. McG.

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Pickworth, Chas N.—. The **Slide Rule.** 12th ed. 118 pp 34 ills. Table. 5 x 7¼ Cloth. $1.00. VN, Whittaker & Co. and Emmott & Co, London

vonVega, Baron - **Logarithmic** Tables and Numbers and **Trigonometrical** Functions. Translated by W. L. F. Fischer, 603 pp. 6 x 9 Cloth. $2.00 Half Morocco $2.50. VN

Tables. See also under subject in question, as Surveying, 526.97, etc.

512 Algebra.

Chrystal, G.—. Algebra

Wentworth, G. A.— Complete Algebra. 525 pp. 12mo Half Morocco. $1.72. G.

513 Geometry.

Chauvenet, Wm—. Geometry L.

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Analytical Geometry, see 516

Descriptive Geometry, see 515.

Drawing, see 740.

514 Trigonometry.

Buchanan, A H.— Plane and Spherical Geometry 5¾ x 9¼. Cloth. $1.00. W

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